

|| MATHEMATICAL MODELLING AND DESIGN OF A THREE-DIMENSIONAL
GEODETIC NETWORK FOR LOCALISED EARTH DEFORMATION //

by
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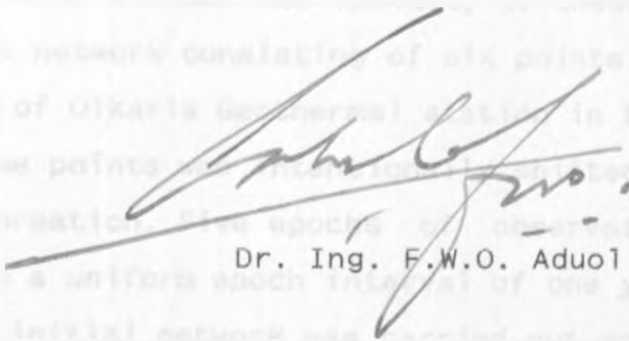
DECLARATIONS

This thesis is my original work and has not been presented for a degree in any other university.

S.M. Musyoka



This thesis has been submitted for examination with my approval as University supervisor.



Dr. Ing. F.W.O. Aduol

ABSTRACT

Mathematical models, within the framework of integrated geodetic networks for localised three dimensional geodetic monitoring networks are presented. The network design aspects have also been considered.

The development of these mathematical models was based on the kinematic estimation model of geodetic network adjustment using the integrated geodetic approach. The network design aspect considered was the weight problem for each of the various observables used. These observables were astronomic latitude, astronomic longitude, astronomic azimuth, vertical angles, horizontal directions, spatial distances, gravity differences and gravity potential differences. The basic parameters computed were the network coordinates, the point velocities and accelerations of the unstable points.

In order to test the validity of these mathematical models, a test network consisting of six points, derived from an old map of Oikaria Geothermal station in Kenya, was used. One of these points was intentionally shifted so as to cause network deformation. Five epochs of observations were considered; with a uniform epoch interval of one year. The adjustment of the initial network was carried out on the basis of a free network, whereas the rest were computed as fixed. The numerical study was entirely carried out by computer simulation.

Through the models adopted it was possible to estimate both network coordinates and point velocities of the object network with at least two epochs of observations while accelerations required at least three observation epochs. These requirements were in line with the theoretical aspects of the models. The estimated velocities were consistent with the shifts that were introduced into the network.

The results also showed that a small proportion of astronomic azimuth observations were needed while gravity differences were not required.

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INTRODUCTION.

It is a well known fact that certain types of terrain are not at rest, but are slowly moving, thereby causing positions of points located on them to change. Some of the factors causing these movements are crustal deformations, volcanic activity, variation of ground water level, mining activities, and construction of large engineering structures.

Generally, the deformation may be classified into two types: crustal and localised earth deformations. Crustal earth deformations take relatively long periods of time to show any appreciable ground shifts, whereas localised earth deformations tend to be of relatively short frequency so that they can be noticed in much shorter periods of time. Factors contributing to localised earth deformations include engineering construction works and mining. As a safety measure, and also as a guide for future planning in a given area suspected to be unstable, the deformation of the ground need to be monitored as to seek to detect any deformations thereof.

There are various ways of monitoring earth deformations. These include geodetic techniques as well as photogrammetric methods [e.g. Shortis, 1983]. Of these methods, geodetic methods of monitoring earth deformations have found wide application because of the advantage in that they allow monitoring of relative movements to very high accuracies [Ashkenazi, 1980]. These methods have however began to be applied more extensively in the last few years as reported in [Cooper 1987]. In this study it is aimed to make a further

contribution to the subject of monitoring localized deformations using geodetic methods.

1.1 The statement of the problem.

In monitoring of any deformation, a network of precisely coordinated points would be set up on the structure suspected to be unstable. Periodic coordinations of these points are then made and the results compared to find out if any major discrepancies between the new results and the old ones do exist.

It is rarely possible for the surveyor to observe directly the set of the final required elements, i.e. the network coordinates. Instead, quantities relating to the network points such as angles and distances between the stations are normally measured and then related to the desired quantities (i.e. the coordinates) through mathematical relationships. Such geodetic monitoring network of points must of necessity be precise, since the suspected ground shifts are normally of very small magnitudes.

The approach adopted by many surveyors to monitor earth deformations has been to establish separate horizontal and vertical networks in the conventional spirit of one and two dimensional geodetic networks [e.g. Forster 1990, Richardson 1981, Ashkar et al 1980, Black et al 1980]. In such case the deformation monitoring system is separated into horizontal deformation (two dimensional network) and vertical deformation (from one dimensional network). Whereas this arrangement has proved quite successful for most usual surveying tasks, such as in topographic mapping, it has certain weaknesses associated with it.

The neglect of the influence of certain systematic effects on the parameters such as deflection of the vertical, particularly in mountainous regions [Grafarend, 1988], refraction influences and effect of variation of the gravity field in general may significantly distort the network. Also the deterioration of azimuth within the network should not be ignored.

To overcome these difficulties, a three-dimensional adjustment should be adopted to provide a system of precisely coordinated points in three-dimensional space, in line with the observational model. For a three dimensional adjustment, observations need not be directly transformed into the reference ellipsoid, and the azimuths are controlled implicitly.

In order to facilitate the computation of a rigorous three-dimensional network, the quantities that would ordinarily be observed comprise horizontal directions, angles, spatial distances, vertical angles, astronomic latitudes, astronomic longitudes, astronomic azimuths, gravity potential differences and gravity intensity differences. All these observations are incorporated into a single adjustment process within the physical gravity field in which they will have been measured. For each station the three coordinates, either in cartesian coordinate system or the curvilinear system, are obtained together with the deflection of the vertical parameters. Other auxiliary data such as the refraction coefficients are also estimated.

Following the above discussion, this study will principally aim at setting up suitable mathematical models that would be needed for the establishment of a three-dimensional geodetic network for the monitoring of localised earth deformations.

Further, a simulated network will be designed for the purposes of demonstrating the pertinent mathematical models.

1.2 State of the Art.

Deformation measurements and the analysis of movements are an essential task in the field of engineering. Considerable work done so far in deformation surveys has been reported in the 'Proceedings of the Symposia on Deformation' (Commission 6-Engineering Surveying of the International Federation of Surveyors, FIG). Since the establishment of this Commission by FIG, the subject has received considerable contributions from various authors. Some of the publications which addressed the problem of detection of deformation were by, amongst others, van Mierlo (1975), (1975a), Brunner(1979), Niemeier(1981), Koch et al (1981), Chen et al (1983), van Mierlo(1981), Pelzer(1977), Ashkenazi et al (1980), Chen et al (1990). A number of geodetic monitoring networks have been established on the basis of principles discussed in these papers.

Kelly (1983) reports on the monitoring surveys at Loy Yang (Australia) while Murnane(1983) details the aspects of network design and analysis at the Winneke reservoir (Australia) monitoring surveys. Crosilla et al (1986) report on a study carried out to monitor current crustal deformations in a local area (Friuli) in Italy. Relative gravimeter observations for monitoring vertical motions along the Boccono Fault in Venezuela have been described by Drew (1989). Recently, Biacs et al (1990) prepared a PC-based program system for adjustment and deformation analysis of precise engineering and monitoring networks, which they successfully applied on the Paddle River surveys and the Olympic Oval monitoring network in Canada.

In deformation monitoring, it is generally assumed that measurements can be made very quickly with respect to the speed of deformation and that these measurements are made at an epoch (instant of time). When this assumption does not hold, then time factor must also appear in the model as a fourth parameter. Papo and Perelmuter (1984) suggested inclusion of velocities and accelerations of points in the functional model. Aduol and Schaffrin (1990) extended on the idea of inclusion of velocities and accelerations to the kinematic model of deformation monitoring.

To this end, deformation is defined not only to mean change of shape but also to include scale changes, rotations and shifts. In using geodetic networks to monitor deformation, the deformation parameters are derived from changes of coordinates that might have taken place. Unfortunately, the coordinates are datum dependent and the choice of fixed reference datum may be hard to obtain. If one is able to identify some points as fixed and retain their coordinates at every epoch of observation then this is called an absolute monitoring network. If on the other hand, all points in the network are likely to undergo deformation, then this is a relative monitoring network. Since no points in the monitoring network can be said to be stable unless measurements confirm it, then the adjustment must be carried out on the basis of free network [Chen et al ,1990].

The free network adjustment has been discussed in various publications. These include Grafarend and Schaffrin (1974), Perelmuter (1979), and Mittermayer (1972). One advantage of free network analysis is that no point is kept fixed, and the datum is defined through the approximate coordinates of the proposed network.

The idea of computing a geodetic network in three dimensions may be attributed to Bruns, who suggested the computation of a triangulation net in space in 1878 [Heiskanen & Moritz, 1967]. More studies followed later and it was shown that most of the problems encountered in adjusting separated networks, such as reduction of observations onto the reference spheroid could be avoided. Aduol(1981) in his study on optimal design of a three dimensional geodetic network based on simulated data observed that among the commonly observed values of vertical angles, distances, horizontal directions respectively angles, must also be included one each of astronomic observations of azimuth, latitude and longitude. Also with inclusion of gravimetric data, the number of astronomic positions could be reduced. Observation equations for computation in three dimensional in integrated networks are presented in [Aduol 1989].

1.3 Organisation of the report.

In Chapter Two, the theoretical aspects of the parameter estimation models are discussed. Presented in Chapter Three are the necessary observation equations that were used in the adjustment process. The various coordinate systems that are required in the study together with their transformations are also discussed.

The network simulation and results of simulation are presented in Chapter Four. In Chapter Five are presented the computations and the results of these computations. The results are discussed in Chapter Six and major conclusions made in Chapter Seven. The notation used here is defined in the text of the report.

ESTIMATION MODELS FOR LOCAL DEFORMATION MONITORING.2.1 General Models.

For the estimation of the unknown parameters, the linear least squares model is here adopted. The general basis for this estimation will be the simple linear model commonly referred to as the simple Gauss-Markov model. In the following sections we shall therefore consider the estimation of the parameters under the simple linear model as the basic model.

2.1.1 The Simple Gauss-Markov Model.

If A be an $n \times u$ matrix of known coefficients and of full column rank, x a $u \times 1$ vector of unknown parameters to be estimated, y an $n \times 1$ vector of observed values, then the simple Gauss-Markov model may be represented in the form

$$y = Ax + \varepsilon_y ; D(y) = \sigma_o^2 W_{yy}^{-1} = D(\varepsilon_y) = 0, \quad (2.1)$$

for ε_y being an $n \times 1$ vector of observational errors, W_{yy} is a known $n \times n$ positive definite weight matrix of the observed values in vector y and σ_o^2 is a variance component (also called variance of unit weight) of the observations. The least squares estimate of x , \hat{x} can be shown to be

$$\hat{x} = (A'WA)^{-1} A'Wy, \quad (2.2)$$

with

$$D(\hat{x}) = \sigma_o^2 (A'WA)^{-1} =: \sum_{xx} \quad (2.3)$$

and also

$$E[\hat{x}] = x \quad (2.4)$$

From (2.4), it is noted that \hat{x} is an unbiased estimate of x . In fact it can be shown that \hat{x} is the best linear unbiased estimate of x .

The simple linear model under the Gauss-Markov model requires that the normal equation matrix has full rank. In case of rank deficiency, which is usually the case with survey networks, it has to be overcome in some way.

2.1.2 The Simple Gauss-Markov Model with exact restrictions

One way to overcome the rank defect in the Gauss-Markov model defined in equation (2.1) is to set up some exact restrictions in the form

$$r = R x , \quad (2.5)$$

where R is a $c \times m$ design restriction matrix while r is a $c \times 1$ vector of restrictions.

Grouping equations (2.1) and (2.5) in matrix form, one obtains

$$\begin{bmatrix} y \\ r \end{bmatrix} = \begin{bmatrix} A \\ R \end{bmatrix} x + \begin{bmatrix} \varepsilon_y \\ 0 \end{bmatrix} , \quad (2.6)$$

The next step is to minimise the quadratic norm, $\varepsilon_y' W \varepsilon_y$ under the restriction (2.5). The Lagrange function L is formulated thus

$$L = \varepsilon_y' W \varepsilon_y - 2\lambda(Rx - r) \quad (2.7)$$

where λ is a $c \times 1$ vector of Lagrange multipliers. The system of the normal equations matrix then takes the form:

The rank of A is $q < m$ while that of R is $c = m$; therefore to make N regular the restrictions are incorporated as

$$\bar{N} = N + R'R \quad (2.9)$$

The inverse of the normal equation matrix is obtained from [e.g. Schaffrin, 1984] or in [Aduol 1989] as

$$\begin{bmatrix} N & R' \\ R & 0 \end{bmatrix}^{-1} = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} \quad (2.10)$$

with $F_{11} = \bar{N}^{-1} - (\bar{N}^{-1}R')(RN^{-1}R')^{-1}(RN^{-1})$ (2.11a)

$$F_{12} = (\bar{N}^{-1}R')(RN^{-1}R')^{-1} \quad (2.11b)$$

$$F_{21} = (RN^{-1}R')^{-1}RN^{-1} = F_{12}' \quad (2.11c)$$

$$F_{22} = I - (RN^{-1}R')^{-1} \quad (2.11d)$$

and

$$x = F_{11}A'Wy + F_{12}r \quad (2.12a)$$

$$D(x) = \sum_{xx} = \sigma_0^2 F_{11}A'WAF_{11}' \quad (2.12b)$$

In a survey network these restrictions may take the form of fixed control points used to coordinate new points. In this case the quality of the network deteriorates further away as the new points are separated from the control points as shown in Figures 2.1a and 2.1b [Niemeier, 1985], for a two dimensional network. From these two figures, it is noted that the distribution of the control points must be chosen

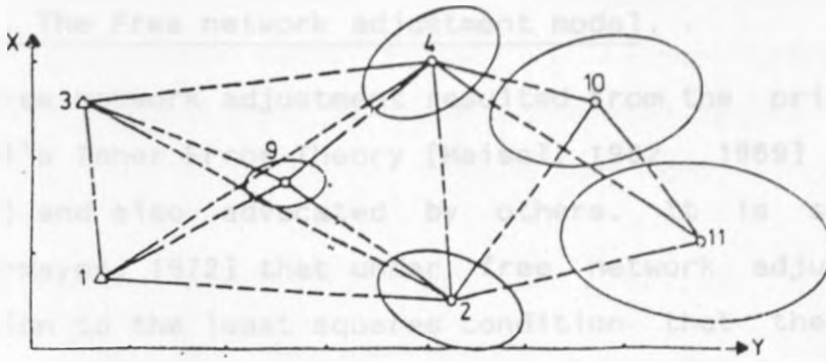


Fig. 2.1a Minimal constraint adjustment. The fixed points are 1 and 3 (after [Niemeier, 1985])

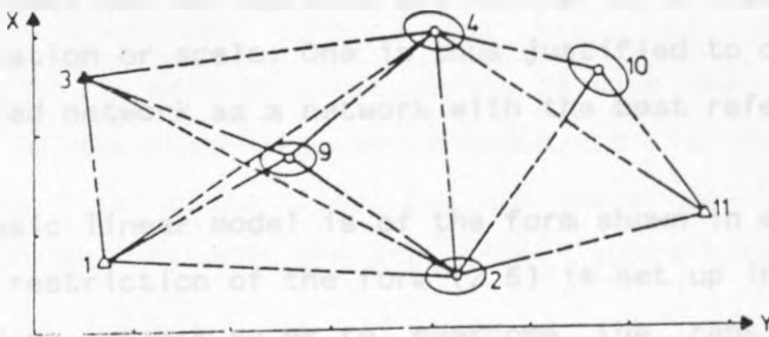


Fig 2.1b Overconstrained adjustment. The fixed points are 1, 3 and 11 (after [Niemeier, 1985])

carefully. The control network should also be of high precision.

2.1.3. The Free network adjustment model.

The free network adjustment resulted from the principles of Meissl's Inner Error Theory [Meissl, 1962, 1969] in Meissl (1932) and also advocated by others. It is shown [e.g. Mittermayer, 1972] that under free network adjustment, in addition to the least squares condition that the quadratic norm $x'Wx$ be a minimum, that one of the conditions

$$x'x = \text{minimum} \quad \text{or} \quad (2.13)$$

$$\text{trace}(D(x)) = \text{minimum}, \quad (2.14)$$

be imposed on the network. From equations (2.13) and (2.14) we have that neither the length of the correction vector nor the sum of the variances resulting from a free network adjustment can be improved any further by a change of origin, orientation or scale. One is thus justified to define a free adjusted network as a network with the best reference datum.

The basic linear model is of the form shown in equation (2.1) and a restriction of the form (2.5) is set up in the solution of a free network so as to overcome the rank defect. The choice of the restriction design matrix is made such that R will be a matrix whose columns are made up of the normalized eigenvectors of those eigenvalues in the normal equations matrix which have values equal to zero due to rank defect in N [e.g. Aduol, 1990]. Usually, R is denoted by G and has properties that

$$NG = 0 \quad (2.15)$$

$$G'G = 0 \quad (2.16)$$

The constraint equation for adjustment of a free network is

$$G'x = 0 \quad (2.17)$$

Various forms of the G matrix for different observation and network types have been listed by Illner (1985). For a three-dimensional case in which horizontal angles have been observed (i.e defects = 7), G is of the form

$$G' = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & \dots & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & \dots & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & \dots & 0 & 0 & 1 \\ 0 & Z_l & -Y_l & 0 & Z_l & -Y_l & \dots & 0 & Z_l & -Y_l \\ -Z_l & 0 & X_l & -Z_l & 0 & X_l & \dots & -Z_l & 0 & X_l \\ Y_l & -X_l & 0 & Y_l & -X_l & 0 & \dots & Y_l & -X_l & 0 \\ X_l & Y_l & Z_l & X_l & Y_l & Z_l & \dots & X_l & Y_l & Z_l \end{bmatrix}$$

$$(2.18)$$

If distances be observed, the seventh row is deleted since observed distances control the scale of the network. Similarly, if azimuth observations be made then the fourth, fifth and the sixth rows are deleted. It is here mentioned that the final coordinates, but not the shape of the adjusted network, depends upon the provisional coordinates.

Noting that the normal equation matrix N is singular, the problem of adjusting for free networks becomes principally one of overcoming the rank defect in N. Several approaches to the solution of N have been suggested by, among others, Grafarend and Schaffrin (1974), Perelmutter, (1979); Cooper, (1980); Brunner, (1979); Chen et al (1990) etc. The main approaches are through the use of generalized inverse matrices and the similarity transformations.

2.2 The Integrated Model

The integrated adjustment models involve both geometrical

observations and gravity field data. Reference to integrated models is made to Grafarend and Richter (1978), Grafarend (1979), Aduol (1989) and others. In most usual surveying practices, one uses the geometrical observations of distances, horizontal directions or angles and vertical angles to solve for the network without regard to the direction of the plumbline at each network station. The integrated models incorporate gravity data into the adjustment to allow computation of the directions of the plumbline at the network points together with other parameters.

The basic equation in an integrated adjustment model may be represented in the form

$$\tilde{Y} = f(X_0) + f'(X)\Delta X + \delta f(X) + \epsilon_Y \quad (2.19)$$

where \tilde{Y} is a vector of observables, $f(X)$ is a vector representing the values computed using the model function and ΔX is a vector of the corresponding parameters. The vector $\delta f(X)$ is the vector of disturbances, such as the deflection of the vertical parameters. The vector ϵ_Y is the vector of random errors in the observable vector Y .

2.3 Estimation models for Deformation Monitoring.

2.3.1 General Models

One approach to the analysis of repeatedly measured networks to detect movement is to estimate individual coordinate vectors for each epoch. The functional model of this set is [Niemeier, 1981]

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_k \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & \dots & A_{1k} \\ A_{21} & A_{22} & & A_{2k} \\ \vdots & & & \vdots \\ A_{k1} & A_{k2} & & A_{kk} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_k \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_k \end{bmatrix} \quad (2.20)$$

with y_i being an $(n_i, 1)$ vector of observations, ε_i an $(n_i, 1)$ vector of residuals, A_{ij} an (n_i, u_i) coefficient of configuration matrix. x_i is a $(u_i, 1)$ vector of estimates for the parameters of the network. e.g. coordinate points, n_i is the number of observations in the i -th epoch, u_i is the number of parameters in the i -th epoch and k , the number of epochs. The stochastic model is given by

$$K_{yy} = \sigma_o^2 Q_{yy} = \sigma_o^2 \begin{bmatrix} Q_{11} & Q_{12} & \dots & Q_{1k} \\ Q_{21} & Q_{22} & \dots & Q_{2k} \\ \vdots & & & \vdots \\ Q_{k1} & Q_{k2} & \dots & Q_{kk} \end{bmatrix} \quad (2.21)$$

with K_{yy} as the variance-covariance matrix for the observation of all epochs; σ_o^2 is the variance of unit weight, valid for all epochs; Q_{yy} is a cofactor matrix of the observations of all epochs; Q_{ij} is a cofactor matrix corresponding to the observation vectors y_i and y_j .

The main solution for each epoch is given by [e.g. Brunner, 1979]

$$\hat{x} = (A'WA)^+ A'Wy \quad (2.22a)$$

$$Q_{xi}^+ = (A'WA) \quad (2.22b)$$

where $+$ indicates the Moore-Penrose inverse [Bjerhammar, 1973]. This solution has minimum norm and a cofactor matrix of minimum trace; in fact it is a free

network solution as described in the previous section.

To detect whether any motion has occurred between the epochs, the global testing is carried out by computing the variance of unit weight.

The estimation for the variance of unit weight σ_{ok}^2 , which is a global quantity for the accuracy of the epoch is computed as

$$\sigma_{ok}^2 = \frac{e' W e}{n-u} \quad (2.23)$$

The hypothesis,

$$H_0 : E [\sigma_{o1}^2] = E [\sigma_{o2}^2] = \dots \dots \dots E [\sigma_{ok}^2] \quad (2.24)$$

$$H_A : E [\sigma_{o1}^2] \neq E [\sigma_{o2}^2] \neq \dots \dots \dots E [\sigma_{ok}^2] \quad (2.25)$$

may be set up. If the hypothesis H_0 is accepted, then the conclusion may be that no movements of the station coordinates have occurred.

A better estimable quantity for the precision of the epochs being compared is obtained if one sums up the single quantities of each epoch [Grundig et al, 1985, Niemeier, 1981]

$$\sigma_o^2 = \frac{e' W e_l + e' W e_j}{r_l + r_j} \quad (2.26)$$

with $r_1 + r_2$ being the degrees of freedom.

This computed value σ_o^2 corresponds to a common adjustment of the two epochs in which the variables of one of the epochs are not considered identical to those of the other epoch.

A deformation vector d , for any pair of observation epochs consisting of coordinate differences, can be set up as

$$d = x_i - x_j \quad (2.27)$$

The quadratic form $d'Wd$, and the quantity Ω^2 can be computed for the purpose of testing the validity of the assumed conditions [Grundig et al, 1985].

$$\Omega^2 = \frac{d'W_{dd}d}{h}, \quad W_{dd} = (Q_i + Q_j)^+ \quad (2.28)$$

with $h = m - r_d$, with m being the number of conditions r_d , is the rank deficiency of the variance-covariance matrix. The quantities Ω^2 and $\hat{\sigma}_o^2$ are both statistically independent [Grundig et al 1985] and can therefore be tested against each other. The test statistic given in Grundig (1985) is

$$F^* = \frac{\Omega^2}{\hat{\sigma}_o^2} \quad (2.29)$$

If the quantity F^* fits the Fischer distribution, i.e.

$$P(F^* < F_{1-\alpha, f_1, f_2} | H_o) = 1-\alpha \quad (2.30)$$

with $1-\alpha$ = level of significance
 $f_1 = h$ and $f_2 = r_i + r_j$ are degrees of freedom,
then the null hypothesis is accepted.

2.3.2 The Simple Kinematic Model

Reference to the simple kinematic model is made to [Aduol and Schaffrin 1990]. The basic concepts of this model are discussed here below.

Let us take G to be the function relating the geometric and the physical parameters (x_1, x_2, \dots, x_k) so that the relationship is represented as $G(x_1, x_2, \dots, x_k)$. For a more general case, let the function G_i at epoch i be nonlinear so that linearising it about a point, one writes

$$G(x_1, x_2, \dots, x_n) = G_{0i}(x_1, x_2, \dots, x_n) + \Delta G(x_1, x_2, \dots, x_n) \quad (2.31)$$

simplified as

$$G_i = G_{0i} + \Delta G_i \quad (2.32)$$

with

$$\Delta G = \frac{\partial G}{\partial x_1} \Delta x_1 + \frac{\partial G}{\partial x_2} \Delta x_2 + \dots + \frac{\partial G}{\partial x_n} \Delta x_n \quad (2.33)$$

and

$$x_i = x_{0i} + \Delta x_i$$

x_{0i} is the approximate value for x_i and Δx_i is the small correction due to nonlinearity of G .

Introducing a time factor in equation (2.31), and considering the initial epoch of observation to have been made at a time $t = t_0$, i.e. after $\Delta t = t - t_0$ has elapsed, then the relationship G at the i -th epoch, may be obtained from the function G as

$$G_i = G_0 + \frac{\partial G}{\partial t} \Delta t + \frac{1}{2} \frac{\partial^2 G}{\partial t^2} \Delta t^2, \quad (2.34)$$

after considering up to second order terms. To estimate the point velocities and accelerations, one sets the partial derivative of the displacement with respect to time and manipulates the result as follows:

$$\frac{\partial G}{\partial t} = \frac{\partial G}{\partial x_1} \cdot \frac{\partial x_1}{\partial t} + \frac{\partial G}{\partial x_2} \cdot \frac{\partial x_2}{\partial t} + \dots + \frac{\partial G}{\partial x_n} \cdot \frac{\partial x_n}{\partial t}$$

and

$$\begin{aligned} \frac{d'G}{dt} &= \frac{\partial}{\partial t} \left[\frac{\partial G}{\partial t} \right] \\ &= \frac{\partial}{\partial t} \left[\frac{\partial G}{\partial x_1} \cdot \frac{\partial x_1}{\partial t} + \frac{\partial G}{\partial x_2} \cdot \frac{\partial x_2}{\partial t} + \dots + \frac{\partial G}{\partial x_k} \cdot \frac{\partial x_k}{\partial t} \right] \end{aligned} \quad (2.35)$$

where $\frac{\partial x_i}{\partial t}$ represents the coordinate velocity and

$$\frac{\partial}{\partial t} \left[\frac{\partial x_i}{\partial t} \right] \text{ is a coordinate acceleration.}$$

Now, taking into consideration that G_i is nonlinear (for a general case) then (2.34) would be rewritten as

$$\begin{aligned} G_i &= G'_i + \frac{\partial G}{\partial x_1} \Delta x_1 + \frac{\partial G}{\partial x_2} \Delta x_2 + \dots + \frac{\partial G}{\partial x_k} \Delta x_k + \\ &+ \left[\frac{\partial G}{\partial x_1} \cdot \frac{\partial x_1}{\partial t} + \frac{\partial G}{\partial x_2} \cdot \frac{\partial x_2}{\partial t} + \dots + \frac{\partial G}{\partial x_k} \cdot \frac{\partial x_k}{\partial t} \right] \Delta t \\ &+ \frac{1}{2} \left[\frac{\partial G}{\partial x_1} \cdot \frac{\partial^2 x_1}{\partial t^2} + \frac{\partial G}{\partial x_2} \cdot \frac{\partial^2 x_2}{\partial t^2} + \dots + \frac{\partial G}{\partial x_k} \cdot \frac{\partial^2 x_k}{\partial t^2} \right] \Delta t^2 \end{aligned} \quad (2.36)$$

One then considers the vector of the observation G_i^e , so that $G_i^e = E[G_i]$. Associated with this vector is the observational error ϵ_i , so that $E[\epsilon_i] = 0$. These vectors can be represented

$$G_i^e = G_i + \epsilon \quad (2.37)$$

using the notation

$$\dot{x} = \frac{dx}{dt}, \quad \ddot{x} = \frac{d}{dt} \left(\frac{dx}{dt} \right)$$

to represent the velocity and acceleration respectively
Equations (2.36) and (2.37) may be related as,

$$\begin{aligned} G_i^o - G_{oi} &= \frac{\partial G}{\partial x_1} \Delta x_1 + \frac{\partial G}{\partial x_2} \Delta x_2 + \dots + \frac{\partial G}{\partial x_k} \Delta x_k + \\ &+ \frac{\partial G}{\partial \dot{x}_1} \Delta t \dot{x}_1 + \frac{\partial G}{\partial \dot{x}_2} \Delta t \dot{x}_2 + \dots + \frac{\partial G}{\partial \dot{x}_k} \Delta t \dot{x}_k + \\ &+ \frac{1}{2} \frac{\partial G}{\partial x_1} \Delta t^2 \ddot{x}_1 + \frac{1}{2} \frac{\partial G}{\partial x_2} \Delta t^2 \ddot{x}_2 + \dots + \frac{1}{2} \frac{\partial G}{\partial x_k} \Delta t^2 \ddot{x}_k + \\ &+ \varepsilon_i \end{aligned} \tag{2.39}$$

Equation (2.38) is the general linearised observation equation for the kinematic estimation of the parameters.

In the kinematic estimation model, one is able to estimate the network coordinates during the initial epoch. During the next epoch of observation (i.e the first epoch) this model can estimate both the network coordinates and the point velocities. A second observation epoch would enable estimation of network coordinates, point velocities and point accelerations. More observation epochs would strengthen the estimation of the above parameters. A diagrammatic representation of this hierarchical estimation of parameters is shown in Figure 2.2. From the theory of this estimation model, the network coordinates are referred to the initial epoch observations (i.e they do not change). Any movements are detected implicitly through the estimated velocities.

2.4 Concluding remarks

The simple Gauss-Markov model of section 2.1.1 requires that sufficient points of the network be known a priori and absolutely in order to solve for the network. In a monitoring case, only the object network can be solved in this manner assuming that the reference points used are taken to be of fixed.

The estimation model 2.1.3 of free network case seems favourable for solution of the reference network as no network points need to be known a priori. It would also seem favourable to adjust the object network on the basis of a free network defining the datum over all points of the reference network.

EPOCH 1	POSITIONS		
EPOCH 2	POSITIONS	VELOCITIES	
EPOCH 3	POSITIONS	VELOCITIES	ACCELERATIONS
EPOCH 4	POSITIONS	VELOCITIES	ACCELERATIONS
EPOCH 5	POSITIONS	VELOCITIES	ACCELERATIONS

Fig. 2.2 The parameters computed in a kinematic estimation model for five observation epochs.

The integrated model of adjustment discussed above seems favourable in those areas where the earth's gravity vector (respectively plumb line) is greatly varying. Such are areas of varying terrain (mountainous regions) and also mining zones. And again for computation of heights derived from vertical angles, the direction of the plumb line should be known.

The general models of section 2.3.1. provide information on whether a network has moved or not between two epochs of observation. If the network has moved, then one is required to carry out a further analysis to detect the particular points that have moved and also to find the magnitude of displacement.

The simple kinematic model provides complete information on the analysis of a monitoring network: the unstable points are identified by the speed of movement and the acceleration is also estimated explicitly.

Putting into consideration the above discussion, the present study adopts the kinematic estimation model using the integrated approach for the solution of a monitoring network for localised earth deformation monitoring.

In monitoring networks where more than one epoch of observations have been made, we note that we are able to estimate not only point positions but also the point velocities and accelerations. The kinematic estimation incorporating the integrated model therefore seems a more suitable estimation model where more than one epoch of observations are made. This approach is adopted in this study.

THE LOCAL THREE-DIMENSIONAL GEODETIC MONITORING NETWORK MODEL

3.1 Coordinate systems.

The coordinate systems that are discussed in this section are those that are relevant to coordination of geodetic network points in three dimensional space. These are astronomic, geocentric and ellipsoidal coordinate systems.

