MODELLING A HIERARCHICAL SYSTEM WITH MULTIPLE ABSORBING STATES

by

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July 30, 2007
DECLARATION

Declaration by the Candidate

I, the undersigned, declare that this project is my original work and has not been presented for a degree in any other University.

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Declaration by the Supervisor

This project has been submitted for examination with my approval as the supervisor.

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ABSTRACT

Previous studies on the education system, which is hierarchical in nature, made use of Markov chain transition models, comprising a single absorbing state. This work considers a Markov chain transition model consisting of multiple absorbing states which takes care of those who attain final maximum grade and those who leave the system before attaining the final grade, which is more realistic.

The purpose of this study is to extend or generalize the original idea of one absorbing state to multiple absorbing states based on grade retention rates, dropout rates, completion rates, survival times and expected length of stay in University education. The work further looks at the merits and demerits of the above considerations, that is, single and multiple absorbing states.
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Chapter 1

GENERAL INTRODUCTION

1.1 INTRODUCTION

Education is a system that is hierarchical in nature and thus amiable to the use of discrete
Markov Chains. A student in the system either transists from one grade to the next higher
grade, repeats the same grade or drops out of the system due to a number of factors such as
sickness, lack of fees, graduation and so on.

In the final grade, a student will either attain a desired maximum qualification, for example a
degree, in a particular education level, repeat the same grade or drop out of the system before
attaining the maximum qualification. Thus, the student ends up as a graduate or a dropout:
these can be referred to as permanent states if we assume that the chance of returning to the
system is small. An absorbing state is a state which once entered becomes a permanent state.

In this study, we model an undergraduate university degree system using the Markov Chain ap-
proach in which graduating and the state at dropping out will constitute the multiple absorbing states of the system.

1.2 LITERATURE REVIEW

Stochastic models have been applied in different hierarchical systems. In the educational field, Gani (1963) proposed a Markovian model to forecast enrolment and degrees awarded in Australian Universities. Thonstad (1967) made use of stochastic models to study enrolments in the Norwegian educational system in his book on educational planning. Uche (1980), applied the Markovian model to the Nigerian educational systems.

The Markovian Chain model has been used to study the Kenyan Primary education system see for example Owino (1982) and Odhiambo and Owino (1985). In these studies, several measures of academic survival for the Kenyan Primary education system were considered and compared. All dropouts from the system were considered as that into one absorbing state. In this study, we shall modify this consideration and reclassify the dropouts into multiple categories based on the status at dropout. Odhiambo and Khogali (1986) studied the Kenyan Primary education system through a cohort analysis. Thus, they followed a group of students joining the system at a particular time until the cohort left the system. Owino and Phillips (1988) compared the retention properties of the Kenyan primary education system between 1964 and 1972 and between 1972 and 1980. The aim here was to justify if the system had been homogeneous over time. It was found that the system was not time homogeneous in the two time periods. Owino and Odhiambo (1994) used a Markovian model to plan the Kenyan Primary education system. In their study, they estimated several capital and human resource
requirements for the system. In addition, Mbugua (2005) used the Markov Chain model to estimate the number of new entrants into the Kenyan Primary education system following the introduction of free primary education.

A more recent study in Kenya using the Markov Chain process was based on grade structured control in a manpower system; see for example, Owino and Bodo (2005). In this study, they derived the maintainable grade structures for an academic department.

In most of these studies, the dropouts have all been grouped into one absorbing state. In our study, we intend to classify the dropouts into several states as stated above.

1.3 OBJECTIVES OF THE STUDY

1.3.1 BROAD OBJECTIVE

In this study, we intend to model an education system with multiple absorbing states comprising of dropouts and graduates, thus relaxing the original idea which grouped all dropouts into one absorbing state.

1.3.2 SPECIFIC OBJECTIVE

The specific objectives of this study include:

i) Modeling an undergraduate university degree programme using a Markov Chain in which graduating and dropping out constitute two absorbing states.

ii) Developing the idea of one absorbing state into multiple absorbing states such that all dropouts who were previously grouped together are now each categorized into states at dropout.
This leads to the multiple dropout system.

1.4 METHODOLOGY

In this study, the theory of Markov Chains will be used; specifically, the theory of Absorbing Markov Chains. A computational software such as Matlab will be used in the analysis of the absorbing Markov Chain with multiple absorbing states. Data for this study was extracted from an undergraduate degree programme in the School of Mathematics, University of Nairobi.

