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Forecasting the Term Structure of Government Bonds in Kenya

1 Introduction

In the last two decades, modelling the term structure of interest rates and using the same to reflect the monetary policy framework in the greater fiscal environment has attracted interest among financial markets players across the globe. Their interest has been the economy, but broken down, it has been the different components in the financial market that include bonds, bills, mortgages and other forms of securities traded in the market. As such, researchers have embarked on a serious review of the yield curve to model the term structure of interest rates, due to its predictive power and forecasting capabilities of future economic states in an economy.

The yield curve has been used over the years to predict and forecast among other economic factors; inflations rates at different periods, real economic output, future possibility of periods of recession, and also potential growth. The yield curve has been used in developed, emerging, and developing economies successfully. However, it has been employed in developed and emerging financial markets than it has been in developing markets. Research shows that the yield curve has been used to model interest rates in American markets; including Colombian and Brazilian market, as well as emerging financial markets like South Africa and India.

Although the concept of forecasting is best discussed from an actuarial perspective, the tenents of Finance and Mathematical Statistics provide very critical grounds for which to begin and build on. Various models have been proposed, especially in the developed and emerging markets. However, few research work has been done in the developing markets that include the Kenyan financial market. Over time, model-based asset pricing and its evolution over the years to bring in modern day dynamics in financial markets has become extremely important. As such, financial institutions offering fixed financial instruments have relied on estimations, modelling and forecasting to build scenarios and come up with plausible and workable solutions to the uncertain environments they operate in, and the anticipation of changes in future. As such, market players have opted to model term structures of interest rates in predicting yields of financial instruments/assets in the financial markets.

The models that have been brought forward in the Kenyan context include the no-arbitrage and equilibrium approaches. However,this study proposes a unique model; Nelson-Siegel (1987). Further, this study considers a reparameterized form of the model called by Diebold and Li (2006) which is an advancement of the Nelson-Siegel model. This is the model to be employed to fit Kenyan data. Additionally, the study considers the same model but include time varying parameters of regression type. To the best of my knowledge, this combination of models bringing in time varying parameters is one of its kind and the first to be conducted in the Kenyan scenario. The models of regression type will then be used for forecasting and the results maybe compared with benchmarking models that successfully worked in other application across the globe.

The term structure of interest rates is best demonstrated in yield curves. A yield curve basically represents the relationship between yields of a financial instrument (a bond in our case) and their term, basically described as the maturity. The term structure basically shows the behavior of interest rates in the short-term, medium-term and the long-term. It is usually a plot of interest rates of bonds against maturity in months or years.

1.1 Concept of Bonds

Gwalani (2015) defines a bond as a debt security instrument that is issued by companies or government with an aim of raising money. A coupon bond pays regular instalments and the principal on maturity date. Das, Ericsson and Kalimipalli (2012), notes that bonds are means of raising funds for the government. Government bonds are instruments of financing a government deficit Ndung'u (2013). Bonds are tools of acquiring loans from individuals and institutions. Bonds are a crucial means of getting funds by the governments. These bonds range from short term to long term bonds depending on the purpose of the bond issuance. Kabua (2011) noted that the first world countries such USA, and European Countries have the best and most complex bond markets.

Jaramillo and Weber (2013) avers that the bond statistics indicates that the world bond market statistics is dominated by the developed countries. For instance America occupied large share of the pie at about 40% of the world value of outstanding domestic bonds; its market comprises mortgage-backed securities, Federal Agency Securities, Corporate Bonds and treasury bills (Myers, 2014). In fact most of Africa's debt is in foreign currency and is therefore considered as not tradable, and very few countries in Africa have a viable bond market. Particularly, for most African countries the bond market is insignificant or non-existent, even though Africa has some of the most heavily indebted countries in the world (Yeboah, 2014). Sub-Saharan African countries in developing local bond markets include the different debt structures and level of market infrastructure. Kenyan government bonds are traded in the Nairobi security exchange market and have maturities ranging from 1-30 years. The continued growth of the bond market is an indicator of the critical role played by the bond market in raising capital for corporates and funds for the government.

1.2 The World Bonds Market

In the developed economies of the world, bonds markets are highly developed. They are also well regulated and considered a critical part of the national economy, and transcends further to influence dynamics in the global economy. They are also highly liquid and well diversified. Players in these markets have undertaken deliberate attempts to carry out research over the years to improve existing products in their markets as well as bring in new ones. In example, the United States of American has one of the most developed market occupying about 40% of the world bond market. Another leader in the segment are bonds in Europe, and the countries in the Euro area. They have strong and stable currencies that are also used as benchmarks and trading currencies in other economies. It is noteworthy that these markets are domiciled in mature democracies with functional governance and regulatory frameworks and are supported by strong research capabilities.

The emerging markets of the world include South Africa, whose bond market is worth about 200 billion USD as of 2018. Although South African bonds market is one of the best in the world ranking highly in liquidity rating, its entire financial market does not match the leaders in the segment. However, it emerges as a leader among the emerging markets category. The South African bonds market provides attractive returns on fixed income instruments, and thus has attracted substantial foreign investments by investors who wish to grow their portfolios. They lack a well-diversified market that is a characteristic of the markets in the developed economies. However, their currencies are relatively stable against major world currencies and are thus tradable.

The developing world bonds market comprise bond markets in the third world countries that are undeveloped. They also use developed and emerging markets as benchmarks for policy formulation and implementation, as well as the currency. This means that shocks or volatilities in the major world currencies affect the performance of currencies in the developing world. In fact, currency strength in these countries are usually measured against the major currencies, in modern day; dollar, euro and pound. Countries in the Sub-Saharan Africa are the most affected by shocks on foreign currencies. Bonds market in these countries are poorly regulated and relatively illiquid compared to their counterparts in developed and emerging economies. These economies are among the highly indebted in the world. The yield on bonds in these markets are highly prone to shocks in the global economy. As such, much of the bonds portfolio as well as debts are held in foreign currencies. Investments in these economies and markets have exhibited overreliance in foreign capital inflows that is mainly considered favorable as it's used to stabilize local currencies.

The bond market lies at the intersection between the real economy and financial markets. With regard to the economy, Christensen Jose and Mussche (2018) point that the risk-free Treasury yield curve responds to shocks in inflation and growth expectations. Becker and Irvishna (2011) asserted that bonds are significantly useful tools of raising funds by companies and governments from both individual and institutional investors. Bonds are an important debt security for governments and corporate institutions. Bond yield is the bond return as given by the summation of the bond price and the capital gain. The rate of return of the bond is crucially beneficial to the investor, since it is the rate of return on their investment (Ben and Castelletti, 2016).

1.3 Modern Day Forecasting Trends

Modern day forecasting widely exploits works done by earlier researchers. Whereas models were developed to solve particular problems, modern day modelling and forecasting seeks to combine at least two models with predetermined factors of interest to come up with a hybrid model, or a superior model that yields results similar to, or superior to the original individual model. In example, researchers have picked on earlier models developed in the 20th century, added possibly time parameters, modelled the same using modern day mathematical/statistical softwares to either get similar or better results. The Nelson-Siegel model to be discussed later in this work has evolved overtime to gain parsimony, flexibility consistency and accuracy in forecasting through a combining scheme of different models. Unlike the original model, the parsimonious model adds to it time varying parameters of regression type.

Therefore, in the concept of forecast combination, a combination of individual forecasts models are set against individual forecast models. As expected, a combination of forecast models is likely to perform better than an individual model, since each model captures particular and precise element in the forecasting process, whereas an individual model will probably offer a one view perspective. Therefore, in modelling the term structures of interest rates today, many researchers across the world have opted to go for combined forecast models, and their results indicate that they perform better than individual models in most cases, yield similar results in few instances and produce worse forecasting results than individual models in very isolated cases.

1.4 Globalization and the Financial Market

In developing and emerging markets, the effects of globalization cannot be ignored. Technocrats in their financial markets are tasked to mitigate possibilities of financial crisis, recessions and various other undesirable events in other economic systems in financial markets that could affect their own. As a result, they set out to model and forecast trends and incidences in the financial market with a view of identifying and reducing their impact.

Financial globalization in developing and emerging markets raises the risk of capital flight, capital inflow and outflow, investment, growth and volatility. Today however, there lacks evidence of modelling effects of globalization in financial markets using term structures.

One unique challenge in the bonds markets for developing and emerging markets is over reliance on foreign currencies to maintain capital reserves. The consequences are that risks of volatility in the foreign currency will affect entire portfolios held in other countries. Thus, exchange rate volatility and highly volatile short maturity financial instruments is experienced.

One would seek to understand the net effect of globalization in the modern day financial market. Whereas globalization is expected to bring about development in research and advancement in technology associated with financial markets, developing countries like Kenya and counterparts in the Sub-Saharan Africa indicate that the region may not have positively tapped into globalization in full. Partly, these economies have largely domesticated foreign theories, modelling and forecasting practices into their economy, whereas the original models were suited for other economies. Secondly, African markets tend to replicate market products of developed economies into their own. A classic example is the derivatives market that is well developed in the American and European market, set to be tested in Kenya the year 2019. Third, the benchmark models of developed countries have also been used as benchmarks for African countries. In example, American long-term bonds modelling have usually used the Random Walk as a benchmark model. Consequently, a similar study conducted in the South Africa market has also used the Random Walk as a benchmark model, and the same for India. This is in spite of the myriad of problems facing Africa, and their uniqueness in that they are associated with poor structures; both in governance, regulatory grounds.