3.1.1 Curvilinear physical coordinates.

This system consists of the astronomic latitude $\bar{\phi}$, astronomic longitude $\bar{\lambda}$ and the orthometric height, H which is a function of the gravity potential W . The orthometric height H is the geometric distance from the surface of the geoid to the point P_i of observation, measured along the gravity vector. The gravity potential W from which H is derived is expressed as

$$W = G \iiint \frac{\rho}{r} (X_a, Y_a, Z_a) dX_a dY_a dZ_a + \frac{1}{2} \omega^2 (X^2 + Y^2) \quad (3.1a)$$

with

$$r = \sqrt{[(X-X_a)^2 + (Y-Y_a)^2 + (Z-Z_a)^2]} \quad (3.1b)$$

where $P(X_a, Y_a, Z_a)$ are the coordinates of the attracting point and $P(X, Y, Z)$ are the coordinates of the observation point. ρ is the density of the attracting material whereas G is the gravitation constant and ω the angular velocity of the earth. The orthometric height H is obtained from

$$H = - \int_{W_g}^{W_p} \frac{dw}{\Gamma} \quad (3.2)$$

in which W_g is the gravity potential at the geoid and W_p , the gravity potential at the standpoint. Γ is the gravity intensity along the vertical through point P_i

3.1.2 Local astronomic coordinate system

The observation point P_i is the origin for this left-handed system. Denoting the three axes by X^* , Y^* and Z^* with the corresponding base vectors E_1^* , E_2^* and E_3^* respectively, a positional vector R^* in this system may be represented as

$$R^* = X^* E_1^* + Y^* E_2^* + Z^* E_3^* \quad (3.3)$$

3.1.3 Ellipsoidal cartesian coordinate system

The origin O of this system is the centre of the reference ellipsoid. The three axes, x, y, z are orthogonal and form a right-handed coordinate system. The corresponding base vectors are f_1, f_2, f_3 .

The axis z coincides with the semi-minor axis of the ellipsoid and is positive in the direction of north.

The axis x is directed such that it passes through an adopted origin of the ellipsoidal equator. The plane xOz would be oriented to be as nearly parallel to the Greenwich meridian as possible.

The axis y completes the right-handed system and is taken positive eastwards.

3.1.4. Ellipsoidal curvilinear coordinate system.

The three coordinates are ellipsoidal latitude φ , ellipsoidal longitude λ and ellipsoidal height h .

The ellipsoidal latitude is the acute angle formed between the geodetic normal at the observation point P_i and the ellipsoidal equatorial plane.

The longitude λ , is the angle formed between ellipsoidal

meridian through P_1 and the plane containing the first and the third base vectors.

The ellipsoidal height h is the distance of the point from the ellipsoidal surface as taken along the ellipsoidal normal. It is reckoned positive towards the zenith. Refer to Figure 3.1

3.1.5 Local ellipsoidal coordinate system

The point of observation P_1 is the origin with the axes x^* , y^* , z^* being orthogonal and left-handed. The corresponding base vectors are e_1^* , e_2^* , e_3^* . A positional vector r , in this system is represented as

$$r = x^* e_1^* + y^* e_2^* - z^* e_3^* \quad (3.4)$$

The axis z^* is taken along the geodetic normal with the positive direction outwards from the reference ellipsoid.

The axis x^* is in the meridian plane and points in the direction of north.

The y^* axis completes the left-handed system and points in the direction of east. See Figure 3.1.

3.1.6 Geocentric cartesian coordinate system.

This is a right-handed cartesian coordinate system whose origin O is at the centre of mass of the earth. The three axes, designated X, Y, Z have the corresponding base vectors F_1, F_2, F_3 . A positional vector R in this system is represented by

$$R = XF_1 + YF_2 + ZF_3 \quad (3.5)$$

The Z axis of this system points towards the mean north pole as defined by the International Polar Motion Service (IPMS).

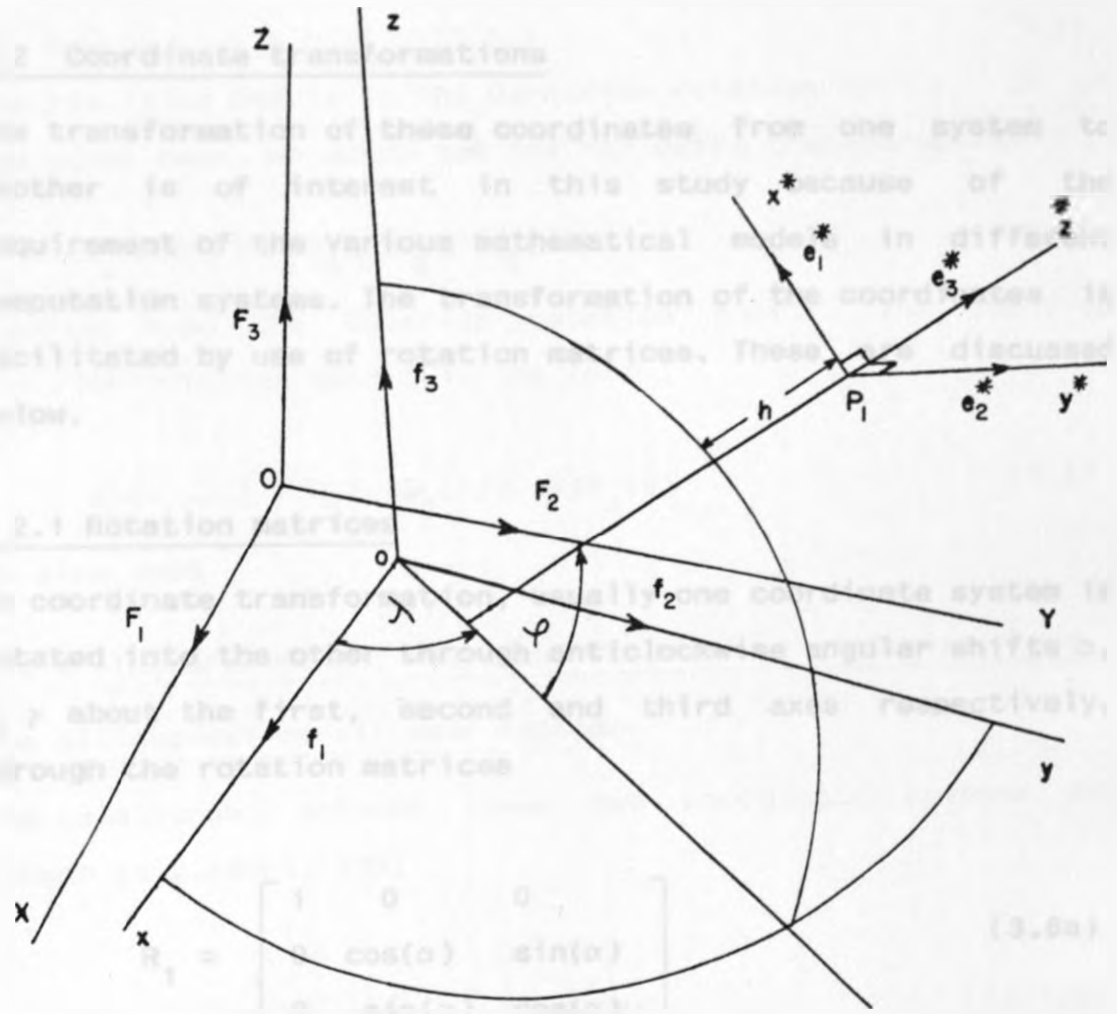
The 3-axis is the same as the 2-axis system, but the 3-axis is perpendicular to the 2-axis. The 3-axis is the same as the 2-axis, but the 3-axis is perpendicular to the 2-axis.

3.2 Coordinate Transformations

The transformation of these coordinates from one system to another is of interest in this study. The requirement of the various mathematical models in different computation systems, the transformation of the coordinates is facilitated by use of rotation matrices. These are discussed below.

3.2.1 Rotation Matrices

In coordinate transformation, the geocentric Cartesian system is rotated into the geocentric ellipsoidal system through angular shifts λ , ϕ , γ about the first, second and third axis respectively through the rotation matrices



$$R_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & \sin(\alpha) \\ 0 & -\sin(\alpha) & \cos(\alpha) \end{bmatrix} \quad (3.6a)$$

$$R_2 = \begin{bmatrix} \cos(\beta) & 0 & -\sin(\beta) \\ 0 & 1 & 0 \\ \sin(\beta) & 0 & \cos(\beta) \end{bmatrix} \quad (3.6b)$$

$$R_3 = \begin{bmatrix} \cos(\gamma) & \sin(\gamma) & 0 \\ -\sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3.6c)$$

Figure 3.1 The geocentric cartesian and ellipsoidal coordinates

where α , β , γ are the angles of rotation about the second, first and third axes respectively. The rotation matrices are defined as follows:

The X axis is in the plane ZOY and is parallel to the Greenwich Meridian. The Y axis completes the right-handed system and is positive eastwards. See also Figure 3.1.

3.2 Coordinate transformations

The transformation of these coordinates from one system to another is of interest in this study because of the requirement of the various mathematical models in different computation systems. The transformation of the coordinates is facilitated by use of rotation matrices. These are discussed below.

3.2.1 Rotation matrices

In coordinate transformation, usually one coordinate system is rotated into the other through anticlockwise angular shifts α , β , γ about the first, second and third axes respectively, through the rotation matrices

$$R_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & \sin(\alpha) \\ 0 & -\sin(\alpha) & \cos(\alpha) \end{bmatrix} \quad (3.6a)$$

$$R_2 = \begin{bmatrix} \cos(\beta) & 0 & -\sin(\beta) \\ 0 & 1 & 0 \\ \sin(\beta) & 0 & \cos(\beta) \end{bmatrix} \quad (3.6b)$$

$$R_3 = \begin{bmatrix} \cos(\gamma) & \sin(\gamma) & 0 \\ -\sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3.6c)$$

where $R_1(\alpha)$ denotes a rotation on the base vector of the first axis through angle α , $R_2(\beta)$ and $R_3(\gamma)$ respectively denote similar shifts through angles β and γ about the second and

third base vectors.

If all the three rotations about the base vectors are carried out as

$$R_c(\alpha, \beta, \gamma) = R_3(\gamma)R_2(\beta)R_1(\alpha) \quad (3.7)$$

the resulting matrix is the Cardanian rotation matrix. If on the other hand, we carry out the following transformation,

$$R_o(\alpha, \beta, \gamma) = R_3(\gamma)R_2(\beta)R_3(\alpha) \quad (3.8)$$

then we have the Eulerian rotation matrix. The modified Eulerian rotation matrix in the form

$$R_E(\alpha, \beta, \gamma) = R_3(\gamma)R_2(\pi/2 - \beta)R_3(\alpha) \quad (3.9)$$

is also used.

3.2.2 Transformation between geodetic cartesian and the ellipsoidal curvilinear systems.

The relationship between these two coordinate systems are common [e.g.Aduol,1989]

$$x = \left[\frac{a}{\sqrt{1-e^2 \sin^2 \varphi}} + h \right] \cos \varphi \cos \lambda \quad (3.10a)$$

$$y = \left[\frac{a}{\sqrt{1-e^2 \sin^2 \varphi}} + h \right] \cos \varphi \sin \lambda \quad (3.10b)$$

$$z = \left[\frac{a(1-e^2)}{\sqrt{1-e^2 \sin^2 \varphi}} + h \right] \sin \varphi \quad (3.10c)$$

The reverse relationship may be found in Cooper (1987).

3.2.3 Geocentric and local astronomic coordinate systems

This transformation is given as [e.g.Aduol,1989]

$$\begin{bmatrix} E_1^* \\ E_2^* \\ E_3^* \end{bmatrix} = R_E(\Lambda, \Phi, 0) \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} \quad (3.11a)$$

where

$$R_E(\Lambda, \Phi, 0) = \begin{bmatrix} \sin\Phi \cos\Lambda & \sin\Phi \sin\Lambda & -\cos\Phi \\ -\sin\Lambda & \cos\Lambda & 0 \\ \cos\Phi \cos\Lambda & \cos\Phi \sin\Lambda & \sin\Phi \end{bmatrix} \quad (3.11b)$$

Thus considering two points $P_I(X_I^*, Y_I^*, Z_I^*)$ and $P_J(X_J^*, Y_J^*, Z_J^*)$ and defining

$$\begin{aligned} X_{IJ}^* &= X_J^* - X_I^* \\ Y_{IJ}^* &= Y_J^* - Y_I^* \\ Z_{IJ}^* &= Z_J^* - Z_I^* \end{aligned} \quad (3.11c)$$

then the corresponding quantities in the geocentric system may be obtained from

$$\begin{bmatrix} X^* \\ Y^* \\ Z^* \end{bmatrix}_{IJ} = R_E(\Lambda, \Phi, 0) \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{IJ} \quad (3.11d)$$

3.2.4 Geocentric and ellipsoidal cartesian systems

This relationship may be represented using the base vectors as

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = R_C(\vartheta_1, \vartheta_2, \vartheta_3) \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} \quad (3.12a)$$

The fully expanded form of $R_C(\vartheta_1, \vartheta_2, \vartheta_3)$ may be found in Aduol

(1989). Usually the rotation angles are very small so that the following rotation matrix results

$$R_c(\vartheta_1, \vartheta_2, \vartheta_3) = \begin{bmatrix} 1 & \vartheta_3 & -\vartheta_2 \\ -\vartheta_3 & 1 & \vartheta_1 \\ \vartheta_2 & -\vartheta_1 & 1 \end{bmatrix} \quad (3.12b)$$

3.2.5 Ellipsoidal cartesian and the ellipsoidal local system

The transformation relationship between these two systems is expressed as

$$\begin{bmatrix} e_1^* \\ e_2^* \\ e_3^* \end{bmatrix} = R_E(\lambda, \phi, 0) \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} \quad (3.13a)$$

so that from equation (3.13a), we have that

$$\begin{bmatrix} x^* \\ y^* \\ z^* \end{bmatrix}_{IJ} = R_E(\lambda, \phi, 0) \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{IJ} \quad (3.13b)$$

with

$$R_E(\lambda, \phi, 0) = \begin{bmatrix} \sin\phi \cos\lambda & \sin\phi \sin\lambda & -\cos\phi \\ -\sin\lambda & \cos\lambda & 0 \\ \cos\phi \cos\lambda & \cos\phi \sin\lambda & \sin\phi \end{bmatrix} \quad (3.13c)$$

3.3. The Observation Equations.

Presented in this section are the observation equations adopted in the establishment of the three dimensional geodetic monitoring network. Some of the mathematical models used are not linear, as required by the procedure of adjustment, and have therefore been linearised. The development of the observation equations is based on the kinematic model [Aduol and Schaffrin 1990], and adopts the integrated approach.

The observation equations developed are for gravity potential, gravity potential difference, gravity intensity, gravity difference, astronomic latitude, astronomic longitude, astronomic azimuth, vertical angle, horizontal direction and spatial distance. The basic deformation parameters that are to be related to the observations are network coordinates in ellipsoidal curvilinear system, φ , λ , h , point velocities and accelerations. Deflection of the vertical parameters, are also estimated. The notation used in this section is the same as that used in the preceding sections in this chapter.

The curvilinear coordinate system is preferred because it is commonly used in map representation and it also relates distances and heights more easily than the cartesian form.

3.3.1 Gravity potential.

The gravity potential W_1 at a point P_1 may be represented as [e.g. Aduol, 1989]

$$W_1 = w_1 + \delta w_1 \quad (3.14a)$$

where w_1 is the model gravity potential and δw_1 is the corresponding gravity potential disturbance. Compare equation (3.14a) with [Heiskanen & Moritz, 1967]

$$W = U + T \quad (3.14b)$$

where W is the geoid potential, U the ellipsoidal (model) potential and T is the disturbing potential.

For an area of limited extent, a radial gravity model may be assumed. Thus

$$w_1 = \frac{GM}{r} \quad (3.15)$$

where G is the gravitational constant, M the mass of the earth and r , the radial distance from the centre of the earth to the

point P_1 . Considering a geocentric coordinate system, r may be expressed as

$$r = (X^2 + Y^2 + Z^2)^{1/2} \Big|_{P_1} \quad (3.16)$$

Let w_{o1} and δw_{o1} be the approximate values at the initial survey epoch, of w_1 and δw_1 respectively. Also let w_i be an observation of w_1 at epoch i with ε_{vi} as an observational error at that epoch; then the following formulation holds,

$$w_i - w_{o1} - \delta w_{o1} = (\Delta w + \Delta \delta w) + (\dot{w} + \delta \dot{w}) \Delta t_i + (\ddot{w} + \delta \ddot{w}) \Delta t_i^2 + \varepsilon_{vi} \quad (3.17)$$

in which $w_1 = w_{o1} + \Delta w_1$ and $\delta w_1 = \delta w_{o1} + \Delta \delta w_1$

The single and the double dot notation represent velocity and acceleration respectively. The Δ notation represents small corrections that are to be added to the approximate values.

Equation (3.17) is the observation equation for gravity potential observed at point P_1 . The parameters are expressed as

$$\Delta w = \frac{\partial w}{\partial \varphi} \Delta \varphi + \frac{\partial w}{\partial \lambda} \Delta \lambda + \frac{\partial w}{\partial h} \Delta h \Big|_{P_1} \quad (3.18a)$$

$$\dot{w} = \frac{\partial w}{\partial \varphi} \dot{\varphi} + \frac{\partial w}{\partial \lambda} \dot{\lambda} + \frac{\partial w}{\partial h} \dot{h} \Big|_{P_1} \quad (3.18b)$$

$$\ddot{w} = \frac{\partial w}{\partial \varphi} \ddot{\varphi} + \frac{\partial w}{\partial \lambda} \ddot{\lambda} + \frac{\partial w}{\partial h} \ddot{h} \Big|_{P_1} \quad (3.18c)$$

where $\Delta \varphi$, $\Delta \lambda$ and Δh are the corrections to be added to the latitude, longitude and height respectively. $\dot{\varphi}$, $\dot{\lambda}$, and \dot{h} are expressed as

$$\dot{\varphi} = \frac{\partial \varphi}{\partial t}, \quad \dot{\lambda} = \frac{\partial \lambda}{\partial t} \quad \text{and} \quad \dot{h} = \frac{\partial h}{\partial t} \quad (3.18d)$$

and $\ddot{\varphi}$, $\ddot{\lambda}$ and \ddot{h} expressed as

$$\ddot{\varphi} = \frac{\partial \dot{\varphi}}{\partial t}, \quad \ddot{\lambda} = \frac{\partial \dot{\lambda}}{\partial t} \quad \text{and} \quad \ddot{h} = \frac{\partial \dot{h}}{\partial t} \quad (3.18e)$$

The expressions necessary for the coordinate transformations are

$$\frac{\partial w}{\partial(\varphi, \lambda, h)} = \frac{\partial(x, y, z)}{\partial(\varphi, \lambda, h)} \cdot \frac{\partial w}{\partial(x, y, z)} \Big|_{P_1} \quad (3.19a)$$

and

$$\frac{\partial w}{\partial(x, y, z)} = \frac{\partial(X, Y, Z)}{\partial(x, y, z)} \cdot \frac{\partial w}{\partial(X, Y, Z)} \Big|_{P_1} \quad (3.19b)$$

where x , y , and z are the station coordinates in local ellipsoidal system while X , Y , Z are the corresponding station coordinates in geocentric cartesian system.

The rotational elements $\vartheta_1, \vartheta_2, \vartheta_3$, between these two coordinate systems relate as [Aduol, 1989]

$$R_c(\vartheta_1, \vartheta_2, \vartheta_3) = \frac{\partial(X, Y, Z)}{\partial(x, y, z)} \quad (3.19c)$$

where R_c is the Cardanian rotation matrix.

For $\vartheta_1, \vartheta_2, \vartheta_3$ being small, then

$$R_c = \begin{bmatrix} 1 & \vartheta_3 & -\vartheta_2 \\ -\vartheta_3 & 1 & \vartheta_1 \\ \vartheta_2 & -\vartheta_1 & 1 \end{bmatrix} \quad (3.19d)$$

Differentiating equation (3.15) with respect to X , Y and Z , the following matrices result,

$$\begin{bmatrix} \frac{\partial w}{\partial \varphi} \\ \frac{\partial w}{\partial \lambda} \\ \frac{\partial w}{\partial h} \end{bmatrix}_{P_1} = \begin{bmatrix} \frac{\partial x}{\partial \varphi} & \frac{\partial y}{\partial \varphi} & \frac{\partial z}{\partial \varphi} \\ \frac{\partial x}{\partial \lambda} & \frac{\partial y}{\partial \lambda} & \frac{\partial z}{\partial \lambda} \\ \frac{\partial x}{\partial h} & \frac{\partial y}{\partial h} & \frac{\partial z}{\partial h} \end{bmatrix}_{P_1} \begin{bmatrix} \frac{\partial w}{\partial x} \\ \frac{\partial w}{\partial y} \\ \frac{\partial w}{\partial z} \end{bmatrix}_{P_1} \quad (3.20a)$$

with

$$\begin{bmatrix} \frac{\partial w}{\partial \varphi_1} \\ \frac{\partial w}{\partial \varphi_2} \\ \frac{\partial w}{\partial \varphi_3} \end{bmatrix}_{P_1} = \begin{bmatrix} \frac{\partial X}{\partial \varphi_1} & \frac{\partial Y}{\partial \varphi_1} & \frac{\partial Z}{\partial \varphi_1} \\ \frac{\partial X}{\partial \varphi_2} & \frac{\partial Y}{\partial \varphi_2} & \frac{\partial Z}{\partial \varphi_2} \\ \frac{\partial X}{\partial \varphi_3} & \frac{\partial Y}{\partial \varphi_3} & \frac{\partial Z}{\partial \varphi_3} \end{bmatrix}_{P_1} \begin{bmatrix} \frac{\partial w}{\partial X} \\ \frac{\partial w}{\partial Y} \\ \frac{\partial w}{\partial Z} \end{bmatrix}_{P_1} \quad (3.20b)$$

and

$$\begin{bmatrix} \frac{\partial w}{\partial x} \\ \frac{\partial w}{\partial y} \\ \frac{\partial w}{\partial z} \end{bmatrix}_{P_1} = \begin{bmatrix} \frac{\partial X}{\partial x} & \frac{\partial Y}{\partial x} & \frac{\partial Z}{\partial x} \\ \frac{\partial X}{\partial y} & \frac{\partial Y}{\partial y} & \frac{\partial Z}{\partial y} \\ \frac{\partial X}{\partial z} & \frac{\partial Y}{\partial z} & \frac{\partial Z}{\partial z} \end{bmatrix}_{P_1} \begin{bmatrix} \frac{\partial w}{\partial X} \\ \frac{\partial w}{\partial Y} \\ \frac{\partial w}{\partial Z} \end{bmatrix}_{P_1} \quad (3.20c)$$

3.3.2 Gravity intensity.

The gravity intensity Γ_1 at a point P_1 may be represented by following expression

$$\Gamma_1 = \sqrt{\left(\frac{\partial w}{\partial X}\right)^2 + \left(\frac{\partial w}{\partial Y}\right)^2 + \left(\frac{\partial w}{\partial Z}\right)^2} \Big|_{P_1} \quad (3.21)$$

Decomposing Γ_1 into a model component γ_1 and a disturbing part $\delta\gamma_1$, then

$$\Gamma_1 = \gamma_1 + \delta\gamma_1 \quad (3.22)$$

where

$$\gamma_1 = \sqrt{\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2} \Big|_{P_1} \quad (3.23)$$

is the gravity intensity for the model gravity. If after Taylor series linearization, the initial values for γ_1 and $\delta\gamma_1$ are γ_{01} and $\delta\gamma_{01}$ respectively, then

$$\gamma_1 = \gamma_{01} + \Delta\gamma_1 \quad \text{and} \quad \delta\gamma_1 = \delta\gamma_{01} + \Delta\delta\gamma_1 \quad (3.24)$$

where $\Delta\gamma$ and $\Delta\delta\gamma$ are respective corrections for γ_{01} and $\delta\gamma_{01}$. Suppose $\tilde{\Gamma}_i$ is an observation for gravity intensity at i th-epoch with an observational error ε_{Ti} then

$$\tilde{\Gamma}_i = \Gamma_i + \varepsilon_{Ti} \quad (3.25)$$

Combining equations (3.22), (3.24) and (3.25), and introducing velocities and accelerations, one gets

$$\begin{aligned} \tilde{\Gamma}_i - \gamma_{01} - \delta\gamma_{01} = & \Delta\gamma + \Delta\delta\gamma + (\dot{\gamma} + \dot{\delta\gamma})\Delta t_i + \\ & + (\ddot{\gamma} + \ddot{\delta\gamma})\Delta t_i^2 + \varepsilon_{Ti} \end{aligned} \quad (3.26)$$

The parameters are expressed as

$$\Delta\gamma = \frac{\partial\gamma}{\partial\varphi} \Delta\varphi + \frac{\partial\gamma}{\partial\lambda} \Delta\lambda + \frac{\partial\gamma}{\partial h} \Delta h \Big|_{P_1} \quad (3.27a)$$

$$\dot{\gamma} = \frac{\partial w}{\partial\varphi} \dot{\varphi} + \frac{\partial w}{\partial\lambda} \dot{\lambda} + \frac{\partial w}{\partial h} \dot{h} \Big|_{P_1} \quad (3.18b)$$

$$\ddot{\gamma} = \frac{\partial w}{\partial\varphi} \ddot{\varphi} + \frac{\partial w}{\partial\lambda} \ddot{\lambda} + \frac{\partial w}{\partial h} \ddot{h} \Big|_{P_1} \quad (3.18c)$$

after having eliminated those parameters that cannot be suitably evaluated in a local network [see also Aduol, 1989]

The differentials are expressed as

$$\frac{\partial\gamma}{\partial(\varphi, \lambda, h)} = \frac{\partial(x, y, z)}{\partial(\varphi, \lambda, h)} \cdot \frac{\partial\gamma}{\partial(x, y, z)} \Big|_{P_1} \quad (3.28a)$$

and

$$\frac{\partial \gamma}{\partial(x,y,z)} = \frac{\partial(X,Y,Z)}{\partial(x,y,z)} \cdot \frac{\partial \gamma}{\partial(X,Y,Z)} \Big|_{P_1} \quad (3.28b)$$

$$= R_c(\vartheta_1, \vartheta_2, \vartheta_3) \cdot \frac{\partial \gamma}{\partial(X,Y,Z)} \Big|_{P_1} \quad (3.28c)$$

Equation (3.26) is the observation equation for gravity intensity in the kinematic estimation model.

3.3.3 Astronomic latitude.

The astronomic latitude, Φ at a point P_1 can be expressed as

$$\Phi_1 = \varphi_1 + \delta\varphi_1 \quad (3.29)$$

where φ_1 is the model part and $\delta\varphi_1$ is the disturbing component. If the ellipsoidal latitude is adopted as the model part, then the disturbing component is the deflection of the vertical in the north-south direction. Thus equation (3.29) may be written as

$$\Phi_1 = \varphi_1 + \xi_1 \quad (3.30)$$

If Φ_{1i} be a realization of Φ_1 and φ_{01} and ξ_{01} be some adopted initial values for φ_1 and ξ_1 respectively, then the following relationship holds:

$$\Phi_{1i} - \varphi_{01} - \xi_{01} = \Delta\varphi_1 + \Delta\xi_1 + \dot{\varphi}_1 \Delta t_i + \ddot{\varphi}_1 \Delta t_i^2 + \varepsilon_\varphi \quad (3.31)$$

with ε_φ as an observational error in Φ_{1i} .

Thus equation (3.31) is the observation equation for astronomic latitude.

3.3.4 Astronomic longitude.

At a point P_1 the astronomic longitude Λ , may be expressed as

$$\Lambda_1 = \lambda_1 + \delta\lambda_1 \quad (3.32)$$

Considering λ_1 as an ellipsoidal longitude, then $\delta\lambda_1$ may be expressed as

$$\delta\lambda_1 = \eta_1 \sec\varphi_1 \quad (3.33)$$

where η_1 is the deflection of the vertical in an east-west direction.

Considering $\tilde{\Lambda}_{1i}$ as an observed value of Λ_1 at epoch i , λ_{o1} as an approximate value for λ_1 and η_{o1} an initial value for η_1 , then

$$\begin{aligned} \tilde{\Lambda}_{1i} - \lambda_{o1} - \eta_{o1} \sec\varphi_1 &= \Delta\lambda_1 + \Delta\eta_1 \sec\varphi_1 + \\ &\quad + \lambda_1 \Delta t_1 + \dot{\lambda}_1 \Delta t_1 + \varepsilon_\Lambda \end{aligned} \quad (3.34)$$

holds. ε_Λ is an observational error in Λ_i .

Equation (3.34) is the observation equation for astronomic longitude.

3.3.5 Astronomic azimuth.

The astronomic azimuth A_{12} from point P_1 to point P_2 may be represented in the form

$$A_{12} = \tan^{-1} \left[\frac{Y_{12}^*}{X_{12}^*} \right] \quad (3.35)$$

Decomposing A_{12} into a model part α_{12} and a disturbing component $\delta\alpha_{12}$, one gets,

$$A_{12} = \alpha_{12} + \delta\alpha_{12} \quad (3.36)$$

with

$$\alpha_{12} = \tan^{-1} \left[\frac{y_{12}^*}{x_{12}^*} \right] \quad (3.37)$$

Suppose that \tilde{A}_{12i} is the observed value for A_{12} at epoch i , α_{o12} and $\delta\alpha_{o12}$ are initial values for α_{12} and $\delta\alpha_{12}$ respectively, adopted for a Taylor series linearization

process, then

$$\alpha_{12} = \alpha_{o12} + \Delta\alpha_{12} \quad \text{and} \quad \delta\alpha_{12} = \delta\alpha_{o12} + \Delta\delta\alpha_{12} \quad (3.38)$$

with $\Delta\alpha_{12}$ and $\Delta\delta\alpha_{12}$ as respective corrections for α_{o12} and $\delta\alpha_{o12}$. Also,

$$\tilde{A}_{12} = A_{12} + \varepsilon_A \quad (3.39)$$

where ε_A is an observational error in \tilde{A}_{12} . Combining equations (3.36), (3.38) and (3.39) and introducing velocity and acceleration, one obtains

$$\begin{aligned} \tilde{A}_{12} - \alpha_{o12} - \delta\alpha_{o12} &= \Delta\alpha_{12} + \Delta\delta\alpha_{12} + \alpha_{12} \Delta t_1 + \\ &+ \alpha_{12} \Delta t_1^2 + \varepsilon_A \end{aligned} \quad (3.40)$$

which is the observation equation for astronomic azimuth.

During the linearization process the differentiation is carried out with respect to the unknown parameters (i.e. coordinates for both points, P_1 and P_2). They are expressed as

$$\begin{aligned} \Delta\alpha_{12} &= \frac{\partial\alpha_{12}}{\partial\varphi_1} \Delta\varphi_1 + \frac{\partial\alpha_{12}}{\partial\lambda_1} \Delta\lambda_1 + \frac{\partial\alpha_{12}}{\partial h_1} \Delta h_1 + \\ &\frac{\partial\alpha_{12}}{\partial\varphi_2} \Delta\varphi_2 + \frac{\partial\alpha_{12}}{\partial\lambda_2} \Delta\lambda_2 + \frac{\partial\alpha_{12}}{\partial h_2} \Delta h_2 \end{aligned} \quad (3.41a)$$

$$\begin{aligned} \dot{\alpha}_{12} &= \frac{\partial\alpha_{12}}{\partial\varphi_1} \dot{\varphi}_1 + \frac{\partial\alpha_{12}}{\partial\lambda_1} \dot{\lambda}_1 + \frac{\partial\alpha_{12}}{\partial h_1} \dot{h}_1 + \\ &\frac{\partial\alpha_{12}}{\partial\varphi_2} \dot{\varphi}_2 + \frac{\partial\alpha_{12}}{\partial\lambda_2} \dot{\lambda}_2 + \frac{\partial\alpha_{12}}{\partial h_2} \dot{h}_2 \end{aligned} \quad (3.41b)$$

$$\begin{aligned} \alpha_{12} = & \frac{\partial \alpha_{12}}{\partial \varphi_1} \varphi_1 + \frac{\partial \alpha_{12}}{\partial \lambda_1} \lambda_1 + \frac{\partial \alpha_{12}}{\partial h_1} h_1 + \\ & \frac{\partial \alpha_{12}}{\partial \varphi_2} \varphi_2 + \frac{\partial \alpha_{12}}{\partial \lambda_2} \lambda_2 + \frac{\partial \alpha_{12}}{\partial h_2} h_2 \end{aligned} \quad (3.41c)$$

$$\Delta \alpha_{12} = \frac{\partial \alpha_{12}}{\partial \xi_1} \Delta \xi_1 + \frac{\partial \alpha_{12}}{\partial \eta_1} \Delta \eta_1 \quad (3.41d)$$

with

$$\frac{\partial \alpha_{12}}{\partial \xi_1} = -\sin \alpha_{12} \tan \beta_{12} \quad (3.41e)$$

and

$$\frac{\partial \alpha_{12}}{\partial \eta_1} = -\tan \varphi_1 - \cos \alpha_{12} \tan \beta_{12} \quad (3.41f)$$

Further expressions for evaluating the coefficients are given as

$$\frac{\partial \alpha}{\partial (\varphi_k, \lambda_k, h_k)} = \frac{\partial (x_k, y_k, z_k)}{\partial (\varphi_k, \lambda_k, h_k)} \cdot \frac{\partial \alpha}{\partial (x_k, y_k, z_k)} \Big|_{k=1,2} \quad (3.42a)$$

$$\frac{\partial \alpha}{\partial (x_k^*, y_k^*, z_k^*)} = \frac{\partial (x_{12}^*, y_{12}^*, z_{12}^*)}{\partial (x_k, y_k, z_k)} \cdot \frac{\partial \alpha}{\partial (x_{12}^*, y_{12}^*, z_{12}^*)} \Big|_{k=1,2} \quad (3.42b)$$

with

$$\frac{\partial (x_{12}^*, y_{12}^*, z_{12}^*)}{\partial (x_2, y_2, z_2)} = - \frac{\partial (x_{12}^*, y_{12}^*, z_{12}^*)}{\partial (x_1, y_1, z_1)} = R_E(\lambda_1, \phi_1, 0) \quad (3.42c)$$

where R_E is the Eulerian rotation matrix.

