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Master Project in Applied Mathematics

Control of Mechanical System by Moving Coordinates and Motion in Fluids, By Applying of Additional Forces and Having Coordinates as a Function of Time

Research Report in Mathematics, Number 14,2020

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Submitted to the School of Mathematics in Partial fulfillment for a degree in Master of Science in Applied Mathematics

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Research Report in Mathematics, Number 14,2020

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Master Thesis

Submitted to the school of Mathematics in Partial fulfilment for a degree in Master of Science in Applied Mathematics

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Abstract.

This thesis is about the qualitative Analysis and model of equations concerned to the control of the mechanical system by moving coordinates and locomotion in a fluid. There are two essential different ways of controlling the mechanical system's motion that is; by applying additional forces and by directly prescribing some of the coordinates as a function of time.

Flettner rotor initiates locomotion of mechanical systems in fluid and by changing the position of the mass center gravity or internal mass, the body can then be moved dependently and can be controlled. There is full stabilization realized at any point of space when the mechanical system subjected to circulation.

When mechanical system is subjected to non-holonomic constraints whereby the asymptotic stability appertaining to non-equilibrium location gets debilitated and transformed to non-asymptotic. By action of holonomic restraints possessing feeble non-holonomic, a system can be stabilized to stable non-asymptotic.

This thesis also model equation of motion for finite-dimensional lagrangian systems and explains the laws of set-valued force that come from the system's interaction with its environs. The laws of a set-valued conditionally rely on geometric form and entities of kinematics.

The dissertation qualitatively analyzes into controllability of bodies dealing with countless or infinite-dimension extension, plunged in fluids with viscosity, and with non-zero vorticity. In particular, we can obtain controllability and stabilization properties for these infinite-measurable extents systems.

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Declaration and Approval

I the undersigned declare that this dissertation is my original work to the best of my knowledge, it has not been submitted in support of an award of a degree in any other university or institution.

Signature   Date

Peter Okoth Olonde

Reg No: I56//11831/2018

In my capacity as a supervisor of the candidate's dissertation, I certify that this dissertation has my approval for submission

Signature   Date

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Dedication

To the Almighty God for being faithful and my family who have always been financially, morally and spiritually supportive. The University of Nairobi has also provided a conducive environment for learning and growth.

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Acknowledgment

I thank God for having given me a healthy life throughout the period I conducted this research. I evince my gratitude to course mates in field of Applied Mathematics who severely gave a lot of assistance, support, and directions during my research that enabled me to seek outstanding scientific sentiments and write this thesis which I still have passion and creed to do more research and enhance the topic.

I feel grateful to my supervisor Dr. James Marita Okwoyo who magnanimously aided my research by perpetually rectifying and pinpointing factual scientific and mathematical speck annotated in my research. Thanks also go to my family who gave me financial support.

Peter Okoth Olonde

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1.0 Introduction

The aim of this project is to conduct qualitative analysis and model equation concerned in control of mechanical system by moving coordinates and motion in fluids, by applying of additional forces and having coordinates as a function of time.

Chapter 1

There has been a lengthy time frame of research on challenges appertaining to forces acting on the mechanical system moving in a perfect fluid. The thesis is about, control of motion of an inflexible system in a perfect fluid. Essentially a lot of outcomes have been achieved through the formulation of a perfect fluid by the likes of Chaplygin [10] and Kirchh of [19]. These modeling checks into a qualitative breakdown of the mechanical system's motion in a perfect fluid, check [9, and 8].

The control hypothesis where moving coordinates that initiate the movement of internal mass or center of mass that subsequently enhanced body's (propulsion), is a validated as captured in [21] which is explicitly expressed in [18,20], even though viscous fluid execute a masterpiece role by giving traction force that controls motion [11]. An alternative body's motion is also achieved by rotation of the Flettner rotor which engages gyrostatic momentum (Kilin and Vetchanin, 2015). Stability through buoyancy explained in [23, 24]. Flettner rotor also controls the system of nonholonomic as exhibited in [5, 6, 7, and 16].

In the motion of mechanical, there is acceptance of occurrence of circulation induced by the fluid velocity existing around the system, this further expounds actions of lift force in the reference to Zhukovskii and Chaplygin. Circulation itself is valued and is induced by gyroscopic forcing aids in alteration of the system's dynamics [9]. The system's motion is controlled with the aid of gyrostat that brings stabilizing effect to the system and alteration of circulation existing around the system as shown in [26]. Notably, circulation initiates the change of position of the internal mass.

It is pragmatic that control of motion of the mechanical system in a free motion can be achieved in [6, 21, 25 by use of Rashevskii-chow hypothesis to substantiate manageability of control of motion as reflected in [1], the aforementioned hypothesis proofs control in free motion as captured in [3]. Furthermore, in [15] there is formulation concerning the substantiation of Poisson stabilization of free motion.

There is proof of motion controllability with aid of linear and circular motion of the center of mass. Surveillance on control of the system's motion introduces challenges concerning speed thus time-optimal control is improvised and expressed in [13, 14], more information about controllability check [17]. In this thesis Integrals of motion's equation substantiate control by rotation of Flettner rotor and circulation around the enhance control of motion

In the details at the final position of trajectory, the system's stabilization mitigates or thwarts the poor effects of circulation. The motion of internal mass should be linear and consistent to get a stable system at a certain position. There should be angular velocity and System's velocity shouldn't have a translation to enhance proper control because the center of mass is bounded.

1.1 Statement of the Problem

The control task is to manipulate the motion of a two-dimensional body in an ideal fluid with the aid of a moving internal mass and a Flettner rotor in the presence of constant circulation around the body. To achieve this, we need to change the position of the internal mass and with the help of a rotating Flettner rotor, the body can be made to move from one point to another.

We determine the complete stability of a body at an arbitrary point of space, in achieving this we require prior knowledge about the influence of nonzero circulation on the motion control, which we ensure that body moves near the given point.

Subsequent task is determining, the influence of non-holonomic constraints on the mechanical system. This is achieved by whereby ensuring asymptotic stability of non-equilibrium position gets weaken and transformed to non-asymptotic by weak non-holonomic constraints, this is when a system is stabilized to stable non-asymptotic.

Another task is controllability and stabilization of bodies with infinite-dimension, immersed in viscous fluids having non-zero vorticity. In achieving this, we need to have an idea about properties for these infinite-measurable extents systems. In a detailed linear system, we get a nonlinear trajectory fully characterized. The control of a linear system admits the solution of inverse dynamics using the structure of a linear system to compute inputs necessary for the performance of a task.

An additional task is the controllability of a body or a chain of bodies with finite-dimensional, immersed in a non-viscous irrotational fluid. To obtain this, we need to know the motion of a body having a finite-dimensional Lagrangian system. This is done by getting some geometric properties that make the system "fit for jumps" so that the equations of motion are linear concerning the time derivative of the control function.

1.2 Objectives and Methodology

The primary aim of the research is to do developed formulation and establish manageability of control of the mechanical system by moving coordinates and motion in fluids using two important concepts, which are;

i. Application of additional forces
ii. Direct prescription of some coordinates as functions of time.

The objective of the thesis considers a diverse extension of theory appertaining to controllability, stabilization, geometric dimensions, and impacts of non-holonomic constraints to the mechanical system, the listed below are also the objectives of the thesis;

i. To seek impact when a mechanical system is subjugated to its non-holonomic constraints.
ii. To determine geometric dimensions that make the system “fit for jump”, to check if the equations of motion are linear with respect to time derivative of control function. This is will be possible by immersing a body in a non-viscous irrotational fluid since the motion can elaborate by the finite-dimensional Lagrangian system.
iii. To probably obtain controllability and stabilization properties for infinite-dimensional systems, when bodies immersed in viscous fluids with non-zero vorticity.