1.5 Problem statement

In the Kenyan financial market, research on bonds is very extensive. However, most of the literature has focused on theoretical and simplistic approach. As such, the component of mathematical modelling of the bonds is very sparse, and where it exists, traditional forecasting methods takes precedence. That could explain the reason why the no-arbitrage models have largely been used in the Kenyan context. Research has also showed that theoretical studies on determinants of long-term yields of Kenyan government bonds is very extensive. The studies seek to explore how the factors; interest rates, inflation rates, credit rating and equity volatility have determined the yield of bonds in the long-term. The studies have relied largely on the ARIMA models. However, there lacks evidence of actual mathematical modelling and forecasting of the factors. This study therefore employs a mathematical approach to modelling and forecasting the term structure of

Kenyan government bonds. To achieve this, the study employs the dynamic Nelson and Siegel model as reassessed by Diebold and Li (2006) to a parsimonious model with time varying parameters of regression type.

1.6 Objectives of the Study

1. To model the term structure of interest rates using the Dynamic Nelson & Siegel model.
2. To forecast the term structure of Government bonds using the dynamic Nelson & Siegel model with time varying parameters.
3. To come up with the best mixture of models suitable for forecasting Kenyan Government Bonds.

1.7 Value of the Study

The results of this study will be beneficial to issuers of government bonds, and by extension, the corporate bonds. The Central Bank of Kenya as the issuer of government bonds in Kenya, will have a new practical perspective of forecasting term structures of government bonds, the behavior and trends in interest rates regimes. It will also provide a first-hand experience on forecasting term structure of interest rates in an environment where economic crimes and poor regulatory framework thrives.

The study will also localize and domesticate valuable forecasting practices successfully employed in developed economies.

To the investor, it will show the behavior of interest rates over time and hence help decide whether to hold short-term, medium-term or long-term maturity bonds for optimum yields at lowest possible risk when building portfolios consisting of risky and risk-free assets.

To researchers and academicians, it will provide an additional model from existing ones, for comparison on the most powerful forecasting models among those tested in Kenya; either individually or as a combination of various models, besides forming a basis for further studies.

The outline of the thesis is as follows:

Chapter 2: This chapter contains the Literature Review, where key papers used in this model are discussed to the extents of their findings and conclusions.

Chapter 3: The methodology, where the models for use in modelling and forecasting are developed.
Chapter 4: Application and discussion of results of the models brought in the methodology section.
Chapter 5: Conclusion and recommendations of the study.

2 Literature Review

Whereas extensive literature on modelling the yield curve is available, very sparse literature has been done on forecasting the term structure, either using the yield curve or otherwise. Modelling the yield curve literature is synonymous with scholars of finance on theoretical basis, whereas the little information on forecasting the same is in the hands of few scholars of statistics, mathematics, and in the recent decades, actuaries. This section will seek to outline the different scholars and their relevant work of forecasting, in line with what this study seeks. In this section therefore, this study explore the various related studies done on the subject of term structure modelling and forecasting, and briefly describe the evolution of relevant and similar works. Whereas other models may be mentioned, it is due to similarity to the Nelson-Siegel model that will be used in modelling and forecasting Kenyan government bond yields.

According to Nelson and Siegel (1987) substantial breakthrough into term structure modelling was achieved between 1950-2000. However, this does not mean that all relevant and superior models were developed during this time, since Diebold and Li (2006) revised the same model in attempt to make it more dynamic and flexible. Most of the earlier models were based on multi-linear regression. The models however were just basic and captured simplistic concepts in modelling and forecasting process. As such, the yield curve modelling was simplified and captured the basic sets of observed shapes of the curve as monotonic, humped and S-shaped. But as we will notice in this study, the yield curves brought about by modelling Kenyan government bonds emerge as; upward sloping and curved, upward sloping, curved and such other descriptions that vary away from the traditional shapes of just monotonic, humped and S-shaped. This was brought about by trends in means of individual months' yields as observed from real market data from the Nairobi Securities Exchange. Nelson and Siegel (1987) also assessed the posterior models in which difference and differential equations were concerned and noted that they estimated forward rates of bonds with exactness. However, they may have lacked in aspects of precise definition of parameters.

Consequently, Nelson and Siegel (1987) enhanced the then differential equations idea to a three factor model with parsimony. Though complex, the model was able to identify the three factors of the model as representative of short-term, medium-term and long-term components of the yield curve. As a result, the Nelson and Siegel model has been noted as a base model that scholars of the yield curve modelling and forecasting can build on, since it incorporates several other models, and is further enhanced over time, including by Svensson (1995) and Diebold and Li (2006) and Christensen et al. (2009).

In some of the subsequent works after Nelson and Siegel (1987), Svensson (1995) assessed the original Nelson-Siegel model to include in a manner that brought in additional component that improved flexibility on the overall model, especially so on extraordinary yield curve shapes and longer terms maturities. Additionally, Diebold and Li (2006) improved the original Nelson-Siegel model to a more dynamic and flexible model that could model modern day yields. Subsequently, he introduced economic sense of latent factor of the yield curve; short-term, medium-term and long-term as coefficients of level, slope and curvature. Diebold and Li (2006) also introduced first order autoregressive model to forecast, and found that the AR(1) model yielded impressive in-sample fitting. Later in 2008, through Diebold et al (2008) the earlier model; Diebold and Li (2006) was reinterpreted and remodeled to reflect systematic risks in a global and country-specific context. As a result, the new model was used to correctly model and forecast yield curves of four currencies of USA, Germany, Japan and the UK. Christensen et al. (2009) also noted that the classic Nelson-Siegel model performed well as an arbitrage free model whereas the Svensson (1995) model best fitted longer maturities.

It is noteworthy that the advancement of the Nelson-Siegel model by Nelson and Siegel (1987) bore the parsimonious dynamic Nelson-Siegel model by Diebold and Li (2006). Their real intention was to come up with a flexible model that could attach time varying parameters of regression type for ease of forecasting. Henceforth, Diebold and Li (2006) described the latent factors of the parsimonious Nelson-Siegel model as level, slope and curvature. This provided a basis for them and scholars who came after, key practical solutions to modelling and forecasting the yield curve. Further, Diebold and Li (2006) introduced autoregressive models to the latent factors of the yield curve to capture the various aspects of the yield curve. From their results, it emerged that the parsimonious model with time varying parameters produced reliable forecasts of the term structure for various time horizons, either short or long and that its results were much accurate than standard forecasting benchmarks employed before. The parsimonious model with regression factors for forecasting produced 1-month-ahead forecasts similar to those of the random walk in the yield curve forecasting, and where the random walk was used as a standard benchmark. Additionally, their work established that parsimonious models are most recommended for out of sample forecasts.

Shu et al. (2018) attempted to model the term structure of the South African government bonds using the parsimonious Nelson-Siegel bonds. Data for the study was acquired from Data stream, covering the period September 2000 to August 2012. Part of the findings of the study is that the dynamic Nelson-Siegel model exhibited good fitting abilities for all maturities.

In regard to the time series factors with the dynamic Nelson-Siegel model, it was established that the dynamic Nelson-Siegel model with VAR-GARCH produced best forecast short term rates, dynamic Nelson-Siegel model with Vector Autoregression (VAR) best predicted short-term rates while the dynamic Nelson Siegel model with Random-Walk (RW) produced best forecasts for the long-term rates. The study also ranked the dynamic Nelson-Siegel model above the random walk in terms of forecasting capabilities.

The study also employed a selection criteria using the Root Mean Square Error (RMSE) to determine the best model. According to the study, at any given horizon, short-term, medium term and long-term, the best forecasting model is one that produced the least error amongst all.

Consequently, the study established that the dynamic Nelson-Siegel with Random Walk forecasting produced the least forecasting errors compared to other models in its class for the long maturities.

Similarly, it was established that the dynamic Nelson-Siegel model with VAR-GARCH models performed significantly better for the 1 year forecast compared to all other models, and thus was deemed suitable to forecast short-term horizons.

Likewise, the observed forecast errors for the dynamic Nelson-Siegel model with RW performed significantly well by producing the least errors in the long maturities horizon, and hence deemed best to forecast long-term maturities. However, the dynamic Nelson-Siegel with ARMA and ARMA-GARCH performed poorly at all maturities. This study could be used as a base/entry study for modelling African markets or as a benchmark for modelling financial market products in Sub-Saharan Africa. Particularly, it could be of use to the markets of Nigeria, which alongside South-Africa, leads in the African bond markets category.

According to a study on combining term structures of interest rate forecasting for Brazil, structural breaks and misspecification biases makes it difficult to find a term structure that dominates all competitors. As such, the study sought to come up with a mixture of models that produced superior forecasting results. The results showed that combining more than one forecasting model yields superior results, especially on accuracy of forecasts compared to any individual model. It was also established that no one individual model would consistently produce superior results on more than one aspect of forecasting. Therefore, it was documented that the challenges that arise from using an individual model could be reduced by use of a mixture of models, and that the challenges arose from among other factors; time horizons, maturity periods and forecast periods. Thus, no one individual model could dominate its competitors on all these grounds. According to the study, results of the combining methods indicate that for each forecast horizon, there exists a mixture of forecasting models that either equals or performs better than any individual forecasting model, and that greater forecast horizons for mixtures of models yielded superior results than any individual model. Thus, the longer the time horizon, the greater the contribution of the mixture of models.

According to Castano (2014) on the dynamic estimation of an interest rate structure in Colombia, he noted that the official estimation of interest rates structure is based on the Nelson-Siegel (1987), where curve fitting is done using real data. However, since it's done only for one day ahead, it becomes difficult to estimate the zero-coupon yield curve. In his work however, he uses the Kalman filter methodology in state-space to estimate the term structure of interest rates, but used the Diebold and Li (2006) to estimate the parameter lambda in the yield curve modelling. Therefore, it was established that the Kalman filter and the dynamic Nelson-Siegel can be combined to come up with better forecasts than for individual forecasts models.

In Belgium financial market, the Nelson-Siegel model has widely been used to fit the term structure of interest rates, mainly due to its ease of linearization. However, in the Belgian case, OLS approach using fixed shape parameter have been observed to behave erratically and exhibit unusually large variances. However, it has been shown that the Nelson-Siegel model can become highly collinear. To solve these problems, they apply ridge regression.