3.3.6 Vertical angle

From a point P_1 to another point P_2 , the vertical angle B_{12} between the two points is represented as

$$B_{12} = \tan^{-1} \left[\frac{Z_{12}^*}{(X_{12}^{*2} + Y_{12}^{*2})^{1/2}} \right] \quad (3.43)$$

The vertical angle may be further decomposed into a model component β_{12} and a disturbing part $\delta\beta_{12}$ such that

$$B_{12} = \beta_{12} + \delta\beta_{12} \quad (3.44)$$

with

$$\beta_{12} = \tan^{-1} \left[\frac{Z_{12}^*}{(x_{12}^{*2} + y_{12}^{*2})^{1/2}} \right] \quad (3.45)$$

Suppose that B_{12} is the observed value free from refraction and B_{12}° is the actual observed value containing effects of refraction δr , then

$$B_{12}^{\circ} = B_{12} + \delta r \quad (3.46)$$

with

$$B_{12} = B_{12} + \varepsilon_B$$

$$= \beta_{12} + \delta\beta_{12} + \varepsilon_B$$

thus

$$B_{12}^{\circ} = \beta_{o12} + \Delta\beta_{12} + \delta\beta_{o12} + \Delta\delta\beta_{12} + \delta r_o + \Delta\delta r + \varepsilon_B \quad (3.47)$$

on taking the approximate values, for β_{12} , $\delta\beta_{12}$ and δr as β_{o12} , $\delta\beta_{o12}$ and δr_o respectively, for use after a Taylor series linearization. On considering the observed angle B_{12}° and incorporating the velocity and acceleration unknowns, we have

$$B_{12}^o - \beta_{o12} - \delta\beta_{o12} - \delta r_o = \Delta\beta_{12} + \Delta\delta\beta_{12} + \Delta\delta r + \beta_{12} \Delta t_i + \beta_{12} \Delta t_i^2 + \epsilon_B \quad (3.48)$$

and $\Delta\beta_{12}$, $\Delta\delta\beta_{12}$ and $\Delta\delta r$ as respective corrections to be added to the initial values β_{o12} , $\delta\beta_{o12}$, and δr_o . ϵ_B is an observational error in B_{12}^o .

Equation (3.48) represents the basic form of the observation equation for vertical angle. The corrections are expressed as differential equations in the unknown parameters (i.e. station coordinates for both points P_1 and P_2 and also the components of the deflection of the vertical at the observing point.

$$\Delta\delta\beta_{12} = \frac{\partial\delta\beta_{12}}{\partial\xi_1} \Delta\xi_1 + \frac{\partial\delta\beta_{12}}{\partial\eta_1} \Delta\eta_1 \quad (3.49)$$

having treated the rotation elements as zero. The coefficients are

$$\frac{\partial\delta\beta_{12}}{\partial\xi_1} = -\cos\alpha_{12} \quad \text{and} \quad \frac{\partial\delta\beta_{12}}{\partial\eta_1} = \sin\alpha_{12} \quad (3.50a)$$

Also

$$\Delta\beta_{12} = \frac{\partial\beta_{12}}{\partial\varphi_1} \Delta\varphi_1 + \frac{\partial\beta_{12}}{\partial\lambda_1} \Delta\lambda_1 + \frac{\partial\beta_{12}}{\partial h_1} \Delta h_1 + \frac{\partial\beta_{12}}{\partial\varphi_2} \Delta\varphi_2 + \frac{\partial\beta_{12}}{\partial\lambda_2} \Delta\lambda_2 + \frac{\partial\beta_{12}}{\partial h_2} \Delta h_2 \quad (3.50b)$$

$$\dot{\beta}_{12} = \frac{\partial \beta_{12}}{\partial \varphi_1} \dot{\varphi}_1 + \frac{\partial \beta_{12}}{\partial \lambda_1} \dot{\lambda}_1 + \frac{\partial \beta_{12}}{\partial h_1} \dot{h}_1 +$$

$$+ \frac{\partial \beta_{12}}{\partial \varphi_2} \dot{\varphi}_2 + \frac{\partial \beta_{12}}{\partial \lambda_2} \dot{\lambda}_2 + \frac{\partial \beta_{12}}{\partial h_2} \dot{h}_2 \quad (3.50c)$$

$$\ddot{\beta}_{12} = \frac{\partial \beta_{12}}{\partial \varphi_1} \ddot{\varphi}_1 + \frac{\partial \beta_{12}}{\partial \lambda_1} \ddot{\lambda}_1 + \frac{\partial \beta_{12}}{\partial h_1} \ddot{h}_1 +$$

$$+ \frac{\partial \beta_{12}}{\partial \varphi_2} \ddot{\varphi}_2 + \frac{\partial \beta_{12}}{\partial \lambda_2} \ddot{\lambda}_2 + \frac{\partial \beta_{12}}{\partial h_2} \ddot{h}_2 \quad (3.50d)$$

with

$$\frac{\partial \beta_{12}}{\partial (\varphi_k, \lambda_k, h_k)} = \frac{\partial (x_k, y_k, z_k)}{\partial (\varphi_k, \lambda_k, h_k)} \cdot \frac{\partial \beta_{12}}{\partial (x_k, y_k, z_k)} \quad k=1,2 \quad (3.51)$$

and

$$\frac{\partial \beta_{12}}{\partial (x_k, y_k, z_k)} = \frac{\partial (x_{12}^*, y_{12}^*, z_{12}^*)}{\partial (x_k, y_k, z_k)} \cdot \frac{\partial \beta_{12}}{\partial (x_{12}^*, y_{12}^*, z_{12}^*)} \quad k=1,2 \quad (3.52)$$

3.3.7 Spatial distance

The spatial distance S_{12} between two points P_1 and P_2 may be expressed mathematically in the form

$$S_{12} = \sqrt{X_{12}^{*2} + Y_{12}^{*2} + Z_{12}^{*2}} \quad (3.53)$$

with the model part s_{12} expressed in the form

$$s_{12} = \sqrt{x_{12}^{*2} + y_{12}^{*2} + z_{12}^{*2}} \quad (3.54)$$

Since distances are not influenced by effects of the gravity

field, the disturbing component becomes equal to zero so that

$$S_{12} = s_{12} \quad (3.55)$$

Further, let $s_{\alpha 12}$ be the approximate value for s_{12} , then

$$s_{12} = s_{\alpha 12} + \Delta s_{12} \quad (3.56)$$

where Δs_{12} is a correction to be added to the initial value $s_{\alpha 12}$. Also suppose a value for spatial distance \tilde{s}_{12i} is observed at epoch i , with a random error ε_s in it, then

$$\tilde{s}_{12i} = s_{12} + \varepsilon_s \quad (3.57)$$

Combining equations (3.56) and (3.57) and including velocity and acceleration one gets,

$$\tilde{s}_{12i} - s_{\alpha 12} = \Delta s_{12} + \dot{s} \Delta t_i + \ddot{s} \Delta t_i^2 + \varepsilon_s \quad (3.58)$$

which becomes the observation equation for distance. The parameters are expressed in the following differential equations,

$$\begin{aligned} \Delta s_{12} = & \frac{\partial s_{12}}{\partial \varphi_1} \Delta \varphi_1 + \frac{\partial s_{12}}{\partial \lambda_1} \Delta \lambda_1 + \frac{\partial s_{12}}{\partial h_1} \Delta h_1 + \\ & + \frac{\partial s_{12}}{\partial \varphi_2} \Delta \varphi_2 + \frac{\partial s_{12}}{\partial \lambda_2} \Delta \lambda_2 + \frac{\partial s_{12}}{\partial h_2} \Delta h_2 \end{aligned} \quad (3.59a)$$

$$\begin{aligned} \dot{\tilde{s}}_{12} = & \frac{\partial s_{12}}{\partial \varphi_1} \dot{\varphi}_1 + \frac{\partial s_{12}}{\partial \lambda_1} \dot{\lambda}_1 + \frac{\partial s_{12}}{\partial h_1} \dot{h}_1 + \\ & + \frac{\partial s_{12}}{\partial \varphi_2} \dot{\varphi}_2 + \frac{\partial s_{12}}{\partial \lambda_2} \dot{\lambda}_2 + \frac{\partial s_{12}}{\partial h_2} \dot{h}_2 \end{aligned} \quad (3.59c)$$

$$\begin{aligned} \ddot{\tilde{s}}_{12} = & \frac{\partial s_{12}}{\partial \varphi_1} \ddot{\varphi}_1 + \frac{\partial s_{12}}{\partial \lambda_1} \ddot{\lambda}_1 + \frac{\partial s_{12}}{\partial h_1} \ddot{h}_1 + \\ & + \frac{\partial s_{12}}{\partial \varphi_2} \ddot{\varphi}_2 + \frac{\partial s_{12}}{\partial \lambda_2} \ddot{\lambda}_2 + \frac{\partial s_{12}}{\partial h_2} \ddot{h}_2 \end{aligned} \quad (3.59d)$$

and also

$$\frac{\partial s_{12}}{\partial(\rho_k, \lambda_k, h_k)} = \frac{\partial(x_k, y_k, z_k)}{\partial(\rho_k, \lambda_k, h_k)} \cdot \frac{\partial s_{12}}{\partial(x_k, y_k, z_k)} \Big|_{k=1,2} \dots\dots(3.60a)$$

and

$$\frac{\partial s_{12}}{\partial(x_k, y_k, z_k)} = \frac{\partial(x_{12}^*, y_{12}^*, z_{12}^*)}{\partial(x_k, y_k, z_k)} \cdot \frac{\partial s_{12}}{\partial(x_{12}^*, y_{12}^*, z_{12}^*)} \Big|_{k=1,2} \dots\dots(3.60b)$$

3.3.8 Horizontal direction

Let the horizontal direction of an observation line $P_1 P_2$ be T_{12} at observation point P_1 . Further let the azimuth of this line be A_{12} : then,

$$T_{12} = A_{12} + \Sigma_T \tag{3.61}$$

where Σ_T is an orientation parameter at the standpoint P_1 . Decomposing the azimuth of this line into a model part α_{12} and a disturbing part $\delta\alpha$ one obtains

$$T_{12} = \alpha_{12} + \delta\alpha_{12} + \Sigma_T \tag{3.62}$$

with α_{12} defined in equation (3.36). Taking initial values for α_{12} and $\delta\alpha_{12}$ we form equation (3.38) and incorporating Σ_T into equation (3.39) we arrive at the modified azimuth equation

$$\tilde{T}_{12i} - \alpha_{12} - \delta\alpha_{12} = \dot{\alpha}_{12} \Delta t_i + \ddot{\alpha}_{12} \Delta t_i^2 + \Sigma_T + \varepsilon_T \tag{3.63}$$

where ε_T is an observational error in the direction

observation T
12i

Equation (3.63) is the observation equation for an observed horizontal direction.

Expression for $\Delta\alpha_{12}$ and $\Delta\delta\alpha_{12}$ are given in equations (3.41). The final coefficients are as provided in equations (3.42).

3.3.9 Gravity potential difference

The difference in gravity potential W_{12} between two points P_1 and P_2 is of the form

$$W_{12} = W_2 - W_1 \quad (3.64)$$

On considering the model and the disturbing components (see also section 3.3.1), we get

$$W_{12} = w_{12} - \delta w_{12} \quad (3.65)$$

where

$$w_{12} = w_2 - w_1 \quad \text{and} \quad \delta w_{12} = \delta w_2 - \delta w_1 \quad (3.66)$$

The observation equation is of the form

$$W_{12i} - w_{o12} - \delta w_{o12} = \Delta w_{12} + \Delta \delta w_{12} + \dot{w}_{12} \Delta t_i + \dot{w}_{12}^2 \Delta t_i^2 + \varepsilon_{v12} \quad (3.67)$$

The parameters are expressed as

$$\begin{aligned} \Delta w_{12} = & \frac{\partial w_{12}}{\partial \varphi_1} \Delta \varphi_1 + \frac{\partial w_{12}}{\partial \lambda_1} \Delta \lambda_1 + \frac{\partial w_{12}}{\partial h_1} \Delta h_1 + \\ & + \frac{\partial w_{12}}{\partial \varphi_2} \Delta \varphi_2 + \frac{\partial w_{12}}{\partial \lambda_2} \Delta \lambda_2 + \frac{\partial w_{12}}{\partial h_2} \Delta h_2 \end{aligned} \quad (3.68a)$$

$$\text{and } \Delta \delta w_{12} = \Delta \delta w_2 - \Delta \delta w_1 \quad (3.68b)$$

$$\begin{aligned} \dot{w}_{12} = & \frac{\partial w_{12}}{\partial \varphi_1} \dot{\varphi}_1 + \frac{\partial w_{12}}{\partial \lambda_1} \dot{\lambda}_1 + \frac{\partial w_{12}}{\partial h_1} \dot{h}_1 + \\ & + \frac{\partial w_{12}}{\partial \varphi_2} \dot{\varphi}_2 + \frac{\partial w_{12}}{\partial \lambda_2} \dot{\lambda}_2 + \frac{\partial w_{12}}{\partial h_2} \dot{h}_2 \end{aligned} \quad (3.68c)$$

$$\begin{aligned} \ddot{w}_{12} = & \frac{\partial w_{12}}{\partial \varphi_1} \ddot{\varphi}_1 + \frac{\partial w_{12}}{\partial \lambda_1} \ddot{\lambda}_1 + \frac{\partial w_{12}}{\partial h_1} \ddot{h}_1 + \\ & + \frac{\partial w_{12}}{\partial \varphi_2} \ddot{\varphi}_2 + \frac{\partial w_{12}}{\partial \lambda_2} \ddot{\lambda}_2 + \frac{\partial w_{12}}{\partial h_2} \ddot{h}_2 \end{aligned} \quad (3.68d)$$

3.3.10 Gravity difference

The difference in gravity Γ_{12} between two points, P_1 and P_2 (see also section 3.3.2) is

$$\Gamma_{12} = \Gamma_2 - \Gamma_1 \quad (3.69)$$

On separating the model portion γ and the disturbing part $\delta\gamma$, one may write

$$\Gamma_{12} = \gamma_{12} + \delta\gamma_{12} \quad (3.70)$$

where

$$\gamma_{12} = \gamma_2 - \gamma_1 \quad \text{and} \quad \delta\gamma_{12} = \delta\gamma_2 - \delta\gamma_1 \quad (3.71)$$

The observation equation is therefore of the form

$$\begin{aligned} \Gamma_{12i} - \gamma_{o12} - \delta\gamma_{o12} = & \Delta\gamma_{12} + \Delta\delta\gamma_{12} + \gamma_{12} \dot{\Delta t}_i + \\ & + \gamma_{12} \ddot{\Delta t}_i^2 + \varepsilon_{\Gamma} \end{aligned} \quad (3.72)$$

The expressions for the parameters are

$$\begin{aligned} \Delta\gamma_{ij} = & \frac{\partial\gamma_{12}}{\partial\varphi_1} \Delta\varphi_1 + \frac{\partial\gamma_{12}}{\partial\lambda_1} \Delta\lambda_1 + \frac{\partial\gamma_{12}}{\partial h_1} \Delta h_1 + \\ & + \frac{\partial\gamma_{12}}{\partial\varphi_2} \Delta\varphi_2 + \frac{\partial\gamma_{12}}{\partial\lambda_2} \Delta\lambda_2 + \frac{\partial\gamma_{12}}{\partial h_2} \Delta h_2 \end{aligned} \quad (3.73a)$$

$$\begin{aligned} \dot{\gamma}_{12} = & \frac{\partial\gamma_{12}}{\partial\varphi_1} \dot{\varphi}_1 + \frac{\partial\gamma_{12}}{\partial\lambda_1} \dot{\lambda}_1 + \frac{\partial\gamma_{12}}{\partial h_1} \dot{h}_1 + \\ & + \frac{\partial\gamma_{12}}{\partial\varphi_2} \dot{\varphi}_2 + \frac{\partial\gamma_{12}}{\partial\lambda_2} \dot{\lambda}_2 + \frac{\partial\gamma_{12}}{\partial h_2} \dot{h}_2 \end{aligned} \quad (3.73b)$$

$$\begin{aligned} \ddot{\gamma}_{12} = & \frac{\partial\gamma_{12}}{\partial\varphi_1} \ddot{\varphi}_1 + \frac{\partial\gamma_{12}}{\partial\lambda_1} \ddot{\lambda}_1 + \frac{\partial\gamma_{12}}{\partial h_1} \ddot{h}_1 + \\ & + \frac{\partial\gamma_{12}}{\partial\varphi_2} \ddot{\varphi}_2 + \frac{\partial\gamma_{12}}{\partial\lambda_2} \ddot{\lambda}_2 + \frac{\partial\gamma_{12}}{\partial h_2} \ddot{h}_2 \end{aligned} \quad (3.73c)$$

and

$$\Delta\delta\gamma_{12} = \Delta\delta\gamma_2 - \Delta\delta\gamma_1 \quad (3.73d)$$

CHAPTER FOUR

THE MONITORING NETWORK

4.1 The Test Network.

The test network was derived from an old map of Olkaria Geothermal station which is situated in the Rift Valley in Kenya, South of Lake Naivasha. A suitable network of points was chosen and their coordinates in the Universal Transverse Mercator System (UTM) scaled off. The geodetic coordinates of these points were then computed from the UTM coordinates by use of conversion tables. The sketch of the network is shown in Figure 4.1.

4.2 Simulation of observations.

The corresponding ellipsoidal cartesian coordinates of the network points were then computed according to equations (3.10). From these coordinates were computed the gravity potential and the gravity intensity at each network point according to equations (3.15) and (3.21) respectively. The potential difference and the gravity difference for any pair of points were subsequently computed.

Further, the ellipsoidal cartesian coordinates were transformed into their corresponding local cartesian values (see section 3.2) from which were computed the spatial distances, vertical angles and the ellipsoidal azimuths for any chosen pair of coordinates.

Using the same notation as in Chapter Three, the various observations were computed as:

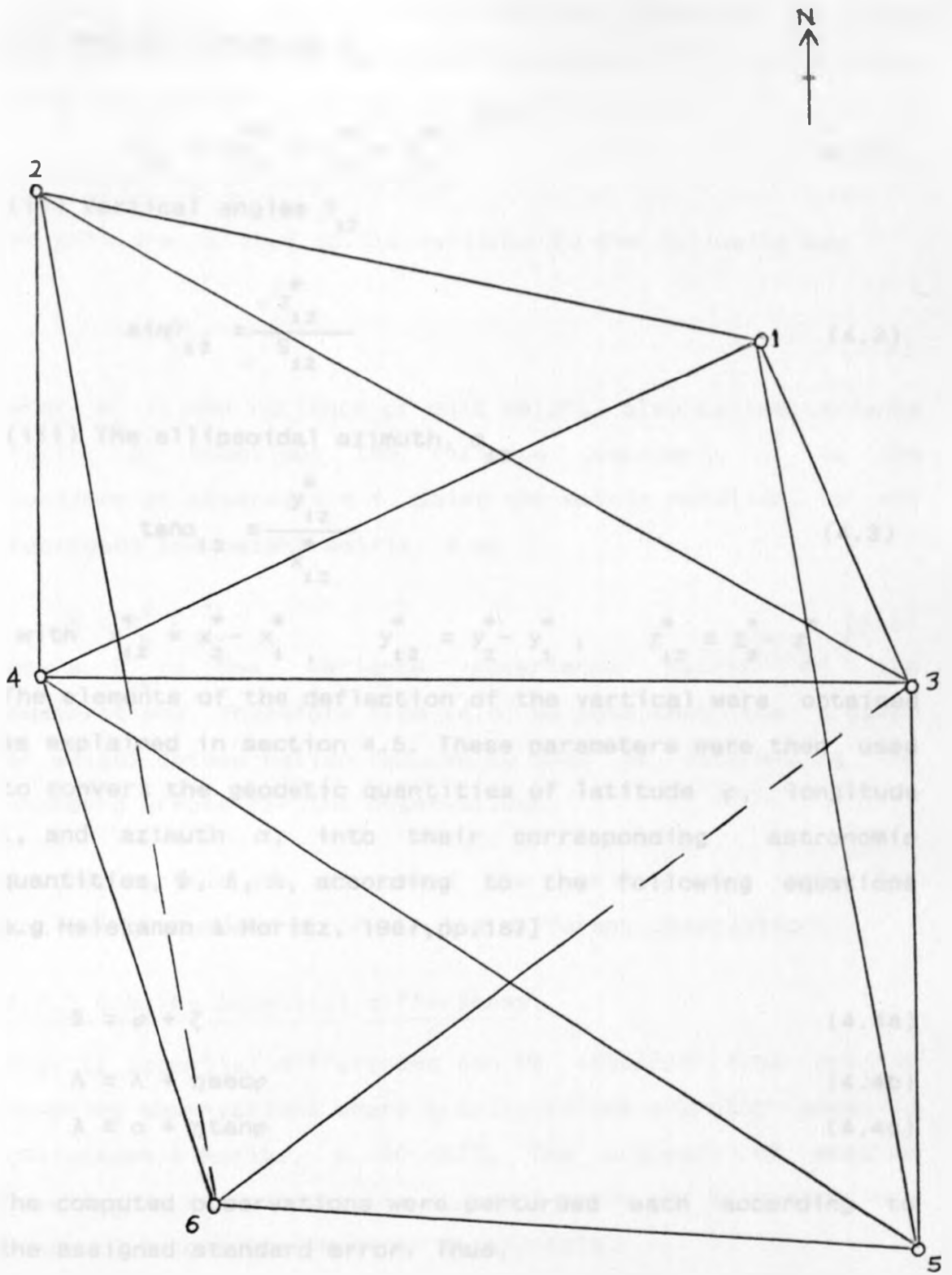


Fig 4.1 The sketch of the network

(i) Spatial distances S_{12}

$$S_{12} = (x_{12}^{*2} + y_{12}^{*2} + z_{12}^{*2})^{1/2} \quad (4.1)$$

(ii) Vertical angles β_{12}

$$\sin \beta_{12} = \frac{z_{12}^*}{S_{12}} \quad (4.2)$$

(iii) The ellipsoidal azimuth, α_{12}

$$\tan \alpha_{12} = \frac{y_{12}^*}{x_{12}^*} \quad (4.3)$$

$$\text{with } x_{12}^* = x_2^* - x_1^*, \quad y_{12}^* = y_2^* - y_1^*, \quad z_{12}^* = z_2^* - z_1^* \quad (4.4)$$

The elements of the deflection of the vertical were obtained as explained in section 4.5. These parameters were then used to convert the geodetic quantities of latitude φ , longitude λ , and azimuth α , into their corresponding astronomic quantities, Φ , Λ , A , according to the following equations e.g Heiskanen & Moritz, 1967, pp.187]

$$\Phi = \varphi + \xi \quad (4.4a)$$

$$\Lambda = \lambda + \eta \sec \varphi \quad (4.4b)$$

$$A = \alpha + \eta \tan \varphi \quad (4.4c)$$

The computed observations were perturbed - each according to the assigned standard error. Thus,

$$y_i = \mu \pm \sigma \cdot z \quad (4.5)$$

where y_i is the perturbed observation, μ the true observation, σ the associated standard error of that observation and z is a random number. The random numbers were generated by a function in the Mainframe Computer (VAX 6310)

and the actual perturbations computed according to (4.5) using a computer program listed in appendix C1. The weights were computed as explained in the next section.

4.3 Weighting of observations.

Weights are related to the variance in the following way

$$w_i = \frac{\sigma_0^2}{\sigma_i^2} \quad (4.6)$$

where σ_0^2 is the variance of unit weight, also called variance factor or sometimes the variance component. σ_i^2 is the variance of observation i . Using the matrix notation, we may represent the weight matrix, W as

$$W = \sigma_0^2 \Sigma_{yy}^{-1} \quad (4.7)$$

where Σ_{yy} is the variance covariance matrix of the observations. Therefore from (4.6) we note that the problem of weight determination reduces to that of determining the standard errors for the observations.

In this section are discussed the various ways used for assigning standard errors to the different observations.

4.3.1 Gravity potential differences.

Gravity potential differences can be obtained from precise leveling observations where gravity values are also measured (Heiskanen & Moritz, p.160-162). The accuracy of precise leveling is quite high. The standard error per kilometre can reach ± 0.3 to 1.0 mm [Mueller et al, 1979].

The United States standard for national vertical control for first order work is $3\text{mm}\sqrt{K}$ for class I, where K is the total leveled distance in kilometres [Mueller et al 1979]. For this first order work, the accuracy of the gravity potential requirement is stated to be $\pm 2 \times 10^{-3} \text{ m s}^{-2}$. From simple

calculation on error propagation and assuming K is not large, the accuracy of the geopotential number obtained is of the order 10^{-3} m s^{-2} .

In this study the a priori standard error for the potential difference was adopted to be $\pm 5 \times 10^{-3} \text{ m s}^{-2}$.

4.3.2 Gravity differences.

The standard errors of the International Gravity Standardisation Net 1971 (IGSN71) is quoted to be less than $\pm 0.1 \times 10^{-5} \text{ ms}^{-2}$ for some values [Torge, 1980] while the global relative standard error of the network scale is $\pm 2 \times 10^{-5} \text{ ms}^{-2}$. In case of gravity difference measurement with gravimeters, a standard error of approximately ± 0.01 to $\pm 0.05 \times 10^{-5} \text{ ms}^{-2}$ can be obtained [Torge, 1980]. This can further be improved to $\pm 5 \times 10^{-6} \text{ ms}^{-2}$ by using LaCoste Romberg gravimeters [MaConnell et al, 1975]. With this wide choice of accuracy, it seems that one would still be within limits if he uses a value of $\pm 1 \times 10^{-7} \text{ ms}^{-2}$ which is used in the present study.

4.3.3 Astronomic latitudes.

The standard error for astronomic latitude is estimated to be $0''.33$ [e.g Robbins, 1976]. Other values quoted are $\pm 0''.25$, and $\pm 0''.2$. A value of $\pm 0''.3$ was used throughout the computations as the standard error for astronomic latitude.

4.3.4 Astronomic longitudes.

Aduol (1981) used the a priori standard error of astronomic longitude as $\pm 0.5 \text{ sec}$. This value had been estimated by Robbins (1976). In Torge (1980) an accuracy of about $\pm 0''.5$ - $1''.0$ is attainable for longitude observations. For the present study the value adopted is $\pm 0''.5$ since the variation of longitudes in the network considered is very small - less than 1° .

4.3.5 Astronomic azimuths

Bomford (1980) estimates the standard error for azimuth as $\pm 1''.0$. Aduol (1981) used a value of $\pm 0''.7$ which he had found consistent with values from various sources [Davies et al, 1971], Ordnance Survey, Stolz (1972), and others. The value of $\pm 0''.7$ is adopted for this study.

4.3.6 Vertical angles

The vertical angle can be observed with a standard error of the random component of about $\pm 0''.4$ to $\pm 0''.6$ [Hradilek, 1984]. However this value may deteriorate to about $\pm 1''.2$ to $3''$ owing to systematic effects. The overall standard error of a vertical angle can be expressed as

$$S_v^2 = S_o^2 + S_r^2 \quad (4.8)$$

where S_v is the standard error of observed vertical angle, S_o and S_r are the effects due to the random and refractive effects. Aduol (1981) used a value of ± 0.001 for standard error of the refraction component which he had found from the results of Hradilek (1973) and Ramsayer (1969). Since the present study is conducted using purely simulated data, it is free from refractive influences and it was found convenient to use a common value of $\pm 1''$ for standard error of the vertical angle.

4.3.7 Spatial distances.

The most precise distances are measured with electronic distance measuring instruments (EDMs), such as a Mekometer ME 3000. The standard error of these instruments consists of two parts, a constant part and the observational part. The observational part is dependent on the length of the measured line whereas the constant part is the same for a particular instrument. Rueger (1983) had estimated the Mekometer

precision as $\pm(0.38\text{mm}+0.35\text{ppm})$.

In this study, distances are considered to have been measured using an instrument of the Mekometer type with the constant part as 0.0004m and the observational part as 1 part per million (ppm). The standard error σ_l is then given by

$$\sigma_l = \pm [0.0004^2 + (1/10^6)^2]^{1/2} \quad (4.9)$$

where l is the length of the observation line.

4.3.8 Horizontal directions.

Using a geodetic theodolite e.g DKM3 Theodolite, horizontal directions can be obtained within $\pm 0''.2$ to $\pm 0''.4$ after station adjustment [Torge,1980]. Bomford (1971) quotes a priori standard errors for horizontal direction range between $\pm 0''.5$ to $\pm 1''.0$. In Aduol (1989) a value of $\pm 0''.7$ had been used.

In the present study, the standard error adopted was $0''.5$ since it is easily attainable and would provide precise results.

4.4 The computer program for simulation of observations.

The observations generated by this program are

- (1) gravity potential difference
- (2) gravity difference
- (3) astronomic latitude
- (4) astronomic longitude
- (5) astronomic azimuth
- (6) vertical angles
- (7) spatial distances
- (8) horizontal directions

The program was written on a mainframe computer (VAX 6310) in FORTRAN language. The program consists of essentially four parts: the first part involves reading the data from a data input file, the second part performs the computation of various observations associated with the observation lines read in the first part. The third part consists of a perturbation routine which transforms the computed data into "field data". The last part consists of various subroutines that are needed. A flow chart for the program is presented in appendix B.2 and the program listing in appendix C1.

4.5 Elements of the deflection of the vertical

The elements of the deflection of the vertical are commonly decomposed into two parts: the north-south component, ξ and the east-west component, η . The mathematical models of Chapter three require that approximate values of these elements be known before the adjustment is made.

The deflection of the vertical elements can be determined from astronomic and geodetic measurements [e.g. Heiskanen & Moritz, 1967 pp.223] as

$$\xi = \Phi - \varphi \quad (4.10a)$$

$$\eta = (\lambda - \Lambda) \cos \varphi \quad (4.10b)$$

or by using the gravity anomalies, Δg as given by the Vening Meinesz formulae [e.g. Heiskanen & Moritz, 1967 pp.114]

$$\xi = \frac{1}{4\pi\gamma} \int_0^\pi \int_0^{2\pi} \Delta g \sin \psi \frac{dS(\psi)}{d(\psi)} \cos \alpha \, d\psi \, d\alpha \quad (4.11a)$$

$$\eta = \frac{1}{4\pi\gamma} \int_0^\pi \int_0^{2\pi} \Delta g \sin \psi \frac{dS(\psi)}{d(\psi)} \sin \alpha \, d\psi \, d\alpha \quad (4.11b)$$

The deflection of the vertical values obtained by using equations (4.11) have the same sign as those of equations

equations (4.11) have the same sign as those of equations (4.10) but differ in that the astrogeodetic deflections are oriented with respect to the geodetic coordinate system. Computations of equations (4.11) require values of gravity anomaly to be known and also the computation is usually lengthy and difficult.

A computer program in FORTRAN for computation of gravimetric quantities from high degree spherical harmonic function is given by Rapp (1982). However this program was not used as the values obtained are of the type (4.11). For this study, the elements of the deflection of the vertical were computed from astronomic coordinates which have been computed from the gravity potential [e.g Ashkenazi 1983]. The values used are shown in tables 5.2.

$$\bar{\varphi} = \tan^{-1} \left[\frac{Z}{\sqrt{(X^2 + Y^2)}} \right] \quad (4.12a)$$

$$\Lambda = \tan^{-1} \left[\frac{Y}{X} \right] \quad (4.12b)$$

Equations (4.10) are then applied.

