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INDICATING ESSENTIALS OF SECONDARY SCHOOL ALGEBRA A COMPARATIVE ANALYSIS OF BRITISH, UNITED STATES AND ENTEBBE PROGRAMS

A Dissertation

Presented to

the Faculty of the School of Education

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In Partial Fulfillment

of the Requirements for the Degree

Doctor of Philosophy

(Mathematics Education)

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by

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CHAPTER I

INTRODUCTION AND STATEMENT OF THE PROBLEM

The movement to reform the teaching of mathematics gains ground steadily; the need to revitalize mathematics teaching is generally accepted but the nature and extent of the reform that is necessary remains a matter of informed and sensitive guesswork.

The development of improved mathematics programs has not been restricted to the United States. A significant illustration of this fact is found in the first report of the International Clearinghouse of Science and Mathematics published in 1966, which is a volume of 291 pages listing the science and mathematics curriculum programs now in progress throughout the world.

Some of the principal protagonists of this rethinking in the United States are: the Commission of Mathematics of the College Entrance Examination Board (CEEB), the University of Illinois Committee on School Mathematics (UICSM), the School Mathematics Study Group (SMSG), and the Secondary School Curriculum Committee of the National

Council of Teachers of Mathematics. In Great Britain, there are the Midlands Mathematics Project (MMP), the Norfield Mathematics Teaching Project (NMTP), and the School Mathematics Project (SMP).

In Africa, the African Education Program seeks to bring to that continent some of the newer and more effective methods of preparing improved school curricula in mathematics and the sciences based on the American experience in school reform over the past decade (183:472). The program applies the methods of curriculum development in conjunction with British and African scholars and teachers for the purpose of developing new course materials indigenous to African needs and relevant to African aspirations.

Two major reports on Secondary School Mathematics, one from England and one from the United States, were published within the years 1958-1959. Each sought to outline the essentials of secondary college preparatory mathematics, its content and methods. These reports were:

1. Teaching Mathematics in Secondary Schools, Ministry of Education Pamphlet No. 36 (London: Her Majesty's Stationery Office, 1958). Paper, vi-154 pp.

2. Report of the Commission on Mathematics, Program for College Preparatory Mathematics (New York: College Entrance Examination Board, 1959), 63 pp.

A reading of both reports revealed that each dealt with essentially the same problems at roughly the same time (less than a year separates the reports) but there were rather sharp differences between them in analyses and recommendations. In some instances, they appeared to be contradictory.

These reports are among the most prominent examples in mathematics education on both continents. They are reviewed in Chapter II. Frank H. Bowles, President, College Entrance Examination Board, stated in forewording his commission's report:

> It is the hope of the Board that, in due course the Commission's recommendations will be integrated with those of other groups to improve the study of mathematics from the first grade to the graduate school. (39:x)

Philip Peak, Acting Dean of the School of Education and Professor of Education at Indiana University, Bloomington, Indiana, writing in the Mathematics Teacher, stated that both reports should be studied together to give the teacher a substantial look at mathematics in the

English-speaking secondary schools, and Bryan Thwaites, Director of the School Mathematics Project (SMP) at Westfield College (University of London), Southampton, England, stated:

> In view of the great changes which have taken place during recent years in mathematics at University level including those changes which are often considered collectively under the term "Modern Mathematics," there is a prima facie case for a critical look at the content of present school syllabi. Research should be instituted without delay to ascertain which part, if any, of traditional syllabi should be omitted as of doubtful value and what might more profitably be included. (31:29)

Some of the controversies in the reform movements in the United States and Europe provide insight into the nature of the problem involved.

In October, 1961, <u>The New York University Alumni News</u> published an article written by one of its professors of mathematics, Morris Kline. In commenting about the experimental programs in high school mathematics, Professor Kline pointed out that he felt the "new" mathematics stressed theory and proof at the expense of other processes by which mathematics is created. He went on to say that the primary value of mathematics in the high school is that it is the language and essential instrument of science (130:1, 3, 8). The appearance of this article started a controversy that occupied the pages of The News until February, 1962. The December, 1961 issue carries Professor Begle's answer to Kline's attack. He emphasized that new concepts and discoveries in the sciences like quanta, nuclei, mass energy and Lorentz transformations need new concepts in mathematics such as vectors, groups and matrices (33:1,8). Professor of physics Morris H. Shamos, in the February, 1962 issue, expressed the opinion that the lack of ability of students to apply basic mathematical concepts to the sciences is due to a mathematics curriculum that does not fire the students' imagination (58:1, 8).

Professor Krzysztof Tatarkiewics of the University of Lublin, Poland, writing for UNESCO states:

> The primary aim of mathematics teaching at primary and secondary levels is to drill the pupil in a limited range of problems. The parts of mathematics he learns before the baccalaureate or secondary school leaving certificate can be set out in a few pages in the case of arithmetic and algebra and in twenty or thirty pages in that of geometry. . . . It is thus of great importance that his mathematics "drill" should include as much as possible of what will be necessary for the applied mathematics he uses, or will form a grounding for university studies. This is possible only if the pupil learns no more than is genuinely essential. (63:48)

In the March, 1962 edition of The American Mathematical Monthly, there appeared an article written by seventy-five mathematicians including Garret Birkhoff of

Harvard, Richard Courant of New York University, H. S. M. Coxter of University of Toronto, Morris Kline of New York University, George Polya of Stanford University and W. W. Sawyer of Wesleyan University. These mathematicians agreed on four premises:

1. It is incorrect to stress content at the expense of pedagogy.
2. Mathematicians may unconsciously assume that young people ought to like what mathematicians like, and that the only worthwhile students are those who have the ability to become mathematicians.
3. There is a need to learn more mathematics than in the past.
4. The teaching of elementary and high school mathematics needs improvement.

Beginning with these premises, the authors list several guide lines for high school mathematics:

1. High school mathematics should provide for the needs of all students, without undue concentration on the needs of future mathematicians.
2. Premature formalization and the premature introduction of abstractions are dangerous. Before these are attempted there must be an adequate background of facts. Challenging concrete applications should accompany the teaching of concepts.

3. Mathematics is the language and essential instrument of the sciences. If mathematics is separated from the sciences, it loses one of its most important sources of interest and motivation.
4. Mathematical thinking is not only deductive reasoning. The student must have experience with some of the more informal processes in order to understand and appreciate the role of formal and rigorous proofs.
5. There are several levels of rigor. The levels should be matched to the student's mathematical experiences and background.
6. The best way to guide the mental development of a student is to allow him to retrace the mental development of the race, i.e., its great line, not the errors of detail.
7. A new curriculum should emphasize unifying general concepts, preceded by concrete preparation and followed by challenging application. (155:189-192)

Nearly everyone agrees that mathematics is an essential part of a general education at the secondary level. So wrote Professor Maurice L. Hartung, Professor of the Teaching of Mathematics at the University of Chicago.

> This happy state of unanimity disappears when an effort is made to give specific answers to questions about content and method. Diversity of opinion is then the rule. Differences exist not only between the views of the mathematicians and those of general educators, but even among the mathematical experts themselves. The problem of determining what mathematics is essential has no neat mathematical solution. The most effective efforts to specify an essential mathematical training are those that give attention both to the behaviors sought and to the content with which they operate. (126:82)

A distinguished former member of His Majesty's Inspectorate wrote in 1931:

> The very last thing I desire to do is to impose on teachers my ideas of methods. Anything in the nature of standardized method in English Schools is unthinkable. The Board of Education, as I know it, never issued decrees in matters affecting the faith and doctrines of our educational system; it confined itself to making suggestions . . . and a method is not a piece of statuary, finished and unalterable, but is an ever changing thing, varying with the particular genius of the teacher who handles it. . . . The method itself counts for something, but what counts for very much more is the life that the craftsman when actually at work breathes into it. (59:vii)

The interested reader would, at this point, realize that differences in programs and the resulting levels of achievement, may be influenced by a number of factors, such as the philosophy and the organization and administration of the educational system. He may then ask, what is the best indicator of the over-all objective for teaching secondary school mathematics? Rudman (1958), when discussing the use of textbooks, stated that textbooks have been designed to perform one function which is to supply a course of study. The textbook is most effective when used for this purpose, and it is an accepted fact that the textbook is the major teaching tool in mathematics classrooms. The content of the textbook used thereby determines in large part what topics are to be taught and the extent to which each will be developed. This belief has been evidenced since the beginning of the century when Reader stated that the best expression of the methods of teaching any branch of the curriculum at any period of its history is revealed in the textbooks of the period.

This study, therefore, suggests that the content of textbooks used determines in large part what topics are to be taught and the treatment expected can be inferred from the appropriate examination papers.

Basic Difficulties

What follows is a listing of six difficulties that have motivated this study:

1. There is great interest in many countries in the possibility of radical changes and improvements in the teaching of mathematics. This trend has a number of contributing causes.
2. There is the broad underlying fact that modern society is making increasing demands on all citizens for simple mathematical skills and an appreciation of numerical significance. Those in executive positions in the large organizations of today are increasingly called upon to make decisions in which quantitative appreciations are essential.

3. The demand for scientists and engineers--all of whom must have sound knowledge and understanding of mathematics in industry and in other branches of economic activity are leading to a demand for more mathematicians with new kinds of skill. All of these demands are creating a need for a reappraisal of the content and methods of school mathematics.

4. Despite the great amount of discussion and study of the problem of mathematics teaching, much of it is not having the desired impact on the schools (26:11). In the last analysis, it is in the schools that action must be taken, and it is there that the significance of the new thinking will be judged.

5. This lag between the new ideas and their effect on the school is, of course, inevitable and perhaps desirable. Nevertheless, it was felt that the works of groups of experts is at best a tentative beginning, since inevitably the textbooks, the experiments, and definitive programs will need to be adapted to the traditions and the needs of the different countries

in which the modernization of mathematics curricula is undertaken.

6. There seems to be a total lack of any kind of systematic identification of secondary mathematics curricula problems and their interrelations and implications for one another.

An enormous amount of entirely new mathematics has been developed in the last fifty years. Some of this mathematics is recommended for the college capable student. But a double question faces the curriculum planners of today. Which one and how many of the new mathematical concepts should be included in the already overcrowded pre-college secondary program?

Statement of Purpose

The problem studied may be stated in general terms as the investigation of the areas of agreement or disagreement in secondary school algebra in England and the United States. These data are then used to help indicate essentials and suggest research pertinent to the Entebbe Program (The African Mathematical Program). More concretely, the objectives were: (1) to discover, by content analysis, the common areas of concern of secondary algebra in England

and the United States; (2) to compare these common areas of concern (the essentials) with the areas of concern in the Entebbe Program; (3) to classify the common areas of concern as "traditional" or "modern"; and (4) to suggest needed research pertinent to the Entebbe Program.

Questions To Be Answered

In order to give even greater specificity to the study, the following questions were posed:

1. Who were the personnel responsible for each program? What were the differences in makeup in these groups?
2. What are the common areas of concern (essentials) of secondary algebra indicated in the selected British and American Programs?
3. What are the apparent areas of disagreement?
4. What are the variations, if any, in recommendations regarding common areas of concern?
5. Are the variations, if any, in recommendations regarding common areas of concern contradictory? If so, can these be accounted for by different national conditions or new information as reported in some study or do they represent differences in point of view?

6. To what extent does the Entebbe Mathematics Program encompass common areas of concern (essentials) as indicated in both programs?

7. What implications do these observations have for the Entebbe Mathematics Program?

Hypotheses

Looking again at the general statement of the problem, four hypotheses for the research were deduced:

1. That there are areas of agreement (essentials) in secondary algebra.

2. That there are procedural variations in some of the essentials between programs.

3. That the Entebbe Mathematics Programs recognizes some of the essentials.

4. That in the areas of disagreement between the British and American programs, the Entebbe Mathematics Program has greater number of elements of the American program than of the British program.

Assumptions

In testing these hypotheses,[1] it was assumed in this research that the data relating to the concern of this study are contained in: (1) official documents--reports, (2) textbooks--most modern, and (3) external examinations--appropriate examination papers. Other publications of the College Entrance Examination Board, Her Majesty's ministry publications and the Educational Services Incorporated, are also regarded as primary source materials.

Delimitations

It was thought wise to carefully delimit the study in two important respects:

1. The study was limited to an analysis of the materials cited under assumptions.
2. The study was limited to secondary college preparatory or academic programs in algebra, since algebra has been the major area of modernization and revision.

Limitations

Two major limitations to the research were recognized.

---

[1]Nonstatistical.

1. If there are essentials to be taught incidentally or perhaps orally which are not indicated in the selected series, there is no way of recording these data.
2. The significance and degree of any differences and contradictions cannot be assessed, except by verbal comparisons and where data are involved.

Organization of the Dissertation

The report of the research is organized into seven chapters. The introduction and statement of the problem is Chapter I. Chapter II is titled "Review of the Literature," and the purpose of this chapter is to set the stage for the reporting of the research proper. In it, an attempt has been made to report such studies as are available in addition to selected writings which, though they may not satisfy the criteria of rigorous research, nevertheless, accurately reflect the state of the field. Chapter III is a detailed presentation of the procedure of the study.

In Chapter IV, an effort has been made to develop the background and the philosophy for the teaching of secondary mathematics in England and the United States by noting the emphasis placed on: (1) the basic laws of learning, (2) the intrinsic values of mathematics and (3) by bringing out the part played by examinations in both universities and secondary schools. Chapter V presents the basis for selection of the textbooks and examinations and in Chapter VI the analysis and comparison of concepts (with emphasis on the essentials) are tabulated as they were found in the programs and evaluated by the jury.

Thus, the reporting arrangement attempts to present the results of the research in such fashion that the reader is permitted to realize the conclusions which may be drawn before they are presented in the last chapter, VII, titled "Summary." This final section, in addition to pulling the report together, reviews the research in relation to the hypotheses being tested.

CHAPTER II

REVIEW OF THE LITERATURE

> The goals of mathematics instruction seem to need a great deal of additional study so that a more nearly precise formulation and an improved basis for fundamental agreement among researchers may be determined. (89:299)

Hancock (1961) studied the evolution of the secondary mathematics curriculum and concluded that the aims of mathematics instruction seemed to change to meet the demands of society. In periods when society saw no pressing need for mathematics, utilitarian aims were stressed; during periods of severe depression, the cultural aims of mathematics were emphasized; however, during times when the needs for mathematics instruction were readily apparent, the main goal of instruction seemed to be to cover as much material as possible.

A frequently cited formulation of objectives is the Check List of twenty-nine questions given in the Guidance Pamphlet in Mathematics. This pamphlet constituted the Final Report of the Commission on Post War Plans of the National Council of Teachers of Mathematics. Of the twenty-nine items, perhaps sixteen, or a little more than half, might be considered as applicable at the subsistence level. The others seem clearly to be above that level. Among these are the items indicated briefly by the following terms: (15) Constructions, (17) Vectors, (23) Using the axioms, (25) Similar triangles and proportion, and (26) Trigonometry.

The concepts and abilities suggested in connection with these items are rather generally believed to be sufficiently pervasive in modern life to warrant the claim that an educated person should have them. A strong argument can be given that one needs at least this much mathematics to read intelligently from newspapers, magazines, and books of the nonfictional type in order to be informed and gain understanding of the world. These concepts and abilities are also needed in the study of other subjects, notably the sciences, and to some extent the social sciences.

At the present time the topics in the above list are commonly taught, in the United States, in a course called General Mathematics, or in Algebra. Often the orientation is toward vocational use or college preparation. If this viewpoint is taken, the behavior sought, and particularly the kind of thinking desired, leans noticeably in the direction of understandings and interpretative abilities, rather than toward skill in operational technique.

Although there are a number of isolated studies of the learning process as it relates to secondary mathematics, perhaps pertinent to this study are the considerations of accepting the child as an active participant in the learning process which was the goal of Hendrix (1961), who showed the distinctions in three processes of learning: the inductive method, the nonverbal awareness method, and the incidental method. Hendrix advocated the nonverbal awareness method, in which there is an emphasis upon the discovery of mathematical principles but not an insistence upon a precise formulation of the principles by the student. Subsequently, in 1963, she noted that the manner in which the UICSM materials had been developed revealed research problems in nonverbal instructional phenomena and in the discovery approach. However, most research groups studying modern mathematics curriculum emphasized clarity and precision of language for both the teacher and the student.

Summarizing some of the work of the University of Illinois Committee on School Mathematics (UICSM) during the past ten years, Hale (1961), pointed out that precision of language was a goal for the teacher and the textbook writer; however, correct action was a characteristic of the good learner in the UICSM program.

The organization of the National Longitudinal Study of Mathematical Abilities (NLSMA) was reported by Cahen (1963). The major purpose of this study is to identify factors that contribute to achievement and problem solving ability in mathematics and to interactions of these factors with various approaches to the learning of mathematics. Cahen's work, when coupled with the extensive investigations reported by Alpert, Stellwagon, and Becker (1963), proved to be significant attempts to find interrelations among the many variables interacting in a learning situation. Donald J. Dessart, writing in the Review of Educational Research (89:307) states that although such studies probably create more questions than answers and do not provide clear-cut, compartmentalized conclusions, they do come to grips with the total problem of learning mathematics in a secondary classroom.