3 Methodology

The essence of modelling; either Statistical, Actuarial or any other mathematical related field is to better understand the process and further be able to tell the behaviour of the process at various points or times (t) under various conditions. Therefore, the models to be introduced in this chapter will allow us to forecast, just as any predictive model will do; the behaviour of bond yields in Kenya at various points in time (t).

In this chapter, the study attempt to use the dynamic Nelson Siegel model to forecast term structure of Kenya Government Bonds. Simply put, this study attempts to forecast how government Bonds yields in Kenya change or generally behave over their term from placement to maturity.

3.1 The Dynamic Nelson-Siegel model as a forecasting model

This work seeks to forecast the term structure of Kenyan government Bond yields by extending original works by Diebold and Li: (2006). To make this possible we add time varying parameters. The Nelson Siegel model to be adopted will include (RW), ARMA, AR and GARCH components as the time varying parameters.

The Nelson- Siegel model has been used over time to forecast yield curves. It is thus very popular with Central Banks, Reserve Banks and other major Financial Regulators in the many markets across developed countries such as USA,UK with developed economies and characteristic deep markets.

Nelson and Siegel (1987) explains a forward rate as a function of maturity(τ); and is given as a solution to a second order differential equation;

$$r(\tau) = \beta_1 + \beta_2 \exp(-\tau\lambda) + \beta_3[(\tau\lambda)\exp(-\tau\lambda)] \tag{1}$$

To obtain a yield curve from a forward curve we integrate the above equation with respect to dx from 0 to τ.

$$y(\tau) = \frac{1}{\tau}\int_0^\tau r(x)dx$$

Hence, the exact solution becomes:

$$y_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1-\exp^{\lambda_t \tau}}{\lambda_t \tau}\right) + \beta_{3t}\left(\frac{1-\exp^{-\lambda_t \tau}}{\lambda_t \tau} - \exp^{-\lambda_t \tau}\right) \quad (2)$$

In this case, $y(\tau)$ becomes the yield at maturity; and parameters are described as :

1. β_{1t} is the level factor
2. β_{2t} is the slope factor
3. β_{3t} is the curvature factor
4. m is the time to maturity
5. λ_t is the decay factor

3.2 Mathematical Estimation of parameter λ

To mathematically estimate the parameter λ, you evaluate the maxima on β_{3t} loading $\frac{1-e^{-\lambda m}}{\lambda m} - e^{-\lambda m}$ and then replace τ with the preferred period, in the short term (between 12-36 months) to get the value of λ. It is noteworthy that small values of λ indicates slow decay and thus associated with long-term maturities. Similarly, large values of parameter λ indicates fast decay and are thus associated with short maturities.

$$y_t(m) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda \tau}}{\lambda \tau}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda \tau}}{\lambda \tau} - \exp^{-\lambda \tau}\right)$$

Consider the parameter loading on; β_{3t}

$$\left\{\frac{1-e^{-\lambda \tau}}{\lambda \tau} - e^{-\lambda \tau}\right\}$$

and differentiate with respect to λ.

From;

$$\frac{1-e^{-\lambda \tau}}{\lambda \tau} - e^{-\lambda \tau} = \frac{1-e^{-\lambda \tau} - e^{-\lambda \tau}\lambda \tau}{\lambda \tau}$$

Using quotient rule:

$$\frac{u'v - v'u}{v^2}$$

$$\frac{1 - e^{-\lambda\tau} - e^{-\lambda\tau}\lambda}{\lambda\tau}$$

$$u' = \tau e^{-\lambda\tau} - [-\tau e^{-\lambda\tau}.\lambda\tau + \tau e^{-\lambda\tau}]$$

$$v' = \tau$$

$$\begin{aligned} u' = \tau e^{-\lambda\tau} + \tau e^{-\lambda\tau}.\lambda\tau - e^{-\lambda\tau} = \tau e^{-\lambda\tau}.\lambda\tau \\ = \lambda\tau e^{-\lambda\tau} \\ = \lambda^2\lambda e^{-\lambda\tau} \end{aligned}$$

$$\frac{\partial L_3}{\partial\lambda} = \frac{\tau^2\lambda e^{-\lambda\tau}\lambda\tau - \tau[1 - e^{-\lambda\tau} - e^{-\lambda\tau}\lambda\tau]}{\lambda^2\tau^2} = 0$$

$$\tau^3\lambda^2 e^{-\lambda\tau} - \tau[1 - e^{-\lambda\tau} - \lambda\tau e^{-\lambda\tau}] = 0$$

$$\tau^2\lambda^2 e^{-\lambda\tau} - [1 - e^{-\lambda\tau} - \lambda\tau e^{-\lambda\tau}] = 0$$

$$\tau^2\lambda^2 e^{-\lambda\tau} - 1 + e^{-\lambda\tau} + \lambda\tau e^{-\lambda\tau} = 0$$

$$\tau^2\lambda^2 - e^{\lambda\tau} + 1 + \lambda\tau = 0$$

$$1 + \lambda\tau + \tau^2\lambda^2 - e^{\lambda\tau} = 0$$

3.3 Dynamic Nelson Siegel Model with time varying parameters.

In this section the study introduces models that forecast the term structure of Kenyan Government bond yields by extending works of Diebold and Li (2006) to bring in time varying parameters. The purpose of bringing in the time varying parametric models is to better understand their nature especially when combined with the Nelson Siegel model.

Let $\tau = 1, 2, 3, \ldots, 97$ be the $N = 97$ maturities in months of discounted bonds with say,£1 face value. Again, let $y_t(\tau_i)$ be their yields to maturity and further let h be the number of months to be forecasted. Therefore the study seek to find $h-$ steps ahead forecast of each maturity τ_i at time t. Thus the study

$$\hat{y}_{t+h}(\tau_i)$$

3.4 Dynamic Nelson Siegel Model with Random Walk.

Random Walk with a drift.

Let

$$y_t(\tau_i) = y_{t-1}(\tau_i) + \varepsilon_t(\tau_i)$$

where $\varepsilon_t(\tau_i)$ is a white noise process.

Then, the $h-$ steps ahead forecasts for the yield will be given as

$$\hat{y}_{t+h}(\tau_i) = y_t(\tau_i)$$

This simply means that a one step forecast is dependent on the yield in the immediate past.

Therefore a dynamic Nelson Siegel (DNS) with Random Walk model (DNS-RW)for h-steps ahead forecasts of the yields and each of the factors are;

$$\hat{y}_{t+h}(m) = \hat{\beta}_{1,t+h} + \hat{\beta}_{2,t+h}\left[\frac{1-e^{-\lambda m}}{\lambda m}\right] + \hat{\beta}_{3,t+h}\left[\frac{1-e^{-\lambda m}}{\lambda m} - e^{-\lambda m}\right],$$

$$\beta_{i,t+h} = a_0 + \sum_{j=1}^{p} a_j \beta_{t-j+1} + \varepsilon_{t+h}, i = 1, 2, 3$$

where

$$\varepsilon_{t+h} \sim N(0, \sigma^2)$$

Dynamic Nelson Siegel with Autoregressive Model

A dynamic Nelson-Siegel with autoregressive model (DNS-AR) is then applied to the time series of the yields and factors to produce $h-$ step- ahead forecasts:

$$\hat{y}_{t+h}(m) = \hat{\beta}_{1,t+h} + \hat{\beta}_{2,t+h}\left[\frac{1-e^{-\lambda m}}{\lambda m}\right] + \hat{\beta}_{3,t+h}\left[\frac{1-e^{-\lambda m}}{\lambda m} - e^{-\lambda m}\right],$$

$$\beta_{i,t+h} = a_0 + \sum_{j=1}^{p} a_j \beta_{t-j+1} + \varepsilon_{t+h}, i = 1,2,3.$$

where $\varepsilon_{t+h} \sim N(0,\sigma^2)$.

3.5 Dynamic Nelson Siegel with Autoregressive Moving Average Model

The yield forecasts based on (DNS-ARMA) factor specifications are:

$$\hat{y}_{t+h}(m) = \hat{\beta}_{1,t+h} + \hat{\beta}_{2,t+h}\left[\frac{1-e^{-\lambda m}}{\lambda m}\right] + \hat{\beta}_{3,t+h}\left[\frac{1-e^{-\lambda m}}{\lambda m} - e^{-\lambda m}\right],$$

$$\beta_{i,t+h} = a_0 + \sum_{j=1}^{p} a_j \beta_{i,t-j+1} + \sum_{k=1}^{q} b_k \varepsilon_{t-k+1}, i = 1,2,3$$

where

$$\varepsilon_{t+h} \sim N(0,\sigma^2)$$

3.5.1 Dynamic-Nelson Siegel with Vector Ansalysis(DNS-VAR)

The (DNS-VAR) model for forecasting is:

$$\hat{y}_{t+h}(m) = \hat{\beta}_{1,t+h} + \hat{\beta}_{2,t+h}\left[\frac{1-e^{-\lambda m}}{\lambda m}\right] + \hat{\beta}_{3,t+h}\left[\frac{1-e^{-\lambda m}}{\lambda m} - e^{-\lambda m}\right]$$

and:

$$\hat{\beta}_{i,1t+h} = a_0 + \sum_{i=1}^{p}\left(a_j \beta_{t-j+1} + b_j \beta_{2,t-j+1} + c_j \beta_{3,t-j+1}\right) + \varepsilon_{t+h}, i = 1,2,3$$

and $\varepsilon_{t+h} \sim N(0,\sigma^2)$

3.6 Dynamic Nelson-Siegel with GARCH(1,1) Model

A dynamic Nelson-Siegel with GARCH(1,1) model first considers the mean equations that follow the $AR(p)$,$ARMA(p,q)$ and $VAR(p)$ processes. It also assumes that the error term ε_{t+h} is distributed $N(0,\sigma^2_{\tau,\tau+h})$ and follows a GARCH(1,1) error term as

$$\sigma^2_{\tau,\tau+h} = \alpha_0 + \alpha_1 \varepsilon^2_{\tau+h-1} + \alpha^2 \sigma^2_{\tau,\tau+h-1}$$

4 Data Analysis and Interpretation

In this section, the study seek to explore and carry out empirical analysis for end-of-month yields for Kenyan Government Bonds; to be used to estimate and forecast. Although Government bonds are issued by the Central Bank of Kenya (acting as the reserve bank), the data on the bonds was sourced from the Nairobi Securities Exchange. The data covered the period September 2008-April 2019, and did not include treasury bills, notes and other short term instruments. It also excluded bonds with maturity periods of less than thirty (30) days. The maturity period of the bonds range from 12 months ($1year$) to 360 Months ($30years$). However, for the purpose of this study we consider bonds with maturities of 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, 144, 156, 168 and 180 months. The data for the period September 2008 -September 2016 will be used for in-sample estimation, while data for the period October 2016 - April 2019 will be used for out-of-sample forecasting.

