DENSIFICATION OF GEODETIC CONTROL NETWORKS UNDER REPRODUCING PARAMETRIC AND STOCHASTIC FIDUCIAL CONSTRAINTS

BY

GODFREY ONYANGO OGONDA

A thesis submitted in partial fulfilment for the degree
of
Master of Science in Surveying
in the University of Nairobi.

July, 2001

DECLARATIONS

This thesis is my original work and has not been presented for a degree in any other university.

15.10.2001

Godfrey Onyango Ogonda

This thesis has been submitted for examination with our approval as university supervisors

16.10.01

Prof. Dr.-Ing. F. W. O. Aduol

17/10/01

Dr.-Ing. S. M. Musyoka

ABSTRACT

A principal consideration in densification of geodetic systems has been the need to incorporate the stochasticity of the datum parameters in the densification process. The special question however is ***how to incorporate the stochasticity of the datum parameters in the estimation of the new parameters while reproducing the datum parameters together with their stochasticity (respectively reproducing parametric and stochastic fiducial constraints).*** A number of approaches addressing the above question have been proposed. These include: static-dynamic, pseudo-dynamic and sub-optimal network fusion approaches. Of these, pseudo-dynamic, static-dynamic and sub-optimal network fusion approaches reproduce the datum parameters and their stochasticity while static and dynamic approaches do not possess the reproducing quality.

The aim in this study was to evaluate the practical applicability, and to establish the suitability, of two densification approaches with the reproducing quality. The static-dynamic and sub-optimal fusion approaches are considered, with a view to identifying their strength and weaknesses as approaches to densification of geodetic systems. The results are compared to establish which of the approaches is best suited for recommendation to be adopted for geodetic densification and under what circumstances. For a general perspective, the non-reproducing techniques namely; static and dynamic approaches are also discussed, evaluated and compared to the two approaches. Although the pseudo-dynamic approach has the reproducing capability, it is not considered since the approach has the drawback in that, on one hand datum parameters are treated as non-stochastic entities, thus fixed, while on the other hand, they are treated as stochastic, resulting in an inconsistent estimation model.

To evaluate these approaches, each of the techniques is used to adjust simulated and real test geodetic networks at two levels of densification. The simulated geodetic network consists of three first order, three second order and nine third order points while the real geodetic network points were extracted from the national geodetic network of Kenya consisting of eleven first order, fifteen second order and ten third order points. For each approach, and at every level of densification on the two networks, the parameters, the standard errors and their corresponding error ellipses were compared against each other.

The results indicate that the datum parameters in the static-dynamic and the sub-optimal fusion approaches are reproduced together with their stochasticity, that is, maintained definitive. Although the datum parameters are reproduced together with their stochasticity, the new point parameters obtained using sub-optimal fusion approach are similar to the parameters obtained using the dynamic approach. That is, it compares to adjusting the network using the dynamic approach and applying a corrective term on the datum parameters to keep them unchanged. The covariance matrices obtained through the two approaches are closer to each other as demonstrated by the confidence error ellipses. The results generally demonstrate that both the static-dynamic and sub-optimal fusion approaches give more realistic estimates of the parameters than the static and dynamic approaches.

ACKNOWLEDGEMENT

I wish to express my sincere gratitude to my supervisors Prof. Dr.–Ing. F. W. O. Aduol and Dr.–Ing. S. M. Musyoka whose tireless efforts and patience greatly contributed towards the completion of this study. Their advice, suggestions and encouragement inspired me greatly. I express my appreciation and gratefulness to you.

I am grateful to the University of Nairobi for offering a scholarship that enabled me undertake the course. Special thanks to Dr. G. C. Mulaku, Chairman Department of Surveying and the entire staff of the Department of Surveying for the assistance and encouragement they offered to me.

Last but not least. I do appreciate the assistance offered to me by the staff of the technical section of Survey of Kenya, specifically to Mr. Muiruri, who readily availed to me the data used in the real test network. Finally, to all who contributed in one way or another and have not been mentioned. I salute you. May God bless you all.

DEDICATION

To my father Narkiso Ogonda Ong'wen

To my mother Mary Ochung' Ogonda

And to my wife Carren Adhiambo

You have given me so much.

This is dedicated to you.

TABLE OF CONTENT

LIST OF TABLES

LIST OF FIGURES

NOTATION

Listed below are the notations used in the text. The page in which the notation first appears is given in parenthesis.

y is the $u \times 1$ vector of observation increments (14)

x is the $m \times 1$ vector of unknown parameters (14)

x_1 is the $c \times 1$ vector of parameters with prior information (15)

x_2 is the $(m-c) \times 1$ vector of parameters without prior information (15)

$\hat{x}_1$ as defined in the text (15)

$\hat{x}_2$ as defined in the text (15)

A is the $n \times m$ coefficient matrix (14)

$[A_1\ A_2]$ as defined in the text (15)

W is the $n \times n$ positive definite weight matrix (14)

e is the $n \times 1$ vector of random observation errors (14)

e_0 is the $c \times 1$ vector of restriction errors (17)

$D(y)$ is the dispersion of y (14)

Σ is the dispersion matrix (14)

Σ_{yy} is the dispersion matrix of the observations (18)

Σ_{zz} is the dispersion matrix of the restrictions (18)

Σ_{ss} as defined in the text (25)

σ_0^2 is the a-priori variance of unit weight (14)

$\hat{\sigma}_0^2$ is the a-posteriori variance of unit weight (15)

$D(e)$ dispersion of e (14)

K is the $c \times 1$ restriction design matrix (17)

Z_0 is the $c \times 1$ vector of restriction parameters (17)

C denotes the covariance (17)

W_z is the weight of the restrictions (17)

W_y is the weight of the observations (17)

n is the number of observations (18)

c is the number of the restrictions (18)

m is the number of unknown parameters (18)

$D(\hat{x})$ dispersion of $\hat{x}$ (21)

$D(\hat{x}_2)$ is the dispersion of $\hat{x}_2$ (15)

$E(\hat{x})$ is the expectation of $\hat{x}$ (15)

$E(\hat{x}_2)$ is the expectation of $\hat{x}_2$ (15)

N is the normal equation matrix (15)

G is the orthogonal matrix made up of eigenvectors (19)

Q_{xx} is the cofactor matrix of the unknown parameters (19)

$tr(Q_{xx})$ is the trace of Q_{xx} (19)

Δx_i is the approximate value of the x_i coordinates (20)

Δy_i is the approximate value of the y_i coordinates (20)

L is the Lagrange function (17)

N_r as defined in the text (22)

λ vector of Lagrange multipliers (17)

$\overline{\sigma}_C$ is the circular probable error (52)

σ_E, σ_N is the standard error in E and N respectively (52)

E, N are the easting and northing co-ordinates respectively (33)

σ_N^2 is the variance of the N (55)

σ_E^2 is the variance of the E (55)

σ_{EN} is the covariance between E and N (55)

ψ is the bearing of a (55)

H_o is the null hypothesis (118)

H_a is the alternative hypothesis (118)

χ_m^2 is the Chi-square test at m degrees of freedom (118)

$F_{m_1,m_2} = \dfrac{\overline{\sigma}_1}{\overline{\sigma}_2}$ is the F-test statistic at m_1, m_2 degrees of freedom for independent samples 1 and 2 (123)

ϕ is the target function (29)

ζ as defined in the text (25)

y_ζ as defined in the text (25)

A_ζ as defined in the text (25)

W_ζ as defined in the text (25)

e_{ζ} as defined in the text (25)

$\Sigma_{\zeta\zeta}$ as defined in the text (28)

Q_{e} as defined in the text (26)

Q_{e_0} as defined in the text (26)

Q_{yy} as defined in the text (26)

σ_{0v} as defined in the text (26)

x_{ζ} as defined in the text (25)

L_1 as defined in the text (30)

L_2 as defined in the text (30)

I_t as defined in the text (30)

I_m as defined in the text (30)

l_1 as defined in the text (30)

l_2 as defined in the text (30)

$\hat{\lambda}_1$ as defined in the text (30)

$\hat{\lambda}_2$ as defined in the text (30)

$\mathfrak{R}$ as defined in the text (30)

1

INTRODUCTION

1.1 The Geodetic Densification Problem

The introduction of new points into an already existing geodetic system of control points is termed densification. The densification process, that is, the measurement of geometrical relations of certain new points to a number of previously estimated control points is a problem commonly encountered by geodesists in their work. The basic guiding principle in geodesy, "from the whole to the part", is as valid today as it was hundreds of years ago. Progress in technology, though, has had an important effect in the amount of data that can be collected and analyzed in geodetic systems. In this context, emphasis has been put in the study of efficient and accurate densification approaches.

The basic problem of geodetic densification is two-fold; firstly the problem of optimal design of a geodetic system and secondly the problem of establishing the most suitable technique for processing and for post analysis of the system. Having achieved the optimal design of a geodetic system in its fundamental configuration, then as a rule, the densification problem arises in that with additional observations, certain new points have to be intercalated into the fundamental network. This generally involves interpolation, or to a lesser extent, extrapolation about the datum (fiducial) points. Densification, as addressed in this report, begins by a given field of datum points at which certain prior information is known, which then is followed by the interpolation of the information at points other than the datum (fiducial) points. The question is then that of how to handle the position values of the already fixed stations.

The densification problem has existed for many years and has been considerably applied to problems in geodesy, hydrography, cadastral surveying, engineering surveying, photogrammetry and recently also in the Global Positioning System (GPS). In Kenya for example, several first order, second order, and third order frameworks of survey stations have been established and permanently marked on the ground via the process of densification using triangulation, traversing and Doppler satellite positioning techniques on the Arc Datum 1960 based on Clarke's 1880 ellipsoid. Research into suitable densification approaches thus becomes clearly relevant to the needs of the geodetic community as also evidenced by the number of studies on the subject by several researchers.

The central concern in both the design and processing of geodetic systems is the formulation of functional relations between unknown parameters and the observables. The unknown parameters are then estimated through an estimator that only succeeds in optimal statistics characteristics if it is unbiased, of minimum variance (best) and invariant from any other geodetic estimation results calculated in the same model. In geodetic practice, in one form, the functional models are linear and several such estimates which satisfy few of the optimal statistics characteristics have been proposed and include for example Best Linear Estimate (BLE), Best Linear Unbiased Estimate (BLUE), Best Linear Minimum Biased Estimate (BLMBE).

In free network adjustment of geodetic systems, the "datum" is normally defined over a number of network points in "free mode", and this results in the problem that neither the dispersion matrix nor the configuration matrix of the system are of full rank. The results therefore are biased estimates where the bias could be a minimum and the unknowns are estimated with minimum variance. This, as noted by *Grafarend [1976]*, is highly preferred in the adjustment of first order and underdetermined geodetic systems where no prior information is available.

In second and higher order densification of geodetic systems, incorporation of prior information in the densification process results in two data sets that need to be fused. Depending on the manner in which the reference data (prior information) is handled in the process, we have the following four possible cases:

- Holding existing prior information as fixed and errorless.
- Treating existing prior information as fixed and errorless but propagating their covariance information.
- Perform weighted parameter adjustment with the existing prior information weighted by their predetermined covariance matrix.
- Holding existing information as fixed stochastic entities.

The conventional approach to densification was performed by over-constraining the geodetic system, in which case, the control point parameters are considered as fixed, non-stochastic, entities in the form of exact restrictions (respectively exact prior information). However the control points should be considered stochastic having been obtained from previous adjustment. Classically, one performs a number of observations on a framework of control parameters and new parameters and then deduces, from the observations, the new parameters on the assumption that the control parameters are known exactly. The approach whereby the control point parameters are considered as errorless has been termed as *hierarchical densification* by *Pelzer [1980]*, as *static densification* by *Cooper [1987]* and *Aduol [1993]* while *Vanicek and Lugoe [1986]* refer to it *as over-constrained adjustment of densification*. The approach, although reproducing the results from the previous adjustment exactly, gives too optimistic results as a consequence of neglecting the variance-covariance matrix of the datum points and is therefore considered rarely optimal among all possible reproducing techniques. The approach is also non-rigorous in the statistical sense.

The proposal to consider the control point parameters as stochastic was already made by the beginning of the twentieth century as noted by *Wolf [1983]*. The new points were estimated on the basis of the static mode, while the stochasticity of the estimated points were computed through error propagation incorporating the stochasticity of the control point parameters *[Aduol 1993]*. Works by *Van Mierlo [1984], Nickerson et al. [1986]* and *Wolf [1984]* refer to this approach as *quasi-hierarchic or pseudo-dynamic*. The quasi-hierarchic (respectively pseudo-dynamic) approach has the disadvantage that the model for parameter estimation and that for stochastic estimation are not consistent. That is, one treats the control point parameters as stochastic on one hand and on the other treats them as non-stochastic *[Aduol, 1993]*. The end result is that the covariance matrix of the new points is updated by a corrective term.

In another approach, the densification is done by considering control point parameters as stochastic and then proceed to estimate all the new points including the control points by combining them in the model the parametric and stochastic constraints. *Vanicek and Lugoe [1986]* noted that, a statistically rigorous way to densification of geodetic systems is to simultaneously adjust both the prior information of the control points and the introduced framework of new points through the imposition of properly weighted constraints on existing control point parameters. This is only achieved through rigorous propagation of the covariance matrix of the fiducial constraints. This approach has been referred to by, among others, *Pelzer*

[1980], El-Hakim [1982] and *Grafarend and Schaffrin [1988]* as *dynamic model*. The result of the rigorous densification of the geodetic systems is that the existing control framework changes position due to the effect of new observations propagated into the existing system. This is thought, as indicated by *Vanicek and Lugoe [1986]*, "to be rather unfortunate from the practical point of view". One is left wondering whether to replace the datum points with the new results or not. This may be justified if it is established that the newly densified framework is more accurate than the existing reference framework as would be the case in observations derived from Very Long Baseline Interferometry (VLBI) for instance, or if blunders or systematic errors are detected in the existing control framework. Although stochastically rigorous, powerful and efficient, the dynamic approach has the drawback that the control point parameters change to new values with every new single point added to the geodetic system. Therefore, if control point parameters are not properly updated, accumulation of inconsistencies may destroy the quality of the geodetic system. This is the problem experienced by the *dynamic* approach to densification, whose practical applicability however is still an open question.

It is evident that there exist considerable differences in quality between the datum points and new points in densification of networks. *Schaffrin [1985]* suggested that it was not appropriate to deal with both the new and datum points in the same manner by simply adding the previous coordinates as "pseudo observations", e.g. as used in the static and dynamic approaches, instead he constructed a more robust method "*the best homogeneously linear (weakly) unbiased predictor*" *(homBLUP)*. This method proved to be robust enough against eventual errors in the prior information without destroying the "homogeneity of the neighbourhood".

Hierarchical models for densification are considered the most appropriate approaches to densification since they keep the parameters of higher order points unchanged. The problem encountered in hierarchical densification however is that of how to incorporate the datum point covariances in the estimation of the new point parameters in the most rigorous and proper way. This is a fact that was already fully recognized by *W. Baarda* in writing specifications for the activities of the Dutch cadastre *[Baarda, et al. 1956]*.

To address the geodetic densification problem, *Aduol [1993]* proposed an approach to densification referred to as the *static-dynamic model*. In this approach, the control points are treated as stochastic restrictions (respectively stochastic prior information) while the control point parameters are reproduced with their covariance matrix as definitive with the concept of a consistent mathematical formulation and thus combines the properties of both *static* and *dynamic* approaches. This approach is referred to as, *estimation with incomplete prior information* by *Theil [1963,1971]* and as *stepwise regression* by *Toutenburg [1975], Bibby and Toutenburg [1977]* and *Toutenburg [1977]*.

In order to overcome the same geodetic densification problem, *Schaffrin [1998]* has proposed the *sub-optimal fusion* approach. In the *sub-optimal fusion* approach the best linear uniformly unbiased estimate (BLUUE) is determined. Like the *static-dynamic* approach, *sub-optimal fusion* approach has the property that the datum parameters are exactly reproduced and their stochasticity incorporated in the rigorous estimation of the new point parameters together with their dispersion matrix. In this study, the performance of the sub-optimal fusion approach as regards the rationale of overcoming the geodetic densification problem of exactly reproducing the datum point parameters together with their stochasticity in the densification process has been investigated.

1.2 Statement of the Problem

In the conventional approach to the densification of the geodetic systems, termed *static* by *Aduol [1993]*, the control points are considered as non-stochastic fixed entities. The consideration of parametric and stochastic fiducial constraints as exact resulted in the estimated new points having less dispersion than they really have, thus giving falsified estimates *[Aduol, 1999]*.

The need to incorporate the stochasticity of the control point parameters in the estimation of the new point parameters led to the development of the dynamic approach in which the control point parameters obtain corrections for their values hence new values are obtained after the adjustment. As a result. the control point parameters change their values dynamically in principle with every single measurement added to the system, that is, the control points move during adjustment. If the control points are not properly updated, accumulation of inconsistencies may destroy the quality of the geodetic system *[Schaffrin, 1998]*.

The principal problem in geodetic densification therefore is, how to incorporate stochasticity of the control points in the estimation of the new points while reproducing the control point parameters together with their dispersion matrix (respectively reproducing parametric and stochastic fiducial constraints). To address this problem some authors have proposed the adoption of the dynamic approach, but to ignore changes to the control point parameters unless they are "significantly" large. In this case, where the changes to the control point parameters are neglected on the basis of whether they are "large" or not, the final parameters adopted cannot be consistent with the mathematical model used for the estimation of the new point parameters.

In the static-dynamic approach proposed by *Aduol [1993]*, the properties of the static and dynamic models are combined, thus the control point parameters are taken as stochastic while at the same time retain their definitiveness. Addressing the same problem, *Schaffrin [1998]* has proposed a densification approach with similar properties as the static-dynamic approach. The approach reproduces control point parameters together with their dispersion matrix while incorporating their stochasticity in the estimation of the new points. *Schaffrin [1998]* has referred to the approach as the *sub-optimal network fusion*, which gives reproducing best linear uniformly unbiased estimates (repro *BLUUE)*.

Densification of geodetic systems calls for use of proper estimation techniques that also give the reliability of the estimated parameters. Studies by *Miima [1997]* indicated that the models that incorporate the *static-dynamic* concept, that is, reproducing the control point parameters while incorporating their stochasticity in the estimation of new point parameters, produce best statistically agreeable results. There is need therefore to study the *sub-optimal network fusion* approach against the *static-dynamic* approach with a view to evaluating its practical applicability in a densification exercise and overall suitability in densification work in general.

In this study, the approaches with reproducing property (i.e. *static-dynamic and sub-optimal network fusion*) are considered. The approaches are compared with each other in view of determining their relative effectiveness to densification of geodetic systems.

The *static* and *dynamic* approaches discussed above have been used to compute the parameters and the results compared to those obtained by the static-dynamic and sub-optimal fusion approaches. It is however noted that the static approach does not possess the reproducing property, that is, it reproduces the control parameters while the variance-covariance matrix vanishes. Similarly the *dynamic* approach does not have the reproducing property in that both the control point parameters and their stochasticity are updated through the densification process.

1.3 Objectives of the Study

The main objective of this study is to demonstrate practical applicability, and to evaluate the suitability, of the *sub-optimal network fusion* approach to densification as proposed by *Schaffrin [1998]*, in relation to the *static-dynamic* densification approach as proposed by *Aduol [1993]*. Through this, it is hoped to gain an insight into the relative effectiveness of these two approaches to densification, i.e. how to incorporate the stochasticity of the control points in the estimation of the new points while reproducing the parametric and stochastic fiducial constraints, and also to establish which of the approaches is best suited for recommendation to be adopted for densification and under what circumstances.

The specific objectives of the study are:

- To obtain densification results of the real and the simulated geodetic systems through the *Static-Dynamic* approach and the *Sub-Optimal network fusion* approach.
- Analyze and compare the accuracy, efficiency and consistency between the prior information and the densification results obtained by the two approaches.
- To determine the reliability and suitability of the two approaches relative to each other.

1.4 Literature Review

With the improvements on positioning technology, several studies have been carried out on densification to provide for more precise and refined control points. The densification concepts though remain unchanged, as stated in *[Aduol 1993] "In the densification of networks we normally have two groups of points to be handled in the parameter estimation. These are the already existing points over which the network datum is defined, and then there are the densification points to be newly coordinated".*

In densification of geodetic systems, the manner in which the prior information is handled classifies the various densification approaches as either *reproducing or non-reproducing* densification techniques. In the conventional approach referred to as *static* approach by among others *Cooper [1987]* and *Aduol [1993]*, and the *hierarchical* approach by, among others, *Pelzer [1980]*, the prior information about the control points are treated as exact non-stochastic entities. The *static* approach has the problem in that, it does not reproduce the stochastic fiducial constraints but rather reproduces the parametric fiducial constraints as exact non-stochastic entities. This approach thus reproduces the control point parameters at the expense of neglecting their corresponding variance-covariance matrices.

The need to consider control point information as stochastic was already recognized as early as 1882 when *W. Jordan* advocated that the control points in the densification of geodetic systems be treated as "correlated observation" *[Wolf 1983]*. The early researchers estimated the new points on the basis of *static* approach, while the stochasticity on the estimated new points were computed through "error propagation" incorporating the errors on the control points *[Aduol 1993]*. Recent works on this approach have termed it as *quasi-hierarchic* or *pseudo-dynamic [Van Mierlo 1984, Nickerson et al. 1986]*.

Theil and Goldberger [1961] highlighted the uncertainties that arise when, during statistical estimation of economic relations, a hypothesis is formulated, and appropriate computation to provide desirable estimates of parameters of the linear relation carried out, only to find out that the estimated income elasticity of some commodity was negative. In their search for a statistical estimator, *Theil and Goldberger [1961]* are quoted as saying *"The difficulty seems to be that the investigator has a prior knowledge which he can not conveniently incorporate in the hypothesis and which he therefore omits. This kind of a prior knowledge, however, is precisely the major source of rejections in hypothesis. It seems clear that it is logically more consistent to incorporate such knowledge in the hypothesis right at the beginning than exclude it from the hypothesis and reject it afterwards when the results contradict the omitted knowledge"*.

Theil and Goldberger [1961] then proposed a model of "mixed" estimation that was an effort to incorporate prior knowledge of coefficients in regression analysis and other linear statistical models. The prior knowledge was formulated in terms of prior estimates of parameters, which were assumed to be biased and to have a moment matrix *[Miima 1997]*. This can be considered as part of the fundamental mathematical formulation of the dynamic model.

Theil [1963] analyzed the use of incomplete prior information in regression analysis by considering the combination of prior and sample information with the fact that both are stochastic but independent of each other. He tested the compatibility of the two and proposed a measure for the relative contribution of sample and prior information to the results of estimation.

Toutenburg [1974] developed an approach for combining stochastic prior information in a vector of regression coefficients with incomplete prior information on the variances of the disturbance terms. This enlarged the general linear regression model to yield a restricted regression model. This work was actually the basis for the *static-dynamic* approach.

Blaha [1974] studied densification networks when fixed parameters were neglected in the variance-covariance propagation with the aim to correct the variance-covariance matrices for the contribution of such uncertainties. This was to be done through considering the general least-squares solution with weighted unknown or some weighted and some unknown parameters, hence providing a more general approach to *hierarchical* densification. His work was an expansion on the work of *Papo [1973]*, who had he proposed a method by which, without altering the values of the adjusted parameters their a-posteriori covariance matrix could be improved by inclusion of the effect of uncertainties in the constants of the adjustment process. The algorithm outlined in *Blaha [1974]*, is a method that permits the propagation of random errors from a previously determined network into the accuracy estimates and solution vector for merged network points without affecting the original network's accuracy or solution vector.

Cooper and Leahy [1978] undertook densification of a geodetic system under two approaches, one, by considering the control points to be fixed absolutely and in the other by regarding the control points as correlated, and hence not fixed. The result of their study indicated that the approach where the control points are treated as correlated observations yielded better results. It has to be noted that this approach has a weakness as pointed out in *[Aduol 1993]* through the statement *"Taken to its ultimate, one has for instance with this that where only a single point is being coordinated by "intersection" with the datum points forming a part of a national geodetic reference system, the single new point would (theoretically) cause all points in the national network to acquire new coordinate values and new stochastic parameters. With this we note that the concept of a national geodetic reference system is effectively lost."*

In *[Koch, 1983a]*, densification of geodetic systems by considering the fixed control points as random variables is discussed. He also addressed a special case of transformation of covariance matrix for the parameters of the control points if the system of the control points was changed during the densification process.

In trying to answer the question of whether to consider the control points as stochastic or as fixed non-stochastic entities, *Van Mierlo [1984]* suggested the adoption of a "compromise solution". He proposed the consideration of fundamental geodetic systems as stochastic so that their covariances are fully taken into account while at the same time they are considered as non-stochastic, in which case they are not corrected by the resulting residuals and the discrepancies are arbitrarily put to zero *[Miima 1997]*. *Wolf [1983]* demonstrated that this approach led to a bias in the residual of the geodetic system, thus giving falsified angles and distances to the given points.

Schaffrin [1984] proposed the adoption of the *dynamic* approach to densification but to ignore the changes to the control point parameters and their stochasticity unless they are "significantly large". In this case *Aduol [1993]* states however, that as long as the changes are merely neglected on the basis of whether they are large or not, the final parameters adopted can not be consistent with the mathematical model adopted for the estimation of the new parameters.

Schaffrin [1985] suggested that it was not appropriate to deal with both the new and old measurements in the same manner by simply adding the previous parameters as "pseudo observations" as in the case of *static* and *dynamic* approaches. Instead he constructed a more robust method called " *the best homogeneously linear (weakly) unbiased predictor*". This proved to be robust enough against eventual errors in the prior information without destroying the "homogeneity of the neighbourhood".

Nickerson et al [1986] performed densification of a second order geodetic control network by *static, dynamic* and *semi-dynamic* approaches. The results indicated, that the *dynamic approach* provided realistic error ellipses for geodetic system densification but resulted in the change of control point parameters. They recommended that "one simply records and not apply the corrections to the control point parameters and use the covariance information provided by the *dynamic* approach". It can be noted that this assumption leads to the distortion in geodetic systems which is not reflected in the confidence ellipses.

Vanicek and Lugoe [1986] suggested that a statistically rigorous densification of geodetic systems must consist of a simultaneous adjustment of both the reference control point framework and the new point framework. *Vanicek and Lugoe [1986]* state that: *"The statistically rigorous way of adjusting the new points, i.e. the densification network, into the* *existing network is to adjust both networks together, using both the 'old' and 'new' observations. Alternatively, the new densification network, including the junction points, can be rigorously adjusted in a phase adjustment mode, where the information from the existing network is propagated into the new phase of the adjustment by (1) using the existing positions of the junction points for initial estimates; and (2) using the inverse of the covariance matrix of these existing positions (as obtained from the previous adjustment) for a prior weight matrix of the junction points".* They noted that, unfortunately, as a result of the rigorous adjustment, the positions of the junction points as well as the old points do change. This however should have been expected as the proposed approach was simply the dynamic densification approach.

Cooper [1987] indicated the necessity to consider the control points as stochastic rather than non-stochastic fixed entities. In his work, the result of *dynamic* approach is that the control point parameters change as a result of the estimation of new points in the geodetic system. This introduces the anomaly of having two sets of parameters for the national control point and so *Cooper [1987]* recommended a re-estimation of parameters and stochasticity of all points in the geodetic system, not just of the new points.

Illner [1988] considered the hierarchical densification of geodetic systems and proposed models for *dynamic*, *static* and *hybrid* approaches to densification

In his study, *Lugoe [1990]* discussed approaches to densification of geometrically strong and geometrically weak geodetic systems and also considered the simultaneous densification and integration. He observed this as being a statistically viable approach as pertaining to densification and integration together.

Aduol [1993] suggested the *static-dynamic* approach to densification, which he then compared to the *static* and *dynamic* approaches to densification of geodetic systems. From the analysis of the respective variance-covariance matrices, strong theoretical and practical qualities of the *static-dynamic* approach against the fully *static* and *dynamic* approaches to densification are demonstrated. The results are based on a simulated network adjustment and he recommended a similar study on a real network to ascertain the results.

Furthering the study carried out by *Aduol [1993]*, *Miima [1997]* states: *"There are basically four densification approaches which have been proposed. The main distinction in the models is* *dependent on how the coordinates of the higher control stations are treated during densification*". *Miima [1997]* considered densification under *static*, *dynamic* and *static-dynamic* approaches of a real two-dimensional geodetic system comprising 15 secondary and 22 tertiary points built on a control system defined by 8 primary points. The resulting parameters, standard errors and standard error ellipses were compared. Analysis of the results indicated that *the static-dynamic* approach gives more realistic estimates than the *static* and *dynamic* approaches. which was in agreement with the results of the study carried out by *Aduol [1993]*.

Schaffrin [1997] noted that the static approach in which the control point parameters are considered fixed, non-stochastic, constraints will rarely be optimal among all possible hierarchical data fusion methods that ought to keep the so called "reference information unchanged". He derived the "optimal reproducing estimator" in the context of geodetic network densification by employing non-Bayesian techniques. Hierarchical estimators have, in contrast, been proposed by Berliner [1996] for time series and by Wilke et al. [1998] for time-space models *[Schaffrin and Cothren, 1999]*.

Schaffrin and Cothren, [1999] noted the need for stability of the reference data that are meant to provide information of such high quality. To avoid their change during the densification process, they outlined a strictly *hierarchical* method in which the estimation procedure is designed to reproduce everything that belongs to a "higher category" and perform an adjustment in the least-squares sense on everything else in the geodetic system. The technique is then compared to the traditional approach based on Helmert's transformation and the non-hierarchical approach through the integration of photogrammetric geodetic systems of substantially different scales. The result of their study indicated that the traditional approach based on Helmert's transformation is non-optimal (in the sense of minimum mean-square error). They also realized that in the *hierarchical* approach, the reference points involved remain unchanged and that the *optimal* method provides the same result for the new points as the *non-hierarchical* approach except for slightly enlarged variance component estimates.

In comparing the free net adjustment, followed by *Helmert transformation* to the *sub-optimal fusion* approach. *Schaffrin [2000]* realized that the latter approach is superior over the former in terms of the Mean-Squared-Error (MSE) risk.

1.5 Organization of the Report

The report is organized into eight chapters. In Chapter Two, linear estimation methods relevant to the study are discussed. In Chapter Three, the densification techniques, both reproducing and non-reproducing, are presented. The general experimental design and the geodetic test networks on which the densification techniques are applied are presented in Chapter Four. Chapter Five outlines the test results of the computations obtained from the four experiments carried out in the study. Analysis of the results is contained in Chapter Six. Discussions are contained in Chapter Seven while the relevant conclusions and recommendations are drawn in Chapter Eight.

2

ESTIMATION METHODS

The data collected from experiments and surveys are normally analyzed and then used to estimate essential parameters using relevant estimation techniques. Though estimation techniques are used so extensively in the theory and application of geodetic densification approaches, it is impossible in this report to discuss all the techniques. This section therefore, is primarily concerned with the *Gauss-Markov model with stochastic restriction* and *free network adjustment* techniques. The techniques are briefly discussed since they are relevant to the estimation of parameters in the present study.

2.1 The Gauss-Markov Model

The simple Gauss-Markov model is the basis for the least-squares estimation technique which minimizes the sum of the squares of the residuals. The model is given as

$$E(y) = Ax, \qquad D(y) = \sigma_0^2 W^{-1} \tag{2.1.1}$$

where y is a $n\times$ vector of observational increments

A is the $n\times$ coefficient matrix

x is the $m\times 1$ vector of unknown parameters

σ_0^2 is the (typically unknown) variance component or variance of unit weight

W is the $n\times n$ positive definite weight matrix

Since y is stochastic then (2.1.1) takes the form

$$y = Ax + e \quad . \quad e \sim\sim (0, \Sigma = \sigma_0^2 W^{-1}, \quad \Sigma \text{ is positive definite} \tag{2.1.2}$$

where e is a $n\times$ vector of random observation errors

$\Sigma := D(e)$ is the corresponding $n\times n$ dispersion matrix

With the assumption that the prior information is given in the form of estimated parameters which, in turn, appear in the linearized observation equations for the estimation of the new points, we obtain the extended Gauss-Markov model as

$$y = A_1 x_1 + A_2 x_2 + e \qquad (2.1.3)$$

where $A := \begin{bmatrix} A_1 & A_2 \end{bmatrix}$ is the $n\times$ coefficient matrix

x_1 is a $c \times 1$ vector of parameters with prior information

x_2 is a $(m-c)\times 1$ vector of parameters without prior information

With the least squares requirement that the sum of weighted squared residuals be minimum, and taking into consideration the weights of the observations, and provided the coefficient matrix A has full column rank, then the estimate of the unknown parameter x is given by

$$\hat{x} = (A'WA)^{-1} A'Wy \qquad (2.1.4)$$

with

$$D(\hat{x}_2) = \hat{\sigma}_0^2 (A'WA)^{-1} \qquad (2.1.5)$$

and

$$\hat{\sigma}_0^2 = \frac{\hat{e}'W\hat{e}}{n-m} \quad , \quad \hat{e} = y - A\hat{x} \qquad (2.1.6)$$

Further

$$E(\hat{x}) = x \qquad (2.1.7)$$

indicating that $\hat{x}$ is an unbiased estimate of x.

Estimation under the Gauss-Markov model is the only possible where the geodetic system has been freed of any datum defects since the model assumes that the resultant normal equation matrix has full rank. Therefore, any datum defects in the system must be overcome before hand.

2.2 Gauss-Markov Model with Restrictions

In the event that the coefficient matrix A is not of full column rank, we would have the case of a Gauss-Markov model with rank defect. In such case $(A'WA)^{-1}$ in (2.1.4) and (2.1.5) above would not exist. This situation arises when for instance the datum for the parameters being adjusted is incompletely defined by observations and restrictions, i.e. the observations do not cater for all the degrees of freedom in the network. *Koch [1987, p212]* states that, when observations are formed, it is necessary to add a set of restrictions, which in effect complete the definition of the datum.

A coordinate system in three-dimensional space necessitates the definition of seven degrees of freedom. If it is defined in shape, this includes one scale, three translation elements and three orientation elements. For a two-dimensional space, four elements, namely, one scale, one orientation and two translations are to be defined. The necessary and sufficient number and type of datum elements can be defined by appropriate combination of measurements. *Cooper [1987]* outlines the number and type of Cartesian coordinate datum elements for two- and three-dimensional spaces which are defined by inclusion of certain measurements in a network. Datum defects due to fewer degrees of freedom than necessary may be overcome through the incorporation of appropriate restrictions. This section discusses the variants of the Gauss-Markov model under different forms of restriction.

2.2.1 Exact Restrictions

Exact restrictions may be incorporated in the geodetic system for two main reasons; firstly, to overcome datum defects and secondly, to fulfill certain physical and geometrical conditions in the model. In general, the Gauss-Markov model with exact restrictions is set up in the form

$$y = Ax + e \qquad (2.2.1)$$

and $$z_0 = Kx \qquad (2.2.2)$$

with (2.2.2) as the exact restriction, in which

z_0 is a $c \times 1$ vector of restriction parameters (non-random)

and K is a $c \times m$ restriction design matrix , $rk = c$

(2.2.2) may also be referred to as exact prior information *[Durbin, 1953]* and *[Aduol, 1993]*.

To determine the estimate of x under the least-squares condition and further fulfilling (2.2.2), the Lagrange function L is used, along with the $c \times 1$ vector λ of Lagrange multipliers

$$L = e'We + 2(Kx - z_0)'\lambda \qquad (2.2.3)$$

with which, under least-squares condition the resulting normal equations take the form

$$\begin{bmatrix} A'WA & K' \\ K & 0 \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{\lambda} \end{bmatrix} = \begin{bmatrix} A'Wy \\ z_0 \end{bmatrix} \qquad (2.2.4)$$

Provided the model (2.2.1) and (2.2.2) is of full rank, the estimates of x and λ may be obtained as

$$\begin{bmatrix} \hat{x} \\ \hat{\lambda} \end{bmatrix} = \begin{bmatrix} A'WA & K' \\ K & 0 \end{bmatrix}^{-1} \begin{bmatrix} A'Wy \\ z_0 \end{bmatrix} \qquad (2.2.5)$$

The inversion of the normal equations matrix may be performed through block matrix techniques, see *Aduol [1999]* and *Schaffrin [1984]*.

2.2.2 Stochastic Restrictions

In this case, the rank defect in the coefficient matrix A is overcome by introducing restrictions together with their stochasticity. The stochastic restrictions are set on in the form

$$z_0 = Kx + e_0 \quad , \quad z_0 \sim (0, \Sigma_z = \sigma_0^2 W_z^{-1}) \quad , \quad C(e, e_0) = 0 \tag{2.2.6}$$

where z_0 is a $c \times 1$ vector of restriction parameters (random),

K is a $c \times l$ restriction design matrix

e_0 is a $c \times m$ error vector of z_0 , $rk \quad K = c$,

Σ_Z is a $c \times c$ covariance matrix of z_0

C denote "covariance"

Taking (2.2.6) together with (2.1.3), *Gauss-Markov with stochastic restrictions* is expressed as

$$\begin{bmatrix} y \\ z_0 \end{bmatrix} = \begin{bmatrix} A_1 & A_2 \\ K_1 & K_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} e \\ e_0 \end{bmatrix} \tag{2.2.7}$$

which on taking

$$\bar{y} = \begin{bmatrix} y \\ z_0 \end{bmatrix}, \quad \bar{A} = \begin{bmatrix} A_1 & A_2 \\ K_1 & K_2 \end{bmatrix}, \quad x := \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \; and \; \bar{e} = \begin{bmatrix} e \\ e_0 \end{bmatrix} \tag{2.2.8}$$

Then (2.2.7) may be written as

$$\bar{y} := \bar{A}x + \bar{e} \; with \; \bar{e} \sim (0, \Sigma_{\bar{y}}) \tag{2.2.9}$$

$$\text{for } D(\bar{e}) = \Sigma_{\bar{y}} = \begin{bmatrix} \Sigma_y & 0 \\ 0 & \Sigma_z \end{bmatrix} \tag{2.2.10}$$

on the assumption that y and z are independent.

From (2.2.10) the combined weight matrix $\overline{W}$ is defined in the form

$$\overline{W} = \Sigma_{\bar{y}}^{-1} = \begin{bmatrix} \Sigma_y^{-1} & 0 \\ 0 & \Sigma_Z^{-1} \end{bmatrix} \tag{2.2.11}$$

provided the inverse exists.

Under the least-squares condition we have that

$$\hat{x} = (\bar{A}'\overline{W}\bar{A})^{-1} \bar{A}\overline{W}\bar{y} \tag{2.2.12}$$

with $\hat{D}(\hat{x}) = \hat{\Sigma} = \hat{\sigma}_0^2(\overline{A}'\overline{WA})^{-1}$ (2.2.13)

where $\hat{\sigma}_0^2$ is the variance of unit weight given in the form

$$\frac{\overline{e}'\overline{We}}{n+c-m} \quad , \quad \overline{e} = \overline{y} - A\hat{x} \tag{2.2.14}$$

for n being the number of observations, c the number of restrictions and m the number of unknowns.

We have

$$E(\hat{x}) = (\overline{A}'\overline{WA})^{-1}(\overline{A}'\overline{WA})x = x \tag{2.2.15}$$

thus demonstrating that $\hat{x}$ is an unbiased estimate of x.

Densification of a geodetic system may be performed under stochastic restrictions where some of the unknown parameters in the system are known a priori with their stochasticity. In such a case, the unknowns may be incorporated into the estimation model as stochastic prior information through the *Gauss-Markov model with stochastic restrictions.*

In the case where restrictions are introduced only to overcome datum defects and define the reference system, we have a *minimally constrained* model *[Mikhail, 1976]. [Koch, 1987].* Where restrictions are more than, but involve, those needed to overcome datum defects, we have an *over-constrained model [Aduol, 1999]*. Under the *over-constrained* models, it may happen that the simple Gauss-Markov model has full rank, in such a case it is called *over-constrained with full rank*. However, if the simple Gauss-Markov model has a rank defect, such that among the restrictions, some go towards overcoming the rank defects, it is referred to as *over-constrained with rank defect.*

2.3 The Free Network Adjustment Technique

Free network adjustment is the technique in which the geodetic system is defined over exact restrictions without however considering any particular parameter (unknown or observable) as fixed. Such a geodetic system is considered free, in that, its geometrical size and shape is determined while remaining essentially independent of the control points.

In free network adjustment, the restriction design matrix in (2.2.2) must be chosen in such a way that the restrictions overcome the rank defect. Such a matrix K will be one whose columns are made up of the normalized eigenvectors of those eigenvalues in the normal equation matrix

$$N = A'WA \tag{2.3.1}$$

which have values equal to zero, due to the rank defect in N. If we denote the special form of K as G', then due to its special form

$$NG = 0 \tag{2.3.2}$$

and $G'G = D \quad , \quad D$ diagonal (2.3.3)

since the columns of G are eigenvectors orthogonal to each other.

Specifically in the free network adjustment, the geodetic system datum is defined over all the approximate values computed to an arbitrary datum. This concept is referred to as "inner solution" and gives unique results. The "inner solution" is the minimum norm in least-squares condition of the singular equations:

$$e'We \Rightarrow \min \tag{2.3.4a}$$

$$x'x \Rightarrow \min \tag{2.3.4b}$$

yielding $tr(Q_{xx}) \Rightarrow \min$ (2.3.4c)

These are the generalized inner constraints with (2.3.4a) as the basic least-squares condition, (2.3.4b) as the scale control that minimizes deviation of the final geodetic values from the approximate values and (2.3.4c) as the inner accuracy that ensures that the accuracy of the estimated values is the best possible *[Schmitt, 1982]*.

For a two-dimensional geodetic system in which only angles have been observed, the corresponding restriction design matrix for n points is given in the form

$$G' = \begin{bmatrix} 1 & 0 & 1 & 0 & . & . & 1 & 0 \\ 0 & 1 & 0 & 1 & . & . & 0 & 1 \\ y_1 & -x_1 & y_2 & -x_2 & . & . & y_n & -x_n \\ x_1 & y_1 & x_2 & y_2 & . & . & x_n & y_n \end{bmatrix} \tag{2.3.5}$$

For conditions (2.3.4) to be fulfilled, it turns out that the restriction equation (2.2.2) with G as already specified above, in fact becomes

$$G'x = 0 \tag{2.3.6}$$

If we take approximate values of x_i and y_i as x_i' and y_i' such that

$$x_i = x'_i + \Delta x_i, \quad y_i = y'_i + \Delta y \tag{2.3.7}$$

then we note from (2.3.5) and (2.3.6) that, the rows in G' respectively establish the condition that

$$\text{(i)} \sum_{i=1}^{n} \Delta x_i = 0 \tag{2.3.8a}$$

$$\text{(ii)} \sum_{i=1}^{n} \Delta y_i = 0 \tag{2.3.8b}$$

$$\text{(iii)} \sum_{i=1}^{n} (x_i \Delta y_i - y_i \Delta x_i) = 0 \tag{2.3.8c}$$

$$\text{(iv)} \sum_{i=1}^{n} (x_i \Delta x_i + y_i \Delta y_i) = 0 \tag{2.3.8d}$$

Conditions (i) and (ii) go towards overcoming translation defects by ensuring that the centre of mass of the geodetic system is maintained at that defined by the approximate values. Condition (iii) is to overcome the rotation defect, by ensuring that the directions in the geodetic system are each changed by as minimum a value as possible, while condition (iv) overcome the scale defect by changing the scale as defined by the approximate values as little as possible.

The restriction design matrix is applied to the normal equations in the form

$$\bar{N} = A'W_y A + GG' \tag{2.3.9}$$

from which obtain

$$F_{11} = \bar{N}^{-1} - \bar{N}^{-1}GG'\bar{N}^{-1} \tag{2.3.10a}$$

$$F_{12} = \bar{N}^{-1}G \tag{2.3.10b}$$

$$F_{21} = G'\bar{N}^{-1} \tag{2.3.10c}$$

$$F_{22} = 0 \tag{2.3.10d}$$

which gives the estimates of the unknowns as

$$\begin{aligned} \hat{x} &= F_{11}A'W_y y + F_{12}Z_0 \\ &= F_{11}A'W_y y \quad for \quad Z_0 = 0 \quad , \end{aligned} \tag{2.3.11}$$

and

$$\hat{D}(\hat{x}) = \hat{\sigma}_0^2 \bar{N}^{-1} A'W_y A\bar{N}^{-1} = \hat{\sigma}_0^2 F_{11} \tag{2.3.12}$$

The result of such a free network adjustment is the consistency and thus the internal precision of the geodetic system that may be checked free of external influences associated with attaching a geodetic system to an absolute control system. This makes the free network adjustment best suited for adjustment of control geodetic systems, as it results in fairly representative estimates of the systems parameters with uniformly distributed accuracies.

Various forms of the restriction matrix, depending on different observation combinations, are listed by *Illner [1985]*. Since the normal equation in (2.3.1) is singular, the free network problem can be viewed as principally overcoming this rank defect. Several approaches to the solution of N are considered in detail by, among others, *Mittermayer [1972], Pope [1973], Grafarend and Schaffrin [1974], Brunner [1979], and Meissl [1982].*

3

GEODETIC DENSIFICATION TECHNIQUES

3.1 Introduction

From the statistical point of view, the classical least-squares adjustment is a procedure to estimate free unknowns and residuals. It is also possible, and usual, to estimate additional parameters, for example the variance of unit weight of the observations to obtain a statement of its accuracy. The problem arises where heterogeneous geodetic observations are to be used together in an adjustment model, for example, in densification, where there are distances and directions observed. Additional problem to this is, to estimate the weight relation between those various observations so as to obtain the accuracy of the various groups of observations in their relation or even correlation. The attempt to solve this task led to the proposal of several densification approaches.

In this chapter, the various approaches to densification are discussed. They are grouped into either reproducing or non-reproducing depending on whether the datum points are affected or not, when the heterogeneous data sets are fused.

3.2 Non-Reproducing Densification Techniques

Densification approaches that do not preserve the control point parameters together with their variance-covariance matrices (respectively do not reproduce every "reference information" that belongs to a "higher category" together with their stochasticity) are termed non-reproducing densification techniques. The two most commonly used approaches in this category, namely the *Static* and the *Dynamic* approaches, are presented in this section based on the works of *Aduol [1993,1999]*.

3.2.1 The Static Densification Approach

The basic concept of this approach to densification is based on the Gauss-Markov model. The Gauss-Markov model with stochastic restrictions is given in (2.1.3) and (2.2.6) as

$$y = A_1x_1 + A_2x_2 + e \tag{3.2.1a}$$

$$z_0 = Kx + e_0 \tag{3.2.1b}$$

In the static densification approach, the datum parameters contained in x_1 are treated as exact prior information thus the representation in (3.2.1b) then becomes

$$z_0 = Kx \quad e_0 = 0, \quad e_0 \sim (0,0) \tag{3.2.2}$$

The full parameter estimation model then becomes

$$y = A_1x_1 - A_2x_2 + e \quad e \sim (0, \sigma_{0y}{}^2 W_y{}^{-1}) = (0, \Sigma_{yy}) \tag{3.2.3a}$$

$$z_0 = K_1x_1 - K_2x_2 \tag{3.2.3b}$$

Under the least-squares condition, the normal equations for the set up take the form

$$\begin{bmatrix} A_1'W_yA_1 & A_1'W_yA_2 & K_1' \\ A_2'W_yA_1 & A_2'W_yA_2 & K_2' \\ K_1 & K_2 & 0 \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{\lambda} \end{bmatrix} = \begin{bmatrix} A_1'W_yy \\ A_2'W_yy \\ Z_0 \end{bmatrix} \tag{3.2.4}$$

where λ is the vector of Lagrange multipliers. From this we obtain the estimate $\hat{x}$ of x is as

$$\hat{x} = (N_r{}^{-1} - N_r{}^{-1}K'R_r{}^{-1}KN_r{}^{-1})A'W + N_r{}^{-1}K'R_r{}^{-1}z_0 \tag{3.2.5a}$$

for $N_r := A'W_yA + K'K$ and $R_r := KN_r{}^{-1}K'$ (3.2.5b)

with $A := \begin{bmatrix} A_1 & A_2 \end{bmatrix}$ and $K := \begin{bmatrix} K_1 & K_2 \end{bmatrix}$ (3.2.5c)

Also the estimate dispersion matrix $D(\hat{x})$ is given as

$$D(\hat{x}) = \hat{\Sigma}_{\hat{x}} = \hat{\sigma}_0{}^2(N_r{}^{-1} - N_r{}^{-1}K'R_r{}^{-1}KN_r{}^{-1})N(N_r{}^{-1} - N_r{}^{-1}K'R_r{}^{-1}KN_r{}^{-1}) \tag{3.2.6a}$$

$$= \hat{\sigma}_0{}^2(N_r{}^{-1} - N_r{}^{-1}K'R_r{}^{-1}KN_r{}^{-1}) \tag{3.2.6b}$$

with $\hat{\sigma}_0{}^2 = \dfrac{\hat{e}'W_y\hat{e}}{n+c-m}$, $\hat{e} = y - A\hat{x}$ (3.2.7)

In the special case that K_1 is non-singular and $K_2 = 0$, we should have from (3.2.3b) that

$$x_1 = K_1^{-1}z_0 \tag{3.2.8}$$

and with (3.2.3a) we obtain that

$$y - A_1K_1^{-1}z_0 = A_2x_2 + e \tag{3.2.9}$$

If in this we set

$$\zeta = y - A_1 K_1^{-1} z_0 \tag{3.2.10}$$

then (3.2.9) becomes a simple Gauss-Markov model in the form

$$\zeta = A_2 x_2 + e \quad e \sim (0, \sigma_0^2 W_y^{-1}) = (0, \Sigma_{yy}) = (0, \Sigma_{\zeta\zeta}) \tag{3.2.11}$$

In which we now have only the unknown sub-vector x_2 appearing and $\Sigma_{\zeta\zeta} = D(\zeta)$.

In practice we normally have the exact prior information of (3.2.3b) comprising only the control point parameters so that the vector x_1 contains only the control point parameters. If we let the exact prior information values of x_1 be ζ_1, then (3.2.3b) becomes simply

$$\zeta_1 = x_1 \tag{3.2.12}$$

being equivalent to taking $z_0 = \zeta_1$ and $K = I_1$; where I_1 is the $c \times c$ identity matrix. With this we now have that

$$\zeta = y - A_1 \zeta_1 \tag{3.2.13}$$

From (3.2.11) the estimate $\hat{x}_2$ of x_2 is obtained in the form

$$\hat{x}_2 = (A_2' W_y A_2)^{-1} A_2' W_y \zeta \tag{3.2.14a}$$

with $\hat{D}(\hat{x}) = \hat{\sigma}_0^2 (A_2' W_y A_2)^{-1}$ (3.2.14b)

for $\hat{\sigma}_0^2 = \dfrac{\hat{e}' W_y \hat{e}}{n - m_2}$, $m_2 = m - c$ (3.2.14c)

It is usual in geodetic computations to start off a parameter estimation process from approximate values of the parameters, normally for linearization purposes. Moreover even in linear models, approximate values are still normally adopted for ease of numerical reasons for example as in levelling. In such cases we could have that

$$x_1 = x_{01} + \Delta x_1 \text{ and } x_2 = x_{02} + \Delta x_2 \tag{3.2.15}$$

In which x_{01} and x_{02} are the approximate values to x_1 and x_2 respectively. In such situation (3.2.1) takes the form

$$\bar{y} = y - A_1 x_{01} - A_2 x_{02} = A_1 \Delta x_1 + A_2 \Delta x_2 + e \quad e \sim (0, \sigma_0^2 W_y^{-1}) \tag{3.2.16a}$$

and $z_0 = K\Delta x + e_0 = Z_0 - K_1 x_{01} - K_2 x_{02}$, $e_0 \sim (0, \Sigma_{ZZ})$ (3.2.16b)

Since the unknown parameters to be estimated are now Δx_1 and Δx_2 from which the estimates of x_1 and x_2 may then be obtained through (3.2.15); and correspondingly (3.2.3b) and (3.2.8) would become

$$\hat{z}_0 = K_1\Delta x_1 + K_2\Delta x_2 \quad and \quad \Delta x_1 = K_1^{-1}\hat{z}_0 \tag{3.2.17}$$

If in (3.2.15) we take $x_1 = x_{01}$, then in (3.2.17) we have that

$$\Delta x_1 = K_1^{-1}\hat{z}_0 = 0$$

(3.2.18)

And following into this we have that (3.2.16) becomes

$$\tilde{y} = A_2\Delta x_2 + e \text{ , } e \sim (0, \sigma_{0y}^2 W_y^{-1}) \tag{3.2.19}$$

wnich is similar to (3.2.11).

From this we obtain the usual estimates

$$\Delta\tilde{x}_2 = (A_2' W_y A_2)^{-1} A_2' W_y \tilde{y} \tag{3.2.20a}$$

$$\hat{D}(\Delta x_2) = \hat{\sigma}_0^2 (A_2' W_y A_2)^{-1} \tag{3.2.20b}$$

and $\hat{\sigma}_0^2 = \dfrac{\hat{e}' W_y \hat{e}}{n - m_2}$, $\hat{e} = \hat{y} - A_2\Delta\hat{x}_2$, $m_2 = m - c$ (3.2.20c)

This is the form of parameter estimation often adopted in the static densification of geodetic systems.

Even though this approach reproduces the control point parameters exactly, in the context of this study, it is termed a non-reproducing technique since the approach ignores the stochasticity of the parameters and thus does not reproduce the prior information variance-covariance matrix. This implicit omission is not justified as the control point parameters themselves could probably have been obtained earlier through an adjustment process and have a variance-covariance matrix associated with them. The effect of neglecting the variance-covariance matrix is considered in depth by *Wolf [1983]*.

3.2.2 The Dynamic Densification Approach

The dynamic densification approach is based on the use of stochastic restrictions within the framework of the model suggested by *J. Durbin [1953]* as represented in (2.2.7), that is

$$y = A_1x_1 + A_2x_2 + e, \quad e \sim (0, \sigma_{0y}^2 W_y^{-1}) = (0, \Sigma_{yy}) \tag{3.2.21a}$$

$$z_0 = K_1x_1 + K_2x_2 + e_0, \qquad e_0 \sim (0, \sigma_{0z}^2 W_Z^{-1}) = (0, \Sigma_{ZZ}) \quad , \quad c(e_1\, e_2) = 0 \tag{3.2.21b}$$

from which it may be written as

$$\begin{bmatrix} y \\ z_0 \end{bmatrix} = \begin{bmatrix} A_1 & A_2 \\ K_1 & K_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} e \\ e_0 \end{bmatrix} \tag{3.2.22}$$

$$y_\varsigma = \begin{bmatrix} y \\ z_0 \end{bmatrix}; \quad A_\varsigma = \begin{bmatrix} A_1 & A_2 \\ K_1 & 0 \end{bmatrix}; \quad x_\varsigma = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}; \quad e_\varsigma = \begin{bmatrix} e \\ e_0 \end{bmatrix} \tag{3.2.23}$$

We have that

$$y_\varsigma = A_\varsigma x_\varsigma + e_\varsigma \quad e_\varsigma \sim (0, \sigma_{oy}^2 W_y^{-1}) \tag{3.2.24}$$

In which, on the assumption that y and Z_0 are independent, we shall now be having

$$E(e_\varsigma) = 0, \quad D(e_\varsigma) = \begin{bmatrix} \sigma_{oy}^2 W_y^{-1} & 0 \\ 0 & \sigma_{oZ}^2 W_Z^{-1} \end{bmatrix} = \Sigma_{\varsigma\varsigma} \tag{3.2.25}$$

The corresponding weight matrix W_ς is given as

$$W_\varsigma = \begin{bmatrix} \sigma_{oy}^2 W_y & 0 \\ 0 & \sigma_{oZ}^2 W_Z \end{bmatrix} = \Sigma_{\varsigma\varsigma}^{-1} \tag{3.2.26}$$

This is the mixed estimation of *Theil and Goldberger [1961]*. From this with the definition $A = [A_1 \quad A_2]$, $K = [K_1 \quad 0]$, we have that

$$\hat{x} = (A_\varsigma' W_\varsigma A_\varsigma)^{-1} A_\varsigma' W_\varsigma y_\varsigma \tag{3.2.27a}$$

$$= (A'\Sigma_{yy}^{-1} A + K'\Sigma_{ZZ}^{-1} K)^{-1} (A'\Sigma_{yy}^{-1} y + K'\Sigma_{zz}^{-1} z_0) \tag{3.2.27b}$$

with

$$D(\hat{x}) = (A'\Sigma_{yy}^{-1} A + K'\Sigma_{zz}^{-1} K)^{-1} = \Sigma_{\hat{x}\hat{x}} \tag{3.2.28}$$

and

$$\hat{\sigma}_{0y}^2 = \frac{\hat{e}' W_y \hat{e}}{trace(W_y Q_e Q_e)} \tag{3.2.29a}$$

$$\hat{\sigma}_{0z}^2 = \frac{\hat{e}_0' W_Z \hat{e}_0}{trace(W_Z Q_{e_0} Q_{e_0})} \tag{3.2.29b}$$

In which

$$Q_{e_0} Q_{e_0} = Q_{zz} - K Q_{\hat{x}\hat{x}} K' - W_z^{-1} - K(A' W_{\varsigma\varsigma} A)^{-1} K' = \sigma_{0z}^{-2} D(\hat{e}_0) = \sigma_{0z}^{-2} (\Sigma_{zz} - K \Sigma_{\hat{x}\hat{x}} K') \tag{3.2.30a}$$

$$Q_e Q_e = Q_{yy} - A Q_{\hat{x}\hat{x}} A' - W_y^{-1} - A(A' W_{\varsigma\varsigma} A)^{-1} A' = \sigma_{0y}^{-2} D(\hat{e}) = \sigma_{0y}^{-2} (\Sigma_{\hat{y}\hat{y}} - A \Sigma_{\hat{x}\hat{x}} A') \tag{3.2.30b}$$

$$e = Q_e Q_e W_y y \qquad \text{[cf. Mikhail 1976]} \tag{3.2.31a}$$

$$e_0 = Q_{e_0} Q_{e_0} W_Z z_0 \tag{3.2.31b}$$

Researchers have proved that the dynamic approach to densification is statistically a better approach as compared to the static approach since the stochasticity of the control point parameters are considered in the densification of the geodetic systems thus providing a more realistic estimation. However, this approach results in change of the control point parameters "dynamically" in principle with every single measurement added to the geodetic system. As a result the concept of a national reference system loses meaning.

3.3 Reproducing Densification Techniques

Contrary to non-reproducing densification techniques, densification approaches that preserve the control point parameters together with their variance covariance matrices (respectively reproduce the parametric and stochastic fiducial constraints) are termed reproducing densification techniques. In this section, two such approaches are discussed namely; *Static-Dynamic* and *Sub-Optimal fusion.*

3.3.1 Static-Dynamic Approach

From (2.2.6) we have the parametric and stochastic fiducial constraints given as

$$z_0 = Kx_1 + e_0 \, , \quad e_0 \sim (0, \sigma_{0z}^2 W_z^{-1}) \, , \; C(e, e_0) = 0 \tag{3.3.1}$$

From equation (3.3.1) we have that if K such that K^{-1} exists then:

$$x_1 = K^{-1}(z_0 - e_0) \tag{3.3.2}$$

replacing (3.3.2) into 3.3.1) we obtain that

$$y = A_1 K^{-1}(z_0 - e_0) + A_2 x_2 + e \quad e \sim (0, \sigma_{0y}^2 W_y^{-1}) \tag{3.3.3}$$

which we may rewrite as

$$y - A_1 K^{-1} z_0 = A_2 x_2 + e - A_1 K^{-1} e_0 \tag{3.3.4}$$

On taking

$$\bar{y} = y - A_1 K^{-1} z_0 \qquad and \qquad \bar{e} = e - A_1 K^{-1} e_0 \tag{3.3.5}$$

(3.3.4) then becomes

$$\bar{y} = A_2 x_2 + \bar{e} \, , \; \bar{e} \sim (0, \sigma_0^2 \Sigma_{\bar{y}\bar{y}}) \tag{3.3.6}$$

and

$$D(\bar{e}) = \sigma_{0y}^2 W_y^{-1} + A_1 K^{-1} \Sigma_{zz} (A_1 K^{-1})' = \Sigma_{\bar{y}\bar{y}} \tag{3.3.7}$$

On using least-squares condition we obtain $\hat{x}_2$ estimate of x_2 as

$$\hat{x}_2 = (A_2' \Sigma_{\bar{y}\bar{y}} A_2)^{-1} A_2 \Sigma_{\bar{y}\bar{y}}^{-1} \bar{y} \tag{3.3.8}$$

and

$$D(\hat{x}_2) = (A_2 \Sigma_{\bar{y}\bar{y}} A_2)^{-1} \tag{3.3.9}$$

$$D(\hat{x}_2) = (A_2' \Sigma_{\bar{y}\bar{y}} A_2)^{-1} \tag{3.3.9}$$

Now, in densification where only the coordinates of the datum points have been collected in x_1, we notice that we shall have $K_1 = I$, i.e an identity matrix of the same dimension as K_1.