1.5 SIGNIFICANCE OF THE STUDY

In most of the previous Markov Chain models used in educational planning, the absorbing state considered was usually only one, implying that graduates and dropouts were grouped together. In this study, the absorbing states are two or more. It, therefore, is assumed that an element of precision is introduced by using multiple absorbing states as compared to using only one absorbing state. Using multiple absorbing states, it is possible to precisely ascertain the ratios of those who drop out due to graduation after attaining the maximum qualification and the ratios of those who drop out before attaining the maximum qualification.
Chapter 2

MODELS WITH MULTIPLE ABSORBING STATES

2.1 INTRODUCTION

The theory of stochastic processes deals with systems which develop in time or space in accordance with probabilistic laws. A state is defined as the possible position occupied by the system; the set of all such possible states being called the state space. An absorbing state is a state which when once entered becomes a permanent state.

An important feature called the Markov property, is that the transition probabilities for the transition from time \( n \) to time \( (n+1) \) depend on the state given to be occupied at time \( n \) and the final state at time \( (n+1) \), but not in addition on what happened before time \( n \).

A matrix \( P = ((P_{ij})) \) of transition probabilities is a stochastic matrix if its elements are
non-negative and all its row sums are unity. If for a stochastic process the last value in the sequence depends on the immediate previous value of the sequence only, then it is a Markov process. Consequently, a Markov chain is a Markov process in discrete time with a discrete state space.

2.2 THE MODEL

Consider a Markov Chain model with $s$ non-absorbing states; 1, 2, 3,...,$s$ corresponding to the grades of the system and $r$ absorbing states corresponding to the various final qualifications. Here, $r + s = N$, thus $N$ is the total number of possible states of the system.

An absorbing state is a state which becomes permanent once it has been entered. This implies that the transition probability between absorbing states should be represented by one, justifying the use of the identity matrix but transition from an absorbing state to a non-absorbing state, which is impossible should be represented by zero; hence the matrix of zeroes.

On the other hand, transitions from non-absorbing states to absorbing states are possible, so are transitions between non-absorbing states. The relevant entries lie between zero and one since they are probabilities. The transition matrix $P$, of the Markov Chain can then be represented in the following canonical form, assuming homogeneity;
where

$I$ is an $rxr$ identity matrix which gives transition probabilities between absorbing states.

$O$ is an $rxs$ matrix of zeroes which gives transition probabilities from an absorbing state to a non-absorbing state.

$R = (r_{ik})$ is an $sxr$ matrix, $r_{ik}$ being the probability that a student who is in grade $i$ at time $(t-1)$ will graduate with final education $k$ at time $t$, $i = 1, 2, ..., s$ and $k = 1, 2, ..., r$.

$Q = (q_{ij})$ is an $sx$s matrix, $q_{ij}$ being the probability that a student who is in grade $i$ at time $(t-1)$ will be in grade $j$ at time $t$; $i, j = 1, 2, ..., s$. 

\[
P = \begin{pmatrix}
1 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & \ldots & 0 \\
0 & 1 & 0 & \ldots & 0 & 0 & 0 & 0 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \ldots & 1 & 0 & 0 & 0 & \ldots & 0 \\
r_{11} & r_{12} & r_{13} & \ldots & r_{1r} & q_{11} & q_{12} & q_{13} & \ldots & q_{1s} \\
r_{21} & r_{22} & r_{23} & \ldots & r_{2r} & q_{21} & q_{22} & q_{23} & \ldots & q_{2s} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
r_{s1} & r_{s2} & r_{s3} & \ldots & r_{sr} & q_{s1} & q_{s2} & q_{s3} & \ldots & q_{ss}
\end{pmatrix}
\]
2.3 THE n-STEP TRANSITION PROBABILITY MATRIX

By the Chapman-Kolmogorov result, the n-step transition probability matrix is given by

\[ P^{(n)} = P^n \]  \hspace{1cm} (2.3)

Consider equation 2.2, then

\[
P^2 = \begin{pmatrix} I & O \\ R + QR & Q^2 \end{pmatrix}
\]  \hspace{1cm} (2.4)

\[
P^3 = \begin{pmatrix} I & O \\ R + QR + Q^2R & Q^3 \end{pmatrix}
\]  \hspace{1cm} (2.5)

\[
P^n = \begin{pmatrix} I & O \\ (I + Q + Q^2 + \ldots + Q^{n-1})R & Q^n \end{pmatrix}
\]  \hspace{1cm} (2.6)