4.6 The free network solution

Since distances and azimuth observations were present, the network needed to be controlled only in the translational elements namely X, Y, Z or φ , λ , h. Therefore the restriction matrix of equation (2.18) becomes

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & \dots & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & \dots & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & \dots & 0 & 0 & 1 \end{bmatrix} \quad (4.13)$$

2. However, in the present computations the datum was defined over approximate coordinates in curvilinear form, that is in ϕ, λ, h . The G matrix was therefore transformed to G_{ϕ} to conform with the datum coordinates as

$$G_{\phi} = GJG' \quad (4.14)$$

where J is a 3x3 coefficient matrix defined as

following table in order of rows:

$$J = \begin{bmatrix} \partial x / \partial \phi & \partial y / \partial \phi & \partial z / \partial \phi \\ \partial x / \partial \lambda & \partial y / \partial \lambda & \partial z / \partial \lambda \\ \partial x / \partial h & \partial y / \partial h & \partial z / \partial h \end{bmatrix} \quad (4.15)$$

(1) second column

(2) first column of the matrix

In the second table, the following matrix is formed. The elements of this matrix are the partial derivatives of the coordinates with respect to the datum coordinates. These elements are as indicated below:

The first column, which has three elements, is the partial derivative of the coordinates with respect to the datum coordinate ϕ . The second column has three elements, the partial derivatives of the coordinates with respect to the datum coordinate λ . The third column has three elements, the partial derivatives of the coordinates with respect to the datum coordinate h . The elements of the matrix are as indicated below:

The first column of the matrix is the partial derivative of the coordinates with respect to the datum coordinate ϕ . The second column has three elements, the partial derivatives of the coordinates with respect to the datum coordinate λ . The third column has three elements, the partial derivatives of the coordinates with respect to the datum coordinate h .

The second column of the matrix is the partial derivative of the coordinates with respect to the datum coordinate λ . The third column has three elements, the partial derivatives of the coordinates with respect to the datum coordinate h .

CHAPTER FIVE

THE NETWORK DESIGN AND COMPUTATIONS

5.1 Introduction

The establishment of a geodetic network includes the following steps in order of execution:

- (1) field reconnaissance
- (2) design of the network
- (3) marking of the points
- (4) carrying out the observations
- (5) network adjustment
- (6) interpretation of the results.

In the second step, of network design, Grafarend (1970) has identified four orders in the design of a survey network. These orders are as outlined below:

The first order, called the Zero Order Design (ZOD) is the search for the optimal datum. The datum is usually determined by the nature of the problem, for example in absolute networks, the reference stations provide for the datum. In the present study the datum is defined through the approximate coordinates that have been used, within the framework of a free network.

The First Order Design (FOD) is the next level and refers to the configuration of the network, where the positioning of the points and the observation plan are to be optimized.

The Second Order Design (SOD) is the weight problem, that is the distribution of various accuracies to the different observations.

The Third Order Design (THOD) refers to the optimal improvement of an already existing network by addition or deletion of points and /or observations. This stage is not considered for the present study.

The need for network design is to minimise "over surveying" as this is labour and time consuming. As already indicated, the actual position locations will not be investigated as these are constrained by other factors beyond this study, such as area topography, geology and station access.

In the approach to the solution of this problem, one recognises the three general criteria [e.g Schmitt, 1990] of precision, reliability and economy. The ideal situation is to express the three criteria as an analytic function which one then optimises. However, owing to the difficulties involved in establishing this general function, particularly in three dimensional networks, the present study adopts a different but satisfactory method.

In this study, the weights that are assigned to the observation values are considered optimal as these are based on wide experiences from various reports on different surveys. However, the main interest in this study is to find the optimal number of observations for each observation type.

The procedure adopted for optimization is by computer simulation method through the variance component estimation as explained in section 5.2.1. This procedure has an advantage in that it permits the possibility of using arbitrary decision criteria for the choice of an optimal design without having to formulate the objective functions in analytic forms.

5.2. Network simulation.

The parameter estimation model of the Gauss-Markov type is given in Chapter Two as

$$y = Ax + \epsilon \quad \text{with} \quad D(y) = D(\epsilon) = \Sigma_{yy} \quad (5.1)$$

The parameter vector is then given by

$$x = (A'WA)^{-1} A'Wy \quad (5.2)$$

and the dispersion Σ_{xx} of x is given by

$$\Sigma_{xx} = \sigma_0^2 (A'WA)^{-1} \quad (5.3a)$$

and the dispersion Σ_{yy} of the estimate y of y is given by

$$\Sigma_{yy} = A \Sigma_{xx} A' \quad (5.3b)$$

and σ_0^2 is the variance of unit weight. Σ_{xx} is the covariance matrix of parameters.

For a survey network, x is the vector of the coordinates or their small corrections, y the vector of observations and A consists of the values computed from the mathematical models using the approximate station locations. The precision of the network is then assessed by analysing Σ_{xx} .

The criterion for assessment used in this study is the size of the absolute error ellipsoids. The values obtained are shown in the tables in section 5.4

Further to the analysis of the variance-covariance matrix Σ_{xx} , a posterior variance of unit weight $\hat{\sigma}_0^2$ is also computed from [e.g. Mikhail, 1976]

$$\hat{\sigma}_0^2 = \frac{\hat{\epsilon}' W \hat{\epsilon}}{(n+r-u)} \quad (5.4)$$

where n is the number of observations, r is the number of restrictions and u is the number of parameters. The a priori value of σ_0^2 is taken as unit.

5.2.1 The variance component estimation

Consider the Gauss Markov model [Aduol, 1989]

$$y = Ax + \varepsilon_y \quad (5.5)$$

with $E(\varepsilon) = 0$, $D(\varepsilon) = \Sigma_{\varepsilon\varepsilon} = D(y) = \Sigma_{yy}$

In order to solve a survey network, one normally observes different types of observations such as distances, angles, directions, vertical angles, azimuths etc. In this case then, one may consider the vector y of (5.5) to consist of several subvectors y_l , with each subvector containing a different observation type. Associated with each subvector y_l , is a variance component for that observation type σ_{ol}^2 . Equation (5.5) would then be represented as

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_k \end{bmatrix} = \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_k \end{bmatrix} x + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_k \end{bmatrix} \quad (5.6)$$

with $\varepsilon \sim (0, \Sigma_{\varepsilon}) = (0, \sigma_{ol}^2 W^{-1})$, $l = 1(1)k$, for k observational types.

The dispersion of y , $D(y)$ is obtained as

$$D(y) = D\left(\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_k \end{bmatrix} \right) = \begin{bmatrix} \sigma_{o1}^2 W_{11}^{-1} & & & 0 \\ & \sigma_{o2}^2 W_{22}^{-1} & & 0 \\ & & \ddots & \\ 0 & & & \sigma_{ok}^2 W_{kk}^{-1} \end{bmatrix} \quad (5.7)$$

the parameters. Therefore the number of measurements for each observation type can be altered depending on its contribution.

Before the number of observations are altered, it is necessary to set some accuracy criteria that the system must achieve. In the next section some precision criteria are reviewed and a choice as to which criterion to use is made.

Of importance is the question of which particular observation should one drop out during the optimization. A possible solution is to ensure that the remaining observations are uniformly distributed as much as is possible.

In this study, the remaining observations were kept uniformly distributed as much as was possible. The observations whose residuals appeared greater, though within acceptable limits, were also eliminated.

With regard to the astronomic observations, namely latitude, longitude and azimuth, special attention was given. The requirement was that these quantities be observed from the same stations as much as possible since this could minimise travelling expenses and again it is sometimes possible to observe these quantities simultaneously.

5.3 The precision criteria.

The quality of the adjusted coordinates of the network are analysed via the network's a posterior variance-covariance matrix according to equation (2.3)

$$D(\hat{x}) = (A' \Sigma_{yy}^{-1} A)^{-1} \cdot \sigma_0^2 \quad (5.15)$$

where σ_0^2 is the variance of unit weight.

5.3.1 Positional standard errors.

The positional standard errors of the parameters (i.e. coordinates) are obtained by simply taking the square roots of the diagonal elements of the variance-covariance matrix, $D(x)$. The size of the standard errors are dependent on the chosen datum [e.g Cross, 1979]. For this reason, the positional standard errors are not quite useful as tools of network analysis. However, in this study, they have been computed since the network was adjusted on the basis of a free network where no coordinates had been fixed (see section 2.1.3).

5.3.2 Spherical standard error and spherical probable error.

Spherical standard error, σ_s is a single measure for three dimensional cases and is given by [Mikhail, 1976]

$$\sigma_s = \frac{1}{3} (\sigma_x + \sigma_y + \sigma_z) \quad (5.16a)$$

where σ_x , σ_y , and σ_z are the standard errors in X, Y, and Z respectively.

Closely associated with σ_s is the spherical probable error SPE defined as [Mikhail, 1976]

$$\text{SPE} = 0.513(\sigma_x + \sigma_y + \sigma_z) \quad (5.16b)$$

for $0.35 \leq (\sigma_z / \sigma_x) \leq 1.0$

5.3.3 Mean radial spherical error (MRSE)

This is defined as [Mikhail, 1976]

$$\text{MRSE} = \sqrt{(\sigma_x^2 + \sigma_y^2 + \sigma_z^2)} \quad (5.17)$$

5.3.4 Standard error ellipsoids.

Using matrix and vector notation and simplifying, the function

$$X' \Sigma^{-1} X = 1 \quad (5.18)$$

where X is the vector of parameters and Σ is the covariance matrix. Equation (5.18) represents the equation of the standard error ellipsoid. The semi axes of this error ellipsoid are the square roots of the eigenvalues of the variance-covariance matrix and their directions are computed from the eigenvectors of those eigenvalues [Hirvonen, 1971].

The probability of a point falling on this standard ellipsoid is 19.9% [Mikhail, 1976]. The advantage of using the error ellipsoid as a tool for network analysis is that the values obtained for the computation of the ellipsoid are derived from the whole set of the variances and covariances unlike in single measure criteria where only the diagonal elements are considered.

5.3.5 Concluding remarks.

From the foregoing, one notes that the single precision criteria uses only part of the information concerning the precision of the network and as such is therefore not a very good precision criteria to use. On the other hand, the error ellipsoids make full use of all information concerning the precision of a network.

Therefore for this study the values of the error ellipsoids are computed and as a single precision criteria the spherical standard error σ_s is also computed. And since the mathematical models being tested are required for precise monitoring networks we set as a general requirement that each

of the axes of the error ellipsoids do not exceed 10mm.

5.4 The Results of computations for initial epoch.

5.4.1 Test I

Test I was the fully observed model and was considered to contain the highest number of observations (not necessarily all possible observations). The present network consisted of 26 possible observation lines for double observations and 6 possible observations for point observations. All possible observations could be used during the computations but since it is not always possible (owing to field conditions) to observe all possible lines the following choice was made: all six point observations and 24 double line observations were assumed to have been possible to observe.

The full model was considered to yield results of highest accuracy. The observations used for this model are shown in Tables 5.1 and the approximate coordinates together with the elements of the deflections of the vertical are shown in Tables 5.2. The results of the computations for this model are given in Tables 5.3, 5.4 and 5.5.

In Table 5.3, the trace refers to the redundant observations as computed according to equation (5.13). The "observations used", here refers to the actual number of observations that were used in contributing towards the estimation of the unknown parameters. The rest of the other observations were only necessary for improving the adjusted observations and are here referred to as "observations left".

Table 5.1 The simulated observations of the full model

(a)

STATION	ASTRONOMIC LATITUDE			ASTRONOMIC LONGITUDE		
1	-0°	51	38 .330	36°	19	18 .023
2	-0	51	13 .523	36	17	59 .570
3	-0	52	08 .561	36	19	34 .531
4	-0	52	05 .417	36	17	58 .453
5	-0	53	10 .269	36	19	32 .273
6	-0	53	03 .861	36	18	15 .211

(b)

LINE	DISTANCE (metres)	HORIZONTAL DIRECTION			VERTICAL ANGLE		
1 2	2550.2932	287°	33	36.38	-3°	-48	-39.68
1 3	1106.5828	151	20	30.06	-15	-41	-10.90
1 4	2609.1804	251	11	46.19	-4	-53	-52.32
1 5	2893.3510	171	11	1.25	-5	-59	-3.52
2 1	2550.2929	107	33	37.91	3	47	17.43
2 3	3397.3061	120	6	5.88	-2	-12	-35.87
2 4	1606.9482	181	14	37.00	-1	-54	-21.31
2 6	3448.4205	171	56	22.06	-1	-16	-48.61
3 1	1106.5826	331	20	29.88	15	40	36.66
3 2	3397.3041	300	6	4.25	2	10	46.10
3 4	2974.3997	271	52	4.44	1	28	2.67
3 5	1910.0090	182	5	50.06	0	-3	-56.02
4 1	2609.1800	71	11	47.25	4	52	28.37
4 2	1606.9473	1	14	37.23	1	53	29.14
4 3	2974.3996	91	52	5.81	-1	-29	-38.39
4 5	3528.4614	124	39	1.75	-1	-17	-41.76
4 6	1880.7889	164	0	12.06	0	-42	-17.70
5 1	2893.3513	351	11	1.38	5	57	30.30
5 3	1910.0093	2	5	49.92	0	2	53.91
5 4	3528.4620	304	39	0.25	1	15	47.57
5 6	2392.4915	274	44	40.69	1	19	41.27
6 3	2991.6447	55	6	41.52	-1	-2	-52.00
6 4	1880.7901	344	0	12.25	0	41	16.29
6 5	2392.4917	94	44	42.13	-1	-20	-58.32

(c)

LINE		ASTRONOMIC AZIMUTH		
1	5	171	11	1.25
2	3	120	6	5.88
5	3	2	5	49.82
6	5	94	44	42.28
5	4	304	38	39.25
1	2	287	33	36.25
3	5	182	5	50.06
4	1	71	11	46.08
4	2	1	15	28.60
4	6	163	59	24.94

(d)

LINE		POTENTIAL DIFFERENCE ($\text{m}^2 \text{s}^{-2}$)	GRAVITY DIFFERENCE (ms^{-2})
1	2	1653.98265760	0.00051767
1	3	2929.62462040	0.00091735
1	4	2176.91591540	0.00068274
1	5	2950.19104020	0.00092570
2	1	-1653.98345620	-0.00051784
2	3	1275.64002700	0.00040046
2	4	522.93370600	0.00016442
2	6	748.62703420	0.00023547
3	1	-2929.62526580	-0.00091849
3	2	-1275.64248020	-0.00039987
3	4	-752.71028380	-0.00023611
3	5	20.56379880	0.00000642
4	1	-2176.91321700	-0.00068228
4	2	-522.93053260	-0.00016419
4	3	752.71046880	0.00023593
4	5	773.27358080	0.00024239
4	6	225.69636540	0.00007033
5	1	-2950.18804320	-0.00092384
5	3	-20.56612000	-0.00000719
5	4	-773.27510320	-0.00024224
5	6	-547.58032160	-0.00017211
6	3	527.01113480	0.00016430
6	4	-225.69667800	-0.00007043
6	5	547.58018520	0.00017257

Table 5.2 Approximate coordinates

(a)

STN	UTM COORDINATES		GEODETIC COORDINATES			HEIGHT h (m)
	X (m)	Y	ϕ	λ		
1	9904334.88	201888.29	-0° 51 59.55	36° 19 18.02		2207.900
2	9904890.75	199460.91	-0 51 34.56	36 17 59.58		2038.900
3	9903189.63	202400.07	-0 52 29.98	36 19 34.54		1908.800
4	9903284.36	199426.75	-0 52 26.83	36 17 58.44		1985.700
5	9901279.96	202331.63	-0 53 32.11	36 19 32.28		1906.900
6	9901476.98	199946.84	-0 53 25.67	36 18 15.21		1962.800

(b)

STN	ELLIPSOIDAL CARTESIAN COORDINATES			DEFL. OF THE VERT.	
	X	Y	Z	ξ	η
1	5140177.718	3778830.147	-95838.194	21.213	0.000
2	5141487.484	3776781.987	-95067.973	21.044	0.000
3	5139622.594	3779056.330	-96768.421	21.421	0.000
4	5141446.194	3776707.306	-96672.831	21.400	0.000
5	5139638.794	3778981.400	-98676.885	21.844	0.000
6	5141097.793	3777095.873	-98479.939	21.800	0.000

TABLE 5.3 The fully observed network.

	type of observation	total no. of obser.	obser. used	obser. left
1.	potential differences	24	4.962 (20.7%)	19.038 (79.3%)
2.	gravity differences	24	0.0 (0.0%)	24 (100%)
3.	astro latitudes	6	5.803 (96.7%)	0.197 (3.3%)
4.	astro longitudes	6	5.775 (96.3%)	0.225 (3.7%)
5.	astro azimuths	10	0.099 (1.0%)	9.901 (99.0%)
6.	vertical angles	24	0.417 (1.7%)	23.583 (98.3%)
7.	spatial distances	24	6.447 (26.9%)	17.553 (73.1%)
8.	horizontal directions	24	9.496 (39.6%)	14.504 (60.4%)

Table 5.4 The estimated coordinates for the full model

STATION	LATITUDE (ρ)			LONGITUDE (λ)			HEIGHT (m) h
1	- 0°	51	59.550	36°	19	18.020	2207.901
2	- 0	51	34.561	36	17	59.581	2038.899
3	- 0	52	29.982	36	19	34.541	1908.799
4	- 0	52	26.821	36	17	58.440	1985.701
5	- 0	53	32.110	36	19	32.281	1906.900
6	- 0	53	25.676	36	18	15.211	1962.799

Table 5.5 The parameters of the error ellipsoids for the full model

station	axes (m)	azimuth (deg)	v. angle (deg)	σ_{ϵ} (m)
1	0.0090	235.6	1.6	0.0065
	0.0020	173.8	-86.6	
	0.0063	145.6	3.0	
2	0.0097	201.8	2.1	0.0069
	0.0022	164.5	-87.4	
	0.0067	111.8	1.6	
3	0.0078	215.8	2.1	0.0058
	0.0019	178.3	-87.3	
	0.0060	125.7	1.6	
4	0.0079	201.1	2.1	0.0057
	0.0018	168.8	-87.5	
	0.0057	111.0	1.3	
5	0.0097	199.5	1.7	0.0068
	0.0020	161.5	-87.8	
	0.0064	109.4	1.3	
6	0.0098	229.2	1.6	0.0072
	0.0023	163.6	-86.0	
	0.0073	139.1	3.6	

5.4.2 Test II

Table 5.3 shows that various observation types contributed differently towards the estimation of the unknown parameters. Clearly, gravity difference observations were not required at all in the adjustment since their contribution was 0%. These

were then eliminated altogether from the adjustment. A low percentage of astronomic azimuth observations was required and therefore azimuth observations were reduced to just about the required number which was one. For practical reasons the astronomic azimuth observations used in this test were two.

The vertical angle observations were not very much needed in the adjustment. For this reason their number was reduced to six. Potential differences were highly used and therefore the full number was considered optimum. Spatial distances and horizontal directions were also used in larger proportions and since these are not difficulty to measure, it was decided that the full number of the observations be used.

Test II was then carried out using the new set of measurements in each of the eight observation types. The results are shown in Tables 5.6, 5.7 and Table 5.8.

Table 5.6 The Optimised network

	type of observation	total no. of obser.	obser. used	obser. left
1.	potential differences	24	4.985 (20.8%)	19.015 (79.2%)
2.	gravity differences	0	-	-
3.	astro latitudes	6	5.918 (98.6%)	0.082 (1.4%)
4.	astro longitudes	6	5.697 (95.0%)	0.303 (5.0%)
5.	astro azimuths	2	0.016 (0.8%)	1.984 (99.2%)
6.	vertical angles	6	0.274 (4.6%)	5.726 (95.4%)
7.	spatial distances	24	6.521 (27.2%)	17.479 (72.8%)
8.	horizontal directions	24	9.589 (40.0%)	14.411 (60.0%)

Table 5.7 The estimated coordinates of the optimised network

STATION	LATITUDE		LONGITUDE			HEIGHT (m)
1	- 0° 51	59.550	36° 19	18.020		2207.900
2	- 0 51	34.561	36 17	59.580		2038.900
3	- 0 52	29.982	36 19	34.540		1908.800
4	- 0 52	26.821	36 17	58.440		1985.700
5	- 0 53	32.110	36 19	32.281		1906.900
6	- 0 53	25.676	36 18	15.211		1962.800

Table 5.8 Parameters of the error ellipsoids for the optimised network

station	axes (m)	azimuth (deg)	v. angle (deg)	σ_s (m)
1	0.0057	245.4	1.6	0.0045
	0.0013	189.6	-87.2	
	0.0051	155.3	2.3	
2	0.0066	184.1	2.2	0.0048
	0.0014	182.6	-87.8	
	0.0049	94.0	0.1	
3	0.0052	348.7	-2.1	0.0041
	0.0012	199.2	-87.5	
	0.0045	78.8	-1.2	
4	0.0055	351.1	-2.0	0.0040
	0.0012	193.3	-87.8	
	0.0041	81.1	-0.8	
5	0.0066	183.1	1.8	0.0047
	0.0014	180.9	-88.2	
	0.0046	93.1	0.1	
6	0.0063	225.9	2.0	0.0050
	0.0016	174.7	-86.9	
	0.0057	135.8	2.4	

5.5 Epoch II results.

In this second epoch of observations, point 4 of the network was deliberately displaced by 0".001 (about 30mm) in latitude, 0".001 in longitude and 15mm in height. Thus point 4 was taken to be the only unstable point of the object network. The task was now to estimate the new point position together with the point velocity including the auxiliary

parameters. At first the network was computed as free network and it was noticed that the displacements that had been injected into point 4 could not be recovered.

The network was then computed as a fixed one with all other points fixed except point 4. The results of these computations are shown in Table 5.9,

Table 5.9 Results of Epoch II observations

	latitude	longitude	height
old coordinates	-0° 52'26".820	36° 17'58".450	1985.700 m
new coordinates	-0 52 26.820	36 17 58.450	1985.700
std. error for new coordinates	0".000712	0".000839	0.0075m
point velocity	-0.00032 "/yr (\approx 10mm)	0.00179 "/yr (\approx 55mm)	-0.0150 m/yr
std. error for point velocity	0.00100 "/yr	0.001188 "/yr	0.0106 m/yr

5.6 Epoch III Results

This is the third epoch of observations. Here, again point 4 of the network was purposely shifted by a further 0".001 of arc in both latitude and longitude and by 15mm in height. A different set of observations was computed and these observations were then used in the kinematic model to estimate the coordinates of point 4, the velocity of movement and also the acceleration.

The network was computed as a fixed one with all other points fixed except station 4. The results of this computation are shown in Table 5.10

Table 5.10 Results of Epoch III observations

	latitude	longitude	height
old coordinates	-0° 52'26".820	36° 17'58".450	1985.700 m
new coordinates	-0 52 26.820	36 17 58.450	1985.700
std. error for new coordinates	0".00593	0".00700	0.06290 m
point velocity	-0.00037 "/yr (\approx 11mm)	0.00175 "/yr (\approx 54mm)	-0.0149 m/yr
std. error for point velocity	0.0084 "/yr	0.0099 "/yr	0.0890 m/yr
point acceleration	-0.00055 "/yr ²	0.00040 "/yr ²	-0.0068m/yr ²
std. error for acceleration	0.00854 "/yr ²	0.01012 "/yr ²	0.08900m/yr ²

5.7 Epoch IV Results

This epoch consisted of the fourth set of observations. Station 4 was again allowed to move by a further amount of 0".001 in longitude only. The new set of observations was simulated and used in the kinematic model to estimate point 4 position, velocity and acceleration.

The computation of the network is carried out on the basis of a fixed network. The results of this computation are shown in Table 5.11

Table 5.11 Results of Epoch IV observations

	latitude	longitude	height
old coordinates	-0° 52'26".820	36° 17'58".450	1985.700 m
new coordinates	-0 52 26.820	36 17 58.450	1985.700
std. error for new coordinates	0".006433	0".00758	0.0687 m
point velocity	-0.00051 "/yr (\approx 16mm)	0.00156 "/yr (\approx 48mm)	-0.0149 m/yr
std. error for point velocity	0.00914 "/yr	0.01078 "/yr	0.0973 m/yr
point acceleration	-0.00006 "/yr ²	0.00170 "/yr ²	-0.0109 m/yr ²
std. error for acceleration	0.00809 "/yr ²	0.00958 "/yr ²	0.08422 m/yr ²

5.8 Epoch V Results.

Station 4 was again allowed to shift further by small amounts of 0".003 in latitude and 10mm in height. The new set of observations was then computed. The kinematic model was used to compute the new set of parameters namely point 4 position, it's velocity and acceleration together with other auxiliary data.

Again, the computation of this network was carried out on the basis of a fixed network. The results of this computation are shown in Table 5.12

Table 5.12 Results of Epoch V observations

	latitude	longitude	height
old coordinates	-0 52'26".820	36 17'58".450	1985.700 m
new coordinates	-0 52 26.820	36 17 58.450	1985.700
std. error for new coordinates	0".00069	0".00082	0.0075m
point velocity	-0.00067 "/yr (\approx 21mm)	0.00134 "/yr (\approx 41mm)	-0.0148 m/yr
std. error for point velocity	0.00099 "/yr	0.00117 "/yr	0.0106 m/yr
point acceleration	-0.00057 "/yr ²	0.0034 "/yr ²	-0.0181 m/yr ²
std. error for acceleration	0.00083 "/yr ²	0.00098 "/yr ²	0.0086 m/yr ²

The observations of the fifth epoch of observation were again computed on the basis of a free network, similar to the initial epoch observations. This static mode of network computation was made for purposes of comparison with the results of the initial epoch. The results obtained are shown in Tables 5.13, 5.14, 5.15 and 5.16.

Table 5.13 Estimated coordinates for Epoch V (fixed mode)

STATION	LATITUDE			LONGITUDE			HEIGHT (m)
1	- 0	51	59.550	36	19	18.020	2207.901
2	- 0	51	34.561	36	17	59.581	2038.899
3	- 0	52	29.982	36	19	34.541	1908.799
4	- 0	52	26.820	36	17	58.450	1985.700
5	- 0	53	32.110	36	19	32.281	1906.900
6	- 0	53	25.676	36	18	15.211	1962.799

TABLE 5.14 The fifth epoch free network results

	type of observation	total no. of obser.	obser. used	obser. left
1.	potential differences	24	4.991 (20.8%)	19.009 (79.2%)
2.	gravity differences	0	-	-
3.	astro latitudes	6	5.964 (99.4%)	0.036 (0.6%)
4.	astro longitudes	6	5.932 (98.9%)	0.068 (1.1%)
5.	astro azimuths	2	0.246 (12.3%)	1.754 (87.7%)
6.	vertical angles	10	0.131 (1.3%)	9.869 (98.7%)
7.	spatial distances	24	8.579 (35.8%)	15.421 (64.2%)
8.	horizontal directions	24	7.158 (29.8%)	16.842 (70.2%)

Table 5.15 Estimated coordinates for Epoch V observations (free network mode)

STATION	LATITUDE			LONGTUDE			HEIGHT (m)
1	- 0°	51	59.548	36°	19	18.022	2207.907
2	- 0	51	34.558	36	17	59.582	2038.907
3	- 0	52	29.981	36	19	34.543	1908.807
4	- 0	52	26.820	36	17	58.459	1985.667
5	- 0	53	32.110	36	19	32.281	1906.907
6	- 0	53	25.674	36	18	15.211	1962.807

Table 5.16 The parameters of the error ellipsoids (free network)

station	axes (m)	azimuth (deg)	v. angle (deg)	σ_s (m)
1	0.0096	191.9	1.2	0.0069
	0.0017	208.9	-88.8	
	0.0069	101.9	-0.4	
2	0.0127	245.4	0.4	0.0083
	0.0018	163.2	-86.9	
	0.0065	155.4	3.1	
3	0.0090	315.3	-0.6	0.0064
	0.0015	203.1	-88.4	
	0.0065	45.3	-1.5	
4	0.0085	310.3	-0.7	0.0062
	0.0015	198.0	-88.1	
	0.0065	40.3	-1.8	
5	0.0121	246.4	0.3	0.0081
	0.0017	163.8	-87.7	
	0.0067	156.4	2.3	
6	0.0123	191.4	1.0	0.0083
	0.0019	159.6	-88.8	
	0.0071	101.4	0.6	

5.9 The Main Computer Program

This section explains the program in broad terms only. The program listings are however given in appendix C2. A flow chart of the program was also prepared to aid the reader in understanding the program and is in appendix B2. Part of the results produced are listed in appendices C3 and C4.

Although separate programs were written for each epoch, these can be combined into one program which would consist of basically two controls: one for free network adjustment and the other for fixed network computation.

The program was written in FORTRAN 77 and prepared on a VAX 6310 Mainframe computer. The storage capacity for this system was large enough to accommodate the program requirements.

The program consists of 16 segments; the main segment and 15 subroutines. The main segment controls the operation of all the other segments and also forms the design matrix, the weight matrix and the observation vector. These segments are

NETWORK: This is the main segment that controls the operation of all other segments.

RTDMS: Converts radians into degrees, minutes and seconds.

NORMAL: Forms the normal equation's matrix from the design matrix and the weight matrix.

EULA: Forms the Eulerian rotation matrix.

MATINV: Inverts the normal equation matrix.

ELLIPSOID: Computes the parameters of the error ellipsoids for the network points.

RADIAN: Converts angular measurements from degrees, minutes and seconds into radians.

AZIMUTH: Computes the azimuth of a line from the coordinate differences.

TRANSP: performs matrix transposition.

ELLOR: Computes the vertical angle for an ellipsoidal radius.

VAR: Computes the variance components for various observation types.

CART: Computes the geocentric coordinates from curvilinear coordinates.

ASSIGN: Assigns the various observation lines to their corresponding codes.

JACCOB: Computes the elements of the matrix of differentials in ϕ , λ and h .

DELTA: Computes the elements of the vector of differentials in gravity potential.

TIMES: Premultiplies two matrices.

The time required in the central processing unit for running the program was within the time limit (30 cpu seconds) set by

CHAPTER SIX

DISCUSSIONS

This chapter discusses the various findings from the report, particularly the experiments and attempts to provide possible explanations to those findings. It is notified here that other discussions are also made within the report under the headings "concluding remarks".

During the optimisation of the network, the gravity difference observations were not required in the estimation of the parameters as shown in Table 5.3. Since these observations are necessary, mainly for estimation of the heights, it therefore meant that the potential difference observations contributed much more towards estimation of heights.

A small proportion of astronomic azimuth observations were "used" in the network while a relatively large proportion of astronomic latitude and astronomic longitude were "used" as seen in Tables 5.3 and 5.6. In fact less than one azimuth observation was needed. Noting that only one azimuth observation is sufficient to orient a network, it therefore meant that the remaining proportion to fulfill this requirement was contributed by the astronomic latitude and astronomic longitude.

A small proportion of vertical angle observations was required contrary to the expectations. Since vertical angles would mainly contribute towards estimation of heights, deflection of the vertical elements and refraction, it seems that the basic parameter, height, got a maximum contribution from potential difference observations. Refraction coefficients were not estimated and therefore this reduced

the requirement of vertical angles. It is suggested here that in real situations, a maximum number of vertical angles would be needed, since real data has other systematic influences such as refraction.

The optimised network turned out to be more precise than the fully observed network, to the contrary. This could be due to the fact that during the optimisation process, those observations with relatively large residuals were discarded.

The proportion of observations that were required towards the estimation of the parameters are fairly the same for both the fully observed and the optimised network as shown in Tables 5.3 and 5.6. From this, one may infer that there could be a basic contributory requirement from each observation type that must go towards estimating the parameters.

Throughout the computation of the network from the second epoch up to the fifth epoch of measurement, the new coordinates of the unstable point remained significantly unaltered. This is in line with the mathematical models used for the network computation since all network coordinates for any epoch are referred to the initial epoch coordinates.

For the second epoch of observation, one is able to estimate velocity if displacement of a point has occurred. Since the mathematical models used estimate velocity, the displacement is obtained implicitly as the product of velocity and the time that has elapsed between the two epochs of measurement.

In order to test the mathematical models, some shifts were introduced as shown in section 5.5. To a fairly good approximation, the models of adjustment recovered these shifts. This was the case for both the second and the third

epoch of observations.

The fourth epoch results were a little different, the model of adjustment was able to recover, to a good approximation, the shifts introduced in both latitude and longitude, but not height. The shift in height appeared exaggerated.

The fifth epoch results also showed that the model was able to recover, approximately, the shifts introduced in longitude and height but not latitude.

It is noted that there is a general trend for the model to slow down the movement of the point in latitude and compensate this movement in both longitude and height components. If a point is arbitrarily displaced in each of the three axes of a coordinate system, it is possible to return the point to its original position by translation on only two of the axes combined with suitable rotations. This could have happened to the adjustment of the network particularly during the fourth and the fifth epochs.

It is also noted that the third epoch of observation was able to estimate the point accelerations. This is possible since there are two velocities between the first and the third epoch of observation. In fact it was expected that the estimation of accelerations would improve as the number of epochs increased since beyond the third epoch of observation, redundancy in acceleration increased. This is the case as shown in Tables 5.9, 5.10, and 5.11.

In order to obtain very precise results for the network, the a priori standard errors of astronomic longitudes had to be improved from $0''.5$ to $0''.3$. This therefore meant that extra care has to be taken when making these observations. This

includes improved methods of observing longitudes.