Then, from (3.3.8) and (3.3.9) we would now have that

$$\hat{x}_2 = [A_2'(\sigma_{0y}^2 W_y^{-1} + A_1 \Sigma_{ZZ} A_1')^{-1} A_2]^{-1} A_2'(\sigma_{0y}^2 W_y^{-1} + A_1 \Sigma_{ZZ} A_1')^{-1}(y - A_1 z_0) \tag{3.3.10}$$

and

$$D(\hat{x}_2) = [A_2'(\sigma_{0y}^2 W_y^{-1} + A_1 \Sigma_{ZZ} A_1')^{-1} A_2]^{-1} \tag{3.3.11}$$

We note that with this model, we are able to estimate densification parameters x_2 by incorporating control point parameters x_1 as stochastic prior information while at the same time keeping x_1 numerically unchanged. This combines the properties of both the static and dynamic models. *Theil [1963,1971]* refers to this model as *estimation with incomplete prior information while Toutenburg [1975], Bibby and Toutenburg [1977]* and *Toutenburg [1977]* refer to it *as stepwise regression.*

3.3.2 Sub-Optimal Fusion Approach

Another approach to densification is the *sub-optimal network fusion model* discussed by *Schaffrin [1998]* in which the best linear uniformly unbiased estimate $\bar{x}$ with the reproducing property is determined. We set the restriction

$$K\bar{x} = z_0 \qquad D(K\bar{x}) = \sigma_{0Z}^2 W_Z^{-1} \tag{3.3.12}$$

This estimator has the linear property that

$$\bar{x} = L_1 y + L_2 z_0 \tag{3.3.13}$$

With L_1 and L_2 as the unknowns to be determined. The estimator is also uniformly unbiased in that

$$x = E(\bar{x}) = L_1 Ax + L_2 Kx \quad \textit{for all } x \in \Re^m \tag{3.3.14a}$$

which implies

$$L_1 A + L_2 K - I_m = 0 \tag{3.3.14b}$$

The model also has the reproducing property in that

$$0 = K\bar{x} - z_0 = (KL_1)\cdot y + (KL_2 - I_l)\cdot z_0 \ \textit{for arbitrary } y \in \Re^n,\ z_0 \in \Re^l \tag{3.3.15}$$

$K \; L_1 \quad L_2 \; = \; 0 \quad I_l$ and thus with (3.3.14b) $A'L_1'K' = 0$ (3.3.16)

The estimate $\bar{x}$ has the best or minimum variance (Mean Square Error) given by

$$D(\bar{x}) = \sigma_{0_y}^2 (L_1 W_y^{-1} L_1') + \sigma_{0_z}^2 (L_2 W_Z^{-1} L_2')$$

$$= \sigma_{0_y}^2 l_1'(I_m \otimes W_y^{-1}) l_1 + \sigma_{0_z}^2 l_2'(I_m \otimes W_Z^{-1}) l_2 \tag{3.3.17}$$

for $l_1 := vec\ L_1'$, $l_2 := vec\ L_2'$; $\otimes$ denotes the Kronecker-Zehfuss Product

To obtain the unknowns L_1 and L_2 in the case $\sigma_{0_y}^2 = \sigma_{0_z}^2$ we set the Lagrange target functions in the form:

$$\phi[l_1 = vec\ L_1', l_2 = vec\ L_2', \lambda_1, \lambda_2] := l_1'(I_m \otimes W_y^{-1}) l_1 + l_2'(I_m \otimes W_Z^{-1}) l_2 + 2\lambda_1'(K \otimes A') l_1 +$$

$$+ 2\lambda_2'[(I_m \otimes A') l_1 + (I_m \otimes K') l_2 - vec\ I_m] = stationary\ l_1, l_2, \lambda_1, \lambda_2 \tag{3.3.18}$$

The Euler-Lagrange necessary conditions then become

$$\tfrac{1}{2} \cdot \tfrac{\partial \phi}{\partial l_1} = (I_m \otimes W_y^{-1}) l_1 + (K' \otimes A)\hat{\lambda}_1 + (I_m \otimes A)\hat{\lambda}_2 = 0 \tag{3.3.19a}$$

$$\tfrac{1}{2} \cdot \tfrac{\partial \phi}{\partial l_2} = (I_m \otimes W_Z^{-1}) l_2 + (I_m \otimes K)\hat{\lambda}_2 = 0 \tag{3.3.19b}$$

$$\tfrac{1}{2} \cdot \tfrac{\partial \phi}{\partial \lambda_1} = (K \otimes A') l_1 \doteq 0 \tag{3.3.19c}$$

$$\tfrac{1}{2} \cdot \tfrac{\partial \phi}{\partial \lambda_2} = (I_m \otimes A') l_1 + (I_m \otimes K') l_2 - Vec\ I_m \doteq 0 \tag{3.3.19d}$$

Sufficient condition for the solution of the Lagrange target functions is.

$$\frac{1}{2} \cdot \frac{\partial^2 \phi}{\partial \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} \partial [l_1' \;\; l_2']} = \begin{bmatrix} I_m \otimes W_v^{-1} & 0 \\ 0 & I_m \otimes W_Z^{-1} \end{bmatrix} \text{ is positive definite}$$

Using the Euler-Lagrange necessary conditions, from equation (3.3.19a) we have that

$$l_1 = -(K' \otimes W_y A)\hat{\lambda}_1 - (I_m \otimes W_y A)\hat{\lambda}_2 \tag{3.3.20}$$

Substituting (3.3.20) into (3.3.19c) we have that

$$0 = (K \otimes A') l_1 = -(KK' \otimes N)\hat{\lambda}_1 - (K \otimes N)\hat{\lambda}_2 \tag{3.3.21}$$

From (3.3.19d) we obtain

$$vec\ I_m = (I_m \otimes K') l_2 - (K' \otimes N)\hat{\lambda}_1 - (I_m \otimes N)\hat{\lambda}_2 \tag{3.3.22}$$

which implies

$$-\hat{\lambda}_2 = vec\ N^{-1} - (I_m \otimes N^{-1}K') l_2 + (K' \otimes I_m)\hat{\lambda}_1 \tag{3.3.23}$$

Substituting (3.3.19c) into (3.3.19d) we obtain

$$(K\otimes K')l_2 = vec\,K' \tag{3.3.24}$$

From (3.3.20) and (3.3.23) we also have that

$$l_1 = vec\,(W_y AN^{-1}) - [I_m \otimes (W_y AN^{-1}K')]l_2 \tag{3.3.25}$$

From (3.3.19b) and (3.3.23) we obtain that

$$l_2 = -(I_m \otimes W_z K)\hat{\lambda}_2 \tag{3.3.26a}$$

$$= vec(W_z KN^{-1}) - [I_m \otimes (W_z KN^{-1}K')]l_2 + (K'\otimes W_z K)\hat{\lambda}_1 \tag{3.3.26b}$$

which implies that

$$[I_m \otimes W_z (W_z^{-1} + KN^{-1}K')]l_2 = vec(W_z KN^{-1}) + (K'\otimes W_z K)\hat{\lambda}_1 \tag{3.3.26c}$$

which further reduces to

$$l_2 = vec[(W_z^{-1} - KN^{-1}K')^{-1}KN^{-1}] + [K'\otimes (W_z^{-1} + KN^{-1}K')^{-1}K]\hat{\lambda}_1 \tag{3.3.27}$$

Substituting (3.3.24) into (3.3.27) we obtain that

$$vec\,K' - vec[K'(W_z^{-1} + KN^{-1}K')^{-1}KN^{-1}K'] = vec[K'(W_z^{-1} + KN^{-1}K')^{-1}W_z^{-1}] \tag{3.3.28a}$$

$$= [KK'\otimes K'(W_z^{-1} + KN^{-1}K')^{-1}K]\hat{\lambda}_1 \tag{3.3.28b}$$

which implies

$$[I_l \otimes KK'(W_z^{-1} + KN^{-1}K')^{-1}]\cdot[vec(KK'W_z)^{-1} - (I_l \otimes K)\hat{\lambda}_1] \tag{3.3.28c}$$

and further reduces to

$$(I_l \otimes K)\hat{\lambda}_1 = vec\,(KK'W_z)^{-1} = vec\,[KK'(KK')^{-1}W_z^{-1}(KK')^{-1}]$$

$$= (I_l \otimes K)\cdot vec\,[K'(KK'W_z KK')^{-1}] \tag{3.3.29}$$

substituting (3.3.29) into (3.3.27) we obtain

$$l_2 = vec\,[(W_z^{-1} - KN^{-1}K')^{-1}KN^{-1}] + [K'\otimes (W_z^{-1} + KN^{-1}K')^{-1}]\cdot vec\,(KK'W_z)^{-1}$$

$$= vec\,[(W_z^{-1} - KN^{-1}K')^{-1}(KN^{-1} + W_z^{-1}(KK')^{-1}K)] = vec\,L_2' \tag{3.3.30}$$

which implies

$$L_2 = [N^{-1}K' - K'(KK')^{-1}W_z^{-1}](W_z^{-1} + KN^{-1}K')^{-1}$$

$$= [N^{-1}K'W_z - K'(KK')^{-1}](I_l + KN^{-1}K'W_z)^{-1}K'(KK')^{-1}$$

$$= [I_m + N^{-1}K'W_z K - I_m + K'(KK')^{-1}K](I_m + N^{-1}K'W_z K)^{-1}K'(KK')^{-1} = \ldots\ldots =$$

$$= K'(KK')^{-1} - [I_m - K'(KK')^{-1}K](N + K'W_z K)^{-1}K'W_z \tag{3.3.31}$$

Similarly substituting (3.3.30) into (3.3.25) we obtain

$$l_1 = vec\,(W_y AN^{-1}) - [I_m \otimes (W_y AN^{-1}K')]\cdot l_2$$

$$= vec\,(W_y AN^{-1}) - vec\,\{W_y AN^{-1}K'(I_l + W_z KN^{-1}K')^{-1}[W_z KN^{-1} + (KK')^{-1}K]\}$$

$$= vec\,\{W_y A(N+K'W_Z K)^{-1}[I_m - K'(KK')^{-1}K]\} = vec\,L_1' \quad (3.3.32)$$

which implies that

$$L_1 = [I_m - L_2 K)N^{-1}A'W_y = [I_m - K'(KK')^{-1}K](N+K'W_Z K)^{-1}A'W_y$$

$$= [I_m - K'(KK')^{-1}K][N^{-1} - N^{-1}K'(W_Z^{-1} + KN^{-1}K')^{-1}KN^{-1}]A'W_y \quad (3.3.33)$$

Substituting (3.3.31) and (3.3.33) into (3.3.13) we obtain the results as

$$\bar{x} = L_1 y + L_2 z_0 = N^{-1}A'W_y y + L_2(z_0 - KN^{-1}A'W_y y)$$

$$= \hat{x}_u + [N^{-1}K' + K'(KK')^{-1}W_Z^{-1}](W_Z^{-1} + KN^{-1}K')^{-1}(z_0 - K\hat{x}_u) = \ldots =$$

$$= \hat{x} + K'(KK')^{-1}(I_l + KN^{-1}K'W_Z)^{-1}\hat{e}_0 = \hat{x} + K'(KK)^{-1}(z_0 - K\hat{x}) \quad (3.3.34)$$

and

$$D(\bar{x}) = \sigma_0^2(L_1 W_y^{-1} L_1') + \sigma_0^2(L_2 W_Z^{-1} L_2') \quad (3.3.35)$$

$$= \sigma_0^2(N^{-1} - L_2 KN^{-1} - N^{-1}K'L_2' + L_2(KN^{-1}K' + W_Z^{-1})L_2'] = \ldots =$$

$$= \sigma_0^2[N^{-1} - N^{-1}K'(W_Z^{-1} + KN^{-1}K')^{-1}KN^{-1} + K'(KK')^{-1}W_Z^{-1}(W_Z^{-1} + KN^{-1}K')^{-1}W_Z^{-1}(KK')^{-1}K]$$

$$= D\{\hat{x}\} + D\{K'(KK')^{-1}(z_0 - K\hat{x})\}$$

Further

$$E(\bar{x}) = x, \qquad z_0 - K\bar{x} = \bar{e}_0 = 0, \qquad D(K\bar{x}) = D(z_0) = \sigma_0^2 W_Z^{-1} \quad (3.3.36)$$

With this approach, we note that control points keep their values while for the new points, the optimal estimates will be obtained. This generalizes (and improves) Helmert's approach

4

EXPERIMENTAL DESIGN

4.1 General Experimental Design

Experimental design refers to the framework or structure of an experiment. *Fisher [1960]* enumerated three principles that should be followed in experimental designs; the *principle of replication* which requires that the experiment should be repeated more than once. the *principle of randomization* which requires protection against extraneous factors and the *principle of local control* in which the range of the variable is made as wide as possible but the causes of the variability should be measured and eliminated from the experimental errors.

There are four types of formal experimental designs; *Completely Randomized* design, *Randomized Block* design, *Latin Square* design and *Factorial* design *[Kothari, 1990]*. Of these, Completely Randomized design (C. R. Design) is the simplest possible design and its procedure of analysis is also easier, e.g. by analysis of variance. The essential characteristic of this design is that subjects are randomly assigned to experimental treatments (or vice versa). The design involves only two principles; viz., the principle of replication and the principle of randomization. *Kothari [1990]* described a simple randomized design using a flow chart given below:

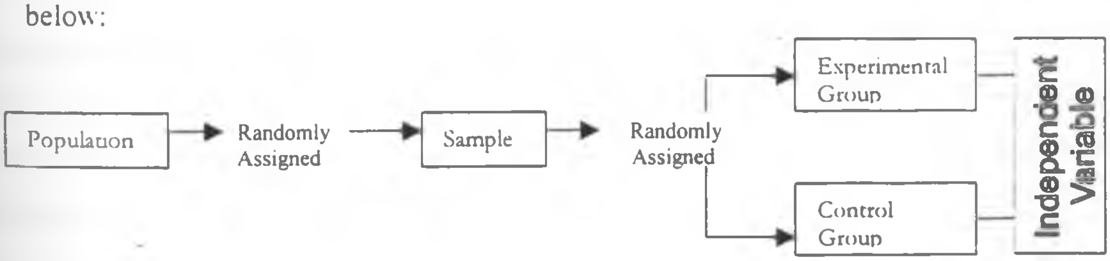


Figure 4.1: Two group Simple Randomized design (After C. Kothari)

In this report, the real network was randomly selected from the geodetic network of Kenya while the simulated network was randomly designed so as to obtain a symmetric triangle. The experimental design can be said to be of the simple randomized experimental design

4.2 The Test Networks

To test and evaluate the densification approaches discussed in chapter three, two types of geodetic networks were adopted. For the real network, data were obtained from Survey of Kenya records consisting of initial observations for the first, second and third orders. The real first order network consisted of eleven points (as shown in Fig. 4.5). The first order network was then densified to fifteen second order points (as shown in Fig. 4.6) and the second order network was further densified to another fifteen third order points (as shown in Fig. 4.7). The data consisted of original field note observations for distances and angles and the adjusted coordinates of the network based on U.T.M projection referred to Clarke's 1880 ellipsoid on the African Arc Datum 1960. The adjusted coordinates were adopted as approximate coordinates in the present study.

A simulated two-dimensional network comprising of 15 points was also adopted for the study. The simulated first order network consisted of three points (as shown in Fig. 4.2). The first order network was then densified to three second order points (as shown in Fig. 4.3) and the second order network was further densified to fifteen third order points (as shown in Fig. 4.4). The observables were horizontal directions and distances. Actual observations were simulated and coordinates of the computed points obtained in order to demonstrate as well as analyze the different densification approaches. For the fundamental network and the first level densification, the horizontal directions were assigned a standard error of ±0.5" while for the second and third order densification levels, the standard errors were taken to be ±1". Distances were assigned a uniform standard error of ±0.03 metres throughout

4.2.1 The Simulated Network

4.2.1.1 First Order Network

The simulated first order network consisted of three stations as shown in Fig. (4.2) with corresponding approximate coordinates and observation data sets given in Tables (4.1) and Table (4.2) respectively. This was considered as the fundamental simulated network upon which the first level and second level simulated network densifications were performed. The fundamental network was defined by adjusting the first order network within the framework of a free network model. The standard error of the network was taken to be ±0.5" based on the report by *Aduol [1981, 1993]*.

Table 4.1: Approximate coordinates for the simulated first order network

POINT	COORDINATES	
	N (m)	E (m)
1	100.000	100.000
2	446.410	300.000
3	100.000	500.000

Table 4.2: Observational data sets for the simulated first order network

Observation Number	Line		Bearing °	'	"	Distance (m)
1	1	2	30	0	0.00	400.006
2	1	3	90	0	10.61	399.991
3	2	3	150	0	0.00	399.993
4	2	1	209	59	53.01	-
5	3	1	270	0	0.00	-
6	3	2	329	59	54.99	-

4.2.1.2 Second Order Network

The simulated second order network consisted of three stations connected onto the simulated first order network described in section 4.2.1.1 (cf. Fig. 4.3). The corresponding observational data sets are as given in Table 4.4 while the approximate coordinates are listed in Table (4.3). In Table (4.3), the first three approximate coordinates were determined through free network adjustment of the simulated first order network. The standard error of the observations was taken to be ± 1".

Table 4.3: Approximate coordinates for the simulated second order network

POINT	COORDINATES	
	N (m)	E (m)
1	99.9956	99.9971
2	446.4087	300.0064
3	100.0057	499.9965
4	273.0500	200.0700
5	273.0500	400.0900
6	100.0600	300.0500

Figure 4.2: The simulated first order network

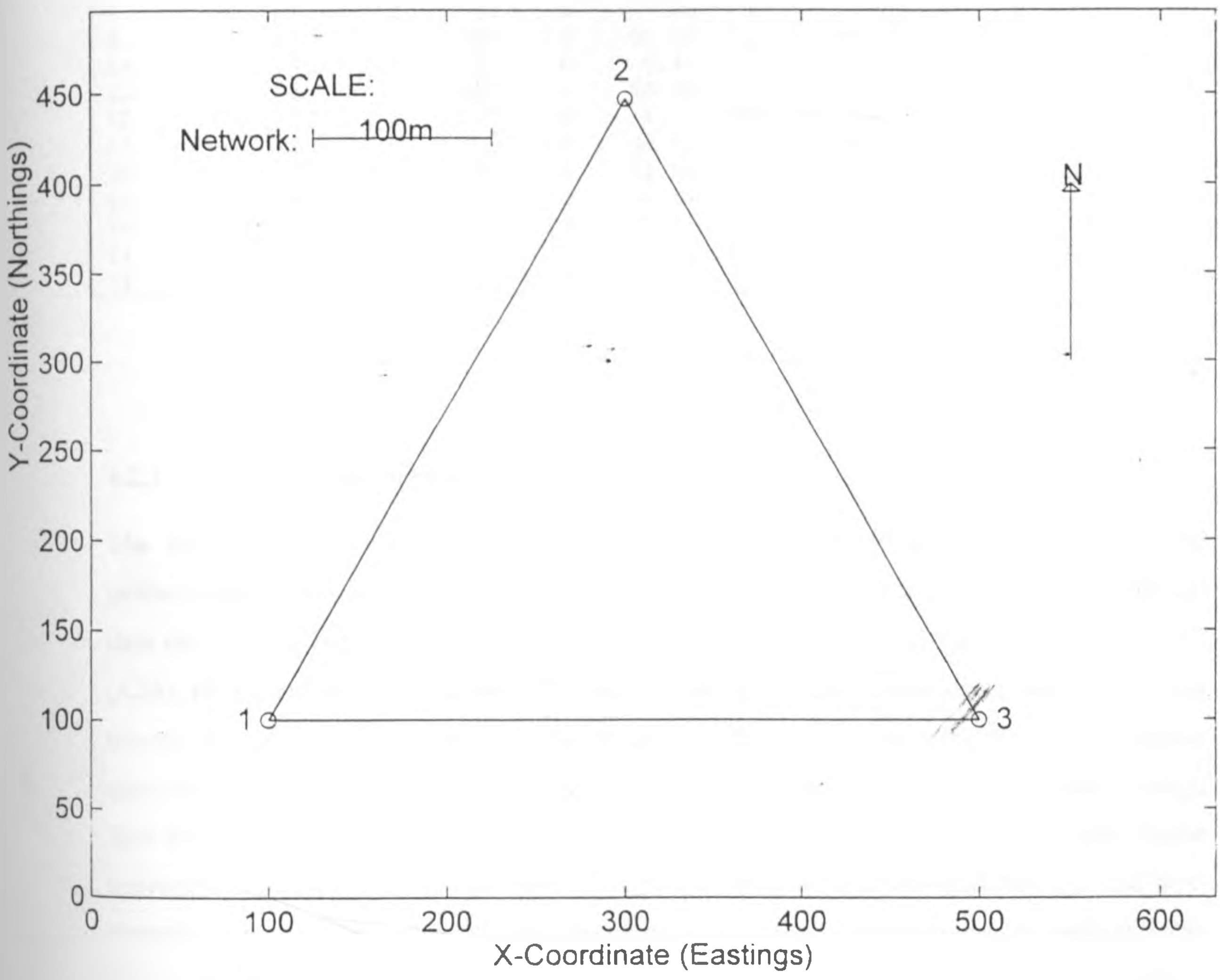


Table 4.4: Observational data sets for the simulated second order network

Observation Number	Line		Bearing °	'	"	Distance (m)
1	1	4	30	0	0.00	200.060
2	1	6	90	0	10.61	200.040
3	2	4	209	59	53.01	200.080
4	2	5	150	0	4.99	199.980
5	3	5	329	59	54.99	200.030
6	3	6	270	0	3.01	199.930
7	4	1	210	0	0.00	-
8	4	2	30	0	7.00	-
9	4	5	89	59	52.62	200.020
10	4	6	150	0	5.42	200.090
11	5	2	329	59	50.99	-
12	5	4	270	0	4.37	-
13	5	6	209	59	58.02	199.920
14	5	3	150	0	13.96	-
15	6	1	270	0	0.00	-
16	6	4	329	59	57.71	-
17	6	5	30	0	6.43	-
18	6	3	89	59	54.98	-

4.2.1.3 Third Order Network

The simulated third order network consisted of nine stations connected onto the simulated second order network described in section 4.2.1.2 (cf. Fig. 4.4). The corresponding observational data sets are as given in Table (4.6) while the approximate coordinates are given in Table (4.5a), (4.5b), (4.5c) and (4.5b). In Table (4.5a), the first six approximate coordinates were determined through first level densification of the simulated second order network using the Static-Dynamic approach while in Table (4.5b), the first six approximate coordinates were determined through first level densification of the simulated second order network using the Sub-Optimal Fusion approach. In Table (4.5c), all the approximate coordinates were determined through first level densification of the simulated second order network using the Dynamic approach while in Table (4.5d), the first six approximate coordinates were determined through first level densification of the simulated second order network using the Static approach.

Figure 4.3: The simulated second order network

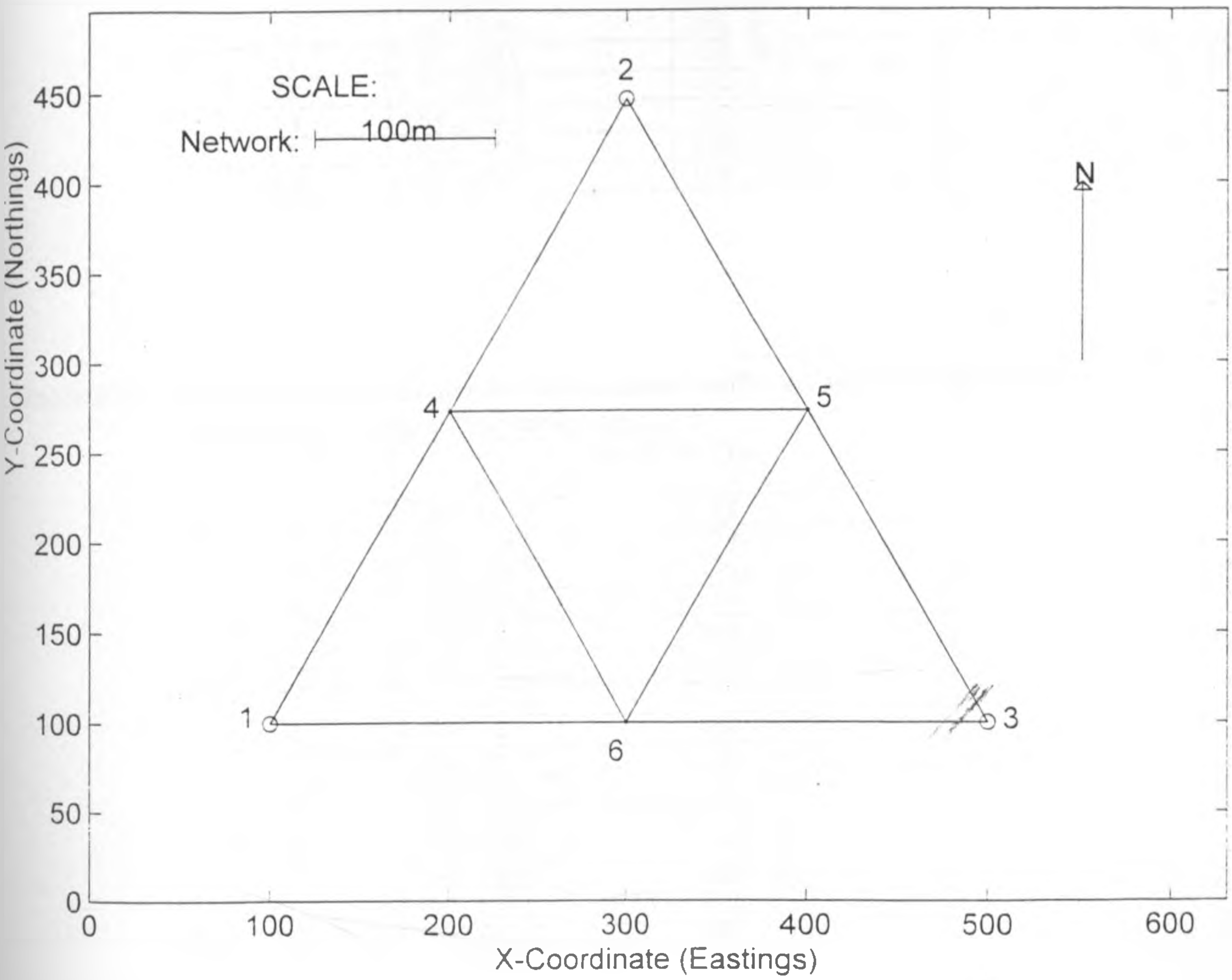


Table 4.5.1: Approximate coordinates for the simulated third order network used in the Static-Dynamic densification approach

POINT	COORDINATES	
	N (m)	E (m)
1	99.9956	99.9971
2	446.4087	300.0064
3	100.0057	499.9965
4	272.8963	200.1378
5	272.8948	400.1745
6	100.1226	300.0987
7	100.0000	200.0000
8	186.6000	150.0000
9	186.6020	250.0000
10	359.8080	250.0000
11	359.8080	350.0000
12	273.2050	300.0000
13	186.6020	450.0000
14	100.0000	400.0000
15	186.6020	350.0000

Table 4.5.2: Approximate coordinates of the simulated third order network used in the Sub-Optimal Fusion densification approach

POINT	COORDINATES	
	N (m)	E (m)
1	99.9956	99.9971
2	446.4087	300.0064
3	100.0057	499.9965
4	272.8809	200.1552
5	272.8795	400.1517
6	100.1419	300.0960
7	100.0000	200.0000
8	186.6000	150.0000
9	186.6020	250.0000
10	359.8080	250.0000
11	359.8080	350.0000
12	273.2050	300.0000
13	186.6020	450.0000
14	100.0000	400.0000
15	186.6020	350.0000

Table 4.5.3: Approximate coordinates of the simulated third order network used in the Dynamic densification approach

POINT	COORDINATES	
	N (m)	E (m)
1	100.0051	100.0179
2	446.3823	300.0025
3	100.0116	499.9775
4	272.8809	200.1552
5	272.8795	400.1517
6	100.1419	300.0960
7	100.0000	200.0000
8	186.6000	150.0000
9	186.6020	250.0000
10	359.8080	250.0000
11	359.8080	350.0000
12	273.2050	300.0000
13	186.6020	450.0000
14	100.0000	400.0000
15	186.6020	350.0000

Table 4.5.4: Approximate coordinates for the simulated third order network used in the Static densification approach

POINT	COORDINATES	
	N m)	E (m)
1	99.9936	99.9971
2	446.4087	300.0064
3	100.0057	499.9965
4	272.9269	200.1238
5	272.9257	400.1571
6	100.1099	300.0885
7	100.0000	200.0000
8	186.6000	150.0000
9	186.6020	250.0000
10	359.8080	250.0000
11	359.8080	350.0000
12	273.2050	300.0000
13	186.6020	450.0000
14	100.0000	400.0000
15	186.6020	350.0000

Table 4.6: Observational data sets for the simulated third order network

Observation Number	Line		Bearing °	'	"	Distance (m)
1	1	8	29	59	54.90	100.060
2	1	7	89	59	53.00	100.040
3	2	10	209	59	57.01	100.080
4	2	11	149	59	58.00	99.980
5	3	13	330	0	0.00	100.030
6	3	14	269	59	52.01	99.930
7	4	8	209	59	53.00	100.020

Figure 4.4: The simulated third order network

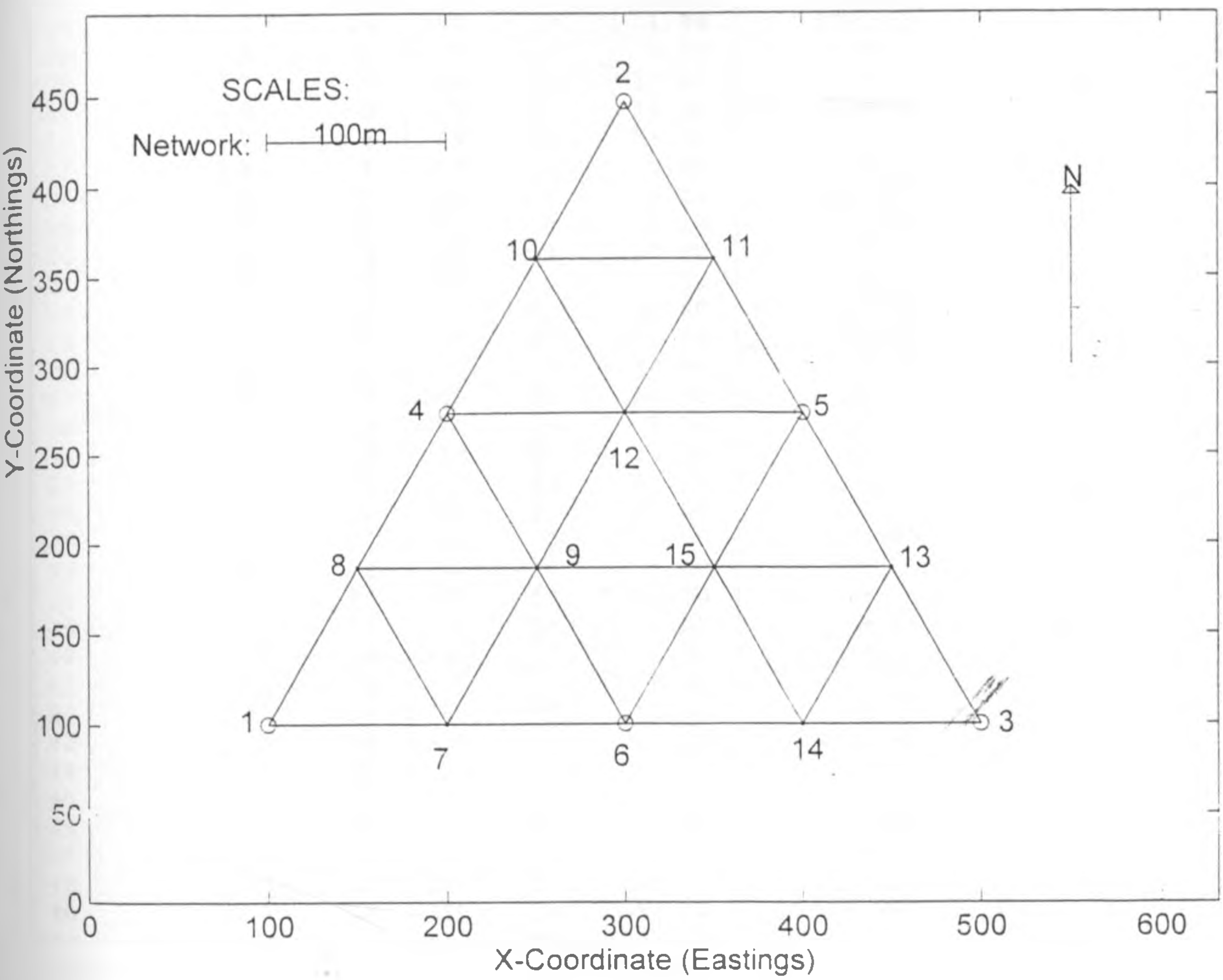


Table 4.6: Continued

Observation Number	Line		Bearing °	'	"	Distance (m)
8	4	9	150	0	4.00	100.040
9	4	12	90	0	2.62	99.970
10	4	10	30	0	8.42	100.030
11	5	11	330	0	2.90	99.890
12	5	12	269	59	54.37	100.080
13	5	15	210	0	5.02	99.940
14	5	13	150	0	3.96	99.990
15	6	7	270	0	9.00	99.980
16	6	9	329	59	54.71	99.930
17	6	15	30	0	6.43	100.020
18	6	14	90	0	1.98	100.030
19	7	1	270	0	0.00	-
20	7	8	330	0	10.00	-
21	7	9	29	59	51.00	99.970
22	7	6	89	59	57.00	-
23	8	1	210	0	0.00	-
24	8	7	150	0	0.10	100.000
25	8	9	90	0	0.10	100.060
26	8	4	29	59	57.90	-
27	9	4	330	0	0.00	-
28	9	12	29	59	53.00	100.040
29	9	15	89	59	55.00	100.010
30	9	6	150	0	10.90	-
31	9	7	209	59	59.00	-
32	9	8	270	0	5.00	-
33	10	2	29	59	55.00	-
34	10	4	209	59	57.00	-
35	10	12	149	59	50.00	-
36	10	11	90	0	5.00	100.000
37	11	2	330	0	1.00	-
38	11	10	269	59	54.00	-
39	11	12	209	59	54.00	-
40	11	5	150	0	1.00	-
41	12	9	210	0	0.00	-
42	12	4	269	59	52.00	-
43	12	10	330	0	0.00	100.060
44	12	11	30	0	0.00	99.980
45	12	5	90	0	0.00	-
46	12	15	149	59	55.00	99.970
47	13	5	330	0	7.00	-
48	13	15	269	59	58.00	-
49	13	14	209	59	58.00	99.960
50	13	3	149	59	53.00	-
51	14	3	90	0	2.00	-
52	14	6	270	0	0.00	-
53	14	15	329	59	58.00	-
54	14	13	29	59	50.10	-
55	15	5	29	59	50.00	-
56	15	12	330	0	2.00	-
57	15	9	269	59	55.90	-
58	15	6	210	0	2.90	-
59	15	14	150	0	4.00	100.050
60	15	13	89	59	50.10	100.030

4.2.2 The Real Network

4.2.2.1 First Order Network

The real first order network consisted of eleven stations as shown in Fig. (4.5) with corresponding approximate coo2dinates and observation data sets given in Tables (4.7) and Table (4.8) respectively. This was considered the fundamental real network upon which the real first level and real second level network densifications were performed. The fundamental network was defined by adjusting the real first order network within the framework of free network model. In the study, all the eleven points were considered approximate and the adjustment done assuming that the measurements were of first order precision with the standard error of the directions taken as $\pm 0.5''$. This was based on a variety of experiences from various studies and field surveys e.g. *Miima [1997], Musyoka [1993] and Aduol [1981].*

Table 4.7: Approximate coordinates for the real first order network

Point	S.K Code	S.K Name	Coordinates N (m)	E (m)
1	Sk56	TARU	9583105.104	513215.911
2	SK60	JIBANA	9576179.214	574648.574
3	SK61	KABANINI	9600981.179	569580.645
4	SK62	KILIFI	9599052.465	594885.489
5	SK63	SOKOKE	9611665.689	589503.327
6	SK64	KILIMANJARO	9624778.596	606329.877
7	SK65	MANGEA	9640229.659	580182.061
8	SK66	KIKUYUNI	9644287.319	609203.330
9	SK67	KOYENI	9661267.194	585576.660
10	SK68	BORE	9660984.693	598816.071
11	SK69	MAGARINI	9661696.939	6[illegible]8181.173

Table 4.8: Observational data sets for the real first order network

Observation Number	Line		Bearing °	'	"	Distance (m)
1	1	2	96	25	56.3	61821.778
2	1	3	72	24	12.7	59131.496
3	1	7	49	32	4.6	88020.944
4	2	1	276	25	56.5	-
5	2	3	348	27	5.0	25314.493
6	2	5	22	42	51.5	38470.110
7	2	4	41	30	1.9	30540.413
8	3	1	252	24	12.9	-
9	3	7	15	06	55.6	40655.120
10	3	5	61	47	43.3	22606.877
11	3	4	94	21	30.9	25378.281
12	3	2	168	27	5.0	-
13	4	2	221	30	1.8	-

Figure 4.5: The real first order network

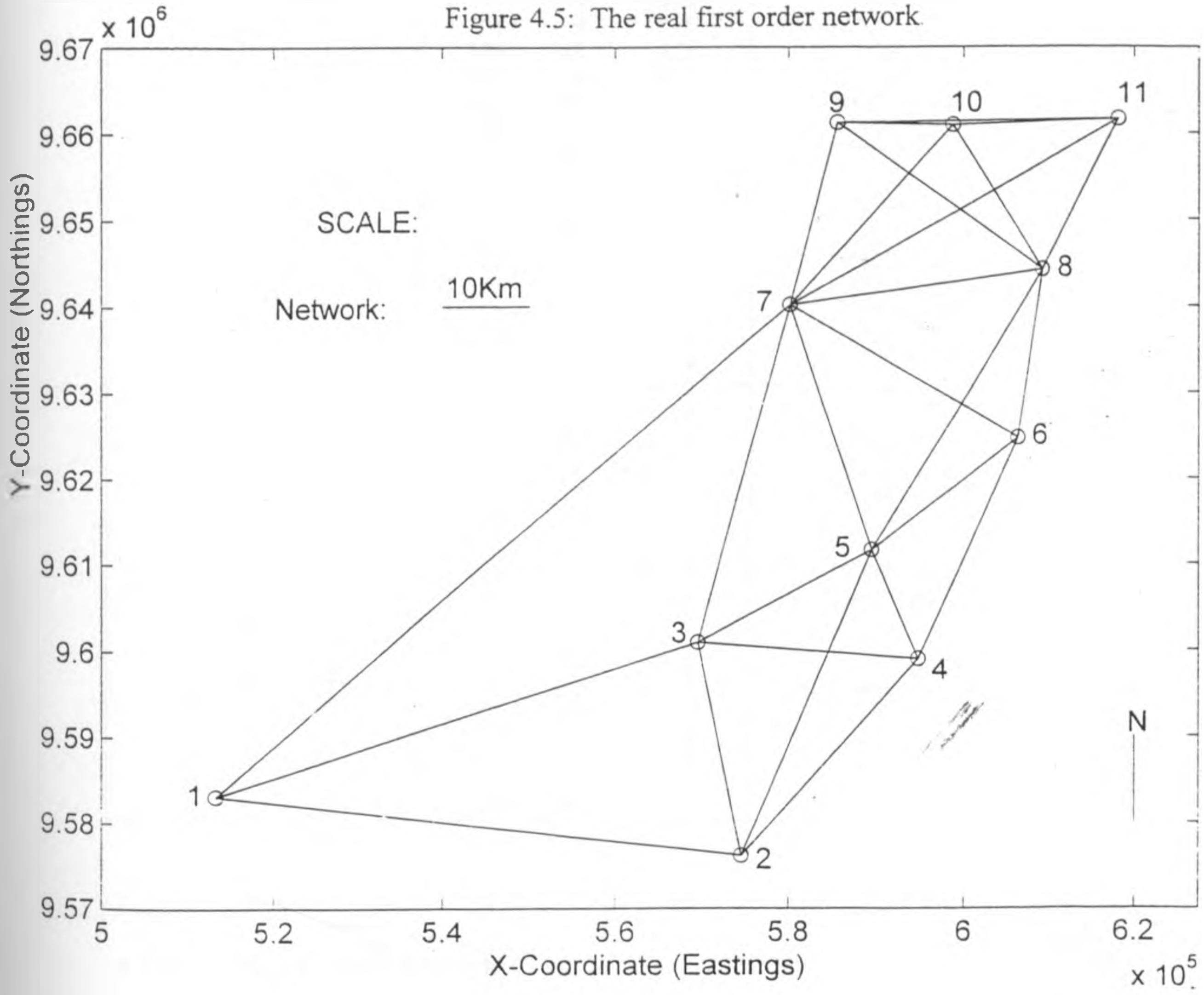


Table 4.8: Continued

Observation Number	Line		Bearing °	'	"	Distance (m)
14	4	3	274	21	31.0	-
15	4	5	336	53	30.0	13713.543
16	4	6	23	58	55.4	28156.771
17	5	4	156	53	30.1	-
18	5	2	202	42	51.7	-
19	5	3	241	47	43.4	-
20	5	7	341	55	37.0	30046.422
21	5	8	31	7	39.1	38108.480
22	5	6	52	4	14.6	21332.688
23	6	4	203	58	55.6	-
24	6	5	232	4	14.7	-
25	6	7	300	34	45.6	30371.764
26	6	8	08	2	4.0	19719.190
27	7	1	229	32	4.6	-
28	7	3	195	6	55.7	-
29	7	5	161	5	37.0	-
30	7	6	120	34	45.7	-
31	7	8	82	2	26.4	29303.530
32	7	11	60	32	9.8	43643.720
33	7	10	41	55	3.7	27892.630
34	7	9	14	22	56.2	21718.190
35	8	6	188	22	44.0	-
36	8	5	211	7	39.0	-
37	8	7	262	2	26.4	-
38	8	9	305	42	13.7	29095.230
39	8	10	328	6	52.8	19664.661
40	8	11	27	16	45.7	19588.160
41	9	7	194	22	56.4	-
42	9	8	125	42	13.7	-
43	9	10	91	13	20.4	13242.450
44	9	11	89	14	41.4	32607.292
45	10	9	271	13	20.5	-
46	10	7	221	55	3.8	-
47	10	8	148	6	52.9	-
48	10	11	87	53	37.1	19378.227
49	11	9	269	14	41.5	-
50	11	10	267	53	37.0	-
51	11	7	240	32	9.9	-
52	11	8	207	16	45.8	-

4.2.2.2 Second Order Network

The real second order network consisted of fifteen stations connected onto the real first order network described in section (4.2.2.1) above. (cf. Fig. 4.6). The corresponding observational data sets are as listed in Table (4.10) while the approximate coordinates are given in Table (4.9). In Table (4.9), the first eleven approximate coordinates were determined through free network adjustment of the real first order network while the rest are obtained from Survey of Kenya. The angular error of the observations was taken to be ± 1".

Table 4.9: Approximate coordinates for the real second order network

Point	S.K Code	Coordinates N (m)	E (m)
1	SKP56	9583104.004	513215.950
2	SKP60	9576179.235	574648.607
3	SKP61	9600981.218	569580.575
4	SKP62	9599052.507	594885.488
5	SKP63	9611665.696	589503.231
6	SKP64	9624778.565	606329.885
7	SKP65	9640229.703	580182.077
8	SKP66	9644287.286	609203.286
9	SKP67	9661267.225	585576.691
10	SKP68	9660984.753	598816.111
11	SKP69	9661696.859	618181.218
12	192.S.4	9651426.870	578082.210
13	192.S.8	9649302.660	592482.850
14	193.S.2	9640093.640	625611.430
15	198.S.12	9614609.390	603055.440
16	192.S.2	9626055.670	593121.870
17	192.S.1	9623694.380	582449.870
18	192.S.3	9625643.250	561292.670
19	197.S.4	9607749.860	545064.590
20	198.S.8	9610759.110	559563.590
21	198.S.9	9609410.820	563591.710
22	198.S.7	9592183.630	576550.570
23	198.S.3	9584543.130	589735.740
24	198.S.5	9585354.590	561756.300
25	197.S.1	9580752.500	538526.380
26	197.S.2	9589557.530	528224.190

Table 4.10: Observational data sets for the real second order network

Observation Number	Line		Bearing °	'	"	Distance (m)
1	12	7	169	22	42.3	11392.335
2	12	9	37	17	34.9	12369.293
3	13	7	233	35	15.5	-
4	13	8	106	41	48.0	-
5	13	9	330	0	19.6	13814.687
6	14	6	231	32	25.3	24623.681
7	14	8	284	20	13.6	16935.563
8	14	11	341	1	11.6	-
9	15	4	207	42	24.5	-
10	15	5	257	44	41.4	-
11	15	6	17	50	53.6	10683.383
12	16	5	194	6	54.6	14837.980
13	16	6	95	31	21.6	113269.516
14	16	8	41	24	51.2	24310.634
15	16	7	317	36	22.6	-
16	17	5	149	36	47.3	-
17	17	3	193	54	57.5	-

Figure 4.6: The real second order network

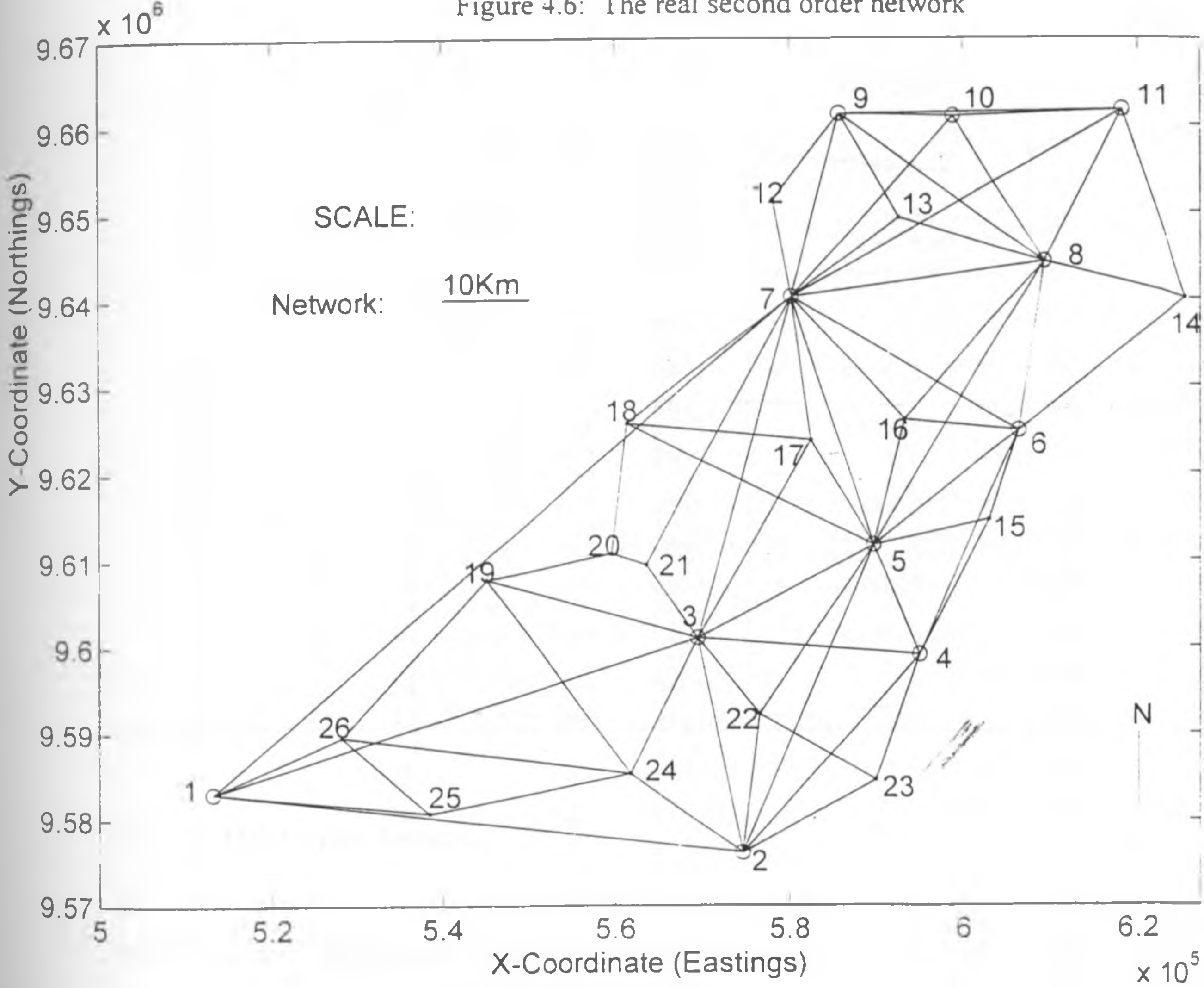


Table 4.10: Continued

Observation Number	Line		Bearing °	'	"	Distance (m)
18	17	18	275	15	46.6	-
19	17	7	352	11	25.9	16690.070
20	18	20	186	37	34.5	14984.326
21	18	7	52	19	28.9	23865.886
22	18	17	95	15	46.0	-
23	18	5	116	21	25.0	31483.464
24	19	6	222	47	24.0	24790.315
25	19	4	143	18	7.5	-
26	19	3	105	26	4.2	-
27	19	20	78	16	29.4	14808.001
28	20	21	108	30	23.4	4247.690
29	20	18	06	37	34.9	-
30	20	19	258	16	29.6	-
31	21	3	144	36	27.9	10340.648
32	21	20	288	30	23.4	-
33	21	7	28	17	39.9	35000.587
34	22	2	270	0	9.3	17894.890
35	22	3	120	5	28.5	15238.942
36	22	5	33	37	5.1	23394.975
37	22	3	321	36	42.0	-
38	23	2	240	59	50.2	-
39	23	4	19	32	28.1	15396.022
40	23	22	300	5	28.4	-
41	24	2	125	26	21.1	15823.986
42	24	3	26	35	50.9	-
43	24	19	323	18	7.5	-
44	24	6	277	8	39.0	33794.283
45	24	5	258	47	39.4	23681.294
46	25	1	275	18	31.7	25419.370
47	25	6	310	31	10.6	13552.337
48	25	4	78	47	38.4	-
49	26	1	246	44	0.1	16336.566
50	26	25	130	31	11.1	-
51	26	24	97	8	39.5	-
52	26	19	42	47	24.4	-

4.2.2.3 Third Order Network

The real third order network consisted of fifteen stations connected onto the real second order network described in section 4.2.2.2 (cf. Fig. 4.7). The corresponding observational data sets are as listed in Table (4.12) while the approximate coordinates are given in Table (4.11a), (4.11b), (4.11c) and (4.11d). In Table (4.11a), the first twenty six approximate coordinates were determined through densification process using the Static-Dynamic approach while in Table (4.11b) the first twenty six approximate coordinates were determined through first level densification process using the Sub-Optimal Fusion approach of the real second order network. In Table (4.11c) all the approximates were determined through first level densification process using the dynamic approach while in Table (4.11d), the first twenty six coordinates were determined through densification process using the static approach. The rest are as obtained from Survey of Kenya.

Table 4.11.1: Approximate coordinates for the real third order network used in the Static-Dynamic densification approach

Point	S.K Code	Coordinates N (m)	E (m)
1	SKP56	9583104.004	513215.950
2	SKP60	9576179.235	574648.607
3	SKP61	9600981.218	569580.575
4	SKP62	9599052.507	594885.488
5	SKP63	9611665.696	589503.231
6	SKP64	9624778.565	606329.885
7	SKP65	9640229.703	580182.077
8	SKP66	9644287.286	609203.286
9	SKP67	9661267.225	585576.691
10	SKP68	9660984.753	598816.111
11	SKP69	9661696.859	618181.218
12	192.S.4	9651426.855	578082.243
13	192.S.8	9649302.681	592482.865
14	193.S.2	9640093.572	625611.395
15	198.S.12	9614609.371	603055.410
16	192.S.2	9626055.619	593121.942
17	192.S.1	9623694.425	582449.886
18	192.S.3	9625643.288	561292.564
19	197.S.4	9607749.690	545064.420
20	198.S.8	9610759.097	559563.423
21	198.S.9	9609410.878	563591.472
22	198.S.7	9592183.544	576550.590
23	198.S.3	9584543.218	589735.830
24	198.S.5	9585354.356	561756.098
25	197.S.1	9580752.250	538526.297
26	197.S.2	9589557.286	528224.012
27	197.T.1	9598477.45	540001.04
28	197.T.4	9590143.01	544871.93
29	198.T.2	9592184.55	558655.59
30	198.T.9	9586646.84	596567.51
31	198.T.5	9593341.46	585647.23
32	198.T.13	9610982.26	577814.01
33	198.T.14	9608655.32	595394.33
34	198.T.20	9619838.45	564763.86
35	192.T.3	9620013.73	589321.42
36	192.T.7	9635147.76	567613.38
37	192.T.18	9630041.92	586400.12
38	192.T.26	9639112.32	594892.10
39	192.T.21	9635021.72	609221.86
40	192.T.30	9649311.34	618081.16
41	192.T.35	9656888.74	608069.420

Table 4.11.2: Approximate coordinates for the real third order network used in the Sub-Optimal Fusion densification approach

Point	S.K Code	Coordinates N (m)	E (m)
1	SKP56	9583104.004	513215.950
2	SKP60	9576179.235	574648.607
3	SKP61	9600981.218	569580.575
4	SKP62	9599052.507	594885.488
5	SKP63	9611665.696	589503.231
6	SKP64	9624778.565	606329.885
7	SKP65	9640229.703	580182.077
8	SKP66	9644287.286	609203.286
9	SKP67	9661267.225	585576.691
10	SKP68	9660984.753	598816.111
11	SKP69	9661696.859	618181.218
12	192.S.4	9651426.813	578082.226
13	192.S.8	9649302.665	592482.859
14	193.S.2	9640093.569	625611.424
15	198.S.12	9614609.393	603055.425
16	192.S.2	9626055.643	593121.948
17	192.S.1	9623694.391	582449.936
18	192.S.3	9625643.222	561292.596
19	197.S.4	9607749.665	545064.452
20	198.S.8	9610759.036	559563.462
21	198.S.9	9609410.818	563591.511
22	198.S.7	9592183.530	576550.637
23	198.S.3	9584543.174	589735.857
24	198.S.5	9585354.357	561756.081
25	197.S.1	9580752.276	538526.270
26	197.S.2	9589557.318	528223.996
27	197.T.1	9598477.45	540001.04
28	197.T.4	9590143.01	544871.93
29	198.T.2	9592184.55	558655.59
30	198.T.9	9586646.84	596567.51
31	198.T.5	9593341.46	585647.23
32	198.T.13	9610982.26	577814.01
33	198.T.14	9608655.32	595394.33
34	198.T.20	9619838.45	564763.86
35	192.T.3	9620013.73	589321.42
36	192.T.7	9635147.76	567613.38
37	192.T.18	9630041.92	586400.12
38	192.T.26	9639112.32	594892.10
39	192.T.21	9635021.72	609221.86
40	192.T.30	9649311.34	618081.16
41	192.T.35	9656888.74	608069.420

Table 4.11.3: Approximate coordinates for real third order network used in the Dynamic densification approach

Point	S.K Code	Coordinates N (m)	E (m)
1	SKP56	9583104.115	513215.937
2	SKP60	9576179.227	574648.570
3	SKP61	9600981.168	569580.640
4	SKP62	9599052.461	594885.487
5	SKP63	9611665.687	589503.315
6	SKP64	9624778.595	606329.877
7	SKP65	9640229.663	580182.066
8	SKP66	9644287.320	609203.331
9	SKP67	9661267.192	585576.661
10	SKP68	9660984.693	598816.071
11	SKP69	9661696.939	618181.173
12	192.S.4	9651426.813	578082.226
13	192.S.8	9649302.665	592482.859
14	193.S.2	9640093.569	625611.424
15	198.S.12	9614609.393	603055.425
16	192.S.2	9626055.643	593121.948
17	192.S.1	9623694.391	582449.936
18	192.S.3	9625643.222	561292.596
19	197.S.4	9607749.665	545064.452
20	198.S.8	9610759.036	559563.462
21	198.S.9	9609410.818	563591.511
22	198.S.7	9592183.530	576550.637
23	198.S.3	9584543.174	589735.857
24	198.S.5	9585354.357	561756.081
25	197.S.1	9580752.276	538526.270
26	197.S.2	9589557.318	528223.996
27	197.T.1	9598477.45	540001.04
28	197.T.4	9590143.01	544871.93
29	198.T.2	9592184.55	558655.59
30	198.T.9	9586646.84	596567.51
31	198.T.5	9593341.46	585647.23
32	198.T.13	9610982.26	577814.01
33	198.T.14	9608655.32	595394.33
34	198.T.20	9619838.45	564763.86
35	192.T.3	9620013.73	589321.42
36	192.T.7	9635147.76	567613.38
37	192.T.18	9630041.92	586400.12
38	192.T.26	9639112.32	594892.10
39	192.T.21	9635021.72	609221.86
40	192.T.30	9649311.34	618081.16
41	192.T.35	9656888.74	608069.420

Table 4.11.4: Approximate coordinates for the real third order network used in the Static densification approach

Point	S.K Code	Coordinates N (m)	E (m)
1	SKP56	9583104.004	513215.950
2	SKP60	9576179.235	574648.607
3	SKP61	9600981.218	569580.575
4	SKP62	9599052.507	594885.488
5	SKP63	9611665.696	589503.231
6	SKP64	9624778.565	606329.885
7	SKP65	9640229.703	580182.077
8	SKP66	9644287.286	609203.286
9	SKP67	9661267.225	585576.691
10	SKP68	9660984.753	598816.111
11	SKP69	9661696.859	618181.218
12	192.S.4	9651426.856	578082.241
13	192.S.8	9649302.679	592482.863
14	193.S.2	9640093.585	625611.399
15	198.S.12	9614609.374	603055.417
16	192.S.2	9626055.626	593121.935
17	192.S.1	9623694.418	582449.886
18	192.S.3	9625643.281	561292.577
19	197.S.4	9607749.704	545064.434
20	198.S.8	9610759.098	559563.437
21	198.S.9	9609410.872	563591.495
22	198.S.7	9592183.549	576550.599
23	198.S.3	9584543.206	589735.832
24	198.S.5	9585354.386	561756.124
25	197.S.1	9580752.276	538526.299
26	197.S.2	9589557.305	528224.023
27	197.T.1	9598477.45	540001.04
28	197.T.4	9590143.01	544871.93
29	198.T.2	9592184.55	558655.59
30	198.T.9	9586646.84	596567.51
31	198.T.5	9593341.46	585647.23
32	198.T.13	9610982.26	577814.01
33	198.T.14	9608655.32	595394.33
34	198.T.20	9619838.45	564763.86
35	192.T.3	9620013.73	589321.42
36	192.T.7	9635147.76	567613.38
37	192.T.18	9630041.92	586400.12
38	192.T.26	9639112.32	594892.10
39	192.T.21	9635021.72	609221.86
40	192.T.30	9649311.34	618081.16
41	192.T.35	9656888.74	608069.420

Figure 4.7: The real third order network

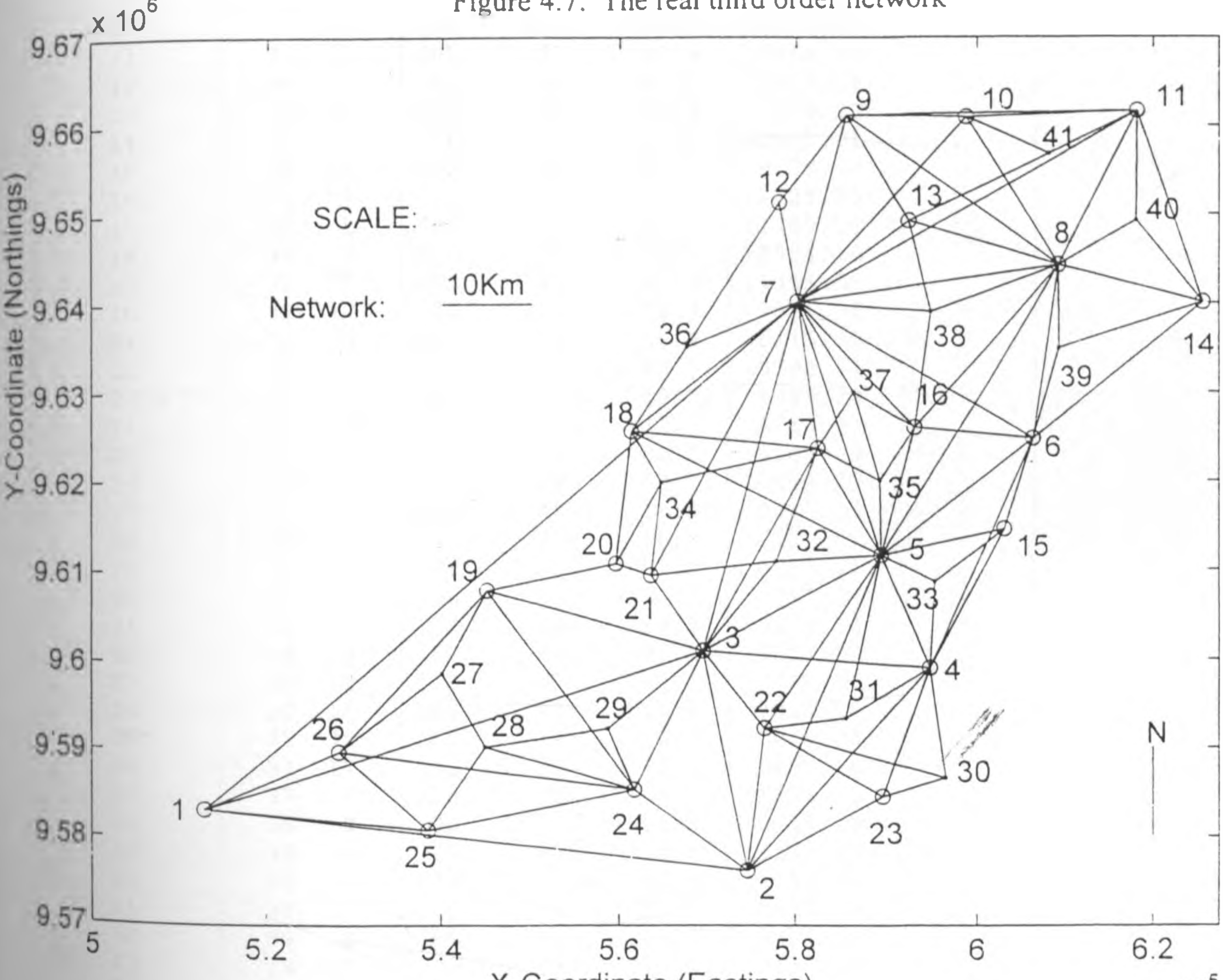


Table 4.12: Observational data sets for the real third order network

Observation Number	Line		Bearing °	′	″	Distance (m)
1	27	26	232	51	31.2	14773.89
2	27	19	28	38	17.9	10564.67
3	27	28	149	41	47.2	9653.42
4	28	27	329	41	49.2	9653.42
5	28	29	81	34	28.8	13934.03
6	28	24	105	50	04.4	17550.11
7	28	25	214	2	52.3	11333.73
8	29	3	51	9	34.8	14026.28
9	29	24	155	35	06.0	7500.98
10	29	28	261	34	28.9	13934.03
11	30	22	285	27	41.4	20768.54
12	30	4	352	16	43.1	12519.18
13	30	23	252	53	06.9	7148.22
14	31	4	58	16	33.0	10861.01
15	31	22	262	44	43.9	9170.04
16	31	5	11	53	00.5	18725.55
17	32	5	86	39	13.9	11709.18
18	32	3	219	27	48.4	12954.16
19	32	21	263	41	41.8	14309.08
20	32	17	20	2	09.3	13531.09
21	33	5	297	4	01.9	6615.69
22	33	15	52	8	46.7	9702.74
23	33	4	183	1	59.5	9616.29
24	34	18	329	7	12.9	6763.58
25	34	17	77	42	03.2	18101.49
26	34	21	186	24	52.7	10493.27
27	34	20	209	48	10.9	10463.23
28	35	16	32	10	16.9	7137.81
29	35	5	178	45	07.4	8350.01
30	35	17	298	10	32.6	7795.22
31	36	12	32	44	39.3	19354.74
32	36	7	67	59	06.6	13557.22
33	36	18	213	37	30.2	11414.36
34	37	16	120	40	11.5	7814.95
35	37	35	163	45	30.3	10445.03
36	37	17	211	53	41.8	7476.30
37	37	7	328	36	09.9	11935.45
38	38	8	70	7	12.5	15218.09
39	38	16	187	43	14.8	13176.15
40	38	7	274	20	37.9	14752.40
41	38	13	346	41	53.6	10471.29
42	39	8	359	53	06.5	9265.58
43	39	14	72	48	16.9	17156.36
44	39	6	195	45	58.9	10642.58
45	40	11	0	27	45.3	12385.92
46	40	14	140	45	14.3	11902.59
47	40	8	240	29	37.9	10200.37
48	41	10	293	52	37.3	10119.34
49	41	11	64	34	08.8	11196.72
50	41	13	244	2	51.2	17334.62

4.3 Methods of Analysis

There are a number of criteria for determination of precision and reliability of a densification process. The precision of a network is the measure of the manner in which it propagates random errors. This is quantified by standard errors of parameters, positional error ellipses and standard errors of adjusted observations. Internal reliability of a network is its ability to have blunders detected. This is quantified by the improvement ratio i.e. the ratio of the a-posteriori standard error of an observation to it's a-priori value included in the inverseweight matrix *[Askenazi, 1980]*. External reliability of an observation is a measure of the effect of an undetected blunder, in that observation, on the estimated parameters and other quantities derived from these parameters. This is quantified by the gamma factor discussed in detail by *Cross [1983]*.