Yield summary statistics.

Table 1. The descriptive statistics of monthly yields for different maturities.

	Mean	Standard Deviation	Maximum	Minimum	$\rho(1)$	$\rho(12)$	$\rho(30)$
12	10.3286371	3.8748071	21.6482	2.9234	0.9209775	-0.0691630	-0.0838703
24	10.9306781	3.6011846	22.3333	3.4496	0.9482592	-0.1105005	-0.0924854
36	11.0999360	3.2008933	19.2695	3.6972	0.9564102	-0.0929253	-0.0834966
48	11.1935562	2.9120848	16.8020	3.8360	0.9557875	-0.0593740	-0.0701369
60	11.4074030	2.5895830	16.4940	4.1640	0.9555411	-0.0513156	-0.0744479
72	11.5366997	2.4466260	16.4544	4.7656	0.9534062	-0.0352877	-0.0651667
84	11.6353166	2.3260696	16.4255	5.1585	0.9502684	-0.0205747	-0.0563589
96	11.7301893	2.2228309	16.4044	5.3923	0.9484911	-0.0237282	-0.0499759
108	11.8446294	2.1419254	16.3965	5.6005	0.9456593	-0.0412628	-0.0415324
120	11.9599780	2.0239156	16.4834	5.7749	0.9325223	0.0067214	-0.0298081
132	12.0549378	1.9863644	16.5503	5.9072	0.9098505	-0.0036632	-0.0238278
144	12.2060788	1.9353337	16.6080	6.0085	0.9056748	-0.0497925	-0.0200139
156	12.3304749	1.8865076	16.6516	6.0585	0.8906528	-0.0767410	-0.0181328
168	12.4660357	1.8110394	16.6943	6.0965	0.8923889	-0.1124252	-0.0055444
180	12.6065518	1.7321964	16.7361	6.3237	0.8951923	-0.1256271	-0.0150938

With reference to the table 1 above, it emerges that the mean values for different maturities ranging from 12-180 months presents a yield curve with a characteristic upward sloping and concave. Better put, the yield curve originate from the 12-month at 10.3286371, and rise steadily to a maximum of 12.6065518 at 180-month maturity. In more developed financial markets with developed and vibrant economies, the curve may begin to fall slightly humped towards the maturity period thus producing an upward sloping and concave curve or studies have showed that the South African bonds (Data available in Data Stream) and Colombian government bonds have exhibited this characteristic. The standard deviation reveals that the short rates are more volatile than longer rates, and that volatility decreases with increase in maturity period. i.e The volatility at 12-month period stands at 3.8748071 and decreases at every month maturity duration to achieve a low of 1.7321964 at 180-month. Therefore, it is imperative that longer rates are less risky than short rates. This could be interpreted that, when an investor wishes to build a portfolio of risky assets and risky-free assets, it is sensible to consider longer maturities for risk-free assets than short maturities. The autocorrelations also show that the yields are highly persistent.

This could also explain why Governments in third world countries and developing markets have opted for long maturity infrastructure bonds and sovereign bonds to funds large infrastructure projects. A case in point is the Standard Gauge Railway project in Kenya that cost an estimated Ksh. 327 Billion, and similar project projects in Tanzania and Ethiopia estimated to cost between UDD 6 Billion USD 8 Billion.

The mean yields reflect typical returns in the unique environment that the Kenyan economy operates in. In many instances, Kenyan Government bonds are issued to either bridge budget deficits or increase liquidity levels to fund the governments' critical operations. In most cases, these set of actions are not guided by market forces of demand and supply, but rather by the desire to increase liquidity for government operations. As a result, the bonds are issued at a relatively higher rate than expected so as to build cash flows over short periods of time. Another notable effect are political declarations by powerful African leaders who prefer to stamp their authority by making decisions to appease the general public (the electorate), most of which are guided by political expedience or fulfil the wishes of the people. A perfect case is the capping of interests (bank lending rates)in Kenya through the parliament. The result has been a straining banking sector that avoids unsecured loans, decrease in loans uptake and slowed economic growth. Consequently, the Central Bank of Kenya in 2019 has proposed abolishment of the interest capping law to spur economic growth.

Most yield curves in developed and emerging economies with characteristic deep and highly liquid financial markets exhibits upward sloping and concave, while the case of Kenya only presents upward sloping normal curves. The curve exhibited by the Kenyan bonds is an indicator that yields on longer term bonds may continue to rise, mainly responding to periods of economic expansion. In this scenario, investors in the bond market expect that yields on longer-maturity bonds become even higher in the future.

As a result, investors release their cash from the short term securities that are volatile in anticipation of cashing in on higher yields from longer-term securities that are less risky.

4.1 Parameter estimation.

4.1.1 The parameter λ_t.

This parameter carries two critical functions in the parameterized dynamic Nelson-Siegel model's estimation process;

i. It governs the exponential decay rate of the yield curve, and

ii. It also governs where the loading on (β_{3t}) (corresponding to the curvature of the yield curve) gains its peak (achieves maximum).

Small values of parameter λ produce slow decay rates and can thus be best used to fit the yield curve at long maturities. Similarly, large values of the parameter produce fast decay rates and are thus best used to fit yield curves at short maturities.

4.1.2 Estimating the parameter λ.

To estimate the value of λ, we consider the loading on the medium term factor (associated with β_{3t}) and determine where it achieves its maximum. For this work, we consider between 24-36 months maturities and average them out to a period of 30 months (as a mean). We then compute the value of λ that maximizes the loading of the β_{3t} at 30-month maturity; to achieve a 0.05977726.

4.1.3 Estimation and interpretation of the latent Dynamic factors β_{1t},β_{2t},β_{3t}

BETAS SUMMARY

Table 2. Estimated β_{1t}, β_{2t} and β_{3t}

	β_{1t}	β_{2t}	β_{3t}
nobs	97.000000	97.000000	97.000000
NAs	0.000000	0.000000	0.000000
Minimum	7.215475	-7.164164	-18.879154
Maximum	16.502528	14.019008	8.755374
1. Quartile	12.266508	-4.886694	-6.312974
3. Quartile	13.761801	-1.406857	0.422924
Mean	12.836853	-2.304029	-3.181372
Median	13.198465	-2.715806	-3.501015
Sum	1245.174776	-223.490775	-308.593039
SE Mean	0.158855	0.415189	0.541851
LCL Mean	12.521529	-3.128172	-4.256937
UCL Mean	13.152177	-1.479885	-2.105806
Variance	2.447774	16.721045	28.479430
Stdev	1.564536	4.089137	5.336612
Skewness	-1.235441	2.384127	-0.134053
Kurtosis	2.063939	6.824618	0.045482

Table 3. Extract of estimated β_{1t}, β_{2t} and β_{3t}

	Minimum	Maximum	Mean	Stdev
β_{1t}	7.215475	16.502528	13.198465	1.564536
β_{2t}	-7.164164	14.019008	-2.715806	4.089137
β_{3t}	-18.879154	8.755374	-3.501015	5.336612

From the table, the mean, min and max of β_{1t} are estimated as 12.836853 , 7.215475 and 16.502528 respectively. From observation, β_{1t} has yielded positive values for the mean, min and max; which resonates positively with Diebold and Li (2006) works on β_{1t} as a parameter in the model. Additionally, β_{2t} has a negative mean. This can be explained in terms of the yield curve that; it bears a positive slope during the sampled period. Further, β_{3t} has a negative mean value, thus yielding a humped shaped yield curve. However, if β_{3t} had had a positive value, the curve would only be slightly humped. It is worth noting that this the factor that determines the curvature of the yield curve.

The Loading on Dynamic Factors The loading on β_{1t} is one (1), a constant. Being a constant, it does not decay to zero in the limits. As such, it is treated as a long term factor for not decaying to zero. The loading of β_{2t} is $\frac{1-e^{-\lambda m}}{\lambda m} - e^{-\lambda m}$. This is a function that begins at one and decays monotonically and quickly to zero. Considering the behaviour of parameter lambda explained earlier on the decay rates, then, the β_{2t} loading is suitable to represent the short term the short term maturity, owing to the rate at which it decays to zero.

The loading on β_{3t} is $\frac{1-e^{-\lambda m}}{\lambda m} - e^{-\lambda m}$. This component has a unique characteristics. First, it begins at zero and thus not short term. It then increases to a point (maximum) and then decays to zero. Therefore, since it decays to zero, it is not considered a long-term component. This term is therefore used to represent the medium term factor.

Having been able to estimate the dynamic factors, we can then plot them on a graph of loadings against maturity in months to graphically represent the behaviour of the curves as level, slope and curvature respectively.

