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RANGE-BASED APPROACH TO VOLATILITY MODELLING AND FORECASTING VALUE-AT-RISK

Research Report in Mathematics, Number 20, 2020

FRANCIS MAKORI NYAMACHE

June 2020

Submitted to the School of Mathematics in partial fulfillment for a degree in Master of Science in Actuarial Science

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Master Thesis
Submitted to the School of Mathematics in partial fulfillment for a degree in Master of Science in Actuarial Science

Submitted to: The Graduate School, University of Nairobi, Kenya

Abstract

The purpose of this thesis is to model and forecast value-at-risk based on range-measuring rather than the commonly acknowledged volatility models that are based on closing prices. The use of close-to-close prices in modelling and forecasting value-at-risk might not capture important intra-day information about the price movement. As a result, crucial price movement information is lost and consequently the model becomes less efficient. This thesis recommends the inclusion or range-measuring, described as the difference between the highest and lowest prices of an underlying stock within a time interval, a day, to compute Value-at-Risk. The project uses data of an NSE-listed and trading company, SASN, between November 2009 and November 2019 on which the predictability of range-based and close-to-close estimates was established. It was observed that the values obtained by range-based models were more accurate than when only the daily closing prices are used. The range-based models successfully capture dynamics of the volatility and achieves improve performance relative to the GARCH-type models. These findings are fairly consistent and can be extended to applications like portfolio optimization.

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Declaration and Approval

I the undersigned declare that this dissertation is my original work and to the best of my knowledge, it has not been submitted in support of an award of a degree in any other university or institution of learning.

Signature Date

FRANCIS M. NYAMACHE
Reg No. I56/12745/2018

In my capacity as a supervisor of the candidate's dissertation, I certify that this dissertation has my approval for submission.

Signature Date

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List of Abbreviations and Acronyms

AIC Akaike Information Criterion

ASMF Average Square Magnitude function

BIC Bayes Information Criterion

CARR Conditional Autoregressive Range

CDF Cumulative distribution function

DCF Conditional Density Function

KCB Kenya Commercial Bank

GARCH Generalized Autoregressive Conditional Heteroscedasticity

ME Mean Error

MSE Mean-Square Error

NSENairobi Stock Exchange

RGARCH Range Generalized Autoregressive Conditional Heteroscedasticity

RTARCH Range Threshold Generalized Autoregressive Conditional Heteroscedasticity

TARCH Threshold Generalized Autoregressive Conditional Heteroscedasticity

QLIKE QLIKE Loss Function

SASN Sasini

VR Violation Ratio

Chapter 1

Introduction

1.1 Background of the Study

Volatility of assets plays a vital role in finance. As a measure of riskiness, volatility is key in asset pricing, derivatives pricing, risk management, and portfolio management. A good understanding of return volatilities and accurate estimates are valuable to practitioners in the financial field as well as players in the financial field including independent investors and traders.

In the recent, a number of banks have collapsed in Kenya as well as retail companies which have significantly impacted the portfolio stability of investment banks and private investors. Poor risk management and lack of proper oversight, regulation, and assessment of the financial sector have been cited as among the major factors that contributed to the collapse of these banks. Gathaiya [2017] established that CBK had put more focus on macro-prudential regulation that relates to factors affecting individual banks while giving less focus on factors affecting stability of the entire financial sector. Accurate volatility estimates are essential in the stability of financial institutions especially banks plagued with non-performing loans. For instance, Dubai Bank was put under receivership because of its deteriorating cash reserve ratio Gathaiya [2017].

Due to such failures of investment banks and financial institutions, the need to model and forecast volatility more accurately has risen gradually, with the aim that accurate volatility estimates can help. The need for accurate volatility estimates has also grown as the financial markets around the world move towards deregulation and globalization. This includes entry into financial markets in developing countries and emerging markets especially by foreign investors who are much interested in the stability of the financial markets and the entire economy. The input into the global risk management models has therefore continued to grow and the threat of global spillover effects enlarges due to the use of inaccurate volatility estimates. Volatility quantifies the dispersion of returns. However, volatility is time-varying and not easy to predict because volatility is not observable, and the volatility forecasts are affected by changing estimates of the levels of volatility at the time period.

Kenya as an emerging market has attracted significant attention from investors and consequently practitioners and researchers in the financial field. Kenya's stock indices have yielded high returns becoming appealing and decoupling in demand from investors. However, the returns are not consistent hence the need to establish a volatility model that is appropriate for the Kenyan stock market. Kenya offers more exciting long-term investment opportunities which encourages the analysis of stock market data with the range-based models to establish whether the range-based model provides more accurate forecasts.

1.1.1 Risk Diversification

Geographic diversification of assets is an important requirement in minimizing risk exposure in stock portfolios or pension funds chou. The low correlation between the emerging markets and developed markets have attracted foreign investors to the emerging markets Sharma et al. [2016]. The classical time series models such as the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) models, stochastics volatility models, and the implied volatility measures like realized volatility are commonly used in forecasting volatility. GARCH-type models are used to model time-varying conditional volatility due to their simplicity and easy approach to estimation ad flexibility in terms of volatility dynamics.

GARCH-type models are developed using data on closing prices, that is, the daily returns. This way, important information about intraday price movement might be neglected. For instance, if the closing price of two consecutive days is equal, the return is be zero. Howver, the changes in prices during the day might be explosive and the typical GARCH model fails to capture this information. Also, the GARCH-type models are based on the moving averages with weights that decay gradually hence slow to adapt the changes in levels of volatility Sharma et al. [2016]. Some studies attempt to overcome this drawback using intraday GARCH models. A simpler approach to modeling the intraday variation is the use of price range. Range is defined as the difference between the highest and lowest prices over a given sampling interval such as daily or weekly. This paper uses the day-to-day variability Sharma et al. [2016]. The estimates obtained using range apporach are more effective compared to estimates obtained using the close-to-close return approach.

1.2 Justification of the Study

Financial institutions including investment banks, individual and corporate investors rely on volatility estimates to make investment decisions. Knowing the accuracy or a measure is crucial. For instance, they use Value-at-Risk to estimate the amount of cash they require to reserve for covering potential losses. While the historical VaR is extremely simple to calculate and explain even to non-risk professionals, it comes with its shortcomings. Banks for instance combine portfolio sensitivity to market changes and probability of a given change in the market to measure market risk. This makes the need for accurate estimates crucial. Any inaccuracies will imply that the institution is not reserving sufficiently for potential losses. Currently, there is no study that has covered range-based volatility and empirically tested using of an NSE-listed and trading company data. This study attempts to fill up the research gap.

In the recent past, banks in Kenya has been put under receivership. [Gathaiya, 2017], established that poor risk management and poor financial sector oversight as among the major reasons contributing collapse of Banks such as Imperial Bank, Dubai Bank, and Chase Bank. A research is required to establish better and more accurate volatility measures enabling players in the financial market have a better oversight and improved risk management strategies. This research, being the first of the kind in Kenya, will set a foundation for further research and give it the required attention.

Hence, evaluation of volatility using NSE-trading company data could provide an outlook of how the Indices and entire economy is performing. It will give an insight into further research and application of the volatility approach recommended in this paper to the entire market.

Table 1.1 shows details of the selected company, category as listed in the NSE, and the stock ticker:

No.	Company Names and Stock Ticker	Category
1.	Sasini Ltd (SASN)	Agricultural

Table 1.1: NSE-listed Company Data

1.3 Statement of the Problem

Accurate volatility estimates are vital in financial applications such as portfolio optimization, derivatives and options pricing. The historical volatility estimators that are commonly used are less accurate. While other markets, developed economies, have tested range-based models and forecasting on the stock prices, there is little research on the same in emerging markets. The widely used and well-established return-based volatility measured used in time series analysis such as development of GARCH-type models suffer inefficiency. The first challenge with historical models arises from the fact that volatility is not directly observable hence need to address the problem of volatility measurement.

Based on the price returns over several days, we obtain volatility of the stock returns which is typically defined as the squared returns or standard deviation of the returns. This approach only helps obtain the average volatility over the sampled time period hence not sufficient since the volatility changes on daily basis. If close-to-close prices are used to estimate volatility on daily basis, the estimate is referred to as the squared daily return. Squared daily return is noisy. Since it is the only that we have, it is commonly used in models such as GARCH-type in which the squared returns are used and processed by applying the moving average. If there is a high frequency of intraday data for whole price process for the day, it is possible to estimate daily volatility. For most financial data, the highest and lowest daily prices are available. Range-based volatility estimation seeks to address the shortcomings of the squared return and standard deviation approaches to estimating volatility.

The growing need for accurate volatility estimators, need for application on empirical data in emerging markets such as Kenya motivated the evaluation whether range provides additional information to the volatility process helpful in improving forecasting compared to the GARCH-type approaches. One benefit of utilizing the range approach is that the financial assets contain useful information about the movement of prices within a given time period whereas the use of squared return only include the closing prices. This is an important feature especially when the market experiences large price swings Chou and Wang [2014]. The interest to study the application of range-based approach to volatility forecasting has grown tremendously in finance literature in developed markets. However, the areas has not be widely explored in emerging markets. The linkages in markets may change due to a financial crisis and volatility is viewed as a vital channel for the changes. Because of market linkages, financial crises significantly spread and impact financial economies on a global scale. We use the range-based volatility model introduced by Chou and Wang [2014] to evaluate changes in volatility dependence structure.

1.4 Hypothesis of the Study

Return based volatility estimation uses the open, close and adjusted close prices to calculate returns while range-based volatility approach uses the difference between highest and lowest stock prices. The open close price can be very low and the open price in the next trading day be high due to market conditions. However, if in the previous trading day there was a price slump during the day, then there might be market players who are significantly impacted if they transacted when the price was at a the lowest.

This study hypothesizes that range-based volatility approach can guard the market players from such price slumps. Another null hypothesis is that range-based volatility approach gives a better estimate of volatility compared to return-based VaR approach. It is also hypothesized that range provides additional intraday information about the movements of stock prices during the day.

1.5 Objectives of the study

Majority of past research studies have focused on return-based volatility estimators evident from their wide application in the finance and investment field. For research studies involving range-based volatility approach, the researchers have focused on developed markets such as Europe and America. Although emergent markets such as Kenya present exciting long-term investment opportunities, there is no research that has empirically tested the Kenyan stock market using the range-based volatility estimators.

1.5.1 General Objective

The general objective of this study is to test the range-based volatility estimation using data of a company listed and trading at the NSE.

1.5.2 Specific Objectives

The Specific objectives of this study are:

1. To test the accuracy of range-based volatility model in the Kenyan market by establishing whether the approach provides additional information.

2. To assess which among the range-based models, RGARCH and CARR is better than the other.

1.6 Significance of the Study

This study uses data from the Kenyan stock market to assess the accuracy of range-based volatility models. An accurate measure of volatility will be significant to the different stakeholders in the financial sector including savings and investment banks and institutions, traders in the stock market, and individual investors. The findings of the study will not only be applicable to the stock exchange but also in the entire financial sector benefiting corporations as well as individuals.

With this study, practitioners in the financial sector can blend its concepts and finding with technology to further financial technology such as development of virtual investment advisors. The findings of the study will benefit practitioners in the financial sector. Banks and financial institutions will use the findings to develop improved risk management strategies. With more accurate value at risk estimates, financial institutions such as banks will be able to allocate sufficient capital for reserves to cover for potential losses.

Chapter 2

Literature Review

It is conventional that asset returns take a normal distribution. However, carrying out a joint multivariate distribution test gives misleading conclusion about the dependence of assets. Because of the variations in behavior of assets returns during different periods than normal periods, proper asset allocation and even hedging may not be enough to protect asset returns against unexpected losses Li and Hong [2011]. Various researchers have studies extreme risks using different return-based volatility models for the stock market while little research exists for range-based models in emerging markets such as Kenya. Various research studies in range-based volatility models have shown better performance than return-based models. Parkinson (1980) in his research on measurement of errors in return-based volatility estimates established that the errors were higher and hence yielded to inefficiency of the model. These findings supported the desire to investigate whether range provides additional information on price movements and how it can be applied in the Kenyan market. In another research conducted by Brandt and Jones [2002] to investigate ranged-based volatility approach to measuring volatility using stochastic models reported improved efficiency and robustness to market microstructure noise.

Andersen et al. [2003] conducted a research on modeling and forecasting realized volatility using high-frequency intraday data. The researchers developed links between realized volatility and conditional variance with the aim of establishing whether the models could predict volatility with higher accuracy and hence helpful in the currency exchange market. By treating volatility as observed rather than latent, the approach facilitated the modeling and forecasting with the use of simple methods that are based on directly observable variables Andersen et al. [2003]. Guided by the general theory for continuous-time arbitrage-free price processes, the researchers developed a forecasting framework for realized volatility and correlation. The study established that the range-based framework produced successful volatility estimates and generally dominating the estimates from conventional GARCH and related approaches. Additionally, the forecasts were well-calibrated and associated with VaR for multivariate foreign exchange applications. Volatility forecasts are not only useful in practical financial decisions but they extend beyond risk modeling and management into sport and derivative asset prices Andersen et al. [2003]. The researchers recommended that further studies explore the gains achieved by simple volatility modeling and forecasting procedures that rely on range.

Research by Brandt and Jones [2006] supports the literature by Andersen et al. [2003]. Brandt and Jones [2006] studying range-based exponential GARCH (EGARCH) established that the range-based model performed better compared to return-based EGARCH models for both in-sample and out-of-sample volatility estimates. In their research, Brandt and Jones combined two-factor EGARCH models with data on the range and used empirical analysis of SP 500 index to investigate model superiority based on efficiency of the forecasts. The researchers established that range incorporated information into EGARCH models which significantly improved in-sample fit and the accuracy of out-sample forecasts of the models. In a two-factor and fractionally integrated EGARCH models, range incorporated volatility asymmetry that dominated both in-sample and out-of-sample forecasts. The research also established that log returns are more likely to give rise to biased estimates than log range returns because return-based log has idiosyncratic noise.

In another research, Petneházi and Gáll [2019] studied how predictable the range-based estimates could be. The results were compared against those obtained using close-to-close returns, commonly applicable in practice. Testing the models using Dow Jones Industrial Average index, the researchers established that the direction of changes in the estimates obtained using range-based models were more predictable than that of the return-based values. Petneházi and Gáll [2019] outlines known properties of volatility which include persistence, mean reversion, asymmetric effect, and influence of exogenous variables. According to these features of volatility, volatility should be forecastable. However, since it cannot be measured, developing reasonable proxies is the best approach. Such proxies include the standard deviation of returns calculated using daily closing prices. However, following the daily closing prices, it is not possible to tell the price movements during the day which are not captured by close prices. The Open, Close, High, and Low price data is readily available unlike high freqeuency data such as minutely or hourly price quotes. Although finding accurate volatility estimates using these daily values only is challenging it is important to ensure accurate estimates are obtained. In this research Petneházi and Gáll [2019] found it easier to obtain volatility estimates using the range-based approach than using close-to-close approach.

Anderson et al. [2015] also studied range-based volatility approach to measure volatility contagion in securitized real estate markets. The researchers used a time-varying ranged-based model to capture information about the dynamics of volatility in a securitized real estate. Anderson et al. [2015] from the empirical analysis established that it is possible that economic crisis in one market, adverse volatility spread and affected linked markets. Using copula functions and volatility model, the researchers were able to explain the pattern of in the extreme tails of the distributions as related to establishing Value-at-Risk. Anderson et al. [2015] found out that there exist linkages between REIT markets worldwide. This implies that, REIT returns in one market such as Europe can affect the REIT returns in the United States of America. Also, there is high correlation of the markets during downturn than up movement of prices and so returns. This enriches our study by showing that based on prior research findings, there is a way markets are related and hence just regional diversification is not enough to cushion assets and investments from risk. Another literature focuses on value of volatility timing in an economy using range-based approach. Chou [2005] used range-based volatility model to investigate the economic value of volatility timing in a mean-variance framework. The researchers also compared the performance of return-based volatility model in both in-sample and out-of-sample volatility timing strategies with the range-based volatility model. The CARR model was used with SP 500 data. The researchers concluded that the range-based approach had more economic value than the return-based model approach. This supports previous research studies.