At the fifth epoch, the network was again computed on the basis of a free network, just as the initial epoch. It was noted that the fifth epoch free network coordinates and the fixed coordinates differ significantly particularly in longitude, and in general all free network coordinates shifted significantly so that one is not able to pinpoint which point was unstable. All the free network heights except for the unstable point 4 were displaced by the same amount, 7mm while point 4 was shifted by about 3mm. This shows that the effect of displacement on point 4 was distributed over all the other points by the model of adjustment. This therefore means that if the network was being computed on the basis of discrete epochs, the varying coordinates of the network (due to deformation) would not provide sufficient information on which points were unstable.

CONCLUSION

This chapter summarises the work done and gives recommendations arising from the findings of the work so far reported.

7.1 Summary

Reported herein are the mathematical models for computation of localised three dimensional geodetic monitoring networks. Also considered is the network design aspect.

The mathematical models described here are based on the kinematic estimation model of geodetic network adjustment within the framework of integrated geodetic networks. The design aspect considered was the weight problem for each of the various observations used. These observations were astronomic latitude, astronomic longitude, astronomic azimuth, vertical angles, spatial distances, horizontal directions, gravity intensity differences and gravity potential differences. The main parameters computed by these models were the network coordinates based on the initial epoch observations, point velocities of the unstable network points together with the acceleration values where possible. Other auxiliary parameters were the deflection of the vertical elements and refraction coefficients.

A test network consisting of six points was computed based on the proposed mathematical models. Since the study was carried out by computer simulation, intentional shifts had to be introduced into one of the network points so as to cause some network deformation. Five observation epochs were made at a uniform interval of one year. The initial (first) epoch observation yielded network coordinates while the second

observation epoch, estimated in addition, point velocities. Third observation epoch together with the succeeding epochs each estimated network coordinates, point velocities and accelerations. Although the velocities varied from epoch to epoch, the network coordinates remained the same as those computed with the initial epoch observations. All these realizations were in support of the theory of the mathematical models used. The displacements computed from the product of velocity and time reflected the magnitudes of the intentional shifts that had been injected into the network to cause deformation.

In order to determine the number of observations needed for each type of observation, individual contribution of each type of observation required in the estimation of the parameters was computed based on the variance component estimation. From this estimated contribution, one was then able to alter the number of observations in each type of observation as necessary. For the test network, it was noted that only few (about one) of azimuth observations was needed in the adjustment of the network. Gravity values were not required in the adjustment while the other observations were needed in varying proportions as shown in Chapter Five. The computation of the initial network was repeated with the optimum number of observations and the accuracy parameters of the network did not deteriorate; an indication that the discarded elements were superfluous and therefore not required.

7.2 Conclusions

In kinematic estimation model using the integrated approach, it is necessary to observe each of astronomic latitude and astronomic longitude at all stations of the network.

Only few (about 1%) of astronomic azimuth observations are necessary in a localised monitoring network when using the kinematic model of adjustment.

Since the optimised network was of comparable accuracy with the fully observed network, it is concluded that the variance component estimation procedure through which the optimisation was made is valid and therefore acceptable.

The kinematic estimation models using the integrated approach proposed here are capable of detecting point shifts of small magnitudes: in this experiment, $0''.002$ in either latitude or longitude and about 10mm in height could be detected. The mathematical models are thus acceptable.

The kinematic estimation models provide means for continuous but pointwise monitoring of ground deformations since velocities and accelerations are estimated. This continuous monitoring can also be extended to prediction of deformation in areas concerned.

7.2 Recommendations

Although the optimisation results require that only few vertical angle observations are necessary to compute the network, it is recommended here that a maximum number of vertical angle observations should be made in order to estimate and subsequently eliminate refractive influences which the test data lacked.

It is recommended that as a further test to the kinematic estimation models, real data should be used and the stochasticity of the reference network should be taken into account.

APPENDIX A

THE PARTIAL DERIVATIVES USED IN THE OBSERVATION EQUATIONS

The notation used is similar to that used in the text. N and M are the ellipsoidal radii of curvature in the normal and in the meridian respectively and e is the first eccentricity of the reference ellipsoid.

$$\frac{\partial x}{\partial \varphi} = -(M + h)\sin\varphi\cos\lambda, \quad \frac{\partial x}{\partial \lambda} = -(N + h)\cos\varphi\sin\lambda, \quad \frac{\partial x}{\partial h} = \cos\varphi\cos\lambda \quad \dots A.1$$

$$\frac{\partial y}{\partial \varphi} = -(M + h)\sin\varphi\sin\lambda, \quad \frac{\partial y}{\partial \lambda} = -(N + h)\cos\varphi\cos\lambda, \quad \frac{\partial y}{\partial h} = \cos\varphi\sin\lambda \quad \dots A.2$$

$$\frac{\partial z}{\partial \varphi} = (M + h)\cos\varphi, \quad \frac{\partial z}{\partial \lambda} = 0, \quad \frac{\partial z}{\partial h} = \sin\varphi \quad A.3$$

$$\frac{\partial w}{\partial X} = -\frac{GMX}{r^3}, \quad \frac{\partial w}{\partial Y} = -\frac{GMY}{r^3}, \quad \frac{\partial w}{\partial Z} = -\frac{GMZ}{r^3} \quad A.4$$

$$\frac{\partial}{\partial X} \left(\frac{\partial w}{\partial X} \right) = -\frac{GM}{r^3} + \frac{3GMX^2}{r^5}$$

$$\frac{\partial}{\partial Y} \left(\frac{\partial w}{\partial X} \right) = \frac{\partial}{\partial X} \left(\frac{\partial w}{\partial Y} \right) = \frac{3GMXY}{r^5}$$

$$\frac{\partial}{\partial Z} \left(\frac{\partial w}{\partial X} \right) = \frac{\partial}{\partial X} \left(\frac{\partial w}{\partial Z} \right) = \frac{3GMXZ}{r^5}$$

$$\frac{\partial}{\partial Y} \left(\frac{\partial w}{\partial Y} \right) = -\frac{GM}{r^3} + \frac{3GMY^2}{r^5} \quad A.5$$

$$\frac{\partial}{\partial Z} \left(\frac{\partial w}{\partial Y} \right) = \frac{\partial}{\partial Y} \left(\frac{\partial w}{\partial Z} \right) = \frac{3GMYZ}{r^5}$$

$$\frac{\partial}{\partial Z} \left(\frac{\partial w}{\partial Z} \right) = -\frac{GM}{r^3} + \frac{3GMZ^2}{r^5}$$

$$\frac{\partial \alpha_{ij}}{\partial x_{ij}^*} = -\frac{y_{ij}^*}{x_{ij}^{*2} + y_{ij}^{*2}}, \quad \frac{\partial \alpha_{ij}}{\partial y_{ij}^*} = -\frac{x_{ij}^*}{x_{ij}^{*2} + y_{ij}^{*2}}, \quad \frac{\partial \alpha_{ij}}{\partial z_{ij}^*} = 0 \quad \text{A.6}$$

$$\frac{\partial \beta_{ij}}{\partial x_{ij}^*} = -\frac{x_{ij}^* z_{ij}^*}{(x_{ij}^{*2} + y_{ij}^{*2} + z_{ij}^{*2})(x_{ij}^{*2} + y_{ij}^{*2})^{1/2}}$$

$$\frac{\partial \beta_{ij}}{\partial y_{ij}^*} = -\frac{y_{ij}^* z_{ij}^*}{(x_{ij}^{*2} + y_{ij}^{*2} + z_{ij}^{*2})(x_{ij}^{*2} + y_{ij}^{*2})^{1/2}} \quad \text{A.7}$$

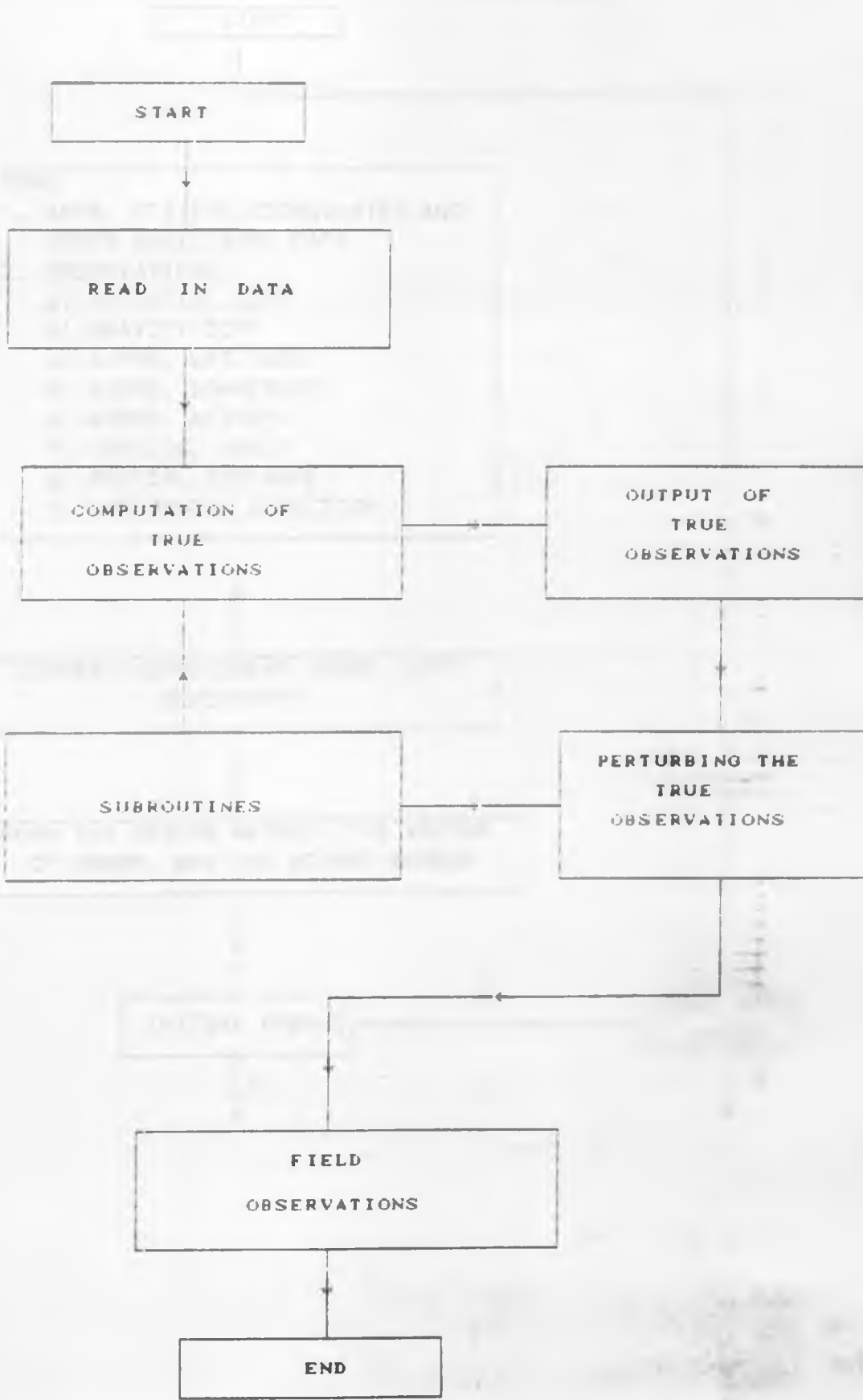
$$\frac{\partial \beta_{ij}}{\partial z_{ij}^*} = \frac{(x_{ij}^{*2} + y_{ij}^{*2})^{1/2}}{(x_{ij}^{*2} + y_{ij}^{*2} + z_{ij}^{*2})}$$

$$\frac{\partial s_{ij}}{\partial x_{ij}^*} = \frac{x_{ij}^*}{s_{ij}} = \cos \alpha_{ij} \cos \beta_{ij}$$

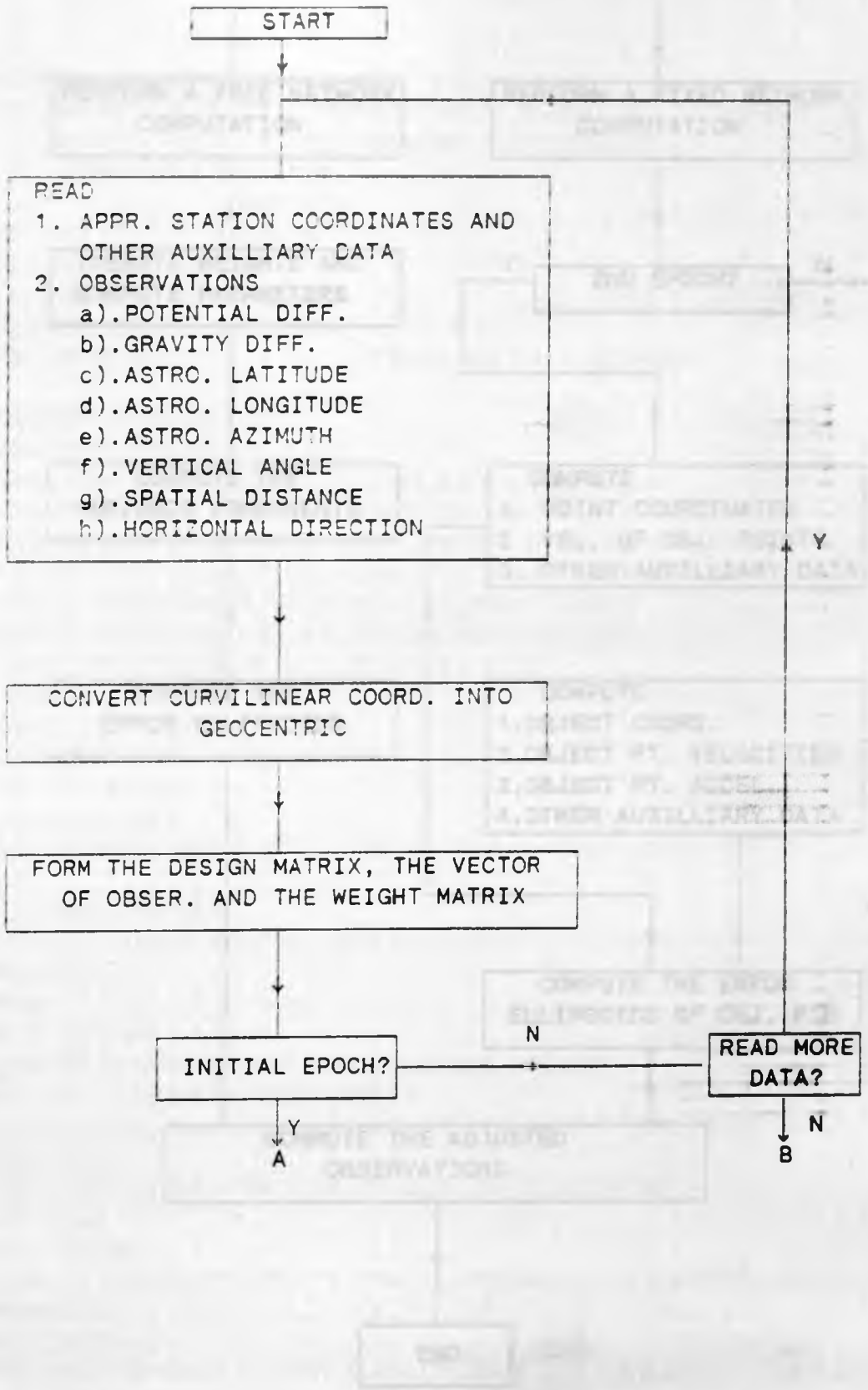
$$\frac{\partial s_{ij}}{\partial y_{ij}^*} = \frac{y_{ij}^*}{s_{ij}} = \sin \alpha_{ij} \cos \beta_{ij} \quad \text{A.8}$$

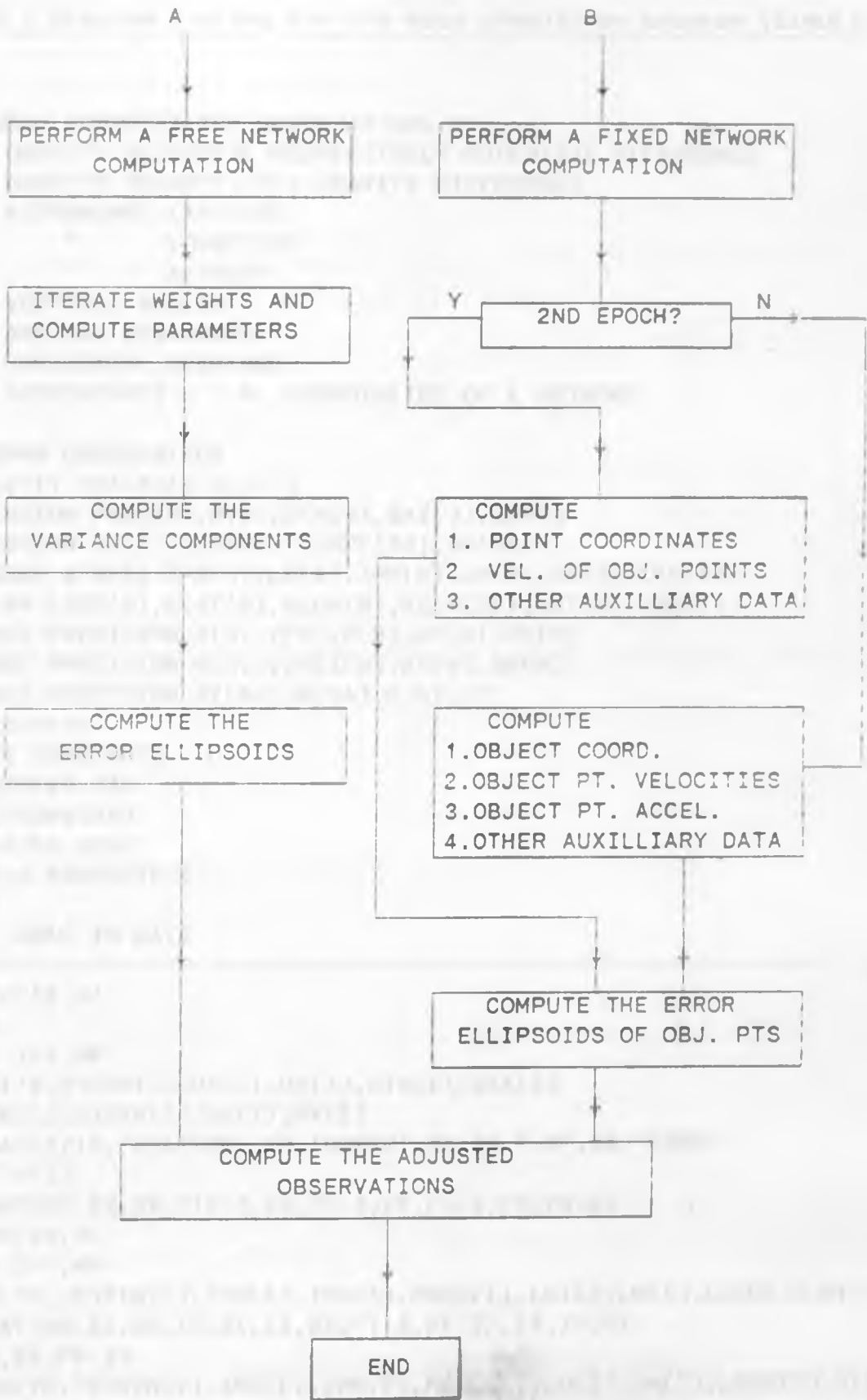
$$\frac{\partial s_{ij}}{\partial z_{ij}^*} = \frac{z_{ij}^*}{s_{ij}} = \sin \beta_{ij}$$

APPENDIX B.1 The Data Simulation Program (SIMUL)



APPENDIX B.2 The Main Program for Adjustment (ADAN)





APPENDIX C PROGRAM LISTINGS AND THE RESULTS

Appendix C.1 program Listing for the data simulation program (SIMUL)

```

C      PROGRAM COMPUTES THE OBSERVATIONS OF:
C      1. GRAVITY POTENTIAL RESPECTIVELY POTENTIAL DIFFERENCE
C      2. GRAVITY RESPECTIVELY GRAVITY DIFFERENCE
C      3. ASTRONOMIC LATITUDE
C      4.      "      LONGITUDE
C      5.      "      AZIMUTH
C      6. VERTICAL ANGLES
C      7. SPATIAL DISTANCES
C      8. HORIZONTAL BEARINGS.
C      FROM APPROXIMATE U.T.M. COORDINATES OF A NETWORK
C
C      PROGRAM OBSERVATION
C      IMPLICIT REAL*8(A-H,O-Z)
C      DIMENSION PSEC(9),H(9),ETA(9),SAI(9),GR(9)
C      DIMENSION GX(3,1),RG(3,1),GPT(54),GD(54)
C      INTEGER STN(9),IPD(9),LD(9),IPM(9),LM(9),KN(54),KF(54)
C      REAL*8 LSEC(9),ALAT(9),ALON(9),AZIM(58),LAT(9),LON(9)
C      DOUBLE PRECISION X(9),Y(9),Z(9),UX(9),UY(9)
C      DOUBLE PRECISION R(3,3),AZI(9),S(54),GP(9)
C      DOUBLE PRECISION AZ(54),B(54),E,C1,C2
C      =====
C      INPUT CONSTANTS
C      A=6378249.145
C      E=0.082483399
C      P=206264.8063
C      GPOT=3.9860047D14
C      N=24
C      A. READ IN DATA
C      -----
C      WRITE(12,4)
C      NP=6
C      DO 1 I=1,NP
C      READ(15,2)STN(I),UX(I),UY(I),ETA(I),SAI(I)
1      WRITE(12,2)STN(I),UX(I),UY(I)
4      FORMAT(//1X,'STATION',2X,'NORTH',2X,'U.T.M',2X,'EAST',
</50('=')
2      FORMAT(2X,I2,3X,F10.2,3X,F9.2,2X,F5.3,2X,F6.4)
C      WRITE(12,3)
C      DO 5 I=1,NP
10     READ(15,10)STN(I),IPD(I),IPM(I),PSEC(I),LD(I),LM(I),LSEC(I),H(I)
10     FORMAT(2X,I2,2X,I2,2X,I3,2X,F7.3,2X,I2,2X,I2,2X,
<F6.3,2X,F8.3)
C      WRITE(12,10)STN(I),IPD(I),IPM(I),PSEC(I),LD(I),LM(I),LSEC(I),H(I)
C      CALL RADIAN(IPD(I),IPM(I),PSEC(I),LAT(I))

```



```

CALL RADIAN(LD(I),LM(I),LSEC(I),LON(I))
C   CONVERT FROM CURVILINEAR GEODETIC TO GEODETIC CARTESIAN.
C1=A/SQRT(1.-(E**2*SIN(LAT(I))**2))
C2=C1+H(I)
X(I)=C2*COS(LAT(I))*COS(LON(I))
Y(I)=C2*COS(LAT(I))*SIN(LON(I))
Z(I)=(C2-C1*E**2)*SIN(LAT(I))
ALAT(I)=(ATAN(Z(I)/DSQRT(X(I)**2+Y(I)**2)))*P
ALON(I)=(ATAN(Y(I)/X(I)))*P
SAI(I)=ALAT(I)-LAT(I)*P
ETA(I)=(ALON(I)-LON(I)*P)/COS(LAT(I))
GR(I)=GPOT/(X(I)**2+Y(I)**2+Z(I)**2)
GP(I)=GPOT/DSQRT(X(I)**2+Y(I)**2+Z(I)**2)
5   CONTINUE
WRITE(12,8)
DO 21 I=1,NP
21  WRITE(12,15)STN(I),X(I),Y(I),Z(I),GR(I),GP(I)
WRITE(12,100)
DO 101 I=1,NP
CALL RTDMS(ALAT(I),MD,MM,SE1)
CALL RTDMS(ALON(I),NL,NM,SE2)
WRITE(12,104)STN(I),MD,MM,SE1,NL,NM,SE2
101 CONTINUE
104 FORMAT(2X,I2,3X,I2,2X,I3,F8.3,3X,I2,2X,I2,2X,F8.3)
100 FORMAT(//2X,'STN',5X,'ASTRO. LAT',9X,'ASTRO. LON',/50('='))
C   READ IN OBSERVATION LINES.
C   -----
DO 20 I=1,N
READ(15,27)KN(I),KF(I)
20  CONTINUE
C
C   C. VERTICAL ANGLES, GEODETIC AZIMUTH AND DISTANCE.
C   -----
DO 50 I=1,N
K1=KN(I)
K2=KF(I)
GX(1,1)=X(K2)-X(K1)
GX(2,1)=Y(K2)-Y(K1)
GX(3,1)=Z(K2)-Z(K1)
CALL ROTAT(LAT(K1),LON(K1),R)
CALL TIMES(R,GX,RG,3,3,1)
S(I)=DSQRT(RG(1,1)**2+RG(2,1)**2+RG(3,1)**2)
B(I)=(ASIN(RG(3,1)/S(I)))*P
CALL AZIMUTH(RG(2,1),RG(1,1),AZ(I))
AZIM(I)=AZ(I)+ETA(I)*TAN(LAT(I))
GPT(I)=GP(K2)-GP(K1)
GD(I)=GR(K2)-GR(K1)
50  CONTINUE
C   OUTPUT RESULTS
C   -----
WRITE(12,85)

```

```

DO 90 I=1,N
CALL RTDMS(AZ(I),IA,IM,SECS)
CALL RTDMS(AZIM(I),NI,MA,SX)
CALL RTDMS(B(I),IB,MB,SE)
WRITE(12,88)KN(I),KF(I),S(I),IA,IM,SECS,IB,MB,SE,GPT(I)
<,GD(I)
88  FORMAT(1X,I2,1X,I2,1X,F9.4,1X,I3,1X,I2,1X,F6.3
<,2X,I3,1X,I3,1X,F7.3,2X,F10.3,2X,F9.7)
90  CONTINUE
85  FORMAT(//2X,'LINE',2X,'DISTANCE',4X,'AZIMUTH',5X,
<7X,'V. ANGLE ',2X,'P.DIFF',2X,'G.DIFF',/70('='))
55  FORMAT(2X,I2,2X,I2,3X,I3,2X,I2,2X,F6.3,3X,I3,2X,I2,2X,F6.3)
45  FORMAT(2X,I2,2X,I2,4X,I3,2X,I2,2X,F6.3)
35  FORMAT(2X,I2,2X,I2,4X,F15.4)
27  FORMAT(2X,I2,2X,I2)
15  FORMAT(2X,I2,2X,F12.3,2X,F12.3,2X,F12.3,2X,F9.7,2X,F14.3)
  8  FORMAT(//1X,'STATION',4X,'X',12X,'Y',10X,'Z',3X,'GRAVITY',
<2X,'G. POTEN.',/70('='))
  3  FORMAT(//2X,'STATION',4X,'LATITUDE',5X,'LONGITUDE',5X,'HEIGHT',/
<60('='))
WRITE(12,1001)
1001 FORMAT(/5X,'STN',10X,'ETA',10X,'SAI',/5X,40('='))
DO 1002 I=1,NP
1002 WRITE(12,1003)I,ETA(I),SAI(I)
1003 FORMAT(3X,I2,3X,F15.5,2X,F15.5)
STOP
END
SUBROUTINE RTDMS(ANG,IDEG,IMIN,SEC)
C CONVERTS RADIANS INTO DEGREES, RADIANS AND SECONDS
IMPLICIT REAL*8(A-H,O-Z)
INTEGER IDEG,IMIN
IDEG=ANG/3600.
IMIN=(ANG-(IDEG*3600))/60.
SEC=(ANG-(IDEG*3600+IMIN*60))
RETURN
END
SUBROUTINE ROTAT(PI,DL,R)
C FORMS THE EULERIAN ROTATION MATRIX
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION R(3,3)
R(1,1)=-1.0*SIN(PI)*COS(DL)
R(1,2)=-1.0*SIN(PI)*SIN(DL)
R(1,3)=COS(PI)
R(2,1)=-1.0*SIN(DL)
R(2,2)=COS(DL)
R(2,3)=0.0
R(3,1)=COS(PI)*COS(DL)
R(3,2)=COS(PI)*SIN(DL)
R(3,3)=SIN(PI)
RETURN
END

```

```

C      SUBROUTINE AZIMUTH(DY,DX,AZ)
C      COMPUTES THE CORRECT AZIMUTH FOR A GEODETIC LINE
      IMPLICIT REAL*8(A-H,O-Z)
      P=206264.8063
      PI=3.141592654
      IF(ABS(DY).GT.ABS(DX))GOTO 2
      AZ=ATAN(DY/DX)
      IF(DX.LT.0.0) GOTO 3
      IF(AZ.GT.0.0) GOTO 8
      AZ=((2.0*PI)+AZ)*P
      GOTO 1
3     AZ=(PI+AZ)*P
      GOTO 1
2     AZ=ATAN(DX/DY)
      IF(DY.LT.0.0) GOTO 4
      AZ=((PI/2.0)-AZ)*P
      GOTO 1
4     AZ=((3.0*PI/2.0)-AZ)*P
1     RETURN
8     AZ=AZ*P
      RETURN
      END
      SUBROUTINE TIMES(A,B,C,II,KK,JJ)
C      PREMULTIPLIES TWO MATRICES
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(II,1),B(KK,1),C(II,1)
      DO 10 I=1,II
      DO 10 J=1,JJ
10    C(I,J)=0.0
      DO 30 I=1,II
      DO 30 K=1,KK
      AA=A(I,K)
      IF(AA.EQ.0.0)GOTO 30
      DO 20 J=1,JJ
20    C(I,J)=C(I,J)+AA*B(K,J)
30    CONTINUE
      RETURN
      END
      SUBROUTINE RADIAN(IDEG,IMIN,SEC,RAD)
C      CONVERTS DEGREES,MINUTES AND SECONDS INTO RADIANS
      IMPLICIT REAL*8(A-H,O-Z)
      TERM1=IMIN/60.0
      TERM2=SEC/3600.0
      RDEG=IDEG
      DEG=RDEG+TERM1+TERM2
      RAD=3.1415926536*DEG/180.0
      RETURN
      END
      SUBROUTINE ATRAN(A,B,M,N)
C      MATRIX TRANSPOSITION

```

```

REAL A(M,N),B(N,M)
DO 30 I=1,N
DO 30 J=1,M
30 B(I,J)=A(J,I)
RETURN
END

```

APPENDIX C.2 Program listing for the main program (ADAN)