Hence, in canonical form,

\[
P^n = \begin{pmatrix} I & O \\ R^n & Q^n \end{pmatrix}
\]  \hspace{1cm} (2.7)

where

$I$ is an $r \times r$ identity matrix which gives transition probabilities between absorbing states in $n$ steps

$O$ is an $r \times s$ matrix of zeroes which gives transition probabilities from absorbing states to non-absorbing states in $n$ steps

$R^n = (r_{ik}^{(n)}) = (I + Q + Q^2 + \ldots + Q^{n-1})R$ is an $s \times r$ matrix which gives the probability that a student who is in grade $i$ will graduate with final education $k$ within $n$ years, $i = 1, 2, \ldots, s$ and $k = 1, 2, \ldots, r$. It is also called the completion ratio.

$Q^n = (q_{ij}^{(n)})$ is an $s \times s$ matrix which gives the probability that a student who is in grade $i$
will be in grade j, n years later; i, j = 1, 2,...,s.

It is important to note that if n represents the number of years, then the transition process matrix within one year is given by $P$, within two years $n$ takes the value 2 therefore the transition process matrix is given by $P^2$ and so on until within $n$ years, it is given by $P^n$.

From equation (2.7), the entries for I and O are the identity matrix and the matrix of zeroes respectively but the entries for $R^n$ and $Q^n$ depend on the value that $n$ takes. Hence, the completion ratio depends on the value of $n$. This completion ratio is the basis of the computations in this work.

### 2.4 THE FUNDAMENTAL MATRIX

From equations (2.6) and (2.7),

$$R^n = (I + Q + Q^2 + ... + Q^{n-1})R$$

(2.8)

The aim here is to reduce equation (2.8) to its simplest form therefore using the expansion,

$$\frac{(1-x^n)}{(1-x)} = 1 + x + ... + x^{n-1}$$

we get that

$$(I - Q)(I + Q + Q^2 + ... + Q^{n-1}) = (I - Q^n)$$

(2.9)

implying that
\[ (I + Q + Q^2 + \ldots + Q^{n-1}) = (I - Q)^{-1}(I - Q^n) \] (2.10)

therefore, substituting for \((I + Q + Q^2 + \ldots + Q^{n-1})\) in \(P_n\) by \((I - Q)^{-1}(I - Q^n)\)

then

\[
P_n = \begin{pmatrix} I & 0 \\ (I - Q)^{-1}(I - Q^n)R & Q^n \end{pmatrix} \] (2.11)

but expanding

\[
(I - Q)(I + Q + Q^2 + \ldots) = (I + Q + Q^2 + Q^3 + \ldots - Q - Q^2 - Q^3 - Q^4 - \ldots) \] (2.12)

\[ = I \] (2.13)

implying that taking limits as \(n\) tends to infinity of equation (2.9)

\[
\lim(n \to \infty)(I - Q)(I + Q + Q^2 + \ldots Q^{n-1}) = \lim(n \to \infty)(I - Q^n) \] (2.14)

hence

\[
\lim(n \to \infty)I = \lim(n \to \infty)I - \lim(n \to \infty)Q^n \] (2.15)

implying that

\[
\lim(n \to \infty)Q^n = 0 \] (2.16)

Which confirms that for an absorbing Markov Chain, all the students must leave the system ultimately; therefore, substituting for \((I - Q^n)\) in \(P_n\) then

\[
\lim(n \to \infty)P_n = \begin{pmatrix} I & 0 \\ (I - Q)^{-1}R & O \end{pmatrix} \] (2.17)
The matrix \( L = (I - Q)^{-1} \) is called the fundamental matrix of the absorbing Markov Chain. The elements \( l_{ij} \) in the fundamental matrix represent the length of time in years that a student spends in grade \( j \) given he was in grade \( i \), before leaving the system.
Chapter 3

APPLICATION OF THE MODEL AND DISCUSSIONS

3.1 INTRODUCTION

An undergraduate university degree programme is a hierarchical system where students either transist to the next grade, repeat the same grade or drop out of the system due to a number of reasons. Thus, movement within the system is stochastic in nature.