As per the methodology, the research done regarding this dissertation was an applied research, meaning it is not new. Instead, many pieces of previous academic research exist about control of mechanical system by moving coordinates and locomotion in fluids. Other extension of the basic theory and some practical were done and analyzed that is “a rolling ball on horizontal table”. This dissertation has possessed a new research form but on an existing research subject. Methodology uses mathematical model, kinematics analysis, and mathematical analysis in which there is formulation of functions

Chapter 2

2.0 Literature Review

Research about the control motion of a body in an ideal fluid with a moving internal mass and an internal rotor in the presence of constant circulation around the body, which was first researched by (Vetchanin and Kilin 2016). The control motion of a rigid body in an ideal fluid is a classical problem of hydrodynamics and has been studied for a long time. Many substantial outcomes were achieved within the hypothetical description of an ideal fluid by Lamb [22] and Steklov [29].

According to the research, changing the position of the internal mass and by rotating the rotor, the body can be made to move to a given point, and discuss the influence of nonzero circulation on the motion control. However, in the presence of circulation around the body the system cannot be completely stabilized at an arbitrary point of space.

Therefore, this thesis which perfectly researches the study of the control of the mechanical system by moving coordinates and locomotion in the fluid by application of additional forces and prescription of the coordinates as a function of time fully controls the motion of the mechanical system motion with complete stabilization at any point.

Also, the thesis further discusses the impacts of non-holonomic constraints on the mechanical system. Review of an article titled "effects of nonholonomic constraints on the mechanical system", written by Porikladnaya Mekhanika (1965). In which the primary result found comprises the concept that whenever ideal nonholonomic restraints are inflicted on mechanical systems the asymptotic stability of the zero-equilibrium position is weakened to nonasymptotic, However, the article does not explain how nonholonomic constraints with weak nonholonomic strengthen a system to nonasymptotic stability. This thesis, therefore, improves the article by expounding that utilizing nonholonomic constraints with weak nonholonomic it is also possible to strengthen a system to nonasymptotic stability.

The article titled "measure differential involvement in Control of motion for finite-dimensional lagrangian systems" written by Zurich (1973), explains the laws of set-valued force that come from the system's interaction with its environment. Laws of a set-valued conditionally rely on geometric form and entities of kinematics. Due to the habituation, this relationship of forces and entities of kinematic are surveyed in detail. Classically, nonpotential unilateral forces are contained by appropriate generalized force directions in the generalized force direction. The weakness of the article is that there is no formulation of impinging situations on positions thus is not explicitly comprehensive. This thesis formulates the impinging on the position where velocity an acceleration levels exactly defined.

A research title, "Infinite-dimensional extension, symmetries and catalog", written by Phillipe, Richard, and Pierre (2007), explain the notion of infinite dimension system central equivalence and flatness matched correspondence between trajectories of systems which is not restricted to control systems described by ordinary differential equations. It can be adjusted to delay differential systems and to partial differential equations with boundary control. The weakness of the research is that there are a lot of technicalities and the visualization is not comprehensive. This thesis, therefore, makes partial differential equation visualizable, less technical, and comprehensive.

The dissertation also checks into controllability and stability of infinite-dimension extension, covering bodies immersed in fluids with viscosity, and with non-zero vorticity. In particular, we can obtain controllability and stabilization properties for these infinite-measurable extents al systems. The thesis expounds a linear system in which the nonlinear trajectory is fully characterized and are controllable. Comprehensive control of the linear system admits the solution of inverse dynamics using the structure of the linear system to compute inputs necessary for the performance of a task.

Chapter 3

3.0 The Mathematical Model

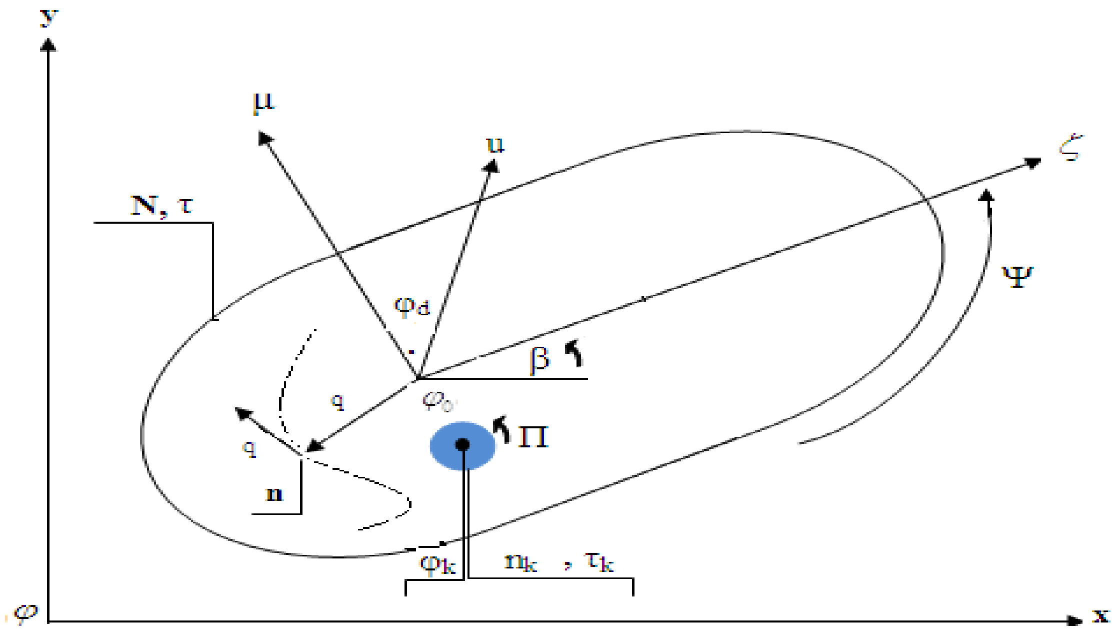

Figure 1: Mechanical system here is a simple body and Flettner rotor.

The system contains a speck[1] of mass **'n'** and fletcher rotor having a mass of $\boldsymbol{n_k}$. Speck motion is limited by the shell motion though traces (arbitrary) smooth trajectory given by $q = [\varsigma(t), \mu(t)]$. Flettner rotor is circular and it is rotating at an angular velocity $\pi(t)$, in detail the system's or body's level of motion is orthogonal to the Flettner rotor axis of rotation that goes via rotor's internal mass. The assumption is made that there exist continuous circulation ψ at the body's peripheral is induced by fluid's velocity; this is derived from the Lagrange hypothesis. The position φ_0 concurs with the system's internal mass at a point whereby the system's position in space is given by vector $k = (x, y)$

[1] Speck is a very small particle, which in this case is contained in a moving mechanical system.

, this is a radius vector. To explain about system's motion, we have two Cartesian coordinates that are a rigid one and a moving coordinate φxy are bounded

$\varphi_0 \zeta \mu$ respectively on the system (check Fig 1). Moving coordinate rotates and makes an angle β with fixed coordinate hence the system's space framework is illustrated by point (k, β), this fully points out the system's location and alignment.

The system's angular velocity is denoted by ψ. At the body's center φ_0, the absolute velocity is represented by $u = (u_1, u_2)$. This velocity appertains to the axes of the moving coordinate. Therefore, this relation of kinematic account is liable:

$$dp = P\psi \, , P = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{1}$$

Here the vector of velocities denoted by $w = (u_1, u_2, \omega)$ and abscissa $p = (x, y, \beta)$ are vectors. Vectors of radius ζ_d and μ_d defines the position φ_d of the internal mass of the system[2] while vectors of radius ζ_k μ_k define the position φ_k of the internal mass of the Flettner rotor.

