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.Pricing of Futures with basis risk

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Nyaribo Miller Nyamari

August 2018

Submitted to the School of Mathematics in partial fulfilment for a degree in Master of Science in Pure Mathematics

Master Project in Mathematics **University of Nairobi**

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.Pricing of Futures with basis risk

Research Report in Mathematics, Number 2[illegible], 2018

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Master of Science Project

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Prepared for The Director
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Abstract

This study into detail presents a new pricing approach of futures . It examines the basis risk by assuming it to have a stochastic behavior as a modified Brownian process The originality of this approach therefore brings into consideration the basis risk as compared to other pricing methods like the Black scholes model . The futures with basis risk pricing model is applied on the times watch S & P 500 stock. From the data, the model is empirically analysed by first calculating the basis risk then pricing the futures.

The study combines the option spot price, volatility of spot return, initial basis, the basis volatility, correlation between the basis and spot return to come up with the futures price. The approach, could help researchers to test the accuracy of the pricing model or their input basis risk, and also can help investor to compare the market with the estimated price to come up with the best ultimate investment decision. The discussion, methodology and testing are focused on the issues of advanced financial modelling.

The findings here evidenced that there is a positive correlation between the futures call option price and the correlation coefficient between basis and spot return. It is however remarkably shown to be negatively related in initial basis values. Thus its concluded that the model is a better version than Black Scholes model as it eliminates the bias experienced in time to maturity ,systematic moneyness and got better prediction power.

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Declaration and Approval

I the undersigned declare that this project report is my original work and to the best of my knowledge, it has not been submitted in support of an award of a degree in any other university or institution of learning.

29/08/2018

Signature Date

NYARIBO MILLER NYAMARI
Reg No. I56/5000/2017

In my capacity as a supervisor of the candidate, I certify that this report has my approval for submission.

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Dedication

This project is dedicated to my parents

Contents

Acknowledgments

I thank the entire school of mathematics fraternity, my friends,classmates and all that i did not mention for their continued and strong support during the period of this study. I also thank my supervisors for the guidance and encouragement.. . . .

Nyaribo Miller Nyamari

Nairobi, 2017.

1 Introduction

1.1 Background

1.1.1 Options and Futures

Whereas options are derivative contracts where the holder may choose to forfeit the contract since the holder has the right to exercise but not the obligation ,futures on the other hand are financial standardized forward contracts where the holder has an obligation to either buy or sell the financial instrument at a predetermined future date and future price. A call option on an underlying asset gives the holder the right to buy the asset at a predetermined price the strike price at a specified time in the future . The put works in the opposite way of the call, allowing the holder to sell the underlying asset at the strike price 1.1.2 Basis risk

This is the risk brought by imperfect hedging, it occurs because the value of a futures contract will not always move in line with the underlying security. Therefore, it's the difference between spot price of the hedged asset and the futures price of the hedging instruments

Basis risk will always occur in futures contract since the asset in the existing position is often not the same as the one underlying the futures. Also in most cases the hedging horizon may not match perfectly with the maturity of futures contract. Therefore any investor trading with futures can't afford ignoring the basis risk, most scholar models like the Black Scholes assumed a risk-free rate of interest which to futures is very unrealistic

1.13 PRICING FUTURES WITH BASIS RISK

Options having been considered to be the most dynamic segment of the security markets since April 1973 , with more than 1 million contacts per day, its worthwhile to state here that there futures price has to be determined with a bit of certainty and accuracy. This can't be done without considering the basis risk as seen before. This has prompted many researchers to do research into the pricing of the futures contract and the underlying security. It therefore made my study to focus on pricing futures with basis risk

1.2 Statement of the problem

Financial derivatives have over the recent years formed major volume of what is traded in the stock exchange markets. As much as Option pricing has been studied extensively in both the academic and trading context with numerous approaches ranging from sophisticated general equilibrium models to ad hoc statistical fits. For this reason, traders of stock exchange markets have become more interested in knowing how futures are priced as in most cases there is a gap between the spot price and the futures price of the underlying asset. The gap looked at carefully is the basis risk itself, many models used widely like Black Scholes don't take basis risk into account. This therefore forms the problem of my study as I try to price the Futures with basis risk taken care of.