Another literature by Jinghong Shu and Jin Zhang focused on testing range estimators of historical volatility by incorporating daily trading range Shu and Zhang [2006]. The researchers computed the mean error (ME) which is the average difference between the estimated variance and the actual variance. Mean-square Error (MSE) was also computed which is the average of the squared error and the relative error which is the percentage difference between the mean estimated variance and the true variance Shu and Zhang [2006]. Efficiency of the estimators was the concern of the researchers. Efficiency was calculated using Parkinson estimator in which it states that the larger the ratio, the more efficient the estimator. Testing range-based models using data of SP 500 index, the researchers established that range estimators performed well when assets followed a continuous geometric Brownian motion. It was noted that, opening jump or a large drift led to differences in range estimators Shu and Zhang [2006]. The researchers also established that range estimators are fairly robust toward effects of non-market factors such as bid-ask bounce and asymmetric information of traders. This study extends the research by including empirical evidence from an emerging market.

Literature by Ripple and Moosa [2009] studying the effect of maturity, trading volume, and open interest on crude oil futures price range-based volatility supports the findings by Shu and Zhang [2006] that range-based approach provides more efficient estimates compared to historical volatility approach. Ripple and Moosa examined the factors that determined the futures prices volatility for crude oil using intraday range-based approach. The study used 131 contract-by-contract analysts and tested the model specification stepwise using non-nested tests. The research established that sudden jumps and drops in the price of assets and stock prices or currency pair price had significant impact on the price movements Ripple and Moosa [2009]. For instance, a drop in price shaped the trader's opinion whether to keep their position open or closed. This affected the trade volumes as well as the behavior of other market participants. Incorporating the difference between highest and lowest price added crucial information helpful in predicting volatility with a higher accuracy.

Akay et al. [2010] supports the findings of Shu and Zhang [2006] by developing a range-based volatility measure for federal fund market. The results showed that range-based models exhibited higher efficiency and were more robust to microstructure noise. Akay et al. [2010], examined the robustness of range estimators used and established that the models were more robust supporting the findings by Parkinson [1980]. The researchers examined Parkinson's range-based volatility estimate in the federal funds market and compared it with return-based standard deviation. Range-based estimates were more accurate in distinguishing liquidity crisis (simultaneous rise and drop in liquidity demand due to factors like negative market shocks) from normal daily trading.

Maciel and Ballini [2017] conducted a study on the accuracy of Value-at-Risk model and forecasting with range-based models relying on the empirical data of SP 500 and IBOVESPA, U.S. and Brazilian economies. The researchers compared GARCH-type approaches and the Conditional autoregressive range (CARR) model. The out-of-sample results showed that range-based volatility models provided a more accurate VaR forecasts than GARCH models Maciel and Ballini [2017]. The researchers established that out-of-sample results indicated that range-based volatility models offered additional informational to the historical GARH and TARCH-type models. Additionally, the models achieved more accurate VaR forecasts when range was included as an exogenous variable in variance equation for both developed economies and developing economies.

The significance of this literature to our study is that it validates the research hypothesis by demonstrating that range adds additional information to volatility modeling. The use of Brazilian and American economies aims at demonstrating volatility contagion as well as applicability of range-based models in different economies. The different literatures have focused on studying range-based approach to estimating volatility in established economies including China, United States of America, and Europe as well as developing economy using Brazilian Stock market data. This research extends the study by modeling and forecasting volatility using the data of Sasini PLC. This helps establish if the range-based approach is more accurate than return-based approach to estimating volatility in developing economies. Most of the literature has focused on developed markets while little attention given to emergent markets. Minimizing risk through regional diversification of assets cannot be fully achieved because risk in one economic market affects returns in another market. This makes the study of volatility in developing economies important. This research contributes to the study of range-based volatility modeling by providing empirical analysis and evidence for the application of range in developing economies.

Chapter 3

Methodology

This section reviews the methods, data, and the performance measurements used in the study. The fundamental concepts of VaR modeling and forecasting are also discussed. It provides an overview of the historical approaches to volatility modeling, GARCH and TARCH-type models as well as range-based volatility, and the Conditional Autoregressive (CARR) methodology.

3.1 Introduction

Volatility is widely applicable in the finance field. As a measure of riskiness, volatility is a key factor in portfolio management, risk management, and option pricing. Volatility quantifies the dispersion of returns. The volatility of assets varies with time. However, volatility is not observable directly and needs to be estimated. Although predictable, forecasting the future volatility levels is challenging. Daily returns based on close to close has been widely accepted as the approach to estimating volatility by computing the dispersion of the returns. However, return-based has drawbacks including inaccuracy of volatility estimates. Range-based volatility approach aims to address the drawbacks of return-based models. Range is defined as the difference between the highest and lowest market prices over a given sampling interval.

Rational investors do no prefer securities with higher volatility resulting in a positive link between risk and the assetsś performance in the future. Empirical evidence however shows that there is a mix when establishing the relationship between volatility and future returns. When the volatility is high, the investors discount the stocks and focus on stocks with higher returns and promising higher returns in the future, at a given volatility levels.

3.2 GARCH and TARCH Models)

The simplest method for modeling returns is given by;

$$r_t = \sigma_t \varepsilon_t \tag{3.2.1}$$

where $r_t = ln(P_t) - ln(P_{t-1})$ is the log return of the asset at the time t. P_t is the price of the asset at time t, while ε_t is independent and identically distributed. That is ε_t $(0,1)$, it is a zero-mean noise. The assumption is thatε_t is normal and σ_t is the volatility of the asset, and that it varies with time. That is, it does not assume constant volatility. The differences in σ_t specifications define the differences in volatility models.

Bollershev (1986) introduced the GARCH model to extend the CARR model pioneered by Engle (1982). CARR model allows the inclusion of conditional variance in the variance equation. GARCH is a widely applicable model in modeling volatility because it is flexible and accurate in modeling known properties of financial assets such as clustering and leptokurtosis.

A $GARCH(p,q)$ model is defined as below;

$$r_t = \sigma_t \varepsilon_t \tag{3.2.2}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i r_{t-1}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2, \tag{3.2.3}$$

whereby: $\omega > 0$ is a constant and α is a coefficient that measures the short term effect of the ε_t on the conditional variance while $\beta_1 \geq 0$ is a coefficient that measures the long-term effect on the conditional variance.

The Threshold ARCH (TARCH) model is an asymmetric approach developed assuming sudden changes in returns of assets leading to varying effects on conditional variance. That is, the response of variance to positive and negative shocks in price is different, thus the conclusion of the asymmetric impact. Glosten et al. [1993] define $TARCH(p,q)$ as

$$r_t = \sigma_t \varepsilon_t \tag{3.2.4}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i r_{t-1}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 + \sum_{i=1}^{p} \gamma_i r_{t-1}^2 I_{t-} \tag{3.2.5}$$

where $I_{t-1} = 1$ if $r_{t-1} < 0$ refers to negative effect and $I_{t-1} = 0$ if $r_{t-1} \geq 0$ refers to positive positive. Gamma parameter, γ_i measures the asymmetric effect or presence of leverage. A positive gamma value indicates the presence of leverage effect and a value of zero for gamma implies asymmetric effect.

3.3 Range-Based Volatility

Different range estimators are considered for modeling volatility. Parkinson [1980] proposed a range-based volatility estimator that includes open and close prices and not just close to close prices. Parkinson [1980] proposed an improved estimator version that used High and Low to calculate range-based returns. This study used Garman-Klass range estimator because it describes the volatility dynamics and is a similar measure to CARR model.

3.3.1 Garman-Klass

The problem with Parkinson volatility estimate is that it fails to take into account the opening and closing price. Markets are most active during the open and close sessions hence failing to include this important information is a non-negligible drawback of the model. At the opening and closing, market factors can contribute to high or low returns due to the number of market participants and number of trades being made. Garman Klass volatility estimate incorporates intraday information. The Garmn Klass model used for estimating range-based return seris is given by

$$\sigma_{GK} = \sqrt{\frac{1}{2T}\sum_{t=1}^{T}\ln(\frac{H_t}{L_t})^2 - \frac{2\ln 2 - 1}{T}\ln(\frac{C_t}{O_t})^2} \tag{3.3.1}$$

Where: T is Number of days in the sample period O_t Opening price on day t

H_t High price on day t

L_t Low price on day t

C_t Close price on day t

This is calculated by starting with the scaling factor which equals to the number of trading days in a year. This study used 252 trading days hence no needs for scaling because the number of trading days is equals to the sample size n, of trading days in the year, $N = n$.

3.3.2 Range-Based GARCH (p,q,s)

Range is the difference between the highest and lowest prices. In this study, range is the difference between the highest and lowest log prices of the stock. It is expressed as H_t for the High price and L_t for the lowest price reached in a logarithm type for the trading day t. Chou and Wang [2014] defines range of log returns as

$$r_t = \ln(H_t) - \ln(L_t) \tag{3.3.2}$$

Two types of range volatility models were used in which realized range was includes as an exogenous variable. Range variable was considered in the traditional GARCH and TARCH models to obtain RGARCH and RTARCH models respectively. The purpose is to establish if range provides additional relevant information to the volatility process, which can be helpful in obtaining better and accurate volatility forecast. The equations can are rewritten below to denote the Range GARCH model $(RGARCH)(p,q,s)$.

$$r_t = \sigma_t \varepsilon_t, \tag{3.3.3}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i r_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 + \sum_{k=1}^{s} \Theta_k R_{t-k}^2, \tag{3.3.4}$$

where Θ_k, for $k = 1$ to s is the parameter that measures the effect of the additional information provided by range to the volatility process.

3.3.3 Ranged-Based TARCH (p,q,s) Model

The Threshold ARCH model with range is denoted as $RTARCH(p,q,s)$

$$r_t = \sigma_t \varepsilon_t \tag{3.3.5}$$

$RTARCH(p,q,s$ is defines as;

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i r_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 + \sum_{i=1}^{p} \gamma_i r_{t-i}^2 I_{t-i} + \sum_{k=1}^{s} \Theta_k R_{t-k}^2 \tag{3.3.6}$$

3.3.4 Conditional Autoregressive $CARR(p,q)$

The second class of range-based model used in this study is the CARR model suggested by Chou [2005]) which entails a special case of multiplicative error model (MEM), proposed by Engle [2002] and extended for GARCH approach. The multiplicative error model is used to model non-negative valued processes like duration and realized volatility. Chou [2005] proposed focus on the price range process rather than log range. Conditional Autoregressive (CAR) models have been widely utilized in analyzing spatial data in the finance field as models for both observable and latent variables. The focus us to unveil and quantify how the quantities of interest change with explanatory variables and detect clusters. This model is based on the concept that the probability of the predicted values at any given time are conditional on the level of adjacent values. Chou [2005] defined price range, R_t, and the $CARR(p,q)$ model as;

$$R_t = h_t \varepsilon_t, \tag{3.3.7}$$

$$h_t = \omega + \sum_{i=1}^{p} \alpha_i R_{t-i} + \sum_{j=1}^{q} \beta_j h_{t-j}, \tag{3.3.8}$$

Whereby h_t which is the value of conditional range up to time t, and the error term ε_t takes a standard normal distribution. The density function $f(*)$ has a mean of 1. Ranged-based CARR approach uses price range (R_t) model process while GARCH models use asset returns (r_t) to model conditional variance. The model has similar assumptions to those of GARCH apprpach in modeling of asset returns and also include time-varying realized variance as an exogenous variable for distinguishing the models.

3.4 Evaluating the Volatility Forecast

The performance of the forecast is evaluated using statistical loss functions. Since true volatility cannot be observed directly, bias may arise among the competing models when estimating and forecasting volatility. the estimation. Maciel and Ballini [2017] compared different loss functions for volatility forecasting and established that mean squared error (MSE) and quasi-likelihood (QLIKE) loss functions cannot be easily affected by extreme values when estimating volatility hence the two loss functions can be used to evaluate the models for volatility forecasts.

MSE symmetrically penalizes forecasting errors while QLIKE is asymmetric hence penalizes under-estimation more than over-estimation. This makes the model more suitable for use in areas such as risk management and forecasting Value-at-Risk where under-estimation can be costlier than over-overestimation Sharma et al. [2016]. MSE functions is defines as;

$$MSE = E(\sigma_t^2 - \widehat{\sigma}_t) \tag{3.4.1}$$

QLIKE Loss function is defined as

$QLIKE = E(\log(\widehat{\sigma}_t^2 + \sigma_t^2 \widehat{\sigma}_t^{-2}$ where $\widehat{\sigma}_t^2$ is the forecasted variance and σ_t^2 is the actual or observed variance. Based on the CARR model, $\widehat{\sigma}_t^2 = \widehat{h}_t^2$ which is the realized variance and computed as,

$\sigma_t^2 = \sum_{j=1}^{\frac{1}{\Delta}} r_{t+j*\Delta,\Delta}^2$,

where;

$$r_{t,\Delta} = \ln(P_t) - \ln(P_{t-\Delta}) \tag{3.4.2}$$

is defined as the sample of the delta-period return where delta can be equal to 1-minute quotations. This study uses 1-day quotations.

For both MSE and QLIKE functions, smaller values indicate higher accuracy of the model. The forecasting measures that are widely applicable in practice do not reveal which model is statistically accurate than the other. Additional testes are required to compared the competing volatility forecasting models to establish which model is better than the other in terms of accuracy. This study utilized the Diebold-Mariano (DM)Diebold and Mariano [1995] Statistic test to assess the null hypothesis of equal accuracy in the forecasts between the competing models. It is assumed that the losses arising in the models i and j are defined in L_{it} and L_{jt} whereby;

$L_t = \sigma_t^2 - \widehat{\sigma}_t^2$

Diebold-Mariano test is used to verify the null hypothesis by testing the equal accuracy hypothesis for the models.

$E(L_t^i) = E(L_t^j)$

The null hypothesis for equal accuracy of the models equation is defines as;

$H_0 : E(d_t) = 0$

The DM test is given by;

$$DM = \frac{\overline{d}}{\sqrt{Var(\overline{d})}} \tag{3.4.3}$$

whereby; $\overline{d} = T^{-1} \sum_{j=1}^{T} d_{t+j}$, T is refers to the total number of forecasts while $\overline{d}$, and $Var(\overline{d}$ is give by the HAC estimator. Diebold and Mariano (1995) established that the test statistic assumes a standard normal distribution.

3.5 Estimating and Validating Value-at-Risk

Risk analysis is utilized in the assessment of the performance of the estimates. Different forecasting methods are assessed for efficiency using an economic criteria. Value-at-Risk (VaR) measure is used to determine the possible market risk value of a financial asset that will be lost over a given time horizon h, at a given significance level $\alpha_V aR$. It can also be used to determine the market value loss on an asset which is not expected to be exceeded with probability $1 - \alpha_V aR$.

VaR is defined as;

$\Pr(r_{t+h} \leq VaR_{t+h}^{\alpha_V aR}) = 1 - \alpha_{VaR}$ Here, α_{VaR}^{-th} quantile of the conditional distribution of returns is the Value-at-Risk. It is described as $VaR_{t+h}^{\alpha^{VaR}} = CDF^{-1}(\alpha_V aR)$. The Conditional Density Function $DCF(*)$ refers to the cumulative distribution function and $CDF^{-1}(*)$ is the inverse. This study considers the time period $h = 1$ which is the daily frequency because daily time-periods give the greatest practical interest for the Kenyan Stock Market.

The parametric VaR at time $t+1$ is defined as;

$VaR_{t+1}^{\alpha_V aR} = \widehat{\sigma}_{t+1} CDF_z^{-1}(\alpha_V aR)$

where; $\widehat{\sigma}_{t+1}$ refers to the predicted volatility at time $t+1$ which can easily be extracted from the model by finding sigma in fitted values. $CDF_z^{-1}(\alpha_V aR)$ refers to the critical value obtained from the normal distribution table at α-confidence level. $VaR_{t+1}^{\alpha_V aR} = \widehat{\sigma}_{t+1} CDF_z^{-1}(\alpha_V aR)$ was obtained using return-based volatility models TARCH and GARCH. Volatility range as an exogenous variable for the models RGARCH and RTARCH models was obtained.