The quality of the geodetic system can be analyzed either before or after the densification process. This is usually achieved through the network's a-posteriori variance-covariance matrix. In practice, the a-posteriori variance-covariance matrix needs to be determined for all planned networks otherwise one can not be sure that the network will fulfil the task for which it has been designed. Similarly, the densified network should have an associated variance-covariance matrix so as to know the uses to which the final parameters can be put.

Irrespective of any sparsity of the design matrix and the weight matrix, the a-posteriori variance-covariance matrix will in general be a full matrix i.e. for every unknown there will be an associated variance and for every pair of unknowns there will be a covariance. In this study therefore, main tool to aid analysis will be the network's a-posteriori variance-covariance matrix as given in (3.3.11) and (3.3.35d). The elements of the matrix are used to determine positional standard errors, error ellipses, circular probable errors (CPE) and standard errors of the derived quantities as discussed below.

4.3.1 Positional Standard Errors

Standard errors of the estimates are obtained by taking the square root of the diagonal elements of the variance-covariance matrix. Hence we quote for each adjusted coordinate a standard error. However in most cases, these standard errors are of limited use because of their dependence on the origin (datum). Clearly, the further away they are from the network fixed points, the larger the positional standard errors will be.

In certain networks, there will be more than one fixed point. For instance in the situation where the third order network is being fitted onto a second order network. In this case, a larger positional error may merely indicate that the nearest fixed point is a long way away. The only situation in which positional errors may be useful is in a free network where the datum is defined over approximate coordinates neither fixing any particular point, for example, in a combined satellite and terrestrial network where there will be no fixed point at all.

4.3.2 Circular Probable Errors

A measure of accuracy for each part instead of the two components of the positional error can be obtained by combining the components, to give a vector sum referred to as "circular probable error" or "radial standard error" *[Mikhail, 1976,pg 33]* or "positional error sphere" for a three-dimensional case *[Aduol, 1981, pg 46]* and is given by:

$$\bar{\sigma}_C = \left[\frac{\sigma_E^{\,2} + \sigma_N^{\,2}}{2}\right]^{\frac{1}{2}} \qquad (4.3.1)$$

For n parameters we have that

$$\bar{\sigma}_C = \left[\frac{\sigma_1^{\,2} + \sigma_2^{\,2} - \cdots\cdots + \sigma_n^{\,2}}{n}\right]^{\frac{1}{2}} \qquad (4.3.2)$$

where σ_i $[i = 1, 2 \ldots\ldots\ldots\ldots\ldots n]$ are the respective standard errors.

4.3.3 Standard Error Ellipses

Standard error ellipses are relevant to two-dimensional networks only. However, extensions to other dimensions are possible although only the three-dimensional case is likely to be practically useful and in which case we talk of an error ellipsoid. There are two types of error ellipses within networks: the first, relative error ellipses, which express the reliability of the position of a point with respect to a neighbouring point. The second is the absolute error ellipse for a point, which reflects how accurately the point has been positioned. In the present study we shall use absolute error ellipses, as the interest of the study is to learn how accurately points are fixed.

The derivation of error ellipse parameters is given in the summary in Appendix C and is given through the equations below.

$$\tan 2\psi_m = \frac{2\sigma_{xy}}{(\sigma_x^2 - \sigma_y^2)} \tag{4.3.3}$$

$$\sigma_m^2 = \sigma^2_{\max} = \tfrac{1}{2}[\sigma_x^2 + \sigma_y^2 + ((\sigma_y^2 - \sigma_x^2)^2 + 4(\sigma_{xy})^2)^{\frac{1}{2}}] \tag{4.3.4a}$$

$$\sigma_n^2 = \sigma^2_{\min} = \tfrac{1}{2}[\sigma_x^2 + \sigma_y^2 - ((\sigma_y^2 - \sigma_x^2)^2 + 4(\sigma_{xy})^2)^{\frac{1}{2}}] \tag{4.3.4b}$$

where σ_x^2 and σ_y^2 are the position variances in two mutually perpendicular directions ψ and $\psi + 90$. Hence we can compute the directions and values of the maximum and minimum variances at any point in the network.

Absolute error ellipses suffer the same disadvantage of being dependent on the origin (datum) as the positional standard errors. However, we can get some important overall network information from the pattern of the error ellipses. For example minor axes pointing approximately towards the origin indicate an overall network weakness in orientation. Major axes pointing towards the origin indicate overall network weakness in scale control.

4.3.4 Mean Shifts

These are the vectors determined from the final adjusted coordinates of points in a network using different methods or under different circumstances. They give a measure of displacement between points, which can be used to analyze networks.

4.4 Numerical Computing

4.4.1 Design of Computation

The fundamental network of the system was defined by adjusting the first order networks within the framework of a free network. The first densification was performed by applying the concept of *static-dynamic* approach on simulated and real networks at the first and second levels. In the static-dynamic densification, the observation data sets in Table (4.4) and (4.10) and the approximate coordinates in Table (4.3) and (4.9) were used for the first level densification. The observation data sets in Table (4.6) and (4.12) and the approximate coordinates in Table (4.5a) and (4.11a) were used for the second level densification. The second densification was performed by applying the concept of *sub-optimal fusion* approach on simulated and real networks at the first and second levels. In the sub-optimal fusion densification, the observation data sets in Table (4.4) and (4.10) and the approximate coordinates in Table (4.3) and (4.9) were used for the first level densification. The observation data sets in Table (4.6) and (4.12) and the approximate coordinates in Table (4.5b) and (4.11b) were used for the second level densification. The third densification was performed by applying the concept of *dynamic* approach on simulated and real networks at the first and second levels. In the dynamic densification, the observation data sets in Table (4.4) and (4.10) and the approximate coordinates in Table (4.3) and (4.9) were used for the first level densification. The observation data sets in Table (4.6) and (4.12) and the approximate coordinates in Table (4.5c) and (4.11c) were used for the second level densification. The fourth densification was performed by applying the concept of *static* approach on simulated and real networks at the first and second levels. In the static densification, the observation data sets in Table (4.4) and (4.10) and the approximate coordinates in Table (4.3) and (4.9) were used for the first level densification. The observation data sets in Table (4.6) and (4.12) and the approximate coordinates in Table (4.5d) and (4.11d) were used for the second level densification.

4.4.2 Computer Programs

The computations were done using programs written in MATLAB which is an acronym for MATrix LABoratory. MATLAB is a technical computing environment for high performance numeric computation and visualization. It is an iteractive system whose basic data element is a matrix that does not require dimensioning thus enhancing the speed of numerical solutions. Separate programs were written for each task i.e. free network adjustment, static-dynamic,static, dynamic and sub-optimal fusion approaches. In this section, two main programs FREE.M and DENSITY.M coded in MATLAB with which the computations were carried out on a personal computer model Pentium 3000 are explained.

4.4.2.1 Program FREE.M

This program performs network adjustment using the concept of free network adjustment (cf. section 2.3) to adjust the primary networks which were used to define the fundamental network on which subsequent densification using static-dynamic and sub-optimal fusion approaches were performed. Flow chart 1 in Appendix A.2 shows the systematic stages of the program.

4.4.2.2 Program DENSITY.M

All the approaches to densification discussed in chapter three are considered. It consist of three modules which are separately written, tested and linked together to form the program.

In the first module, the observation and provisional values are read into the computer memory and the computational matrices are initialized to zero. The flow chart in Appendix A.3 describes the process.

In the second module, the design matrix A, the residual vector of the observations Y and the weight matrix W are formed from the reduced observations in read in module one above. The network is densified depending on the different approaches and levels. The a-posteriori variance of unit weight is also computed.

In the third module, the data obtained in the second module are used in network analysis through determination of variance-covariance matrix from equations (3.2.20), (3.2.28), (3.3.11) and (3.3.35d). The variance-covariance matrix, a-posteriori standard error of the observation data sets and posteriori error ellipses are determined. Finally, the results are output for each densification approach. Appendix A.3 describes the process.

5

THE TEST RESULTS

5.1 Introduction

In this chapter, the results are presented in two main sections. Section 5.2 contains simulated network results while section 5.3 contains real network results. Each section further contains results of the free network adjustment and of the densification process using *static-dynamic*, *sub-optimal fusion*, *dynamic* and *static* approaches. The computed shifts between the final coordinates obtained from the different approaches are also presented in each section. The Tables of results presented in each of the sections consists of estimated correction to the approximate or provisional coordinates $(\Delta E, \Delta N)$ for each network point, their corresponding standard errors (σ_E, σ_N) computed from equations (3.2.28), (3.3.11) and (3.3.35d). Also presented in each table is the parameters for error ellipses viz.: semi-major axis (σ_{max}), semi-minor axis (σ_{min}) and the orientation of the semi-major axis (α) computed from equations (4.3.3), (4.3.4a) and (4.3.4b). These results, together with those listed in Appendix B, are part of the results obtained by running the programs discussed in Chapter Four.

5.2 The Simulated Network

5.2.1 First Order Network Adjustment

The results presented in this section were obtained by adjusting the simulated first order network through the free network technique using the approximate coordinates listed in Table (4.1) and observation data sets listed in Table (4.2). This formed the fundamental network on which the simulated first level and simulated second level densifications were performed. The resulting error ellipses are presented diagrammatically in Fig. 5.1.

Table 5.2.1: Coordinate Corrections and Stochastic Parameters-First Order Simulated Network

POINT	ΔN (m)	ΔE (m)	σ_N (m)	σ_E (m)	σ_{max} (m)	σ_{min} (m)	α ° ' "
1	-0.0044	-0.0029	0.0028	0.0034	0.0036	0.0025	60 0 0.1
2	-0.0013	0.0064	0.0036	0.0025	0.0036	0.0025	00 00 00
3	0.0057	-0.0035	0.0028	0.0034	0.0036	0.0025	299 59 59.9

$\bar{\sigma}_N$=0.00309 $\bar{\sigma}_E$=0.00313 $\bar{\sigma}_C$=0.00311 $\hat{\sigma}_0^2$=0.999667

Table 5.2.2: Estimated Coordinates-First Order Simulated Network.

POINT	PROVISIONAL COORDINATES		ADJUSTED COORDINATES	
	N(m)	E(m)	N(m)	E(m)
1	100.000	100.000	99.9956	99.9971
2	446.410	300.000	446.4087	300.0064
3	100.000	500.000	100.0057	499.9965

5.2.2 Static-Dynamic Densification

The results presented in this section were determined by considering the estimated coordinates of the first order simulated network listed in Tables (5.2.2) as fixed stochastic constraints. The *static-dynamic* approach was then used to densify the first order network by intercalating into it second and third order points giving results for first level densification (cf. section 5.2.2.1) and second level densification (cf. section 5.2.2.2).

5.2.2.1 First Level Densification

In the first level simulated network densification, first order points 1, 2 and 3 were considered as fixed stochastic parameters while second order points 4, 5 and 6 were considered as new points whose coordinates were to be estimated (cf. Figure (4.3)). The observation data sets in Table (4 4) and the approximate coordinates in Table (4 3) were used resulting in parameters listed in Table (5.2.3) and Table (5.2.4) below while error ellipses are given diagrammatically in Figure (5.2).

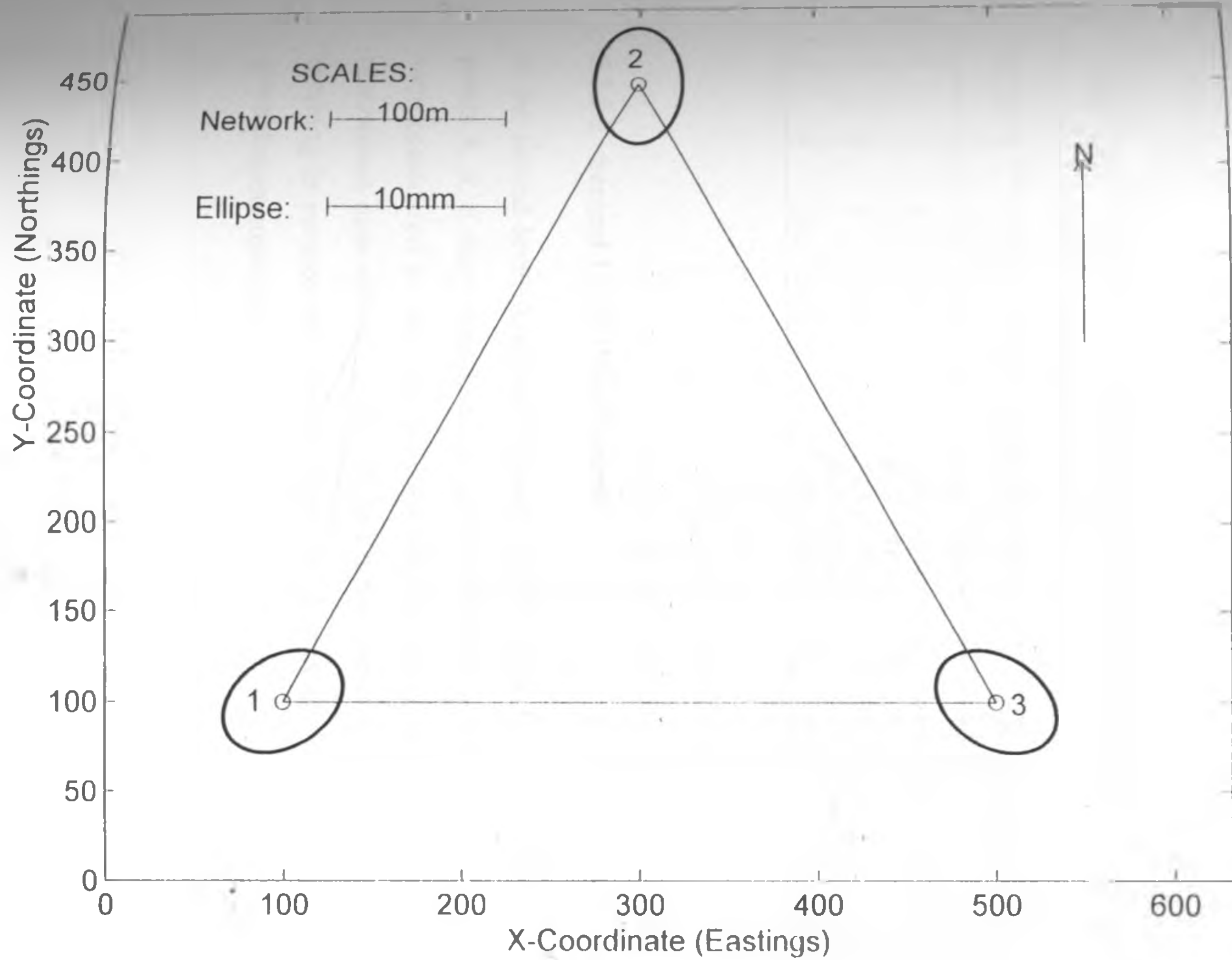


Figure 5.1: Point error ellipses -first order simulated network

Table 5.2.3: Coordinate Corrections and Stochastic Parameters-First Level Simulated Network -Static-Dynamic Approach

POINT	ΔN (m)	ΔE (m)	σ_N (m)	σ_E (m)	σ_{max} (m)	σ_{min} (m)	α ° ' "
1	0.0000	0.0000	0.0028	0.0034	0.0036	0.0025	60 0 0.1
2	0.0000	0.0000	0.0036	0.0025	0.0036	0.0025	0 0 0
3	0.0000	0.0000	0.0028	0.0034	0.0036	0.0025	297 59 59.9
4	-0.1537	0.0678	0.0043	0.0036	0.0046	0.0033	302 54 21.1
5	-0.1552	0.0845	0.0043	0.0036	0.0046	0.0033	57 05 12.4
6	0.0626	0.0487	0.0031	0.0046	0.0046	0.0032	00 01 05.1

$\bar{\sigma}_N$=0.003541 $\bar{\sigma}_E$=0.003570 $\bar{\sigma}_C$=0.003555 $\hat{\sigma}_0^2$=1.004263

Table 5.2.4: Estimated Coordinates-First Level Simulated Network Densification -Static-Dynamic Approach

POINT	PROVISIONAL COORDINATES N(m)	E(m)	ADJUSTED COORDINATES N(m)	E(m)
1	99.9956	99.9971	99.9956	99.9971
2	446.4087	300.0064	446.4087	300.0064
3	100.0057	499.9965	100.0057	499.9965
4	273.0500	200.0700	272.8964	200.1378
5	273.0500	400.0900	272.8948	400.1745
6	100.0600	300.0500	100.1226	300.0987

5.2.2.2 Second Level Densification

In the second level simulated network densification, first order points 1, 2, 3 and second order points 4, 5, 6 were considered as fixed stochastic parameters while third order points 7 to 15 were considered as new points whose coordinates were to be estimated (cf. Figure (4.4)). The observation data sets in Table (4.6) and the approximate coordinates in Table (4.5a) were used resulting in parameters listed in Table (5.2.5) and Table (5.2.6) below while error ellipses are given diagrammatically in Figure (5.3).

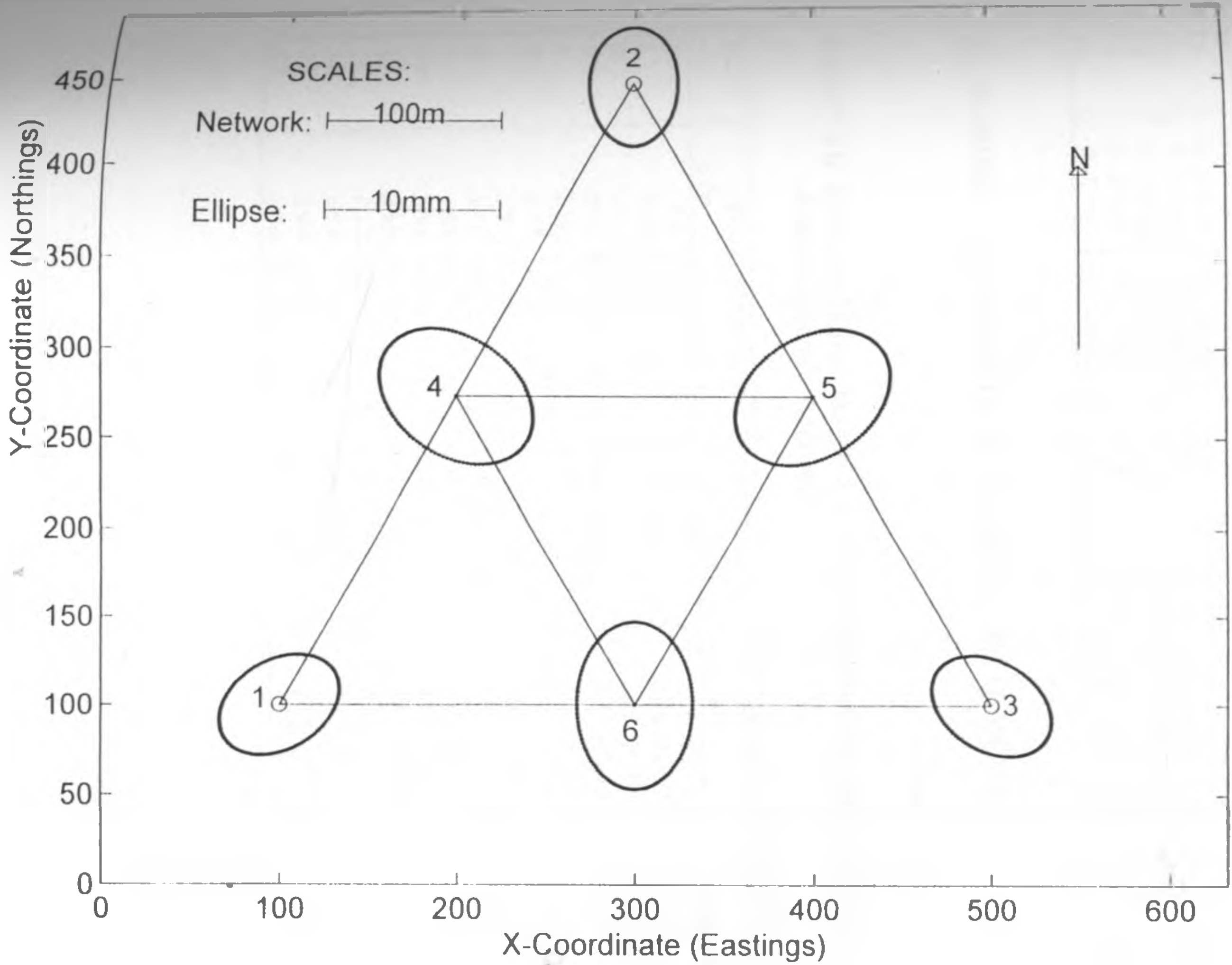


Figure 5.2: Point error ellipses -first level simulated network densification static-dynamic approach

Table 5.2.5:Coordinate Corrections and Stochastic Parameters-Second Level Simulated Network -Static-Dynamic Approach

POINT	ΔN (m)	ΔE (m)	σ_N (m)	σ_E (m)	σ_{max} (m)	σ_{min} (m)	α ° ' "
1	0.0000	0.0000	0.0028	0.0034	0.0036	0.0025	60 00 0.1
2	0.0000	0.0000	0.0036	0.0025	0.0036	0.0025	361 59 60.0
3	0.0000	0.0000	0.0028	0.0034	0.0036	0.0025	298 59 59.9
4	0.0000	0.0000	0.0043	0.0036	0.0046	0.0033	302 54 21.1
5	0.0000	0.0000	0.0043	0.0036	0.0046	0.0033	57 05 12.4
6	0.0000	0.0000	0.0031	0.0046	0.0046	0.0032	00 01 05.1
7	0.0157	-0.0891	0.0019	0.0025	0.0026	0.0019	21 09 18.8
8	0.0385	-0.0934	0.0024	0.0023	0.0026	0.0021	83 38 18.6
9	0.0531	-0.0776	0.0020	0.0020	0.0021	0.0020	37 14 54.5
10	0.0947	-0.0534	0.0026	0.0020	0.0027	0.0020	338 58 23.1
11	0.0974	-0.0630	0.0026	0.0020	0.0027	0.0020	21 02 7.6
12	0.1031	-0.0645	0.0021	0.0019	0.0022	0.0020	359 58 10.0
13	0.0492	-0.0182	0.0023	0.0023	0.0026	0.0021	276 03 59.8
14	0.0049	-0.0398	0.0019	0.0025	0.0026	0.0019	333 23 23.1
15	0.0587	-0.0641	0.0020	0.0020	0.0021	0.0020	323 05 21.1

$\bar{\sigma}_N$=0.002822 $\bar{\sigma}_E$=0.002818 $\bar{\sigma}_C$=0.002820 $\hat{\sigma}_0^2$=1.006233

Table 5.2.6: Estimated Coordinates- Second Level Simulated Network Densification -Static-Dynamic Approach

Pt.	PROVISIONAL COORDINATES N(m)	E(m)	ADJUSTED COORDINATES N(m)	E(m)
1	99.9956	99.9971	99.9956	99.9971
2	446.4087	300.0064	446.4087	300.0064
3	100.0057	499.9965	100.0057	499.9965
4	272.8964	200.1378	272.8964	200.1378
5	272.8948	400.1745	272.8948	400.1745
6	100.1226	300.0987	100.1226	300.0987
7	100.0000	200.0000	100.0157	199.9109
8	186.6000	150.0000	186.6385	149.9066
9	186.6020	250.0000	186.6551	249.9224
10	359.8080	250.0000	359.9027	249.9466
11	359.8080	350.0000	359.9054	349.9370
12	273.2050	300.0000	273.3081	299.9355
13	186.6020	450.0000	186.6512	449.9818
14	100.0000	400.0000	100.0049	399.9602
15	186.6020	350.0000	186.6607	349.9359

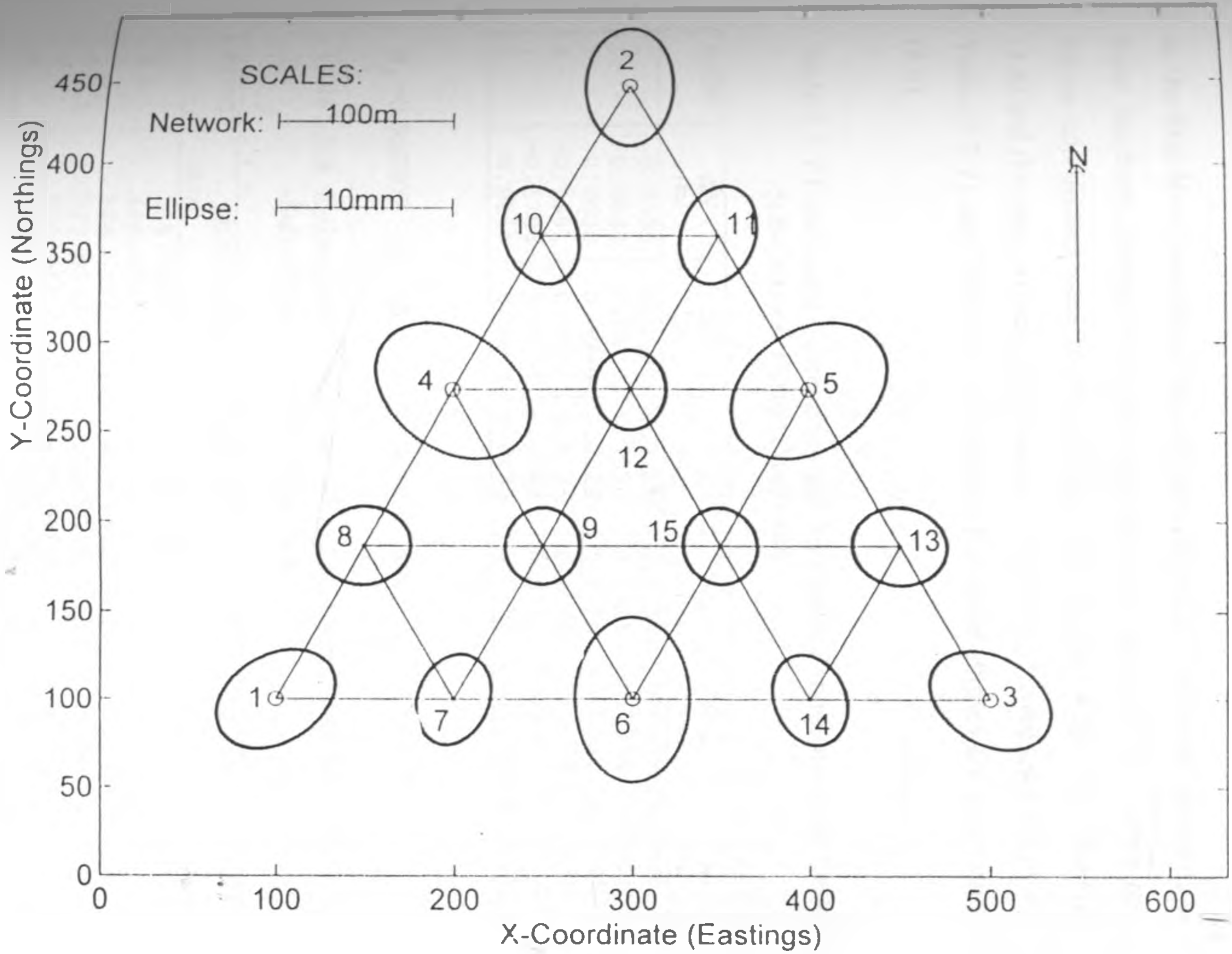


Figure 5.3: Point error ellipses -second level simulated network densification static-dynamic approach

5.2.3 Sub-Optimal Fusion Densification

The results presented in this section were determined by considering the estimated coordinates of the first order simulated network listed in Tables (5.2.2) as fixed stochastic constraints. The *sub-optimal fusion* approach was then used to densify the first order network by intercalating on to the network second and third order points giving results for first level densification {cf. section (5.2.3.1)} and second level densification {cf. section (5.2.3.2)}.

5.2.3.1 First Level Densification

In the first level simulated network densification, first order points 1, 2, 3 were considered as fixed stochastic parameters while second order points 4, 5, 6 were considered as new points whose coordinates were to be estimated (cf. Figure (4.3)). The observation data sets in Table (4.4) and the approximate coordinates in Table (4.3) were used resulting in parameters listed in Table (5.2.7) and Table (5.2.8) below while error ellipses are given diagrammatically in Figure (5.4).

Table 5.2.7: Coordinate Corrections and Stochastic Parameters-First Level Simulated Network -Sub-Optimal Fusion Approach

POINT	ΔN (m)	ΔE (m)	σ_N (m)	σ_E (m)	σ_{max} (m)	σ_{min} (m)	α ° ' "
1	0.0000	0.0000	0.0028	0.0034	0.0036	0.0025	60 0 0.1
2	0.0000	0.0000	0.0036	0.0025	0.0036	0.0025	362 59 60.0
3	0.0000	0.0000	0.0028	0.0034	0.0036	0.0025	299 59 59.9
4	-0.1691	0.0852	0.0048	0.0040	0.0050	0.0036	30[illegible] 54 21.1
5	-0.1705	0.0617	0.0048	0.0040	0.0050	0.0036	57 05 12.4
6	0.0819	0.0460	0.0035	0.0051	0.0050	0.0035	00 01 05.1

$\bar{\sigma}_N$=0.003807 $\bar{\sigma}_E$=0.003816 $\bar{\sigma}_C$=0.003812 $\hat{\sigma}_0^2$=1.003253

Table 5.2.8: Estimated Coordinates- First Level Simulated Network Densification -Sub-Optimal Fusion Approach

POINT	PROVISIONAL COORDINATES		ADJUSTED COORDINATES	
	N (m)	E (m)	N (m)	E (m)
1	99.9956	99.9971	99.9956	99.9971
2	446.4087	300.0064	446.4087	300.0064
3	100.0057	499.9965	100.0057	499.9965
4	273.0500	200.0700	272.8809	200.1552
5	273.0500	400.0900	272.8795	400.1517
6	100.0600	300.0500	100.1419	300.0960

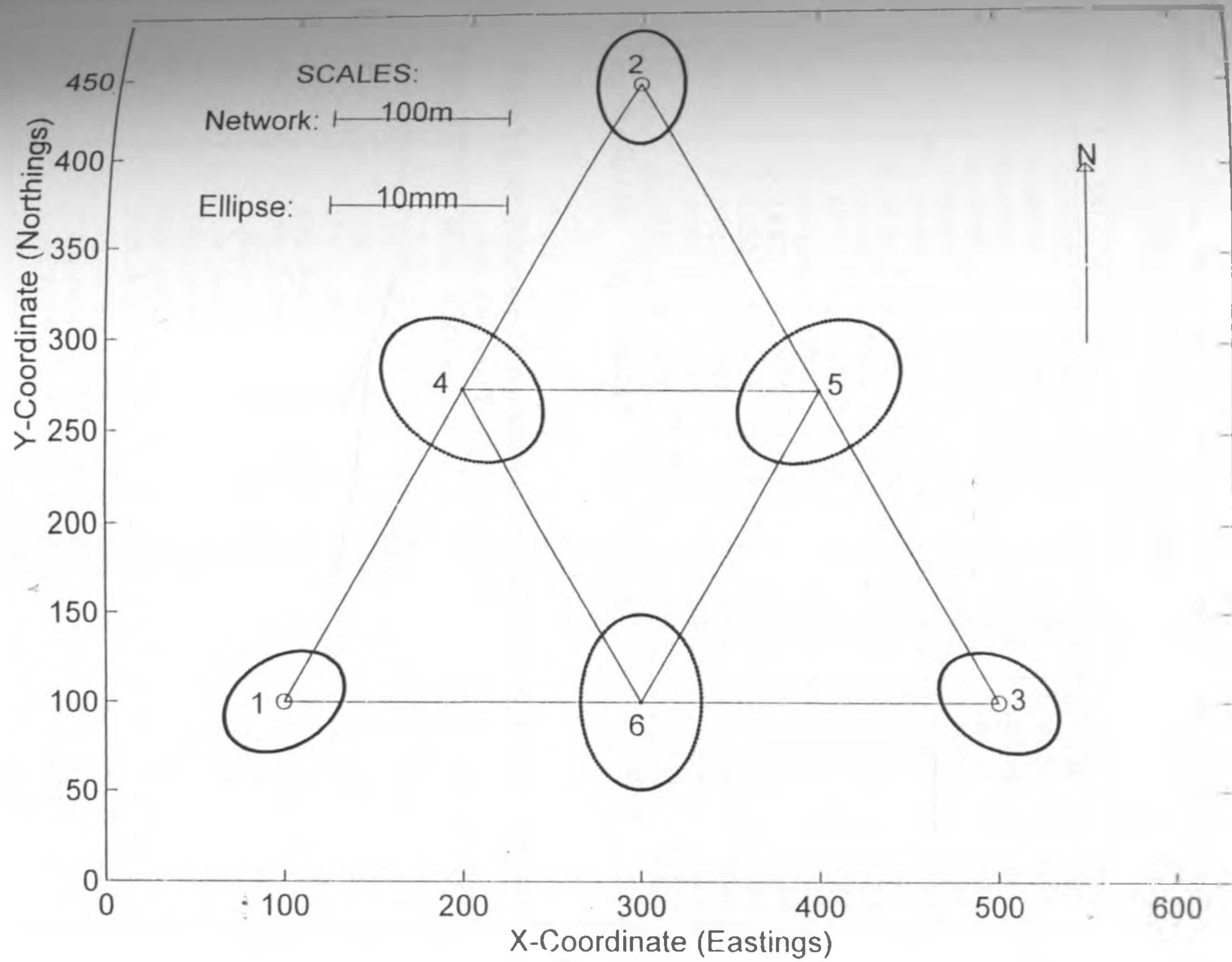


Figure 5.4: Point error ellipses -first level simulated network densification sub-optimal fusion approach

5.2.3.2 Second Level Densification

In the second level simulated network densification, first order points 1, 2, 3 and second order points 4, 5, 6 were considered as fixed stochastic parameters while third order points 7 to 15 were considered as new points whose coordinates were to be estimated {cf. Figure (4.4)}. The observation data sets in Table (4.6) and the approximate coordinates in Table (4.5b) were used resulting in parameters listed in Table (5.2.9) and Table (5.2.10) below while error ellipses are given diagrammatically in Figure (5.5).

Table 5.2.9: Coordinate Corrections and Stochastic Parameters-Second Level Simulated Network -Sub-Optimal Fusion Approach

POINT	ΔN (m)	ΔE (m)	σ_N (m)	σ_E (m)	σ_{max} (m)	σ_{min} (m)	α ° ' ''
1	0.0000	0.0000	0.0028	0.0034	0.0036	0.0025	60 0 0.1
2	0.0000	0.0000	0.0036	0.0025	0.0036	0.0025	03 0 0
3	0.0000	0.0000	0.0028	0.0034	0.0036	0.0025	300 59 59.9
4	0.0000	0.0000	0.0048	0.0040	0.0050	0.0036	302 54 21.1
5	0.0000	0.0000	0.0048	0.0040	0.0050	0.0036	57 05 12.4
6	0.0000	0.0000	0.0035	0.0051	0.0051	0.0035	01 01 05.1
7	0.0232	-0.1308	0.0023	0.0030	0.0030	0.0023	21 39 18.8
8	0.0574	-0.1359	0.0027	0.0028	0.0030	0.0024	83 38 18.6
9	0.0770	-0.1156	0.0024	0.0024	0.0024	0.0023	37 14 54.5
10	0.1403	-0.0776	0.0031	0.0024	0.0031	0.0023	338 58 23.1
11	0.1447	-0.0934	0.0031	0.0024	0.0031	0.0023	21 10 27.6
12	0.1550	-0.0943	0.0026	0.0023	0.0025	0.0023	359 58 10.0
13	0.0722	-0.0264	0.0027	0.0028	0.0030	0.0024	276 03 59.8
14	0.0055	-0.0566	0.0023	0.0030	0.0030	0.0028	333 23 23.1
15	0.0855	-0.0916	0.0024	0.0024	0.0024	0.0023	323 05 21.1

$\bar{\sigma}_N$=0.003158 $\bar{\sigma}_E$=0.003156 $\bar{\sigma}_C$=0.003157 $\hat{\sigma}_0^2$=1.004917

Table 5.2.10: Estimated Coordinates- Second Level Simulated Network Densification -Sub-Optimal Fusion Approach

POINT	PROVISIONAL COORDINATES N (m)	PROVISIONAL COORDINATES E (m)	ADJUSTED COORDINATES N (m)	ADJUSTED COORDINATES E (m)
1	99.9956	99.9971	99.9956	99.9971
2	446.4087	300.0064	446.4087	300.0064
3	100.0057	499.9965	100.0057	499.9965
4	272.8809	200.1552	272.8809	200.1552
5	272.8795	400.1517	272.8795	400.1517
6	100.1419	300.0960	100.1419	300.0960
7	100.0000	200.0000	100.0232	199.8692
8	186.6000	150.0000	186.6574	149.8641
9	186.6020	250.0000	186.6790	249.8844
10	359.8080	250.0000	359.9483	249.9224
11	359.8080	350.0000	359.9527	349.9066
12	273.2050	300.0000	273.3600	299.9057
13	186.6020	450.0000	186.6742	449.9736
14	100.0000	400.0000	100.0055	399.9434
15	186.6020	350.0000	186.6875	349.9084

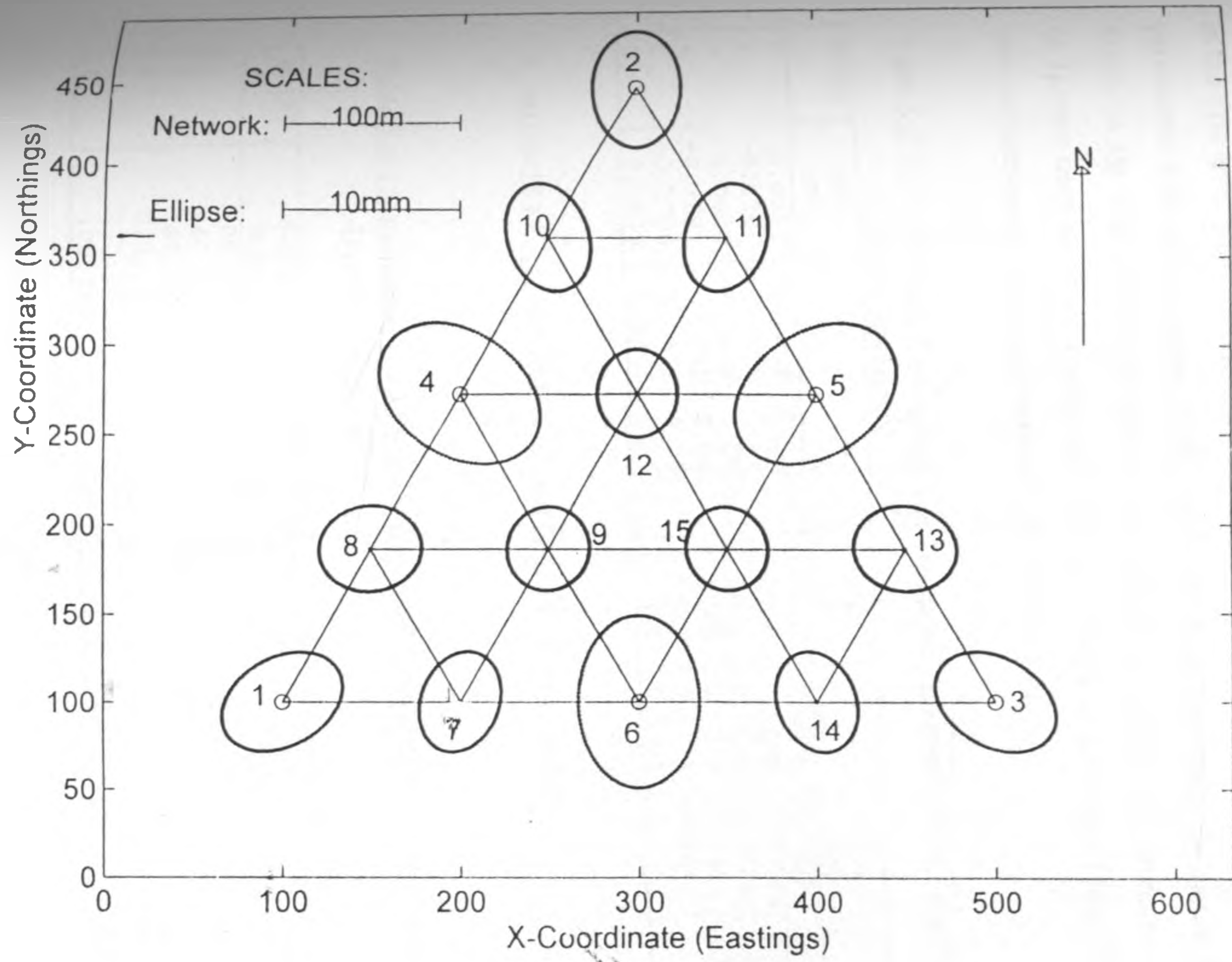


Figure 5.5: Point error ellipses -second level simulated network densification sub-optimal fusion approach

5.2.4 Dynamic Densification

The results presented in this section were determined by considering the estimated coordinates of the first order simulated network listed in Table (5.2.2) as stochastic constraints. The *dynamic* approach was then used to densify the first order network by intercalating on to the network second and third order points giving results for first level densification {c.f. section (5.2.4.1)} and second level densification {c.f. section (5.2.4.2)}.

5.2.4.1 First Level Densification

In the first level simulated network densification, all points 1 to 6 were considered as stochastic parameters with all the points being estimated afresh including the datum points 1, 2, 3. (cf. Figure (4.3)). The observation data sets in Table (4.4) and the approximate coordinates in Table (4.3) were used resulting in parameters listed in Table (5.2.11) and Table (5.2.12) below while error ellipses are given diagrammatically in Figure (5.6).

Table 5.2.11: Coordinate Corrections and Stochastic Parameters-First Level Simulated Network -Dynamic Approach

POINT	ΔN (m)	ΔE (m)	σ_N (m)	σ_E (m)	σ_{max} (m)	σ_{min} (m)	α ° ' "
1	0.0095	0.0208	0.0029	0.0036	0.0036	0.0029	60 0 0.1
2	-0.0264	-0.0039	0.0039	0.0026	0.0039	0.0026	364 59 50.0
3	0.0059	-0.0190	0.0029	0.0036	0.0036	0.0029	301 59 59.9
4	-0.1691	0.0852	0.0055	0.0046	0.0059	0.0041	302 54 21.1
5	-0.1705	0.0617	0.0055	0.0046	0.0059	0.0041	57 05 12.4
6	0.0819	0.0460	0.0040	0.0058	0.0058	0.0040	00 01 05.1

$\bar{\sigma}_N$=0.004253 $\bar{\sigma}_E$=0.004255 $\bar{\sigma}_C$=0.004254 $\hat{\sigma}_0^2$=1.000725

Table 5.2.12: Estimated Coordinates- First Level Simulated Network Densification -Dynamic Approach

POINT	PROVISIONAL COORDINATES N (m)	PROVISIONAL COORDINATES E (m)	ADJUSTED COORDINATES N (m)	ADJUSTED COORDINATES E (m)
1	99.9956	99.9971	100.0051	100.0179
2	446.4087	300.0064	446.3823	300.0025
3	100.0057	499.9965	100.0116	499.9775
4	273.0500	200.0700	272.8809	200.1552
5	273.0500	400.0900	272.8795	400.1517
6	100.0600	300.0500	100.1419	300.0960

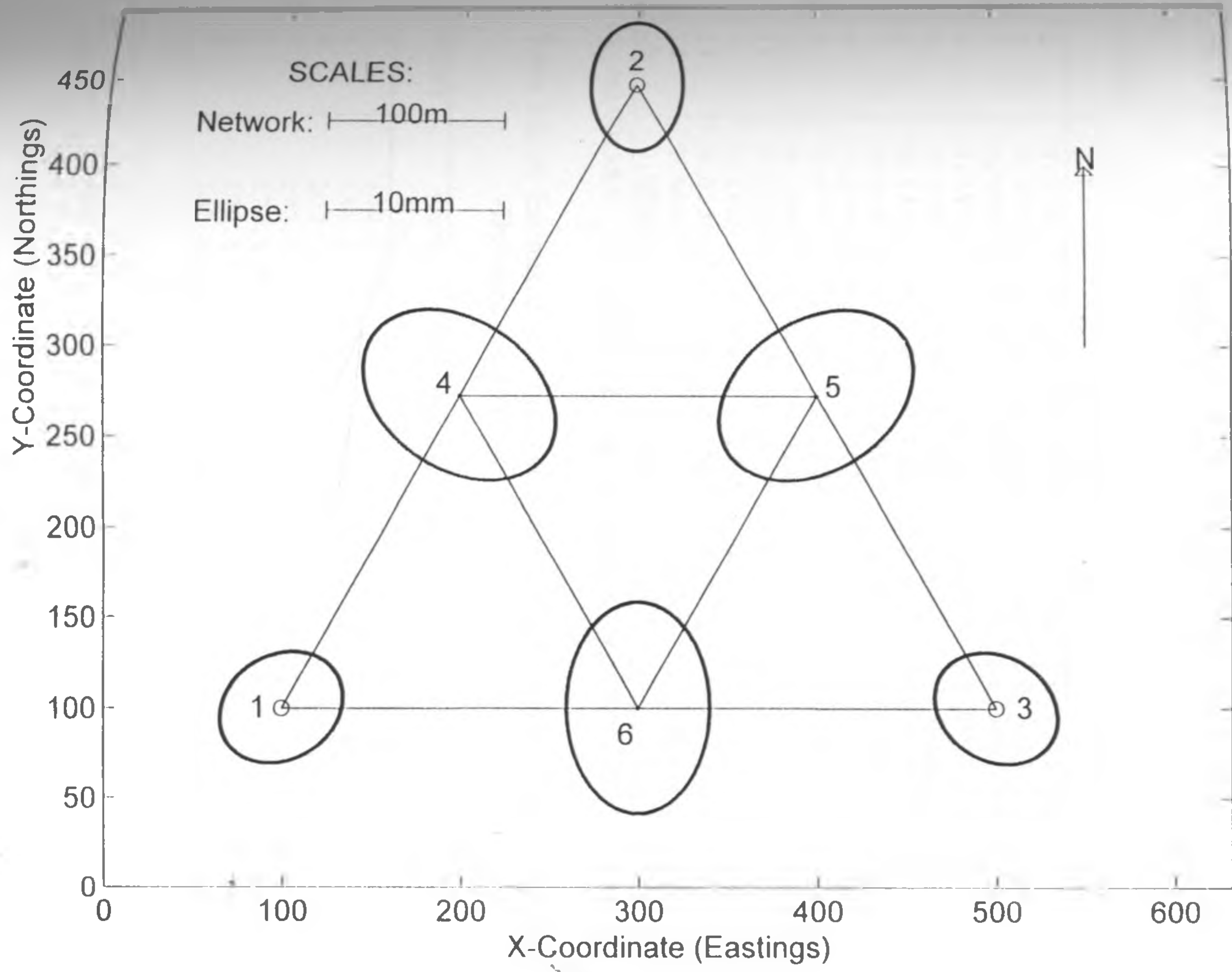


Figure 5.6 Point error ellipses -first level simulated network densification dynamic approach

5.2.4.2 Second Level Densification

In the second level simulated network densification, first order points 1 to 15 were considered as stochastic parameters with all the points being estimated including the datum point 1 to 6. {cf. Figure (4.4)}. The observation data sets in Table (4.6) and the approximate coordinates in Table (4.5c) were used resulting in parameters listed in Table (5.2.13) and Table (5.2.14) below while error ellipses are given diagrammatically in Figure (5.7).

Table 5.2.13: Coordinate Corrections and Stochastic Parameters-Second Level Simulated Network-Dynamic Approach

POINT	ΔN (m)	ΔE (m)	σ_N (m)	σ_E (m)	σ_{max} (m)	σ_{min} (m)	α ° ' "
1	0.0323	0.0973	0.0025	0.0032	0.0036	0.0025	60 0 0.1
2	0.0693	0.0564	0.0035	0.0022	0.0036	0.0025	03 0 0
3	0.0063	0.0034	0.0025	0.0032	0.0036	0.0025	302 59 59.9
4	0.1600	0.0338	0.0029	0.0026	0.0049	0.0035	302 54 21.1
5	0.1400	0.0645	0.0029	0.0026	0.0049	0.0035	57 05 12.4
6	0.1315	0.0109	0.0024	0.0029	0.0049	0.0034	02 01 05.1
7	0.0164	-0.1374	0.0026	0.0034	0.0034	0.0026	21 39 18.8
8	0.0580	-0.1468	0.0030	0.0031	0.0033	0.0027	83 38 18.6
9	0.0775	-0.1180	0.0026	0.0027	0.0027	0.0026	37 14 54.5
10	0.1538	-0.0802	0.0035	0.0027	0.0035	0.0026	338 58 23.1
11	0.1584	-0.0878	0.0035	0.0027	0.0035	0.0026	21 10 27.6
12	0.1615	-0.0931	0.0028	0.0026	0.0028	0.0026	359 58 10.0
13	0.0730	-0.0143	0.0030	0.0031	0.0033	0.0027	276 03 59.8
14	0.0002	-0.0483	0.0026	0.0034	0.0034	0.0025	333 23 23.1
15	0.0862	-0.0871	0.0026	0.0027	0.0027	0.0026	323 05 21.1

$\bar{\sigma}_N$ =0.002883 $\bar{\sigma}_E$ =0.002893 $\bar{\sigma}_c$ =0.002888 $\hat{\sigma}_0^2$=1.002717

Table 5.2.14: Estimated Coordinates- Second Level Simulated Network Densification -Dynamic Approach

POINT	PROVISIONAL COORDINATES N(m)	PROVISIONAL COORDINATES E(m)	ADJUSTED COORDINATES N(m)	ADJUSTED COORDINATES E(m)
1	100.0051	100.0179	100.0374	99.9206
2	446.3823	300.0025	446.4516	299.9461
3	100.0116	499.9775	100.0179	499.9809
4	272.8809	200.1552	272.7209	200.1890
5	272.8795	400.1517	272.7395	400.2162
6	100.1419	300.0960	100.2734	300.0851
7	100.0000	200.0000	100.0164	199.8626
8	186.6000	150.0000	186.6580	149.8532
9	186.6020	250.0000	186.6795	249.8820
10	359.8080	250.0000	359.9618	249.9198
11	359.8080	350.0000	359.9664	349.9122
12	273.2050	300.0000	273.3665	299.9069
13	186.6020	450.0000	186.6750	449.9857
14	100.0000	400.0000	99.9998	399.9517
15	186.6020	350.0000	186.6882	349.9129

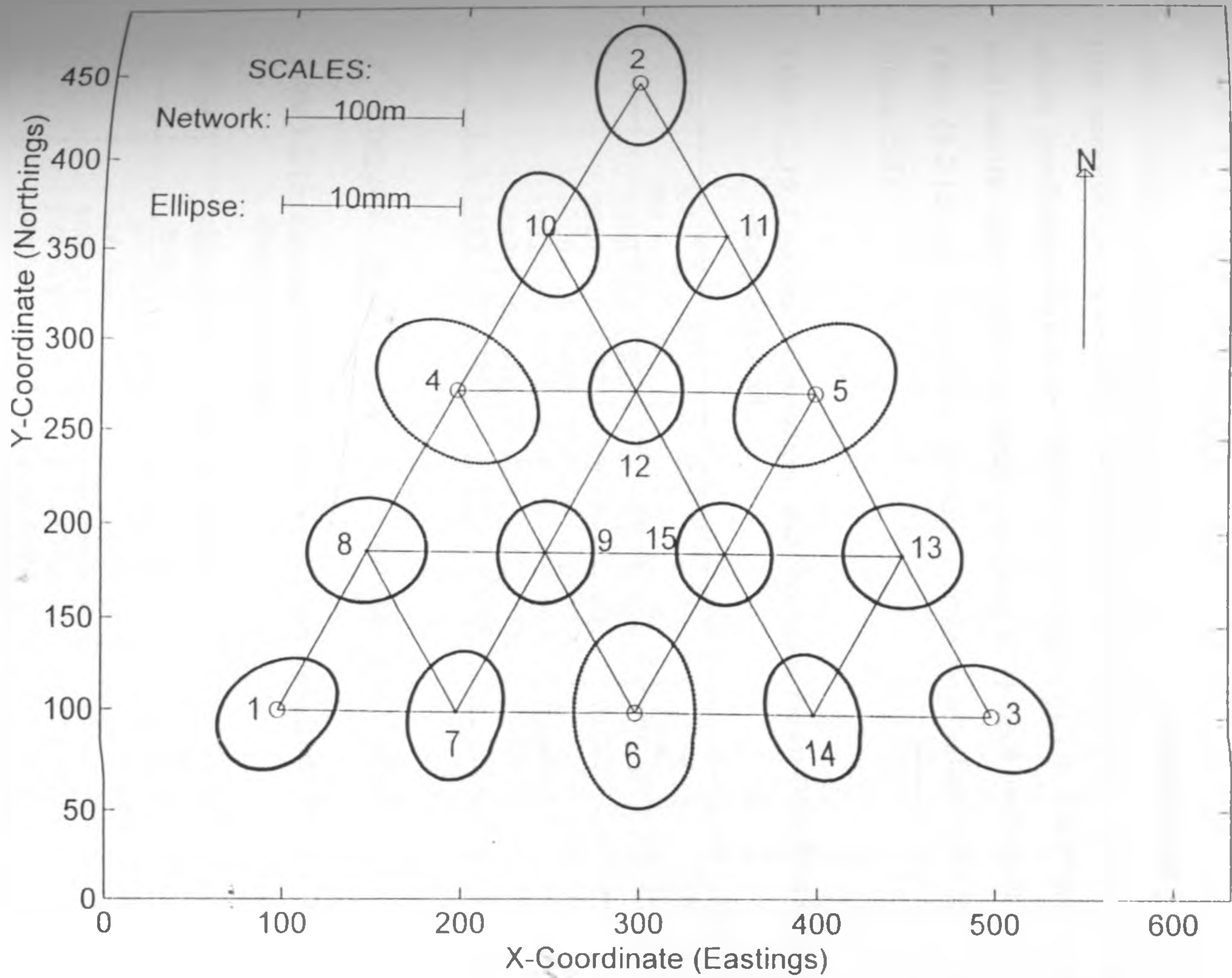


Figure 5.7: Point error ellipses -second level simulated network densification dynamic approach

5.2.5 Static Densification

The results presented in this section were determined by considering the estimated coordinates of the first order simulated network listed in Tables (5.2.2) as fixed non-stochastic entities. The *static* approach was then used to densify the first order networks by intercalating into them second and third order points giving results for first level densification {cf. section (5.2.5.1)} and second level densification {cf. section (5.2.5.2)}.

5.2.5.1 First Level Densification

In the first level simulated network densification, first order points 1, 2, 3 were considered as fixed non-stochastic parameters while second order points 4, 5, 6 were considered as new points whose coordinates were to be estimated (cf. Figure (4.3)). The observation data sets in Table (4.4) and the approximate coordinates in Table (4.3) were used resulting in parameters listed in Table (5.2.15) and Table (5.2.16) below while error ellipses are given diagrammatically in Figure (5.8).

Table 5.2.15: Coordinate Corrections and Stochastic Parameters-First Level Simulated Network -Static Approach

POINT	ΔN (m)	ΔE (m)	σ_N (m)	σ_E (m)	σ_{max} (m)	σ_{min} (m)	α ° ' ''
1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0
2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0
3	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0
4	-0.1231	0.0538	0.0039	0.0031	0.0042	0.0026	299 58 11.4
5	-0.1243	0.0671	0.0039	0.0031	0.0042	0.0026	60 01 27.3
6	0.0499	0.0385	0.0026	0.0042	0.0042	0.0026	00 00 40.4

$\bar{\sigma}_N$=0.002489 $\bar{\sigma}_E$=0.002476 $\bar{\sigma}_c$=0.002484 $\hat{\sigma}_0^2$=1.00400

Table 5.2.16: Estimated Coordinates- First Level Simulated Network Densification -Static Approach

POINT	PROVISIONAL COORDINATES N(m)	PROVISIONAL COORDINATES E(m)	ADJUSTED COORDINATES N(m)	ADJUSTED COORDINATES E(m)
1	99.9956	99.9971	99.9956	99.9971
2	446.4087	300.0064	446.4087	300.0064
3	100.0057	499.9965	100.0057	499.9965
4	273.0500	200.0700	272.9269	200.1238
5	273.0500	400.0900	272.9257	400.1571
6	100.0600	300.0500	100.1099	300.0885

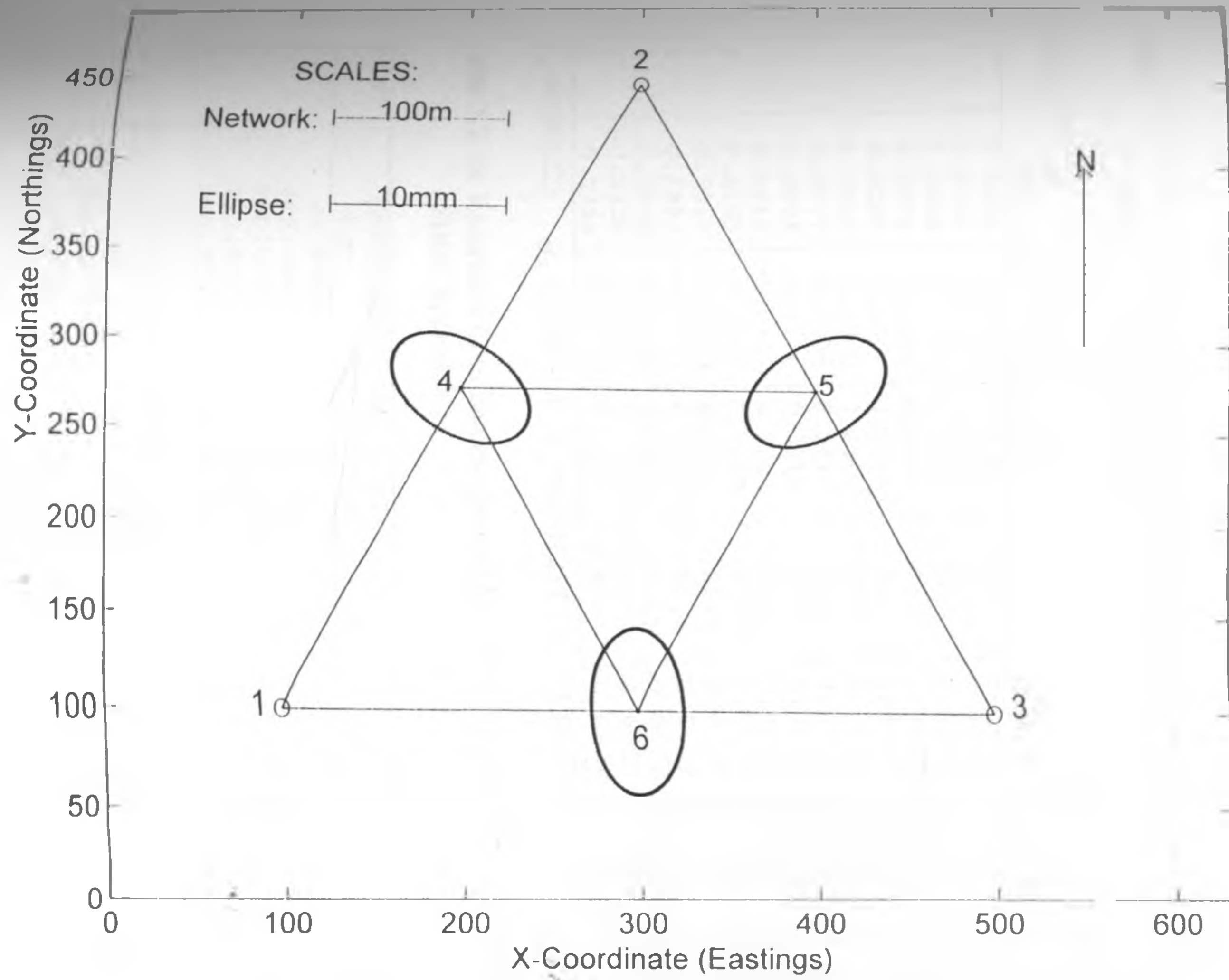


Figure 5.8: Point error ellipses -first level simulated network densification static approach

5.2.5.2 Second Level Densification

In the second level simulated network densification, first order points 1, 2, 3 and second order points 4, 5, 6 were considered as fixed non-stochastic parameters while third order points 7 to 15 were considered as new points whose coordinates were to be estimated (cf. Figure (4.4)). The observation data sets in Table (4.6) and the approximate coordinates in Table (4.5d) were used resulting in parameters listed in Table (5.2.13) and Table (5.2.14) below while error ellipses are given diagrammatically in Figure (5.9).