1.3 Objectives

Main Objective of The Study•

To come up with a pricing model for futures with basis risk

1.3.1 Specific Objective of the study

To test the accuracy of the pricing model on S & P 500 Futures on the market watch

1.4 Research questions

1. How is the model for pricing futures modelled
2. How does it perform compared to Black Scholes.

1.5 Justification

This aims at playing a major crucial role and will bring financial sense to several groups in the financial markets sector . Sectors like the Nairobi Securities Exchange (NSE), Capital Markets Authority (CMA), Investment promotion agents will be great beneficial as they will be able to price futures with more accuracy as compared to before. Lastly the scholars and academicians can pick from there to further research and finally come up with a better internationally accepted futures pricing formula other than the incumbent Black Scholes Formula

1.6 Scope

This study is based on derivative markets Futures and Future options. The study utilizes listed S & P 500 indexed securities.

2 Literature review

2.1 Introduction

Here, the chapter will review the strands of literature review surrounding the pricing of futures. In 1848,Chicago Board of Trade was opened and this began the the history of future markets and up to date they are traded more actively, Due to this financial academics and practioners have become more interested in understanding the pricing of future contracts.

2.2 Literature review

2.2.1 Review of Literature

There have been a number of literature review done on pricing of futures both with different focus and major finding . Fama and French 1987,began it all they came up with two theories of pricing futures one being cost of carry theory and the other the risk premium theory The cost of carry theory explained the difference between current spot price and the future price and it was found out that they were due to forgone interest on commodity storage, warehouse costs & convenience yield in inventory. Risk premium theory on the other hand explained that considered future price normally comprised of a forecast of Future spot price and an expected risk premium.

Kumar Kishore(2002), did an investigation into the empirical price relationship between NSE 50 futures and The NSE 50 index, with an objective to find out whether there is any change in the volatility underlying the index due to introduction of NSE 50 index futures and whether the future price were having sufficient predictive information. It was found out that the volatility in the post futures period is less than the volatility before the introduction of futures indicated by regression. Further it showed that the information coefficient in the post futures period is more than that of the pre-futures. Singh and Guptha (2007), looked into Indian equity arbitrage efficiency of the futures market for a sampled period of 6 months between June 2000 to December 2005. Findings suggested that despite the stable and strong long-run relationship that exists between futures and cash market, there are significant deviations in the future market and its cost of carry. This offers exploitable arbitrage opportunities to the traders. Therefore, mispricing was observed to be a direct function of time to maturity, interest rates stochastic behaviour, short sale restrictions in the cash market, information asymmetry in both futures and cash market and the restricted exposure of institutional traders in the future market.

Ashok Banarji and Bhattacharya (2005), with price relationship between index futures and underlying index spot in an Indian case study. They used the very popular index futures in India Nifty with a major focus on price variation discovery between future contracts market in India and similar markets in other countries. Tring to make to make better price discovery in the spot market and with Granger causality methodology, root need square error and need absolute error were examined. Conclusively they found out that there Indian Nifty Spot price lead to Future prices as much as their futures prices and the underlying spot prices are normal. Shaveta gupta and Sahil (2006),provided historical background and the development of the Indian derivatives and with the study it focused on four needs: risk averse to risk take movement, identifying the current and future prices, entrepreneurial activities catalyzing and a big boom on the volume of savings and investments . They did a graphical analysis of futures and options growth and found out that there is an overwhelming number of people trading in futures compared to options and the gap as been on a rising scale since 2003.and conclusively they agreed that as much there is increasing trading volume in futures, their imperfections in the market are also growing demanding much research on pricing of futures.

