A COMPARISION OF THE MOVING AVERAGE, GARCH (1, 1), AND ARCH VOLATILITY MODELS AS MEASURE OF VOLATILITY OF RETURNS

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DECLARACTION

This research is my original work and has not been presented for a degree award in any other university.

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DEDICATION

To Mum and Dad. I also dedicate this to Salome Thinguri for her moral support. Special dedication to my aunt Phyllis Gimona for her financial assistance.

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I would like to thank the Almighty God for the guidance, protection and direction throughout the study period. I thank God for my supervisors, Dr Mwaniki and Prof Weke for their invaluable time and prudent guidance in the development and production of this work. Much thanks to the entire academic staff of School of Mathematics for their encouragement and support.

Nonetheless, I take full responsibility for the errors and shortcomings in this study.

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ABSTRACT

Many empirical studies have shown strong evidence against some of the underlying assumptions of the Black Scholes Model. However this paper has focussed on the constant value that is assumed for the volatility. Empirical research shows that the volatility of financial asset prices is following a stochastic process and varies through time. This paper has highlighted the different that determine volatility, and some of them act as alternatives or improvement from earlier models.

We have compared three models: ARCH, GARCH and the Moving Average Model. GARCH is a good description of the evolution of the variance process of the asset returns. It provides a better evolution of asset returns than compared to the ARCH model. It also captures volatility clustering quite well.

CHAPTER ONE

INTRODUCTION

1.1 BACKGROUND

An option is a security giving the right to buy or sell the asset subject to certain conditions, within a specified period of time. In general, options can be either American or European, a distinction that has nothing to do with geographical location. American options can be exercised at any time up to the maturity, whereas European options can only be exercised at the maturity. The price that is paid for the asset when the option is exercised is known as the strike price or exercise price. The last day at which an option can be exercised is known as the maturity date or expiration date.

There are two basic types of options. A call option gives the holder of the option the right to buy an asset by the maturity date for the strike price. A put option gives the holder of the option the right to sell an asset by the maturity date for the strike price. Nevertheless, European options are generally easier to analyze than American options, and some of the properties of an American option are usually derived from those of its European counterpart.

The first options were used in ancient Greece to speculate on the olive harvest. Earlier studies tell of a man who purchased the right to use olive presses. It was mid-winter, and the owner of the olive presses was happy to sell the right to use the olive presses during the harvest season. It generated income for the olive press owner during the off season. The man purchasing the rights ensured that he would have use of the presses during the busy season. If the olive harvest was really good, the purchaser might be able to even resell his right to use the olive presses for a profit. The use of the options is the same.

1.1.1 BIRTH OF THE MODERN OPTION

The established financial markets have had options available for decades. The options contract was originally an "over the counter" product. This meant that only people with specialized needs and information tended to engage in the purchase and sale of options. This original options contract was not standardized in its terms or conditions. There was also no secondary market for options and no way to properly and consistently assess the value of the options contract.

In 1973, the modern financial options market came into existence. The Chicago Board of Trade (CBOT) opened the Chicago Board Options Exchange (CBOE). The CBOE instituted a new "exchange traded options contract". This contract was standardized in its terms and conditions. An options buyer and seller no longer had to sit down and negotiate terms of the contract every time he or she sought to buy an option. Thus, the CBOE could publish quoted options prices for the first time, and could establish a market maker system to make sure that there was a secondary or resell market for options.

At the same time, the Options Clearing Corporation was formed to make sure that the contract would be honored by all members. Lastly, the whole process came under the regulatory control of the Securities and Exchange Commission. Thus, the trading of the modern option, "exchange traded options contract" had begun. On the first day the contracts traded, April 26, 1973, a total of 911 contracts were traded. Since that time, options trading has grown enormously. In 2007, there were over 2.8 billion contracts cleared by the Options Clearing Corporation.

Options are now widely traded in variety of financial instruments: from stocks and bonds to exchange-traded funds, commodities and currency futures.

1.2 USES OF OPTIONS

There are two main reasons for using options: to speculate and to hedge. Speculation is described as the betting on the movement of a security. Speculation is the territory in which the big money is made and lost. The use of options in this manner is the reason options have the reputation of being risky. This is because when you buy an option, you have to be correct in determining not only the direction of the stock's movement, but also the magnitude and the timing of this movement. To succeed, you must correctly predict whether a stock will go up or down, and you have to be right about how much the price will change as well as the time frame it will take for all this to happen.

The other function of options is hedging. This is the making an investment to reduce the risk of adverse price movements in an asset. This is an area of risk management. The main objective of risk management is to assess risk and develop strategies that minimize it. This is because bond prices, stock prices, current rates and interest rates fluctuate and thereby creating risk. Also the diverse range of potential underlying assets and payoffs alternatives lead to a huge variety of option contracts available to be traded on the markets.

The financial market provides a number of financial instruments which began as theory of option pricing in the 1900 when French mathematician Louis Bachelier deduced an option pricing formula based on the assumption that stock prices follow a Brownian motion with zero drift. However, the Black-Scholes (1973) option pricing model laid the foundation for a new era of option valuation theory.

In 1973, Fischer Black and Myron Scholes proposed a formula to price for the stock options, which made a major breakthrough in the field of mathematical finance. Later on, Robert Merton, student of Fisher Black, published a paper expanding the mathematical understanding of the options pricing model and coined the term "Black-Scholes" options pricing model. The model has had a huge influence on the way that trader's price and hedge options. Finally, Merton and Scholes received the 1997 Nobel Prize in Economics for this and related work. Though ineligible for the prize because of his death in 1995, Black was mentioned as a contributor by the Swedish academy.

1.3 THE BLACK SCHOLES MODEL

1.3.1 ASSUMPTIONS FOR BLACK SCHOLES MODEL

Before we derive the formula for the value of an option in terms of price of the stock, we should first create an "ideal condition" in the market both for the stock and for the option. As written is Black & Scholes (1973).

1. The short-term interest rate is known and is constant through time.
2. The stock price follows a random walk in continuous time with a variance rate proportional to the square of the stock price. Thus, the distribution of possible stock prices at the end of any finite interval is lognormal. The variance rate of the return on the stock is constant.
3. The stock pays no dividends or other distributions.
4. The option is "European", that is, it can only be exercised at maturity.
5. There are no transaction costs in buying or selling the stock or the option.
6. It is possible to borrow any fraction of the price of a security to buy it or hold it, at the short-term interest rate.
7. There are no penalties to short selling. A seller who does not own a security will simply accept the price of the security from a buyer, and will agree to settle with the buyer on some future date by paying him an amount equal to the price of the security on that date.

1.3.2 DERIVATION OF THE BLACK SCHOLES FORMULA

We derive the Black-Scholes formula within a self-finance portfolio using Black-Scholes partial differential equation approach based on Hull (2008).

1.3.2.1 GENERALIZED WIENER PROCESS

The drift rate is known as the mean change per unit time for a stochastic process and the variance rate is the variance per unit time. The basic Wiener process, dw, has a drift rate of zero and a variance rate of 1.0. The drift rate of zero means that the expected value of w at any future time is equal to its current value while the variance rate of 1.0 means that the variance of the change in w is a time interval of length T equals T. Then, a generalized Wiener process for variance x is defined in terms of dw as:

$$dw = mdt + ndw \qquad \text{1.1}$$

Where m and n are constants.

1.3.2.2 THE PROCESS FOR A STOCK PRICE

We assume the price process of a non-dividend-paying stock is a stochastic process which follows a generalized Wiener process with a constant expected drift rate and a constant variance rate.

Obviously, the assumption of constant expected drift rate is inappropriate and needs to be replaced by the assumption that the expected return is constant. If S is the stock price at time t, then the expected drift rate in S should be assumed to be μS for some constant parameter μ. This means that in a short interval of time, Δt, the expected increase in S is $\mu S \Delta t$. The parameter μ is the expected rate of return on the stock.

We should also consider the volatility of a stock price. A reasonable assumption is that the variability of the percentage return in a short period of time, Δt, is the same regardless of the stock price. This suggests that the standard deviation of the change in short period of time Δt should be proportional to the stock price and leads to the model:

$$dS = \mu S dt + \sigma S dw \qquad \text{1.2}$$

where σ is the volatility of the stock price, μ is its expected rate of return, and w is a generalized Wiener process.

