"RELATIVITY IN COMPLEX SPACETIME"

by

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DECLARATION

This work is an original thesis prepared in fulfillment of the requirement for the Degree of Doctor of Philosophy (Ph.D.) in Physics at the University of Nairobi and has not been submitted before anywhere for assessment.

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DEDICATION

Dedicated to my parents:

Henry Awuor

&

Helen Ong'ute

whose complexions I bear in reality and in my imaginations.

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The aim and content of this thesis are briefly described in the introduction. My debt to those who have investigated on the same subject, notably, Johannes Kepler (1571-1630), Sir Isaac Newton (1642-1727), Max Planck (1858-1947), Albert Einstein (1879-1955), Niels Bohr (1885-1962) and Louis de Broglie (1892-1987), on whose inspiring formulations I have leant for justification, and as the basis of my confidence.

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ABSTRACT

A general theory of Relativity has been developed on the basis of a complexified spacetime and its scale equivalence principle. The results are in agreement with the familiar Schwarzschild solution (as derived from Einstein's relativity formulations) [Adler et el., (1965)] [1] and with the well confirmed Newtonian law of Gravitation.

On the very large scale, a composite curvature spacetime cosmological model is obtained in which an Euclidean horizon replaces the Event horizon in the determination of local scale evolution of spacetime.

The complexified relativistic formulation calls to question the constancy of G (Newton's universal gravitational constant), the weak Cosmological Principle which prescribes a requirement of homogeneity in the distribution of cosmological matter [Narlikar, (1978)] [2], as well as the proper meaning of the black hole radius. Finally, some quantum considerations of gravity as well as of the Big Bang are propounded as plausible physical probabilities.

CONTENTS

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FIGURE CAPTIONS

....it must be conceded that a theory has an important advantage if its basic concepts and fundamental hypotheses are 'close to experience', ... Yet more and more, as the depth of our knowledge increases, we must give up this advantage in our quest for logical simplicity and uniformity in the foundations of physical theory....

ALBERT EINSTEIN - Scientific American, 1950.

INTRODUCTION

Throughout the history of physics, many attempts have been made to explain physical phenomena by geometrical arguments. Geometry grew in ancient Greece as the science of plane and solid figures [McKenzie, (1960)][3]. The construction of a figure was perceived to bring out points, which one then naturally regards as pre-existing - at least potentially - in a continuous medium (called space) that provides so to speak, the material from which the figures are made. Understandably, geometry came to be conceived of as the study of this medium, the science of points and their relation in space. In other words, space was considered as the repository of all points required for carrying out the admissible construction of geometry.

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Later, space came to be identified as the divine presence in which matter is placed, thus setting the stage for natural philosophy and geometrical principles mainly attributed to Newton and based on Euclid's geometric construction. In the 19th century, bold yet entirely plausible variations of the established principles and methods of geometric thinking gave rise to a whole crop of systems, that furnished either generalizations and extensions of, or alternatives to, the geometry of Euclid [Torretti, (1983)][4]. The choice of a physical geometry, among the many possibilities offered by mathematics, was either a matter of fact, to be resolved by experimental reasoning, or a mere matter of agreement.

In the third quarter of the century, Bernhard Riemann (1826-1862) sought to extend the two dimensional Gaussian geometry and to classify physical space within the vast genus of structured sets or 'manifolds' of which it is an instance. This was the basis of Riemann's theory of metric manifolds which Albert Einstein adopted as the right approach to physical geometry, in his theory of gravitation or, as he preferred to call it, the General Theory of Relativity [Einstein, (1920)][5]. However, a localized approximation to General Relativity, known as the Special Theory of

Relativity, which had earlier been developed by Einstein as a form of a non-Newtonian physics, was only properly interpreted on the basis of non-Euclidean geometry discovered by Hermann Minkowski, in which time and space are unified as a single chronogeometric structure [Einstein, (1920)] [5]. This, so called, Minkowski geometry defines a uniquely affine and rather special case of the Riemann geometry.

Minkowski's significant contribution towards the formal development of the theory of relativity was his recognition that the four-dimensional space-time continuum on which Albert Einstein had founded his relativistic electrodynamics of moving bodies was none other than the theory of invariants of a definite group of linear transformations, namely, the Lorentz group. This, he achieved by a replacement of the usual real time coordinate by a proportional magnitude of imaginary time. Under these conditions, the natural laws satisfying the demands of the Special Theory of Relativity assume mathematical forms, in which the time coordinate plays exactly the same role as the three space coordinates in Euclidean geometry. This was the dawn of four-dimensional (and indeed of higher dimensional) spacetime as a physical geometric construction.

The idea of many dimensions has been a common feature of much recent theoretical speculations, especially on quantum gravity, on string theory and on supergravity theory, but of course it does pose the intriguing question of why the extra dimensions are not 'seen' in the ebb and flow of daily life? This is usually answered by supposing that they are 'curled up on themselves' in a very small circle (presumably of Planck length size). The question whether the global topological structures of the extra dimensions is exactly a set of circles, or whether it is something more complex, is currently a matter of some debate; but the general idea seems to work, provided that the extra dimension are spatial: trying to have more than one time dimension has not been considered as very productive [Isham, (1989)] [6].

Unlike in Newtonian mechanics where the inexorable notions of determinism and clarity of description are preserved, quantum physics abolishes clear-cut trajectories and introduces a probabilistic fitfulness into nature. The resulting elusive quality bestowed on physical reality has been the subject of rather disputed interpretations especially in quantum gravity. An important implication of quantizing gravity is the persistence of the more central question; whether a fundamental revision of spacetime concepts should precede attempts to quantize the gravitational field, or whether it should emerge 'after the event'. The latter has been preferred by most researchers, and takes advantage of a great body of theoretical and experimental (particle physics) material against which new ideas can be developed and tested. However, the former, more iconoclastic, option also has its attractions which may be dated back to Minkowski's geometric construction, and forms the main leitmotiv of this thesis.

Although Minkowski's geometry is characterized by the introduction of an imaginary time coordinate, it made an even more profound contribution regarding the viability of complex numbers in the interpretation of physical geometry. The significance of this complex notion understandably motivates a more pragmatic complexification of the Minkowskian spacetime, which is the main subject of Chapter I. In this case the extra spatial dimensions assume imaginary scales and only qualify for physical interpretation in terms of interactive energy.

In science, a theory is considered better if it minimizes the number of fundamental generalizations (postulates) required in the description (or explanation) of physical phenomena. That this is the case, and advantage of a fully complexified spacetime over Minkowskian spacetime is quantitatively evident (in Chapter I) since the Weak Equivalence Principle accredited to Einstein [Adler, (1965)][1] reduces to a non-trivial mathematical solution, based on the energy conservation principle, as well as on a dimensional scale equivalence which is proposed here as a more fundamental principle of equivalence.

A geometric modification necessarily demands a reformulation of the concepts of motion and consequently of gravity. Albert Einstein had emphasized this implication in his attempts to reduce the theory of gravity to geometry. In Chapter II, it is shown that the foregoing two principles (of scale equivalence and of energy conservation) suffice in the reformulation of Relativity (Special and General) which is in consonance with both the Newtonian and Einstenian forms.

On the very large length scales, the a relationship between the complex spacetime and matter provides a composite spacetime curvature model of the universe in which a flat (Euclidean) horizon separates a closed Newtonian region from the outer (open) bounds (influenced by antigravity). This is the theme of Chapter III, in which the energy density profile clearly deviates from homogeneity. The complexified scheme further distinguishes itself as an unequivocal relativity formulation that questions the constancy of G (Newton's gravitational constant), the metric structure of gravity, as well as the proper meaning of the black hole radius. These are expounded in Chapter IV.

Finally, some quantum considerations of gravity as well as of the Big Bang are propounded here as Appendix A and Appendix B respectively.

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CHAPTER I

COMPLEX SPACETIME

The purpose of this chapter is to convey a complexified spacetime possibility as an extension of Minkowski's geometric configuration, whereby the extra spatial dimensions assume imaginary scales, while the additional temporal coordinate is real. In addition to the Principle of Energy Conservation, a requirement of scale equivalence is introduced as a basis for the description of all physical interactions and consequently reduces the Weak Equivalence Principle to a non-trivial energy solution.

(I.1) Basic Formulations

Consider the non-Euclidean spacetime of general relativity described by a four general coordinates χ_ℓ with a Minkowskian line-element [Isham, (1989)][6] written in a quadratic form as

$$dS_M^2 = \eta_{j\ell}\, d\chi_j\, d\chi_\ell, \qquad (j,\ell = 0,1,2,3) \qquad (I.1.1)$$

where $\eta_{j\ell}$ are the metric functions, χ_0 points in the *imaginary* timelike direction, while χ_1, χ_2 and χ_3 are *real* spatial coordinates. The above four coordinate description is evidently incomplete in generality unless χ_ℓ is fully complexified. In order to investigate the implications of such a complexified spacetime structure, let us define an eight complexified general coordinates

$$\chi_\ell^{(\alpha)} = i^\alpha \chi_\ell, \qquad (I.1.2)$$

where $\alpha = 0,1$ while i is the imaginary unit, so that the line-element is complexified as

$$dS_c^2 = i^{\alpha+\alpha'} dS_M^2, \qquad (I.1.3)$$

where $\alpha' = 0,1$. In this case the expression for velocity under the summation convention becomes

$$v_{\alpha\alpha'} = \sum_{\alpha,\alpha'} \frac{\partial \chi_{\ell\neq 0}^{(\alpha)}}{\partial \chi_0^{(\alpha')}}$$
$$= \left(\frac{v_{00}v_{01}^2}{v_{01}^2 + v_{00}^2} + \frac{v_{10}^2 v_{11}}{v_{11}^2 + v_{10}^2} \right) + i \left(\frac{v_{10}v_{11}^2}{v_{11}^2 + v_{10}^2} - \frac{v_{00}^2 v_{01}}{v_{01}^2 + v_{00}^2} \right) \qquad (I.1.4)$$

Since the significance of an appropriate physical geometry lies in its interpretation of energy, we here introduce the concept of energy in relation to velocity as a scalar product of the form

$$E_T = \xi(v_{\alpha\alpha'} . v_{\alpha\alpha'}), \qquad (I.1.5)$$

where E_T is the total energy over the entire spacetime while ξ is a proportionality ratio. Thus the global essence of the complex spacetime geometry should manifest itself via the real and imaginary components of the total energy of the universe. In addition, the concept of global conservation of energy demands that the total energy of the universe be zero [Hawking, (1988)][7], that is

$$E_T = 0, \qquad (I.1.6)$$

which produces immediately that

$$v_{00} = -v_{11} \quad \text{and} \quad v_{01} = v_{10}. \qquad (I.1.7)$$

(I.2) Scale Equivalence Principle

There are infinite sets of relations of Eqn (I.1.7) that satisfy Eqn (I.1.6) and are thus intractable except under the special condition of scale equivalence, in which case

$$|v_{00}| = |v_{01}|, \qquad (I.2.1)$$

so that Eqn (I.1.5) becomes

$$E_T = \xi \left[(\frac{v_{00}}{2} + \frac{v_{11}}{2}) \pm i(\frac{v_{00}}{2} + \frac{v_{11}}{2}) \right]^2$$
$$= \xi \left[\sum_{\alpha,\alpha'} \frac{v_{00}^{(\alpha)} v_{\alpha'\alpha'}^{(\alpha)}}{2} \pm \frac{i}{2} \left(\sum_{\alpha'} v_{\alpha'\alpha'}^{(0)} \right)^2 \right], \qquad (I.2.2)$$

where

$$v^{(\alpha)}_{\alpha'\alpha'} = i^{\alpha} v_{\alpha'\alpha'} \qquad (I.2.3)$$

in analogy with Eqn (I.1.2). Eqn (I.2.2) is the expected form of a dynamical energy formulation, that is, in consonance with classical kinematics, and is guaranteed by the uniqueness of the scale equivalence contained in Eqn (I.2.1) which may be expressed as a principle in a more concise and qualitative form as follows;

> 'The energy of all physical interactions are representable on a complex spacetime system in which the scales of the real and of the imaginary coordinates are equivalent',

that is, when the scalar $|\chi_{\ell}^{(\alpha)}|$ is independent of α.