Below is a plot of the three (3) factors described as the level, slope and curvature respectively, as well as the time series plots for the three factor loadings.

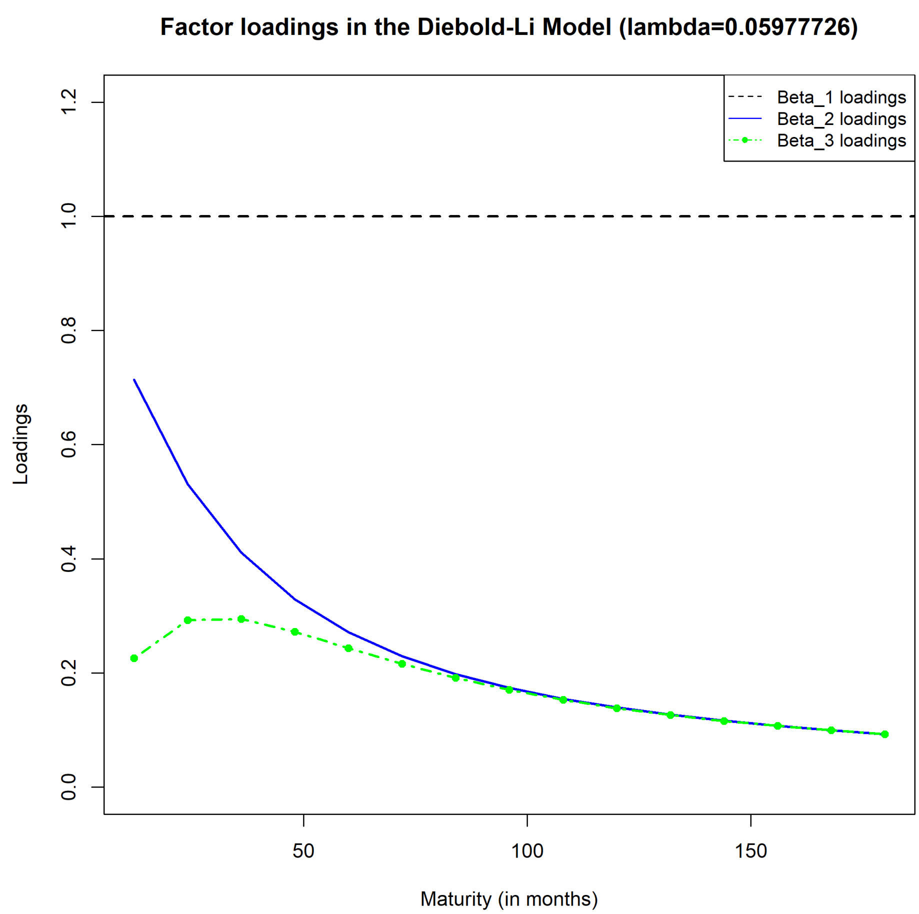

Figure 1. A plot of the level,slope and curvature factors of the Dynamic Nelson-Siegel model.

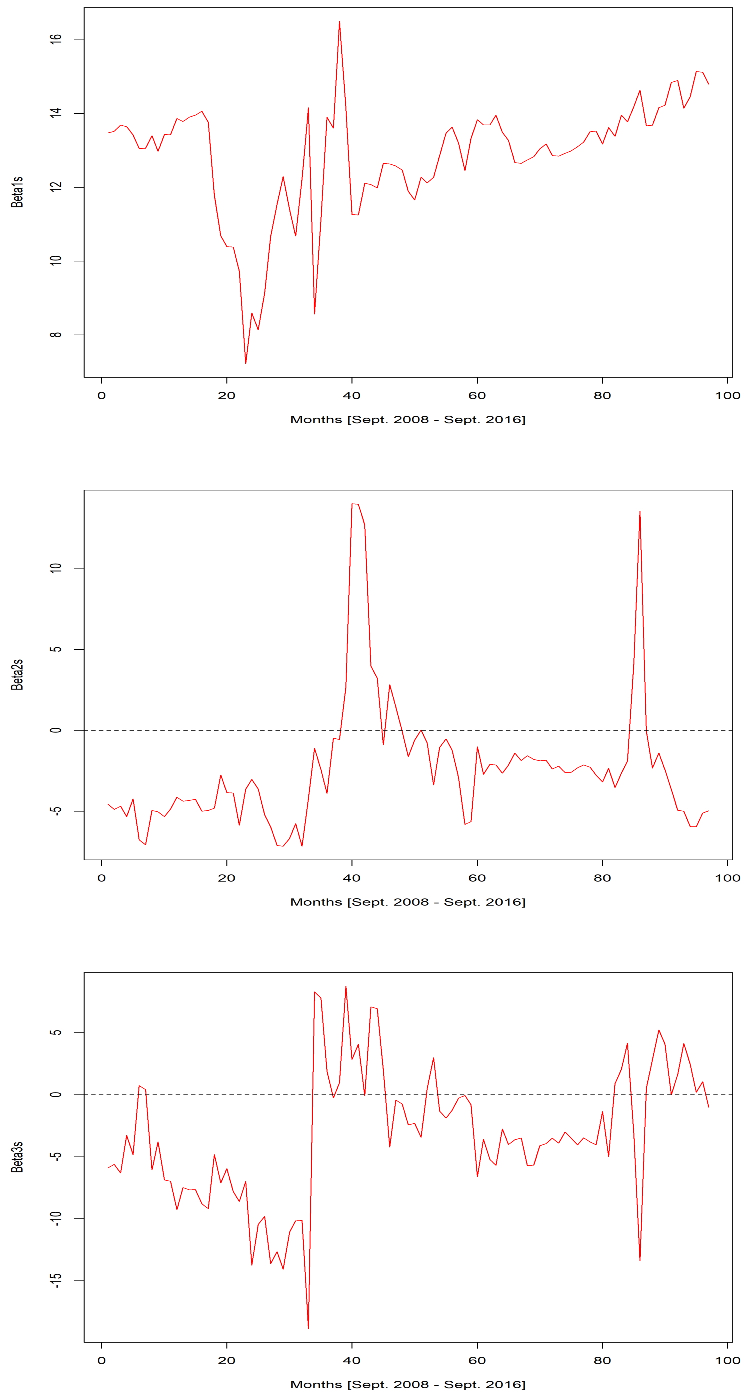

Figure 2. Time series plots for the three factor loadings; $\beta_{1t}, \beta_{2t}, \beta_{3t}$

4.2 Fitting the yield curve using the dynamic Nelson-Siegel model.

In this section, we fit a yield curve estimated with the Nelson-Siegel Model against the average actual yield curve from real data.

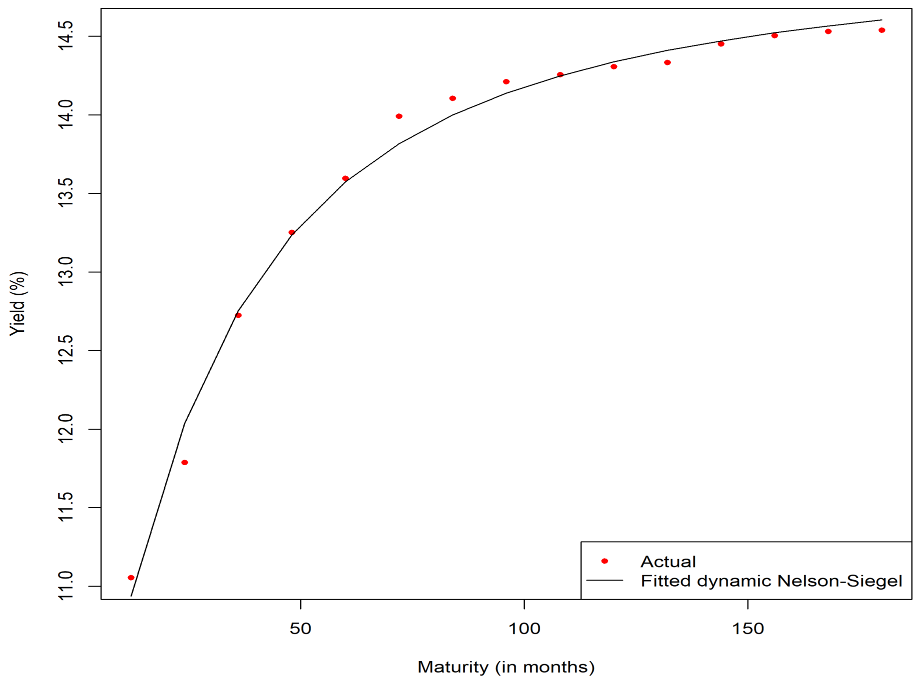

Figure 3. Plot of actual real data from bond yields and fitted plot based on the Nelson-Siegel model to come up with average yiled curves.

From observations, the curves are to a large extent similar. It is notable that the two curves for all periods sampled for time to maturity exhibit an increasing slope and concave. Therefore, this implies that the dynamic Nelson-Siegel model adequately capture and reflect the actual yields of Kenyan Government bonds, as well as it fits the yield curves of the Kenyan bond markets appropriately.

4.3 Fitting the yield curve errors.

The means of the residuals of the model (errors) are fitted and reveals that; errors stand at -0.1460577 for 12-month and 0.2974199 for the 180-month maturities. This result indicate that the dynamic Nelson-Siegel model do not perform well for short and long bond maturities.