3.1 Prescription of Coordinates as Functions of Time

Flettner action, the center of the system's moving coordinate, the Mechanical system's kinetic energy is expressed to the extent of a certain function of time as shown below.

$$E = 0.5((C \times \psi), \psi) + (\mathrm{v}, \psi),$$

$$C = \begin{pmatrix} c_1 & 0 & \square \\ 0 & c_2 & l \\ \square & l & d \end{pmatrix} , v = \begin{pmatrix} n\zeta' \\ n\mu' \\ n(\zeta\mu' - \zeta'\mu) + \tau_k \pi \end{pmatrix} \tag{2}$$

$$c_1 = N + n + n_r + \gamma_1,$$

[2] The term system referred to the body in motion

$$c_2 = N + n + n_k + \gamma_{2,}$$

$$d = N(\zeta_d^2 + u_d^2) + \tau + n(\zeta^2 + u^2) + n_k(\zeta_k^2 + u_k^2) + \tau_k + \gamma_6,$$

$$h = -nu, l = n\zeta$$

3.2 Additional Forces

External circular motion or circulation around the body's peripheral induces additional forces thus making to have an equation of motion of the system considered as below.

$$\frac{d}{dt}\left(\frac{\delta E}{\delta u_1}\right) = \psi \frac{\delta E}{\delta u_2} - \gamma u_2 - \xi\psi,$$

$$\frac{d}{dt}\left(\frac{\delta E}{\delta u_2}\right) = (-1) \times \psi \times \frac{\delta E}{\delta u_1} + (\gamma \times u_1) + (\chi \times \psi), \tag{3}$$

$$\frac{d}{dt}\left(\frac{\delta E}{\delta \psi}\right) = u_2 \frac{\delta E}{\delta u_1} - u_1 \frac{\delta E}{\delta u_2} + \xi u_1 - \chi u_{2,}$$

Unequal sided body moving in fluid experiences acting forces and the coefficient related to the system of the body(10) are $\boldsymbol{\eta}$ $\gamma = (\varpi \times \Psi)$,u, $\zeta = (\varpi \times \Psi \times \eta)$and $X = (\varpi \times \Psi \times u)$ the previous derivatives (3) can simply be illustrated with aid of Poincare equations in (2) above.

$$\frac{d}{dt}\left(\frac{\delta G}{\delta u_1}\right) = \psi \frac{\delta G}{\delta u_2} + \frac{\sin\beta}{\tan\beta}\frac{\delta G}{\delta x} + \sin\beta \frac{\delta G}{\delta y},$$

$$\frac{d}{dt}\left(\frac{\delta G}{\delta u_2}\right) = -\psi \frac{\delta G}{\delta u_1} - \sin\beta \frac{\delta G}{\delta x} + \frac{\sin\beta}{\tan\beta}\frac{\delta G}{\delta y}, \tag{4}$$

$$\frac{d}{dt}\left(\frac{\delta G}{\delta \psi}\right) = u_2 \frac{\delta G}{\delta u_1} - u_1 \frac{\delta G}{\delta u_2} + \frac{\delta G}{\delta \beta}$$

System mass is taken as the summation of body mass **N** moving in an ideal fluid, speck mass **n** (mass of a particle in the body)**,** and Flettner rotor having mass $\boldsymbol{n_k}$. Product of system mass and gravitational force result in an acting force. This force acts perpendicularly and when divided by area of the body which is in contact with fluid, it exacts a pressure, then the pressure weakens viscous drag[3] between the mechanical system and the fluid. Hence the velocity of the mechanical system moving in the ideal fluid is not limited by viscous drag. Going to further extend of putting down equations in Lagrangian form as expressed below;

$$G = 0.5((C\times\psi),\psi)+(a,\psi)+(v,\psi),$$
$$a = \begin{pmatrix} -0.5\gamma(x\sin\beta - y\cos\beta) \\ -0.5\gamma(x\cos\beta + y\sin\beta) \\ -\chi(x\sin\beta - y\cos\beta) - \xi(x\cos\beta + y\sin\beta) \end{pmatrix} \tag{5}$$

Combination of equations (1) and (4) create an enclosed compound of six differential equations

$$\frac{d}{dt}\left(c_1u_1 + h\psi + n\zeta^{'}\right) = \psi\left(c_2u_2 + l\psi + n\mu^{'}\right) - \gamma u_2 - \zeta\psi,$$
$$\frac{d}{dt}\left(c_2u_2 + l\psi + n\mu^{'}\right) = -\psi\left(c_1u_1 + h\psi + n\zeta^{'}\right) + \gamma u_1 + \chi\psi,$$
$$\frac{d}{dt}\left(hu_1 + lu_2 + D\psi + n\left(\zeta\mu^{'} - \mu\zeta^{'}\right) + \tau_k\Pi\right) = u_2\left(c_1u_1 + h\psi + n\dot{\zeta}\right) - u_1\left(c_2u_2 + l\psi + n\dot{\mu}\right) + \xi u_1 - \chi u_2,$$
$$\frac{dx}{dt} = u_1\cos\beta - u_2\sin\beta, \qquad \frac{dy}{dt} = u_1\sin\beta + u_2\cos\beta, \qquad \frac{d\beta}{dt} = \psi \tag{6}$$

The motion of the mechanical system is fully annotated in the variables u_1, u_2, ψ, x, y, and β . In a scenario where the mechanical system is in free motion that is $\zeta^{'} = \mu^{'} = 0, \Pi = 0,$ then equation (6) above, mathematically acknowledge first integrals in equations 9 and 10.

$$q_x = \left(\frac{\partial G}{\partial u_1} - \chi\right)\frac{\sin\beta}{\tan\beta} - \left(\frac{\partial G}{\partial u_2} - \xi\right)\sin\beta + (0.5\gamma)y,$$
$$q_y = \left(\frac{\partial G}{\partial u_1} - \chi\right)\sin\beta + \left(\frac{\partial G}{\partial u_2} - \xi\right)\frac{\sin\beta}{\tan\beta} - (0.5\gamma)x \tag{7}$$
$$R = xq_y - yq_x + \frac{\partial G}{\partial\psi} + 0.5\gamma\left(x^2 + y^2\right) - a_3.$$

When the motion of a mechanical system is controlled, then the equation above is expressed as below.

---

[3] *is the frictional force between moving mechanical system and fluid. It limits the velocity of the motion of the system in the fluid*

$$q_x = (c_1 u_1 + h\psi + v_1 - \chi)\frac{\sin\beta}{\tan\beta} - (c_2 u_2 + l\psi + v_2 - \xi)\sin\beta + \gamma y, .$$

$$q_y = (c_1 u_1 + h\psi + v_1 - \chi)\sin\beta + (c_2 u_2 + l\psi + v_2 - \xi)\frac{\sin\beta}{\tan\beta} - \gamma x,$$

$$R = hu_1 + lu_2 + D\psi + v_3 + 0.5\gamma(x^2 + y^2) + xq_y - yq_x. \tag{8}$$

The body's angular and linear momentum speck appertain to the integrals q_x , q_y , and R. these also control specks of locomotion of a body in a fluid which is summarized as integrals for the mechanical system having internal masses in motion (21).

We take into account that equations (6) having the differentiation of (Π') and it has no angular velocity (Π), thereafter the rotation of the Flettner rotor is at consistent angular velocity as never affects the dynamic of the mechanical system.

3.3 Controllability of Mechanical System's Motion

With aid of the Rashevskii-Chow hypothesis expressed in [1, 12, and 27] substantiates chances of control of motion on a rigid set of initial integrals in the mechanical system with the prevalence of free motion[4] [3].The hypothesis needs a compact set of steady Poisson speck at every point for drift in the mechanical system phase, provided there is fullness of linear span of the vector fields, the velocities U_1, U_2, and ψ as shown in equations below.

$$(c_1 u_1 + h\psi - \chi)^2 + (c_2 u_2 + l\psi - \xi)^2 = (q_x - \gamma y)^2 + (q_y + \gamma x)^2 . \tag{9}$$

In equation (9) it's prominent that curved domain bounds the free motion, in which domain's location and size rely on magnitude of the kinetic energy of the system E, sets of integrated functions q_x and q_y, geometrical of body structure and circular motion.

