University of Nairobi

School of Mathematics

ISSN: 2410-1397

Master Dissertation in Actuarial Science

Longevity Risk and Private Pension Funds: Analysis of Longevity Risk Using the Renshaw-Haberman Model

Research Report in Mathematics, Number XX, 2018

Wanjiru Kimondo

December 2018

Submitted to the School of Mathematics in partial fulfilment for a degree in Master of Science in Actuarial Science

Longevity Risk and Private Pension Funds: Analysis of Longevity Risk Using the Renshaw-Haberman Model

Research Report in Mathematics, Number XX, 2018

Wanjiru Kimondo

School of Mathematics
College of Biological and Physical sciences
Chiromo, off Riverside Drive
30197-00100 Nairobi, Kenya

Master of Science Project
Submitted to the School of Mathematics in partial fulfilment for a degree in Master of Science in Actuarial Science

Prepared for The Director
Board Postgraduate Studies
University of Nairobi

Monitored by Director, School of Mathematics

Abstract

Longevity risk is a main topic of study abroad and has just started being a point of analysis in third world countries. Due to the increase in life expectance there has been a strain on the national pension funds and life assurance holding firms. A lot of research has been done on Lee-Carter models of longevity risk and there has been evidence in almost every research that longevity risk exists in third world countries' populations. However, little research has been done on the effects of cohorts on longevity risk in the same regions.

A cohort study is a study on a group of individuals with something like one specific shared experience within a certain time period. The most well-known example is a "birth cohort", that is, the people in the group are born in the same time or amid a similar era. In a prospective study, we are examining a group of individuals over time by observing patterns of effects or outcomes due to the cohort differences within the group. This study focused on modelling the effects of longevity risk and measuring the effects of age-period cohorts. The study used the Renshaw-Haberman to model the effects of longevity on mortality for Kenya (period: 1970-2010).

A time-varying mortality index is forecasted in an ARIMA framework and is used to generate projected life expectancies at normal retirement age. The study then modelled the life expectance for the next 50 years from 2010. The effects of longevity risk on the annuities reserves and pension positions.

The study found that the longevity risk exists and that an increase in life expectance results in an increase in the cost of pay-outs for pensioners in the market. The study also found that an improvement in age specific cohort risk results in a higher life expectation and lower mortality risk.

Dissertations in Mathematics at the University of Nairobi, Kenya.
ISSN 2410-1397: Research Report in Mathematics

DISTRIBUTOR: School of Mathematics, University of Nairobi, Kenya

Declaration and Approval

I the undersigned declare that this dissertation is my original work and to the best of my knowledge. It has not been submitted in any other Kenyan University or any other institution of learning for an award of any degree.

Signature Date

Wanjiru Kimondo
Reg No. I56/83856/2016

In my capacity as a supervisor of the candidate's dissertation, I certify that this dissertation has my approval for submission.

Signature Date

Professor J.A.M. Otieno
School of Mathematics,
University of Nairobi,
Box 30197, 00100 Nairobi, Kenya.

Dedication

This project is dedicated to God and my family.

Contents

Acknowledgments

I am overwhelmed in all humbleness and gratefulness to acknowledge my debt to all those who have helped me move these ideas in this project well above the level of simplicity and into something concrete.

First and foremost, I would like to thank the Almighty God who helped me complete this project successfully and in time.

Secondly, I would like to express my gratitude to my supervisor, Prof. J.A.M. Otieno. His attitude and substance of genius which went a long way into the planning and development of this project. His willingness to give his time so generously has been very much appreciated.

In addition, we would like to also offer my appreciation to Prof. Weke who has been very supportive in successfully completing my project.

Last, but not least, I wish to express my love and gratitude to my beloved family and my close friends for their understanding and endless love through the duration of my studies.

Kimondo Wanjiru

Nairobi, 2018.

1 INTRODUCTION

1.1 Background Information

Since the 20th century, the human mortality trend has been observed to be decelerating. Improved mortality rates over the years results in higher levels of life expectancy. This trend then introduced longevity risk which is described as the risk that an individual, or group of individuals, will live longer than anticipated.

The field of mortality and longevity risks and in particular the accurate forecasting and finan-cial management of such risks has become a topic of great interest to academics, actuaries and financial professionals, (Edwards & Munhenga, 2011). Mortality enhancements around the globe are putting pressure on governments, pension scheme providers, life insurance offices and people to manage the longevity risk they encounter, (Kathyuka, et al, 2014).

The older generation consume a growing share of their resources as they age in the decades ahead (Marleen, et al, 2014). It is perceived that this will strain the company's balance sheets and government's pension schemes who have been coming up with arrangements that will result in financial constraints with aging. It is however noted in (Taruvinga & Gatawa, 2010) that these arrangements have been underestimating the populations' life expectancy hence leading to forecasts that are not accurate.

Unforeseen longevity past the standard estimates will obviously be advantageous for individuals and the society. This however has an adverse effect as it causes a major financial risk for governments and defined benefits pension providers who will be liable to pay out more in benefits and pensions than anticipated, (Shaw, 1994). It might likewise be a budgetary hazard for people who could fall short on retirement assets themselves. These risks build gradually with time and in the event they are not tended it could lead to large negative effects on already depleted balance sheets which will affect financial stability, (Marleen, et al, 2014).

1.1.1 Types of Pension Plans

A defined contribution (DC) plan is a retirement plan whereby either the employer, employee or both contribute an agreed amount on a regular basis to the pension scheme. In DC plans, the retirement benefits future may fluctuate on the basis of investment earnings.

Individuals on a DC pension plans will be totally exposed to the risks. For organizations, DC plans appear as a good solution for reducing the company's pension risks taking into account of longevity risks. While there is some reality to this view, it overlooks potential effects from inadequacies in other solutions (Madsen, C. and Tans, M., 2011).

A defined benefit pension (DB) plan is a pension plan that promises a predetermined pension settlement, a lump-sum or a combination of both upon retirement. This pension plan is sponsored by an employer or a sponsor. The pension payment is usually determined by an equation which depends on the worker's compensation history, employment duration and age, rather than a defined benefit which depends directly on an individual.

DB Plans have all the risk on the company providing the retirement benefits and in a situation where policy holders live longer, the fund might not be sufficient to cater for the extra years lived. This cost has to be met by the company.

While a large number of the present workers may have been wanting to retire between the ages of 50 and 60, the pattern to provide DC plans instead of DB plans will likely imply that employees will have to work for longer and increase their savings. This in itself is an indirect method for increasing the standard retirement age, leaving employees to choose when they will retire. The more you work, the more your retirement will be comfortable. Organizations should be well prepared for this move in workers conduct and expectations. An example of this is the recent increase of minimum retirement ages for lecturers in Kenyan public universities from 60 years to 70 years, an increase of 10 years which is very significant.

Retirement ages and financing assumptions for pension schemes and other retirement remunerations do not yet completely mirror the effect of longer

lived populations. Accordingly, as researchers say, the three pillars of pensions – state-run pension plans, organization sponsored plans and private retirement savings – are experiencing unprecedented difficulties.

In the first pillar, state pensions are based on the framework of 'pay-as-you-go', which makes them very vulnerable. State pensions also referred to as national pension utilize tax income from younger workers with the end goal to pay the older generation. In Kenya, there is a large population between the ages 0-35. Thus there is a large working population. An example of a national retirement plan in Kenya is the National Social Security Fund (NSSF) which is a service organization which gives standardized savings security to every single Kenyan worker in all sectors by receiving their contributions, managing the scheme funds, processing and eventually paying out benefits to entitled members or dependants.

In the second pillar, company-sponsored retirement plans have it such that the employer also contributes towards the employees' retirement. These are an example of defined contribution (DC) pension scheme which uses the contributions and any interest earned on investment of the contributions to pay the retirees. In spite of the fact DC plans are, by definition, not subject to funding requirements, beneficiaries still expect them to be appropriately supported with the end goal to have the capacity to pay for their retirement. As longevity increases, more funds should be saved.

After assessing the longevity risk, the benefit provider (or sponsor) should then look for appropriate solutions. The process of mitigating longevity risk can be referred to as de-risking. To do this, the identified longevity risk should be correctly measured and priced. Longevity risk is just one of several risks faced by pension plans and it should not be addressed in isolation. Other pension risks include interest rate risks, equity risk, and inflation among others.

Marleen, et al, (2014), proposed that capital markets can provide vehicles to hedge longevity risk adequately and trans¬fer this risk from those unable to manage it to those who have the capacity to estimate its effect in exchange of increased returns, for instance, life insurance offices.

New capital market solutions have been innovated to enable transfer of longevity risks. These include longevity (or survivor) bonds, longevity (or survivor) swaps and mortality (or q-) forward contracts. The price estimates for these products requires very accurate forecasts of mortality rates, (Marleen, et al, 2014).

1.1.2 Mortality and Longevity Risks

DB pension schemes, companies, governments, insurers, reinsurers, long-term healthcare providers and individuals are all subjected to the uncertainties associated with increased life expectancy. Although some of these uncertainties can be diversified by aggregating individual lives into large groups, they cannot be eliminated completely and can have significant economic consequences. These uncertainties are usually referred to as "longevity risk", or "mortality risk" (depending on the context), and have become a vital concern for DB pension schemes and the respective interested parties.