RESIDUALS SUMMARY STATS

	Mean	Standard Deviation	Maximum	Minimum
12	-0.1460577	0.1930214	0.3599348	-1.2997648
24	0.2487802	0.4061927	2.7948326	-0.7528687
36	0.1459062	0.1862732	1.0784607	-0.4850746
48	-0.0205216	0.2645939	0.4067031	-1.6320044
60	-0.0304981	0.2094977	0.8600960	-0.8317749
72	-0.0858744	0.1665940	0.4954903	-0.7128371
84	-0.1372983	0.1758057	0.2488893	-0.6623764
96	-0.1641078	0.1962488	0.8296018	-0.6562163
108	-0.1488908	0.3090022	2.0144000	-0.6008290
120	-0.1152014	0.1822156	0.5855563	-0.5507038
132	-0.0881846	0.1796935	0.1259703	-1.0138846
144	0.0057778	0.1168625	0.2221463	-0.6315758
156	0.0815162	0.1687064	0.4468869	-0.9496686
168	0.1752344	0.1932532	0.6731202	-0.2643270
180	0.2794199	0.2438398	0.8633692	-0.1995713

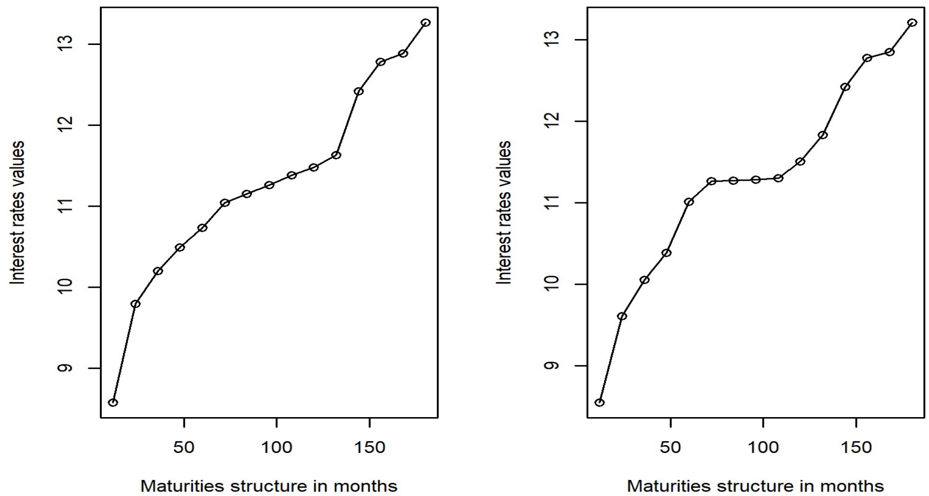

Figure 4. Estimated yield curves for 2 and 3 years maturity periods representing short-term maturities.

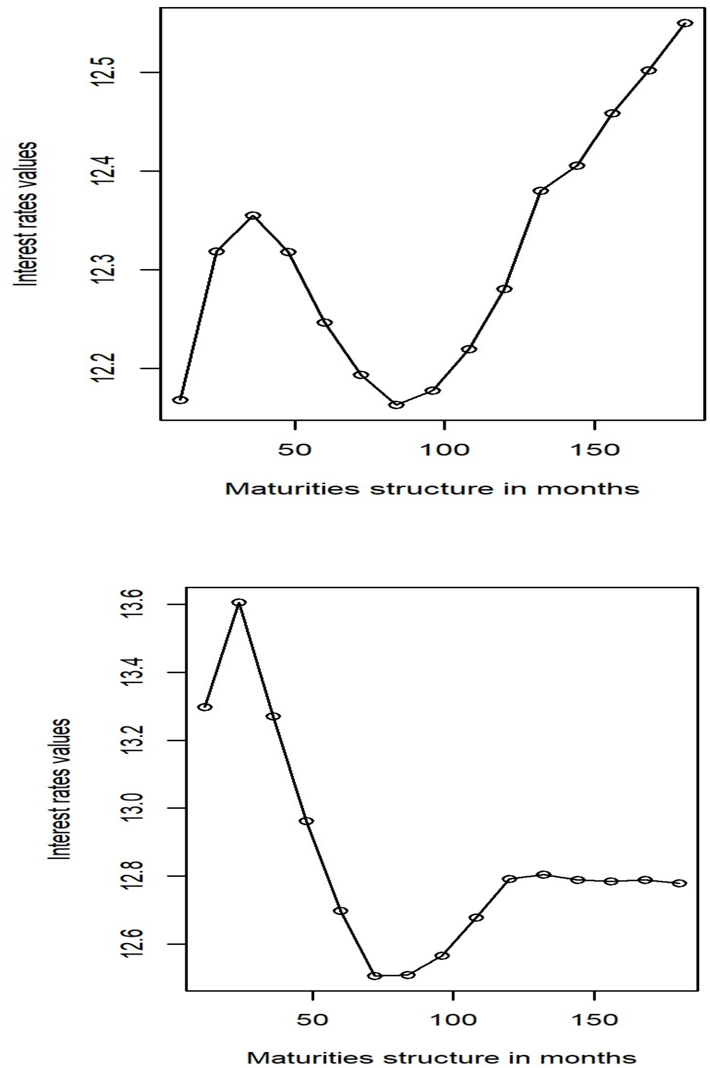

Figure 5. Estimated yield curves for 6 and 7 years maturity periods representing medium-term maturities.

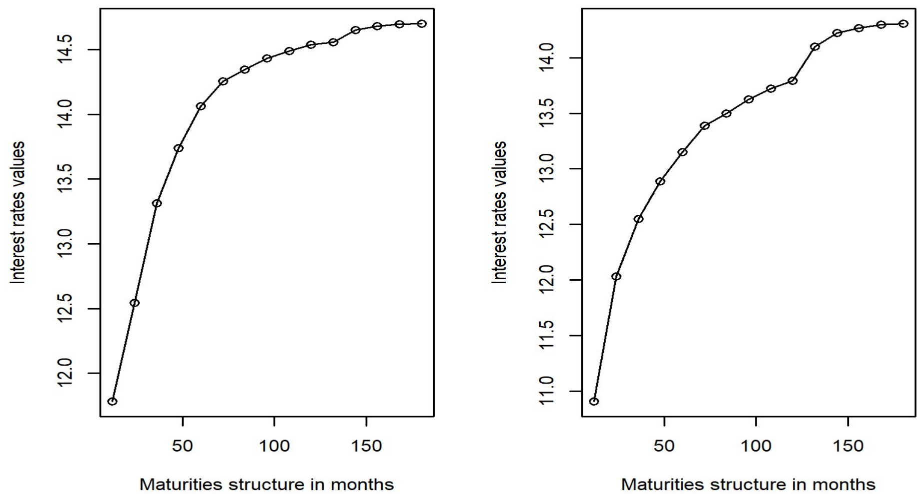

Figure 6. Estimated yield curves for 14 and 15 years maturity periods representing long-term maturities.

4.4 Forecasting results

In this section, we provide forecasting results for different maturities with the dynamic-Siegel model with the time series models. The data for out-of-sample forecasting is for period of October 2016 to April 2019, and the forecasting horizons are $1-, 6-$, and $12-$ month ahead forecasts. Therefore, we will consider h-steps ahead as $h = 1, h = 6$ and $h = 12$. The maturities of the data to be used remains at 12 months, 60 and 120 months and 180 months, representing the short-term, medium and long-term maturities respectively.

The forecasts also utilize the Root Mean Square Error as a criteria to identify the performance and superiority of a model, against its competitors. In this study, the model with the least RMSE emerges the best forecast model for the time horizon, while those that yield large values of RMSE will be considered as poor forecasting models.

4.4.1 The h=1 forecasts for 12-, 60-, 120-, 180- month maturity.

From the descriptive statistics, the DNS-VAR-GARCH forecasts the 12-month maturity 1-month ahead most accurately. The DNS-VAR and DNS-RW random walk also produce favourable results. The DNS-ARMA-GARCH posted the worst forecasting results for the 1-month forecast of 12-month horizon. For the 60-month maturity forecasts, the DNS-RW forecasted most accurately, while DNS-VAR and DNS-AR provided close and competitive results. The same case applied for the 60-month maturities. The DNS-AR and DNS-VAR produced similar results and most accurately forecasted the long-term maturities for 1-month forecast.
Comparatively, the DNS-RW performs best for all maturities than competing models.

Root Mean Square Measure of Forecasting errors for various Models

Dynamic Nelson-Siegel (DNS) with corresponding Regressive Model.	12	60	120	180
$h = 1-$ month				
DNS-RW	0.4737	0.3481	0.3453	0.4699
DNS-AR	0.4744	0.3625	0.3569	0.4673
DNS-ARMA	0.5141	0.4290	0.3777	0.5132
DNS-VAR	0.4800	0.3548	0.3511	0.4899
DNS-AR-GARCH	0.4769	0.3709	0.3769	0.4750
DNS-ARMA-GARCH	0.5222	0.4018	0.3749	0.4792
DNS-VAR-GARCH	0.4684	0.3653	0.3599	0.4899

4.4.2 The h=6 forecasts for 12-, 60-, 120-, 180- month maturity.

The DNS-VAR-GARCH performs best for the short-term maturities. The DNS-AR-GARCH and DNS-RW also yield impressive results for the same horizon. For the medium-term maturities, the DNS-VAR performs best, while the DNS-RW posts best forecasting results for the long-term maturities.
Overall, the DNS-RW yields superiors for all maturities for 6-steps ahead forecasts, and the DNS-ARMA-GARCH performed worst across all maturities.

Root Mean Square Measure of Forecasting errors for various Models

Dynamic Nelson-Siegel (DNS) with corresponding Regressive Model.	12	60	120	180
$h = 6-$ month				
DNS-RW	1.0969	0.6235	0.05259	0.6409
DNS-AR	1.0950	0.6306	0.5201	0.6500
DNS-ARMA	4.0411	4.2832	3.3470	2.7467
DNS-VAR	1.0726	0.5762	0.4893	0.6498
DNS-AR-GARCH	1.0945	0.6481	0.549	0.6960
DNS-ARMA-GARCH	5.5835	3.4745	3.432	3.8409
DNS-VAR-GARCH	1.0324	0.5780	0.5521	0.6783

4.4.3 The h=12 forecasts for 12-, 60-, 120-, 180- month maturity.

DNS-VAR-GARCH performs best in forecast in the short-term maturities, while the DNS-VAR and DNS-AR performs well for medium-term maturities. Similarly, the DNS-RW performs well for long-term maturities.