Perhaps the best known intensive work on conceptual learning in mathematics published within this decade, is the work of Gagne and his associates. They took a more rigorous approach than did others to the general problem of how to identify, and how to form a sequence of the elements of knowledge in a program. In a series of related studies, Gagne (1962), Gagne and Bassler (1963), Gagne and Brown (1961), Gagne and Paradise (1961), and Gagne and others (1962) investigated various topics in mathematics concerning the nature, structure, and sequence of subordinate knowledge requirements and the effects of ability, method of response guidance, and degree of repetition.

The general approach in these studies was to identify a criterion task, such as solving linear equations or adding integers, and then to ask what subskills, if learned and retained, would enable the student to perform the task. For example, what kinds of subskills would transfer to the task? By repeatedly asking and answering this question, first for general subskills, then for specific subskills, and eventually for the level of basic known abilities of the students entering a course, Gagne and his associates developed a pyramid-shaped hierarchy of knowledge requirements. Once an appropriate sequence for the program was arranged, various conditions of practice could be investigated and compared.

An interesting finding of one of the experiments in this series (Gagne and Bassler, 1963) was that the criterion-task performance remained at a high level nine weeks after training, even though some of the subordinate knowledges were forgotten. This outcome does not mean that the subordinate knowledges were not necessary to master the task, but it probably does mean that a higher-order process of consolidation took place--a consolidation illustrating the principle that the criterion-task itself is not necessarily the appropriate unit for teaching. Conversely, items needed for teaching purposes are not necessarily permanently retained even when it can be shown that the criterion of learning was reached. This experiment further indicates the significance of a careful delineation of and agreement concerning the objectives of mathematics instruction for all secondary school students.

In a different category can be placed surveys on practices and trends, such as those of The Organization for European Economic Cooperation, Office for Scientific and Technical Personnel (1961), which surveyed practices and trends in school mathematics in its member countries and in the United States, and McLean (1960), who surveyed the status of integrated algebra-geometry courses in California and sought to determine the acceptability of such courses to teachers and college directors of admission. Integrated courses were not commonly found, teachers disagreed as to the value of such courses, and colleges generally accepted such courses except for science majors. It was suggested that integrated algebra-geometry courses be offered only as a second track in the college-preparatory mathematics curriculum (197:281).

The analysis of modern and conventional programs constitutes a different category of investigation. The National Council of Teachers of Mathematics (1963) provided an analysis of the new mathematics programs based upon the criteria of social applications, structure, vocabulary, methods, concepts versus skills, proofs, and evaluation.

Howard F. Fehr (1959) conducted a study of seventeen countries and gave an over-all view of mathematics instruction in all the countries concerned with regards to (1) the material or subject matter included in the program; (2) the school organization and the sequential arrangement.

of subject matter either by years of instruction, grades one through nine or ten, or by age, six years to, but not including, sixteen years; also the time allotted to mathematics instruction; (3) the selection, promotion and segregation of pupils into special classes--particularly those classes designated as preparatory to university entrance; (4) the methods of instruction with special reference to desired goals of pupil achievement; (5) the preparation of teachers of mathematics; (6) the systems of examination; and (7) the directions and trends that instruction is taking with regard to philosophical, cultural, and psychological aspects of learning.

Kemeny (1963), reporting for the International Commission on Mathematical Instruction, presented a summary of reports from representatives of twenty-one nations on attempts to modernize mathematics teaching. Most important was the general agreement that much of traditional mathematics should be taught from a modern point of view. However, as far as the details of these recommendations are concerned, there is considerable disagreement. The stress in the algebra study at age fourteen or fifteen is on the solution of equations, first the simple equation in one unknown, then two equations in two unknowns (or three unknowns), and finally the quadratic equation. Insofar as the reports show, the emphasis is on tricks and formulas and not on proofs (123:182).

Rajaratnam (1957) concluded that new ideas of variables, function, equation, and equality were mixed with outworn and erroneous ideas and terminology in the elementary algebra books she surveyed, and Dominy (1962) conducted A Comparative Analysis of European and American Elementary School Mathematics Textbook Programs. This study presents information based on an analysis of the actual content found in textbook programs selected as being representative of the elementary school mathematics program followed in the United States as compared to programs representative of those being followed in England, France, the German Federal Republic, and Union of Soviet Socialist Republics, and outlines recommendations for arithmetic in the United States elementary school mathematics program. Her analytical design was primarily used in designing the analysis of this study.

Despite considerable emphasis on mathematics preparation and curriculum changes, few investigations have reported implications for college entrance and preparation. Yet, of considerable interest to those who want to improve the curriculum, is the adequate preparation of students for specific objectives.

G. B. Smith (1958), analyzing the preparation of 1,124 freshmen entering the University of Kansas in 1956, found that 29 per cent of the men and 5 per cent of the women had four or more years of mathematics. Forty-seven per cent of arts and science students, 81 per cent of engineering students, and 28 per cent of fine arts students had three years. Keedy (1958), using questionnaire returns from 134 engineering schools, learned that thirty-eight required solid geometry for entrance; he concluded that solid geometry was not significant in relation to the requirements for entrance to engineering.

Brant (1960), followed up Keedy's study by asking fifty-one schools with some kind of solid geometry requirement if they would accept a one-year course of plane, solid, and coordinate geometry. In the few instances where a solid geometry requirement still existed in the vast majority of remaining courses, a fused course would be accepted. Thus while three-dimensional concepts were still judged important, solid geometry, as such, was an uncommon requirement (81:281).

It would appear worthwhile to devote some effort to the comparative study of events which precede these two processes of concept learning and utilization; in other words, to examine the question, "Which concepts (or concept sequences) which are presently in modern recommended textbooks are essential for the objectives of today's youth?" There is the related question, too, of the extent to which "modern" mathematical concepts which are considered fundamental to the understanding, appreciation, and utilization of secondary mathematics, have permeated the curriculum. This is the general framework of present study.

Another frequently cited formulation of essentials of secondary mathematics is the bulletin on Mathematical Needs of Prospective Students in the College of Engineering of the University of Illinois (1951) which states:

> This section lists topics in secondary mathematics, an understanding of which is considered to be indispensable. (113:85)

This study was motivated by the desire of many educators to know exactly what competencies their students need and the increasing need for mathematical competencies in the applications of scientific advances in modern society. Although many items in the list use the term concept explicitly, ("concept of similarity"), all involve concepts implicitly and it is assumed that the mathematical needs for prospective engineers are essentially the same as the need for students preparing for study in any area requiring courses in college mathematics.

Professor Bruce E. Meserve of the University of Illinois noted that until the desired level or depth of understanding of the concepts and the necessary degrees of skill are more precisely defined, teachers in the high schools will be unable to determine how much emphasis is to be given a topic. He points out that the sample test items supplied in the supplementary bulletin are of help in this respect (144:87). Still a careful analysis and exposition of the behavior desired would add greatly to the ultimate effectiveness of the project. The fact is that the topics in the University of Illinois list represent the traditional approach to mathematics in secondary schools and colleges. Briefly, this criticism is to the effect that the list, which contains ninety-seven topics, seems to ignore the point of view of modern mathematics, and the belief that traditional courses put too much emphasis upon certain topics, of which the solution of oblique triangles and the law of tangents in trigonometry may be cited as examples. Meanwhile fundamental concepts, such as class or set, and modern techniques, such as those involved in statistical studies, are neglected.

Teaching Mathematics in Secondary Schools, Ministry of Education Report--England

This bulletin is an official publication of the British Ministry of Education. It was developed and written by a commission for the Board of Education in 1958. It has the features of both a course of study and a syllabus. It attempts to develop a background and a philosophy for the teaching of mathematics. Chapter I presents the history of mathematics in England and shows why the present situation exists. The book brings out the part played by examinations in both universities and secondary schools (as indicated in Chapters II and III) and by the basic laws of learning. It is interesting to note the emphasis placed on intrinsic values of mathematics. The authors recommend grouping according to ability with the abler probing more deeply. They encourage discussion and discourage working papers set in previous examinations.

Specifically, the authors recommend that the syllabus should start with number experiences and lead to abstractions with understandings all along the way. Rules formulated for students have no place in the early study of mathematics. The method used by a child attempting something new is an important part of instruction.

In the opinion of the authors of this bulletin, some topics that need rethinking are the LCM, order of operations, groups of nonrelated fractions, compound quantities, and checking by complex methods. These topics perhaps use up more time in the classroom than they should. More emphasis is needed on concepts of quantity, the relationships which exist between items, and principles which are applicable to extensions. Arithmetic, algebra, and geometry should implement each other and lead to calculus, coordinate geometry, and trigonometry. This book sets trigonometry as a unifying factor in mathematics and would have the student well acquainted with it in secondary school. Ideas of locus, proof not based on construction, and three-dimensional geometry along with two-dimensional geometry are recommended. The place of mechanics in relation to mathematics should be a part of the program.

Chapter V discusses the sixth form and how to teach students at this level. It states that the universities may have had undue influence in examinations and methods of instruction. Teaching should not be lecturing but discussing, raising questions, and posing problems. The students at this level not specializing in mathematics should still study it as a part of a good education.

Chapter VI deals with mathematics for the ordinary pupils. It emphasizes the challenge in this direction and the great contribution to be made. Several alternatives are mentioned including lengthening time, less rigorous examinations or different content. "Standards" are variables and need to be considered in terms of their applications.

Finally, the commission recommended that the mathematics classroom needs the right atmosphere, a teacher with the right attitude, and equipment which is functional rather than complex. The commission emphasized that variations of methods between schools should be considered and change in method just for change should not take place. An undue amount of time is probably spent on review. Instead, this should be done as needed. Drill follows understanding. Students need training in learning on their own from all sorts of sources.

Report of the Commission on Mathematics (CEEB)--United States

> The Commission on Mathematics of the College Entrance Examination Board, appointed in 1955, grew out of the concern of the mathematics examiners that the Board's tests were not reflecting fully and appropriately the emerging programs of mathematics instruction in forward-looking college preparatory schools, both public and private, and moreover, that the standard curriculum taught in most secondary schools was sadly out of date. The Commission was formed to consider broadly the secondary school college preparatory curriculum and to make recommendations looking towards its modernization, modification, and improvement. It seeks to secure the introduction in American secondary schools generally of a mathematics program oriented to the needs of the second half of the twentieth century. These needs are vastly more extensive than was the case when the present traditional curriculum took form. . . . Moreover, mathematics itself has changed both in content and, more important, in the point of view from which it is regarded by mathematicians. (139:19)

The Commission pointed out six specific areas in the present curriculum which need revision:

1. Too much attention is given, particularly in algebra, to routine manipulation in artificial situations, and not enough emphasis is laid on fundamental concepts.

2. Deductive reasoning is taught chiefly in connection with plane and solid geometry, and its application to other parts of mathematics is largely ignored. Its use in algebra and trigonometry should be expanded.

3. Too often, the usual geometry course consists of rote memorization of sequences of theorems and fails to explain the deductive process clearly.

4. Many topics which are now included were important at one time for applied science, but now have become obsolete. These should be replaced by topics of current importance. Examples of obsolete topics are: extensive solution of triangles by logarithms, deductive methods in solid geometry, and Horner's Method for finding the roots of a polynomial.

5. Examples of modern subjects which might be included are: descriptive statistics, statis-

tical inference, elementary properties of sets, and the basic ideas of modern algebra. Many of these topics are more elementary than topics now in our curriculum.

6. Mathematics is too often presented as a series of isolated tricks so that students get no view of the subject as a whole, and do not realize its position as a creative endeavor in our civilization.

Mathematics, the Committee stated, is a different subject today than it was a generation ago, its applications are vastly more extensive, and its essential nature is now considered to be entirely different than was the case heretofore. Thus to meet the manifold social needs of the second half of the twentieth century--the needs of mathematics itself, of physical science, of social science, of technology, of industry--requires a curriculum revised in content, but even more basically revised in point of view. The most important point to be made with respect to the actual details of revision is that any proposals should be based on a careful analysis of the curricular implications of such matters as were set forth in the preceding section, and should focus the suitability of the course content on the needs of the student.

The Commission on Mathematics is convinced that curricular revision will be successful in producing a high school mathematics curriculum oriented to the needs of the present and the future only if three principles are heeded.

1. The proposals must be based on the existing curriculum, and must consist of modification, modernization, and improvement of the present pattern, rather than its discontinuance and replacement by entirely new content.
2. The point of view of modern mathematics must be used as a guide in determining the modifications to be made. This point of view is well stated by W. W. Sawyer, who wrote: "The mathematician of older times asked, 'Can I find a trick to solve this problem?' If he could not find a trick today, he looked for one tomorrow. But today one no longer assumes that a trick need exist at all. He asks rather, 'Is there any reason to suppose that this problem can be solved with the means at hand? Can it be broken up into simpler problems? What is it that makes a problem soluble, and how can it be

tested for solubility?' He tries to discover the nature of the problem he is dealing with."

3. Changes to be proposed must be sufficiently far reaching so that the modified curriculum is truly oriented to present and future needs, but not so radical as to be beyond the competence of the available teaching staff. However, a willingness upon the part of school officials to provide means for teachers to participate in programs of in-service education must be assumed.

The Commission has, therefore, consciously attempted to formulate a program that in itself constitutes an appropriate part of liberal or general education, and students who have studied the recommended curriculum should have developed such maturity, power, and understanding as to be ready for mathematics on a true college level. And, finally, in no case is it assumed that students in this program are "gifted" or exceptionally talented in mathematics. The Commission's proposals constitute recommendations for the revision of the high-school curriculum to be followed by the average, normal, or ordinary college-bound student; indeed, as an optimum by all such students for three years.

The specific changes suggested by the Commission are set forth in detail in its report. They are accompanied by both exposition and supporting argument. Here, the suggestions concerning algebra can be summarized, as follows:

Algebra:

Little change in the actual content of elementary and intermediate algebra is envisaged, but a fundamentally altered point of view is regarded as absolutely essential. Algebra must be treated as a study of mathematical structure, rather than only as the development of manipulative skill in one particular mathematical system. Provision should be made for experience in deductive reasoning in algebra as well as in geometry.

The introduction of the point of view advocated by the Commission will require that teachers familiarize themselves with certain concepts not hitherto ordinarily included in their college training, in particular the notions of set statements, variable relations, and functions, as these are formulated in modern mathematics.

The curriculum envisaged by the Commission prepares for more advanced work, but only by the appropriate means of seeking to develop sufficient mathematical understanding, power, and maturity through the study of pertinent secondary school mathematics that the graduate will be ready to advance to the study of collegiate mathematics.

Finally, although the program has been developed for college preparatory students, many of the Commission's proposals are also appropriate for the so-called "general" mathematics courses. Particularly is this true of the Commission's strong emphasis on point of view as even more important than content. For creative teaching and a curriculum that permits and encourages it is of the utmost importance no matter what the ability or objective of the pupil.

The following summary of the Commission's thinking will appropriately summarize what has been said.

For College-Capable Students, the Commission on Mathematics Presents a Nine-Point Program to Meet Contemporary Needs

1. Strong preparation, both in concepts and in skills, for college mathematics at the level of calculus and analytic geometry.
2. Understanding of the nature and role of deductive reasoning in algebra, as well as in geometry.

3. Appreciation of mathematical structure ("patterns")--for example, properties of natural, rational, real, and complex numbers.
4. Judicious use of unifying ideas--sets, functions, and relations.
5. Treatment of inequalities along with equations.
6. Introduction of coordinates and vectors in plane geometry and in trigonometry.
7. Space perception and essentials of solid geometry incorporated with plane geometry.
8. Twelfth grade mathematics centered on elementary functions--polynomial, exponential, circular.
9. Additional twelfth grade material recommended: either introductory probability and statistical reasoning, or an introduction to modern algebra (fields and groups).

Some Common Elements in New Mathematics Curricula

All of the new programs attempt to avoid the presentation of new materials as a series of unrelated topics. Instead, they stress unifying themes or ideas in mathematics such as structure, operations and their inverses, measurement, graphical representation, systems of numeration, properties of numbers, the development of the real number system, statistical inference, language and notation of sets, logical deductions, and valid generalizations (67:26). A comprehensive discussion of unifying ideas appears in the Twenty-fourth Yearbook of the National Council of Teachers of Mathematics, "The Growth of Mathematical Ideas, Grades K-12."

The emphasis on these unifying ideas has resulted in the introduction of words and ideas from college mathematics. For example, the introduction of the notation of sets has involved using symbols and words normally reserved for the college level. It has been the experience of many teachers that the set ideas and language are helpful in explaining many other fundamental mathematical concepts. The structure of mathematics is a basic concept that seems to lend itself to description through the language of sets. Although discovery and an emphasis on the meaning of mathematical operations are not basic mathematical concepts per se, they are characteristics common to the new mathematics programs.