PROGRAM NETWORK

```

IMPLICIT REAL*8(A-H,O-Z)
DIMENSION AT(17,550),ATW(17,550),W(550,550),ER(550,1)
DIMENSION ETW(1,17),ADX(550,17),A(550,17)
DIMENSION AAT(550,550),YC(550),P(550),AX(550,1)
DIMENSION ERR(550,1),SW(550,550),SET(550,550),STD(550)
DIMENSION PN(550),SEY(550),SEE(550),YL(550)
DIMENSION HT(6),HT1(6),EZ(3,1),RZ(3,1),OX1(6),OY1(6),OZ1(6)
DIMENSION DW(3,1),RC(3,3),RW(3,1),DJ(3,3)
DIMENSION ATWA(17,17),AINV(17,17),ATY(17,1)
DIMENSION DX(17,1),COVX(17,17),CC(17,17)
DIMENSION G(3,19),WEIN(48)
DIMENSION BB(17,17),F11(17,17),BV(17,17),WEN(3,3)
DIMENSION FT(17,17),FF(17,17)
DIMENSION Q(3,3),D(3,3),VAL(3),R(3),F1T(17,17)
DIMENSION RI(3,1),DI(3,1),DA(3,1),EC(3,1),QT(3,3)
DIMENSION RA(3,1),DS(3,1),DB(3,1),T(3),RJ(3,1),GPD1(24)
DIMENSION GPD(24),DIST1(24),EH1(24),ES1(24),GEO1(24),BR1(6)
DIMENSION RL1(24),AS1(4),VIN1(24),SPT1(24),DIR1(24)
DIMENSION DIST(24),RE(3,3),ETWE(1,1),EP1(24),EG1(24),EB1(6)
DIMENSION EH(24),ES(24),WCO(3,3),WE(3,3),WET(3,3),QX(3,3)
DIMENSION EA1(4)
DIMENSION GEO(24),GRAV(24),BR(6),RL(10),AS(4),VIN(24),SPT(24)
DIMENSION DIR(24),EP(24),EG(24),EB(6),EL(10),EA(4),EV(24)
DIMENSION QEE(24),SE(24),WP(24,24),XE(6),YE(6),ZE(6)
DIMENSION ZX(6),ZY(6),ZZ(6),OX(6),OY(6),OZ(6),DFX(6)
DIMENSION DFY(6),YLO(6),DFZ(6)
DIMENSION ZR(6),ZU(6),ZJ(6),WG(3,18),YP(24),YG(24),YB(24)
DIMENSION YA(4),YV(24),YD(24),YH(24),PS(24),GS(24),BS(10)
DIMENSION TS(10),AST(4),SVE(24),SDP(24),SHD(24)
DIMENSION HT3(6),BLAT3(6),BLON3(6),ETA3(6),SAI3(6),DIST3(24)
DIMENSION HOR3(24),VER3(24),AZ3(6),GPD3(24)
DIMENSION HOR4(24),VER4(24),BLAT4(6),BLON4(6),GPD4(24),AZ4(6)
DIMENSION DIST4(24)
INTEGER KH(24),JH(24),JV(24),KV(24),KH1(24),JH1(24)
INTEGER KVI(24),KII(24),KJ(24),HDEG,HMIN,VDEG,VMIN
INTEGER KI(6),JP(24),KN(24),KF(24),AD,AM,J9(10),KK(6),NZ1(6)
INTEGER NZ(6),NC(6),KG(24),JG(24),KI1(6),KA(2),KA1(2),NC1(6)
INTEGER KN1(6),KF1(6),J91(6),KK1(6),JP1(24),KV1(24),KII1(24)
INTEGER JV1(24),KI2(6),NC2(6),NZ2(6),KII2(24),JP2(24),KH2(24)
INTEGER JH2(24),KV2(24),JV2(24),J92(2),KA2(2),KA3(6),J93(6)
DIMENSION BLAT1(6),BLON1(6),ETA1(6),SAI1(6),HOR1(24),VER1(24)
DIMENSION HT2(6),OX2(6),OY2(6),OZ2(6),XAC(6),YAC(6),ZAC(6)
DIMENSION BLAT2(6),BLON2(6),SAI2(6),ETA2(6),DIST2(24),HOR2(24)
DIMENSION VER2(24),GPD2(24),AZ2(2)
REAL*8 LONSEC,LATSEC,LATI(6),LONG(6),LATI2(6),LONG2(6)
REAL*8 TE1,T1,T3,SXYZ,LATI1(6),LONG1(6),LONG3(6),LATI3(6)

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7540 FORMAT(//20X,'INPUT ITERATION')
      READ(*,7541)LEE
7541 FORMAT(I1)
      WRITE(4,36)
36  FORMAT(//2X,'STN',4X,'LATITUDE',8X,'LONGITUDE',5X,'HEIGHT')
      WRITE(4,37)
37  FORMAT(6X,'DEG MIN SEC',4X,'DEG MIN SEC',9X,'m',
</2X,70('='))
      DO 5 I=1,NP
      READ(27,6)IP,LATDEG,LATMIN,LATSEC,LONDEG,LONMIN,LONSEC,H
      HT(I)=H
      KI(I)=IP
      WRITE(4,6)KI(I),LATDEG,LATMIN,LATSEC,LONDEG,LONMIN,LONSEC,HT(I)
      CALL RADIAN(LATDEG,LATMIN,LATSEC,LATI(I))
      CALL RADIAN(LONDEG,LONMIN,LONSEC,LONG(I))
C
C  CONVERT SPHERICAL COORD. INTO EQUIVALENT GEODETIC ONES.
      CALL CART(LATI(I),LONG(I),HT(I),X(I),Y(I),Z(I))
      OX(I)=X(KI(I))
      OY(I)=Y(KI(I))
      OZ(I)=Z(KI(I))
5  CONTINUE
      DO 1115 I=1,NP
      READ(27,6)IP,LATDEG,LATMIN,LATSEC,LONDEG,LONMIN,LONSEC,H
      HT1(I)=H
      KI1(I)=IP
      CALL RADIAN(LATDEG,LATMIN,LATSEC,LATI1(I))
      CALL RADIAN(LONDEG,LONMIN,LONSEC,LONG1(I))
6  FORMAT(3X,I1,3X,I2,2X,I3,2X,F7.3,3X,I2,2X,I2,2X,F6.3,
<2X,F8.3)
C  CONVERT SPHERICAL COORD. INTO EQUIVALENT GEODETIC ONES.
      CALL CART(LATI1(I),LONG1(I),HT1(I),XE(I),YE(I),ZE(I))
      OX1(I)=XE(KI1(I))
      OY1(I)=YE(KI1(I))
      OZ1(I)=ZE(KI1(I))
1115 CONTINUE
      DO 7801 I=1,NP
      READ(27,6)IP,LATDEG,LATMIN,LATSEC,LONDEG,LONMIN,LONSEC,H
      HT2(I)=H
      KI2(I)=IP
      WRITE(4,6)KI2(I),LATDEG,LATMIN,LATSEC,LONDEG,LONMIN,LONSEC,HT2(I)
      CALL RADIAN(LATDEG,LATMIN,LATSEC,LATI2(I))
      CALL RADIAN(LONDEG,LONMIN,LONSEC,LONG2(I))
C  CONVERT SPHERICAL COORD. INTO EQUIVALENT GEODETIC ONES.
      CALL CART(LATI2(I),LONG2(I),HT2(I),XAC(I),YAC(I),ZAC(I))
      OX2(I)=XAC(KI2(I))
      OY2(I)=YAC(KI2(I))
      OZ2(I)=ZAC(KI2(I))
7801 CONTINUE
      DO 8300 I=1,NP
      READ(27,6)IP,LATDEG,LATMIN,LATSEC,LONDEG,LONMIN,LONSEC,H

```

```

HT3(I)=H
KI2(I)=IP
WRITE(4,6)KI2(I),LATDEG,LATMIN,LATSEC,LONDEG,LONMIN,LONSEC,HT3(I)
CALL RADIANT(LATDEG,LATMIN,LATSEC,LATI3(I))
CALL RADIANT(LONDEG,LONMIN,LONSEC,LONG3(I))
C CONVERT SPHERICAL COORD. INTO EQUIVALENT GEODETIC ONES.
CALL CART(LATI3(I),LONG3(I),HT3(I),XAC(I),YAC(I),ZAC(I))
OX2(I)=XAC(KI2(I))
OY2(I)=YAC(KI2(I))
OZ2(I)=ZAC(KI2(I))
8300 CONTINUE
DO 63 I=1,NB
READ(27,916)NC(I),LATD,LATM,TSEC
CALL RADIANT(LATD,LATM,TSEC,BLAT(I))
63 CONTINUE
DO 1163 I=1,NB
READ(27,916)NC1(I),LATD,LATM,TSEC
CALL RADIANT(LATD,LATM,TSEC,BLAT1(I))
1163 CONTINUE
DO 6802 I=1,NB
READ(27,916)NC2(I),LATD,LATM,TSEC
CALL RADIANT(LATD,LATM,TSEC,BLAT2(I))
6802 CONTINUE
DO 8301 I=1,NB
READ(27,916)NC2(I),LATD,LATM,TSEC
CALL RADIANT(LATD,LATM,TSEC,BLAT3(I))
8301 CONTINUE
DO 8401 I=1,NB
READ(27,916)NC2(I),LATD,LATM,TSEC
CALL RADIANT(LATD,LATM,TSEC,BLAT4(I))
8401 CONTINUE
C
DO 3070 I=1,NP
READ(27,3071)NC(I),SAI(I),ETA(I)
3071 FORMAT(2X,I1,2X,F6.3,2X,F6.3)
3070 CONTINUE
DO 307 I=1,NP
READ(27,3071)NC1(I),SAI1(I),ETA1(I)
307 CONTINUE
DO 7803 I=1,NP
READ(27,3071)NC2(I),SAI2(I),ETA2(I)
7803 CONTINUE
DO 8302 I=1,NP
READ(27,3071)NC2(I),SAI3(I),ETA3(I)
8302 CONTINUE
C
DO 3023 I=1,NL
READ(27,917)NZ(I),LOND,LONM,DSEC
917 FORMAT(3X,I1,3X,I2,2X,I2,2X,F6.3)
CALL RADIANT(LOND,LONM,DSEC,BLON(I))
3023 CONTINUE

```



```

DO 302 I=1,NL
READ(27,917)NZ1(I),LOND,LONM,DSEC
CALL RADIAN(LOND,LONM,DSEC,BLON1(I))
302 CONTINUE
DO 7804 I=1,NL
READ(27,917)NZ2(I),LOND,LONM,DSEC
CALL RADIAN(LOND,LONM,DSEC,BLON2(I))
7804 CONTINUE
DO 8303 I=1,NL
READ(27,917)NZ2(I),LOND,LONM,DSEC
CALL RADIAN(LOND,LONM,DSEC,BLON3(I))
8303 CONTINUE
DO 8403 I=1,NL
READ(27,917)NZ2(I),LOND,LONM,DSEC
CALL RADIAN(LOND,LONM,DSEC,BLON4(I))
8403 CONTINUE
C
916 FORMAT(3X,I1,3X,I2,2X,I3,2X,F7.3)
WRITE(4,29)
29 FORMAT(//2X,'STN',12X,'X',16X,'Y',17X,'Z',/2X,55('='))
DO 14 I=1,NP
WRITE(4,27)KI(I),X(I),Y(I),Z(I)
14 CONTINUE
DO 1411 I=1,NP
WRITE(4,27)KI1(I),XE(I),YE(I),ZE(I)
1411 CONTINUE
DO 7805 I=1,NP
WRITE(4,27)KI2(I),XAC(I),YAC(I),ZAC(I)
7805 CONTINUE
27 FORMAT(2X,I2,3(2X,F16.3))
18 FORMAT(2X,I1,2X,I1,2X,F11.3)
DO 11 I=1,ND
READ(27,9)KII(I),JP(I),DIST(I)
11 CONTINUE
DO 1111 I=1,ND
READ(27,9)KII1(I),JP1(I),DIST1(I)
1111 CONTINUE
DO 7806 I=1,ND
READ(27,9)KII2(I),JP2(I),DIST2(I)
7806 CONTINUE
DO 8305 I=1,ND
READ(27,9)KII2(I),JP2(I),DIST3(I)
8305 CONTINUE
DO 8405 I=1,ND
READ(27,9)KII2(I),JP2(I),DIST4(I)
8405 CONTINUE
9 FORMAT(3X,I1,3X,I1,4X,F9.4)
C
DO 3001 I=1,NH
READ(27,3002)KH(I),JH(I),HDEG,HMIN,HSEC
CALL RADIAN(HDEG,HMIN,HSEC,HOR(I))

```

```

3002 FORMAT(3X,I1,3X,I1,3X,I3,2X,I2,2X,F5.2)
3001 CONTINUE
      DO 301 I=1,NH
        READ(27,3002)KH1(I),JH1(I),HDEG,HMIN,HSEC
        CALL RADIAN(HDEG,HMIN,HSEC,HOR1(I))
301 CONTINUE
      DO 7807 I=1,NH
        READ(27,3002)KH2(I),JH2(I),HDEG,HMIN,HSEC
        CALL RADIAN(HDEG,HMIN,HSEC,HOR2(I))
7807 CONTINUE
      DO 8306 I=1,NH
        READ(27,3002)KH2(I),JH2(I),HDEG,HMIN,HSEC
        CALL RADIAN(HDEG,HMIN,HSEC,HOR3(I))
8306 CONTINUE
      DO 8406 I=1,NH
        READ(27,3002)KH2(I),JH2(I),HDEG,HMIN,HSEC
        CALL RADIAN(HDEG,HMIN,HSEC,HOR4(I))
8406 CONTINUE
      IF(NV.EQ.0)GOTO 5120
      DO 3003 I=1,NV
        READ(27,3004)KV(I),JV(I),VDEG,VMIN,VSEC
        CALL RADIAN(VDEG,VMIN,VSEC,VER(I))
3004 FORMAT(3X,I1,3X,I1,3X,I3,2X,I3,2X,F6.2)
3003 CONTINUE
      DO 303 I=1,NV
        READ(27,3004)KV1(I),JV1(I),VDEG,VMIN,VSEC
        CALL RADIAN(VDEG,VMIN,VSEC,VER1(I))
303 CONTINUE
      DO 7808 I=1,NV
        READ(27,3004)KV2(I),JV2(I),VDEG,VMIN,VSEC
        CALL RADIAN(VDEG,VMIN,VSEC,VER2(I))
7808 CONTINUE
      DO 8307 I=1,NV
        READ(27,3004)KV2(I),JV2(I),VDEG,VMIN,VSEC
        CALL RADIAN(VDEG,VMIN,VSEC,VER3(I))
8307 CONTINUE
      DO 8407 I=1,NV
        READ(27,3004)KV2(I),JV2(I),VDEG,VMIN,VSEC
        CALL RADIAN(VDEG,VMIN,VSEC,VER4(I))
8407 CONTINUE
C
5120 DO 2390 I=1,LP
      READ(27,679)KII(I),JP(I),GPD(I)
2390 CONTINUE
      DO 239 I=1,LP
        READ(27,679)KII1(I),JP1(I),GPD1(I)
239 CONTINUE
      DO 8308 I=1,LP
        READ(27,679)KII2(I),JP2(I),GPD2(I)
8308 CONTINUE
      DO 6160 I=1,LP

```

```

      READ(27,679)KII2(I),JP2(I),GPD3(I)
6160 CONTINUE
      DO 8408 I=1,LP
      READ(27,679)KII2(I),JP2(I),GPD4(I)
8408 CONTINUE
      679 FORMAT(2X,I1,2X,I1,6X,F14.8)
3801 FORMAT(2X,I1,2X,I1,9X,F11.8)
      DO 2345 II=1,NA
      READ(27,1234)J9(II),KA(II),JJ,MM,SS
      CALL RADIAN(JJ,MM,SS,AZ(II))
2345 CONTINUE
      DO 234 II=1,NA
      READ(27,1234)J91(II),KA1(II),JJ,MM,SS
      CALL RADIAN(JJ,MM,SS,AZ1(II))
234 CONTINUE
      DO 7810 II=1,NA
      READ(27,1234)J92(II),KA2(II),JJ,MM,SS
      CALL RADIAN(JJ,MM,SS,AZ2(II))
7810 CONTINUE
      DO 8309 II=1,NA
      READ(27,1234)J93(II),KA3(II),JJ,MM,SS
      CALL RADIAN(JJ,MM,SS,AZ3(II))
8309 CONTINUE
      DO 8409 II=1,NA
      READ(27,1234)J93(II),KA3(II),JJ,MM,SS
      CALL RADIAN(JJ,MM,SS,AZ4(II))
8409 CONTINUE
1234 FORMAT(2X,I1,2X,I1,2X,I3,2X,I2,2X,F6.3)
C   LET THE NUMBER OF POINTS IN THE NETWORK BE 'NP' AND
C   THE NUMBER OF UNKNOWNNS BE ,NU
C   1. NO. OF DISTANCE OBSERVATIONS,ND
C   2. NO. OF VERTICAL ANGLE OBSERVATIONS, NV
C   3. NO. OF AZIMUTH OBSERVATIONS,NA
C
C   -----
C   INITIALIZE THE DESIGN MATRIX,A.
C   -----
1000 DO 100 I=1,NT
      DO 100 J=1,NU
      A(I,J)=0.0
100 CONTINUE
C
C   FORMING THE DESIGN MATRIX AND THE VECTOR OF OBSERVATIONS 'Y'
C   -----
C   1. COEFFICIENTS FOR GRAVITY POTENTIAL DIFFERENCE
C   -----
      IF(KOUNTA.EQ.0)GOTO 1001
      DO 98 I=1,NP
      CALL CART(LATI(I),LONG(I),HT(I),X(I),Y(I),Z(I))
98 CONTINUE
      KOUNTA=0

```

```

1001 K=1
      PRINT*, 'POTENTIAL COEFFICIENTS'
      LAC=0
      KAY=0
      MAC=0
      ICODE=0
6002 DO 105 I=1,LP
      IF(LAC.EQ.1)GOTO 6150
      IF(MAC.EQ.1)GOTO 8501
      IF(ICODE.EQ.1)GOTO 6005
      K1=KII(I)
      K2=JP(I)
      CALL ASSIGN(K1,X,Y,Z,LATI, LONG,HT,6,XR,YR,ZC,PAR,CIR,H)
      CALL ASSIGN(K2,X,Y,Z,LATI, LONG,HT,6,XR2,YR2,ZC2,PAR2,CIR2,H2)
      S1=DSQRT(XR**2+YR**2+ZC**2)
      S2=DSQRT(XR2**2+YR2**2+ZC2**2)
      GPS=GPD(I)
      GOTO 6004
6005 K1=KII1(I)
      K2=JP1(I)
      CALL ASSIGN(K1,X,Y,Z,LATI, LONG,HT,6,XR,YR,ZC,PAR,CIR,H)
      CALL ASSIGN(K2,X,Y,Z,LATI, LONG,HT,6,XR2,YR2,
<ZC2,PAR2,CIR2,H2)
      S1=DSQRT(XR**2+YR**2+ZC**2)
      S2=DSQRT(XR2**2+YR2**2+ZC2**2)
      GPS=GPD1(I)
      GOTO 6004
8501 K1=KII2(I)
      K2=JP2(I)
      CALL ASSIGN(K1,X,Y,Z,LATI, LONG,HT,6,XR,YR,ZC,PAR,CIR,H)
      CALL ASSIGN(K2,X,Y,Z,LATI, LONG,HT,6,XR2,YR2,ZC2,PAR2,CIR2,H2)
      S1=DSQRT(XR**2+YR**2+ZC**2)
      S2=DSQRT(XR2**2+YR2**2+ZC2**2)
      GPS=GPD2(I)
      GOTO 6004
6150 K1=KII2(I)
      K2=JP2(I)
      CALL ASSIGN(K1,X,Y,Z,LATI, LONG,HT,6,XR,YR,ZC,PAR,CIR,H)
      CALL ASSIGN(K2,X,Y,Z,LATI, LONG,HT,6,XR2,YR2,ZC2,PAR2,CIR2,H2)
      S1=DSQRT(XR**2+YR**2+ZC**2)
      S2=DSQRT(XR2**2+YR2**2+ZC2**2)
      GPS=GPD3(I)
6004 CALL DELTA(XR,YR,ZC,1,DW)
      DO 111 IN=1,3
      111 DW(IN,1)=-1.0*DW(IN,1)
      CALL JACCOB(PAR,H,CIR,DJ)
      CALL TIMES(DJ,DW,RW,3,3,1)
      IF(K1.NE.4)GOTO 6006
      J=1
      A(K,J)=RW(1,1)/RHO
      J=2

```

```

A(K,J)=RW(2,1)/RHO
J=3
A(K,J)=RW(3,1)
IF (ICODE.NE.1)GOTO 6006
J=LN+1
A(K,J)=RW(1,1)*TIME/RHO
J=LN+2
A(K,J)=RW(2,1)*TIME/RHO
J=LN+3
A(K,J)=RW(3,1)*TIME
IF(MAC.NE.1)GOTO 6006
J=LA+1
A(K,J)=RW(1,1)*TIM/RHO
J=LA+2
A(K,J)=RW(2,1)*TIM/RHO
J=LA+3
A(K,J)=RW(3,1)*TIM
6006 IF(K2.NE.4) GOTO 6001
CALL DELTA(XR2,YR2,ZC2,1,DW)
CALL JACCOB(PAR2,H2,CIR2,DJ)
CALL TIMES(DJ,DW,RW,3,3,1)
J=1
A(K,J)=RW(1,1)/RHO
J=2
A(K,J)=RW(2,1)/RHO
J=3
A(K,J)=RW(3,1)
7501 IF(ICODE.NE.1)GOTO 6001
J=LN+1
A(K,J)=RW(1,1)*TIME/RHO
J=LN+2
A(K,J)=RW(2,1)*TIME/RHO
J=LN+3
A(K,J)=RW(3,1)*TIME
IF(MAC.NE.1)GOTO 6001
J=LA+1
A(K,J)=RW(1,1)*TIM/RHO
J=LA+2
A(K,J)=RW(2,1)*TIM/RHO
J=LA+3
A(K,J)=RW(3,1)*TIM
6001 YC(K)=GM*(S1-S2)/(S1*S2)
YL(K)=(GPS-GM*(S1-S2)/(S1*S2))
P(K)=0.005
K=K+1
105 CONTINUE
IF(KAY.EQ.1)GOTO 6003
IF(LAC.EQ.1)GOTO 6341
IF(MAC.EQ.1) GOTO 6100
IF(ICODE.EQ.1)GOTO 8502
ICODE=1

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      K=LP+1
      GOTO 6002
8502 MAC=1
      K=2*LP+1
      GOTO 6002
6100 LAC=1
      K=3*LP+1
      GOTO 6002
6341 K=4*LP+1
      KAY=1
      DO 6342 I=1,LP
6342 GPD3(I)=GPD4(I)
      GOTO 6002
C 3. COEFFICIENTS FOR LATITUDE OBSERVATIONS.
C -----
6003 PRINT*, 'LATITUDE COEFFICIENTS'
      ICODE=0
      MAC=0
      LAC=0
      KAY=0
      K=5*LP+1
6013 DO 115 I=1,NB
      IF(LAC.EQ.1) GOTO 6151
      IF(MAC.EQ.1) GOTO 8503
      IF(ICODE.EQ.1)GOTO 6010
      M=NC(I)
      YC(K)=LATI(I)*RHO-SAI(I)
      YL(K)=BLAT(I)*RHO-LATI(I)*RHO-SAI(I)
      IF(M.NE.4) GOTO 6900
      GOTO 6011
6010 M=NC1(I)
      YC(K)=LATI(I)*RHO-SAI(I)
      YL(K)=BLAT1(I)*RHO-LATI(I)*RHO-SAI(I)
      IF(M.NE.4) GOTO 6900
      GOTO 6011
8503 M=NC1(I)
      YC(K)=LATI(I)*RHO-SAI(I)
      YL(K)=BLAT2(I)*RHO-LATI(I)*RHO-SAI(I)
      IF(M.NE.4) GOTO 6900
      GOTO 6011
6151 M=NC1(I)
      YC(K)=LATI(I)*RHO-SAI(I)
      YL(K)=BLAT3(I)*RHO-LATI(I)*RHO-SAI(I)
      IF(M.NE.4) GOTO 6900
6011 J=1
      A(K,J)=1.0
7503 IF(ICODE.NE.1)GOTO 6012
      J=LN+1
      A(K,J)=TIME
      IF(MAC.NE.1)GOTO 6012
      J=LA+1

```

```

A(K,J)=TIM
2 J=LU+1
A(K,J)=1.0
0 P(K)=0.3
K=K+1
CONTINUE
IF(KAY.EQ.1)GOTO 6014
IF(LAC.EQ.1)GOTO 6343
IF(MAC.EQ.1)GOTO 6101
IF(ICODE.EQ.1)GOTO 8504
ICODE=1
LU=3
K=5*LP+NB+1
GOTO 6013
4 MAC=1
K=5*LP+2*NB+1
GOTO 6013
1 LAC=1
K=5*LP+3*NB+1
GOTO 6013
3 KAY=1
K=5*LP+4*NB+1
DO 7330 I=1,NB
) BLAT3(I)=BLAT4(I)
GOTO 6013
COEFFICIENTS FOR LONGITUDE OBSERVATIONS.
-----

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```

) PRINT=,'LONGITUDE COEFFICIENTS'
ICODE=0
LAC=0
MAC=0
KAY=0
LU=3
K=5*(LP+NB)+1
) DO 120 I=1,NL
IF(LAC.EQ.1)GOTO 6152
IF(MAC.EQ.1)GOTO 8505
IF(ICODE.EQ.1)GOTO 6015
M=NZ(I)
YC(K)=LONG(M)*RHO-ETA(M)/COS(LATI(M))
YL(K)=(BLON(M)-LONG(M))*RHO-ETA(M)/COS(LATI(M))
IF(M.NE.4)GOTO 6051
GOTO 6016
M=NZ(I)
YC(K)=LONG(M)*RHO-ETA(M)/COS(LATI(M))
YL(K)=(BLON1(M)-LONG(M))*RHO-ETA(M)/COS(LATI(M))
IF(M.NE.4)GOTO 6051
GOTO 6016
M=NZ2(I)
YC(K)=LONG(M)*RHO-ETA(M)/COS(LATI(M))
YL(K)=(BLON2(M)-LONG(M))*RHO-ETA(M)/COS(LATI(M))

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        IF(M.NE.4)GOTO 6051
        GOTO 6016
6152 M=NZ2(I)
        YC(K)=LONG(M)*RHO-ETA(M)/COS(LATI(M))
        YL(K)=(BLON3(M)-LONG(M))*RHO-ETA(M)/COS(LATI(M))
        IF(M.NE.4)GOTO 6051
6016 J=2
        A(K,J)=1.0
        J=LU+2
        A(K,J)=1.0/COS(LATI(M))
7504 IF(ICODE.NE.1)GOTO 6051
        J=LN+2
        A(K,J)=TIME
        J=LU+2
        A(K,J)=1.0/COS(LATI(M))
        IF(MAC.NE.1)GOTO 6051
        J=LA+1
        A(K,J)=TIM
        J=LU+2
        A(K,J)=1.0/COS(LATI(M))
6051 P(K)=0.3
        K=K+1
120 CONTINUE
        IF(KAY.EQ.1)GOTO 6018
        IF(LAC.EQ.1)GOTO 6345
        IF(MAC.EQ.1) GOTO 6102
        IF(ICODE.EQ.1)GOTO 8506
        LU=3
        K=5*(LP+NB)+NL+1
        ICODE=1
        GOTO 6019
8506 MAC=1
        K=5*(LP+NB)+2*NL+1
        GOTO 6019
6102 LAC=1
        K=5*(LP+NB)+3*NL+1
        GOTO 6019
6345 KAY=1
        K=5*(LP+NB)+4*NL+1
        DO 7331 I=1,NL
7331 BLON3(I)=BLON4(I)
        GOTO 6019
6018 PRINT*, 'AZIMUTH COEFFICIENTS'
        ICODE=0
        LAC=0
        MAC=0
        KAY=0
        LU=3
C 5. COEFFICIENTS FOR OBSERVED AZIMUTH.
C -----
C LET 'NA' BE THE NO. OF OBSERVED LINES FOR ASTRONOMIC AZIMUTH.

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      K=5*(LP+NG+NL+NB)+1
6024 DO 125 I=1,NA
      IF(LAC.EQ.1)GOTO 6153
      IF(MAC.EQ.1) GOTO 8507
      IF(ICODE.EQ.1)GOTO 6020
      L1=J9(I)
      L2=KA(I)
      CALL ASSIGN(L1,X,Y,Z,LATI, LONG,HT,6,XR,YR,ZC,PAR,CIR,H)
      CALL ASSIGN(L2,X,Y,Z,LATI, LONG,HT,6,XR2,YR2,ZC2,PAR2,CIR2,H2)
      SAA=SAI(I)
      TEE=ETA(I)
      AZIM=AZ(I)
      GOTO 6021
6020 L1=J91(I)
      L2=KA1(I)
      CALL ASSIGN(L1,X,Y,Z,LATI, LONG,HT,6,XR,YR,ZC,PAR,CIR,H)
      CALL ASSIGN(L2,X,Y,Z,LATI, LONG,HT,6,XR2,
<YR2,ZC2,PAR2,CIR2,H2)
      SAA=SAI(I)
      TEE=ETA(I)
      AZIM=AZ1(I)
      GOTO 6021
8507 L1=J92(I)
      L2=KA2(I)
      CALL ASSIGN(L1,X,Y,Z,LATI, LONG,HT,6,XR,YR,ZC,PAR,CIR,H)
      CALL ASSIGN(L2,X,Y,Z,LATI, LONG,HT,6,XR2,
<YR2,ZC2,PAR2,CIR2,H2)
      SAA=SAI(I)
      TEE=ETA(I)
      AZIM=AZ2(I)
      GOTO 6021
6153 L1=J93(I)
      L2=KA3(I)
      CALL ASSIGN(L1,X,Y,Z,LATI, LONG,HT,6,XR,YR,ZC,PAR,CIR,H)
      CALL ASSIGN(L2,X,Y,Z,LATI, LONG,HT,6,XR2,
<YR2,ZC2,PAR2,CIR2,H2)
      SAA=SAI(I)
      TEE=ETA(I)
      AZIM=AZ3(I)
6021 RZ(1,1)=XR2-XR
      RZ(2,1)=YR2-YR
      RZ(3,1)=ZC2-ZC
      CALL EULA(PAR,CIR,RE)
      CALL TIMES(RE,RZ,EZ,3,3,1)
C MATRIX OF DIFFERENTIALS OF AZIMUTH, DA
      S2=EZ(1,1)**2+EZ(2,1)**2
      SP2=DSQRT(S2+EZ(3,1)**2)
      DA(1,1)=-1.0*EZ(2,1)/S2
      DA(2,1)=EZ(1,1)/S2
      DA(3,1)=0.0
      CALL EULA(PAR,CIR,RE,2)

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DO 131 IC=1,3
DO 131 JC=1,3
131 RE(IC,JC)=-1.0*RE(IC,JC)
CALL TIMES (RE,DA,RA,3,3,1)
CALL JACCOB(PAR,H,CIR,DJ)
CALL TIMES(DJ,RA,RJ,3,3,1)
IF(L1.NE.4)GOTO 6022
J=1
A(K,J)=RJ(1,1)
J=2
A(K,J)=RJ(2,1)
J=3
A(K,J)=RJ(3,1)*RHO
7505 IF(ICODE.NE.1)GOTO 6022
J=LN+1
A(K,J)=RJ(1,1)*TIME
J=LN+2
A(K,J)=RJ(2,1)*TIME
J=LN+3
A(K,J)=RJ(3,1)*TIME*RHO
IF(MAC.NE.1) GOTO 6022
J=LA+1
A(K,J)=RJ(1,1)*TIM
J=LA+2
A(K,J)=RJ(2,1)*TIM
J=LA+3
A(K,J)=RJ(3,1)*TIM*RHO
6022 IF(L2.NE.4)GOTO 6023
CALL EULA(PAR,CIR,RE,2)
CALL TIMES(RE,DA,RA,3,3,1)
CALL JACCOB(PAR2,H2,CIR2,DJ)
CALL TIMES(DJ,RA,RJ,3,3,1)
J=1
A(K,J)=RJ(1,1)
J=2
A(K,J)=RJ(2,1)
J=3
A(K,J)=RJ(3,1)*RHO
7506 IF(ICODE.NE.1)GOTO 6023
J=LN+1
A(K,J)=RJ(1,1)*TIME
J=LN+2
A(K,J)=RJ(2,1)*TIME
J=LN+3
A(K,J)=RJ(3,1)*TIME*RHO
IF(MAC.NE.1) GOTO 6023
J=LA+1
A(K,J)=RJ(1,1)*TIM
J=LA+2
A(K,J)=RJ(2,1)*TIM
J=LA+3

```

```

      A(K,J)=RJ(3,1)*TIM*RHO
6023 IF(L1.EQ.4) GOTO 6700
      IF(L2.NE.4) GOTO 6901
6700 J=LU+1
      A(K,J)=-1.0*SIN(AZIM)*TAN(ASIN(EZ(3,1)/SP2))
      T2=A(K,J)
      J=LU+2
      A(K,J)=(-1.0*TAN(PAR)-COS(AZIM)*TAN(ASIN(EZ(3,1)/SP2)))
      T3=A(K,J)
6901 T2=-1.0*SIN(AZIM)*TAN(ASIN(EZ(3,1)/SP2))
      T3=(-1.0*TAN(PAR)-COS(AZIM)*TAN(ASIN(EZ(3,1)/SP2)))
      CALL AZIMUTH(EZ(2,1),EZ(1,1),BRG)
      YC(K)=BRG-T2*SAA-T3*TEE
      YL(K)=(AZIM*RHO-BRG)-T2*SAA-T3*TEE
      P(K)=0.7
      K=K+1
125  CONTINUE
      IF(KAY.EQ.1) GOTO 6025
      IF(LAC.EQ.1) GOTO 6346
      IF(MAC.EQ.1) GOTO 6103
      IF(ICODE.EQ.1)GOTO 8508
      ICODE=1
      LU=3
      K=5*(LP+NB+NL)+NA+1
      GOTO 6024
8508 MAC=1
      K=5*(LP+NB+NL)+2*NA+1
      LU=3
      GOTO 6024
6103 LAC=1
      K=5*(LP+NB+NL)+3*NA+1
      GOTO 6024
6346 KAY=1
      K=5*(LP+NB+NL)+4*NA+1
      DO 7332 I=1,NA
7332 AZ3(I)=AZ4(I)
      GOTO 6024
6025 PRINT*, 'VERTICAL ANGLE COEFFICIENTS'
      ICODE=0
      LAC=0
      MAC=0
      KAY=0
      LU=3
C 6.  COEFFICIENTS FOR VERTICAL ANGLES.
C -----
      K=5*(LP+NA+NL+NB)+1
6034 DO 130 I=1,NV
      IF(LAC.EQ.1)GOTO 6154
      IF(MAC.EQ.1) GOTO 8509
      IF(ICODE.EQ.1)GOTO 6030
      MI=KV(I)