1.3.2.3 ITÔ'S LEMMA

The Itô's Lemma, was presented by K. Itô in 1951. In order to understand Itô's formula in its most simple form, we start with a Taylor expansion to the lowest orders for a function of two variables $F(t,S)$:

$$dF(t,S) = \frac{\partial F}{\partial t}dt + \frac{\partial F}{\partial S}dS + \frac{1}{2}\frac{\partial^2 F}{\partial t^2}(dt)^2 + \frac{\partial^2 F}{\partial t \partial S}dtds + \frac{1}{2}\frac{\partial^2 F}{\partial S^2}(dS)^2 + \cdots$$

Where S is described by stochastic process given by:

$dS = \mu(x,t)dt + \sigma(x,t)dW$ 1.3

W is Wiener Process with a property $(dW)^2 = dt$

$(dS)^2 = \mu^2(x,t)(dt)^2 + \sigma^2(x,t)(dW)^2 + 2\mu(x,t)\sigma(x,t)dtdW = \sigma^2(x,t)dt$....................1.4

Substituting equation 1.3 and 1.4 into the Taylor's expansion above yields:

$$dF(t,S) = \frac{\partial F}{\partial t}dt + \frac{\partial F}{\partial S}(\mu(x,t)dt + \sigma(x,t)dW) + \frac{1}{2}\frac{\partial^2 F}{\partial S^2}(dS)^2$$

$$= \frac{\partial F}{\partial t}dt + \frac{\partial F}{\partial S}(\mu(x,t)dt + \sigma(x,t)dW) + \frac{1}{2}\sigma^2(x,t)\frac{\partial^2 F}{\partial S^2}$$

$= \left(\frac{\partial F}{\partial t} + \mu(x,t)\frac{\partial F}{\partial S} + \frac{1}{2}\sigma^2(x,t)\frac{\partial^2 F}{\partial S^2}\right)dt + \sigma(x,t)\frac{\partial F}{\partial S}dW$ 1.5

Which is the Itô's formula.

The generalized expression is given by:

$dF(t,S_1 ... S_n) = \frac{\partial F}{\partial t}dt + \sum_{i=1}^{n}\frac{\partial F}{\partial S_i}dS_i + \frac{1}{2}\sum_{i,j=1}^{n}\frac{\partial^2}{\partial S_i \partial S_j}\sigma_i\,\sigma_j dt$..1.6

1.3.2.4 VOLATILITY

The volatility is a measure of the uncertainty about the return provided by the stock. It can be defined as the standard deviation of the return provided by the stock in 1 year when the return is expressed using continuous compounding.

In this study, I will concentrate on finding the volatility of a stock by estimating volatility from historical data. Therefore, in order to estimate the volatility of a stock empirically, we should obverse the stock price at fixed intervals of time (*e.g.* daily, weekly, or monthly).

We define:

$n+1$: number of observations

S_i: stock price at the ith interval, with $i = 0, 1, \ldots, n$

t: length of time interval in years

and let $a_i = \ln\left(\frac{S_i}{S_{i-1}}\right)$ where $i = 1, 2, \ldots, n$.

the usual estimate, σ, of the standard derivation of the a_i is given by:

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{i-1}^{n}(a_i - \bar{a})^2} \quad \text{.......1.7}$$

or

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\bar{a}^2 - \frac{1}{n(n-1)}\left(\sum_{i=1}^{n}a_i\right)^2} \quad \text{.......1.8}$$

Where $\bar{a}$ is the mean of a_i.

It's not easy to choose an appropriate value for *n*. Generally, more data lead to more accuracy, but the volatility does change over time and data that are too old may not be relevant for predicting the future volatility. A compromise that seems to work reasonably well is to use closing prices from daily data over the most recent 90 to 180 days. An often-used rule of thumb is to set *n* equal to the number of days to which the volatility is to be applied. Thus, if the volatility estimate is to be used to value a 2-year option, daily data for the last 2 years are used.

1.3.2.5 DERIVATION OF THE BLACK-SCHOLES-MERTON DIFFERENTIAL EQUATION

This relies on one market containing a risk-free bond B that pays an interest rate r and a stock S. The price of the stock follows a Wiener process (Geometrical Brownian Motion) with a constant drift $\alpha$and a stochastic term σSdW, where σ is the volatility. The two securities are given by

$$\begin{cases} dB(t) = r \cdot B(t)dt \\ B(0) = 1 \end{cases} \quad \text{...1.9}$$

On rearranging 1.9 yields $B(t) = e^{rt}$

$$\begin{cases} dS(t) = \alpha \cdot S(t)dt + \sigma \cdot S(t)dW(t) \\ S(0) = s \end{cases}$$

The initial condition of the bond is 1 and the initial stock price is s. We now consider a portfolio h of the bond and the stock: $h = (h^0 h^1)$, where h holds the number of each security. After the comparison between relative self-finance portfolio and the derivation from Itô's Lemma get:

$$\frac{dF}{dt} + rS\frac{dF}{dS} + \frac{1}{2}\sigma^2 S^2 \frac{d^2F}{dS^2} = rF \quad \text{...1.10}$$

This is the Black-Scholes partial differential equation. Note that this equation is independent from α. In a risk neutral word, we can explain the terms in the partial differential equation as:

$\frac{dF}{dt}$ the change of value with respect to time t.

$rS\frac{dF}{ds}$ the change of value with respect to underlying price

$\frac{1}{2}\sigma^2 S^2 \frac{d^2F}{ds^2}$ the change of value with respect to volatility

rF the expected change of value of derivative security.

1.3.2.6 THE SOLUTION OF THE BLACK-SCHOLES-MERTON PARTIAL DIFFERENTIAL EQUATION

For one European option with strike price *K*, the price of this option F_T at maturity *T* will be $F_T = max\{S_T - K, 0\}$ which is called the boundary condition of the Black-Scholes partial differential equation. Therefore, we have

$$\begin{cases} F_t + rSF_s + \frac{1}{2}\sigma^2 S^2 F_{ss} - rF = 0 \\ F_T = max\{S_T - K, 0\} \end{cases} \quad \text{.......} 1.11$$

We assume that $F(t, S_t)$ is a solution to the partial differential equation above, where

$$\begin{cases} dS = r \cdot Sdt + \sigma \cdot SdV \\ S(t) = s \end{cases}$$

Where r is the risk-free interest rate and V is a Wiener process under risk neutral probability measure Q.

Under risk neutral probability measure Q, the stochastic part vanishes and we get:

$$F(t, S) = e^{-r(T-t)} E^Q_{t-s}[max\{S_T - K, 0\}] \text{.......} 1.12$$

We can write 1.12 as

$$\Pi[t, S] = s \cdot N\left(-Z_0 + \sigma \cdot \sqrt{T-t}\right) - K \cdot e^{-r(T-t)} N(-Z_0) \text{.......} 1.13$$

1.13 is equal to: $= s \cdot N(d_1) - K \cdot e^{-r(T-t)} N(d_2)$

Where

$$\begin{cases} d_1 = \dfrac{\ln\left\{\frac{S}{K}\right\} + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma \cdot \sqrt{T-t}} \\ d_2 = d_1 - \sigma \cdot \sqrt{T-T} \end{cases}$$

this is the final expression of the Black-Scholes formula. Here, *s* is the initial underlying price, *K*

is the strike price of the option, σ is the volatility on the market, *r* is the current interest rate on the market, *t* is the initial time and *T* is the maturity. We therefore substitute the data into the formula and find the corresponding option price under Black-Scholes model.

1.4 CRITICISM OF IMPLICIT PROPERTIES OF BLACK-SCHOLES

Despite the Black Scholes model's popularity and wide spread use, the model is built on some non-real life assumptions.

Firstly, the BSM assumes a geometric Brownian motion model. This implies that the series of first differences of the log prices must be uncorrelated. For example the S&P 500 as a whole, observed over several decades, daily from 1 July 1962 to 29 Dec 1995, there are in fact small but statistically significant correlations in the differences of the logs at short time lags (Hull 2002). At its core, neither people nor a model can consistently predict the direction of the market or an individual stock. The Black Scholes theorem assumes stocks move in a manner referred to as a random walk; random walk means that at any given moment in time, the price of the underlying stock can go up or down with the same probability. However, this assumption does not hold as stock prices are determined by many factors that cannot be assigned the same probability in the way they will affect the movement of stock prices. Moreover, the price of a stock in time $t+1$ is independent from the price in time t; the martingale property of Brownian motion. There may not be a single source or factor driving two assets even if one is a derivative of the other as is stated in the martingale representation theorem.

Secondly, the model assumes that log normally distributed underlying stock prices are normally distributed. However, as observed by Clark, asset returns have a finite variance and semi-heavy tails contrary to stable distributions like log normal with infinite variance and heavy tails. As noted by Hull, experience has shown that returns are leptokurtic, i.e., have much more of a tendency to exhibit outliers than would be the case if they were normally distributed. An example is provided by the returns on the S&P 500 series. There is overwhelming evidence that the returns are not normal, but instead have a leptokurtic (i.e., long-tailed) distribution.