All of the new programs emphasize the structure of mathematics. It is reflected in the careful development of mathematics as a deductive system (67:26). The emphasis is on the basic principles or properties common to all systems of mathematics. There has been a tendency to look at the characteristics of each mathematical model separately. This has resulted in the students learning many seemingly unrelated facts. In the new curricula, students are encouraged to discover general laws and principles. An example is presented in the Appendix.

An attempt has been made in this chapter to report such studies as are available in addition to selected writings which, though they may not satisfy the criteria of rigorous research, nevertheless accurately reflect the state of the field.

CHAPTER III

THE PROCEDURE OF THE STUDY

A comparative analysis such as the one described herein is made possible only by having data pertaining to the actual content found in the selected textbooks perused and recorded in a like manner. Such a study could reveal errors in thinking, but even if it proved nothing, what it might disprove could be of utmost importance.

If "the essence of mathematics is economy of thought and expression," (60:62) then the success of a college preparatory mathematics program is determined to an appreciable degree by the provisions made, content-wise, for a systematic, sequential development of basic knowledges and understandings. Certain concepts must of necessity be considered basic and greatest achievement can be anticipated only when the student's attention is focused on these concepts.

This study, based on a documentary analysis of what <u>is</u> included in selected textbook programs falls into the category of descriptive research. It only deals with actual content, therefore, even though it will be possible to compare the concepts and their order of appearance in the corresponding programs at any particular level, the reader is reminded that one cannot, with any degree of validity, indicate whether a child has learned to compute by using the process in a mechanical manner or whether the mathematical operations are performed with any specific degree of understanding.

Listing of Possible Essentials

The textbooks were read for the purpose of obtaining data as to the concepts included, and also for noting major developmental steps which occurred in each textbook series. This preliminary investigation indicated that a college teacher of freshmen, working into a freshman class, cannot tell whether they have had a traditional mathematics program, SMSG, UICSM, Ball State, something else, or a mixture of all these. He cannot even count on their using the same mathematics vocabulary. While the consequences of this variety are yet to be fully explored, articulation with the high school remains a difficult problem.

It is also important from the college preparatory standpoint, that CUPM . . . The Committee on the

Undergraduate Program in Mathematics, decide:

> That the first two years of analysis should be the same, regardless of the intended career of the student. That is, the freshman calculus course should be the same whether the student is going into the social sciences, the physical sciences, graduate work in mathematics, teaching mathematics, or any other field. (58:2)

These college curricular changes have also made desirable a digest of at least the minimum content expected of high school mathematics programs, so that teachers can have adequate specific information as they prepare their students for professional education in any of the fields.

It was noted earlier, that the primary purpose of the bulletin, . . . Mathematical Needs of Prospective Students . . . published by the college of engineering of the University of Illinois in 1951 and revised in 1958, was to satisfy this last need by listing topics in algebra, geometry, and trigonometry, which the entering engineering student at the University of Illinois is expected to understand and be able to apply. While the course organization and content of engineering curricula are far from being identical to other college preparatory curricula, there is sufficient similarity in college preparatory algebra content to support the utilization of the fifty-seven algebraic concepts on the Illinois list as the basis for classifying the concepts indicated in each textbook examined. The thirty-six basic algebraic concepts which were listed as minimum essentials are:

1. Signs of aggregation; _viz_., parenthese, brackets, braces, _et al_., and their use.
2. Rational numbers; i.e., the integers and the functions.
3. Fundamental operations with rational numbers.
4. Fundamental operations with algebraic fractions.
5. Fundamental operations with polynomials.
6. Common special products; _viz_., $a(b + c)$, $(a + b)(a - b)$, $(a \pm b)^2$, and $(a + b)(c + d)$, emphasizing the distributive law.
7. Factoring; _viz_., $ab + ac$, $a^2 \pm 2ab + b^2$, $a^2 - b^2$, $ax^2 + bx + c$ based on the distributive law.
8. Laws of exponents, including negative and fractional exponents.
9. Solution of linear equations having numerical and/or literal coefficients.
10. Solution of a system of linear equations.
11. Determinants, their evaluation by minors, and their use in solving systems of linear

equations.

12. Variation, direct and inverse.

13. Function and functional notation. Representation of a function by a table of corresponding values, by a graph, and, where possible, by an equation or verbal statement.

14. Properties of a linear function; viz., rate of change, graph, slope, and y-intercept of the graph.

15. The quadratic equation: derivation of the quadratic formula; solution by formula and, where appropriate, by factoring.

16. Irrational numbers and fundamental operations with these numbers.

17. Real numbers and fundamental operations with these numbers.

18. Complex numbers and fundamental operations with these numbers.

19. Quadratic polynomials in one variable--standard form, graph, location of maximum or minimum by completing the square; nature of roots, and expressions for the sum and product of the roots of a quadratic equation.

20. Common quadratic equations in two variables.

21. Solution of a system of two quadratic equations.

22. Solution of verbal problems by algebraic methods.

23. Solution of equations in which the unknown occurs under a radical sign.

24. Binomial theorem with positive integral exponents.

25. Scientific notation or standard-form numbers--e.g., 2.54×10^3, 1.2×10^{-4}.

26. Principles of computation with logarithms.

*27. Change of the base of logarithms.

*28. Solution of exponential and logarithmic equations.

*29. Factor theorem.

*30. Finding the rational roots of higher degree equations of the form $f(x) = 0$ where $f(x)$ is a polynomial in x.

*31. Sketching of the graphs of higher degree polynomials.

*32. Approximating the irrational roots of higher degree equations, preferably by the method of interpolation.

*33. Arithmetic progressions.

*34. Geometric progressions, both finite and infinite.

35. Properties of the relation of equality.

36. Properties of the relation of inequality.

The concepts marked with asterisk were those ordinarily studied in advanced (college) algebra. Students who had had an understanding of all concepts except those so marked, began with college algebra and trigonometry as their first mathematics courses in college. These students took more than the minimum time to complete any of the engineering curricular.

An additional twenty-one algebraic concepts were listed as supplementary concepts. They are:

1. Extraction of square roots.
2. Binomial theorem with fractional and negative exponents.
3. Permutations.
4. Combinations.
5. Probability.
6. Multiplication and division of complex numbers in polar form.
7. De Moivre's theorem.

8. Exponential form of a complex number.
9. Ordered pair form of a complex number.
10. Set element of a set, designation of a set by description and listing; set-builder.
11. Subset, proper subset.
12. Empty set, complement of a set.
13. Operations on sets: union and intersection.
14. Ordered pair of numbers, set of ordered pairs of numbers, cartesian set.
15. Open sentences, statements.
16. Relation as a set of ordered pairs of numbers.
17. Further development of the function concept, i.e., as a set of ordered pairs of numbers in which each element of the domain is paired with one and only one element of the range; inverse of a function.
18. Descriptive statistics: measures of central tendency and simple measures of dispersion.
19. Properties of a number field, examples of fields.
20. Circular functions of real numbers, certain inverse circular functions; viz., arc sin, arc cos, arc tan.

21. Derivative of a polynomial, inverse of a derivative.

Concerning these supplementary concepts the committee stated:

> Some topics are not sufficiently fundamental to be classified as indispensable. It is recommended however, that the subject listed below be studied if there is time available for the whole group or for individual students whose rate of learning warrants supplementary work. (52:12)

There would be little argument among mathematicians, that a good college preparatory algebra textbook program, should provide the necessary foundations for the development of an understanding of these fifty-seven concepts, which are hereafter referred to as Possible Essentials of College Preparatory Secondary School Algebra.

Textbook Content Analysis

In order to aid understanding as well as to give added meaning to the data compiled in this study, all analyses of content are made according to the age level of the pupils who would normally be using a particular textbook under the operational set-up of the program being represented.

Because of the subject limitation arbitrarily chosen for this study (as stated in Chapter I), only that portion of each series made up of the textbooks designated for use from the time the formal study of algebra is begun, until the child reaches college entrance age in the United States, or an equivalent age (the "O" level), in England, is included.

To collect data in a uniform manner, each textbook listed for use at a specific level was marked with a numeral which correspond to the age of the pupils normally using the particular book in the country for which it was published.

Each program (context of the textbooks) was then divided into sections. An arbitrary decision was made to separate each program into ten equal parts, regardless of the number of pages contained in any particular series. Each part, therefore, represented one-tenth (0.1) of the instructional program for a child in any specific country (190:83).

In the case of the English series (SMP), where the series (from 13 to the "O" level), is published in five volumes, the total number of pages for all five volumes was divided into ten parts as agreed upon originally. The same procedure was followed with the American textbooks, which contained work for two years of instruction in two volumes.

The total number of pages in both volumes was divided into ten equal sections.

Only volumes one, two and three of the Entebbe series were available. These three volumes make up one-half of the planned four year course. The other one-half (for secondary C three, and secondary C four) was expected to become available during summer 1967. The total number of pages in volumes one, two, and three was divided into five equal sections, thereby keeping the number of parts for each program equal to ten agreed upon originally.

A code was used when recording data on all charts comparing specific content of the various programs. A semicolon separates the numeral which tells the age of the pupil using the book from the numeral which indicates the tenth-part of the program where some content is first presented. By way of illustration, if a possible essential or developmental step initially appeared in the third- tenth part of a series normally used by twelve-year-olds, it would be assigned a classification code on the chart of (12;3), thus assuring uniformity of approach and maintaining consistency.

Examination Content Analyses

Since the purpose of this study was to identify the "Essentials" (so that no important concept was excluded), and to compare these essentials with concepts in the Entebbe Program, a system of exhibiting the relationship between essentials in the English program and essentials in the United States program seemed desirable.

Five individuals, two members of the faculty of the school of education of the University of Southern California, two secondary school mathematics teachers, and the assistant supervisor of secondary school mathematics, of the Los Angeles City Schools, cooperated in the classification of the Essentials. These individuals were selected because of their knowledge of, and interest in, secondary mathematics education. Their occupational classifications and professional qualifications are presented in Appendix C. They are hereafter referred to as jurors.

A short questionnaire was designed to solicit the personal opinions of the five jurors, on the classification of the essentials. Their responses were requested on two items which were stated as follows:

1. To help provide an over-all picture of this list of Essentials of Secondary School Algebra,

please classify each concept as "modern" or "traditional."

2. Classify each item in each set of examinations, with respect to the required abilities indicated. Please do not omit any item and do not give a dual classification to any item.

	Item Number
A. Understanding of Basic Concepts	
B. Computational Skill	
C. Ability of Application of Basic Concepts	

<u>Illustrations of Essentials</u>

A sample list of objectives, prepared by the National Council of Teachers of Mathematics was presented and used to illustrate (by textbook examples) the essentials.

<u>Comparison of Entebbe Materials with the Essentials</u>

Each Entebbe test item was tallied with its related concept in the Augmented List of Possible Essentials, to produce a graphical comparison of Entebbe Items and the Essentials.

Summary of the Chapter

To assure uniformity of approach and to maintain consistency, certain rules of procedure were followed.

1. A chart was developed listing the possible essentials (page 130), and space provided for recording data according to the age-part-of-series plan described.
2. As all three series did not include all of the same concepts, any concept appearing in any series, but not already one of the possible essentials, was added to the list forming the Augmented List of Possible Essentials.
3. Unless otherwise stated, the age-part-of-series notation indicates the time when the particular concept first appears in the series.
4. Where the development of a concept has been carried from one age level to another, or repeated as review material at the next level, only the time of the initial development was noted.
5. Where a concept does not appear in the series analyzed, a dash is used to indicate absence of data. In such cases the reader can conclude

that *if* the concept appears at all in the particular series, it does so at an earlier or later age-level.

6. A short questionnaire was designed to solicit the personal opinions of jurors, on the classification of essentials and the analysis of examination content.

7. Essentials were classified as modern or traditional, by jurors. The list of essentials was compared with the content of each examination, and each item was classified with respect to the required abilities suggested in the questionnaire.

8. Examples were used to illustrate the essentials.

9. Each Entebbe test item was compared with the essentials.

CHAPTER IV

SURVEY OF PHILOSOPHY, ADMINISTRATION AND ORGANIZATION

The differences in philosophy, administration and organization between the schools of England and those of the United States are great enough to affect very appreciably the courses and methods of teaching in the two countries. Yet, history contains many examples of international exchange of ideas, principles and practices in education. It is doubtful that one can find answers to educational problems by aping the schools of other nations, since all the detailed aspects which go to make up an educational system cannot be transferred intact from one environment to another.

The formal education given children reflects what a society values and needs; therefore, any educational system, in total or in part, must be viewed within the framework of the country it serves and with some understanding of the purposes and values of the prevailing social system. Thus, the study of foreign systems means a critical approach and a challenge to one's philosophy ~~and~~, therefore, a clearer analysis of the background and basis underlying the educational system of one's own country. With this in mind, the idea that making comparisons may be dangerous must be stressed. Erroneous conclusions are apt to be drawn due to one's inability to discriminate between those things which are truly comparable and those representing values of a particular society.

To focus attention on one aspect of education, in this case secondary mathematics, does not mean that other dimensions can be forgotten, and it is also true that if the secondary school mathematics program is to be seen in its proper perspective, an overview of the entire secondary school program is required.

England

The School Year

The school year begins in September, and continues until late in July. It is divided into three "terms" separated by vacations at Christmas and at Easter, each lasting about three weeks, and a summer vacation of six or seven weeks. Mid-term is usually marked by a long week-end of three or four days' duration. The average number of days of actual school attendance per year is 200.

Length of School Life

Full-time education is compulsory from the age of five, and it is customary for children to start school at the beginning of the term in which their fifth birthday falls. The school leaving age of fifteen is to be raised to sixteen in 1970-71. Pupils reaching fifteen between September and January may not leave before Easter, those reaching fifteen between February and August remain till the end of the summer term. Pupils may stay at school till the age of nineteen.

Administration

England has a national system of education that is locally administered. Although in law the execution by local education authorities of the national policy is under the control and direction of the Secretary of State for Education and Science, in practice the administration of education is a partnership conducted by consent between the Secretary of State, the authorities and the teachers and also, where their interests are affected, the various voluntary bodies. Another special feature is the degree of freedom enjoyed by local educational authorities and schools in the management of their affairs. The only statutory requirements laid down by the Education Act of 1944 are that the school day must begin with an act of corporate worship and that the curriculum must include religious instruction (35:1). In detail, therefore, there is much diversity of practice among schools (even among schools in the area of the same local authority) in the planning of the timetable, in the content of syllabuses of work, in the textbooks used and in the teaching methods adopted. On the other hand, there are also broad similarities of practice. These arise from the influences of professional opinion and of public examinations.

The Two Stages

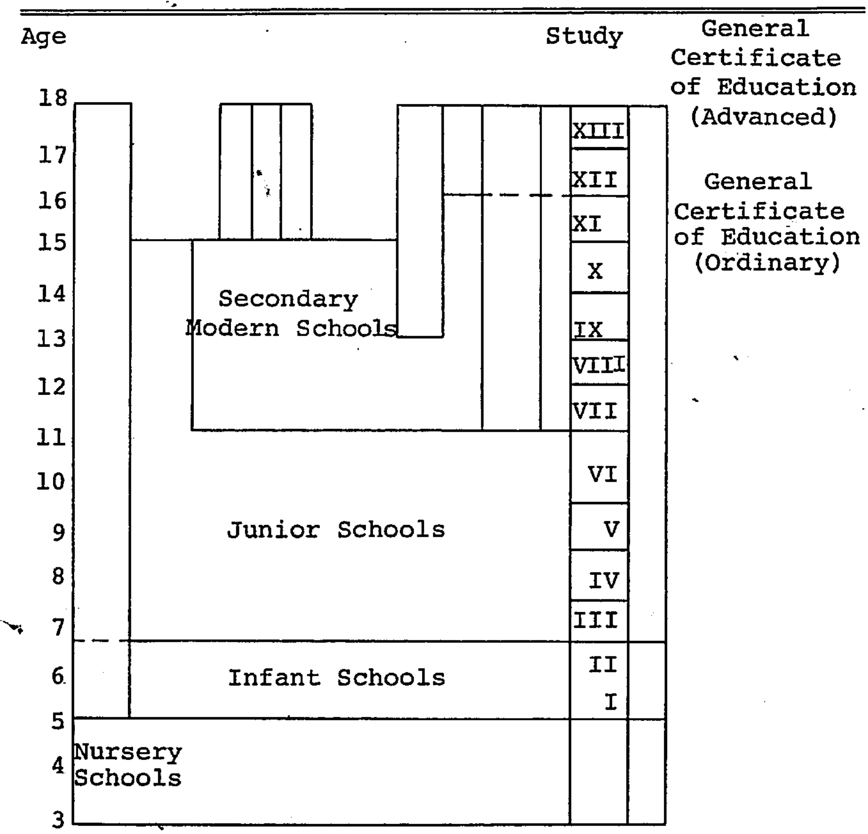
As in the United States, so in Britain, the period of full-time education is divided into two stages, "primary" and "secondary." In England, a child normally completes the primary stage at the end of the school year in which his eleventh birthday falls. This point corresponds approximately to the end of the American fifth grade (35:1). During this primary stage, a British child may attend one school, or particularly in urban areas, two, these being an infant school and a junior school. If he begins in an infant school, he will transfer to the junior school at the end of the school year in which he becomes seven years old. Upon leaving the junior school he goes to a secondary school of one type or another. The secondary stage is not, at present, usually characterized by the division, so often found in the American system, between junior high schools and senior high schools. However, the organization of secondary education is under review in many areas (35:2).