2.2.2 Conclusion

There is a short coming in pricing basis risk in all the literatures forementioned. This results into the largely noted deviations between the spot price and the futures price . Alternative methods have been put into play but still basis risk is not covered into detail Its remarkablely seen that there is a significant number of empirical literature on Black Scholes the incumbent modelling formula. This however assumes many factors among them the risk free rate assumption,which is therefore inconsistent with the real statistical properties of futures in the trading exchange forums This promptly me to do further research into the pricing of futures

3 Methodology

The chapter discusses the methodology used in this study. It commences with the data source, software used in analysis, the model for pricing the futures with basis risk, parameter estimation, significance of parameters and finally the test of performance of the model used in the study.

3.1 Data

We used the market watch website and selected the S & P 500 index futures for the period of one year from Jan 2008 to Dec2008. S & P 500 futures were most preferred since from the existing literature Guo (2000) gave them priority and being the most liquidable in the American market, they got a lot of impact

3.2 Data analysis software

R software was used for the better part of the analysis part and for its effectiveness in testing the model for the study.

3.3 The Model

3.3.1 Definition of terms

Modified Brownian bridge process
This is a stochastic process same as the Brownian motion process except that at reaches a specific point at a specified time with definite sure probability of one. This in our study solves the no arbitrage assumption as at that time the spot price and the futures price will be zero. And to the topic the basis risk being the difference between the spot price and futures price will therefore be zero at that time too.

Geometric Brownian motion
This is a process with the following equation;

$$dS(t) = (r - \delta)S(t)dt + \sigma_s S(t)dW_s^Q(t) \quad (1)$$

From the equation in our study under the spot martingale measure Q, the underlying security will follow the process with a continuous dividend yield d, constant instantaneous drift r and volatility σ_s . One dimensional standard Brownian motion is assumed to be W_s^Q with probability space (Ω, F, P, Q).

Basis risk $Z(t,u)$,the basis risk is the log futures price with maturity date u subtracted from log spot prices as shown

$$Z(t,U) = lnF(t,U) - lnS(t) \tag{2}$$

Notably $Z(t,u)$ will be a normal distribution considering the fact that the other parameters are lognormal.

3.3.2 The model derivation and parameter definitions

The model is derived in the following steps; **Step 1.basis risk derivation** Given time interval $(0,U)$,0 being the beginning time and U maturity time,the basis is derived by the modified Brownian bridge process where the following terms are assumed $Z(0,U) = lnF(0,U)\check{}lnS(0)$. Deriving under spot martingale measure it gives the following;

$$dZ(t,U) = \frac{-Z(t,U)}{U-t}dt + \sigma_z(t,U)[\rho dW_s^Q(t) + \sqrt{(1-\rho^2)}dW_s^Q] \tag{3}$$

Where the parameters $dZ(t,U)$ is the basis volatility,ρ the correlation coefficient for the underlying security and basis,dW_s^Q the one dimensional Brownian motion, With this $E(dW_s^Q dW_s^Q) = 0$ and $E(dsdz) = \rho dt$

Solving equation … 3 gives;

$$Z(t,U) = \frac{(U-t)Z(0,U)}{U} + (U-t)\int_0^t \frac{\rho_z(\upsilon,U}{U-\upsilon}[\rho dW_s(\upsilon) + \sqrt{(1-\rho^2)}dW_z(\upsilon)] \tag{4}$$

Step two; pricing the futures with the basis risk

On finding the basis risk as in step one the next step in our model derivation is to now derive the equation for pricing the futures with the very basis risk found in step one; Our final equation will be as follows;