```

```

MJ=JV(I)
CALL ASSIGN(MI,X,Y,Z,LATI,LONG,HT,6,XR,YR,ZC,PAR,CIR,H)
CALL ASSIGN(MJ,X,Y,Z,LATI,LONG,HT,6,XR2,YR2,ZC2,PAR2,CIR2,H2)
VERT=VER(I)
HORIZ=HOR(MI)
GOTO 6031
6030 MI=KV1(I)
MJ=JV1(I)
CALL ASSIGN(MI,X,Y,Z,LATI,LONG,HT,6,XR,YR,ZC,PAR,CIR,H)
CALL ASSIGN(MJ,X,Y,Z,LATI,LONG,HT,6,XR2,YR2,
<ZC2,PAR2,CIR2,H2)
VERT=VER1(I)
GOTO 6031
8509 MI=KV2(I)
MJ=JV2(I)
CALL ASSIGN(MI,X,Y,Z,LATI,LONG,HT,6,XR,YR,ZC,PAR,CIR,H)
CALL ASSIGN(MJ,X,Y,Z,LATI,LONG,HT,6,XR2,YR2,
<ZC2,PAR2,CIR2,H2)
VERT=VER2(I)
GOTO 6031
6154 MI=KV2(I)
MJ=JV2(I)
CALL ASSIGN(MI,X,Y,Z,LATI,LONG,HT,6,XR,YR,ZC,PAR,CIR,H)
CALL ASSIGN(MJ,X,Y,Z,LATI,LONG,HT,6,XR2,YR2,
<ZC2,PAR2,CIR2,H2)
VERT=VER3(I)
6031 RZ(1,1)=XR2-XR
RZ(2,1)=YR2-YR
RZ(3,1)=ZC2-ZC
CALL EULA(PAR,CIR,RE,2)
CALL TIMES(RE,RZ,EZ,3,3,1)
C MATRIX OF DIFFERENTIALS OF VERTICAL ANGLES,DB
SP2=EZ(1,1)**2+EZ(2,1)**2+EZ(3,1)**2
S2=DSQRT(EZ(1,1)**2+EZ(2,1)**2)
DB(1,1)=-1.0*EZ(3,1)*EZ(1,1)/(SP2*S2)
DB(2,1)=-1.0*EZ(3,1)*EZ(2,1)/(SP2*S2)
DB(3,1)=S2/SP2
CALL EULA(PAR,CIR,RE,2)
DO 141 IC=1,3
DO 141 JC=1,3
141 RE(IC,JC)=-1.0*RE(IC,JC)
CALL TIMES(RE,DB,RA,3,3,1)
CALL JACCOB(PAR,H,CIR,DJ)
CALL TIMES(DJ,RA,RJ,3,3,1)
IF(MI.NE.4)GOTO 6032
J=1
A(K,J)=RJ(1,1)
J=2
A(K,J)=RJ(2,1)
J=3
A(K,J)=RJ(3,1)*RHO

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```

7507 IF(ICODE.NE.1)GOTO 6032
      J=LN+1
      A(K,J)=RJ(1,1)*TIME
      J=LN+2
      A(K,J)=RJ(2,1)*TIME
      J=LN+3
      A(K,J)=RJ(3,1)*TIME*RHO
      IF(MAC.NE.1) GOTO 6032
      J=LA+1
      A(K,J)=RJ(1,1)*TIM
      J=LA+2
      A(K,J)=RJ(2,1)*TIM
      J=LA+3
      A(K,J)=RJ(3,1)*TIM*RHO
6032 IF(MJ.NE.4)GOTO 6033
      CALL EULA(PAR,CIR,RE,2)
      CALL TIMES(RE,DB,RA,3,3,1)
      CALL JACCOB(PAR2,H2,CIR2,DJ)
      CALL TIMES(DJ,RA,RJ,3,3,1)
      J=1
      A(K,J)=RJ(1,1)
      J=2
      A(K,J)=RJ(2,1)
      J=3
      A(K,J)=RJ(3,1)*RHO
7508 IF(ICODE.NE.1)GOTO 6033
      J=LN+1
      A(K,J)=RJ(1,1)*TIME
      J=LN+2
      A(K,J)=RJ(2,1)*TIME
      J=LN+3
      A(K,J)=RJ(3,1)*TIME*RHO
      IF(MAC.NE.1)GOTO 6033
      J=LA+1
      A(K,J)=RJ(1,1)*TIM
      J=LA+2
      A(K,J)=RJ(2,1)*TIM
      J=LA+3
      A(K,J)=RJ(3,1)*TIM*RHO
6033 IF(MI.EQ.4) GOTO 6701
      IF(MJ.NE.4) GOTO 6902
6701 J=LU+1
      A(K,J)=-COS(HORIZ)
      J=J+1
      A(K,J)=SIN(HORIZ)
6902 YC(K)=ASIN((EZ(3,1)/DSQRT(SP2)))
      YL(K)=VERT-ASIN((EZ(3,1)/DSQRT(SP2)))
      P(K)=1.0
      K=K+1
130  CONTINUE
      IF(KAY.EQ.1)GOTO 6035

```

```

IF(LAC.EQ.1) GOTO 6348
IF(MAC.EQ.1) GOTO 6104
IF(ICODE.EQ.1)GOTO 8510
ICODE=1
LU=3
K=5*(LP+NA+NL+NB)+NV+1
GOTO 6034
8510 MAC=1
K=5*(LP+NA+NL+NB)+2*NV+1
LU=3
GOTO 6034
6104 LAC=1
K=5*(LP+NA+NL+NB)+3*NV+1
GOTO 6034
6348 KAY=1
K=5*(LP+NA+NL+NB)+4*NV+1
DO 7333 I=1,NV
7333 VER3(I)=VER4(I)
GOTO 6034
6035 PRINT*, 'SPATIAL DISTANCE COEFFICIENTS'
ICODE=0
LAC=0
MAC=0
KAY=0
C 7. COEFFICIENTS FOR SPATIAL DISTANCE.
C -----
C LET NO. OF DISTANCE OBSERVATIONS BE 'ND'
6049 K=5*(LP+NA+NL+NB+NV+NG)+1
6041 DO 135 I=1,ND
IF(LAC.EQ.1) GOTO 6155
IF(MAC.EQ.1)GOTO 8511
IF(ICODE.EQ.1)GOTO 6036
MI=KII(I)
MJ=JP(I)
CALL ASSIGN(MI,X,Y,Z,LATI, LONG,HT,6,XR,YR,ZC,PAR,CIR,H)
CALL ASSIGN(MJ,X,Y,Z,LATI, LONG,HT,6,XR2,YR2,ZC2,PAR2,CIR2,H2)
STE=DIST(I)
GOTO 6037
6036 MI=KII(I)
MJ=JP(I)
CALL ASSIGN(MI,X,Y,Z,LATI, LONG,HT,6,XR,YR,ZC,PAR,CIR,H)
CALL ASSIGN(MJ,X,Y,Z,LATI, LONG,HT,6,XR2,
<YR2,ZC2,PAR2,CIR2,H2)
STE=DIST1(I)
GOTO 6037
8511 MI=KII2(I)
MJ=JP2(I)
CALL ASSIGN(MI,X,Y,Z,LATI, LONG,HT,6,XR,YR,ZC,PAR,CIR,H)
CALL ASSIGN(MJ,X,Y,Z,LATI, LONG,HT,6,XR2,
<YR2,ZC2,PAR2,CIR2,H2)
STE=DIST2(I)

```

```

        GOTO 6037
6155 MI=KII2(I)
        MJ=JP2(I)
        CALL ASSIGN(MI,X,Y,Z,LATI,LONG,HT,6,XR,YR,ZC,PAR,CIR,H)
        CALL ASSIGN(MJ,X,Y,Z,LATI,LONG,HT,6,XR2,
<YR2,ZC2,PAR2,CIR2,H2)
        STE=DIST3(I)
6037 RZ(1,1)=XR2-XR
        RZ(2,1)=YR2-YR
        RZ(3,1)=ZC2-ZC
        CALL EULA(PAR,CIR,RE)
        CALL TIMES(RE,RZ,EZ,3,3,1)
C MATRIX OF DIFFERENTIALS OF SPATIAL DISTANCE
        S2=DSQRT(EZ(1,1)**2+EZ(2,1)**2+EZ(3,1)**2)
        DS(1,1)=EZ(1,1)/S2
        DS(2,1)=EZ(2,1)/S2
        DS(3,1)=EZ(3,1)/S2
        CALL EULA(PAR,CIR,RE)
        DO 142 IC=1,3
        DO 142 JC=1,3
142 RE(IC,JC)=-1.0*RE(IC,JC)
        CALL TIMES(RE,DS,RA,3,3,1)
        CALL JACCOB(PAR,H,CIR,DJ)
        CALL TIMES(DJ,RA,RJ,3,3,1)
        IF(MI.NE.4)GOTO 6038
        J=1
        A(K,J)=RJ(1,1)/RHO
        J=2
        A(K,J)=RJ(2,1)/RHO
        J=3
        A(K,J)=RJ(3,1)
7509 IF(ICODE.NE.1)GOTO 6038
        J=LN+1
        A(K,J)=RJ(1,1)*TIME/RHO
        J=LN+2
        A(K,J)=RJ(2,1)*TIME/RHO
        J=LN+3
        A(K,J)=RJ(3,1)*TIME
        IF(MAC.NE.1) GOTO 6038
        J=LA+1
        A(K,J)=RJ(1,1)*TIM/RHO
        J=LA+2
        A(K,J)=RJ(2,1)*TIM/RHO
        J=LA+3
        A(K,J)=RJ(3,1)*TIM
6038 IF(MJ.NE.4) GOTO 6039
        CALL EULA(PAR,CIR,RE)
        CALL TIMES(RE,DS,RA,3,3,1)
        CALL JACCOB(PAR2,H2,CIR2,DJ)
        CALL TIMES(DJ,RA,RJ,3,3,1)
        J=1

```

```

      A(K,J)=RJ(1,1)/RHO
      J=2
      A(K,J)=RJ(2,1)/RHO
      J=3
      A(K,J)=RJ(3,1)
7510 IF(ICODE.NE.1)GOTO 6039
      J=LN+1
      A(K,J)=RJ(1,1)*TIME/RHO
      J=LN+2
      A(K,J)=RJ(2,1)*TIME/RHO
      J=LN+3
      A(K,J)=RJ(3,1)*TIME
      IF(MAC.NE.1) GOTO 6039
      J=LA+1
      A(K,J)=RJ(1,1)*TIM/RHO
      J=LA+2
      A(K,J)=RJ(2,1)*TIM/RHO
      J=LA+3
      A(K,J)=RJ(3,1)*TIM
6039 YC(K)=S2
      YL(K)=(STE-S2)
      P(K)=DSQRT((0.0004**2)+(STE*0.40D-06)**2)
      K=K+1
135  CONTINUE
      IF(KAY.EQ.1)GOTO 6040
      IF(LAC.EQ.1)GOTO 6349
      IF(MAC.EQ.1)GOTO 6105
      IF(ICODE.EQ.1)GOTO 8512
      LU=3
      ICODE=1
      K=5*(LP+NB+NL+NV+NA+NG)+ND+1
      GOTO 6041
8512 MAC=1
      K=5*(LP+NB+NL+NV+NA+NG)+2*ND+1
      LU=3
      GOTO 6041
6105 LAC=1
      K=5*(LP+NB+NL+NV+NA+NG)+3*ND+1
      LU=3
      GOTO 6041
6349 KAY=1
      K=5*(LP+NB+NL+NV+NA+NG)+4*ND+1
      DO 7334 I=1,ND
7334 DIST3(I)=DIST4(I)
      GOTO 6041
C      8. COEFFICIENTS FOR HORIZONTAL ANGLE.
C      -----
6040 PRINT*, 'HORIZONTAL ANGLE COEFFICIENTS'
      ICODE=0
      LAC=0
      KAY=0

```



```

MAC=0
LU=3
K=5*(LP+NG+NA+NL+NB+NV+ND)+1
6047 DO 145 I=1,NH
      IF(LAC.EQ.1)GOTO 6156
      IF(MAC.EQ.1)GOTO 8513
      IF(ICODE.EQ.1)GOTO 6042
      MI=KH(I)
      MJ=JH(I)
      CALL ASSIGN(MI,X,Y,Z,LATI, LONG,HT,6,XR,YR,ZC,PAR,CIR,H)
      CALL ASSIGN(MJ,X,Y,Z,LATI, LONG,HT,6,XR2,YR2,ZC2,PAR2,CIR2,H2)
      SAA=SAI(I)
      TEE=ETA(I)
      HORIZ=HOR(I)
      GOTO 6043
6042 MI=KH1(I)
      MJ=JH1(I)
      CALL ASSIGN(MI,X,Y,Z,LATI, LONG,HT,6,XR,YR,ZC,PAR,CIR,H)
      CALL ASSIGN(MJ,X,Y,Z,LATI, LONG,HT,6,XR2,YR2,ZC2,PAR2,CIR2,H2)
      SAA=SAI(I)
      TEE=ETA(I)
      HORIZ=HOR1(I)
      GOTO 6043
8513 MI=KH2(I)
      MJ=JH2(I)
      CALL ASSIGN(MI,X,Y,Z,LATI, LONG,HT,6,XR,YR,ZC,PAR,CIR,H)
      CALL ASSIGN(MJ,X,Y,Z,LATI, LONG,HT,6,XR2,YR2,ZC2,PAR2,CIR2,H2)
      SAA=SAI(I)
      TEE=ETA(I)
      HORIZ=HOR2(I)
      GOTO 6043
6156 MI=KH2(I)
      MJ=JH2(I)
      CALL ASSIGN(MI,X,Y,Z,LATI, LONG,HT,6,XR,YR,ZC,PAR,CIR,H)
      CALL ASSIGN(MJ,X,Y,Z,LATI, LONG,HT,6,XR2,YR2,ZC2,PAR2,CIR2,H2)
      SAA=SAI(I)
      TEE=ETA(I)
      HORIZ=HOR3(I)
6043 RZ(1,1)=XR2-XR
      RZ(2,1)=YR2-YR
      RZ(3,1)=ZC2-ZC
      CALL EULA(PAR,CIR,RE)
      CALL TIMES(RE,RZ,EZ,3,3,1)
      S2=EZ(1,1)**2+EZ(2,1)**2
      SP2=DSQRT(S2+EZ(3,1)**2)
      DA(1,1)=EZ(2,1)/S2
      DA(2,1)=EZ(1,1)/S2
      DA(3,1)=0.0
      CALL EULA(PAR,CIR,RE,2)
      DO 146 IC=1,3
      DO 146 JC=1,3

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146 RE(IC,JC)=-1.0*RE(IC,JC)
CALL TIMES(RE,DA,RA,3,3,1)
CALL JACCOB(PAR,H,CIR,DJ)
CALL TIMES(DJ,RA,RJ,3,3,1)
IF(MI.NE.4)GOTO 6044
J=1
A(K,J)=RJ(1,1)
J=2
A(K,J)=RJ(2,1)
J=3
A(K,J)=RJ(3,1)*RHO
7511 IF(ICODE.NE.1)GOTO 6044
J=LN+1
A(K,J)=RJ(1,1)*TIME
J=LN+2
A(K,J)=RJ(2,1)*TIME
J=LN+3
A(K,J)=RJ(3,1)*TIME*RHO
IF(MAC.NE.1) GOTO 6044
J=LA+1
A(K,J)=RJ(1,1)*TIM
J=LA+2
A(K,J)=RJ(2,1)*TIM
J=LA+3
A(K,J)=RJ(3,1)*TIM*RHO
6044 IF(MJ.NE.4)GOTO 6045
CALL EULA(PAR,CIR,RE,2)
CALL JACCOB(PAR2,H2,CIR2,DJ)
CALL TIMES(DJ,RA,RJ,3,3,1)
IF(ICODE.EQ.1)GOTO 7512
J=1
A(K,J)=RJ(1,1)
J=2
A(K,J)=RJ(2,1)
J=3
A(K,J)=RJ(3,1)*RHO
7512 IF(ICODE.NE.1)GOTO 6045
J=LN+1
A(K,J)=RJ(1,1)*TIME
J=LN+2
A(K,J)=RJ(2,1)*TIME
J=LN+3
A(K,J)=RJ(3,1)*TIME*RHO
IF(MAC.NE.1)GOTO 6045
J=LA+1
A(K,J)=RJ(1,1)*TIM
J=LA+2
A(K,J)=RJ(2,1)*TIM
J=LA+3
A(K,J)=RJ(3,1)*TIM*RHO
6045 IF(MI.EQ.4) GOTO 6702

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      IF(MJ.NE.4) GOTO 6903
6702 J=LU+1
      A(K,J)=(-1.0*SIN(HORIZ)*TAN(ASIN(EZ(3,1)/SP2)))
      T6=A(K,J)
      A(K,J+1)=(-1.0*TAN(PAR)-COS(HORIZ)*TAN
<(ASIN(EZ(3,1)/SP2)))
      T7=A(K,J)
6903 J=KOR+MI
C      ORIENTATION PARAMETER
C      -----
      A(K,J)=1.0
6705 T6=(-1.0*SIN(HORIZ)*TAN(ASIN(EZ(3,1)/SP2)))
      T7=(-1.0*TAN(PAR)-COS(HORIZ)*TAN
<(ASIN(EZ(3,1)/SP2)))
      CALL AZIMUTH(EZ(2,1),EZ(1,1),ANG)
      YC(K)=ANG-T6*SAA-T7*TEE
      YL(K)=(HORIZ)*RHO-ANG-T6*SAA-T7*TEE
      P(K)=0.5
      K=K+1
145  CONTINUE
      IF(KAY.EQ.1) GOTO 6046
      IF(LAC.EQ.1)GOTO 7335
      IF(MAC.EQ.1)GOTO 6106
      IF(ICODE.EQ.1)GOTO 8514
      ICODE=1
      LU=3
      KOR=5
      K=5*(LP+NB+NA+NV+NG+ND+NL)+NH+1
      GOTO 6047
8514 MAC=1
      K=5*(LP+NB+NA+NV+NG+ND+NL)+2*NH+1
      GOTO 6047
6106 LAC=1
      K=5*(LP+NB+NA+NV+NG+ND+NL)+3*NH+1
      GOTO 6047
7335 KAY=1
      K=5*(LP+NB+NA+NV+NG+ND+NL)+4*NH+1
      DO 7336 I=1,NH
7336 HOR3(I)=HOR4(I)
      GOTO 6047
6046 IF(KOUNTA.GT.0)GOTO 1002
      DO 123 I=1,NT
      P(I)=P(I)**2
7000 FORMAT(3X,I3,E25.9)
123  CONTINUE
      PRINT*, 'THE WEIGHT MATRIX'
6911 FORMAT(2X,I3,2X,E25.9)
1009 KOUNTA=0
      DO 222 I1=1,NT
      DO 222 J1=1,NT
      IF(I1.EQ.J1)GOTO 81

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      W(I1,J1)=0.0
      GOTO 222
81   IF(KOT.NE.1)GOTO 212
      P(I1)=PN(I1)
212  W(I1,I1)=SIG2/(P(I1)+ER(I1,1)**2)
      PN(I1)=W(I1,I1)
222  CONTINUE
      KOT=0
      GOTO 109

1004 L=2
1002 DO 101 I=1,NT
      DO 102 J=1,NT
      IF(J.EQ.I)GOTO 103
      W(I,J)=0.
      GOTO 102
103  W(I,I)=PN(I)
102  CONTINUE
101  CONTINUE
C    PARAMETER ESTIMATION
C    -----
109  CALL NORMAL(A,W,ATW,ATWA,NU,NT)
      CALL MATINV(ATWA,AINV,NU)
      CALL TIMES(ATW,YL,ATY,NU,NT,1)
      CALL TIMES(AINV,ATY,DX,NU,NU,1)
      CALL TIMES(A,DX,AX,NT,NU,1)
      DO 522 I=1,NT
      ER(I,1)=YL(I)-AX(I,1)
522  CONTINUE
545  FORMAT(3X,I3,2X,F25.6,2X,F25.6)
      CALL NORMAL(ER,W,ETW,ETWE,1,NT)
      SIG02=ETWE(1,1)/(NT-NU)
      IF(KOT.NE.1)GOTO 2222
      WRITE(4,887)KOUNTA
887  FORMAT(/5X,'ITERATION NO.',4X,I2)
      WRITE(4,530)SIG02
2222 SIG2=SIG02
C    COMPUTE THE COVARIANCE MATRIX
C    -----
      DO 528 I=1,NU
      DO 528 J=1,NU
528  COVX(I,J)=SIG02*AINV(I,J)
C    SCALING THE NEW WEIGHTS
C    -----
      DO 192 I=1,NT
      PN(I)=PN(I)/SIG2
      SEY(I)=1./PN(I)
      SEE(I)=SIG2*(SEY(I))
      ER(I,1)=ER(I,1)*PN(I)*SEE(I)
192  CONTINUE
      KOT=1
      KOUNTA=KOUNTA+1

```

```

IF(KOUNTA.NE.2)GOTO 1002
C INCREMENT OLD VALUES
C -----
LATI(4)=(DX(1,1)/RHO)+LATI(4)
LONG(4)=(DX(2,1)/RHO)+LONG(4)
HT(4)=DX(3,1)+HT(4)
SAI(4)=SAI(4)+DX(4,1)
NUM=NUM+1
ETA(4)=ETA(4)+DX(5,1)
NUM=NUM+1
DO 797 I=1,NH
MI=KII(I)
J=5+MI
HOR(MI)=HOR(MI)+DX(J,1)/RHO
797 CONTINUE
C -----
ICOUNT=ICOUNT+1
WRITE(4,6897)
6897 FORMAT(/4X,'THE PARAMETER MATRIX')
WRITE(4,9000)(DX(I,1),I=1,NU)
9000 FORMAT(E25.6)
C OUTPUT RESULTS
C -----
C POINT COVARIANCE MATRIX
C -----
I=4
L1=1
L2=2
L3=3
Q(1,1)=COVX(L1,L1)
Q(1,2)=COVX(L1,L2)
Q(1,3)=COVX(L1,L3)
Q(2,1)=COVX(L2,L1)
Q(2,2)=COVX(L2,L2)
Q(2,3)=COVX(L2,L3)
Q(3,1)=COVX(L3,L1)
Q(3,2)=COVX(L3,L2)
Q(3,3)=COVX(L3,L3)
CALL JACCOB(LATI(4),HT(4),LONG(4),WE)
WE(1,1)=WE(1,1)/RHO
WE(1,2)=WE(1,2)/RHO
WE(1,3)=WE(1,3)/RHO
WE(2,1)=WE(2,1)/RHO
WE(2,2)=WE(2,2)/RHO
WE(2,3)=WE(2,3)/RHO
CALL TRANSP(WE,WET,3,3)
CALL TIMES(WET,Q,QX,3,3,3)
CALL TIMES(QX,WE,WCO,3,3,3)
WRITE(4,333)I
333 FORMAT(/4X,'STATION NO.',I,/4X,40('='))
WRITE(4,1028)

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1028 FORMAT(3X,'Q IN LOCAL ELLIPSOIDAL SYSTEM')
      WRITE(4,334)((WCO(N,M),M=1,3),N=1,3)
334  FORMAT(3F16.9)
      CALL ELLIPSOID(WCO,LATI(4),LONG(4))
C
530  FORMAT(/2X,'THE COMPUTED VARIANCE OF UNIT WEIGHT=',F20.4)
C    THE NEW COORDINATES
C    -----
      WRITE(4,656)
656  FORMAT(/10X,'THE COMPUTED COORDINATES',/10X,40('='),/3X,'STN'
      ,4X,'LATITUDE',7X,'LONGITUDE',10X,'HEIGHT',/3X,60('-'))
      WRITE(4,37)
      I=4
      ZR(4)=LATI(4)
      ZU(4)=LONG(4)
      ZJ(4)=HT(4)
      CALL RTDMS(LATI(4),LA,LM,SL)
      CALL RTDMS(LONG(4),LO,LOM,SLO)
      WRITE(4,767)I,LA,LM,SL,LO,LOM,SLO,HT(I)
      SL=DSQRT(COVX(1,1))
      SLO=DSQRT(COVX(2,2))
      SH=DSQRT(COVX(3,3))
      WRITE(4,8547)
8547 FORMAT(/5X,'THE STANDARD ERRORS',/5X,30('='))
      WRITE(4,8549)SL,SLO,SH
8549 FORMAT(/4X,F15.6,'"',/4X,F15.6,'"',/4X,F15.6,'m')
767  FORMAT(2X,I1,2(3X,I3,2X,I3,2X,F7.3),3X,F10.3)
      WRITE(4,401)
401  FORMAT(/6X,'THE FINAL CARTESIAN COORDINATES',/2X,'STN',7X,'X',
      <12X,'Y',8X,'Z',12X,'DX',9X,'DY',9X,'DZ',/70('='))
      I=4
      CALL CART(ZR(I),ZU(I),ZJ(I),ZX(I),ZY(I),ZZ(I))
      DFX(4)=ZX(4)-OX(4)
      DFY(4)=ZY(4)-OY(4)
      DFZ(4)=ZZ(4)-OZ(4)
      WRITE(4,399)4,ZX(4),ZY(4),ZZ(4),DFX(4),DFY(4),DFZ(4)
399  FORMAT(1X,I1,1X,F12.2,1X,F12.2,1X,F12.2,1X,3F10.3)
5457 FORMAT(4X,F7.3,8X,F9.4,8X,F7.3,8X,F9.4)
      WRITE(4,7560)
7560 FORMAT(/10X,'THE POINT VELOCITIES',/10X,20('='),/1X,'STN',2X,
      <' "/year',3X,' "/year',2X,'m/year',7X,'STANDARD ERRORS',
      </1X,70('-'))
      J=12
      DO 7561 I=1,1
      BE1=DX(J,1)
      SER1=DSQRT(COVX(J,J))
      J=J+1
      BE2=DX(J,1)
      SER2=DSQRT(COVX(J,J))
      J=J+1
      BE3=DX(J,1)

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```

SER3=DSQRT(COVX(J,J))
J=J+1
WRITE(4,7562)I,BE1,BE2,BE3,SER1,SER2,SER3
7561 CONTINUE
7562 FORMAT(2X,I2,3F10.5,3X,3F10.5)

WRITE(4,8560)
8560 FORMAT(/10X,'THE POINT ACCELERATIONS',/10X,20('='),/1X,'STN',2X,
<'"/year sq.',3X,'"/year sq.',2X,'m/year sq.',7X,'STANDARD ERRORS'
<,/1X,70('-'))
J=15
DO 8561 I=1,1
BE1=DX(J,1)
SER1=DSQRT(COVX(J,J))
J=J+1
BE2=DX(J,1)
SER2=DSQRT(COVX(J,J))
J=J+1
BE3=DX(J,1)
SER3=DSQRT(COVX(J,J))
J=J+1
WRITE(4,8562)I,BE1,BE2,BE3,SER1,SER2,SER3
8561 CONTINUE
8562 FORMAT(2X,I2,3F10.5,3X,3F10.5)
444 STOP
END

```

C
C
C

```

SUBROUTINE DELTA(X,Y,Z,NTERM,DW)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION DW(3,1)

```

C SUBROUTINE FOR FORMING THE VECTOR OF DIFFERENTIALS IN GRAVITY
C POTENTIAL.
C 1. 'GM' IS THE PRODUCT OF GRAVITATIONAL CONSTANT AND EARTH'S
C MASS

```

GM=3.9860047E14
RHO=206264.8063
R=(X**2+Y**2+Z**2)**0.5
IF(NTERM.NE.1)GOTO 30
DW(1,1)=-1.0*(GM/R**3)*X
DW(2,1)=-1.0*(GM/R**3)*Y
DW(3,1)=-1.0*(GM/R**3)*Z
GOTO 40
30 DW(1,1)=-2.0*(GM/R**4)*X
DW(2,1)=(-2.0*GM/R**4)*Y
DW(3,1)=-2.0*(GM/R**4)*Z
40 RETURN
END

```

C

```

SUBROUTINE JACCOB(PI,H,DL,DJ)

```

```

      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION DJ(3,3)
C     SUBROUTINE FOR FORMING THE DERIVATIVES IN PI, LABDA AND h
C     1. 'A' IS THE SEMI MAJOR RADIUS OF THE EARTH.
C     2. 'E' IS THE ECCENTRICITY OF THE EARTH.
C     3. 'TN' AND 'TM' RADII OF CURVATURE OF THE ELLIPSOID.
C     4. 'H' IS THE ELLIPSOIDAL HEIGHT.
      A=6378249.145
      E=0.082483399
      TN=A/(1.0-(E*SIN(PI))**2)**0.5
      TM=A*(1.0-E**2)/(1.0-(E*SIN(PI))**2)**1.5
      SM=TM+H
      SN=TN+H
      DJ(1,1)=-1.0*SM*SIN(PI)*COS(DL)
      DJ(1,2)=-1.0*SM*SIN(PI)*SIN(DL)
      DJ(1,3)=SM*COS(PI)
      DJ(2,1)=-1.0*SN*COS(PI)*SIN(DL)
      DJ(2,2)=SN*COS(PI)*COS(DL)
      DJ(2,3)=0.0
      DJ(3,1)=COS(PI)*COS(DL)
      DJ(3,2)=COS(PI)*SIN(DL)
      DJ(3,3)=SIN(PI)
      RETURN
      END

```

```

C-----
      SUBROUTINE NORMAL(A,ST,ATK,FK,NN,MM)
      IMPLICIT REAL*8(A-H,O-Z)
C     FORMS THE NORMAL EQUATIONS MATRIX
      DIMENSION FK(NN,1),A(MM,1),ST(MM,1),ATK(NN,1)
      DO 10 I=1,NN
      DO 10 J=1,MM
10    ATK(I,J)=0.0
      DO 30 K=1,MM
      DO 30 I=1,NN
      AA=A(K,I)
      IF(AA.EQ.0.)GOTO 30
      DO 20 J=1,MM
20    ATK(I,J)=ATK(I,J)+AA*ST(K,J)
30    CONTINUE
      DO 40 I=1,NN
      DO 40 J=1,NN
40    FK(I,J)=0.0
C
      DO 60 K=1,MM
      DO 60 J=1,NN
      AA=A(K,J)
      IF(AA.EQ.0.0)GOTO 60
      DO 50 I=J,NN
50    FK(I,J)=FK(I,J)+ATK(I,K)*AA
60    CONTINUE
C

```



```

      DO 70 I=1,NN
      DO 70 J=1,NN
70    FK(I,J)=FK(J,I)
      RETURN
      END

```

C-----

```

C
C      SUBROUTINE RTDMS(ANG, IDEG, IMIN, SEC)
C      CONVERTS RADIANS TO DEGREES MINUTES AND SECONDS
C      IMPLICIT REAL*8(A-H,O-Z)
      ANG=ANG*206264.8063
      IDEG=ANG/3600
      IMIN=(ANG-(IDEG*3600))/60
      SEC=(ANG-(IDEG*3600+IMIN*60))
      RETURN
      END

```

```

C
C      SUBROUTINE EULA(PI, DL, RE, ITY)
C      IMPLICIT REAL*8(A-H,O-Z)
C      DIMENSION RE(3,3)
C      SUBROUTINE CONSTRUCTS THE EULERIAN ROTATION MATRIX.
      RE(1,1)=-1.*SIN(PI)*COS(DL)
      RE(1,2)=-1.*SIN(PI)*SIN(DL)
      RE(1,3)=COS(PI)
      RE(2,1)=-1.0*SIN(DL)
      RE(2,2)=COS(DL)
      RE(2,3)=0.0
      RE(3,1)=COS(PI)*COS(DL)
      RE(3,2)=COS(PI)*SIN(DL)
      RE(3,3)=SIN(PI)
      RETURN
      END

```

```

C
C      SUBROUTINE RADIAN( IDEG, IMIN, SEC, RAD)
C      CONVERTS DEGREES, MINUTES AND SECONDS INTO RADIANS
      TERM1=IMIN/60.0
      TERM2=SEC/3600.0
      RDEG=IDEG
      DEG=RDEG+TERM1+TERM2
      RAD=3.1415926536*DEG/180.0
      RETURN
      END

```

```

C
C      SUBROUTINE TIMES(A,B,C, II, KK, JJ)
C      IMPLICIT REAL*8(A-H,O-Z)
C      DIMENSION A(II,1), B(KK,1), C(II,1)
      DO 10 I=1, II
      DO 10 J=1, JJ
10    C(I,J)=0.0
      DO 30 I=1, II
      DO 30 K=1, KK