Secondary Schools

It is the duty of the local education authorities to provide secondary education in whatever form appears to be appropriate to the local circumstances. The differences between types of school are becoming less and less distinct but many authorities in England provide secondary education of two broad types, "grammar" and "modern," usually in separate schools. Some authorities also have selective secondary technical schools and bilateral and multilateral schools, which are nonselective and provide within a single institution all forms of secondary education; they are generally larger than most other types of secondary school. (See Table 1.)

TABLE 1

EDUCATIONAL SYSTEM IN ENGLAND[a,b]



[a]Adapted from: Edmund J. King, <u>Other Schools and Ours</u> (New York: Rinehart & Co., Inc., 1958), p. 71.

[b]Table shows the levels within the general organization of the English school system.

Each local education authority decides the method to be used in its area to determine which form of secondary education will best suit individual children, and this has hitherto usually taken the form of a test in the Spring of the year in which the children attain the age of eleven. It has become known as the 11+ examination because it affects the form of secondary education the child follows when he is over eleven years of age, and the same test is taken by most of the children in the authority's area who are due for transfer. The form of the examination has usually been such as to measure general aptitudes together with attainment in arithmetic and in English. Many authorities are, however, experimenting with other methods of selection, often involving much greater reliance on primary school assessment of the children's potentialities together with the results of tests taken over the last year or two of the child's primary school life.

Less than one-quarter of the children in England are subsequently allocated to secondary grammar or secondary technical schools, although the proportion varies from area to area. The remainder mostly attend secondary modern schools (35:3).

Grammar Schools

The curriculum of the secondary grammar school is broadly based at lower form levels but is specialized by the time pupils reach the sixth form, where the work is closely related to requirements for university entrance. These requirements are expressed in terms of a student's success in the public examination, set in England by certain university boards, known as the Examination for the General Certificate of Education, and often referred to as "the G. C. E." Students usually take the G. C. E. for the first time at about the age of sixteen when they offer for examination at Ordinary level most of the subjects they have been studying for the past four or five years. Those who remain at school for a further two years offer for examination at Advanced level the principal subjects they have continued to study during that time, usually three in number.

G. C. E. results are used to determine suitability not only for the universities but also for training colleges for teachers for certain courses in technical colleges and for certain professions.

About 17 per cent of grammar school pupils go on to universities, and well over one-half the university entrants in any one year are drawn from grammar school pupils (35:5).

Modern Schools

Secondary modern schools provide the broad general education required by the majority of students who leave school at fifteen. The subjects of the curriculum are the same as those for the grammar school except that the number of students who study a foreign language is small and more time is given to industrial arts for boys and home economics for girls. The teaching methods and the content of syllabuses are varied so that they match, as nearly as is practicable, the wide range of ability among the students. Age-groups are devided into forms according to ability and to a lesser extent, in the later stages according to choice of subjects.

In 1965, an examination with a nonacademic approach, known as the Certificate of Secondary Education, was introduced; it is designed particularly for sixteen-year-old students for whom the G. C. E. is unsuitable (35:6).

Other Types of Secondary Schools

There are still secondary technical schools in some areas, mainly in large towns. They are becoming more and more like grammar schools with a bias toward scientific and practical subjects.

There are also Independent Schools which provide for about 6 per cent of the total school population in England. They vary widely in type, from the small kindergarten school, often conducted in a private house, to the famous "Public Schools." The curriculum of public schools is very much the same as that of grammar schools and their students take the same public examinations.

The allowance of time for mathematics between the ages of eleven and fifteen years inclusive is 3½ to four hours weekly of instruction and about 1½ hours of homework. In the sixth form the time varies enormously between one school and another; it may be as little as seven hours weekly or as much as fifteen hours, according to the amount of science taken. The school week may consist of thirty-five periods each of forty-five minutes; in general, a teacher will be in action for about thirty of these unless he is head of a department. There are no official textbooks, as it is an accepted principle of education in England that the teaching staff of a school are the appropriate people to decide which are the best textbooks to use. The broad aim is to give each child an education suited to his age, aptitude and ability, without inquiring too closely just what fulfills these conditions.

For mathematics and languages the students are frequently "re-set" into differently graded classes, to allow for the differences in aptitude which pupils manifest for mathematics or languages compared with literary subjects.

Mathematics is taught as far as possible as a unified subject with algebra and geometry, starting in the first year and taught concurrently, usually by the same teacher (81:388).

United States

The United States exemplifies a decentralized system of education. Although there is a central national office of education--the United States Office of Health, Education and Welfare--each state is sovereign in regard to its educational program, thereby determining the length of compulsory schooling, making its own school laws, setting standards for teacher training, and so on. Because of the decentralization of American education, it is difficult to show that a "system" of secondary education exists in the United States.

The usual age of beginning school in the United

States is six years, though kindergartens exist in many places for children under that age. The legal minimum number of years for school attendance range from eight years in seven of the fifty states, to nine, ten, eleven and twelve in other states. Most children in the United States attend school for about twelve years, and a considerable portion continue for longer periods. (See Table 2.)

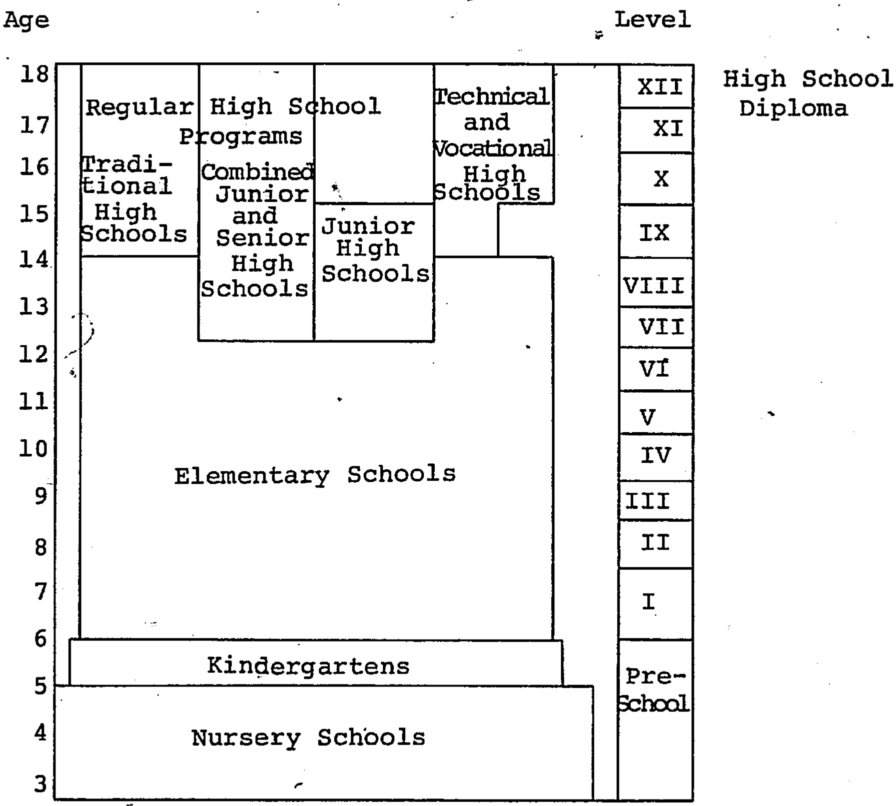
The number of actual days of school attendance is 156 for a national average; however, many of the states require 180 to 190 days (123:124). The schools are in session five days per week for, from five and one-half to six hours per day.

The traditional four-year high school enrolls about 18 per cent of all the secondary pupils, the divided junior-senior high school about 50 per cent, and most of the remainder are in undivided junior-senior six-year schools. Approximately 90 per cent of all fourteen to seventeen year olds are enrolled in some type of secondary school.

Present day American thought conceives of secondary education as a program concerned with all adolescent youth. Secondary education is understood to encompass the period of life beginning with the onset of adolescence and

TABLE 2

EDUCATIONAL SYSTEM IN THE UNITED STATES[a,b]



[a]Adapted from Edmund J. King, Other Schools and Ours (New York: Rinehart & Co., Inc., 1958), p. 109.

[b]Table shows the levels within the general organization of the United States school system.

continuing through the post-adolescent period--an age range from about twelve to twenty. Educationally, this means from the beginning of the seventh grade through the junior college or through the sophomore year in the university. Thus junior colleges are frequently discussed as essentially secondary in nature, although they cover the first two years of university work.

The comprehensive high school is a distinctive feature of American secondary education. It includes in its offerings all types of special education, combined in the same school. All students take the required core course, such as English, social studies, science, and mathematics; as electives they choose the courses required to meet the needs of commercial, technical, agricultural or home economics training. There is a basic belief that it advances the democratic ideal to include all secondary students in the same high schools. This approach has gone far to solve the problem of parity of esteem for the various courses, which is of concern today to secondary education in England.

Before an analysis of the mathematical content is made, certain general differences between school systems must be considered. In general, students preparing to go to college in the United States have: (1) an average age of eighteen after twelve years of school (not including kindergarten), (2) a nine-month school year, (3) a five day week, (4) one period of mathematics a day (35-50 minutes).

Secondary School Mathematics

As a result of the report of the Commission on Mathematics of the College Entrance Examination Board in 1959 and the experimental curriculum projects such as the School Mathematics Study Group, the program in secondary school mathematics is now generally as follows:

> School Year 7. Study of an informal structure of whole number; the algorisms for computation; the positive rational numbers; applications to percent, distance, area, and volume; informal geometry including constructions; and introduction to algebra.
>
> School Year 8. Study of an informal structure of the rational numbers; the pythagorean theorem; irrational numbers; the real number line; negative numbers; solution of simple equations and inequalities; finite number systems; graphs and statistics; indirect measurement and numerical trigonometry; introduction to deduction and proof.
>
> School Year 9. A year of study of elementary algebra from a more formal study of number systems; sets and operations; operations on polynomials and rational expressions; solution of equations and inequalities; use of deduction and proof.
>
> School Year 10. A year of strong deductive axiomatic geometry, plane and solid, using the properties of real numbers; the introduction and use of rectangular coordinates.

School Year 11. A year of extended in algebra, including a formal study of the system of real numbers; linear, quadratic, rational, exponential, logarithmic, angle, and circular functions; inverse functions; the related algebraic solution of equations and inequalities and transformations of these functions.

School Year 12. Although the work of this year is not standardized, the course usually includes a continuation of the study of algebraic functions, limits and continuity, probability and statistical inference; matrix algebra, including simple vector spaces; and an extension of trigonometry and solid geometry. (67:36, 37)

For the abler students, the above program is completed by the end of School Year 11. The twelfth year is then usually devoted to a study of analysis, including analytic geometry and differential and integral calculus. These students then take the Advanced Placement Examination of the College Entrance Examination Board whereby they may receive college credit and/or advanced placement depending on the policy of the particular college. Changing college programs, particularly in the freshman year, have focused the attention of secondary schools on preparing their students to make the transition to college study.

Summary

National differences condition, to a large extent, the development of education. There are basic similarities and fundamental differences between English and American education. To know what these are and why they exist may be of decided importance as one reviews one's educational endeavors, especially if such educational endeavors (like the Entebbe Mathematics Program) are dependent on both the British and the American educational achievements.

CHAPTER V

SELECTED TEXTBOOKS AND EXAMINATIONS

The aim of the textbook is to provide the best approach to a given subject--best at least in terms of the opinion of those who are directly concerned with the instructional program, be it leading educators who are specialists in the field, or as is the case of many foreign nations, those who are in a position to dictate what should be included in a particular program.

The choice of mathematics textbooks is dependent upon regulations concerning school textbooks in general. In England, there is free choice of any available textbooks which seem best to meet the needs of the ongoing program of the school. In the United States, choices depend to a degree on a multiplicity of directives; the decision being made according to the rulings of the various states and local boards in an independent manner.

Basis for Selection

The validity of the results of a study like the present one, which utilizes the judgmental content in textbooks and examinations, depends largely on the validity of the analytic process itself. The criterion of usefulness was assumed to be a major one (but not the only one) in this study. Therefore, the selection of textbooks for this study was guided by the following criteria:

1. The need to select a series which represents the current point of view (one that is presently in use in the school system) concerning the teaching of algebra--a sound program which is not unduly influenced by "extreme" points of view.
2. A perusal of available secondary school algebra programs offered by the various educational publishers.
3. Data included in Roehr's study (161:25).
4. A consideration of contemporary programs with a modern point of view.

Textbooks and Examinations Included in This Study

England

Because many textbook series are available in England and the head teacher of any school has freedom of selection, it might seem invidious to single out the so-called best algebra series. The problem was solved with the cooperation of Dr. Bryant Thwaites who indicated two modern mathematics series now in experimental use in English schools. It should be noted that:

> Although there is a great deal of individual experimentation in England, it is a matter of regret that--apart from the S.M.P., only the Midlands Mathematical Experiments, under the direction of Mr. Cyril S. Hope of Worcester Training College, publishes such reports. (177:49)

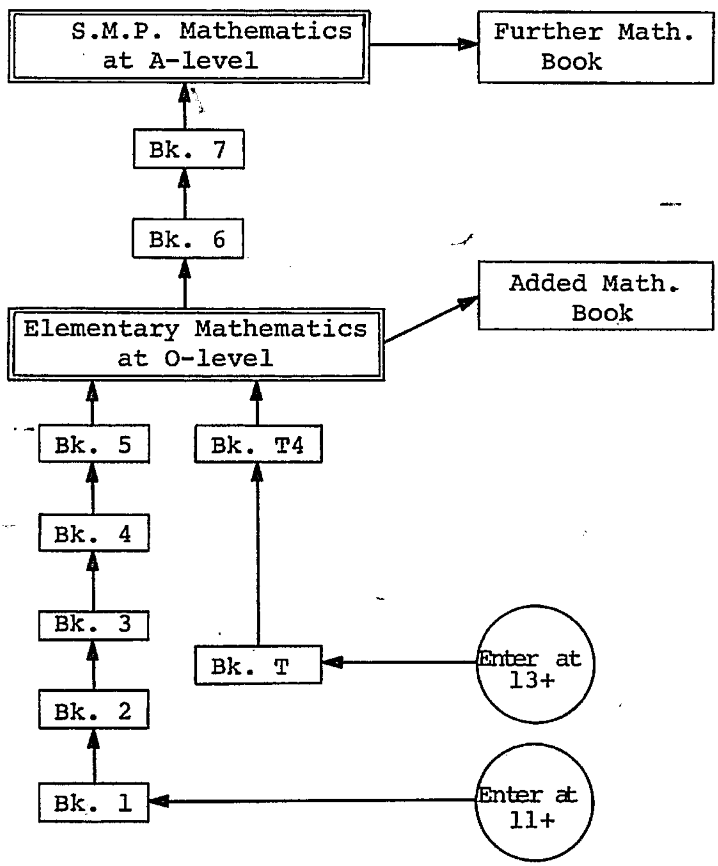
Furthermore the Southampton Mathematical Conference 1961 was held to bring together mathematicians from schools, universities and industry in order to consider problems of secondary mathematics education. This conference concluded that there is an urgent need for textbooks which present the subject from a modern point of view, and Dr. Thwaites observed that no member of the conference was able to name a single British school textbook which does so (31:30).

The present plan of the School Mathematics Project (S.M.P.) texts is as indicated in Table 3 (61:5).

The School Mathematics Project itself (Dr. Thwaites indicated) could be regarded as being a direct result of the 1961 conference. Books 1, 2, 3, and 4 were analyzed. Since Book 5 is not yet in print the outline provided in

TABLE 3

PRESENT PLAN OF THE SCHOOL MATHEMATICS PROJECT TEXTS[a]



[a]Adapted from: The School Mathematics Project. Director's Report, 1964/65.

the Director's Report 1964/65 (61:13) was used. The five books together form the complete O-level course.

A very complete and helpful teachers' guide for Book T4 provided a chapter-by-chapter commentary on the published version of Book T4 and a helpful direction in the analysis of the selected series.