In the next section, it shall be shown how the Weak Equivalence Principle is contained in the above principle of Scale Equivalence.

(I.3) The Weak Equivalence Principle

The Principle of Equivalence (EP):- 'that the extent to which a piece of matter produces, or reacts to a gravitational field is determined by its inertial mass', has historically played an important role in the development of gravitational theory. Isaac Newton regarded this principle as the cornerstone of mechanics and made it an empirical issue by carrying out experiments using pendular to verify it with modest precision, [Will, (1988)][8]

One consequence of the EP is the mass-independence of the acceleration of a particle in a gravitational field. This elementary form of EP is known as the Weak Equivalence Principle (WEP). Another implication is contained in a much more powerful and far-reaching EP known as the Einstein EP (EEP), which Einstein used in 1907 as the basis of General Relativity (more precisely, in asserting that gravitation is a curved spacetime phenomenon) [Einstein, (1956)][9].

A derivation of the WEP by a plausible interpretation of Eqn (I.2.2) is presented here, based on an idea advanced by Laue [Torretti, (1983)][10] in which we consider each distinct term as an expression of interactive energy in a particular physical domain. So far there are four separate and distinct physical domains distinguished in physics as the gravitational, the electromagnetic, the strong and the weak interactions.

If we identify the real part of Eqn (I.2.2) with $\alpha' = 0$ as the gravitational interaction energy E_G then we may write

$$E_G = \sum_{\alpha} \xi_\alpha \frac{v_{00}^{(\alpha)2}}{2} \qquad (I.3.1)$$

where ξ_0 and ξ_1 are the inertial and gravitational masses respectively.

If we assume that $\chi_{\ell \neq 0}^{(\alpha)}$ are at least twice-differentiable functions of $\chi_0^{(\alpha')}$ then from elementary calculus we can write

$$v_{\alpha\alpha'}^{(b)2} = 2 g_{\alpha\alpha'}^{(b)} \chi_{\ell \neq 0}^{(\alpha)}, \qquad (I.3.2)$$

where $g_{\alpha\alpha'}^{(b)}$ are accelerations associated with interaction energies of kinetic and potential types, with b=0,1.

It follows at once from Eqn (I.3.1) that if Eqn (I.1.6) holds independently in the gravitational domain, then

$$g_{00}^{(0)} \chi_{\ell \neq 0}^{(\alpha)} (\xi_0 - \xi_1) = 0, \qquad (I.3.3)$$

where $g_{00}^{(0)}$ and $\chi_{\ell \neq 0}^{(0)}$ need not vanish. Thus we obtain a non-trivial solution to Eqn (I.3.3) as

$$\xi_0 = \xi_1, \qquad (I.3.4)$$

which is a statement of the Weak Equivalence Principle [Torretti, 1983)][4], hitherto considered as a fundamental postulate of spacetime curvature [Einstein, 1920)][5]. This relation has been established experimentally by Eotvost and by Dicke [Adler et. al, (1965)][1], and has been tested, for the Earth, by the observations of the Moon-Earth orbit using laser ranging [Narlikar, (1978)][11] By Eqn (I.3.4) above, it has been shown that for a complex spacetime in which the principle of energy conservation is upheld, the Weak Equivalence Principle (so far considered as a postulate) is reduced to a non trivial mathematical solution based on the unique and apparently more fundamental principle of Scale Equivalence.

Finally we may surmise that interactive energies in all physical domains of reality are reducible to the form of Eqn (I.3.1) while the imaginary term in Eqn (I.2.2) may be attributed to antimatter, in which case we arrive at a more complete form of the Strong Equivalence Principle [Will, (1988)][8].

CHAPTER II

RELATIVITY

The development of General Relativity (GR) has been one of the greatest intellectual adventure of our time. The basal idea that is principal to the subject of Relativity is that every motion must only be considered as a relative motion [Einstein, (1920)] [5]. Although very successful, Einstein's theory of relativity is often criticized for giving, without justification, a central theoretical role to the propagation of light, in that it founds the concept of time upon the law of propagation of light (The Light Principle) [Torretti, (1983)] [4], [Savickas, (1994)] [12].

As an alternative formulation of the General Theory of Relativity, the complexified spacetime of chapter 1 will be adopted here in a derivation of quantitative formulae for gravitational acceleration and the Lorentz transformations of Special Relativity, without an explicit reference to the Light Principle. In order to show some compatibility of the complex spacetime formulation of GR with Einstein's GR, a derivation of Schwarzschild line-element has been presented in section (II.4), while sections (II.5) and (II.6) provide derivations of Kepler's third law and of Newton's gravitational law respectively.

(II.1) Gravitational Acceleration

Consider a total gravitational interaction energy E_G as expressed in (I.3.1). Under the principle of energy conservation defined by Eqn (I.1.6) and the Weak Equivalence Principle given by Eqn (I.3.4), the E_G independently vanishes so that Eqn (I.3.1) becomes

$$\sum_{\alpha} v_{00}^{(\alpha)2} = 0 \qquad (II.1.1)$$

$$\Rightarrow v_{00}^{(0)2} = -2g_{00}^{(1)}\chi_{\ell\neq 0}^{(0)}, \qquad (II.1.2)$$

where $g_{00}^{(1)}$ is the normal component of acceleration for a particle moving a long a path whose radius of curvature is $\chi_{\ell\neq 0}^{(0)}$.

In the familiar centripetal form, Eqn (II.1.2) becomes,

$$g_{00}^{(1)} = -\frac{V_g^2}{\chi_{\ell\neq 0}^{(0)}}, \qquad (II.1.3)$$

where

$$V_g = \pm\frac{v_{00}^{(0)}}{|\sqrt{2}|}. \qquad (II.1.4)$$

Thus Eqn (II.1.3) presents the desired form of gravitational acceleration, which is clearly consistent with the isotropic Newtonian kinematics, provided $\chi_{\ell\neq 0}^{(0)}$ is considered as a unit length.

(II.2) Special Relativity

The formulations of Special Relativity relied heavily on the law of propagation of light postulated by Einstein, in the derivation of the linearized Lorentz transformations. An attempt to derive the Lorentz transformations from Relativity Principle alone, without assuming anything about the velocity of light or any other specific phenomenon was given by a Russian Mathematical Physicist W. Von Ignatowsky. However, he wrongly maintains that the Principle of Reciprocity (if inertial frame F' moves with a velocity V in F then F moves with a velocity $-V$ in F') is a consequence of the Principle of Relativity [Torretti, (1983)] [13].

In this section the centripetal form of gravity given by Eqn (II.1.3) will be shown to prescribe an invariant velocity in the form of Eqn (II.1.4), which is necessary for the determination of Lorentz transformations.

From Eqn (I.1.4) we can write;

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$$V_g = \frac{d\chi_{\ell\neq 0}^{(0)}}{d\chi_0^{(0)}} \qquad (II.2.1)$$

so that Eqn (II.1.4) becomes

$$d\chi_{\ell\neq 0}^{(0)} \pm |V_g| d\chi_0^{(0)} = 0. \qquad (II.2.2)$$

A scalar transformation of Eqn (II.2.2) presents three possibilities as follows;

$$\beta(d\chi^{(0)}_{\ell\neq 0} \pm |V_g| d\chi^{(0)}_0 = 0) \equiv \begin{cases} d\chi^{(0)'}_{\ell\neq 0} \pm |V'_g| d\chi^{(0)'}_0 = 0 & \text{........(II.2.3a)} \\ d\chi^{(0)'}_{\ell\neq 0} \pm |V'_g| d\chi^{(0)}_0 = 0 & \text{........(II.2.3b)} \\ d\chi^{(0)'}_{\ell\neq 0} \pm |V_g| d\chi^{(0)'}_0 = 0 & \text{........(II.2.3c)} \end{cases}$$

where β is a scalar multiplier.

In conformity with the centripetal description in Eqn (II.1.3), we here give physical meaning to the three transformations by a suitable interpretation of their rigid 'disc-like' possibilities:

(a) Eqn (II.2.3a) is only satisfied for all non zero scalars $\beta \neq 0$ if $|V_g| = 0$ and $|d\chi^{(0)}_\ell| = 0$. This is a stationary case and the transformation does not represent energy exchange or interaction in the form of relative motion for the disc.

(b) Eqn (II.2.3b) describes Einstein's rigid rotating disc which (as he proved) is representative of a non-Lorentzian transformation [Einstein, (1920)][5].

(c) Eqn (II.2.3c) describes a rotation of a 'semi-rigid' disc (SRD) consisting of concentric circular rings (Fig 1.) each rotating at a constant tangential speed V_g in which case the two parts of the transformation may be written out as;

$$d\chi^{(0)'}_{\ell\neq 0} + |V_g| d\chi^{(0)'}_0 = \beta_1(d\chi^{(0)}_{\ell\neq 0} + |V_g| d\chi^{(0)}_0), \qquad (II.2.4a)$$

and

$$d\chi^{(0)'}_{\ell\neq 0} - |V_g| d\chi^{(0)'}_0 = \beta_2(d\chi^{(0)}_{\ell\neq 0} - |V_g| d\chi^{(0)}_0), \qquad (II.2.4b)$$

where β_1 and β_2 are scalar constants.

From equations (II.2.4a), (II.2.4b) and (II.1.4) we can proceed by the usual method of Special Relativity (which is satisfied at localized regions or at regions far removed from the disc center)[Einstein, (1920)][5], to obtain the Lorentz transformations as;

$$d\chi_{\ell\neq 0}^{(0)'} = \frac{d\chi_{\ell\neq 0}^{(0)} - v d\chi_0^{(0)}}{\left(1 - \frac{v^2}{V_g^2}\right)^{\frac{1}{2}}} \qquad (II.2.5a)$$

and

$$d\chi_0^{(0)'} = \frac{d\chi_0^{(0)} - \frac{v}{V_g^2} d\chi_{\ell\neq 0}^{(0)}}{\left(1 - \frac{v^2}{V_g^2}\right)^{\frac{1}{2}}} \qquad (II.2.5b)$$

where v is the relative speed defined by the transformation, that is, between two rings in Fig 1. Clearly, $0 \leq v \leq V_g$ in real spacetime.

The invariance of V_g is consistently redolent of Einstein's postulate on the constancy of the speed of light, that is, the Light Principle [Torretti, (1983)][4] and thus guarantees the validity of the Lorentz transformations above. In addition it suffices to appreciate the value of Eqn (II.1.4), that generates the Special Relativistic results in Eqns (II.2.5a) and (II.2.5b), without any additional postulates in contrast to those inherent in Einstein's derivation based on the Light Principle.

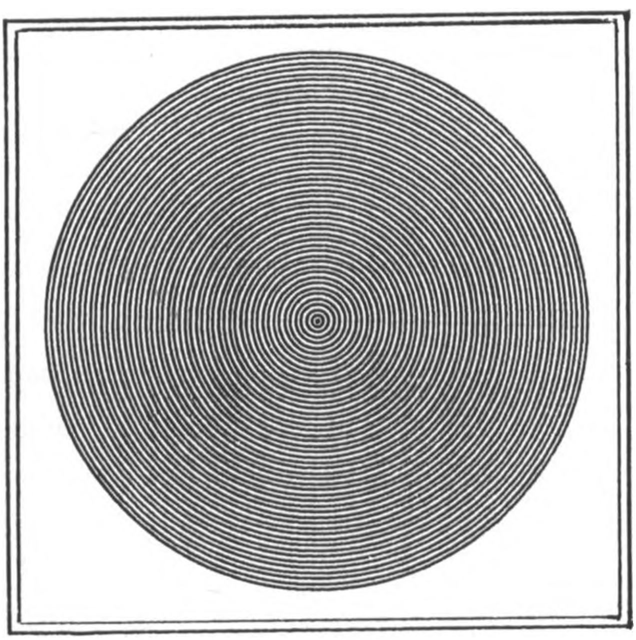

Fig 1. 'Semi-Rigid' disc (SRD) model comprising concentric rings.