Thirdly, the model assumes a constant volatility. However, ever since the 1987 and 2008 stock market crash, this assumption has proven false. While volatility can be relatively constant in very short term periods, it is never constant in the long term. In other words, it is often found that for financial time series, after taking logs (if needed) and first differences, the level of volatility seems to change with time. Often, periods of high volatility follow immediately after a large change (often downward) in the level of the original series. It may take quite some time for this heightened volatility to subside. For example, the plot of differences of the logs of the S&P 500 shows very long periods of high volatility interspersed with periods of relative calm. This type of pattern is often referred to as volatility clustering (Hull 2002). Consequently, more recent option valuation models substitute Black-Scholes's constant volatility with a stochastic process generated estimates. However, given its simplicity and mathematical tractability as compared to some of its more recent variations, the Black-Scholes model continues to be in widespread use.

Fourthly, Black-Scholes model assumes that interest rates are constant and known. This assumption is also unrealistic. The model uses the risk-free rate to represent this constant and known rate. While the Kenyan Government Treasury Bill 90 day rate can be used as a part of the model, there is no such thing as a risk-free rate. The most recent recession of 2008 indicates that even government securities can have default risk as is the case of Greece which was downgraded. Furthermore, treasury rates can and do change in times of increased volatility.

Fifthly, the model assumes that the underlying stock does not pay dividends during the option's life. However, this assumption does not apply in all, or actually, most cases since most public companies pay dividends to their shareholders. This assumption relates to the basic Black-Scholes formula, and typically the model is adjusted by subtracting the discounted value of a future dividend from stock prices to account for dividends.

Finally, one of the most significant assumptions of the theorem is its assumption that there are no fees for buying and selling options and stocks and no barriers to trading. However, this is hardly the case in the real world. More crucially, the model assumes that markets are perfectly liquid and it is possible to purchase or sell any amount of stock or options or their fractions at any given time. This assumption is implausible. For one thing, as demonstrated by the events of 1987, 1998, 2007-2008 markets are not perfectly liquid. Moreover, in most instances, due to company policies, or other factors, investors are limited by the amount of money they can invest and, it is not possible to sell fractions of options.

Given the above criticisms the focus of this paper will be mainly in relation to the constant volatility assumption. Volatility is a measure of the dispersion of an asset price about its mean level over a fixed time interval. Careful modeling of an asset's volatility is crucial for the valuation of options and of portfolios containing options or securities with implicit options as well as for the success of many trading strategies involving options.

1.5 STATEMENT OF THE PROBLEM

Many empirical studies have shown strong evidence against some of the underlying assumptions of the Black-Scholes Model. However my main focus will be of particular interest in the constant value that is assumed for the volatility. However, empirical research shows that the volatility of financial asset prices is following a stochastic process and varies through time. It means that while other properties of an option such as exercise price, time to maturity, current price of underlying asset; can be observed directly from the market, the return volatility is the uncertainty factor in the Black Scholes model. As volatility increases, the probability that stock price will rise or fall increases, which in response will also increase the value of both call and put options. Return volatility thus plays a major role in option pricing. Therefore, accurate measures and good forecasts of volatility are critical for option pricing theories as well as trading strategies. At present, there have been many models developed to determine volatility, and some of them act as alternatives or improvement from earlier models.

The family of GARCH models is an example, starting from the autoregressive conditional heteroskedasticity (ARCH) model of Engle (1982). We therefore try to compare the suitability of the ARCH and the GARCH model as a superior measure of volatility to the historical volatility that is used for the Black Scholes Model.

1.6 OBJECTIVES

General Objectve: To highlight the various time dependent volatility models

Specific Objective: To determine the superiority between the GARCH and ARCH in explaining the volatility of return

CHAPTER TWO

LITERATURE REVIEW

INTRODUCTION

Modeling volatility is challenging because volatility in financial and commodity markets appears to be highly unpredictable. There has been a proliferation of volatility specifications since the original, simple constant volatility assumption of the famous option pricing model developed by Fischer Black and Myron S. Scholes (1973). This model is based on historical volatility however it should be based on future volatility which is difficult to estimate.

Suppose that the value of the asset at the end of day *i* is S_i. Define $u_i$as the percentage change of asset price between the end of day i-1 and the end of day *i*, so that: $u_i = \frac{S_i - S_{i-1}}{S_{i-1}}$ or $u_i = \ln \frac{S_i}{S_{i-1}}$

Is the log relative prices for $i = 1, 2, \ldots, n$ with n the number of returns in the historical sample. The historical volatility estimate is then given by

$\sigma^2 = \frac{1}{n-1} \sum_{i=1}^{n} (u_i - \bar{u})^2$ 2.1

In the equation above, $\bar{u}$ is the mean of the *ui*'s:

$$\bar{u} = \frac{1}{n} \sum_{i=1}^{n} u_{n-i}$$

$\bar{u}$ is usually close to zero (especially for daily data). This gives noncentered volatility estimate given by:

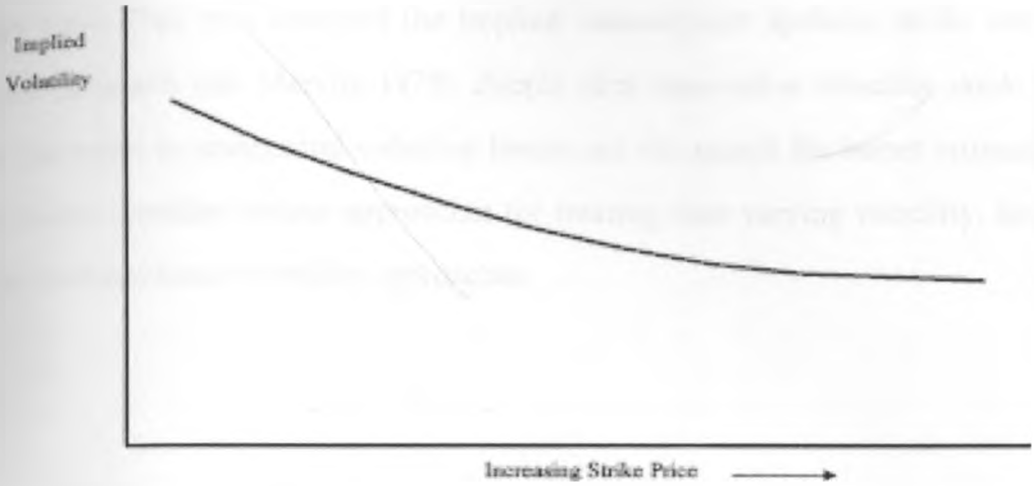
$\sigma^2 = \frac{1}{n} \sum_{j=1}^{n} {u_j}^2$ 2.2

While the historical volatility of an asset return is readily computed from observed asset returns, this measure may be an inaccurate estimate of the future volatility expected to prevail over the life of an option. The future volatility is unobservable and may differ from the historical volatility. Hence, unlike the other parameters that are important for pricing options, the volatility input has to be modeled. In addition the Black-Scholes model assumes that volatility is constant. However evidence points to volatility being time-varying (Bollerslev et. al 1992) and also the variation may be random or stochastic. Randomness means that future volatility cannot be readily predicted using current and past information.

2.1 VOLATILITY SMILE

For the Black Scholes model, the only input that is unobservable is the future volatility of the underlying asset. One way to determine this volatility is to select a value that equates the theoretical Black Scholes price of the option to the observed market price. This value is often referred to as the implied (or implicit) volatility of the option. Under the Black Scholes model, implied volatilities from options should be the same regardless of which option is used to compute the volatility. However, in practice, this is usually not the case. Different options (in terms of strike prices and maturities) on the same asset yield different implied volatilities, outcomes that are inconsistent with the Black Scholes model.

The pattern of the Black Scholes implied volatilities with respect to strike prices has become known as the volatility smile. The existence of a smile also means that if only one volatility is used to price options with different strikes, pricing errors will be systematically related to strikes. The smile has also been shown to depend on options' maturities. Therefore the existence of the smile is an indication of the inadequacy of the constant volatility Black Scholes model.



