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Numerical Solution for Hydromagnetic Steady Flow of Liquid Between Two Parallel Plates Under Applied Pressure Gradient When Upper Plate Is Moving With Constant Velocity Under the Influence of Inclined Magnetic Field

Research Report in Mathematics, Number 33, 2019

DAVID KARANJA KAGAI

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Submitted to the School of Mathematics in partial fulfilment for a degree in Master of Science in Applied Mathematics

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Master Thesis
Submitted to the School of Mathematics in partial fulfilment for a degree in Master of Science in Applied Mathematics

Submitted to: The Graduate School, University of Nairobi, Kenya

Abstract

The current paper explores the steady laminar flow of viscous in compressible fluid between two parallel infinite plates when a pressure gradient which is constant is imposed on the system and the top plate is also stimulating with continual velocity and lower plate is held stationery under the influence of magnetic field.

The guiding partial differential equation is changed into ordinary differential equation and the solved numerically using central difference method and analytically using laplace transformation method done by Singh,2014.

The effect of the arising important parameter on flow characteristics has been discussed through tables A comparative study has been taken into account between existing results and present work and it has been found to be in excellent harmony

The numerical expression for fluid velocity at a different inclination has been shown on the table and graph which shows that the increase of inclination of magnetic field there is a decrease in velocity profile.

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Declaration and Approval

I the undersigned declare that this dissertation is my original work and to the best of my knowledge, it has not been submitted in support of an award of a degree in any other university or institution of learning.

Signature Date

DAVID KARANJA KAGAI
Reg No. I56/8572/2017

In my capacity as a supervisor of the candidate's dissertation, I certify that this dissertation has my approval for submission.

Signature Date

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Dedication

This project is dedicated to my parents: Julius Kagai and Grace Mukami And lastly to my entire family

Contents

Acknowledgments

To start with, I thank the almighty God for taking me this far,conferring me with strength ,ability and opportunity to undertake this research study ,Without His blessing i wouldn't have achieved this .

Toward my journey of this degree i met a pillar of support in my guide Prof C.B.Singh,who gave endless guidance and encouragement throughout the entire course of this work and for ensuring i stay in line without deviating from heart of my research . Without his advice this research would not have succeeded and i shall forever be grateful to him for his backing.

I acknowledge my sincere recognition to the University of Nairobi through the School of Mathematics for the opportunity awarded to me to pursue this studies,without which, I could not have made to this far.

I take pleasure in recognizing the University of Nairobi lectures ;Dr.Charles Nyandwi , Prof.Ogana ,Prof Pokhariyal Ganesh ,Dr were for taking me through this course on various course unit

I have great pleasure in endorsing my obligation to my classmates at university of Nairobi;Collins Kiragu ,Sheila Kariaga,Pretty Cynthia Nyarara ,Lewis Kivuva,Flora Mwikali ,Julius Ngugi, Felix Murithi and Eric Ombasa in making sure that the fire kept burning and being available anytime i needed motivation and propelling me on this research . Their groundwork and dependable ideas have made great contribution in the entire thesis.

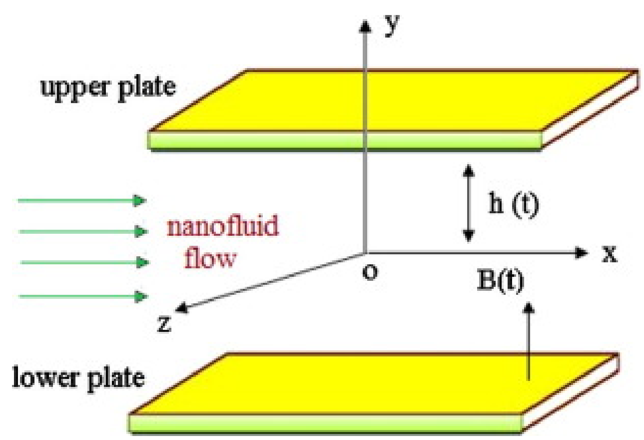
To my family members,I take this chance to express the deep gratitude from the bottom of my heart to my beloved parents,grandparents and my sibling for their love and continuous support both spiritually and materially

David Karanja Kagai

Nairobi, 2019.

1 Introduction

The outline of the thesis is as follows:
Magneto hydrodynamics (MHD) is a very long word for a very old principle ,the same principle that derive all of of our electric motors and electric generators .Effectively when a charge particle moves it is asssociated with electric field and by convention we use the right hand rule which state that "to find the direction of the magnetic force on a positive moving charge,the thumb of the fingers in the direction of b and the force F is directed perpendicular to the right hand palm". magnetic field can induce current in a moving conductive fluid such as plasma,ionized gases liquid metal.



Magneto hydrodynamics (MHD) is as a result of interaction of two branches namely electromagnetic theory and fluid dynamic. Assuming that steady flow condition has been attained,consider an electrically conducting fluid having a velocity vector V , at right angle to this we apply a magnetic field B .As a result of interaction of the two fields,electrically field denoted by E is induced at right angle to both V and B this electrically field is given by $E = V \times B$.
if we assume that the fluid is isotropic we can denote its electrical conductivity by the scalar quantity σ. By Ohms law,the density of the current J induced in the conducting fluid is given by $J{=}\alpha\ E$, i.e., for stationary condition. Simultaneously occurring with the induced current is the induced electromotive force $F{=}J \times B\ Sin\ \alpha$,

where α is an angle of inclination of magnetic field with the horizontal.

"A familiar model which is normally studied in this thesis consists of an infinitely long channel of constant cross - section with a uniform static magnetic field applied transverse to the axis of the channel. The walls of channel are either insulators, conductors or a combination of insulators and conductors depending on the intended application. For example, in the MHD generator and pump, the channel cross-section is normally circular with conducting walls"(Singh, 1993).

1.1 Recent research

Serclif (1956) has studied the steady motion of electrically conducting fluid in pipes under transverse magnetic field.

Drake (1965) has considered the flow in a channel due to periodic pressure gradient and solved by the method of separation of variables.

Singh and Ram(1977) considered the laminar flow of an electrically conducting fluid through a channel in the presence of a transverse magnetic field under the influence of a periodic pressure gradient and solved it by Laplace transform. Ram et al. (1984) have considered Hall effects on heat and mass transfer flow through porous medium.

Simonura (1991) considered magnetohydrodynamic turbulent channel flow under a uniform transverse magnetic field.

Kazuyuki (1992) discussed inertia effects in two dimensional MHD channel flow.

Pop and Watanabe (1995) considered the Hall effects on boundary layer flow over a continuous moving flat plate.

Ram(1995)discussed effects of Hall and ion slip currents on free convective heat generating flow in a rotating fluid.

Singh(1998)considered unsteady magnetohydrodynamic flow of liquid through a channel under variable pressure gradient and solved it by method of Laplace transform.

Singh (2000)considered unsteady flow of liquid through a channel with pressure gradient changing exponentially under the influence of inclined magnetic field and solved it by method of Laplace transform.

Singh(2014) has considered Hydro-magnetic Steady Flow of Liquid Between Two Parallel Infinite Plates Under Applied Pressure Gradient when Upper Plate is Moving with Constant Velocity Under the Influence of Inclined Magnetic Field and solved it Laplace transform method.

2 Chapter 2

2.1 Preliminaries

In this section, we will discuss properties of fluid,classification of fluid flow phenomena,dimensional analysis.

2.2 Definition

Fluid :A fluid can be defined as any substance that deform continuously when subjected to a shear stress

2.2.1 Properties of fluid

viscosity

This is a property of fluid by ideal of which a fluid will offer resistance to shear stress

Newtonian law of viscosity

This is just a fluid property by virtue of which a fluid offer resistance.

Non-Newtonian fluid

This are type of fluid that fails to follow newtonians viscosity law(i.e the fluid do not offer any resistance)
*viscocity will change subjected to force either on more liquid or solid.

Pressure

This is the Normal force exerted by a fluid per unit area

Density

This is the quantity of matter contained in a unit volume of a substance

Specific Weight

It is weight per unit volume

Specific gravity

This is the ratio of its weight to that of an equal volume of water at a specific temperature usually at 4°C

2.2.2 Types of force acting on a moving fluid

Inertia force

Inertia force is equal to the mass and acceleration of the moving fluid

$F_i = \rho A V^2$

Elastic force

Elastic force can be defined as the product of elastic stress and area of the flow

$F_e = KA$ **=Area x Elastic stress**

Viscous force

Viscous force can be said to be equal to the shear stress as a result of viscosity and surface area of the fluid flow.