Table 5.2.17: Coordinate Corrections and Stochastic Parameters-Second Level Simulated Network - Static Approach

POINT	ΔN (m)	ΔE (m)	σ_N (m)	σ_E (m)	σ_{max} (m)	σ_{min} (m)	α ° ' "
1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0
2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0
3	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0
4	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0
5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0
6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0
7	0.0277	-0.1129	0.0013	0.0020	0.0020	0.0013	09 07 52.9
8	0.0304	-0.1136	0.0018	0.0015	0.0020	0.0012	290 46 26.5
9	0.0642	-0.0937	0.0014	0.0013	0.0014	0.0012	60 25 00.2
10	0.1465	-0.0367	0.0019	0.0014	0.0020	0.0013	309 14 21.4
11	0.1516	-0.0915	0.0019	0.0014	0.0020	0.0013	50 55 08.4
12	0.1957	-0.0733	0.0012	0.0014	0.0014	0.0012	359 57 27.9
13	0.0503	-0.0121	0.0018	0.0015	0.0020	0.0012	69 00 07.6
14	-0.0338	-0.0368	0.0013	0.0020	0.0020	0.0012	350 55 41.6
15	0.0759	-0.0739	0.0014	0.0013	0.0014	0.0012	299 28 22.2

$\bar{\sigma}_N = 0.001223$ $\bar{\sigma}_E = 0.001204$ $\bar{\sigma}_C = 0.001214$ $\hat{\sigma}_0^2 = 1.002736$

Table 5.2.18: Estimated Coordinates- Second Level Simulated Network Densification -Static Approach

POINT	PROVISIONAL COORDINATES N (m)	E (m)	ADJUSTED COORDINATES N (m)	E (m)
1	99.9956	99.9971	99.9956	99.9971
2	446.4087	300.0064	446.4087	300.0064
3	100.0057	499.9965	100.0057	499.9965
4	272.9269	200.1238	272.9269	200.1238
5	272.9257	400.1571	272.9257	400.1571
6	100.1099	300.0885	100.1099	300.0885
7	100.0000	200.0000	99.9723	199.8871
8	186.6000	150.0000	186.6304	149.8864
9	186.6020	250.0000	186.6662	249.9063
10	359.8080	250.0000	359.9545	249.9633
11	359.8080	350.0000	359.9596	349.9085
12	273.2050	300.0000	273.4007	299.9267
13	186.6020	450.0000	186.6523	449.9879
14	100.0000	400.0000	99.9662	399.9632
15	186.6020	350.0000	186.6779	349.9261

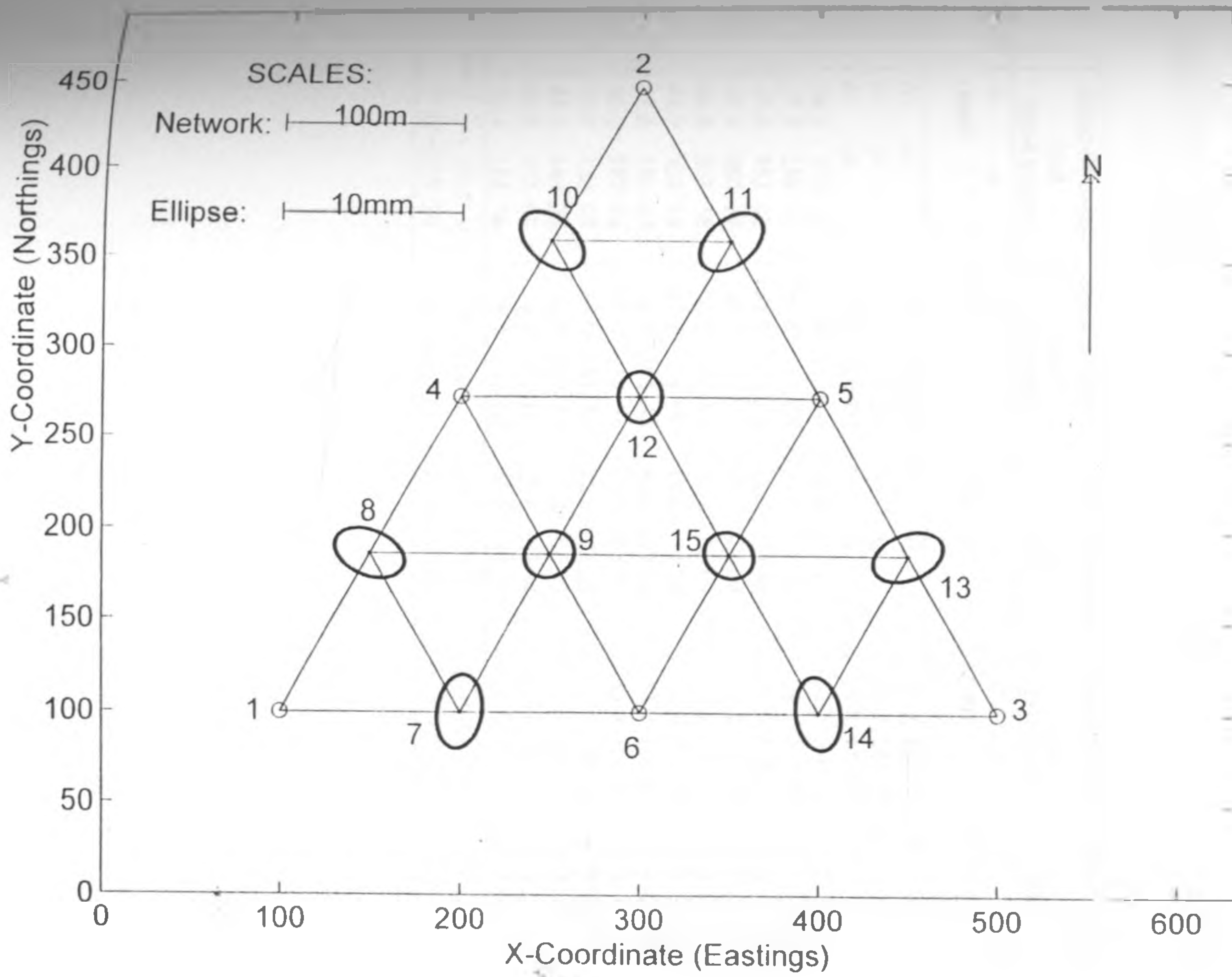


Figure 5.9: Point error ellipses -second level simulated network densification static approach

5.2.6 Computed Shifts

The final coordinates obtained from the static-dynamic, sub-optimal Fusion, dynamic and static densifications were compared to each other and the results tabulated as shown in Table (5.2.19). Figures (5.10 to 5.15) depict these shifts graphically. In the Tables, δ is the magnitude of the shift in the two sets of coordinates for each point while α is the bearing of the shift. Also, $\bar{\delta}$ and $\bar{\alpha}$ are the mean of δ and α for the respective densification approaches.

Table 5.2.19: Shifts between estimated parameters for the Experiments (Simulated Network)

	Static-Dynamic and Sub-Optimal		Static-Dynamic and Dynamic		Static-Dynamic and Static		Sub-Optimal and Dynamic		Sub-Optimal and Static		Dynamic and Static	
St	δ (mm)	α (° ')	δ (mm)	α (° ')	δ (mm)	α (° ')	δ (mm)	α (° ')	δ (mm)	α (° ')	δ (mm)	α (° ')
1	0	0	87.2	118 39	0	0	87.2	118 39	0	0	81.5	239 09
2	0	0	74.0	125 26	0	0	74.0	125 26	0	0	74.0	305 26
3	0	0	19.8	128 02	0	0	19.8	128 02	0	0	19.8	308 02
4	24.2	134 10	0.8	343 44	33.6	155 21	163.5	348 4	55.7	145 41	216.1	162 25
5	27.3	236 39	160.8	344 58	35.5	150 36	154.1	335 16	46.5	186 40	195.3	16 23
6	19.5	352 02	151.4	174 51	16.3	38 46	131.9	175 16	32.9	13 11	163.5	358 48
7	42.4	280 12	48.3	90 50	49.5	28 44	9.5	44 09	54.0	340 37	50.4	330 57
8	46.5	293 58	56.8	110 04	21.8	68 09	10.9	93 09	35.0	320 27	43.2	309 44
9	44.9	302 10	47.2	121 08	19.6	124 35	2.5	101 46	25.4	300 18	27.7	298 42
10	51.6	332 03	64.9	155 36	54.4	197 52	13.7	169 06	41.4	261 23	44.1	279 32
11	56.2	327 16	65.8	157 53	61.2	152 16	14.8	202 14	7.2	195 24	7.7	28 33
12	59.8	330 08	65.0	153 54	93.0	174 34	6.6	190 28	45.8	207 18	39.5	210 04
13	24.4	340 23	24.1	189 18	6.2	100 13	12.1	266 13	26.2	326 51	22.8	354 28
14	16.8	272 03	9.9	59 02	38.8	355 34	10.1	304 29	44.0	333 16	35.5	341 06
15	38.4	314 16	35.9	140 05	19.8	150 20	4.6	261 09	20.1	298 28	16.7	307 58
Mean	$\bar{\delta}$= 37.7	$\bar{\alpha}$=275 20	$\bar{\delta}$= 60.8	$\bar{\alpha}$=268 32	$\bar{\delta}$= 37.5	$\bar{\alpha}$=253 00	$\bar{\delta}$=47.7	$\bar{\alpha}$=343 26	$\bar{\delta}$= 36.2	$\bar{\alpha}$= 49 34	$\bar{\delta}$= 69.2	$\bar{\alpha}$=253 17

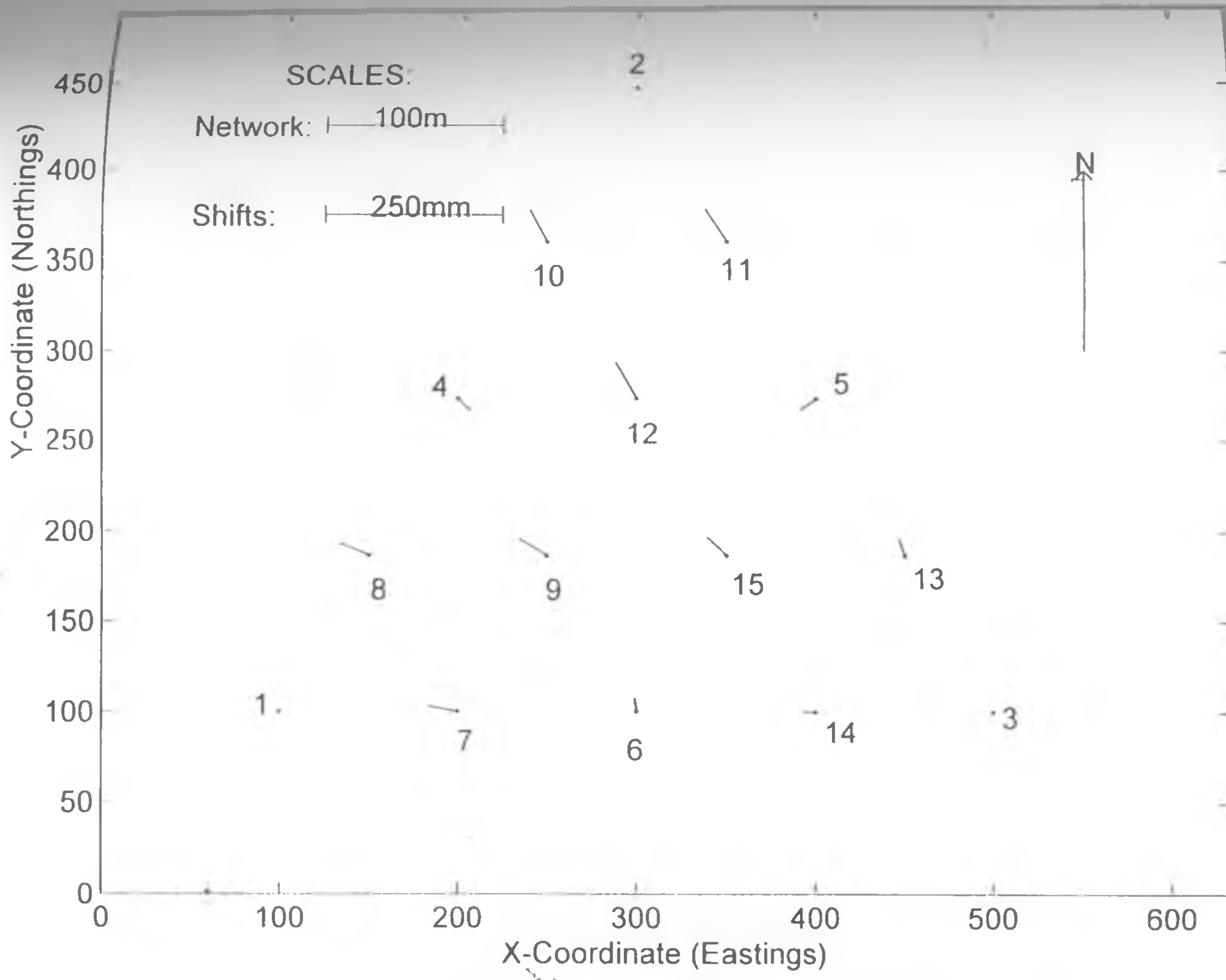


Figure 5.10 Coordinate shifts of sub-optimal fusion approach with respect to static-dynamic approach

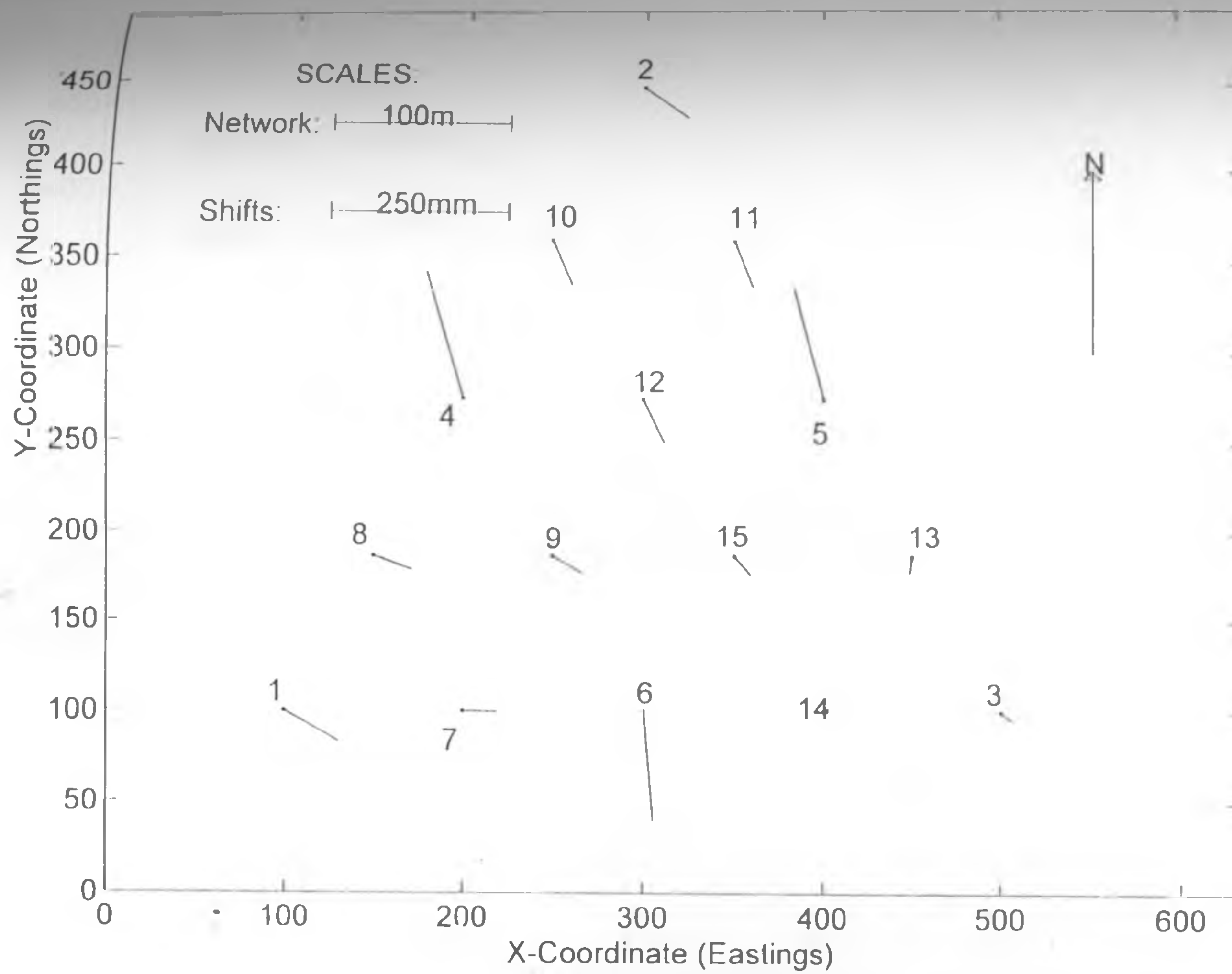


Figure 5.11: Coordinate shifts of dynamic approach with respect to static-dynamic approach

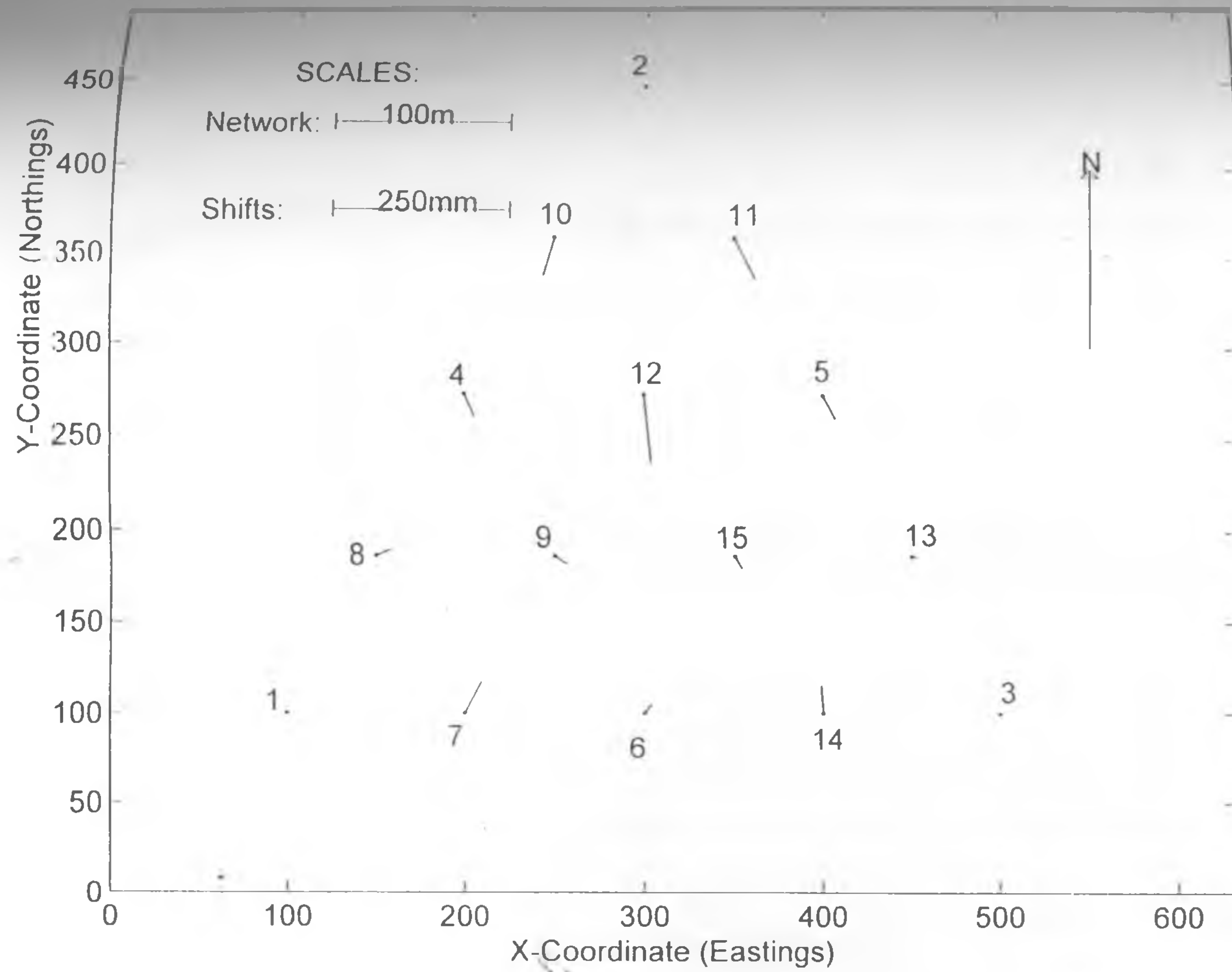


Figure 5.12: Coordinate shifts of static approach with respect to static-dynamic approach

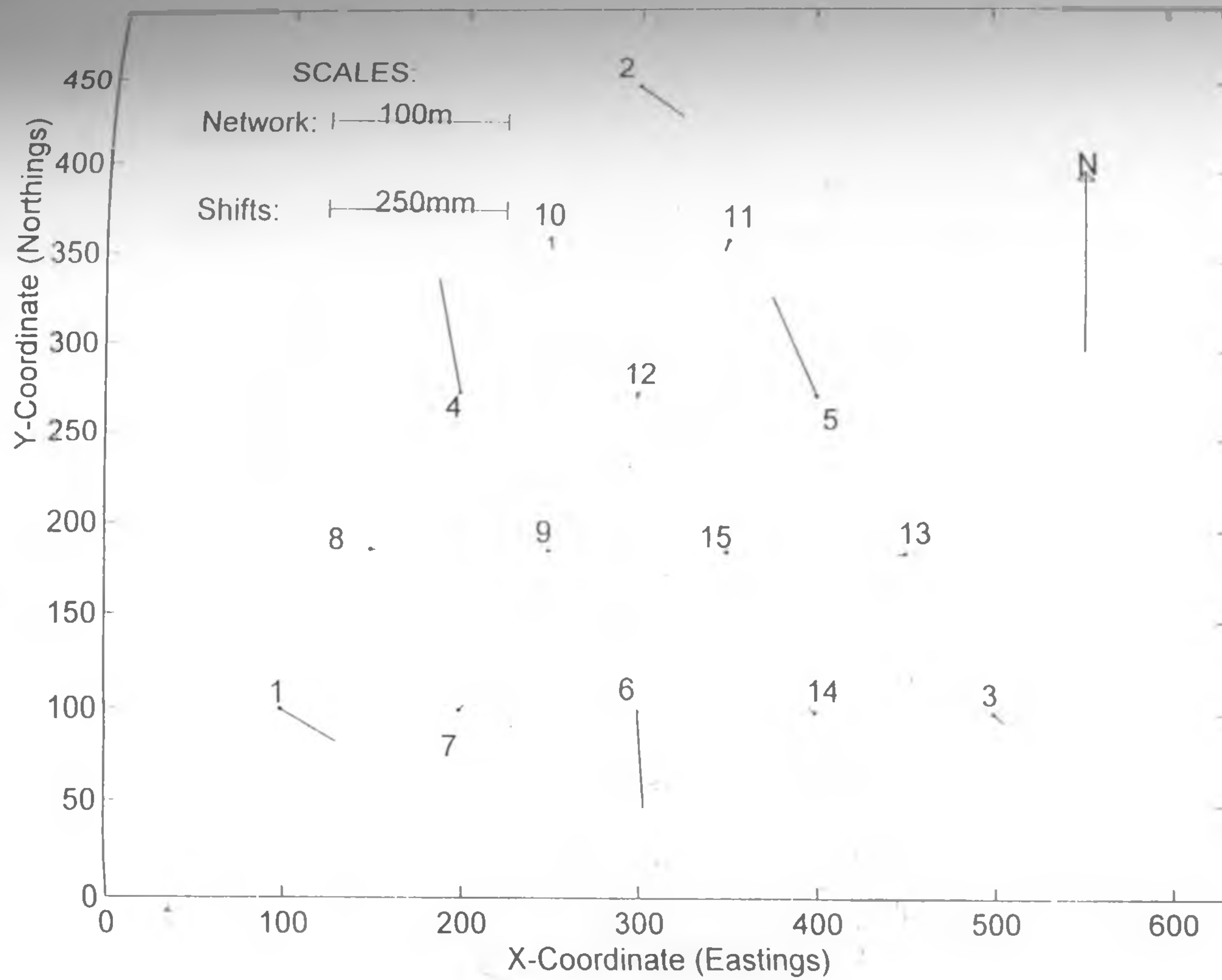


Figure 5.13: Coordinate shifts of dynamic approach with respect to sub-optimal fusion approach

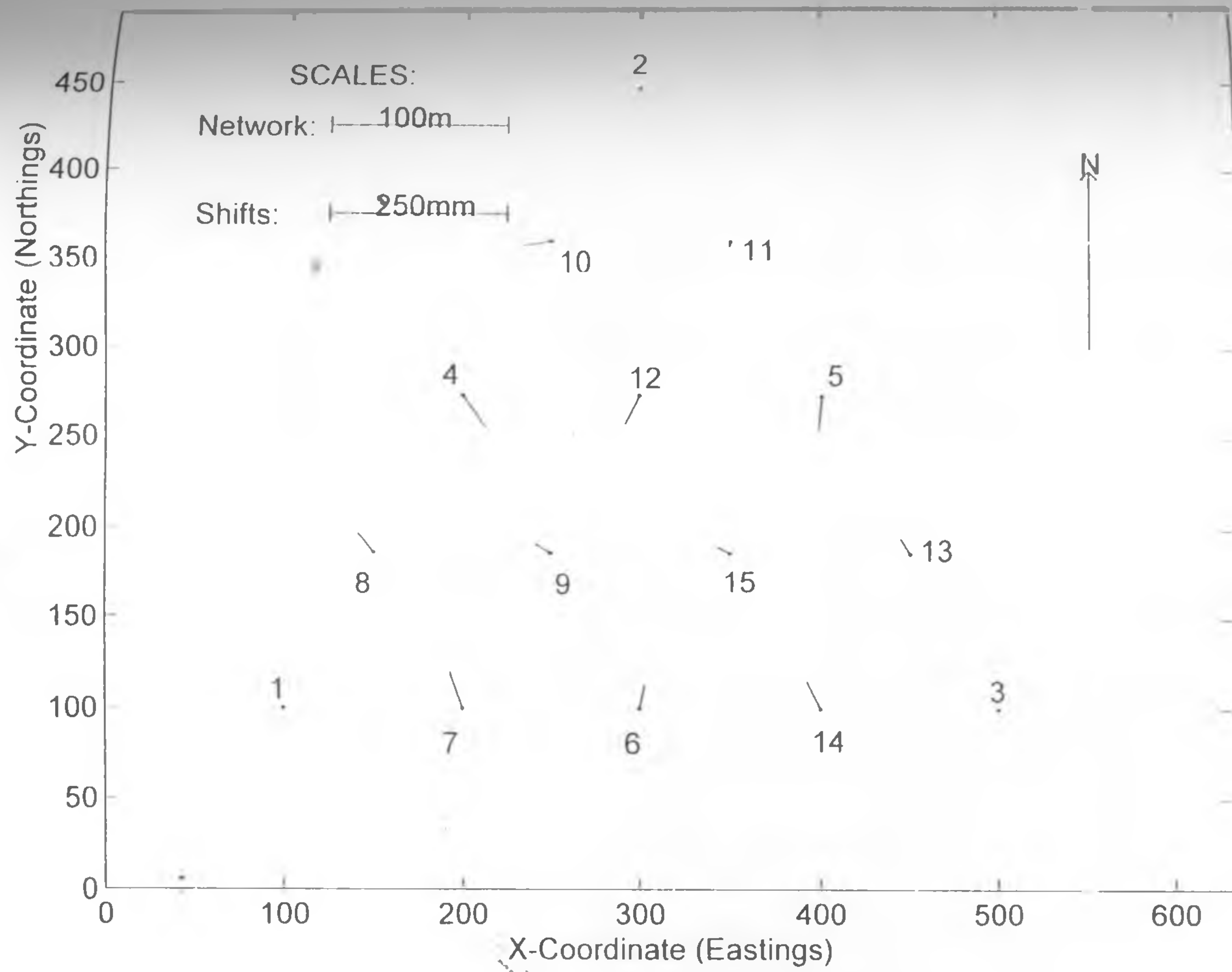


Figure 5.14: Coordinate shifts of static approach with respect to sub-optimal fusion approach

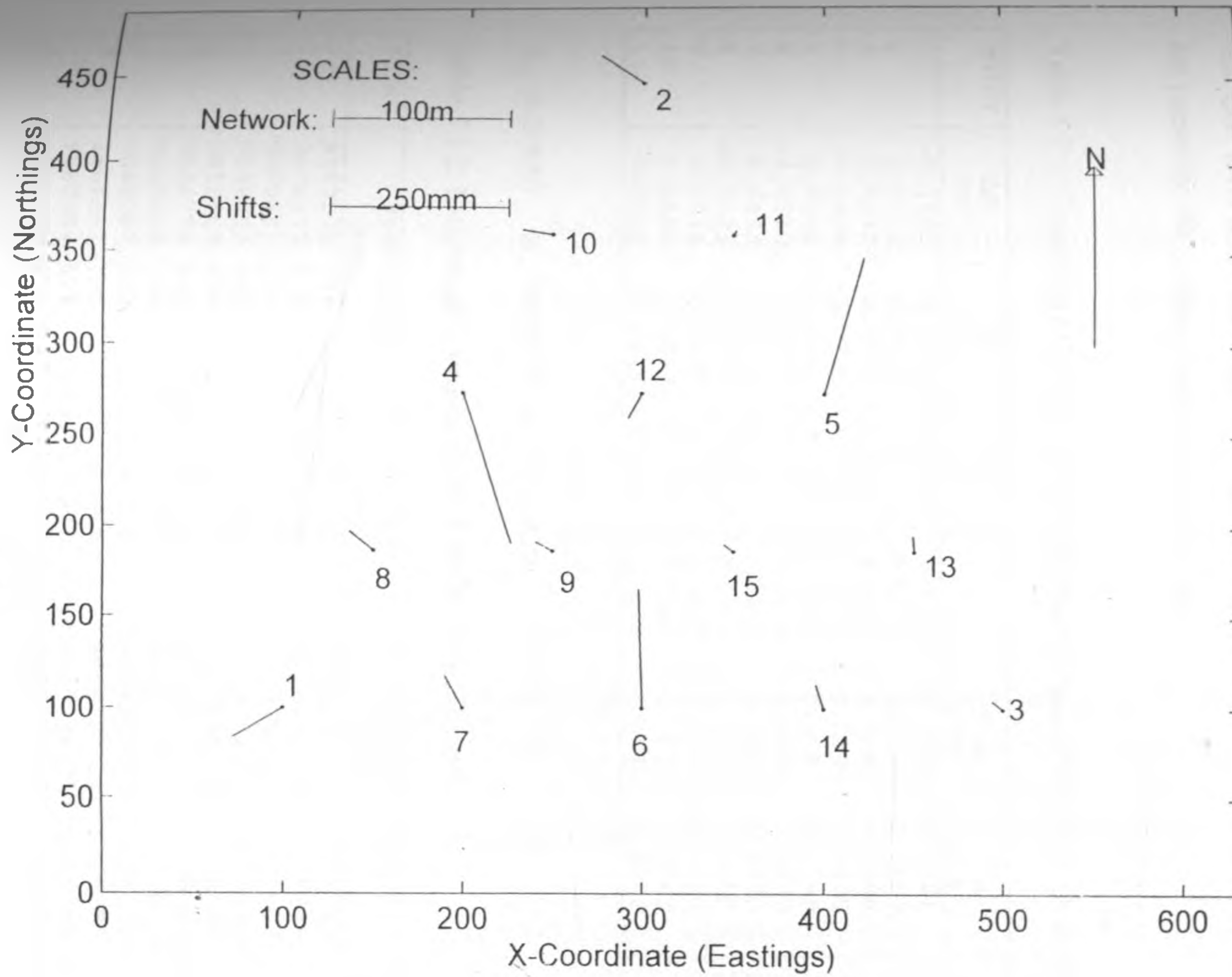


Figure 5.15: Coordinate shifts of static approach with respect to dynamic approach

5.3 The Real Network

5.3.1 First Order Network Adjustment

The results presented in this section were obtained by adjusting the real first order network through the free network technique using the approximate coordinates listed in Table (4.7) and observation data sets listed in Table (4.8). This formed the fundamental network on which the real first level and real second level densifications were performed. The resulting error ellipses are presented diagrammatically in Figure (5.16).

Table 5.3.1: Coordinate Corrections and Stochastic Parameters-First Order Real Network

POINT	ΔN (m)	ΔE (m)	σ_V (m)	σ_E (m)	σ_{max} (m)	σ_{min} (m)	α ° ' "
1	-0.1003	0.0389	0.0166	0.0125	0.0169	0.0121	29 21 57.1
2	0.0214	0.0328	0.0109	0.0119	0.0126	0.0100	291 58 8.1
3	0.0393	-0.0698	0.0099	0.0093	0.0105	0.0086	71 14 37.2
4	0.0420	-0.0009	0.0083	0.0093	0.0095	0.0080	309 20 32.9
5	0.0068	-0.0961	0.0074	0.0082	0.0087	0.0069	303 40 52.7
6	-0.0310	0.0077	0.0075	0.0091	0.0093	0.0073	319 09 10.3
7	0.0438	0.0159	0.0072	0.0075	0.0077	0.0070	290 17 24.7
8	-0.0329	-0.0437	0.0057	0.0070	0.0070	0.0057	352 25 4.2
9	0.0314	0.0305	0.0078	0.0073	0.0078	0.0073	353 35 36.1
10	0.0596	0.0399	0.0071	0.0067	0.0071	0.0067	01 58 46.1
11	-0.0802	0.0448	0.0085	0.0078	0.0090	0.0072	68 12 59.1

$\bar{\sigma}_V$=0.00880 $\bar{\sigma}_E$=0.00878 $\bar{\sigma}_C$=0.00879 $\hat{\sigma}_0^2$=1.003744

Table 5.3.2: Estimated Coordinates-First Order Real Network

POINT	PROVISIONAL COORDINATES		ADJUSTED COORDINATES	
	N (m)	E (m)	N (m)	E (m)
1	9583105.104	513215.911	9583105.004	513215.950
2	9576179.214	574648.574	9576179.235	574648.607
3	9600981.179	569580.645	9600981.218	569580.575
4	9599052.465	594885.489	9599052.507	594885.488
5	9611665.689	589503.327	9611665.696	589503.231
6	9624778.596	606329.877	9624778.565	606329.885
7	9640229.659	580182.061	9640229.703	580182.077
8	9644287.319	609203.330	9644287.286	609203.286
9	9661267.194	585576.660	9661267.225	585576.691
10	9660984.693	598816.071	9660984.753	598816.111
11	9661696.939	618181.173	9661696.859	618181.218

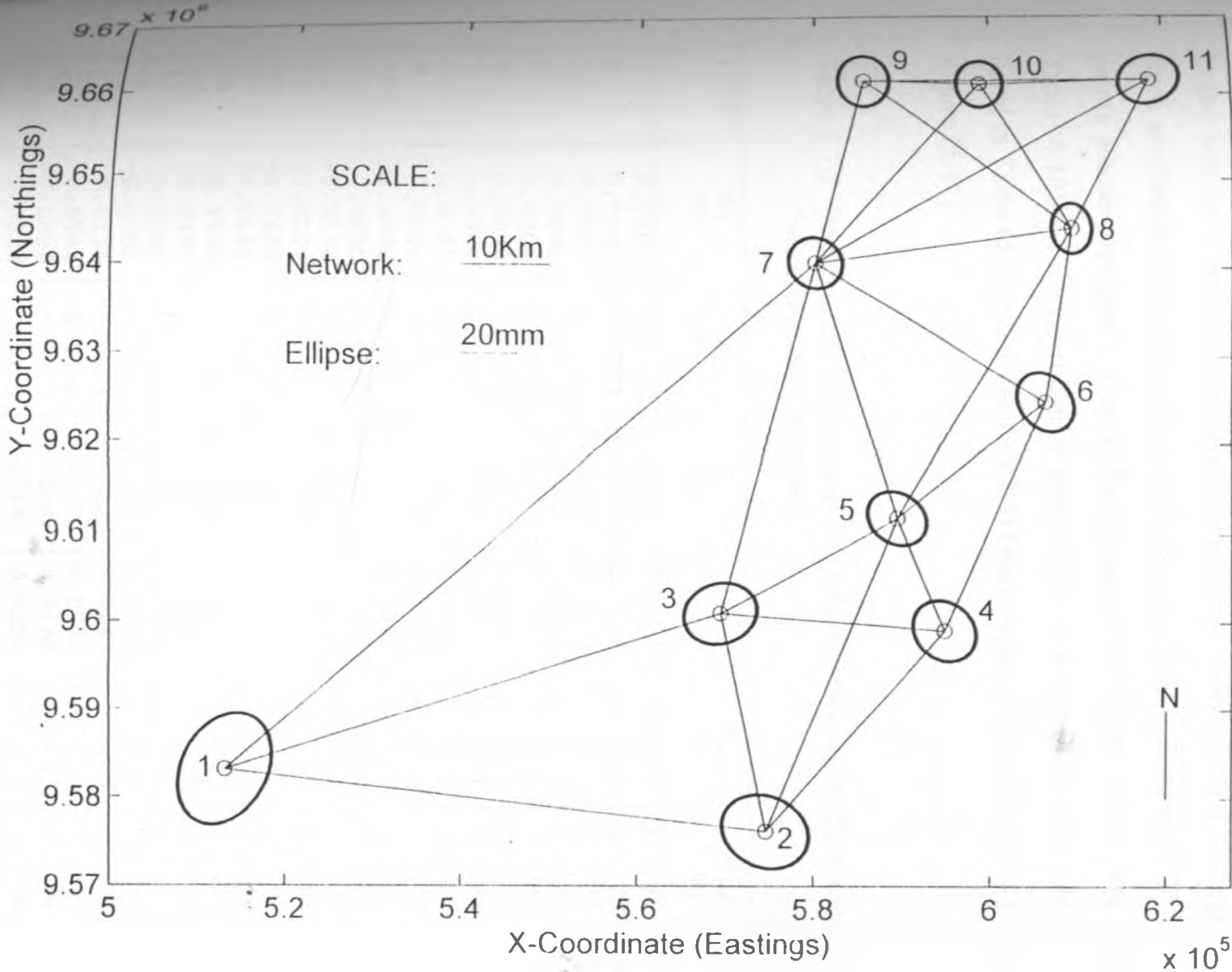


Figure 5.16: Point error ellipses -first order real network

5.3.2 Static-Dynamic Densification

The results presented in this section were determined by considering the estimated coordinates of the first order real network listed in Tables (5.3.2) as fixed stochastic constraints. The *static-dynamic* approach was then used to densify the first order network by intercalating into it second and third order points giving results for first level densification (cf. section 5.3.2.1) and second level densification (cf. section 5.3.2.2).

5.3.2.1 First Level Densification

In the first level real network densification, first order points 1 through 11 were considered as fixed stochastic parameters while second order points 12 through 26 were considered as new points whose coordinates were to be estimated (cf. Figure (4.6)). The observation data sets in Table (4.10) and the approximate coordinates in Table (4.9) were used resulting in parameters listed in Table (5.3.3) and Table (5.3.4) below while error ellipses are given diagrammatically in Figure (5.17).

Table 5.3.3: Coordinate Corrections and Stochastic Parameters-First Level Real Network -Static-Dynamic Approach

POINT	ΔN (m)	ΔE (m)	σ_N (m)	σ_E (m)	σ_{max} (m)	σ_{min} (m)	α ° ' "
1	0.0000	0.0000	0.0166	0.0125	0.0169	0.0121	29 21 57.1
2	0.0000	0.0000	0.0109	0.0119	0.0126	0.0100	291 58 8.1
3	0.0000	0.0000	0.0099	0.0093	0.0105	0.0086	74 14 37.2
4	0.0000	0.0000	0.0083	0.0093	0.0095	0.0080	310 20 32.9
5	0.0000	0.0000	0.0074	0.0082	0.0087	0.0069	304 40 52.7
6	0.0000	0.0000	0.0075	0.0091	0.0093	0.0073	319 9 10.3
7	0.0000	0.0000	0.0072	0.0075	0.0077	0.0070	290 17 24.7
8	0.0000	0.0000	0.0057	0.0070	0.0070	0.0057	352 25 4.2
9	0.0000	0.0000	0.0078	0.0073	0.0078	0.0073	353 35 36.1
10	0.0000	0.0000	0.0071	0.0067	0.0071	0.0067	01 58 46.1
11	0.0000	0.0000	0.0085	0.0078	0.0090	0.0072	68 12 59.1
12	-0.0154	0.0333	0.0145	0.0255	0.0259	0.0137	336 19 44.5
13	0.0214	0.0152	0.0336	0.0537	0.0605	0.0186	58 02 31.8
14	-0.0680	-0.0347	0.0361	0.0188	0.0361	0.0188	05 05 58.4
15	-0.0189	-0.0296	0.0201	0.0432	0.0445	0.0167	328 40 22.2
16	-0.0509	0.0724	0.0179	0.0176	0.0200	0.0150	85 28 52.6
17	0.0453	0.0162	0.0212	0.0531	0.0533	0.0206	11 12 15.0
18	0.0382	-0.1063	0.0233	0.0242	0.0258	0.0216	282 57 17.3
19	-0.1701	-0.1703	0.0397	0.0303	0.0428	0.0256	55 35 22.0
20	-0.0131	-0.1671	0.0202	0.0241	0.0248	0.0193	44 09 40.3
21	0.0585	-0.2382	0.0182	0.0235	0.0250	0.0163	51 48 14.8
22	-0.0856	0.0201	0.0204	0.0366	0.0385	0.0166	320 13 12.4
23	0.0880	0.0905	0.0218	0.0390	0.0403	0.0194	327 26 08.2
24	-0.2337	-0.2025	0.0203	0.0183	0.0203	0.0183	351 35 15.5
25	-0.2501	-0.0828	0.0414	0.0215	0.0413	0.0215	359 28 03.5
26	-0.2443	-0.1778	0.0348	0.0195	0.0355	0.0182	27 08 07.5

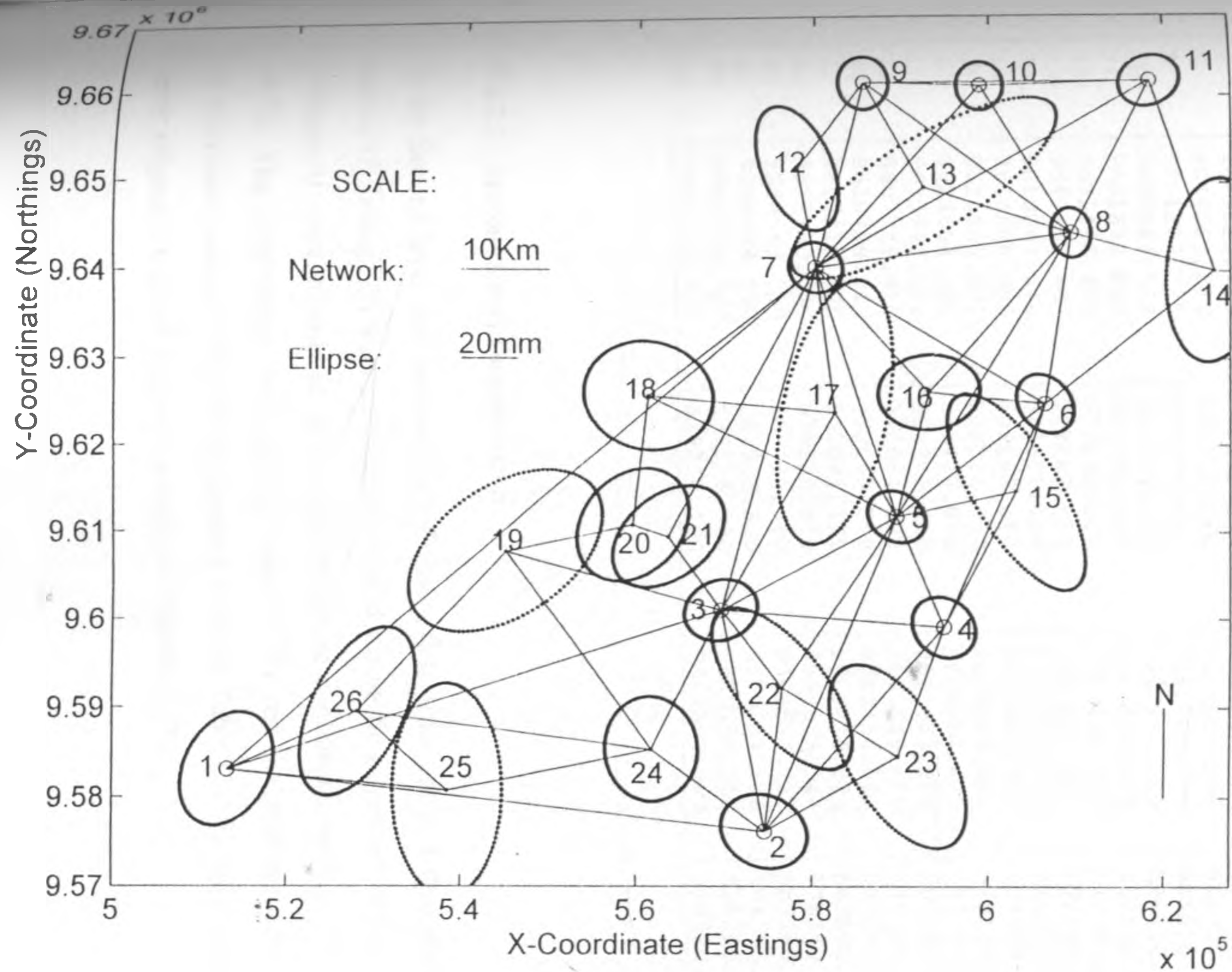


Figure 5.17: Point error ellipses -first level real network densification static-dynamic approach

$\overline{\sigma}_V$ =0.02134 $\overline{\sigma}_E$ =0.02515 $\overline{\sigma}_C$ =0.02332 $\hat{\sigma}_0^2$ =1.009214

Table 5.3.4: Estimated Coordinates- First Level Real Network Densification -Static-Dynamic Approach

POINT	PROVISIONAL COORDINATES		ADJUSTED COORDINATES	
	N(m)	E(m)	N(m)	E(m)
1	9583105.004	513215.950	9583105.004	513215.950
2	9576179.235	574648.607	9576179.235	574648.607
3	9600981.218	569580.575	9600981.218	569580.575
4	9599052.507	594885.488	9599052.507	594885.488
5	9611665.696	589503.231	9611665.696	589503.231
6	9624778.565	606329.885	9624778.565	606329.885
7	9640229.703	580182.077	9640229.703	580182.077
8	9644287.286	609203.286	9644287.286	609203.286
9	9661267.225	585576.691	9661267.225	585576.691
10	9660984.753	598816.111	9660984.753	598816.111
11	9661696.859	618181.218	9661696.859	618181.218
12	9651426.870	578082.210	9651426.855	578082.243
13	9649302.660	592482.850	9649302.681	592482.865
14	9640093.640	625611.430	9640093.572	625611.395
15	9614609.390	603055.440	9614609.371	603055.410
16	9626055.670	593121.870	9626055.619	593121.942
17	9623694.380	582449.870	9623694.425	582449.886
18	9625643.250	561292.670	9625543.288	561292.564
19	9607749.860	545064.590	9607749.690	545064.420
20	9610759.110	559563.590	9610759.097	559563.423
21	9609410.820	563591.710	9609410.878	563591.472
22	9592183.630	576550.570	9592183.544	576550.590
23	9584543.130	589735.740	9584543.218	589735.830
24	9585354.590	561756.300	9585354.356	561756.098
25	9580752.500	538526.380	9580752.250	538526.297
26	9589557.530	528224.190	9589557.286	528224.012

5.3.2.2 Second Level Densification

In the Second level real network densification, first order points 1 through 11 and second order points 12 through 26 were considered as fixed stochastic parameters while third order points 27 through 41 were considered as new points whose coordinates were to be estimated (cf. Figure (4.7)). The observation data sets in Table (4.12) and the approximate coordinates in Table (4.11a) were used resulting in parameters listed in Table (5.3.5) and Table (5.3.6) below while error ellipses are given diagrammatically in Figure (5.18).

Table 5.3.5: Coordinate Corrections and Stochastic Parameters-Second Level Real Network -Static-Dynamic Approach

POINT	ΔN (m)	ΔE (m)	σ_N (m)	σ_E (m)	σ_{max} (m)	σ_{min} (m)	α ° ' "
1	0.0000	0.0000	0.0166	0.0125	0.0169	0.0121	29 21 57.1
2	0.0000	0.0000	0.0109	0.0119	0.0126	0.0100	291 58 8.1
3	0.0000	0.0000	0.0099	0.0093	0.0105	0.0086	71 14 37.2
4	0.0000	0.0000	0.0083	0.0093	0.0095	0.0080	311 20 32.9
5	0.0000	0.0000	0.0074	0.0082	0.0087	0.0069	305 40 52.7
6	0.0000	0.0000	0.0075	0.0091	0.0093	0.0073	319 9 10.3
7	0.0000	0.0000	0.0072	0.0075	0.0077	0.0070	290 17 24.7
8	0.0000	0.0000	0.0057	0.0070	0.0070	0.0057	352 25 4.2
9	0.0000	0.0000	0.0078	0.0073	0.0078	0.0073	353 35 36.1
10	0.0000	0.0000	0.0071	0.0067	0.0071	0.0067	01 58 46.1
11	0.0000	0.0000	0.0085	0.0078	0.0090	0.0072	68 12 59.1
12	0.0000	0.0000	0.0145	0.0255	0.0259	0.0137	336 19 44.5
13	0.0000	0.0000	0.0336	0.0537	0.0605	0.0186	58 02 31.8
14	0.0000	0.0000	0.0361	0.0188	0.0361	0.0188	05 05 58.4
15	0.0000	0.0000	0.0201	0.0432	0.0445	0.0167	328 40 22.2
16	0.0000	0.0000	0.0179	0.0176	0.0200	0.0150	85 28 52.6
17	0.0000	0.0000	0.0212	0.0531	0.0533	0.0206	11 12 15.0
18	0.0000	0.0000	0.0233	0.0242	0.0258	0.0216	282 57 17.3
19	0.0000	0.0000	0.0397	0.0303	0.0428	0.0256	55 35 22.0
20	0.0000	0.0000	0.0202	0.0241	0.0248	0.0193	44 09 40.3
21	0.0000	0.0000	0.0182	0.0235	0.0250	0.0163	51 48 14.8
22	0.0000	0.0000	0.0204	0.0366	0.0385	0.0166	320 13 12.4
23	0.0000	0.0000	0.0218	0.0390	0.0403	0.0194	327 26 08.2
24	0.0000	0.0000	0.0203	0.0183	0.0203	0.0183	351 35 15.5
25	0.0000	0.0000	0.0414	0.0215	0.0413	0.0215	359 28 03.5
26	0.0000	0.0000	0.0348	0.0195	0.0355	0.0182	27 08 07.5
27	-0.0087	0.0163	0.0277	0.0209	0.0277	0.0209	11 10 29.1
28	-0.0114	0.0113	0.0282	0.0167	0.0282	0.0166	354 37 35.0
29	-0.0145	0.0089	0.0169	0.0163	0.0185	0.0144	82 13 41.8
30	0.0005	-0.0017	0.0320	0.0239	0.0320	0.0240	02 41 03.9
31	-0.0054	0.0010	0.0183	0.0207	0.0243	0.0131	283 04 27.7
32	0.0009	-0.0108	0.0220	0.0200	0.0259	0.0146	[illegible]79 19 59.7
33	0.0041	-0.0032	0.0125	0.0142	0.0154	0.0110	69 14 07.4
34	-0.0039	-0.0036	0.0132	0.0210	0.0211	0.0130	348 24 05.9
35	0.0032	0.0139	0.0134	0.0171	0.0171	0.0134	08 43 13.2
36	-0.0183	0.0151	0.0258	0.0197	0.0294	0.0138	65 42 31.4
37	0.0027	0.0180	0.0126	0.0202	0.0206	0.0119	27 17 37.2
38	0.0035	0.0019	0.0181	0.0126	0.0185	0.0120	32 50 51.4
39	0.0791	-0.0208	0.0619	0.0211	0.0623	0.0200	13 23 18.9
40	-0.0045	-0.0335	0.0155	0.0399	0.0399	0.0154	01 03 11.8
41	0.0195	0.0088	0.0331	0.0381	0.0412	0.0290	294 38 59.6

$\bar{\sigma}_V$=0.023355 $\bar{\sigma}_E$=0.024296 $\bar{\sigma}_C$=0.023830 $\hat{\sigma}_0^2$=1.009889

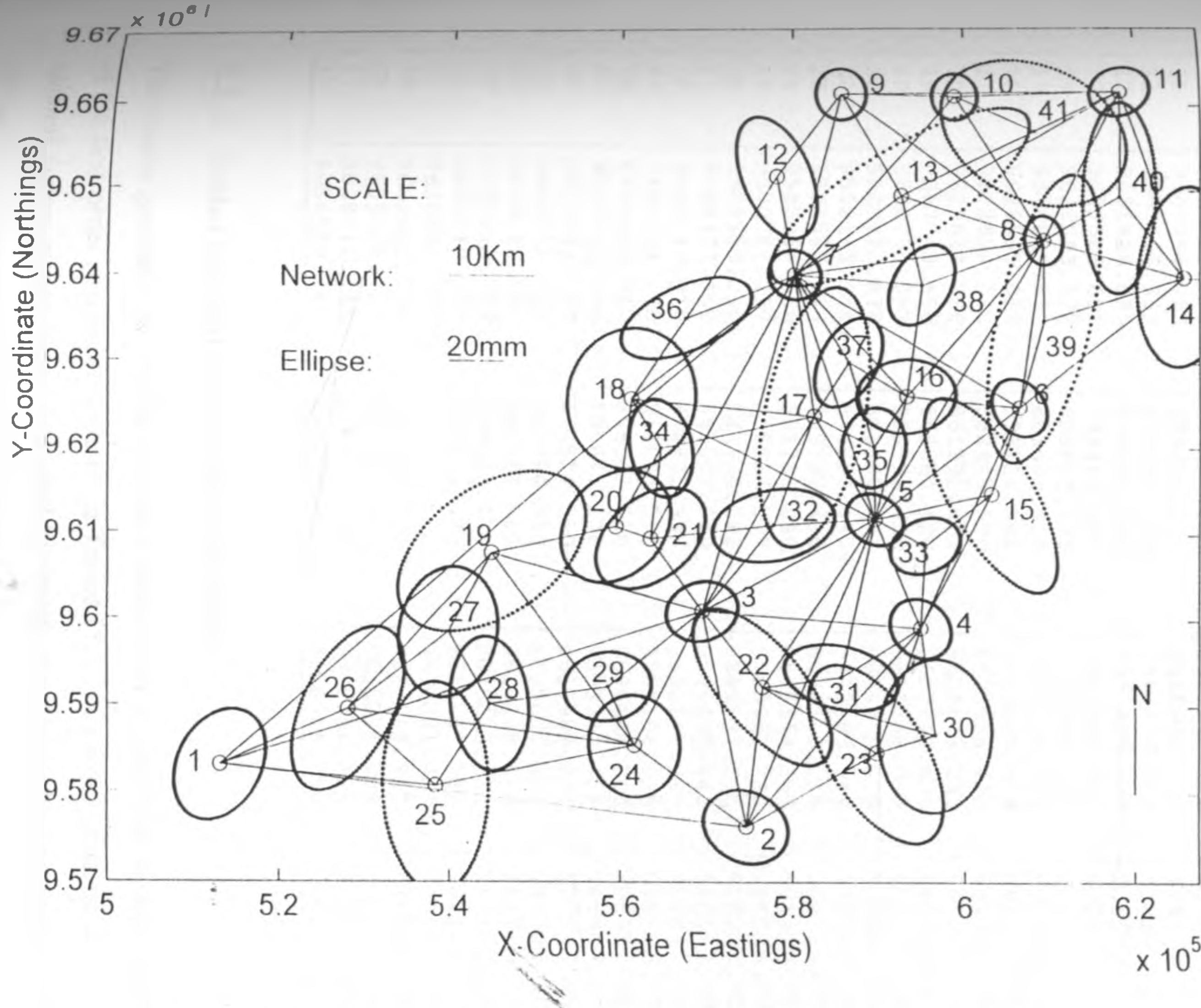


Figure 5.18: Point error ellipses -second level real network densification static-dynamic approach

Table 5.3.6: Estimated Coordinates- Second Level Real Network Densification -Static-Dynamic Approach

POINT	PROVISIONAL COORDINATES		ADJUSTED COORDINATES	
	N (m)	E (m)	N (m)	E (m)
1	9583105.004	513215.950	9583105.004	513215.950
2	9576179.235	574648.607	9576179.235	574648.607
3	9600981.218	569580.575	9600981.218	569580.575
4	9599052.507	594885.488	9599052.507	594885.488
5	9611665.696	589503.231	9611665.696	589503.231
6	9624778.565	606329.885	9624778.565	606329.885
7	9640229.703	580182.077	9640229.703	580182.077
8	9644287.286	609203.286	9644287.286	609203.286
9	9661267.225	585576.691	9661267.225	585576.691
10	9660984.753	598816.111	9660984.753	598816.111
11	9661696.859	618181.218	9661696.859	618181.218
12	9651426.855	578082.243	9651426.855	578082.243
13	9649302.681	592482.865	9649302.681	592482.865
14	9640093.572	625611.395	9640093.572	625611.395
15	9614609.371	603055.410	9614609.371	603055.410
16	9626055.619	593121.942	9626055.619	593121.942
17	9623694.425	582449.886	9623694.425	582449.886
18	9625643.288	561292.564	9625643.288	561292.564
19	9607749.690	545064.420	9607749.690	545064.420
20	9610759.097	559563.423	9610759.097	559563.423
21	9609410.878	563591.472	9609410.878	563591.472
22	9592183.544	576550.590	9592183.544	576550.590
23	9584543.218	589735.830	9584543.218	589735.830
24	9585354.356	561756.098	9585354.356	561756.098
25	9580752.250	538526.297	9580752.250	538526.297
26	9589557.286	528224.012	9589557.286	528224.012
27	9598477.450	540001.040	9598477.441	540001.056
28	9590143.010	544871.930	9590142.999	544871.941
29	9592184.550	558655.590	9592184.535	558655.599
30	9586646.840	596567.510	9586646.840	596567.508
31	9593341.460	585647.230	9593341.455	585647.231
32	9610982.260	577814.010	9610982.261	577813.999
33	9608655.320	595394.330	9608655.324	595394.327
34	9619838.450	564763.860	9619838.446	564763.856
35	9620013.730	589321.420	9620013.733	589321.434
36	9635147.760	567613.380	9635147.742	567613.395
37	9630041.920	586400.120	9630041.923	586400.138
38	9639112.320	594892.100	9639112.323	594892.102
39	9635021.720	609221.860	9635021.799	609221.839
40	9649311.340	618081.160	9649311.335	618081.126
41	9656888.740	608069.420	9656888.759	608069.429

5.3.3 Sub-Optimal Fusion Densification

The results presented in this section were determined by considering the estimated coordinates of the first order real network listed in Tables (5.3.2) as fixed stochastic constraints. The *sub-optimal fusion* approach was then used to densify the first order network by intercalating on to the network second and third order points giving results for first level densification {cf. section (5.3.3.1)} and second level densification {cf. section (5.3.3.2)}.

5.3.3.1 First Level Densification

In the first level real network densification, first order points 1 through 11 were considered as fixed stochastic parameters while second order points 12 through 26 were considered as new points whose coordinates were to be estimated {cf. Figure (4.6)}. The observation data sets in Table (4.10) and the approximate coordinates in Table (4.9b) were used resulting in parameters listed in Table (5.3.7) and Table (5.3.8) below while error ellipses are given diagrammatically in Figure (5.19).

Table 5.3.7: Coordinate Corrections and Stochastic Parameters-First Level Real Network -Sub-Optimal Fusion Approach

POINT	ΔN (m)	ΔE (m)	σ_N (m)	σ_E (m)	σ_{max} (m)	σ_{min} (m)	α ° ' "
1	0.0000	0.0000	0.0166	0.0125	0.0169	0.0121	29 21 57.1
2	0.0000	0.0000	0.0109	0.0119	0.0126	0.0100	291 58 8.1
3	0.0000	0.0000	0.0099	0.0093	0.0105	0.0086	71 14 37.2
4	0.0000	0.0000	0.0083	0.0093	0.0095	0.0080	312 20 32.9
5	0.0000	0.0000	0.0074	0.0082	0.0087	0.0069	306 40 52.7
6	0.0000	0.0000	0.0075	0.0091	0.0093	0.0073	319 9 10.3
7	0.0000	0.0000	0.0072	0.0075	0.0077	0.0070	290 17 24.7
8	0.0000	0.0000	0.0057	0.0070	0.0070	0.0057	352 25 4.2
9	0.0000	0.0000	0.0078	0.0073	0.0078	0.0073	353 35 36.1
10	0.0000	0.0000	0.0071	0.0067	0.0071	0.0067	01 58 46.1
11	0.0000	0.0000	0.0085	0.0078	0.0090	0.0072	68 12 59.1
12	-0.0571	0.0158	0.0134	0.0237	0.0240	0.0127	336 19 44.5
13	0.0051	0.0094	0.0312	0.0498	0.0561	0.0173	58 02 31.8
14	-0.0711	-0.0058	0.0335	0.0175	0.0335	0.0174	05 05 58.4
15	0.0034	-0.0152	0.0186	0.0401	0.0414	0.0155	328 40 22.2
16	-0.0268	0.0777	0.0166	0.0163	0.0186	0.0139	85 28 52.6
17	0.0109	0.0655	0.0197	0.0493	0.0495	0.0191	11 12 15.0
18	-0.0280	-0.0737	0.0216	0.0225	0.0239	0.0200	282 57 17.3
19	-0.1947	-0.1383	0.0369	0.0281	0.0398	0.0238	55 35 22.0
20	-0.0740	-0.1282	0.0187	0.0224	0.0230	0.0179	44 09 40.3
21	-0.0016	-0.1985	0.0169	0.0218	0.0232	0.0151	51 48 14.8
22	-0.1004	0.0667	0.0189	0.0340	0.0357	0.0154	320 13 12.4
23	0.0439	0.1172	0.0203	0.0362	0.0373	0.0180	327 26 08.2
24	-0.2327	-0.2191	0.0188	0.0170	0.0188	0.0170	351 35 15.5
25	-0.2244	-0.1095	0.0384	0.0200	0.0383	0.0200	359 28 03.5
26	-0.2118	-0.1940	0.0323	0.0181	0.0329	0.0169	27 08 07.5

$\bar{\sigma}_N$=0.01993 $\bar{\sigma}_E$=0.02345 $\bar{\sigma}_C$=0.021760 $\hat{\sigma}_0^2$=1.006536

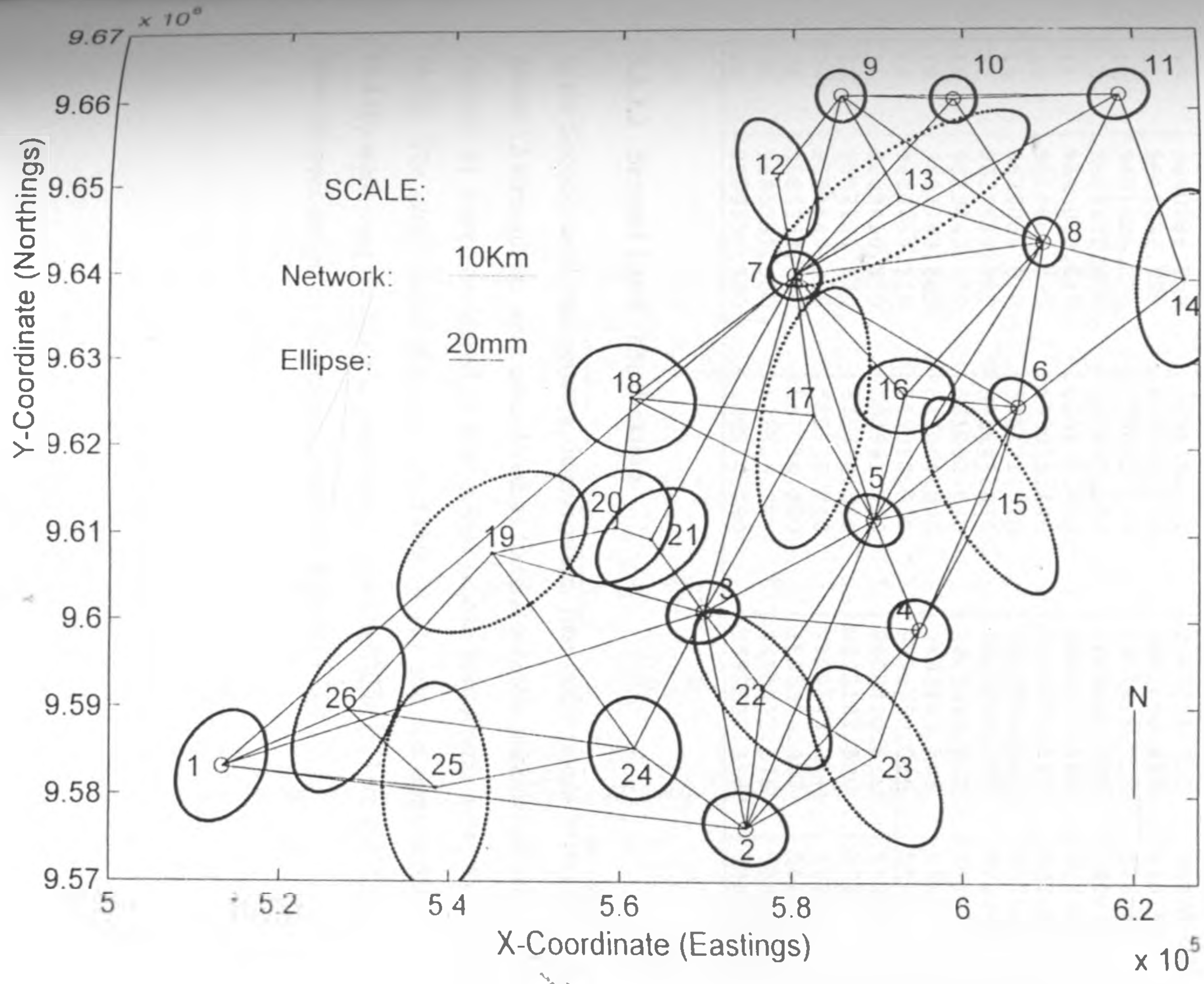


Figure 5.19: Point error ellipses -first level real network densification sub-optimal fusion approach

Table 5.3.8: Estimated Coordinates- First Level Real Network Densification -Sub-Optimal Fusion Approach

POINT	PROVISIONAL COORDINATES		ADJUSTED COORDINATEN(m)	
	N(m)	E(m)	N(m)	E(m)
1	9583105.004	513215.950	9583105.004	513215.950
2	9576179.235	574648.607	9576179.235	574648.607
3	9600981.218	569580.575	9600981.218	569580.575
4	9599052.507	594885.488	9599052.507	594885.488
5	9611665.696	589503.231	9611665.696	589503.231
6	9624778.565	606329.885	9624778.565	606329.885
7	9640229.703	580182.077	9640229.703	580182.077
8	9644287.286	609203.286	9644287.286	609203.286
9	9661267.225	585576.691	9661267.225	585576.691
10	9660984.753	598816.111	9660984.753	598816.111
11	9661696.859	618181.218	9661696.859	618181.218
12	9651426.870	578082.210	9651426.813	578082.226
13	9649302.660	592482.850	9649302.665	592482.859
14	9640093.640	625611.430	9640093.569	625611.424
15	9614609.390	603055.440	9614609.393	603055.425
16	9626055.670	593121.870	9626055.643	593121.948
17	9623694.380	582449.870	9623694.391	582449.936
18	9625643.250	561292.670	9625643.222	561292.596
19	9607749.860	545064.590	9607749.665	545064.452
20	9610759.110	559563.590	9610759.036	559563.462
21	9609410.820	563591.710	9609410.818	563591.511
22	9592183.630	576550.570	9592183.530	576550.637
23	9584543.130	589735.740	9584543.174	589735.857
24	9585354.590	561756.300	9585354.357	561756.081
25	9580752.500	538526.380	9580752.276	538526.270
26	9589557.530	528224.190	9589557.318	528223.996

5.3.3.2 Second Level Densification

In the Second level real network densification, first order points 1 through 11 and second order points 12 through 26 were considered as fixed stochastic parameters while third order points 27 through 41 were considered as new points whose coordinates were to be estimated (cf. Figure (4.7)). The observation data sets in Table (4.12) and the approximate coordinates in Table (4.11b) were used resulting in parameters listed in Table (5.3.9) and Table (5.3.10) below while error ellipses are given diagrammatically in Figure (5.20).