This procedurally and mathematically is derived as below;

$$dZ(t,U) = \frac{-Z(t,U)}{U-t}dt + \sigma_z(t,U)dW^*(t) \tag{5}$$

Where,

$$dW^*(t) = \rho dW_s(t) + \sqrt{1-\rho^2}dW_z(t)$$

Thus,

$$Z(t,U) = X(t)[Z(0,U) + \int_0^t \frac{\alpha(u)*-\delta(u)y(u)}{X(u)}du + \int_0^t \frac{y(u)}{X(u)}dW^(*)(u)] \tag{6}$$

Where $X(t)$ is stochastic exponential SDE's and as follow:

$$dX(t) = \beta(t)X(t)dt + \delta(t)X(t)dW^*(t)$$

From 5 and 6,we have;

$$\alpha(t) = 0,$$
$$\beta(t) = \frac{-1}{U-t}$$
$$y(t) = \sigma_z(t,U)$$
$$\delta(t) = 0$$

Finding the mean and variance of the pricing model equation,this is what we find Mean of Pricing equation,Variance of pricing equation,basis mean,correlation between the basis and the underlying security are as follows;

$$E_Q\left[F(t,U)\right] \equiv F(0,U)\exp\left[(r-\delta)t+\mu_{BASIS}(0,t)\right]$$

$$V_Q\left[F(t,U)\right] \equiv \left[F(0,U)\right]^2 \exp\left[2(r-\delta)t+2\mu_{BASIS}(0,t)\right]\left(\exp\left[\sigma^2_{SZ}(0,t,U)\right]-1\right)$$

where

$$\mu_{BASIS}(0,t)=\frac{-t\times Z(0,U)}{U}+\frac{1}{2}\int_0^t\left[\frac{2\rho(U-t)\sigma_Z(v,U)\sigma_S}{U-v}+\frac{(U-t)^2\sigma^2_Z(v,U)}{(U-v)^2}\right]dv$$

$$\sigma_{SZ}(0,t,U)^2=\int_0^t\left[\sigma^2_S+\frac{2\rho(U-t)\sigma_S\sigma_Z(v,U)}{U-v}+\frac{(U-t)^2\sigma^2_Z(v,U)}{(U-v)^2}\right]dv$$

Hedging process by short selling a basis This can only be achieved if a call option with a basis risk is assumed to be a martingale under spot martingale measure whereas the call price with basis risk is a function of F and t,C basis=C 9F,T). A riskless hedge is therefore created merton(1973). Mathematically proofing it results to;

$$G(t) = -C(t) + nF(T,U) \tag{7}$$

Where $F(t,U) = S(t)e^{Z(t,U)}$ Solving by Ito's calculus

$$\begin{aligned} dG(t) = \{&(n-C_F)F(t,U)[(r-\delta)-\frac{Z(t,U)}{U-t}+\rho\sigma_S\sigma_Z(t,U)+\frac{1}{2}\sigma^2_Z(t,U)] \\ &-[C_t-\frac{1}{2}C_{FF}F^2(t,U)\sigma_{SZ}(0,t,U)^2]\}dt+(n-C_F[(\sigma_S+\rho\sigma_Z(t,U))dw^S_0(t) \\ &+\sigma_z(t,U)\sqrt{1-\rho^2}dw^S_0(t)] \end{aligned} \tag{8}$$

Therefore under no arbitrage assumption, the hedging portfolio earns a riskless interest rate when n=cF The no arbitrage futures pricing equation with basis risk is therefore same as Black scholes PDE and is as follows:

$$C_F rF + C_t + \frac{1}{2}C_{FF}F^2\sigma^2_{SZ}(0,t,U) = rC \tag{9}$$

The ultimate futures with basis risk pricing model equation With the fore mentioned steps it all leads to the main objective of our study i.e. finding the futures with basis risk pricing model. The model is therefore as below; This is what we will use in analyzing

$$C_0^{Basis} = F(0,U)\exp\left[-\delta T + \mu_{BASIS}(0,T)\right] N(d_1^{BASIS}) - ke^{-rT} N(d_2^{BASIS})$$

Where

$$d_1^{BASIS} = \frac{\ln\frac{F(0,U)}{K} + (r-\delta)T + \mu_{BASIS}(0,T) + \frac{1}{2}\sigma_{SZ}^2\left(0,T,U\right)}{\sqrt{\sigma_{SZ}\left(0,T,U\right)^2}}$$

(15)

$$d_2^{BASIS} = d_1^{BASIS} - \sqrt{\sigma_{SZ}\left(0,T,U\right)^2}$$

the data from the market watch so as we proof its efficiency and working practicability.