Rubinstein (1985) reports that short-maturity out of the money calls on equities have market prices that are much higher than the Black Scholes model would predict. On the other hand, since the stock market crash of 1987, the volatility smile has had a persistent shape, especially when derived from equity index option prices as the strike price of index equity options increases, their implied volatilities decrease. Thus, an out of the money put (or in the money call) option has a greater implied volatility than an in-the-money put (or out of the money call) of equivalent maturity. Because the option price, for calls or puts, increases as volatility rises, higher option prices are associated with higher implied volatilities. Thus, relatively high out of the money put prices are mirrored in high implied volatilities for those options.

2.2 EMPIRICAL RESEARCH

Since Black & Scholes published their formula, a lot of empirical research has been undertaken to compare the Black & Scholes model price of options to market prices (Rowley 1987). Much of this research focused on the volatility because this is the only parameter that needs to be estimated. People started to realize that historical and implied volatility changes through time. They also observed the implied variance rate declines as the exercise price increases (Macbeth and Mervile 1979). People thus observed a volatility skew. This then started the quest to understand volatility better and the search for better estimates thereof. We therefore consider various approaches for treating time varying volatility; deterministic volatility and stochastic volatility approaches.

2.2.1 DETERMINISTIC VOLATILITY

The simplest relaxation of the constant volatility assumption is to allow volatility to depend on its past in such a way that future volatility can be perfectly predicted from its history and possibly other observable information. Suppose the variance of asset returns σ_{t+1}^2 is described by the following equation:

$$\sigma_{t+1}^2 = \theta + K\sigma_t^2 \qquad 2.3$$

The future volatility depends on a constant and a constant proportion of the last period's volatility. In this case, the constant variance of the asset returns in the Black Scholes formula can be replaced by the average variance that is expected to prevail from time *t* until time *T*, which is approximately given by $\frac{1}{T-t}\sum_{u=t}^{T}\sigma_u^2$ and the Black Scholes formula can continue to be used.

A more general case specifies volatility as a function of other information known to market participants. One alternative of this kind presents volatility as a function of the level of the asset price $\sigma(S)$. One particular model of this type, known as the constant elasticity of variance (CEV) model, in which volatility is proportional to the level of the stock price raised to a power, appeared early in the option pricing literature (Cox and Steve Ross 1976). However, the CEV model proved not to be free of pricing biases (David Bates 1994). A more recent variation on this volatility specification was developed by Rubinstein (1994). Instead of assuming a particular form of the volatility function, Rubinstein's method effectively infers the dependence of volatility on the level of the asset price from traded options at all available strike prices. He calls the model "implied binomial trees" because the implied risk-neutral distribution (which depends on the volatility) of the asset price at maturity is inferred from option prices by constructing a binomial tree for movements of the asset price. Related models have been proposed by Emanuel Derman and Iraz Kani (1994), Bruno Dupire (1994), and David Shimko (1993).

In a recent empirical test of deterministic volatility models, including binomial tree approaches, Bernard et. al (1996) show that the Black Scholes model does a better job of predicting future option prices. The option delta, which is derived from an option pricing model and measures the sensitivity of the option price to changes in the underlying asset price, can be used to specify positions in options that offset underlying asset price movements in a portfolio. The authors demonstrate that the Black Scholes model resulted in better hedges than those from models based on deterministic volatility functions. The authors note that one reason for the better performance of the Black Scholes model is that errors, from various sources, in quoted option prices distort parameter estimates for deterministic volatility models and consequently degrade these models' predictions.

2.2.2.1 ARCH MODELS.

Autoregressive conditional heteroscedasticity (ARCH) models for volatility are a type of deterministic volatility specification that makes use of information on past prices to update the current asset volatility. The term autoregressive in ARCH refers to the element of persistence in the modeled volatility, and the term conditional heteroscedasticity describes he presumed dependence of current volatility on the level of volatility realized in the past. ARCH models provide a well established quantitative method for estimating and updating volatility.

ARCH models were introduced by Robert F. Engle (1982) for general statistical time-series modeling. An ARCH model makes the variance that will prevail one step ahead of the current time a weighted average of past squared asset returns, instead of equally weighted squared returns, as is done typically to compute variance. ARCH places greater weight on more recent squared returns than on more distant squared returns; consequently, ARCH models are able to capture volatility clustering, which refers to the observed tendency of high-volatility or low-volatility periods to group together.

Some features of ARCH models also make them attractive compared with many other types of option pricing models that allow for time varying volatility. In an ARCH model, the variance is driven by a function of the same random variable that determines the evolution of the returns. In other words, the random source that affects the statistical behavior of returns and volatility through time is the same. As a result, volatility can be estimated directly from the time series of observed returns on an asset. In contrast, the direct estimation of volatility from the returns process is very difficult using stochastic volatility models.

There are many different types of ARCH models that have a wide variety of applications in macroeconomics and finance. In finance, the two most popular ARCH processes are generalized ARCH (GARCH) (Bollerslev 1986) and exponential GARCH (EGARCH) (Daniel B. Nelson 1991). Researchers have tended mostly to use the GARCH process and its variations for option pricing.

The GARCH (p, q) model is specified as:

$$\sigma_n{}^2 = \omega + \sum_{i=1}^{q} \alpha_i \mu_{n-i}{}^2 + \sum_{i=1}^{p} \beta_i \sigma_{n-i}{}^2 \quad \text{......} 2.4$$

In this equation, $\omega > 0$, $\alpha_i \geq 0$, and $\beta_i \geq 0$ hold.

As in ARCH models, variables p and q are the order of dependency. The distinction of GARCH model is that the conditional variance is specified not only as a linear function of past sample variances, but also including lagged conditional variances to enter the equation as well.

The simplest GARCH model is GARCH(1,1) model, which is expressed as:

$$\sigma_i{}^2 = \omega + \alpha \mu_{i-1}{}^2 + \beta \sigma_{i-1}{}^2 \quad \text{......} 2.5$$

The weights assigned to both conditional and unconditional volatility - γ, α, and β must sum to one. For a stable GARCH(1,1) process, $\alpha + \beta < 1$ is required, otherwise the weight applied to the long-term variance is negative. An interesting empirical finding is that in financial series, particularly in daily series, $\alpha + \beta$ is often close to one.

The popularity of GARCH (1, 1) may be explained by three observations. First, the model has only four parameters and these can be estimated easily. Second, it provides an explanation of the major stylized facts for daily returns. Third, it is often found that the volatility forecasts for this specification have similar accuracy to forecasts from more complicated specifications. Initially, we assume conditional normal distributions following Bollerslev (1986) and Taylor (1986), who independently defined and derived properties of the GARCH (1, 1) model.

Although GARCH captures the evolution of the variance process of asset returns quite well, it turns out that there is no easily computable formula, like the Black-Scholes formula, for

European option pricing under a GARCH volatility process. Instead, computer intensive methods are used to simulate the returns and the volatility under the risk-neutral distribution in order to compute European option prices and hedge ratios (Kaushik and Ng 1993; Duan 1995). Some researchers substitute the expected average variance from a GARCH model for the variance input in the Black-Scholes formula (Engle et al 1994). However, the Black Scholes formula does not hold if the variance of asset returns follows a GARCH process; such a substitution is theoretically inconsistent but may work in practice.

Engle et al. (1994) compared the trading profits resulting from a particular trading rule by using two alternatives for the variance forecasts needed for Black Scholes: the variance forecast from a GARCH model and the variance forecast in the form of the Black Scholes implied volatility from a previous period. In their experiment using S&P 500 index options, Engle, Kane, and Noh produced greater hypothetical trading profits using the GARCH volatility forecast than they did using the Black-Scholes implied volatility.

2.2.1.3 EXPONENTIALLY WEIGHTED MOMENTS MODELS.

Hobson and Rogers (1996) propose a new type of option pricing model for time varying volatility that also has the potential to match the observed volatility smile. Their mathematical specification allows past asset price movements to feed back into current volatility. This characteristic is similar to the GARCH model in terms of a similar feedback effect; however, the type of feedback can be much more general than encountered in standard GARCH models. Also like GARCH, but unlike standard stochastic volatility models, there is only one source of uncertainty that drives both the asset price and its volatility.

The Hobson-Rogers model captures past asset price volatility through an offset function. The feedback relationship is primarily embodied in the functional dependence of the volatility on the offset function. The intuition behind the offset function is apparent from its form:

$$S_t^{(m)} = \sum_{u=1}^{\infty} \theta e^{-\theta u}(Z_t - Z_{t-u})^m \quad \text{.......... 2.6}$$

Where $S_t^{(m)}$ is the value of the function at time t and m is the order of the function.