Root Mean Square Measure of Forecasting errors for various Models

Dynamic Nelson-Siegel (DNS) with corresponding Regressive Model.	12	60	120	180
$h = 12-$ month				
DNS-RW	1.7110	0.6680	0.4831	0.5303
DNS-AR	1.6550	0.5545	0.4611	0.6173
DNS-ARMA	154.9089	93.3425	50.9789	41.5467
DNS-VAR	1.6070	0.5733	0.5221	0.7073
DNS-AR-GARCH	1.5743	0.6282	0.699	0.8873
DNS-ARMA-GARCH	176.6578	96.7890	61.8364	42.8725
DNS-VAR-GARCH	1.3890	0.6842	0.6763	0.8367

Root Mean Square Measure of Forecasting errors for various Models

Dynamic Nelson-Siegel (DNS) with corresponding Regressive Model.	12	60	120	180
$h = 1-$ month				
DNS-RW	1.0969	0.6235	0.05259	0.6409
DNS-AR	1.0950	0.6306	0.5201	0.6500
DNS-ARMA	4.0411	4.2832	3.3470	2.7467
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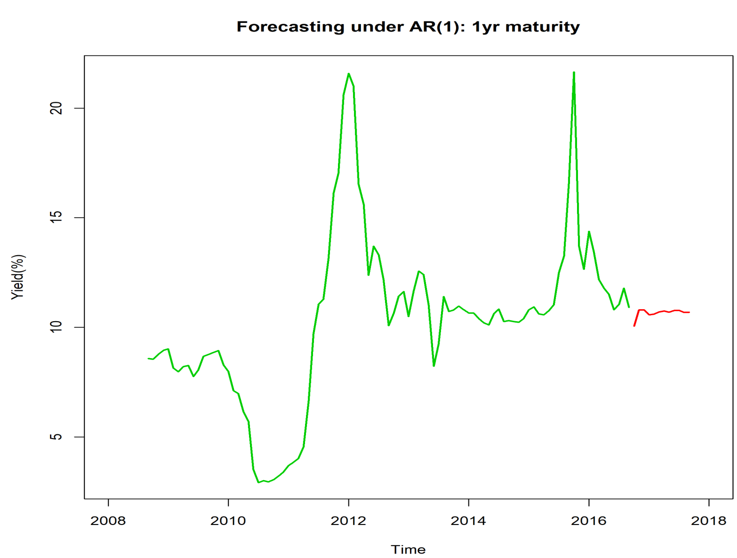
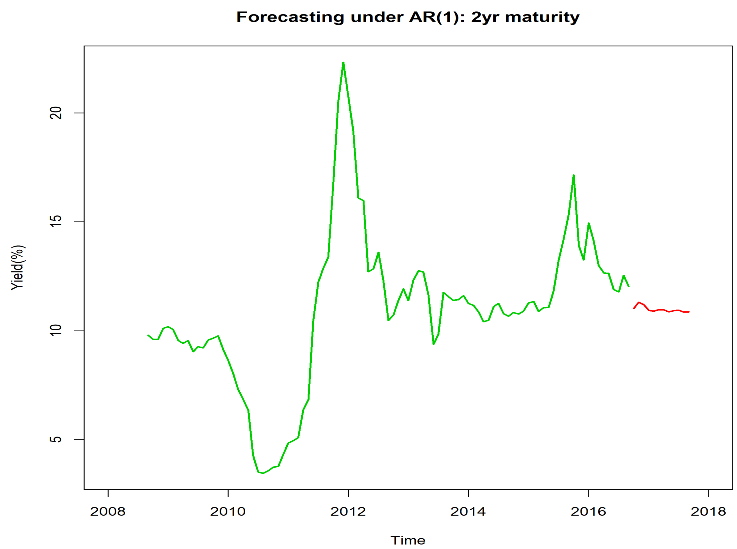
DNS-VAR-GARCH	1.0324	0.5780	0.5521	0.6783
$h = 12-$ month				
DNS-RW	1.7110	0.6680	0.4831	0.5303
DNS-AR	1.6550	0.5545	0.4611	0.6173
DNS-ARMA	154.9089	93.3425	50.9789	41.5467
DNS-VAR	1.6070	0.5733	0.5221	0.7073
DNS-AR-GARCH	1.5743	0.6282	0.699	0.8873
DNS-ARMA-GARCH	176.6578	96.7890	61.8364	42.8725
DNS-VAR-GARCH	1.3890	0.6842	0.6763	0.8367

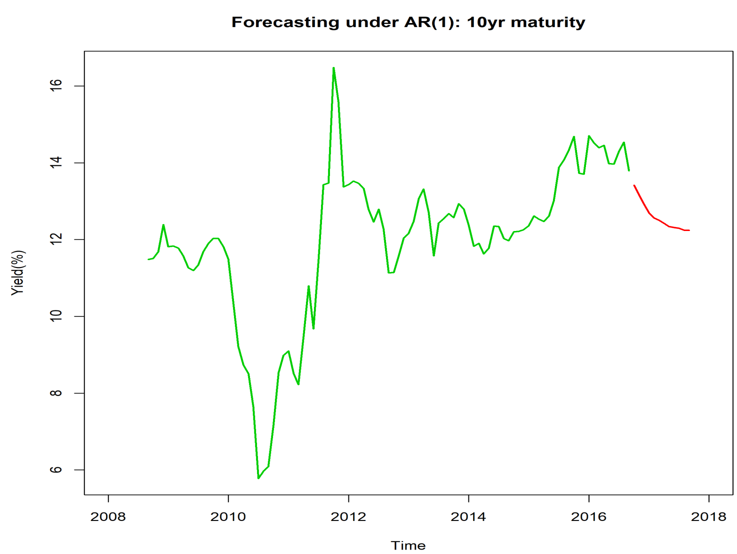
5 Conclusion

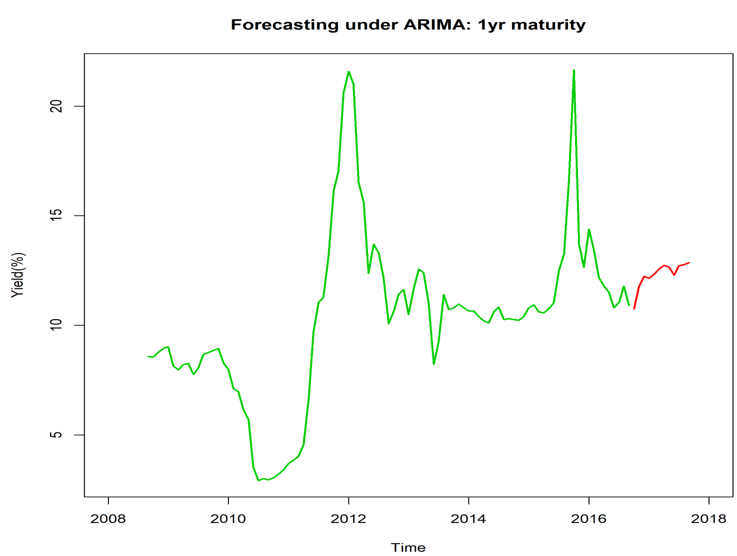
This study focused on the forecasting of the term structure of Kenyan bond yields. From the analysis of the 1-, 6-, and 12-month ahead forecasts, it is observed that the Dynamic Nelson model with Random Walk performed exceptionally well for all maturities, and best for long-term maturities. Thus, in the Kenyan context, it may be recommended as a benchmark model for all maturities. As the study had indicated earlier that the most recommended combining scheme of models is one that consistently produce superior results over different horizons for different maturities. In this study, the DNS-RW does so, and is thus recommended as a benchmark model for forecasting bond yields in Kenya. In our study, the DNS-RW is the superior model suitable to forecast Kenyan bond yields. On the other hand, the DNS-ARMA and DNS-ARMA-GARCH performs extremely poorly at all maturities for 6- and 12-month ahead forecasts. The results also show that the Dynamic-Siegel model is suitable to forecast yields for all maturities in the short-term, for 1-month ahead forecasts.

Term structure modelling and forecasting is indeed and important part monetary policy, and guides investment decision at various levels during issuance of bonds. This is because it divulge away from the traditional forecasting using the yield curve, while at the same time carrying with it the same components of the yield curve, to bring in superior results than what a basic yield curve would produce. The results of the of this study serve as evidence that term structure modelling can indeed work efficiently in third world financial markets as it would in developed and emerging markets, and in this case, forecasting the term structure of government bond yields in Kenya using the Dynamic Siegel model.