$F_v = \tau \frac{du}{dy} A = \mu \frac{U}{d} A$

Pressure force

This is as a result of product of pressure intensity and flow area $F_p = pA$

Gravity force

Product of acceleration and gravitation force is said to be the gravity force
$F_g = \rho ALg$

Surface tension force

Surface tension is the product of of surface tension and the length of the surface of the flowing fluid
$F_s = \sigma$d

2.2.3 Classification of fluid flow phenomena

Steady and unsteady flow

"Steady flow can be defined as A flow whose flow state expressed by velocity, pressure, density, etc., at any position, does not vary with time,while unsteady flow is a flow whose flow state does vary or change with time is called an unsteady flow"(Nakayama,1998).

Lamina flow and turbulent flow

This can be described as which the flow follows a smooth path and also the path do not interfere with other, we refer this flow as lamina flow(particle flow along well defined stream line) while the flow which the flow is irregular is referred to as the turbulent flow

Uniform flow and Non uniform flow

Uniform flow can be defined as the flow in which velocity at any point does not change with respect to space while Non uniform flow is the flow which velocity changes at any given time with respect to time

Compressible Flows and In-compressible

Compressible flow is the flow which density vary at some point (density is not constant) incompressible flow the density is said to be constant at any point

Rotational flow and irrotational flow

Rotational flow is defined as flow in which fluid particle rotate on their own axis while flowing along a streamline

Irrotational flow is defined as flow in which fluid particle do not rotate on their own axis while flowing along a streamline or a path

2.3 DIMENSION ANALYSIS

Buckingham pi Theorem

Non-dimensional parameters are as a result of dimensional analysis for the experimental data of unknow flow problem.This non -dimensional parameter are reffered to as pi terms and may be be grouped and expressed in functional forms this was illustrated by Edgar Buckingham (1867-1940)

Buckingham pi theorem, states that if an equation involving k variables is dimensionally homogeneous, then it can be reduced to a relationship among (k -r) independent dimensionless products, where r is the minimum number of reference dimensions required to describe the variable. For a physical system, involving k variables, the functional relation of variables can be written mathematically as, $y = f(X_1 , X_2,X_3 ...., X_k)$

$$\pi_1 = \varphi(\pi_2, \pi_2, \pi_{k-r})$$

How to determine pi terms

Step 1

Write down all the variables involved in the problem defined and ensure they are all independent in nature to minimize variables needed to describe the structure

Step 2 Each variables should be expressed in terms of the basic dimension $(M, L, TORF, L, T)$

Step 3

"Apply the buckingham pi theorem which claims that the number of pi terms is equal to (k-r),where k is number of variables and r is the number of reference dimension required to describe the variables" (Watanabe, 1995)

Step 4 Select the variables which can be joined to lead to pi terms (ie duplicating variables equals the number of reference dimension)

Step 5 "The pi terms are formed by multiplying one of the non-repeating variables by the product of the repeating variables each raised to an exponent that will make the combination dimensionless.Usually denoted as x_i, x_1^a, x_i^b, x_3^c the exponents ab c , and are determined so that the combination is dimensionless"(Watanabe, 1995).

step 6 For the remaining non -repeating variables ,repeat step 5

step 7

Substitute the common dimension (M, L, T) of the terms which are not constant into the pi terms to ensure that all the pi terms are dimensionless

2.4 NON DIMENSIONAL NUMBERS IN FLUID DYNAMICS

DYNAMICALLY SIMILAR Two system are claimed to be dynamically similar when the several force acting on corresponding fluid element have the same ratio to one another in both system ,so that the path followed by the corresponding elements in the two system will be geometrically similar

GEOMETRICALLY SIMILAR Two system are dynamically similar when the ratio of corresponding length in the two system is constant so that one is scale model of the other

Let L be the characteristic length in the system under consideration ,for example the diameter of a pipe ,and t a typical time ,then the mass of an element is proportion to $\rho\ l^3$ and its acceleartion to $\frac{L}{t^2}$,so that

$$\text{inertial force } \alpha \text{ mass x acceleration}$$

$$\text{inertial force } \alpha \rho l^3 \text{ x } \frac{l}{t^2}$$

$$\text{inertial force } \alpha \rho l^2 \text{ x } (\frac{l}{t})^2$$

$$\text{since v} \alpha\ \frac{l}{t}$$

$$\text{inertial force } \alpha \rho l^2 v^2$$

REYNOLDS NUMBER,Re

Reynolds number is defined as the ratio of inertia force to that of the viscous force.This number is used for indication of whether a fluid flow through a body is turbulent
Thus for viscous resistance the requirement for dynamically similarity in the two system is equality of Reynolds number

PROOF

If the motion is controlled by viscous resistance ,the flow in the two system will be dynamically similar if both the ratio inertia force and the viscous force are similar

Viscous force α viscous shear stress x area

Viscous force α μ x velocity gradient x l^2

but velocity gradient α $\frac{v}{l}$
Viscous force α $\mu\frac{v}{l}$ x l^2

Viscous force α μ v x l^2

$$\frac{inertiaforce}{viscousforce} \alpha \frac{\rho vl^2}{\mu vl} \alpha \frac{\rho vl}{\mu}$$

FROUDE NUMBER,Fr

This dimensionless number is used for indication of the effect or the gravity influence on a fluid two system are dynamically similar for wave resistance if the froude number is the same for both system

proof

For wave resistance the resisting force is due to gravity

Gravity force = masss x gravitational acceleration

Gravity force α ρl^3 x g

$$\frac{inertiaforce}{gravityforce} \alpha \frac{\rho l^2 v^2}{\rho l^3 g}$$

$$\frac{inertia force}{gravity force} \alpha \frac{v}{\sqrt{lg}}$$

which is froude number

MACH NUMBER,Ma

For elastic compression of a fluid the elastic force is dependent on the bulk module k of the fluid
The Bulk modulus can be defined as the elastic properties of a liquid or solid when under pressure on both surface
The compressibility effects dynamical similarity is attained when the mach number is the same in both system

proof

Elastic force $\alpha\ K\ l^2$

Thus

$$\frac{inertia}{surface tension force} \alpha \frac{\rho v^2 l^2}{k l^2}$$

This is the mach number

WEBER NUMBER ,We

This is a dimensionless number that is used for analyzing the fluid flow in case an interface takes place between two different fluids. Weber number is used as the ratio between both the inertia force and the surface tension,and is used to indicate whether the surface tension is dominant for surface tension effect if T is the surface tension per unit length , surface tension force αTl

And

$$\frac{inertiaforce}{surfacetension}\ \alpha\ \frac{\rho v^2 l^2}{Tl}$$

HARTMANN NUMBER ,Ha

This is a dimension less number that is used to indicate the ratio between the magnetic viscosity and the ordinary viscosity in the flow (This is the ratio of magnetic force to the viscous force)

Its ussually denoted as

$$Ha= BL\sqrt{\frac{\sigma}{\mu}}$$

B is the magnetic field

L is the characteristic length scale

σ is the electrical conductivity

μ is the dynamic viscosity

Hartman number is usually encountered in fluid flow through a magnetic fields

2.5 Numerical Method

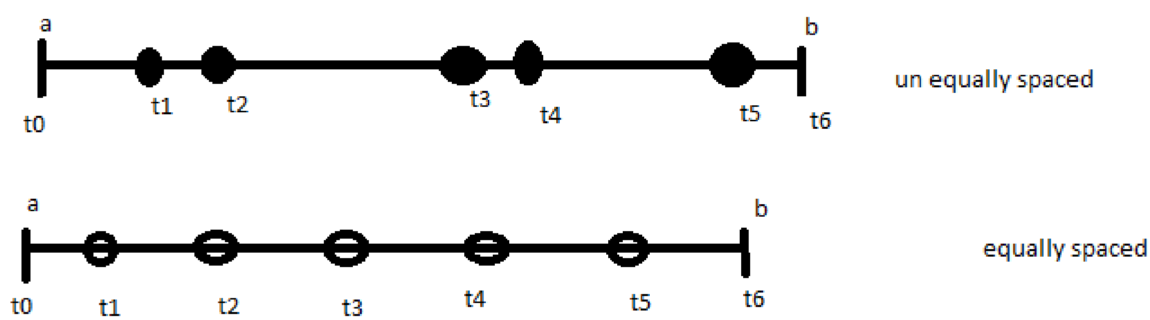
Central Difference Method

To develop a scheme for solving an equation such as

$$\frac{dy}{dt} = f(t,y); y(t_0) = y_0$$

For this equation to be solved numerically, we will have to make some assumption that
i) The solution exist
ii)The solution is unique

We seek a solution maybe in the interval [a,b]



let a be t_0 and divide this into some sub interval.This points are called mesh points or grid point .

The mesh point can be equally spaced, for example
$t_1 = t_0+h$
$t_2 = t_1+h$

The general rule is ; If you divide $[a,b]$into x sub intervals and $a = t_0$,then the mesh point are $t_0 t_1 t_2 ... t_n$.