This function simply weights deviations of a transformed current price Z_t (a "discounted" logarithm of the price) from its value u periods ago, $(Z_t - Z_{t-u})$, raised to the power m. The power applied to the deviation, or order of the offset function, is technically the statistical moment of the offset that is employed. For example, a first-order offset function (m = 1) considers the deviation itself, whereas a second-order offset function takes the squares of those deviations and therefore consists of a measure related to the variances of those deviations. The weighting is done by an exponential function that through the parameter θ places more or less importance on the past relative to the present. A high value for θ implies that recently experienced changes in the asset price have a much greater impact on volatility (and the drift) than more distant past shocks. This weighting is similar to the treatment of past return shocks in ARCH modeling. A low θ gives relatively more weight to the past shocks. The persistence of

past shocks θ can be estimated indirectly from options prices.

The feedback mechanism in this model works primarily through the asset price volatility, which can take any number of functional forms. Hobson and Rogers consider a simple version of the offset function, with m = 1, can give option prices that when substituted into the Black Scholes equation generate a volatility smile in implied Black Scholes volatilities evaluated at different strike prices, reflecting the smile observed in actual markets. The model's ability to trace out a smile is suggestive and may indicate the model's potential to match actual prices well.

2.3 STOCHASTIC VOLATILITY

Stochastic volatility implies that the future level of the volatility cannot be perfectly predicted using information available today. The popularity of stochastic volatility in option pricing grew out of the fact that distributions of the asset returns exhibit fatter tails than those of the normal distribution (Mandelbrot 1963; Fama 1965). Stochastic volatility models can be consistent with fat tails of the return distribution. The occurrence of fat tails would imply, for example, that out of the money options would be underpriced by the Black Scholes model, which assumes that returns are normally distributed. Stochastic volatility models could also be an alternative explanation for skewness of the return distribution. Despite the relative complexity of stochastic volatility models, they have been popular with researchers, and additional justification for these models has recently come to light in the literature on asymmetric information about the future asset price and its impact on traded options. In a stochastic volatility model, volatility is driven by a random source that is different from the random source driving the asset returns process, although the two random sources may be correlated with each other.

In contrast to a deterministic volatility model in which the investor incurs only the risk from a randomly evolving asset price, in a stochastic volatility environment, an investor in the options market bears the additional risk of a randomly evolving volatility. In a deterministic volatility model, an investor can hedge the risk from the asset price by trading an option and a risk-free asset based on a risk exposure computed using an option pricing formula (Cox and Rubinstein 1985). However, with a random volatility process, there are two sources of risk (the risk from the asset price and the volatility risk); a risk-free portfolio cannot be created as in the Black Scholes model. After hedging, there is a residual risk that stems from the random nature of the volatility process. Since there is no traded asset whose payoff is a known function of the volatility, volatility risk cannot be perfectly hedged. In order to bear this volatility risk, rational investors would demand a "volatility risk" premium, which has to be factored into option prices and hedge ratios.

A feature of stochastic-volatility models that is not shared by deterministic volatility models is that the price of an option can change without any change in the level of the asset price. The reason is that the option price is driven by two random variables: the asset rice and its volatility. In stochastic-volatility models, these two variables may not be perfectly correlated, implying that the expected volatility over the life of the option may change without any change in the asset price. The change in volatility alone can cause the option price to change. Most stochastic-volatility models assume that volatility is mean reverting. That is, although volatility varies from day to day, there is a presumed long-run level toward which volatility settles in the absence of additional shocks. The evidence for this phenomenon is especially strong in markets for interest rate derivatives (Litterman et al. 1991; Amin and Morton 1994).

Stochastic-volatility models can be classified into two broad categories: those that lack closed form solutions for European options and those that have closed form solutions.

2.3.1 STOCHASTIC VOLATILITY OPTION MODELS WITHOUT CLOSED FORM SOLUTION.

Hull and White (1987), Scott (1987), and Wiggins (1987) were among the first to develop option pricing models based on stochastic volatility. Hull and White as well as Scott made the questionable assumption that the risk premium of volatility is zero, that is, the volatility risk is not priced in the options market and that volatility is uncorrelated with the returns of the underlying asset. Wiggins, who also assumed a zero volatility risk premium, found that the estimated option values under stochastic volatility were not significantly different from Black Scholes values, except for long maturity options. For equity options, Lamoureux and Lastarapes (1993) offer evidence against the assumption of a zero volatility risk premium. For currency options, Melino and Turnbull (1992) found that a random volatility model yields option prices that are in closer agreement with the observed option prices than those of the Black Scholes model. While the numerical methods and computers currently available allow computation of these stochastic volatility option prices, they are still largely impractical. As a result, these stochastic volatility models may not currently be useful for practitioners. Nevertheless, development of stochastic volatility models continues as researchers attempt to find more tractable models.

2.3.2 STOCHASTIC VOLATILITY MODELS WITH CLOSED FORM SOLUTIONS.

Stein and Stein (1991) develop a European option pricing model under stochastic volatility that is somewhat easier to evaluate than the models without the closed form solutions and are also less computationally expensive. The authors make the unrealistic assumption of zero correlation between the volatility process and the returns of the underlying asset.

Heston (1993) was the first to develop a stochastic volatility option pricing model for European equity and currency options that can be easily implemented, is computationally inexpensive, and allows for any arbitrary correlation between asset returns and volatility. The model gives closed form solutions for option prices.

In this model, the asset returns r_t and the variance σ_t^2 are assumed to evolve through time as

$$r_t = \mu + \sigma_t \in_{1,t}$$

and

$$\sigma_t^2 = \sigma_{t-1}^2 + K(\theta - \sigma_{t-1}^2) + \gamma\sigma_t \in_{2,t} \qquad 2.5$$

respectively, where $\in_{1,t}$ and $\in_{2,t}$ are two standard normal random variables that could be correlated with each another, either positively or negatively, with a correlation coefficient, ρ. Equivalently, this coefficient also measures the correlation between the return of the asset and the volatility process. In this model, the variance evolves through time in such a way that its long run average level is measured by θ and the speed with which it is pulled toward this long run mean is measured by K, also known as the mean reversion coefficient. The variable γ is a measure of the volatility of variance. If γ is zero, the model simplifies to a time varying deterministic volatility model. The particular nature of the process ensures that volatility "reflects" away from zero: if volatility ever becomes zero, then the non zero K ensures that volatility will become positive.

However σ_t^2 in this model is not directly comparable to the implied variance from the Black-Scholes model. The reason is that σ_t^2 represents the instantaneous variance (at time t), whereas the implied variance in the Black-Scholes model is the average expected variance through the life of an option and need not equal the instantaneous variance if the model is not true. In Heston's model, the average expected variance during the life of an option is a function of the instantaneous variance, the long-run average variance, the speed with which the instantaneous variance adjusts, and the time to expiration of the option.

The option price and hedge ratios in Heston's model are functions not only of the parameters that appear in the Black-Scholes formula but also of K, θ, ρ, γ and an additional parameter, ω. The parameter ω is a constant such that $\omega\sigma_t^2$ measures the risk premium of volatility. The volatility risk premium is assumed to be directly proportional to the level of the volatility. The need for an assumption about the form of the volatility risk premium is a weakness of any stochastic volatility model because the form of the volatility risk premium cannot be deduced from the weak assumption that all investors prefer more wealth to less wealth, but requires assumptions on investor tolerance toward risk. In this model, the form of the volatility-risk premium is crucial because it enables the derivation of the closed form solutions for option prices and hedge ratios. However, it should not be interpreted as a weakness of this model vis-à-vis other stochastic volatility models of option prices because others make the even stronger and less plausible assumption that the risk premium of volatility is zero.

When volatility is stochastic, as it probably is in the real world, hedging using the Black Scholes model does not result in risk free positions. A stochastic-volatility model may do a better job of hedging against price and volatility risks. Nandi (1996) finds that for S&P 500 index options the returns of a hedge portfolio constructed using Heston's stochastic-volatility model come closer to matching a risk-free return through time better than hedge portfolio returns obtained using the Black-Scholes model.

2.3.3 VOLATILITY JUMPS.

All the time-varying volatility models that have been discussed so far assume that the volatility of the underlying asset as well as its price evolves "smoothly," though randomly, through time:

there are no jumps in the volatility process. However, a likely cause of financial market volatility is the arrival of information and its subsequent incorporation into asset prices through trading. To the extent that information arrives in discrete lumps, it is possible that volatility shifts between episodes of low and high volatility. For example, uncertainty about an impending news release (concerning some macroeconomic variable, like an anticipated change in interest rates) may cause the volatility of an asset price to rise. However, after a few rounds of trading, with the information having been incorporated into asset prices, volatility may revert back to its previous level. To account for jumps like those in the example, Vasantlilak Naik (1993) develops a pricing model for European options in which volatility switches between low and high levels. Each level is expected to last for a certain period of time that is not known beforehand. One version of his model assumes that the risk from the volatility jumps is not priced by market participants. The model takes the same parameters that enter the Black Scholes formula as well as additional parameters such as the probabilities of jumps from one level to another level, given that volatility is currently in a particular level. Naik finds that short maturity options are much more sensitive to volatility shifts than long-maturity options. The reason is that, over a long period of time, expected upward and downward jumps in volatility are canceled by each other, resulting in a volatility that is close to the normal level.