Despite the fact that the terms "longevity risk" and "mortality risk" are frequently used interchangeably, they are in reality inverse sides of a similar coin. Though longevity is identified with the length of life, mortality relates to the death rate. Therefore longevity risk describes the risk that an individual, or group of individuals, will live longer than anticipated, while mortality risk is usually used referred to as the risk that an individual, or group of individuals, will live, in total, shorter than anticipated, that is, their mortality will be higher than anticipated.

According to Edwards, et al (2011) a stochastic approach is is a better approach to assess the uncertainty and associated longevity risks adequately by modelling life expectancy and mortality as it attaches probabilities in different forecasts.

1.2 Notation, Definition and Terminologies

We will define $q(x,t)$ to be the underlying aggregate death rate at age x in year t. This is an unobservable rate. What we do observe relies upon how, for instance, national statistics offices record deaths and population sizes. However, in numerous nations we observe the crude death rate,$m_c(x,t)$,

which is the number of deaths, $D(x,t)$, aged x last birthday at the date of death, during year t, divided by the Exposure, $E(x,t)$ (the average population aged x last birthday during year t).

Different models have been developed to measure the effects of longevity risk on reserves of annuities, life policies and pensions. The earliest and generally famous one stochastic factor discrete-time model is the Lee and Carter (1992) which postulates that the true underlying death rate $m_{x,t}$=-$log(1-q_{x,t})$. This implies that longevity risk is not affected by cohorts i.e. changes in age specific demographic parameters, lower infant mortality rates. The Renshaw and Haberman (2006) was an extension of the Lee-Carter Model as it includes cohort effects. This modification to the Lee-Carter model was done to capture the effects that could be attributed to the year of birth $t-x$.

Other models were built based on the Lee-Carter (1992) model and the Renshaw-Haberman (2006) models. Currie (2006) proposes to be a simplified version of the Renshaw and Haberman (2006) model, where the age, period and cohort effects influence mortality rates independently.

The P-Spline approach is essentially a penalized fitting process using basis splines (Currie et al., 2004; CMI 2006). Basis splines are a set of basic functions which are constructed from cubic splines. Through the optimization of a penalized likelihood or regression function, they are fitted to the underlying data. According to Currie et al. (2004), the P-splines model is shown to fit the mortality data better due to the local nature of the parameters it is able to adapt more readily to variability in the mortality rates. It does this with fewer parameters than the Lee Carter model. The P-splines model also provides a much lighter forecast of future mortality than the Lee Carter model. These models have been successfully utilized in the developed countries which include, USA, English and Wales and Sweden.

Due to the 2030 vision on improved health care and general improvement in life, Kenya will have a higher life expectancy rate. As a result unanticipated mortality improvements will be of most prominent significance at higher ages and will cause providers of annuities and life insurance offices to suffer losses in their life business. The issue is that retired people are as of now

living much longer than expected. Thus, life insurers and retirement benefit providers are paying out longer than expected resulting in diminished profit margins. This will be a big coat to the pension companies 12 years from now.

Previous studies have mainly concentrated on developed countries only, due to the relatively large proportion of the population living beyond retirement age. As a result, the financial impact of the demographic changes in developing countries on retirement benefits has been ignored and may pose future problems for companies dealing in these benefits as well as the state managed NSSF.

The decrease in AIDS epidemic has been an especially crucial factor for Africa. Life expectancy for Africans actually decreased in the 1990s due to the ravages of Aids, (WHO, 2015). Globally, children born in 2015 had a life expectancy of 71.4 years.

The Society of Actuaries in 2012, found that if the average life expectancy in a population were to unexpectedly increase by four or five years, then liabilities to pensions could increase by as much as 15% to 20% over an extended period of time. The Society of Actuaries further provides frequent updates to its mortality improvement factors, but the challenge for pension companies is that timing and magnitude are uncertain. No one knows when the next major change to life expectancy will occur.

Edwards & Munhenga, (2011) postulates that with regards to getting ready for retirement income security, planning monetary assets or even creating shareholder value in public organizations, increasing life expectancy is resulting in weakening accounting reports and increasing financial risk for governments and individuals.

1.3 Problem Statement

Extensive research has been done on forecasting the effects of longevity using the Lee-Carter model. A time-varying mortality index is forecasted in an ARIMA framework which is then used to produce life expectancies projections at normal retirement age. We also apply Lee-Carter mortality projection techniques to the evaluation of retirement costs.

Lee, R. (2002) established that the LC method is a valuable and fitting approach to extrapolate the historical patterns in the level and age distribution of mortality. Conversely, extrapolation may not generally be a sensible procedure to employ.

The method assumes a pattern of change in the age distribution of mortality, such that the mortality rates decline at different ages maintaining the same ratios after some time. But in practice, the general speed of decline at different ages may vary. Horiuchi & Wilmoth (1995) points that the mortality rates at old ages were observed in Sweden to decline more gradually than at other ages, yet in recent decades they have come to decline more rapidly. In U.S.A, it is observed that there has been a slowdown in mortality declines for ages 5 to 50 relative to the older and younger ages. Therefore this method cannot consider such shifts into account.

According to Cairns *et al.*, (2009), the Lee-Carter model is unable to produce a nontrivial correlation structure between the year-on-year changes in mortality rates at different ages. The LC model diminishes the mortality development over time for all ages to one single time trend, the mortality index. However, mortality as a function of one single time trend implies perfect correlation between changes in mortality at all ages which does not seem biologically reasonable.

The Lee-Carter method does not incorporate cohort effects in forecasted mortality rates. The incorporation of cohort effect is a desirable property of any stochastic mortality model. The failure to include cohort effects in a population where such effects are present leads to inaccurate forecasts of mortality.

The RH (2006) model is a modification of the LC (1992) model that includes the cohort effects of mortality forecasting, provides an effective solution to the mentioned limitations.

1.4 Research Objectives

The research seeks to examine how longevity risk can influence defined benefits pension schemes. In this regard, the research seeks to provide the outcomes of estimating the Renshaw and Haberman (2006) model for Kenya.

Furthermore, the research seeks to:

1 Analyze the uncertainty of future mortality and life expectancy outcomes

2 Estimate future longevity risk.

3 Estimate future annuity pay-out value in the future

1.5 Scope of the Study

The study was investigating the impact of longevity risk on retirement benefits in developing countries. This study was limited to retirement benefit providers, for example, pension funds and insurance firms providing pension retirement covers.

The researchers believed that Kenya would provide a suitable representation of data and would therefore give reliable findings and results. We were able to establish this through a correlation test with a developed country, U.S.A, where this study has been undertaken several times.

1.6 Validity of the Study

The research focuses on the effects of longevity risk in the Kenyan pension market and annuity providers market. This will help protect insurers, regulators, pension providers against longevity risk. Forecasts of the duration the population may live are vital on the grounds that they advise government's long-term forecasts of the incurred costs of public pensions and other planning assumptions for a maturing society. Insurance companies and benefit providers additionally forecasts the costs of annuity and other products, in light of their policyholders or members. Actuaries are involved

in both these areas. The modelling and forecasting of the mortality rates is the key point in estimating the process of mortality-linked securities that facilitates the emergence of liquid markets.

2 LITERATURE REVIEW

2.1 Introduction

In this part, we summarize key developments accomplished so far in analysis of longevity risk, mortality modelling and forecasting. Longevity risk is now considered as one of the world's most pressing financial risks as there appears to be no slowdown in trend of improving life expectancy.

2.2 Longevity Risk and Pensions

According to Jones, (2013), longevity risk affects; *(1)* governments who need to fund guaranteed retired individuals through pensions and healthcare from a shrinking tax base, *(2)* corporate sponsors who financially support retirement and medical coverage commitments to former employees accrued over numerous years and, *(3)* people who may have decreased or no ability to depend on governments or corporate sponsors to finance retirement.

The effect of the longevity risk on living benefits must be precisely faced. Reinsurance arrangements and capital allocation can avail appropriate tools to face this risk, (Edwards & Munhenga, 2011). Nevertheless, the issue of "finding" the longevity risk by means of a possible sharing between, say, the annuity provider and the annuitant ought to be considered.

Schoeni & Ofstedal, (2010) summarises the key topics of research on the demography of ageing. These are population patterns (including mortality); global comparisons; the financial aspects of ageing and the elements of wellbeing in later life. There has been a global decline in fertility and mortality rates from higher levels to lower levels and as a result, the world's population is growing older. Population aging is being experienced all over the world. Developing nations are undergoing population aging at the fastest rates with developed countries having had encountered it before. They likewise talk about the aging of the older generation. The portion of the older generation over the age of 80 is growing more quickly than the older generation. This also varies among different countries and regions

in the world. Longevity has occurred in the past half a century and this has been encouraged to a large extent by eradication of infectious diseases and effective treatment and management of chronic diseases, (Edwards & Munhenga, 2011). However, with the share of older people in the world increasing, countries face troublesome difficulties, with regard to health care, retirement frameworks and labour market supply. Older persons consume a larger amount of health care services, medications and extended life implies that they are drawing retirement and other seniority benefits for longer timeframes than previously. Moreover, as the extent of older individuals relative to those in their working year's increases, national security and pension programs that depend on the taxes of current workers to pay retirees' benefits (a "pay-as-you-go" scheme) turn out to be progressively unsustainable, (Kathyuka, et al, 2014).

Recent patterns in mortality lead to the utilization of projected survival models when pricing and reserving for life annuities and other long-term living benefits, (Rizzuto & Orsini, 2012). Several projection models have been proposed and are applied in actuarial practice. Nonetheless, the future mortality trend is random and henceforth, whatever type of model is adopted, systematic deviations from the forecasted mortality may happen. At that point, a "model" (or a "parameter") risk emerges, which is plainly a non-pooling risk. Changes in the mortality trend refer to both young and old ages. When the random mortality trend at old ages is concerned, it is usually referred to as "longevity risk", (Rizzuto & Orsini, 2012).