[4] *is a motion of mechanical system that cannot be controlled or are debilitated and its non-zero motion, drift is same as free motion*

In the integration of (1) and (4) concerning Poincare recurrence hypothesis (2) then it's achieved that drift is Poisson steady. Then, after this, we research on the fullness of the vector field's linear span and related exchangers.

Control of motion of the mechanical system with aid of internal mass that's moves (arbitrary) within the body is validated. Therefore, the addition of the Flettner rotor to (an analogous) mechanical system then it is obvious that motion can be controlled. In this part, we substantiate control of motion of two situations whereby the following restraining conditions are subjected to the possibility of movement of the center of mass;

a) Non-moving or rigid center of mass.

b) Center of mass moving curve-wise.

3.4 **Control of motion of a system when internal mass is rigid**

The thesis checks into control of motion of the mechanical system, taking into account the altering rotation of the Flettner rotor, internal mass ignored because it is rigid. Various equations of motion are annotated and expressed as manifested below by first commencing with $\zeta^{'} = \mu^{'} = 0$. Having motion's equations in [4] and very initial integrals in [8] as below:

$$\frac{d}{dt}(c_1 u_1) + \frac{d}{dt}(\square\psi) = c_2 u_2 \psi + l\psi^2 - (\gamma u_1 + \zeta\psi),$$

$$\frac{d}{dt}(c_2 u_2) + \frac{d}{dt}(l\psi) = -\psi c_1 u_1 - \psi^2 \square + \gamma u_1 + x\psi,$$

$$\frac{d}{dt}(\square u_1 + l u_2 + d\psi + \tau_k \pi) = u_2(c_1 u_1 + \square\psi) - u_1(c_2 u_2 + l\psi) + \xi u_1 - x u_2$$

(10)

Then the equations below follow:

$$q_x = (c_1 v_1 + \square\psi - \chi)\frac{sin\,\beta}{tan\,\beta} - (c_2 u_2 + l\psi - \xi)\,sin\,\beta + \gamma y,$$

$$q_y = (c_1 u_1 + \square\psi - \chi)\,sin\,\beta + (c_2 u_2 + l\psi - \xi)\frac{sin\,\beta}{tan\,\beta} - \gamma x,$$

$R = \square u_1 + l u_2 + d\psi + \tau_k \pi + \frac{\gamma}{2}(x^2 + y^2) + x q_y - y q_x$`````````

(11)

We get velocities from integral functions of q_x, q_y and function R as;

$$\boldsymbol{\psi} = \frac{\mathbf{1}}{\boldsymbol{c}}\begin{pmatrix} \bar{x}\frac{\sin\beta}{\tan\beta} + \bar{y}\sin\beta + \chi \\ -\bar{x}\sin\beta + \bar{y}\frac{\sin\beta}{\tan\beta} + \xi \\ H - (2\gamma)^{-1}(\bar{x}^2 + \bar{y}^2) - \tau_k\pi \end{pmatrix}$$

(12)

In which $\bar{x} = q_x - \gamma_y$, $\bar{y} = q_y + \gamma_x$, $H = 0.5\gamma(2R\gamma + q_x^2 + q_y^2)$, sequentially velocities in the matrix (12) is replaced with k[6] in (1). Thereafter equation appertaining body's motion is achieved as the body is at a rigid point on integrals set: q_x, q_y and R. The equation responsible to motion control and it is outstanding linear form is expressed below;

$$d/dp = U_0(p) + U_1(\pi), \tag{13}$$

$$U_0(p) = z\left(\bar{x}\cos\beta + \bar{y}\sin\beta + \chi, -\bar{x}\sin\beta + \bar{y}\cos\beta + \xi, H - (0.5\gamma)\left(\bar{x}^2 + \bar{y}^2\right)\right)^E$$

$$U_1 = Z(0,0,-\tau_k)^E, Z = \frac{p}{C},$$

[6]*kinematic similarities occur when velocity at any position in the framework flow is rationally related by a fixed scale factor to the velocity at the homologous point in the perfect flow while thinking it is preserving the similar flow streamline shape.*

Flettner rotor[7] rotates at an angular velocity Π which is regarded as the control of motion, the vector U_0 relates to drift as vector field U_1 concerns for of control activity. Notably, the Flettner rotor initiates

the locomotion of the mechanical system in a fluid. Checking into vectors field with some in brackets: $U_0, U_1, U_2 = [U_0\, U_1]$, $U_3 = [U_2, U_1]$.

(14)

Rank appertaining to the linear span of a vector that is $U_0U_1U_3$ equivalent to three on every point, but not on the plane expressed below

$$-\frac{4}{2}\left(-c_2^2+c_1^2\right)\left(\left(\bar{x}^2-\bar{y}^2\right)\frac{\sin^2\beta}{\tan\beta}^2+\overline{xy}\left(-1+2\cos^2\beta\right)\right)+\left(2c_1^2-c_1c_2\right)\xi\left(\bar{y}\sin\beta+\bar{x}\frac{\sin\beta}{\tan\beta}\right)$$

$$+\left(2c_2^2-c_1c_2\right)x\left(\frac{\bar{x}\cos\beta\tan^2\beta-\bar{y}\sin\beta}{\tan\beta}\right)=0 \tag{15}$$

Alternatively, a linear span of the vector fields $\mathbf{U}_1$, $\mathbf{U}_2$, $\mathbf{U}_3$ has a rank equivalent to three in every surface but not on this plane below;

$$c_1^2\left(\xi^2+c_2^2x^2+\frac{4}{2}\left(c_1-c_2\right)^2\left(\bar{x}^2+\bar{y}^2\right)+3c_1\left(c_2-c_1\right)\right)\xi\left(\bar{x}\sin\beta-\bar{y}\frac{\sin\beta}{\tan\beta}\right)+3\left(c_2^2-c_1c_2\right)\chi\left(\bar{y}\sin\beta+\bar{x}\cos\beta\right)=0$$

(16)

Remark

Equations [15] and [16] are true whenever the moving coordinate is selected in a way that matrix C becomes a diagonal matrix as C=diagonal (c_1, c_2, d). The planes of (15) and (16) intersect on the curves;

$$\bar{x}=\left(I_1+I_2\right)\cos\beta\tan\beta+\left(I_1-I_2\right)\frac{\sin\beta}{\tan\beta}$$

---

[7]. *Notably, the Flettner rotor is a kind of a rotor that initiates locomotion of the mechanical system in a fluid.*

$$\bar{y} = (-I_1 - I_2)\frac{\sin\beta}{\tan\beta} - (-I_1 + I_2)\cos\beta\tan\beta \tag{17}$$

The solutions to equations are I_1 and I_2

$$\left(16\!\left/2\right.|c_2 - c_1|I_1 + 3(c_1\xi + c_2 x)\right)^2 + \left(16\!\left/2\right.|c_2 - c_1|I_2 + 6\!\left/2\right.(c_1\xi - c_2 x)\right)^2 - \frac{4}{2}\left(c_1^2\xi^2 + c_2^2 x^2\right) = 0$$

$$\left(I_1 + \frac{c_2(2c_2 - c_1)x + c_1(2c_1 - c_2)\xi}{2(4)(c_2^2 - c_1^2)}\right)^2 - \left(I_2 - \frac{c_2(2c_2 - c_1)x - c_1(2c_1 - c_2)\xi}{2(4)(c_2^2 - c_1^2)}\right)^2 - \frac{c_1c_2(2c_2 - c_1)(2c_1 - c_2)x\xi}{4(c_2^2 - c_1^2)} = 0 \tag{18}$$

Therefore, measurable extent[8] appertaining to vector fields' linear span within configuration space Ꟈ is equivalent to three in all points but not on the line of curvatures check [17]. This is because the aforementioned curvatures are planes of double dimension two, hypothesis below attest to be true.