3.3.3 Parameter estimation

The parameters used in the model as seen in the ultimate pricing model equation are the underlying stock volatility σ_s, volatility for the basis dz, correlation coefficient between the basis and the underlying security p, and the initial basis Z(0,U). Black Scholes model is used to find the underlying stock volatility σ_s. And all of the other parameters fore mentioned are estimated from the S &P 500 Index market watch realized data

3.3.4 Significance of Parameters

The p value will be used to show the significance of the parameters used. When $0 > \rho$, the value of call price with basis risk is increasing with the increase of the volatility of basis. If $0 < \rho$, the value of call price with basis risk is decreasing with the increase of the volatility of basis.

3.3.5 Test of Performance of the model

To test the performance of the model . we try to compare it with Black Scholes Model figures . Using our R software we shall come up with the Mean Error, Mean of Absolute Error, Root Mean Square Error . Further index point error and percentage errors will be used to assess the efficiency of our model. This will be found as follows,index point error will be the difference between the model price and the actual price while the percentage error is the index point divided by the model found out price. As in;

$$e_{index} = C_{model} - C_{actual} \tag{10}$$

$$e_{percentage} = \frac{e_{index}}{C_{model}} * 100\% \tag{11}$$

4 Data analysis and Results

4.1 Data

data for S& P 500 futures for a period of one year is obtained from the ivesting .com website and i trimmed and cleaned it.data on futures options is obtained from the website in the same same manner.

4.2 Data preparation

Test of the model is performed with investing.com S &P 500 futures and future options from January 1, 2017 to Jan, 2018. We chose the data based on two considerations as Lim and Guo (2000). This is because the options have great liquidity in American. Secondly, S &P 500 index, S &P 500 futures, and S &P 500 futures options are used wildly in existing literature. Both the underlying futures and the options on futures expired by jan 2018.

4.3 Data analysis software

R software and R codecs used in the modeliing are as follows

4.4 General analysis of the data

First,We Match the futures options price with the nearest corresponding futures price preceding the option transactions. On investing.com both the futures and options prices are quoted in index points. The jan 2017 to jan 2018 futures prices in the sample period is on a decreasing rate and it ranges from 2680.75 to 2243.5. Similarly the options strike price of the similar period ranges from 900 to 1500. Though the range sould not move widely as this strike price will lead to large forecasting error. However the actually response to the efficiency of models in all situations is kept at bay. 2040 call option prices were identified in the sample. The cleaning part involved doing the common ststistical filtering rules to get the right data for the analysis in R software. This among many involved ignoring all call prices less than 0.5 for price discreteness as per investing.com S & P 500 futures options tick size for S &P 500 futures of 0.05. Continuous price movements are assumed in most prior methods. This contrasts the real world ticks in prices movements. The rule has been advanced by many scholars including Guo (2000) with their S &P 500 futures option data. The filtered sample consists of 1800 call prices. This can be seen in figure 1 below;