This model has not been empirically tested and therefore cannot yet be evaluated against other stochastic volatility models. In general, jump models can be difficult to verify empirically because jumps occur infrequently. The parameters of such models may be imprecisely estimated using relatively small historical data series on option prices or underlying asset prices.

CHAPTER THREE

RESEARCH METHODOLOGY

3.1 INTRODUCTION

Several volatility models have been developed to try and explain the major stylized facts for asset returns. It is natural to seek tests that can decide which of these models provides the best description of asset returns. We therefore compare three models; ARCH model, GARCH (1,1) and the moving average model.

3.2 MODEL SPECIFICATION

3.2.1 ARCH MODEL

Many models are developed that correspond to stochastic volatility process characteristic. One widely known model is the ARCH model, introduced by Engle (1982). This model is setting unconditional volatility constant, while allowing the conditional volatility to change over time. This conditional volatility is subsequently altered by past returns. The ARCH (q) model is specified as:

$$\sigma_n{}^2 = \omega + \sum_{i=1}^{q} \alpha_i \mu_{n-i}{}^2 \qquad 3.1$$

The variable ω is: $\omega = \gamma \cdot V_L$

In this equation, $\omega > 0$, and $\alpha i \geq 0$ hold. V_L is the long-run average variance rate and γ is the weight assigned to V_L. Daily return μ_i is calculated using $\frac{S_i - S_{i-1}}{S_{i-1}}$. The variable q is the order of dependency to past returns.

An assumption underlying this model is that volatility is changing over time and there is tendency that a large error will be likely followed by a large error and a small error followed by a small error. The variable q is the period the conditional variance depends on. The larger the variable q, the longer is the period of volatility clustering.

3.2.2 GARCH(1,1)

The simplest GARCH model is GARCH(1,1) model, which is expressed as:

$\sigma_i^{\,2} = \omega + \alpha\mu_{i-1}^{\,2} + \beta\sigma_{i-1}^{\,2}$ 3.2

The weights assigned to both conditional and unconditional volatility - γ, α, and β must sum to one. For a stable GARCH(1,1) process, $\alpha + \beta < 1$ is required, otherwise the weight applied to the long-term variance is negative. An interesting empirical finding is that in financial series, particularly in daily series, $\alpha + \beta$ is often close to one.

3.2.3 MOVING AVERAGE MODEL

Suppose that the value of the asset at the end of day *i* is S_i. Define u_i as the percentage change of asset price between the end of day i-1 and the end of day *i*, so that: $u_i = \frac{S_i - S_{i-1}}{S_{i-1}}$

The unbiased estimate of one-day volatility, using *ui* of *m* days before today, is

$\sigma_n^{\,2} = \frac{1}{m-1}\sum_{i=1}^{m}(u_{n-1} - \bar{u})^2$ 3.3

In the equation above, $\bar{u}$ is the mean of the *ui*'s:

$$\bar{u} = \frac{1}{m}\sum_{i=1}^{m} u_{n-i}$$

A simplified approach to estimate volatility is given by

$\sigma_n^{\,2} = \frac{1}{m}\sum_{i=1}^{m} u_{n-1}^{\,2}$ 3.4

We will use the above simple equation to estimate volatility for the Moving Average model. The amount of historical days used to estimate volatility should be determined carefully.

Some expect that more data would lead to better precision, but data that are too old might be unrelated to predict the future.

CHAPTER FOUR

4.1 DATA ANALYSIS, INTERPRETATION AND DISCUSSION OF RESULTS

Data used for implementation consists of daily close prices from NSE 20 share index from the period September 2011 to September 2012, which are collected for the purpose of estimating volatility. Information will be sourced from the Nairobi Stock Exchange.

From the below analysis it seems the GARCH is a good description of the evolution of the variance process of the asset returns. It provides a better evolution of asset returns than compared to the ARCH model. It also captures volatility clustering quite well. A disadvantage of the GARCH model is that it is computationally demanding. The moving average model is used as a benchmark for the comparison of both models.

Table 4.1 ARCH MODEL RESULTS

u_i	Coefficient	Standard Error	z	P > IzI	[95%confidence interval]	
u_i constant	0.0012239	0.0003316	3.69	0.000	0.000574	0.0018738
ARCH(1)	0.0122273	0.0702186	0.17	0.862	-0.1253987	0.1498532
Constant	0.0000225	2.38e-06	9.46	0.000	0.000179	0.0000272

Source: Own Calculations

Its graphical representation is given by Graph 4.2

Figure 4.1: ARCH(1,1) RESULTS

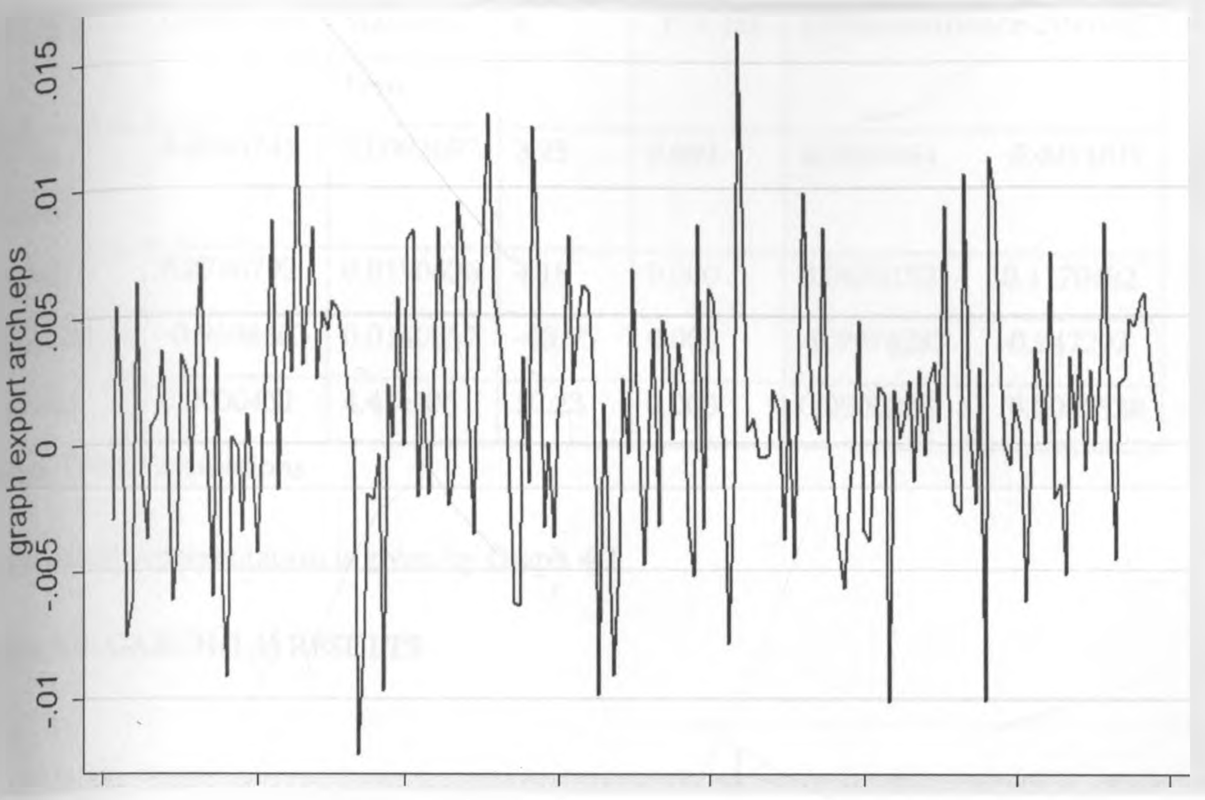


TABLE 4.2: GARCH(1,1) RESULTS

u_i	Coefficient	Standard Error	z	P > IzI	[95%confidence interval]	
u_i constant	0.0008741	0.0002692	3.25	0.001	0.0003464	0.0014017
ARCH(1)	0.0796792	0.0190636	4.18	0.000	0.0423152	0.1170432
GARCH(1,1)	-0.9698603	0.0140657	-68.95	0.000	-0.9974287	-0.942292
Constant	0.0000451	4.41e-06	10.23	0.000	0.0000365	0.0000538