A body moving in an ideal fluid with circulation around it with the first velocity can be moved from one point to another by rotation of the Flettner rotor. This notion makes the control of motion to be logic. The hypothesis of Rashevskii-Chow establishes control by utilization of motion onward the vector fields in oscillating time that is forth and back. If free-motion prevails then there is the possibility of motion onward it in forward time (Kilin and Verchanin, 2015).

With aid of perpetual occurrence quality of progressive path or trajectories there is the application of motion in backward time, though proration of the period that continuously occurs within a system may be intricately challenged and if the period is lengthy hence putting up such kind of control is cumbersome.

For preferred kind of control with a prevalence of circulation around the body is a prerequisite for control of motion of the mechanical system. It is possible to exhibit. $\Psi = 0$ The system displayed

[8] *A measurable extent is a set of coordinates specifying the position of a point or sizes same as extensive magnitude or generally dimension*

by expression (13) cannot be controlled in the notion of Rashevskii-Chow. Alternatively, from expression (13) circulation initiated by free motion which doesn't fully get indemnified by rotation of Flettner rotor. Then what ensues is that we take into account the compounded model of control by Flettner rotor and motion of the internal mass.

3.5 Control of motion as internal mass moves along a specific curve

Let's propose that motion of the center of mass moves is specifically onward a curve $q=(\zeta(z),\mu(z))$, the curve has a constant z, below is a matrix form illustration of velocities emanating from integrals equation (8).

$$\psi=\frac{1}{C}\begin{pmatrix}\bar{x}\frac{\sin\beta}{\tan\beta}+\bar{y}\sin\beta+\chi-n\frac{d\zeta}{dz}z' \\ -\bar{x}\sin\beta+\bar{y}\frac{\sin\beta}{\tan\beta}+\xi-n\frac{d\mu}{dz}z' \\ H-(2\gamma)^{-1}(\overline{x}^2+\overline{y}^2)-\left(\zeta n\frac{d\mu}{dz}-\mu n\frac{d\gamma}{dz}\right)z'-\tau_k\Pi\end{pmatrix}$$

(19)

We get the system's equation of motion of integrals (8) by replacing equation (19) with relative kinematic expression in (1). The achieved equation relies on the mass position of **z** and velocity z' on a line of curvature, on the same line achieving linear controllability which is equations of motions this is got by application of expression [4].

$$ds=v_0(s)+v_1(z)dz+v_2\Pi,$$

(20)

$$u_0(s)=K\left(\overline{x}\frac{\sin\beta}{\tan\beta}+\overline{y}\sin\beta+\chi,\frac{\sin\beta}{\tan\beta}(-\overline{x}\tan\beta+\overline{y})+\xi,H-(2\gamma)^{-1}(\overline{x}^2+\overline{y}^2),0\right)^E$$

$$u_1(z)=K$$

Within facet space Ꞑ there is the existence of vector $s=(x,y,\beta,z)^E$. It's factual that the velocity of the mechanical system moving onward a curve z' plus the Flettner rotor s' angular velocity Π

ensures the controllability of the mechanical system. In this case coordinates of the center of mass do not contribute to controllability. Free motion is confirmed by vector field U_0 as vector fields U_1 and U_2 are aligned to activities concerned for control of motion. Let's check into these vector fields:

U_1, U_2,

$U_3 = [U_1, U_2]$,

$U_4 = [U_1, U_3]$,

$U_5 = [U_2, U_3]$,

$U_6 = [U_1, U_4]$ (21)

The above vector fields except for $\mathbf{V_O}$ exhibit fullness of their linear span as we also substantiate motion control in the absence of circulation. It is important to check either motion of canter of mass is onward a curve or a line;

1. The reciprocating motion of the center of mass on a straight line is parameterized as shown below

$$\zeta = r_1 \sin z \qquad \mu = r_2 \sin z \tag{22}$$

and taking note that constants r_1, r_2, doesn't disappear at one time while the vector field U_1, U_2, U_4, U_5 depend on the condition that $z = \frac{\pi}{2}\ and \frac{3\pi}{2}$ (23)

The condition of one-dimensional dependence emanating from the equation (23) we get that, $C \sin \beta + D \sin \beta = 0,$ (24)

C and **D** rely on (24) independent variables of the mechanical system. Four ranks are appertaining to vector field linear span (21) which exists in the facet space Ꞓ but not on the plane with double measurable extent as in (23) and (24). Therefore, these hypotheses are true. The body can move in an ideal fluid at a given velocity by responding motion, these are; rotation of Flettner rotor and by the circular motion of the center of mass. The motion is independent to circulation at the body's peripheral

We the given variable to Circular motion of the center of mass as expression shown below$\zeta = k_0 \frac{\sin z}{\tan z}, \ \mu = k_0 \cos z \tan z, \ k_0 \geq 0.$ (25)

The vector field $U_1, U_2, U_3,$ and U_5 are autonomous in space Ꞡ hence the theorem (1.2) above is true.

3.6 A Skier Controlling His Motion

In this case, we consider mechanism a skier use when skiing on a skate to control his speed. The control mechanism involves raising and lowering internal mass or barycentre[9] in the process of skiing on a surface with different undulating[10] topography. We model control derivative equations and then check into jump function with impulsive motion (Lind and Sanders,1996).

Skater carrying a skier has a total weight of "skier" plus the weight of skate itself. In the process of skiing, the skier controls the motion of the skate by raising or lowering his barycenter or internal mass. This is achieved by the skier reactions specifically up and down movements of his body thus changing internal mass position.

On the steep sloppy surface of the snow, there is a high magnitude of gravitational force which influence the high speed of the skate thus skier moderates the speed by lowering barycenter or internal mass which tend to reduce the influence of gravitational force, meaning if skier lowers barycenter, the internal mass gets away from center hence less magnitude of centripetal force and skiing speed reduced. This is done by moving the whole body up and enhances stability by using skiing sticks.

[9] Barycenter is the internal mass or center of mass of the body.
[10] Undulating means sloppy surface, as per the thesis, it is the slope of snowcapped surfaces or topography (landscape).

When the skier gets onto a snow surface of a gentle slope, the magnitude of gravitational force is lessening and the skate's speed is reducing hence the skier raises barycenter or internal mass which moves close to center and centripetal force influence intensify of kinetic energy thus then skiing motion speed increases. Raise of barycenter is done by moving the whole body downwards.

The rate of the body's reaction is not equivalent to the speed of ice molecules displacement or generally speed of the skate. Analytical stability initiated by the change of mass position and speed aids in motion control when skiing. When the skier's weight reacts at a rate equivalent to the speed of inclination of a skier then it limits analytical stability. This scenario happens to liquid with low viscosity, for instance, water and skate with a low natural frequency.

3.7 Additional Forces Aids Control of Skiers' Motion

The mass of the skier under the influence of gravity introduces weight and in this case weight of a skier, the due force of gravity is expressed as $G_w = G_m + G_{let} + G_a$. Where Ga denotes the force that accelerates the skier and G_w denotes weight due to gravity, there exists another force represented as G_{flad}which is expressed as $G_{flad} = G_m + G_{let}, G_{flad}$ is denoting effective force,

The G_{flad} is a skier load simply exerted on the snow and requited influenced by snow. The Skier needs evade skidding by balancing his or her weight making center of mass and the ski intersects on a straight line which must be at 180^0 or parallel to direction traced by the vector of force G_{flad}

$$G_{flad} = G_w - G_a = G_m + G_{let} \tag{26}$$

Then we subsequently achieve

$$G_{flad} = \begin{bmatrix} -1 & 1 & 1 \\ 1 & 1 & \sin\varphi \\ \sin\theta & \sin\varphi & -1 \end{bmatrix} \times \begin{bmatrix} \sin\theta & \sin\theta & \sin\theta \\ \sin\varphi & \cos\theta & \sin\varphi \\ \cos\varphi & \sin\varphi & 1 \end{bmatrix} = (m \times g) \begin{bmatrix} -\sin\theta & \sin\varphi & \cos\varphi \\ \sin\theta & \cos\theta & \sin^2\varphi \\ \sin^2\theta & \sin^2\varphi & -1 \end{bmatrix}$$