Table 5.3.9: Coordinate Corrections and Stochastic Parameters-Second Level Real Network -Sub-Optimal Fusion Approach

POINT	ΔN (m)	ΔE (m)	σ_N (m)	σ_E (m)	σ_{max} (m)	σ_{min} (m)	α ° ' "
1	0.0000	0.0000	0.0166	0.0125	0.0169	0.0121	29 21 57.1
2	0.0000	0.0000	0.0109	0.0119	0.0126	0.0100	291 58 8.1
3	0.0000	0.0000	0.0099	0.0093	0.0105	0.0086	71 14 37.2
4	0.0000	0.0000	0.0083	0.0093	0.0095	0.0080	313 20 32.9
5	0.0000	0.0000	0.0074	0.0082	0.0087	0.0069	307 40 52.7
6	0.0000	0.0000	0.0075	0.0091	0.0093	0.0073	319 9 10.3
7	0.0000	0.0000	0.0072	0.0075	0.0077	0.0070	290 17 24.7
8	0.0000	0.0000	0.0057	0.0070	0.0070	0.0057	352 25 4.2
9	0.0000	0.0000	0.0078	0.0073	0.0078	0.0073	353 35 36.1
10	0.0000	0.0000	0.0071	0.0067	0.0071	0.0067	01 58 46.1
11	0.0000	0.0000	0.0085	0.0078	0.0090	0.0072	68 12 59.1
12	0.0000	0.0000	0.0134	0.0237	0.0240	0.0127	336 19 44.5
13	0.0000	0.0000	0.0312	0.0498	0.0561	0.0173	58 02 31.8
14	0.0000	0.0000	0.0335	0.0175	0.0335	0.0174	05 05 58.4
15	0.0000	0.0000	0.0186	0.0401	0.0413	0.0155	328 40 22.2
16	0.0000	0.0000	0.0166	0.0163	0.0186	0.0140	85 28 52.6
17	0.0000	0.0000	0.0197	0.0493	0.0495	0.0191	11 12 15.0
18	0.0000	0.0000	0.0216	0.0225	0.0239	0.0200	282 57 17.3
19	0.0000	0.0000	0.0369	0.0281	0.0397	0.0238	55 35 22.0
20	0.0000	0.0000	0.0187	0.0224	0.0230	0.0179	44 09 40.3
21	0.0000	0.0000	0.0169	0.0218	0.0231	0.0151	51 48 14.8
22	0.0000	0.0000	0.0189	0.0340	0.0357	0.0154	320 13 12.4
23	0.0000	0.0000	0.0203	0.0362	0.0373	0.0180	327 26 08.2
24	0.0000	0.0000	0.0188	0.0170	0.0188	0.0169	351 35 15.5
25	0.0000	0.0000	0.0384	0.0200	0.0384	0.0200	359 28 03.5
26	0.0000	0.0000	0.0323	0.0181	0.0329	0.0169	28 08 07.5
27	0.2265	0.3338	0.0260	0.0197	0.0261	0.0196	12 10 29.1
28	0.1243	0.1794	0.0265	0.0157	0.0265	0.0156	354 7 35.0
29	0.2070	0.0410	0.0159	0.0154	0.0174	0.0135	82 13 41.8
30	-0.0224	-0.1603	0.0301	0.0225	0.0301	0.0226	02 41 03.9
31	-0.0046	-0.0144	0.0172	0.0195	0.0228	0.0123	283 04 27.7
32	-0.0965	0.2880	0.0207	0.0188	0.0244	0.0137	79 19 59.7
33	-0.0324	0.0676	0.0118	0.0133	0.0145	0.0103	69 14 07.4
34	0.0092	0.0848	0.0124	0.0198	0.0200	0.0123	348 24 05.9
35	-0.0241	0.0547	0.0126	0.0161	0.0161	0.0126	08 43 13.2
36	0.1173	-0.0382	0.0243	0.0181	0.0276	0.0129	65 42 31.4
37	-0.0380	0.0167	0.0119	0.0185	0.0194	0.0112	27 17 37.2
38	0.0604	0.0142	0.0170	0.0119	0.0174	0.0113	32 50 51.4
39	0.1788	0.0276	0.0583	0.0199	0.0586	0.0188	13 23 18.9
40	0.0646	-0.0609	0.0145	0.0376	0.0375	0.0145	01 03 11.8
41	0.0301	-0.0557	0.0311	0.0358	0.0388	0.0273	294 38 59.6

$\bar{\sigma}_N$=0.021886 $\bar{\sigma}_E$=0.022705 $\bar{\sigma}_C$=0.022299 $\hat{\sigma}_0^2$=1.009277

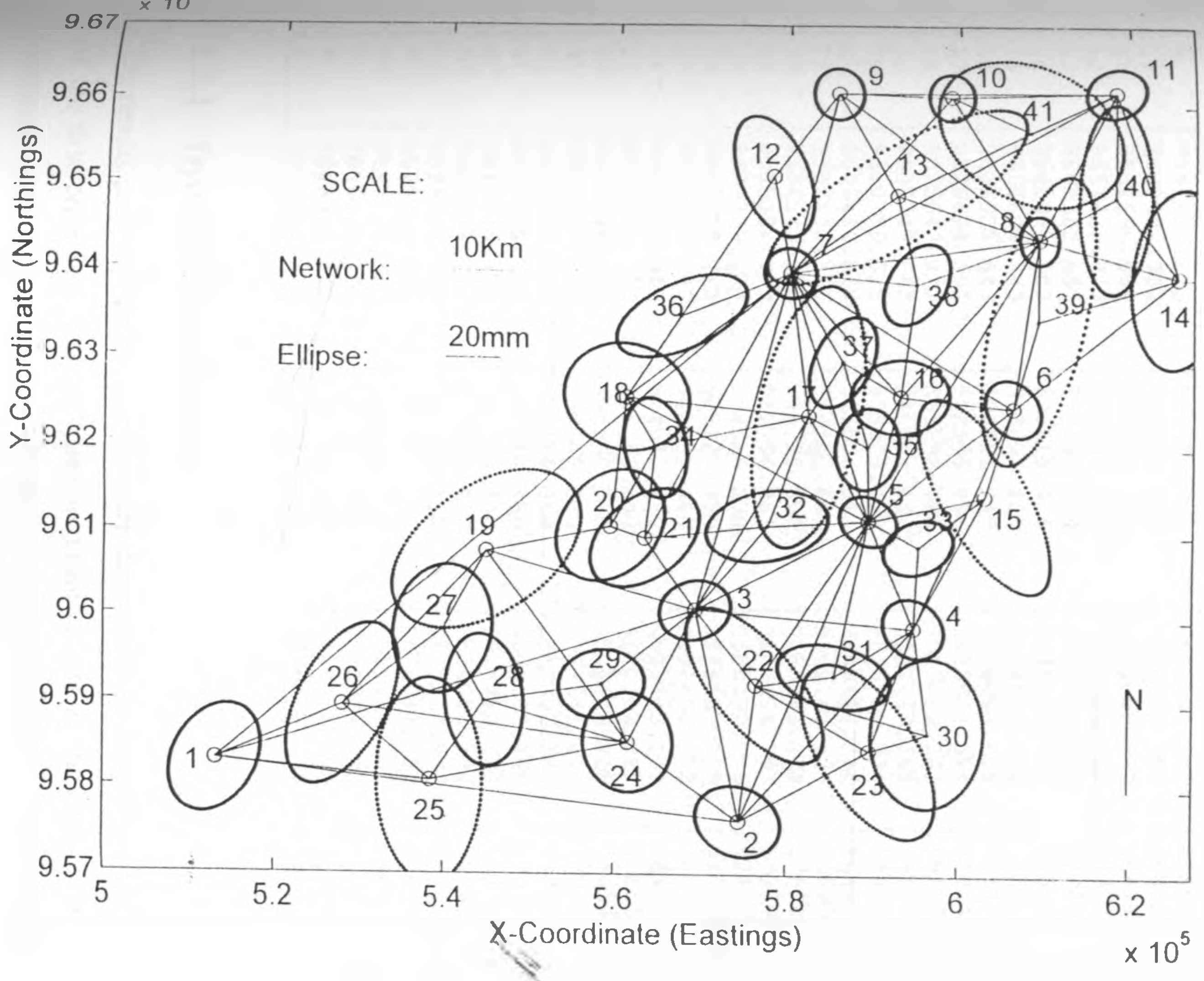


Figure 5.20: Point error ellipses -second level real network densification sub-optimal fusion approach

Table 5.3.10: Estimated Coordinates- Second Level Real Network Densification -Sub-Optimal Fusion Approach

POINT	PROVISIONAL COORDINATES N (m)	E (m)	ADJUSTED COORDINATES N (m)	E (m)
1	9583104.004	513215.950	9583105.004	513215.950
2	9576179.235	574648.607	9576179.235	574648.607
3	9600981.218	569580.575	9600981.218	569580.575
4	9599052.507	594885.488	9599052.507	594885.488
5	9611665.696	589503.231	9611665.696	589503.231
6	9624778.565	606329.885	9624778.565	606329.885
7	9640229.703	580182.077	9640229.703	580182.077
8	9644287.286	609203.286	9644287.286	609203.286
9	9661267.225	585576.691	9661267.225	585576.691
10	9660984.753	598816.111	9660984.753	598816.111
11	9661696.859	618181.218	9661696.859	618181.218
12	9651426.813	578082.226	9651426.813	578082.226
13	9649302.665	592482.859	9649302.665	592482.859
14	9640093.569	625611.424	9640093.569	625611.424
15	9614609.393	603055.425	9614609.393	603055.425
16	9626055.643	593121.948	9626055.643	593121.948
17	9623694.391	582449.936	9623694.391	582449.936
18	9625643.222	561292.596	9625643.222	561292.596
19	9607749.665	545064.452	9607749.665	545064.452
20	9610759.036	559563.462	9610759.036	559563.462
21	9609410.818	563591.511	9609410.818	563591.511
22	9592183.530	576550.637	9592183.530	576550.637
23	9584543.174	589735.852	9584543.174	589735.857
24	9585354.357	561756.081	9585354.357	561756.081
25	9580752.276	538526.270	9580752.276	538526.270
26	9589557.218	528223.996	9589557.318	528223.996
27	9598477.450	540001.040	9598477.677	540001.374
28	9590143.010	544871.930	9590143.134	544872.109
29	9592184.550	558655.590	9592184.757	558655.631
30	9586646.840	596567.510	9586646.818	596567.350
31	9593341.460	585647.230	9593341.455	585647.216
32	9610982.260	577814.010	9610982.164	577814.298
33	9608655.320	595394.330	9608655.288	595394.3[illegible]8
34	9619838.450	564763.860	9619838.459	564763.945
35	9620013.730	589321.420	9620013.706	589321.475
36	9635147.760	567613.380	9635147.877	567613.342
37	9630041.920	586400.120	9630041.882	586400.137
38	9639112.320	594892.100	9639112.380	594892.114
39	9635021.720	609221.860	9635021.899	609221.888
40	9649311.340	618081.160	9649311.405	618081.099
41	9656888.740	608069.420	9656888.770	608069.364

5.3.4 Dynamic Densification

The results presented in this section were determined by considering the estimated coordinates of the first order real network listed in Table (5.3.2) as stochastic constraints. The *dynamic* approach was then used to densify the first order network by intercalating on to the network second and third order points giving results for first level densification {c.f. section (5.3.4.1)} and second level densification {c.f. section (5.3.4.2)}.

5.3.4.1 First Level Densification

In the first level real network densification, all points 1 through 26 were considered as stochastic parameters with all of them being re-estimated including the datum points 1 through 11. {cf. Figure (4.6)}. The observation data sets in Table (4.10) and the approximate coordinates in Table (4.9) were used resulting in parameters listed in Table (5.3.11) and Table (5.3.12) below while error ellipses are given diagrammatically in Figure (5.21).

Table 5.3.11: Coordinate Corrections and Stochastic Parameters-First Level Real Network -Dynamic Approach

POINT	ΔN (m)	ΔE (m)	σ_N (m)	σ_E (m)	σ_{max} (m)	σ_{min} (m)	α ° ' "
1	0.1106	-0.0128	0.0179	0.0129	0.0179	0.0129	29 21 57.1
2	-0.0081	-0.0372	0.0114	0.0126	0.0126	0.0114	291 58 8.1
3	-0.0498	0.0652	0.0104	0.0100	0.0105	0.0100	71 14 37.2
4	-0.0458	-0.0007	0.0091	0.0102	0.0102	0.0091	314 20 32.9
5	-0.0087	0.0843	0.0079	0.0088	0.0088	0.0076	308 40 52.7
6	0.0299	-0.0075	0.0082	0.0098	0.0098	0.0082	319 9 10.3
7	-0.0400	-0.0108	0.0077	0.0082	0.0082	0.0077	290 17 24.7
8	0.0335	0.0447	0.0063	0.0077	0.0077	0.0063	352 25 4.2
9	-0.0329	-0.0305	0.0085	0.0081	0.0085	0.0081	353 35 36.1
10	-0.0596	-0.0399	0.0079	0.0074	0.0079	0.0074	01 58 46.1
11	0.0801	-0.0449	0.0094	0.0087	0.0094	0.0087	68 12 59.1
12	-0.0571	0.0158	0.0154	0.0272	0.0277	0.0146	336 19 44.5
13	0.0051	0.0094	0.0359	0.0574	0.0647	0.0199	58 02 31.8
14	-0.0711	-0.0058	0.0386	0.0202	0.0386	0.0201	05 05 58.4
15	0.0034	-0.0152	0.0215	0.0461	0.0476	0.0179	328 40 22.2
16	-0.0268	0.0777	0.0192	0.0187	0.0214	0.0161	85 28 52.6
17	0.0109	0.0655	0.0226	0.0568	0.0570	0.0220	11 12 15.0
18	-0.0280	-0.0737	0.0249	0.0259	0.0275	0.0231	282 57 17.3
19	-0.1947	-0.1383	0.0425	0.0323	0.0458	0.0274	55 35 22.0
20	-0.0740	-0.1282	0.0216	0.0257	0.0265	0.0206	44 09 40.3
21	-0.0016	-0.1985	0.0195	0.0252	0.0267	0.0174	51 48 14.8
22	-0.1004	0.0667	0.0218	0.0391	0.0411	0.0178	320 13 12.4
23	0.0439	0.1172	0.0233	0.0417	0.0430	0.0208	327 26 08.2
24	-0.2327	-0.2191	0.0217	0.0195	0.0217	0.0195	351 35 15.5
25	-0.2244	-0.1095	0.0442	0.0230	0.0442	0.0230	359 28 03.5
26	-0.2118	-0.1940	0.0372	0.0209	0.0380	0.0195	27 08 07.5

$\bar{\sigma}_N = 0.022829$ $\bar{\sigma}_E = 0.026885$ $\bar{\sigma}_C = 0.024939$ $\hat{\sigma}_0^2 = 1.003964$

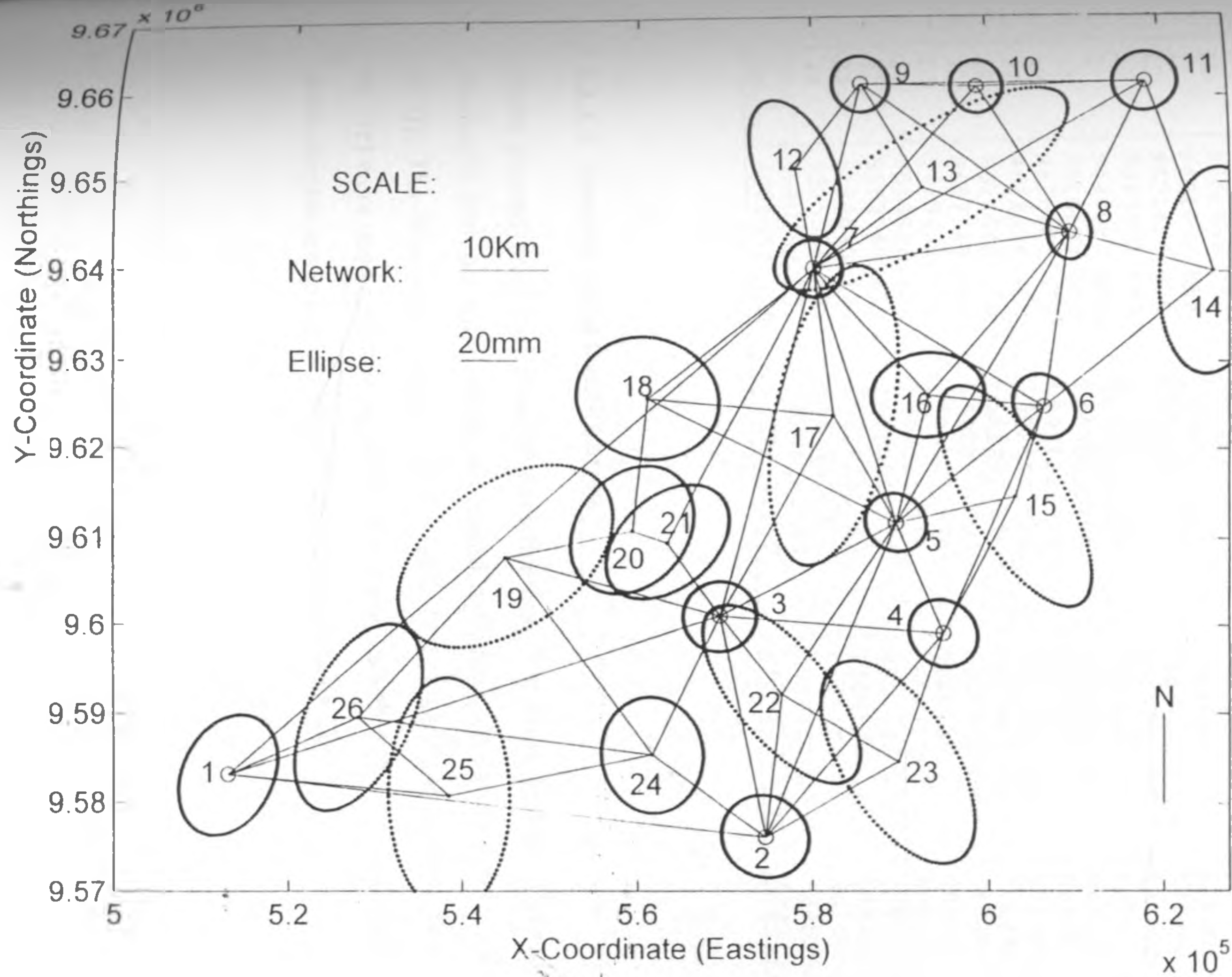


Figure 5.21: Point error ellipses -first level real network densification dynamic approach

Table 5.3.12: Estimated Coordinates- First Level Real Network Densification -Dynamic Approach

POINT	PROVISIONAL COORDINATES N(m)	PROVISIONAL COORDINATES E(m)	ADJUSTED COORDINATES N(m)	ADJUSTED COORDINATES E(m)
1	9583105.004	513215.950	9583105.115	513215.937
2	9576179.235	574648.607	9576179.227	574648.570
3	9600981.218	569580.575	9600981.168	569580.640
4	9599052.507	594885.488	9599052.461	594885.487
5	9611665.696	589503.231	9611665.687	589503.315
6	9624778.565	606329.885	9624778.595	606329.877
7	9640229.703	580182.077	9640229.663	580182.066
8	9644287.286	609203.286	9644287.320	609203.331
9	9661267.225	585576.691	9661267.192	585576.661
10	9660984.753	598816.111	9660984.693	598816.071
11	9661696.859	618181.218	9661696.939	618181.173
12	9651426.870	578082.210	9651426.813	578082.226
13	9649302.660	592482.850	9649302.665	592482.859
14	9640093.640	625611.430	9640093.569	625611.424
15	9614609.390	603055.440	9614609.393	603055.425
16	9626055.670	593121.870	9626055.643	593121.948
17	9623694.380	582449.870	9623694.391	582449.936
18	9625643.250	561292.670	9625643.222	561292.596
19	9607749.860	545064.590	9607749.665	545064.452
20	9610759.110	559563.590	9610759.036	559563.462
21	9609410.820	563591.710	9609410.818	563591.511
22	9592183.630	576550.570	9592183.530	576550.637
23	9584543.130	589735.740	9584543.174	589735.857
24	9585354.590	561756.300	9585354.357	561756.081
25	9580752.500	538526.380	9580752.276	538526.270
26	9589557.530	528224.190	9589557.318	528223.996

5.3.4.2 Second Level Densification

In the Second level real network densification, first order points 1 through 41 were considered stochastic with all points being estimated including the datum points 1 through 26 (cf. Figure (4.7)). The observation data sets in Table (4.12) and the approximate coordinates in Table (4.11c) were used resulting in parameters listed in Table (5.3.13) and Table (5.3.14) below while error ellipses are given diagrammatically in Figure (5.22).

Table 5.3.13: Coordinate Corrections and Stochastic Parameters-Second Level Real Network -Dynamic Approach

POINT	ΔN (m)	ΔE (m)	σ_N (m)	σ_E (m)	σ_{max} (m)	σ_{min} (m)	α ° ' "
1	0.0103	-0.0389	0.0177	0.0133	0.0177	0.0133	29 21 57.1
2	-0.0214	-0.0328	0.0116	0.0127	0.0127	0.0116	291 58 8.1
3	-0.0214	0.0895	0.0101	0.0095	0.0102	0.0094	71 14 37.2
4	-0.0391	0.0049	0.0085	0.0096	0.0096	0.0084	315 20 32.9
5	-0.0070	0.0903	0.0075	0.0084	0.0084	0.0075	309 40 52.7
6	0.0310	-0.0077	0.0080	0.0096	0.0096	0.0080	319 9 10.3
7	-0.0394	-0.0085	0.0074	0.0075	0.0075	0.0074	290 17 24.7
8	0.0337	0.0345	0.0060	0.0072	0.0072	0.0060	352 25 4.2
9	-0.0314	-0.0305	0.0083	0.0078	0.0083	0.0078	353 35 36.1
10	-0.0602	-0.0401	0.0075	0.0071	0.0075	0.0071	01 58 46.1
11	0.0750	-0.0442	0.0089	0.0083	0.0089	0.0082	68 12 59.1
12	0.0560	-0.0105	0.0148	0.0256	0.0256	0.0148	336 19 44.5
13	0.0015	0.0219	0.0320	0.0372	0.0380	0.0311	58 02 31.8
14	0.0918	0.0111	0.0351	0.0179	0.0351	0.0179	05 05 58.4
15	-0.0149	0.0564	0.0199	0.0396	0.0398	0.0196	328 40 22.2
16	0.0262	-0.0559	0.0149	0.0161	0.0163	0.0147	85 28 52.6
17	-0.0171	0.0007	0.0165	0.0220	0.0221	0.0163	11 12 15.0
18	0.0264	0.0576	0.0173	0.0196	0.0198	0.0171	282 57 17.3
19	0.1921	0.1410	0.0392	0.0287	0.0395	0.0282	55 35 22.0
20	0.0344	0.1162	0.0171	0.0221	0.0225	0.0166	44 09 40.3
21	0.0162	0.1312	0.0148	0.0194	0.0195	0.0147	51 48 14.8
22	0.0919	-0.0856	0.0192	0.0294	0.0294	0.0191	320 13 12.4
23	-0.0412	-0.0650	0.0219	0.0290	0.0290	0.0218	327 26 08.2
24	0.1846	0.1866	0.0186	0.0164	0.0187	0.0163	351 35 15.5
25	0.2326	0.1062	0.0400	0.0213	0.0401	0.0212	359 28 03.5
26	0.1926	0.1839	0.0319	0.0189	0.0319	0.0188	29 08 07.5
27	0.1490	0.1410	0.0324	0.0246	0.0325	0.0245	13 10 29.1
28	0.1284	0.1131	0.0330	0.0195	0.0331	0.0195	354 7 35.0
29	0.1219	0.0795	0.0198	0.0191	0.0217	0.0169	82 13 41.8
30	0.0028	-0.0663	0.0375	0.0281	0.0375	0.0281	02 41 03.9
31	0.0319	-0.0659	0.0114	0.0242	0.0284	0.0153	283 04 27.7
32	-0.0175	0.1251	0.0258	0.0234	0.0304	0.0171	79 19 59.7
33	-0.0831	0.1223	0.0147	0.0165	0.0180	0.0129	69 14 07.4
34	-0.0242	0.1065	0.0154	0.0247	0.0248	0.0153	348 24 05.9
35	-0.0185	0.0555	0.0157	0.0200	0.0200	0.0157	08 43 13.2
36	0.0251	-0.0407	0.0302	0.0231	0.0344	0.0161	65 42 31.4
37	-0.0338	0.0281	0.0147	0.0237	0.0242	0.0140	27 17 37.2
38	0.0509	0.0307	0.0211	0.0148	0.0217	0.0234	32 50 51.4
39	0.1965	0.0003	0.0726	0.0247	0.0073	0.0181	13 23 18.9
40	0.1379	-0.0095	0.0181	0.0468	0.0468	0.0340	01 03 11.8
41	0.0183	0.0061	0.0388	0.0446	0.0483	0.0273	294 38 59.6

$\bar{\sigma}_N$=0.024693 $\bar{\sigma}_E$=0.022717 $\bar{\sigma}_C$=0.023726 $\hat{\sigma}_0^2$=1.008111

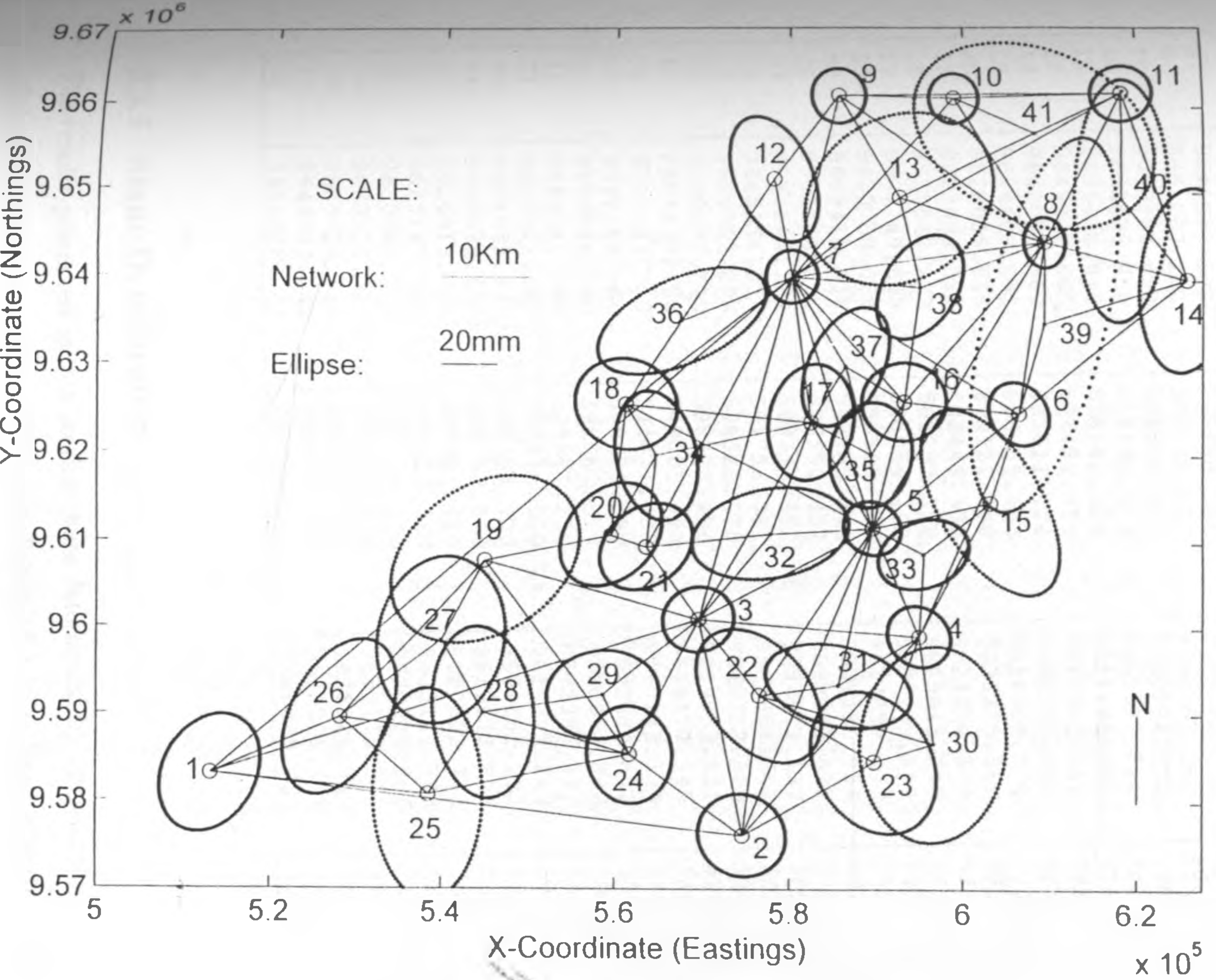


Figure 5.22: Point error ellipses -second level real network densification dynamic approach

Table 5.3.14: Estimated Coordinates- Second Level Real Network Densification
-Dynamic Approach

POINT	PROVISIONAL COORDINATES N(m)	PROVISIONAL COORDINATES E(m)	ADJUSTED COORDINATES N(m)	ADJUSTED COORDINATES E(m)
1	9583104.114	513215.937	9583104.214	513215.898
2	9576179.227	574648.570	9576179.206	574648.537
3	9600981.168	569580.640	9600981.147	569580.730
4	9599052.461	594885.487	9599052.422	594885.492
5	9611665.687	589503.315	9611665.680	589503.405
6	9624778.595	606329.877	9624778.626	606329.869
7	9640229.663	580182.066	9640229.624	580182.057
8	9644287.320	609203.331	9644287.354	609203.366
9	9661267.192	585576.661	9661267.161	585576.630
10	9660984.693	598816.071	9660984.633	598816.031
11	9661696.939	618181.173	9661697.014	618181.129
12	9651426.813	578082.226	9651426.869	578082.216
13	9649302.665	592482.859	9649302.667	592482.881
14	9640093.569	625611.424	9640093.661	625611.435
15	9614609.393	603055.425	9614609.378	603055.481
16	9626055.643	593121.948	9626055.669	593121.892
17	9623694.391	582449.936	9623694.374	582449.937
18	9625643.222	561292.596	9625643.248	561292.654
19	9607749.665	545064.452	9607749.857	545064.593
20	9610759.036	559563.462	9610759.070	559563.578
21	9609410.818	563591.511	9609410.834	563591.642
22	9592183.530	576550.637	9592183.622	576550.551
23	9584543.174	589735.852	9584543.133	589735.787
24	9585354.357	561756.081	9585354.542	561756.268
25	9580752.276	538526.270	9580752.509	538526.376
26	9589557.218	528223.996	9589557.411	528224.180
27	9598477.450	540001.040	9598477.599	540001.181
28	9590143.010	544871.930	9590143.138	544872.043
29	9592184.550	558655.590	9592184.672	558655.669
30	9586646.840	596567.510	9586646.843	596567.444
31	9593341.460	585647.230	9593341.492	585647.164
32	9610982.260	577814.010	9610982.243	577814.135
33	9608655.320	595394.330	9608655.237	595394.482
34	9619838.450	564763.860	9619838.426	564763.967
35	9620013.730	589321.420	9620013.711	589321.475
36	9635147.760	567613.380	9635147.785	567613.339
37	9630041.920	586400.120	9630041.886	586400.148
38	9639112.320	594892.100	9639112.371	594892.131
39	9635021.720	609221.360	9635021.917	609221.860
40	9649311.340	618081.160	9649311.478	618081.150
41	9656888.740	608069.420	9656888.758	608069.426

5.3.5 Static Densification

The results presented in this section were determined by considering the estimated coordinates of the first order real network listed in Tables (5.3.2) as fixed non-stochastic entities. The *static* approach was then used to densify the first order networks by intercalating into them second and third order points giving results for first level densification {cf. section (5.3.5.1)} and second level densification {cf. section (5.3.5.2)}.

5.3.5.1 First Level Densification

In the first level real network densification, first order points 1 through 11 were considered as fixed non-stochastic parameters while second order points 12 through 26 were considered as new points whose coordinates were to be estimated (cf. Figure (4.6)). The observation data sets in Table (4.10) and the approximate coordinates in Table (4.9) were used resulting in parameters listed in Table (5.3.15) and Table (5.3.16) below while error ellipses are given diagrammatically in Figure (5.23).

Table 5.3.15: Coordinate Corrections and Stochastic Parameters-First Level Real Network -Static Approach

POINT	ΔN (m)	ΔE (m)	σ_N (m)	σ_E (m)	σ_{max} (m)	σ_{min} (m)	α ° ' ''
1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0
2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0
3	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0
4	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0
5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0
6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0
7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0
8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0
9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0
10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0
11	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0
12	-0.0140	0.0308	0.0118	0.0212	0.0216	0.0111	336 12 24.7
13	0.0193	0.0125	0.0295	0.0477	0.0540	0.0154	58 18 52.5
14	-0.0553	-0.0311	0.0312	0.0160	0.0312	0.0160	04 22 11.1
15	-0.0159	-0.0234	0.0170	0.0381	0.0395	0.0134	327 29 38.6
16	-0.0438	0.0653	0.0150	0.0141	0.0163	0.0125	75 27 44.6
17	0.0384	0.0162	0.0180	0.0473	0.0476	0.0174	11 39 49.7
18	0.0313	-0.0933	0.0202	0.0211	0.0225	0.0186	284 08 41.4
19	-0.1564	-0.1560	0.0348	0.0263	0.0376	0.0221	56 00 09.5
20	-0.0118	-0.1532	0.0170	0.0205	0.0215	0.0158	51 27 24.7
21	0.0517	-0.2147	0.0151	0.0200	0.0217	0.0125	57 08 49.3
22	-0.0806	0.0292	0.0165	0.0319	0.0333	0.0136	324 03 50.6
23	0.0756	0.0925	0.0182	0.0337	0.0349	0.0160	327 07 12.4
24	-0.2037	-0.1762	0.0164	0.0146	0.0167	0.0144	333 33 39.6
25	-0.2244	-0.0809	0.0364	0.0176	0.0365	0.0176	359 13 35.4
26	-0.2248	-0.1670	0.0301	0.0155	0.0308	0.0139	28 39 51.4

$\bar{\sigma}_N$=0.017607 $\bar{\sigma}_E$=0.021251 $\bar{\sigma}_C$=0.019514 $\hat{\sigma}_0^2$=1.004632

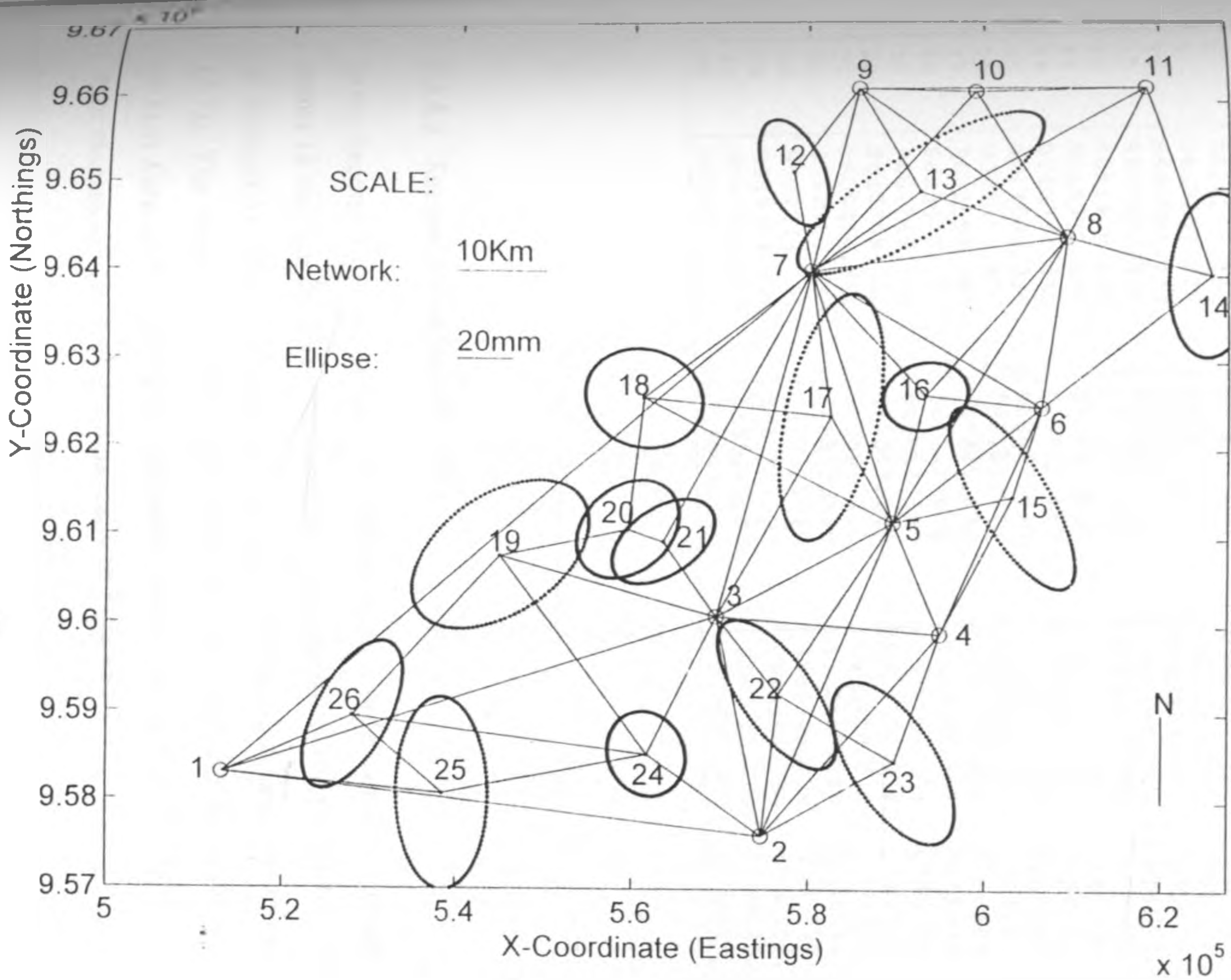


Figure 5.23: Point error ellipses -first level real network densification static approach

Table 5.3.16: Estimated Coordinates- First Level Real Network Densification -Static Approach

POINT	PROVISIONAL COORDINATES N(m)	E(m)	ADJUSTED COORDINATES N(m)	E(m)
1	9583105.004	513215.950	9583105.004	513215.950
2	9576179.235	574648.607	9576179.235	574648.607
3	9600981.218	569580.575	9600981.218	569580.575
4	9599052.507	594885.488	9599052.507	594885.488
5	9611665.696	589503.231	9611665.696	589503.231
6	9624778.565	606329.885	9624778.565	606329.885
7	9640229.703	580182.077	9640229.703	580182.077
8	9644287.286	609203.286	9644287.286	609203.286
9	9661267.225	585576.691	9661267.225	585576.691
10	9660984.753	598816.111	9660984.753	598816.111
11	9661696.859	618181.218	9661696.859	618181.218
12	9651426.870	578082.210	9651426.856	578082.241
13	9649302.660	592482.850	9649302.679	592482.863
14	9640093.640	625611.430	9640093.585	625611.399
15	9614609.390	603055.440	9614609.374	603055.417
16	9626055.670	593121.870	9626055.626	593121.935
17	9623694.380	582449.870	9623694.418	582449.886
18	9625643.250	561292.670	9625643.281	561292.577
19	9607749.860	545064.590	9607749.704	545064.434
20	9610759.110	559563.590	9610759.098	559563.437
21	9609410.820	563591.710	9609410.872	563591.495
22	9592183.630	576550.570	9592183.549	576550.599
23	9584543.130	589735.740	9584543.206	589735.832
24	9585354.590	561756.300	9585354.386	561756.124
25	9580752.500	538526.380	9580752.276	538526.299
26	9589557.530	528224.190	9589557.305	528224.023

5.3.5.2 Second Level Densification

In the Second level real network densification, first order points 1 through 11 and second order points 12 through 26 were considered as fixed non-stochastic parameters while third order points 27 through 41 were considered as new points whose coordinates were to be estimated (cf. Figure (4.7)). The observation data sets in Table (4.12) and the approximate coordinates in Table (4.11d) were used resulting in parameters listed in Table (5.3.17) and Table (5.3.18) below while error ellipses are given diagrammatically in Figure (5.24).

Table 5.3.17: Coordinate Corrections and Stochastic Parameters-Second Level Real Network -Static Approach

POINT	ΔN (m)	ΔE (m)	σ_N (m)	σ_E (m)	σ_{max} (m)	σ_{min} (m)	α °	'	"
1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0121	00		
2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0100	00		
3	0.0000	0.0000	0.0000	0.0000	0.0000	0.0086	00		
4	0.0000	0.0000	0.0000	0.0000	0.0000	0.0080	00		
5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0069	00		
6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0073	00		
7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0070	00		
8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0057	00		
9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0073	00		
10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0067	00		
11	0.0000	0.0000	0.0000	0.0000	0.0000	0.0072	00		
12	0.0000	0.0000	0.0000	0.0000	0.0000	0.0137	00		
13	0.0000	0.0000	0.0000	0.0000	0.0000	0.0186	00		
14	0.0000	0.0000	0.0000	0.0000	0.0000	0.0188	00		
15	0.0000	0.0000	0.0000	0.0000	0.0000	0.0167	00		
16	0.0000	0.0000	0.0000	0.0000	0.0000	0.0150	00		
17	0.0000	0.0000	0.0000	0.0000	0.0000	0.0206	00		
18	0.0000	0.0000	0.0000	0.0000	0.0000	0.0216	00		
19	0.0000	0.0000	0.0000	0.0000	0.0000	0.0256	00		
20	0.0000	0.0000	0.0000	0.0000	0.0000	0.0193	00		
21	0.0000	0.0000	0.0000	0.0000	0.0000	0.0163	00		
22	0.0000	0.0000	0.0000	0.0000	0.0000	0.0166	00		
23	0.0000	0.0000	0.0000	0.0000	0.0000	0.0194	00		
24	0.0000	0.0000	0.0000	0.0000	0.0000	0.0183	00		
25	0.0000	0.0000	0.0000	0.0000	0.0000	0.0215	00		
26	0.0000	0.0000	0.0000	0.0000	0.0000	0.0182	00		
27	-0.0014	0.0159	0.0166	0.0125	0.0223	0.0144	52	55	14.8
28	-0.0075	0.0050	0.0109	0.0119	0.0243	0.0127	345	44	29.8
29	0.0032	-0.0035	0.0099	0.0093	0.0125	0.0104	30	28	57.2
30	-0.0006	-0.0006	0.0083	0.0093	0.0243	0.0098	20	07	54.8
31	-0.0040	0.0012	0.0074	0.0082	0.0221	0.0114	276	35	29.1
32	-0.0050	0.0079	0.0075	0.0091	0.0213	0.0113	60	34	17.7
33	0.0026	-0.0015	0.0072	0.0075	0.0133	0.0092	74	38	26.1
34	-0.0037	0.0037	0.0057	0.0070	0.0135	0.0076	03	11	55.3
35	0.0005	0.0062	0.0078	0.0073	0.0097	0.0091	29	07	45.1
36	-0.0185	0.0067	0.0071	0.0067	0.0249	0.0105	81	41	46.5
37	-0.0018	0.0034	0.0085	0.0078	0.0135	0.0085	351	41	47.1
38	0.0018	0.0010	0.0139	0.0245	0.0145	0.0107	48	36	47.9
39	0.0438	-0.0105	0.0323	0.0516	0.0548	0.0156	22	31	13.9
40	-0.0025	-0.0244	0.0347	0.0181	0.0379	0.0135	359	57	15.7
41	0.0111	-0.0031	0.0193	0.0415	0.0355	0.0170	63	11	41.4

$\bar{\sigma}_N$=0.009568 $\bar{\sigma}_E$=0.012295 $\bar{\sigma}_C$=0.011016 $\hat{\sigma}_0^2$=1.000323

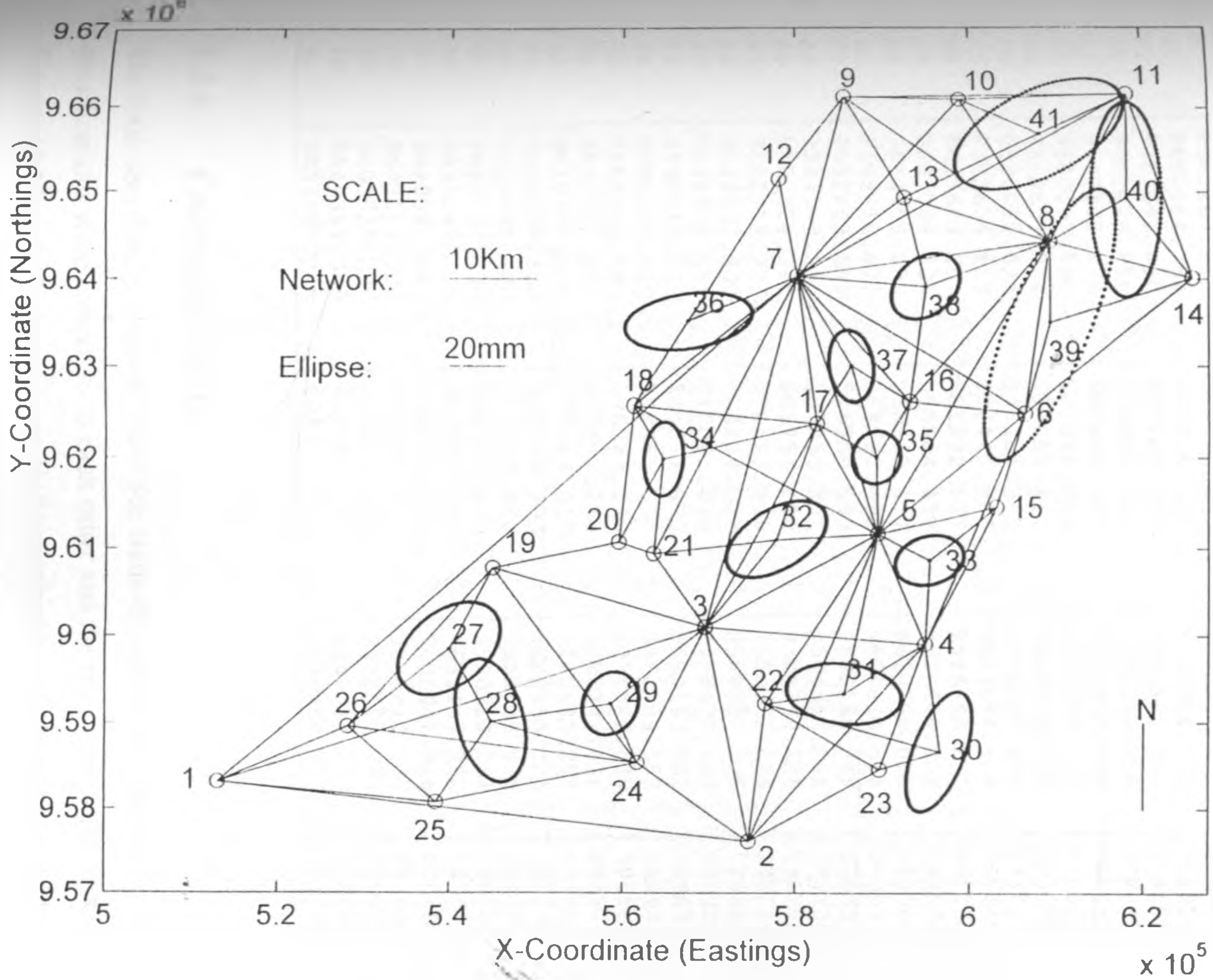


Figure 5.24: Point error ellipses -second level real network densification static approach

Table 5.3.18: Estimated Coordinates- Second Level Real Network Densification -Static Approach)

POINT	PROVISIONAL COORDINATES N(m)	PROVISIONAL COORDINATES E(m)	ADJUSTED COORDINATES N(m)	ADJUSTED COORDINATES E(m)
1	9583105.004	513215.950	9583105.004	513215.950
2	9576179.235	574648.607	9576179.235	574648.607
3	9600981.218	569580.575	9600981.218	569580.575
4	9599052.507	594885.488	9599052.507	594885.488
5	9611665.696	589503.231	9611665.696	589503.231
6	9624778.565	606329.885	9624778.565	606329.885
7	9640229.703	580182.077	9640229.703	580182.077
8	9644287.286	609203.286	9644287.286	609203.286
9	9661267.225	585576.691	9661267.225	585576.691
10	9660984.753	598816.111	9660984.753	598816.111
11	9661696.859	618181.218	9661696.859	618181.218
12	9651426.856	578082.241	9651426.856	578082.241
13	9649302.679	592482.863	9649302.679	592482.863
14	9640093.585	625611.399	9640093.585	625611.399
15	9614609.374	603055.417	9614609.374	603055.417
16	9626055.626	593121.935	9626055.626	593121.935
17	9623694.418	582449.886	9623694.418	582449.886
18	9625643.281	561292.577	9625643.281	561292.577
19	9607749.704	545064.434	9607749.704	545064.434
20	9610759.098	559563.437	9610759.098	559563.437
21	9609410.872	563591.495	9609410.872	563591.495
22	9592183.549	576550.599	9592183.549	576550.599
23	9584543.206	589735.832	9584543.206	589735.832
24	9585354.386	561756.124	9585354.386	561756.124
25	9580752.276	538526.299	9580752.276	538526.299
26	9589557.305	528224.023	9589557.305	528224.023
27	9598477.450	540001.040	9598477.449	540001.056
28	9590143.010	544871.930	9590143.002	544871.935
29	9592184.550	558655.590	9592184.553	558655.587
30	9586646.840	596567.510	9586646.839	596567.509
31	9593341.460	585647.230	9593341.456	585647.231
32	9610982.260	577814.010	9610982.255	577814.018
33	9608655.320	595394.330	9608655.323	595394.329
34	9619838.450	564763.860	9619838.446	564763.864
35	9620013.730	589321.420	9620013.731	589321.426
36	9635147.760	567613.380	9635147.752	567613.387
37	9630041.920	586400.120	9630041.918	586400.123
38	9639112.320	594892.100	9639112.322	594892.101
39	9635021.720	609221.860	9635021.764	609221.849
40	9649311.340	618081.160	9649311.338	618081.134
41	9656888.740	608069.420	9656888.751	608069.417

5.3.6 Computed Shifts

The final coordinates obtained from the static-dynamic, sub-optimal Fusion, dynamic and static densifications were compared to each other and the results tabulated as shown in Table (5.3.19). Figures (5.25 to 5.30) depict these shifts graphically. In the table, δ is the magnitude of the shift in the two sets of coordinates for each point while α is the bearing of the shift. Also, $\overline{\delta}$ and $\overline{\alpha}$ are the mean of δ and α for the respective densification approaches.

Table 5.3.19: Shifts between estimated parameters for the Experiments (Real Network)

	Static-Dynamic and Sub-Optimal		Static-Dynamic and Dynamic		Static-Dynamic and Static		Sub-Optimal and Dynamic		Sub-Optimal and Static		Dynamic and Static	
St	δ (mm)	α (° ')	δ (mm)	α (° ')	δ (mm)	α (° ')	δ (mm)	α (° ')	δ (mm)	α (° ')	δ (mm)	α (° ')
1	0	0	216.3	166 05	0	0	216.3	166 05	0	0	216.3	166 05
2	0	0	75.8	67 30	0	0	75.8	67 30	0	0	75.8	67 30
3	0	0	170.5	294 37	0	0	170.5	294 37	0	0	170.5	294 37
4	0	0	85.1	357 18	0	0	85.1	357 18	0	0	85.1	357 18
5	0	0	174.7	275 15	0	0	174.7	275 15	0	0	174.7	275 15
6	0	0	63.1	165 18	0	0	63.1	165 18	0	0	63.1	165 18
7	0	0	81.5	14 12	0	0	81.5	14 12	0	0	81.5	14 12
8	0	0	105.0	229 38	0	0	105.0	229 38	0	0	105.0	229 38
9	0	0	88.4	43 37	0	0	88.4	43 37	0	0	88.4	43 37
10	0	0	144.2	33 41	0	0	144.2	33 41	0	0	144.2	33 41
11	0	0	178.7	150 08	0	0	178.7	150 08	0	0	178.7	150 08
12	45.3	22 02	30.4	117 24	2.2	116 34	56.9	169 52	45.5	199 14	28.2	297 28
13	17.1	20 33	21.3	311 11	2.8	45 00	22.1	264 48	14.6	195 57	21.6	123 41
14	29.2	275 54	97.6	204 12	13.6	162 54	92.7	186 49	29.7	122 37	84.1	25 21
15	26.6	214 17	71.3	264 22	7.6	113 12	58.0	284 59	20.6	22 50	64.1	86 25
16	24.7	194 02	70.7	135 00	9.9	135 00	61.7	114 54	21.4	37 24	60.8	315 00
17	60.5	304 13	72.1	315 00	7.0	00 00	17.0	356 38	56.8	118 22	67.3	130 47
18	73.3	334 08	98.5	293 58	14.8	298 18	63.6	245 51	62.0	162 09	83.8	113 12
19	40.6	307 59	240.4	226 01	19.8	225 00	238.2	216 18	42.9	155 13	220.7	46 06
20	69.9	326 05	57.3	279 53	14.0	265 55	120.9	253 40	66.8	158 02	143.8	101 14
21	71.6	326 58	175.6	284 31	23.8	284 37	132.0	263 02	56.3	163 30	151.8	104 30
22	49.0	286 35	87.2	153 26	10.3	240 57	125.9	223 04	42.5	116 34	87.4	326 40
23	51.6	328 28	95.2	26 50	12.2	350 32	81.1	59 38	40.6	142 00	85.8	211 39
24	17.0	93 22	252.0	222 26	39.7	220 55	263.0	225 18	51.9	303 59	212.3	42 43
25	37.5	133 55	177.5	206 25	26.1	184 24	255.9	204 28	29.0	270 00	245.4	18 17
26	35.8	153 26	209.4	233 21	21.9	210 04	206.2	243 11	30.0	295 43	189.4	55 58
27	396.0	233 25	201.5	218 21	8.0	180 00	208.2	67 59	391.3	54 22	195.3	39 48
28	215.5	231 13	172.4	216 16	6.7	116 34	66.1	93 28	218.4	52 49	173.7	38 27
29	224.3	351 48	153.8	207 04	21.6	146 19	93.1	335 55	208.7	12 10	144.5	34 34
30	159.5	82 04	64.1	92 41	1.4	315 00	97.3	255 06	160.4	97 31	65.1	273 31
31	15.0	90 00	76.5	118 54	1.0	180 00	63.8	125 26	15.0	266 12	76.0	298 15
32	314.2	287 58	136.2	277 36	19.9	287 32	81.1	115 51	294.4	108 00	117.6	95 51
33	79.6	296 53	152.3	304 50	1.0	00 00	74.2	313 22	77.4	116 54	150.1	124 58
34	89.9	261 41	112.8	280 13	8.0	270 00	39.7	326 19	82.0	80 53	104.9	100 59
35	49.1	56 38	46.5	298 13	8.2	75 58	5.0	180 00	55.0	117 02	52.9	112 12
36	145.0	158 34	70.6	127 31	12.8	141 20	92.0	01 52	132.8	340 12	58.2	304 30
37	41.0	01 24	38.3	344 53	15.8	71 34	11.7	250 01	38.6	158 45	40.6	142 00
38	58.2	191 53	56.1	211 08	1.4	45 00	19.2	297 54	59.4	12 38	57.4	31 29
39	111.3	206 06	119.8	190 05	36.4	344 03	33.3	122 44	140.5	16 07	153.4	04 07
40	75.0	21 05	145.0	189 32	10.4	286 42	89.1	214 56	76.5	331 05	140.7	05 43
41	69.9	99 36	3.2	71 34	14.4	56 19	63.2	280 57	56.3	289 43	11.4	52 07
Mean	$\bar{\delta}$ = 89.8	$\bar{\alpha}$ = 171 10	$\bar{\delta}$ = 114.4	$\bar{\alpha}$ = 300 10	$\bar{\delta}$ = 13.1	$\bar{\alpha}$ = 23 48	$\bar{\delta}$ = 102.8	$\bar{\alpha}$ = 285 48	$\bar{\delta}$ = 87.2	$\bar{\alpha}$ = 197 57	$\bar{\delta}$ = 113.9	$\bar{\alpha}$ = 112 47

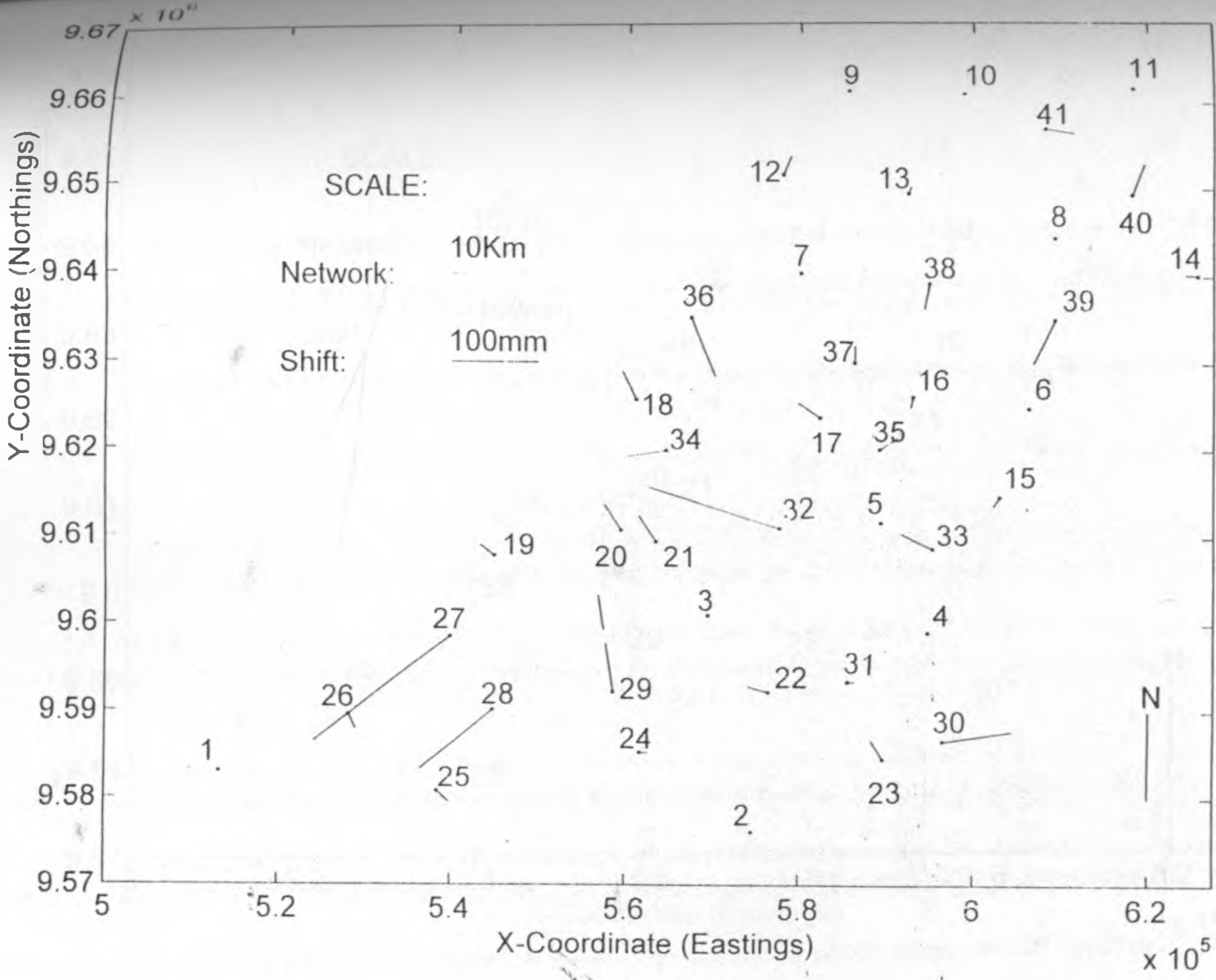


Figure 5.25: Coordinate shifts of sub-optimal fusion approach with respect to static-dynamic approach