2.2.1 Valuation of Longevity Risk

Analysing longevity and its effects, forecasting of future mortality rates is key for any pension firms, (Taruvinga & Gatawa, 2010). People have always had an interest in the human lifespan. Forecasting mortality trends has a long history in demography and actuarial science. Demographers used mortality forecasts for population projections while actuaries used then to project cash flows and assess premium and reserves in life offices and pension annuities. Some organizations used the mortality forecasts to support their policy decisions, (Trauth & Reimer-Hommel, 2000).

Early mortality models were deterministic and did not consider the potential future improvements in mortality rates. They assumed future mortality would behave the same way as current and past data. Earlier models involved fitting a parametric curve mortality data. Abraham De Moivre was probably the first person to mathematically model mortality in 1725. He suggested that:

$l_x = k(1 - \frac{x}{86})$ for $12 \leq x \leq 86$

Where l_x is the number of individuals who are still alive at age x last birthday from a total number of Individuals, l_0 and k is a normalizing constant. The assumption applied in this model is that all individuals ought to be dead by age 86.

Over the past twenty years, various new methodologies were produced with the end goal to forecast mortality using stochastic models. Stochastic models appear to be more appealing than the prior deterministic models, since they include a confidence error to each estimate.

Lee and Carter (1992) shows how the LC model was used to forecast mortality rates in U.S.A. This model has been generally used for both demographic and actuarial applications because, firstly, it produced satisfactory fits and forecasts of mortality rates for different nations. For instance, the Lee Carter model was used in Japan, Austria, Australia, Belgium and the Nordic countries. Secondly, the Lee-Carter model structure permits the construction of confidence intervals related to mortality projections. In spite of its reasonable performance, the LC model had a few constraints (Lee 2000) which caused negative responses. Because of this, new stochastic models were produced with the most remarkable models being the Renshaw and Haberman (2006) and Cairns et.al models (2006, 2007, and 2008).

Stochastic models vary significantly according to a number of key components: number of sources of randomness driving mortality improvements at different ages, assumptions of smoothness in the age and period dimensions, incorporation or not of cohort effects and the estimation method used. The research will centre on the use of the RH model to model longevity risk as it incorporates the cohort effects in the population.

A cohort effect is a variation in health status because of different elements that each age cohort in a population is exposed to as environment and society are changing, (Gustafsson, 2011). A cohort study is a study on a group of individuals with something like one specific shared experience within a certain time period, (Rizzuto & Orsini, 2012). The most well-known example is a "birth cohort", that is, the people in the group are born in the same time or amid a similar era (Gustafsson, 2011). In a prospective study, which is the centre of this research, we are examining a group of individuals over time by observing patterns of effects or outcomes due to the cohort differences within the group. Then again, for a review examine information is gathered dependent on certain result from past records. Regardless, a cohort study (prospective or retrospective) can be said to be the last link in the chain to confirm a link between disease and exposure.

2.3 Stochastic Models for Measuring Longevity Risks

Previous researches and studies used different models to model mortality and survival statistics of populations.

Suppose we have:

* $\beta_x^{(1)}$ An age specific parameter; the set α_x, x=0,1,... reflects the general shape of the mortality schedule.
* $\beta_x^{(2)}$ defines the speed of mortality rate by age to changes through time, specified by $k_t^{(2)}$
* The period effect $k_t^{(1)}$=1 and $k_t^{(2)}$ follows a one-dimensional random walk with drift:

$$k_t^{(2)} = k_{t-1}^{(2)} + \mu + C\xi_{x,t}^{(2)}$$

in which μ is a constant drift term, C is a constant volatility and $\xi_{x,t}^{(2)}$ is a one-dimensional i.i.d N (0,1) error. The time trend of $k_t^{(2)}$ signifies the general speed of mortality improvement

* $\delta_{x,t}$ is the error term, which has no long term trend
* $\gamma_{t-x}^{(i)}$represents the cohort effect

Then we have the following models:

2.3.1 Lee–Carter Model

Ronald Lee and Lawrence Carter (1992) first presented the Lee–Carter (LC) model which was out of their work in the late 1980s and early 1990s attempting to use inverse projection of rates in historical demography, (Lee & Carter, 1992). The United States Social Security Administration, US Census Bureau, and United Nations have used this model in their projections. It has turned into the most widely used mortality forecasting model globally.

There have been extensions to the Lee–Carter model, most taking account for missing years, correlated male and female populations and large scale coherency in populations that share a mortality regime (western Europe, for example), (Renshaw & Haberman, 2006).

This model is the still the most popular one stochastic factor discrete-time model. Lee and Carter (1992) postulates that the true underlying death rate, $m_{x,t}$=- log $(1 - q_{x,t})$, satisfies the following equation:

$$\log m_{x,t} = \beta_x^{(1)} + \beta_x^{(2)} k_t^{(2)} + \delta_{x,t}$$

Where:

$\gamma_{t-x}^{(1)} = \gamma_{t-x}^{(2)}$=1 , that is, the model has no cohort effect

2.3.2 Renshaw and Haberman Model

Renshaw and Haberman (2006) generalized the Lee–Carter Method to capture the effects of cohorts. This modification to the Lee-Carter model was done by simply including a cohort factor $\gamma_{t-x}^{(3)}$ to capture effects that could be attributed to the year of birth t-x.

This model postulates that $m_{x,t}$ satisfies:

$$\log m_{x,t} = \beta_x^{(1)} + \beta_x^{(2)} k_t^{(2)} + \beta_x^{(3)} \gamma_{t-x}^{(3)}$$

Where period effect $k_t^{(2)}$ follows (3.4) and $\gamma_c^{(3)}$ is a cohort effect where c=t-x is the year of birth. They postulated that cohort effect $\gamma_c^{(3)}$ is modeled as an ARIMA (1, 1, 0) process that is independent of $k_t^{(2)}$, i.e.

$$\Delta\gamma_c^{(3)} = \mu_\gamma + \alpha_\gamma\left(\Delta\gamma_{c-1}^{(3)} - \mu_\gamma\right) + \alpha_\gamma Z_c^{(\gamma)}$$

Chapter 3 provides a more detailed discussion of the Renshaw and Haberman model.

2.3.3 Currie Age-Period-Cohort Model

Currie (2006), simplified the Renshaw and Haberman model, where the age, period and cohort effects influence mortality rates independently.

It was modelled as follows:

$$\log m_{x,t} = \beta_x^{(1)} + k_t^{(2)} + \gamma_{t-x}^{(3)}$$

2.3.4 P-Splines

According to Currie et al (2004), basis splines are a set of basic functions constructed from cubic splines. To estimate the parameters they maximised the following regression equation:

$$\log m_{x,t} = \sum_{i,j} \theta_{i,j} B_{i,j}^{\alpha y}(x,t)$$

Where $B_{i,j}^{\alpha y}(x,t)$ pre-specified basis are functions with regularly-spaced knots and θ_{ij} are parameters to be estimated.

2.3.5 Cairns-Blake-Dowd 1 Model

This is the original CBD model developed by Cairns, et al. (2006). The model postulates that mortality rates $q_{x,t}$ satisfy:

$$\text{logit } q_{x,t} = k_t^{(1)} + (x - \bar{x})k_t^{(2)}$$

Where, $\beta_x^{(1)} = 1$, $\beta_x^{(2)} = (x - \bar{x})$ and $\bar{x} = \frac{1}{n_\alpha}\sum_i x_i$ is the mean age over the range of ages used in the analysis.

2.3.6 Cairns-Blake-Dowd 3 Model

This is the third generalization of the original CBD model. According to Cairns et al., (2009), the impact of the cohort effect $\gamma_c^{(3)}$ for any specific cohort diminishes over time that is, $\beta_x^{(3)}$ decreases with x instead of remaining constant. This leads to

$$\text{logit } q_{x,t} = \beta_x^{(1)}k_t^{(1)} + \beta_x^{(2)}k_t^{(2)} + \beta_x^{(3)}\gamma_{t-k}^{(3)}$$

where

$\beta_x^{(1)}$ =1, $\beta_x^{(2)} = (x - \bar{x})$, $\beta_x^{(3)} = (x_c - x)$

for some constant parameter x_c to be estimated. This result is

$$\text{logit } q_{x,t} = k_t^{(1)} + k_t^{(2)}(x - \bar{x}) + \gamma_{t-x}^{(3)}(x_c - x)$$

2.4 Conceptual Framework

Prospective life tables give a view on the future development of mortality rates. Surely, in most developed countries longevity has been improving for a number of decades and a basic take at the standard life tables is more restrictive and can underestimate the genuine advancement of future mortality. The prospective life table offers a better view of the mortality advancement.

Mortality data on the population as a whole is of limited use unless it can be broken down into subpopulations, in which individuals are grouped according to common characteristics. The common characteristics among members of such subpopulations provide greater insight into the drivers of mortality and greater confidence in forecasting longevity.

Outlined below are the state variables involved in stochastic mortality models.

(i)Age Effects ($\beta_x^{(i)}$)

The age effects are either, non-parametric and estimated from historical data or assume some particular functional form. Mortality forecasts were performed within the same range of ages which made it unnecessary in this thesis to simulate or extrapolate the age effects.