Suppose we evaluate this equation at t_n

$$\left(\frac{dy}{dt}\right)t_n = f(t_n, y_n)$$

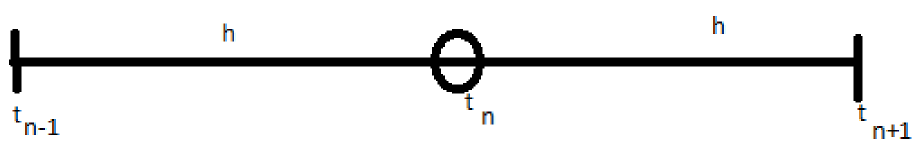
Approximate this and express derivatives in terms of t , t_n , t_{n-1} . This will help to get $y(t_n)$, $y(t_{n-1})$............

2.5.1 Single step

Single step :This involves using a one step backward or expressing the value at its immediate points.

2.5.2 Multiple step

Multiple step It involves expressing the value at its immediate multiple points Lets consider a typical mesh points t_n



Approximating the derivative at t_n, we consider a function

$$y(t+h) = y(t) + \frac{hy'(t)}{1!} + \frac{h^2y''(t)}{2!} + \frac{h^3y'''(t)}{3!}..$$

$$y(t-h) = y(t) - \frac{hy'(t)}{1!} + \frac{h^2y''(t)}{2!} - \frac{h^3y'''(t)}{3!}...$$

There are differents way of approximating

i)**Forward difference approximation**

$$y'(t) = \frac{y(t+h) - y(t)}{h} - \frac{h}{2!}y''(t) - \frac{h^2}{3!}y'''(t)$$

ii)Backward Difference Approximation.

$$y'(t) = \frac{y(t) - y(t-h)}{h} + \frac{h}{2!}y''(t) - \frac{h^2}{3!}y'''(t) + \ldots$$

And use

$$y'(t) = \frac{y(t_n) - y(t_{n-1})}{h}$$

iii)Central difference approximation

Subtracting the first equation from the second equation we get

$$y'(t) = \frac{y(t+h) - y(t-h)}{2h} + \frac{h^2}{3!}$$

$$y'(t) = \frac{y(t+h) - y(t-h)}{2h} + 0h^2$$

Note :Numerical equation reduces a differential equation to a difference equation.

2.6 Boundary-value problem(B.V.P)

This are the values of the differential equation solved subject to specified condition at two or more points.

2.6.1 Second Order B.v.p

1)Boundary condition of first kind $y(a) = \gamma_1$, $y(b) = \gamma_2$

2)Boundary condition of the second kind $y'(a) = \gamma_1$, $y'(b) = \gamma_2$

3)Boundary condition of the 3^{rd} kind $\alpha_1(a)$+$\alpha_2(b)$

Steps for solving a BVP

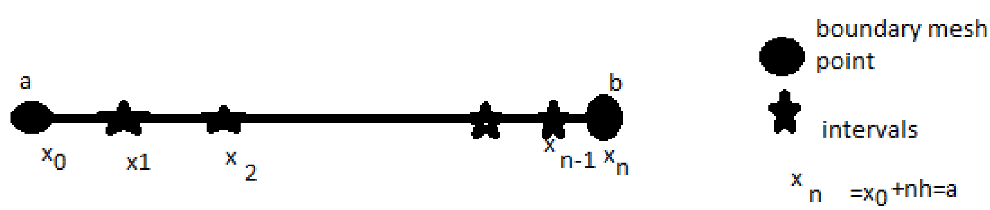
Given an equation

$$y''(x) + q_1(x)y'(x) + q_0y(x) = r(x)$$

Which is solved in subject

to $y(a) = \gamma_1$, $y(b) = \gamma_2$

Divide $[a,b]$ into N equal sub intervals each of width h



Using central difference

$$\frac{y_{n+1} - 2y_n + y_{n-1}}{h^2} + q_{1n}\left(\frac{y_{n+1} - y_{n-1}}{2h}\right) + q_{0n}y_n = \gamma_n$$

Multiply bot side by h^2.

$$y_{n+1} - 2y_n + y_{n-1} + hq_{1n}\left(\frac{y_{n+1} - y_{n-1}}{2}\right) + h^2q_{0n}y_n = h^2\gamma_n$$

Arranging the terms together

$$(1 - \frac{1}{2}hq_{1h})y_{n-1} + (h^2q_{01} - 2)y_n + (1 + \frac{hq_{1n}}{2})y_{n+1} = h^2r_n$$

Appling the boundary conditions and substituting accordingly we have n=1

$$(h^2q_{01}-2)y_1+(1+\frac{1}{2}hq_{11})y_2$$

n=2

$$(1-\frac{1}{2}hq_{1,2})y_1+(h^2q_{02}-2)y_2+(1+\frac{1}{2}hq_{1,2})y_3 0=h^2r_2$$

n=3

$$(1-\frac{1}{2}hq_{1,3})y_1+(h^2q_{0,3}-2)y_3+(1+\frac{1}{2}hq_{1,3})y_4 0=h^2r_3$$

n=N-2

$$0+(1-\frac{1}{2}hq_{1,N-2})y_{N-3}+(h^2q_{0N-2}-2)y_{N-2}+(1+\frac{1}{2}hq_{1,N-2})y_{N-1}=h^2r_{N-2}$$

n=N-1

$$0+(1-\frac{1}{2}hq_{1,N-1})y_{N-2}+(h^2q_{0,N-1}-2)y_{N-1}=h^2r_{N-1}-(1+\frac{1}{2}hq_{1,N-1}\gamma_2$$

let

b_i=$(h^2q_{0,1}-2)$;i=2 ,3.....N-1

c_i=$1+\frac{1}{2}hq_{1i}$.;i=1,2....N-1

d_i=h^2r_1-(1-$\frac{1}{2}hq_{11})\gamma_1$;i=2,3...

a_i=$1-\frac{1}{2}hq_{1i}$;i=2,3....

$$
\begin{bmatrix}
b_1 & c_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
a_2 & b_2 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & a_3 & b_3 & c_3 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & a_4 & b_4 & c_3 & 0 & 0 & 0 & 0 \\
. & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . \\
0 & 0 & 0 & 0 & 0 & a_{N-3} & b_{N-3} & c_{N-3} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & a_{N-2} & b_{N-2} & c_{N-2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{N-1} & b_{N-1}
\end{bmatrix}
\begin{bmatrix}
y_1 \\ y_2 \\ y_3 \\ y_4 \\ . \\ . \\ y_N-3 \\ y_N-2 \\ y_N-1
\end{bmatrix}
=
\begin{bmatrix}
d_1 \\ d_2 \\ d_3 \\ d_4 \\ . \\ . \\ d_{N-3} \\ d_{N-2} \\ d_{N-1}
\end{bmatrix}
$$

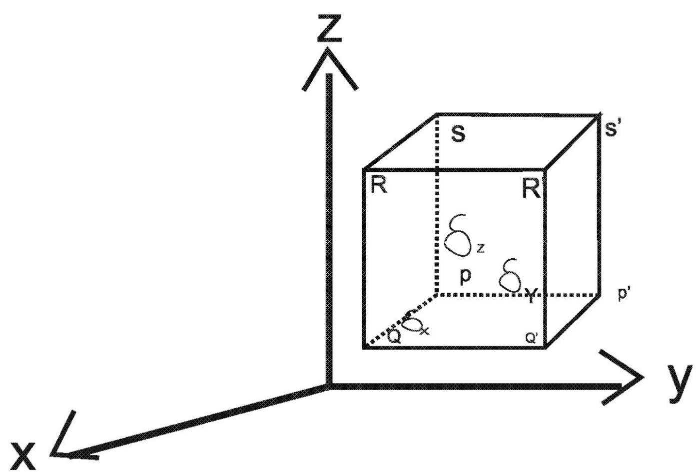
$A\vec{y} = \vec{d}$ A is the metric of coefficient

2.7 EQUATION OF CONTINUITY IN CARTESIAN COORDINATES

Consider a flow field in a fluid flow and let there be a fluid particle at f_z $(x, y, z)$and also let ρ (x, y, z) be the density of the fluid particle at p at any time t.

The velocity component at point p given by u,v,w parallel to the rectangular coordinate axis

Construct a smooth parallelopiped with edges ∂x ,∂y,∂z which are parallel to their coordinates axes having p at one angular point as shown.



we need to note that

1)Mass of fluid will pass through the face $PSS'P'$ and leave at the opposite face $QQ'RR'$.

2)Mass of fluid will will pass also through the face $PQRS$ and leave at face $P'Q'R'S'$.

3)Mass of fluid will will pass also through the face $PP'QQ'$ and leave at face $RSS'R'$.