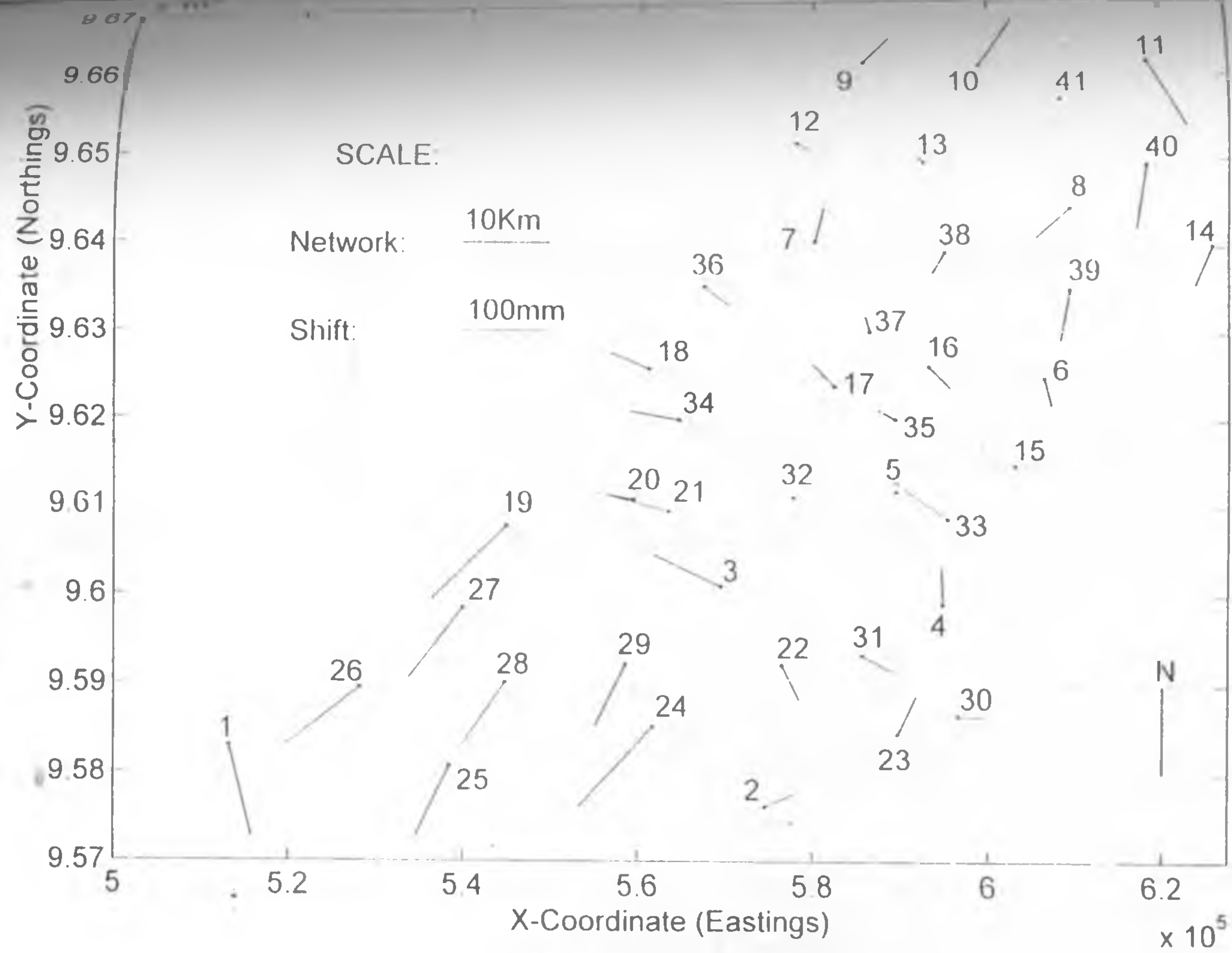


Figure 5.26: Coordinate shifts of dynamic approach with respect to static-dynamic approach

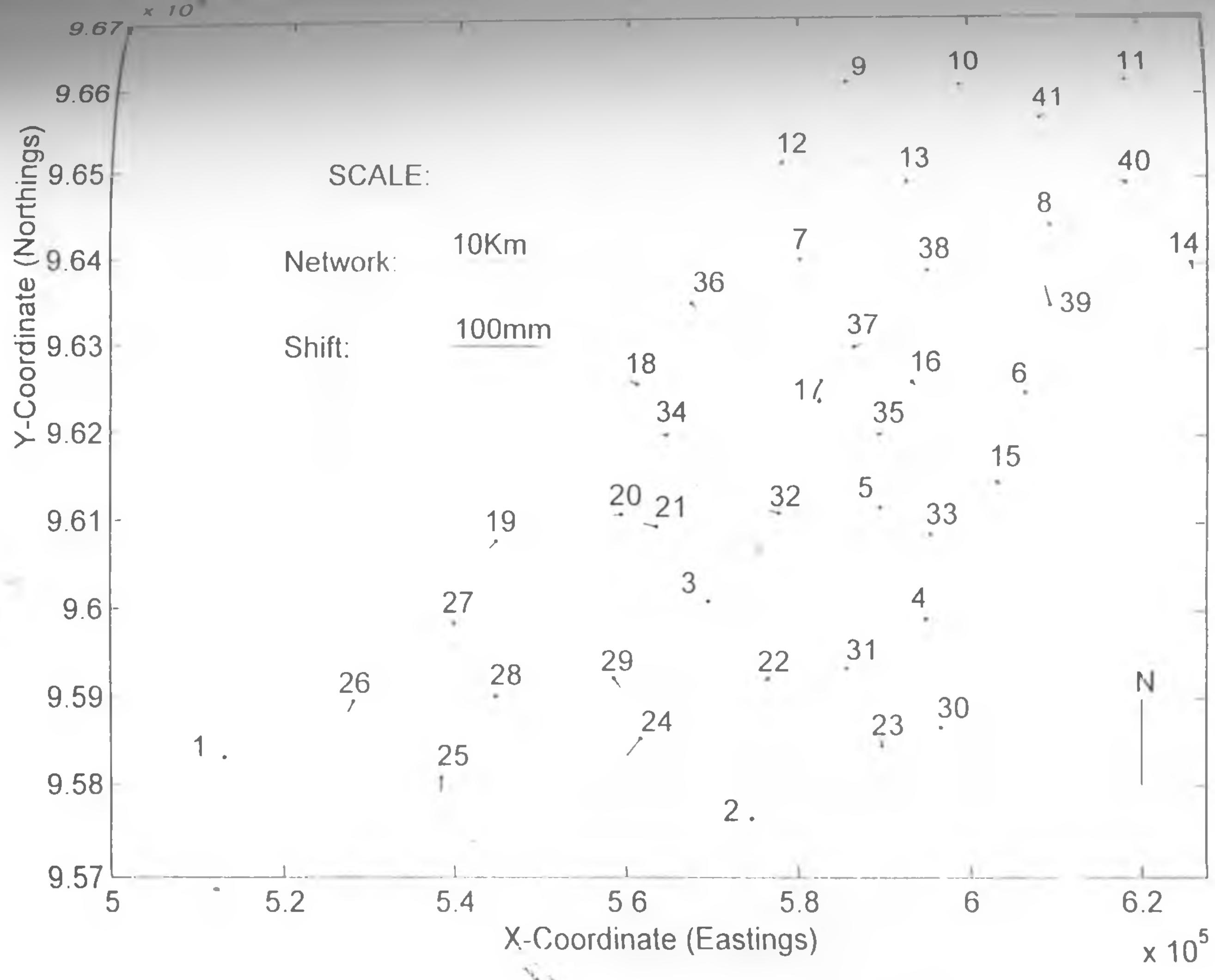


Figure 5.27: Coordinate shifts of static approach with respect to static-dynamic approach

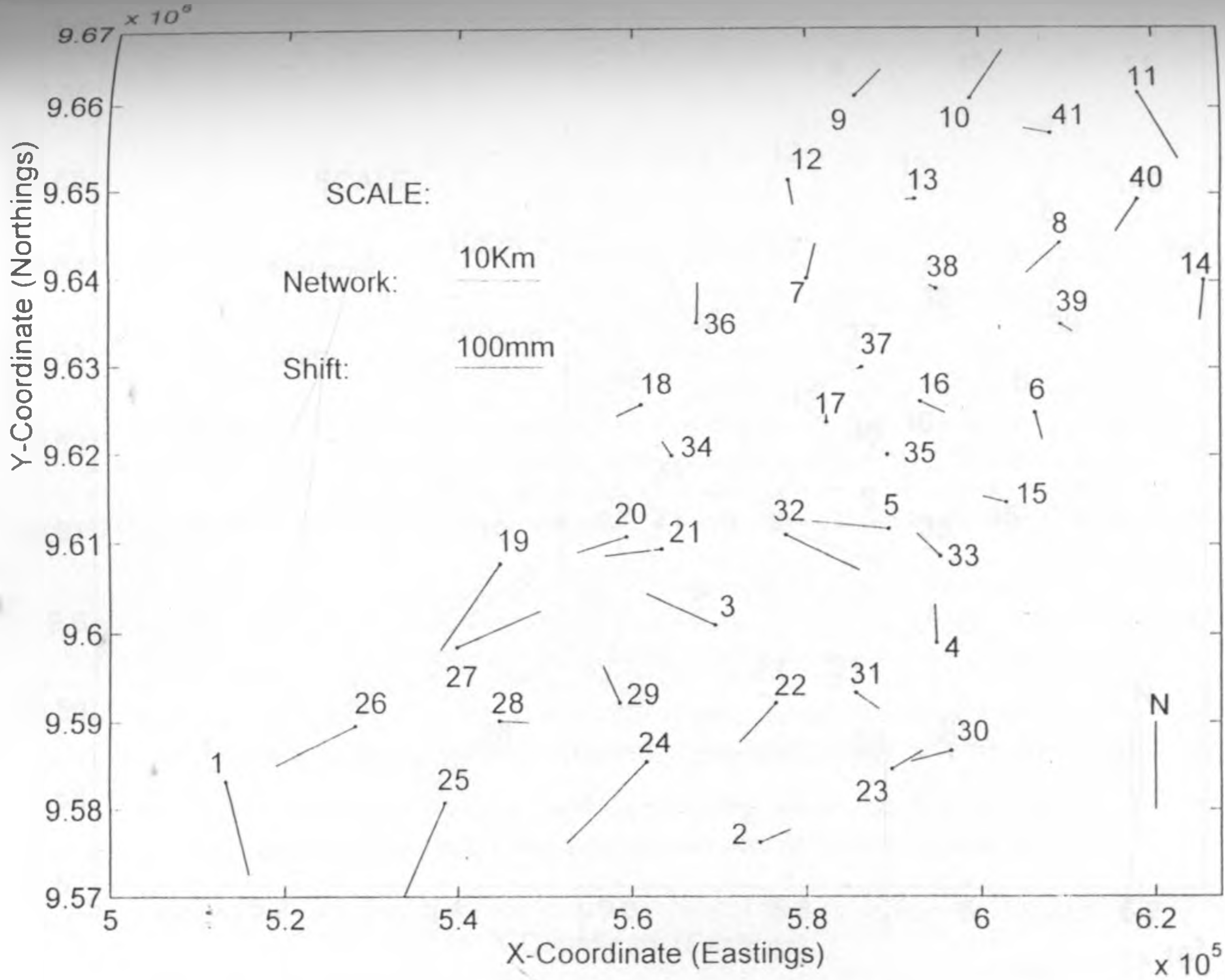


Figure 5.28: Coordinate shifts of dynamic approach with respect to sub-optimal fusion approach

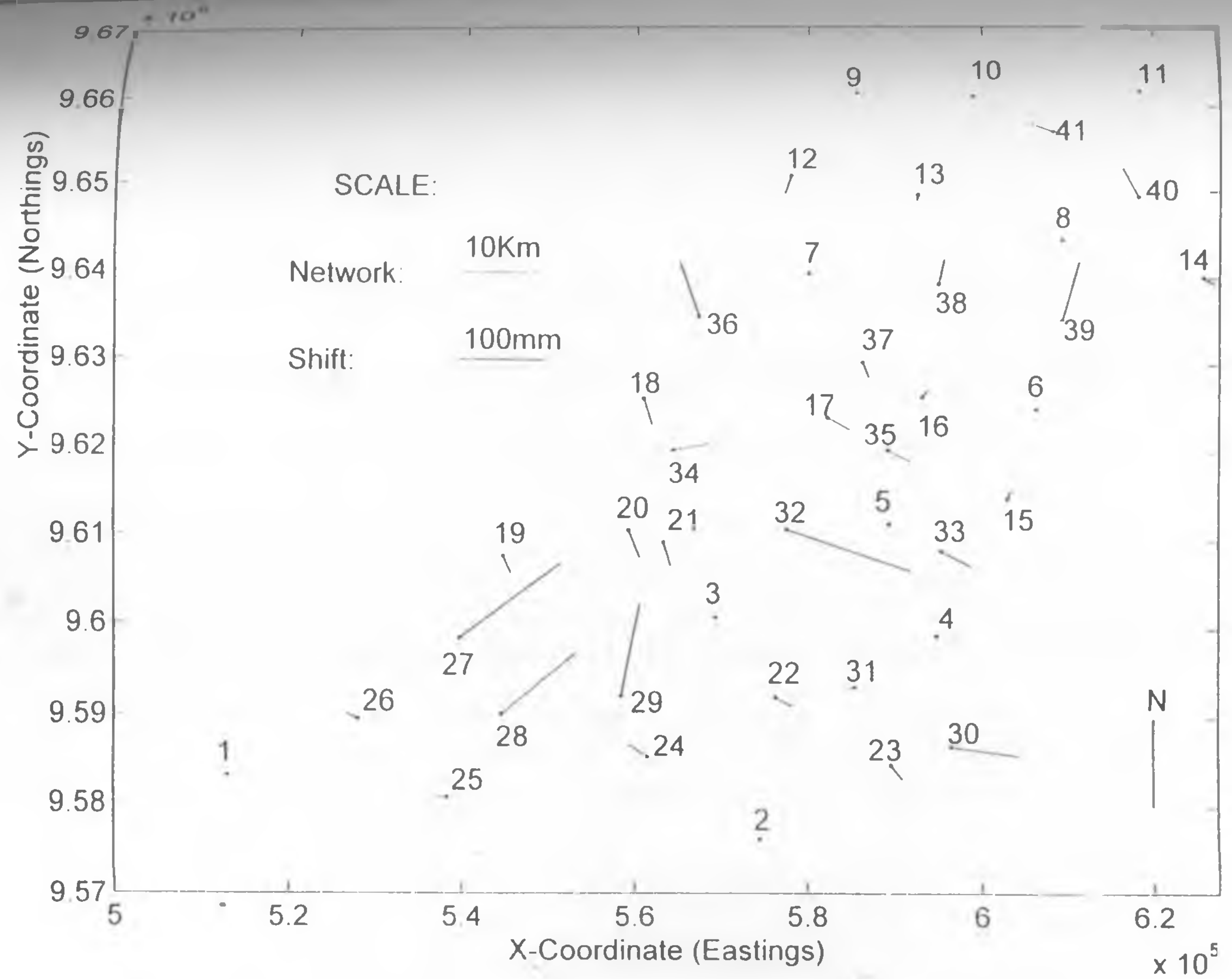


Figure 5.29: Coordinate shifts of static approach with respect to sub-optimal fusion approach

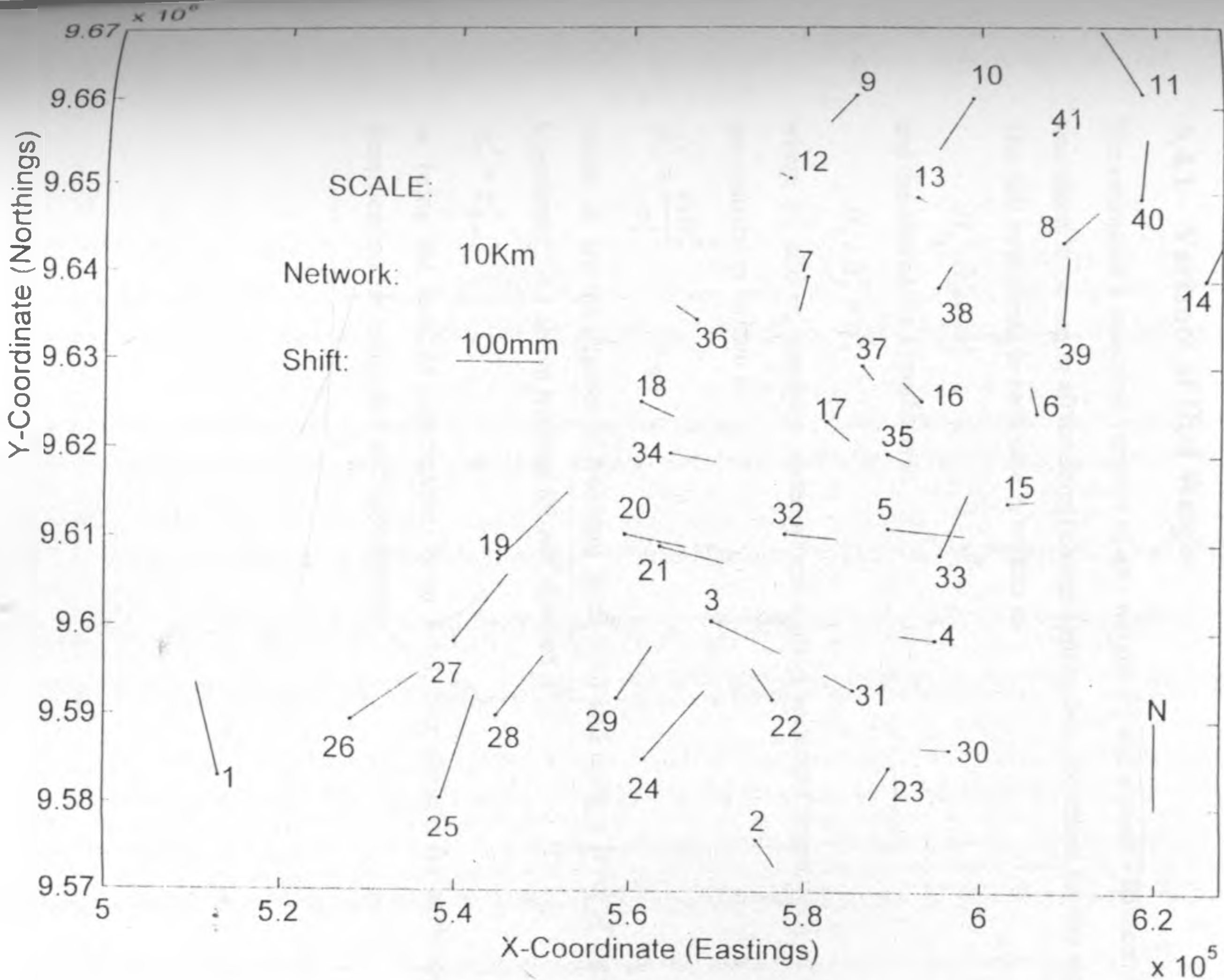


Figure 5.30: Coordinate shifts of static approach with respect to dynamic approach

5.4 Analysis of Results

In section (4.3), a number of precision and reliability criteria for densification network analysis have been discussed. The estimated corrections to the provisional parameters which have been computed through the use of static-dynamic, sub-optimal fusion, dynamic and static approaches are presented in Chapter Five together with the standard errors, error ellipses and displacement components in Tables (5.2.1) to (5.3.30). In this chapter, the results presented in Chapter Five are analyzed.

5.4.1 Variance of Unit Weight

The estimated a posteriori variance of unit weight $\hat{\sigma}_0^2$ and a priori variance of unit weight, which was taken to be one in all the densification approaches, are tested for any significant difference. The null hypothesis in each case is written as

$$H_o : \hat{\sigma}_0^2 = \sigma_0^2 \tag{5.4.1}$$

and the alternative hypothesis as

$$H_a : \hat{\sigma}_0^2 \neq \sigma_0^2 \tag{5.4.2}$$

where $\hat{\sigma}_0^2$ and σ_0^2 are the a posteriori and a priori variances respectively. Using the χ^2 test, the test statistic is written as

$$\chi_m^2 = \frac{m\hat{\sigma}_0^2}{\sigma_0^2} \tag{5.4.3}$$

where m are the degrees of freedom. With 5.4.3 and using a level of significance of 5%, the hypothesis 5.4.1 above is tested for and rejected if

$$\chi_m^2 > \chi_{\frac{\alpha}{2},m}^2 \tag{5.4.4}$$

α being the level of significance. From 5.4.3, test statistic for each approach and level of densification are computed and tabled below.

Table 5.4.1: Computed statistical values for χ^2 test of Simulated network densification

	First Level Densification		Second Level Densification	
STATIC-DYNAMIC DENSIFICATION	$\chi^2_{15} = 15.064$	Accepted	$\chi^2_{60} = 60.374$	Accepted
	$\chi^2_{0.025,15} = 28.488$		$\chi^2_{0.025,60} = 84.893$	
SUB-OPTIMAL FUSION DENSIFICATION	$\chi^2_{15} = 15.049$	Accepted	$\chi^2_{60} = 60.295$	Accepted
	$\chi^2_{0.025,15} = 28.488$		$\chi^2_{0.025,60} = 84.893$	
DYNAMIC DENSIFICATION	$\chi^2_{15} = 15.020$	Accepted	$\chi^2_{60} = 60.163$	Accepted
	$\chi^2_{0.025,15} = 28.488$		$\chi^2_{0.025,60} = 84.893$	
STATIC DENSIFICATION	$\chi^2_{21} = 21.084$	Accepted	$\chi^2_{72} = 72.197$	Accepted
	$\chi^2_{0.025,21} = 36.556$		$\chi^2_{0.025,72} = 99.276$	

Table 5.4.2: Computed statistical values for χ^2 test of the Real network densification

	First Level Densification		Second Level Densification	
STATIC-DYNAMIC DENSIFICATION	$\chi^2_{28} = 28.258$	Accepted	$\chi^2_{18} = 18.178$	Accepted
	$\chi^2_{0.025,28} = 45.628$		$\chi^2_{0.025,18} = 32.551$	
SUB-OPTIMAL FUSION DENSIFICATION	$\chi^2_{28} = 28.183$	Accepted	$\chi^2_{18} = 18.167$	Accepted
	$\chi^2_{0.025,28} = 45.628$		$\chi^2_{0.025,18} = 32.551$	
DYNAMIC DENSIFICATION	$\chi^2_{28} = 28.111$	Accepted	$\chi^2_{18} = 18.146$	Accepted
	$\chi^2_{0.025,28} = 45.628$		$\chi^2_{0.025,18} = 32.551$	
STATIC DENSIFICATION	$\chi^2_{50} = 50.232$	Accepted	$\chi^2_{70} = 70.024$	Accepted
	$\chi^2_{0.025,50} = 72.906$		$\chi^2_{0.025,70} = 96.879$	

These results indicate that the null hypothesis (6.1) is accepted which implies no significance difference between a posteriori and a priori variances of unit weight. From the Tables above, the null hypothesis for the χ^2_m test was accepted for all the experiments on real and simulated networks at both levels of densification. This is a clear indication that the estimated a posteriori variances of unit weight from the densification approaches are statistically equal in the estimation and assumed to be unity.

The acceptance of the null hypothesis indicates that the estimation processes were correctly done and more specifically that the a priori variance of unit weight was correctly chosen and further all the four approaches relate the unknown parameters completely and correctly.

Although in all cases the null hypothesis is acceptable, it has to be observed that for individual statistical estimates, the more close the test statistic is to the tabulated values the more the estimate is considered reliable. An examination of the values of χ^2_m and $\chi^2_{\frac{\alpha}{2},m}$ determined, static densification are closest followed by static-dynamic densification then sub-optimal fusion densification and finally dynamic densification.

It is expected that the densification results of the real and simulated networks by using the static approach appear as the best because the approach is based on the assumption that higher order points are fixed and non-stochastic. This, as we know, is not true since these higher order points were themselves determined from previous adjustment and therefore has stochasticity associated to them.

The results of static-dynamic and sub-optimal fusion densifications have the second closest values while those of dynamic densification have the largest difference. This is attributed to the fact that for static-dynamic and sub-optimal fusion approaches, the datum parameters are maintained definitive. The results determined from static-dynamic approach as compared to those obtained from sub-optimal fusion approach, seem to be closest what might indicate that the static-dynamic approach is superior. Although strictly the difference is insignificant. It is though observed that these two approaches give results that lie between those obtained by static and dynamic approaches and vary from each other slightly. The values obtained from dynamic approach have the largest difference. It is therefore concluded that the static-dynamic and sub-optimal approaches are both viable approaches to densification though the analysis of variances indicate former is superior to the latter.

5.4.2 Standard Errors

The standard errors for static densification are generally smaller followed by those for static-dynamic densification, then for sub-optimal densification and lastly for dynamic densification. The results of static densification seem to be more accurate from the analysis of the standard errors but, as in the case of variances analyzed above, this is because the datum parameters are considered as fixed non-stochastic entities. The standard errors from static-dynamic and sub-optimal fusion approaches lie between those obtained from static and dynamic approaches and the most important observation is that they are close to each other.

5.4.3 Error Ellipses

Point error ellipses resulting from free network adjustment of the simulated and real networks are depicted in Figures (5.1) and (5.16). Figures (5.2), (5.3), (5.17) and (5.18) depict point error ellipses for the first and second level densification of the simulated and real network using static-dynamic approach. Error ellipses resulting from the first and second level densification of the simulated and real networks using sub-optimal fusion approach are depicted in Figures (5.4), (5.5), (5.19) and (5.20) and those resulting from dynamic approach are depicted in Figures (5.6), (5.7), (5.21) and (5.22). Lastly error ellipses resulting from the first and second level densification of the real and simulated networks using static approach are depicted in Figures (5.8), (5.9), (5.23) and (5.24).

In the first level real network densification using static approach, network points 1 to 11 do not have error ellipses while in the second level real network densification, network points 1 to 26 do not have error ellipses. Correspondingly, in the first level simulated network densification points 1 to 3 do not have error ellipses while in the second level densification, points 1 to 6 do not have error ellipses. This, as expected, is due to the fact that static approach do not consider the stochasticity of the datum parameters and thus consider them to be accurate with error ellipses equal to zeros.

Datum point error ellipses for the first level densification using static-dynamic approaches are similar to those obtained, for the same points, in first order free network adjustment. Also, datum point error ellipses for the second level densification using static-dynamic approaches are similar to those obtained, for the same points, in first level densification. This indicate that the static-dynamic and sub-optimal fusion approach are capable of exactly reproducing the datum point parameters together with their stochasticity.

Error ellipses resulting from densification using static approach are the smallest followed by those obtained using static-dynamic and sub-optimal fusion approaches and lastly by those obtained using dynamic approach The implication of the error ellipse of a point is a space in which there is 0.394 probability that the estimated point lies inside, and therefore the smaller the ellipse the more accurate the point is placed. Though it need to be realized that the error ellipses resulting from static approach is attributed to the fundamental concept of considering the datum parameters as fixed non-stochastic entities and therefore not representative enough.

Static-dynamic and sub-optimal approaches which incorporate the stochasticity of the datum parameters and yield second best results to the static approach, are thus considered to be most reliable estimates. It is however indeed noticed that, in all the approaches used in densification of the simulated network, the error ellipses are symmetric.

5.4.4 Efficiency of Estimates

From the discussions in section (5.4.1) and (5.4.2) coupled with the results obtained by *Miima [1997]*, results of static-dynamic densification can be considered to be the best overall. On the basis of this, the computed $\bar{\sigma}_E$, $\bar{\sigma}_N$ and $\bar{\sigma}_C$ of the sub-optimal fusion, dynamic and static densifications are tested for any significant difference from the values determined for static-dynamic densification.

The null hypothesis H_o and its alternative H_a are stated respectively as:

$$H_o : \bar{\sigma}_s^2 = \bar{\sigma}_t^2 \qquad (5.4.5a)$$

$$H_o : \bar{\sigma}_s^2 > \bar{\sigma}_t^2 \qquad (5.4.5b)$$

where $\bar{\sigma}_s^2$ is taken as the standard factor computed from static-dynamic densification while $\bar{\sigma}_t^2$ is the factor being tested. The test statistic in this case is defined as:

$$F_{m_1,m_2} = \frac{\bar{\sigma}_s^2}{\bar{\sigma}_t^2} \qquad (5.4.6)$$

and the null hypothesis is rejected if

$$F_{m_1,m_2} > F_{\alpha,m_1,m_2} \qquad (5.4.7)$$

where α, m_1 and m_2 are the level of significance and degrees of freedom for the sample s and t respectively. Using a level of significance of 5% and with (5.4.7), one obtains values given in the tables below.

Table 5.4.3: Computed values of the test statistics for $\overline{\sigma}_N$ (Simulated network densification)

	First Level Densification		Second Level Densification	
STATIC-DYNAMIC	-		-	
SUB-OPTIMAL FUSION	$F_{15,15} = 1.0751$ $F_{0.025,15,15} = 2.4325$	Accepted	$F_{60,60} = 1.1191$ $F_{0.025,60,60} = 1.3900$	Accepted
DYNAMIC	$F_{15,15} = 1.2011$ $F_{0.025,15,15} = 2.4325$	Accepted	$F_{60,60} = 1.0218$ $F_{0.025,60,60} = 1.3900$	Accepted
STATIC	$F_{15,21} = 0.7030$ $F_{0.025,15,21} = 2.3375$	Accepted	$F_{60,72} = 0.4334$ $F_{0.025,60,72} = 1.3620$	Accepted

Table 5.4.4: Computed values of the test statistics for $\overline{\sigma}_E$ (Simulated network densification)

	First Level Densification		Second Level Densification	
STATIC-DYNAMIC	-		-	
SUB-OPTIMAL FUSION	$F_{15,15} = 1.421$ $F_{0.025,15,15} = 2.043$	Accepted	$F_{60,60} = 1.398$ $F_{0.025,60,60} = 2.098$	Accepted
DYNAMIC	$F_{15,15} = 0.985$ $F_{0.025,15,15} = 2.336$	Accepted	$F_{60,60} = 1.453$ $F_{0.025,60,60} = 2.122$	Accepted
STATIC	$F_{15,21} = 1.654$ $F_{0.025,15,21} = 2.43$	Accepted	$F_{60,72} = 1.378$ $F_{0.025,60,72} = 2.765$	Accepted

Table 5.4.5: Computed values of the test statistics for $\overline{\sigma}_C$ (Simulated network densification)

	First Level Densification		Second Level Densification	
STATIC-DYNAMIC	-		-	
SUB-OPTIMAL FUSION	$F_{15,15} = 1.222$ $F_{0.025,15,15} = 2.077$	Accepted	$F_{60,60} = 1.331$ $F_{0.025,60,60} = 2.076$	Accepted
DYNAMIC	$F_{15,15} = 1.236$ $F_{0.025,15,15} = 2.345$	Accepted	$F_{60,60} = 1.334$ $F_{0.025,60,60} = 2.068$	Accepted
STATIC	$F_{15,21} = 1.221$ $F_{0.025,15,21} = 2.376$	Accepted	$F_{60,72} = 1.340$ $F_{0.025,60,72} = 2.098$	Accepted

Table 5.4.6: Computed values of the test statistics for $\overline{\sigma}_N$ (Real network densification)

	First Level Densification		Second Level Densification	
STATIC-DYNAMIC	-		-	
SUB-OPTIMAL FUSION	$F_{28.28} = 1.0707$	Accepted	$F_{18.18} = 1.0671$	Accepted
	$F_{0.025.28.28} = 2.098$		$F_{0.025.18.18} = 2.188$	
DYNAMIC	$F_{28.28} = 1.0697$	Accepted	$F_{18.18} = 1.0573$	Accepted
	$F_{0.025.28.28} = 2.277$		$F_{0.025.18.18} = 2.087$	
STATIC	$F_{28.50} = 0.8251$	Accepted	$F_{18.70} = 0.4097$	Accepted
	$F_{0.025.28.50} = 2.333$		$F_{0.025.18.70} = 2.098$	

Table 5.4.7: Computed values of the test statistics for $\overline{\sigma}_E$ (Real network densification)

	First Level Densification		Second Level Densification	
STATIC-DYNAMIC	-		-	
SUB-OPTIMAL FUSION	$F_{28.28} = 1.0725$	Accepted	$F_{18.18} = 1.0701$	Accepted
	$F_{0.025.28.28} = 2.121$		$F_{0.025.18.18} = 2.122$	
DYNAMIC	$F_{28.28} = 1.0690$	Accepted	$F_{18.18} = 1.0695$	Accepted
	$F_{0.025.28.28} = 2.098$		$F_{0.025.18.18} = 2.098$	
STATIC	$F_{28.50} = 0.8450$	Accepted	$F_{18.70} = 0.5060$	Accepted
	$F_{0.025.28.50} = 2.099$		$F_{0.025.18.70} = 2.133$	

Table 5.4.8: Computed values of the test statistics for $\overline{\sigma}_C$ (Real network densification)

	First Level Densification		Second Level Densification	
STATIC-DYNAMIC	-		-	
SUB-OPTIMAL FUSION	$F_{28.28} = 1.0717$	Accepted	$F_{18.18} = 1.0687$	Accepted
	$F_{0.025.28.28} = 1.6500$		$F_{0.025.18.18} = 2.2450$	
DYNAMIC	$F_{28.28} = 1.0694$	Accepted	$F_{18.18} = 1.0044$	Accepted
	$F_{0.025.28.28} = 1.6500$		$F_{0.025.18.18} = 2.2450$	
STATIC	$F_{28.50} = 0.3368$	Accepted	$F_{18.70} = 0.4623$	Accepted
	$F_{0.025.28.50} = 1.4500$		$F_{0.025.18.70} = 1.9200$	

5.4.5 Co-ordinate Shifts

Computed shift differences in magnitude and direction between the estimated values of the densification approaches are presented in Tables (5.2.19) and (5.3.19). The shifts resulting from simulated networks are depicted graphically in Figures (5.10 to 5.15) while those resulting from real networks are depicted in Figures (5.25) to (5.30). It noticed that parameters estimated from both the static-dynamic and sub-optimal fusion approaches are closer to those estimated from static approach. This may be attributed to the fact that all these approaches keep the datum parameters unchanged. The coordinate shifts between static-dynamic and sub-optimal fusion are the second closes followed by the shifts between sub-optimal fusion and dynamic densifications. The shifts between static-dynamic and dynamic comes last in this comparison.

6

DISCUSSION

Performance of the static-dynamic, sub-optimal fusion, dynamic and static approaches is discussed in this chapter, based mainly on the results in Chapter Five and the analysis of the results in Chapter Six. The discussion is presented in two parts. In the first part presented in section (6.1), the results for static-dynamic, sub-optimal fusion, dynamic and static densifications are discussed at the first level of densification while in the second part, presented in section (6.2), the results for static-dynamic, sub-optimal fusion, dynamic and static densifications are discussed at the second level of densification.

6.1 The First Level of Densification

First level of densification was performed by intercalation of the second order points into the first order points {cf. Figure (4.3) and Figure (4.6)}. The results of the first level densification of both real and simulated networks indicate that the standard errors as estimated for the new points in the dynamic approach are generally very close to those estimated in the static-dynamic and sub-optimal fusion approaches. In fact, as realized by *Aduol [1993]*, if the standard errors are multiplied by the factor $\frac{\sigma_0}{\hat{\sigma}_0}$, then practically one obtains equal standard errors from dynamic, static-dynamic and sub-optimal fusion approaches. This implies therefore that if standard errors are obtained by taking $\hat{\sigma}_0{}^2$ same as $\sigma_0{}^2$ then the standard errors obtained from the dynamic, static-dynamic and sub-optimal approaches are all equal.

The set of standard errors of the estimated parameters through use of the static-dynamic and the sub-optimal fusion approaches were very close to those estimated through the use of the dynamic approach in both the real and simulated network densifications. The computed average standard errors for the four experiments, though statistical tests show that there were no significant differences between them, their magnitude varied significantly. The average standard errors are $\pm$23.32 mm, $\pm$21.76 mm, $\pm$24.94 mm, and $\pm$19.51 mm for the static-dynamic, the sub-optimal fusion, the dynamic and the static densifications respectively in the first level real network densification while in the first level simulated network densification they are $\pm$3.55 mm, $\pm$3.81 mm, $\pm$4.25 mm and $\pm$2.48 mm for static-dynamic, sub-optimal fusion, dynamic and static densifications respectively. Correspondingly, the average standard errors are $\pm$23.83 mm, $\pm$22.30 mm, $\pm$23.72 mm and $\pm$11.02 mm for the static-dynamic, the sub-optimal fusion, the dynamic and the static densifications respectively in the second level real network densification while in the second level simulated network densification they are $\pm$2.82 mm, $\pm$3.16 mm, $\pm$2.89 mm and $\pm$1.21 mm for the static-dynamic, the sub-optimal fusion, the dynamic and the static densifications respectively.

In the static-dynamic and sub-optimal fusion approaches, the first level densification was undertaken, with the datum stations 1 to 11 in the real network and datum stations 1 to 3 in simulated network case being treated as fixed stochastic entities. This is the reason why these points have their error ellipses similar to the error ellipses obtained in the first order network adjustment by use of the free network adjustment technique. {cf. Figures (5.2), (5.4), (5.17) and (5.19)}. Similarly, the second level densification was undertaken, with the datum stations 1 to 26 in the real network and datum stations 1 to 6 in simulated network case being treated as fixed stochastic entities. This is the reason why these points have their error ellipses similar to the error ellipses obtained in the first level real and simulated network densifications by use of the static-dynamic and the sub-optimal fusion approaches. {cf. Figures (5.3), (5.5), (5.18) and (5.20)}. This is the essence of the reproducing property depicted by the static-dynamic and the sub-optimal fusion approaches

It is noted that in the static approach, the first level of densification was undertaken with the datum points 1 to 11 in the real network and datum points 1 to 3 in the simulated network being treated as fixed non-stochastic entities. Therefore, as expected, the error ellipses of these points are equal to zero. Similarly, in the second level densification, datum points 1 to 26 in the real network and datum points 1 to 6 in the simulated network have their error ellipses equal to zero.

In the dynamic approach, the densification was performed by treating the datum parameters as stochastic entities (i.e. the use of the dynamic approach) resulting in larger error ellipses compared to both static-dynamic and sub-optimal fusion approaches. In the dynamic densification, all points have error ellipses that are different from the previously determined error ellipses and therefore lacks the reproducing capability {cf. Figures (5.6), (5.7), (5.21) and (5.22)}. It is though noticeable that all point error ellipses for the datum stations are relatively smaller compared to those that were determined in previous adjustment. The point error ellipses in the second level densification are smaller compared to point error ellipses in the first level densification determined earlier. Thus the point error ellipses and standard errors of the estimated parameters in the dynamic approach tend to improve or become smaller with the increase in densification levels. This, as noted by *Aduol [1993]*, may be attributed to the increase in redundancy in the geodetic system with addition of new observations. It is though important to note that despite the rigorous nature of this approach to densification, due to incorporation of the stochasticity of the datum points in the densification, the datum points are estimated anew resulting in a technical handicap as the concept of control for national geodetic references loses meaning.

The apparent similarity in the behaviour of error ellipses in the dynamic, static-dynamic and sub-optimal fusion approaches both at the first level and second level densification of the real and simulated networks is to be expected since the dynamic, the static-dynamic and the sub-optimal fusion approaches are similar in principle. The differences in the results obtained are due to the mode of application i.e. which datum parameters are kept fixed in the static-dynamic and sub-optimal cases. In fact, the basis of the sub-optimal fusion approach is the estimation of the new point parameters according to the dynamic approach while applying a corrective term on the datum points parameters to keep them unchanged. This is clearly illustrated by the comparison of shifts of points from their position after phasing to their position as a result of dynamic and sub-optimal fusion approaches presented in Figures (5.13) and (5.28).

6.2 The Second Level of Densification

At second level of densification, the results of the first level of densification were densified into a third order network. From Tables (5.2.5), (5.3.5), (5.2.9), (5.3.9), (5.2.13) and (5.3.13) the static-dynamic and sub-optimal fusion approaches yielded relatively smaller error ellipses as compared to the dynamic approach. It is however noted that, as in the first level of densification, error ellipses resulting from the static approach are the smallest. It is observed that the order of accuracy of determined parameters at this level is similar to that of first level of densification. However it is noticed that numerical values of accuracies decrease at this level for all approaches which indicate a general improvement of accuracy in the higher order geodetic systems. This was not expected as it is expected that the accuracy should reduce with increase in densification levels. This can be attributed to the increase in redundancy in the geodetic systems.

From section 5.4.1 the analysis indicates that the estimated a-posteriori variance of unit weight for all the approaches at all levels of densification were statistically equal to the a-priori variance of unit weight. This therefore indicates that the estimation processes were correctly carried out and specifically that the approaches relate to unknown parameters completely and correctly.

The efficiency of estimates tested in section 5.4.4 indicated no significant differences between the computed circular probable error for all the experiments at both level of densification. While this signified the validity of the estimates determined, it is attributed to the fact that the parameters used as estimates in the study were in fact computed final point coordinates from Survey of Kenya records in the case of the real network while the parameters for the simulated network were close enough to the expected values. This therefore resulted in the closeness of the determined estimates which has resulted in the determined accuracy being very close in magnitude.

6.3 Concluding Remarks

The dynamic, static-dynamic and sub-optimal fusion approaches all incorporate stochasticity of the datum parameters in the estimation of new point parameters except that in the static-dynamic and the sub-optimal fusion approaches the datum parameters are maintained definitive while in the dynamic approach all the parameters are estimated afresh. It is the lack of reproducing characteristic that makes the dynamic approach relatively unattractive in practical application in situations requiring the maintenance of the datum. The static approach to densification does not incorporate the stochasticity of the datum point parameters in the estimation of new point parameters and is thus considered to lack the reproducing property.

Using the analysis of the computations, classification of the viability of the approaches to densification of geodetic networks would be the static approach, followed by the static-dynamic approach then the sub-optimal approach and lastly the dynamic approach. But the static densification approach has the special problem that datum points are 'forced' to be exact when they in fact are not. Though normally applied, the approach has the effect of turning out results which are too optimistic in terms of their stochasticity; especially if the a-priori variance of unit weight is adopted. As a result, the static-dynamic and sub-optimal fusion approaches are considered to be more acceptable than the dynamic and static approaches. The dynamic approach though, may be preferred in densification of geodetic systems in certain circumstances for instance in isolated precise engineering networks in which there is no need to fix the datum points. Such networks would include those used in the analysis of deformation of engineering structures like dams. The dynamic approach is also used in the adjustment of scientific research networks such as those used in crustal deformation monitoring.

7

CONCLUSION AND RECOMMENDATIONS

Summary of the work done is presented in this chapter together with the findings and recommendations arising from the study.

7.1 Preamble

The main objective of the study was to demonstrate practical applicability and to evaluate the suitability of the sub-optimal fusion approach to densification in relation to the static, dynamic and static-dynamic approaches.

In order for the objective to be realized, static-dynamic, sub-optimal fusion, dynamic and static approaches to densification were used to estimate parameters for two networks, one of which was extracted from a part of the Kenyan geodetic network, dubbed the real network, and the other was a simulated network. The real network consists of eleven first order, fifteen second order and fifteen third order points while the simulated network consists of three first order, three second order and nine third order points. {cf. section (4.2)}. The networks were densified under each of the four densification approaches considered.

The results of the four approaches are close to one another in the real network case and this may be attributed to the fact that the approximate coordinates were actually adjusted final coordinates from Survey of Kenya records office. In the simulated network results, the closeness may be attributed to smaller dispersions of the observation from their adjusted values. In the dynamic approach, datum points are estimated afresh while in the static-dynamic and sub-optimal fusion, the datum parameters are held definitive.

7.2 Conclusions

The use of the dynamic approach incorporates the stochasticity of the datum parameters which is the first condition of choice of a viable densification approach. Unfortunately, the approach results in all the datum parameters being re-estimated afresh and thus fails the second condition of choice, that is to reproduce the datum parameters. This has led to the conclusion that the dynamic approach cannot be effectively used in cases where datum parameters have to be reproduced as is the case with any national reference datum. The approach though can be used in special situations where the datum needs not be maintained.

The static approach does not incorporate the stochasticity of the datum parameters in the estimation of the new point parameters and therefore fails the first condition of choice of a viable densification approach. Even though the approach results in datum parameters being retained fixed, it is considered to fail the second condition too.

The results indicate that the datum parameters obtained in the static-dynamic and the sub-optimal fusion approaches are reproduced together with their stochasticity, that is maintained definitive. Although the sub-optimal fusion approach has the reproducing property, the new point parameters obtained are exactly similar to the parameters obtained using dynamic approach. That is, it compares to performing network densification using the dynamic approach and applying a corrective term on the datum parameters to keep them unchanged. The approach therefore accomplishes the short falls of the dynamic approach.

It can therefore be concluded that both static-dynamic and sub-optimal fusion approaches can be effectively considered as the best suited approaches to the densification of geodetic systems.

7.3 Recommendations

From the comparisons of the dynamic, static-dynamic and sub-optimal fusion approaches to densification of geodetic networks and the analysis of the results obtained, it is recommended that the static-dynamic and sub-optimal fusion are both viable approaches to densification of geodetic networks.

Also as a further test to the four densification approaches studied, it is recommended that they be subjected to the densification of three and four dimensional geodetic systems.

REFERENCES

1. **Aduol, F. W. O.**, (1981). Optimal design for a three-dimensional terrestrial geodetic network. Thesis submitted for the degree of Master of Science (Surveying) in the University of Nairobi.
2. **Aduol, F. W. O.**, (1993). A Static-Dynamic model for densification of geodetic networks. Allgemeine Vermessungs-Nachrichten, International Edition 1993, pp. 20-25.
3. **Aduol, F. W. O.**, (1999). Lecture notes (unpublished). M. Sc. Class notes for FSP402 course. Department of Surveying, University of Nairobi.
4. **Askenazi**, V. and **Grist, S.**, (1980). Criterion for Optimatization: A Pratical Assessment of the Free Network Adjustment. Bulletin Geodesia e Scienzia Affini, Vol33, No. 1.
5. **Askenazi,** V. and **Grist, S.**, (1981). Intercomparison of 3-D Geodetic network adjustment models. Proceedings of the International Symposium on Geodetic and Computations of the International Association of Geodesy, Munich.
6. **Baarda, W.**, **Groot, D. DE**, and **Harkink, F.**, (1956). Handleiding voor de technische werkzaamheden van het kadaster (Manual for the technical activities of cadastral services in the Netherlands) 's-Gravenhage.
7. **Berliner, L. M.**, (1996). Hierarchical Bayesian time series models in maximum Entopy and Bayesian Methods; K. Hanson/R. Silver ed., Kluwer: Dordrecht, pg 15-22
8. **Bibby, J.**, and **Toutenburg, H.**, (1977). Prediction and Improved estimation in linear models. John Wiley & Sons, Chichester – New York – Brisbane – Toronto, pp. 180.
9. **Blaha, G.**, (1974). Influence de l'incertitude des parameters tenus fixes dans une compensation sur la propagation des variances-covariances. Bulletin Geodesic, No. 113, pp. 307-315.
10. **Brunner, F. K.,** (1979). On the analysis of geodetic networks for the determination of the incremental strain tensor. Survey Review, XXV, Vol. 192, April 1979, pp. 56-67.
11. **Cooper, M. A. R.**, and **Leahy, F. J.**, (1978). An adjustment of a second order network. Survey Review, Vol. 24, No. 187, pp. 224-233.
12. **Cooper, M. A. R.**, (1987). Control Surveys in Civil Engineering. Collins Professional and Technical Books, William Collins & Sons Co. Ltd. (Grafton Street, London W1X 3LA)
13. **Cross, P. A.**, (1983). Advanced least squares applied to position-fixing. Working paper No. 6, Department of Land Surveying, North East London Polytechnic, pp. 205.
14. **Durbin, J.**, (1953). A note on regression when there is extraneous information about one of the coefficients. Journal of American Statistical Association, No. 48, pp. 799-808.

15. **El-Hakim, S. F.**,(1982). Results from a precise photogrammetric densification of an urban control network. Canadian Surveyor, Vol. 36, No. 2, pp. 165-172.

16. **Fisher, R. A.**, (1960). The Design of Experiments. 7th ed., Hafner-Publishing Co., New York.

17. **Grafarend, E. W.**, and **Schaffrin, B.**, (1974). Unbiased Free Net Adjustment. Survey Review, XXII, Vol. 171.

18. **Grafarend, E. W.**, (1976). Geodetic Application of Stochastic Processes. Physics of the Earth and Planetary Interiors 12, pp 151-179.

19. **Grafarend, E. W.**, and **Schaffrin, B.**, (1988). Von der statischen zur dynamischen Auffassung geodatischer Netze. Zeitschrift fur Vermessungswesen, 113, Nr. 2, pp. 80-103.

20. **Illner, M.**, (1985). Datumsfestlegung in Freien Netzen. Deutsche Geodaestische Kommission. Reihe C. Heft. Nr. 309, pg. 9.

21. **Illner, M.**, (1988). Differential models for sequential optimization of geodetic networks. Manuscripta geodetica, Vol. 13, No. 5, pp. 306-315.

22. **Koch, K. R.**, (1987). Parameter Estimation and Hypothesis Testing in Linear Models. Springer-Verlag. Berlin Heidelberg.

23. **Koch, K. R.**, (1983a). Rechenverfahren bei der Einschaltung von Punkten in ein trigonometrisches Netz, AVN, 90. Nr. 33, pp 99-107.

24. **Kothari, C. R.**, (1990). Research Methodology: Methods and Techniques.2nd ed., Wiley Eastern Limited. New Delhi.

25. **Lugoe, F. N.**, (1990). Rigorous Mathematical models for the densification and integration of geodetic networks. Bulletin Geodesique, No. 64, pp. 219-229.

26. **Meiss, P.,** (1982). A Modern Approach. Mittellungen der Geodestischen Institute der Technischen Universtat, Graz. Folge 43.

27. **Miima, J. B** ,(1997). An Evaluation of the Static, Dynamic and Static-Dynamic Geodetic Densification models on a part of the Kenyan Geodetic Network. Thesis submitted for the degree of Master of Science (Surveying) in the University of Nairobi.

28. **Mikhail, E.**, (1976). Observations and Least-Squares. University Press of America, Lanham, New York, London.

29. **Mittermayer, E.**, (1972). A generalisation of the least-squares method for adjustment of free networks. Bulletin Geodesique, Vol. 4, pp 139-157.

30. **Musyoka, S. M.**,(1993). Mathematical modelling and design of a three dimensional geodetic network for localised earth deformation. Thesis submitted for the degree of Master of Science (Surveying) in the University of Nairobi.

31. **Nickerson, B. G.**, **Knight, W. R.**, **Voon, A.**, and **Caldwel, R.**, (1986). Horizontal geodetic network densification. Canadian Surveyor, Vol. 40, No. 1, pp. 13-22.
32. **Papo,** (1973). Zur Analyse geodatischer Deformationmessungen-Deutsche geodatische
33. **Pelzer, H.**, (1980). Dynamische oder hierarchische Netze? In: H. Pelzer (Ed.); Geodatische in Landes-und Ingenieurvermessung. Konrad Wittwer, Stuttgart.
34. **Pope, A. J.**, (1973). Two Approaches to non-linear Least-Squares Adjustments. Canadian Surveyor, vol. 28, No. 5, pp 663-669.
35. **Schaffrin, B.**, (1984). On the use of prior information in free networks (in German):in: K. Rinner et al. (eds.), Ingenieurvermessung '84, Dummler: Bonn, B6/1-15.
36. **Schaffrin, B.**, (1985). Robust alternatives for network densification. Scientific bulletins of Stanislaw staszic University of Mining and Metallurgy Cracow, No. 1024, pp. 18-29.
37. **Schaffrin, B.**, (1997). On suboptimal geodetic network fusion. Poster paper, IAG General Meeting, Rio de Janeiro, Brazil, September.
38. **Schaffrin, B.**, (1998). Advanced Network Densification Methods, Internal report of the Department of Geodetic Science, Ohio State University, Columbus, Ohio.
39. **Schaffrin, B.**, and **Cothren, J.**, (1999). Hierarchical data fusion with photogrammetric applications. Paper submitted to the ISPRS Journal of Photogrammetry and Remote Sensing, April.
40. **Schaffrin, B.**, (2000). Reproducing Estimators via Least-Squares: An Optimal Alternative to the Helmert Transformation. Paper dedicated to Erick Grafarends at his 60^{th} birthday, Festschrift E. Grafarend.
41. **Schmitt, G.,** (1982). Optimisation of Geodetic Networks. Reviews of Geophysics and Space Physics, Vol. 20, No. 4, pp. 877-884.
42. **Theil, H.**, and **Goldberger, A. S.,** (1961). On pure and mixed statistical estimation in economics. International Economic Review, No. 2. pp. 65-78.
43. **Theil, H.**, (1963). On the use of incomplete prior information in regression analysis. Journal of the American Statistical Association, Vol. 58, pp. 401-414.
44. **Theil, H.**, (1971). Principles of Econometrics. John Wiley & Sons, Santa Barbara. New York, London, Sydney, Toronto.
45. **Toutenburg, H.**, (1974). The use of mixed prior information in regression analysis.
46. **Toutenburg, H.**, (1975). Vorhersage in linearen Modellen. Akademic Verlag, Berlin, pp. 176.
47. **Toutenburg, H.**, (1977).

48. **Vanicek, P.**, and **Lugoe, F. N.**, (1986). Rigorous densification of horizontal networks. Journal of Surveying Engineering, ASCE, Vol. 112, No. 1, pp. 18-29.

49. **Van Mierlo, J.**, (1984). Inner precision of a densification network. Allgemeine Vermessungs-Nachrichten, International Supplement, 1, pp. 40-46.

50. **Wilke, C. K. , Berliner, L. M.** and **Cressie, N.,** (1998). Hierarchical Bayesian Space Time Models. Environmental and Ecological Statistics, to appear, Schaffrin (1998).

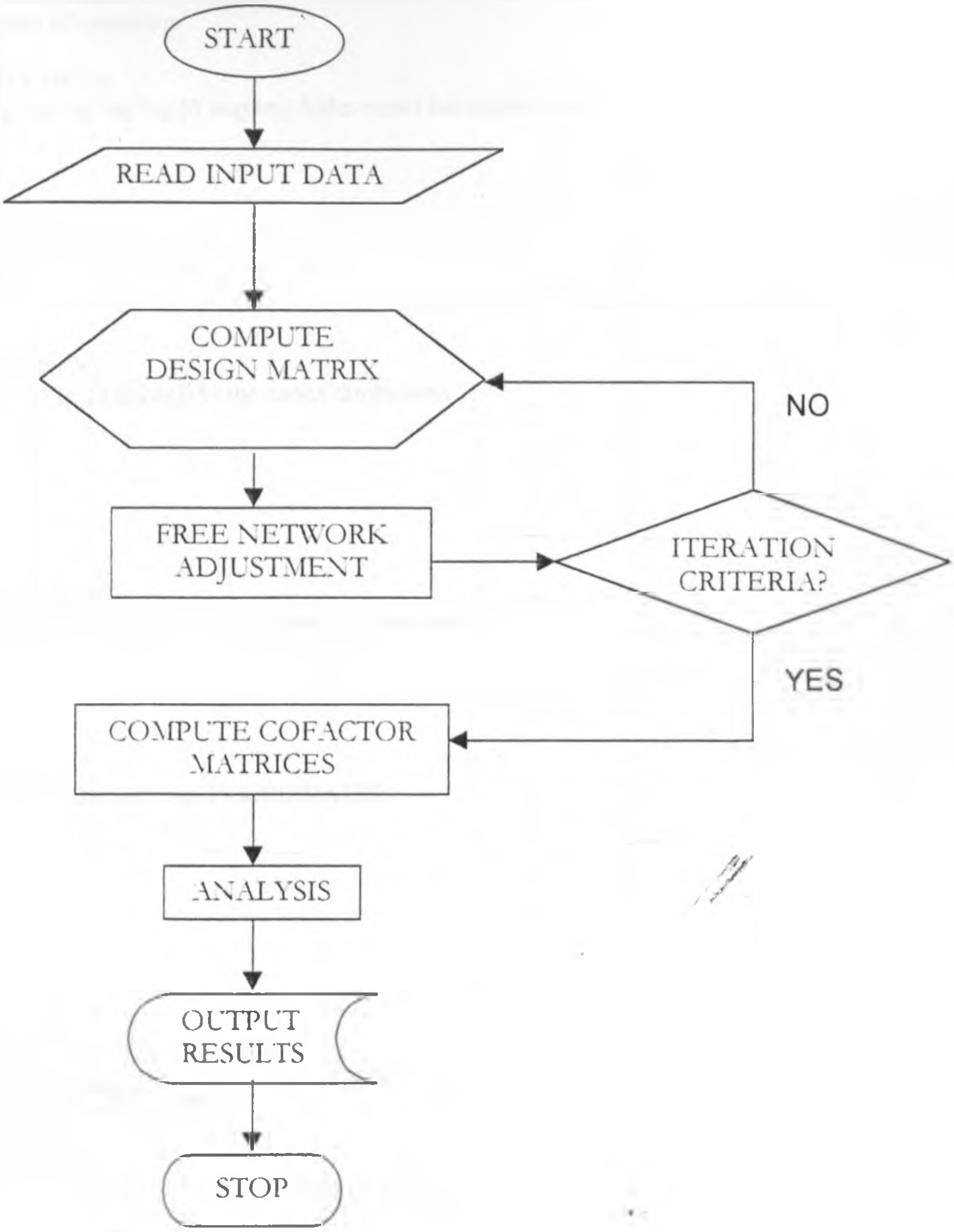
51. **Wolf, H.**, (1983). On the densification problem of geodetic networks. University of Uppsala, Institute of Geophysics, Department of Geodesy, Report No. 19, pp. 227-283.

52. **Wolf, H.**, (1984). Zur Praxis der Punkteinschaltungen. Allgemeine Vermessungs-Nachrichten, 91, Nr.. 11-12, pp. 432-440.