(II.3) General Relativity

In his formulations of the general principle of relativity as a theory of gravitation, Einstein was motivated by the realization that the physical interpretation of space and time on a rigid rotating disc, and hence in the presence of (gravitational) acceleration, are in conflict with those of non-accelerated inertial systems, and consequently of Special Relativity and its Lorentz Transformations. His final result of GR was a geometrization of gravity in a theory consonant with the local validity of Special Relativity, in which the gravitational action of matter on matter was mediated by the spacetime geometry.

Since Eqn (II.2.3c) generates the desired Lorentz transformations, the SRD interpretation will be adopted here as the basis of General Relativity, that is, a theory of gravity that is consistent with Special Relativity.

Consider a section of the SRD model (Fig 1) which satisfies Eqn (II.1.3). By change of notation, such that, $\chi^{(0)}_{\ell\neq 0} \rightarrow r$, and $g^{(1)}_{00} \rightarrow g$ we end up with

$$g_{11} = -\frac{V_g^2}{r_1} \quad \text{and} \quad g_{22} = -\frac{V_g^2}{r_2}, \qquad (II.3.1)$$

as accelerations produced by particles located at the two rings of radii r_1 and r_2 respectively as indicated in Fig 2.

Fig 2 shows that in a given period of time t, point particles at radii r_1 and r_2 each moving at a constant speed V_g, sweep out angles θ_1 and θ_2 respectively. Now suppose an acceleration g_{21} of magnitude $|g_{22}|$ is produced by a particle located at a radius r_1 and moving at a speed of V_2 such that it takes the same time t to cover the angle θ_2, then we can write

$$\begin{aligned} g_{21} &= -\frac{V_2^2}{r_1} \\ &= -\frac{V_g^2}{r_1}\left(\frac{\theta_2}{\theta_1}\right)^2 \end{aligned} \qquad (II.3.2)$$

Due to the constancy of V_g the length of arcs swept out by the tips of r_1

and r_2 in equal times are necessarily of equal lengths, so that

$$\frac{\theta_2}{\theta_1} = \frac{r_1}{r_2}. \tag{II.3.3}$$

Substituting Eqn (II.3.3) into Eqn (II.3.2) gives

$$g_{21} = -\frac{V_g^2}{r_2^2} r_1 \tag{II.3.4}$$

which is in agreement with the Newtonian inverse square law, with r_1 as the reference unit of distance. In general we have to know our reference unit r_1 so that g_{j1} may be determined at any other distance r_j from the gravitating body, in the form

$$g_{j1} = -\frac{V_g^2}{r_j^2} r_1 \tag{II.3.5a}$$

This is a statement of general relativity; an expression of acceleration due to gravity that is consistent with special relativity whose consequence is the assertion that every particle in the presence of a gravitational field g_{j1} moves in a curve whose radius of curvature is r_j. This qualifies Einstein's geodesic postulate: that every particle moves in a straight line even in the presence of gravity (except that the space is curved). Using Eqn (II.1.4), we obtain a more explicit form of Eqn (II.3.5a) as

$$g^{(1)}_{\alpha\alpha'} = -\frac{v^{(0)2}_{\alpha\alpha'}}{2} \frac{\hat{\chi}^{(\alpha)}_{\ell\neq 0}}{\chi^{(\alpha)2}_{\ell\neq 0}} \tag{II.3.5b}$$

where $\hat{\chi}^{(\alpha)}_{\ell\neq 0}$ is a unit spatial dimension. Since Eqn (II.3.5b) is based on a complex spacetime background, we shall refer to it as a complex general relativity reformulation.

Clearly, the results above provide a mathematically simpler version of GR, thus overcoming a fundamental difficulty encountered with Einstein's GR; the lack of an exact general solution to the problem of motion, largely due to implicit mathematical complications of the theory in the establishment of how gravity curves spacetime.

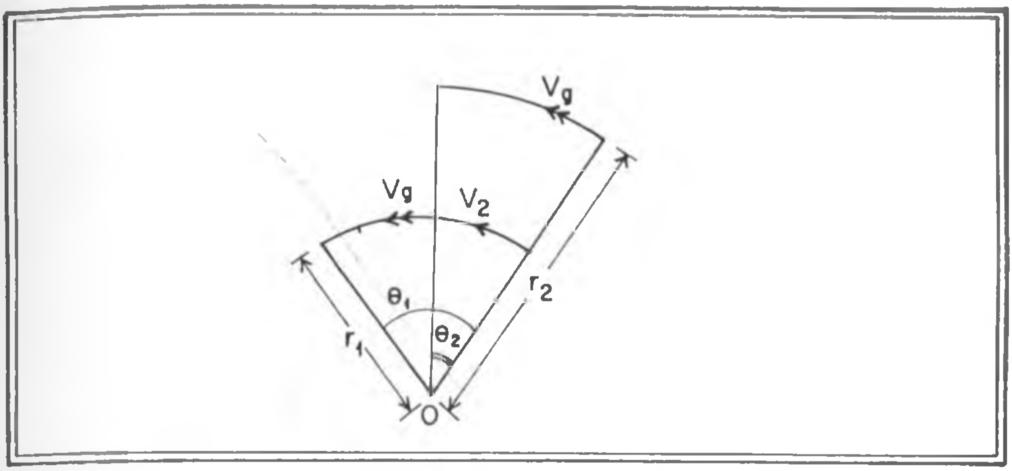

Fig 2. Lorentzian dynamics on a SRD model.

(II.4) Schwarzschild Line-element

Due to the lack of an exact general solution to the problem of motion in Einstein's general relativity, various approximation methods (like Schwarzschild solution) are used to impose restrictions on the form of the matter tensor, that the gravitational field equations do actually determine in free fall. Indeed the only very reliable experimental verification of Einstein's field equations is based on the Schwarzschild line- element. [Torretti, (1983)][4]. An alternative derivation of the Schwarzschild solution is here presented via the SRD model as a pointer to the versatility and simplifications accrued from its formulations of gravity on a spacetime background defined by $\chi^{(0)}_{\ell\neq 0}$ and $\chi^{(0)}_0$. Consider Eqn (I.1.2) from which the Schwarzschild line-element may be expressed as

$$\begin{aligned} dS^2_S &= V^2_g d\chi^{(0)2}_0 - d\chi^{(0)2}_{\ell\neq 0} \\ &= V^2_g d\chi^{(0)'2}_0 - d\chi^{(0)'2}_{\ell\neq 0} \end{aligned} \qquad (II.4.1)$$

in accordance with the generalized Lorentz transformation where $\chi^{(0)}_{\ell\neq 0} = r_j$ and V_g are mutually orthogonal as in Fig 2. Eqn (II.2.5b) gives the value of $d\chi^{(0)'}_0$ relative to the origin where $d\chi^{(0)}_{\ell\neq 0} = 0$. If the starting time is set to zero, that is, $d\chi^{(0)}_0 = 0$ then Eqn (II.2.5a) gives the relative length contrition of each ring so that Eqn (II.4.1) becomes

$$dS^2_S = V^2_g\left(1 - \frac{v^2}{V^2_g}\right)d\chi^{(0)2}_0 - \left(1 - \frac{v^2}{V^2_g}\right)^{-1} d\chi^{(0)2}_{\ell\neq 0}. \qquad (II.4.2)$$

For small v, we obtain a first order approximation as

$$\begin{aligned} dS_S^2 &\approx \left(1-\frac{v^2}{V_g^2}\right)V_g^2 d\chi_0^{(0)2} - \left(1+\frac{v^2}{V_g^2}\right)d\chi_{\ell\neq 0}^{(0)2} \\ &\approx \left(1-\frac{2r_c}{\chi_{\ell\neq 0}^{(0)}}\right)V_g^2 d\chi_0^{(0)2} - \left(1+\frac{2r_c}{\chi_{\ell\neq 0}^{(0)}}\right)d\chi_{\ell\neq 0}^{(0)2} \end{aligned} \quad (II.4.3)$$

where the replacement

$$\frac{v^2}{V_g^2} = \frac{2r_c}{\chi_{\ell\neq 0}^{(0)}} \quad (II.4.4)$$

is justified by equating Eqn (II.3.1) to (II.3.2), with r_c as a constant. Eqn (II.4.3) is the isotropic form of the conformal Schwarzschild solution presented by Adler et al., from an alternative derivation based on Einstein's GR and tensor analysis, in which case $2r_c$ is the Schwarzschild radius. It is on the basis of Eqn (II.4.3) that the well-known discrepancy in classical mechanics concerning the perihelic motion of the planet Mercury was satisfactorily resolved [Adler et al., (1965)][1].

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(II.5) Kepler's Third Law

Kepler's third law is also called the law of periods and its derivation begins from Eqn (II.3.1) and Fig 2, in which case we may write

$$g_{jj} = -\frac{V_g^2}{r_j} = -\left(\frac{2\pi r_j}{T_j}\right)^2 \frac{1}{r_j} = -\frac{4\pi^2}{T_j^2} r_j, \quad (II.5.1)$$

where T_j is the period of rotation for a particle moving with constant speed V_g at a constant radius r_j. We shall refer to T_j as the gravitational period at r_j.

By equating Eqn (II.3.5a) to Eqn (II.5.1) ie when $|g_{jj}| = |g_{j1}|$ according to Eqn (II.3.2), we obtain

$$\frac{r_j^3}{T_j^2} = \frac{V_g^2}{4\pi^2} r_1 \quad (II.5.2a)$$

This is Kepler's third law with r_1 as the reference unit of distance.

In terms of the general complex spacetime co-ordinate notation, Eqn (II.5.2a) becomes

$$\frac{\chi^{(\alpha)3}_{\ell\neq 0}}{\chi^{(\alpha')2}_{0}} = \frac{v^{(0)2}_{\alpha\alpha'}}{8\pi^2}\hat{\chi}^{(\alpha)}_{\ell\neq 0} \qquad (II.5.2b)$$

Historically Newton arrived at the gravitational law on the basis of Johanness Kepler's astronomical laws. More specifically, it is the application of Kepler's third law, to the orbit of the moon about the Earth that gave support to Newton's theory of gravity.

(II.6) Newton's Gravitational Law

In order to show that the formulation of gravitation based on the SRD model bears consonance with the quantitative outcomes of Newtonian gravitational law, reference is made of Eqn (II.3.5a) which may be rewritten as

$$g_{j1} = -\left(\frac{2\pi r_1}{T_1}\right)^2 \frac{r_1}{r_j^2} \qquad (II.6.1a)$$

If we assume a symmetric and homogeneous mass distribution so that the mass M_1 of the potential source located at O (in Fig 2) is given by

$$M_1 = \frac{4\pi r_1^3}{3}\rho_M \qquad (II.6.2)$$

where ρ_M is mass density, then Eqn (II.6.1a) becomes

$$g_{j1} = -\frac{4\pi^2}{T_1^2}\left(\frac{3M_1}{4\pi\rho_M}\right)\frac{1}{r_j^2} = -\frac{G_{j1}M_1}{r_j^2}, \qquad (II.6.1b)$$

which is the Newtonian gravitational acceleration, with

$$G_{j1} = \frac{3\pi}{T_1^2\rho_M} \qquad (II.6.3)$$

as the gravitational parameter assumed as a universal proportionality constant in the Newtonian formalism. From the derivation above, G_{j1} is clearly dependent on the variations of density ρ_M and of the reference scale determined by the period T_1 at the surface of the gravitating mass.

