UNIVERSITY OF NAIROBI

Enterprise Risk Management In Black Scholes Option Pricing Model Using Risk Hedging Techniques

BY

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Abstract

Risk is the uncertainty associated with future outcome or event. Option traders face a risk of incurring losses as a result of unfavorable changes of parameters in the option pricing model.

Options are financial derivatives which derive their value from underlying assets.

Options became popular in 1973 after Fisher Black, Myron Scholes and Robert developed the Black Scholes option pricing model. Risk factors called Greeks are derived from this model. Greeks measure the sensitivity of the value of option to the changes in parameter values in the model holding other parameters fixed. These Greeks include: Delta, Gamma, Rho, Theta and Vega.Enterprise risk management (ERM) manages risks to be within the organization's risk appetite.

ERM deals with risks in holistic basis.Developing an ERM framework on option portfolios will help in risk management during option trading.Risk hedging techniques form a large part of ERM framework.These techniques include:Delta hedging, Gamma hedging, Vega hedging and Theta hedging.

Keywords: **Risk, Option, Enterprise Risk Management (ERM), Greek, Delta, Gamma, Vega, Theta, Rho, Hedging, derivative.**

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Declaration and Approval

I the undersigned declare that this dissertation is my original work and to the best of my knowledge, it has not been submitted in support of an award of a degree in any other university or institution of learning.

20th June,2022

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In my capacity as a supervisor of the candidate's dissertation, I certify that this dissertation has my approval for submission.

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Dedication

I would like to dedicate this project to my family and friends for their constant emotional and physical support.

Contents

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First and foremost, praises and thanks to the almighty God for being my strength and guide from the start of the project until its successful completion.

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1 INTRODUCTION

1.1 Risk, Enterprise Risk and Enterprise risk management

Risk is the uncertainty associated with future events or outcome (Banks, 2004). Entities face various types of risks which include strategic risks, operational risks, hazard risks and pricing risks. The aim of every organization is to manage these risks by coming up with a risk management framework which depend solely the amount of uncertainty the organization is willing to accept. This is called risk appetite.

Uncertainty brings both risk and opportunities. The role of the organization is to deal with the risk at hand and use the opportunity available to enhance value.

Pricing of European options using black Scholes model has some risk factors in it called Greeks. The risk factors include Delta, Gamma, Vega, Rho and Theta.

Risk management has been practiced for years. Imagine somebody burning fire at night to keep wild animals away. Early lenders reduced risk of loan defaulters by limiting the amount loaned to individuals and restricting loans to those considered not able to repay them.

Risk management is the identification, evaluation and prioritization of risks followed by coordinated and economical application of resources to minimize, monitor and control probability or impact of unfortunate events or to maximize realization of opportunities (defined in ISO 31000 as effect of uncertainty on objectives).

Risk management entails: Risk Identification, Risk description, Risk estimation, Risk evaluation, Risk reporting, Communication and Risk treatment.

The Committee of Sponsoring Organization (COSO) developed an Enterprise Risk Management framework in September, 2004. This framework is used to date.

Enterprise risks are risks which put the business operations in jeopardy. Examples of enterprise risks include: Compliance risks, Strategic risks, Operational risks, financial risks and Reputation risks.

COSO defines Enterprise risk management (ERM) as a process, affected by an entity's board of directors,management and other personnel, applied in strategy setting and across enterprise designed to identify potential events that may affect the entity and manage risk to be within its risk appetite to provide reasonable assurance regarding the achievement of entity's objectives.

ERM deals with risks in a holistic basis (Sweeting, 2011). It deals with many risks at ago by identifying the relationships among the risks instead of focusing on individual risks.

Dealing with risks in holistic basis enables the organization to easily identify risks and act fast as required. This enhances value of the organization.

Effective risk management maximizes value by mitigating the risk as much as possible. It enables an organization attain its objective in terms of risk management. ERM enhances total compliance to the law and regulations and effective reporting hence avoiding reputation risks to the organization.

The Objectives of ERM include:
1. Aligning risk appetite and strategy.
2. Enhancing response decision.
3. Reducing operational surprises and losses.
4. Identifying and managing multiple and cross-enterprise risks.
5. Seizing available opportunities to maximize value.

ERM supports value creation by enabling the management to deal effectively with potential future events that create uncertainty.
An ERM framework should define essential components,suggest a common language, and provide clear direction and guidance for enterprise risk management.

In Kenya ERM is not fully developed and utilized and hence increase in risks to organizations making the traditional risks to constantly evolve (Kimotho, 2015).
Nakumatt and Uchumi are some of the firms in Kenya which closed down due to poor ERM management frameworks.

1.1.1 Options

Options are financial derivatives deriving their values from the underlying assets.
Options are financial derivatives that gives the right to the holder but not the obligation to buy or sell an underlying stock at a pre-defined strike price K within a certain time period.
There are two types of options;
Call options which gives the holder the right to buy the underlying asset by a certain date for a given price.
Put option which gives the holder the right to sell the underlying asset by a certain date for a certain price.
Options can take two positions, long or short position. In long position an investor has already bought and owns the share of stock while in the short position an investor owes stock to another person but has not actually bought them yet.
The act of selling the option is referred to as writing off the option.

Risks Associated with Options includes:
1. Spot risk-are directional risks associated with movement of the spot price of an option. These movements take place due to ever changing market conditions.
2. Volatility risks- They give indications to the likelihood of an instrument to move away from its present price.
3. Informational Risk-They result from imperfect degree of information in the market.
4. Credit Risks-It is loss incurred on a contact if the counterpart fails to honour option contract of engagements.

Put-Call Parity

There exists an important relationship between the prices of European put and call options that have the same strike price and same time of maturity.
Put-Call parity shows that the value of a European call with a certain exercise price and exercise date can be deducted from the value of a European put with the same exercise price and time to maturity.
If there is a deviation from the put –call parity then it would result in an arbitrage opportunity and traders would take advantage of this opportunity to make riskless profit.
Put –Call parity is important because it eliminates the possibility of arbitrage which enables traders to make profits with no risks.
Consider a European Call;
$C(T,S_T) = max(S_T - K, 0)$, where K is the strike price, S is the exercise price and T is time to maturity.
For a European Put;
$P(T,S_T) = Max(K - S_T)$.
Finding the relationship between the two,
$C_T - P_T = S_T - K$
Considering three cases,
Case 1: at time T, $S_T = K$(trivial case)

Case 2:If at time T, $S_T < K$,implies,
$C(T,S_T) - P(T,S_T) = max(S_T - K, 0) - max(K - S_T, 0)$
$0 - max(K - S_T, 0) = S_T - K$

Case 3:If at time T, $S_T > K$,implies,
$C(T,S_T) - P(T,S_T) = max(S_T - K, 0) - max(K - S_T, 0)$
$max(K - S_T, 0) - 0 = S_T - K$

In general, at maturity time T,
$C(T,S_T) - P(T,S_T) = S_T - K$,therefore at time t,
$C(t,s) = S + P(t,s) - e^{r(T-t)}K$ or
$C_t = S + P_t - e^{-r(T-t)}K$
Where:
C_t is the average of the bid and ask prices for the call option.
P_t is the average of the bid and ask prices for the put option.
K is the strike price.
r is the interest rate.
$(T-t)$ is the time period under observation.

European Option Pricing
European options can be priced using Binomial model, Monte Carlo and Black Scholes model.
Black Scholes model is commonly used in pricing European options even today.
Early discussions on pricing of options dates back to Bachelier (1900) and Keynes (1930).
Option pricing issue remained dormant until 1960's when Samuelson in 1965 considered the problem. He however failed to solve the option pricing problem.
The breakthrough was achieved by economists Fisher Morgan, Myron Scholes and Robert Merton.
They developed the famous Black Scholes model in 1973.
The model focuses in valuing European options which allows exercise on a single date.
They developed a closed form solution for the price of the European option on a common stock.
According to Elroy Dimson, the underlying idea is that an investor could exactly replicate the pay-off of the option by trading at each point in the time in the stock and the riskless bond.
This trading strategy should be self-financing, have an initial cost (strike price) but then require other inflows or outflows until the terminal date when the payoff should exactly match the payoff of the option.
Black Sholes model is used in only pricing European options. It cannot be used to price American options because these options can be exercised in any time before maturity.
Black Scholes model uses partial derivative equations. This model is based on the principle of hedging and focusses on eliminating risks associated with volatility of underlying assets and stock options.
The model is based on six variables: Volatility, Underlying strike price, Exercise price, Risk free rate, Time to maturity and type of option.
Assumptions of Black Scholes model
1. *Lognormal distribution.* The Black Scholes model assumes that stock prices follow a lognormal distribution.
2. *No dividends.* The model assumes that stocks do not pay dividends or returns.

3. *Random walk*. The stock market is highly volatile hence a state of random walk is assumed.
4. *Expiration date*. The model assumes that an option can only be exercise on its expiration date.
5.*Frictionless Market*. No transaction costs involved.
6. *Risk Free Interest rate*. The rate of interest is assumed to be constant.
7. *No Arbitrage*. It assumes there is no arbitrage opportunity to make riskless profit.

Limitations of Black Scholes model

1. It is limited only to Europeans Market (assumes options can only be exercised on expiration).
2. Risk free interest rate is rarely true in the current market. The current market is highly volatile.
3. The frictionless market assumed is never true. Transaction costs, taxes and commissions are involved during trading.
4. The no return assumption rarely exist in the current market.

Despite these limitations,Black Scholes model has been used to date to price European options.Greeks are derived from this model.

1.1.2 Risk Measures(Greeks)

Greeks measure the sensitivity of the values of a derivative product (Option) to changes in parameters value holding other parameters fixed.
Greeks include:

Delta

It measures the changes in option premium based on how the price of the underlying security changes. It provides means of projecting option price changes based on option correlation with price of underlying asset.

Gamma

It represents how delta of options change with one point change in the price of underlying stock.
Gamma projects possible future price movements.

Theta

It measures how the time value of an option erodes throughout the option life.

Vega
It is a metric for implied volatility.
It measures the effect of change in volatility for the underlying asset on option value.

Rho
It considers interest rate that is available in the market. This interest rate is usually fixed for a long period.

1.1.3 Risk Hedging

Risk Hedging can be defined as a strategy for reducing exposure to an investment risk. One can hedge the risk of an investment by taking an offsetting position in another investment. The hedging parameters in European options include delta, gamma, Vega, Rho and theta
Risk hedging technique developed will involve a portfolio with at most three stocks.

1.2 Statement of Problem

Black Scholes Model have been used over years to price European options. The pricing has some risk factors called Greeks which affect the price of European options.

The problem is to address these risk factors and risks associated with them whenever Black Scholes pricing model parameters changes (underling stock price, time and volatility) and how to deal with them using ERM framework to maximize the option value. Risk hedging techniques using these risk factors will minimize the risks during option trading.