Examinations

General Certificate Examination
Ordinary Level
School Mathematics Project
June, 1964

Elementary Mathematics 1-30 Questions (2½ hours)

Elementary Mathematics 2-10 Questions (2½ hours)

United States

The series chosen was one of several which might have been selected as "representative" of the secondary school algebra program (with a modern point of view) followed in the United States.

> The authors and editorial advisers have studied the recommendations of many groups, including the Commission on Mathematics of the College Entrance Examination Board (CEEB) and the School Mathematics Study Group (SMSG), which have sought to improve the mathematics programs in the schools. (49:3)

Thus specific textbook series were selected as representative of those being used in England and the

United States with a modern point of view. In each case the information and/or criteria given in the preceding section was used in making the selection.

Dolciani, Mary P., Berman, Simon L., and Freilich, Julius. Modern Algebra Structure and Method. Boston: Houghton Mifflin Co., 1965.

Book 1--used in the ninth school year by fifteen year olds, 551 pages.

Book 2--used in the eleventh school year by seventeen year olds, 605 pages.

A very complete and helpful teacher's guide accompanies each book in the series.

Examinations

Progress Tests to Accompany Modern Algebra Structure and methods.

Book 1--Comprehensive Test, Chapters 1 through 14. 49 Questions.

Book 2--Comprehensive Test, Chapters 1 through 5. 30 Questions.

Comprehensive Test, Chapters 6 through 9. 30 Questions.

Comprehensive Test, Chapters 10 through 12. 15 Questions.

Comprehensive Test, Chapters 13 through 16. 30 Questions.

The Entebbe Mathematics Series

Secondary I --Preliminary Edition
Student Text: Three volumes
Teachers' Guide: Three volumes

Secondary II --Preliminary Edition
Student Text: Three volumes
Teachers' Guide: Three volumes

Secondary III--Preliminary Edition
Student Text: Algebra--One volume
Teachers' Guide: Algebra--One volume

Secondary One is for the first year secondary school in which most pupils are about thirteen years old, Secondary Two for the next year, and so on. Age equivalents should not, however, be taken too literally, as many children do not begin school until after age six (41:30).

Examinations

African Education Study, Entebbe Mathematics.

Sample Secondary One Mathematics Test. 1964. 40 Questions.

Secondary One Mathematics Test No. 1A. 1964. 40 Questions.

Secondary One Mathematics Test No. 2A. 1964. 40 Questions.

Secondary Two Mathematics Test No. 1. 1965. 40 Questions.

Secondary Two Mathematics Test No. 2. 1965. 40 Questions.

Secondary Two Mathematics Test No. 3. 1965.
40 Questions.

Secondary Two Mathematics Test No. 4. 1965.
40 Questions.

At the time of writing these were the only tests available for the Entebbe series.

In each case the appropriate examination indicated by the author was selected for analysis because the expected treatment of the content of the syllabus may be inferred from the appropriate examination paper.

CHAPTER VI

ESSENTIALS OF SECONDARY SCHOOL ALGEBRA

Even a cursory appraisal of the secondary school mathematics textbook programs of England and the United States, is sufficient for the examiner to realize that certain topics are regarded as basic to a sound program. There are variations in the age (or grade) placement and sequence of the prescribed content, and in the actual concepts themselves. In fact, the scope and rigor of each program, up to any selected grade level, is determined by these differences.

Classification of Essentials

Appendix B presents data on 104 concepts (An Augmented List of Possible Essentials) from which thirty-five were selected as essentials, because they appeared in both the English and the United States Textbook series.

TABLE 4

COMMON CONCEPTS (ESSENTIALS) IN THE ENGLISH AND UNITED STATES SECONDARY SCHOOL MATHEMATICS TEXTBOOK PROGRAMS, CLASSIFIED AS MODERN (M), OR TRADITIONAL (T)

1. Ratio and proportion. (T)

2. Solution of problems involving measurements, e.g., addition of lengths expressed in feet and inches, calculation of areas and volumes, addition or subtraction of angles. (T)

3. Preparation and interpretation of statistical graphs; viz., bar, circle, and line.

4. Fundamental operations with polynomials. (T)

5. Common special products; viz., $a(b + c)$, $(a + b)(a - b)$, $(a \pm b)^2$, and $(a + b)(c - d)$, emphasizing the distributive law. (M)

6. Factoring; viz., $ab + ac$, $a^2 \pm 2ab + b^2$, $a^2 - b^2$, $ax^2 + bx + c$ based on the distributive law. (M)

7. Solution of a system of linear equations. (T)

8. Determinants, their evaluation by minors, and their use in solving systems of linear equations. (T)

9. Variation, direct, inverse and joint.

10. Function and functional notation. Representation of a function by a table of corresponding values, by a graph, and where possible by an equation or verbal statement. (M)

11. Properties of a linear function; viz., rate of change, graph, slope, and y-intercept of the graph.

TABLE 4--Continued

12.	Quadratic polynomials in one variable, location of maximum or minimum, nature of roots, expressions for the sum and product of the roots of a quadratic equation.		
13.	Scientific notation or standard-form of numbers, e.g., 2.54×10^3, 1.2×10^{-4}.		
14.	Principles of computation with logarithms.		
15.	Solution of exponential and logarithmic equations.		
16.	Finding the rational roots of higher degree equations of the form f(x) = 0 where f(x) is a polynomial in x.	(M)	
17.	Permutations.		(T)
18.	Combinations.		(T)
19.	Probability.		
20.	Sets, element of a set, designation of a set by description and listing set-builder.	(M)	
21.	Subset, proper subset.	(M)	
22.	Empty set, complement of a set.	(M)	
23.	Operations on sets: union and intersection.	(M)	
24.	Ordered pair of numbers, set of ordered pairs of numbers, cartesian set.		
25.	Open sentences, statements.		
26.	Descriptive statistics: measures of central tendency and simple measures of dispersion.		

TABLE 4--Continued

27.	Equations, Inequalities, and Problem Solving.		
28.	Inequalities and special graphs.	(M)	
29.	Quadratic Inequalities. Relations between roots and coefficients.		
30.	Graphing quadratic Relations.		
31.	Evaluating and applying Trigonometry functions.		
32.	Vectors and Matrices.	(M)	
33.	Matrix Algebra, matrices and transformation.		
34.	Frequency distributions. The histogram, normal distribution, cumulative frequency curve.	(M)	
35.	Radicals and exponents.		(T)
36.	Percentage problems.		(T)
37.	Different number bases.	(M)	
38.	Domain and Range of definition of a function.	(M)	
39.	Absolute values.	(M)	
40.	Solution of verbal problems by Algebraic methods.		
41.	Solution of linear equations having numerical and/or literal coefficients.		

Each concept in Table 4 was classified as modern (M), or traditional (T), as indicated in Chapter III. The very pertinent question, how this minimum list can be used, is later discussed in Chapter VII. Although such a collection of concepts does not provide adequate information as to the required student's understandings, nevertheless, by an analysis of appropriate tests, one can gain some additional insight into the area of student's understanding of the processes involved.

In Table 5, the test items are classified into three categories, (A) multiple-choice form, (B) completion form with simple numerical values or algebraic variables for answer and (C) completion form with some written work.

TABLE 5

CLASSIFICATION OF ENGLISH AND UNITED STATES TEST ITEMS WITH RESPECT TO SUGGESTED ANSWER FORMS A, B, AND C

	England		United States	
	Frequency	%	Frequency	%
A. Multiple-choice	5	10	15	25.3
B. Filling in with numerals or algebraic variables	13	26	5	8.3
C. Written work	32	64	40	66.4

All of the questions in the Entebbe series were multiple choice questions. On the contrary, two out of three questions in the English and American series were of the completion form.

In Table 6, the items are classified according to whether their questions are based on (A) the student's understanding of basic mathematical concepts, (B) the student's computational skill, or (C) the student's ability to apply basic mathematical principles. In the case of a lack of consensus on the classification of any item, that item was not classified.

TABLE 6

CLASSIFICATION OF ENGLISH, UNITED STATES AND ENTEBBE TEST ITEMS WITH RESPECT TO REQUIRED ABILITIES

	England		U. S.		Entebbe	
	Freq.	%	Freq.	%	Freq.	%
A. Understanding of basic concepts	9	18	19	31.6	34	28.2
B. Computational skill	3	6	7	11.7	3	2
C. Ability of Application of basic concepts	7	14	9	15	13	10.8
D. No unanimous opinion	31	62	25	41.7	70	58.5

In Tables 5 and 6, the test items are classified with respect to (1) the form of answer required of the student, and (2) the abilities required to answer the questions. It is interesting to note that 62 per cent of items in the English tests, 41 per cent of items in the United States series and 58 per cent of the test items in the Entebbe series were not classified because the jurors lacked unanimous opinions in each instance. More discussion considering the contents of these items will be presented in the comparison of the essentials.

Comparison of Essentials

Major emphasis in curriculum construction and evaluation is usually placed on the objectives of instruction. Therefore the above evaluation of test items mirror some of the many aspects of learning in the classroom, such as the relative emphasis placed on understanding, reasoning, and problem solving. Figure 1 is a bivariate distribution of essentials in the English and in the United States programs according to the age-part-of-series plan described in Chapter III.

Figure 2 is a matching of points (a matching of concepts), using both axes of Figure 1, with the axes

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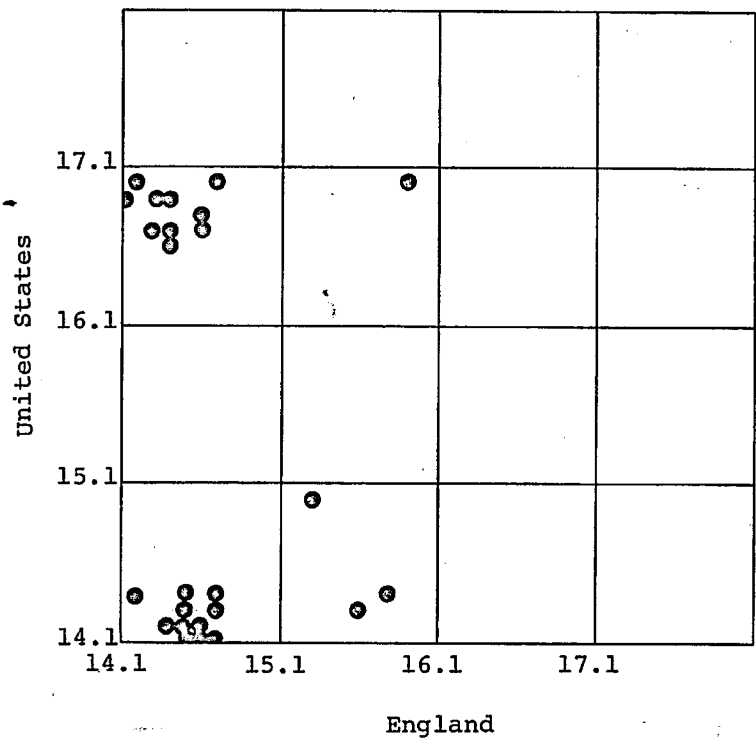


Figure 1.*--A bivariate distribution of the essentials.

*Any interpretation of this graph must be made with the understanding that full-time education is compulsory in England from the age of five, and that the school leaving age is fifteen.

The usual age of beginning school in the United States is six years, though kindergartens exist in many places for children under that age. The legal minimum number of years for school attendance range from eight years in seven of the fifty states, to nine, ten, eleven and twelve in other states. Most children in the United States attend school for about twelve years.

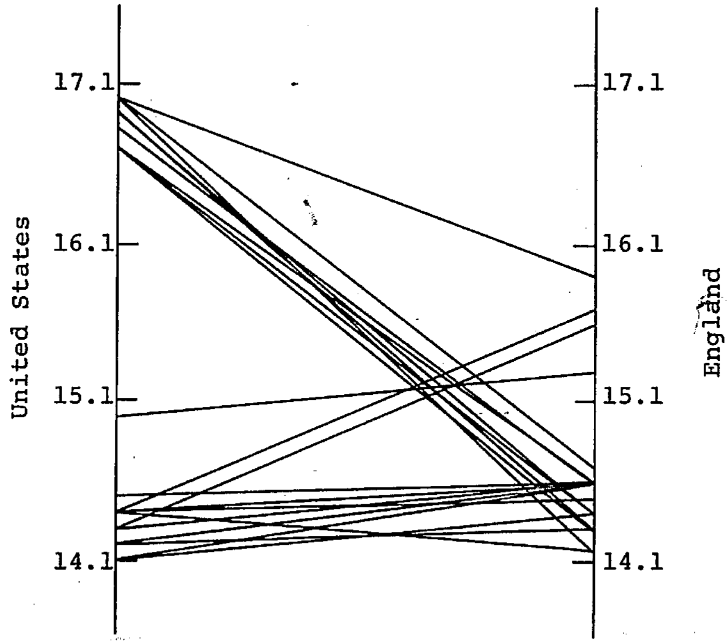


Figure 2.--A two-way matching of common algebraic concepts (a matching of the essentials).

placed in parallel as an aid to visual comparison. It is a two-way matching of common algebraic concepts, and more specifically it is a matching of the essentials.

It may be observed that Figures 1 and 2 present exactly the same data. Both graphs present comparable pictures of the essentials at a quick glance, and each is a reinforcement of the other.

Since the available Entebbe series represented only half of the entire series (as indicated in Chapter V), it was not to be expected that the entire list of essentials presented in Table 4, could be identified in the partial Entebbe series that was studied. To provide a comparative perspective of the essentials and the Entebbe program, Table 7 presents common concepts observed in the three series, and Figure 3 exhibits a comparison of these concepts at a glance.

By illustrating the varied types of items devised as samples of each concept, it was hoped that the reader will perceive the variety of information a student can obtain, if careful construction and judicious use is made of items representing the essentials.

TABLE 7

SOME CONCEPTS COMMON TO THE ENGLISH, AMERICAN AND ENTEBBE SERIES

1.	Ratio and proportion.	(T)
2.	Preparation and interpretation of statistical graphs; <u>viz</u>., bar, circle, and line.	
3.	Fundamental operations with polynomials.	(T)
4.	Common special products; <u>viz</u>., $a(b + c)$, $(a + b)(a - b)$, $(a \pm b)^2$, and $(a + b)(c - d)$, emphasizing the distributive law.	(M)
5.	Factoring; <u>viz</u>., $ab + ac$, $a^2 \pm 2ab + b^2$, $a^2 - b^2$, $ax^2 + bx + c$ based on the distributive law.	(M)
6.	Solution of a system of linear equations.	(T)
7.	Function and functional notation. Representation of a function by a table of corresponding values, by a graph, and where possible by an equation or verbal statement.	(M)
8.	Properties of a linear function; <u>viz</u>., rate of change, graph, slope, and y-intercept of the graph.	
9.	Quadratic polynomials in one variable, location of maximum or minimum, nature of roots, expressions for the sum and product of the roots of a quadratic equation.	
10.	Principles of computation with logarithms.	
11.	Solution of exponential and logarithmic equations.	

TABLE 7--Continued

12.	Finding the rational roots of higher degree equations of the form f(x) = 0 where f(x) is a polynomial in x.	(M)
13.	Sets, element of a set, designation of a set by description and listing set-builder.	(M)
14.	Subset, proper subset.	(M)
15.	Operations on sets, union and intersection.	(M)
16.	Ordered pair of numbers, set of ordered pairs of numbers, cartesian set.	
17.	Open sentences, statements.	
18.	Equations, inequalities, and problem solving.	
19.	Inequalities and special graphs.	(M)
20.	Graphing quadratic relations.	
21.	Evaluating and applying Trigonometry functions.	
22.	Solution of verbal problems by Algebraic methods.	
23.	Solution of linear equations having numerical and/or literal coefficients.	

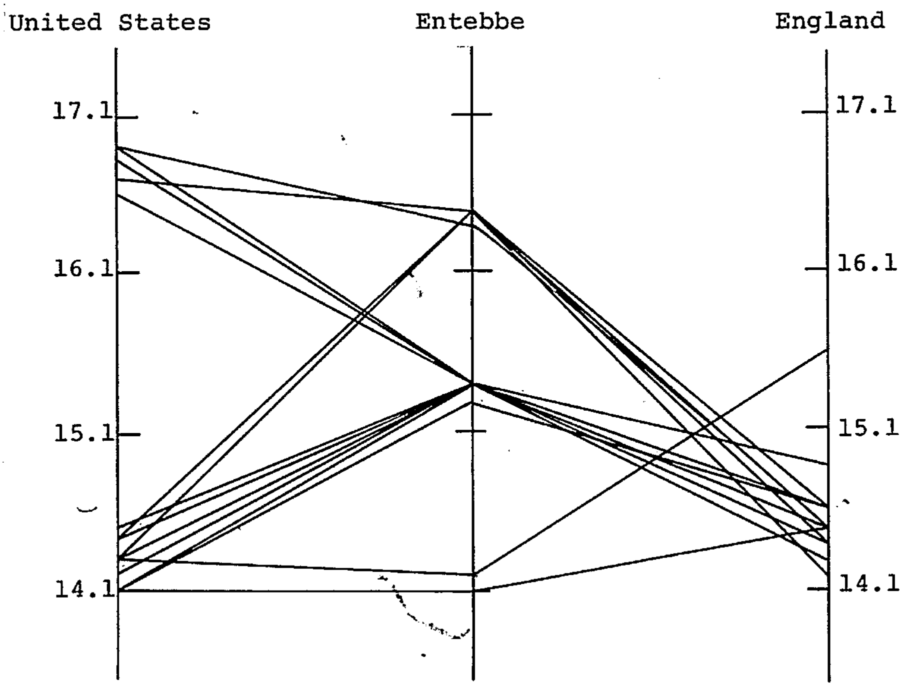


Figure 3.--A three way matching of common algebraic concepts in the English, United States and the Entebbe programs.