In the general complex coordinates we have

$$g^{(1)}_{\alpha\alpha'} = -\frac{4\pi^2}{\hat{\chi}_0^{(\alpha')2}}\left(\frac{3M_1}{4\pi\rho_M}\right)\frac{1}{\chi^{(\alpha)2}_{\ell\neq 0}} \tag{II.6.4}$$

and

$$G_{\alpha'1} = \frac{3\pi}{\chi_0^{(\alpha')2}\rho_M} \tag{II.6.5}$$

where we have assumed that ρ_M is a non-imaginary quantity. The acceleration $g^{(1)}_{\alpha\alpha'}$ points in the radial direction of unit displacement $\hat{\chi}^{(0)}_{\ell\neq 0}$.

In the above consideration, the SRD model, has been established as a basis of an alternative non-tensorial derivation of a General Theory of Relativity, that is, by suitable physical interpretation of the linearized Lorentz transformation. The resulting simplicity is an advantage over the Einsten's GR whose mathematical difficulties have largely hindered the establishment of an exact general solution to the problem of motion.

Since the SRD formulation of GR has submitted itself into compliance with the Newtonian theory of gravity as well as with Einstein's GR, the underlying complex spacetime background seems justified as a realistic and viable description of an ubiquitous interactive physical geometry.

CHAPTER III

COSMOLOGY

Although the universe has been widely presumed to be of singular curvature on the large scale, a possibility for a non-singular (composite) model has been developed here as a viable description of our observable universe based on a complexified spacetime background. The concept of antigravity arises here as an interpretation of the energy conservation principle. The results favour a universe of infinite existence.

(III.1) A Composite Spacetime Curvature Model

An important problem in the current theory of gravity is the search for a realistic theoretical model for the distribution and dynamics of cosmological matter and consequently of spacetime curvature. A theoretical concept or model is acceptable if it maintains credibility and compatibility with observational evidences. One such theory is the Big Bang Hypothesis described by the Friedmann equation [Silk, (1980)] [14], which together with the Cosmological Principle is often used as the basis of a standard procedure in the determination of spacetime curvature K.

The Big Bang theory provides three distinct curvatures determined by $K = \pm 1, 0$. Since the Friedmann scheme presumes that the Universe is of singular curvature, there ought to be some principle that picks out one K and hence one model to represent the entire universe. In the absence of such a principle, it is understandable that the Cosmological Principle be called to question; and the answer is to be sought within the precincts of energy conservation.

A composite spacetime curvature (CSC) model of the universe in which all $K = \pm 1, 0.$ are encountered within the large scale spatial range, is developed here as a more realistic consequence of the energy conservation principle.

Consider the expression for velocity in complexified spacetime given by Eqn (I.1.4) as;

$$v_{\alpha\alpha'} = \left(\frac{v_{00}v_{01}^2}{v_{01}^2+v_{00}^2} + \frac{v_{10}^2 v_{11}}{v_{11}^2+v_{10}^2}\right) + i\left(\frac{v_{10}v_{11}^2}{v_{11}^2+v_{10}^2} - \frac{v_{00}^2 v_{01}}{v_{01}^2+v_{00}^2}\right). \quad (III.1.1)$$

For simplicity we rewrite Eqn (III.1.1) as

$$v_{\alpha\alpha'} = v_R + iv_I \quad (III.1.2)$$

where v_R and v_I derive their meaning from the brackets in Eqn (III.1.1). In this case the total interaction energy E_T over the entire spacetime may be expressed as a scalar product of velocities, in the form,

$$E_T = \xi[v_R^2 + (iv_I)^2 + 2iv_R v_I] \quad (III.1.3)$$

If we consider the real part of Eqn (III.1.3) as representative of all observable energy E_0 then

$$E_0 = \xi[v_R^2 + (iv_I)^2] \quad (III.1.4)$$

where E_0 may assume either potential or kinetic forms, while ξ is determined in each domain of physical interactions.

From the principle of Conservation of Energy, it is apparent that different forms of energy are inherently indistinguishable so that we may express E_0 in a fully singular potential form as

$$E_0 = 2\xi g_M^{eff} \chi_{\ell\neq 0}^{(0)} \quad (III.1.5)$$

where g_M^{eff} is the effective acceleration in a dynamical universe of total mass M. Thus Eqn (III.1.4) becomes

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$$g_M^{eff} = \frac{H^2 \chi_{\ell\neq 0}^{(0)}}{2} - g_M^N \quad (III.1.6)$$

where g_M^N is the Newtonian gravitational acceleration associated with mass M and velocity v_I, while H is the Hubble constant defined by the Hubble law;

$$v_R = H\chi_{\ell\neq 0}^{(0)} \quad (III.1.7)$$

with reference to a gravitating mass centered at $\chi^{(0)}_{\ell\neq0} = 0$. Consider a case when $g^{eff}_M = 0$ which occurs at a flat (Euclidean) Horizon μ defineed by $\chi^{(0)}_{\ell\neq0} = \mu$, so that

$$2GM = H^2\mu^3, \qquad (III.1.8)$$

where G is the (Newtonian) universal gravitational constant. Thus Eqn (III.1.6) becomes

$$g^{eff}_M = \frac{H^2}{2\chi^{(0)2}_{\ell\neq0}}[\chi^{(0)3}_{\ell\neq0} - \mu^3]. \qquad (III.1.9)$$

This gives the potential profile of the large scale gravitational structure of the universe. At any instant, when H is constant, Equation (III.1.9) gives a profile shown in Fig 3 [Buers et al. (1998)][15] (Ref. Appendix C). In this scheme, all the three spatial curvatures appear within a single universe, that is within the range:

$$0 < \chi_{\ell\neq0} < \infty. \qquad (III.1.10)$$

Hence spacetime is;

(i) Closed (with attractive gravity, ie $K = +1$) when,

$$g^{eff}_M < 0, \text{ for all } |\chi^{(0)}_{\ell\neq0}| < \mu. \qquad (III.1.11)$$

(ii) Flat (with no net gravity, ie $K = 0$) when,

$$g^{eff}_M = 0, \text{ for all } |\chi^{(0)}_{\ell\neq0}| = \mu. \qquad (III.1.12)$$

(iii) Open (with repulsive antigravity, ie $K = -1$) when,

$$g^{eff}_M > 0, \text{ for all } |\chi^{(0)}_{\ell\neq0}| > \mu. \qquad (III.1.13)$$

This is the essence of a composite spacetime curvature (CSC) model.

Unlike the Event horizon which is fixed by the Cosmic Censorship Hypothesis, the (Euclidean) horizon μ introduced here is time-dependent and scales as $\chi^{(0)\frac{2}{3}}_0$, according to the Hubble law. It is a property of a gravitational potential source in an expanding Universe and is directly proportional to $M_0^{\frac{1}{3}}$, where M_0 is the non-relativistic mass of the source. On the other hand μ is independent of the mass of a particle or galaxy in the potential field of the source, that is by ignoring perturbation effects [Anninos et al., (1996)] [16]. In a static universe where the Hubble velocity $v_R \rightarrow 0$, the horizon $\mu \rightarrow \infty$. This is a Newtonian approximation where the universe is infinitely old and is bounded by the long range gravity. For all finite times, the Hubble expansion reduces μ to finite values provided M_0 is finite.

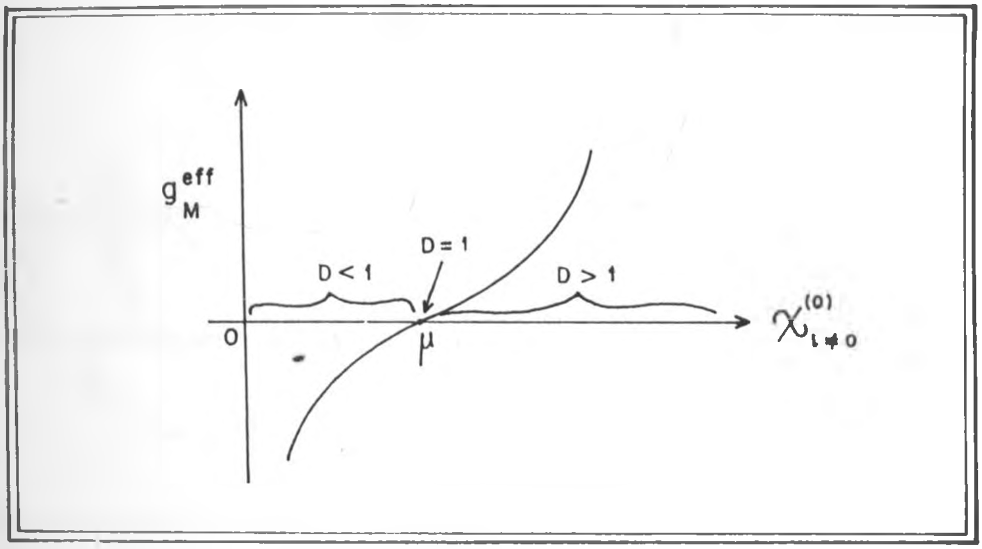

Fig 3. Effective gravity profile for the CSC model.

(III.2) The Observable Universe

An appropriate location of our observable universe is here determined on a complex velocity plane, by applying the energy conservation principle.

From the energy conservation principle, we may write

$$v_R^2 + (iv_I)^2 = v^2 \qquad (III.2.1)$$

so that Eqn (III.1.4) takes a fully singular kinetic form as

$$E_0 = \xi v^2 \qquad (III.2.2)$$

with v as an inertial velocity. From the Hubble law we can write

$$v_R = Dv_I \qquad (III.2.3)$$

where

$$D = -\frac{1}{2}\frac{dv_R}{dv_I}, \qquad (III.2.4)$$

so that the inertial velocity reduces to

$$v = \sqrt{D^2 - 1}\left(\frac{v_{10}v_{11}^2}{v_{11}^2 + v_{10}^2} - \frac{v_{00}^2 v_{01}}{v_{01}^2 + v_{00}^2}\right). \qquad (III.2.5)$$

This is a kinetic description of the composite spacetime structure in a non-empty space (with non-vanishing v_I). The profile satisfies three possibilities:

(i) closed spacetime when $D < 1$,

(ii) Flat space time when $D = 1$, and

(iii) Open spacetime when $D > 1$.

By application of elementary calculus Eqn (III.2.5) can be rewritten as

$$g_M^{eff} = (D^2 - 1)g_M^N \qquad (III.2.6)$$

where g_M^N is a Newtonian gravitational acceleration while g_M^{eff} is the resultant (effective) acceleration of the universe at a given value of D. We then consider that $D = 1$ when $\chi^{(0)}_{\ell\neq 0} = \mu$ since equation (III.1.9) and (III.2.6) are descriptions of the same system.

On a complex velocity plane given as Fig 4, the total energy in the CSC model presents asymptotic approach to both axes in the first quadrant. If we consider a universe of finite and conserved energy content then we must demand that v^2 be a constant. This describes an arc of a circle centered at the origin. The two curves coincide uniquely when

$$v_R = v_I \qquad (III.2.7)$$

which defines a flat spacetime. It is this region of coincidence that is conjectured to determine the local position of our observable universe (indicated as HOME in Fig 4) in the CSC model.