The absolute value of G_{flad} is given as shown below;

$$\left\|G_{flad}\right\| = \left(G_{let}^2 + G_M^2\right)^{0.5} = \left(\left(1-\cos^2\theta\right)\sin^2\varphi + 1\right)^{0.5} \times mass \times gravity \tag{27}$$

Figure 2 below demonstrates that in the process of skiing a path traced an angle φ on the horizontal plane with a line on which force G_a operates. Skier operative weight force is denoted by G_{flad}. A force orthogonal to the slope of the ski is represented by G_m. There is an inner operating force on the sloping plane called lateral force is denoted as G_{let}. The angle between forces G_{flad} and G_m is the same as the angle of skier's inclination to the ski 's slope and is given as ϑ. To evade skidding skier's feet and his internal mass must intersect on the same line with G_{flad}

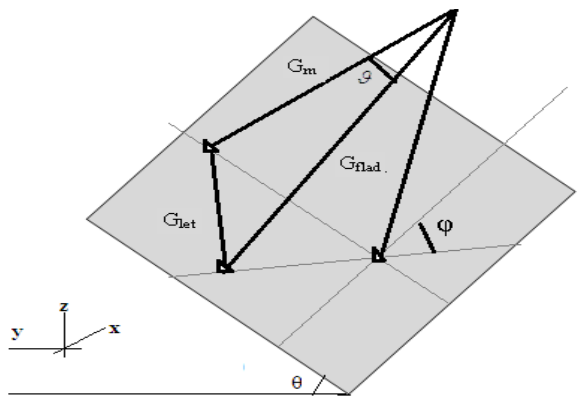

Figure 2

The angle of inclination ϑ lies in this frame $0^0 \le \vartheta \le \pi/2$, the said angle ϑ is associated with a path that is radius which relies on the tilt of skier mathematically equation manifest relationship θ φ shown below.

$$\cos\vartheta = \left(G_{flad} \times G_m\right)\left(\left\|G_{flad}\right\| \times \left\|G_m\right\|\right)^{-1} = \frac{\cos\theta}{\left(\cos^2\theta + \cos^2\varphi\sin^2\theta\right)^{0.5}} \tag{28}$$

Then we proceed and multiply both sides of the equation by this trigonometric identity $\frac{\sin x}{\cos^2 x} = (1-x)^{0.5}$ and as a result, we get $\tan\vartheta$ and on the other side it is narrowed down to $\frac{\sin\theta}{\cos\theta}\left|\frac{\sin\varphi}{\tan\varphi}\right|$.

Another factual mathematical observation on G_{flad}, G_m, and G_{let} is that they are planar vectors, meaning they are on one plane since G_m is orthogonal to the slope of the ski as vector $G_{let}$is generated by the slop, knowing that the vectors are orthogonal (check figure 2). Subsequently, achieve

$$\frac{\sin\vartheta}{\cos\vartheta} = \frac{\|G_{flad}\|}{\|G_m\|} = mass \times gravity \times \tan\theta\cos\theta\left|\frac{\sin\varphi}{\tan\varphi}\right| \div m \times g \times \frac{\sin\theta}{\tan\theta} = \frac{\sin\theta}{\cos\theta}\left|\frac{\sin\varphi}{\tan\varphi}\right|. \tag{29}$$

Chapter 4

4.0 Influence of non-Holonomic constraints

In section, we discuss a mechanical system is subjected to non-holonomic[11] constraints whereby the asymptotic stability appertaining to non-equilibrium position get weaken and transformed to non-asymptotic by weak non-holonomic constraints, a system can be stabilized to stable non-asymptotic. For more explanation, we take an instance of a ball rolling on a horizontal table, and at the very same time, the table is shifted horizontally to any d direction. In the motion of a ball, there is a particle whose motion is constrained to be on the surface of the ball which simply means this is a holonomic constraint when the table is shifted, the mechanical system is subject to non-homonymic constraints thus this particle on the ball can fall off under the influence of shifting force (Svinin, Morinaga, and Yamamoto, 2012).

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[11] *A non-holonomic system is a perpetual and complete flow of variable or parameters managing the system through which a mechanical system may be changed from a certain state to another state*

The ball is to be rolled on a table and it is assumed that the ball has a three-measurable extent which is the right-angled Cartesian coordinate frame. The table is flat and the origin point is marked on it, furthermore, the x-axis and y-axis also inscribed on it.

The ball has one unit radius, we mark a point **D** in red, this point denotes the diameter of the ball again the plane is right-angled to the diameter and it is located at center **A** of the ball, which defines a big circle associated with point **D**, on this big circle we mark another point K green, Position on the ball is z=0 plane and point **D** coincides with the origin, A is at x=0, y=0, z=1 and **K** are at x=1, y=0, and z=1 that is **K** protrude to the positive x-axis. This is the initial orientation of the ball

The rolling ball is on a fully closed line z=0 planes, it is not preconditioning for the said line to be connected, which may not make a ball to skid and cannot twist. Point **A** get back to x=0, y=0, z=1.Point **D** doesn't now coincide with the origin and point **K** doesn't protrude onto the positive x-axis. Actually, by taking the best fit line, the ball gets again another orientation from previous orientation to any likelihood orientation of the ball having **A** positioned at x = 0, y = 0, z = 1. Thus, the system is non-holonomic which may be denoted by the unique and double quaternion **p** and **−p** and if used onto the points denoting the ball, takes points **D** and **K** to their new locations (Svinin, Morinaga, Yamamoto, 2012).

4.1 Effects of non-holonomic constraints on a System

Let's have an equation of motion of mechanical system expressed to "m" terms using abscissa of Lagrangian $\boldsymbol{p_1}, \boldsymbol{p_2}, \boldsymbol{p_3}, \boldsymbol{p_4} \dots \dots .. \boldsymbol{p_m}$, this motion is subjected to some dispelling forces. The equation can now be expressed as below;

$$\sum_{j=1}^{m}\left(\frac{d}{dt}\left(\frac{\partial E}{\partial p_j}\right)-\frac{\partial E}{\partial p_j}+\frac{\partial U}{\partial p_j}+\frac{\partial \square}{\partial p_j}\right)\partial p_j = 0 \qquad (30)$$

Where E is energy in motion of the system, U is the system's energy at rest and the (dissipation) function is depicted by letter **h** that can be expressed in velocities as$dP_1 . dP_2 . dP_3 . dP_4 \ldots\ldots . dP_m : U = 0.5\left(\sum_{jk}^{m} a_{jk}\, dP_j dP_k\right)$. Apparently (E) and (U) denote kinetic energy and potential energy respectively and have (holomorphic) expression as shown below.

$$0.5(4E) = \sum_{j=1}^{m} d^2 p_j + \sum_{jk=1}^{m} C_{jk}\, dp_j \dot{d} p_k \quad 0.5(4U) = \sum_{j=1}^{m} \gamma_j P_j^2 + \sum_{jk=1}^{m} \theta_{jk} P_j P_k$$

(31)

Further explanation γ_j are stable parameters coefficients and axes p_j (j=1, 2, 3, 4, 5, 6…., m) has C_{jk}, θ_{jk} as their functions, disappearing at $p_1 = p_2 = p_3 = p_4 = p_{5=\cdots.} p_m = 0$. From expression (30) we achieve equations of Lagrange. Having non-negative $\gamma_j (j = 1,2,3,4,,\ldots\ldots m)$ then non-balancing points; $p_1 = p_2 = p_3 = p_4 \ldots . p_m = 0$ becomes line with a curve that is steady provided there is full dispersion. The mechanical system is subjected to linear **n** which is non-moving and non-holonomic constraints as manifested below.

$$\frac{dp}{du} = \sum_{r=n+1}^{m} D_l^u (p_1 p_2 p_3 p_4 p_5 p_6 \ldots p_m) d\, \dot{p}_r \qquad (u = (1,2,3,4,5, \ldots\ldots, . n)$$

(32)

Then proceed on to research about the steadiness of the system's unbalancing point. Subsequently get that, equation (30) composed of non-holonomic components (33) preludes equations of Boltzmann-Hamel.

$$\frac{d}{dt}\left(\frac{\partial \overline{E}}{\partial a_l}\right) + \sum_{j=1}^{m} \frac{\partial E}{\partial p_j}\left(B_l^j\right) + \sum_{j=1}^{m} \frac{\partial u}{\partial p_j}\left(B_l^j\right) + \sum_{jk=1}^{m} \frac{\partial \overline{E}}{\partial a_j}\left(\lambda_l^j a_j\right) = -\frac{\partial \overline{\square}}{\partial a_l} (l = n+1, \ldots, m),)$$

(33)

From the equation we get Ricci-Hamel constants, these areλ_{kl}^{j}. The Kinematic speck is a function of abscissa p_1, p_2, $p_3 \ldots p_m$, plus speck t_1, t_2, $t_3 \ldots t_n$, and equivalent to the element of non-holonomic.