Figure 1. Descriptive statistics

	$T \geq 90$		$90 > T \geq 50$		$T < 50$		All Maturity	
Moneyness F/K	Basiscall	BLcall	Basiscall	BLcall	Basiscall	BLcall	Basiscall	BLcall
<0.97								
Number	413		413		251		1239	
Mean	7.931	5.523	7.931	5.523	4.892	4.044	7.702	5.479
SD	4.549	3.521	4.549	3.521	2.863	2.433	4.899	3.712
097-100								
Number	297		297		321		835	
Mean	22.261	16.958	22.261	16.958	11.673	9.977	19.239	14.953
SD	5.727	4.923	5.727	4.923	8.745	7.696	9.206	7.173
100-103								
Number	254		254		285		721	
Mean	40.894	32.782	40.894	32.782	26.988	24.146	36.154	29.787
SD	7.237	6.381	7.237	6.381	10.669	9.726	11.350	8.930
1.03-1.06								
Number	152		152		155		387	
Mean	64.300	54.151	64.300	54.151	53.345	50.029	60.879	52.941
SD	8.212	7.607	8.212	7.607	8.943	8.623	10.321	8.148
$\geq$ 1.06								
Number	299		299		346		806	
Mean	170.658	157.952	170.657	157.952	183.332	179.069	170.334	160.806
SD	81.597	81.677	81.597	81.677	95.367	95.342	85.176	85.918
All Moneyness								
Number	1415		1415		1358		3999	
Mean	57.262	50.218	57.262	50.218	62.372	59.718	53.300	47.859
SD	71.913	68.939	71.913	68.939	86.954	86.024	72.289	70.268

S &P 500 assumes dividend yield of range 1.7% to 1.5% .1.7% estimate is ued in our calculation We use. Risk free rates for s & p 500 is given in the website There are time-to-maturity and 5 moneyness groups as seen. Treasury Bill riskfree rate and yield for american market is asssumed by doing an average of all the discounted quotes.

4.4.1 Parameter Estimation

There were un observable paremeters that needed estimations as forementioned in my literature review for the modelling of the model successfully. After estimating they are assumed to be constant throughout the entire period of 1 year There are four parameters that i estimated in my model this include

- initial basis risk from futures data
- variance of the underlying stock
- variance of the basis
- correlation coefficient between the basis and the underlying security

The variance of the underlying security is found from the incumbent Black Scholes model formulations whereas the rest of the parameters are estimated from the investing.com cleaned data.

4.4.2 Test of Performance of The model

This can be observed in the table below

Figure 2. Test of performance of the model Continued

	T ≥90		90> *T* ≥ 50		*T* <50		All Maturity	
Moneyness *F*/*K*	Basiscall	BLcall	Basiscall	BLcall	Basiscall	BLcall	Basiscall	BLcall
				Panel A: Index-Point Error				
<0.97								
Mean	-0.912	-3.519	1.624	-0.790	2.969	1.930	0.718	-1.505
MAE	2.388	3.727	2.477	2.015	3.126	2.331	2.566	2.872
RMSE	3.009	4.808	3.391	2.794	4.123	2.994	3.387	3.890
097-1.00								
Mean	-1.749	-8.473	2.081	-3.222	1.892	0.195	1.013	-3.273
MAE	3.900	8.473	3.715	4.941	3.013	2.450	3.493	4.901
RMSE	4.192	9.471	4.869	5.429	4.192	3.081	4.444	6.119
1.00-1.03								
Mean	-2.935	-12.387	1.520	-6.591	0.857	-1.985	0.133	-6.233
MAE	4.647	12.387	3.310	7.365	2.468	2.863	3.315	6.853
RMSE	5.435	13.325	4.303	7.838	3.530	3.470	4.348	8.439
1.03-1.06								
Mean	-1.952	-14.066	1.794	-8.356	1.403	-1.913	0.785	-7.152
MAE	3.847	14.066	3.264	8.610	2.584	2.809	3.132	7.598
RMSE	4.346	14.683	4.154	9.310	3.443	3.616	3.940	9.354
□ 1.06								
Mean	0.392	-14.554	3.926	-8.781	1.472	-2.788	2.167	-7.362
MAE	3.027	14.554	3.997	8.781	1.697	2.816	2.816	8.867
RMSE	3.519	14.924	4.381	9.006	2.210	3.340	3.419	8.909
All Moneyness								
Mean	-1.265	-7.944	2.203	-4.840	1.712	-0.942	0.972	-4.468
MAE	3.183	8.042	3.290	5.724	2.535	2.649	3.009	5.464
RMSE	3.843	9.916	4.189	6.776	3.537	3.281	3.887	7.172
				Panel B: Percentaget Error				
<0.97								
Mean	-0.170	-0.805	0.169	-0.253	0.323	0.142	0.043	-0.429
MAE	0.374	0.870	0.324	0.439	0.467	0.485	0.376	0.648
RMSE	0.578	1.226	0.407	0.638	0.626	0.690	0.538	0.964