Historical simulation was used to perform non-parametric VaR estimates. The historical VaR estimation focuses on developing a cumulative distribution function (CDF) for asset returns over time. Historical simulation does not take a particular distribution for the asset returns unlike in the parametric VaR models. Additionally, the assumption is that the returns of assets is independent and identically-distributed, a claim which the use of data and analysis refutes by showing that the returns are not independent and also exhibit patterns like clustering. Historical simulation approach assigns all returns equal weights throughout the period under consideration. As a result, it requires assessing the VaR forecasts for accuracy. The Violation ratio (VR) and the average square magnitude function are used to assess the performance of VaR forecasting models. Violation Ratio is defined as the percentage difference in exceedance. That is, how higher the actual losses or actual VaR was compared to the maximum estimated loss or VaR. This is defined as;

$VR = \frac{1}{T} \sum_{t=1}^{T} \delta_t$

whereby: $\delta_t = 1$ for $t_t < VaR_t$ and $\delta = 0$ for $r_t \geq VaR_t$. When VaR_t is the one-step-ahead forecasted VaR for day t, and T is the total number of observations in the sample. In this study, T is 20 and 100. Note that a lower VR does not always imply better performance. A violation of $\alpha_V aR\%$ is expected if the confidence level used in estimating VaR is $(1 - \alpha_V aR)\%$ If the Volatility Ratio VR is lower or greater than the violation, $\alpha_V aR\%$, it implied that the VaR has been overestimated or underestimated which infers low accuracy in the model which could result in practical implications. . For instance, investors or practitioners can change their investment positions based on VaR alert-based strategies.

3.5.1 Average Square Magnitude Function (ASMF)

The ASMF function takes into account the amount of possible default to measure the cost of exception in the model. It measures the impact of the exceptions on the accuracy of the model. ASMF is computed as;

$$ASMF = \frac{1}{\vartheta}\sum_{t=1}^{\vartheta}\xi_t. \tag{3.5.1}$$

ϑ is the number of exceptions model in the model. $\xi_t = (r_t - VaR_t)^2$ when $r_t < VaR_t$ and $\xi_t = 0$ for $r_t \geq VaR_t$

ASMF makes it possible to distinguish between models that exhibit similar or identical rates. Lower values of ASMF and VR imply higher accuracy of the forecasting model because VaR estimates the potential loss. The accuracy of VaR is significant in making financial and investment decisions. Statistical tests are required to verify the validity of VaR estimates because VaR estimation makes restrictive assumptions.

The measures used are Unconditional and conditional average tests. Kupiec [1995] suggested the used of unconditional coverage test LR_{uc} to evaluate the statistical consistency of unconditional coverage rate at level of confidence that the VaR model prescribes. This study prescribes $\alpha = 1\%$.

The failure probability is hypothesized for each trial $(\widehat{pi})$ that is equal to the probability specified by the model, $(\alpha_V aR)$.

A failure occurs when the forecasted VaR is less that the realized loss hence it cannot cover it. The test statistics for the unconditional coverage LR Test is given by;

$$LR_{uc} = -2\ln[\frac{\alpha_{VaR}^{forecat}}{(1-\alpha_{VaR})^{T-f}}(\widehat{\pi})f(1-(\widehat{\pi}))^{T-f}]\ \chi_1^2 \tag{3.5.2}$$

whereby; $\widehat{\pi} = f/T$, which is the failure rate. $\alpha_V aR$ is the maximum likelihood estimate and $f = \sum_{t=1}^{T}\delta_t$ defines a Bernoulli r.v representing the number of violations for the observations, T. The number of violations is tested against the hypothesis that the failure rate is not equal to $\alpha_V aR$ enabling the verification whether the observed violation rate is statistically consistent with the level of significance defined earlier for the model.

The unconditional coverage likelihood ratio (LR_{uc})can only reject a model if it overestimates or understimates the actual VaR but it cannot determine if the exceptions in the model are distributed randomly. It is important for the exceptions in the model when estimating VaR not to be correlated over time. The conditional coverage test (LR_{cc}) evaluates the serial independence of donditional average in the models. Coverage test considers a quantile-of-loss VaR measure and defines the exceedance process I_t, where I_t is 0 if the actual loss is less than or equal to the VaR estimate, and I_t is 1 if the actual loss exceeds VaR estimate.

Chapter 4

Data Analysis and Results

4.1 Introduction

We consider the highest, lowest, opening and closing daily prices of an NSE-listed and trading company for the period 2009 to 2019. Realized volatility is used as an unbiased estimator that is also more efficient compared to squared return when the log prices follow Brownian Motion. Realized volatility is defined as the sum of squared high-frequency returns with a day. It provides mode information while avoiding data analysis complication.

4.2 Descriptive Statistics

The Table 4.1 details the descriptive statistics for SASN for the 10-year period. This includes close-to-close returns, range-based returns, Parkinson volatility, and Garman Klass volatility.

Statistics	Close-Close	Range-Based	Parkinson	Garman-Klass
Min.	-0.2029408	0.00000	0.00000	0.000000
1st Qu.	-0.0155954	0.01242	0.00746	0.006323
Median	0.0000000	0.03050	0.01832	0.016932
Mean	0.0003366	0.03634	0.02182	0.020967
3rd Qu.	0.0176773	0.05260	0.03159	0.030110
Max.	0.2271197	0.18648	0.11199	0.124373
Standard deviation	0.03528	0.03072	0.01845	0.01921
Skewness	0.0637107	1.015109	1.015109	1.252754
Kurtosis	2.771516	1.037721	1.037721	1.73966

Table 4.1: Descriptive Statistics SASN returns & Volatility

The mean for close-to-close approach is around zero while the range-based return is

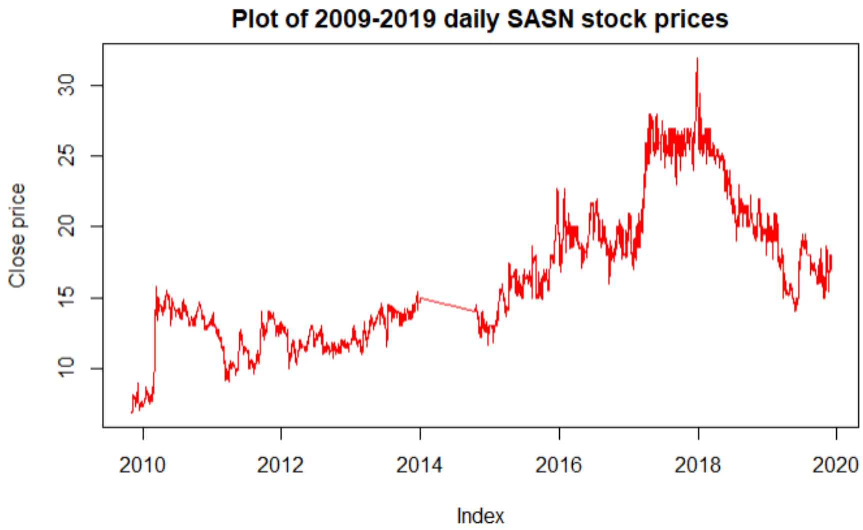

Figure 4.2.1: The time series plot above shows clustering characteristics of returns. High in certain periods and low in certain periods, evolving over the period under study exhibiting a continuous manner and hence volatility. Use of log returns will help achieve stationarity.

0.03. The mean for both Parkinson and Garman Klass volatilities is almost equal, 0.02182 and 0.020967 respectively. The standard deviation for close-to-close returns and range-based returns is around 0.03 with range-based approach having a slightly lower standard deviation. The kurtosis and skewness are positive indicating a leptokurtic distribution. Leptokurtic distribution is crucial in VaR forecasting and modeling because it gives up to three Kurtosis Degiannakis and Livada [2013]. The positive skew and positive mean value imply positive expected returns with positive surprises on the upside. The reason for the positive skew can be due to high trading activities at the open and close of trading days. There is a higher probability of extreme outlier values, stock prices. This can be attributed to the jumps at the open and close. For instance, during the open, more traders participate and when it is near close, depending on the performance of the market. If the traders in the market experienced low prices, they would want to close their positions so that they do not suffer further losses. Similarly, if in the previous day the market was performing poorly, the traders will cautiously enter positions at the open of the succeeding day.

For the volatility range series, they have mean value of about 3 percent, 2 percent and 2 percent respectively. However, the volatility ranges exhibited higher kurtosis and skewness compared to the return series. This is expected when measuring variance.

From the ACF tests, it is observed that the plot decays to zero meaning that the shock affects the process permanently.

Observing the autocorrelation functions (ACFs) and the Ljung-Box Q statistics for returns and range series as shown in the different Ljung-Box plots below reveals higher levels of persistence for range-based returns compared to close-to-close return series. This confirms the use of CARR in range volatility estimation. Figures below show the Ljung-Box plots and ACFs for the different range series.

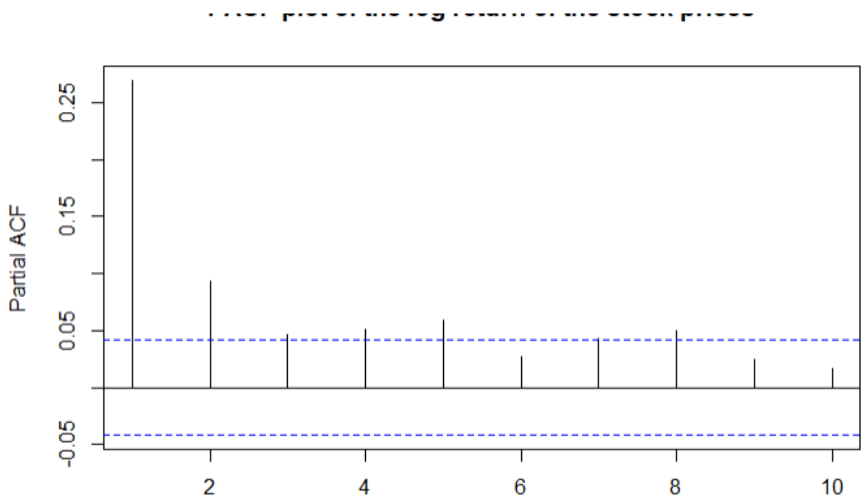

Figure 4.2.2: Ljung-Box Q Statistics ACF range series

Observing the autocorrelation functions (ACFs) and the Ljung-Box Q statistics for returns and range series as shown in the different Ljung-Box plots below reveals higher levels of persistence for range-based returns compared to close-to-close return series. This confirms the use of CARR in range volatility estimation. Figures below show the Ljung-Box plots and ACFs for the different range and return series.

The Figure below shows the Ljung-Box plot for the return series.

It is hypothesized that there is no autocorrelation in the stock returns. The LJung Box tests for log returns show that the returns are not correlated as the p-values are greater than 0.005 hence we fail to reject the null hypothesis of no autocorrelation. It also shows an ARCH effect on the LJung Box tests for both squared and absolute values.

4.2.1 Close-to-close and Range volatility Series Plots

The figures below show the daily returns and range volatility series for SASN for the period 2009 to 2019. It is observable that there is volatility cluster in the series. The figure below shows the range series returns for SASN.

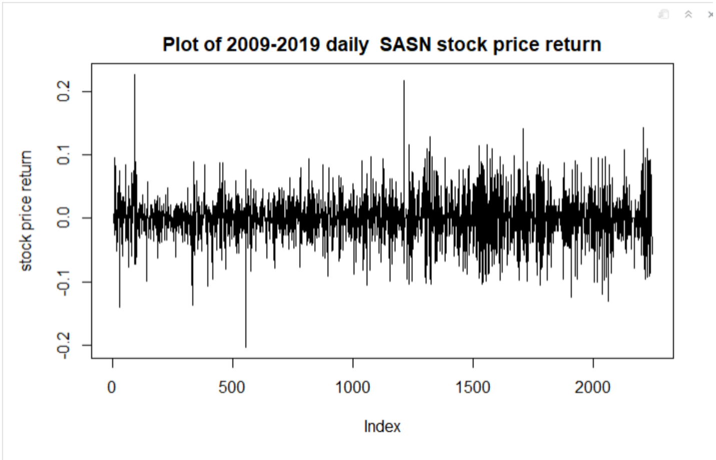

Figure 4.2.3: SASN Range Series Returns

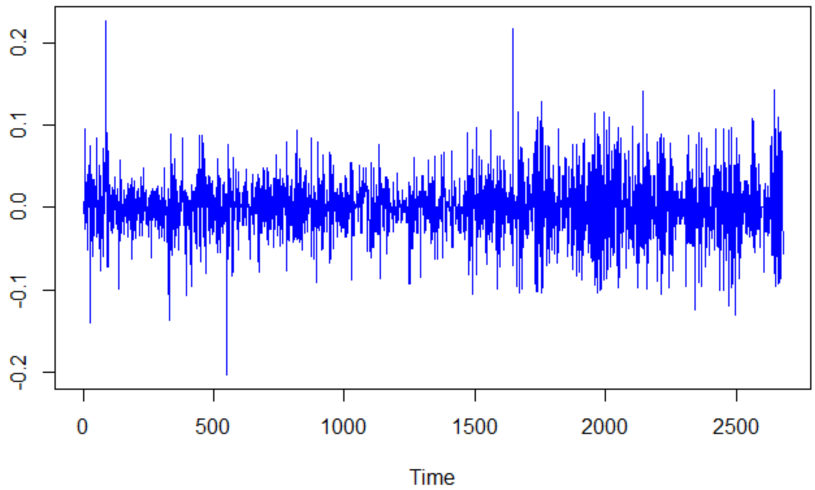

Figure 4.2.4: SASN Daily Series Returns

The figure below shows the close-to-close volatility series for SASN.

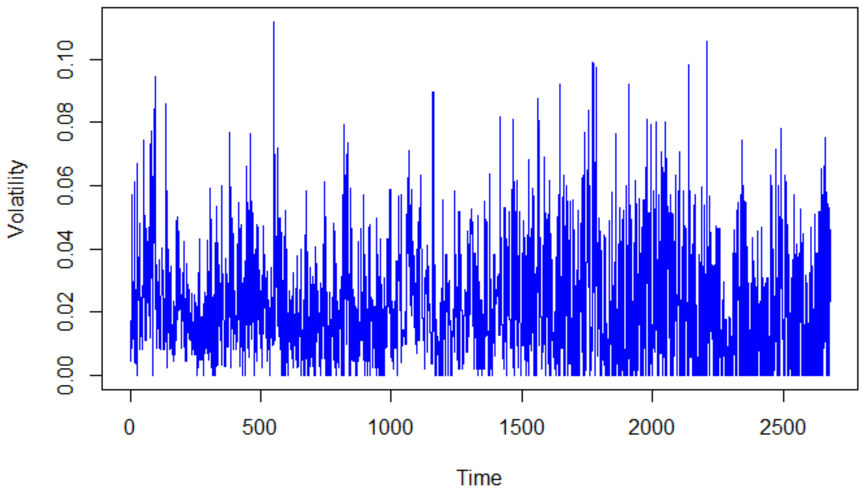

Figure 4.2.5: SASN Close-to-Close Volatility Series Returns

The figure below shows the Garman Klass Volatility series for SASN

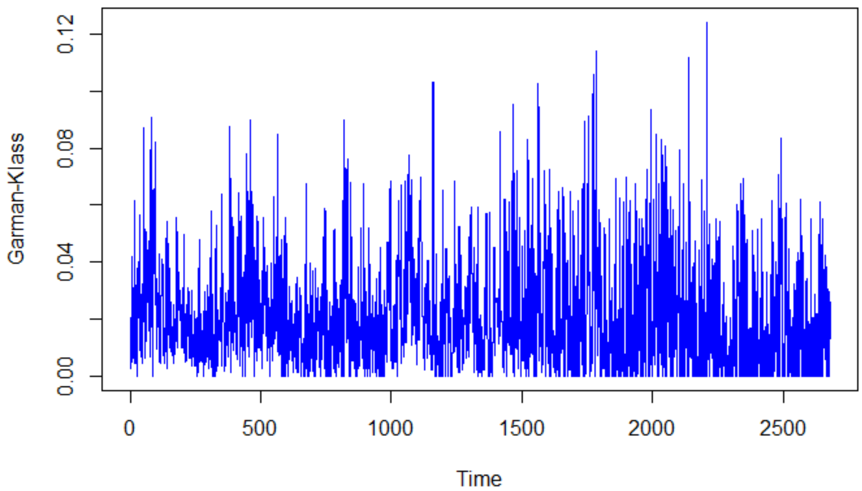

Figure 4.2.6: SASN Garman Klass Volatility Series

All the series return plots show clustering.