1.3 Objectives

General objectives
1.To develop an ERM framework to reduce risks associated with Greeks during trading of options priced using Black Scholes model.
2.To investigate how Greeks affect the price of options during trading and come up with a framework to mitigate the risks associated with these Greeks.
The specific objectives include:
1. To determine how Greeks (delta, Gamma, Vega, Rho, Theta) affect price of options during pricing using Black Scholes model.
2. To develop an ERM framework to minimize risks associated with Greeks using Risk hedging techniques.

1.4 Significance of Study

This study provides more attention towards the option market making huge losses due to failure in recognizing the effects Greeks have on option pricing.

The study will help investors introduce an ERM framework to reduce risks associated with Greeks by practicing risk hedging techniques during option trading.

The investors will study the movement of option prices in respect to Risk factors before deciding on optimal investment decision.
The study will help the academia field research more on ERM and Risk hedging during option pricing using Black Scholes pricing model.

2 LITERATURE REVIEW

Risk is the uncertainity concerning the occurrence of an event. According to Paul Embretch (2005), risk relates strongly to uncertainity and hence to the notion of randomness. In his book, Financial Enterprise Risk Management, Sweeting (2011), defined risk as 'a quantifiable probability associated with a particular outcome or range of outcomes ,conversely it can refer to a non-quantifiable possibility of gains or losses associated with different future events or even possibility of adverse outcomes.

Risk management has been practiced over years to minimize and mitigate risks. Imagine during early periods somebody burning fire at night to keep wild animals away, this was an example of risk management. Early lenders reduced risks of loan defaulters by limiting the amount loaned to an individual and restricting loans to those perceived most likely not to repay them.

Risk management initially focused on pure risks (those with either loss or no loss). According to Simithson (1998), the basic tools of financial risk management are forwards, futures, swaps and options.

Enterprise Risk Management is an important tool for any enterprise. Sweeting (2011), defines ERM as the management of all risks on holistic basis both quantifiable and non-quantifiable. He presents the fact that ERM involves recognizing the context, identifying the risk assessing and comparing the risk appetite, deciding on the extent to which the risks are managed, taking the appropriate action and reporting on and reviewing the actions taken.

Sweeting uses the following tools to identify risk: SWOT (Strength, Weaknesses and Opportunities) analysis which entails identifying strengths, weaknesses and opportunities of a given risk, risk checklist and risk triggered questions. His risk identification techniques include: Brainstorming unstructured group discussions, Interviews, Independent group analysis and surveys. After Identifying, analyzing and assessing the risk, an organization can remove the risk, reduce the risk, transfer the risk or accept the risk.

According to Sweeting, ERM is formalized into a process with details on how to accomplish each stage leading to an ERM framework. ERM requires the presence of a central risk function headed by a chief risk officer.

Sweeting also explains the importance Risk management and ERM. He states that managing risks help reduce the volatility of organization's returns. This increases the value of the firm by reducing risk bankruptcy and perhaps tax liability. Improved risk management can lead to a better trade-off between risk and return. ERM is flexible and can help a firm react quickly to emerging risks.

Option pricing also involves risk management and enterprise risk management. Kumar et al (2018), states that an effective security market provides three principles: Opportunities trading equities, debt securities and derivative products. For the purpose of risk management and trading, the pricing theories of stock options have occupied important place in derivative market.
Options are financial derivative contracts that give the right to the holder, but not an obligation to buy or sell an underlying stock a pre-defined strike price within a certain period. According to Kumar (2018), financial derivatives are innovative instruments in financial market. The financial instruments includes: Options, Swaps and futures. He further states that derivatives have a great deal of use in risk management. A judicial use derivative in right proportion enables a corporate manager to minimize a risk and optimize return.
There exists two types of option: American options and European options. The American options can be exercised before maturity and European options can only be exercised on maturity. Our main interest is the European options.
European options can be priced using Monte Carlo simulations, Binomial model pricing and Black Scholes pricing model.
Podlozhnyuk et al (2008), describes how options can be priced using Monte Carlo simulations and noted that Monte Carlo simulations are effective and efficient for small path counts.
Podlozhnyuk (2008), explains how options are priced using binomial model. They note that the binomial model represents price evolution of the options underlying asset as the binomial tree of all possible prices at equally-spaced time steps from today under the assumption that at each step the price can only move up and down at a fixed rate with respective possibilities P_u and P_d.
Black Scholes Model of pricing European options is widely use to date. Rubinstein (1994), states that Black Scholes option pricing model is most widely used formula.
Kumar et al (2018), further states that Black Scholes (1973) option pricing model is a landmark in history of financial modelling. The model is used for non-dividend paying stock.
Clifford (1976), states that Fisher Black and Myron Scholes provided the first explicit general equilibrium solution to the option pricing problems.
Black Scholes option pricing model have some various assumptions.The assumptions include:Frictionless market, risk free rate of interest, no dividend payment and no arbitrage opportunity.
Nations (2012), states some Black Scholes option pricing model assumptions. The assumptions include: Stock prices exhibit a geometric Brownian motion characteristics, the volatility is constant, there are no transaction costs or taxes, trade is continuous, there are no limits in short selling, no dividends and the risk free rate is constant.
This model also assumes there are no arbitrage opportunities. This is confirmed by Mullaney (2009) in his work. He states that Black Scholes pricing model is derived by no arbitrage assumption.

This model has some drawbacks and limitations despite being used for pricing European options. Jankova (2018), discusses the drawbacks and limitations of Black Scholes option pricing model especially the constant volatility assumption made. He addresses the problem of volatility. The constant volatility assumption is not true in the current market. The underling market volatility is expressed by range σ^2 or standard deviation denoted by σ. A high parameter value indicates considerably more significant price fluctuation as well as a high degree of uncertainty of achieving a desired return. Volatility determination is crucial for correct option pricing.Teneng (2011) also discusses the same limitations.
Yalincak (2012), also discusses the limitations of Black Scholes option pricing model. The limitations include: It is limited only to Europeans Market, Risk free interest rate is rarely constant (the market is highly volatile), transaction costs, taxes and commissions are involved during trading and there is no return (dividend) assumption rarely exist in the current market.

There are risk factors in Black Scholes option pricing model. These risk factors are referred to as Greeks. In order to determine how sensitive options are to certain types of risks, one should look at Greeks options (Hull, 2002).

Kumar (2018), studies Greeks and there significance in managing various types of risks associated with option contracts.

Risk management procedure when pricing options involves risk hedging. Hedging is a way to insure against financial risks via taking an offsetting position to the one in an asset. Examples of hedging techniques include delta hedging, gamma hedging, gamma-delta hedging, Rho hedging, Vega hedging and theta hedging.
Jankova (2018) states that presenting derivative contracts represents a large portion of financial market and they are mostly used as a basic instrument for hedging various risks.

In the journal, Options, Greeks and Risk Management, Paunovic indicates that a portfolio is delta hedged if the delta of the portfolio is zero. This indicates that there is no loss incurred in case the price of an underlying asset moves by a single unit.

Taylor Francis (2001), describes hedging option techniques. They state that proper risk management of option portfolios requires the measurement and analysis of the sensitivity of the option positions value to changes in determining variables. The hedging involves hedging parameters called Greeks. The hedging parameters include delta hedging, gamma hedging, theta hedging, Vega hedging and rho hedging. Volatility plays a

vital role in computation of hedging parameters. Gamma measures the change in hedge ratio for a small change in price of underlying asset.
Gamma is important in risk management. It provides to sellers of insurance a measure of their exposure to a sudden jump in the price of underlying asset.

3 METHODOLOGY

3.1 Ito lemma

Let X be an Ito process given by $dX_t = \mu_t dt + \sigma_t dW_t$ where μ and σ are adapted processes representing drift and volatility respectively.
Let f be a $C^{1,2}$-function such that $f(t,x) \in C^{1,2}([0,\infty) \times \mathbb{R})$ and define Ito process Z by $Z_t = f(t,X_t)$ and $dZ_t = \frac{\partial f}{\partial t}dt + \frac{\partial f}{\partial x}dx + \frac{1}{2}\frac{\partial^2}{\partial x^2}(dx)^2$.
By replacing the value of $dX_t = \mu_t dt + \sigma_t dW_t$.The Ito proposition is given by
$\frac{\partial f}{\partial t}dt + \mu\frac{\partial f}{\partial x}dx + \frac{1}{2}\sigma^2\frac{\partial^2 f}{\partial x^2}(dx)^2$ with the following rules;
$(dt)^2 = 0$, $dt.dw = 0$ and $dt.dw = dt$.
Using the rules,
$df(t,x(t)) = (\frac{\partial f}{\partial t} + \mu\frac{\partial f}{\partial x} + \frac{1}{2}\sigma^2\frac{\partial^2 f}{\partial x^2})dt + \frac{\partial f}{\partial t}dW_t$.

3.2 Geometric Brownian Motion

Let X_t be a diffusion process with a stochastic differential equation $dX_t = \mu X_t dt + \sigma X_t dW_t$.
Solving this process using Ito lemma.
Letting $f(t,x) = \log_e X, f_x = \frac{1}{x}, f_{xx} = \frac{-1}{x^2}$.
$df = f_t dt + f_x dx + \frac{1}{2}(dx)^2$
$dlogX_t = 0 + \frac{1}{x}(\mu x dt + \sigma x dW_t) + \frac{-1}{x^2}(\sigma^2 x^2 (dW_t)^2)$
$\int_0^T (dlogX_t) = (\mu - \frac{1}{2}\sigma^2)dt + \sigma dW_t$
$X_T = X_0 exp((\mu - \frac{1}{2}\sigma^2)(T-t) + \sigma(W_T - W_t))$
$LogX_T = logX_0 + (\mu - \frac{1}{2}\sigma^2)(T-t) + \sigma(W_T - W_t)$
X_T is lognomally distributed.