Illustration of Essentials

Most teachers of mathematics will agree that there are many outcomes that may be expected from a study of mathematics. It has been the point of view of this chapter that if such a wide range of objectives exists, then the role of the essentials in curriculum construction and evaluation, must be mirrored in "the Objectives of Instruction." Since many sets of objectives may be developed; there is of course, no commonly accepted list. However, in order to have a framework of reference, a sample list of objectives, prepared by the National Council of Teachers of Mathematics (22:72) is presented below.

The student should:

...have a knowledge and understanding of mathematical processes, facts, and concepts;
...have skill in computing with understanding, accuracy, and efficiency;
...have the ability to use general problem-solving technique;
...understand the logical structure of mathematics and the nature of proof;
...use mathematical concepts and processes to discover new generalizations and applications;
...recognize and appreciate the role of mathematics in society;
...develop study habits essential for independent progress in mathematics;
...develop reading skill and vocabulary essential for progress in mathematics;
...demonstrate such mental traits as creativity, imagination, curiosity, and visualization;
...develop attitudes that lead to appreciation, confidence, respect, initiative, and independence.

The first five of these objectives are used as categories for this illustration of the general level and character of knowledge expected. No preference is implied for any specific illustration.

Although a single illustrative item may cover a number of objectives and sample a variety of concepts, no attempt is made to cover all possible objectives . . . an impossible task . . . or all of the essentials identified in this study. These items are intended to be illustrative only.

Illustration I: (School Mathematics Project, Book 3, Part 3, pages 70 and 71)

Objective: ...have a knowledge and understanding of mathematical processes, facts, and concepts.

<u>Three Important Identities in the Algebra of Numbers</u>

a. <u>An identity for $(a + b)^2$</u>

Figure 4 will remind you of the reason for believing that $(a + b)^2 = a^2 + 2ab + b^2$.

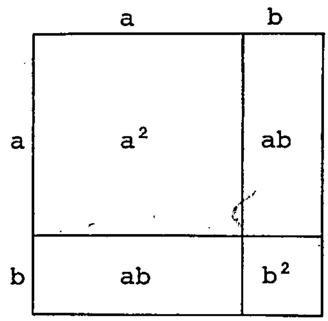


Figure 4

Example

Calculate 2.005^2

$$\begin{aligned} 2.005^2 &= (2 + 0.005)^2 \\ &= 2^2 + 2 \times 2 \times 0.005 + 0.005^2 \\ &= 4 + 0.02 + 0.000025 \\ &= 4.020025. \end{aligned}$$

For most purposes 4.02 or 4.020 would be quite accurate enough. The final term can then be omitted altogether.

b. An identity for $(a - b)^2$

In Figure 5 a square of side a has had two strips of width b marked on it. The area of the square outlined is therefore $(a - b)^2$. It may also be considered to be formed from the original square with the two strips of area ab removed. As this means that the shaded square, of area b^2, has been taken off twice, it has to be added on again so $(a - b)^2 = a^2 - 2ab + b^2$.

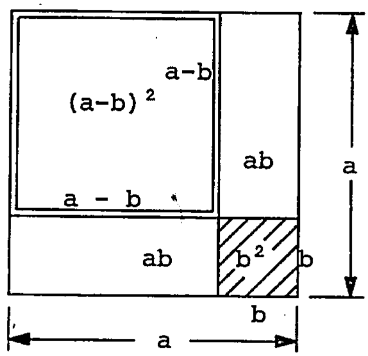


Figure 5

Example

The distance in ft. fallen by a dropped stone in t sec. is given approximately by the formula $d = 16t^2$. A stone is dropped down a pit shaft and the thump comes $4\frac{4}{5}$ sec. later on a stop watch. How deep is the pit?

$$d = 16 \times (4\tfrac{4}{5})^2$$

$$16(5 - \tfrac{1}{5})^2$$

$$16(5^2 - 2 \times 5 \times \tfrac{1}{5} + \tfrac{1}{5^2})$$

$16(25 - 2)$ approximately, (ignoring $16 \times \frac{1}{5^2}$).

368.

The pit is about 370 ft. deep, two significant figures being quite enough in view of the inaccuracies of both the formula and the measuring.

c. An identity for (a + b)(a - b)

Figure 6 will remind you of the reason for believing that $(a + b)(a - b) = a^2 - b^2$.

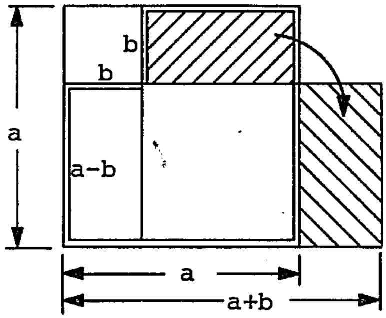


Figure 6

Illustration II: (School Mathematics Project Specimen page for Book 1. Chapter 14, page 250)

Objective: ...have skill in computing with understanding, accuracy, and efficiency.

Flow Diagrams

Simple flow diagrams were used to illustrate the method of using an addition slide rule. Flow diagrams are also useful in arithmetic problems. They provide representation of the plan that goes through your head before you start to calculate.

Example

A girl buys 3 bars of chocolate costing 6½d. each. She has 4s. 7½d. in her purse. If humbugs cost 10½d. per quarter, how many quarters can she buy?

This is a fairly simple problem, and the thought processes can be set down in a simple flow diagram as shown in Figure 7.

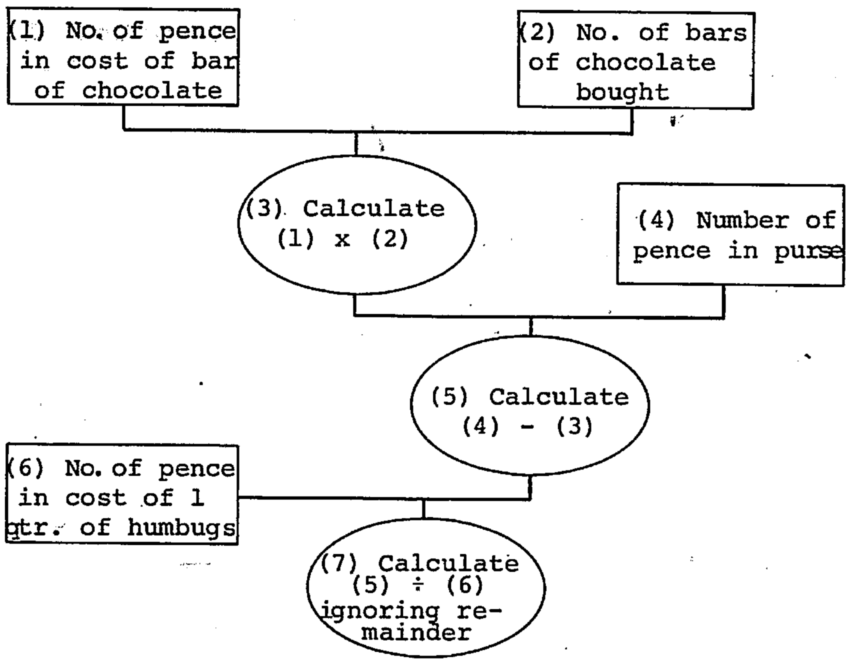


Figure 7

Illustration III: (School Mathematics Project. Book 3, Part 3, pages 64-65)

Objective: ...have the ability to use general problem-solving technique.

<u>Inside the Function Machine</u>

The function f: $x \rightarrow 3x + 4$ can equally be described by the relation $x' = 3x + 4$, where x' is the member of the range corresponding to a member x of the domain, that is the input, x' is the output. We say x' is the <u>subject</u> of this relation (compare the use of the word in grammar). Figure 8 shows you the inside of a function machine, f, engaged in mapping $x \to x'$.

<u>How the Inverse Machine Works</u>. The inverse machine f^{-1} takes x' and maps it back onto x by applying the operations inverse to mult. by 3 and add 4 in the reverse order. It is shown in action in Figure 9.

There is no need to draw the machine, of course. In fact one will find it most convenient to lay it out as in the example below as a <u>flow diagram</u>. When one has had plenty of practice the instructions can be omitted from the flow diagram and, later, one will find that one can write down f^{-1} without needing to write out f.

<u>Example</u>

Find the function inverse to $x \to \frac{x}{4} - 3$.

The corresponding relation is $x' = \frac{x}{4} - 3$.

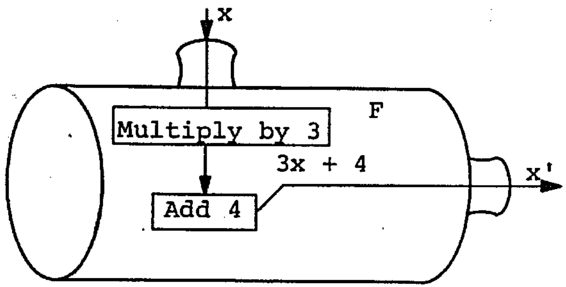


Figure 8

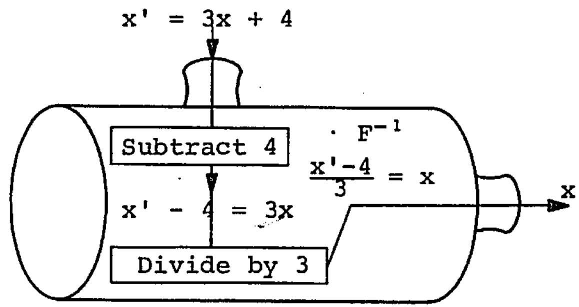
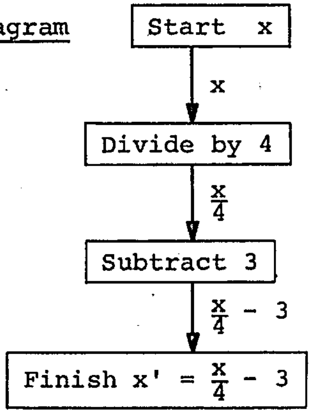
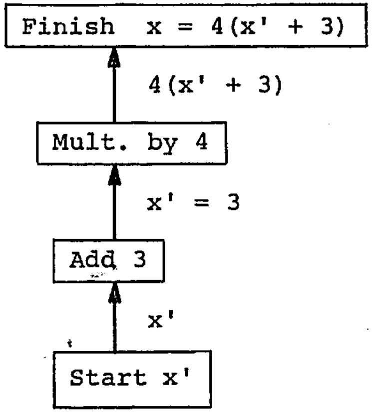


Figure 9

Flow diagram for f



Flow diagram for f^{-1}



This gives $x' - 4(x' + 3)$ or $x - 4(x + 3)$.

Illustration IV: (School Mathematics Program. Book 3, Part 1, pages 20-21)

Objective: ...understand the logical structure of mathematics and the nature of proof.

Summary

Letters denote operations. If A, B are operations and P is a figure or object then A(P) means 'the operation A carried out on the object P'. BA(P) means 'the operation B carried out on the result of operation A on P'. The object P may be omitted. Some pairs of operations are commutative, that is, AB(P) = BA(P), but not all. All operations are associative, that is, (AB)C = A(BC).

One operation on the result of another is called the product of the operations.

The identity operation (I) leaves a figure unchanged.

An operation R combined with its inverse (written R^{-1}) leaves a figure unchanged, that is, $RR^{-1} = R^{-1} = I$ for all R.

In Figure 10 flags A and C are directly congruent, while flags A and B are oppositely congruent.

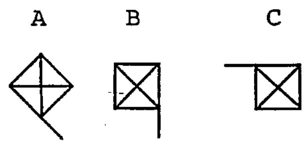


Figure 10

The possible single operations equivalent to products of operations are shown in Tables 8 and 9.

Because "The Nature of Proof" was not identified as one of the concepts in the list of essentials, no effort was made to illustrate it.

Illustration V: (Dolciani Book 2, pages 108-109)

Objective: ...use mathematical concepts and processes to discover new generalizations and applications.

<u>Extra for Experts</u>

<u>Linear Programming</u>. Simultaneous linear inequalities appear in decision problems in applied mathematics. Consider the following situation:

To decide how much wheat and corn to produce on his acreage, a farmer analyzed the requirements for producing each grain. He found that the production of 100 bushels of corn required 2.5 acres of land, $70 in capital, 2 hours of labor in August, and 2 hours of labor in September. To produce 100 bushels of wheat, he needed 5 acres of land,

TABLE 8

POSSIBLE SINGLE OPERATIONS EQUIVALENT TO PRODUCT BY TRANSLATIONS AND ROTATIONS

		Operation performed first		
		Translation	Rotation through Θ about P	Half turn about P
Operation performed second	Translation	Translation (or identity if $T_2 = T_1^{-1}$)	Rotation through Θ about point other than P	Half turn about point other than P
	Rotation through Φ about P	Rotation through Φ about point other than P	Rotation of Θ = Φ about P Identity if Θ = −0	Rotation of Φ + 180° about P Identity if Φ = 180°
	Rotation of Θ about Q	Rotation of Θ about point other than Q	Translation	Rotation of 180° − Θ about point other than Q

TABLE 9

POSSIBLE SINGLE OPERATIONS EQUIVALENT TO PRODUCTS OF REFLECTIONS

		Operation performed first
		Reflection in given line
Operation performed second	Reflection in same line	Identity
	Reflection in parallel line	Translation through twice distance between lines
	Reflection in perpendicular line	Half turn about point of intersection of lines
	Reflection in line at angle θ	Rotation of 20 about point of intersection of lines

$50 in capital, 4 hours of labor in August, and 10 hours of labor in September. Available to him were 100 acres of land, $2,100 in capital, 200 hours of labor in August, and 160 hours of labor in September. If 100 bushels of corn brought a return of $150 and 100 bushels of wheat $250, how should he have divided his production between corn and wheat to make the dollar return as large as possible? The data of the problem are arranged in Table 10.

TABLE 10

DATA FOR LINEAR PROGRAMMING PROBLEM

	Input Requirements Per 100 Bushels		Available Material
	Corn	Wheat	
Land (acre)	2.5	5	100
Capital ($)	70	50	2,100
Aug. labor (hr.)	2	4	200
Sept. labor (hr.)	2	10	160
Value of output of 100 bushels ($)	150	250	

Let x = the number of hundreds of bushels of corn produced;

y = the number of hundreds of bushels of wheat produced.

If R denotes the total return in dollars, then R = 150x + 250y.

The farmer had to maximize R (find its largest value) subject to these inequalities (constraints):

1. $2.5x + 5y \leq 100$
2. $70x + 50y \leq 2100$
3. $2x + 4y \leq 200$
4. $2x + 10y \leq 160$

} The total amount of each input cannot exceed the material available.

5. $x \geq 0$
6. $y \geq 0$

} The farmer cannot produce a negative number of bushels of either grain.

The graph of the intersection of the solution sets of these inequalities is shown as the shaded region in Figure 11 and is called the feasible region, because the coordinates of each of its points satisfy all the constraints. The boundary of the graph is called a convex polygon, and the intersection itself is called a convex set. Because the constraints, as well as R, are linear in x and y, this is called a linear programming problem.

For any value of R, such as R = 3000, the graph of the return equation $3000 = 150x + 250y$ is a straight line. For a greater value of R, say R = 7500, the graph of the return equation, is a line parallel to the graph of R = 3000, but with a larger y-intercept. Different values of R give a family of parallel lines each having y-intercept $\frac{R}{250}$.