Figure 3. Test of performance of the model

	$T \geq 90$		$90 > T \geq 50$		$T < 50$		All Maturity	
Moneyness F/K	Basiscall	BLcall	Basiscall	BLcall	Basiscall	BLcall	Basiscall	BLcall
097-1.00								
Mean	-0.064	-0.435	0.070	-0.238	-0.098	-0.395	-0.029	-0.350
MAE	0.150	0.435	0.162	0.317	0.412	0.586	0.255	0.451
RMSE	0.161	0.484	0.201	0.360	0.725	1.220	0.472	0.824
1.00-1.03								
Mean	-0.067	-0.364	0.030	-0.216	0.011	-0.085	-0.002	-0.212
MAE	0.107	0.364	0.082	0.237	0.093	0.119	0.093	0.231
RMSE	0.124	0.392	0.105	0.258	0.129	0.144	0.120	0.274
1.03-1.06								
Mean	-0.029	-0.254	0.026	-0.160	0.023	-0.043	0.012	-0.136
MAE	0.038	0.170	0.033	0.108	0.024	0.030	0.051	0.144
RMSE	0.052	0.218	0.053	0.146	0.051	0.064	0.064	0.176
□ 1.06								
Mean	-0.004	-0.141	0.025	-0.073	0.009	-0.020	0.012	-0.064
MAE	0.024	0.141	0.026	0.073	0.011	0.021	0.018	0.062
RMSE	0.029	0.157	0.029	0.084	0.017	0.028	0.025	0.089
All Moneyness								
Mean	-0.104	-0.546	0.077	-0.195	0.044	-0.100	0.010	-0.271
MAE	0.225	0.577	0.154	0.270	0.212	0.270	0.195	0.364
RMSE	0.405	0.882	0.244	0.404	0.448	0.669	0.373	0.669

In every time-to-maturity group, the Black model underprices (shown by a negative value) the option prices. T he farther the maturity is, the severer the options underprices. In every moneyness groups, the Black's model also generally underpriced the options. In index-point terms, the degree of underpricing is proportional to the moneyness. In percentage term, the degree of underpricing, however, is opposite proportional to moneyness. A few groups are excluded from the consistent result. In the first subtotal group in Table II, where F/K< 0.97, the Black model outperforms the basis risk model in 90> T 50 and T <50 subgroups. But both models have large mean errors in F/K< 0.97 subtotal group. In particular, the Black's model has mean error of -80.5% basis model is 32.3% wheret< 50 due to deep out of money scenarios We also found that the relative content between the MAE and its mean error in each subgroup of index-point terms was larger for the basis risk model than the Black's model. For example, in the seventeen subgroup in Table II (Panel A), where T90 and F/K1.06, the Black's model generates an MAE of 14.554 and a

Mean of -14.554, while the basis risk model has 3.207 and 0.392, respectively. If we take MAE/Mean as Lim and Guo (2000), we obtain 1.0 for Black's model and 8.2 for the basis risk model. This can be interpreted as the evidence of the Black model's bias. When most of the errors are of the same sign in each subgroups, the mean error in index-point terms will be close to its MAE. The basis risk model's prices are more distributed around the actual prices, so the mean error is much smaller than MAE in index-point terms.