The whole amount of mass of fluid which passes in through the face $pssp'$

$$\text{Total mass in} =(\rho\ \partial y\ \partial z)$$

$$=f(x,y,z)dxdz$$

Mass of fluid which passes out via the opposite face $QRR'Q'$

$$\text{Total mass out} =f(x+\delta x,y,z) \text{ per unit time}$$

$$=f(x,y,z)+\ \delta x\ \frac{\partial f}{\partial x}\ (x,y,z)$$

Expanding this using Taylor series expansion The increase of mass of fluid per unit time through the face is

$$=f(x,y,z)-[f(x,y,z)+\ \delta x\ \frac{\partial f}{\partial x}\ (x,y,z)]$$

$$=-\delta x\ \frac{\partial f}{\partial x}\ (x,y,z)]$$

$$=-\frac{\partial}{\partial x}(\rho u)\ \partial x\ \partial y\ \partial z$$

Similarly the total increase in mass through the face PQRS and P'Q'R'S'

$$=-\frac{\partial}{\partial x}(\rho V)\ \partial x\ \partial y\ \partial z$$

The amount of mass per unit time with element from the flow via the face PP'QQ' and RSS'R' Will be given by

$$=-\frac{\partial}{\partial x}(\rho \mathrm{w})\, \partial x\, \partial y\, \partial z$$

Applying the law of conservation of mass which state

"The rate of increase of the mass of fluid within an element must be equal to the rate of mass through into the element"

$$\frac{\partial}{\partial t}\rho \partial x \partial y \partial z = -\left(\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial x}(\rho v)\frac{\partial}{\partial x}(\rho w)\right)\partial x \partial y \partial z$$

$$= \frac{\partial}{\partial t}\rho = -\left(\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial x}(\rho v)\frac{\partial}{\partial x}(\rho w)\right)$$

$$= \frac{\partial}{\partial t}\rho + \left(\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial x}(\rho v)\frac{\partial}{\partial x}(\rho w)\right) = 0$$

Which is equation of continuity in Cartesian coordinates system which may be given as

$$\frac{\partial}{\partial t}\rho + \rho\frac{\partial u}{\partial x} + \rho\frac{\partial v}{\partial y} + \rho\frac{\partial w}{\partial z} + \mathrm{u}\,\frac{\partial}{\partial x}\rho + \mathrm{v}\,\frac{\partial}{\partial y}\rho + \mathrm{w}\,\frac{\partial}{\partial z}\rho = 0$$

$$= \left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)\rho + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)$$

Then by the definition of of material derivative given by

$$\frac{D}{Dx} = \left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)$$

$$= \frac{D}{Dx}\rho + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)$$

Taking the fluid to be in compressible the density must be constant (ie ρ =constant) Then the above equation reduces to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

which refers to the equation of continuity for an incompressible fluid in cartesian

Dealing with two dimensional we have

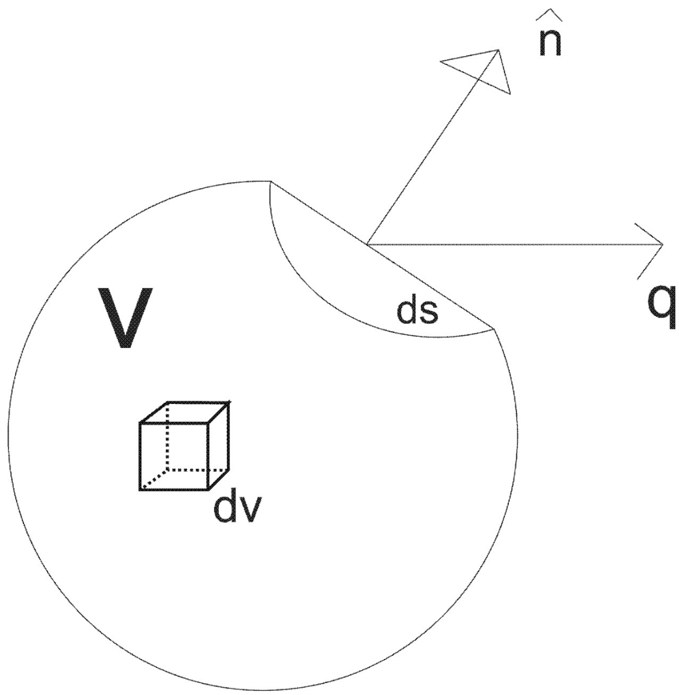
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

2.8 *EQUATION OF MOTION*

Let there be a non -viscous fluid occupying a certain region,in this region at v be the volume enclosed by the same fluid particles at all time

within s let dv be the volume element surrounding a fluid particle p with density ρ

The Mass $=\rho\ dv$ of this element remain constant through out, then ;
$\vec{q}$ being the velocity of the fluid particles.



The momentum of M

$$M = \iiint_v \rho\vec{q}dv$$

The interval takes place over the entire volume v

Let p present the pressure at a point of the surface element ds with an outward normal $\hat{n}$

Total surface force is therefore

-

$$\iint_s p\hat{n}\, ds = -\iiint_v \nabla p dv \text{ by Gauss formular}$$

$\vec{F}$ represents the external force per unit mass acting on the fluid so that the total force acting on the fluid within surface s at any time t is

$$\iiint_v \vec{F} \rho\, dv$$

The total force acting on volume v is

$$\iiint_v \vec{F}\rho\, dv \text{-} \iiint_v \nabla p dv$$

According to the newton second law ,"the rate of change of linear momentum is equal to the total acting on the mass fluid "

$$\frac{Dm}{Dt} = \iiint_v [\rho\vec{F} - \nabla p] dv$$

$$= \iiint_v \frac{Dq}{Dt}\rho\ dv + \iiint_v \vec{q}\frac{D}{Dt}(\rho dv$$

$$= \iiint_v [\rho\vec{F} - \nabla\rho] dv$$

$$\iiint_v \frac{Dq}{Dt}\rho dv = \iiint_v [\rho\vec{F} - \nabla\rho] dv$$

Note $\frac{D}{Dt}(\rho dv)$=0

since ρdv is constant

The volume V is taken as an arbitrary volume of the fluid in the area considered

$$\frac{Dq}{Dt} = \vec{F} \text{-} X \frac{\nabla p}{\rho}$$

which is the euler equation

But $\frac{Dq}{Dt}=\frac{\partial \vec{q}}{\partial t}$ $+(q \cdot \nabla)$ $\vec{q}$

Therefore the euler equation is expressed as

$\frac{\partial \vec{q}}{\partial t}$ $+(q \cdot \nabla)$ $\vec{q}=\vec{F}$ - $\frac{1}{\rho}\nabla p$

Applying the vector identity

$\nabla(\vec{A} \cdot \vec{B})=(\vec{B}\cdot)\vec{A})+(\vec{A} \cdot \nabla)\vec{B}+\vec{B} \times (\nabla \times \vec{A})+\vec{A} \times (\nabla \times \vec{B})$

From this we can show that euler equation can be written in the form

$\frac{\partial \vec{q}}{\partial t}+ \nabla\frac{\vec{q^2}}{2}-\vec{q} \times curl\vec{q}=\vec{F}-\frac{1}{rho}\nabla p$

$\nabla[\vec{q} \cdot \vec{q}]$
$=(\vec{q} \cdot \nabla)\vec{q})+(\vec{q} \cdot \nabla)\vec{q})+\vec{q} \times (\nabla \times \vec{q})+\vec{q} \times (\nabla \times \vec{q})$

such that

$\nabla(q^2)$=2$(\vec{q} \cdot \nabla)\vec{q})$ +2$\vec{q} \times (\nabla \times \vec{q})$

$(\vec{q}\nabla)\vec{q}$ =$\frac{\nabla(\vec{q^2}}{2}$- q$\times(\nabla \times \vec{q})$

using this we have

$\frac{\partial q}{\partial t}$ $+\frac{\nabla q^2}{2}$ $-\vec{q} \times curl\vec{q}$= F-$\frac{1}{\rho}\nabla p$

Corollary if $\vec{q}$= u$\hat{i}$+v$\hat{j}$+w$\hat{k}$

$$\vec{F}=x\hat{i}+y\hat{j}+z\hat{k}i$$

Then the equation of motion takes the form

$$\frac{D}{Dt}[u\hat{i}+v\hat{j}+w\hat{k}]=(x\hat{i}+y\hat{j}+z\hat{k})-\frac{1}{\rho}\ (\hat{i}\frac{\partial p}{\partial x}+\hat{j}\frac{\partial p}{\partial y}+\hat{k}\frac{\partial p}{\partial z})$$

equating the coefficient of $(\hat{i}\ ,\hat{j},\hat{k})$

$$\frac{Du}{Dt}= X-\frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$\frac{Dv}{Dt}= Y-\frac{1}{\rho}\frac{\partial p}{\partial y}$$

$$\frac{Dw}{Dt}= Z-\frac{1}{\rho}\frac{\partial p}{\partial z}$$

This is called the Eulers equation of motion in cartesian plane

3 Defining the problem

3.1 THE GOVERNING EQUATION

The equation of continuity is given by

$$\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)=0 \quad (1)$$

u, v are the components of velocity of the fluid in the x, y direction
The equation of motion