```

```

AA=A(I,K)
IF(AA.EQ.0.)GOTO 30
DO 20 J=1,JJ
20 C(I,J)=C(I,J)+AA*B(K,J)
30 CONTINUE
RETURN
END

```

C

```

SUBROUTINE AZIMUTH(DY,DX,AZ)
C SUBROUTINE TO FIT AZIMUTH IN THE CORECT QUADRANT
IMPLICIT REAL*8(A-H,O-Z)
P=206264.8063
PI=3.141592654
IF(ABS(DY).GT.ABS(DX))GOTO 2
AZ=ATAN(DY/DX)
IF(DY.LT.0.0) GOTO 3
IF(AZ.GT.0.0) GOTO 8
AZ=(PI+AZ)*P
GOTO 1
3 IF(DY.LT.0.)GOTO 5
AZ=(PI+AZ)*P
GOTO 1
5 IF(AZ.GT.0.0) GOTO 10
AZ=(2*PI+AZ)*P
GOTO 1
10 AZ=(PI+AZ)*P
GOTO 1
2 AZ=ATAN(DX/DY)
IF(DY.LT.0.0) GOTO 4
AZ=((PI/2.0)-AZ)*P
GOTO 1
4 AZ=((3.0*PI/2.0)-AZ)*P
1 RETURN
8 AZ=AZ*P
RETURN
END

```

```

SUBROUTINE TRANSP(A,B,M,N)
C PERFORMS MATRIX TRANSPOSITION
REAL*8 A(M,N),B(N,M)
DO 30 I=1,N
DO 30 J=1,M
30 B(I,J)=A(J,I)
RETURN
END

```

C

```

SUBROUTINE MATINV(A,AINV,N)
C INVERTS A SQUARE SYMMETRIC MATRIX
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(N,N),AINV(N,N),B(80,160)
DO 11 I=1,N

```

```

DO 11 J=1,N
11 B(I,J)=A(I,J)
   J1=N+1
   J2=2*N
DO 12 I=1,N
DO 12 J=J1,J2
12 B(I,J)=0.0
DO 13 I=1,N
   J=I+N
13 B(I,J)=1.0
DO 610 K=1,N
   KP1=K+1
   IF(K.EQ.N)GOTO 500
   L=K
DO 400 I=KP1,N
400 IF(ABS(B(I,K)).GT.ABS(B(L,K)))L=I
   IF(L.EQ.K)GOTO 500
DO 410 J=K,J2
   TEMP=B(K,J)
   B(K,J)=B(L,J)
410 B(L,J)=TEMP
500 DO 501 J=KP1,J2
501 B(K,J)=B(K,J)/B(K,K)
   IF(K.EQ.1)GOTO 600
   KM1=K-1
DO 510 I=1,KM1
DO 510 J=KP1,J2
510 B(I,J)=B(I,J)-B(I,K)*B(K,J)
   IF(K.EQ.N)GOTO 700
600 DO 610 I=KP1,N
DO 610 J=KP1,J2
610 B(I,J)=B(I,J)-B(I,K)*B(K,J)
700 DO 701 I=1,N
DO 701 J=1,N
   K=J+N
701 AINV(I,J)=B(I,K)
   RETURN
   END

```

C

```

SUBROUTINE ELLIPSOID(Q,LAT,LON)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION Q(3,3),D(3,3),VAL(3),R(3)
REAL LAT,LON

```

C

```

COMPUTES THE ELEMENTS OF THE POSITIONAL ELLIPSOIDS
RK4(C1,C3,AL)=C1+(2.0*C3*COS(AL))
PI=3.1415926536
D(1,1)=(Q(2,2)*Q(3,3))-(Q(3,2)*Q(2,3))
D(1,2)=(Q(3,1)*Q(2,3))-(Q(2,1)*Q(3,3))
D(1,3)=(Q(2,1)*Q(3,2))-(Q(3,1)*Q(2,2))
D(2,2)=(Q(1,1)*Q(3,3))-(Q(3,1)*Q(1,3))
D(2,3)=(Q(3,1)*Q(1,2))-(Q(1,1)*Q(3,2))

```

```

D(3,3)=(Q(1,1)*Q(2,2))- (Q(2,1)*Q(1,2))
DET=(Q(1,1)*D(1,1))+ (Q(1,2)*D(1,2))+ (Q(1,3)*D(1,3))
RK1=(Q(1,1)+Q(2,2)+Q(3,3))/3.0
RK2=(D(1,1)+D(2,2)+D(3,3))/3.0
RK3=DSQRT((RK1**2)-RK2)
COSPHI=(DET+(2.0*(RK1**3))- (3.0*RK1*RK2))/(2.0*(RK3**3))
PHI=ACOS(COSPHI)
ALPHA=PHI/3.0
VAL(1)=RK4(RK1,RK3,ALPHA)
PHI=PHI+(2.0*PI)
ALPHA=PHI/3.0
VAL(2)=RK4(RK1,RK3,ALPHA)
PHI=PHI-(4.0*PI)
ALPHA=PHI/3.0
VAL(3)=RK4(RK1,RK3,ALPHA)
DO 1 I=1,3,1
R(I)=DSQRT(VAL(I))
U=D(2,3)+(VAL(I)*Q(2,3))
V=D(1,3)+(VAL(I)*Q(1,3))
W=D(1,2)+(VAL(I)*Q(1,2))
USQI=1./(U**2)
VSQI=1./(V**2)
WSQI=1./(W**2)
G=DSQRT(USQI+VSQI+WSQI)
VECTX=1.0/(G*U)
VECTY=1.0/(G*V)
VECTZ=1.0/(G*W)
SINLAT=SIN(LAT)
COSLAT=COS(LAT)
SINLON=SIN(LON)
COSLON=COS(LON)
ETA=- (SINLON*VECTX)+ (COSLON*VECTY)
XI=- (SINLAT*COSLON*VECTX)- (SINLAT*SINLON*VECTY)+ (COSLAT*VECTZ)
ZETA=(COSLAT*COSLON*VECTX)+ (COSLAT*SINLON*VECTY)+ (SINLAT*VECTZ)
CALL ELLOR(ETA,XI,ZETA,AZ,DIST,VA)
AZER=AZ/3600.0
VAER=180.0*VA/PI
WRITE(4,11)R(I)
11  FORMAT(11X,F7.4)
WRITE(4,12)ETA
WRITE(4,12)XI
WRITE(4,12)ZETA
12  FORMAT(17X,2F8.3)
WRITE(4,14)DIST,AZER,VAER
14  FORMAT(34X,F5.3,2F7.1)
1   CONTINUE
ERSQ=(VAL(1)+VAL(2)+VAL(3))/3.0
ER=DSQRT(ERSQ)
WRITE(4,11)ER
RETURN
END

```

```

SUBROUTINE ELLOR(DU,DV,DW,AZ,DIST,VA)
  IMPLICIT REAL*8(A-H,O-Z)
C   COMPUTES THE DISTANCE AND VERTICAL ANGLE FOR AN ELLIPSOIDAL
C   RADIUS
  DX=DU
  DY=DV
  CALL AZIMUTH(DY,DX,AZ)
  DUSQ=DU**2
  DVSQ=DV**2
  DWSQ=DW**2
  DIST=DSQRT(DUSQ+DVSQ+DWSQ)
  VA=ASIN(DW/DIST)
  RETURN
END
SUBROUTINE CART(LATI,LONG,HT,X,Y,Z)
C   COMPUTES EQUIVALENT CARTESIAN COORDINATES FROM GEODETIC COORD.
  IMPLICIT REAL*8(A-H,O-Z)
  REAL*8 LATI,LONG
  E=0.082483399
  R1=6378249.145
  CONST=R1/((1.0-E**2*SIN(LATI)**2)**0.5)
  CONST1=CONST+HT
  X=CONST1*COS(LATI)*COS(LONG)
  Y=CONST1*COS(LATI)*SIN(LONG)
  Z=((1.0-E**2)*CONST+HT)*SIN(LATI)
  RETURN
END
SUBROUTINE ASSIGN(K,X,Y,Z,LATI,LONG,HT,I,X1,Y1,Z1,P1,C1,H1)
C   ASSIGNS OBSERVATION VALUES TO CORRESPONDING OBSERVATION LINES
  IMPLICIT REAL*8(A-H,O-Z)
  DIMENSION X(6),Y(6),Z(6),HT(6)
  REAL*8 LATI(6),LONG(6)
  X1=X(K)
  Y1=Y(K)
  Z1=Z(K)
  P1=LATI(K)
  C1=LONG(K)
  H1=HT(K)
  RETURN
END

```

Appendix C.3 Results of program ADAN for 2nd to 5th epochs

THE SECOND OBSERVATION EPOCH SHIFT POINT 4: +0".001 +0".001 -0.015m

```

=====
STN   LATITUDE          LONGITUDE          HEIGHT
      DEG  MIN  SEC      DEG  MIN  SEC      m
=====
  1   0  -51  -59.550   36  19  18.020   2207.900
  2   0  -51  -34.560   36  17  59.580   2038.900
  3   0  -52  -29.980   36  19  34.540   1908.800
  4   0  -52  -26.820   36  17  58.450   1985.700
  5   0  -53  -32.110   36  19  32.280   1906.900
  6   0  -53  -25.670   36  18  15.210   1962.800
=====

```

```

=====
STN   X                Y                Z
=====
  1   5140177.718      3778830.147      -95838.194
  2   5141487.484      3776781.987      -95067.973
  3   5139622.594      3779056.330      -96768.421
  4   5141445.748      3776707.922      -96672.519
  5   5139638.794      3778981.400      -98676.885
  6   5141097.793      3777095.873      -98479.939
=====

```

ITERATION NO. 1

THE COMPUTED VARIANCE OF UNIT WEIGHT= 1.0000

THE PARAMETER MATRIX

```

-0.162414E-03
 0.194043E-02
 0.213237E-04
 0.240054E+01
-0.256511E+01
 0.520936E+01
 0.176306E+01
 0.456964E+01
 0.402373E+01
-0.693091E+00
 0.141810E+02
-0.317134E-03
 0.179296E-02
-0.149987E-01

```

STATION NO. 4

Q IN LOCAL ELLIPSOIDAL SYSTEM

```

0.000308710   -0.000305877   -0.000100939
-0.000305877    0.000421543    0.000142844
-0.000100939    0.000142844    0.000478788
 0.0279
-0.863

```

-0.503
 0.050
 1.000 210.3 2.9
 0.0073
 -0.071
 0.021
 -0.997
 1.000 163.1 -85.8
 0.0194
 -0.501
 0.864
 0.054
 1.000 120.1 3.1
 0.0201

THE COMPUTED COORDINATES

```

=====
STN      LATITUDE      LONGITUDE      HEIGHT
-----
      DEG  MIN  SEC      DEG  MIN  SEC      m
=====
4        0  -52  -26.820    36   17   58.450    1985.700
  
```

THE STANDARD ERRORS

```

=====
0.000712"
0.000839"
0.007508m
  
```

THE FINAL CARTESIAN COORDINATES

```

=====
STN      X              Y              Z              DX              DY              DZ
-----
4        5141445.71    3776707.97    -96672.52    -0.036          0.048          -0.005
  
```

THE POINT VELOCITIES

```

=====
STN      "/year      "/year      m/year      STANDARD ERRORS
-----
1        -0.00032     0.00179     -0.01500     0.00100     0.00118     0.01062
  
```

THE THIRD OBSERVATION EPOCH SHIFT POINT 4 BY 0".001 0".001 0.015m

```

=====
STN      LATITUDE      LONGITUDE      HEIGHT
-----
      DEG  MIN  SEC      DEG  MIN  SEC      m
=====
1        0  -51  -59.550    36   19   18.020    2207.900
2        0  -51  -34.560    36   17   59.580    2038.900
3        0  -52  -29.980    36   19   34.540    1908.800
4        0  -52  -26.820    36   17   58.450    1985.700
5        0  -53  -32.110    36   19   32.280    1906.900
6        0  -53  -25.670    36   18   15.210    1962.800
  
```

STN	X	Y	Z
1	5140177.718	3778830.147	-95838.194
2	5141487.484	3776781.987	-95067.973
3	5139622.594	3779056.330	-96768.421
4	5141445.748	3776707.922	-96672.519
5	5139638.794	3778981.400	-98676.885
6	5141097.793	3777095.873	-98479.939

ITERATION NO. 1

THE COMPUTED VARIANCE OF UNIT WEIGHT= 1.0000

THE PARAMETER MATRIX

- 0.937652E-04
- 0.203636E-02
- 0.681441E-04
- 0.939907E+00
- 0.157634E+01
- 0.474522E+01
- 0.106830E+01
- 0.438625E+01
- 0.142885E+01
- 0.760663E+00
- 0.146292E+02
- 0.373867E-03
- 0.174512E-02
- 0.148478E-01
- 0.549882E-03
- 0.398151E-03
- 0.675480E-02

STATION NO. 4

Q IN LOCAL ELLIPSOIDAL SYSTEM

- 0.021502968 -0.021245828 -0.006897009
- 0.021245828 0.029286165 0.009815166
- 0.006897009 0.009815166 0.033210669

0.2322

-0.865

-0.500

0.051

1.000 210.0 2.9

0.0613

-0.072

0.022

-0.997

1.000 162.6 -85.7

0.1623

-0.497

0.866

0.055

1.000 119.9 3.2

0.1673

THE COMPUTED COORDINATES

```

=====
STN   LATITUDE           LONGITUDE           HEIGHT
-----
      DEG  MIN  SEC      DEG  MIN  SEC      m
=====
4     0  -52 -26.820    36  17  58.450    1985.700

```

THE STANDARD ERRORS

```

=====
0.005932"
0.006997"
0.062900m

```

THE FINAL CARTESIAN COORDINATES

```

=====
STN   X                Y                Z                DX                DY                DZ
-----
4     5141445.71    3776707.97    -96672.52    -0.037    0.051    -0.00

```

THE POINT VELOCITIES

```

=====
STN   "/year   "/year   m/year           STANDARD ERRORS
-----
1     -0.00037   0.00175  -0.01485    0.00839   0.00990   0.08895

```

THE POINT ACCELERATIONS

```

=====
STN   "/year sq.   "/year sq.   m/year sq.           STANDARD ERRORS
-----
1     -0.00055    0.00040   -0.00675    0.00854   0.01012   0.08895

```

THE FOURTH OBSERVATION EPOCH shift 0.000 0.001 0.000

=====

```

=====
STN   LATITUDE           LONGITUDE           HEIGHT
      DEG  MIN  SEC      DEG  MIN  SEC      m
-----
1     0  -51 -59.550    36  19  18.020    2207.900
2     0  -51 -34.560    36  17  59.580    2038.900
3     0  -52 -29.980    36  19  34.540    1908.800
4     0  -52 -26.820    36  17  58.450    1985.700
5     0  -53 -32.110    36  19  32.280    1906.900
6     0  -53 -25.670    36  18  15.210    1962.800

```

```

=====
STN   X                Y                Z
-----
1     5140177.718    3778830.147    -95838.194
2     5141487.484    3776781.987    -95067.973
3     5139622.594    3779056.330    -96768.421
4     5141445.748    3776707.922    -96672.519
5     5139638.794    3778981.400    -98676.885
6     5141097.793    3777095.873    -98479.939

```

ITERATION NO. 1

THE COMPUTED VARIANCE OF UNIT WEIGHT=

1.0000

THE PARAMETER MATRIX

0.495913E-04
 0.223514E-02
 -0.133901E-04
 0.673132E+01
 0.129610E+02
 0.474246E+01
 0.288008E+00
 0.427240E+01
 0.799852E+00
 -0.600489E+00
 0.155249E+02
 -0.510793E-03
 0.156058E-02
 -0.149020E-01
 -0.593583E-04
 0.169835E-02
 -0.109419E-01

STATION NO. 4

Q IN LOCAL ELLIPSOIDAL SYSTEM

0.025347779 -0.024903745 -0.007902752
 -0.024903745 0.034325268 0.011254923
 -0.007902752 0.011254923 0.039062729

0.2510

-0.866
 -0.496
 0.052

1.000 209.8 3.0

0.0670

-0.073
 0.022
 -0.997

1.000 163.0 -85.6

0.1768

-0.494
 0.868
 0.055

1.000 119.6 3.2

0.1814

THE COMPUTED COORDINATES

STN LATITUDE LONGITUDE HEIGHT

DEG MIN SEC DEG MIN SEC m

4 0 -52 -26.820 36 17 58.450 1985.700

THE STANDARD ERRORS

=====

0.006433"

0.007579"

0.068769m

THE FINAL CARTESIAN COORDINATES

STN	X	Y	Z	DX	DY	DZ
4	5141445.71	3776707.98	-96672.52	-0.041	0.056	0.000

THE POINT VELOCITIES

=====

STN	"/year	"/year	m/year	STANDARD ERRORS		
1	-0.00051	0.00156	-0.01490	0.00914	0.01078	0.09725

THE POINT ACCELERATIONS

=====

STN	"/year sq.	"/year sq.	m/year sq.	STANDARD ERRORS		
1	-0.00006	0.00170	-0.01094	0.00809	0.00958	0.08422

THE FIFTH OBSERVATION EPOCH -0 52 26.825 36 17 58.453 1985.660

=====

STN	LATITUDE			LONGITUDE			HEIGHT
	DEG	MIN	SEC	DEG	MIN	SEC	m
1	0	-51	-59.550	36	19	18.020	2207.900
2	0	-51	-34.560	36	17	59.580	2038.900
3	0	-52	-29.980	36	19	34.540	1908.800
4	0	-52	-26.820	36	17	58.450	1985.700
5	0	-53	-32.110	36	19	32.280	1906.900
6	0	-53	-25.670	36	18	15.210	1962.800

STN	X			Y			Z
1	5140177.718			3778830.147			-95838.194
2	5141487.484			3776781.987			-95067.973
3	5139622.594			3779056.330			-96768.421
4	5141445.748			3776707.922			-96672.519
5	5139638.794			3778981.400			-98676.885
6	5141097.793			3777095.873			-98479.939

ITERATION NO. 1

THE COMPUTED VARIANCE OF UNIT WEIGHT= 1.0000

THE PARAMETER MATRIX

0.205463E-03

0.247809E-02

-0.721011E-04

0.326110E-01

0.814623E-01

THE STANDARD ERRORS

=====

0.006433"

0.007579"

0.068769m

THE FINAL CARTESIAN COORDINATES

STN	X	Y	Z	DX	DY	DZ
4	5141445.71	3776707.98	-96672.52	-0.041	0.056	0.000

THE POINT VELOCITIES

=====

STN	"/year	"/year	m/year	STANDARD ERRORS		
1	-0.00051	0.00156	-0.01490	0.00914	0.01078	0.09725

THE POINT ACCELERATIONS

=====

STN	"/year sq.	"/year sq.	m/year sq.	STANDARD ERRORS		
1	-0.00006	0.00170	-0.01094	0.00809	0.00958	0.08422

THE FIFTH OBSERVATION EPOCH -0 52 26.825 36 17 58.453 1985.660

=====

STN	LATITUDE			LONGITUDE			HEIGHT
	DEG	MIN	SEC	DEG	MIN	SEC	m
1	0	-51	-59.550	36	19	18.020	2207.900
2	0	-51	-34.560	36	17	59.580	2038.900
3	0	-52	-29.980	36	19	34.540	1908.800
4	0	-52	-26.820	36	17	58.450	1985.700
5	0	-53	-32.110	36	19	32.280	1906.900
6	0	-53	-25.670	36	18	15.210	1962.800

STN	X	Y	Z
1	5140177.718	3778830.147	-95838.194
2	5141487.484	3776781.987	-95067.973
3	5139622.594	3779056.330	-96768.421
4	5141445.748	3776707.922	-96672.519
5	5139638.794	3778981.400	-98676.885
6	5141097.793	3777095.873	-98479.939

ITERATION NO. 1

THE COMPUTED VARIANCE OF UNIT WEIGHT= 1.0000

THE PARAMETER MATRIX

0.205463E-03

0.247809E-02

-0.721011E-04

0.326110E-01

0.814623E-01

0.446347E+01
 -0.367304E+00
 0.438317E+01
 -0.142869E+01
 -0.396087E+00
 0.162211E+02
 -0.667329E-03
 0.133512E-02
 -0.148415E-01
 -0.572049E-03
 0.339013E-02
 -0.181030E-01

STATION NO. 4

=====

Q IN LOCAL ELLIPSOIDAL SYSTEM

0.000295420 -0.000288706 -0.000089633
 -0.000288706 0.000397893 0.000127748
 -0.000089633 0.000127748 0.000454313

0.0270

-0.868
 -0.493
 0.053

1.000 209.6 3.1

0.0073

-0.074
 0.022
 -0.997

1.000 163.5 -85.6

0.0191

-0.491
 0.870
 0.055

1.000 119.4 3.2

0.0196

THE COMPUTED COORDINATES

=====

STN	LATITUDE			LONGITUDE			HEIGHT
-----	----------	--	--	-----------	--	--	--------

	DEG	MIN	SEC	DEG	MIN	SEC	m
--	-----	-----	-----	-----	-----	-----	---

4	0	-52	-26.820	36	17	58.450	1985.700
---	---	-----	---------	----	----	--------	----------

THE STANDARD ERRORS

=====

0.000694"
 0.000816"
 0.007472m

THE FINAL CARTESIAN COORDINATES

STN	X	Y	Z	DX	DY	DZ
-----	---	---	---	----	----	----

4	5141445.70	3776707.98	-96672.51	-0.045	0.062	0.006
---	------------	------------	-----------	--------	-------	-------

THE POINT VELOCITIES

=====

STN	"/year	"/year	m/year	STANDARD ERRORS		
1	-0.00067	0.00134	-0.01484	0.00099	0.00117	0.01057

THE POINT ACCELERATIONS

=====

STN	"/year sq.	"/year sq.	m/year sq.	STANDARD ERRORS		
1	-0.00057	0.00339	-0.01810	0.00083	0.00098	0.00863

Appendix C.4 Results for the 5th epoch observations -static mode.

APPROX. COORDINATES

=====

STN	LATITUDE			LONGITUDE			HEIGHT
	DEG	MIN	SEC	DEG	MIN	SEC	m
1	0	-51	-59.550	36	19	18.020	2207.900
2	0	-51	-34.560	36	17	59.580	2038.900
3	0	-52	-29.980	36	19	34.540	1908.800
4	0	-52	-26.820	36	17	58.450	1985.700
5	0	-53	-32.110	36	19	32.280	1906.900
6	0	-53	-25.670	36	18	15.210	1962.800

STN	X	Y	Z
1	5140177.718	3778830.147	-95838.194
2	5141487.484	3776781.987	-95067.973
3	5139622.594	3779056.330	-96768.421
4	5141445.748	3776707.922	-96672.519
5	5139638.794	3778981.400	-98676.885
6	5141097.793	3777095.873	-98479.939

ITERATION NO. 0

THE COMPUTED VARIANCE OF UNIT WEIGHT= 119.2633
ITERATION NO. 1

THE COMPUTED VARIANCE OF UNIT WEIGHT= 1.0002
ITERATION NO. 2

THE COMPUTED VARIANCE OF UNIT WEIGHT= 1.0000
POTENTIAL DIFFERENCE

RESIDUAL	SCALED WEIGHTS
-0.002141112	0.2928267E+06
-0.002798314	0.2928267E+06
-0.000376274	0.2928267E+06
-0.000327732	0.2928267E+06
0.001515449	0.2928267E+06
-0.002112975	0.2928267E+06
0.002170707	0.2928267E+06
-0.000288810	0.2928267E+06
0.002276538	0.2928267E+06
0.000167338	0.2928267E+06
0.001170601	0.2928267E+06
0.000341433	0.2928267E+06
0.002566614	0.2928267E+06
0.000364097	0.2928267E+06

-0.001017982	0.2928267E+06
-0.001214829	0.2928267E+06
-0.000065738	0.2928267E+06
0.002692556	0.2928267E+06
-0.002206315	0.2928267E+06
-0.000021611	0.2928267E+06
-0.001298766	0.2928267E+06
-0.003082253	0.2928267E+06
-0.000178538	0.2928267E+06
0.001194223	0.2928267E+06

VTVP	TRACE	SIG	USED %	LEFT %	TOTAL OBS
19.00967	19.00967	1.000000	20.79	79.21	24
			4.99	19.01	

GRAVITY DIFFERENCE

LATITUDE

RESIDUAL		SCALED WEIGHTS			
0.000433501	0.5916238E+05				
0.000097484	0.5916238E+05				
-0.000150904	0.5916238E+05				
0.000621908	0.5916238E+05				
0.000006709	0.5916238E+05				
-0.000000554	0.5916238E+05				
VTVP	TRACE	SIG	USED %	LEFT %	TOTAL OBS
0.03591	0.03591	1.000000	99.40	0.60	6
			5.96	0.04	

LONGITUDE

RESIDUAL		SCALED WEIGHTS			
-0.001619188	0.2331721E+05				
-0.000142556	0.2331721E+05				
0.000486859	0.2331721E+05				
0.000017785	0.2331721E+05				
-0.000207360	0.2331721E+05				
0.000000108	0.2331721E+05				
VTVP	TRACE	SIG	USED %	LEFT %	TOTAL OBS
0.06814	0.06814	1.000000	98.86	1.14	6
			5.93	0.07	

AZIMUTH

RESIDUAL		SCALED WEIGHTS			
-3.349599301	0.6928144E-01				
-3.755113928	0.6928144E-01				
VTVP	TRACE	SIG	USED %	LEFT %	TOTAL OBS

VTVP	TRACE	SIG	USED %	LEFT %	TOTAL OBS
15.42075	15.42075	1.000000	35.75	64.25	24
			8.58	15.42	

HORIZONTAL ANGLES

RESIDUAL	SCALED WEIGHTS
3.887806217	0.3267808E-01
5.988433892	0.3267808E-01
-11.282736542	0.3267808E-01
1.240950812	0.3267808E-01
2.476789219	0.3267808E-01
1.730231188	0.3267808E-01
-2.428832706	0.3267808E-01
-1.720254942	0.3267808E-01
0.877918862	0.3267808E-01
5.474905426	0.3267808E-01
-7.935941697	0.3267808E-01
1.790696897	0.3267808E-01
-5.354395844	0.3267808E-01
6.431937560	0.3267808E-01
-3.143935201	0.3267808E-01
0.275145846	0.3267808E-01
1.096713229	0.3267808E-01
0.747392590	0.3267808E-01
-0.287696825	0.3267808E-01
-6.906005938	0.3267808E-01
5.902223708	0.3267808E-01
1.931485871	0.3267808E-01
-5.863235880	0.3267808E-01
4.265538065	0.3267808E-01

VTVP	TRACE	SIG	USED %	LEFT %	TOTAL OBS
16.84261	16.84261	1.000000	29.82	70.18	24
			7.16	16.84	

TOTAL DEGREES OF FREEDOM = 63.000

THE PARAMETER MATRIX

- 0.220787E-02
- 0.251431E-02
- 0.686813E-02
- 0.213882E-02
- 0.199689E-03
- 0.658815E-02
- 0.555428E-03
- 0.873539E-03
- 0.650018E-02
- 0.290186E-03

0.780316E-02
 -0.333338E-01
 -0.575273E-04
 0.172115E-03
 0.668253E-02
 -0.400984E-02
 -0.973674E-02
 0.669465E-02
 -0.990954E-01
 0.573519E-01
 -0.126818E-01
 0.294236E-01
 0.318926E-01
 -0.510808E-01
 -0.549894E-01
 -0.308006E-01
 0.259972E-02
 -0.762330E-03
 0.319878E-01
 -0.702656E-03
 -0.273208E+01
 -0.105534E+01
 -0.668494E+01
 -0.134454E+02
 -0.375114E+01
 -0.161323E+01

STATION NO. 1

=====

Q FOR PHI, LAMBDA, H

0.000000053	0.000000009	-0.000000021
0.000000009	0.000000093	-0.000000055
-0.000000021	-0.000000055	0.000002837

Q IN LOCAL ELLIPSOIDAL SYSTEM

0.000034695	-0.000041765	-0.000005095
-0.000041765	0.000057488	0.000007036
-0.000005095	0.000007036	0.000050133

0.0096

-0.978
 -0.207
 0.020

1.000 191.9 1.2

0.0017

-0.019
 -0.010
 -1.000

1.000 208.9 -88.8

0.0069

-0.207
 0.978
 -0.006

1.000 101.9 -0.4

0.0069

STATION NO. 2

=====

Q FOR PHI, LAMBDA, H

0.000000149	0.000000047	-0.000000006
0.000000047	0.000000066	-0.000000077
-0.000000006	-0.000000077	0.000003237

Q IN LOCAL ELLIPSOIDAL SYSTEM

0.000025744	-0.000028902	-0.000025121
-0.000028902	0.000040290	0.000037371
-0.000025121	0.000037371	0.000140384

0.0127

-0.416
-0.909
0.007

1.000 245.4 0.4

0.0018

-0.052
0.016
-0.999

1.000 163.2 -86.9

0.0065

-0.908
0.416
0.053

1.000 155.4 3.1

0.0083

STATION NO. 3

=====

Q FOR PHI, LAMBDA, H

0.000000065	-0.000000020	-0.000000005
-0.000000020	0.000000064	-0.000000042
-0.000000005	-0.000000042	0.000002421

Q IN LOCAL ELLIPSOIDAL SYSTEM

0.000024612	-0.000028561	0.000012114
-0.000028561	0.000039091	-0.000015212
0.000012114	-0.000015212	0.000060867

0.0090

0.711
-0.703
-0.011

1.000 315.3 -0.6

0.0015

-0.026
-0.011
-1.000

1.000 203.1 -88.4

0.0065

0.703
0.711
-0.026

1.000 45.3 -1.5

0.0064

STATION NO. 4

Q FOR PHI, LAMBDA, H

0.0000000063	-0.000000015	-0.000000004
0.0000000015	0.000000057	-0.000000049
0.0000000004	-0.000000049	0.000002159
IN LOCAL ELLIPSOIDAL SYSTEM		
0.000022262	-0.000025609	0.000009193
0.000025609	0.000034654	-0.000011260
0.000009193	-0.000011260	0.000059604

0.0085

0.647
-0.763
-0.013

1.000 310.3 -0.7

0.0015

-0.032
-0.010
-0.999

1.000 198.0 -88.1

0.0065

0.762
0.647
-0.031

1.000 40.3 -1.8

0.0062

STATION NO. 5

Q FOR PHI, LAMBDA, H

0.000000138	0.000000039	0.000000000
0.000000039	0.000000064	-0.000000061
0.000000000	-0.000000061	0.000002820
IN LOCAL ELLIPSOIDAL SYSTEM		
0.000024559	-0.000028237	-0.000020594
0.000028237	0.000039501	0.000031337
0.000020594	0.000031337	0.000130358

0.0121

-0.401
-0.916
0.005

1.000 246.4 0.3

0.0017

-0.039
0.011
-0.999

1.000 163.8 -87.7

0.0067

-0.915
0.401

0.041

1.000 156.4 2.3

0.0081

STATION NO. 6

=====

Q FOR PHI, LAMBDA, H

0.000000058	0.000000021	0.000000000
0.000000021	0.000000154	-0.000000086
0.000000000	-0.000000086	0.000003775

Q IN LOCAL ELLIPSOIDAL SYSTEM

0.000056472	-0.000069359	-0.000010949
-0.000069359	0.000094991	0.000016224
-0.000010949	0.000016224	0.000054792

0.0123

-0.980
-0.198
0.018

1.000 191.4 1.0

0.0019

-0.019
0.007
-1.000

1.000 159.6 -88.8

0.0071

-0.197
0.980
0.011

1.000 101.4 0.6

0.0083

THE COMPUTED COORDINATES

=====

STN	LATITUDE			LONGITUDE			HEIGHT
-----	----------	--	--	-----------	--	--	--------

	DEG	MIN	SEC	DEG	MIN	SEC	m
--	-----	-----	-----	-----	-----	-----	---

1	0	-51	-59.548	36	19	18.022	2207.907
2	0	-51	-34.558	36	17	59.582	2038.907
3	0	-52	-29.981	36	19	34.543	1908.807
4	0	-52	-26.820	36	17	58.459	1985.667
5	0	-53	-32.110	36	19	32.281	1906.907
6	0	-53	-25.674	36	18	15.211	1962.807

THE FINAL CARTESIAN COORDINATES

STN	X	Y	Z	DX	DY	DZ
-----	---	---	---	----	----	----

1	5140177.68	3778830.21	-95838.13	-0.040	0.067	0.062
2	5141487.49	3776782.00	-95067.91	0.002	0.009	0.066
3	5139622.61	3779056.31	-96768.44	0.021	-0.018	-0.017
4	5141445.58	3776708.10	-96672.51	-0.170	0.175	0.009
5	5139638.80	3778981.41	-98676.89	0.002	0.008	-0.002
6	5141097.98	3777095.63	-98480.06	0.182	-0.240	-0.123

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