(ii)Period Effects ($k_t^{(i)}$)

The elements of the period impact have been widely driven using random-walk processes as far back as the presentation of the first Lee-Carter (1992) model. The method behind this model has been enhanced by subsequent authors with the end goal to improve the fit and place the model on more secure statistical foundations, for instance, Brouhns et al. (2002), Booth et al. (2002), Czado et al. (2005), and de Jong and Tickle (2006)).

For each of the three models, the period effects are more precisely described. For example, in the third model (CBD Model), the dynamics of its period effects are driven using a multivariate random walk with drift and correlated innovations to drive the dynamics of the period effect:

$$k_t^{(i)} = k_{t-1}^{(i)} + \mu_k^{(i)} + \sigma_k^{(i)} + Z_k^{(i)}(t)$$

Where $\mu_k^{(i)}$ are the drifts, $\sigma_k^{(i)}$ are the volatilities and $Z_k^{(i)}(t)$ are the standard normal innovations that are correlated across the i components but independent through time.

(iii) Cohort Effects ($\gamma_c^{(i)}$)

A cohort is a group of lives who were born during the same particular time span. Cohort effects exist when mortality rates for a group of birth years reduce systematically faster or slower than the neighbouring cohorts. The cohort effects are estimated for years of birth C_0 to C_1, where the year of birth is equal to t-x, hence the cohort effect for the i^th component is $\gamma_{t-x}^{(i)}$.

The specification of a dynamic process which drives the cohort effect is the principal challenge faced in building a suitable stochastic mortality model. The usual starting point is the assumption that for a given model, the dynamics of the cohort effect are independent of the period effect $k^{(i)}$.

Every models described are subjected to the same underlying assumption that the age, period, and cohort effects are qualitatively different in nature and henceforth should be modelled in different ways. They specifically recognize a randomness in mortality rates at each age from one year onto the next, maybe caused by local environmental factors for example, a heat-wave, which is not observed between adjoining ages.

However, CBD models differ from Lee Carter model and Renshaw and Haberman model in that the LC model assumes a functional relationship between mortality rates over adjacent ages within the same year which results in smoothed mortality rates.

2.5 Desirable Properties of Stochastic Mortality Models

CMI (2005, 2006) and Cairns et al., (2009) describes the desirable properties for a desired model which include;

(a) *Parsimony*: Models which have fewer parameters are better than models with more. All the models describes have a large number of parameters which implies that none of them can be describes as parsimonious according to Cairns et al., (2009. However, comparison can be done whereby the models that have fewer parameters can be said to be more parsimonious than those that have more parameters.

(b) *Transparency*:This property looks into how a model is user friendly to ensure that users understands the model as well as the respective workings. This help with eliminating the use of a given model being used inappropriately which may result in errors in mortality forecasting. There is also subjectivity in transparency since different user have different level of understanding, therefore, this makes it difficult to analyse transparency in all models.

(c) *The ability to generate sample paths for the underlying (and unobservable) death rates,* $m(x,t)$:This process is very necessary especially for tasks like pricing longevity-linked financial instruments and developing hedging strategies (Blake et al. 2006). None of the models described fails on this criterion.

(d) *Cohort effects incorporation*:If the researcher believes there is a presence of cohort effects then it is important to use a model that incorporates cohort effects.

(e) *Robustness*:This looks into the robustness of the model's parameter estimates relative to changes in the period of data used to fit a given model. It is therefore an important property since a model is said to be robust if its parameter estimates will not charge even when a shorter period is applied. If the estimates experience a given jump resulting in a qualitative different solution when a shorter period is applied then it is evident to report that the given model yields results that lack robustness.

(f) *The ability to produce a nontrivial correlation structure from one year to the next given changes in mortality rates at different ages*:A correlation structure is said to be trivial when a perfect correlation is experienced between changes in mortality rates from one year onto the next. This is seen in the LC model when there is a single time process. The RH model additionally has a trivial correlation structure for the same reason except at the youngest age. The CBD Models however do not allow for a trivial correlation structure since they all incorporate two or more underlying period risk factors.

Vital additional properties can be assessed only when we fit the model to the available data:

* The ability of goodness of fit to the data which produces forecasts that are consistent.
* Robust parameter estimates relative to the data used.
* Given that these models are being used to forecast future mortality rates, the outcomes ought to be "biologically reasonable"

2.6 Model Selection

As already discussed, there are numerous methodologies and models that have been proposed to project mortality rates. Choosing the method to be used ought to rely upon the data and their reliability, the resources accessible for the research for which the projection is required. When all is said and done, no mortality projection basis can ever be considered "correct" or "perfect".

Many of the projection methods talked about the potential drawbacks. For instance, and as supported by Brouhns et al., (2002), univariate extrapolation of the parameters of a mortality model can be misleading, and keeping in mind that a multivariate time series model show that the parameters is possible which can result in computational intractability.

It is worth noting that regardless of the type of method used for projection, several problems arise when projecting mortality at exceptionally old ages, specifically, due to the inaccuracies in the availed data and fluctuation because of the small exposures to risk. Modellers should review the qualitative trade-off between simplicity and accuracy.

The Renshaw and Haberman (RH) model is an improvement of the Lee Carter (LC) model as it provides effective solutions to its limitations by including cohort effects. This showed that it performs better than the LC model. Unfortunately, we recognize issues with the robustness of parameter estimates under the RH model, raising doubt about its reasonableness for forecasting.

For this project, the Renshaw and Haberman (2006) model was used in projecting mortality rates. The next chapter describes the Renshaw and Haberman (2006) model in more detail.

3 Research Methodology

3.1 Introduction

In this part, we will summarize the model for analysing the data using the Renshaw-Haberman model.

3.2 Fitting and Application of the Renshaw-Haberman Model

3.2.1 Data

In order to incorporate the cohort effect the research will apply the Renshaw-Haberman model to the available data. The research will pick a particular time interval and a particular age to check whether there are any indication of cohort effects for those ages.

3.2.2 Definition of Parameters

- $q_{x,t}$ -The central death rate for age x at time t
- $m_{x,t}$ -Describes the logarithmically transformed age specific central death rate
- a_x- The average of,$m_{x,t}$ over time t, which describes the describing the general mortality trend at different ages
- k_t - A time-trend index of the general mortality level at different times. k_t captures the most essential pattern in death rates at all ages. Since the mortality is a decreasing factor, we can anticipate that this index will also decrease.
- $b_x^{(1)}$ - Deviations from the age whenk_t varies. $b_x^{(1)}$ is an age-particular constant which depict the relative speed of mortality changes at each age, when k_t is changing. The model takes into account both positive and negative estimations of b_x. A negative value of b_x demonstrates to us that the mortality rate for a particular age is raising with increasing time.

However, in practice, this normally does not make a difference in the long run (Lee and Carter, 1992). At the point when the model is adjusted over a period that is sufficiently long, for the most part it has similar characteristics (Lee and Miller, 2001), with a few special cases, for example some of the European and Central nations (Scherp, 2007).

- Y_{t-x} - A random cohort effect that is a function of the year of birth, t-x.
- $b_x^{(0)}$ - Deviations from the age when Y_{t-x} varies.
- $\xi_{x,t}$ - The error term, including systematic as well as purely random deviations.

3.2.3 The Renshaw-Haberman Formulae

Renshaw Haberman (2006) extended the Lee-Carter model by introducing the first stochastic model with the cohort effect as follows:

$$\log\left(q_{x,t}\right) = m_{x,t} = \alpha_x + b_x^{(1)} k_t + b_x^{(0)} Y_{t-x}$$

Where, α_x is the main age mortality trend, $b_x^{(1)}$ and $b_x^{(0)}$ are parameters which estimate the corresponding interactions with age k_t which is a random period effect and Y_{t-x} a random cohort effect that is a function of the year of birth, t-x. The restrictions the study will use to estimate the parameters are:

$k_t = 0, \sum_x b_x^{(0)} = 1; Y_{t-x} = 0, \sum_x b_x^{(1)} = 1$

3.2.4 Parameter Estimation for the Renshaw-Haberman Model

Let the random variable $d_{x,t}$ denote the number of deaths in a population at age x and time t. We can estimate the central mortality rate $q_{x,t}$ as:

$$\widehat{q}_{x,t} = \frac{d_{x,t}}{e_{x,t}}$$

Where, $d_{x,t}$ represent the number of deaths and represent the matching central exposure for any given subgroup. We let define the combination of age and period, i.e. the cohort year. To get the best estimates we approach a seven-step method).

1. To estimate the parameters in the Renshaw-Haberman model we use pre-programmed software for R named Life Metrics. As in the Lee-Carter model we start to estimate the fixed age effects, but here we use the singular value decomposition (SVD) method to find the least squares solution.

$$\alpha_x = \frac{1}{T} \sum_t m_{x,t}$$

2. After that we will attempt to get suitable initial values:

$$b_x^{(1)} = b_x^{(0)} = \frac{1}{K}$$

   Estimate the simplified period-cohort predictor, with the constraints that $b_x^{(1,0)}$=1 to get initial values for $k_t$and γ_z.