3.3 Partial Differential Equation

Exploring the connection that exists between stochastic differential equation (SDE) and certain parabolic differential equations.
Using the scalar function $\mu(t,x)$, $\sigma(t,x)$ and $\Phi(x)$, a function F is formed which satisfies the following boundary value problem on (0,T)XR.
$\frac{\partial F}{\partial t} + \mu(t,x)\frac{\partial F(t,x)}{\partial x} + \frac{1}{2}\sigma^2\frac{\partial^2 F(t,x)}{\partial x^2} = 0$
$F(T,X) = \Phi(X)$
Solving using Ito formula I get,
$F(T,X) = E(F(T,X_T))$ which is the Feyman K$\hat{a}c$ stochastic representation.Assuming that

F is a solution to the boundary value problem;
$F_t + \mu F_x + \frac{1}{2}\sigma^2 F_{xx} = rF$
F has a representation ;
$F(t,r) = \exp(-r(T-t))E(\Phi(X_T))$ where X satisfies the stochastic differential equation.
$dX_t = \mu dt + \sigma dW_t, X_t = x$

3.3.1 Black Scholes Partial Differential Equation

Let the price of an underlying process V(s,t) be the value derivative.(s is the exercise price and t is time to maturity).A portfolio say π is formed by selling the derivative and buying Δ units of the underlying risky asset.The value of the portfolio is;
$\pi(t)$=V(t)-ΔS(t)
Introducing derivative both sides;
dπ=d(V-ΔS)
Let V=V(s,t) where S_t satisfy the following stochastic differential equation;
$dS = \mu S dt + \sigma s dW_t$
Solving the stochastic differential equation using Ito Lemma , the following is obtained;
$d\pi = (V_t + \frac{1}{2}\sigma^2 V_{ss} dt)$
A portfolio should be different than the risk free alternative, let;
$r\pi dt = d\pi$ and $\pi = V - \Delta S$
$V_t + r\Delta S + \frac{1}{2}\sigma^2 S^2 V_{ss} = rV$
$V(t,s) = max(s-t)$ which is the call option.
Solving Black Scholes PDE using Feyman Kâc stochastic representation theorem,
$V(t,s) = e^{-r(T-t)}E(V(S,T))$
$dS_t = rsdt + \sigma s dW_t$ which is a Geometric Brownian motion.
$S_T e^{(r-\frac{1}{2}\sigma^2)(T-t)+\sigma(W_T-W_t)}$
$V(t,s) = e^{-r(T-t)}E(max(S_T - K, 0))$, K is the strike price.
Solving,
$d_1 = \frac{ln(\frac{S_t}{K})+(r+\frac{1}{2}\sigma^2)\tau}{\sigma^2\sqrt{\tau}}$
$d_2 = \frac{ln(\frac{S_t}{K})+(r+\frac{1}{2}\sigma^2)\tau}{\sigma^2\sqrt{\tau}} = d_1 - \sigma\sqrt{\tau}$
$N(.)$ is the cumulative density function of normal dstribution.
$\tau = T - t$
$C_t = Max(S_t - K, O)$
$C_t = S_t N(d_1) - Ke^{-r\tau}N(d_2)$
By Put-Call parity relationship,
$P_t = Ke^{-r\tau}N(-d_2) - S_t N(-d_1)$

Assumption of the model

1. *Lognormal distribution*. The Black Scholes model assumes that stock prices follow a lognormal distribution.
2. *No dividends*. The model assumes that stocks do not pay dividends or returns.
3. *Random walk*. The stock market is highly volatile hence a state of random walk is assumed.
4. *Expiration date*. The model assumes that an option can only be exercise on its expiration date.
5. *Frictionless Market*. No transaction costs involved.
6. *Risk Free Interest rate*. The rate of interest is assumed to be constant.
7. *No Arbitrage*. It assumes there is no arbitrage opportunity to make riskless profit.

3.4 Option Greeks

Option Greeks are financial measures of sensitivity of an option's price to its underlying determining parameters .The parameters include; volatility, time and price of underlying stock.
Greeks are derived and graphically shown how they behave as the maturity time approaches.

3.4.1 Option Delta

It measures the changes in option premium based on how the price of underlying security changes.
For a European call option on a non-dividend stock,the Δ can be shown as $\Delta = N(d_1)$ and derived as follows;
$\Delta = N(d_1) + S_t \frac{\partial N(d_1)}{\partial S_t} - Ke^{-r\tau} \frac{\partial N(d_2)}{\partial S_t}$
$\Delta = N(d_1) + S_t \frac{\partial N(d_1)}{\partial d_1} \frac{\partial d_1}{\partial S_t} - Ke^{-rt} \frac{\partial N_2}{\partial d_2} \frac{\partial d_2}{\partial S_t}$
$\Delta = N(d_1) + S_t \frac{1}{\sqrt{2\pi}} e^{\frac{-d_1^2}{2}} \frac{1}{S_t \sigma \sqrt{\tau}} - Ke^{-r\tau} \frac{1}{\sqrt{2\pi}} e^{\frac{-d_1^2}{2}} \frac{S_t}{K} e^{r\tau} \frac{1}{S_t \sigma \sqrt{\tau}}$
$\Delta = N(d_1)$
For a European put option the delta can be showed as;
$\Delta = N(d_1) - 1$
It is derived as follows;
$\Delta = \frac{\partial P_t}{\partial S_t} = Ke^{-r\tau} \frac{\partial N(-d_2)}{\partial S_t} - N(-d_1) - S_t \frac{\partial N(-d_1)}{\partial S_t}$
$\Delta = N(d_1) - 1$

3.4.2 Option Gamma

Gamma refers to the rate of change of delta.
For a European call option on a non dividend stock, gamma can be shown as;
$\Gamma = \frac{1}{S_t \sigma \sqrt{\tau}} N^{'}(d_1)$
The derivation is as follows;
$\Gamma = \frac{\partial^2 C_t}{\partial S_t^2} = \frac{\partial}{\partial S_t} \frac{\partial C_t}{\partial S_t}$
$\Gamma = \frac{\partial N(d_1)}{\partial d_1} \frac{\partial d_1}{\partial S_t}$
$\Gamma = \frac{1}{S_t \sigma \sqrt{\tau}} N^{'}(d_1)$
For a European put option the gamma can be showed as;
$\Gamma = \frac{1}{S_t \sigma \sqrt{\tau}} N^{'}(d_1)$
The derivation is as follows;
$\Gamma = \frac{\partial^2 P_t}{\partial S_t^2} = \frac{\partial}{\partial S_t} \frac{\partial P_t}{\partial S_t}$
$\Gamma = \frac{\partial N(d_1 - 1)}{\partial d_1} \frac{\partial d_1}{\partial S_t}$

$\Gamma = \frac{1}{S_t \sigma \sqrt{\tau}} N^{'}(d_1)$

3.4.3 Option Theta

It measures how the time value of an option erodes throughout the Option life.
For a European call option on a non-dividend stock, theta can be written as:
$\Theta = -\frac{S_t \sigma}{2\sqrt{\tau}} N^{'} d_1 - rKe^{-r\tau} N(d_2)$
The derivation is;
$\Theta = -S_t \frac{\partial N(d_1)}{\partial \tau} + -rKe^{-r\tau} \frac{\partial N(d_2)}{\partial \tau}$
$\Theta = -S_t \frac{\partial N(d_1)}{\partial d_1} \frac{\partial d_1}{\partial \tau} + -rKe^{-r\tau} \frac{\partial N(d_2)}{\partial d_2} \frac{\partial d_2}{\partial \tau}$
$\Theta = -\frac{S_t \sigma}{2\sqrt{\tau}} N^{'} d_1 - rKe^{-r\tau} N(d_2)$
For a European put option on a non-dividend stock, theta can be shown as;
$\Theta = -\frac{S_t \sigma}{2\sqrt{\tau}} N^{'} d_1 + rKe^{-r\tau} N(-d_2)$
The derivation is;
$\Theta = -\frac{\partial P_t}{\partial \tau} = -(-r)Ke^{-r\tau} N(-d_2) - Ke^{-r\tau} \frac{\partial N(-d_2)}{\partial \tau} + S_t \frac{\partial N(-d_1)}{\partial \tau}$
$\Theta = -(-r)Ke^{-r\partial\tau}(1 - N(d_2)) - Ke^{-r\tau} \frac{\partial 1-(N(d_2))}{\partial d_2} \frac{\partial d_2}{\partial \tau} + S_t \frac{\partial 1-(N(d_1))}{\partial d_1} \frac{\partial d_1}{\partial \tau}$
$\Theta = -\frac{S_t \sigma}{2\sqrt{\tau}} N^{'} d_1 + rKe^{-r\tau} N(-d_2)$

3.4.4 Option Vega

It is a metric for implied volatility.
It measures the effect of change in Volatility for the underlying asset on option value.
For a European call option on a non-dividend stock , Vega can be shown as;
$\nu = S_t \sqrt{\tau} N^{'}(d_1)$
The derivation is;
$\nu = \frac{\partial C_t}{\partial \sigma} = S_t \frac{\partial N(d_1)}{\partial \sigma} - Ke^{-r\tau} \frac{\partial N(d_2)}{\partial \sigma}$
$\nu = S_t \frac{\partial N(d_1)}{\partial d_1} \frac{\partial d_1}{\partial \sigma} - Ke^{-r\tau} \frac{\partial N(d_2)}{\partial d_1} \frac{\partial d_2}{\partial \sigma}$
$\nu = S_t \frac{1}{\sqrt{2\pi}} e^{\frac{-d_1^2}{2}} \frac{\sigma^2 \tau^{\frac{3}{2}} - [ln\frac{S_t}{k} + (r + \frac{\sigma^2}{2})\tau]\tau^{\frac{1}{2}}}{\sigma^2 \tau}$
$\nu = S_t \sqrt{\tau} N^{'}(d_1)$
For European put on a non-dividend stock,Vega can be shown as;
$\nu = S_t \sqrt{\tau} N^{'}(d_1)$
The derivation is;
$\nu = \frac{\partial P_t}{\partial \sigma} = -S_t \frac{\partial N(-d_1)}{\partial \sigma} + Ke^{-r\tau} \frac{\partial N(-d_2)}{\partial \sigma}$
$\nu = -S_t \frac{\partial (1 - N(d_1))}{\partial d_1} \frac{\partial d_1}{\partial \sigma} + Ke^{-r\tau} \frac{\partial (1 - N(d_2))}{\partial d_1} \frac{\partial d_2}{\partial \sigma}$
$\nu = S_t \sqrt{\tau} N^{'}(d_1)$

3.4.5 Option Rho

It is defined as the rate of change of the option price in respect to the interest rate.
For both the European call and put option the rho is obtained by getting the first partial derivative of the call or put option with respect to riskless interest rate.
The European call option rho is given by;
$\rho = K\tau e^{-r\tau}N(d_2)$
The European put option rho is given by;
$\rho = -K\tau e^{-r\tau}N(-d_2)$

3.4.6 Shares

Shares are unit of ownership in a corporation or a firm. Shares are denoted by S and there value is basically the value of the underlying stock price. A share has a delta of 1 and gamma of zero as shown;
$\frac{d}{ds}S = 1$ which is the delta and $\frac{d^2}{ds^2}S = 1$ which is the gamma.

3.5 Moneyness

This is the strike price position of an option relative to the market value of underlying stock. An option can be in the money (ITM), out of money (OTM) or at the money (ATM). The following table summarizes the moneyness.

	PUT	CALL
In the money	The price of the underlying stock is *less than* the strike price of option	The price of the underlying stock is *greater than* the strike price of option
Out of money	The price of the underlying stock is *greater than* the strike price of option	The price of the underlying stock is *less than* the strike price of option
At the money	The price of the underlying stock is *equal to* the strike price of option	The price of the underlying stock is *equal to* the strike price of option

Figure 1. Moneyness

The intrinsic value of an option is obtained by finding the difference between the strike price of an option and the price of an underlying stock.
The moneyness of an option is obtained by getting the difference between the underlying stock price and the strike price and the dividing the result by underlying stock price.