Source: Own Calculations

Its graphical representation is given by Graph 4.2

Figure 4.2: GARCH(1,1) RESULTS

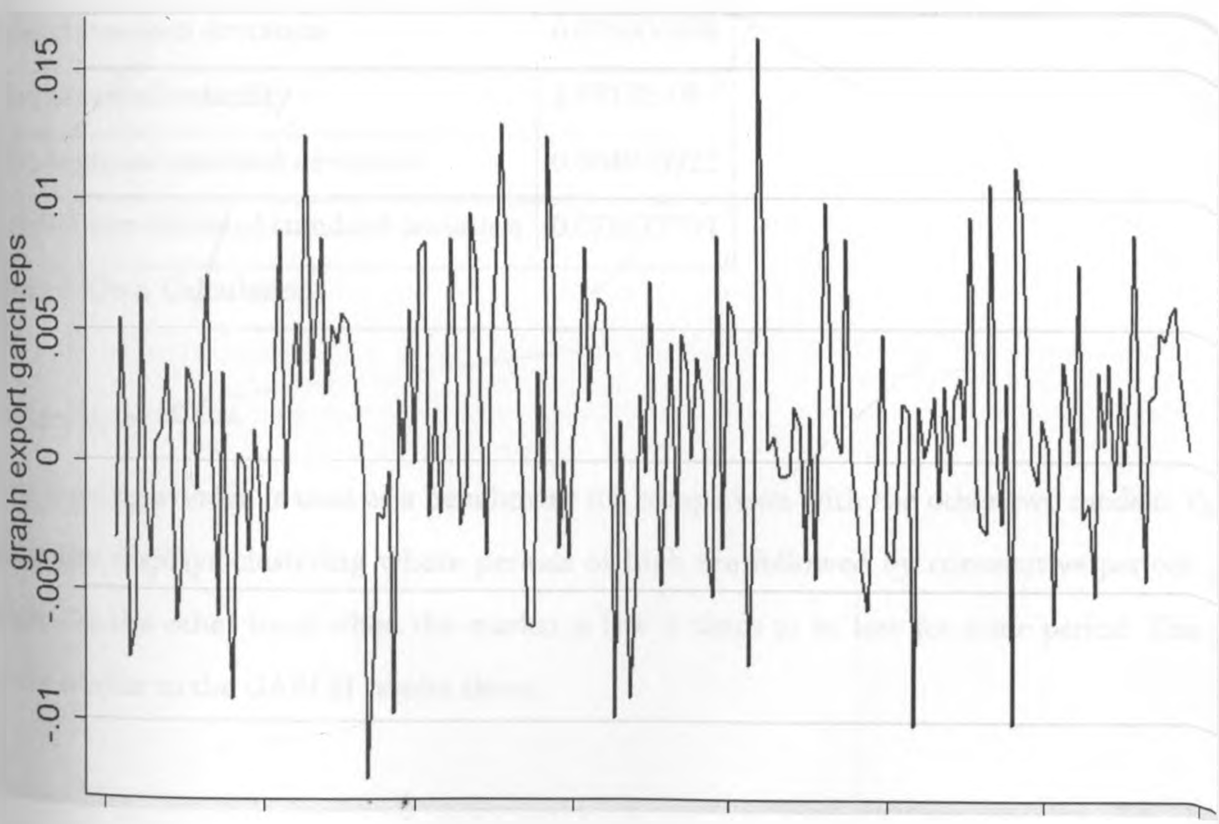


Figure 4.3 MOVING AVERAGE GRAPH RESULTS

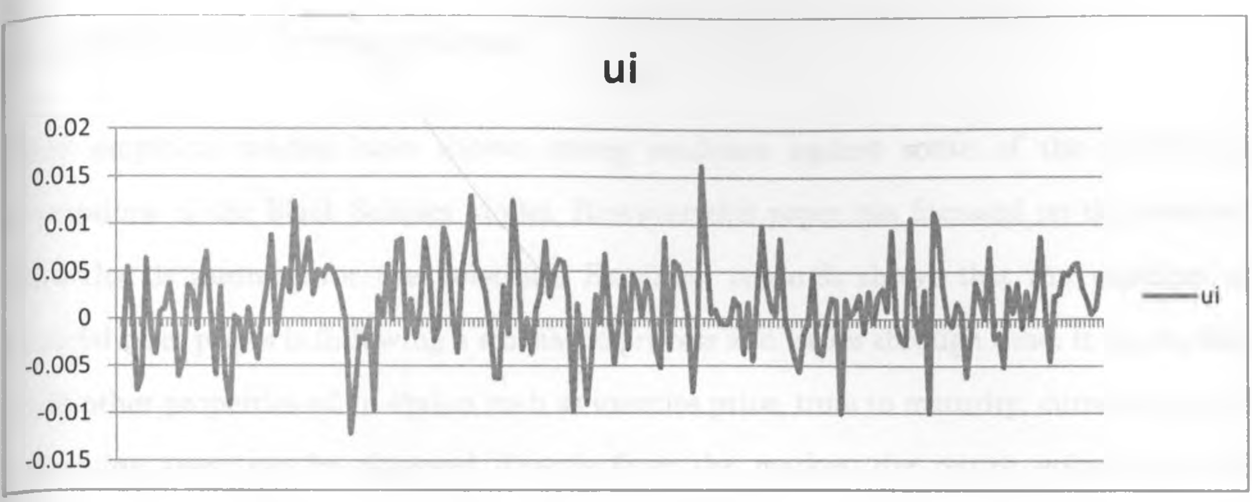


Table 4.3: Moving Average Results

Average	0.001223286
Standard deviation	0.004787981
annual standard deviation	0.076006838
Non centered volatility	2.4312E-05
Non centered standard deviation	0.004930722
Annual non centered standard deviation	0.078272791

Source: Own Calculations

Where $u_i = \frac{S_i - S_{i-1}}{S_{i-1}}$

The moving average is used as a benchmark for comparison with the other two models. The volatility displays clustering where periods of high are followed by consecutive periods of high. On the other hand when the market is low it tends to be low for some period. This is quite similar to the GARCH results above.

CHAPTER FIVE

5.1 SUMMARY AND CONCLUSIONS

Many empirical studies have shown strong evidence against some of the underlying assumptions of the Black Scholes Model. However this paper has focussed on the constant value that is assumed for the volatility. Empirical research shows that the volatility of financial asset prices is following a stochastic process and varies through time. It means that while other properties of an option such as exercise price, time to maturity, current price of underlying asset; can be observed directly from the market, the return volatility is the uncertainty factor in the Black Scholes model. Accurate measures and good forecasts of volatility are critical for option pricing theories as well as trading strategies. This study has highlighted the different measures that determine volatility, and some of them act as alternatives or improvement from earlier models.

We have compared three models: ARCH, GARCH and the Moving Average Model. GARCH is a good description of the evolution of the variance process of the asset returns. It provides a better evolution of asset returns than compared to the ARCH model. It also captures volatility clustering quite well. A disadvantage of the GARCH model is that it is computationally demanding. Therefore the development of tractable time dependent volatility models as well as more efficient methods of model parameter estimation are an area of intensive research.

5.2 FURTHER AREAS OF RESEARCH

The study can be extended to include other criticisms in the assumptions of the Black Scholes Model.

It can also be extended to investigate the effect of the volatility employed in the option price given that the Kenyan derivative market is at its infancy. As stated, Nairobi securities and derivatives market is to be established soon by the Capitals markets authority as part of their capitals markets master plan (CMMP) for Kenya's securities market for the next 5 – 10 years.

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APPNENDIX 1: BASIC DATA

The NSE 20 share index and change of the index for each day are displayed.