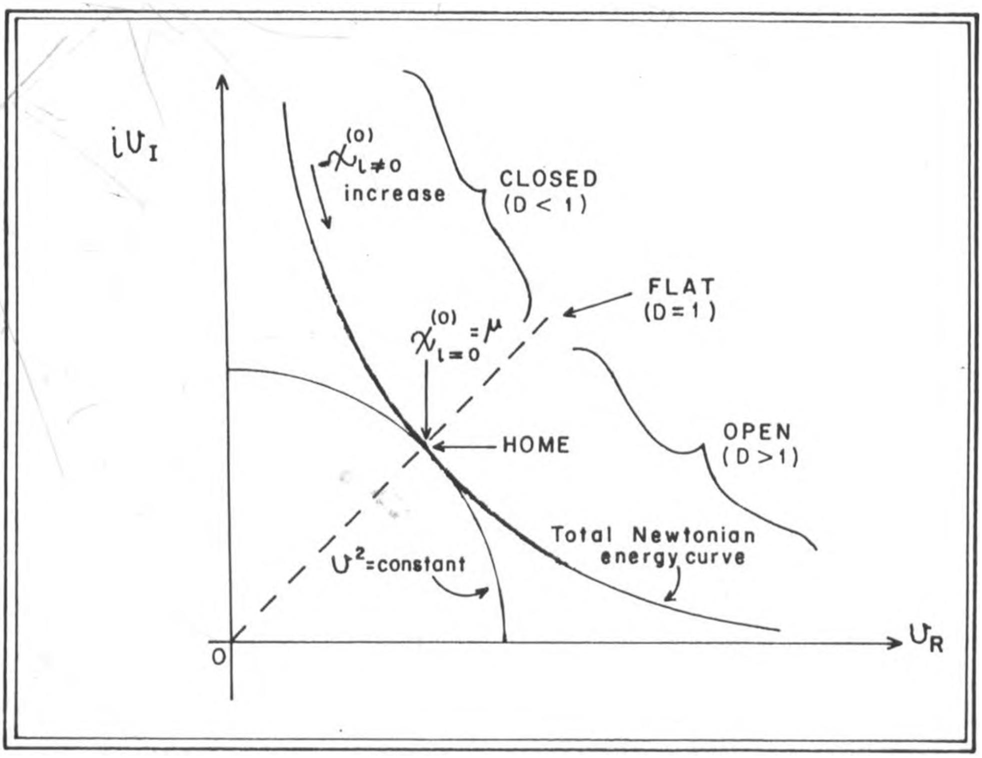

Fig 4. Energy curves on the complex velocity plane of the CSC model.

Although our position in the complex velocity plane is somehow anthropic, it viably provides for the observed homogeneity of our universe. [Hawking (1988)] [7] Further it has been shown that in a complexified spacetime in which kinetic and potential energies combine into a singular form, the spacetime curvature is necessarily composite in the real large scale spatial range. The CSC Model is compatible with Gautreau's (1996) [17] results as has been shown by Buers et al., (1998) [15] (Ref. Appendix C).

(III.3) The Fate of the Universe

From spherically symmetric consideration, a potential source of Schwarzschild radius r_s has an average density

$$\rho_s = \frac{3}{2A_s} \tag{III.3.1}$$

where A_s is the non-decreasing surface area of the event horizon, in the general relativistic units. Similarly,

$$\rho_\mu = \frac{3}{2A_u} \tag{III.3.2}$$

where A_u is the surface area of the universe marked by the Big Bang photons. From the composite spacetime curvature model we obtain

$$\rho_s = \rho_\mu \frac{A_u}{A_s} \tag{III.3.3}$$

Alternatively, the average density of the universe may be expressed as

$$\begin{aligned} \rho_u &= \frac{\text{Total mass of the Universe}}{\text{Total volume of the Universe}} \\ &= \rho_\mu \frac{r_s}{r_u} \end{aligned} \tag{III.3.4}$$

where r_u is the radius of the universe. By combining Eqns (III.3.3) and (III.3.4) to eliminating $\frac{r_s}{r_u}$ we obtain

$$\rho_s \rho_u^2 = \rho_\mu^3 \tag{III.3.5}$$

as a characteristic density profile equation in a composite spacetime curvature model [Buers et al. (1998)] [15].

Our CSC model presents an infinitely closed universe when $\mu = \infty$, that is, in the infinite future. However, since ρ_s is fixed by the Cosmic Censorship Hypothesis, therefore ρ_u scales as $\rho_{\mu}^{\frac{3}{2}}$. It is clear that ρ_u is infinite at $\chi_0^{(0)} = 0$ but decreases to zero thereafter as $\chi_0^{(0)} \to \infty$. Consequently, and in accordance with the energy conservation principle, the universe becomes more open with epoch due to its expansion. This contradiction arises from the use of average density that incorrectly introduces the notion of homogeneity (Cosmological Principle) within the radius r_u of the universe. The proper baryonic radius of the universe is less than r_u, thus giving a closed universe that expands forever. This agrees with Tipler's observation that in Newtonian cosmology, a matter dominated closed universe could expand for ever, as spatial topology is not the determining feature in the dynamics of Newtonian cosmology as it is in general relativity [Tipler, (1996)] [18].

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On the overall, the universe effectively assumes an infinite life span. An inherent self-similarity of the composite structure on the small scale, similar to the independently nucleated superfluid regions considered by G.R. Pickett and his group [Bauerle et al., (1995)] [19], implies an enhancement of galaxy formation with epoch as opposed to the contrary prediction attributed to expansion in Friedman models.

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CHAPTER IV

APPLICATIONS

(IV.1) The Variation of G

In his formulations of an inverse square law of gravity, Newton assumed that the proportionality parameter G, is a universal gravitational constant, that has the same value everywhere for all matter. In Einstein's general relativity, G occurs as a coupling constant between matter and gravity so that the invariance of the action under general coordinate transformations clearly prohibits its variation with space or time [Narlikar, (1983)][2].

However, both Newton's theory and Einsten's GR are unjustified in their reference to point masses and to singularities respectively, as proper representatives of ordinary matter. This, of course, does not in the least compromise the existence of such configurations, especially for Black Holes. But in Physics, nature has to be understood, as far as possible, purely in terms of itself, and idealism must always be seen as so. Thus a non-singular description of matter, and consequently a variable G, are certainly more desirable in a realistic theory of gravity.

The possibility of a variable gravitational G has been considered in Dirac Cosmology [Dirac, (1973)][20] and in the Brans-Dicke theory [Dicke, (1964)][21] or the Hoyle-Narlikar theory [Hoyle et al., (1964)][22] which are Machian in character. Cosmological solutions to Brans-Dicke theory lead to a general result that G decreases with epoch, the models being of the Big Bang type. There are some indications from the celestial mechanics of the Sun-Earth-Moon system that G decreases with epoch [Flandern (1975)[23], Muller (1976)[24]] at a rate comparable to the Hubble constant H, that is,

$$\dot{G} = -\delta G H, \qquad (IV.1.1)$$

where $\dot{G}$ is the rate of change of G with respect to cosmic time T_C and

δ is a dimensionless number of the order of or a fraction of unity. Thus the fractional change of G at the present epoch is approximately a few parts in 10^{11} years.

In a Newtonian framework, the effective gravitational field due to the total mass of a spherical and homogeneous particle is experienced only at radii greater than or equal to the particle radius. From Eqn (II.6.3), the expression for G_{j1} at the surface of a gravitational source mass is given by

$$G_{j1} = \frac{3V_g^2}{4\pi r_1^2 \rho_M} \qquad (IV.1.2)$$

where ρ_M is the mass density, r_1 is the radius of the mass, while V_g is a constant speed. In this case, masses of same density will have a G_{j1} that decreases with increase in radius r_j.

For the entire universe of radius r_u we consider the density to decrease with expansion so that Eqn (IV.1.2) becomes

$$G_{j1} = \frac{V_g^3 T_1}{2\pi M_1} \qquad (IV.1.3)$$

where mass M_1 of the universe may be assumed as a constant provided $r_1 = r_u$. Thus from Eqns (IV.1.2) and (IV.1.3) we obtain

$$\rho_M = \frac{6\pi^2 M_1}{(T_1 V_g)^3} \qquad (IV.1.4)$$

where we have assumed a spherically symmetric mass distribution. Clearly we have a G_{j1} that increases with epoch for the large scale universe.

If we define cosmic time by the expression

$$T_C = f_1 T_1 \qquad (IV.1.5)$$

where f_1 is a dimensionless surface parameter then from Eqn (IV.1.3) we have

$$G_{j1} = \frac{V_g^3}{2\pi f_1 M_1} \qquad (IV.1.6)$$

so that

$$\frac{\dot{G}_{j1}}{G_{j1}} = \frac{1}{f_1 T_1} \qquad (IV.1.7)$$

Therefore

$$\dot{G}_{j1} = G_{j1} H \qquad (IV.1.8)$$

where H is the reciprocal of the cosmic time T_C. Eqn (IV.1.8) agrees with Brans-Dicke theory when $\delta = -1$ in Eqn (IV.1.1).

Although we have used the particle surface, of radius r_1 as the gravitational reference, both exterior and interior reference solutions are desirable for local and large scale probes respectively.

An interior solution arises when $r_j < r_1$ in which case the effective gravitating mass M_j is less than M_1. If the interior mass density is a constant then from Eqn (II.6.1b) we have

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$$g_{j1} = -\frac{3\pi}{T_1^2 \rho_M}\left(\frac{r_0}{r_j}\right)^3 \frac{M_j}{r_j^2} = G_{j1}\frac{M_j}{r_j^2} \qquad (IV.1.9)$$

where r_0 is the actual radius of the reference mass M_1. When $r_0 = r_j = r_1$ then Eqn (IV.1.9) takes the exact form of Eqn (II.6.1b). In general however

$$G_{j1} = \frac{3\pi}{T_i^2 \rho_M}\left(\frac{r_0}{r_j}\right)^3 \qquad (IV.1.10)$$

so that G_{ji} vanishes in the absence of matter, that is, when $r_0 = 0$. Since an expansion of the universe preserves M_1 and decreases ρ_M therefore G_{j1} increases with epoch, at a given value of r_j. If $r_0 < r_j < r_1$ then we obtain $g_{j1} > g_{11}$. For some fixed M_j, we find that g_{j1} is proportional to ρ_M in agreement with Einsten's GR.

An exterior solution occurs when $r_j > r_1$. such that M_j is always equal to M_1. Hence we obtain $g_{j1} < g_{11}$.

In the foregoing analysis, a possible variation of Newton's universal gravitational 'constant' has been deduced from the SRD model of General Relativity, in which the value of G_{j1} at the surface of a particle decreases with increasing radii, provided the density is kept constant. The implied results provide a quantitative distinction between the SRD model of GR and other theories of gravity.

Although measurements of Van Flandern (1975)[23] and those by Muller (1976)[24] support a negative $\frac{\dot{G}}{G}$ our result in Eqn (A.5) gives a positive value. The discrepancy can only be resolved by accurate observatory (or experimental) data since the error bars in Flandern's work are too large so that $\dot{G} \geq 0$ cannot yet be ruled out [Narlikhar, (1978)][2]. The expression of G in Hoyle-Narlikar theory gives a positive value, that is, $\dot{G} > 0$ in agreement with our SRD formulation. A most important consequence of a varying G is the relegation of Einstein's GR to a non adequate group of theories since in it G cannot be deduced and has to be put into action principle on an ad-hoc basis.

(IV.2) Gravity as a Metric Phenomenon

In General Relativity, the physical force of gravity is linked with the non-Lorentzian nature of Riemann space. More precisely, GR presents spacetime that is curved back on the mass causing the distortion. This equivalence means that the geodesic equations of motion can be used as equations of motion for particles in a gravitational field. In the Composite Spacetime Curvature Model, this equivalence is a temporal limiting case of a more complicated structural equivalence.

In finite cosmic time the effective gravitational acceleration is given by Eqn (III.1.9) and Fig 3 of Chapter III. The figure implies a metric geometry of the form given in Fig 5 below. Thus in general,

$$g_M^{metric}(\pm\chi_{\ell\neq 0}^{(0)}) = \mp|g_M^{eff}(\pm\chi_{\ell\neq 0}^{(0)})|, \qquad (IV.2.1)$$

so that the equivalence of gravity to metric curvature occurs only within the range

$$|\chi_{\ell\neq 0}^{(0)}| \leq \mu. \qquad (IV.2.2)$$

In a very old universe $\mu \rightarrow \infty$ since μ scales as $\chi_0^{(0)\frac{2}{3}}$ so that the equivalence is of an infinite range. This is the limiting basis of Einstein's General Relativity, in which gravity assumes static structure with the entire universe as a closed $(g_M^{eff} < 0)$ region.