4.2.2 GARCH-type Modeling

In GARCH and TARCH modeling, the number of lags are p and q, while CARR has three lags, p, q, and s. The Range GARCH and Range TARCH models also have p, q, and s. The model estimates were carried out with the consideration of $p = q = s = 1$ thus obtaining improved accuracy and fewer number of parameters. We can test

Parameter	RGARCH	GARCH	CARR (1,1)	TARCH(1,1)	RTARCH(1,1)
ω	-0.00018	0.00023	0.00039	-0.000221	0.00083
α	0.163041	0.239382	0.57226	-0.53658	0.17597
β	0.62931	0.584423	0.48479	0.52402	0.73334
γ				0.02905	0.12321
θ	0.002	-	-	-	0.0016
LogLikelihood	5760.99	5396.59	5497.209	5545.72	5814.203
AIC	-4.2964	-4.0243	-4.0980	-4.1342	-4.3346
BIC	-4.2832	-4.0111	-4.0804	-4.1166	-4.3282

Table 4.2: The Table details return and range-based volatility model estimates for SASINI, 2009-2019

the different models for validity based on the parameter estimates. For TARCH and RTARCH models, they had an $\alpha+\gamma$ greater than zero and $p-value=0.05$ for testing statistical significance. γ is greater than 0 hence show of leverage effect. $\alpha+\beta$ is greater than 0 hence volatility persistence which can be confirmed from the return and range-series plots.

The Table 4.2 shows the estimates for return and range-based volatility models for SASN. The ω value in each model is significant hence appropriate to conclude that the models are affected by news.

From the parameter estimates in Table 4.2, it is only the asymmetric models TARCH and RTARCH which do not exhibit effect of past squared returns from the α. It is negatively related to volatility while in the symmetric models GARCH, RGARCH, and CARR α is positive indicative positive relation to volatility. The value of β for the Range CARR model is the lowest compared to the other models. This indicates short term memory in its volatility process compared to the other models. RTARCH has the longest memory in its volatility process. In the models, γ measures the leverage effect. A value of γ that is greater than 0 indicates that leverage effect exists while a γ value not equal to zero indicates asymmetry Maciel and Ballini [2017]. Leverage effects means that the volatility of the assets respond to negative and positive returns. The Θ parameter in the RTARCH and RGARCH models show that the models provide more information to volatility modeling for SASN stock than the other models. This answers the research hypothesis whether range provides more information to volatility modeling than using historical approach. Akaike Information Criterion (AIC) and Bayesian Information Criterion are used to assess simplicity of the models Degiannakis and Livada [2013]. Lower AIC and BIC values indicate that the model is the best-performing volatility compared to the others based on the two criteria. In the analysis from the table results, RTARCH model had the lowest AIC and BIC, and then RGARCH model.

Models	MSE	QLIKE
CARR(1,1)	0.000103	-7.5378
RTARCH (1,1,1)	0.0009398	-7.00708
RGARCH(1,1,1)	0.0009496	-7.0218
TARCH (1,1)	0.0009704	-
GARCH (1,1)	0.000917398	-6.69765

Table 4.3: At 5 % significance level. MSE & QLIKE assessment for volatility forecasting, ranked from best-performing to least

4.3 MSE and QLIKE Loss Functions

The performance of the volatility models is assessed using MSE and QLIKE loss function. Realized volatility calculated using 1-day quotations of SASN data was used in which the period 2009 to 2014 was take as the out-of-sample volatility forecasting. The last observation was removed to ensure all observations are of the same size. MSE and QLIKE loss functions for the 4 models was computers. Lower MSE and QLIKE values indicate higher performance of the model. From the analysis, range-based models, RGARCH, CARR, and RTARCH provided lower QLIKE and MSE values compared to their counterparts in which close-to-close returns approach was followed, GARCH and TARCCH models. This observation was expected because standard GARCH-type models have limited information which includes only the daily returns. TARCH model had the highest loss function values while CARR outperformed RGARCH based on MSE and QLIKE criteria. TARCH and RTACH models was best-performing volatility forecast compared to GARCH and RGARCH. However, RGARCH and RTARCH models provided crucial information to the process of volatility because they performed better in forecasts than GARCH and TARCH. Based on QLIKE and MSE values, CARR model gave the best results, that is low loss function values.

From the analysis, CARR model was the best-performing in overall because it had the lowest MSE and QLIKE Loss functions. RGARCH, RTARCH and CARR performed better than TARCH and GARCH as seen by the MSE and QLIKE valuess.

4.4 Diebold-Mariano Test

Diebold-Mariano test was conducted to assess the equivalence of accuracy of the different model pairs. The DM tests are statistically significant at 5% for CARR Model when compared with the critical value of 1.96. $\alpha = 0.05$ is divided by two because the test is two sided hence the upper is 0.975. Based on DM test, CARR model outperforms GARCH model. The improved performance of CARR model may be attributed to the fact that it uses range instead of return-based volatility like GARCH models.

Models	TARCH	RGARCH	RTARCH	CARR
GARCH	0.42802	-6.8313	-6.5644	-6.9087
TARCH	-	-	-6.7111	-6.9879
RGARCH	-	-	-3.9096	-8.3576
RTARCH	-	-	-	-8.0802

Table 4.4: At 5% significance level. Diebold-Mariano Test for volatility forecasting

Measure	RGARCH	GARCH	CARR	TARCH	RTARCH
VR (%)	4.6890	5.962	4.652	5.795	5.0690
ASMF	0.00134	0.0028	0.00119	0.0564	0.0023
LR_{uc}	0.1254	0.3347	4.56532*	5.5670*	1.2859*
LR_{cc}	0.12605	0.60332*	6.60891*	5.6742*	1.2918*

Table 4.5: The Table details the Violation rate, ASMF, unconditional and conditional Likelihood ratio test

4.4.1 Violation Rate, ASMF, Conditional and Unconditional LR

The performance of the volatility models was also compared based on how best they performed in VaR forecasting. This is because VaR forecasting is important for making economic or financial assessments Degiannakis and Livada [2013].

Investors and other practitioners in the financial sectors rely on VaR forecasts. Degiannakis and Livada [2013] established that a VaR estimate should meet the unconditional coverage, and the independence and conditional coverage conditions for it to be valid. One-step-ahead VaR forecasts were conducted at 95 percent confidence level and the forecasts evaluated in terms of Average Magnitude Square Function and Volatility ratio. ASMF measures the extent to which VaR exceeds. Historical simulation, based on close-to-close returns was used as benchmark to compare return- and ranged-based volatility estimates by the ARCH-type models and CARR.

The out-of-sample VaR backtesting shows that all the models generated a valid VaR forecast based on Unconditional Coverage LR and conditional coverage LR. Historical simulation is the only model that did not show valid VaR because its Violation Rate and ASMF exceeded VaR by far. This implies lower accuracy of the model by either overestimation or underestimation.

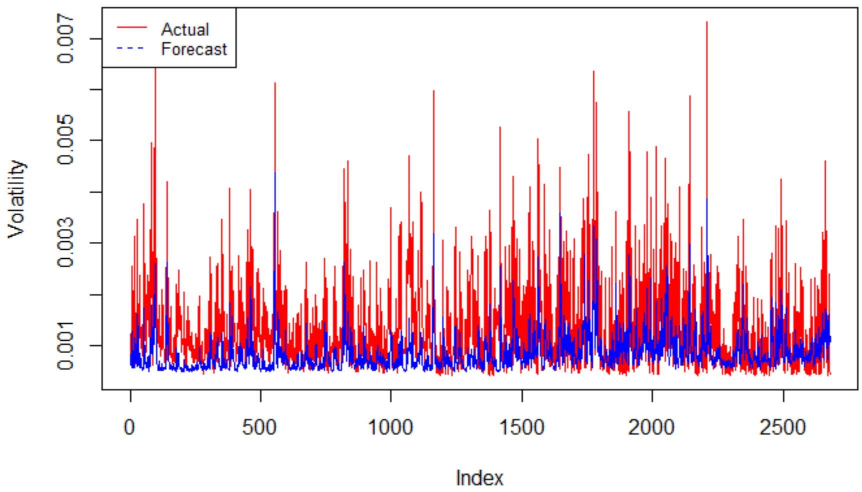

Figure 4.4.1: Actual & Forecasted Volatility Plot, RGARCH Model

The 1-step-ahead volatility forecast using RGARCH shows that the volatility forecasts were slightly lower than the actual.

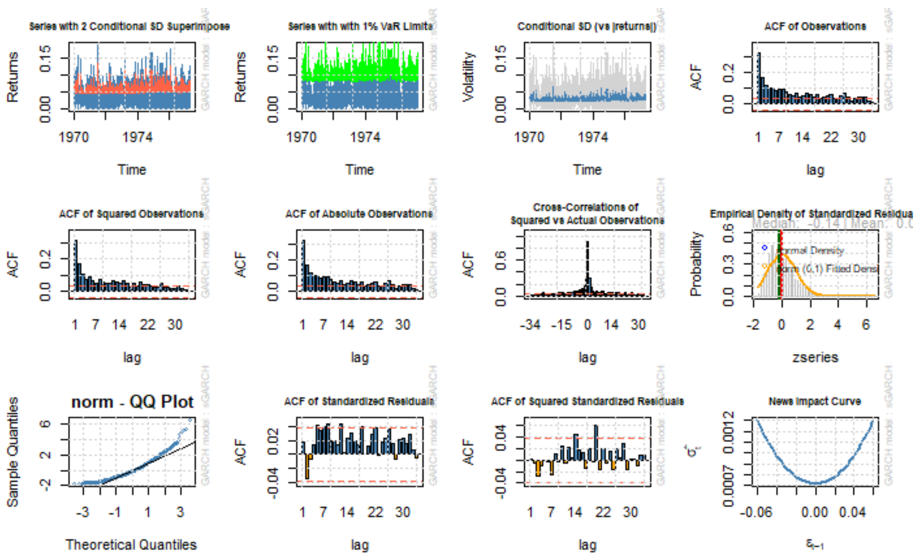

Figure 4.4.2: RGARCH Model fitting plots. Different gaphs we obtained for 4-years ahead forecasts

The plots for testing for normalitty, autocorrelation ACF, news impact curve,and skewness.

The graph below shows the volatility forecasts fore the competing models.

Range-based volatility models such as had lower violation rates and ASMF loss function values. According to the coverage tests, CARR has significantly lower failure rate. This implies that the model overestimated VaR values. The practical implication

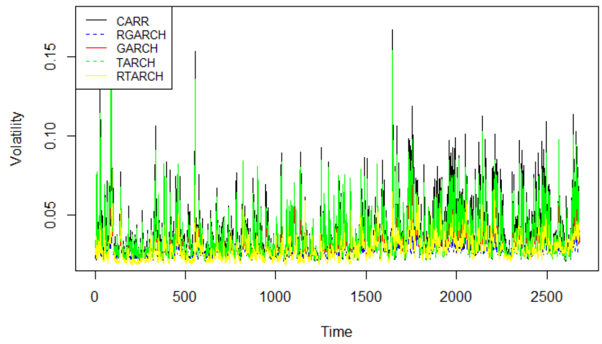

Figure 4.4.3: Comparison of 1-step-ahead volatility forecasts for the competing models

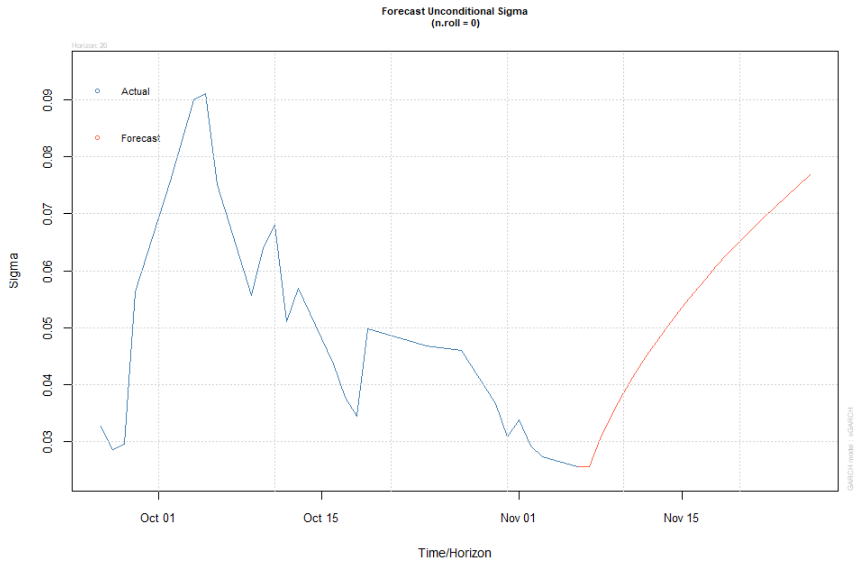

Figure 4.4.4: CARR Model, 1-step-ahead forecasts

of this results is that the risk-averse investors may end up taking unnecessary position. RTARCH and RGARCH more accurately estimated VaR at the 5% expected failure rate. These models also exhibit improved VaR forecasts hence higher accuracy compared to GARCH and TARCH models. We can conclude that range provided additional information which improved the models.

The plot below shows the VaR forecasts for CARR Model

The plot below shows the VaR forecasts with RGARCH Model

4.5 Interpretation of Results

The Range-based CARR model proved to be the most efficient volatility in general. Also, RGARCH model performed better than the other GARCH-type and TARCH

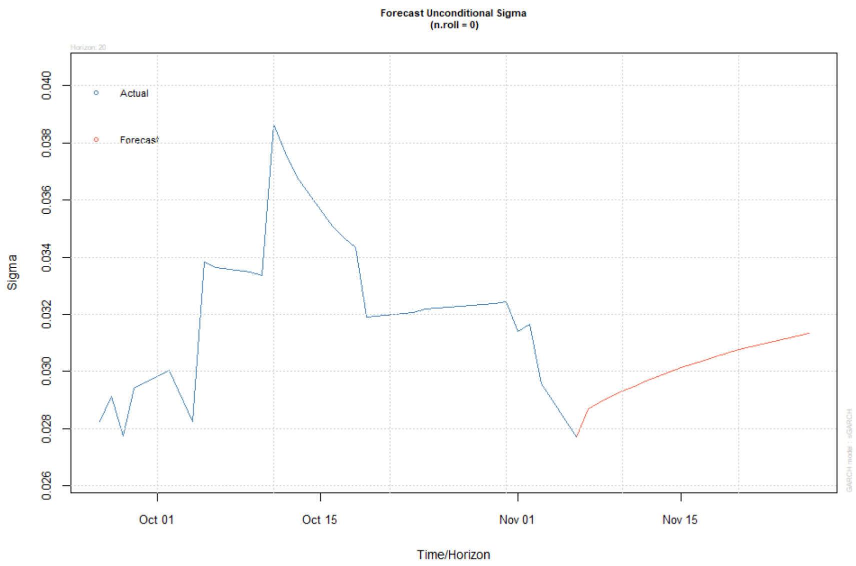

Figure 4.4.5: RGARCH Model, 1-step-ahead forecasts

models. However, CARR model was inefficient in an upward trend scenario, that is, when the volatility is low and market is rising making RGARCH model suitable for such market scenario. The objectives of this study was to test range-based approach to modeling and forecasting VaR by establishing whether range provides additional information, testing range-based models using NSE-trading company data, and assessing efficiency of the competing Range-Based models RGARCH, RTARCH, and CARR. This study has met all the objectives.

Range was computed using open, close, high and low, which is a Garman-Klass range-based approach to computing modeling and forecasting stability. It was established that range-based models RGARCH and RTARCH had an additional parameter Θ. This demonstrated that range provided additional and crucial information to the volatility process.

Various methods were used to assess model efficiency for the competing models including Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC), Mean Square Error (MSE) and QLIKE loss functions, Diebold-Mariano (DM) Test, Average Square Magnitude Function (ASMF), Violation Rates (VR) Unconditional and Conditional Likelihood ratio test.