	NSE 20	u_i	u_i^2
1	3212.86		
2	3203.35	-0.00295998	8.76148E-06
3	3220.74	0.005428692	2.94707E-05
4	3224.87	0.001282314	1.64433E-06
5	3200.46	-0.007569297	5.72943E-05
6	3180.55	-0.006220981	3.87006E-05
7	3200.8	0.006366823	4.05364E-05
8	3196.86	-0.001230942	1.51522E-06
9	3184.92	-0.003734915	1.39496E-05
10	3187.22	0.000722153	5.21505E-07
11	3190.78	0.001116961	1.2476E-06
12	3202.57	0.003695021	1.36532E-05
13	3204.76	0.000683826	4.67618E-07
14	3185.14	-0.006122143	3.74806E-05
15	3171.63	-0.004241572	1.79909E-05
16	3182.88	0.003547072	1.25817E-05
17	3191.72	0.002777359	7.71372E-06
18	3188.23	-0.001093454	1.19564E-06
19	3202.34	0.004425653	1.95864E-05
20	3224.89	0.007041726	4.95859E-05
21	3224.18	-0.000220163	4.84715E-08
22	3205.01	-0.005945698	3.53513E-05
23	3215.7	0.003335403	1.11249E-05
24	3196.7	-0.005908511	3.49105E-05
25	3167.49	-0.009137548	8.34948E-05
26	3168.27	0.000246252	6.06399E-08
27	3167.87	-0.000126252	1.59395E-08
28	3156.87	-0.003472365	1.20573E-05
29	3160.51	0.001153041	1.3295E-06
30	3156.19	-0.001366868	1.86833E-06
31	3142.74	-0.004261467	1.81601E-05
32	3143.9	0.000369105	1.36238E-07

33	3154.46	0.003358885	1.12821E-05
34	3182.14	0.008774877	7.69985E-05
35	3176.36	-0.001816388	3.29926E-06
36	3183.01	0.002093591	4.38312E-06
37	3199.67	0.005234039	2.73952E-05
38	3208.63	0.002800289	7.84162E-06
39	3248.4	0.012394698	0.000153629
40	3258.43	0.003087674	9.53373E-06
41	3275.87	0.005352271	2.86468E-05
42	3303.75	0.008510716	7.24323E-05
43	3312.15	0.002542565	6.46464E-06
44	3329.16	0.005135637	2.63748E-05
45	3343.96	0.004445566	1.97631E-05
46	3362.59	0.005571239	3.10387E-05
47	3380.27	0.005257852	2.7645E-05
48	3394.29	0.004147598	1.72026E-05
49	3401.6	0.002153617	4.63807E-06
50	3399.97	-0.000479186	2.29619E-07
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52	3332.89	-0.007654975	5.85986E-05
53	3326.35	-0.001962261	3.85047E-06
54	3318.95	-0.002224661	4.94912E-06
55	3317.62	-0.000400729	1.60584E-07
56	3285.51	-0.009678625	9.36758E-05
57	3293.1	0.002310144	5.33676E-06
58	3293.91	0.000245969	6.05007E-08
59	3312.85	0.005750005	3.30626E-05
60	3312.56	-8.75379E-05	7.66289E-09
61	3339.27	0.00806325	6.5016E-05
62	3367.23	0.008373088	7.01086E-05
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66	3363.72	0.00107139	1.14788E-06
67	3392.23	0.008475735	7.18381E-05
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69	3400.48	-0.002411477	5.81522E-06
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71	3429.02	0.009476482	8.98037E-05
72	3454.34	0.007384034	5.4524E-05
73	3456.35	0.000581877	3.38581E-07
74	3443.94	-0.003590493	1.28916E-05
75	3461.19	0.005008798	2.50881E-05

76	3489.24	0.008104149	6.56772E-05
77	3534.27	0.012905389	0.000166549
78	3554.46	0.005712637	3.26342E-05
79	3571.2	0.004709576	2.21801E-05
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81	3579.57	-0.000491438	2.41511E-07
82	3557.13	-0.006268909	3.92992E-05
83	3534.53	-0.006353437	4.03662E-05
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85	3541.07	-0.001576131	2.48419E-06
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87	3611.1	0.007246619	5.25135E-05
88	3599.13	-0.003314779	1.09878E-05
89	3599.18	1.38922E-05	1.92995E-10
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93	3628.64	0.008143182	6.63114E-05
94	3637.08	0.00232594	5.41E-06
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107	3653.29	0.000668338	4.46675E-07
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109	3634.82	-0.00028604	8.18187E-08
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112	3657.01	0.004822144	2.32531E-05
113	3670.18	0.003601303	1.29694E-05
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115	3685.36	0.003980113	1.58413E-05
116	3694.23	0.002406821	5.79278E-06
117	3682.23	-0.003248309	1.05515E-05
118	3663.11	-0.005192506	2.69621E-05

119	3694.55	0.008582871	7.36657E-05
120	3682.24	-0.003331935	1.11018E-05
121	3704.7	0.006099548	3.72045E-05
122	3725.55	0.005627986	3.16742E-05
123	3738.15	0.003382051	1.14383E-05
124	3739	0.000227385	5.1704E-08
125	3709.84	-0.007798877	6.08225E-05
126	3703.94	-0.001590365	2.52926E-06
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128	3790.07	0.006950219	4.83055E-05
129	3791.79	0.000453817	2.0595E-07
130	3795.32	0.000930959	8.66684E-07
131	3793.32	-0.000526965	2.77692E-07
132	3791.06	-0.000595784	3.54959E-07
133	3789.33	-0.000456337	2.08243E-07
134	3797.4	0.002129664	4.53547E-06
135	3802.96	0.00146416	2.14376E-06
136	3788.64	-0.003765488	1.41789E-05
137	3795.1	0.001705097	2.90736E-06
138	3778.1	-0.00447946	2.00656E-05
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140	3825.93	0.009874568	9.75071E-05
141	3840.36	0.003771632	1.42252E-05
142	3844.61	0.001106667	1.22471E-06
143	3845.93	0.000343338	1.17881E-07
144	3878.49	0.008466093	7.16747E-05
145	3878.52	7.73497E-06	5.98297E-11
146	3870.51	-0.002065221	4.26514E-06
147	3854.28	-0.004193246	1.75833E-05
148	3832.42	-0.005671617	3.21672E-05
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151	3843.58	0.0048365	2.33917E-05
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153	3815.44	-0.003863988	1.49304E-05
154	3815.1	-8.91116E-05	7.94088E-09
155	3823.49	0.002199156	4.83629E-06
156	3831.01	0.00196679	3.86826E-06
157	3792.22	-0.010125267	0.000102521
158	3800.23	0.002112219	4.46147E-06
159	3801.03	0.000210514	4.4316E-08
160	3804.54	0.000923434	8.5273E-07
161	3814.1	0.002512787	6.3141E-06

162	3808.47	-0.001476102	2.17888E-06
163	3819.45	0.002883048	8.31196E-06
164	3817.7	-0.000458181	2.0993E-07
165	3826.89	0.002407209	5.79465E-06
166	3839.12	0.003195807	1.02132E-05
167	3842.38	0.000849153	7.21061E-07
168	3878.13	0.009304129	8.65668E-05
169	3875.11	-0.000778726	6.06414E-07
170	3865.76	-0.002412835	5.82177E-06
171	3855.14	-0.002747196	7.54709E-06
172	3895.86	0.010562522	0.000111567
173	3897.45	0.000408126	1.66566E-07
174	3888.14	-0.002388741	5.70609E-06
175	3899.62	0.002952569	8.71766E-06
176	3860.41	-0.010054826	0.0001011
177	3903.72	0.011219016	0.000125866
178	3941.1	0.009575482	9.16899E-05
179	3953.84	0.0032326	1.04497E-05
180	3953.53	-7.84048E-05	6.14731E-09
181	3950.18	-0.000847344	7.17992E-07
182	3956.54	0.001610053	2.59227E-06
183	3959.1	0.00064703	4.18648E-07
184	3934.52	-0.006208482	3.85452E-05
185	3927.44	-0.001799457	3.23805E-06
186	3942.4	0.003809097	1.45092E-05
187	3950.97	0.002173803	4.72542E-06
188	3950.9	-1.77172E-05	3.13898E-10
189	3980.53	0.007499557	5.62434E-05
190	3972.03	-0.002135394	4.55991E-06
191	3965.75	-0.001581056	2.49974E-06
192	3945.25	-0.005169262	2.67213E-05
193	3958.62	0.003388885	1.14845E-05
194	3961.05	0.00061385	3.76812E-07
195	3975.79	0.003721236	1.38476E-05
196	3971.68	-0.001033757	1.06865E-06
197	3983.16	0.002890464	8.35478E-06
198	3982.94	-5.52325E-05	3.05063E-09
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201	4032.41	0.000722174	5.21535E-07
202	4014.03	-0.004558068	2.0776E-05
203	4023.55	0.002371681	5.62487E-06
204	4034.07	0.002614607	6.83617E-06

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206	4072.5	0.004615434	2.13022E-05
207	4095.26	0.005588705	3.12336E-05
208	4119.5	0.005919038	3.5035E-05
209	4132.91	0.003255249	1.05966E-05
210	4141.23	0.002013109	4.05261E-06
211	4143.35	0.000511925	2.62067E-07