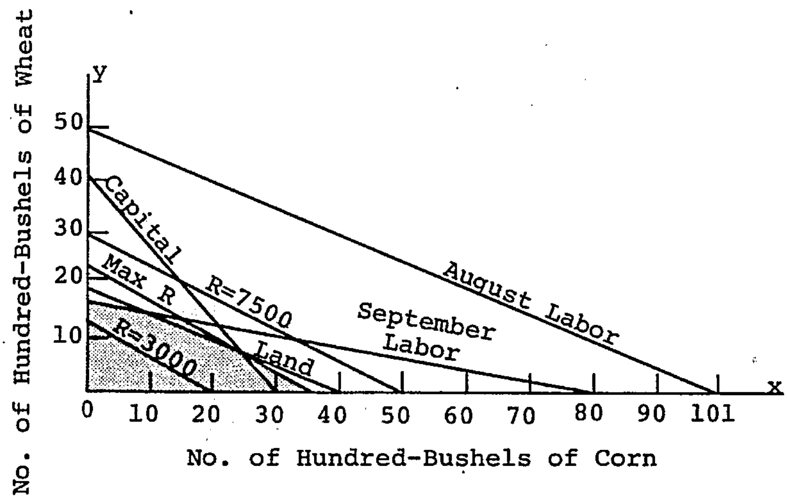


Figure 11

As the Figure suggests, the line of the family having maximum y-intercept and, therefore, maximum R, and containing a point of the feasible region must intersect the region at a vertex. It can be shown in general that whenever a linear expression such as $150x + 250y$ is evaluated over a convex set, it must take on its maximum value at one of the vertices of the polygon bounding the set, and its minimum value at another vertex. By substitution one finds that the expression for R takes on the following values at the vertices, whose coordinates are found by solving simultaneously the equations of adjacent sides of the polygon.

Curiosity about this fact may easily lead students to investigate further.[1]

Thus, over the feasible region the maximum for R is $5611\frac{1}{9}$, and this occurs at the point C. The minimum for R is 0 and occurs at the origin; that is, when he plants nothing. Consequently, to maximize his return, the farmer should have produced $\frac{22,000}{9}$ or about 2,444 bushels of corn and $\frac{7000}{9}$ or about 778 bushels of wheat. His return is then about $5,611.11. (See Table 11.)

This topic provides excellent motivation for the study of linear systems because it is of current importance in applications and is responsible for much research activity.

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[1]See Mary P. Dolciani, Simon L. Berman, and Julius Freilich, Modern Algebra Structure and Method, Book 1 (Boston: Houghton Mifflin Co., 1965), pp. K, L (following page 356).

TABLE 11

SIMULTANEOUS EQUATIONS FOR THE LINEAR PROGRAMMING PROBLEM INDICATING R (THE TOTAL RETURN IN DOLLARS) AND V (THE VERTICES OF THE CONVEX SETS

Vertex	150x + 250y	R
0(0,0)	150(0) + 250(0) = 0 + 0	0
A(0,16)	150(0) + 250(16) = 0 + 4000	4000
$B\left(\frac{40}{3},\frac{40}{3}\right)$	$150\left(\frac{40}{3}\right) + 250\left(\frac{40}{3}\right) = 400\left(\frac{40}{3}\right)$	$5333\ \frac{1}{3}$
$C\left(\frac{220}{9},\frac{70}{9}\right)$	$150\left(\frac{220}{9}\right) + 250\left(\frac{70}{9}\right) = \frac{50,5000}{9}$	$5611\ \frac{1}{9}$
D(30,0)	150(30) + 250(0) = 4500 + 0	4500

Summary of the Chapter

In this chapter, forty-one concepts of secondary school algebra were identified as essentials because they appeared in both the English and the American textbook series, analyzed in this study. These essentials were subsequently classified as modern or traditional by a jury of individuals, who were experienced mathematics educators. Thirty-four per cent of the essentials were identified as modern.

A study was made of the features of appropriate examinations. It was observed that 64 per cent of the items in the English series, and 66 per cent of those in the American series, were of the completing form with some written work. All of the items in the Entebbe series were of the multiple choice variety. Furthermore, 62 per cent of the items in the English series, 42 per cent of the items in the American series, and 58 per cent of the items in the African series were not classified with respect to required abilities, because the jurors were of divided opinions.

Graphical comparisons of the common elements in each series were subsequently constructed, and illustrations of the essentials, based on objectives of instruction were presented.

Illustration I feature the following essentials:

2. Solution of problems involving measurements, e.g., addition of lengths expressed in feet and inches, calculation of areas and volumes, addition or subtraction of angles. (T)

3. Preparation and interpretation of statistical graphs; viz., bar, circle, and line.

27. Equations, inequalities, and problem solving.

35. Radicals and exponents.

40. Solution of verbal problems by algebraic methods.

41. Solution of linear equations having numerical and/or literal coefficients.

Illustration II feature the following essentials:

27. Equations, inequalities, and problem solving.

37. Different number bases. (M)

40. Solution of verbal problems by algebraic methods.

Illustration III feature the following essentials:

4. Fundamental operations with polynomials. (T)

5. Common special products; viz., $a(b + c)$, $(a + b)(a - b)$, $(a \pm b)^2$, and $(a + b)(c - d)$, emphasizing the distributive law. (M)

6. Factoring; viz., ab + ac, $a^2 \pm 2ab + b^2$, $a^2 - b^2$, $ax^2 + bx + c$ based on the distributive law. (M)

10. Function and functional notation. Representation of a function by a table of corresponding values, by a graph, and where possible by an equation or verbal statement. (M)

37. Different number bases. (M)

40. Solution of verbal problems by algebraic methods.

41. Solution of linear equations having numerical and/or literal coefficients.

Illustration IV feature the following essentials:

10. Function and functional notation. Representation of a function by a table of corresponding values, by graph, and where possible by an equation or verbal statement.

25. Open sentences, statements.

37. Different number bases. (M)

40. Solution of verbal problems by algebraic methods.

Illustration V feature the following essentials:

2. Solution of problems involving measurements, e.g., addition of lengths expressed in feet and inches, calculation of areas and volumes, addition or subtraction of angles.

3. Preparation and interpretation of statistical graphs; viz., bar, circle, and line.

5. Common special products; viz., $a(b + c)$, $(a + b)(a - b)$, $(a \pm b)^2$, and $(a + b)(c - d)$, emphasizing the distributive law. (M)

6. Factoring; viz., $ab - ac$, $a^2 \pm 2ab + b^2$, $a^2 - b^2$, $ax^2 + bx + c^2$ based on the distributive law. (M)

7. Solution of a system of linear equations. (T)

10. Function and functional notation. Representation of function by a table of corresponding values, by a graph, and where possible by an equation or verbal statement. (M)

11. Properties of a linear function; viz., rate of change, graph, slope, and y-intercept of the graph.

20. Sets, element of a set, designation of a set by description and listing set-builder. (M)

21. Subset, proper subset. (M)

22. Empty set, complement of a set. (M)

23. Operations on sets; union and intersection. (M)

24. Ordered pair of numbers, set of ordered pairs of numbers, cartesian set.

25. Open sentences, statements.

27. Equations, inequalities, and problem solving.

28. Inequalities and special graphs. (M)

37. Different number bases. (M)

40. Solution of verbal problems by algebraic methods.

41. Solution of linear equations having numerical and/or literal coefficients.

CHAPTER VII

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

This study was made because of a basic difficulty facing teachers and administrators of secondary college preparatory mathematics. The difficulty stems from the fact that the amount of mathematical knowledge is rapidly expanding in every direction. The discovery of new mathematics coupled with its many applications to science, engineering and other human endeavors, have dictated college curricula changes, and are forcing a re-examination of the content of secondary college preparatory mathematics. This applies in particular, to secondary mathematics in foreign college preparatory institutions, whose students seek college and university training in British and American institutions. This study was made to discover a digest of at least the minimum content expected of secondary college preparatory algebra programs so that teachers and administrators can have adequate specific information as they prepare their students for professional education. Then, based upon these essentials, the research of others and the objectives of secondary mathematics education, a comparison of the essentials was made, illustrations presented, and implications for the Entebbe mathematics program outlined.

The Problem

The basic problem is stated in the following question: What are the areas of agreement or disagreement in secondary school algebra in England and the United States? In this respect, it was necessary to find what concepts formed the body of prevailing secondary school algebra programs in England and the United States, and to compare these concepts. It was necessary, also, to determine how these concepts could be related to the Entebbe mathematics program.

Procedure of the Study

A review of official documents and reports was made to determine the status of secondary mathematics. The same sources were searched for recent trends in the curriculum, and the weight of opinion on the content of the curriculum. A review was made of dissertations relating to modern (experimental) and traditional secondary mathematics, and to the content of the curriculum. Additional dissertations on concept learning in mathematics were studied. Sources in the literature of comparative education were examined for findings in secondary mathematics related to England.

Three recent textbooks series that are pioneering efforts to bring modern mathematics into the secondary curriculum (one from the United States, one from England, and one from the Entebbe program) were selected. The content of these textbooks was analyzed in terms of basic mathematical concepts. A list of essentials was determined by selecting common concepts in both the United States and the Entebbe series. Graphical comparisons of the essentials were made, to indicate the extent of agreement or disagreement in sequence of presentation. The concepts of modern mathematics in this list of essentials were identified. Appropriate examination papers were analyzed and their items compared with the list of essentials. A sample list of objectives, prepared by the National Council of Teachers of Mathematics was presented and used to illustrate the essentials. Subsequently, a graphical comparison of Entebbe items and the essentials was constructed.

Based upon the findings in the foregoing part of the study, implications for the Entebbe mathematics program were drawn.

Summary of the Findings

In the course of the study leading up to the comparison of Entebbe items and the essentials, the following findings were obtained:

1. The English report (Teaching Mathematics in Secondary Schools) condemned questions concerning the order of fractional operations.
2. Comprehensive lists of formulae and definitions and other mathematics tables are provided for students taking "O-level" and "A-level" mathematics examinations.

Hypothesis 1: That there are areas of agreement (essentials) in secondary school mathematics.

Table 4 lists such essentials and suggests that 35 per cent of these essentials may be classified as modern mathematics. Figures 1 and 2 are comparable illustrations of these essentials at a glance. Thus, the first hypothesis was supported.

Hypothesis 2: That there are procedural variations in some of the essentials between programs.

Figure 3 indicates the different age placement of the essentials in different programs. The illustrations of the essentials in Chapter VI shows some of the procedural variations in those concepts that were illustrated. Thus, the second hypothesis was supported.

<u>Hypothesis 3</u>: That the Entebbe Mathematics Program recognizes some of the essentials.

Table 7 lists twenty common concepts in the English, United States, and Entebbe Series, and Figure 3 presents a three way age placement matching of these common concepts. Thus, the third hypothesis was supported.

Hypothesis 4: That in the areas of disagreement between the English and United States programs, the Entebbe program has a greater number of elements of the American program than of the English.

From Appendix B, the Augmented List of Possible Essentials it is observed that:

1. Forty-eight per cent of Entebbe test items were identified in the English textbook series.
2. Forty-eight per cent of Entebbe test items were identified in the United States textbook series examined.
3. Forty-eight per cent of Entebbe test items were identified in the list of Essentials.

In a strict sense, this hypothesis was not supported.

Along with comparing English and United States textbook programs with Entebbe program in general, the data included in Tables 5 and 6 show that:

1. Two out of three items in the English and American tests were of the completion form.
2. All of the questions in the Entebbe series were multiple choice questions.
3. Eighteen per cent of the English test items required the student's understanding of basic mathematical concepts.
4. Sixty-two per cent of the English test items lacked unanimous interpretation, and were left unclassified.
5. Thirty-two per cent of the United States test items required the student's understanding of basic mathematical concepts.
6. Forty-two per cent of the United States test items lacked unanimous interpretation, and were left unclassified.
7. Twenty-eight per cent of the Entebbe test items required the student's understanding of basic mathematical concepts.

8. Fifty-eight per cent of the Entebbe test items lacked unanimous interpretation and were left unclassified.

Finally, it is observed that the English program provided:

1. O-level examination tables for students. They include comprehensive lists of formulae and definitions.
2. A-level examination tables for students published in March, 1966, they include comprehensive lists of formulae and definitions.

Conclusions

The findings which have been established in this study warrant the following conclusions:

1. That there are areas of agreement (essentials) in secondary school mathematics that should occupy an increasingly important place in curricula construction and evaluation.
2. That there are procedural variations in some of the essentials between programs. Such variations as the age-placement of concepts suggests possible investigative material for determining optimum placement of the essentials.

3. That the Entebbe Mathematics Program recognizes some of these essentials, and guided by expert opinions, presents them in the language of the people.

4. That in the areas of disagreement between the English and United States programs, the Entebbe program showed no difference.

5. That the English and United States test items probe more often for students' understandings than does the Entebbe test items.

6. That some test items are defiant to classification as either modern or traditional, possibly because of the different definitions of individual jurors.

7. That some test items cannot be classified with respect to ability required to solve the problem, possibly because of the wide coverage of concepts in such items.

8. That some items are advisedly left out of the curriculum. Examples of such items are:

   a) complicated fractions which not only constitute an unnecessary burden on the weaker pupils but have not the saving grace of being useful to the future specialist (202:62).

b) Groups of fractions such as

$$2\frac{1}{8} \times 1\frac{3}{4} + \frac{7}{8} \text{ of } 3\frac{1}{3}$$

are still widely taught, with unofficial rules for the resolution of the ambiguity.

Concerning such questions on the order of fractional operations, Her Majesty's Commission also stated that:

> It is preposterous that such questions (even with whole numbers instead of fractions) appear in secondary school selection tests and consequently in the work of junior schools. Such an example could not occur in practice because no one could risk being misunderstood. (202:50)

9. Some items are unamimously considered of value. Such an item is statistics whose value for all secondary school pupils is no longer in doubt, and programs like the English, the United States and the Entebbe programs introduce it early, but in a form which encourages effort to interpret sets of figures and assess the validity of other interpretations, instead of merely doing routine calculations. Informal ideas of probability also appear and lead to simple numerical problems.

10. Some attention must be given to the A-level tables which were used in A-level examinations in England in 1966, because both of the reports (201 and 202) which were summarized in Chapter II, indicated that one major objective of secondary mathematics education is, that at all levels of instruction, more emphasis should be placed upon pupil discovery and reasoning, reinforced by greater precision of expression.

Like the O-level tables, both tables are novel in that they include comprehensive lists of formulae and definitions. If the burden of memorizing can thus be eased, the examinations can, it is hoped, better test the candidates' range of understanding. A copy of the Elementary Tables is the content of Appendix E.

Teachers readily test such things as speed in mechanical arithmetic, memorizing of formulae and procedures, but seldom the ability to recognize a problem, to transcribe it in mathematical form and to transpose the mathematical form into a practical program, all of which are necessary in assessing mathematical ability. The use of such tables mean that the teacher's methods and approach are very much affected.

Recommendations and Implications

In view of the findings and conclusions of this study, and the objectives that were previously stated, the following recommendations and implications were made:

1. That teachers or any personnel who advise students on education and career choices, apply the list of essentials to direct the study of those who contemplate entering a college or university. Since many related fields (such as science, mathematics, and economics) require about the same high school preparation in mathematics, it is expected that college preparatory algebra will contain many students for whom the list can serve as a guide.
2. American teachers, it seems, prefer to emphasize the logical aspect and make abstract definitions at an earlier stage, and keep away from certain concepts, such as vectors, until the pupils are mature enough to understand their definition in abstract terms. This may make the emphasis ideal for gifted pupils.

3. The trend in England seems to be away from emphasizing the logical aspect and making abstract definitions at an early age. Progress all the time is from practical illustrations by generalizations to the building of simple-systems; but the systems are not set out in abstract form until quite a late stage. The emphasis is on the operations and process rather than any strict logical arrangement until the logic emerges naturally.

4. The Entebbe program like the United States program emphasizes the structure of mathematics at an early stage. This is reflected in the careful development of mathematics as a deductive system. They stress unifying themes such as structure, operations and their inverses, measurement, graphical representation, systems of numerations, properties of numbers, the development of the real number system, and the language and notation of sets.

This list of essentials is a minimum list. It would be unfortunate if the list becomes the maximum content of any college preparatory program. The Illinois

Committee suggested that in order to provide the students with a thorough and broad understanding of mathematics, it is necessary to go beyond any list of essentials. The teacher should encourage each student to progress as far as his capabilities will allow. Stated in other terms:

1. The essentials will provide the teacher with an hierarchy of content from which he can outline activities as these items relate to those objectives which the teacher has chosen. This may mean that individual teachers can be helped to preplan for individualization of instruction.
2. For a student who is considering a college preparatory algebra program the list can be presented to illustrate the kind of knowledge, understanding and skill he must have to succeed.
3. For a foreign student, his parents, and the adviser should consider seriously and as objectively as possible the information available to them in the light of what is required for success.

The idea of a mathematics laboratory or measurement room suggested in the English report is in experimental practice in some United States schools. To draw on its advantages, it is hoped that for future teachers such accommodations will be forthcoming.

Findings of the first large-scale international comparison of how well students learn mathematics were reported in Phi Delta Kappan of April, 1967. It stated that Howard Fehr of Teachers College Columbia University made the following observations:

1. That in all countries a high correlation was found between high achievement in mathematics and the presence of male teachers in the classroom.
2. That U. S. teachers generally know less mathematics than teachers of the other countries studied.
3. That at the very top level in the production of scientists the U. S. does as well as any country in student mathematics achievement.
4. That there are two crucial factors in student mathematical achievement: (a) the curriculum--what do we want the child to learn? and (2) the teacher.