	ITM	OTM	ATM
PUT/CALL	Intrinsic value is positive	Intrinsic value is zero	Intrinsic value is zero.

Figure 2. Moneyness

3.6 Risk Management with Greeks(Hedging)

The basic idea of portfolio hedging is that the value of a portfolio can be made invariant to the factors affecting it.
Greeks are best tools for building portfolios despite market conditions. Understanding Greeks will help a lot in minimizing risks.
Greeks from Black Scholes model can be used to show how to create strategies that profit from the options time to maturity, volatility and risk free interest rate.
Consider a portfolio V with n different types of European call options,
$V = n_1C_1 + n_2C_2 + ... + n_kC_k$ where,
S is the underlying stock price and n_k is the number of call options.
The sensitivity of the portfolio is given by first derivative with respect to S (delta).
$\frac{\partial V}{\partial S} = n_1\frac{\partial C_1}{\partial S} + n_2\frac{\partial C_2}{\partial S} + ... + n_K\frac{\partial C_K}{\partial S}$.
The aim of hedging is to ensure that the value of portfolio remain constant whenever S changes such that $\frac{\partial V}{\partial S} = 0$.

3.6.1 Delta Hedging

A delta edged portfolio has a value of it delta being zero. Delta hedging is effective for slight movements in price of underlying stock. If the price of underlying stock changes by a small amount, the option price should change by delta times amount of change. Suppose a market holds n_1 contracts of a European call options with delta Δ_1 and the price of underlying stock move up by one unit the risk exposure will be $n_1\Delta$ .In order to mitigate this risk by delta hedging, the investor will buy $n_1\Delta_1$ number of shares since the delta of a share is 1.The number of initial call contracts is negative for the market since the investor is writing off the contracts.
By delta hedging such that;
$-n_1\Delta_1 + n_1\Delta_1 = 0$
Assume an investor owns 10,000 call options with a strike price of 50, and underlying stock price of 50, volatility of 0.5 and time to maturity 0.1923 years then the delta will be 0.554 computed using Black Scholes model. The value these call options will be 4.498×10000 as computed using black Scholes model. If the price of the underlying stock move up by a single unit to 51, then a loss of $((51-50) \times 0.554 \times 10000) - 10000 \times (5.070 - 4.498) = -180$ will be incurred by the investor hence there is need to hedge this risk by purchasing 5540 contracts of shares such that with the movement an investor will earn 5540 from shares to neutralize the loss since shares have a delta of 1.
If we create a delta edged portfolio, it is going to be invariant to changes in the underlying stock.
$\Delta = n_1\Delta_1 + n_2\Delta_2 + ... + n_k\Delta_k = 0$
Δ is the cumulative delta of portfolio,Δ_k is the delta of k^{th} option and n_k is the number of contracts of option k.

3.6.2 Gamma Hedging

A portfolio is said to be gamma hedged if the gamma of a portfolio is equal to zero.
$\Gamma = n_1\Gamma_1 + n_2\Gamma_2 + ... + n_k\Gamma_k$
Γ is the cumulative gamma of portfolio,Γ_k is the gamma of k^{th} option and n_k is the number of contracts of option k.
Suppose a market holds n_1 contracts of a European call options with delta Δ_1 and gamma Γ_1 .If the stock price move by large unit the risk exposure is $n_1\Gamma_1$for the market. I gamma hedge the risk incurred by buying n_2 contracts of another European call option with delta Δ_2 and gamma Γ_2.
The interest is to ensure that $-n_1\Gamma_1 + n_2\Gamma_2 = 0$
$n_2 = \frac{n_1\Gamma_1}{\Gamma_2}$
n_2 are the number of contracts of new call to be bought for Gamma hedging.
If the underlying stock make a little move for example by a single unit, we will get a slight loss from a call option. If the underlying stock make a bigger move for example by ten

units, a more significant loss will be made.
The delta hedged position has a considerable risk exposure for large movements in the underlying stock. This is where gamma hedging becomes important because it improves the quality of the delta hedge.

3.6.3 Delta and Gamma Hedging

In order to hedge both delta and gamma, I have to ensure both delta and gamma portfolio is equal to zero. One of the assets in the portfolio is the stock with a delta of 1 and gamma of 0.
$n_1\Delta_1 + n_2\Delta_2 + n_3\Delta_3 = 0$
$n_1\Gamma_1 + n_2\Gamma_2 + n_3\Gamma_1 = 0$
Suppose a market holds n_1 contracts of a European call options with delta Δ_1 and gamma Γ_1 .If there is a small movement in the stock price upwards the exposure risk is $n_1\Delta_1$ and if the stock price move by large unit upwards the risk exposure is $n_1\Gamma_1$ for the market. The investor will gamma hedge the risk incurred by buying n_2 contracts of another European call option with delta Δ_2 and gamma Γ_2 . The delta of a share is 1 and the gamma is zero. Gamma hedging is done by solving the following simultaneous equation. The value of n_1 is known and the number of initial call contracts is negative for the market $(-n_1)$ since the investor is writing off the contract.
$n \times 1 - n_1\Delta_1 + n_2\Delta_2 = 0$
$-n_1\Gamma_1 + n_2\Gamma_2 = 0$
$n_2 = \frac{n_1\Gamma_1}{\Gamma_2}$, these are the number of contracts of new call to be bought for Gamma Hedging.
$n = (n_1\Delta_1 - \Delta_2\frac{n_1\Gamma_1}{\Gamma_2})$, these are number of shares to be bought for delta hedging.

				POSITION GREEK			
		PERCENTAGE GRE		TRADE		CUMULATIVE	
	QUANTITY	DELTA	GAMMA	DELTA	GAMMA	DELTA	GAMMA
SHORT CALLS	n_1	Δ_1	Γ_1	$n_1\Delta_1$	$n_1\Gamma_1$	$n_1\Delta_1$	$n_1\Gamma_1$
NEUTRALIZE GAMMA WITH CALLS	n_2	Δ_2	Γ_2	$n_2\Delta_2$	$n_2\Gamma_2$	$n_2\Delta_2$	$n_2\Gamma_2$
						$n_1\Delta_1 + n_2\Delta_2$	$n_1\Gamma_1 + n_2\Gamma_2$
NEUTRALIZE DELTA WITH SHARES	n	1					

Figure 3. Delta and Gamma hedging

3.6.4 Delta,Gamma and Vega Hedging

Suppose a market holds n_1 contracts of a European call options with delta Δ_1 , gamma Γ_1 and Vega v_1 .If there is a small movement in in the stock price upwards the exposure risk is $n_1\Delta_1$ and if the stock price move by large unit the risk exposure is $n_1\Gamma_1$ for the market and risk exposure for Vega is $n_1 v_1$. The investor will gamma hedge and Vega hedge the risk incurred by buying n_2 contracts of another European call option with delta Δ_2 , gamma Γ_2 and Vega v_2 and n_3 contracts of another European put option with delta Δ_3 , gamma Γ_3 and Vega v_3 . The delta of stock is 1 and the gamma is zero. The following simultaneous equations are solved. The value of n_1 is known and the number of initial call contracts is negative for the market $(-n_1)$ since the investor is writing off the contract.

$-n_1\Gamma_1 + n_2\Gamma_2 + n_3\Gamma_3 = 0$, Gamma Hedging

$-n_1 v_1 + n_2 v_2 + n_3 v_3 = 0$, Vega Hedging

This leads to two simultaneous equations to be solved.

$n_2\Gamma_2 + n_3\Gamma_3 = n_1\Gamma_1$

$n_2 v_2 + n_3 v_3 = n_1 v_1$

$n = -(-n_1\Delta_1 + n_2\Delta_2 + n_3\Delta_3)$, which is the number of shares bought for delta hedging

By solving for the simultaneous equation, the values of n_2 and n_3 are obtained such that;

$-n_1\Gamma_1 + n_2\Gamma_2 + n_3\Gamma_3 = 0$, Gamma hedged

$-n_1 v_1 + n_2 v_2 + n_3 v_3 = 0$, Vega Hedging

$n - n_1\Delta_1 + n_2\Delta_2 + n_3\Delta_3 = 0$, delta hedged

						Position Greek				
		Percentage Greek			Trade			Cumulative		
	Quantity	Delta	Gamma	Vega	Delta	Gamma	Vega	Delta	Gamma	Vega
Option 1	n_1	Δ_1	Γ_1	V_1	$-n_1\Delta_1$	$-n_1\Gamma_1$	$-n_1V_1$	$-n_1\Delta_1$	$-n_1\Gamma_1$	$-n_1V_1$
Option 2	n_2	Δ_2	Γ_2	V_2	$n_2\Delta_2$	$n_2\Gamma_2$	n_2V_2	$n_2\Delta_2$	$n_2\Gamma_2$	n_2V_2
Option 3	n_3	Δ_3	Γ_3	V_3	$n_3\Delta_3$	$n_3\Gamma_3$	n_3V_3	$n_3\Delta_3$	$n_3\Gamma_3$	n_3V_3
Neutralize delta with shares	n	1						$-n_1\Delta_1+n_2\Delta_2 + n_3\Delta_3$	0	0

Figure 4. Delta and Gamma hedging

3.6.5 Theta Hedging

Theta refers to the rate of decline in the value of an option due to passage of time. This is called time decay. This indicates that an option losses value as it moves closer to maturity provided that other pricing components are held constant. An option profitability (intrinsic value) decreases as it gets closer to maturity. Time decay is unfavorable for option buyers since time works against long option holders and time decay is favorable to investor who writes options since written options becomes less valuable as expiration date approaches.
Assume an investor purchases an OTM call option with strike price 1,150 for 5 and the underlying stock is trading at 1,125 and the expiration is 5 days with a theta of 1 per day. This means that the value of the option will be reducing by 1 per day so that the value will be zero at expiration. An option holder can avoid such a loss by ensuring that the underlying price will be at at least 1155 after the five days such that the intrinsic value will be 1,155-1,150=5. This will offset the loss due to theta or time decay.

4 DATA ANALYSIS,RESULTS AND DISCUSSIONS

4.1 Data

The data for this analysis is obtained from Standard and Poor 500 index (S&P 500 index). It is a stock market index tracking the performance of 500 large companies listed on stock exchange in US. The companies include Amazon, Apple etc. S&P 500 index was founded on March 4, 1957 and is operated by S&P Jones index. The data contain daily values and expires on 6/05/2022. It contains the strike prices for put and call options, the Greek value and the underlying stock value which is 4476.58.It also contain the moneyness of the options.
Another set of data is obtained from National Association of Securities Automated Quotations Stock Market (NASDAQ).It is owned by NASDAQ Inc. founded in February 8, 1971.
The data contain daily values and expires 6/05/2022. It contains the strike prices for put and call options, the Greek value and the underlying stock value which is 220.
The analysis is done using the R software.