All models had a positive ω indicating that they are affected by news with the CARR model having the highest ω value. The low α value in TARCH and RTARCH models indicated that the models are not heavily affected by past squared returns. Positive α is an indication of positive relation to volatility. The low β value of the CARR model indicates short term memory in the volatility process. That is, the effect of past volatility on the current market volatility decreases as we move further into the past.

Volatility a hundred days ago has lesser influence on today's volatility than volatility from 50 or lesser time period.

Minimum AIC and BIC values are used as selection criteria. Models with high log-likelihood have low AIC values hence superior goodness-of-fit. AIC assesses the model that adequately describes an unknown while BIC is used to find the true model among the competing volatility models.

QLIKE and MSE are selected because they are robust loss functions, hence less affected by the most extreme observations in the sample. MSE loss function relies on the usual forecast error $\widehat{(}\sigma)^2 - h$. Lower MSE and QLIKE values imply a better model. From Table 4.3, the MSE and QLIKE loss values were obtained at 5 percent confidence level. It was observed that CARR model had the lowest MSE and QLIKE values followed by RTARCH and RGARCH models with GARCH model performing least.

The DM test assess equivalence of accuracy among competing models. At 5 percent level of significance, CARR model was observed to have the lowest DM statistic values compared with corresponding model pairs. Violation rates, ASMF and Conditional and unconditional likelihood ratio tests were carried out on the competing models based on the one-step-ahead VaR forecasts. Range-based volatility models had lower violation and ASMF values indicating lower failure r ates hence better performance. ASMF measures VaR exceedance, that is how far the forecasts exceeded realized VaR. At 95 percent confidence level, range-based models perform better than close-to-close return-based models. CARR model outperforms the other RGARCH models based on most of the measures for selecting suitable model.

Chapter 5

Conclusions and Recommendations

5.1 Conclusions

Based on the various model valuation criteria, the criteria indicate that forecasting error of CARR (1,1) is low than that of GARCH (1,1). The conclusion is that CARR model outperforms the GARCH model. The range-based models, RGARCH, RTARCH, and CARR support Chou [2005] proposition that the range provides more information than return. CARR (1,1) provides a sharper volatility forecasts than range-based GARCH and TARCH models.

The goal of this study was to provide a simple and highly effective approach for forecasting the volatility of returns by using range rather than the daily returns. It was the study's objective to determine the research hypothesis whether range and the price movements during the day affect the returns and hence use of range-based volatility approach to model and forecast volatility. Also, the aim was to establish applicability of the method by conducting empirical analysis of an NSE-listed and trading company, SASN.

On empirical analysis, it was established that using information contained in range significantly improves the accuracy of the volatility estimates. GARCH model performs better than CARR model when the volatilities are lower and the market is rising based on symmetric and asymmetric error statistics. Therefore, in downward trend, volatility is higher and the CARR model is more appropriate hence preferable.

However, in upward trend, it is crucial to use all daily information. That is, open, high, low, and close to determine and efficient volatility measure because using only opening and closing prices may wrongly conclude the volatility estimate. We can conclude that range-based volatility estimates provide additional information to forecasting volatility hence more accurate than return-based volatility estimates.

5.2 Recommendations

5.3 Room for Further Research

The results are not highly optimized and stand for purposes of comparison. Achieving higher accuracy could require large input data. Future research should explore the degree of accuracy to which the volatility estimates can be forecasted with large data input. We only used the daily price range data to make forecasts of a day ahead and this has its shortcomings which include the small data set used for making relatively long period forecasts. It would be worth to explore the predictability of realized volatilities obtained from intraday data. This study recommends further research to include long-term forecasting models, address volatility patterns such as crisis scenarios and application to trading strategies.

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R Notebook

```r
library(quantmod)
```

```
## Loading required package: xts

## Loading required package: zoo

##
## Attaching package: 'zoo'

## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric

## Loading required package: TTR

## Registered S3 method overwritten by 'quantmod':
##   method            from
##   as.zoo.data.frame zoo

## Version 0.4-0 included new data defaults. See ?getSymbols.
```

```r
library(xts)
library(rvest)
```

```
## Loading required package: xml2
```

```r
library(tidyverse)
```

```
## v ggplot2 3.3.2     v purrr   0.3.4
## v tibble  3.0.3     v dplyr   1.0.0
## v tidyr   1.1.0     v stringr 1.4.0
## v readr   1.3.1     v forcats 0.5.0

## -- Conflicts ---------------------------------------------------
## x dplyr::filter() masks stats::filter()
## x dplyr::first()          masks xts::first()
## x readr::guess_encoding() masks rvest::guess_encoding()
## x dplyr::lag()            masks stats::lag()
## x dplyr::last()           masks xts::last()
## x purrr::pluck()          masks rvest::pluck()
```

```r
library(stringr)
library(forcats)
library(lubridate)
```

```
##
## Attaching package: 'lubridate'

## The following objects are masked from 'package:base':
##
##     date, intersect, setdiff, union
```

```r
library(plotly)
```

```
##
## Attaching package: 'plotly'

## The following object is masked from 'package:ggplot2':
##
##     last_plot

## The following object is masked from 'package:stats':
##
##     filter

## The following object is masked from 'package:graphics':
##
##     layout
```

```r
library(dplyr)
library(PerformanceAnalytics)
```

```
##
## Attaching package: 'PerformanceAnalytics'

## The following object is masked from 'package:graphics':
##
##     legend
```

```r
library(quantmod)
library(rugarch)
```

```
## Loading required package: parallel

##
## Attaching package: 'rugarch'

## The following object is masked from 'package:purrr':
##
##     reduce

## The following object is masked from 'package:stats':
##
##     sigma
```

```r
library(rmgarch)
```

```
##
## Attaching package: 'rmgarch'

## The following objects are masked from 'package:dplyr':
##
##     first, last

## The following objects are masked from 'package:xts':
##
##     first, last
```

```r
library(ggfortify)
library(changepoint)
```

```
##
## Attaching package: 'changepoint'

## The following object is masked from 'package:rugarch':
##
##     likelihood
```

```r
library(strucchange)
```

```
## Loading required package: sandwich

##
## Attaching package: 'strucchange'

## The following object is masked from 'package:stringr':
##
##     boundary
```

```r
library(ggpmisc)
library(ModelMetrics)
```

```
##
## Attaching package: 'ModelMetrics'

## The following object is masked from 'package:base':
##
##     kappa
```

```r
library(pastecs)
```

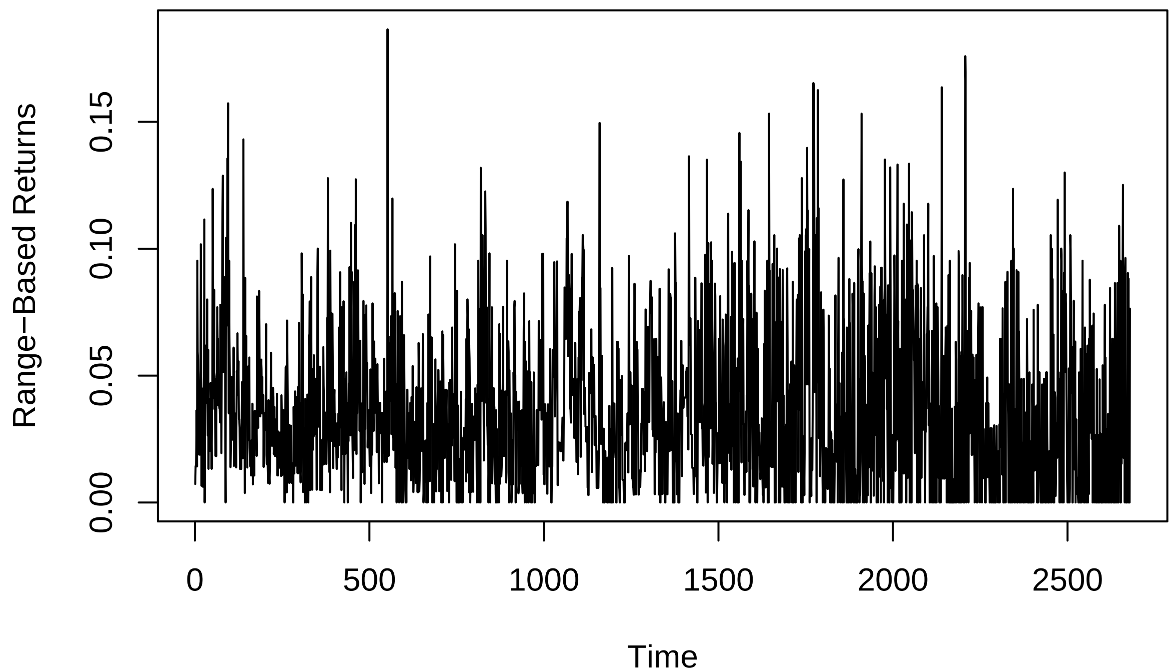
```
##
## Attaching package: 'pastecs'

## The following objects are masked from 'package:rmgarch':
##
##     first, last

## The following objects are masked from 'package:dplyr':
##
##     first, last

## The following object is masked from 'package:tidyr':
##
##     extract

## The following objects are masked from 'package:xts':
##
##     first, last
```

```r
library(MCS)
library(MatrixCorrelation)
```

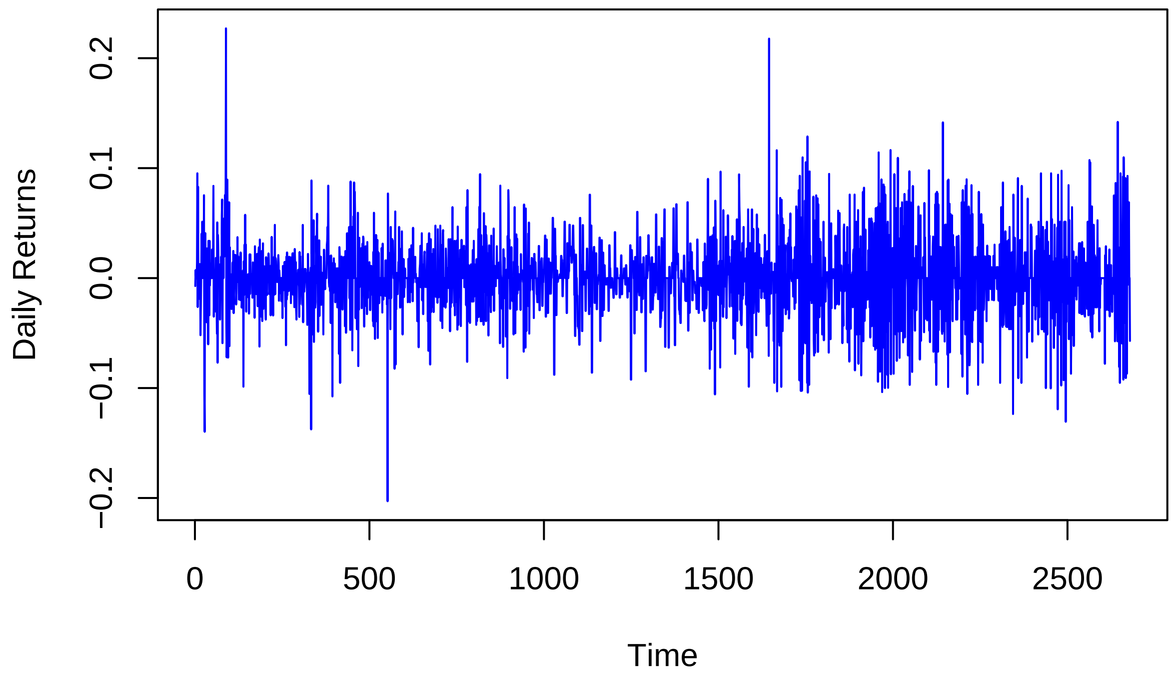
```
##
## Attaching package: 'MatrixCorrelation'

## The following object is masked from 'package:TTR':
##
##     SMI
```

```r
library(multDM)
```

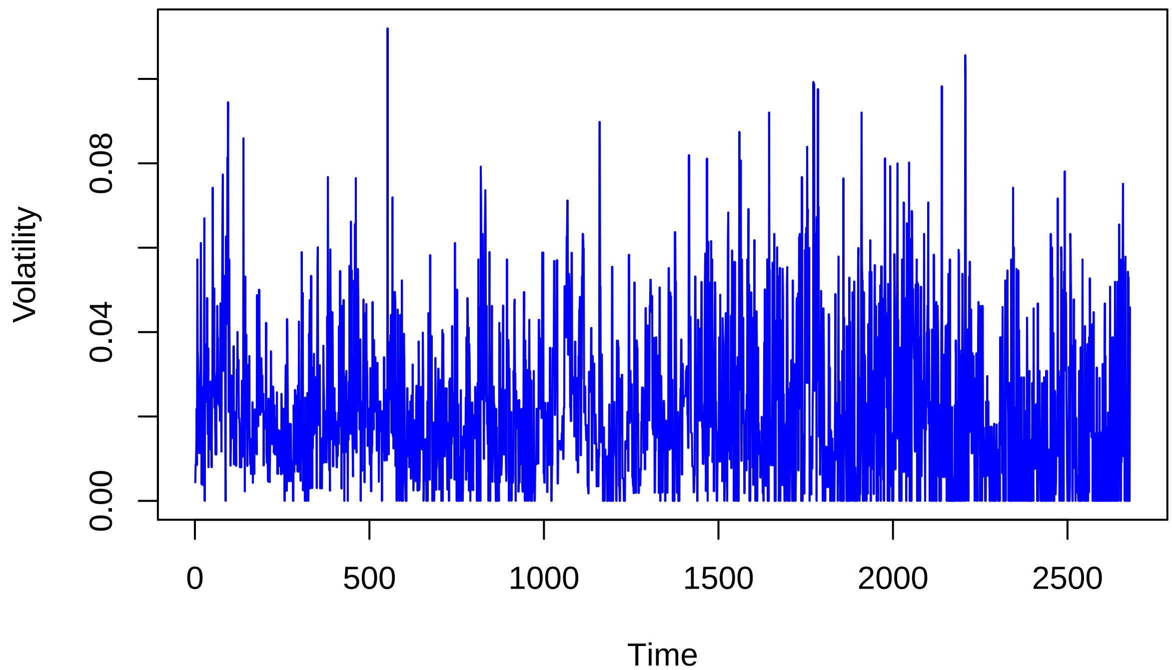
```r
#Upload Data
setwd("C:/Users/USER/Desktop/Resources Project")
mydata <-read.csv("mydata1.csv")
attach(mydata)
#plot Range Returns
plot.ts(mydata$SASN.RANGE.Returns,ylab="Range-Based Returns")
```




```r
# plot close-close returns
plot.ts(SASN.Return.based,col="Blue", ylab="Daily Returns")
```




```
# plot Parkinson Volatility
plot.ts(mydata$Parkinson,col="Blue",ylab="Volatility")
```




```
# plot Garman-Klass Volatility
#plot.ts(mydata$Garman.Klass, col="Blue",ylab="Garman-Klass")
```

```
# create data frame
rSASN <- data.frame(Date,SASN.Return.based,SASN.RANGE.Returns,mydata$Parkinson,Garman.Klass)
# Rename the COlumns
names(rSASN)[1] <- "Date"
names(rSASN)[2] <- "Close-Close Returns"
names(rSASN)[3] <-  "Range-Based Returns"
names(rSASN)[4] <- "Parkinson Volatility"
names(rSASN)[5] <- "Garman-Klass Volatility"
#head(rSASN)
```

```
cols <-c("Close-Close Returns","Range-Based Returns","Parkinson Volatility","Garman-Klass Volatility")
#summary(rSASN[cols])
```

```
# Detailed Summary Statistics
skewness(rSASN[cols])
```

```
##          Close-Close Returns Range-Based Returns Parkinson Volatility
## Skewness           0.0637107            1.015109             1.015109
##          Garman-Klass Volatility
## Skewness                1.252754
```