$t_u = dp_u - \sum_{l=n+1}^{m} D_r^u \dot{p}_r$ (u=1, 2, 3, 4, 5, 6… n);

(34)

$t_{n+1} \ldots \ldots t_m$, are probably equalizes velocities that is $t_r = \dot{dp}_r$ (r=n+1… m); dp_j in (35)

Kinetic energy$\overline{E}$ substituted by an aspect of motion, this is shown as below

$dp_j = \sum_{k=1}^{m} B_i^i t_j$ (j=1, 2, 3, 4, 5….m)

(36)

Having composed the equation of Boltzmann-Hamel we express $t_u = 0$ (u=1, 2, 3, 4, 5… n). We remove velocities $p_i'$from dispersion function and get function **h.** We get that$\overline{\boldsymbol{h}}$ is a negative state of velocities hence cannot be dispersion function.

it follows that forces of dispersion on the system can be influenced by non-holonomic constraints decline, then forces hence debilitate and destabilize the system. As the ball roll without slipping, it is said to be a holonomic constraint with force opposing motion. This force is typically frictional force, and this is illustrated below,

$H = -H_{\square k}(\pm)(u - u_0)$,

$H_{\square k}$ is a constant and $\mathbf{v_0}$ denotes plane velocity and **v** denotes the velocity of the regional point of contact. If the ball doesn't slip then non-holonomic constraints decline and produce resisting force that stable the mechanical system at the border domain.

Having coordinates $y_{n+1} \ldots y_n$, then the system's stability can be investigated using a coefficient of stability$\overline{\gamma}_l$. There is a disturbance of $\overline{\gamma}_l \rhd 0$stability when coefficient$\overline{\gamma}_l \leq 0$. Constant coefficient ($e_{kz}$) in kinetic energy depending on non-holonomic ($C_l^{;0u}$) influence of the Coefficient stability$\overline{\gamma}_l$.

Weak holonomic in holonomic constraints help in obtaining stability referring to $y_{n+1}\ldots y_m$, coordinates. Stable holonomic doesn't influence coefficient of stability $\gamma_l$therefore ultimately the system doesn’t-get strengthen.

The effects of non-holonomic constraints can be illustrated. We consider coefficients associated with constraints equation we then proceed in power expansion of equation in coordinate-wise and by first starting with zero-order terms and note it that these constraints are non-holonomic having weak non-holonomic and when the coefficients either starts with first or higher terms, the constraints are non-holonomic with strong non-holonomic.

4.2 Motion for Finite-Dimension Lagrangian System

A body or a chain of bodies, immersed in a non-viscous irrotational fluid, the motion can be described by a finite-dimensional Lagrangian system. We get some geometric properties that make the system "fit for jumps" so that the equations of motion are linear concerning the time derivative of the control function (Yunt and Glocker, 2008)**.**

The non-viscous and irrotational fluid is an ideal fluid and body or chain of bodies in this fluid is in motion that can be a finite-dimensional system in which Lagrangian dynamics are manifested by considering derivative or differential measure whereby the force of control on velocity and acceleration is incorporated.

Actions of the Lagrangian system in values belonging to the force of control is made credible by the differential measure. Finite-dimensional Lagrangian's control has some of its conditions derived. In the Lagrangian system, there are impulses introduced by control action which are internal confining lines on time circle.

There is a manifestation of fit for a jump when the high magnitude of control forces is applied to the dynamical system, fit for the jump are exhibited as external forces that alter the system's path of motion severely progress. There is also a consideration of numeric expression for optimal trajectories that belong to the finite-dimensional Lagrangian system with variables.

Control of the mechanical system has non-smooth dynamics, and the mechanical system can be expressed in derivative or differential equations with the provision of contact status. Integration is carried out to determine the measured extent of derivative involved. 2^{nd} order derivatives in dynamics give differential equations expressed using algebraic speck. Geometric properties that render the system "fit for jumps", are tangent lines, normal line, relative angles between links, and velocity (Yunt and Glocker, 2008).

There comprehensive explanation of the forces from the domain of interaction of the mechanical system and the non-viscous irrotational fluid. Control motion is explicitly captured in the referral of Gauss principle which we get two expressions. One depicts expression appertaining to Gauss principle in free motion's impact and the other involves impulsive forces in control of motion.

The force imposes a law that is dependent conditionally on a kinematical and geometrical speck. The direction of general force incorporates unilateral force with aid of vector **f** as expressed by the equation of Lagrange for smith dynamics or motion

$$\frac{d}{dt}\left(\frac{\partial E}{\partial v}\right)^{E} - \left(\frac{\partial E}{\partial p}\right)^{E} + \left(\frac{\partial U}{\partial p}\right)^{E} - f = 0 \qquad (37)$$

In this case, E (p, v) is the summation of kinetic energy in the same mechanical system there is the sum of all smooth potential energy denoted by U (p). Vector force **f** brings control into motion equations and for contact on position, velocity, and acceleration there is equation expression using indexes of acceleration and body's velocity as illustrated below.

$\tau_L = \{1,2,3,4,S\}$,

$\tau_Z = \{j \in \tau_L | l_{vj} = 0\}$,

$\tau_M = \{j \in \tau_L | l_{vj} = 0, \lambda_{vj} = 0\}$.

(38)

The index$\boldsymbol{\tau_L}$ represents contacts on the level position of mechanical system which are not smooth and summation, totaling to **S**. Index τ_Z is closed contacts onto mechanical system's position level totaling to **R.** Contacts having velocity and distance normal contacts equivalent to zero is represented by $\tau_M$totaling to **N.**

We get that Vector contact distance and valued component of force are simply related by Kinematics which is tangential and normal For the realization of closing contact vector l_v(p) denotes normal distances between non-negative inflexible bodies.$\lambda_v$is normal contact velocity and $\gamma_z$depicts tangential contact velocity and all are annotated respectively in the expression below.

$$\lambda_v = \psi_v^E(p)v, \tag{39}$$

$$\lambda_z = \psi_z^E(p)v,$$

Where $\lambda_v$is gotten from the sum of the time derivative of l_v (p). Accelerations of contact which are tangential and normal are expressed as below

$$d\gamma_v = \psi_V^E(p)dv + w_v(p, v),$$

$$d\lambda_z = \psi_Z^E(p)dv + w_z(p, v). \tag{40}$$

Acceleration has these indexes

$$A_{vj} = \{\gamma_{vj} | \gamma_{vj} \geq 0, \forall j \in \tau_L\},$$

$A_{zJ(\gamma vj)} = \{\gamma_{zj} | \quad |\gamma_{zj} \quad | \leq \eta_j \gamma_{vj,} \forall j \in \tau_M\}$,

(41)

We have normal and tangential contact force depicted by $\gamma_{vj}$and γ_{vj}respectively and at contact **j** the coefficient of friction is represented byη_j. The Law of friction appertaining to isotopic is explained. The mechanical system imposed on spatial friction and unilateral contact forces without impacts are expressed below

$$N(p)dv - f(p,v) - \psi_z(p)\gamma_z - \psi_v(p)\gamma_v - D(p)I = 0$$

$$d\lambda v_j \in Ma_{vj}\left(\gamma_{vj}\right), \forall j \in I_M \qquad (42)$$

$$-d\gamma_{zj} \in Ma_{zj(\gamma vj)}\left(\gamma_{zj}\right), \forall j \in I_M$$

4.3 Control and Stability for Infinite Dimensional Systems

We check into controllability of bodies with infinite-dimension, plunged or immersed in viscous fluids having non-zero vorticity. In particular, we can obtain controllability and stabilization properties for these infinite-measurable extents al systems.