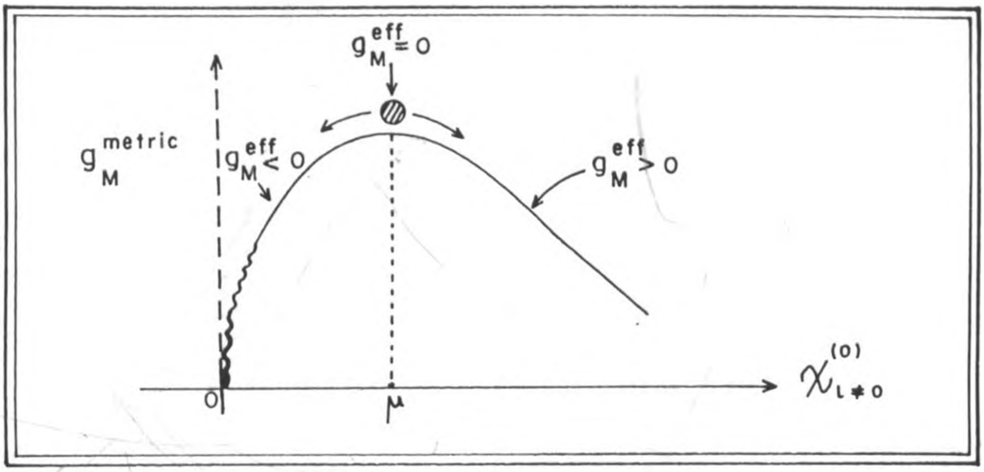

Fig 5. The metric curvature about a finite mass in the CSC model.

(IV.3) The Black Hole Radius

Newton's formulations of gravity does not provide for interactions of particles that are cut off from the Universe such as Black Holes. Every particle is viewed as a dimensionless singularity and gravity is considered as an attractive and long range force that permeates everywhere. However, Einstein's GR predicts the possibility of Black Holes and of singularities but cannot determine their fate.

From the Composite Spacetime Curvature model, a Black Hole (BH) singularity is predicted whose metric curvature is in the form of Fig 5, with the condition that

$$\mu > \chi^{(0)}_{\ell\neq 0} \rightarrow 0 \qquad (IV.3.1)$$

Thus a BH singularity in this case exists within a closed region of its own spacetime, that is, it is not 'naked' [Penrose, (1973)][25]. Consequently, a BH cannot explode as far as an external observer (to whom $\chi^{(0)}_0 > 0$) is concerned. However the BH may evaporate by the Hawking radiation mechanism. By similar arguments, the future universal Big Crunch would not re-explode since $|\mu| \neq 0$, unless the temporal dimensions would also crunch.

In a standard form the BH radius (or Schwarzschild radius) r_s is considered as the Event Horizon beyond which an external observer would not communicate with the interior of a BH [Droz et al., (1996)][26]. For a Black Hole of mass M we have

$$r_s = \frac{2GM}{c^2} \qquad (IV.3.2)$$

where G is the local gravitational constant while c is the speed of light. From Eqns (I.3.1) and (I.3.2) in Chapter I, we can write

$$v^{(1)2}_{00} = -\frac{2GM}{r} \qquad (IV.3.3)$$

where r is the reference radius about the Black Hole. Hence,

$$\frac{v^{(1)2}_{00}}{c^2} = \frac{r_s}{r} \qquad (IV.3.4)$$

Also from Eqn (II.1.2)

$$\begin{aligned} v_{00}^{(1)2} &= -v_{00}^{(0)2} \\ &= -2V_g^2 \end{aligned} \tag{IV.3.5}$$

so that

$$\frac{V_g^2}{c^2} = \frac{r_s}{2r} \tag{IV.3.6}$$

Therefore

$$V_g \approx 0.7c\sqrt{\frac{r_s}{r}}. \tag{IV.3.7}$$

When $r = r_s$, that is, at the Event Horizon, we obtain

$$V_g \approx 0.7c. \tag{IV.3.8}$$

This means that in the configuration of Fig 2 a particle would still manage to circulate the Black Hole at the Schwarzschild radius, at speeds less than c. This contrasts with the popular Newtonian argument for black holes as developed by Laplace [Hawking et al., (1973)] [27].

In the CSC model, there is an ultimate horizon (hereafter refered to as a Classical Horizon), where our classical ideas of physics breaks down, which occurs if $V_g = \pm c$, that is, when

$$r = \frac{r_s}{2} = r_c \tag{IV.3.9}$$

In general, the Composite Spacetime Curvature model gives

$$r_c = \frac{H^2 \mu^3}{2c^2} \tag{IV.3.10}$$

In Einstein's GR, as given by Eqn (II.4.3), r_c is assumed as a meaningless constant of integration [Adler et al., (1965)] [1] or, in general relativistic units, as the mass of the Black Hole [Penrose, (1969)] [28]. But from the foregoing analysis it is identified as the Classical Horizon which is the lowest quantum gravity level (as we shall see in Appendix B), while the Schwarzschild radius is the first excited gravity level.

CONCLUSION

In this thesis, a complex spacetime geometry has been adopted as a logical structure which together with the principle of energy conservation and of scale equivalence has accounted for the weak equivalence principle (WEP). A dynamical analysis of the WEP has lead to a Lorentzian SRD model whose formulations and results are consistent with the observations based on Newton's Gravitational Theory and Einstein's General Theory of Relativity, albeit in entirely different spacetime configuration. This provides a striking confirmation of Poincare's[3] contention that a physical theory is a free creation of the mind rather than a unique induction from observations. Although its gravitational deductions are logical, they need not necessarily correspond with experience and are thus just as uncertain when applied to reality as those of any other empirical theory.

On the very largest length scales, a Composite Spacetime Curvature (CSC) cosmological model has been developed in which antigravity features on the outer bounds of the universe. It is this antigravity that qualifies as the physical cause of the Big Bang explosion due to inflationary quantum fluctuations in the initial global singularity (Appendix A).

Finally, the thesis presents a perspective of quantum gravity with a generalized de Broglie relation (Appendix B), thus provinding a generalized criterion for a possible unification of all the four forces of nature into a single physical theory.

APPENDICES

Appendix A: Quantum Big Bang Probability

During the Newtonian era (before Einstein's GR), the universe was considered as static and the Anthropic Principle was widely accepted, that is, 'things are as they are because they are as they are' - or God made them so. Hence the expansion of the universe or more precisely the Big Bang was not known.

Later, Einstein came up with his equations of GR which predicted that the universe was either expanding or contracting. He therefore added the cosmological term that had a repulsive gravitational effect, to the equations that relate the mass and energy in the universe to the curvature of spacetime. Thus the negative curvature of spacetime produced by the cosmological term canceled the positive curvature of spacetime by the mass and energy in the universe. In this way, he obtained a static model of the universe that continued forever in the same state. Einstein later dropped the cosmological term on the confirmation, by Hubble (in 1929), that the universe is expanding (and referred to it as 'the greatest mistake of my life'). Neither did he appreciate much the fact that if matter caused spacetime to curve in on itself, then a large enough portion of mass could curve a region in on itself so much that it would effectively cut itself off from the rest of the universe to form a singularity, that is, a place where spacetime has a beginning or end. However, other workers like Roger Penrose and Stephen Hawking [Torretti, (1985)][4] have used GR to affirm that the universe must have had a beginning from a singularity by a process or explosion popularly called the Big Bang, and that huge stars can collapse (or end) into a singularity called a Black Hole. However no explanation has been presented on the cause of the Big Bang since Einstein's GR cannot predict what comes out of a singularity. Neither does it predict the fate of a Black Hole conclusively [Turner, (1996)][29].

In the CSC model the idea of repulsive gravity (or antigravity) is introduced which unlike Einstein's cosmological term, does not exactly cancel the effects of gravity throughout space, except at $\chi^{(0)}_{\ell\neq 0} = \mu$. At the beginning when $\chi^{(0)}_0 = 0$ and hence $\mu = 0$, the CSC model gives a singularity at $\chi^{(0)}_{\ell\neq 0} = 0$ which exists in the open region of its own spacetime where antigravity dominates. Consequently, it is the repulsive action of antigravity on this 'naked' singularity, that is propounded here as the cause of the Big Bang explosion.

Globally, the effective gravitational acceleration may be written in accordance with Eqn (III.1.9) as

$$g^{eff}_M = g^{eff}_0 \left[1 - \left(\frac{\mu}{\chi^{(0)}_{\ell\neq 0}} \right)^3 \right] \qquad (A.1)$$

For the Big Bang explosion to occur, a logical initial requirement is a dominant and infinite antigravity, that is,

$$g^{eff}_M = +\infty \qquad (A.2)$$

Previously the Big Bang has been assumed only to occur under arbitrary conditions determined by $\chi^{(0)}_\ell = 0$.

However Eqn (A.2) is also satisfied by two other conditions, that is,

$$\chi^{(0)}_0 = 0, \text{ for all } \chi^{(0)}_{\ell\neq 0} >> 0 \qquad (A.3)$$

and

$$\chi^{(0)}_{\ell\neq 0} = 0, \text{ for all } |\chi^{(1)}_0| >> 0 \qquad (A.4)$$

where the non-vanishing parameters are surmised to arise as quantum fluctuations in the initial singularity via the Uncertainty Principle as an inflationary process. Thus the probability of the Big Bang takes the non deterministic form of Fig 6.

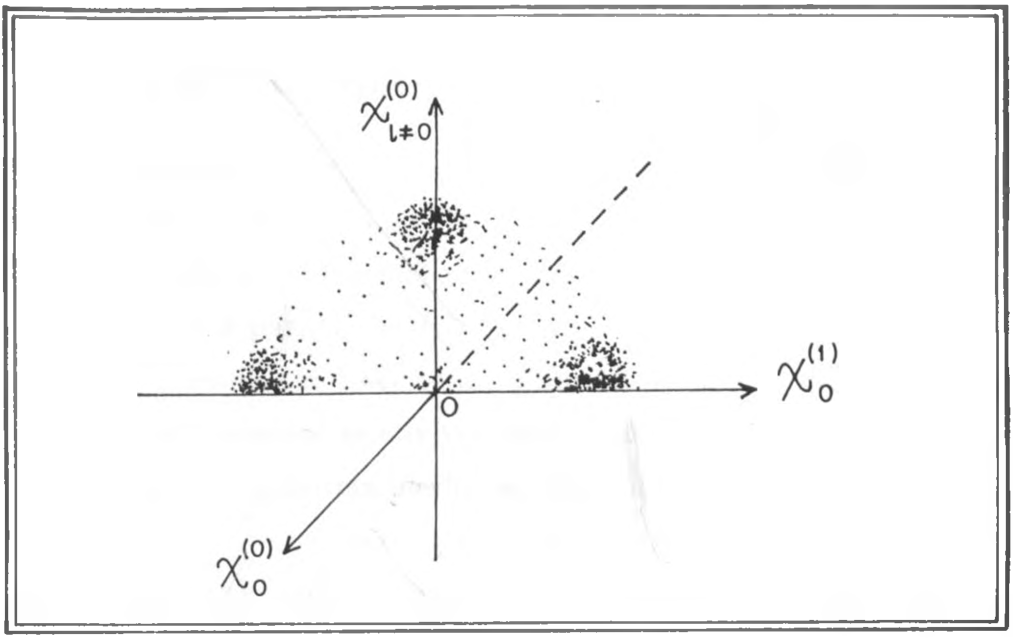

Fig 6. Quantum probability density for the Big Bang origin.

Appendix B: Quantum Gravity

Quantum mechanics has successfully been used to describe the partial theories that govern the weak, the strong and the electromagnetic forces. In order to find a theory that unifies gravity with the other forces, it is desirable that a partial theory of gravity be compatible with quantum mechanics through the Uncertainty Principle. This has not been possible with Einsten's general relativity which is purely 'classical' and is thus inconsistent with quantum mechanics [Hawking, (1988)][7]. In this section we shall explicate a theory of quantum gravity on the basis of the complex GR already developed in Chapter II. The results reveal some quantitative generalizations to matter waves.