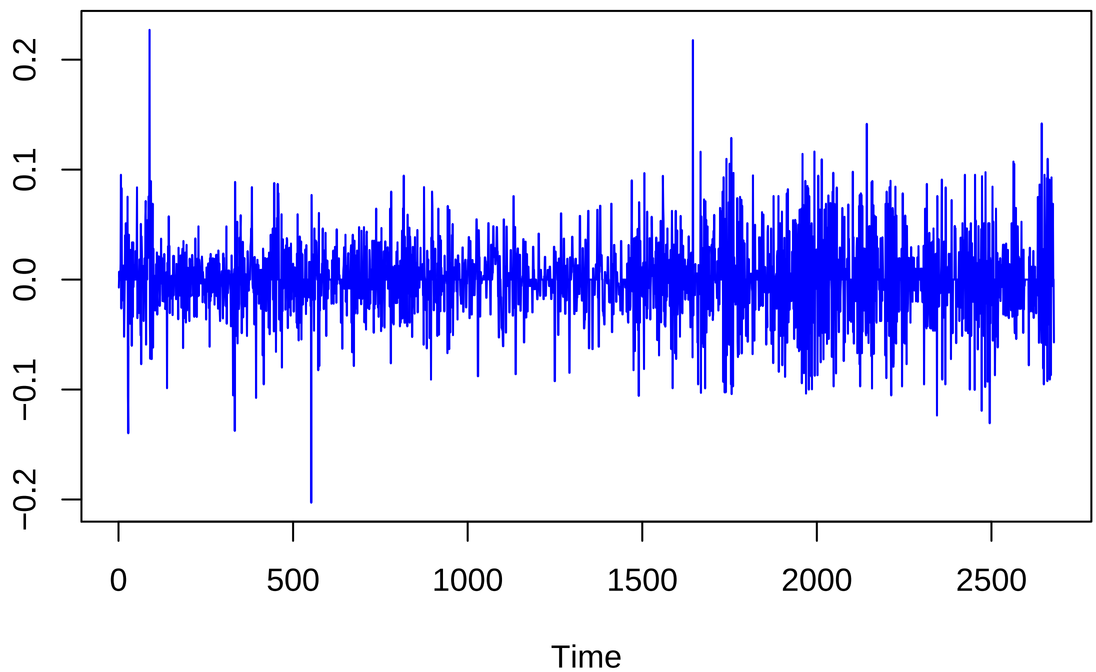
```
kurtosis(rSASN[cols])
```

```
##                 Close-Close Returns Range-Based Returns Parkinson Volatility
## Excess Kurtosis            2.771516            1.037721             1.037721
```

```
##                  Garman-Klass Volatility
## Excess Kurtosis                  1.73966
```

```r
# Descriptive Statistics
#stat.desc(rSASN[cols], norm = "TRUE")
```

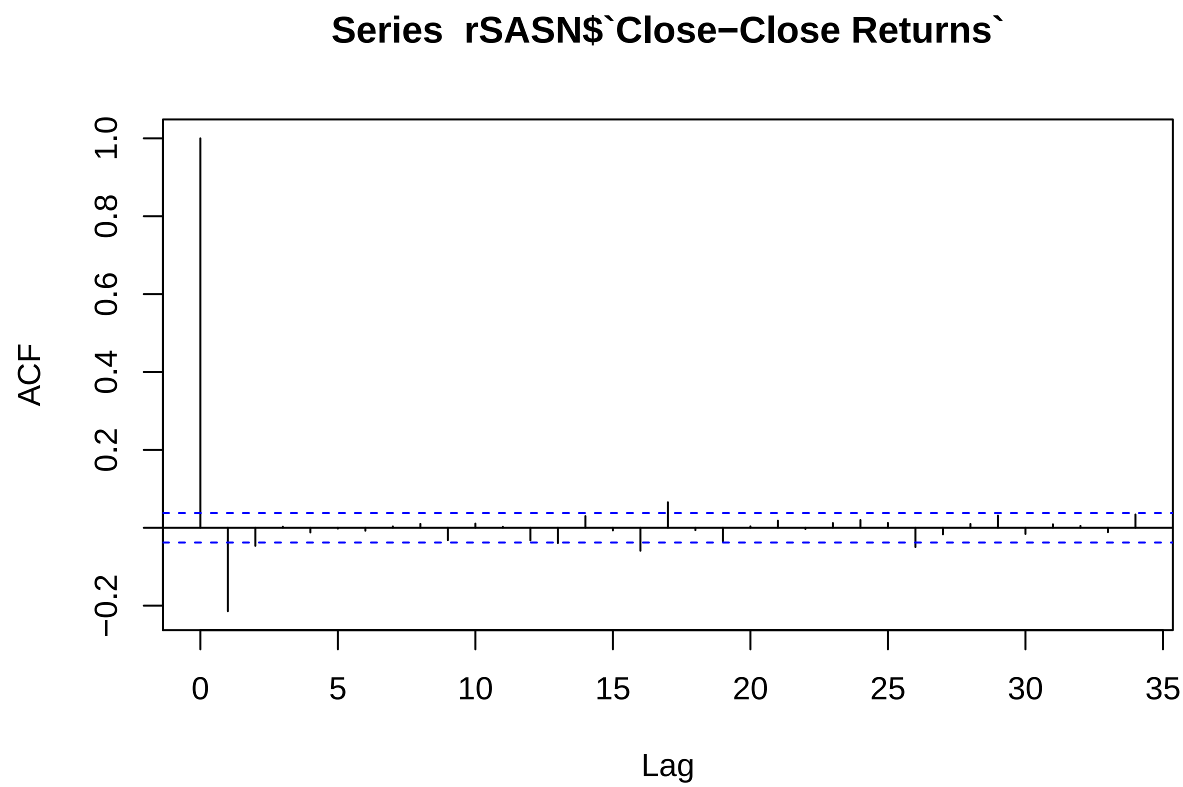
```r
#Plot Time Series Returns and Volatility with date a-axis
plot.ts(rSASN$`Close-Close Returns`,col="Blue",ylab="")
```




```r
Box.test(rSASN$`Close-Close Returns`, type="Ljung-Box")
```

```
##
##  Box-Ljung test
##
## data:  rSASN$`Close-Close Returns`
## X-squared = 123.21, df = 1, p-value < 2.2e-16
```

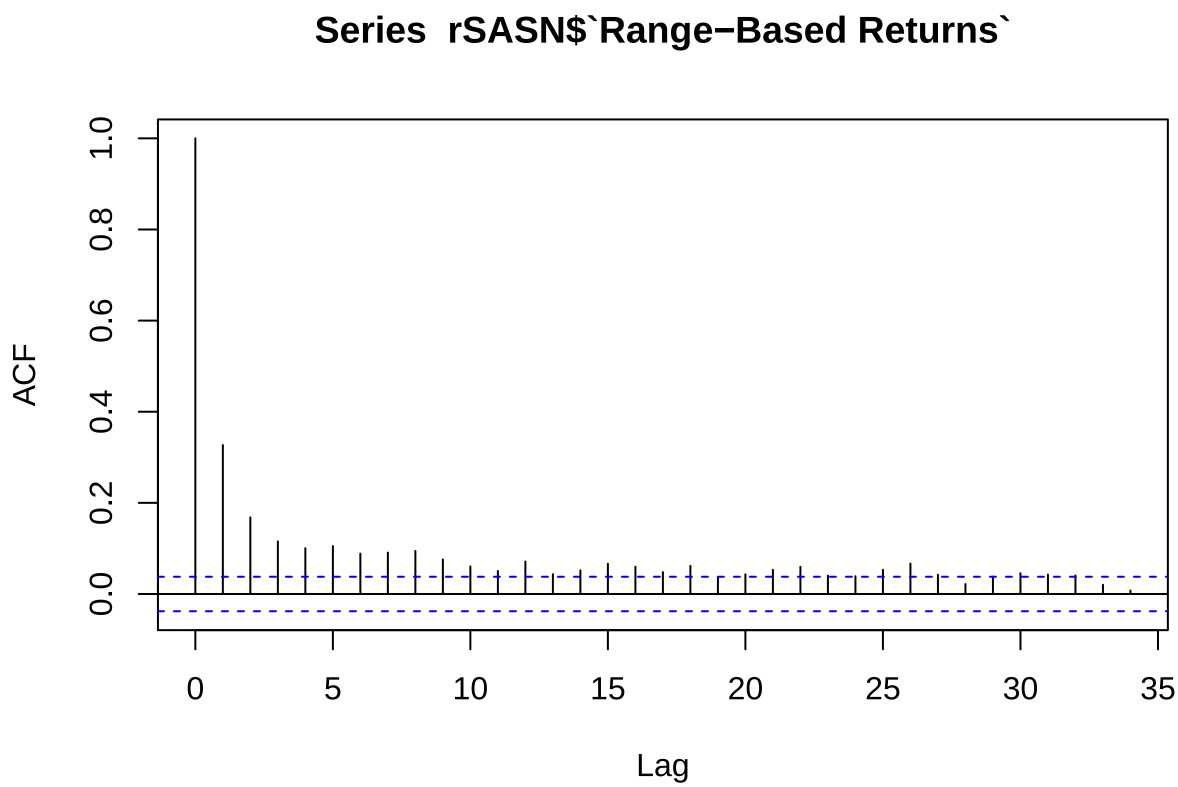
```r
acf(rSASN$`Close-Close Returns`)
```




```r
Box.test(rSASN$`Range-Based Returns`, type="Ljung-Box")
```

```
##
##  Box-Ljung test
##
## data:  rSASN$`Range-Based Returns`
## X-squared = 286.43, df = 1, p-value < 2.2e-16
```

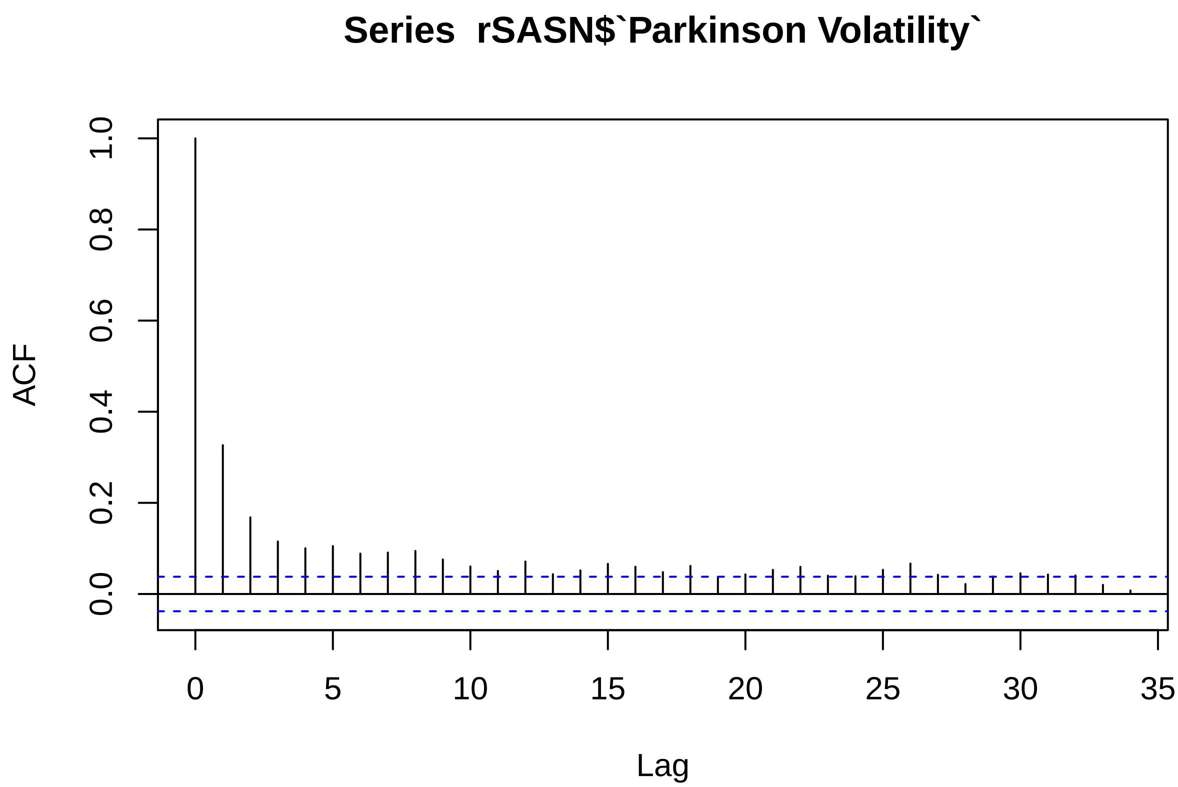
```r
acf(rSASN$`Range-Based Returns`)
```




```r
Box.test(rSASN$`Parkinson Volatility`, type="Ljung-Box")
```

```
##
##  Box-Ljung test
##
## data:  rSASN$`Parkinson Volatility`
## X-squared = 286.43, df = 1, p-value < 2.2e-16
```

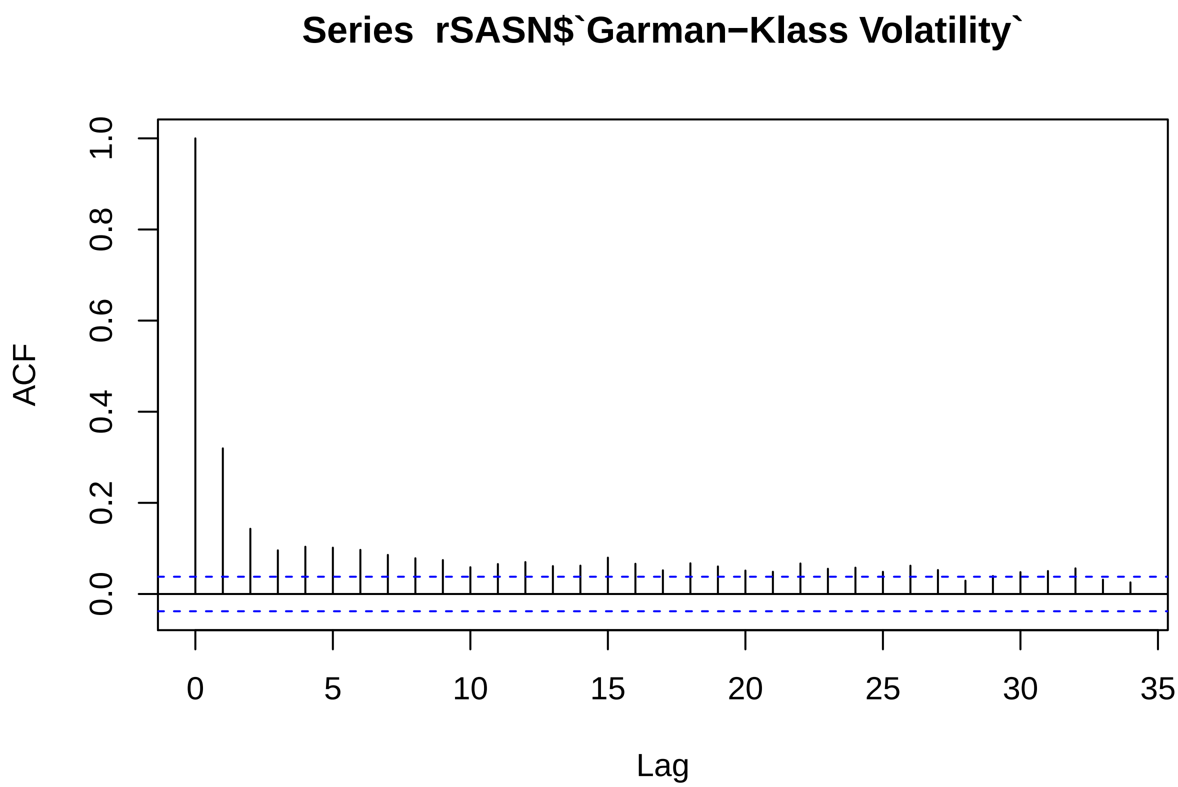
```r
acf(rSASN$`Parkinson Volatility`)
```




```r
Box.test(rSASN$`Garman-Klass Volatility`, type="Ljung-Box")
```

```
##
##  Box-Ljung test
##
## data:  rSASN$`Garman-Klass Volatility`
## X-squared = 274.04, df = 1, p-value < 2.2e-16
```