   Calculate the adapted values $\hat{\gamma}(\hat{a_z}, \hat{b}_x^{(1)}, \hat{b}_x^{(0)}, \hat{k}_t, \hat{\gamma}_z)$

   Calculate the deviance $(y_{x,t}, \hat{y}_{x,t})$

3. We continue by updating the parameter:

$$\hat{y}_z = \hat{y}_z + \frac{\sum_x 2w(y-\hat{y})}{\sum_x 2w(\hat{b}_x^{(1)})^2 \hat{y}}$$

Where,w is either 0 for every empty data cells and 1 for every non-empty data cell

Then we shift the updated parameter such that $\hat{y}_z = \hat{y}_z - \hat{y}_1$

Calculate the adapted values $\hat{\gamma}(\hat{a}_z, \hat{b}_x^{(1)}, \hat{b}_x^{(0)}, \widehat{k}_t, \hat{\gamma}_z)$

Calculate the deviance $(y_{x,t}, \hat{y}_{x,t})$

4. Update parameter $\hat{b}_x^{(1)}$

$$\hat{b}_x^{(1)} = \hat{b}_x^{(1)} + \frac{\sum_x 2w(y-\hat{y})}{\sum_x 2w(\hat{y}_z)^2\hat{y}}$$

Calculate the adapted values $\hat{\gamma}(\hat{a}_z, \hat{b}_x^{(1)}, \hat{b}_x^{(0)}, \hat{k}_t, \hat{\gamma}_z)$

Calculate the deviance $(y_{x,t}, \hat{y}_{x,t})$

5. Update parameter $\hat{k}_t$

$$\hat{k}_z = \hat{k}_z + \frac{\sum_x 2w(y-\hat{y})}{\sum_x 2w(\hat{b}_x^{(0)})^2\hat{y}}$$

Then we shift the updated parameter such that $\hat{k}_t = \hat{k}_t - \hat{k}_1$

Calculate the adapted values $\hat{\gamma}(\hat{a}_z, \hat{b}_x^{(1)}, \hat{b}_x^{(0)}, \hat{k}_t, \hat{\gamma}_z)$

Calculate the deviance $(y_{x,t}, \hat{y}_{x,t})$

6. Update parameter $\hat{b}_x^{(0)}$

$$\hat{b}_x^{(0)} = \hat{b}_x^{(0)} + \frac{\sum_x 2w(y-\hat{y})}{\sum_x 2w(\hat{k}_z)^2\hat{y}}$$

Calculate the adapted values $\hat{\gamma}(\hat{a}_z, \hat{b}_x^{(1)}, \hat{b}_x^{(0)}, \hat{k}_t, \hat{\gamma}_z)$

Calculate the deviance $(y_{x,t}, \hat{y}_{x,t})$

7. Control the divergent convergence

$$\Delta D = D - D_u$$

Where, D is the deviance from step 3 and D_u is the update deviance at step 6 If $\Delta D > 1 * 10^{(-6)}$ => go to step 3

Stop iterate process when $\Delta D \approx 0$ and take the adapted parameters as the ML estimates to the observed data.

Alternatively, stop if $\Delta D<0$ for 5 updating cycles in a row and consider using other starting values or declare the iterations non-convergent

8. When convergence is achieved, rescale the new interaction parameters,

$\hat{b}_x^{(1)}$, $\hat{b}_x^{(0)}$,$\hat{k}_t$ and $\hat{\gamma}_t$

$\hat{b}_x^{(1)} = \frac{\hat{b}_x^{(1)}}{\sum_x \hat{b}_x^{(1)}}$, $\hat{b}_x^{(0)} = \frac{\hat{b}_x^{(0)}}{\sum_x \hat{b}_x^{(0)}}$, $\hat{k}_t = \hat{k}_t * (\sum_x \hat{b}_x^{(0)})$

In order to satisfy the age-period Lee-Carter model constraints:

$\sum_x \hat{b}_x^{(0)} = \sum_x \hat{b}_x^{(1)} = 1$ and $\sum_x k_t = 0$

3.2.5 Forecasts for the Renshaw-Haberman Model α_x and β_x

For us to forecast future mortality rates, Lee and Carter assumes that $a_x, b_x^{(1)}$ and $b_x^{(0)}$ remain constant over time and the time trend $\hat{k}_t$ is intrinsically seen as a stochastic process. The time trend $\hat{k}_t$ is additionally assumed to be independent Y_{t-x}. Lee et al., (1992) suggest using the following random walk with drift model for $\hat{k}_t$:

$$\hat{k}_t = \hat{k}_{t-1} + \theta + C\xi_t \quad \textbf{(1)}$$

in which θ is a constant drift term, C is a constant volatility and ξ_t is a one-dimensional i.i.d N(0,1) error.

An appropriate ARIMA (p,d,q) model for the mortality index k_t is found by carrying out the standard Box and Jenkins methodology (identification-estimation-diagnosis). In general an ARIMA(0,1,0) with drift is found to be appropriate, however, other ARIMA forms give a better fit to some data (Brouhns et al. 2002). As indicated in Booth et al. (2006), ARIMA(0,1,0) is a sensible choice in the situations where there is a stable linear tendency in the yearly mortality improvements, however would be inappropriate for the cases characterised by regular dynamic changes in slope (i.e. non-linear). All things considered, the authors have discovered that this model has performed well in numerous large data applications, even when a more complex model may have been shown by the shape of the period effects.

After discovering an appropriate ARIMA model the variable, the mortality index k_t can be forecasted. Let $\hat{k}_{t_n+s}$ denote the s-period ahead forecast of the mortality index. Then in case of the Poisson Lee-Carter model, the expected value of future death count is given by

$$E[D_{x,t_n+s}] = E_{x,t_n+s}\hat{m}_{x,t_n+s} \tag{2}$$

Where is E_{x,t_n+s} the future exposure and is the $\hat{m}_{x,t_n+s}$ forecast of future death rate with

$$\hat{m}_{x,t_n+s} = exp(\hat{\alpha}_x + \hat{\beta}_x\hat{k}_{x,t_n+s}) \tag{3}$$

Using $\hat{m}_{x,t_n+s}$ we can calculate other quantities of interest, such as life expectancies, life annuity premiums, etc.

3.2.6 Software Analysis of the Renshaw-Haberman Model Using R

The main calculation for the study is to forecast future mortality using survival analysis. To get even better estimates and to forecast mortality we proceed by using the statistical program R. Since the selected methodology involves iterations, it may not be possible to get good parameter values for the Renshaw- Haberman method using Excel. The codes used to get the results are summarised in section: Appendix B.

4 Data Analysis

4.1 Data Collection

4.1.1 Mortality Data Sources in Kenya

There are 3 main data collection sources:

1. Mortality data from specific pensions plans, life insurance and annuity providers

The data is mainly collected from policyholders' files. The data mainly comes from signup forms and claims or maturity activations. The data provide best sources for forecasting future mortality for these populations and projecting future insolvency risks. This data source is not public and is usually subjective to the specific policy provider's needs.

2. Aggregate mortality data compiled by industry bodies

The data is collected by bodies that manage different bodies or govern different insurers and pension providers in a specific geolocation i.e. countries, unions or even counties/provinces. The data is largely inconsistence in that the bodies that provide data year by are different period by period. The industry bodies are not actively collecting data as they focus mainly on the delivery methods, pricing and liquidity but have no data analysis tools to collect data effectively.

3. National population mortality data

Kenya has one of the largest population basis risks in the world this is due to poor data collection by the hospitals and the registrar of death. Furthermore, the data is normally not published for the public to use. Efforts are being put by the government to have the data published and refined.

4.1.2 Sample Data

National population data was used in the research to analyse the Renshaw - Haberman model. Due to the unavailability of data in Kenya, the research used historical raw data from the USA. The data is covering ages 65-90 for reference years 1970-1985 and ages 65-95 for reference years 1986-2010.

The set of data is contained in the file: *sampleData.r*. The Mortality experience in the USA in these years has similarities with the data in Kenya in the same period and we conducted a correlation test to determine whether there exists a linear relationship between the two mortality experiences as well as measure the direction of the relationship.