Basic Formulations

Consider a test particle of mass m which moves from a ring of radius r_i to that of r_f in the gravitational configuration of Fig 2. Since V_g is an invariant quantity, an arbitrary radius r_j gives

$$V_g = 2\pi r_j \nu_j \tag{B.1}$$

where ν_j is the gravitational frequency corresponding to the gravitational period T_j.

By making use of Eqn (II.3.5a), the change in the gravitational potential energy of the particle is given by

$$\begin{aligned} \Delta E_{fi} &= E_i - E_f \\ &= m g_i r_i - m g_f r_f \\ &= m V_g^2 \left(\frac{1}{r_f} - \frac{1}{r_i}\right)\hat{r} \\ &= 2\pi \hat{r} m V_g (\nu_f - \nu_i) \\ &= 2\pi \hat{r} m V_g \Delta \nu_{fi} \end{aligned} \tag{B.2}$$

where $\hat{r}$ is a common reference radius of unit length. Hence we can write,

$$\Delta E_{fi} = h_r \Delta \nu_{fi} \tag{B.3}$$

which is the Planck's formular for energy quantization in which the parameter h_r that is,

$$h_r = 2\pi\hat{r}mV_g \qquad (B.4)$$

corresoponds to the Planck's constant h. However, unlike h which is a universal constant, h_r is dependent on the mass m of the test particle as well as on the reference dimension $\hat{r}$. In order to determine that h_r is a Planck-like parameter, we will hereafter consider its relativistic approximations in the special and the general regimes.

In special relativity, V_g is the magnitude of an invariant velocity vector $\vec{V}_g$ of the test particle so that its linear momentum $\vec{p}$ has a magnitude

$$p = mV_g \qquad (B.5)$$

Substituting Eqn (B.5) into Eqn (B.4) gives

$$h_{\hat{r}} = 2\pi\hat{r}p \qquad (B.6)$$

From wave mechanics [Feyman, (1963)][30], the test particle may be viewed to traverse a path of wavelength $\lambda_{\hat{r}}$ about a reference ring of radius $\hat{r}$ such that

$$2\pi\hat{r} = n\lambda_r \qquad (B.7)$$

where, $n = 1, 2, ...$

Eqns (B.6) and (B.7) combine to give

$$\lambda_r = \frac{h_{\hat{r}}}{np} \qquad (B.8)$$

which is a general form of the de Broglie relation.

When $n = 1$, Eqn (B.8) gives the usual de Broglie wavelength λ_{dB} in which case the wavenumber k assumes an exact equivalence with the curvature k_r of the reference ring. In general,

$$k = nk_{\hat{r}} \qquad (B.9)$$

so that Eqn (B.8) becomes

$$\lambda_{dB} = n\lambda_r \qquad (B.10)$$

This corresponds to the familiar optical formulations in which case n is the refractive index of the material characteristic of $\lambda_{\hat{r}}$.

The de Broglie wavelength prescribes a radius r_{dB} so that in analogy with Eqn (B.7) we can write

$$2\pi r_{dB} = n\lambda_{dB} \qquad (B.11)$$

On substituting Eqns (B.7) and (B.11) into Eqn (B.10) we obtain,

$$r_{dB} = n\hat{r} \qquad (B.12)$$

which is a statement of space quantization.

In general relativity, V_g is the magnitude of a constantly rotating velocity vector so that the test particle has an angular momentum given by,

$$\Omega_{\hat{r}} = mV_g\hat{r} \qquad (B.13)$$

which together with Eqn (B.4) gives

$$\Omega_r = \hbar_{\hat{r}} \qquad (B.14)$$

At the de Broglie radius r_{dB} the angular momentum of the test particle is given by

$$\begin{aligned} \Omega_{dB} &= mV_g r_{dB} \\ &= n\Omega_r \qquad (B.15) \\ &= n\hbar_{\hat{r}} \end{aligned}$$

which is the Bohr formular for the quantization of angular momentum. Thus h_r evidently qualifies as a Planck constant for some fixed m. This is the vital link between general relativity and quantum mechanics. In atomic considerations, m is usually the mass of the electron.

Consistency with Bohr's Correspondence Principle

A general expression for the gravitational acceleration at a distance r_{dB} about a potential source is given by Eqn (II.3.5a) as

$$g_{dB} = -\frac{V_g^2 \hat{r}}{r_{dB}^2} \qquad (B.16)$$

In view of Eqn (B.12), that is, space quantization, Eqn (B.16) becomes

$$g_{dB} = \frac{g_{\hat{r}}}{n^2} \qquad (B.17)$$

where g_r is the gravitational acceleration at the reference ring. Eqn (B.17) is in agreement with Bohr's Correspondence Principle as is evident in Fig 7.

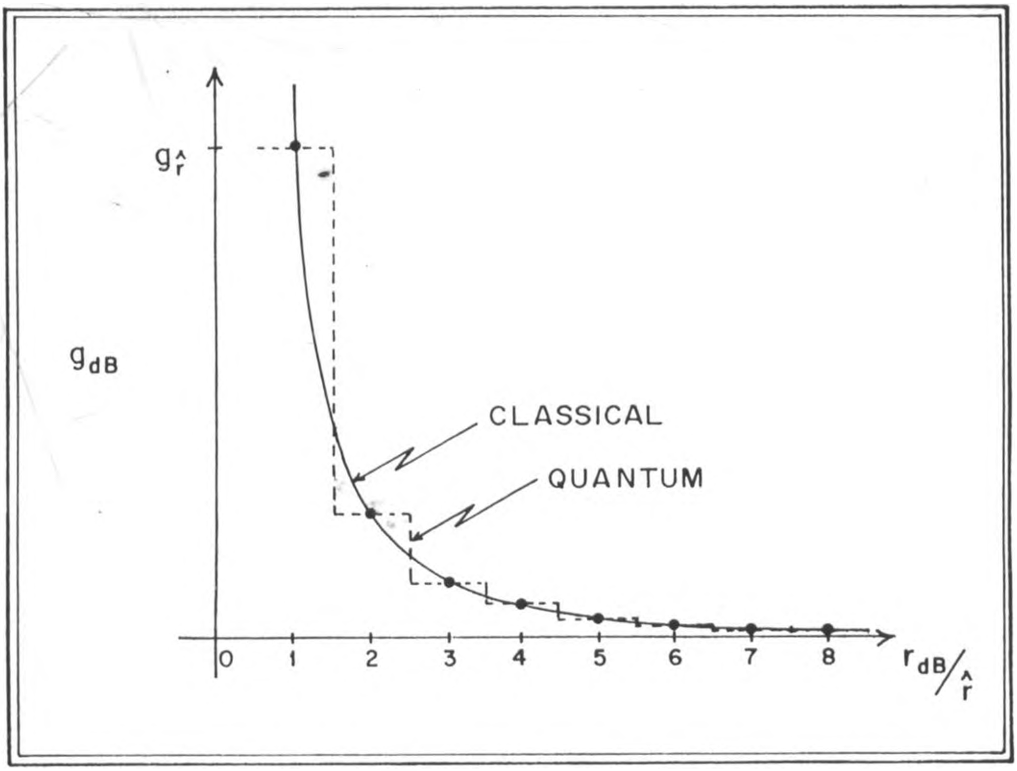

Fig 7. Quantum gravity profile in correspondence with classical gravity.

In the Newtonian picture only the mass of the potential source within the radius $\hat{r}$ contributes to the effective field experienced by the test particle. If r_0 is the radius of the potential source, then $\hat{r} \geq r_0$, satisfies the Newtonian requirement. Hence the quantum effects are substantial when r_0 is minimal, that is, when $g_{\hat{r}}$ is maximum. If $\hat{r} = r_c$, where r_c is the Classical Horizon defined by Eqn (IV.3.9), then the space quantization criterion of Eqn (B.12) becomes

$$r_{dB} = nr_c \qquad (B.18)$$

so that r_c is the gravitational ground level, while the Schwarzschild radius is the first excited gravitational quantum level corresponding to $n = 2$.

From the foregoing analysis, the gravitational energy change may thus be considered to occur with a concomitant exchange of gravitons [Isham, (1975)][31] in accordance with Eqn (B.3) and with a blackbody spectral profile.

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A COMPOSITE SPACE–TIME CURVATURE MODEL

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We develop a curvature specification scheme in which the constraint effects of a localized gravitational potential source on the Hubble expansion comprises the characteristics of the large scale dynamics of the entire universe. Our result is a composite curvature model which has a dynamical Euclidean horizon (of zero curvature) that provides an alternative to the event horizon in the determination of the local scale evolution of space–time.

1. Introduction

An important problem in the current theory of gravity is the search for a realistic model for the distribution and dynamics of cosmological matter. To date this search has been fruitless as only very simple solutions based on the cosmological principle have been investigated. As a consequence, cosmologists worldwide have not yet reached a general consensus as to the fate of the universe (whether it is open or closed). The main cause of these inadequacies is the inability to specify the value of space–time curvature in a realistic theoretical model, though extreme mathematical complications have also hindered much progress in this area.[1]

The standard procedure in the determination of space–time curvature begins with the basic Friedmann equation,[2] governing the evolution of a particle located at a distance R from the center of a homogeneous and isotropic sphere of matter, i.e.

$$\frac{1}{R^2}\left(\frac{dR}{dt}\right)^2 = \frac{8\pi G\rho}{3} - \frac{k}{R^2}, \tag{1.1}$$

where ρ is the reset mass density of the potential source, while $k = \pm 1, 0$ is the time-independent curvature constant.[3]

Unfortunately, we cannot yet specify the spatial curvature of the universe from this Friedmann equation. The difficulty arises from the use of the cosmological principle that asserts the validity of homogeneity and isotropy in the derivation of Eq. (1.1). On the other hand, precise measurements of the rate of expansion and that at which the expansion is slowing down due to gravity, have not been conclusively determined.[4]

An attempt to overcome the above-mentioned difficulty is provided in Tipler's procedure for a reformulated Newtonian gravity theory,[5] in which space–time curvature is determined by the local symmetry conditions, i.e. gravity is no longer a global "action at a distance". Matter locally tells space–time how to curve.

In this letter we shall extend the local symmetry conditions to show how the inadequacy may be overcome by introducing a Euclidean horizon in a dynamically composite representation of space–time curvature in which the constraint effects of a localized gravitational potential source on the Hubble expansion comprises the characteristics of the large scale dynamics of the entire universe. In this scenario, the long-range gravitational field will be considered to be continuous across the boundaries for regions of different curvatures so that we need not seek solutions which are true for all coordinate values, but only for coordinate values representative of the region under consideration. In Sec. 2 we reformulate the basic Friedmann equation by redefining the gravitational potential in terms of a Euclidean horizon whose role in curvature specification is presented in Sec. 3. In order to provide much insight into the essence of the composite space–time curvature and its consequence on the fate of the entire universe, a density profile is derived in Sec. 4. Finally we present a brief discussion and some figures.

2. Basic Formulations

In a pressure-free region, the dynamical energy E_D for a particle of mass m_g in the neighborhood of a localized gravitational potential source may be expressed as

$$E_D = \frac{1}{2} m_g \beta^2 , \tag{2.1}$$

where β is the dynamical velocity. In general,

$$\begin{aligned} E_D &\approx K.e|_{\text{kinetic}} + K.e|_{\text{potential}} + K.e|_{\text{electromagnetic}} \\ &\approx \frac{1}{2} m_g \nu_H^2 + \frac{1}{2} m_g \nu_B^2 + \frac{1}{2} m_g \nu_{em}^2 , \end{aligned} \tag{2.2}$$

where ν_H, ν_B and ν_{em} are kinetic, potential and electromagnetic velocities, respectively. In the absence of electromagnetic fields, and for a neutral particle, ν_{em}^2 vanishes.