The key assumptions made in the undergraduate system are;

i) transition from one grade to another is strictly a one step movement to the next higher grade only,

ii) movement to a lower grade is non-existant.
3.2 INITIAL TRANSITION MATRIX

Let the states of the undergraduate university system be denoted by the integers 1, 2, ..., N at times \( t = 0, 1, 2, ... \). Let \( P_{ij} \) denote the probability that a student in grade \( i \) at time \( (t-1) \) will be in grade \( j \) at time \( t \). This gives rise to a transition matrix \( P = ((P_{ij})) \); \( i, j = 1, 2, ..., N \), then, assuming time homogeneity, equation (2.2) holds. The diagonal elements of \( Q = (q_{ij}) \), are probabilities of an individual repeating grade \( i \), \( i = 1, 2, ..., s \). Let \( n_{ij}(t) \) represent the number of students in grade \( i \) at time \( (t-1) \) who will be in grade \( j \) at time \( t \), also, let \( n_{i}(t-1) \) represent the number of students in grade \( i \) at time \( (t-1) \), then assuming the multinomial distribution, the transition probabilities are estimated from

\[
p_{ij} = \frac{n_{ij}(t)}{n_{i}(t-1)} \tag{3.1}
\]

where \( i, j = 1, 2, ..., N \). This is the proportion of students who were in grade \( i \) at time \( (t-1) \) who end up being in grade \( j \) at time \( t \).

3.3 INITIAL TRANSITION PROCESSES FOR THE SYSTEM

The data for this study was extracted from Bachelor of Science, Actuarial Science in the School of Mathematics, University of Nairobi. Assuming time homogeneity, students enrolments in grades I, II, III and IV for the year 2004 and enrolments for the same students in grades II, III and IV for the year 2005 were as shown in the following Table 3.1.
### Table 3.1 Student enrolments 2004/2005

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>75</td>
<td>67</td>
<td>2</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>II</td>
<td>85</td>
<td>77</td>
<td>1</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>III</td>
<td>49</td>
<td>42</td>
<td>1</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>IV</td>
<td>33</td>
<td>28</td>
<td>4</td>
<td>-</td>
<td>1</td>
</tr>
</tbody>
</table>

Key: G represents Grade, E represents Enrolment, PR represents Proceeded, R represents Repeated. PD represents Passed and Dropped out and D represents Discontinued.

It was noted that:

i) A majority of the students proceeded to the next grade.

ii) A few students repeated the same grade.

iii) Some students dropped out of a particular grade after passing all the courses offered in the grade.

iv) Some students were discontinued from a particular grade due to failure in the courses offered.

Using data from Table 3.1, the following initial transitional matrices were obtained.

#### 3.3.1 INITIAL TRANSITION PROCESS WITH SINGLE ABSORBING STATE

In this case, dropouts who comprised both of students who successfully completed a particular grade and students who were discontinued from the same grade were grouped together. So,
the dropout proportions for students who were in grades I, II, III and IV were \((6/75) = 0.0800\), \((7/85) = 0.0824\), \((6/49) = 0.1224\) and \((29/33) = 0.8788\), respectively. This gives rise to the \(R\) component of the matrix in equation (2.2).

The elements of the \(Q\) component in equation (2.2) represent transition probabilities between the grades. From Table 3.1, the enrolment of students in grade I in 2004 was 75; out of which 67 students proceeded to grade II, 2 repeated grade I, 4 passed in all the courses offered in grade I but dropped out of the system thereafter and finally, 2 were discontinued from grade I due to failure in the courses offered.

Considering the rows and columns in the \(Q\) matrix, position 1,1 (one, one) represents the proportion of students who repeated grade I, specifically, \((2/75) = 0.0267\) and position 1,2 (one, two), represents the proportion of students who proceeded to grade II from grade I, that is, \((67/75) = 0.8933\). The same concept is applied to obtain the relevant proportions of the students who were originally in grades II, III and IV to obtain the elements of the \(Q\) matrix. This \(Q\) matrix remains invariant irrespective of the number of absorbing states under consideration.