As one re-evaluates any system one real criticism is sure to be that of the topics to be taught and the re-shuffling of some of the older topics rendered less necessary by the development of more important and/or broader mathematical ideas, critically analyzed experimental studies, along with a critical analysis of wide observations of practice, are the more desirable bases for conclusions.

An answer then to the question, "What should be taught?" can be given only in terms of what is best for children in the country under consideration and not in terms of what can be taught, or what some other country includes in its program.

The American system, taken by and large, does tend to have the advantage in breadth while the Europeans may score with greater depth. The American child gets the depth later, so that by the time he has completed the secondary program the differences in scope are less great than one may have assumed.

Finally, it is recommended that more time should be devoted to improving the quality and quantity of secondary school mathematics teachers, because the success of mathematics teaching depends upon understanding and providing successful practical remedies for the difficulties that students encounter. More mathematical experiences should be introduced at an earlier age level . . . with both the theoretical and practical approaches to solving problems.

It must therefore be clearly understood that:

1. The major demands of any curricular revision fall on the shoulder of the teachers. In appreciation of this fact, the essentials may be utilized in a manner that makes reasonable demands on teachers, allowing them to implement the recommendations gradually as their training permits.

2. "Indicating Essentials of Secondary School Algebra," suggests, that resulting preparation of students in algebra for college placement ought to be a wiser one than if the preparation were made without considering the essentials.

Recommendations for Further Study

Any study uncovers matters which call for further study. The following recommendations for further study are made:

1. An identification of mathematical concepts essential for college preparatory students and the designing of sequences that can be adapted to the curriculum of different schools, the levels of ability of the students, the times available for the course and the special preferences of the instructor.

Such sequences should show the interdependence of the essentials, and should be usable to plan many other sequences or to introduce variations into any sequence chosen by a student. Content selection and placement in relation to the objectives of mathematics and the objectives of the student, then becomes the responsibility of the teacher.

2. Insofar as the Entebbe mathematics program is concerned, the implications of an extensive concept analysis points to the need for subsequent longitudinal studies in which there would be some continuity in the sequences followed by groups of students. Such longitudinal studies could be designed to provide comparative information after one, two, three and more years of study in prescribed sequences.

Each participating school system has need to evaluate the Entebbe materials in its own setting. This would also be true of all the other new mathematics programs.

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APPENDIX A

A DISCOVERY EXAMPLE

APPENDIX A

A DISCOVERY EXAMPLE

In the new curricula, students are encouraged to discover general laws and principles. As an example they may be asked to consider the following figures (68:27):

> In Figure 1, there are two symbols in the body of the table and a sign of operation in the corner. A means that an object is turned half way around (180°), and B means the object is turned clear around (360°). The → shows the order; A → B = A means the object is first turned 180° and then turned 360°, leaving the object in the same position as if turned 180°:

→	A	B
A	B	A
B	A	B

Figure 1

> When the order of turns is reversed, one discovers that:

$$A \rightarrow B = A$$

$$B \rightarrow A = A$$

$$\text{or } A \rightarrow B = B \rightarrow A$$

One sees, then, that the order does not affect the result--a half turn followed by a full turn leaves the object in the same position as a full turn followed by a half turn, and one knows that the commutative property applies for this operation. It can also be observed that:

$$A \rightarrow B = A$$

$$B \rightarrow B = B$$

or that B has the identity property. In this case, A and B do not represent numbers, and the operation represented by → is not one in arithmetic.

In Figure 2, there are again two symbols in the table and a sign of operation in the corner. In this case, the symbols, Δ and O, represent two different abstract ideas; they do not represent horses or dollars.

→	Δ	O
Δ	O	Δ
O	Δ	O

Figure 2

The → represents a rule that is expressed in the figure. That is, if Δ is paired with Δ, the result is O, which is written as Δ → Δ = O. A study of the table reveals:

$$\Delta \rightarrow O = \Delta$$

$$O \rightarrow \Delta = \Delta$$

$$\text{or } \Delta \rightarrow O = O \rightarrow \Delta$$

The order of operation does not affect the result, and this operation has the commutative property. Also, one sees that:

$$\mathrm{O} \nrightarrow \mathrm{O} = \mathrm{O}$$

$$\Delta \nrightarrow \mathrm{O} = \Delta$$

If the circle is used with the circle, the result is the circle; if it is used with the triangle, the result is the triangle. The circle, then, is the identity element with respect to this operation. Considering both figures, Figure 1 is concerned with moving an object, and Figure 2 is concerned with abstract ideas. Both have two properties in common. Figure 2 represents a miniature mathematical system; Figure 1 is a model or application of the system.

The properties of a mathematical system are fundamental and enduring; the models or applications change as the needs of the society change.

APPENDIX B

AUGMENTED LIST OF POSSIBLE ESSENTIALS

APPENDIX B

AUGMENTED LIST OF POSSIBLE ESSENTIALS

		U. S.	Entebbe	England	Frequency
1.	Ratio and proportion.	14; 3	14; 2	15; 6	16
2.	Interpolation.	14; 4			
3.	Measurement, common units of measure, precision of measurement, significant digits, and rounding.	16; 9	14; 1		2
4.	Conversion of units in a measurement of a physical magnitude.		14; 2		1
5.	Solution of problems involving measurements, e.g., addition of lengths expressed in feet and inches, calculation of areas and volumes, addition or subtraction of angles.	14; 2	14; 2	14; 3	21
6.	Scale drawing.				
7.	Constant, variable, parameter.	14; 1			
8.	Preparation and interpretation of statistical graphs; viz., bar, circle, and line.	14; 4	16; 5	14; 2	
9.	Signs of aggregation; viz., parentheses, brackets, braces, et al., and their use.	14; 1	14; 3		3
10.	Rational numbers; i.e., the integers and the fractions.	14; 1	14; 3		16
11.	Fundamental operations with rational numbers.	14; 1	14; 3		12
12.	Fundamental operations with polynomials.	14; 1	15; 4	14; 6	
13.	Fundamental operations with algebraic fractions.	14; 3	16; 5		
14.	Common special products; viz., $a(b + c)$, $(a + b)(a - b)$, $(a \pm b)^2$, and $(a + b)(c + d)$, emphasizing the distributive law.	14; 3	15; 4	14; 6	3
15.	Factoring; viz., $ab + ac$, $a^2 \pm 2ab + b^2$, $a^2 - b^2$, $ax^2 + bx + c$, based on the distributive law.	14; 3	15; 4	14; 6	
16.	Laws of exponents, including negative and fractional exponents.	16; 7	15; 4		1
17.	Solution of a system of linear equations.	14; 4	15; 4	14; 5	1
18.	Solution of linear equations having numerical and/or literal coefficients.	14; 2	15; 4		36
19.	Determinants, their evaluation by minors, and their use in solving systems of linear equations.	14; 10		15; 7	
20.	Variation, direct, inverse and joint.	14; 4		15; 7	1
21.	Function and functional notation. Representation of a function by a table of corresponding values, by graph, and, possible by an equation or verbal statement.	14; 2	15; 4	14; 6	7

APPENDIX B--Continued

		U. S.	Entebbe	England	Frequency
22.	Properties of a linear function; viz., rate of change, graph, slope, and y-intercept of the graph.	14; 4	15; 4	14; 6	8
23.	The quadratic equation: derivation of the quadratic formula; solution by formula and, where appropriate, by factoring.	14; 5	15; 4		
24.	Irrational numbers and fundamental operations with these numbers.	14; 4	15; 4		2
25.	Real numbers and fundamental operations with these numbers.	14; 1	15; 4		7
26.	Complex numbers and fundamental operations with these numbers.	16; 9			
27.	Quadratic polynomials in one variable, location of maximum or minimum by completing the square; nature of roots, and expressions for the sum and product of the roots of a quadratic equation.	14; 5	15; 4	14; 6	1
28.	Common quadratic equations in two variables.				
29.	Solution of a system of two quadratic equations.	16; 8	15; 4		
30.	Solution of verbal problems by algebraic methods.	14; 1	15; 4	15; 1	17
31.	Solution of equations in which the unknown occurs under a radical sign.	14; 4	16; 4		4
32.	Binomial theorem with positive integral exponents.	16; 10			
33.	Scientific notation or standard-form of numbers--e.g., 2.54×10^3, 1.2×10^{-4}.	16; 7	13; 1	14; 4	5
34.	Principles of computation with logarithms.	16; 9	15; 4	14; 3	
35.	Change of the base of logarithms.	16; 9	16; 4		
36.	Solution of exponential and logarithmic equations.	16; 9	16; 4	14; 4	
37.	Factor theorem.	16; 7	15; 4		
38.	Finding the rational roots of higher degree equations of the form $f(x) = 0$ where $f(x)$ is a polynomial in x.	16; 7	16; 5	14; 6	
39.	Sketching of the graphs of higher degree polynomials.	16; 8			
40.	Approximating the irrational roots of higher degree equations, preferably by the method of interpolation.				
41.	Arithmetic progressions.	16; 10	16; 5		
42.	Geometric progressions, both finite and infinite.	16; 10	16; 5		
43.	Properties of the relation of equality.	14; 1	15; 4		1
44.	Properties of the relation of inequality.	14; 1	15; 4		2
45.	Extraction of square roots. Geometric interpretation.	14; 4	14; 3		1

APPENDIX B--Continued

		U. S.	Entebbe	England	Frequency
46.	Binomial theorem with fractional and negative exponents.	16; 10			
47.	Permutations.	16; 10		15; 9	
48.	Combinations.	16; 10		15; 9	
49.	Probability.	16; 10		15; 9	
50.	Multiplication and division of complex numbers in polar form.	16; 9			
51.	De Moivre's theorem.				
52.	Exponential form of a complex number.				
53.	Ordered pair form of a complex number.				
54.	Sets, elements of a set, designation of a set by description and listing set-builder.	14; 1	14; 3	14; 4	10
55.	Subset, proper subset.	14; 1	14; 3	14; 4	1
56.	Empty set, complement of a set.	14; 1	14; 3	14; 4	4
57.	Operations on sets; union and intersection.	16; 6	14; 3	14; 4	5
58.	Ordered pair of numbers, set of ordered pairs of numbers, cartesian set.	14; 3	16; 5	14; 5	3
59.	Open sentences, statements.	14; 1	15; 3	14; 6	1
60.	Relation as a set of ordered pair of numbers.	14; 4	16; 5		
61.	Descriptive statistics; measures of central tendency and simple measures of dispersion.	16; 10		14; 2	
62.	Properties of a number field, examples fields.			16; 10	
63.	Derivative of a polynomial, inverse of a derivative.				
64.	Extending the number line, operating with directed numbers, absolute values, and directed numbers.	14; 2	14; 3		
65.	Equations, inequalities, and problem solving.	14; 2	16; 5	14; 5	25
66.	Inequalities and special graphs.	14; 4	16; 5	14; 5	3
67.	The concept of proof.	16; 6			
68.	Axioms for real numbers, and field properties.	16; 6	14; 2		13
69.	Radicals.	16; 8	15; 4		2
70.	Quadratic inequalities.	16; 8		14; 6	
71.	Coordinates and distance in a plane.	16; 8			
72.	Graphing quadratic relations.	16; 8	15; 4	14; 6	
73.	Evaluating and applying trigonometry functions.	16; 9	14; 3	14; 4	3
74.	Vectors and matrices.	16; 9		14; 1	
75.	Trigonometric identities and formulas.	16; 9			

APPENDIX B--Continued

		U. S.	Entebbe	England	Frequency
76.	Mathematics induction.	16; 10			
77.	The fundamental theorem of algebra.	16; 10			
78.	Matrix algebra; matrices and transformations.	16; 10		14; 7	
79.	The theorem of the factors of zero.		15; 4	14; 6	
80.	Graph of $y = 2^x$.		15; 4	14; 4	
81.	Asymptotes, symmetry; oblique, horizontal and vertical.		16; 5		
82.	Isometries; rotation, reflection, translation.			14; 1	
83.	Gradients, function and contour.			14; 1	
84.	Rate of change of functions.			14; 1	
85.	Topology (Euler's relation $F - E + V = 2$), simple networks.			14; 2	
86.	Connectivity.			14; 2	
87.	Matrices and networks.			14; 2	
88.	Duality with network examples.			14; 2	
89.	Computer and programming.			14; 2	
90.	Frequency distributions. Histogram, normal distribution, cumulative frequency curve.		16; 10	14; 3	
91.	Correlation, scatter diagrams, line of best fit.			14; 3	
92.	Shearing.			14; 3	
93.	Areas and matrices.			14; 3	
94.	Limits of accuracy and percentage error.		14; 3	14; 4	2
95.	Linear programming.			14; 5	
96.	Vectors in three dimensions.			14; 6	
97.	Inverse functions.			14; 6	
98.	Estimating areas.		14; 3	15; 8	1
99.	The trapezium rule.			15; 8	
100.	Fundamental trigonometry (measuring heights).			14; 5	
101.	Percentage problems.				3
102.	Absolute values.				4
103.	Domain and range of definition of a function.				4
104.	Different number bases.				11
105.	Solution of verbal problems by algebraic methods.	14; 1	15; 1	15; 1	17
106.	Solution of linear equations having numerical and/or literal coefficients.	14; 4	15; 1	14; 5	36

APPENDIX C

LIST OF JURORS

APPENDIX C

LIST OF JURORS

1. Mr. George Wilson
   Supervisor of Student Teaching
   Mathematics-Science
   U. S. C.

2. Mr. Hardie Boyce
   Mathematics Teacher
   Los Angeles City Schools

3. Mr. Leroy C. Pool
   Mathematics Department, Chairman
   Hamilton High School
   Los Angeles City Schools

4. Mr. Sidney Sharron
   Sec. Mathematics Consultant
   Los Angeles City Schools

5. Mr. Walter Markert
   Sec. Mathematics Consultant
   Teacher Education
   U. S. C.

APPENDIX D

ENTEBBE MATHEMATICS SERIES

ENTEBBE MATHEMATICS SERIES

Primary One
Revised Preliminary Edition
Pupil Book: One volume
Teachers' Guide: Two volumes

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Pupil Book: Two volumes
Teachers' Guide: Two volumes

Primary Three
Preliminary Edition
Pupil Book: Two volumes
Teachers' Guide: Two volumes

Primary Four
Preliminary Edition
Pupil Book: One volume
Teachers' Guide: One volume

Primary Five
Preliminary Edition
Pupil Book: One volume
Teachers' Guide: One volume

Primary Six
Preliminary Edition
To be written at 1967 Workshop

Primary Seven
Preliminary Edition
To be written at 1968 Workshop

Entebbe Mathematics Teachers' Handbook, Primary I-III, Preliminary Edition

Entebbe Mathematics Teachers' Handbook, Primary IV-VII, Preliminary Edition, to be written at 1967 Workshop

FIVE YEAR COURSE

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Preliminary Edition
Student Text: One volume
Teachers' Guide: Three volumes

Secondary Two
Preliminary Edition
Student Text: Three volumes
Teachers' Guide: Three volumes

Secondary Three
Preliminary Edition
Student Text: Algebra—One volume
Geometry—One volume
Teachers' Guide: Algebra—One volume
Geometry—One volume

Secondary Four
Preliminary Edition
Student Text: Algebra—One volume
Geometry—One volume
Teachers' Guide: Algebra—One volume
Geometry—One volume

Secondary Five
Preliminary Edition
Student Text: One volume
Teachers' Guide: One volume

FOUR YEAR COURSE

Secondary C One
Preliminary Edition
Student Text: Algebra—One volume
Geometry—One volume
Teachers' Guide: Algebra—One volume
Geometry—One volume

Secondary C Two
Preliminary Edition
Student Text: Algebra—One volume (same book)
Geometry—One volume
Teachers' Guide: Algebra—One volume (same book)
Geometry—One volume

Secondary C Three
Preliminary Edition
Student Text: Algebra—One volume (same book) — available May 1967
Geometry—One volume
Teachers' Guide: Algebra—One volume (same book)
Geometry—One volume

Secondary C Four
Preliminary Edition
Student Text: One volume
Teachers' Guide: One volume
(same books) — available summer 1967

Additional Mathematics (O level), Preliminary Edition, to be written at 1967 Workshop

Advanced Mathematics (A level), Preliminary Edition, to be written at 1968 Workshop

Basic Concepts of Mathematics, an Introductory Text for Teachers

Volume I —Revised Preliminary Edition—*Structure of Arithmetic*
Volume II —Revised Preliminary Edition—*Structure of Arithmetic*
Volume III—Preliminary Edition—*Foundations of Geometry*
Volume IV—Preliminary Edition—*Measurement, Functions, and Probability*

APPENDIX E

ELEMENTARY TABLES

PLEASE NOTE:

Appendix E, page 168
"The School Mathematics
Project, Elementary Tables"
not microfilmed at request of
author. This is available for
consultation at University of
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