Assuming that the cosmic censorship hypothesis holds such that any spherically symmetric gravitational potential source is associated with an event horizon of Schwarzschild type, then in conformity with the classical interpretation that the second term in Eq. (2.3) gives a gravitational potential energy[6,9] in real space, we can write

$$\nu_B(\varepsilon) = i\sqrt{\frac{\varepsilon}{R}} = iN(R) , \tag{2.3}$$

for the potential velocity, where ε is the radius of the source's event horizon and $N(R)$ is real (see Fig. 1).

Substituting Eq. (2.3) into (2.2) gives

$$\beta^2 = \nu_H^2 - \frac{\varepsilon}{R} , \tag{2.4}$$

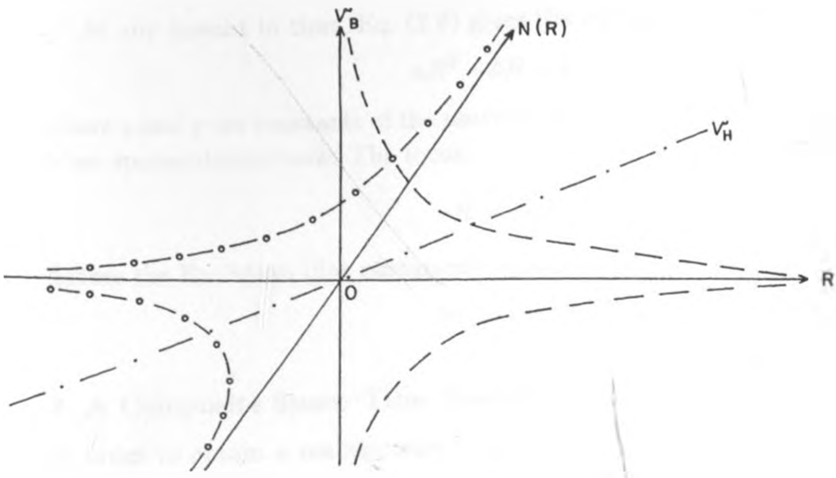

Fig. 1. Dynamical velocity profile. ————, —o—o— and —·—·— represent the imaginary ν_B in real space ($R > 0$), the real ν_B in virtual space ($R < 0$) and the real ν_H (Hubble velocity), about ($R = 0$), respectively.

in which we consider gravity as a constraint to the Hubble velocity ν_H that varies according to the Hubble expansion law, i.e.

$$\nu_H = HR\,. \tag{2.5}$$

For extremely high densities e.g. inside neutron stars, situations arise that are impossible to mimic in terrestrial laboratory.[7] On the other hand, the low densities encountered in most galaxies means that the Schwarzschild radius is often hidden inside their particle surface,[8] thus hindering the verification of Eq. (2.4). Our procedure is to overcome this difficulty by redefining the potential velocity in analogy with Eq. (2.3) as,

$$\nu_B(\mu) = i\sqrt{\frac{\mu}{R}}\,, \tag{2.6}$$

where μ is the Euclidean horizon at which the dynamical energy vanishes and satisfies the Hubble law of the form:

$$\nu_\mu = H\mu = [H\varepsilon]^{1/3}\,, \tag{2.7}$$

where we have used the simplification that all velocities are in the units of c (the speed of light) as in Gautreau's work.[9] Equation (2.4) therefore becomes

$$\beta^2 = H^2R^2 - \mu^3\frac{H^2}{R}\,, \tag{2.8}$$

which is a form of Friedmann equation in which μ is the boundary that separates a region of closed curvature from that of open curvature.

At any instant in time, Eq. (2.8) gives the energy conservation formula

$$aR^3 - bR - q = 0\,, \tag{2.9}$$

where a and q are constants of the universe in the entire space $-\infty \leq R \leq \infty$, while b has spatial dependence. The locus,

$$R = \left[\frac{q}{a}\right]^{1/3}, \tag{2.10}$$

defines the Euclidean (flat curvature) boundary μ at which

$$N(R) = \nu_H(R)\,. \tag{2.11}$$

3. A Composite Space–Time Curvature

In order to obtain a realistic solution for Eq. (2.9), it is necessary to impose two conditions:

(a) The spatial condition:

$$R \geq R_0\,, \tag{3.1}$$

where R_0 is the proper (baryonic) radius of the source mass M_0 considered as spherically symmetric about $R = 0$. This condition is in conformity with the familiar Newtonian force law.

(b) The big bang singularity requirement:

$$R = 0 \quad \text{when} \quad H = \infty\,. \tag{3.2}$$

From Eq. (2.9) a singularity is clearly present at $R = 0$ for all H since the curvature becomes infinitely large as deduced from

$$\beta^2 = \begin{cases} +\infty|_{R=-0}\,, & \text{for infinite recession.} \\ -\infty|_{R=+0}\,, & \text{for infinite attraction.} \end{cases} \tag{3.3}$$

Thus the metric structure is undefined at $R = 0$ as expressed by the geodesic incompleteness in Fig. 2. This is the basis for the assumption that the cosmic censorship hypothesis holds.

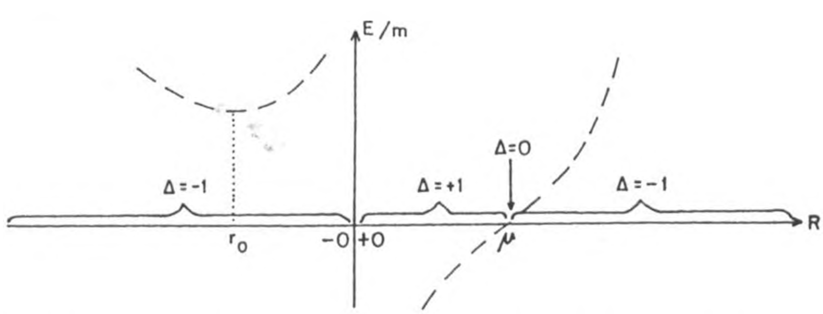

Fig. 2. Variation of the dynamical "energy per unit mass" in space $r_0 = M_0/(3\sqrt{2})$.

In the lower limit of Eq. (3.1) when $R = R_0$, the energy conversation (2.9) becomes the fully relativistic Friedmann equation for a homogeneous and isotropic universe:

$$\frac{1}{R^2}\left(\frac{dR}{dt}\right)^2 = \frac{8\pi G\rho_0}{3} - \frac{k}{R^2}\,, \tag{3.4}$$

where

$$\rho_0 = \frac{3M_0}{4\pi R^3}\,, \tag{3.5}$$

is the time-dependent average rest mass density of the potential source, while $k = \pm 1, 0$ is the time-independent curvature constant.

By direct integration of Eq. (3.4) for $k = 0$, often referred to in literature as Einstein–de Sitter universe, Gautreau[9] has obtained

$$\rho_0 \equiv \rho_0^G = \frac{1}{6\pi t^2}\,, \tag{3.6}$$

as the density for a marginally bound universe. This is the same result obtained by Silk.[2]

However, for $b = 0$, Eq. (2.9) gives

$$\rho_0 \equiv \rho_0^B = \frac{9}{4}\rho_0^G\,, \tag{3.7}$$

which is infinite at the big bang ($t = 0$) and decreases thereafter.

For $k = +1$ (the bounded universe), Gautreau has obtained

$$\rho_{\min}^G = \rho_0^B\,, \tag{3.8}$$

as the minimum density at the time of maximum expansion when $\mu = R_0$.

Further interpretation of Gautreau's unbounded galaxy world line implies that $\mu = R_0$ is the minimum radius for an unbounded universe ($k = -1$) giving a maximum density of

$$\rho_{\max}^G = \rho_0^B\,. \tag{3.9}$$

As opposed to the expectation that $\mu = R_0$ when $R = R_0$, Eqs. (3.6) and (3.7) give μ, as

$$\mu = \left[\frac{4}{9}\right]^{1/3} R_0\,, \tag{3.10}$$

which lies inside the Einstein–de Sitter universe. This situation is, however, not allowed by the topological condition of Eq. (3.1) in real space. An important consequence of our model is the resolution of this topological inconsistency. This is achieved by interpreting the universe as comprising space–time curvature in composite form that satisfies Eqs. (3.6), (3.8) and (3.9) as

$$\Delta = \begin{cases} +1 & \text{for } +0 < R < \mu \\ 0 & \text{for } R = \mu \\ -1 & \text{for } R > \mu,\ R < -0\,. \end{cases} \tag{3.11}$$

Thus the composite curvature Δ presents an interior region collapsing in upon itself with $k = +1$ and an exterior region with $k = -1$, such that the two solutions match continuously across the collapsing boundary where $k = 0$ as illustrated in Fig. 2. This is the composite curvature specification scheme.

The interpretation of Gautreau's equations given above represents a configuration that occurs under the condition that $\mu = \varepsilon$. However, it presents the composite picture much more readily and convincingly.

4. Average Densities

From spherically symmetric consideration, a potential source of Schwarzschild radius ε has an average density

$$\rho_\varepsilon = \frac{3}{2A_\varepsilon}\,, \tag{4.1}$$

where A_ε is the nondecreasing surface area of the event horizon. Similarly

$$\rho_\mu = \frac{3}{2A_u}\,, \tag{4.2}$$

where A_u is the surface area of the universe marked by the big bang photons. Since

$$\rho_\mu = \frac{9}{4}\rho_0^G\,, \tag{4.3}$$

we obtain the relation

$$\rho_\varepsilon = \rho_\mu \frac{A_u}{A_\varepsilon}\,. \tag{4.4}$$

Alternatively, the average density of the universe may be expressed as

$$\rho_u = \frac{\text{total mass}}{\text{total volume}} \frac{M_0^u}{V^u} = \rho_u \frac{\varepsilon}{R_u}\,. \tag{4.5}$$

Combining Eqs. (4.4) and (4.5) gives

$$\rho_\varepsilon \rho_u^2 = \rho_\mu^3\,, \tag{4.6}$$

as a characteristic density profile equation in a composite curvature space–time.

Since ρ_ε is fixed by the cosmic censorship hypothesis, therefore ρ_u scales as $\rho_\mu^{3/2}$. It is clear that ρ_u is infinite at $t = 0$ but decreases to zero thereafter as $t \to \infty$.

5. Discussion

Unlike the event horizon ε which is fixed by the cosmic censorship hypothesis, the Euclidean horizon μ is time-dependent and scales as $t^{2/3}$, according to the Hubble law. This is a property of a gravitational potential source in an expanding Universe and is directly proportional to $M_0^{1/3}$, where M_0 is the nonrelativistic mass of the potential source. On the other hand, μ is independent of the mass m_g of the test particle or galaxy in the potential field of the source, that is, by ignoring perturbation effects.[1]

In a static universe where the Hubble velocity $\nu_H \to 0$ and the horizon $\mu \to \infty$, this is a Newtonian approximation where the universe is infinitely bounded by the long range gravity. For all finite times, the Hubble expansion reduces μ to finite values provided that ε is finite.

Our formulation presents an infinitely closed universe where $R_0 = R_u = \mu = \infty$ in the infinite future. However, the interpretation of Eq. (4.5), that the average density $\rho_u \to 0$ as $t \to \infty$ provides for a fully open universe. This contradiction arises from the use of average density that incorrectly introduces the notion of homogeneity (cosmological principle) within the radius R_u of the universe. The proper (baryonic) radius of the potential source of mass $M_0 = M_0^u$ has a radius R_0 ($< R_u$) giving a closed universe that expands forever. This agrees with Tipler's observation that in Newtonian cosmology, a matter dominated closed universe could expand forever, as spatial topology is not the determining feature in the dynamics of Newtonian cosmology as it is in general relativity.[5]

Equation (2.8) has the advantage that the replacement of the Schwarzschild (Black hole) radius ε in Eq. (2.4) with a Euclidean horizon μ, provides the possibility that astronomical measurements can be made (without the threat of a black hole) that fully describe the dynamics of the universe. Further, the specification of ν_H and ν_B at any position R, implies that the value of energy per unit mass E_D/m can be determined. This is an advantage over Newtonian cosmology.

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