Forecasting under AR(1): 2yr maturity
Yield(%)
5
10
15
20
2008
2010
2012
2014
2016
2018
Time


Forecasting under AR(1): 10yr maturity
Yield(%)
6
8
10
12
14
16
2008
2010
2012
2014
2016
2018
Time


Forecasting under ARIMA: 1yr maturity
Yield(%)
5
10
15
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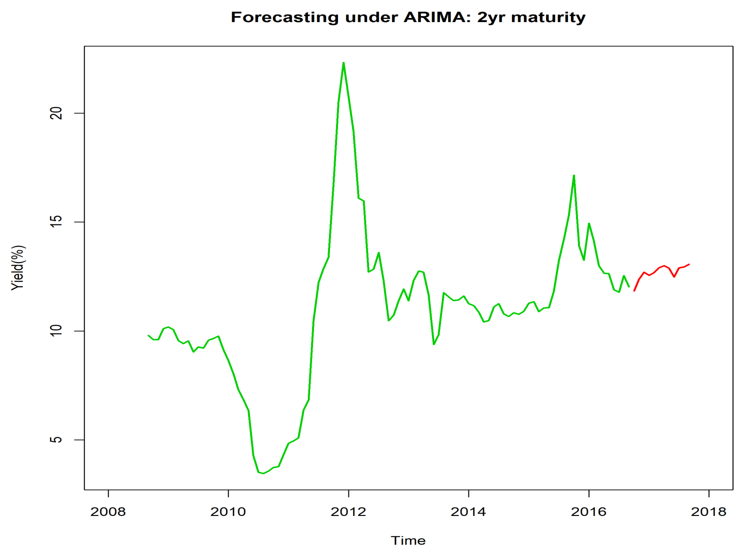
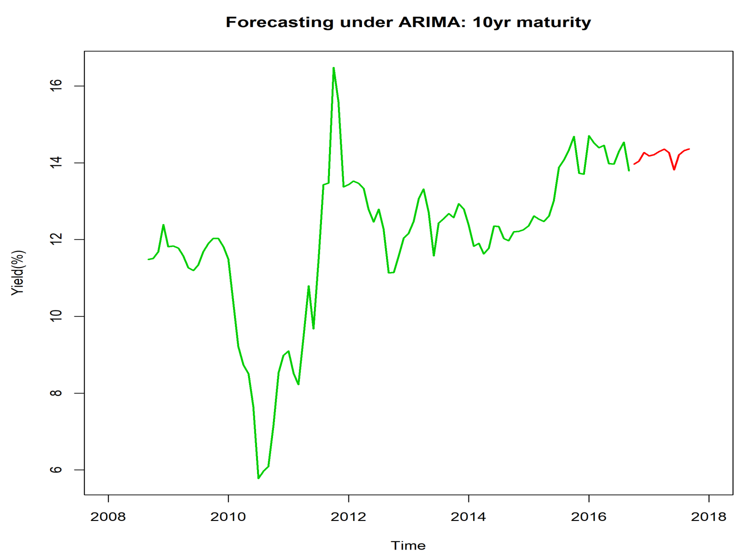
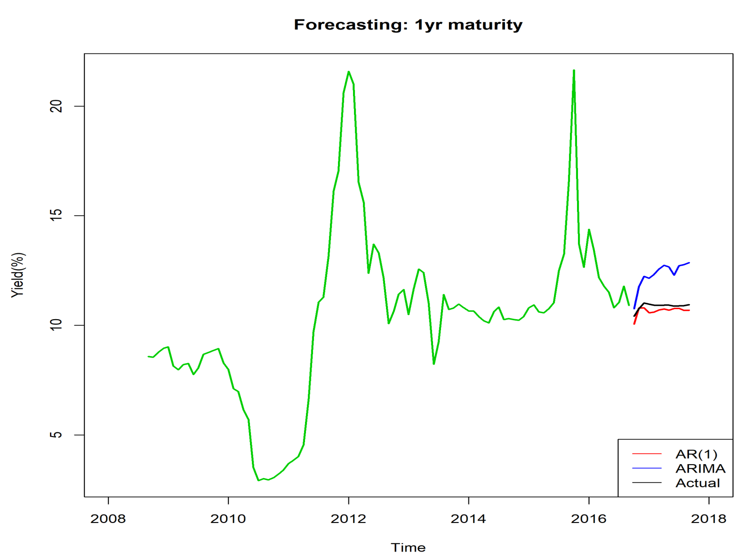
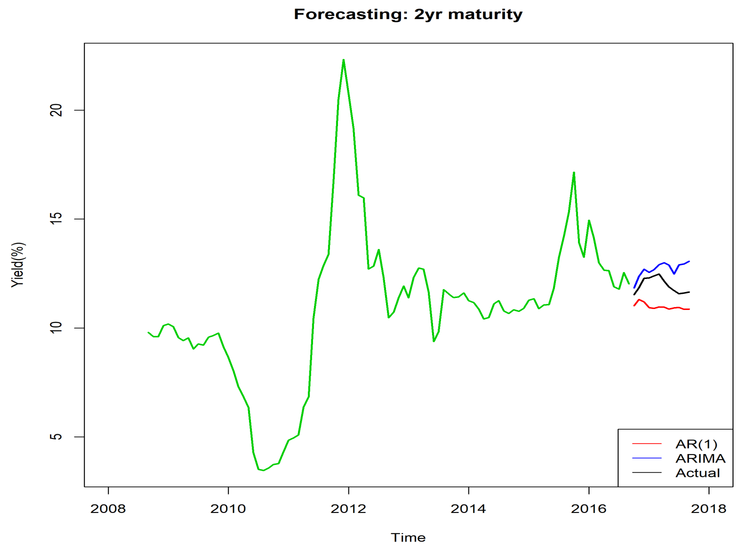
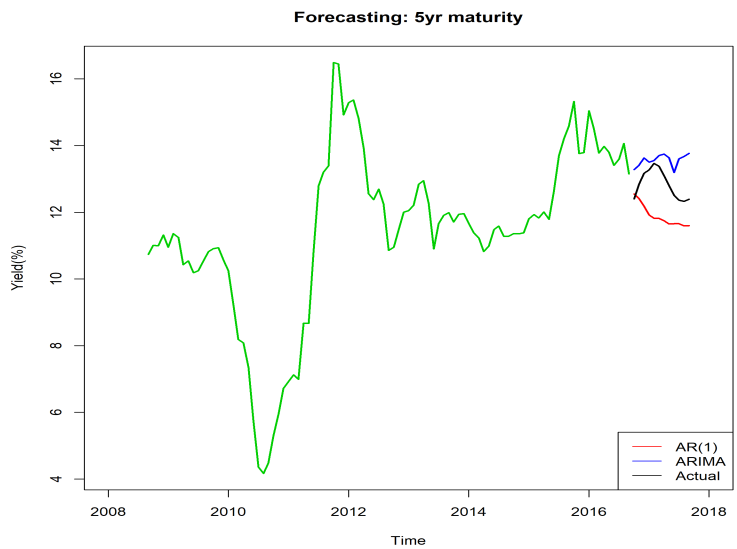
2008
2010
2012
2014
2016
2018
Time

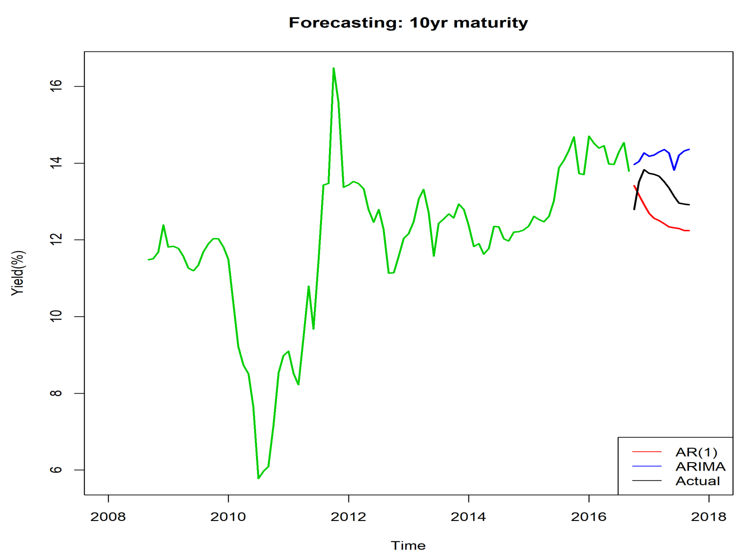

Forecasting under ARIMA: 2yr maturity
Yield(%)
5
10
15
20
2008
2010
2012
2014
2016
2018
Time

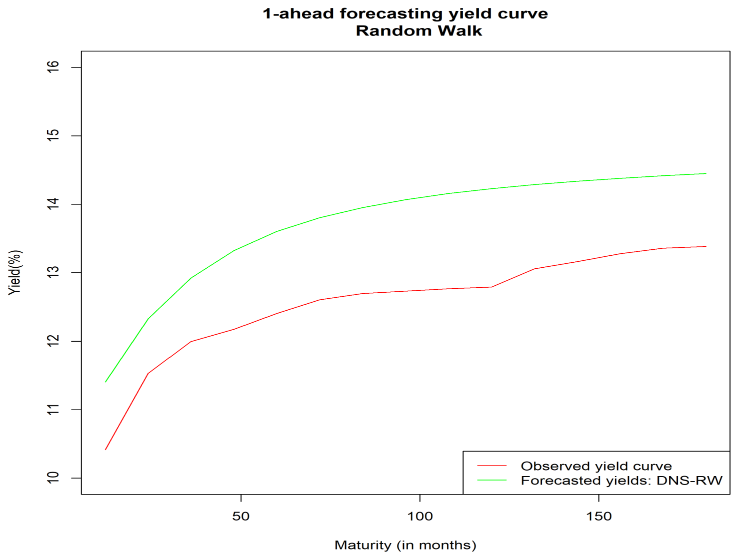

Forecasting under ARIMA: 10yr maturity
Yield(%)
6
8
10
12
14
16
2008
2010
2012
2014
2016
2018
Time


Forecasting: 1yr maturity
Yield(%)
5
10
15
20
AR(1)
ARIMA
Actual
2008
2010
2012
2014
2016
2018
Time


Forecasting: 2yr maturity
Yield(%)
Time
AR(1)
ARIMA
Actual


Forecasting: 5yr maturity
Yield(%)
Time
AR(1)
ARIMA
Actual


Forecasting: 10yr maturity
Yield(%)
Time
AR(1)
ARIMA
Actual


1-ahead forecasting yield curve
Random Walk
Yield(%)
10
11
12
13
14
15
16
50
100
150
Maturity (in months)
Observed yield curve
Forecasted yields: DNS-RW