4.1.3 Correlation Analysis

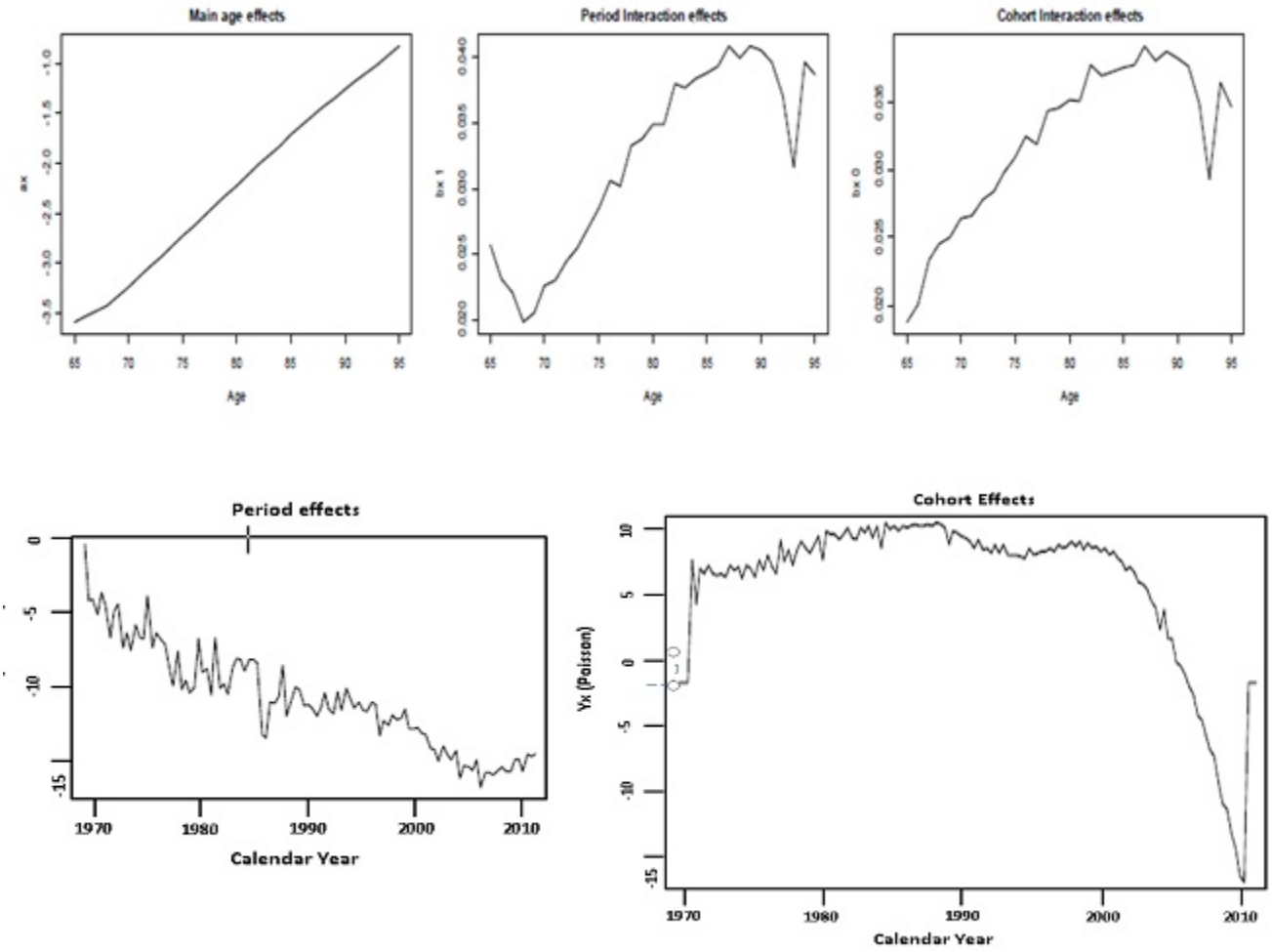
The USA-Kenya correlation coefficient results for the male and female populations are **0.98765** and **0.95667** respectively, that is, the coefficients tend to 1 as the data sets increases. Thus, there is strong correlation between the two mortality data sets have a strong relationship. The r-codes for the analysis are in Appendix B.

4.2 Fitting the Renshaw-Haberman model using R

The main purpose for fitting the model is to predict future mortality which is forecasting life expectancy. To get the best estimates we will use the statistical program R. We will also analyse the residual plot to test the validity of fitting the model.

4.2.1 Parameter Estimation

Figure 1. Age-Period-Cohort Regression: USA data 1970-2010 (male)



4.2.2 Residual Analysis

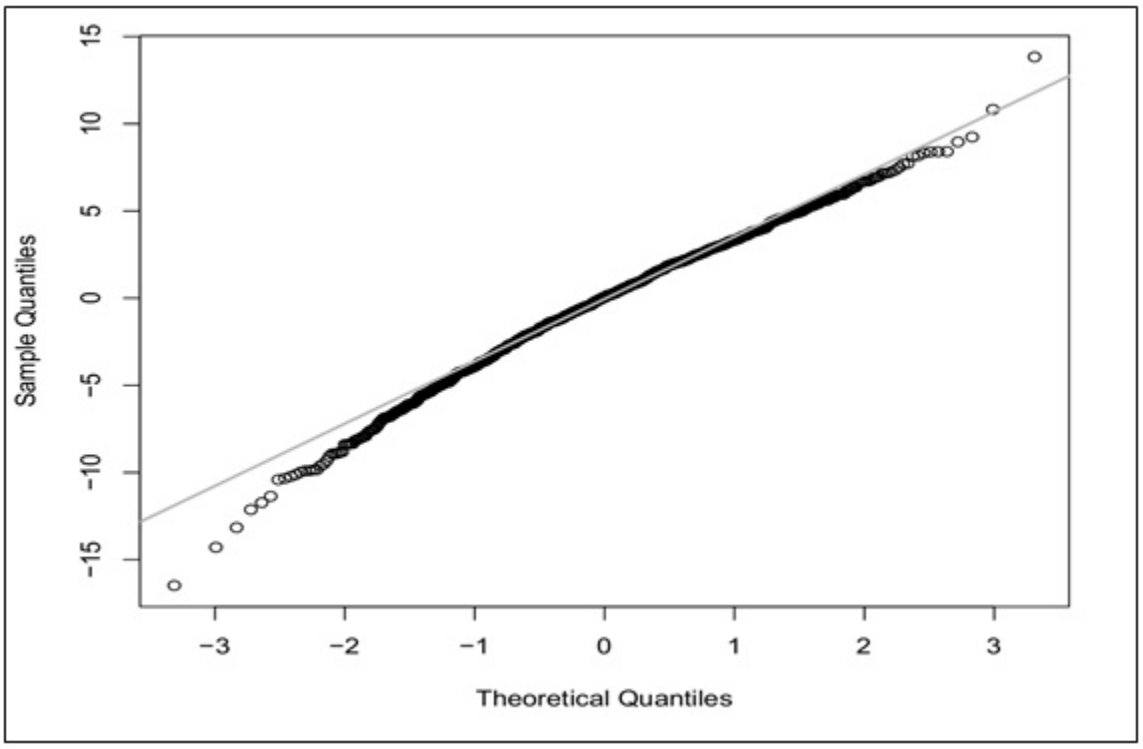
In order to determine the validity of the Renshaw-Haberman model has a relatively good-fit to the data used in the study. We analysed the error terms $\varepsilon_{x,t}$. The $\varepsilon_{x,t} \sim N(o, \sigma_\varepsilon^2)$, i.e. it has a zero mean Gaussian distribution and it reflects the mortality data set influences captured by the model. The error should also have a relatively small variance.

Table 1. Residual Analysis Results

Term	Value
Mean	0.00987
Skewness	-0.10187
Kurtosis	0.600394

The variance is captured in the appendix. We can observe that the mean is a value close to zero. The residuals are slightly skewed to the left while the kurtosis statistic is low implying that residuals have a flat top near the mean rather than a sharp peak relative to a normal distribution. To further test the residuals we used a qq-plot to see if the residuals meet the conditions of the hypothesis that the Renshaw and Haberman model follows a normal distribution.

Figure 2. QQ Plot for the Residual Data



The qq line is 45°, however the data has heavy tails, thus the dataset doesn't adhere to the assumption that we made above but this is largely due to the long period of data analysis. According to Trauth & Reimer-Hommel, (2000), the error terms for the Renshaw-Haberman model have thicker tails then the normal distribution and is slightly skewed. This is largely due to

the number of years understudy i.e. 40 years for the period understudy (1970-2010).

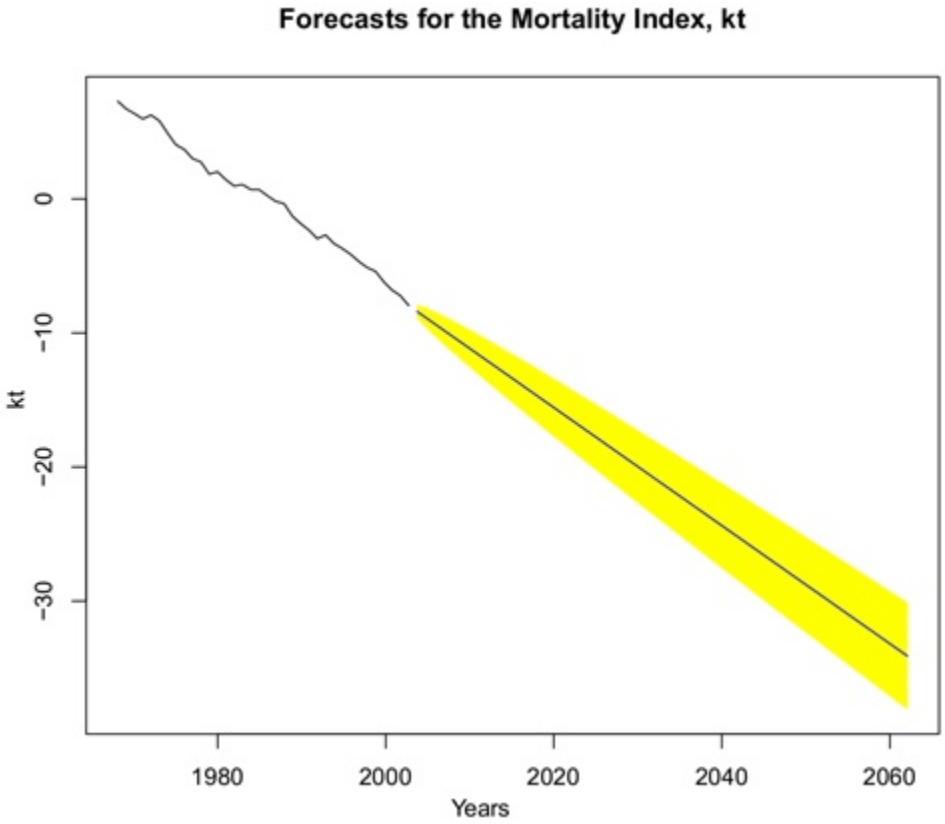
4.3 Reserving with Renshaw-Haberman

4.3.1 Forecasting

In section 4.2.1 the study used historical data of the time trend index of the general mortality index, k_t. The study performed a linear regression and assumed that mortality will follow the same curve the following 50 years i.e. period (2011, 2060). By using different time periods the study will get predictions of future life expectation.