4.2 Moneyness

The moneyness of an option is obtained by getting the difference between the underlying stock price and the strike price and then dividing the result by underlying stock price.
If the strike price increases and price of underlying stock does not change as maturity time approaches, the moneyness of a call option reduces as time to maturity approaches and the moneyness of a put option increase as the maturity time approaches. The graphs below from S&P 500 index shows this.
The moneyness of both the call option and the put option is zero when the option is at the money. (The strike price and price of underlying stock are equal).

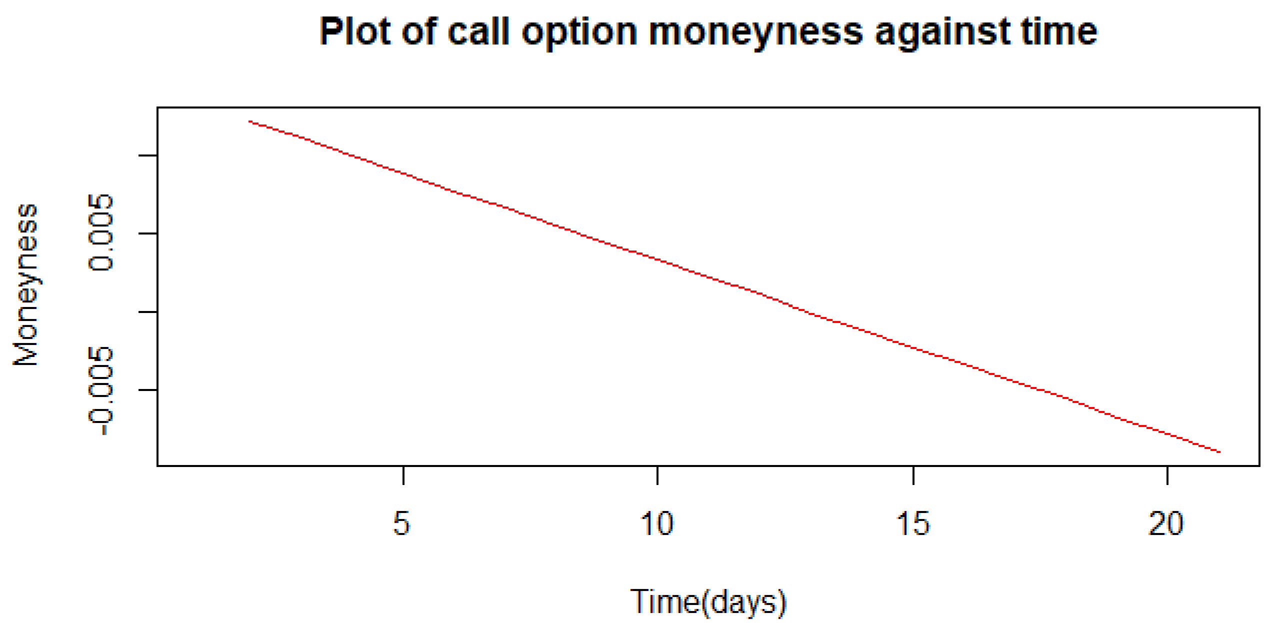

Figure 5. Call option moneyness

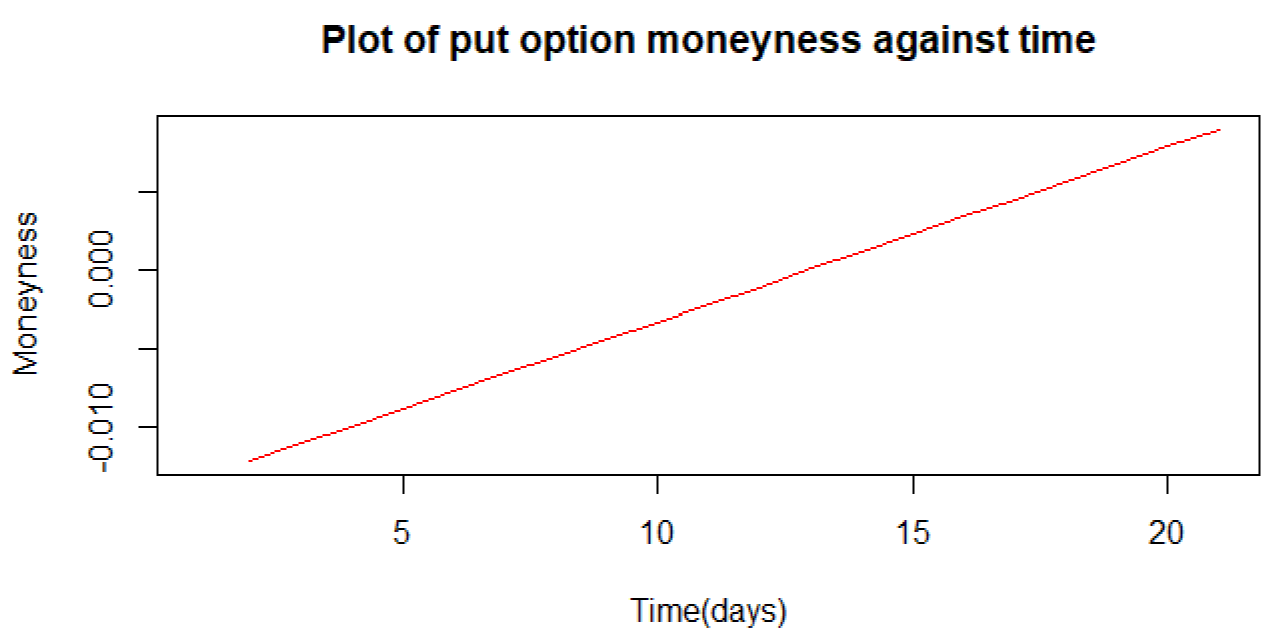

Figure 6. Put Option moneyness

4.3 Delta Hedging

The delta of an option decreases as an option approaches maturity because as maturity approaches an option further gets out of money. Below is a graph plotted using NASDAQ data.

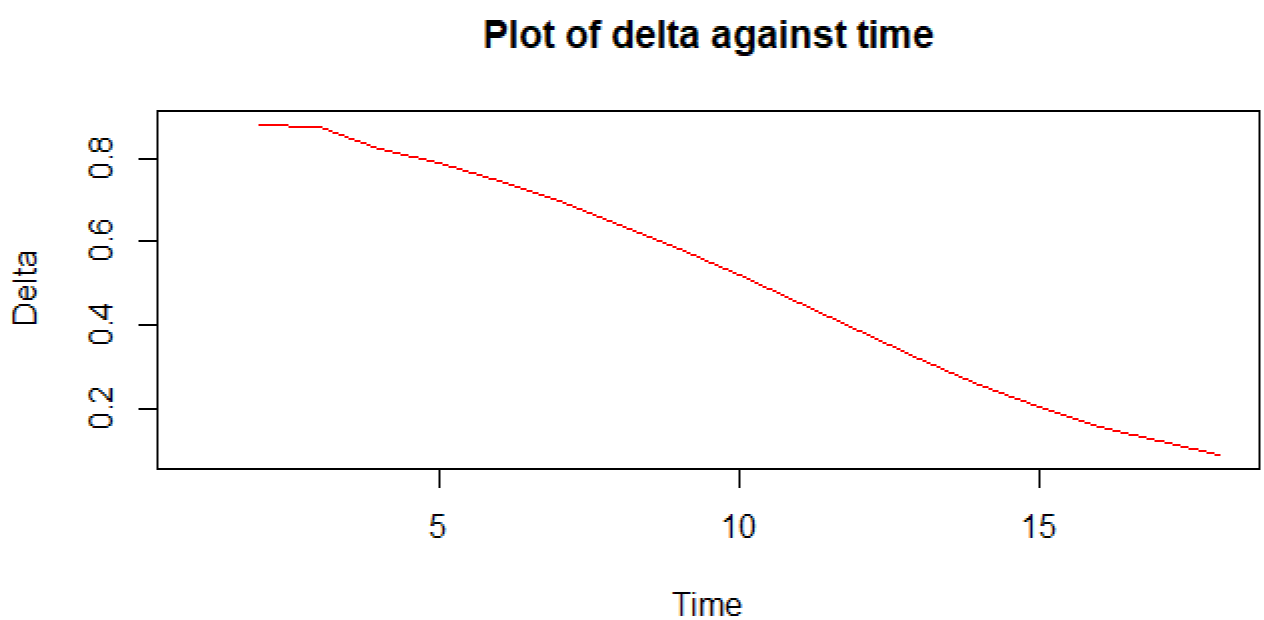

Figure 7. Delta plot against time

Assume an investor is writing off 100,000 call options each with a delta of 0.87967(from NASDAQ delta data). If the underlying stock price rises by a single unit, the investor is exposed to a risk of 87967$(0.87967 \times 100,000)$. This risk can be hedged by the investor purchasing 87967 units of shares with cumulative delta of 87967$(87,967 \times 1)$ since shares has a delta of 1.The shares will give the investor again of 87,967 to offset the loss incurred on the options. In this case the risk will be completely hedged since the cumulative delta will be zero. From the data, the delta value has gown down to 0.8757 and now the investor is exposed to a loss of 87,570 in case the price of the underlying price rises by a single unit. The investor already owns 87,967 shares and 87,570 of these shares will offset the loss leaving the investor with an option of selling the extra 370 shares. This trend will go on until the option reaches maturity.

At maturity the investor will have an option of selling 12,185 units of shares.All the losses as a result of increase of price of underlying stock have been hedged successfully.(Negative sign indicates surplus shares to be sold)

Delta	Cumulative delta	Shares purchased
0.87967	87967	87967
0.87570	87570	-397
0.82244	82244	-5326
0.78825	78825	-3419
0.74491	74491	-4334
0.69622	69622	-4869
0.64088	64088	-5534
0.58163	58163	-5925
0.51875	51875	-6288
0.45186	45186	-6689
0.38513	38513	-6673
0.32173	32173	-6340
0.25970	25970	-6203
0.20507	20507	-5463
0.15778	15778	-4729
0.12185	12185	-3593
0.09070	9070	-12185

Table 1. Delta hedging table

4.4 Gamma Hedging

Gamma hedging improves the quality of a delta hedge and prevents any significance loss in case the price of an underlying stock moves upwards by bigger units for example 10 units. Gamma is at its highest when an option is at the money and it is lowest when it further away from the money. At the money, an option has the highest gamma because delta is most sensitive to underlying price changes .The option is extremely sensitive to stock movements in out of the money and in the money.
The graph below illustrates this. (NASDAQ Gamma).