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+v[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}]+\frac{f_x}{\rho} \quad (2)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+v[\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}]+\frac{f_y}{\rho} \quad (3)$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+v[\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}]+\frac{f_z}{\rho} \quad (4)$$

The components of $\vec{J}\times\vec{B}\ sin\ \alpha$ are F_X,F_Y and F_z in the x, y, z direction simultaneously

Taking in consideration of a two dimensional flow thus equation (1) reduces

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (5)$$

Since the plate are of infinite length we deduce that the flow is through the x-axis and it depends on y.

$$\frac{\partial u}{\partial x}=0 \quad (6)$$

Also taking the assumption that the flow is of a steady flow , the flow variable do not depend on time thus equation (3)and (4) can be expressed as

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial x} + v\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \frac{f_x}{\rho} \tag{7}$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial y} + v\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \frac{f_y}{\rho} \tag{8}$$

Using equation (5) and (6) and also putting in mind that there is no flow in the y -direction equation (7) and equation (8) may now be written as

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial x} + v\left(\frac{\partial^2 u}{\partial x^2}\right) + \frac{f_x}{\rho} \tag{9}$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{f_y}{\rho} \tag{10}$$

Since there is no component of body force in the y-direction f_y=f_z=0 as v=w=0 and $f_x = \vec{J} \times \vec{B} \sin\alpha$. Equation (9) and (10) can be written as

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial x} + v\left(\frac{\partial^2 u}{\partial x^2}\right) + \frac{\vec{J} \times \vec{B}}{\rho} sin\alpha \tag{11}$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial y} \tag{12}$$

Equation (12) shows that pressure is not dependent on y

We also know that

$$\vec{J} = \sigma\vec{E}$$

$$\vec{E} = \vec{U} \times \vec{B} Sin\alpha$$

$\vec{U}$ represents the fluid velocity through the x-axis in the direction of fluid flow

$\vec{J} \times \vec{B}$ sin α=$\sigma[(\vec{U} \times \vec{B} \sin\alpha) \times \vec{B} \sin\alpha]$

=$\sigma[(\vec{U}.\vec{B} \sin\alpha).\vec{B} \sin\alpha - (\vec{B} \sin\alpha.\vec{B} \sin\alpha)\vec{U}]$

Having $\vec{U}$ and $\vec{B}$ *sin* α to be perpendicular vectors we have

$\vec{U} \cdot \vec{B}$ *sin* α=0

This will yield to

$\vec{J}\times \vec{B}$ *Sin* α=-σ B^2 $\vec{U}$ $sin^2\alpha$

Also

$$\frac{\vec{J}X\vec{B}}{\rho} Sin\alpha = -\frac{\sigma B^2 \vec{U}}{\rho} \quad \textbf{(13)}$$

Thus equation(13) reduces equation of motion (11) to

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial x} + v\left(\frac{\partial^2 u}{\partial x^2}\right) + \frac{\sigma B^2 \vec{U}}{\rho} sin^2\alpha \tag{14}$$

3.1.1 NON DIMENSIONLIZING

Singh (1993) introduced the following non dimension quantities
$y' = \frac{y}{a}$, $x' = \frac{x}{a}$, $p' = \frac{pa^2}{\rho v^2}$, $u' = u\frac{a}{v}$
using the above quantities we have

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial u'} \frac{\partial u'}{\partial y'} \frac{\partial y'}{\partial y}$$

$$\frac{\partial u}{\partial y} = \frac{v}{a^2}\frac{\partial u'}{\partial y'} \tag{15}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{v}{a^3}\frac{\partial^2 u'}{\partial y'^2} \tag{16}$$

$$\frac{\partial p}{\partial x} = \frac{\rho v^2}{a^3}\frac{\partial p'}{\partial x'} \tag{17}$$

$$\frac{\partial p}{\partial y} = \frac{\partial p}{\partial p'}\frac{\partial p'}{\partial y'}\frac{\partial y'}{\partial y} \tag{18}$$

Substituting this value in equation(12) and equation(14)

$$\frac{\partial p'}{\partial y'} = 0 \tag{19}$$

and

$$-\frac{1}{\rho}\frac{\rho v^2}{a}\frac{\partial p'}{\partial x}+v\frac{v}{a^3}\frac{\partial u'}{\partial y'^2}-\frac{\sigma b^2}{\rho}\frac{v}{a}u' sin^2\alpha=0 \tag{20}$$

Dropping the primes

$$\frac{v}{a^3}\left[-\frac{\partial p}{\partial x}+\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B^2}{\rho}\frac{a^2}{v}u' sin^2\alpha\right]=0 \tag{21}$$

Equation 26 above can be expressed as

$$-\frac{\partial p}{\partial x}+\frac{\partial^2 u}{\partial y^2}-(ha)^2 usin^2\alpha=0 \tag{22}$$

where $(ha)^2 = \frac{\sigma B^2}{\rho}\ \frac{a^2}{v}$
$ha = Ba\sqrt{\frac{\sigma}{\mu}}\ \sin\alpha$

where
$(ha)^2 = \frac{\sigma B^2}{\rho}\ \frac{a^2}{v}$

thus

$ha = Ba\sqrt{\frac{\sigma}{u}}\ sin\alpha$

$\mu = \rho v$

ha Is known as the Hartman number which is directly propotional to the magnetic field B
And for simplicity we can denote Hartman number *ha* with M

Differentiating equation (22) we have

$$\frac{\partial^2 p}{\partial x^2} = 0 \tag{23}$$

This can also be written as

$$\frac{d^2 p}{dx^2} = 0 \tag{24}$$

since p does not depend on y

Clearly

$$\frac{dp}{dx} = -p \tag{25}$$

p is just a constant

From this we can take ordinary derivative of the equation of motion instead of partial derivatives

$$\frac{d^2 u}{dy^2} - M^2 u sin^2 \alpha = -p \tag{26}$$

Which is our problem to solve using a numerical method

The above equation can be written as

$$u'' - M^2 sin^2 \alpha u = -p \tag{27}$$

Applying the central difference approximation
we have the following equation

$$\frac{u_{n+1} - 2u_n + u_{n-1}}{h^2} - M^2 sin^2 \alpha u_n = -p \tag{28}$$

multiply both side by h^2

$$u_{n+1} - 2u_n + u_{n-1} - h^2M^2 sin^2\alpha u_n = -ph^2 \quad (29)$$

Taking α= 90, $sin^2\alpha$=1

$$u_{n+1} - 2u_n + u_{n-1} - h^2M^2 u_n = -ph^2 \quad (30)$$

writing the terms together

$$u_{n+1} - (2 + h^2M^2)u_n + u_{n-1} = -ph^2 \quad (31)$$

Take n to be no of of step and h to be the step size ie(n=10 and h=0.2)

which is solved under the boundary condition

u=0 when y=-1
u=U when y =+1

$$u_{n+1} - (2 + \frac{1}{25}M^2)u_n + u_{n-1} = -\frac{1}{25}p \quad (32)$$

Taking the first case M=1

$$u_{n+1} - (\frac{51}{25})u_n + u_{n-1} = -\frac{1}{25}p \quad (33)$$

taking n=1

$$u_2 - (\frac{51}{25})u_1 + u_0 = -\frac{1}{25}p$$

$$u_2 - (\frac{51}{25})u_1 = -\frac{1}{25}p \tag{34}$$

n=2

$$u_3 - \frac{51}{25}u_2 + u_1 = -\frac{1}{25}p \tag{35}$$

n=3

$$u_4 - \frac{51}{25}u_3 + u_2 = -\frac{1}{25}p$$

$$u_4 - \frac{51}{25}u_3 + u_2 = -\frac{1}{25}p \tag{36}$$

n=4

$$u_5 - \frac{51}{25}u_4 + u_3 = -\frac{1}{25}p \tag{37}$$

n=5

$$u_6 - \frac{51}{25}u_5 + u_4 = -\frac{1}{25}p \tag{38}$$

n=6

$$u_7 - \frac{51}{25}u_6 + u_5 = -\frac{1}{25}p \tag{39}$$

n=7

$$u_8 - \frac{51}{25}u_7 + u_6 = -\frac{1}{25}p \tag{40}$$

n=8

$$u_9 - \frac{51}{25}u_8 + u_7 = -\frac{1}{25}p \tag{41}$$

n=9

$$-\frac{51}{25}u_9+u_8=-\frac{1}{25}p-U \tag{42}$$

Take p=U As velocity U of upper plate is constant and from (25) p is non dimensional pressure which is also constant