The expectation of life for retirees is projected to improve in the future as predicted by the Renshaw-Haberman. Figure 4 shows an illustration of the forecasted mortality index over a 60 year prediction horizon. The predicted mortality index indicates a general drop in future mortality rates. Improved life expectancy implies that retirees will live for longer than expected, therefore longevity risk is present.

Figure 3. Mortality index Forecasts



The probability of dying was calculated using the Renshaw-Haberman formula as follows:

$$q_{x,t} = e^{(a_x + b_x^{(1)} k_t + b_x^{(0)} Y_{t-x})}$$

The simulation will be then be used to estimate the life expectancy of for a person aged 60, in the year 2010 at time intervals (2015, 2020, 2025, . . . , 2050, 2055, 2060), as follows:

$$e_{x,t} = \int_0^\infty ({}_t p_x) dt = \int_0^\infty (1 - {}_t q_x) dt$$

Figure 4. Summary of Life Expectance at age 60 in year 2010

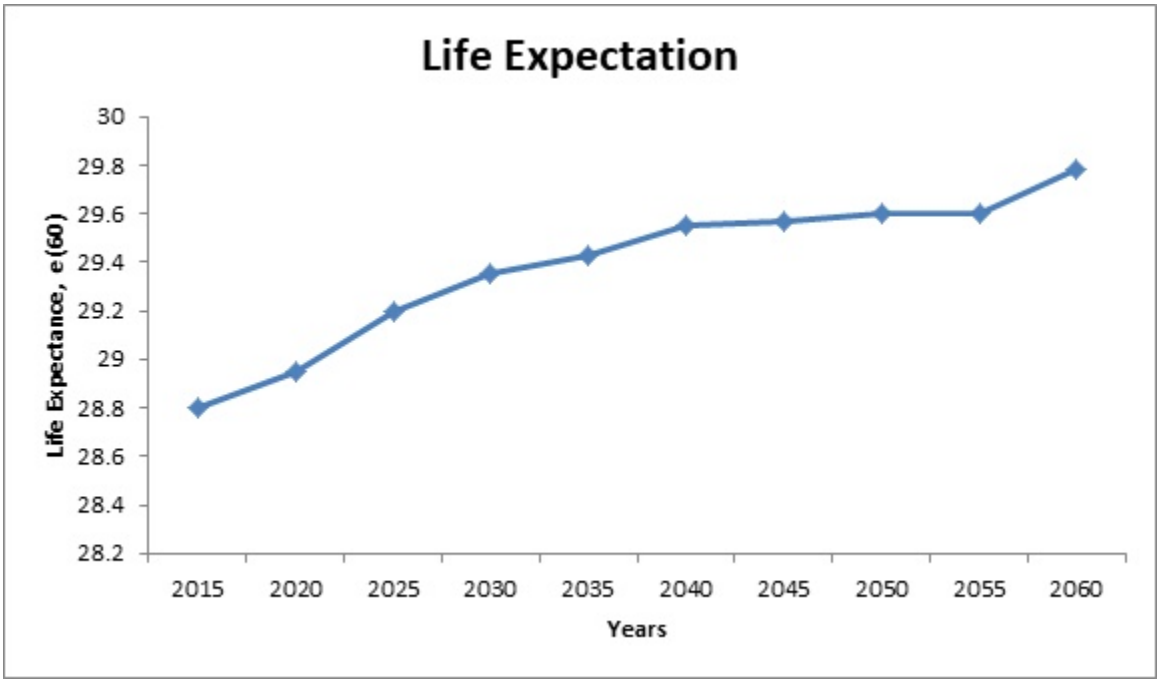


Figure 4 shows that the life expectancy will increase by a steady factor of approximately 15% for the next 60 years from 2010. This is largely due to the increase in health care and the low infant mortality recorded in the year 2010 i.e. the cohort analysis year.

The results will then be used to compute the mean annuity present value of annuities as follows:

$$\bar{\alpha}_x = \int_0^\infty (v_t^t p_x) dt = \int_0^\infty [v^t (1 -_t q_x)] dt$$

The following assumptions were made in calculating the actuarial mean present value (MPV) of a yearly annuity of 1 monetary unit payable on retirement:

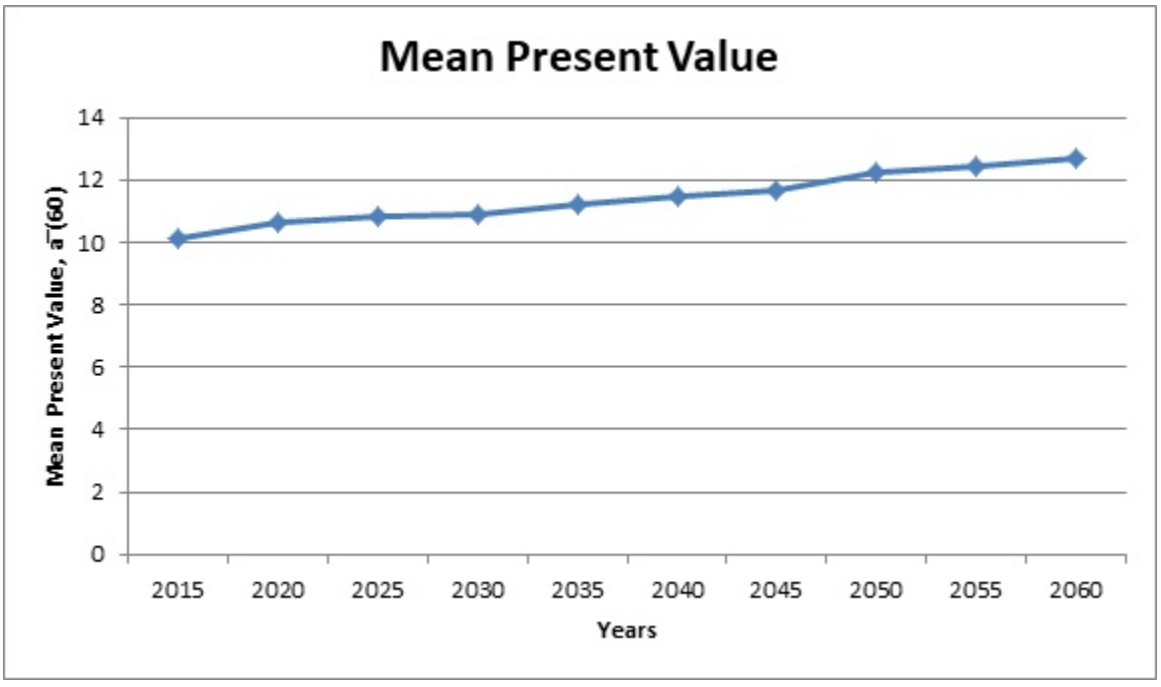
- The retirement age, 'x', is 60 at year 2010
- The payments are made continuously as people retire at any given time
- The risk free interest rate, 'i', is at 9 % as per the CBK rate for 2018, October
- Inflation rate, 'r', is at 5.53% as KNBS in 2018

Table 2. Summary of Life Expectancy and MPV of Annuities as at year 2010

Year	$e_{x,t}$	$\bar{a}_x$
2015	28.80	10.140
2020	28.95	10.614
2025	29.20	10.817
2030	29.35	10.912
2035	29.43	11.233
2040	29.55	11.446
2045	29.57	11.644
2050	29.60	12.256
2055	29.60	12.403
2060	29.78	12.667

These results are also displayed in the below Figure 5

Figure 5. Summary MPV of Annuities as at year 2010



From figure 5 due to the increase in life expectance we have increased MPV at the same period, which is normally not the case for life annuities.

The Renshaw and Haberman model forecasts show an improvement of life expectancy at retirement age for persons born in recent years. The actuarial present value of a yearly annuity of 1 monetary unit payable to retirees born recently is also calculated to be higher. The Renshaw and Haberman mortality projections imply that the cost of life insurance and pension annuities in future is expected to increase due to the reduction of mortality.

5 Conclusions & Recommendations

5.1 Results Summary

The main objective for the study was to examine the effects of DB pension liabilities caused by longevity risk. To assess this risk, a stochastic approach was found ideal since it allows probabilities to be attached in different forecasts.

5.1.1 Future Mortality & Life Expectation

Due to the low child mortality in year 2010 the population increase resulted in higher expectation rate for the population in the same year. The future mortality will result in higher life expectation. This postulates that the longevity risks for pension companies are going to be high in the next 3 decades.

5.1.2 Renshaw-Haberman Model Future Pay-Out Values for the Annuities

The model did not fit the data as the error terms were not normally distributed to therefore the model failed the goodness of fit test. However, the model was viable for the study because the anomaly was largely due to the large data set. A larger data set with one of the parameters set as time will have a high variability around the mean.

The cohort effect of the research was birth and the better the infant mortality resulted in a higher expectation of life for the population when they reach the retirement age of 60 as per the research. The lower the infant life mortality the higher survival rates of retirees in the future.

The Renshaw-Haberman model isolates the cohort effects and models mortality. It also provides the data analytics with a clear understanding of the effects of the specific cohort.

The Renshaw-Haberman (RH) modification to the Lee-Carter model incorporates the cohort effect, hence, provides the best fit. Despite this strength, the model raised problem in relation to the robustness of its parameter estimates which questions its suitability for forecasting.