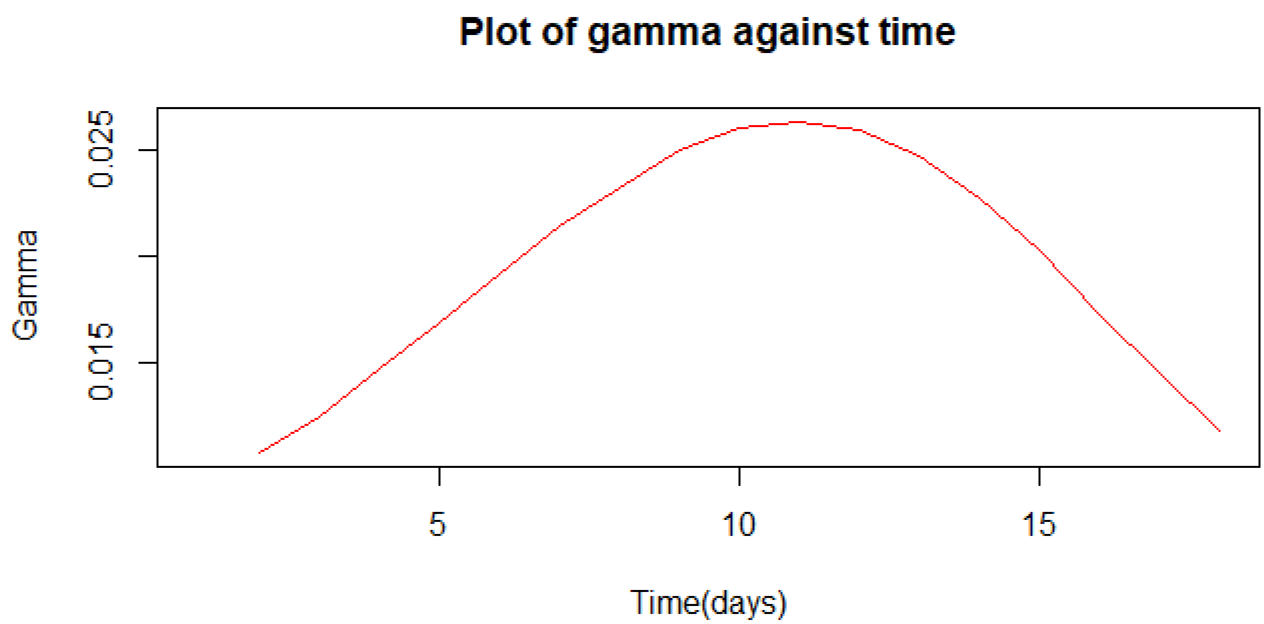

Figure 8. Gamma plot against time

Assume an investor is writing off a call options with a delta of 0.87967 and Gamma of 0.01071 (from NASDAQ data). In case there is huge movement in the price of underlying stock, the investor will be exposed to huge losses. These losses can be mitigated by gamma hedging. The investor will have to purchase given number of options such that the cumulative gamma will be zero. The cumulative gamma before purchase is $-0.01071 \times 1 = -0.01071$ since the investor has only one call option and the investor is writing off the option.
The investor decides to purchase n_2 contracts of a second call option with a delta of 0.59795 and gamma of 0.00210. (The second delta should be lower than the first delta since delta itself represents a risk). The aim of the investor is to find the value of n_2 .
$-0.01071 \times 1 + 0.00210 \times n_2 = 0$ for the gamma to be zero.

$$n_2 = \frac{n_1\Gamma_1}{\Gamma_2} = \frac{1 \times 0.01071}{0.00210} = 5.1$$

The investor will have to purchase 5.1 contacts of call option to completely gamma hedge making the cumulative gamma to be zero.

4.5 Delta and Gamma Hedging

Delta gamma hedging ensures an investor is not exposed to risks resulting from both small and large movement of prices of underlying stock.
Assume an investor is writing off 10,000 call options each with a delta of 0.87967 and gamma of 0.01071 each (from NASDAQ data). The investor is required to gamma hedge first by purchasing n_2 contracts of a call option with delta 0.69622 and gamma 0.02137. (The second delta should be lower than the first delta since delta itself represents risk). The aim of the investor is to find the value of n_2.
$n_1 = 10,000$, $\Delta_1 = 0.87967$, $\Gamma_1 = 0.01071$, $\Delta_2 = 0.69622$, $\Gamma_2 = 0.02137$
The cumulative position delta of the first call option is given by $-n_1\Delta_1$ and the cumulative position gamma is $-n_1\Gamma_1$.

$-n_1\Delta_1 = -10,000 \times 0.87967 = -8796.7$
$-n_1\Gamma_1 = -10,000 \times 0.01071 = -107.1$
The aim of the investor is to find the value of n_2 such that the cumulative gamma is zero.
$-0.01071 \times 10,000 + 0.02137 \times n_2 = 0$, for the gamma to be zero.
$n_2 = \frac{n_1\Gamma_1}{\Gamma_2} = \frac{10,000 \times 0.01071}{0.02137} = 5,011.70$
The investor will have to purchase 5,011.70 contracts of call option to completely gamma hedge making the cumulative gamma to be zero.
The cumulative delta will become,
$-10,000 \times 0.87967 + 5011.70 \times 0.69622 = -5,307.5$
In order to completely delta hedge the investor is required to purchase 5307.5 units of shares with a total delta of 5,307.24 $(5,307.5 \times 1)$ and gamma 0. The investor will now have a portfolio with the following cumulative delta and cumulative gamma;
Cumulative delta
$-10,000 \times 0.87967 + 5011.70 \times 0.69622 + 5307.5 \times 1 = 0$ completely delta hedged.
Cumulative gamma
$-10,000 \times 0.01071 + 5011.70 \times 0.02137 + 5307.5 \times 0 = 0$ completely gamma hedged.

				POSITION GREEK				
		PERCENTAGE GREEK		TRADE		CUMULATIVE		
	QUANTITY	DELTA	GAMMA	DELTA	GAMMA	DELTA	GAMMA	
CALLS	10,000	0.87967	0.01071	8796.7	107.1	-8796.7	-107.1	
NEUTRALIZE GAMMA WITH CALLS	5011.7	0.69622	0.02137	3489.2	107.1	3489.2	107.1	
						-5307.5		
NEUTRALIZE DELTA WITH SHARES	*5307.5*	1				5307.5		
					Cumulative	0	0	

Figure 9. Delta Gamma hedging

A scenario may arise where the cumulative delta is positive before delta hedging. Here an investor may delta hedge by purchasing a put option with a very small gamma since the delta of a put option is always negative. Given the following data from NASADAQ and S&P 500, we can both delta and gamma hedge as follows;

$n_1 = 10,000$, $\Delta_1 = 0.87967$, $\Gamma_1 = 0.01071$, $\Delta_2 = 0.59795$, $\Gamma_2 = 0.00210$

The cumulative position delta of the first call option is given by $-n_1\Delta_1$ and the cumulative position gamma is $-n_1\Gamma_1$.

$-n_1\Delta_1 = -10,000 \times 0.87967 = -8796.7$

$-n_1\Gamma_1 = -10,000 \times 0.01071 = -107.1$

The aim of the investor is to find the value of n_2.

$-0.01071 \times 10,000 + 0.00210 \times n_2 = 0$ for the gamma to be zero.

$n_2 = \frac{n_1\Gamma_1}{\Gamma_2} = \frac{10,000 \times 0.01071}{0.00210} = 51,000$

The investor will have to purchase 51,000 contacts of call option to completely gamma hedge making the cumulative gamma to be zero.

The cumulative delta will become,

$-10,000 \times 0.87967 + 51,000 \times 0.59795 = 21,698.75$

In order to completely delta hedge the investor is required to purchase n units of put options with a total delta of -21,698.75 and very small or negligible gamma value.

In this case we assume an investor purchases n units of a put option with a delta of -0.39909 and gamma of 0.00216 (S&P 500 INDEX);

Then, $-0.39909n = 21698.75$

$n = 54,370.56$

The investor will now have a portfolio with the following cumulative delta and cumulative gamma;

Cumulative delta

$-10,000 \times 0.87967 + 51000 \times 0.59795 + 54,370.56 \times -0.39909 = 0$,completely delta hedged.

Cumulative gamma

$-10,000 \times 0.01071 + 51,000 \times 0.02137 + 54,370 \times 0.00216 = 1,100$

The cumulative gamma is real small the fact that we are dealing with a portfolio with 115,370 units (10,000 units of call option, 51,000 units of call option and 54,370 units of put option).

4.6 Delta,Gamma and Vega Hedging

Vega hedging/Vega neutral is a method of managing risk in options trading by establishing a hedge against the implied volatility of the underlying asset. Delta hedging, gamma hedging and Vega hedging can be done together in a portfolio.
Suppose an investor is writing off 10,000 call options each with a delta of 0.64088, gamma of 0.02323 and Vega of 0.17336. (NASDAQ Greek data).
The investor need to both gamma and Vega hedge then finally delta hedge. The investor purchases n_2 contracts of a call option with a delta of 0.12185, gamma of 0.01453 and Vega of 0.09383.
The investor further purchases n_3 contracts of a put option with a delta of -0.54662, gamma of 0.02640 and Vega of 0.18382. The aim of the investor is to obtain the values of n_2 and n_3 to ensure that gamma and Vega hedging is completely done. The investor has the following;

	Quantity	Delta	Gamma	Vega
Option 1	10,000	0.6000	0.2000	0.1633
Option 2	n_2	0.1218	0.1002	0.1002
Option 3	n_3	-0.8123	0.3012	0.2010

Table 2. Delta,Gamma and Vega values.

Cumulative gamma $-0.2000 \times 10,000 + 0.1002n_2 + 0.3012n_3 = 0$ for complete gamma hedging
Cumulative Vega $-0.1633 \times 10,000 + 0.1002n_2 + 0.2010n_3 = 0$ for complete Vega hedging
This leads to two simultaneous equations to be solved;
$0.1002n_2 + 0.0.3012n_3 = 2000$
$0.1002n_2 + 0.2010n_3 = 1633$
Solving the simultaneous equations,the values of n_2 and n_3 are obtained.
$n_2 = 8950.1237$
$n_3 = 3662.6747$
Purchasing 8,950.1237 contracts of call option and 3,662.6747 contracts of put option will make both the cumulative gamma and cumulative Vega be zero.
The investor now needs to delta hedge,
Cumulative delta $-10,000 \times 0.6000 + 8,950.1237 \times 0.1218 + 3,662.6747 \times -0.8123 = -7,885.0656$
In order to completely delta hedge the investor is required to purchase 7,885.0656 contracts of shares with a total delta of 7,885.0656 $(7,885.0656 \times 1)$ and gamma 0.
The investor will now have a portfolio with the following cumulative delta, cumulative

gamma and cumulative Vega;
Cumulative delta
$-10,000 \times 0.6000 + 8,950.1237 \times 0.1218 + 3,662.6747 \times -0.8123 + 7,885.0656 \times 1 = 0$ completely delta hedged.
Cumulative gamma
$-10,000 \times 0.2000 + 8,950.1237 \times 0.1002 + 3,662.6747 \times 0.3012 = 0$ completely gamma hedged.
Cumulative Vega
$-10,000 \times 0.1633 + 8,950.1237 \times 0.1002 + 3,662.6747.0.2010 = 0$ completely Vega hedged.