4.4.3 Goodness of Fit

The MAE of the Black model in grand total group of index-point terms is 5.464, while it is 3.009 for the basis risk model. In percentage terms, the MAE is 36.4% black is 19.4%. The RMSE statistics are consistent with this. The basis risk model dominates the Black's in most of the groups. According to the empirical test, we make a short conclusion here. The smaller bias and the better goodness of fit in the empirical test for the basis risk model over the Black's shows that there is evidence of basis risk in the S &P 500 futures options. The basis risk model is a better specification than the model without basis risk

5 Conclusion and recommendation

5.1 Conclusion

The numerical test shows that the futures call option price is positively related with the correlation coefficient between basis and spot return but is a negatively in the initial basis value. Meanwhile, the sign of correlation coefficient determines the relationship between the basis volatility and the futures call price. This theoretical superiority has been empirically tested by S &P 500 futures options daily data. Comparing with the Black's model, the empirical test shows clear evidence supporting the occurrence of basis risk in futures options on stock index. Our model outperforms the Black's model by producing smaller bias and better goodness of fit. It not only eliminates systematic moneyness and time-to-maturity biases produced by Black model, but also has better prediction power. In overall sample data, the mean errors in terms of index and percentage are 0.973% and 1.0% for our model-4.468 and black 27.1%

5.2 Recommendation

5.3 Future Research

Appendices

A •

B •

$$F(t,U) = S(t)\exp[Z(t,U)] \qquad \text{(B-1)}$$

Using (4) and the solution of (1), the Eq. (5) is derived.

By (B-1), *F(0,U)* can be obtained, and substituted into (5). The transformed function for (5) turns out to be:

$$F(0,U) = F(0,U)\exp\left[(r-\delta-\frac{1}{2}\sigma_S^2)t - \frac{tZ(0,U)}{U} + \int_0^t \sigma(v)\bullet dW^Q(v)\right] \qquad \text{(B-2)}$$

Where

$$\sigma(v) = \begin{bmatrix} \sigma_S + \dfrac{(U-t)\rho\sigma_Z(v,U)}{U-u} \\ \dfrac{(U-t)\sqrt{1-\rho^2}\,\sigma_Z(v,U)}{U-u} \end{bmatrix}, \text{ and } \quad dW^Q(v) = \begin{bmatrix} dW_S^Q(v) \\ dW_Z^Q(v) \end{bmatrix}$$

C •

$$\frac{dF(t,U)}{F(t,U)} = \left[(r-\delta) - \frac{Z(t,U)}{U-t} + \rho\sigma_S\sigma_Z(t,U) + \frac{1}{2}\sigma_Z(t,U)^2\right]dt + \left(\sigma_S + \rho\sigma_Z(t,U)\right)dW_S^Q(t)$$

$$+\sigma_Z(t,U)\sqrt{1-\rho^2}\,dW_Z^Q(t) \quad \text{(C-1)}$$

By assuming the call option with basis risk is a function of *F* and *t*. Applied Itos' lemma, we obtain

$$dC = \left\{C_F F(t,U)\left[(r-\delta) - \frac{Z(t,U)}{U-t} + \rho\sigma_S\sigma_Z(t,U) + \frac{1}{2}\sigma_Z(t,U)^2\right] + \frac{1}{2}C_{FF}\left[F^2(t,U)\left(\sigma_S^2 + 2\rho\sigma_S\sigma_Z(t,U) + \sigma_Z(t,U)^2\right)\right] + C_t\right\}dt$$

$$\left(\sigma_S + \rho\sigma_Z(t,U)\right)dW_S^Q(t) + \sigma_Z(t,U)\sqrt{1-\rho^2}\,dW_Z^Q(t) \quad \text{(C-2)}$$

From (10) we know

$$dG(t) = -dC(t) + ndF(t,U) \quad \text{(C-3)}$$

Substitute (C-1) and (C-2) into (C-3), Equation (11) is computed.