The thesis expounds a linear system in which the nonlinear trajectory is fully characterized and are controllable. Comprehensive control of the linear system admits the solution of inverse dynamics using the structure of the linear system to compute inputs necessary for the performance of a task (Yunt and Glocker, 2008).

Inversion achieved by getting the right inputs to enhance a control system actually from a state to another one. For the occurrence of stability, there must be a function balancing right or required aims of operation with stability and effort producing maximum control to the system. Inversion of dynamics will always assume the system's dynamics are certain and fixed.

Besides, the idea of a linear system can be extended to various systems of parameters having boundary control essential in controlling linear systems. Control of the mechanical system is more accurate when a smooth infinite dimension manifold having the advantage of vector filed.

4.4 Vectors in Infinite Dimension and System Control

We regard bodies immersed in a fluid with viscosity, and with non-zero vorticity as a mechanical system with differential equations as shown below.

$\omega = \dot{y} = \frac{\partial u}{\partial y} - \frac{\partial u}{\partial x} = \square(y),$ y∈Y⊂ $\mathbb{K}^m$ (43)

Comprehensively paired (Y, h), in this case, letter X is a set of $\mathbb{K}^m$ and it is an open set and on X [a] smooth vector field is given as h. We get that the solution of (45) is mapping $e \to y(e)$in a way that y (e) =h y(e) $\forall e \geq 0$. Noting better that when a smooth function$y \to f(x)$ on Y and$e \to y(e)$, by definition is a trajectory then; $\frac{d}{de} f\big(y(e)\big) = \frac{\partial f}{\partial y}\big(y(e)\big).\dot{y}(e) = \frac{\partial f}{\partial y}\big(y(e)\big).\square\big(x(e)\big) \quad \forall e \geq 0.$

From this equation what ensues is an aggregation of derivative that is the mapping derivative illustrated as shown; $y \to \frac{\partial f}{\partial y}(y).\square(y)$

This sum of derivative (mapping) may be called "time-derivative" of "**f**" checking into control system there is identical elucidation for a vector field and "space" $\dot{y} = \square(y, v),$ (44)

Having derivative h smooth on $Y \times V \subset \mathbb{K}^m \times \mathbb{K}^n$ as a subset, on Y we get that his infinite vector field collection but not a vector field then parameterized by v as$v \in V$, the mapping vector field on Y is manifested as below$y \to \square_v(y) = f(y, v)$.

A solution of equation (46) relates well with a smooth solution of it with aid of smooth solution of (46) which is mapping$e \to \big(y(e), v(e)\big)$ having values in $Y \times V$as

$\dot{y}(e) = h\big(y(e), v(e)\big)\forall e \geq 0,$

Taking note of infinite mapping $e \to \zeta(e) = (y(e), v(e), \dot{v}(e), \ldots\ldots)$. As the values in $Y \times V \times \mathbb{K}_n^\infty$, and $\mathbb{K}_n^\infty = \mathbb{K}n \times \mathbb{K}n$ …, represents the product of many countable numbers of copies of $\mathbb{K}n$.The position of $\mathbb{K}_n^\infty$is in the system o$(v^1, v^2, \ldots.)$ $u^j \in \mathbb{K}n$.The said mapping satisfies.

$\boldsymbol{\zeta(e) = \big(h\big(y(e), v(e)\big), \dot{v}(e), \ddot{v}(e), \ldots\ldots\big) \; \forall e \geq 0},$

Then the infinite vector field trajectory

$(y, v, v^1, \ldots\ldots) \to H(y, v, v^1, \ldots\ldots) = (\square(y, v), v^1, v^2, \ldots\ldots\ldots)$
Is prominently on $Y \times V \times \mathbb{K}_{n}^{\infty}$ and for any mapping $e \to \zeta(e) =$

$(y(e), v(e), v^1(e), \ldots\ldots\ldots)$and this is the solution of this infinite vector field which ultimately expressed as (y (e), v (e), v (e)…) having $\dot{y}(e) = \square\big(y(e), v(e)\big)$, then this represents the solution of (46). H is a vector field and doesn't so far give parameters to a group of vector fields (Yunt and Glocker, 2008).

In the same case, equation (46) which is the control system is is the data for $Y \times V \times \mathbb{K}_{n}^{\infty}$ in conjunction with smooth vector field H. It is seen that same as non-controllable scenario it has illustration as “time-derivative” of a smooth function $(y, v, v^1, ..) \to f(y, v, v^1, \ldots . v^r)$ relying on many and a certain number of variables by $\dot{f}(y, v, v^1, \ldots, v^{r+1}) = Bf.H$

$$\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{y}}.\boldsymbol{h}(\boldsymbol{y}, \boldsymbol{v}) + \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{v}}.\boldsymbol{v}^{\mathbf{1}} + \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{v}^{\mathbf{1}}}.\boldsymbol{v}^{\mathbf{2}} + \ldots$$

The totaling expressed above is certain or finite since function f rely on certain many variables.

Chapter 5

5.0 Conclusion

In the research, the motion of a body which is hydro-dynamically asymmetric together with the rotation of the Fletcher rotor is fully controllable with aid of moving coordinates that change the position of the internal mass, which subsequently changes circular motion of internal mass and mechanical system's angular momentum. The main purpose of the Flettner rotor is to induce a motion mechanical system.

In a wide range of countless time interval, the controllability of the system makes up the free motion drift simply by the circular motion of internal mass and rotation of the Flettner rotor especially when they exist within a circular domain. The body structure and body domain's velocity define radius and central point. The drift outside this domain utilizes the motion of the center of mass to get covered within a known time interval. In this conclusion we suggest solutions to various problems by denotation of the very physical devices these remedies are;

a) For full body's stability at any point space at the infinite interval time frame, there should be perfectly design of basic and practicable patterns motion of the center of mass

b) Construction of detailed control of the mechanical system to enhance the system's motion from a space point to another space point.

c) Construction of mechanical system controls by altering circulation around the body as per the smooth law actually when the mechanical system's motion does not have first integrals.

We conclude that when a mechanical system is subjected to non-holonomic constraints, controllability can be achieved whereby the asymptotic stability appertaining to non-equilibrium position get weaken and transformed to non-asymptotic by weak non-holonomic constraints, thus the system gets stabilized.

System immersed in non-viscous irrotational fluid is controllable, in which the motion is expressed by a finite-dimensional lagrangian system. We found out that actions of the Lagrangian system in values belonging to the force of control is made credible by the differential measure.

Finite-dimensional Lagrangian's control has some of its conditions derived. In the Lagrangian system, there are impulses introduced by control action which are internal confining lines on time circle.

Another objective of the thesis that has been achieved, is controllability and stability properties of bodies with infinite-dimensional systems immerse in viscous fluid with non-zero vorticity whereby we get nonlinear trajectory is controllable and comprehensive control of the linear system admits the solution of inverse dynamics using the structure of the linear system to compute inputs necessary for the performance of controlling task.

Future Research

The investigation of controlled motion leads to the operation speed problem. In future time I therefore recommend to research on construction of time-optimal controls, I also advice any researcher to conduct an investigative study on the stated topic.

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