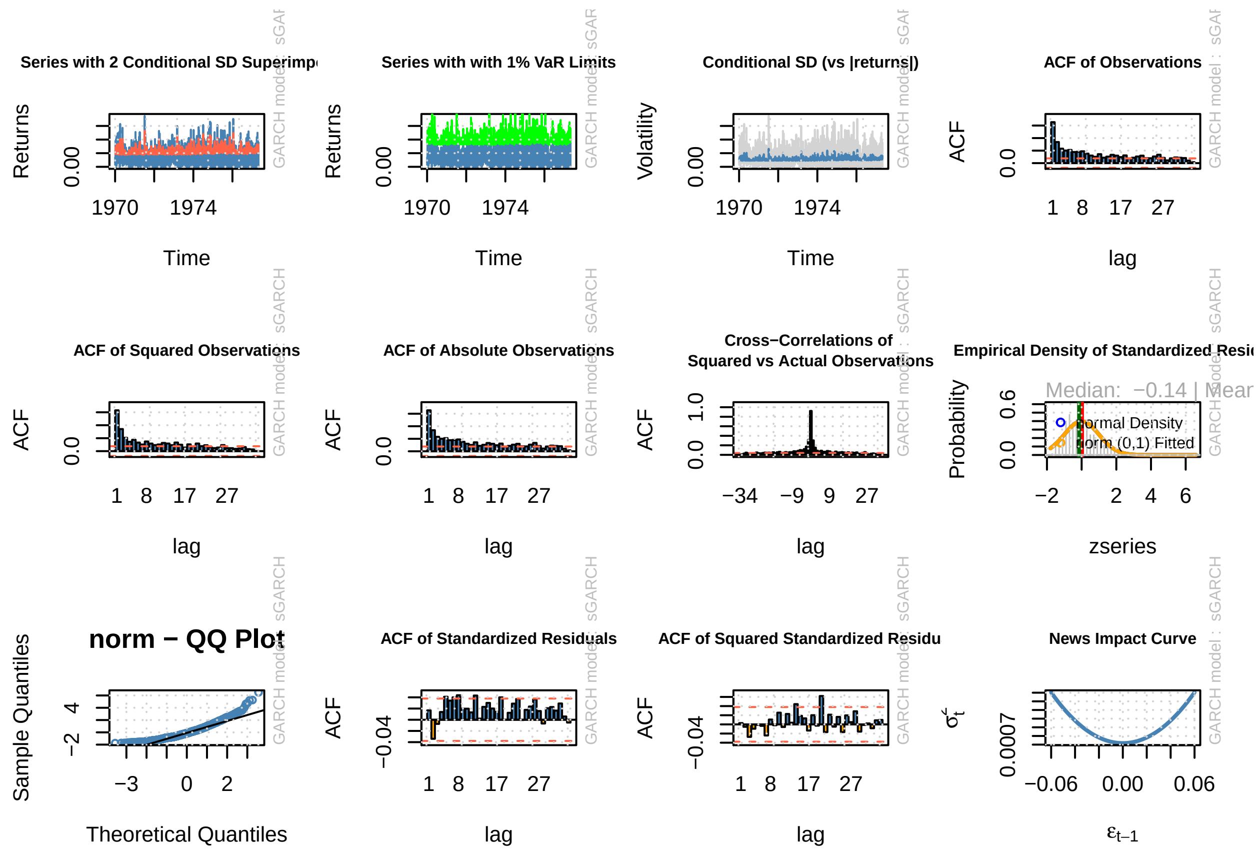
```r
acf(rSASN$`Garman-Klass Volatility`)
```




```r
# Assign model to standard specification, Range GARCH(1,1)
ug_spec = ugarchspec()
# GARCH Estimation using SASN data
sasn_ugfit = ugarchfit(spec = ug_spec, data = SASN.RANGE.Returns)
#sasn_ugfit
```

```r
plot(sasn_ugfit, which = "all")
```

```
##
## please wait...calculating quantiles...
```



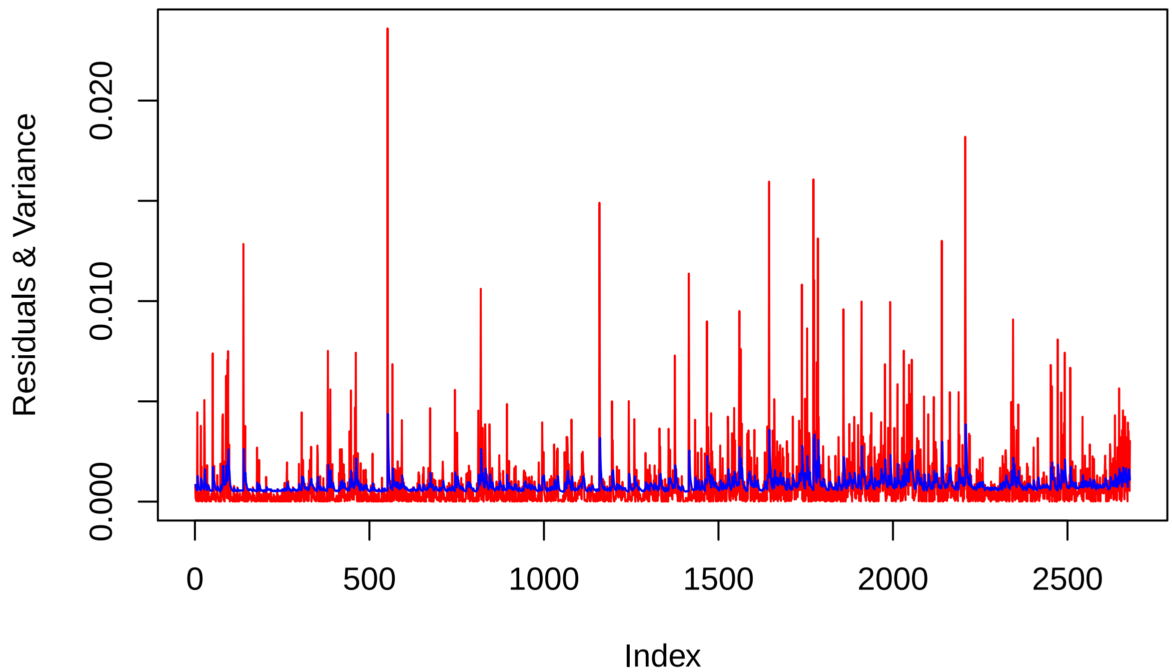

```r
# Check Elements
paste("Elements in the @model slot")
```

```
## [1] "Elements in the @model slot"
```

```r
#names(sasn_ugfit@model)
#names(sasn_ugfit@fit)
```

```r
# Extract estimated coefficients
#sasn_ugfit@fit$coef
```

```r
sasn_var <- sasn_ugfit@fit$var   # Assign estimated conditional variances
sasn_res2 <- (sasn_ugfit@fit$residuals)^2   # assign the estimated squared residuals
#plot squared residuals and conditional variance
plot(sasn_res2, type = "l",col="Red",xlab="Index", ylab ="Residuals & Variance")
lines(sasn_var, col = "blue")
```




```r
# Forecast conditional volatility or square root of conditional variance, for 2678
sasn_predict <- ugarchforecast(sasn_ugfit, n.ahead = 100)
#sasn_predict
```

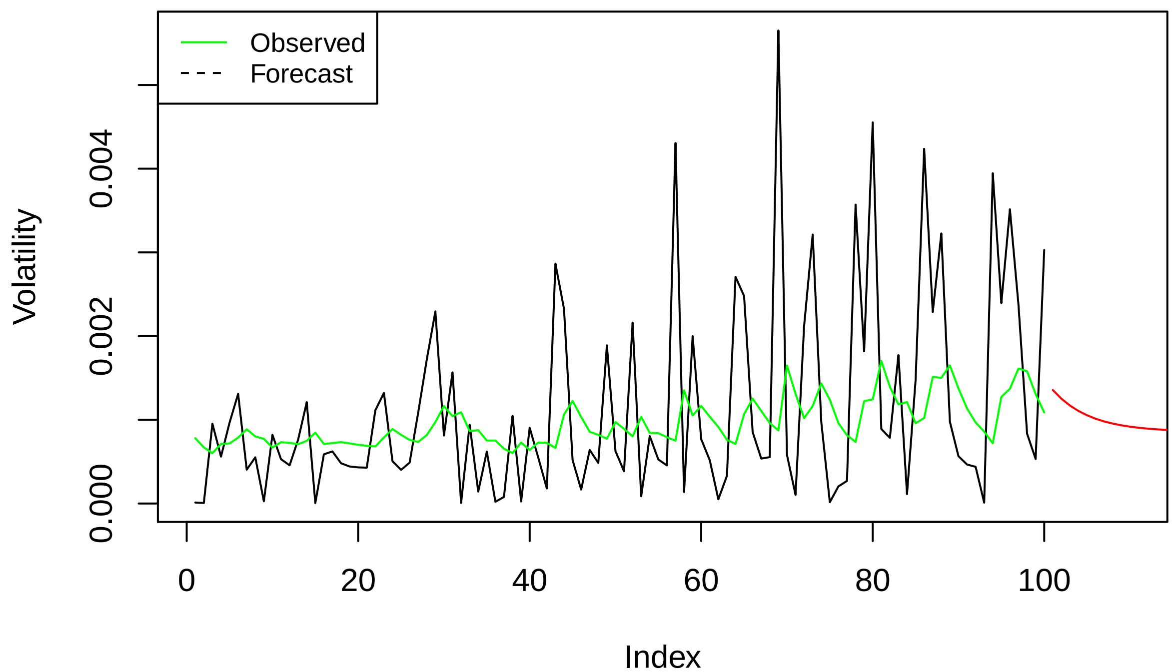
```r
Sasn_spredict <- sasn_predict@forecast$sigmaFor
#plot(Sasn_spredict, type = "l",main = "1-step-ahead Volatility Forecast",ylab="Volatility Forecast")
```

```r
# plot sigma squared
sasn_pred1<-ts(sasn_ugfit@fit$sigma^2)
#plot.ts(sasn_pred1,col="Blue",ylab="Variance")
```

```r
ug_spec = ugarchspec()
# GARCH Estimation using SASN data, Historical
sasn_ugfit_h <- ugarchfit(spec = ug_spec, data = SASN.Return.based)
# plot fitted values
sasn_predf<-sasn_ugfit@fit$fitted.values
#plot.ts(sasn_predf, col="Blue",ylab="")
```

```r
# Plot the fprrecasts with the last 100 observations in the estimation.
# Last 100 observations in variance
sasn_var1 <- c(tail(sasn_var,100),rep(NA,10))
# Get last 100 observations of residual values
sasn_res2t <- c(tail(sasn_res2,100),rep(NA,10))
# Assign predicted values
Sasn_spredict <- c(rep(NA,100),(Sasn_spredict)^2)
# Display the observations
plot(sasn_res2t, type = "l",ylab="Volatility")
```

```
lines(Sasn_spredict, col = "Red")
# 100 days, 1-step-ahead volatility
lines(sasn_var1, col = "green")
legend("topleft", legend=c("Observed","Forecast"), col=c("Green","Black"),lty=1:2,cex = 0.8)
```



Determining MSE and QLIKE for GARCH Model Above

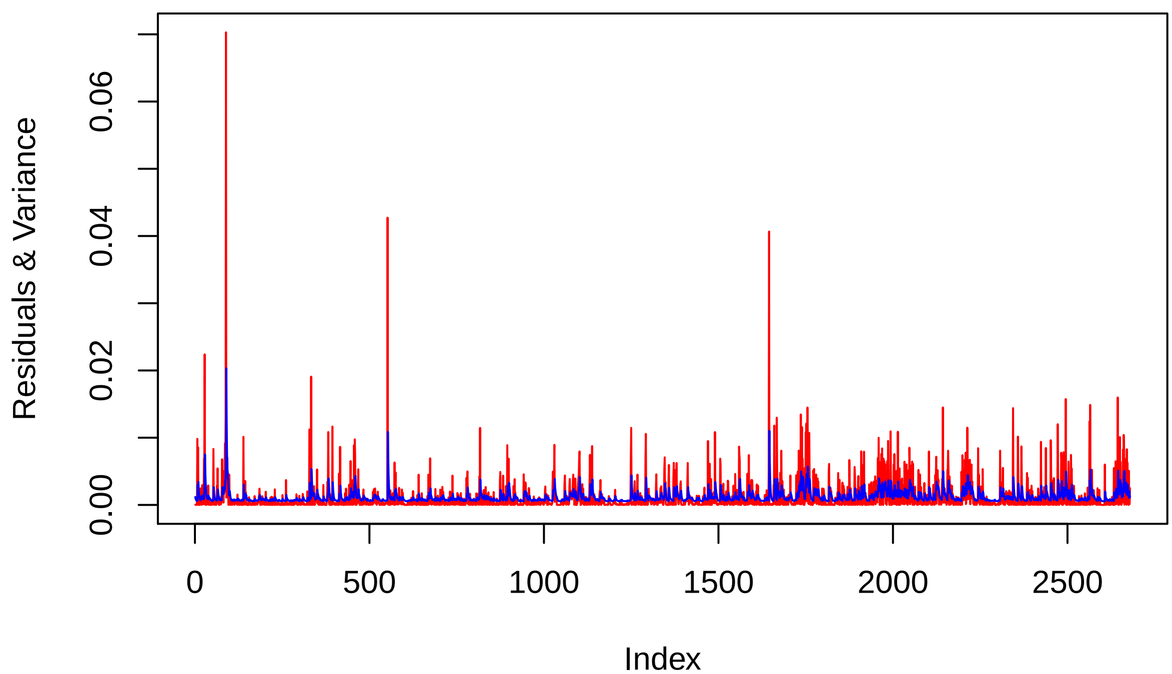
```
sasn_range.retuns <-SASN.RANGE.Returns
#str(sasn_range.retuns)
sasn_range.predict<-ugarchboot(sasn_ugfit, n.ahead = 2679,method=c("Partial","Full")[1])
#str(sasn_range.predict)
#str(sasn_range.retuns)
r.mse<-mse(rSASN$`Range-Based Returns`,sasn_ugfit@fit$sigma)
#mse(rSASN$`Range-Based Returns`,ugfit@fit$sigma)
# install.packages("MCS")
r.qlike<- LossVol(rSASN$`Range-Based Returns`,sasn_ugfit@fit$sigma,which="QLIKE")
r_qlike<-r.qlike[1]
# length(Garman.Klass)
# length(ugfit@fit$sigma)
Loss.Functions<-data.frame(r.mse,r_qlike)
names(Loss.Functions)[1] <- "MSE"
names(Loss.Functions)[2] <- "QLIKE"
#Loss.Functions
```

```
# computing Ratio Volatility
rgarch.RV<-RV( sasn_ugfit@fit$fitted.values,sasn_ugfit@fit$sigma)
#rgarch.RV
```

ESTIMATING USING GARCH with Historical Return

```
# Assign model to standard specification, GARCH(1,1)
ug_spec = ugarchspec()
# GARCH Estimation using SASN data, Historical
sasn_ugfit_h <- ugarchfit(spec = ug_spec, data = SASN.Return.based)
#sasn_ugfit_h
```

```
#for the sasn_ugfit_h Model that uses historical returns
sasn_var.h <- sasn_ugfit_h@fit$var   # Assign estimated conditional variances
sasn_res2.h <- (sasn_ugfit_h@fit$residuals)^2   # assign the estimated squared residuals
#plot squared residuals and conditional variance
plot(sasn_res2.h, type = "l",col="Red",xlab="Index", ylab ="Residuals & Variance")
lines(sasn_var.h, col = "blue")
```