						Position Greek	
		Percentage Greek			Cumulative		
	Quantity	Delta	Gamma	Vega	Delta	Gamma	Vega
Option 1	-10000	0.6000	0.2000	0.1633	-6000	-2000	-1633
Option 2	8950	0.1218	0.1002	0.1002	1090	898	898
Option 3	3663	-0.8123	0.3012	0.2010	-2975	1103	736
Neutralize delta with shares	7885	1			-7885	0	0

Figure 10. Delta,Gamma and Vega hedging

4.7 Theta Hedging

Theta refers to the rate of decline in the value of an option due to passage of time. This is called time decay. This indicates that an option losses value as it moves closer to maturity provided that other pricing components are held constant. An option profitability (intrinsic value) decreases as it gets closer to maturity. This can explained by the moneyness graphs.

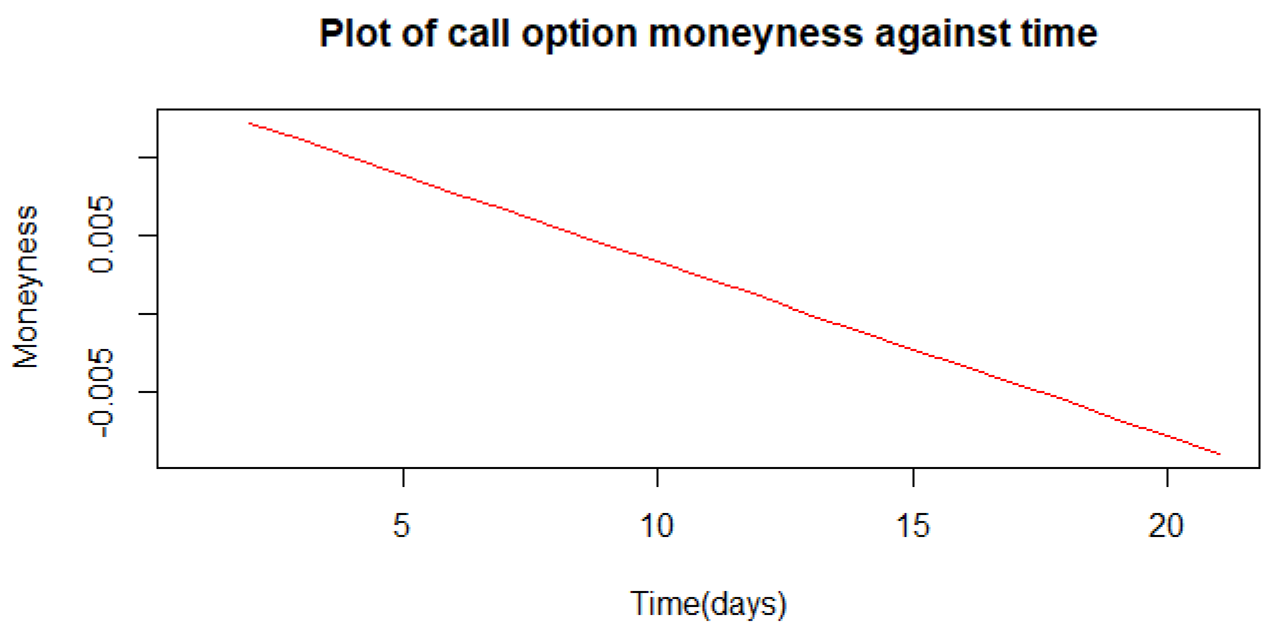

Figure 11. Call option moneyness

Theta loss increases as an option approaches expiration.
The value of theta tend to increase as expiration time decrease and then accelerates when expiration time approaches. Theta is highest when the option is at the money since the intrinsic value of the option here is zero. (Strike price is equal to the underlying price).
Time decay is unfavorable for option buyers since time works against long option holders and time decay is favorable to investor who writes options since written options becomes less valuable as expiration date approaches.

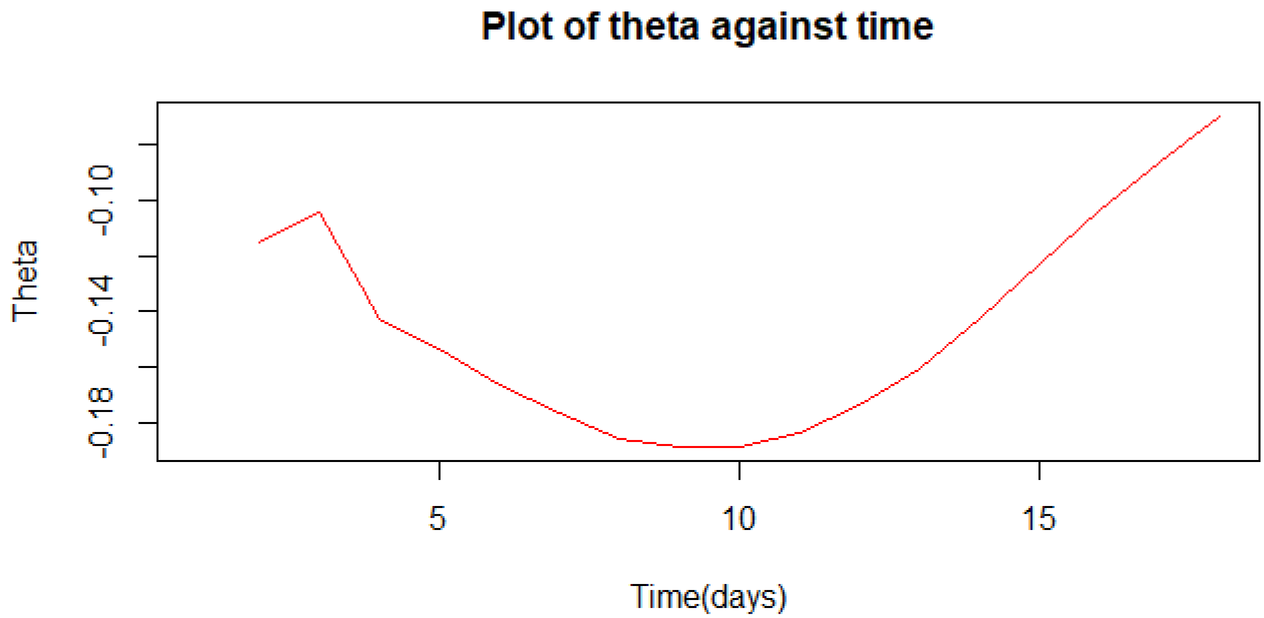

Figure 12. Plot of theta

5 CONCLUSION

Option market is always a changing market. These constant changes leads to unexpected risk exposures. Investors should therefore practice risk hedging to minimize the risks they are exposed to and this will in turn maximize returns during option trading.
It is essential and advisable for investors to fully understand Greeks and their roles in enterprise risk management.A small change in underlying stock price makes the option price to change by the delta values times the amount of change.A higher gamma indicates the accelerated option value change.Vega is important in measuring the sensitivity of option price to volatility changes.Increase in volatility increases the option price and vice versa.Rho plays an important role in determining the impact of interest rate change on option prices.A unit percentage increase in interest rate increases the option price by the rho value. Greeks are essential tools for building market portfolios and hence the risks which portfolios are exposed to are dealt with in holistic basis. Dealing with risks in holistic basis reduces the cost of risk management and one is able to deal with many risks at ago, this is enterprise risk management. Developing a framework to enable an investor introduce delta hedging, gamma hedging and Vega hedging simultaneously is an example of enterprise risk management.
Portfolio building encourages diversification.The investors are able to spread and offset the risks in different investments.
Introduction of risk management strategies in which hedging is part of will enable the option market minimize risks associated with slight or huge movements of prices of underlying stock.
Greeks derived from the Black Scholes option pricing model will help an investor understand the effects of changes in volatility, interest rate, underlying stock price and strike price on the general price of options. This will give an investor an insight on how to trade on options.

5.1 Future Research

This study has been carried out limited to a portfolio with three stocks only. I recommend a similar study to be done based on portfolios with at least four stocks. This will provide an insight on how to hedge risks involving large portfolios with many stocks.

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5.2 Appendix

```
setwd("C:\\Users\\Peter Mokua\\Desktop\\MSC PROJECT")
Moneyness<-read.csv("Moneyness.CSV", sep=",", header=TRUE)
dim(Moneyness)
attach(Moneyness)
head(Moneyness)
tail(Moneyness)
names(Moneyness)
Moneynesscall<-subset(Moneyness,select = c(2))
Moneynessput<-subset(Moneyness,select = c(9))
Moneynessput
Moneynesscall
Moneynesscall1<-ts(Moneynesscall, frequency=1)
Moneynessput1<-ts(Moneynessput, frequency=1)
plot(Moneynesscall,type = 'l',col='red',ylab='Moneyness',xlab='Time(days)')
plot(Moneynessput,type = 'l' ,col='red',ylab='Moneyness',xlab='Time(days)')
```

Figure 13. R codes

```
setwd("C:\\Users\\Peter Mokua\\Desktop\\MSC PROJECT")
NASDAQGREEKS<-read.csv("NASDAQGREEKS.CSV", sep=",", header=TRUE)
dim(NASDAQGREEKS)
attach(NASDAQGREEKS)
head(NASDAQGREEKS)
tail(NASDAQGREEKS)
names(NASDAQGREEKS)
Delta<-subset(NASDAQGREEKS,select = c(1))
Theta<-subset(NASDAQGREEKS,select = c(4))
Theta
Delta
Delta1<-ts(Delta, frequency=1)
Theta1<-ts(Theta, frequency=1)
plot(Delta,type = 'l',main='Plot of delta against time',col='red',ylab='Delta',xlab='Time')
plot(Delta1,col='red')
Gamma<-subset(NASDAQGREEKS,select = c(2))
Gamma
Gamma1<-ts(Gamma, frequency=1)
plot(Gamma1,col='red')
plot(Gamma,type = 'l',main='Plot of gamma against time',col='red',ylab='Gamma',xlab='Time(days)')
plot(Theta,type = 'l',main='Plot of theta against time',col='red',ylab='Theta',xlab='Time(days)')
```