Thus, the initial transitional matrix with single absorbing state, \(P_i\), assuming time homogeneity is;
The R matrix, \( r \), is therefore, a column vector incorporating only one absorbing state, since in the element \( r_{ik} \), \( k \) takes value 1 only, hereby representing dropping out only. Therefore,

\[
P_1 = \begin{pmatrix}
1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0.0800 & 0.0267 & 0.8933 & 0.0000 & 0.0000 \\
0.0824 & 0.0000 & 0.0118 & 0.9058 & 0.0000 \\
0.1224 & 0.0000 & 0.0000 & 0.0205 & 0.8571 \\
0.8788 & 0.0000 & 0.0000 & 0.0000 & 0.1212
\end{pmatrix}
\] (3.2)

The R matrix, \( r \), is therefore, a column vector incorporating only one absorbing state, since in the element \( r_{ik} \), \( k \) takes value 1 only, hereby representing dropping out only. Therefore,

\[
\begin{pmatrix}
0.0800 \\
0.0824 \\
0.1224 \\
0.8788
\end{pmatrix}
\] (3.3)

where, for example, \( r_{21} = 0.0824 \) implies that the probability of a student dropping out from grade II is 0.0824; obtained by dividing 7 students by 85 students from Table 3.1.

### 3.3.2 Initial Transition Process with Double Absorbing States

Dropouts from a particular grade who comprised of students who were discontinued from the grade as well as students who successfully completed the grade were grouped together. Thus, the proportions obtained in section 3.3.1 for students who were in grades I, II and III hold. On the other hand, the proportions of students who were discontinued from grade IV and students who graduated were separately determined. Hence, the proportion of students who graduated is \((28/33) = 0.8484\) and the proportion of students who were discontinued is \((1/33) = 0.0303\).
Thus, the initial transitional matrix with double absorbing states, \( P_2 \), assuming time homo-
geneity is:

\[
P_2 = \begin{pmatrix}
1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0.0800 & 0.0000 & 0.0267 & 0.8933 & 0.0000 & 0.0000 \\
0.0824 & 0.0000 & 0.0000 & 0.0118 & 0.9058 & 0.0000 \\
0.1224 & 0.0000 & 0.0000 & 0.0000 & 0.0205 & 0.8571 \\
0.0304 & 0.8484 & 0.0000 & 0.0000 & 0.0000 & 0.1212 \\
\end{pmatrix}
\]

In this case, the \( R \) matrix, namely \( R_2 \), is therefore of order 4x2, since in the element \( r_{ik} \),
k takes absorbing state values 1 and 2. Let absorbing state 1 represent dropping out before
attaining the maximum qualification and absorbing state 2 represent graduating after attaining
the maximum qualification. Hence,

\[
R_2 = \begin{pmatrix}
0.0800 & 0.0000 \\
0.0824 & 0.0000 \\
0.1224 & 0.0000 \\
0.0304 & 0.8484 \\
\end{pmatrix}
\]

where, for example, \( r_{41} = 0.0304 \) implies that the probability of a student dropping out
from grade IV without attaining the maximum qualification is 0.0304 whereas \( r_{12} = 0.8484 \)
implies that the probability of a student graduating from grade IV after attaining the maximum
qualification is 0.8484.
3.3.3 INITIAL TRANSITION PROCESS WITH MULTIPLE ABSORBING STATES

In the multiple absorbing states model, we consider proportions of students who dropped out from a particular grade after successful completion of the courses offered in the grade, proportions of students who dropped out as a result of discontinuation from a particular grade and proportions of students who graduated from the system. For example, using data from Table 3.1, the proportion of students who were in grade I and who dropped out after successful completion of the same grade is \((4/75) = 0.0533\). On the other hand, the proportion of students who were discontinued from grade I is \((2/75) = 0.0267\). Similar computations provide the relevant entries for students who were in grades II, III and IV.

Thus, the initial transitional matrix with multiple absorbing states, \(P_5\), assuming time homogeneity is:

\[
P_5 = \begin{pmatrix}
1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 \\
0.0533 & 0.0000 & 0.0000 & 0.0000 & 0.0267 & 0.0267 & 0.8933 \\
0.0000 & 0.0471 & 0.0000 & 0.0000 & 0.0353 & 0.0000 & 0.0118 & 0.9058 \\
0.0000 & 0.0000 & 0.0816 & 0.0000 & 0.0408 & 0.0000 & 0.0205 & 0.8571 \\
0.0000 & 0.0000 & 0.8484 & 0.0304 & 0.0000 & 0.0000 & 0.0000 & 0.1212 \\
\end{pmatrix}
\]

(3.6)

The \(R\) matrix, \(R_5\), is of order 4x5, since in the element \(r_{ik}\), \(k\) takes values 1, 2, 3, 4 and
5. Absorbing state 1 represents dropping out after attaining grade I education level, the same applying to absorbing states 2, 3 and 4 while absorbing state 5 represents being discontinued from a particular grade.