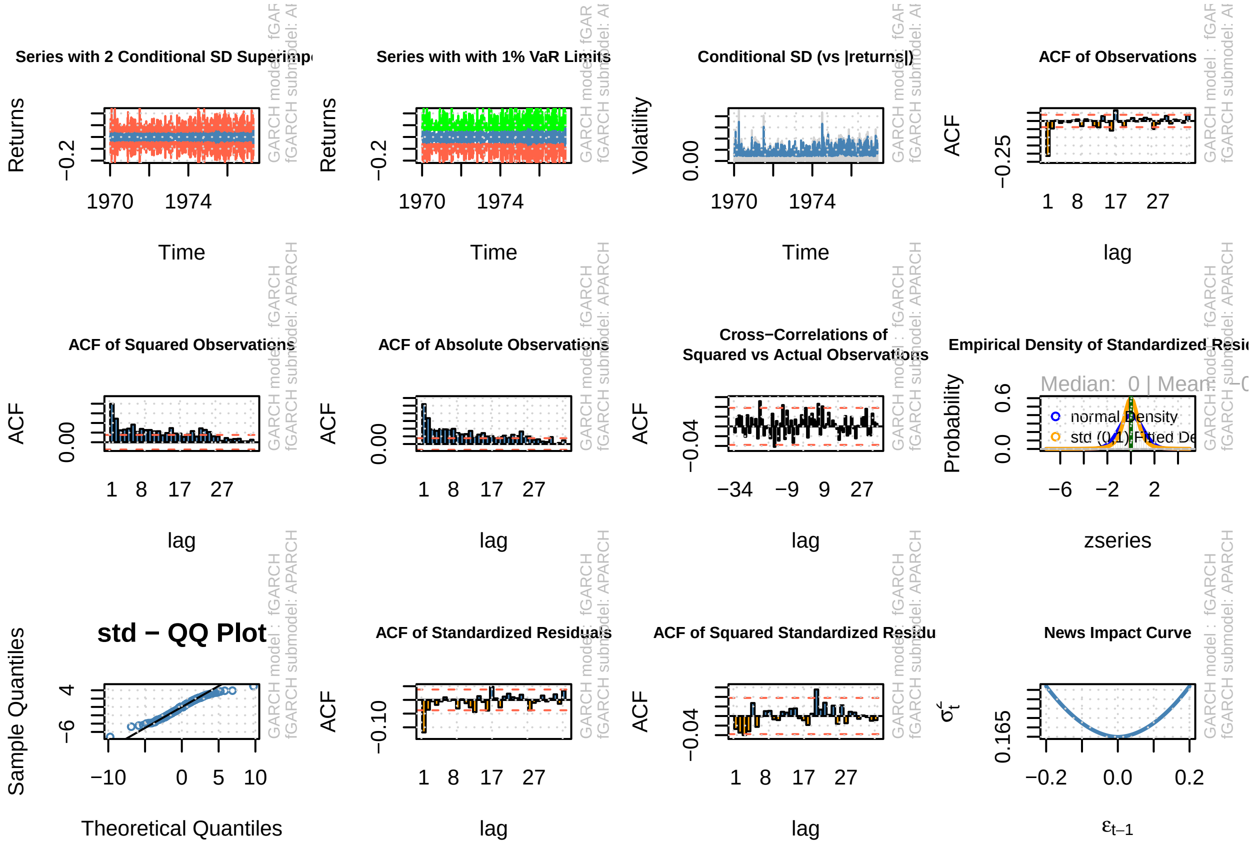
USING GARCH(1,1,1) CARR Model

```
garchMod_t <- ugarchspec(
          variance.model=list(model="fGARCH",
                              garchOrder=c(1,1),
                              submodel="APARCH"),
           mean.model=list(armaOrder=c(0,0),
                          include.mean=TRUE,
                          archm=TRUE,
                          archpow=2
                          ),
           distribution.model="std"
                           )
```

```
garchFit1 <- ugarchfit(spec=garchMod_t, data=SASN.Return.based)
#garchFit1
```

```
plot(garchFit1, which="all")
```

```
##
## please wait...calculating quantiles...
```

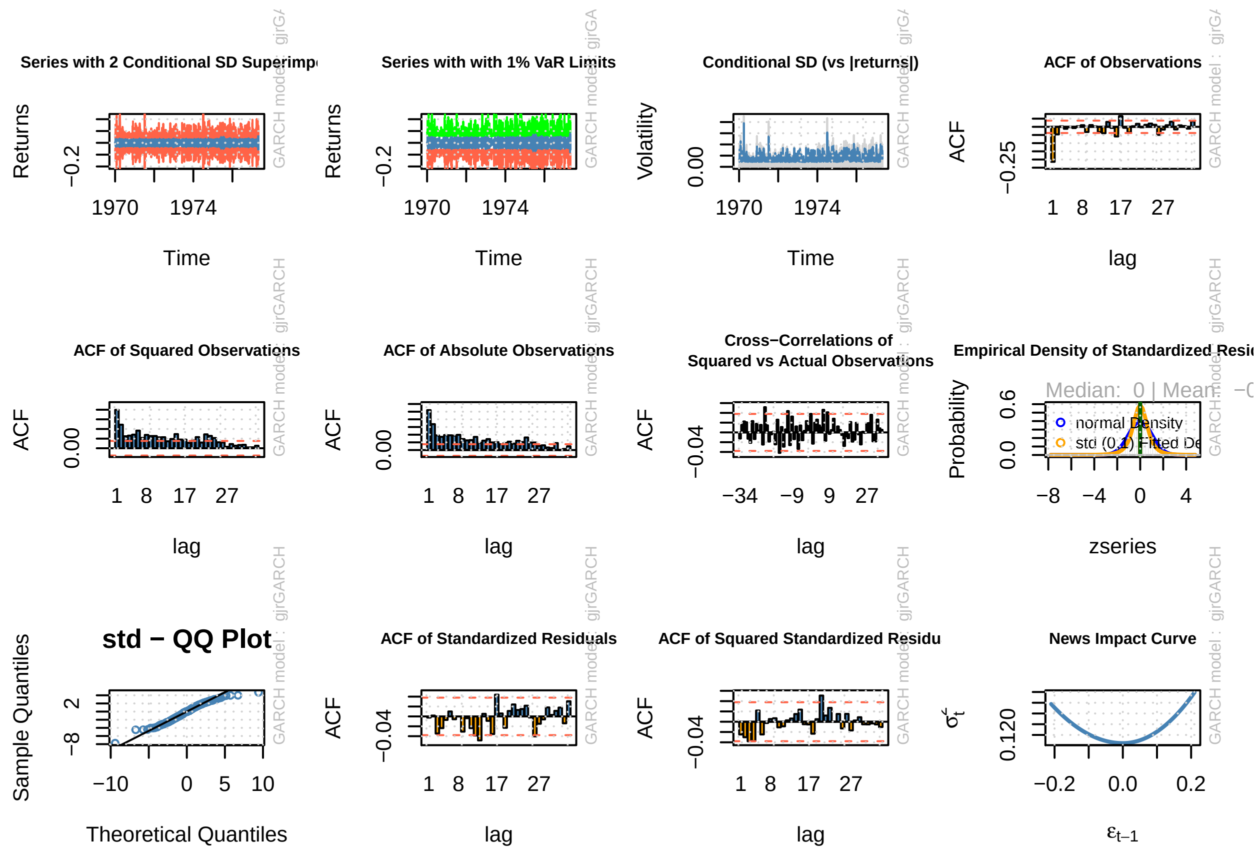


Modeling with TARCH SASN Data

```
library(rugarch)
spec_tgarch<-ugarchspec(variance.model = list(model="gjrGARCH", garchOrder=c(1,1)), mean.model=list(arma
sasn_tgarchfit<-ugarchfit(spec = spec_tgarch,data = SASN.Return.based)
#sasn_tgarchfit
```

```
# plot & display model sasn_tgarchfit
plot(sasn_tgarchfit, which = "all")
```

```
##
## please wait...calculating quantiles...
```

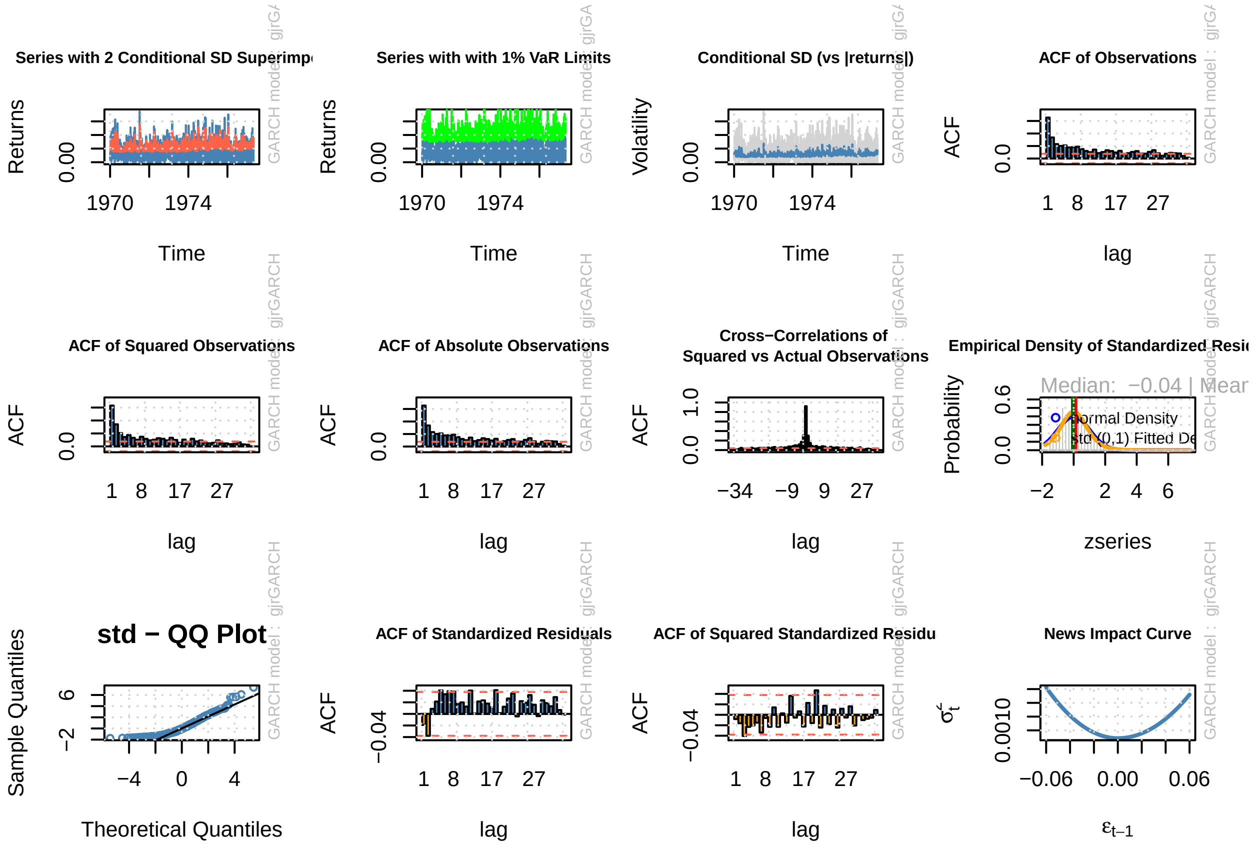


Modeling RTARCH

```
# Using Range-based
sasn_tgarchfit.range<-ugarchfit(spec = spec_tgarch,data = SASN.RANGE.Returns)
#sasn_tgarchfit.range
```

```
# plot RTARCH Range-based model
plot(sasn_tgarchfit.range,which="all")
```

```
##
## please wait...calculating quantiles...
```




```
tgarch.forecast<-sasn_tgarchfit.range@fit$residuals
tarch.forecast<-sasn_tgarchfit@fit$residuals
rgarch.forecast<-sasn_ugfit@fit$residuals
garch.forecast<-sasn_ugfit_h@fit$residuals
carr.forecast<-garchFit1@fit$residuals
```

```
rgarch.mse<-mse(rSASN$`Range-Based Returns`,sasn_ugfit@fit$sigma)
rgarch.qlike<- LossVol(rSASN$`Range-Based Returns`,sasn_ugfit@fit$sigma,which="QLIKE")
garch.mse<-mse(rSASN$`Range-Based Returns`,sasn_ugfit_h@fit$sigma)
garch.qlike<-LossVol(rSASN$`Range-Based Returns`,sasn_ugfit_h@fit$sigma,which="QLIKE")
tarch.mse<-mse(rSASN$`Range-Based Returns`,sasn_tgarchfit@fit$sigma)
tarch.qlike<-LossVol(rSASN$`Range-Based Returns`,sasn_tgarchfit@fit$sigma,which="QLIKE")
rtarch.mse<-mse(rSASN$`Range-Based Returns`,sasn_tgarchfit.range@fit$sigma)
rtarch.qlike<-LossVol(rSASN$`Range-Based Returns`,sasn_tgarchfit.range@fit$sigma,which="QLIKE")
carr.mse<-mse(rSASN$`Range-Based Returns`,garchFit1@fit$sigma)
carr.qlike<-LossVol(rSASN$`Range-Based Returns`,garchFit1@fit$sigma,which="QLIKE")
#All.LossFunctions<-data.frame(rgarch.mse,rgarch.qlike,garch.mse,garch.qlike,tarch.mse,tarch.qlike,rtarch.mse,rtarch.qlike,carr.mse,carr.qlike)
#All.LossFunctions
```

```
# Computing Average Square Magnitude Function (ASMF)
rgarch.ASMF<-sum(sasn_ugfit@fit$residuals)
rgarch.ASMF
```

```
## [1] 2.048433
```

```
#computing RV and ASMF for GARCH Model
garch.RV<-RV(rSASN$`Range-Based Returns`,sasn_ugfit_h@fit$sigma)
garch.ASMF<-sum(sasn_ugfit_h@fit$se.coef)
```

```r
#garch.RV
#garch.ASMF
#computing RV and ASMF For CARR model
carr.RV<-RV(rSASN$`Range-Based Returns`,garchFit1@fit$sigma)
carr.ASMF<-sum(garchFit1@fit$se.coef)
#carr.RV
#carr.ASMF
# computing RV and ASMF for TARCH
tarch.Rv<-RV(rSASN$`Range-Based Returns`,sasn_tgarchfit@fit$sigma)
tarch.ASMF<-sum(sasn_tgarchfit@fit$se.coef)
#tarch.Rv
#tarch.ASMF
#computing RV and ASMF for RTARCH
rtarch.RV <-RV(rSASN$`Range-Based Returns`,sasn_tgarchfit.range@fit$sigma)
rtarch.ASMF <- sum(sasn_tgarchfit.range@fit$se.coef)
#rtarch.RV
#rtarch.ASMF
```

Determining Diebold-Mariano

```r
#install.packages("multDM")
#GARCH vs TARCH
#garch.tarch<-dm.test(garch.forecast,tarch.forecast,h=1)
#garch.tarch
DM.test(garch.forecast,tarch.forecast,rSASN$`Range-Based Returns`,loss.type = "SE",h=1)
```

```
##
##  Diebold-Mariano test
##
## data:  garch.forecast and tarch.forecast and rSASN$`Range-Based Returns`
## statistic = -6.6657, forecast horizon = 1, p-value = 2.634e-11
## alternative hypothesis: Forecast f1 and f2 have different accuracy.
```

```
#GARCH vs RGARCH
garch.rgarch<-DM.test(garch.forecast,rgarch.forecast,rSASN$`Range-Based Returns`,loss.type = "SE",h=1)
#garch.rgarch
#GARCH vs RTARCh
garch.rtarch<-DM.test(garch.forecast,tgarch.forecast,rSASN$`Range-Based Returns`,loss.type = "SE",h=1)
#garch.rtarch
#GARCH vs CARR
garch.carr <- DM.test(garch.forecast,carr.forecast,rSASN$`Range-Based Returns`,loss.type = "SE",h=1)
#garch.carr
#TARCH vs RGARCH
tarch.rgarch <-DM.test(tarch.forecast,rgarch.forecast,rSASN$`Range-Based Returns`,loss.type = "SE",h=1)
#tarch.rgarch
#TARCH vs RTARCH
tarch.rtarch<-DM.test(tarch.forecast,tgarch.forecast,rSASN$`Range-Based Returns`,loss.type = "SE",h=1)
#tarch.rtarch
#RGARCH vs RTARCH
rgarch.rtarch<-DM.test(rgarch.forecast,tgarch.forecast,rSASN$`Range-Based Returns`,loss.type = "SE",h=1)
#rgarch.rtarch
#RGARCH vs CARR
rgarch.carr<-DM.test(rgarch.forecast,carr.forecast,rSASN$`Range-Based Returns`,loss.type = "SE",h=1)
#rgarch.carr
#RTARCH vs CARR
rtarch.carr<-DM.test(tgarch.forecast,carr.forecast,rSASN$`Range-Based Returns`,loss.type = "SE",h=1)
#rtarch.carr
```

Appendices

R Codes

R Notebook