5.2 Limitations of the Study

The data which was used for the study didn't belong to the Kenyan populations and thus any changes in the cohort effects might not be the best representative of the Kenyan cohort effects. The period of analysis is in the past; hence the research will only be important for academic purposes and not for practical industry use in the Kenyan market.

5.3 Recommendations of the Study

The data used in the study is the USA, thus for future studies, data from actual developing countries should be used even though similarities in the data exist between the countries. Shortcomings may be encountered as a result of using the US data to forecast the Kenyan scenario. For instance, U.S. may experience a greater longevity risk than Kenya due to their medical advancements, better lifestyle and other factors.

Longevity risk needs a very efficient model to anticipate the expected costs that will be incurred if the risk occurs hence the best and the most recommended model is the Cairns-Blake-Dowd (2008) Model (CBD-4).

According to this model, for any specific cohort its cohort effect impact reduces over time opposed to remaining constant like what the other models suggest. This is one of the major reasons as to why the CBD-4 model is highly recommended for calculating the longevity risk. Another reason is that it assumes a functional relationship between mortality rates in adjacent ages.

Bibliography

[1] Booth, H., and Tickle, L., (2008). "Mortality modelling and forecasting: A review of methods". *The Australian Demographic Social Research Institute*.

[2] Booth, H., Maindonald, J., Smith, L. (2002). "Applying Lee-Carter Under Conditions of Variable Mortality Decline". *Population Studies*, 56: 325-336.

[3] Brockmann, H., Gjonça, A., and Maier, H. (2000). "Old-age mortality in Germany prior to and after reunification". *Demographic research*, 3 (1).

[4] Brouhns, N., Denuit, M., and Vermunt, J.K. (2002). "A Poisson log-bilinear approach to the construction of projected lifetables". *Insurance: Mathematics and Economics*, 31(3): 373-393.

[5] Butt, Z. and Haberman, S. (2010). "**ilc**: A Collection of R Functions for Fitting a Class of Lee-Carter Mortality Models using Iterative Fitting Algorithms". *Actuarial Research Paper* 190, City University, London, UK.

[6] Cairns, A. J. G., Blake, D., Dowd, K., Coughlan, G.D, Epstein, D., Ong, A. and Balevich, I. (2009). "A Quantitative Comparison of Stochastic Mortality Models Using Data from England and Wales and the United States". *North American Actuarial Journal*, 13(1): 1-35.

[7] Carter, L.R. and Prskawetz, A. (2001). "Examining Structural Shifts in Mortality Using the Lee-Carter Method". *Working Paper WP2001-007*, Max Planck Institute for Demographic Research, Rostock, Germany.

[8] Case, A. C and Deaton, A. (2003). "Broken Down by Work and Sex: How our Health Declines". *NBER Working Paper No. 9821*, National Bureau of Economic Research.

[9] Case, A. C and Paxson, C. (2010). "Causes and consequences of early-life health". *Demography 47* (Supplement): S65-S85.

[10] Continuous Mortality Investigation (2007). "Stochastic projection methodologies: Further progress and P-Spline model features, example results and implications". *Working Paper 20 (revised)*, Institute of Actuaries and Faculty of Actuaries.

[11] Continuous Mortality Investigation (2009). "A prototype mortality projections model: Part one – An outline of the proposed approach". *Working Paper 38*, Institute of Actuaries and Faculty of Actuaries.

[12] Continuous Mortality Investigation (2009). "A prototype mortality projections model: Part two – Detailed analysis". *Working Paper 39*, Institute of Actuaries and Faculty of Actuaries.

[13] Costa, D. L., (2005). "Causes of Improving Health and Longevity at Older Ages: *A Review of the explanations." Genus*, 61(1): 21–38.

[14] Currie, I. D., Durban, M. And Eilers, P. H. C. (2004). "Smoothing and Forecasting Mortality Rates". *Statistical Modelling*, 4: 279–98.

[15] Edwards , T., Munhenga, C. (2011). Longevity Risk. In T. Edwards, & C. Munhenga, *Actuarial Evaluation of Risk in Pensions* (pp. 24-30). Harare: University of Zimbabwe.

[16] Gustafsson, M. (2011). *Cohort Effects in Swedish mortality and their Effects on Technical Provisions for Longevity Risk*. Universitet Stockholm, Department of Mathemetical Statistics. Stockholm: University of Stockholm Press. Retrieved October 12, 2018, from http://www.maths.su.se/matstat.

[17] Lee, R., & Carter, L. (1992, September). Modelling and Forecasting the Time Series of U.S. Mortality. *Journal of the American Statistical Association, 87, 659-671.*

[18] Rizzuto, D., & Orsini, N. (2012). Lifestyle, Social Factors, and Survival after age 75: Population Based Study. *Clinical Research ed.*(e5568).

[19] Shaw, C. (1994). Accuracy and uncertainty of the national population projections for the United Kingdom. *Population Trends(77)*, pp. 24-32.

[20] Taruvinga, T.,& Gatawa, I. (2010). Understanding Population Dynamics in Africa. *UZ Weekly*, III, pp. 50-62.

[21] Trauth, T., & Reimer-Hommel, P. (2000). Challenges and Solutions for the Management of Longevity Risk. In M. Frenkel, U. Hommel, & M. Rudolf, *Risk and Management: Challenge and Opportunity (pp. 85-100)*. Springer.

Appendix

Appendix A: Stochastic Mortality Models Formulae

Model	Formula
Lee Carter (1992)	$\log mx,t = \beta_x^{(1)} + \beta_x^{(2)} k_t^{(2)}$
Renshaw and Haberman (2006)	$\log m(x,t) = \beta_x^{(1)} + \beta_x^{(2)} k_t^{(2)} + \beta_x^{(3)} \gamma_{t-x}^{(3)}$
Currie (2006)	$\log m(x,t) = \beta_x^{(1)} + n_\alpha^{-1} k_t^{(2)} + n_\alpha^{-1} \gamma_{t-x}^{(3)}$
Currie, Durban, and Eilers (2004)	$\log m(x,t) = \sum_{i,j} \theta_{i,j} \beta_{i,j}^{\alpha y}(x,t)$
CBD-1	$\text{logit } q(x,t) = k_t^{(1)} + (x-\bar{x}) k_t^{(2)}$
CBD-2	$\text{logit } q(x,t) = k_t^{(1)} + (x-\bar{x}) k_t^{(2)} + \gamma_{t-x}^{(3)}$
CBD-3	$\text{logit } q(x,t) = k_t^{(1)} + (x-\bar{x}) k_t^{(2)} + ((x-\bar{x})^2 - \bar{\sigma}_x^2) + \gamma_{t-x}^{4}$
CBD-4	$\text{logit} q(x,t) = \beta_x^{(1)} k_t^{(1)} + \beta_x^{(2)} k_t^{(2)} + \beta_x^{(3)} \gamma_{t-x}^{(3)}$

Appendix B: R-Codes Snippets

The study used the Lifemetrics data set imbedded in the R codes library:

1. Data Correlation Analysis (Kenya and USA mortality tables)

```
#Kenya office mortality rates for 2001-2003 Males
> KEUSAMort <- read.csv (file = "KenyaUSAMort.csv", head=TRUE, sep = ",")
> ke_male_qx <- KEUSAMort$keOmales_qx
> us_male_qx <- KEUSAMort$USAmales_qx
> ke_female_qx <- KEUSAMort$keOfemales_qx
> us_female_qx <- KEUSAMort$USAfemales_qx
> #Correlation test - Males
> cor(ke_male_qx,us_male_qx)
> #Correlation test - Females
> cor(ke_female_qx,us_female_qx)
```

2. Fitting the Renshaw-Haberman Model

The data for the model was standardised to a Lee carter model and the inbuilt data analytic code was used: “fit701…”

```
#Load the source code and the list qdata.usa which contains the input data
> source('fitModels.r')
> source('simModels.r')
> source('sampleData.r')
> > #fit701 requires the following variables as input
> #    x – a vector of ages of length n, e.g. 60, 61, 62,...
> #    y – a vector of years of length m, e.g. 1968, 1969, 1970,...
> #    etx – m by n array of exposures, row 1 corresponds to year 1 (y[1]),
#column 1 corresponds to age 1 (x[1])
> #    dtx – m by n array of deaths, corresponding to the exposures
> #    wa – m by n array of 0/1 weights.
#loading up data from the list qdata.usa
#These variables are the required input parameters needed for the fit functions
> x=qdata.usa$x
> y=qdata.usa$y
> etx=qdata.usa$etx
> dtx=qdata.usa$dtx
> wa=qdata.usa$wa
 ### FITTING THE LEE CARTER MODEL ####
> # To fit a model call the corresponding function passing in
> # vectors and arrays extracted from the input list qdata.usa
> res=fit701(x,y,etx,dtx,wa)
> #to return a list containing all the relevant output from the
> #estimation procedure
> names(res)
```

Appendix C: Statistical Data

1. Summary of MPV and Expectation of Life for Women

Year	$e_{x,t}$	$\bar{a}_x$
2015	28.80	10.140
2020	28.95	10.614
2025	29.20	10.817
2030	29.35	10.912
2035	29.43	11.233
2040	29.55	11.446
2045	29.57	11.644
2050	29.60	12.256
2055	29.60	12.403
2060	29.78	12.667

2. Variance for Residuals (after every 10 years)

Year	Variance
1970	3.455
1975	5.698
1980	2.908
1985	1.345
1990	0.234
1995	56.890
2000	2.678
2005	29.698
2010	34.567