Figure 14. R codes

Calls										Puts									
Strike	Last	Theor.	IV	Delta	Gamma	Theta	Vega	Rho	Avg IV	Strike	Last	Theor.	IV	Delta	Gamma	Theta	Vega	Rho	Avg IV
4,435.00	81.14	95.70	19.60%	0.59795	0.00210	-2.12382	3.62519	1.13215	18.11%	4,435.00	68.43	51.90	19.03%	-0.39909	0.00216	-2.21691	3.61955	-0.80638	18.11%
4,440.00	88.86	92.90	19.62%	0.58722	0.00212	-2.14269	3.64886	1.11230	18.11%	4,440.00	52.78	53.20	18.81%	-0.40919	0.00220	-2.20713	3.64248	-0.82678	18.11%
4,445.00	83.23	89.55	19.48%	0.57696	0.00214	-2.14039	3.66887	1.09363	18.11%	4,445.00	56.00	54.90	18.69%	-0.41991	0.00223	-2.20745	3.66412	-0.84857	18.11%
4,450.00	78.13	86.25	19.34%	0.56651	0.00217	-2.13662	3.68667	1.07456	18.11%	4,450.00	60.64	56.70	18.58%	-0.43085	0.00225	-2.20777	3.68337	-0.87085	18.11%
4,455.00	75.08	82.30	19.01%	0.55627	0.00221	-2.10906	3.70159	1.05619	18.11%	4,455.00	60.20	58.30	18.40%	-0.44181	0.00229	-2.19841	3.69978	-0.89307	18.11%
4,460.00	78.92	79.70	19.03%	0.54510	0.00222	-2.12122	3.71503	1.03538	18.11%	4,460.00	68.87	60.10	18.26%	-0.45309	0.00231	-2.19211	3.71368	-0.91600	18.11%
4,465.00	66.85	75.70	18.66%	0.53430	0.00227	-2.08587	3.72519	1.01594	18.11%	4,465.00	78.30	62.20	18.19%	-0.46463	0.00233	-2.19181	3.72479	-0.93959	18.11%
4,470.00	60.06	73.45	18.74%	0.52288	0.00227	-2.10212	3.73296	0.99449	18.11%	4,470.00	74.05	63.80	17.97%	-0.47621	0.00236	-2.17228	3.73278	-0.96303	18.11%
4,475.00	61.10	70.15	18.53%	0.51148	0.00229	-2.08199	3.73765	0.97357	18.11%	4,475.00	74.10	65.75	17.82%	-0.48808	0.00238	-2.16016	3.73768	-0.98718	18.11%
4,480.00	68.10	67.30	18.42%	0.49992	0.00231	-2.07237	3.73927	0.95211	18.11%	4,480.00	67.76	67.76	17.68%	-0.50013	0.00241	-2.14622	3.73927	-1.01173	18.11%
4,485.00	60.18	64.45	18.30%	0.48822	0.00232	-2.05916	3.73772	0.93038	18.11%	4,485.00	69.86	69.60	17.47%	-0.51246	0.00243	-2.12323	3.73736	-1.03675	18.11%
4,490.00	53.80	61.45	18.12%	0.47626	0.00234	-2.03756	3.73280	0.90821	18.11%	4,490.00	72.53	71.70	17.32%	-0.52494	0.00245	-2.10448	3.73179	-1.06216	18.11%
4,495.00	45.70	57.85	17.77%	0.46360	0.00238	-1.99327	3.72392	0.88493	18.11%	4,495.00	97.30	73.95	17.19%	-0.53752	0.00246	-2.08649	3.72245	-1.08786	18.11%
4,500.00	50.07	55.85	17.83%	0.45195	0.00237	-1.99550	3.71242	0.86295	18.11%	4,500.00	83.40	76.45	17.12%	-0.55005	0.00246	-2.07242	3.70943	-1.11356	18.11%
4,505.00	48.30	52.35	17.46%	0.43867	0.00241	-1.94621	3.69536	0.83840	18.11%	4,505.00	101.30	78.15	16.80%	-0.56370	0.00250	-2.02874	3.69103	-1.14110	18.11%
4,510.00	45.12	50.50	17.53%	0.42707	0.00239	-1.94568	3.67704	0.81643	18.11%	4,510.00	77.16	80.85	16.74%	-0.57639	0.00249	-2.01283	3.66991	-1.16721	18.11%
4,515.00	37.10	47.90	17.38%	0.41440	0.00239	-1.91710	3.65335	0.79270	18.11%	4,515.00	104.30	83.25	16.58%	-0.58969	0.00250	-1.98301	3.64364	-1.19438	18.11%
4,520.00	34.30	45.00	17.12%	0.40096	0.00241	-1.87437	3.62399	0.76758	18.11%	4,520.00	111.36	85.75	16.43%	-0.60308	0.00250	-1.95211	3.61290	-1.22176	18.11%
4,525.00	39.72	43.00	17.09%	0.38891	0.00239	-1.85734	3.59393	0.74479	18.11%	4,525.00	90.00	88.10	16.22%	-0.61705	0.00251	-1.91164	3.57615	-1.25023	18.11%
4,530.00	37.10	40.70	16.97%	0.37615	0.00238	-1.82621	3.55824	0.72074	18.11%	4,530.00	156.28	91.20	16.19%	-0.62959	0.00249	-1.89231	3.53909	-1.27621	18.11%

Figure 15. S&P Greeks

Calls							Puts						
Strike	Moneyness	Bid	Midpoint	Ask	Last	IV	Strike	Moneyness	Bid	Midpoint	Ask	Last	IV
4,425.00	1.22%	99.50	99.75	100.00	81.62	0.20	4,425.00	-1.22%	49.70	49.90	50.10	49.05	0.19
4,430.00	1.11%	96.10	96.30	96.50	85.03	0.20	4,430.00	-1.11%	51.20	51.40	51.60	55.69	0.19
4,435.00	1.00%	91.90	92.10	92.30	81.14	0.19	4,435.00	-1.00%	53.40	53.60	53.80	68.43	0.19
4,440.00	0.89%	89.00	89.30	89.60	92.84	0.19	4,440.00	-0.89%	54.40	54.60	54.80	52.78	0.19
4,445.00	0.77%	85.60	85.95	86.30	83.23	0.19	4,445.00	-0.77%	56.10	56.30	56.50	56.30	0.19
4,450.00	0.66%	82.20	83.30	84.40	84.98	0.19	4,450.00	-0.66%	57.80	58.05	58.30	58.32	0.18
4,455.00	0.55%	78.80	79.00	79.20	75.08	0.19	4,455.00	-0.55%	59.60	59.80	60.00	60.20	0.18
4,460.00	0.44%	76.10	76.35	76.60	79.65	0.19	4,460.00	-0.44%	61.40	61.60	61.80	60.75	0.18
4,465.00	0.33%	73.20	73.40	73.60	66.85	0.19	4,465.00	-0.33%	63.60	63.85	64.10	78.30	0.18
4,470.00	0.22%	70.10	70.30	70.50	60.06	0.19	4,470.00	-0.22%	65.20	65.35	65.50	74.05	0.18
4,475.00	0.11%	66.90	67.15	67.40	70.70	0.18	4,475.00	-0.11%	64.50	66.35	68.20	74.10	0.17
4,480.00	-0.01%	64.10	64.35	64.60	67.70	0.18	4,480.00	0.01%	69.20	69.40	69.60	66.16	0.18
4,485.00	-0.12%	61.20	61.40	61.60	60.18	0.18	4,485.00	0.12%	71.30	71.50	71.70	69.86	0.17
4,490.00	-0.23%	58.40	58.60	58.80	53.80	0.18	4,490.00	0.23%	73.50	73.70	73.90	73.74	0.17
4,495.00	-0.34%	55.40	55.60	55.80	45.70	0.18	4,495.00	0.34%	75.70	75.85	76.00	97.30	0.17
4,500.00	-0.45%	52.80	53.00	53.20	55.91	0.18	4,500.00	0.45%	78.60	78.80	79.00	83.40	0.17
4,505.00	-0.56%	50.10	50.35	50.60	48.30	0.17	4,505.00	0.56%	80.30	80.50	80.70	101.30	0.17
4,510.00	-0.68%	47.60	47.85	48.10	50.51	0.17	4,510.00	0.68%	82.70	82.95	83.20	77.16	0.17
4,515.00	-0.79%	44.20	45.15	46.10	37.10	0.17	4,515.00	0.79%	85.20	85.45	85.70	104.30	0.16
4,520.00	-0.90%	42.40	42.60	42.80	44.49	0.17	4,520.00	0.90%	87.80	87.95	88.10	111.36	0.16

Figure 16. Moneyness data

DELTA	GAMMA	RHO	THETA	VEGA	VOLATILITY	STRIKE	DELTA	GAMMA	RHO	THETA	VEGA	VOLATILITY
0.87967	0.01071	0.07657	-0.11545	0.09282	0.40249	200.00	-0.12068	0.01073	-0.01231	-0.11581	0.09318	0.40351
0.87570	0.01241	0.07733	-0.10439	0.09498	0.35541	202.50	-0.14486	0.01256	-0.01479	-0.12717	0.10569	0.39065
0.82244	0.01466	0.07292	-0.14227	0.12056	0.38190	205.00	-0.17559	0.01467	-0.01794	-0.14008	0.11985	0.37952
0.78825	0.01687	0.07051	-0.15333	0.13427	0.36957	207.50	-0.21128	0.01693	-0.02161	-0.15209	0.13418	0.36810
0.74491	0.01912	0.06715	-0.16643	0.14892	0.36170	210.00	-0.25327	0.01926	-0.02593	-0.16363	0.14844	0.35806
0.69622	0.02137	0.06322	-0.17650	0.16214	0.35236	212.50	-0.30238	0.02149	-0.03102	-0.17433	0.16188	0.34988
0.64088	0.02323	0.05857	-0.18563	0.17336	0.34665	215.00	-0.35676	0.02321	-0.03670	-0.18431	0.17308	0.34606
0.58163	0.02502	0.0535	-0.18816	0.18114	0.33630	217.50	-0.41808	0.02501	-0.04308	-0.18745	0.18114	0.33640
0.51875	0.02602	0.04799	-0.18839	0.18487	0.32996	220.00	-0.48225	0.02605	-0.04984	-0.18738	0.18486	0.32966
0.45186	0.02636	0.042	-0.18364	0.18371	0.32370	222.50	-0.54662	0.02640	-0.05668	-0.18260	0.18382	0.32323
0.38513	0.02598	0.03597	-0.17365	0.17734	0.31710	225.00	-0.61479	0.02602	-0.06397	-0.17238	0.17731	0.31656
0.32173	0.02471	0.030	-0.16048	0.16630	0.31255	227.50	-0.67509	0.02458	-0.07060	-0.16152	0.16695	0.31529
0.25970	0.02274	0.02443	-0.14264	0.15038	0.30723	230.00	-0.73669	0.02267	-0.07740	-0.14391	0.15140	0.31015
0.20507	0.02025	0.01936	-0.12310	0.13185	0.30244	232.50	-0.78644	0.02017	-0.08314	-0.12828	0.13496	0.31057
0.15778	0.01724	0.01492	-0.10407	0.11179	0.30159	235.00	-0.84207	0.01729	-0.08944	-0.10241	0.11175	0.30030
0.12185	0.01453	0.01156	-0.08684	0.09383	0.29985	237.50	-0.87309	0.01456	-0.09348	-0.09016	0.09633	0.30739
0.09070	0.01176	0.00862	-0.06999	0.07578	0.29924	240.00	-0.92605	0.01080	-0.09957	-0.05440	0.06473	0.27874

Figure 17. Nasdaq Greeks