Equation 33-42 may be written as

u_2 - $(\frac{51}{25})u_1$ =-$\frac{1}{25}U)$

u_3 - $\frac{51}{25}u_2$+u_1=$-\frac{1}{25}U$

u_4- $\frac{51}{25}u_3$+u_2=$-\frac{1}{25}U$

u_5 - $\frac{51}{25}u_4$+u_3=$-\frac{1}{25}U$

u_6 - $\frac{51}{25}u_5$+u_4=$-\frac{1}{25}U$

u_7 - $\frac{51}{25}u_6$+u_5=$-\frac{1}{25}U$

u_8 - $\frac{51}{25}u_7$+u_6=$-\frac{1}{25}U$

u_9 - $\frac{51}{25}u_8$+u_7=$-\frac{1}{25}U$

- $\frac{51}{25}u_9$+u_8=$-\frac{26}{25}U$

Which can be written in matrix form as

$$\begin{bmatrix} -\frac{51}{25} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -\frac{51}{25} & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -\frac{51}{25} & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -\frac{51}{25} & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\frac{51}{25} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -\frac{51}{25} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -\frac{51}{25} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{51}{25} & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{51}{25} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \\ u_9 \end{bmatrix} = \begin{bmatrix} -\frac{1}{25}U \\ -\frac{1}{25}U \\ -\frac{1}{25}U \\ -\frac{1}{25}U \\ -\frac{1}{25}U \\ -\frac{1}{25}U \\ -\frac{1}{25}U \\ -\frac{1}{25}U \\ -\frac{26}{25}U \end{bmatrix}$$

Using row reduction formula the above matrix can be reduced to

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \\ u_9 \end{bmatrix} = \begin{bmatrix} 0.1886U \\ 0.3446U \\ 0.4745U \\ 0.5834U \\ 0.6756U \\ 0.7548U \\ 0.8242U \\ 0.8866U \\ 0.9444U \end{bmatrix}$$

$\frac{u_1}{U} = 0.1886$; $\frac{u_2}{U} = 0.3446$; $\frac{u_3}{U} = 0.4745$

$\frac{u_4}{U} = 0.5834$; $\frac{u_5}{U} = 0.6756$; $\frac{u_6}{U} = 0.7548$

$\frac{u_7}{U} = 0.8242$; $\frac{u_8}{U} = 0.8866$; $\frac{u_9}{U} = 0.9444$

CASE 2 M=1;
Taking α=60
$sin^2(60)$=$\frac{3}{4}$

$u_{n+1}-2u_n+u_{n-1}$ - $h^2M^2sin^2\alpha u_n$=-ph^2

$u_{n+1}-2u_n+u_{n-1}$ - $\frac{3}{100}u_n$=-p$\frac{1}{25}$

u_{n+1}-$\frac{203}{100}u_n$+u_{n-1} =-$\frac{1}{25}$ p

P=U

n=1 : u_2 - $(\frac{203}{100})u_1$ =-$\frac{1}{25}U)$

n=2 : u_1 - $\frac{203}{100}u_2$+u_3=$-\frac{1}{25}U$

n=3 : u_2- $\frac{203}{100}u_3$+u_4=$-\frac{1}{25}U$

n=4 : u_3 - $\frac{203}{100}u_4$+u_5=$-\frac{1}{25}U$

n=5 : u_4 - $\frac{203}{100}u_5$+u_6=$-\frac{1}{25}U$

n=6 : u_5 - $\frac{203}{100}u_6$+u_7=$-\frac{1}{25}U$

n=7 :u_6 - $\frac{203}{100}u_7$+u_8=$-\frac{1}{25}U$

n=8 : u_7 - $\frac{203}{100}u_8$+u_9=$-\frac{1}{25}U$

n=9 : u_8- $\frac{203}{100}u_9$=$-\frac{26}{25}U$

Which can be written as

$$\begin{bmatrix} -\frac{203}{100} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -\frac{203}{100} & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -\frac{203}{100} & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -\frac{203}{100} & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\frac{203}{100} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -\frac{203}{100} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -\frac{203}{100} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{203}{100} & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{203}{100} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \\ u_9 \end{bmatrix} = \begin{bmatrix} -\frac{1}{25}U \\ -\frac{1}{25}U \\ -\frac{1}{25}U \\ -\frac{1}{25}U \\ -\frac{1}{25}U \\ -\frac{1}{25}U \\ -\frac{1}{25}U \\ -\frac{1}{25}U \\ -\frac{26}{25}U \end{bmatrix}$$

Using row reduction formula the above matrix can be reduced to

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \\ u_9 \end{bmatrix} = \begin{bmatrix} 0.2056U \\ 0.3774U \\ 0.5206U \\ 0.6393U \\ 0.7372U \\ 0.8173U \\ 0.8818U \\ 0.9328U \\ 0.9718U \end{bmatrix}$$

U just a constant

$\frac{u_1}{U}$=0.2056 ;; $\frac{u_2}{U}$=0.3774 ; $\frac{u_3}{U}$=0.5206

$\frac{u_4}{U}$=0.6393 ; $\frac{u_5}{U}$=0.7372 ; $\frac{u_6}{U}$=0.8173

$\frac{u_7}{U}$=0.8818; $\frac{u_8}{U}$=0.9328; $\frac{u_9}{U}$=0.9718

CASE 3 Taking α=45
$sin^2(45)$=$\frac{1}{2}$

$u_{n+1}-2u_n+u_{n-1}$ - $h^2M^2sin^2\alpha u_n$=-ph^2

$u_{n+1}-2u_n+u_{n-1}$ - $\frac{1}{50}u_n$=-p$\frac{1}{25}$

u_{n+1}-$\frac{101}{50}u_n$+u_{n-1} =-$\frac{1}{25}$ p

P=U

n=1 : u_2 - $(\frac{101}{50})u_1$ =-$\frac{1}{25}U)$

n=2 : u_1 - $\frac{101}{50}u_2$+u_3=$-\frac{1}{25}U$

n=3 :u_2 - $\frac{101}{50}u_3$+u_4=$-\frac{1}{25}U$

n=4 : u_3 - $\frac{101}{50}u_4$+u_5=$-\frac{1}{25}U$

n=5 : u_4 - $\frac{101}{50}u_5$+u_6=$-\frac{1}{25}U$

n=6 :u_5 - $\frac{101}{50}u_6$+u_7=$-\frac{1}{25}U$

n=7 : u_6 - $\frac{101}{50}u_7$+u_8=$-\frac{1}{25}U$

n=8 : u_7 - $\frac{101}{50}u_8$+u_9=$-\frac{1}{25}U$

n=9 : u_8- $\frac{101}{50}u_9$=$-\frac{26}{25}U$

which can be writen as

$$\begin{bmatrix} -\frac{101}{50} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -\frac{101}{50} & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -\frac{101}{50} & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -\frac{101}{50} & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\frac{101}{50} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -\frac{101}{50} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -\frac{101}{50} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{101}{50} & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{101}{50} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \\ u_9 \end{bmatrix} = \begin{bmatrix} -\frac{1}{25}U \\ -\frac{1}{25}U \\ -\frac{1}{25}U \\ -\frac{1}{25}U \\ -\frac{1}{25}U \\ -\frac{1}{25}U \\ -\frac{1}{25}U \\ -\frac{1}{25}U \\ -\frac{26}{25}U \end{bmatrix}$$

Using row reduction the above matrix can be reduced to

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \\ u_9 \end{bmatrix} = \begin{bmatrix} 0.2259U \\ 0.4163U \\ 0.5751U \\ 0.7053U \\ 0.8097U \\ 0.8902U \\ 0.9486U \\ 0.9859U \\ 1.0029U \end{bmatrix}$$