Appendices

APPENDIX A: PROGRAM LISTINGS AND FLOWCHART DIAGRAMS

APPENDIX A.1: FREE.M PROGRAM FLOWCHART



APPENDIX A.2: FREE M PROGRAM

```
clear all
PROGRAM TO PERFORM FREE NETWORK ADJUSTMENT
angobs =? % number of angles observed
disobs =? % number of distances observed
totobs = ? % number of total observations
points=?   % number of network points
new=?      % number of unknown parameters = twice number of network points
cond=?     % number of conditions

fid=fopen('angobs1A.txt','r');
RA=fscanf(fid,'%g %g %g %g %g',[5 angobs]) %the matrix has angobs rows
fclose(fid)
for i=1:angobs
  kp1(i)=RA(1,i);
  kp2(i)=RA(2,i);
  ideg(i)=RA(3,i);
  min(i)=RA(4,i);
  sec(i)=RA(5,i);
end
fid=fopen('disobs1A.txt','r');
B=fscanf(fid,'%g %g %g',[3 disobs]) % the matrix disobs rows
fclose(fid)
for i=1:disobs
  ip1(i)=B(1,i);
  ip2(i)=B(2,i);
  dist(i)=B(3,i);
end
fid=fopen('coord1A.txt','r');
C=fscanf(fid,'%g %g %g',[3 points]) % the matrix has point rows
fclose(fid)
for i=1:points
  kp(i)=C(1,i);
  y(i)=C(2,i);
  x(i)=C(3,i);
  %STORING THE PROVISIONAL COORDINATES
  x0(i)=x(i);
  y0(i)=y(i);
end
ITERATE=0;
ii=1;
VUW(ii)=1;
VUW(ii+1)=0;
fid=fopen('output1A.txt','w')
fprintf(fid,'        PROVISIONAL VALUES\n')
fprintf(fid,' POINT   Y-COORDINATES     X-COORDINATES\n')
fprintf(fid,'%4.0f %16.3f %18.3f\n',C)
while (abs(VUW(ii)-VUW(ii+1))<=0.001)
  ITERATION=ITERATION+1;
 %FORMATION OF DESIGN MATRIX A1 FOR THE DISTANCES
 for i=1:disobs
   j1=ip1(i);
   j2=ip2(i);
  denom1(i)=sqrt((x(j2)-x(j1))^2+(y(j2)-y(j1))^2);
  j3=j1*2-1;
  j4=j1*2;
  A1(i,j3)=(x(j1)-x(j2))/denom1(i);
  A1(i,j4)=(y(j1)-y(j2))/denom1(i);
  j5=j2*2-1;
```

```
  j6=j2*2;
  A1(i,j5)=(x(j2)-x(j1))/denom1(i);
  A1(i,j6)=(y(j2)-y(j1))/denom1(i);
  Y1(i)=dist(i)-denom1(i);
  W1(i,i)=VUW(ii)^2/((0.003^2)+(dist(i)*0.000001)^2);
end
%FORMATION OF DESIGN MATRIX A2 FOR THE DIRECTIONS
sin1=206264.80626
for i=1:angobs
  m1=kp1(i);
  m2=kp2(i);
  denom2(i)=(x(m2)-x(m1))^2+(y(m2)-y(m1))^2;
  m3=m1*2-1;
  m4=m1*2;
  A2(i,m3)=(y(m2)-y(m1))/denom2(i)*sin1;
  A2(i,m4)=(x(m1)-x(m2))/denom2(i)*sin1;
  m5=m2*2-1;
  m6=m2*2;
  A2(i,m5)=(y(m1)-y(m2))/denom2(i)*sin1;
  A2(i,m6)=(x(m2)-x(m1))/denom2(i)*sin1;
  ANG0(i)=(ideg(i)+min(i)/60.0+sec(i)/3600.0);
  de1(i)=x(m2)-x(m1);
  dn1(i)=y(m2)-y(m1);
  bear(i)=quad(dn1(i),de1(i));
  ANG1(i)=bear(i)*180/pi;
  Y2(i)=(ANG0(i)-ANG1(i))*3600.0;
  W2(i,i)=VUW(ii)^2/0.5^2;
end
%FORMATION OF THE COMPLETE DESIGN MATRIX,WEIGHT MATRIX AND Y-VECTOR
for i=1:totobs
  for j=1:new
    if(i<=disobs)
      A(i,j)=A1(i,j);
      Y(i,1)=Y1(i);
      W(i,i)=W1(i,i);
    else
      kk=i-disobs
      A(i,j)=A2(kk,j);
      Y(i,1)=Y2(kk);
      W(i,i)=W2(kk,kk);
    end
  end
end
%FORMATION OF THE RESTRICTION MATRIX
for i=1:4
  for j=1:new
    G(i,j)=0.0;
  end
end
for i=1:points
  n1=2*i-1;
  n2=2*i;
  G(1,n1)=1.0;
  G(2,n2)=1.0;
  G(3,n1)=y(i)-9624020.73;
  G(3,n2)=-1.0*(x(i)-585465.74);
  G(4,n1)=x(i)-585465.74;
  G(4,n2)=y(i)-9624020.73;
end
RNI=inv(A'*W*A+G'*G);
```

```
F11=(RNI-RNI*G'*G*RNI);
 DX=F11*A'*W*Y;
 E=Y-A*DX;
 for i=1:78
   if(i<=26)
      E1(i,1)=E(i,1);
    else
      m=i-26;
      E2(m,1)=E(i,1);
   end
 end
 %COMPUTING VARIANCE COMPONENTS
 W1I=inv(W1);
 W2I=inv(W2);
 QQE1=W1*(W1I);%-A1/(A1'*W1*A1)*A1');
 QQE2=W2*(W2I);%-A2/(A2'*W2*A2)*A2');
 TRACE1=trace(QQE1);
 TRACE2=trace(QQE2);
 VC1(ii)=(E1'*W1*E1)/TRACE1;
 VC2(ii)=(E2'*W2*E2)/TRACE2;
 VUW(ii+1)=(E1'*W1*E1+E2'*W2*E2)/(TRACE1+TRACE2);
 COVX=VUW(ii+1)*(F11*A'*W*A*F11');
 COVY=A*COVX*A';
  ii=ii+1
End % end while
%ADJUSTING THE OBSERVATIONS
for i=1:78
  if(i<=26)
    Adist(i)=dist(i)-E(i,1);
  else
    l1=i-26
    ANG(l1)=ANG0(l1)*3600.0-E(i,1);
    idegf(l1)=fix(ANG(l1)/3600.0);
    rmin(l1)=(ANG(l1)/3600.0-idegf(l1))*60.0;
    minf(l1)=fix(rmin(l1));
    secf(l1)=(rmin(l1)-minf(l1))*60.0;
  end
end
%STANDARD ERROR OF THE OBSERVATIONS
for i=1:78
  OBE(i)=sqrt(abs(COVY(i,i)));
end
%UPDATING THE PROVISIONAL VALUES
for i=1:11
  jj=2*i-1;
  mm=2*i;
  x(i)=x(i)+DX(jj);
  y(i)=y(i)+DX(mm);
% correction to provisional values
  CX(i)=x0(i)-x(i);
  CY(i)=y0(i)-y(i);
end
%STANDARD ERROR OF THE ADJUSTED PARAMETERS
for i=1:22
  STD(i)=sqrt(COVX(i,i));
end
%COMPUTING THE ERROR ELLIPSE PARAMETERS
for i=1:11
  k=2*i-1;
  VARX(i)=COVX(k,k);
```

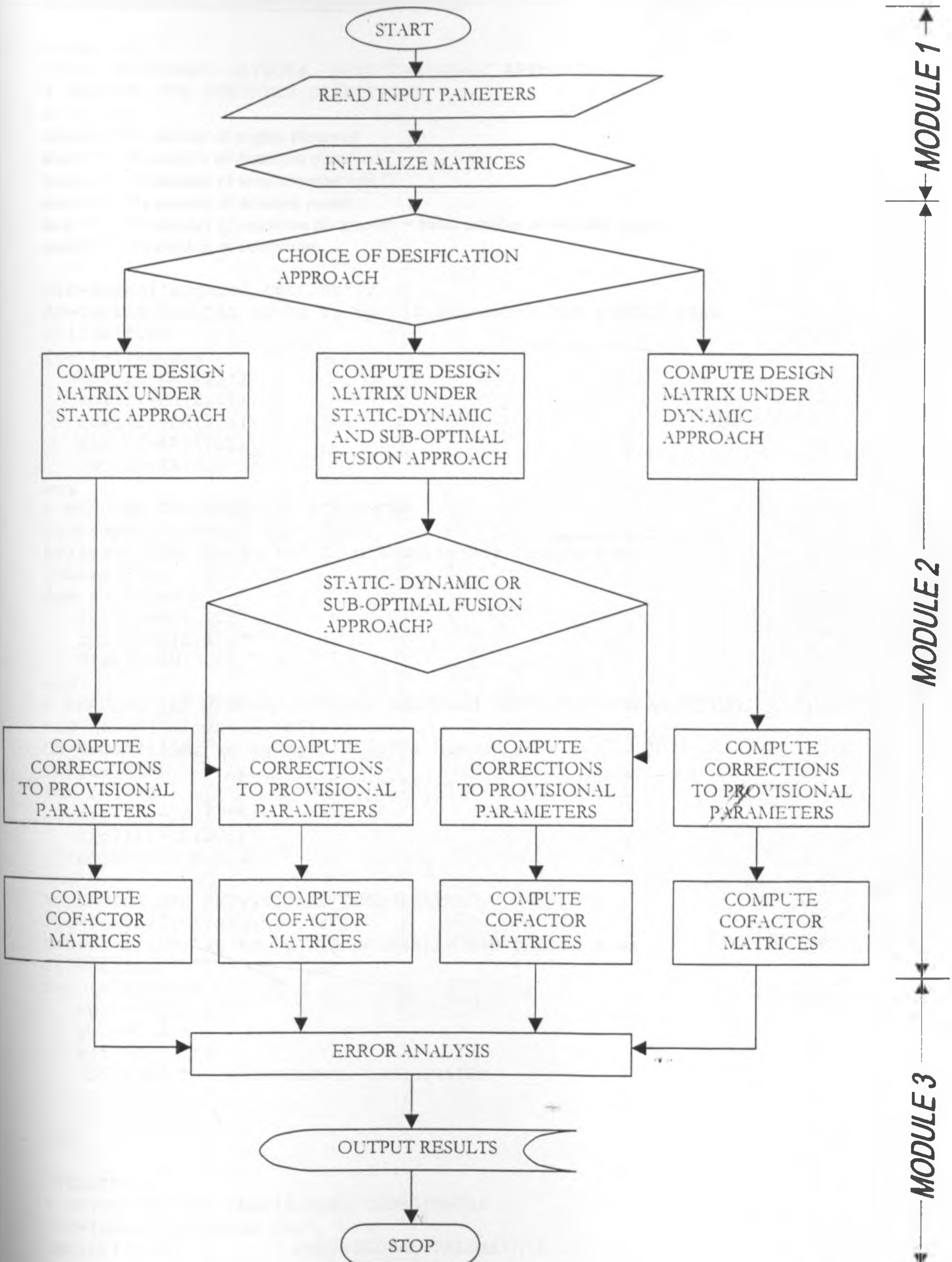
```
  m=2*i;
  VARY(i)=COVX(m,m);
  COVXY(i)=COVX(k,m);
  a(i)=(0.5*(VARX(i)+VARY(i))+(0.25*(VARX(i)-VARY(i))^2+COVXY(i)^2)^0.5)^0.5;
  b(i)=(0.5*(VARX(i)+VARY(i))-(0.25*(VARX(i)-VARY(i))^2+COVXY(i)^2)^0.5)^0.5;
  theta(i)=atan((2*COVXY(i))/(VARX(i)-VARY(i)));
  if(theta(i)<=0)
    theta(i)=2*pi+theta(i);
  end
  direct(i)=theta(i)*180.0/pi;
  PE(i)=((VARX(i)+VARY(i))/2.0)^0.5;
  degt(i)=fix(direct(i));
  rrr=(direct(i)-degt(i))*60.0;
  mint(i)=fix(rrr);
  sect(i)=(rrr-mint(i))*60.0;
end
% NETWORK MEAN ERROR
  TRACE3=trace(COVX);
NWERROR=sqrt(TRACE3/22);
ETWE1=E1'*W1*E1;
ETWE2=E2'*W2*E2;
fprintf(fid,' ITERATION = %4.0f VUW= %10.6f\n',ITERATE,VC1(ii)+VC2(ii))

% ****************************************
% OUTPUTS
%******************************************
fprintf(fid,'            RESULTS\n ')
fprintf(fid,' DISTANCE   :  TRACE1= %8.3f ETWE1= %8.3f\n',TRACE1,ETWE1)
fprintf(fid,'  DIRECTIONS :  TRACE2= %8.3f ETWE2= %8.3f\n',TRACE2,ETWE2)
fprintf(fid,' POINT   Y-COORD    X-COORD    STD-Y   STD-X   RESIDUAL-Y RESIDUAL-X\n')
fprintf(fid,'          (m)        (m)        (m)     (m)      (m)       (m)\n')
for i=1:11
  ll=i*2-1;
  mm=i*2;
fprintf(fid,'%5.0f %12.3f %12.3f %8.4f %7.4f %10.4f %12.4f\n',kp(i),y(i),x(i),STD(mm),STD(ll),CY(i),CX(i))
end
fprintf(fid,'    ADJUSTED OBSERVATIONS\n')
fprintf(fid,'    ADJUSTED DISTANCES\n')
fprintf(fid,' RAY       OBSERVED-DIST     ADJUSTED-DIST    STD-ERR   RESIDUALS\n')
fprintf(fid,'              (m)             (m)            (m)       (m)\n')
for i=1:26
  fprintf(fid,'%2.0f %2.0f %17.3f %17.3f %11.3f %11.3f\n',ip1(i),ip2(i),dist(i),Adist(i),OBE(i),E(i,1))
end
fprintf(fid,'    ADJUSTED DIRECTIONS\n')
fprintf(fid,' RAY   OBSERVED-DIRECT    ADJUSTED-DIRECT    STD-ERR   RESIDUALS\n')
fprintf(fid,'      (           " )   (          " )    (")        (")\n')
for i=1:52
  l=i+26
  fprintf(fid,'%2.0f %2.0f %5.0f %4.0f %7.2f %5.0f %4.0f %7.2f %9.3f
%11.2f\n',kp1(i),kp2(i),ideg(i),min(i),sec(i),idegf(i),minf(i),secf(i),OBE(l),E(l,1))
end
fprintf(fid,'   ERROR ELLIPSE PARAMETERS\n')
fprintf(fid,'POINT  SEMI-MAJOR AXIS(a)    SEMI-MINOR AXIS(b)    DIRECTION\n')
for i=1:11
fprintf(fid,'%3.0f %12.6f %25.6f %8.0f %4.0f %7.2f\n',i,a(i),b(i),degt(i),mint(i),sect(i))
end
fprintf(fid,' NETWORK ERROR=%10.5f\n',NWERROR)
ITERATE=ITERATE+1;
end %end ii
```

```
%=======================================
%PLOTTING THE ERROR ELLIPSES
%=======================================
plot(x,y,'ro')
hold;
axis([500000 627419 9570000 9670000])
xlabel('X-Coordinate (Eastings)')
ylabel('Y-Coordinate (Northings)')
title('REAL PRIMARY NETWORK')
for i=1:26
  m1=ip1(i);
  m2=ip2(i);
  xx=[x(m1) x(m2)];
  yy=[y(m1),y(m2)];
  plot(xx,yy,'g-')
end
for k=1:11
  n=60;
  theta=(-n:2:n)/n*pi;
  %THE POINT ELLIPSES
  xxx=a(k)*cos(theta)*10e5*0.4;
  yyy=b(k)*sin(theta)*10e5*0.4;
  %THE ROTATION MATRIX
  g(k)=degt(k)+mint(k)/60+sect(k)/3600;
  phi=(g(k)/180)*pi;
  R(1,1)=sin(phi);
  R(1,2)=cos(phi);
  R(2,1)=cos(phi);
  R(2,2)=-sin(phi);
  %ROTATE THE ELLIPSE ACCORDING TO THE ANGLE OF INCLINATION
  for i=1:n+1
    for j=1:n+1
      xy=[xxx(1,j) yyy(1,j)];
      RXY=R*xy';
      xR(i,j)=RXY(1);
      yR(i,j)=RXY(2);
    end
  end
  xxx=xR;
  yyy=yR;
  %TRANSLATE THE FIGURE
  xxp=x(k);
  yyp=y(k);
  ox=xxp*ones(n+1);
  oy=yyp*ones(n+1);
  xxx=(xxx+ox); %scaling
  yyy=(yyy+oy); %scaling
  plot(xxx,yyy,'r.');
end
% NETWORK NUMBERING
gtext('1')
gtext('2')
gtext('3')
gtext('4')
gtext('5')
gtext('6')
gtext('7')
gtext('8')
gtext('9')
gtext('10')
```

```
gtext('11')
% DRAWING SCALE BAR
text(520000,9640000,'Network:')
sx1=[540000 550000];
sy1=[9640000 9640000];
plot(sx1,sy1,'b-');
gtext('10Km')
text(520000,9630000,'Ellipse:')
sx2=[540000 546400];
sy2=[9630000 9630000];
plot(sx2,sy2,'b-');
gtext('20mm')
% DRAWING DIRECTION BAR
bx=[620000 620000];
by=[9580000 9590000];
plot(bx,by,'b-')
gtext('N')
text(525000,9650000,'SCALE:')
fclose(fid)
```

APPENDIX A.3: DENSITY.M PROGRAM FLOWCHART



APPENDIX A.4: DENSITY.M PROGRAM

```
clear all
%REAL SECONDARY NETWORK (STATIC-DYNAMIC APPROACH)
% READING THE OBSERVED DIRECTIONS
echo off
angobs =? % number of angles observed
disobs =? % number of distances observed
totobs = ? % number of total observations
points=?   % number of network points
new=?      % number of unknown parameters = twice number of network points
cond=?     % number of conditions

fid=fopen('angobs2.txt','r');
RA=fscanf(fid,'%g %g %g %g %g',[5 angobs])% has angobs rows
fclose(fid)
for i=1:angobs
   kp1(i)=RA(1,i);
   kp2(i)=RA(2,i);
   ideg(i)=RA(3,i);
   min(i)=RA(4,i);
   sec(i)=RA(5,i);
end
% READING THE OBSERVED DISTANCES
fid=fopen('disobs2.txt','r');
B=fscanf(fid,'%g %g %g',[3 disobs])% has disobs rows
fclose(fid)
for i=1:disobs
   ip1(i)=B(1,i);
   ip2(i)=B(2,i);
   dist(i)=B(3,i);
end
% READING THE PRIMARY NETWORK OBSERVED DISTANCES FOR PLOTTING
fid=fopen('disobs1A.txt','r');
CB=fscanf(fid,'%g %g %g',[3 26])% has 26 rows
fclose(fid)
for i=1:26
   cip1(i)=CB(1,i);
   cip2(i)=CB(2,i);
   cdist(i)=CB(3,i);
end
% READING THE PROVISIONAL COORDINATES
fid=fopen('coord2.txt','r');
C=fscanf(fid,'%g %g %g',[3 points])% has points rows
fclose(fid)
for i=1:points
   kp(i)=C(1,i);
   y(i)=C(2,i);
   x(i)=C(3,i);
   %STORING THE PROVISIONAL COORDINATES
   x0(i)=x(i);
   y0(i)=y(i);
end

ITERATE=0;
% OUTPUTING THE PROVISIONAL COORDINATES
fid=fopen('output2a.txt','w')
fprintf(fid,'          PROVISIONAL VALUES\n')
fprintf(fid,' POINT   Y-COORDINATES     X-COORDINATES\n')
fprintf(fid,'%4.0f %16.3f %18.3f\n',C)
VUW(1)=1
```

```
%FORMATION OF DESIGN MATRIX A1 FOR THE DISTANCES
for i=1:disobs
   j1=ip1(i);
   j2=ip2(i);
   denom1(i)=sqrt((x(j2)-x(j1))^2+(y(j2)-y(j1))^2);
   j3=j1*2-1;
   j4=j1*2;
   A1(i,j3)=(x(j1)-x(j2))/denom1(i);
   A1(i,j4)=(y(j1)-y(j2))/denom1(i);
   j5=j2*2-1;
   j6=j2*2;
   A1(i,j5)=(x(j2)-x(j1))/denom1(i);
   A1(i,j6)=(y(j2)-y(j1))/denom1(i);
   Y1(i)=dist(i)-denom1(i);
   W1(i,i)=VUW(ii)^2/((0.01^2)+(dist(i)*0.000001)^2);
end
%FORMATION OF DESIGN MATRIX A2 FOR THE DIRECTIONS
sin1=206264.80626
for i=1:angobs
   m1=kp1(i);
   m2=kp2(i);
   denom2(i)=((x(m2)-x(m1))^2+(y(m2)-y(m1))^2);
   m3=m1*2-1;
   m4=m1*2;
   A2(i,m3)=(y(m2)-y(m1))/denom2(i);
   A2(i,m4)=(x(m1)-x(m2))/denom2(i);
   m5=m2*2-1;
   m6=m2*2;
   A2(i,m5)=(y(m1)-y(m2))/denom2(i);
   A2(i,m6)=(x(m2)-x(m1))/denom2(i);
   ANG0(i)=(ideg(i)+min(i)/60.0+sec(i)/3600.0);
   de1(i)=x(m2)-x(m1);
   dn1(i)=y(m2)-y(m1);
   bear(i)=quad(dn1(i),de1(i));
   ANG1(i)=bear(i)*180/pi;
   Y2(i)=(ANG0(i)-ANG1(i))*pi/180.0;
   W2(i,i)=VUW(ii)^2/(1.0*pi/648000)^2;
end
%FORMATION OF THE COMPLETE DESIGN MATRIX,WEIGHT MATRIX AND Y-VECTOR
for i=1:totobs
   for j=1:angobs
      if(i<=disobs)
         A(i,j)=A1(i,j);
         Y(i,1)=Y1(i);
         W(i,i)=W1(i,i);
      else
         kk=i-disobs
         A(i,j)=A2(kk,j);
         Y(i,1)=Y2(kk);
         W(i,i)=W2(kk,kk);
      end
   end
end
%FORMATION OF THE DESIGN MATRIX FOR THE FIDUCIAL POINTS WITH & WITHOUT PRIOR
INFORMATION (A3&A4)
for i=1:totobs
   for j=1:angobs
      if(j<=cond)
         % with prior information
         A3(i,j)=A(i,j);
```

```
      else
         %without prior information
         kk=j-cond;
         A4(i,kk)=A(i,j);
      end
   end
end
%PRIOR INFORMATION
%standard error of the fiducial points
std=[0.0125 0.0166 0.0119 0.0109 0.0093 0.0099 0.0093 0.0083 0.0082 0.0074
0.0091 0.0075 0.0075 0.0072 0.007 0.0057 0.0073 0.0078 0.0067 0.0071 0.0078
0.0085]
%observational error of the fiducial points
E3=[-0.0389 0.1003 -0.0328 -0.0214 0.0698 -0.0393 0.0009 -0.042 0.0961 -
0.0068 -0.0077 0.031 -0.0159 -0.0438 0.0437 0.0329 -0.0305 -0.0314 -0.0399 -
0.0596 -0.0448 0.0802]
%MANIPULATION FOR THE STATIC-DYNAMIC APPROACH
a=[0.016857 0.012603 0.010512 0.009571 0.008686 0.009283 0.007735 0.007007
0.007839 0.007062 0.008967]
b=[0.012139 0.010018 0.008578 0.008035 0.006867 0.007263 0.006954 0.005729
0.007254 0.006689 0.0072]
direct=[29.36586389 291.9689194 71.24367778 309.342461111 297.681297222
319.52852777 290.290191666 352.417841666 353.593372222 1.979458333
68.216416666]
for i=1:cond
   Wr(i,i)=VUW(ii)^2/(std(i))^2;
   WrI(i,i)=(std(i))^2/VUW(ii)^2;
end
for i=1:points
   mm=2*i-1;
   nn=2*i;
   r(mm,1)=x(i)+E3(mm);
   r(nn,1)=y(i)+E3(nn);
end
for i=1:totobs
   WI(i,i)=1.0/W(i,i);
end
QQQ=inv(WI+A3*WrI*A3');
DX=inv(A4'*QQQ*A4)*(A4'*QQQ*Y);
E=Y-A4*DX;
for i=1:totobs
   if(i<=disobs)
      E1(i,1)=E(i,1);
   else
      m=i-disobs;
      E2(m,1)=E(i,1);
   end
end
for i=1:cond
   EE3(1,i)=E3(i);
end
E3=EE3'
%COMPUTING VARIANCE COMPONENTS
W1I=inv(W1);
W2I=inv(W2);
QQE1=W1*(W1I);%-(A1/(A1'*W1*A1)*A1'));
QQE2=W2*(W2I);%-(A2/(A2'*W2*A2)*A2'));
QQE3=Wr*WrI;
TRACE1=trace(QQE1);
TRACE2=trace(QQE2);
TRACE3=trace(QQE3);
```

```
VC1=(E1'*W1*E1)/TRACE1;
VC2=(E2'*W2*E2)/TRACE2;
VC3=(E3'*Wr*E3)/TRACE3;
VUW(ii+1)=((E1'*W1*E1)+(E2'*W2*E2)+(E3'*Wr*E3))/(TRACE1+TRACE2+TRACE3);
COVX=inv(A4'*QQQ*A4);
%formation of the complete covariance matrix
for i=1:angobs
   if(i<=cond)
      COV(i,i)=(std(i))^2;
   else
      kk=i-cond;
      COV(i,i)=COVX(kk,kk);
   end
end
COVY=A*COV*A';
%ADJUSTING THE OBSERVATIONS
for i=1:totobs
   if(i<=cond)
      Adist(i)=dist(i)-E(i,1);
   else
      l1=i-cond
      ANG(l1)=ANG0(l1)*pi/180.0-E(i,1);
      ANG1(l1)=ANG(l1)*180/pi;
      idegf(l1)=fix(ANG1(l1));
      rmin(l1)=(ANG1(l1)-idegf(l1))*60.0;
      minf(l1)=fix(rmin(l1));
      secf(l1)=(rmin(l1)-minf(l1))*60.0;
   end
end
%STANDARD ERROR OF THE OBSERVATIONS
for i=1:totobs
   if(i<=disobs)
    OBE(i)=sqrt(abs(COVY(i,i)));
   else
    OBE(i)=sqrt(abs(COVY(i,i)))*sin1;
   end
end
%UPDATING THE PROVISIONAL VALUES
for i=1:points
   j=i-cond/2;
   jj=2*j-1;
   mm=2*j;
   x(i)=x(i)+DX(jj);
   y(i)=y(i)+DX(mm);
% correction to provisional values
   CX(i)=x0(i)-x(i);
   CY(i)=y0(i)-y(i);
end
%STANDARD ERROR OF THE ADJUSTED PARAMETERS
for i=1:angobs
   STD(i)=sqrt(COV(i,i));
end
%COMPUTING THE ERROR ELLIPSE PARAMETERS
for i=1:points
   j=i-cond/2;
   k=2*j-1;
   VARX(i)=COVX(k,k);
   m=2*j;
   VARY(i)=COVX(m,m);
   COVXY(i)=COVX(k,m);
```

```
   a(i)=(0.5*(VARX(i)+VARY(i))+(0.25*(VARX(i)-
VARY(i))^2+COVXY(i)^2)^0.5)^0.5;
   b(i)=(0.5*(VARX(i)+VARY(i))-(0.25*(VARX(i)-
VARY(i))^2+COVXY(i)^2)^0.5)^0.5;
   theta(i)=atan((2*COVXY(i))/(VARX(i)-VARY(i)));
   if(theta(i)<=0)
      theta(i)=2*pi+theta(i);
   end
   direct(i)=theta(i)*180.0/pi;
   PE(i)=((VARX(i)+VARY(i))/2.0)^0.5;
end
for i=1:points
   degt(i)=fix(direct(i));
   rrr=(direct(i)-degt(i))*60.0;
   mint(i)=fix(rrr);
   sect(i)=(rrr-mint(i))*60.0;
end
% NETWORK MEAN ERROR
TRACE3=trace(COV);
NWERROR=sqrt(TRACE3/58);
ETWE1=E1'*W1*E1;
ETWE2=E2'*W2*E2;
fprintf(fid,' ITERATION = %4.0f VUW= %10.6f\n',ITERATE,VUW(ii))
end %end while
% ****************************************
% OUTPUTS
%****************************************
fprintf(fid,'                RESULTS\n ')
fprintf(fid,' DISTANCE   :  TRACE1= %8.3f ETWE1= %8.3f\n',TRACE1,ETWE1)
fprintf(fid,'  DIRECTIONS :  TRACE2= %8.3f ETWE2= %8.3f\n',TRACE2,ETWE2)
fprintf(fid,' POINT    Y-COORD     X-COORD      STD-Y    STD-X   RESIDUAL-Y
RESIDUAL-X\n')
for i=1:points
   ll=i*2-1;
   mm=i*2;
fprintf(fid,'%5.0f %12.3f %12.3f %8.4f %8.4f %10.4f
%11.4f\n',kp(i),y(i),x(i),STD(mm),STD(ll),CY(i),CX(i))
end
fprintf(fid,'     ADJUSTED OBSERVATIONS\n')
fprintf(fid,'     ADJUSTED DISTANCES\n')
fprintf(fid,' RAY        OBSERVED-DIST     ADJUSTED-DIST    STD-ERR
RESIDUALS\n')
for i=1:disobs
   fprintf(fid,'%2.0f %2.0f %17.3f %17.3f %11.4f
%11.4f\n',ip1(i),ip2(i),dist(i),Adist(i),OBE(i),E(i,1))
end
fprintf(fid,'     ADJUSTED DIRECTIONS\n')
fprintf(fid,' RAY    OBSERVED-DIRECT     ADJUSTED-DIRECT    STD-ERR
RESIDUALS(")\n')
for i=1:angobs
   l=i+disobs
   fprintf(fid,'%2.0f %2.0f %5.0f %4.0f %7.2f %5.0f %4.0f %7.2f %9.4f
%11.2f\n',kp1(i),kp2(i),ideg(i),min(i),sec(i),idegf(i),minf(i),secf(i),OBE(l)
,E(l,1)*sin1)
end
fprintf(fid,'    ERROR ELLIPSE PARAMETERS\n')
fprintf(fid,'POINT  SEMI-MAJOR AXIS(a)      SEMI-MINOR AXIS(b)
DIRECTION\n')
for i=1:points
fprintf(fid,'%3.0f %12.6f %25.6f %8.0f %4.0f
%7.2f\n',i,a(i),b(i),degt(i),mint(i),sect(i))
```

```
end
fprintf(fid,' NETWORK ERROR= %10.5f\n',NWERROR)
ITERATE=ITERATE+1;


%==========================================
%PLOTTING THE ERROR ELLIPSES
%==========================================
for i=1:points
 cy(i)=y(i);
 cx(i)=x(i);
end
for i=1:points
 l=i-cond/2;
 ny(l)=y(i);
 nx(l)=x(i);
end
plot(cx,cy,'ro')
hold
plot(nx,ny,'b.')
axis([500000 627419 9570000 9670000])
xlabel('X-Coordinate (Eastings)')
ylabel('Y-Coordinate (Northings)')
%if(jj==1)
  title('Figure 6.3: Real Network Point Error Ellipses (Static-Dynamic
Approach)')
 %else
  %title('SIMULATED SECONDARY NETWORK')
%end
for i=1:cond/2
   m1=cip1(i);
   m2=cip2(i);
   xx1=[x(m1) x(m2)];
   yy1=[y(m1),y(m2)];
   plot(xx1,yy1,'r-')
end

for i=1:angobs
   m1=kp1(i);
   m2=kp2(i);
   xx2=[x(m1) x(m2)];
   yy2=[y(m1),y(m2)];
   plot(xx2,yy2,'g--')
end

for k=1:points
   n=120;
   theta=(-n:2:n)/n*pi;
   %THE POINT ELLIPSES
      xxx=a(k)*cos(theta)*10e5*0.4;
      yyy=b(k)*sin(theta)*10e5*0.4;
   %THE ROTATION MATRIX
   g(k)=degt(k)+mint(k)/60+sect(k)/3600;
   phi=(g(k)/180)*pi;
   R(1,1)=sin(phi);
   R(1,2)=cos(phi);
   R(2,1)=cos(phi);
   R(2,2)=-sin(phi);
  %ROTATE THE ELLIPSE ACCORDING TO THE ANGLE OF INCLINATION
   for i=1:n+1
      for j=1:n+1
```

```
            xy=[xxx(1,j) yyy(1,j)];
            RXY=R*xy';
            xR(i,j)=RXY(1);
            yR(i,j)=RXY(2);
        end
    end
    xxx=xR;
    yyy=yR;
    %TRANSLATE THE FIGURE
    xxp=x(k);
    yyp=y(k);
    ox=xxp*ones(n+1);
    oy=yyp*ones(n+1);
    xxx=(xxx+ox); %scaling
    yyy=(yyy+oy); %scaling
    plot(xxx,yyy,'b.');
end
% NETWORK NUMBERING
for i=1:points
  point(i)=i;
end
text(x,y,'point')
% DRAWING SCALE BAR
text(520000,9640000,'Network:')
sx1=[540000 550000];
sy1=[9640000 9640000];
plot(sx1,sy1,'b-');
gtext('10Km')
text(520000,9630000,'Ellipse:')
sx2=[540000 546400];
sy2=[9630000 9630000];
plot(sx2,sy2,'b-');
gtext('20mm')
%DRAWING DIRECTION BAR
bx=[620000 620000];
by=[9580000 9590000];
plot(bx,by,'b-')
gtext('N')
text(525000,9650000,'SCALE:')
fclose(fid)
end
```

APPENDIX A.5: SUBFUNCTION QUAD.M

THIS SUBFUNCTION PERFORMS A DATUM JOIN BETWEEN TWO POINTS AND RETURNS THE BEARING IN RADIANS

```
function[bear]=quad(dn,de)
thita=atan(abs(de/dn))
if(dn>0)
   if(de>0)
      bear=thita;
   else
      bear=2*pi-thita;
   end
else
   if(de>0)
      bear=pi-thita;
   else
      bear=pi+thita;
   end
end
if(dn==0)
   if(de>0)
      bear=pi/2;
   else
      bear=3.0*pi/2.0;
   end
end
if(de==0)
   if(dn>0)
      bear=0.0;
   else
      bear=pi;
   end
end
```

APPENDIX B: RESULTS OF THE STUDY

APPENDIX B.1: RESULTS OF THE FREE NETWORK ADJUSTMENT

APPENDIX B.1.1: Results of the real network freenet adjustment

FIRST ORDER REAL NETWORK ADJUSTMENT

ITERATION = 3 VUW= 1.0374396

RESULTS

DISTANCE : TRACE1= 26.000 ETWE1= 0.124
DIRECTIONS : TRACE2= 52.000 ETWE2= 0.032

ADJUSTED OBSERVATIONS

ADJUSTED DISTANCES

RAY		OBSERVED-DIST (m)	ADJUSTED-DIST (m)	STD-ERR (m)	RESIDUALS (m)
1	2	61821.778	61821.834	0.028	-0.056
1	3	59131.496	59131.502	0.023	-0.006
1	7	88020.944	88020.916	0.019	0.028
2	3	25314.493	25314.468	0.015	0.025
2	5	38470.110	38470.147	0.014	-0.037
2	4	30540.413	30540.441	0.015	-0.028
3	7	40655.120	40655.057	0.015	0.063
3	5	22606.877	22606.890	0.012	-0.013
3	4	25378.281	25378.267	0.013	0.014
4	5	13713.543	13713.537	0.009	0.006
4	6	28156.771	28156.836	0.013	-0.065
5	7	30046.422	30046.414	0.014	0.008
5	8	38108.480	38108.545	0.012	-0.065
5	6	21332.688	21332.657	0.011	0.031
6	7	30371.764	30371.771	0.015	-0.007
6	8	19719.190	19719.197	0.011	-0.007
7	8	29303.530	29303.543	0.013	-0.013
7	11	43643.720	43643.734	0.013	-0.014
7	10	27892.630	27892.613	0.012	0.017
7	9	21718.190	21718.179	0.012	0.011
8	9	29095.230	29095.289	0.012	-0.059
8	10	19664.661	19664.639	0.011	0.022
8	11	19588.160	19588.179	0.011	-0.019
9	10	13242.450	13242.429	0.008	0.021
9	11	32607.292	32607.348	0.012	-0.056
10	11	19378.227	19378.194	0.011	0.033

ADJUSTED DIRECTIONS

RAY		OBSERVED-DIRECT (°	'	")	ADJUSTED-DIRECT (°	'	")	STD-ERR (")	RESIDUALS (")
1	2	96	25	56.33	96	25	56.46	0.104	-0.13
1	3	72	24	12.70	72	24	12.99	0.110	-0.29
1	7	49	32	4.60	49	32	4.68	0.055	-0.08
2	1	276	25	56.50	276	25	56.46	0.104	0.04
2	3	348	27	5.00	348	27	5.23	0.146	-0.23
2	5	22	42	51.50	22	42	51.76	0.100	-0.26
2	4	41	30	1.98	41	30	1.90	0.124	0.08
3	1	252	24	12.97	252	24	12.99	0.110	-0.02
3	7	15	6	55.60	15	6	55.54	0.086	0.06

3	5	61	47	43.31	61	47	43.27	0.133	0.04
3	4	94	21	30.95	94	21	30.94	0.120	0.01
3	2	168	27	5.03	168	27	5.23	0.146	-0.20
4	2	221	30	1.88	221	30	1.90	0.124	-0.02
4	3	274	21	31.00	274	21	30.94	0.120	0.06
4	5	336	53	30.00	336	53	30.50	0.175	-0.50
4	6	23	58	55.48	23	58	55.54	0.118	-0.06
5	4	156	53	30.13	156	53	30.50	0.175	-0.37
5	2	202	42	51.70	202	42	51.76	0.100	-0.06
5	3	241	47	43.42	241	47	43.27	0.133	0.15
5	7	341	55	37.04	341	55	36.86	0.101	0.18
5	8	31	7	39.11	31	7	39.01	0.087	0.10
5	6	52	4	14.61	52	4	14.56	0.141	0.05
6	4	203	58	55.65	203	58	55.54	0.118	0.11
6	5	232	4	14.75	232	4	14.56	0.141	0.19
6	7	300	34	45.60	300	34	45.60	0.096	0.00
6	8	8	22	44.00	8	22	44.19	0.159	-0.19
7	1	229	32	4.63	229	32	4.68	0.055	-0.05
7	3	195	6	55.72	195	6	55.54	0.086	0.18
7	5	161	55	37.08	161	55	36.86	0.101	0.22
7	6	120	34	45.72	120	34	45.60	0.096	0.12
7	8	82	2	26.49	82	2	26.33	0.090	0.16
7	11	60	32	9.83	60	32	9.82	0.079	0.01
7	10	41	55	3.70	41	55	3.77	0.104	-0.07
7	9	14	22	56.21	14	22	56.36	0.131	-0.15
8	6	188	22	44.06	188	22	44.19	0.159	-0.13
8	5	211	7	39.04	211	7	39.01	0.087	0.03
8	7	262	2	26.45	262	2	26.33	0.090	0.12
8	9	305	42	13.74	305	42	13.62	0.096	0.12
8	10	328	6	52.87	328	6	52.64	0.129	0.23
8	11	27	16	45.76	27	16	45.66	0.145	0.10
9	7	194	22	56.41	194	22	56.36	0.131	0.05
9	8	125	42	13.81	125	42	13.62	0.096	0.19
9	10	91	13	20.49	91	13	20.72	0.187	-0.23
9	11	89	14	41.48	89	14	41.37	0.100	0.11
10	9	271	13	20.55	271	13	20.72	0.187	-0.17
10	7	221	55	3.80	221	55	3.77	0.104	0.03
10	8	148	6	52.90	148	6	52.64	0.129	0.26
10	11	87	53	37.12	87	53	36.75	0.152	0.37
11	9	269	14	41.53	269	14	41.37	0.100	0.16
11	10	267	53	37.01	267	53	36.75	0.152	0.26
11	7	240	32	9.98	240	32	9.82	0.079	0.16
11	8	207	16	45.78	207	16	45.66	0.145	0.12

APPENDIX **B.1.2**: Results of the simulated network freenet adjustment

FIRST ORDER SIMULATED NETWORK ADJUSTMENT

ITERATION = 3 VUW= 0.947944

RESULTS

DISTANCE :	TRACE1= 3.000	ETWE1=	2.763 VC1=	3.254
DIRECTIONS :	TRACE2= 6.000	ETWE2=	5.515 VC2=	62.253

ADJUSTED OBSERVATIONS

ADJUSTED DISTANCES

RAY		OBSERVED-DIST (m)	ADJUSTED-DIST (m)	STD-ERR (m)	RESIDUALS (m)
1	2	400.0060	400.0071	0.0063	-0.0011
1	3	399.9910	399.9994	0.0063	-0.0084
2	3	399.9930	399.9888	0.0063	0.0042

ADJUSTED DIRECTIONS

RAY		OBSERVED-DIRECT (		")	ADJUSTED-DIRECT (		")	STD-ERR (")	RESIDUALS (")
1	2	30	0	0.00	29	59	56.71	2.2322	3.29
1	3	90	0	10.61	90	0	5.23	2.2322	5.38
2	3	150	0	0.00	149	59	57.40	2.2322	2.60
2	1	209	59	53.01	209	59	56.71	2.2322	-3.70
3	1	270	0	0.00	270	0	5.23	2.2322	-5.23
3	2	329	59	54.99	329	59	57.40	2.2322	-2.41

APPENDIX B.2: RESULTS FOR STATIC-DYNAMIC DENSIFICATION

APPENDIX B.2.1: Results of the first level real network densification using static-dynamic approach

SECOND ORDER REAL NETWORK ADJUSTMENT
(STATIC-DYNAMIC APPROACH)

ITERATION = 2 VUW= 1.000000

RESULTS

DISTANCE : TRACE1= 30.000 ETWE1= 115.349
DIRECTIONS : TRACE2= 52.000 ETWE2= 91.836

ADJUSTED OBSERVATIONS

ADJUSTED DISTANCES

RAY		OBSERVED-DIST	ADJUSTED-DIST	STD-ERR	RESIDUALS
12	7	11392.335	11392.344	0.0161	-0.0091
12	9	12369.293	12369.302	0.0200	-0.0092
13	9	13814.687	13814.686	0.0388	0.0012
14	6	24623.681	24623.689	0.0272	-0.0084
14	8	16935.563	16935.563	0.0207	0.0002
15	6	10683.383	10683.384	0.0236	-0.0008
16	5	14837.980	14837.957	0.0188	0.0233
16	6	13269.516	13269.537	0.0192	-0.0211
16	8	24310.634	24310.559	0.0182	0.0747
17	7	16690.070	16690.068	0.0225	0.0021
18	20	14984.326	14984.294	0.0297	0.0316
18	7	23865.886	23865.817	0.0241	0.0693
18	5	31483.464	31483.565	0.0245	-0.1005

19 26	24790.315	24790.379	0.0441	-0.0641
19 20	14808.001	14808.026	0.0374	-0.0247
20 21	4247.690	4247.690	0.0318	0.0000
21 3	10340.648	10340.625	0.0217	0.0226
21 7	35000.587	35000.688	0.0202	-0.1005
22 2	16116.890	16116.931	0.0227	-0.0408
22 23	15238.942	15238.935	0.0468	0.0066
22 5	23394.975	23394.981	0.0265	-0.0060
23 4	15396.022	15396.053	0.0249	-0.0307
24 2	15823.986	15824.021	0.0216	-0.0353
24 3	17476.302	17476.306	0.0215	-0.0036
24 26	33794.283	33794.458	0.0260	-0.1746
24 25	23681.294	23681.280	0.0280	0.0143
25 1	25419.370	25419.371	0.0244	-0.0010
25 26	13552.337	13552.333	0.0399	0.0039
26 1	16336.566	16336.670	0.0255	-0.1036
26 24	33794.482	33794.458	0.0260	0.0244

<u>ADJUSTED DIRECTIONS</u>

RAY	OBSERVED-DIRECT			ADJUSTED-DIRECT			STD-ERR	RESIDUALS(")
12 7	169	22	42.30	169	22	41.61	0.4589	0.69
12 9	37	17	34.90	37	17	35.92	0.3753	-1.02
13 7	233	35	16.50	233	35	16.17	0.5511	0.33
13 8	106	41	48.00	106	41	48.42	0.4111	-0.42
13 9	330	0	20.65	330	0	20.86	0.7183	-0.21
14 6	231	32	25.31	231	32	25.00	0.2560	0.31
14 8	284	20	12.64	284	20	12.01	0.4196	0.63
14 11	341	1	11.63	341	1	10.98	0.1985	0.65
15 4	207	42	24.54	207	42	25.21	0.4565	-0.67
15 5	257	44	42.41	257	44	42.07	0.3293	0.34
15 6	17	50	53.63	17	50	53.84	0.7900	-0.21
16 5	194	6	54.60	194	6	54.50	0.2606	0.10
16 6	95	31	21.62	95	31	22.96	0.2922	-1.34
16 8	41	24	51.23	41	24	52.20	0.1545	-0.97
16 7	317	36	22.62	317	36	23.04	0.1994	-0.42
17 5	149	36	47.34	149	36	48.27	0.6795	-0.93
17 3	209	32	9.51	209	32	9.39	0.3675	0.12
17 18	275	15	46.63	275	15	46.55	0.2969	0.08
17 7	352	11	25.96	352	11	26.78	0.6330	-0.82
18 20	186	37	34.51	186	37	33.96	0.4517	0.55
18 7	52	19	28.91	52	19	27.66	0.2065	1.25
18 17	95	15	46.03	95	15	46.55	0.2969	-0.52
18 5	116	21	25.06	116	21	25.81	0.1561	-0.75
19 26	222	47	24.01	222	47	24.46	0.3565	-0.45
19 24	143	18	7.51	143	18	6.69	0.2762	0.82
19 3	105	26	4.12	105	26	5.35	0.3155	-1.23
19 20	78	16	29.43	78	16	31.41	0.5937	-1.98
20 21	108	30	23.41	108	30	25.22	1.3038	-1.81
20 18	6	37	34.92	6	37	33.96	0.4517	0.96
20 19	258	16	29.64	258	16	31.41	0.5937	-1.77
21 3	144	36	27.98	144	36	31.51	0.4612	-3.53
21 20	288	30	23.42	288	30	25.22	1.3038	-1.80
21 7	28	17	39.99	28	17	38.41	0.1347	1.58
22 2	186	46	39.33	186	46	37.71	0.4734	1.62
22 23	120	5	28.54	120	5	31.18	0.4847	-2.64
22 5	33	37	5.13	33	37	5.24	0.2844	-0.11
22 3	321	36	42.00	321	36	40.90	0.5812	1.10
23 2	240	59	50.20	240	59	50.35	0.3364	-0.15
23 4	19	32	28.15	19	32	28.61	0.4986	-0.46
23 22	300	5	28.42	300	5	31.18	0.4847	-2.76

24	2	125	26	21.12	125	26	25.00	0.2865	-3.88
24	3	26	35	49.90	26	35	48.99	0.2397	0.91
24	19	323	18	7.51	323	18	6.69	0.2762	0.82
24	26	277	8	39.00	277	8	39.31	0.2355	-0.31
24	25	258	47	39.42	258	47	39.48	0.3812	-0.06
25	1	275	18	29.72	275	18	26.41	0.3486	3.31
25	26	310	31	10.60	310	31	11.80	0.6619	-1.20
25	24	78	47	38.42	78	47	39.48	0.3812	-1.06
26	1	246	43	54.13	246	43	54.52	0.4478	-0.39
26	25	130	31	11.12	130	31	11.80	0.6619	-0.68
26	24	97	8	39.56	97	8	39.31	0.2355	0.25
26	19	42	47	24.43	42	47	24.46	0.3565	-0.03

APPENDIX B.2.2: Results of the first level simulated network densification using static-dynamic approach

SECOND ORDER SIMULATED NETWORK ADJUSTMENT
(STATIC-DYNAMIC APPROACH)

ITERATION = 2 VUW= 0.855268

RESULTS

DISTANCE :	TRACE1= 9.000	ETWE1= 8.701	VC1= 1.300
DIRECTIONS :	TRACE2= 18.000	ETWE2= 20.207	VC2= 1.345
RESTRICTIONS :	TRACE3= 0.000	ETWE3= 8.278	VC3= 2.514

ADJUSTED OBSERVATIONS

ADJUSTED DISTANCES

RAY		OBSERVED-DIST	ADJUSTED-DIST	STD-ERR	RESIDUALS
1	4	200.0600	200.3131	0.0052	0.2531
1	6	200.0400	199.9784	0.0058	-0.0616
2	4	200.0800	199.9594	0.0054	-0.1206
2	5	199.9800	199.6085	0.0054	-0.3715
3	5	200.0300	200.4226	0.0052	0.3926
3	6	199.9300	199.9622	0.0058	0.0322
4	5	200.0200	200.0034	0.0052	-0.0166
4	6	200.0900	200.5731	0.0056	0.4831
5	6	199.9200	200.1768	0.0056	0.2568

ADJUSTED DIRECTIONS

RAY		OBSERVED-DIRECT			ADJUSTED-DIRECT			STD-ERR	RESIDUALS(")
1	4	30	0	0.00	29	59	57.11	5.2363	-2.89
1	6	90	0	10.61	90	0	23.13	4.4107	12.52
2	4	209	59	53.01	209	59	42.26	4.9482	-10.75
2	5	150	0	4.99	150	0	10.01	4.9470	5.02
3	5	329	59	54.99	329	59	44.79	5.2384	-10.20
3	6	270	0	3.01	270	0	14.56	4.4130	11.55
4	1	210	0	0.00	209	59	57.11	5.2363	-2.89
4	2	30	0	7.00	30	0	10.24	4.9482	3.24
4	5	89	59	52.62	89	59	46.73	6.4613	-5.89
4	6	150	0	5.42	150	0	9.34	6.0438	3.92

5	2	329	59	50.90	329	59	41.83	4.9470	-9.07
5	4	270	0	4.37	270	0	10.23	6.4613	5.86
5	6	209	59	58.02	209	59	53.76	6.0427	-4.26
5	3	150	0	13.96	150	0	22.73	5.2384	8.77
6	1	270	0	0.00	270	0	1.91	4.4107	1.91
6	4	329	59	57.71	329	59	53.92	6.0438	-3.79
6	5	30	0	6.43	30	0	10.58	6.0427	4.15
6	3	89	59	54.98	89	59	58.50	4.4130	3.52

APPENDIX B.2.3: Results of the second level real network densification using static-dynamic approach

THIRD ORDER REAL NETWORK ADJUSTMENT
(STATIC-DYNAMIC APPROACH)

ITERATION = 2 VUW= 1.089

RESULTS

DISTANCE : TRACE1= 30.000 ETWE1= 6.926
DIRECTIONS : TRACE2= 50.000 ETWE2= 24.452

ADJUSTED OBSERVATIONS

ADJUSTED DISTANCES

RAY		OBSERVED-DIST	ADJUSTED-DIST	STD-ERR	RESIDUALS
27	26	14773.890	14773.894	0.0332	-0.0040
27	28	9653.420	9653.417	0.0337	0.0033
28	29	13934.030	13934.026	0.0216	0.0037
28	24	17550.110	17550.095	0.0241	0.0146
29	3	14026.280	14026.287	0.0179	-0.0070
29	24	7500.980	7500.963	0.0246	0.0170
30	22	20768.540	20768.536	0.0411	0.0036
30	23	7148.220	7148.220	0.0429	-0.0005
31	4	10861.010	10861.009	0.0205	0.0012
31	5	18725.550	18725.559	0.0184	-0.0090
32	3	12954.160	12954.155	0.0218	0.0050
32	21	14309.080	14309.071	0.0291	0.0087
33	5	6615.690	6615.690	0.0150	0.0004
33	4	9616.290	9616.289	0.0142	0.0011
34	18	6763.580	6763.583	0.0268	-0.0034
34	21	10493.270	10493.267	0.0214	0.0025
34	20	10463.230	10463.226	0.0249	0.0045
35	16	7137.810	7137.804	0.0217	0.0060
35	17	7795.220	7795.233	0.0484	-0.0133
36	7	13557.220	13557.216	0.0204	0.0044
36	18	11414.360	11414.357	0.0317	0.0030
37	35	10445.030	10445.025	0.0176	0.0047
37	17	7476.300	7476.310	0.0349	-0.0104
37	7	11935.450	11935.458	0.0156	-0.0077
38	8	15218.090	15218.089	0.0141	0.0012
38	16	13176.150	13176.153	0.0238	-0.0026
38	7	14752.400	14752.402	0.0138	-0.0022
39	14	17156.360	17156.352	0.0321	0.0079

40 11	12385.920	12385.928	0.0165	-0.0080
41 13	17334.620	17334.635	0.0593	-0.0146

ADJUSTED DIRECTIONS

RAY	OBSERVED-DIRECT			ADJUSTED-DIRECT			STD-ERR	RESIDUALS(")
27 26	232	51	31.26	232	51	32.02	0.5201	-0.76
27 19	28	38	17.97	28	38	17.33	0.7361	0.64
27 28	149	41	47.26	149	41	48.14	0.5988	-0.88
28 27	329	41	49.25	329	41	48.14	0.5988	1.11
28 29	81	34	28.89	81	34	29.95	0.4442	-1.06
28 24	105	50	4.45	105	50	3.54	0.3739	0.91
28 25	214	2	52.29	214	2	53.00	0.6201	-0.71
29 3	51	9	34.82	51	9	34.07	0.2662	0.75
29 24	155	35	6.04	155	35	4.99	0.6444	1.05
29 28	261	34	28.99	261	34	29.95	0.4442	-0.96
30 22	285	27	41.43	285	27	41.43	0.3556	0.00
30 4	352	16	43.10	352	16	43.07	0.3965	0.03
30 23	252	53	6.97	252	53	7.00	1.0626	-0.03
31 4	58	16	33.02	58	16	33.13	0.3686	-0.11
31 22	262	44	43.89	262	44	44.76	0.5865	-0.87
31 5	11	53	0.49	11	53	0.51	0.2268	-0.02
32 5	86	39	13.99	86	39	13.96	0.3793	0.03
32 3	219	27	48.46	219	27	47.60	0.3405	0.86
32 21	263	41	41.82	263	41	42.85	0.3860	-1.03
32 17	20	2	9.29	20	2	9.14	0.7920	0.15
33 5	297	4	1.97	297	4	2.06	0.4386	-0.09
33 15	52	8	46.69	52	8	46.58	0.6813	0.11
33 4	183	1	59.52	183	1	59.60	0.3422	-0.08
34 18	329	7	12.99	329	7	13.85	0.8871	-0.86
34 17	77	42	3.25	77	42	2.29	0.2954	0.96
34 21	186	24	52.69	186	24	53.75	0.5830	-1.06
34 20	209	48	10.99	209	48	11.01	0.5632	-0.02
35 16	32	10	16.87	32	10	16.16	0.6512	0.71
35 5	178	45	7.49	178	45	8.14	0.4371	-0.65
35 17	298	10	32.64	298	10	31.89	0.8689	0.75
36 12	32	44	39.26	32	44	40.50	0.3156	-1.24
36 7	67	59	6.55	67	59	5.90	0.3667	0.65
36 18	213	37	30.21	213	37	30.80	0.5509	-0.59
37 16	120	40	11.50	120	40	10.20	0.5793	1.30
37 35	163	45	30.30	163	45	31.22	0.4702	-0.92
37 17	211	53	41.82	211	53	42.43	1.3186	-0.61
37 7	328	36	9.94	328	36	9.23	0.3199	0.71
38 8	70	7	12.51	70	7	11.47	0.2319	1.04
38 16	187	43	14.82	187	43	14.79	0.3218	0.03
38 7	274	20	37.98	274	20	38.03	0.2523	-0.05
38 13	346	41	53.56	346	41	53.62	1.0271	-0.06
39 8	359	53	6.52	359	53	6.06	0.4590	0.46
39 14	72	48	16.88	72	48	16.90	0.7708	-0.02
39 6	195	45	58.94	195	45	58.75	0.5008	0.19
40 11	0	27	45.30	0	27	45.74	0.6242	-0.44
40 14	140	45	14.38	140	45	13.88	0.6856	0.50
40 8	240	29	37.98	240	29	39.23	0.4588	-1.25
41 10	293	52	37.34	293	52	36.78	0.6513	0.56
41 11	64	34	8.77	64	34	8.51	0.5963	0.26
41 13	244	2	51.24	244	2	51.41	0.5748	-0.17

APPENDIX B.2.4: Results of the second level simulated network densification using static-dynamic approach

THIRD ORDER SIMULATED NETWORK ADJUSTMENT (STATIC-DYNAMIC APPROACH)

ITERATION = 2 VUW= 0.811081

RESULTS

DISTANCE	: TRACE1=	30.000	ETWE1=	4.266	VC1=	0.142
DIRECTIONS	: TRACE2=	60.000	ETWE2=	64.997	VC2=	1.600
RESTRICTIONS	: TRACE3=	0.000	ETWE3=	28.908	VC3=	2.209

ADJUSTED OBSERVATIONS

ADJUSTED DISTANCES

RAY	OBSERVED-DIST	ADJUSTED-DIST	STD-ERR	RESIDUALS
1 7	100.0400	99.9138	0.0043	0.1262
1 8	100.0600	99.9897	0.0038	0.0703
2 10	100.0800	99.9462	0.0042	0.1338
2 11	99.9800	99.8794	0.0042	0.1006
3 13	100.0300	100.0445	0.0038	-0.0145
3 14	99.9300	100.0363	0.0043	-0.1063
4 8	100.0200	99.8177	0.0052	0.2023
4 9	100.0400	99.5793	0.0051	0.4607
4 10	100.0300	100.2548	0.0053	-0.2248
4 12	99.9700	99.7985	0.0045	0.1715
5 11	99.8900	100.4721	0.0053	-0.5821
5 12	100.0800	100.2398	0.0045	-0.1598
5 13	99.9900	99.5928	0.0052	0.3972
5 15	99.9400	99.8010	0.0051	0.1390
6 7	99.9800	100.1879	0.0057	-0.2079
6 9	99.9300	100.0277	0.0045	-0.0977
6 14	100.0300	99.8615	0.0057	0.1685
6 15	100.0200	99.8628	0.0045	0.1572
7 9	99.9700	100.0376	0.0030	-0.0676
8 7	100.0000	100.0197	0.0032	-0.0197
8 9	100.0600	100.0157	0.0032	0.0443
9 12	100.0400	100.0502	0.0030	-0.0102
9 15	100.0100	100.0135	0.0030	-0.0035
10 11	100.0000	99.9904	0.0030	0.0096
12 10	100.0600	99.9876	0.0034	0.0724
12 11	99.9800	99.9963	0.0034	-0.0163
12 15	99.9700	100.0390	0.0030	-0.0690
13 14	99.9600	100.0487	0.0032	-0.0887
15 14	100.0500	100.0583	0.0030	-0.0083
15 13	100.0300	100.0459	0.0032	-0.0159

ADJUSTED DIRECTIONS

RAY	OBSERVED-DIRECT			ADJUSTED-DIRECT			STD-ERR	RESIDUALS(")
1 8	29	59	54.90	30	0	16.38	8.3621	-21.48
1 7	89	59	53.00	89	59	56.04	7.1340	-3.04
2 10	209	59	57.01	209	59	39.34	7.4573	17.67
2 11	149	59	58.00	149	59	57.95	7.4581	0.05
3 13	330	0	0.00	329	59	58.17	8.3616	1.83

3	14	269	59	52.01	269	59	50.63	7.1338		1.38
4	8	209	59	53.00	210	0	29.33	10.0363		-36.33
4	9	150	0	4.00	149	59	47.99	9.7562		16.01
4	12	90	0	2.62	89	59	19.93	10.9300		42.69
4	10	30	0	8.42	29	59	30.43	9.8465		37.99
5	11	330	0	2.90	330	0	4.67	9.8367		-1.77
5	12	269	59	54.37	270	0	37.62	10.8959		-43.25
5	15	210	0	5.02	210	0	50.19	9.7449		-45.17
5	13	150	0	3.96	150	0	4.61	10.0478		-0.65
6	7	270	0	9.00	269	59	39.57	8.3380		29.43
6	9	329	59	54.71	329	59	33.26	10.7264		21.45
6	15	30	0	6.43	30	0	18.38	10.7415		-11.95
6	14	90	0	1.98	90	0	28.09	8.3539		-26.11
8	1	210	0	0.00	210	0	20.97	8.3621		-20.97
8	7	150	0	0.10	149	59	58.24	7.1633		1.86
8	9	90	0	0.10	90	0	2.69	6.4747		-2.59
8	4	29	59	57.90	30	0	33.74	10.0363		-35.84
7	1	270	0	0.00	270	0	2.34	7.1340		-2.34
7	8	330	0	10.00	330	0	7.15	7.1633		2.85
7	9	29	59	51.00	29	59	53.76	6.7515		-2.76
7	6	89	59	57.00	89	59	28.77	8.3380		28.23
9	4	330	0	0.00	329	59	44.39	9.7562		15.61
9	12	29	59	53.00	29	59	56.46	6.1143		-3.46
9	15	89	59	55.00	89	59	56.65	6.0021		-1.65
9	6	150	0	10.90	149	59	47.83	10.7264		23.07
9	7	209	59	59.00	210	0	0.96	6.7515		-1.96
9	8	270	0	5.00	270	0	7.10	6.4747		-2.10
12	9	210	0	0.00	210	0	2.76	6.1143		-2.76
12	4	269	59	52.00	269	59	10.37	10.9300		41.63
12	10	330	0	0.00	329	59	58.92	6.3476		1.08
12	11	30	0	0.00	29	59	59.09	6.3484		0.91
12	5	90	0	0.00	90	0	42.69	10.8959		-42.69
12	15	149	59	55.00	149	59	51.04	6.1150		3.96
10	2	29	59	55.00	29	59	37.53	7.4573		17.47
10	4	209	59	57.00	209	59	20.15	9.8465		36.85
10	12	149	59	50.00	149	59	49.92	6.3476		0.08
10	11	90	0	5.00	90	0	5.05	7.9949		-0.05
11	2	330	0	1.00	330	0	0.65	7.4581		0.35
11	10	269	59	54.00	269	59	55.15	7.9949		-1.15
11	12	209	59	54.00	209	59	53.69	6.3484		0.31
11	5	150	0	1.00	150	0	2.96	9.8367		-1.96
15	5	29	59	50.00	30	0	36.68	9.7449		-46.68
15	12	330	0	2.00	329	59	57.34	6.1150		4.66
15	9	269	59	55.90	269	59	57.46	6.0021		-1.56
15	6	210	0	2.90	210	0	15.21	10.7415		-12.31
15	14	150	0	4.00	150	0	2.33	6.7515		1.67
15	13	89	59	50.10	89	59	49.13	6.4757		0.97
13	5	330	0	7.00	330	0	7.34	10.0478		-0.34
13	15	269	59	58.00	269	59	56.24	6.4757		1.76
13	14	209	59	58.00	209	59	58.97	7.1610		-0.97
13	3	149	59	53.00	149	59	51.87	8.3616		1.13
14	3	90	0	2.00	89	59	59.62	7.1338		2.38
14	6	270	0	0.00	270	0	26.30	8.3539		-26.30
14	15	329	59	58.00	329	59	56.93	6.7515		1.07
14	13	29	59	50.10	29	59	51.86	7.1610		-1.76

APPENDIX B.3: RESULTS FOR SUB-OPTIMAL FUSION DENSIFICATION

APPENDIX B.3.1: Results of the first level real network densification using sub-optimal fusion approach

SECOND ORDER REAL NETWORK ADJUSTMENT
(SUB-OPTIMAL FUSION APPROACH)

ITERATION = 2 VUW= 1.000000

RESULTS

DISTANCE : TRACE1= 30.000 ETWE1= 190.811
DIRECTIONS : TRACE2= 52.000 ETWE2= 86.793

ADJUSTED OBSERVATIONS

ADJUSTED DISTANCES

RAY		OBSERVED-DIST	ADJUSTED-DIST	STD-ERR	RESIDUALS
12	7	11392.335	11392.306	0.0143	0.029
12	9	12369.293	12369.346	0.0148	-0.053
13	9	13814.687	13814.697	0.0166	-0.010
14	6	24623.681	24623.710	0.0243	-0.029
14	8	16935.563	16935.592	0.0190	-0.029
15	6	10683.383	10683.358	0.0145	0.025
16	5	14837.980	14837.981	0.0155	-0.001
16	6	13269.516	13269.534	0.0157	-0.018
16	8	24310.634	24310.538	0.0151	0.096
17	7	16690.070	16690.109	0.0188	-0.039
18	20	14984.326	14984.288	0.0164	0.038
18	7	23865.886	23865.831	0.0205	0.055
18	5	31483.464	31483.506	0.0246	-0.042
19	26	24790.315	24790.370	0.0229	-0.055
19	20	14808.001	14808.025	0.0170	-0.024
20	21	4247.690	4247.690	0.0106	0.000
21	3	10340.648	10340.553	0.0139	0.095
21	7	35000.587	35000.722	0.0214	-0.135
22	2	16116.890	16116.922	0.0166	-0.032
22	23	15238.942	15238.933	0.0177	0.009
22	5	23394.975	23394.968	0.0181	0.007
23	4	15396.022	15396.085	0.0173	-0.063
24	2	15823.986	15824.036	0.0169	-0.050
24	3	17476.302	17476.312	0.0182	-0.010
24	26	33794.283	33794.461	0.0190	-0.178
24	25	23681.294	23681.285	0.0200	0.009
25	1	25419.370	25419.342	0.0203	0.028
25	26	13552.337	13552.330	0.0158	0.007
26	1	16336.566	16336.668	0.0168	-0.102
26	24	33794.482	33794.461	0.0190	0.021

ADJUSTED DIRECTIONS

RAY		OBSERVED-DIRECT			ADJUSTED-DIRECT			STD-ERR	RESIDUALS(")
12	7	169	22	42.30	169	22	42.07	0.4396	0.23
12	9	37	17	34.90	37	17	36.11	0.4013	-1.21
13	7	233	35	16.50	233	35	16.04	0.2754	0.46
13	8	106	41	48.00	106	41	48.62	0.5162	-0.62

13	9	330	0	20.65	330	0	20.66	0.8656	-0.01
14	6	231	32	25.31	231	32	24.83	0.2616	0.48
14	8	284	20	12.64	284	20	12.06	0.4146	0.58
14	11	341	1	11.63	341	1	11.22	0.1927	0.41
15	4	207	42	24.54	207	42	25.18	0.4994	-0.64
15	5	257	44	42.41	257	44	42.35	0.3778	0.06
15	6	17	50	53.63	17	50	53.97	0.8211	-0.34
16	5	194	6	54.60	194	6	54.51	0.2719	0.09
16	6	95	31	21.62	95	31	22.58	0.2826	-0.96
16	8	41	24	51.23	41	24	52.10	0.1717	-0.87
16	7	317	36	22.62	317	36	23.26	0.1736	-0.64
17	5	149	36	47.34	149	36	47.90	0.7007	-0.56
17	3	209	32	9.51	209	32	8.92	0.3496	0.59
17	18	275	15	46.63	275	15	46.87	0.2823	-0.24
17	7	352	11	25.96	352	11	27.33	0.6342	-1.37
18	20	186	37	34.51	186	37	34.04	0.4104	0.47
18	7	52	19	28.91	52	19	28.29	0.2190	0.62
18	17	95	15	46.03	95	15	46.87	0.2823	-0.84
18	5	116	21	25.06	116	21	26.10	0.1456	-1.04
19	26	222	47	24.01	222	47	23.85	0.3632	0.16
19	24	143	18	7.51	143	18	6.52	0.2145	0.99
19	3	105	26	4.12	105	26	5.47	0.2818	-1.35
19	20	78	16	29.43	78	16	30.89	0.5550	-1.46
20	21	108	30	23.41	108	30	25.27	0.6649	-1.86
20	18	6	37	34.92	6	37	34.04	0.4104	0.88
20	19	258	16	29.64	258	16	30.89	0.5550	-1.25
21	3	144	36	27.98	144	36	31.56	0.4936	-3.58
21	20	288	30	23.42	288	30	25.27	0.6649	-1.85
21	7	28	17	39.99	28	17	38.78	0.1190	1.21
22	2	186	46	39.33	186	46	37.09	0.4767	2.24
22	23	120	5	28.54	120	5	30.70	0.3216	-2.16
22	5	33	37	5.13	33	37	5.66	0.3242	-0.53
22	3	321	36	42.00	321	36	41.40	0.4587	0.60
23	2	240	59	50.20	240	59	49.73	0.3610	0.47
23	4	19	32	28.15	19	32	29.14	0.5247	-0.99
23	22	300	5	28.42	300	5	30.70	0.3216	-2.28
24	2	125	26	21.12	125	26	25.12	0.2842	-4.00
24	3	26	35	49.90	26	35	48.31	0.2348	1.09
24	19	323	18	7.51	323	18	6.52	0.2145	0.99
24	26	277	8	39.00	277	8	39.11	0.2134	-0.11
24	25	258	47	39.42	258	47	39.25	0.3421	0.17
25	1	275	18	29.72	275	18	26.60	0.3295	3.12
25	26	310	31	10.60	310	31	11.62	0.4325	-1.02
25	24	78	47	38.42	78	47	39.25	0.3421	-0.83
26	1	246	43	54.13	246	43	54.97	0.4352	-0.84
26	25	130	31	11.12	130	31	11.62	0.4325	-0.50
26	24	97	8	39.56	97	8	39.11	0.2134	0.45
26	19	42	47	24.43	42	47	23.85	0.3632	0.58

APPENDIX **B.3.2**: Results of the first level simulated network densification using sub-optimal fusion

SECOND ORDER SIMULATED NETWORK ADJUSTMENT
(SUB-OPTIMAL FUSION APPROACH)

ITERATION = 2 VUW= 0.935921

RESULTS

DISTANCE :	TRACE1=	9.000	ETWE1=	8.777	VC1=	0.753
DIRECTIONS :	TRACE2=	18.000	ETWE2=	16.708	VC2=	2.039
RESTRICTIONS :	TRACE3=	0.000	ETWE3=	8.278	VC3=	2.514

ADJUSTED OBSERVATIONS

ADJUSTED DISTANCES

RAY		OBSERVED-DIST	ADJUSTED-DIST	STD-ERR	RESIDUALS
1	4	200.0600	199.6483	0.0053	0.4117
1	6	200.0400	200.1119	0.0054	-0.0719
2	4	200.0800	200.2265	0.0054	-0.1465
2	5	199.9800	200.5482	0.0054	-0.5682
3	5	200.0300	199.4195	0.0053	0.6105
3	6	199.9300	199.9170	0.0054	0.0130
4	5	200.0200	199.9965	0.0046	0.0235
4	6	200.0900	199.2806	0.0046	0.8094
5	6	199.9200	199.5369	0.0046	0.3831

ADJUSTED DIRECTIONS

RAY		OBSERVED-DIRECT			ADJUSTED-DIRECT			STD-ERR	RESIDUALS("
1	4	30	0	0.00	29	59	57.94	3.7048	2.06
1	6	90	0	10.61	90	0	11.35	3.5314	-0.74
2	4	209	59	53.01	209	59	56.43	3.5488	-3.42
2	5	150	0	4.99	150	0	1.67	3.5483	3.32
3	5	329	59	54.99	329	59	58.83	3.7060	-3.84
3	6	270	0	3.01	269	59	59.86	3.5325	3.15
4	1	210	0	0.00	209	59	57.94	3.7048	2.06
4	2	30	0	7.00	30	0	9.02	3.5488	-2.02
4	5	89	59	52.62	89	59	53.21	4.1041	-0.59
4	6	150	0	5.42	150	0	5.02	4.1041	0.40
5	2	329	59	50.90	329	59	48.99	3.5483	1.91
5	4	270	0	4.37	270	0	3.79	4.1041	0.58
5	6	209	59	58.02	209	59	58.45	4.1041	-0.43
5	3	150	0	13.96	150	0	15.91	3.7060	1.95
6	1	270	0	0.00	270	0	1.80	3.5314	-1.80
6	4	329	59	57.71	329	59	58.09	4.1041	-0.38
6	5	30	0	6.43	30	0	6.02	4.1041	0.41
6	3	89	59	54.98	89	59	52.64	3.5325	2.34

APPENDIX **B.3.3**: Results of the second level real network densification using sub-optimal fusion

THIRD ORDER REAL NETWORK ADJUSTMENT
(SUB-OPTIMAL FUSION APPROACH)

ITERATION = 2 VUW= 1.0024

RESULTS

DISTANCE :	TRACE1=	30.000	ETWE1=	427.703
DIRECTIONS :	TRACE2=	50.000	ETWE2=	462.847

ADJUSTED OBSERVATIONS

ADJUSTED DISTANCES

RAY	OBSERVED-DIST	ADJUSTED-DIST	STD-ERR	RESIDUALS
27 26	14773.890	14774.000	0.0173	-0.110
27 28	9653.420	9653.427	0.0113	-0.007
28 29	13934.030	13933.904	0.0124	0.126
28 24	17550.110	17550.101	0.0149	0.009
29 3	14026.280	14026.123	0.0127	0.157
29 24	7500.980	7501.022	0.0121	-0.042
30 22	20768.540	20768.432	0.0252	0.108
30 23	7148.220	7148.174	0.0229	0.046
31 4	10861.010	10861.021	0.0120	-0.011
31 5	18725.550	18725.561	0.0144	-0.011
32 3	12954.160	12954.270	0.0128	-0.110
32 21	14309.080	14309.127	0.0174	-0.047
33 5	6615.690	6615.769	0.0097	-0.079
33 4	9616.290	9616.256	0.0107	0.034
34 18	6763.580	6763.531	0.0154	0.049
34 21	10493.270	10493.321	0.0139	-0.051
34 20	10463.230	10463.187	0.0152	0.043
35 16	7137.810	7137.810	0.0126	0.000
35 17	7795.220	7795.275	0.0365	-0.055
36 7	13557.220	13557.214	0.0128	0.006
36 18	11414.360	11414.413	0.0177	-0.053
37 35	10445.030	10445.024	0.0103	0.006
37 17	7476.300	7476.322	0.0243	-0.022
37 7	11935.450	11935.492	0.0114	-0.042
38 8	15218.090	15218.058	0.0112	0.032
38 16	13176.150	13176.170	0.0154	-0.020
38 7	14752.400	14752.410	0.0118	-0.010
39 14	17156.360	17156.330	0.0170	0.030
40 11	12385.920	12385.859	0.0128	0.061
41 13	17334.620	17334.604	0.0343	0.016

ADJUSTED DIRECTIONS

RAY	OBSERVED-DIRECT			ADJUSTED-DIRECT			STD-ERR	RESIDUALS "
27 26	232	51	31.26	232	51	33.18	0.4277	-1.92
27 19	28	38	17.97	28	38	21.90	0.5851	-3.93
27 28	149	41	47.26	149	41	44.31	0.4504	2.95
28 27	329	41	49.25	329	41	44.31	0.4504	4.94
28 29	81	34	28.89	81	34	31.50	0.3747	-2.61
28 24	105	50	4.45	105	49	58.17	0.3297	6.28
28 25	214	2	52.29	214	2	53.15	0.5014	-0.86
29 3	51	9	34.82	51	9	31.83	0.2727	2.99
29 24	155	35	6.04	155	34	53.94	0.4752	12.10
29 28	261	34	28.99	261	34	31.50	0.3747	-2.51
30 22	285	27	41.43	285	27	41.56	0.3316	-0.13
30 4	352	16	43.10	352	16	40.43	0.3789	2.67
30 23	252	53	6.97	252	53	6.05	0.7934	0.92
31 4	58	16	33.02	58	16	32.96	0.4150	0.06
31 22	262	44	43.89	262	44	46.80	0.5035	-2.91
31 5	11	53	0.49	11	53	0.35	0.2377	0.14
32 5	86	39	13.99	86	39	15.98	0.3762	-1.99
32 3	219	27	48.46	219	27	42.94	0.3869	5.52
32 21	263	41	41.82	263	41	39.77	0.3563	2.05
32 17	20	2	9.29	20	2	13.93	0.6875	-4.64
33 5	297	4	1.97	297	4	2.05	0.4370	-0.08

33	15	52	8	46.69	52	8	48.19	0.5463	-1.50
33	4	183	1	59.52	183	1	58.04	0.3115	1.48
34	18	329	7	12.99	329	7	18.54	0.6896	-5.55
34	17	77	42	3.25	77	42	2.82	0.2589	0.43
34	21	186	24	52.69	186	24	47.27	0.4962	5.42
34	20	209	48	10.99	209	48	6.90	0.4675	4.09
35	16	32	10	16.87	32	10	15.03	0.5285	1.84
35	5	178	45	7.49	178	45	7.15	0.4093	0.34
35	17	298	10	32.64	298	10	30.51	0.6687	2.13
36	12	32	44	39.26	32	44	38.86	0.3203	0.40
36	7	67	59	6.55	67	59	3.68	0.4109	2.87
36	18	213	37	30.21	213	37	30.98	0.5468	-0.77
37	16	120	40	11.50	120	40	10.95	0.4504	0.55
37	35	163	45	30.30	163	45	32.10	0.3530	-1.80
37	17	211	53	41.82	211	53	41.59	1.0290	0.23
37	7	328	36	9.94	328	36	8.85	0.3304	1.09
38	8	70	7	12.51	70	7	10.81	0.2379	1.70
38	16	187	43	14.82	187	43	15.95	0.2839	-1.13
38	7	274	20	37.98	274	20	38.84	0.2389	-0.86
38	13	346	41	53.56	346	41	53.73	0.8406	-0.17
39	8	359	53	6.52	359	53	7.14	0.4408	-0.62
39	14	72	48	16.88	72	48	15.27	0.6500	1.61
39	6	195	45	58.94	195	45	58.37	0.5400	0.57
40	11	0	27	45.30	0	27	45.28	0.6101	0.02
40	14	140	45	14.38	140	45	12.28	0.5684	2.10
40	8	240	29	37.98	240	29	40.72	0.4435	-2.74
41	10	293	52	37.34	293	52	36.45	0.5523	0.89
41	11	64	34	8.77	64	34	7.82	0.6447	0.95
41	13	244	2	51.24	244	2	51.71	0.4993	-0.47

APPENDIX **B.3.4**: Results of the second level simulated network densification using sub-optimal fusion

THIRD ORDER SIMULATED NETWORK ADJUSTMENT
(SUB-OPTIMAL FUSION APPROACH)

ITERATION = 2 VUW= 0.986242

RESULTS

DISTANCE :	TRACE1=	30.000	ETWE1=	5.408	VC1=	0.180
DIRECTIONS :	TRACE2=	60.000	ETWE2=	59.504	VC2=	1.575
RESTRICTIONS :	TRACE3=	0.000	ETWE3=	25.477	VC3=	0.870

ADJUSTED OBSERVATIONS

ADJUSTED DISTANCES

RAY		OBSERVED-DIST	ADJUSTED-DIST	STD-ERR	RESIDUALS
1	7	100.0400	99.8721	0.0037	0.1679
1	8	100.0600	99.9849	0.0034	0.0751
2	10	100.0800	99.9189	0.0036	0.1611
2	11	99.9800	99.8231	0.0036	0.1569
3	13	100.0300	100.0686	0.0034	-0.0386
3	14	99.9300	100.0531	0.0037	-0.1231
4	8	100.0200	99.8181	0.0048	0.2019

4 9	100.0400	99.5176	0.0045	0.5224
4 10	100.0300	100.2870	0.0048	-0.2570
4 12	99.9700	99.7515	0.0040	0.2185
5 11	99.8900	100.5302	0.0048	-0.6402
5 12	100.0800	100.2471	0.0040	-0.1671
5 13	99.9900	99.5669	0.0048	0.4231
5 15	99.9400	99.7669	0.0045	0.1731
6 7	99.9800	100.2268	0.0053	-0.2468
6 9	99.9300	100.0493	0.0040	-0.1193
6 14	100.0300	99.8475	0.0053	0.1825
6 15	100.0200	99.8569	0.0040	0.1631
7 9	99.9700	100.0537	0.0024	-0.0837
8 7	100.0000	100.0300	0.0025	-0.0300
8 9	100.0600	100.0203	0.0024	0.0397
9 12	100.0400	100.0786	0.0023	-0.0386
9 15	100.0100	100.0240	0.0023	-0.0140
10 11	100.0000	99.9842	0.0025	0.0158
12 10	100.0600	99.9793	0.0024	0.0807
12 11	99.9800	99.9920	0.0024	-0.0120
12 15	99.9700	100.0619	0.0023	-0.0919
13 14	99.9600	100.0724	0.0025	-0.1124
15 14	100.0500	100.0863	0.0024	-0.0363
15 13	100.0300	100.0652	0.0024	-0.0352

ADJUSTED DIRECTIONS

RAY	OBSERVED-DIRECT			ADJUSTED-DIRECT			STD-ERR	RESIDUALS("
1 8	29	59	54.90	29	59	24.70	5.9021	30.20
1 7	89	59	53.00	89	59	48.21	5.2531	4.79
2 10	209	59	57.01	210	0	25.34	5.1278	-28.33
2 11	149	59	58.00	149	59	59.76	5.1279	-1.76
3 13	330	0	0.00	330	0	2.73	5.9031	-2.73
3 14	269	59	52.01	269	59	53.15	5.2534	-1.14
4 8	209	59	53.00	210	0	23.25	8.1620	-30.25
4 9	150	0	4.00	150	0	16.75	8.0949	-12.75
4 12	90	0	2.62	89	59	30.54	9.2220	32.08
4 10	30	0	8.42	29	59	40.20	8.1219	28.22
5 11	330	0	2.90	330	0	1.14	8.1142	1.76
5 12	269	59	54.37	270	0	26.23	9.1973	-31.86
5 15	210	0	5.02	210	0	30.28	8.0880	-25.26
5 13	150	0	3.96	150	0	1.21	8.1684	2.75
6 7	270	0	9.00	270	0	13.82	6.8056	-4.82
6 9	329	59	54.71	329	59	42.03	9.1915	12.68
6 15	30	0	6.43	29	59	41.20	9.2033	25.23
6 14	90	0	1.98	90	0	0.85	6.8161	1.13
8 1	210	0	0.00	209	59	29.80	5.9021	30.20
8 7	150	0	0.10	150	0	2.72	3.9740	-2.62
8 9	90	0	0.10	89	59	56.07	3.9726	4.03
8 4	29	59	57.90	30	0	28.15	8.1620	-30.25
7 1	270	0	0.00	269	59	55.21	5.2531	4.79
7 8	330	0	10.00	330	0	12.62	3.9740	-2.62
7 9	29	59	51.00	29	59	48.16	3.9733	2.84
7 6	89	59	57.00	90	0	1.82	6.8056	-4.82
9 4	330	0	0.00	330	0	12.75	8.0949	-12.75
9 12	29	59	53.00	29	59	48.75	3.6127	4.25
9 15	89	59	55.00	89	59	53.24	3.6188	1.76
9 6	150	0	10.90	149	59	58.22	9.1915	12.68
9 7	209	59	59.00	209	59	56.16	3.9733	2.84
9 8	270	0	5.00	270	0	0.97	3.9726	4.03
10 2	29	59	55.00	30	0	23.33	5.1278	-28.33
10 4	209	59	57.00	209	59	28.78	8.1219	28.22

10 12	149	59	50.00	149	59	51.48	3.9714	-1.48
10 11	90	0	5.00	90	0	4.08	3.9727	0.92
11 2	330	0	1.00	330	0	2.76	5.1279	-1.76
11 10	269	59	54.00	269	59	53.08	3.9727	0.92
11 12	209	59	54.00	209	59	55.22	3.9725	-1.22
11 5	150	0	1.00	149	59	59.24	8.1142	1.76
12 9	210	0	0.00	209	59	55.75	3.6127	4.25
12 4	269	59	52.00	269	59	19.92	9.2220	32.08
12 10	330	0	0.00	330	0	1.48	3.9714	-1.48
12 11	30	0	0.00	30	0	1.22	3.9725	-1.22
12 5	90	0	0.00	90	0	31.86	9.1973	-31.86
12 15	149	59	55.00	150	0	1.68	3.6128	-6.68
13 5	330	0	7.00	330	0	4.25	8.1684	2.75
13 15	269	59	58.00	270	0	0.74	3.9736	-2.74
13 14	209	59	58.00	209	59	56.51	3.9738	1.49
13 3	149	59	53.00	149	59	55.73	5.9031	-2.73
14 3	90	0	2.00	90	0	3.14	5.2534	-1.14
14 6	270	0	0.00	269	59	58.87	6.8161	1.13
14 15	329	59	58.00	329	59	59.99	3.9731	-1.99
14 13	29	59	50.10	29	59	48.61	3.9738	1.49
15 5	29	59	50.00	30	0	15.26	8.0880	-25.26
15 12	330	0	2.00	330	0	8.68	3.6128	-6.68
15 9	269	59	55.90	269	59	54.14	3.6188	1.76
15 6	210	0	2.90	209	59	37.67	9.2033	25.23
15 14	150	0	4.00	150	0	5.99	3.9731	-1.99
15 13	89	59	50.10	89	59	52.84	3.9736	-2.74

APPENDIX B.4: RESULTS FOR DYNAMIC DENSIFICATION

APPENDIX **B.4.1**: Results of the first level real network densification using dynamic approach

SECOND ORDER REAL NETWORK ADJUSTMENT
(DYNAMIC ADJUSTMENT)

ITERATION = 0 VUW= 1.00938

RESULTS

DISTANCE : TRACE1= 30.000 ETWE1= 19.093
DIRECTIONS : TRACE2= 52.000 ETWE2= 10.738

ADJUSTED OBSERVATIONS

ADJUSTED DISTANCES

RAY	OBSERVED-DIST	ADJUSTED-DIST	STD-ERR	RESIDUALS
12 7	11392.335	11392.344	0.0158	-0.009
12 9	12369.293	12369.301	0.0164	-0.008
13 9	13814.687	13814.684	0.0184	0.003
14 6	24623.681	24623.697	0.0269	-0.016
14 8	16935.563	16935.557	0.0211	0.006
15 6	10683.383	10683.384	0.0160	-0.001
16 5	14837.980	14837.969	0.0171	0.011
16 6	13269.516	13269.524	0.0173	-0.008
16 8	24310.634	24310.592	0.0167	0.042
17 7	16690.070	16690.071	0.0209	-0.001

18 20	14984.326	14984.288	0.0183	0.038
18 7	23865.886	23865.798	0.0227	0.088
18 5	31483.464	31483.585	0.0273	-0.121
19 26	24790.315	24790.370	0.0255	-0.055
19 20	14808.001	14808.025	0.0188	-0.024
20 21	4247.690	4247.690	0.0118	0.000
21 3	10340.648	10340.632	0.0151	0.016
21 7	35000.587	35000.681	0.0237	-0.094
22 2	16116.890	16116.934	0.0180	-0.044
22 23	15238.942	15238.933	0.0197	0.009
22 5	23394.975	23395.007	0.0199	-0.032
23 4	15396.022	15396.042	0.0192	-0.020
24 2	15823.986	15824.010	0.0182	-0.024
24 3	17476.302	17476.297	0.0199	0.005
24 26	33794.283	33794.461	0.0212	-0.178
24 25	23681.294	23681.285	0.0222	0.009
25 1	25419.370	25419.365	0.0221	0.005
25 26	13552.337	13552.330	0.0176	0.007
26 1	16336.566	16336.636	0.0179	-0.070
26 24	33794.482	33794.461	0.0212	0.021

ADJUSTED DIRECTIONS

RAY	OBSERVED-DIRECT			ADJUSTED-DIRECT			STD-ERR	RESIDUALS(")
12 7	169	22	42.30	169	22	41.74	0.4876	0.56
12 9	37	17	34.90	37	17	36.18	0.4457	-1.28
13 7	233	35	16.50	233	35	16.39	0.3053	0.11
13 8	106	41	48.00	106	41	49.16	0.5737	-1.16
13 9	330	0	20.65	330	0	21.30	0.9621	-0.65
14 6	231	32	25.31	231	32	24.59	0.2904	0.72
14 8	284	20	12.64	284	20	11.53	0.4607	1.11
14 11	341	1	11.63	341	1	11.37	0.2141	0.26
15 4	207	42	24.54	207	42	25.42	0.5550	-0.88
15 5	257	44	42.41	257	44	42.74	0.4187	-0.33
15 6	17	50	53.63	17	50	54.29	0.9118	-0.66
16 5	194	6	54.60	194	6	55.67	0.3009	-1.07
16 6	95	31	21.62	95	31	23.03	0.3135	-1.41
16 8	41	24	51.23	41	24	52.00	0.1907	-0.77
16 7	317	36	22.62	317	36	23.64	0.1916	-1.02
17 5	149	36	47.34	149	36	48.91	0.7781	-1.57
17 3	209	32	9.51	209	32	9.56	0.3882	-0.05
17 18	275	15	46.63	275	15	46.87	0.3139	-0.24
17 7	352	11	25.96	352	11	27.53	0.7048	-1.57
18 20	186	37	34.51	186	37	34.04	0.4563	0.47
18 7	52	19	28.91	52	19	28.07	0.2430	0.84
18 17	95	15	46.03	95	15	46.87	0.3139	-0.84
18 5	116	21	25.06	116	21	26.30	0.1610	-1.24
19 26	222	47	24.01	222	47	23.85	0.4038	0.16
19 24	143	18	7.51	143	18	6.52	0.2385	0.99
19 3	105	26	4.12	105	26	5.22	0.3120	-1.10
19 20	78	16	29.43	78	16	30.89	0.6170	-1.46
20 21	108	30	23.41	108	30	25.27	0.7392	-1.86
20 18	6	37	34.92	6	37	34.04	0.4563	0.88
20 19	258	16	29.64	258	16	30.89	0.6170	-1.25
21 3	144	36	27.98	144	36	32.04	0.5453	-4.06
21 20	288	30	23.42	288	30	25.27	0.7392	-1.85
21 7	28	17	39.99	28	17	38.72	0.1320	1.27
22 2	186	46	39.33	186	46	36.63	0.5274	2.70
22 23	120	5	28.54	120	5	30.70	0.3575	-2.16
22 5	33	37	5.13	33	37	5.00	0.3600	0.13
22 3	321	36	42.00	321	36	41.03	0.5066	0.97

23	2	240	59	50.20	240	59	49.60	0.3981	0.60
23	4	19	32	28.15	19	32	28.95	0.5831	-0.80
23	22	300	5	28.42	300	5	30.70	0.3575	-2.28
24	2	125	26	21.12	125	26	24.75	0.3119	-3.63
24	3	26	35	49.90	26	35	47.86	0.2595	2.04
24	19	323	18	7.51	323	18	6.52	0.2385	0.99
24	26	277	8	39.00	277	8	39.11	0.2372	-0.11
24	25	258	47	39.42	258	47	39.25	0.3804	0.17
25	1	275	18	29.72	275	18	25.71	0.3643	4.01
25	26	310	31	10.60	310	31	11.62	0.4808	-1.02
25	24	78	47	38.42	78	47	39.25	0.3804	-0.83
26	1	246	43	54.13	246	43	53.63	0.4812	0.50
26	25	130	31	11.12	130	31	11.62	0.4808	-0.50
26	24	97	8	39.56	97	8	39.11	0.2372	0.45
26	19	42	47	24.43	42	47	23.85	0.4038	0.58

APPENDIX **B.4.2**: Results of the first level simulated network densification using dynamic approach

SECOND ORDER SIMULATED NETWORK ADJUSTMENT
(DYNAMIC APPROACH)

ITERATION = 2 VUW= 1.183034

RESULTS

DISTANCE :	TRACE1=	9.000	ETWE1=	6.755	VC1=	0.754
DIRECTIONS :	TRACE2=	18.000	ETWE2=	78.488	VC2=	4.360
RESTRICTIONS :	TRACE3=	0.000	ETWE3=	15.085	VC3=	2.514

ADJUSTED OBSERVATIONS

ADJUSTED DISTANCES

RAY		OBSERVED-DIST	ADJUSTED-DIST	STD-ERR	RESIDUALS
1	4	200.0600	199.6296	0.0062	0.4304
1	6	200.0400	200.0911	0.0063	-0.0511
2	4	200.0800	200.2017	0.0063	-0.1217
2	5	199.9800	200.5273	0.0063	-0.5473
3	5	200.0300	199.4049	0.0062	0.6251
3	6	199.9300	199.8980	0.0063	0.0320
4	5	200.0200	199.9965	0.0055	0.0235
4	6	200.0900	199.2806	0.0055	0.8094
5	6	199.9200	199.5369	0.0055	0.3831

ADJUSTED DIRECTIONS

RAY		OBSERVED-DIRECT			ADJUSTED-DIRECT			STD-ERR	RESIDUALS(")
1	4	30	0	0.00	29	59	59.30	4.0128	0.70
1	6	90	0	10.61	90	0	10.36	3.9495	0.25
2	4	209	59	53.01	209	59	55.42	4.0110	-2.41
2	5	150	0	4.99	150	0	3.38	4.0104	1.61
3	5	329	59	54.99	329	59	57.44	4.0139	-2.45
3	6	270	0	3.01	270	0	0.48	3.9508	2.53
4	1	210	0	0.00	209	59	59.30	4.0128	0.70

4	2	30	0	7.00	30	0	8.01	4.0110	-1.01
4	5	89	59	52.62	89	59	53.21	4.8832	-0.59
4	6	150	0	5.42	150	0	5.02	4.8832	0.40
5	2	329	59	50.90	329	59	50.70	4.0104	0.20
5	4	270	0	4.37	270	0	3.79	4.8832	0.58
5	6	209	59	58.02	209	59	58.45	4.8832	-0.43
5	3	150	0	13.96	150	0	14.52	4.0139	-0.56
6	1	270	0	0.00	270	0	0.82	3.9495	-0.82
6	4	329	59	57.71	329	59	58.09	4.8832	-0.38
6	5	30	0	6.43	30	0	6.02	4.8832	0.41
6	3	89	59	54.98	89	59	53.25	3.9508	1.73

APPENDIX B.4.3: Results of the second level real network densification using dynamic approach

THIRD ORDER REAL NETWORK ADJUSTMENT
(DYNAMIC APPROACH)

ITERATION = 3 VUW= 1.062579

RESULTS

DISTANCE : TRACE1= 30.000 ETWE1= 53.927
DIRECTIONS : TRACE2= 50.000 ETWE2= 88.962

ADJUSTED OBSERVATIONS

ADJUSTED DISTANCES

RAY		OBSERVED-DIST	ADJUSTED-DIST	STD-ERR	RESIDUALS
27	26	14773.890	14773.879	0.0179	0.011
27	28	9653.420	9653.421	0.0145	-0.001
28	29	13934.030	13933.995	0.0158	0.035
28	24	17550.110	17550.148	0.0168	-0.038
29	3	14026.280	14026.222	0.0157	0.058
29	24	7500.980	7500.959	0.0127	0.021
30	22	20768.540	20768.531	0.0223	0.009
30	23	7148.220	7148.226	0.0128	-0.006
31	4	10861.010	10861.005	0.0151	0.005
31	5	18725.550	18725.556	0.0183	-0.006
32	3	12954.160	12954.184	0.0159	-0.024
32	21	14309.080	14309.040	0.0169	0.040
33	5	6615.690	6615.679	0.0121	0.011
33	4	9616.290	9616.293	0.0134	-0.003
34	18	6763.580	6763.577	0.0121	0.003
34	21	10493.270	10493.284	0.0134	-0.014
34	20	10463.230	10463.209	0.0138	0.021
35	16	7137.810	7137.816	0.0120	-0.006
35	17	7795.220	7795.211	0.0126	0.009
36	7	13557.220	13557.203	0.0161	0.017
36	18	11414.360	11414.346	0.0150	0.014
37	35	10445.030	10445.020	0.0131	0.010
37	17	7476.300	7476.301	0.0124	-0.001
37	7	11935.450	11935.436	0.0143	0.014
38	8	15218.090	15218.143	0.0141	-0.053
38	16	13176.150	13176.160	0.0164	-0.010

38 7	14752.400	14752.441	0.0149	-0.041
39 14	17156.360	17156.362	0.0199	-0.002
40 11	12385.920	12385.940	0.0162	-0.020
41 13	17334.620	17334.624	0.0208	-0.004

ADJUSTED DIRECTIONS

RAY		OBSERVED-DIRECT			ADJUSTED-DIRECT			STD-ERR	RESIDUALS(")
27	26	232	51	31.26	232	51	31.51	0.5194	-0.25
27	19	28	38	17.97	28	38	18.16	0.6793	-0.19
27	28	149	41	47.26	149	41	47.52	0.5748	-0.26
28	27	329	41	49.25	329	41	47.52	0.5748	1.73
28	29	81	34	28.89	81	34	29.97	0.4782	-1.08
28	24	105	50	4.45	105	50	4.36	0.4050	0.09
28	25	214	2	52.29	214	2	52.80	0.5963	-0.51
29	3	51	9	34.82	51	9	33.26	0.3468	1.56
29	24	155	35	6.04	155	35	8.85	0.5441	-2.81
29	28	261	34	28.99	261	34	29.97	0.4782	-0.98
30	22	285	27	41.43	285	27	40.62	0.4137	0.81
30	4	352	16	43.10	352	16	41.91	0.4817	1.19
30	23	252	53	6.97	252	53	6.79	0.9712	0.18
31	4	58	16	33.02	58	16	31.91	0.5273	1.11
31	22	262	44	43.89	262	44	43.05	0.6076	0.84
31	5	11	53	0.49	11	52	59.64	0.3023	0.85
32	5	86	39	13.99	86	39	14.45	0.4782	-0.46
32	3	219	27	48.46	219	27	45.75	0.4928	2.71
32	21	263	41	41.82	263	41	41.42	0.4248	0.40
32	17	20	2	9.29	20	2	11.98	0.4971	-2.69
33	5	297	4	1.97	297	4	1.28	0.5525	0.69
33	15	52	8	46.69	52	8	48.52	0.6025	-1.83
33	4	183	1	59.52	183	1	56.93	0.3939	2.59
34	18	329	7	12.99	329	7	14.29	0.7264	-1.30
34	17	77	42	3.25	77	42	3.09	0.2595	0.16
34	21	186	24	52.69	186	24	53.19	0.5696	-0.50
34	20	209	48	10.99	209	48	9.32	0.5708	1.67
35	16	32	10	16.87	32	10	19.06	0.6321	-2.19
35	5	178	45	7.49	178	45	7.29	0.5191	0.20
35	17	298	10	32.64	298	10	32.12	0.4987	0.52
36	12	32	44	39.26	32	44	40.25	0.4051	-0.99
36	7	67	59	6.55	67	59	4.96	0.5233	1.59
36	18	213	37	30.21	213	37	31.53	0.6435	-1.32
37	16	120	40	11.50	120	40	10.11	0.4999	1.39
37	35	163	45	30.30	163	45	31.90	0.4504	-1.60
37	17	211	53	41.82	211	53	40.26	0.5368	1.56
37	7	328	36	9.94	328	36	9.01	0.4195	0.93
38	8	70	7	12.51	70	7	11.05	0.3033	1.46
38	16	187	43	14.82	187	43	13.48	0.3404	1.34
38	7	274	20	37.98	274	20	38.71	0.3035	-0.73
38	13	346	41	53.56	346	41	53.77	0.7167	-0.21
39	8	359	53	6.52	359	53	6.77	0.5605	-0.25
39	14	72	48	16.88	72	48	16.77	0.8192	0.11
39	6	195	45	58.94	195	45	58.97	0.6886	-0.03
40	11	0	27	45.30	0	27	46.11	0.7784	-0.81
40	14	140	45	14.38	140	45	12.80	0.7026	1.58
40	8	240	29	37.98	240	29	41.40	0.5655	-3.42
41	10	293	52	37.34	293	52	36.74	0.7046	0.60
41	11	64	34	8.77	64	34	8.42	0.8222	0.35
41	13	244	2	51.24	244	2	51.36	0.6000	-0.12

APPENDIX **B.4.4**: Results of the second level simulated network densification using dynamic approach

THIRD ORDER SIMULATED NETWORK ADJUSTMENT
(DYNAMIC APPROACH)

ITERATION = 2 VUW= 1.090861

RESULTS

DISTANCE :	TRACE1=	30.000	ETWE1=	8.554	VC1=	0.285
DIRECTIONS :	TRACE2=	60.000	ETWE2=	28.379	VC2=	0.473
RESTRICTIONS :	TRACE3=	0.000	ETWE3=	38.442	VC3=	9.870

ADJUSTED OBSERVATIONS

ADJUSTED DISTANCES

RAY	OBSERVED-DIST	ADJUSTED-DIST	STD-ERR	RESIDUALS
1 7	100.0400	99.9420	0.0036	0.0980
1 8	100.0600	99.9820	0.0036	0.0780
2 10	100.0800	99.9156	0.0037	0.1644
2 11	99.9800	99.8815	0.0037	0.0985
3 13	100.0300	100.0448	0.0036	-0.0148
3 14	99.9300	100.0291	0.0036	-0.0991
4 8	100.0200	99.7018	0.0033	0.3182
4 9	100.0400	99.3606	0.0026	0.6794
4 10	100.0300	100.4194	0.0033	-0.3894
4 12	99.9700	99.7195	0.0026	0.2505
5 11	99.8900	100.6928	0.0033	-0.8028
5 12	100.0800	100.3108	0.0026	-0.2308
5 13	99.9900	99.4188	0.0033	0.5712
5 15	99.9400	99.6754	0.0026	0.2646
6 7	99.9800	100.2228	0.0034	-0.2428
6 9	99.9300	99.9318	0.0026	-0.0018
6 14	100.0300	99.8669	0.0034	0.1631
6 15	100.0200	99.7513	0.0026	0.2687
7 9	99.9700	100.0621	0.0028	-0.0921
8 7	100.0000	100.0385	0.0029	-0.0385
8 9	100.0600	100.0288	0.0028	0.0312
9 12	100.0400	100.0856	0.0026	-0.0456
9 15	100.0100	100.0309	0.0026	-0.0209
10 11	100.0000	99.9924	0.0029	0.0076
12 10	100.0600	99.9873	0.0028	0.0727
12 11	99.9800	100.0004	0.0028	-0.0204
12 15	99.9700	100.0685	0.0026	-0.0985
13 14	99.9600	100.0799	0.0029	-0.1199
15 14	100.0500	100.0938	0.0028	-0.0438
15 13	100.0300	100.0728	0.0028	-0.0428

ADJUSTED DIRECTIONS

RAY	OBSERVED-DIRECT			ADJUSTED-DIRECT			STD-ERR	RESIDUALS(")
1 8	29	59	54.90	29	59	43.41	4.4529	11.49
1 7	89	59	53.00	89	59	56.27	4.3684	-3.27
2 10	209	59	57.01	210	0	9.98	4.4158	-12.97
2 11	149	59	58.00	149	59	54.43	4.4158	3.57
3 13	330	0	0.00	330	0	3.71	4.4525	-3.71

3	14	269	59	52.01	269	59	50.66	4.3680	1.35
4	8	209	59	53.00	210	0	47.91	4.2665	-54.91
4	9	150	0	4.00	150	0	6.62	4.3473	-2.62
4	12	90	0	2.62	89	58	56.13	4.3623	66.49
4	10	30	0	8.42	29	59	15.94	4.2518	52.48
5	11	330	0	2.90	330	0	6.43	4.2509	-3.53
5	12	269	59	54.37	270	0	56.36	4.3586	-61.99
5	15	210	0	5.02	210	0	55.61	4.3462	-50.59
5	13	150	0	3.96	149	59	56.03	4.2688	7.93
6	7	270	0	9.00	269	59	45.32	4.1959	23.68
6	9	329	59	54.71	329	59	29.98	4.3595	24.73
6	15	30	0	6.43	29	59	57.47	4.3612	8.96
6	14	90	0	1.98	90	0	29.18	4.1981	-27.20
8	1	210	0	0.00	209	59	48.51	4.4529	11.49
8	7	150	0	0.10	150	0	2.71	4.6045	-2.61
8	9	90	0	0.10	89	59	56.08	4.6028	4.02
8	4	29	59	57.90	30	0	52.81	4.2665	-54.91
7	1	270	0	0.00	270	0	3.27	4.3684	-3.27
7	8	330	0	10.00	330	0	12.61	4.6045	-2.61
7	9	29	59	51.00	29	59	48.17	4.6037	2.83
7	6	89	59	57.00	89	59	33.32	4.1959	23.68
9	4	330	0	0.00	330	0	2.62	4.3473	-2.62
9	12	29	59	53.00	29	59	48.80	4.1858	4.20
9	15	89	59	55.00	89	59	53.20	4.1929	1.80
9	6	150	0	10.90	149	59	46.17	4.3595	24.73
9	7	209	59	59.00	209	59	56.17	4.6037	2.83
9	8	270	0	3.00	270	0	0.98	4.6028	4.02
12	9	210	0	0.00	209	59	55.80	4.1858	4.20
12	4	269	59	52.00	269	58	45.51	4.3623	66.49
12	10	330	0	0.00	330	0	1.51	4.6014	-1.51
12	11	30	0	0.00	30	0	1.26	4.6028	-1.26
12	5	90	0	0.00	90	1	1.99	4.3586	-61.99
12	15	149	59	55.00	150	0	1.69	4.1860	-6.69
10	2	29	59	55.00	30	0	7.97	4.4158	-12.97
10	4	209	59	57.00	209	59	4.52	4.2518	52.48
10	12	149	59	50.00	149	59	51.51	4.6014	-1.51
10	11	90	0	3.00	90	0	4.05	4.6030	0.95
11	2	330	0	1.00	329	59	57.43	4.4158	3.57
11	10	269	59	54.00	269	59	53.05	4.6030	0.95
11	12	209	59	54.00	209	59	55.26	4.6028	-1.26
11	5	150	0	1.00	150	0	4.53	4.2509	-3.53
15	5	29	59	50.00	30	0	40.59	4.3462	-50.59
15	12	330	0	2.00	330	0	8.69	4.1860	-6.69
15	9	269	59	55.90	269	59	54.10	4.1929	1.80
15	6	210	0	2.90	209	59	53.94	4.3612	8.96
15	14	150	0	4.00	150	0	5.98	4.6034	-1.98
15	13	89	59	50.10	89	59	52.83	4.6040	-2.73
13	5	330	0	7.00	329	59	59.07	4.2688	7.93
13	15	269	59	58.00	270	0	0.73	4.6040	-2.73
13	14	209	59	58.00	209	59	56.52	4.6042	1.48
13	3	149	59	53.00	149	59	56.71	4.4525	-3.71
14	3	90	0	2.00	90	0	0.65	4.3680	1.35
14	6	270	0	0.00	270	0	27.20	4.1981	-27.20
14	15	329	59	58.00	329	59	59.98	4.6034	-1.98
14	13	29	59	50.10	29	59	48.62	4.6042	1.48

APPENDIX B.5: RESULTS FOR STATIC DENSIFICATION

APPENDIX B.5.1: Results of the first level real network densification using static approach

SECOND ORDER REAL NETWORK ADJUSTMENT
(STATIC APPROACH)

ITERATION = 0 VUW= 1.06678

RESULTS

DISTANCE : TRACE1= 30.000 ETWE1= 14.178
DIRECTIONS : TRACE2= 52.000 ETWE2= 7.066

ADJUSTED OBSERVATIONS

ADJUSTED DISTANCES

RAY	OBSERVED-DIST	ADJUSTED-DIST	STD-ERR	RESIDUALS
12 7	11392.335	11392.346	0.0142	-0.0110
12 9	12369.293	12369.303	0.0176	-0.0096
13 9	13814.687	13814.686	0.0358	0.0006
14 6	24623.681	24623.700	0.0246	-0.0191
14 8	16935.563	16935.563	0.0187	-0.0002
15 6	10683.383	10683.379	0.0213	0.0040
16 5	14837.980	14837.962	0.0167	0.0181
16 6	13269.516	13269.545	0.0168	-0.0288
16 8	24310.634	24310.559	0.0159	0.0755
17 7	16690.070	16690.075	0.0203	-0.0048
18 20	14984.326	14984.286	0.0264	0.0398
18 7	23865.886	23865.811	0.0220	0.0753
18 5	31483.464	31483.550	0.0224	-0.0858
19 26	24790.315	24790.377	0.0396	-0.0622
19 20	14808.001	14808.023	0.0336	-0.0217
20 21	4247.690	4247.702	0.0281	-0.0117
21 3	10340.648	10340.606	0.0195	0.0417
21 7	35000.587	35000.682	0.0179	-0.0953
22 2	16116.890	16116.937	0.0201	-0.0468
22 23	15238.942	15238.938	0.0420	0.0040
22 5	23394.975	23394.972	0.0237	0.0031
23 4	15396.022	15396.064	0.0222	-0.0418
24 2	15823.986	15824.017	0.0191	-0.0313
24 3	17476.302	17476.267	0.0188	0.0351
24 26	33794.283	33794.472	0.0216	-0.1886
24 25	23681.294	23681.304	0.0237	-0.0104
25 1	25419.370	25419.371	0.0218	-0.0005
25 26	13552.337	13552.322	0.0355	0.0147
26 1	16336.566	16336.687	0.0228	-0.1213
26 24	33794.482	33794.472	0.0216	0.0104

ADJUSTED DIRECTIONS

RAY	OBSERVED-DIRECT			ADJUSTED-DIRECT			STD-ERR	RESIDUALS (")
12 7	169	22	42.30	169	22	41.65	0.4031	0.65
12 9	37	17	34.90	37	17	35.87	0.3300	-0.97
13 7	233	35	16.50	233	35	16.17	0.5088	0.33

13	8	106	41	48.00	106	41	48.45	0.3778	-0.45
13	9	330	0	20.65	330	0	20.81	0.6649	-0.16
14	6	231	32	25.31	231	32	25.06	0.2313	0.25
14	8	284	20	12.64	284	20	12.17	0.3778	0.47
14	11	341	1	11.63	341	1	11.05	0.1793	0.58
15	4	207	42	24.54	207	42	25.16	0.4201	-0.62
15	5	257	44	42.41	257	44	42.10	0.2957	0.31
15	6	17	50	53.63	17	50	53.94	0.7276	-0.31
16	5	194	6	54.60	194	6	54.62	0.2267	-0.02
16	6	95	31	21.62	95	31	22.86	0.2609	-1.24
16	8	41	24	51.23	41	24	52.11	0.1346	-0.88
16	7	317	36	22.62	317	36	23.04	0.1748	-0.42
17	5	149	36	47.34	149	36	48.32	0.6302	-0.98
17	3	209	32	9.51	209	32	9.36	0.3412	0.15
17	18	275	15	46.63	275	15	46.53	0.2653	0.10
17	7	352	11	25.96	352	11	26.77	0.5878	-0.81
18	20	186	37	34.51	186	37	33.96	0.4048	0.55
18	7	52	19	28.91	52	19	27.78	0.1882	1.13
18	17	95	15	46.03	95	15	46.53	0.2653	-0.50
18	5	116	21	25.06	116	21	25.81	0.1422	-0.75
19	26	222	47	24.01	222	47	24.41	0.3199	-0.40
19	24	143	18	7.51	143	18	6.84	0.2460	0.67
19	3	105	26	4.12	105	26	5.21	0.2889	-1.09
19	20	78	16	29.43	78	16	31.24	0.5363	-1.81
20	21	108	30	23.41	108	30	25.00	1.1370	-1.59
20	18	6	37	34.92	6	37	33.96	0.4048	0.96
20	19	258	16	29.64	258	16	31.24	0.5363	-1.60
21	3	144	36	27.98	144	36	31.21	0.4144	-3.23
21	20	288	30	23.42	288	30	25.00	1.1370	-1.58
21	7	28	17	39.99	28	17	38.55	0.1201	1.44
22	2	186	46	39.33	186	46	37.60	0.4339	1.73
22	23	120	5	28.54	120	5	30.93	0.4271	-2.39
22	5	33	37	5.13	33	37	5.28	0.2575	-0.15
22	3	321	36	42.00	321	36	41.08	0.5268	0.92
23	2	240	59	50.20	240	59	50.21	0.3041	-0.01
23	4	19	32	28.15	19	32	28.69	0.4510	-0.54
23	22	300	5	28.42	300	5	30.93	0.4271	-2.51
24	2	125	26	21.12	125	26	24.48	0.2527	-3.36
24	3	26	35	49.90	26	35	49.11	0.2086	0.79
24	19	323	18	7.51	323	18	6.84	0.2460	0.67
24	26	277	8	39.00	277	8	39.38	0.2080	-0.38
24	25	258	47	39.42	258	47	39.47	0.3429	-0.05
25	1	275	18	29.72	275	18	26.62	0.3234	3.10
25	26	310	31	10.60	310	31	11.78	0.5933	-1.18
25	24	78	47	38.42	78	47	39.47	0.3429	-1.05
26	1	246	43	54.13	246	43	54.69	0.4107	-0.56
26	25	130	31	11.12	130	31	11.78	0.5933	-0.66
26	24	97	8	39.56	97	8	39.38	0.2080	0.18
26	19	42	47	24.43	42	47	24.41	0.3199	0.02

APPENDIX B.5.2: Results of the first level simulated network densification using static approach

SECOND ORDER SIMULATED NETWORK ADJUSTMENT
(STATIC APPROACH)

ITERATION = 2 VUW= 1.093478

RESULTS

DISTANCE :	TRACE1= 9.000	ETWE1=	2.220	VC1=	0.247
DIRECTIONS :	TRACE2= 18.000	ETWE2=	195.819	VC2=	10.879
RESTRICTIONS :	TRACE3= 0.000	ETWE3=	15.085	VC3=	2.514

<u>ADJUSTED OBSERVATIONS</u>

ADJUSTED DISTANCES

RAY		OBSERVED-DIST	ADJUSTED-DIST	STD-ERR	RESIDUALS
1	4	200.0600	200.2937	0.0042	0.2337
1	6	200.0400	199.9886	0.0042	-0.0514
2	4	200.0800	199.9789	0.0042	-0.1011
2	5	199.9800	199.6440	0.0042	-0.3360
3	5	200.0300	200.3872	0.0042	0.3572
3	6	199.9300	199.9519	0.0042	0.0219
4	5	200.0200	200.0067	0.0040	-0.0133
4	6	200.0900	200.5337	0.0040	0.4437
5	6	199.9200	200.1427	0.0040	0.2227

ADJUSTED DIRECTIONS

RAY		OBSERVED-DIRECT			ADJUSTED-DIRECT			STD-ERR	RESIDUALS (")
1	4	30	0	0.00	29	59	28.74	2.6845	-31.26
1	6	90	0	10.61	90	0	36.15	2.6825	25.54
2	4	209	59	53.01	210	0	10.57	2.6819	17.56
2	5	150	0	4.99	150	0	9.60	2.6808	4.61
3	5	329	59	54.99	329	59	45.20	2.6856	-9.79
3	6	270	0	3.01	270	0	1.52	2.6839	-1.49
4	1	210	0	0.00	209	59	28.74	2.6845	-31.26
4	2	30	0	7.00	30	0	38.55	2.6819	31.55
4	5	89	59	52.62	89	59	46.43	3.7944	-6.19
4	6	150	0	5.42	150	0	28.26	3.7944	22.84
5	2	329	59	50.90	329	59	41.42	2.6808	-9.48
5	4	270	0	4.37	270	0	9.93	3.7944	5.56
5	6	209	59	58.02	209	59	24.86	3.7944	-33.16
5	3	150	0	13.96	150	0	23.14	2.6856	9.18
6	1	270	0	0.00	270	0	14.93	2.6825	14.93
6	4	329	59	57.71	330	0	12.84	3.7944	15.13
6	5	30	0	6.43	29	59	41.68	3.7944	-24.75
6	3	89	59	54.98	89	59	45.46	2.6839	-9.52

<u>APPENDIX **B.5.3**: Results of the second level real network densification using static approach</u>

THIRD ORDER REAL NETWORK ADJUSTMENT
(STATIC APPROACH)

ITERATION = 3 VUW= 1.032345

<u>RESULTS</u>

DISTANCE :	TRACE1= 30.000	ETWE1=	4.323
DIRECTIONS :	TRACE2= 50.000	ETWE2=	34.100

ADJUSTED OBSERVATIONS

ADJUSTED DISTANCES

RAY	OBSERVED-DIST	ADJUSTED-DIST	STD-ERR	RESIDUALS
27 26	14773.890	14773.878	0.0147	0.0122
27 28	9653.420	9653.417	0.0120	0.0032
28 29	13934.030	13934.022	0.0128	0.0076
28 24	17550.110	17550.119	0.0131	-0.0093
29 3	14026.280	14026.286	0.0122	-0.0055
29 24	7500.980	7500.968	0.0105	0.0123
30 22	20768.540	20768.530	0.0137	0.0096
30 23	7148.220	7148.223	0.0102	-0.0029
31 4	10861.010	10861.008	0.0126	0.0021
31 5	18725.550	18725.558	0.0148	-0.0076
32 3	12954.160	12954.162	0.0129	-0.0024
32 21	14309.080	14309.067	0.0135	0.0128
33 5	6615.690	6615.692	0.0101	-0.0018
33 4	9616.290	9616.288	0.0111	0.0024
34 18	6763.580	6763.574	0.0094	0.0057
34 21	10493.270	10493.272	0.0078	-0.0019
34 20	10463.230	10463.221	0.0096	0.0085
35 16	7137.810	7137.813	0.0094	-0.0026
35 17	7795.220	7795.224	0.0094	-0.0045
36 7	13557.220	13557.220	0.0128	0.0003
36 18	11414.360	11414.359	0.0121	0.0008
37 35	10445.030	10445.025	0.0099	0.0045
37 17	7476.300	7476.305	0.0098	-0.0048
37 7	11935.450	11935.454	0.0105	-0.0039
38 8	15218.090	15218.090	0.0107	-0.0003
38 16	13176.150	13176.145	0.0136	0.0052
38 7	14752.400	14752.401	0.0117	-0.0014
39 14	17156.360	17156.360	0.0165	-0.0005
40 11	12385.920	12385.926	0.0135	-0.0058
41 13	17334.620	17334.623	0.0173	-0.0029

ADJUSTED DIRECTIONS

RAY	OBSERVED-DIRECT			ADJUSTED-DIRECT			STD-ERR	RESIDUALS
27 26	232	51	31.26	232	51	32.23	0.3082	-0.97
27 19	28	38	17.97	28	38	17.37	0.3914	0.60
27 28	149	41	47.26	149	41	47.99	0.4283	-0.73
28 27	329	41	49.25	329	41	47.99	0.4283	1.26
28 29	81	34	28.89	81	34	30.17	0.3628	-1.28
28 24	105	50	4.45	105	50	3.09	0.2829	1.36
28 25	214	2	52.29	214	2	53.37	0.2876	-1.08
29 3	51	9	34.82	51	9	33.76	0.1588	1.06
29 24	155	35	6.04	155	35	4.10	0.3426	1.94
29 28	261	34	28.99	261	34	30.17	0.3628	-1.18
30 22	285	27	41.43	285	27	41.49	0.2221	-0.06
30 4	352	16	43.10	352	16	43.09	0.1619	0.01
30 23	252	53	6.97	252	53	6.61	0.6980	0.36
31 4	58	16	33.02	58	16	33.11	0.4077	-0.09
31 22	262	44	43.89	262	44	44.88	0.4115	-0.99
31 5	11	53	0.49	11	53	0.51	0.2206	-0.02
32 5	86	39	13.99	86	39	14.08	0.3458	-0.09
32 3	219	27	48.46	219	27	47.31	0.3236	1.15
32 21	263	41	41.82	263	41	42.61	0.2877	-0.79
32 17	20	2	9.29	20	2	9.47	0.2726	-0.18

33 5	297	4	1.97	297	4	2.05	0.3933	-0.08
33 15	52	8	46.69	52	8	46.67	0.1951	0.02
33 4	183	1	59.52	183	1	59.56	0.2517	-0.04
34 18	329	7	12.99	329	7	14.27	0.3760	-1.28
34 17	77	42	3.25	77	42	2.38	0.0898	0.87
34 21	186	24	52.69	186	24	53.15	0.2632	-0.46
34 20	209	48	10.99	209	48	10.66	0.2399	0.33
35 16	32	10	16.87	32	10	15.73	0.2716	1.14
35 5	178	45	7.49	178	45	8.34	0.2387	-0.85
35 17	298	10	32.64	298	10	31.56	0.2486	1.08
36 12	32	44	39.26	32	44	40.34	0.2562	-1.08
36 7	67	59	6.55	67	59	5.71	0.3618	0.84
36 18	213	37	30.21	213	37	30.76	0.4360	-0.55
37 16	120	40	11.50	120	40	10.43	0.2560	1.07
37 35	163	45	30.30	163	45	31.36	0.3108	-1.06
37 17	211	53	41.82	211	53	42.61	0.3473	-0.79
37 7	328	36	9.94	328	36	8.98	0.2082	0.96
38 8	70	7	12.51	70	7	11.49	0.1968	1.02
38 16	187	43	14.82	187	43	14.93	0.1859	-0.11
38 7	274	20	37.98	274	20	38.00	0.1922	-0.02
38 13	346	41	53.56	346	41	53.54	0.2132	0.02
39 8	359	53	6.52	359	53	6.28	0.4145	0.24
39 14	72	48	16.88	72	48	17.20	0.6559	-0.32
39 6	195	45	58.94	195	45	58.37	0.5528	0.57
40 11	0	27	45.30	0	27	45.89	0.6312	-0.59
40 14	140	45	14.38	140	45	13.54	0.5297	0.84
40 8	240	29	37.98	240	29	39.18	0.4464	-1.20
41 10	293	52	37.34	293	52	36.52	0.5004	0.82
41 11	64	34	8.77	64	34	8.56	0.6518	0.21
41 13	244	2	51.24	244	2	51.37	0.4213	-0.13

APPENDIX **B.5.4**: Results of the second level simulated network densification using static approach

THIRD ORDER SIMULATED NETWORK ADJUSTMENT
(STATIC APPROACH)

ITERATION = 2 VUW= 1.0464495

RESULTS

DISTANCE :	TRACE1=	30.000	ETWE1=	4.591	VC1=	0.153
DIRECTIONS :	TRACE2=	60.000	ETWE2=	82.517	VC2=	9.375
RESTRICTIONS :	TRACE3=	0.000	ETWE3=	30.514	VC3=	69.209

ADJUSTED OBSERVATIONS

ADJUSTED DISTANCES

RAY	OBSERVED-DIST	ADJUSTED-DIST	STD-ERR	RESIDUALS
1 7	100.0400	99.8900	0.0020	0.1500
1 8	100.0600	99.9726	0.0020	0.0874
2 10	100.0800	99.8931	0.0020	0.1869
2 11	99.9800	99.8182	0.0020	0.1618
3 13	100.0300	100.0425	0.0020	-0.0125
3 14	99.9300	100.0333	0.0020	-0.1033

4 8	100.0200	99.8542	0.0020	0.1658
4 9	100.0400	99.5952	0.0014	0.4448
4 10	100.0300	100.2884	0.0020	-0.2584
4 12	99.9700	99.8038	0.0014	0.1662
5 11	99.8900	100.4979	0.0020	-0.6079
5 12	100.0800	100.2314	0.0014	-0.1514
5 13	99.9900	99.6303	0.0020	0.3597
5 15	99.9400	99.8090	0.0014	0.1310
6 7	99.9800	100.2015	0.0020	-0.2215
6 9	99.9300	100.0512	0.0014	-0.1212
6 14	100.0300	99.8748	0.0020	0.1552
6 15	100.0200	99.8889	0.0014	0.1311
7 9	99.9700	100.0887	0.0018	-0.1187
8 7	100.0000	100.0485	0.0020	-0.0485
8 9	100.0600	100.0199	0.0018	0.0401
9 12	100.0400	100.1244	0.0017	-0.0844
9 15	100.0100	100.0198	0.0017	-0.0098
10 11	100.0000	99.9452	0.0020	0.0548
12 10	100.0600	99.9395	0.0018	0.1205
12 11	99.9800	99.9531	0.0018	0.0269
12 15	99.9700	100.1039	0.0017	-0.1339
13 14	99.9600	100.0847	0.0020	-0.1247
15 14	100.0500	100.1131	0.0018	-0.0631
15 13	100.0300	100.0618	0.0018	-0.0318

ADJUSTED DIPECTIONS

RAY	OBSERVED-DIRECT			ADJUSTED-DIRECT			STD-ERR	RESIDUALS(")
1 8	29	59	54.90	30	0	19.17	2.5881	-24.27
1 7	89	59	53.00	89	59	47.07	2.5911	5.93
2 10	209	59	57.01	209	59	36.98	2.5924	20.03
2 11	149	59	58.00	149	59	58.44	2.5939	-0.44
3 13	330	0	0.00	329	59	56.96	2.5861	3.04
3 14	269	59	52.01	269	59	58.62	2.5890	-6.61
4 8	209	59	53.00	210	0	20.87	2.5919	-27.87
4 9	150	0	4.00	149	59	46.92	2.5417	17.08
4 12	90	0	2.62	89	59	45.38	2.5416	17.24
4 10	30	0	8.42	29	59	38.40	2.5869	30.02
5 11	330	0	2.90	330	0	4.11	2.5855	-1.21
5 12	269	59	54.37	270	0	12.20	2.5345	-17.83
5 15	210	0	5.02	210	0	40.36	2.5387	-35.34
5 13	150	0	3.96	150	0	5.90	2.5942	-1.94
6 7	270	0	9.00	269	59	51.14	2.5885	17.86
6 9	329	59	54.71	329	59	38.12	2.5353	16.59
6 15	30	0	6.43	30	0	22.41	2.5381	-15.98
6 14	90	0	1.95	90	0	17.47	2.5908	-15.49
8 1	210	0	0.00	210	0	23.76	2.5881	-23.76
8 7	150	0	0.10	149	59	53.96	3.5259	6.14
8 9	90	0	0.10	90	0	6.64	3.5250	-6.54
8 4	29	59	57.90	30	0	25.28	2.5919	-27.38
7 1	270	0	0.00	269	59	53.37	2.5911	6.63
7 8	330	0	10.00	330	0	2.87	3.5259	7.13
7 9	29	59	51.00	29	59	58.01	3.5255	-7.01
7 6	89	59	57.00	89	59	40.34	2.5885	16.66
9 4	330	0	0.00	329	59	43.32	2.5417	16.68
9 12	29	59	53.00	30	0	3.58	3.0655	-10.58
9 15	89	59	55.00	89	59	57.91	3.0643	-2.91
9 6	150	0	10.90	149	59	52.69	2.5353	18.21
9 7	209	59	59.00	210	0	5.21	3.5255	-6.21
9 8	270	0	5.00	270	0	11.05	3.5250	-6.05
12 9	210	0	0.00	210	0	9.88	3.0655	-9.88

12	4	269	59	52.00	269	59	35.82	2.5416	16.18
12	10	330	0	0.00	329	59	58.58	3.5251	1.42
12	11	30	0	0.00	29	59	58.66	3.5258	1.34
12	5	90	0	0.00	90	0	17.27	2.5345	-17.27
12	15	149	59	55.00	149	59	43.09	3.0656	11.91
10	2	29	59	55.00	29	59	35.17	2.5924	19.83
10	4	209	59	57.00	209	59	28.12	2.5869	28.88
10	12	149	59	50.00	149	59	49.58	3.5251	0.42
10	11	90	0	5.00	90	0	5.55	3.5259	-0.55
11	2	330	0	1.00	330	0	1.14	2.5939	-0.14
11	10	269	59	54.00	269	59	55.65	3.5259	-1.65
11	12	209	59	54.00	209	59	53.26	3.5258	0.74
11	5	150	0	1.00	150	0	2.40	2.5855	-1.40
15	5	29	59	50.00	30	0	26.84	2.5387	-36.84
15	12	330	0	2.00	329	59	49.39	3.0656	12.61
15	9	269	59	55.90	269	59	58.72	3.0643	-2.82
15	6	210	0	2.90	210	0	19.24	2.5381	-16.34
15	14	150	0	4.00	149	59	58.86	3.5251	5.14
15	13	89	59	50.10	89	59	45.82	3.5254	4.28
13	5	330	0	7.00	330	0	8.63	2.5942	-1.63
13	15	269	59	58.00	269	59	52.93	3.5254	5.07
13	14	209	59	58.00	210	0	2.53	3.5255	-4.53
13	3	149	59	53.00	149	59	50.66	2.5861	2.34
14	3	90	0	2.00	90	0	7.61	2.5890	-5.61
14	6	270	0	0.00	270	0	15.69	2.5908	-15.69
14	15	329	59	58.00	329	59	53.46	3.5251	4.54
14	13	29	59	50.10	29	59	55.42	3.5255	-5.32

APPENDIX C: DERIVATION OF ERROR ELLIPSE PARAMETERS

Given the position variances σ_x^2 and σ_y^2 in two mutually perpendicular directions (typically in the N-S and E-W) and the covariance σ_{xy}^2 we may ask the question, which are the variances σ_m^2 and σ_n^2 in two other mutually perpendicular directions with azimuths ψ and $\psi + 90^0$ respectively?

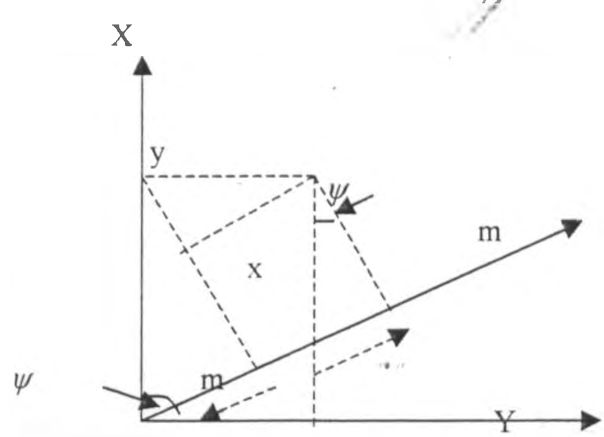
It is well known that

$m = x \cos\psi + y \sin\psi$

$n = x \cos(\psi + 90) + y \sin(\psi + 90)$

Applying Gauss's Propagation

of Error Law we have



Positional Error Ellipse parameter Derivations

$$\sigma_m^2 = \cos^2\psi\,\sigma_x^2 + \sin^2\psi\,\sigma_y^2 + 2\cos\psi\sin\psi\,\sigma_{xy} \quad (1)$$

$$\sigma_n^2 = \sin^2\psi\,\sigma_x^2 + \cos^2\psi\,\sigma_y^2 - 2\cos\psi\sin\psi\,\sigma_{xy} \quad (2)$$

Now it is interesting to consider in which direction is σ_m^2 a maximum and what is the value of that maximum. For maximum or minimum

$$\frac{\partial(\sigma_m^2)}{\partial\psi} = 0 \tag{3}$$

Hence differentiating (4.3.3)

$$\therefore 0 = -2\cos\psi_m \sin\psi_m \sigma_x^2 + 2\sin\psi_m \cos\psi_m \sigma_y^2 + 2(\cos^2\psi_m - \sin^2\psi_m)\sigma_x$$

$$= -\sin 2\psi_m \sigma_x^2 + \sin 2\psi_m \sigma_y^2 + 2\cos 2\psi_m \sigma_{xy} \tag{4}$$

giving

$$\tan 2\psi_m = \frac{2\sigma_{xy}}{(\sigma_x^2 - \sigma_y^2)} \tag{5}$$

substituting this into (4.3.1) and (4.3.2) and manipulating gives

$$\sigma_m^2 = \sigma^2{}_{\max} = \tfrac{1}{2}[\sigma_x^2 + \sigma_y^2 + ((\sigma_y^2 - \sigma_x^2)^2 + 4(\sigma_{xy})^2)^{\frac{1}{2}}] \tag{6a}$$

$$\sigma_n^2 = \sigma^2{}_{\min} = \tfrac{1}{2}[\sigma_x^2 + \sigma_y^2 - ((\sigma_y^2 - \sigma_x^2)^2 + 4(\sigma_{xy})^2)^{\frac{1}{2}}] \tag{6b}$$

Hence we can compute the directions and values of the maximum and minimum variances at any point in the network.