$\frac{u_1}{U}$ = 0.2259 ; $\frac{u_2}{U}$ = 0.4163 ; $\frac{u_3}{U}$ = 0.5751

$\frac{u_4}{U}$ = 0.7053 ; $\frac{u_5}{U}$ = 0.8097 ; $\frac{u_6}{U}$ = 0.8902

$\frac{u_7}{U}$ = 0.9486 ; $\frac{u_8}{U}$ = 0.9859 ; $\frac{u_9}{U}$ = 1.0029

CASE 4 M=1;
Taking α=30
$sin^2(30)$=$\frac{1}{4}$

$u_{n+1}-2u_n+u_{n-1}$ - $h^2M^2sin^2\alpha u_n$=-ph^2

$u_{n+1}-2u_n+u_{n-1}$ - $\frac{1}{100}u_n$=-p$\frac{1}{25}$

u_{n+1}-$\frac{201}{100}u_n$+u_{n-1} =-$\frac{1}{25}$ p

P=U

n=1 :u_2 - $(\frac{201}{100})u_1$ =-$\frac{1}{25}U)$

n=2 : u_1 - $\frac{201}{100}u_2$+u_2=$-\frac{1}{25}U$

n=3 : u_2- $\frac{201}{100}u_3$+u_4=$-\frac{1}{25}U$

n=4 : u_3 - $\frac{201}{100}u_4$+u_5=$-\frac{1}{25}U$

n=5 : u_4 - $\frac{201}{100}u_5$+u_6=$-\frac{1}{25}U$

n=6 : u_5 - $\frac{201}{100}u_6$+u_7=$-\frac{1}{25}U$

n=7 : u_6 - $\frac{201}{100}u_7$+u_8=$-\frac{1}{25}U$

n=8 : u_7 - $\frac{201}{100}u_8$+u_9=$-\frac{1}{25}U$

n=9 : u_8 - $\frac{201}{100}u_9$=$-\frac{26}{25}U$

which can be writen as

$$\begin{bmatrix} -\frac{201}{100} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -\frac{201}{100} & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -\frac{201}{100} & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -\frac{201}{100} & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\frac{201}{100} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -\frac{201}{100} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -\frac{201}{100} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{201}{100} & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{201}{100} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \\ u_9 \end{bmatrix} = \begin{bmatrix} -\frac{1}{25}U \\ -\frac{1}{25}U \\ -\frac{1}{25}U \\ -\frac{1}{25}U \\ -\frac{1}{25}U \\ -\frac{1}{25}U \\ -\frac{1}{25}U \\ -\frac{1}{25}U \\ -\frac{26}{25}U \end{bmatrix}$$

Using row reduction the above matrix can be reduced to

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \\ u_9 \end{bmatrix} = \begin{bmatrix} 0.2503U \\ 0.4630U \\ 0.6404U \\ 0.7842U \\ 0.8958U \\ 00.9764U \\ 1.0268U \\ 1.0474U \\ 1.0385U \end{bmatrix}$$

$\frac{u_1}{U}$ = 0.2503 ; $\frac{u_2}{U}$ = 0.4630 ; $\frac{u_3}{U}$ = 0.6404

$\frac{u_4}{U}$ = 0.7842 ; $\frac{u_5}{U}$ = 0.8958 ; $\frac{u_6}{U}$ = 00.9764

$\frac{u_7}{U}$ = 1.0268 ; $\frac{u_8}{U}$ = 1.0474 ; $\frac{u_9}{U}$ = 1.0385

Using the above formula to calculate m=1.5 m= 2 ,the following tables will be obtained

m=1.5

Y	excact	numerical	error	analytical	numerical	error
-1	0	0	0	0	0	0
-0.8	0.1888	0.1886	0.001059322	0.1703	0.2056	0.0353
-0.6	0.345	0.3446	0.00115942	0.3156	0.3774	0.0618
-0.4	0.4749	0.4745	0.000842283	0.4404	0.5206	0.0802
-0.2	0.5838	0.5834	0.000685166	0.5483	0.6393	0.091
0	0.676	0.6756	0.000591716	0.6426	0.7372	0.0946
0.2	0.7551	0.7548	0.000397298	0.7262	0.8173	0.0911
0.4	0.8245	0.8242	0.000363857	0.8015	0.8818	0.0803
0.6	0.8867	0.8866	0.000112778	0.8709	0.9328	0.0619
0.8	0.9445	0.9444	0.000105876	0.9364	0.9718	0.0354
1	1	1	0	1	1	0
	α=90				α=60	

.

Y	analyrical	numerical	error	analytical	numerical	error
-1	0	0	0	0	0	0
-0.8	0.1497	0.2259	0.0762	0.1265	0.2503	0.1238
-0.6	0.2824	0.4163	0.1339	0.2443	0.463	0.2187
-0.4	0.4007	0.5751	0.1744	0.3545	0.6404	0.2859
-0.2	0.507	0.7053	0.1983	0.4583	0.7842	0.3259
0	0.6034	0.8097	0.2063	0.5566	0.8958	0.3392
0.2	0.6918	0.8902	0.1984	0.6505	0.9764	0.3259
0.4	0.7741	0.9486	0.1745	0.7409	1.0268	0.2859
0.6	0.8519	0.9859	0.134	0.8287	1.0474	0.2187
0.8	0.9267	1.0029	0.0762	0.9148	1.0385	0.1237
1	1	1	0	1	1	0
	α=45				α=30	

y	Analytical 90degree	numerical	error	Analytical 60degree	numerical	error
-1	0	0	0	0	0	0
-0.8	0.2607	0.1325	0.1282	0.2317	0.1531	0.0786
-0.6	0.4544	0.237	0.2174	0.4113	0.2765	0.1348
-0.4	0.5985	0.3228	0.2757	0.5509	0.3786	0.1723
-0.2	0.7063	0.3976	0.3087	0.66	0.4663	0.1937
0	0.7875	0.4682	0.3193	0.7461	0.5454	0.2007
0.2	0.8493	0.5409	0.3084	0.8149	0.6214	0.1935
0.4	0.8975	0.6223	0.2752	0.8712	0.6993	0.1719
0.6	0.9364	0.7198	0.2166	0.9187	0.7844	0.1343
0.8	0.9696	0.842	0.1276	0.9607	0.8824	0.0783
	m=1.5				m=1.5	

.

y	Analytical 45 degree	numerical	error	Analytical 30 degree	numerical	error
-1	0	0	0	0	0	0
-0.8	0.1974	0.181	0.0164	0.1551	0.2205	0.0654
-0.6	0.3585	0.3301	0.0284	0.2911	0.4059	0.1148
-0.4	0.4906	0.4541	0.0365	0.4111	0.5605	0.1494
-0.2	0.5998	0.5585	0.0413	0.5179	0.6877	0.1698
0	0.6908	0.648	0.0428	0.6138	0.7904	0.1766
0.2	0.7679	0.7268	0.0411	0.701	0.8709	0.1699
0.4	0.8345	0.7982	0.0363	0.7815	0.9309	0.1494
0.6	0.8937	0.8655	0.0282	0.857	0.9719	0.1149
0.8	0.948	0.9318	0.0162	0.9293	0.9948	0.0655
			m=1.5			

y	ANALYTIC 90DEGREE	Numerical	error	ANALYTIC 60DEGREE	Numerical	error
-1	0	0	0	0	0	0
-0.8	0.33	0.0936	0.2364	0.2935	0.1124	0.1999
-0.6	0.5513	0.1621	0.3892	0.5013	0.1982	0.3392
-0.4	0.6998	0.2166	0.4832	0.6487	0.2679	0.4321
-0.2	0.7997	0.2657	0.534	0.7535	0.3296	0.4878
0	0.8671	0.3173	0.5498	0.8284	0.391	0.5111
0.2	0.913	0.3797	0.5333	0.8826	0.4592	0.5029
0.4	0.9447	0.4628	0.4819	0.9225	0.5426	0.4597
0.6	0.9675	0.58	0.3875	0.953	0.6511	0.373
0.8	0.9849	0.75	0.2349	0.9779	0.7977	0.2279
1	1	1	0	1	1	0

.

y	ANALYTIC 45DEGREE	Numerical	error	ANALYTIC 30DEGREE	Numerical	error
-1	0	0	0	0	0	0
-0.8	0.2484	0.1409	0.1075	0.1888	0.1886	0.0002
-0.6	0.4362	0.2532	0.183	0.345	0.3446	0.0004
-0.4	0.5786	0.3457	0.2329	0.4749	0.4745	0.0004
-0.2	0.6871	0.4258	0.2613	0.5838	0.5834	0.0004
0	0.7705	0.5	0.2705	0.676	0.6756	0.0004
0.2	0.8353	0.5742	0.2611	0.7551	0.7548	0.0003
0.4	0.8868	0.6543	0.2325	0.8245	0.8242	0.0003
0.6	0.9293	0.7468	0.1825	0.8867	0.8866	1E-04
0.8	0.966	0.8591	0.1069	0.9445	0.9444	1E-04
1	1		1	1	1	0
		m=2				

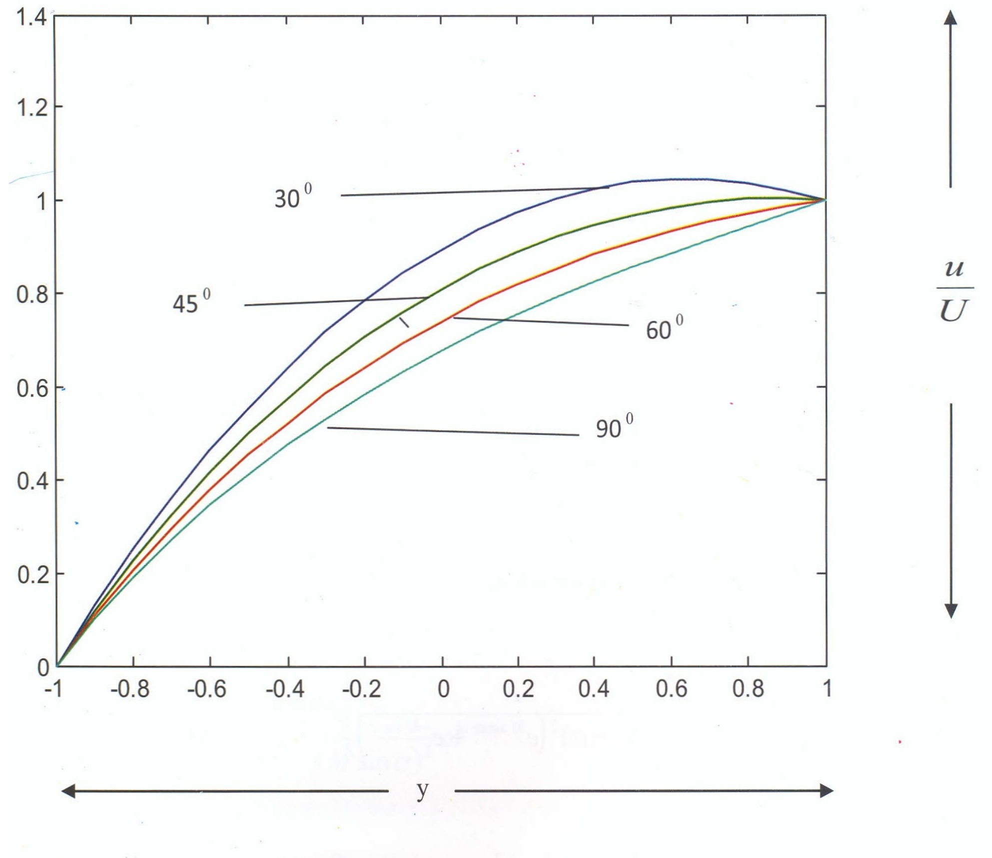
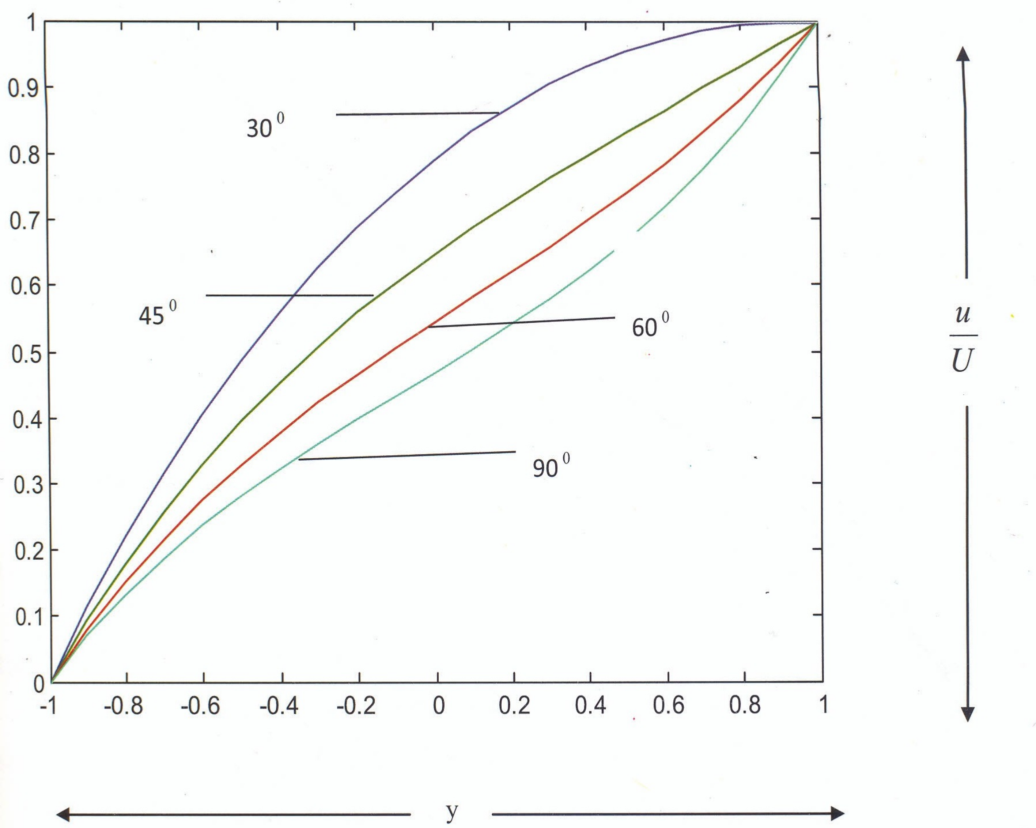


Figure 1: *M = 1.0*



1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
30⁰
45⁰
60⁰
90⁰
u/U
y

Figure 2: *M=1.5*

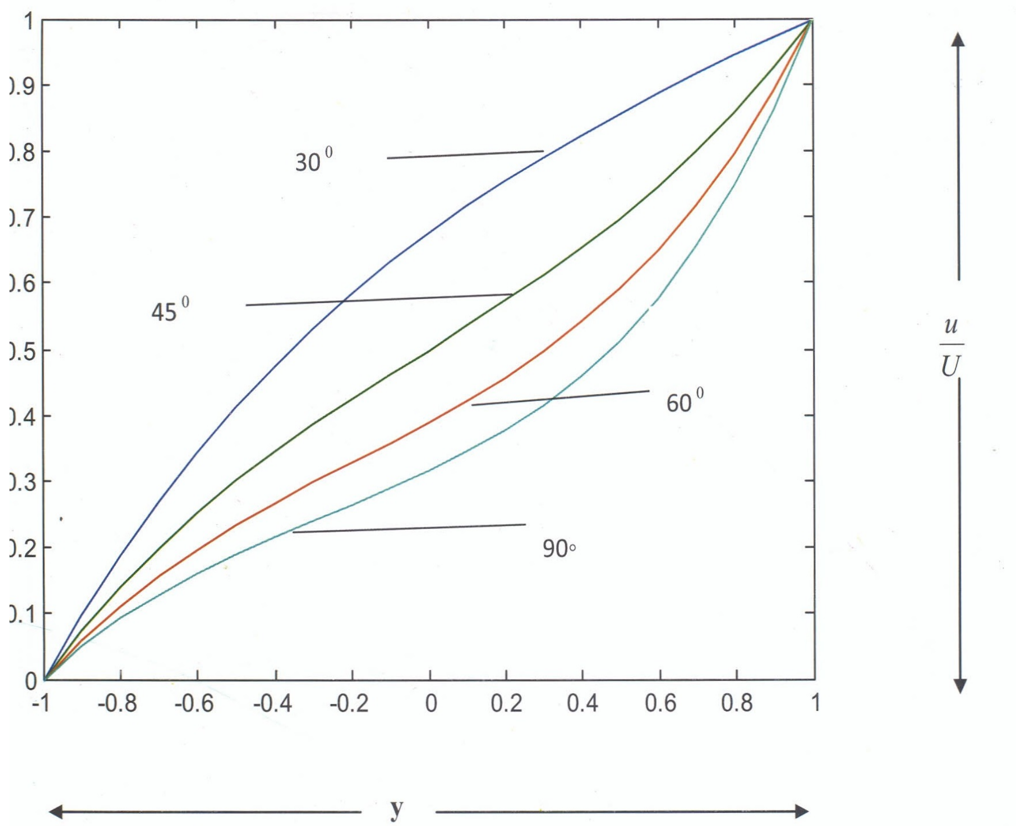


Figure 3: *M = 2.0*

4 DISCUSSION OF RESULTS AND APPLICATIONS

The problem has been solved by the central difference method with 10 equal mesh point and step size of 0.2 An analytic expression for the velocity of fluid particle has been obtained. It is clear from equation (34) that if we take applied pressure gradient P = 0 then we get problem of Singh (2007) as a particular case of present problem. table 1 to 6 are calculated at the inclinations of 30 degree,45 degree,60 degree and 90 degree . It is clear from the tables that velocity decreases as the strength of magnetic field is increased. Also evident is the fact that with the increase of inclination of magnetic field, there is a decrease in the velocity profile. Again velocity profiles at 90 degree gives us the steady hydro magnetic flow of viscous in compressible fluid under applied pressure gradient and when upper plate is also moving with constant velocity under the influence of transverse magnetic field as a particular case of present problem. It is evident from (34) that when U = 0 then the problem of magneto hydrodynamic steady flow of liquid between parallel plates Singh (1993) can be obtained as a particular case of this problem. The results obtained here can be applied to the designs and operations of MHD generator, MHD pump, electromagnetic flow meter, and to crude oil

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