In this case,

\[
R_5 = \begin{pmatrix}
0.0533 & 0.0000 & 0.0000 & 0.0000 & 0.0267 \\
0.0000 & 0.0471 & 0.0000 & 0.0000 & 0.0353 \\
0.0000 & 0.0000 & 0.0816 & 0.0000 & 0.0408 \\
0.0000 & 0.0000 & 0.0000 & 0.8484 & 0.0304
\end{pmatrix}
\]

where, for example, \( r_{22} = 0.0471 \) implies that the probability of a student dropping out of grade II after passing in all the courses offered is 0.0471.

3.4 COMPLETION RATES

The dropout rate \( n \) years later from grade \( i \) is given by

\[
r_{ik}^{(n)} = \sum_{j=1}^{s} q_{ij}^{(n-1)} r_{jk}
\]

where \( i, j = 1, \ldots, s \) and \( k = 1, \ldots, r \). Note that \( q_{ij}^{(n-1)} \) is the probability that a student in grade \( i \) will be in grade \( j \), \((n-1)\) years later and \( r_{jk} \) is the probability that a student in grade \( j \) at time \((t-1)\) graduates with final education \( k \) at time \( t \). Actually, \( r_{ik}^{(n)} \) is the \((i,k)^{th}\) element of the product \( Q^{(n-1)} R \).

Hence, the cumulative dropout rate within \( x \) years, from grade \( i \) is given by

\[
r_{ik}^{(x)} = \sum_{n=1}^{x} r_{ik}^{(n)}
\]
where, \( i = 1, \ldots, s \) and \( k = 1, \ldots, r \)

Again, \( r_{ik}^{(x)} \) is the \((i,k)^{th}\) element of the matrix sum and product of

\[(I + Q + Q^2 + \ldots + Q^{x-1})R\]

### 3.4.1 COMPLETION RATES UNDER SINGLE ABSORBING STATE

Considering the single absorbing state model, the completion rate is the \((i,k)^{th}\) element of

\[(I + Q + Q^2 + \ldots + Q^{x-1})r\]

and can be represented in vector notation as:

\[r^{(n)} = (I + Q + Q^2 + \ldots + Q^{x-1})r\]  \hspace{1cm} (3.10)

Table 3.2 (overleaf) is a summary of the completion rates within \( x \) years using a single absorbing state model.
Table 3.2 Completion rates within x years using a Single absorbing state model

<table>
<thead>
<tr>
<th>Years (x)</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0800</td>
<td>0.0824</td>
<td>0.1224</td>
<td>0.8788</td>
</tr>
<tr>
<td>2</td>
<td>0.1557</td>
<td>0.1942</td>
<td>0.8781</td>
<td>0.9853</td>
</tr>
<tr>
<td>3</td>
<td>0.2577</td>
<td>0.8801</td>
<td>0.9849</td>
<td>0.9982</td>
</tr>
<tr>
<td>4</td>
<td>0.8731</td>
<td>0.9849</td>
<td>0.9982</td>
<td>0.9998</td>
</tr>
<tr>
<td>5</td>
<td>0.9831</td>
<td>0.9982</td>
<td>0.9998</td>
<td>1.0000</td>
</tr>
<tr>
<td>6</td>
<td>0.9979</td>
<td>0.9998</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>7</td>
<td>0.9997</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>8</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>9</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>10</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
</tbody>
</table>
Inferring from Table 3.2, the completion rate for students who were in grade I within the first two years, by the year 2006, was 15.57 percent of the students. The completion rate is expected to be 25.77 percent of the students by the year 2007. This is because the dropouts from the system comprised only of the students who had not attained the maximum qualification. Moreover, by the year 2008, it is expected that 87.31 percent of the students will drop out of the system since most of the students who were in grade I are expected to graduate from the system four years later. Eventually, students who repeated particular grades are expected to continue dropping out of the system such that by the year 2012, all grade I students will have dropped out of the system.

Considering students who were in grade II, 19.42 percent of the students had dropped out of the system by the year 2006. They were the students who had not attained maximum qualification but within three years, that is by 2007, 88.01 percent of the students are expected to drop out of the system because three years down the line, most of the grade II students will graduate from the system. It is expected that by the year 2011, all grade II students will have dropped out of the system.

Deducing further, by the year 2005, 12.24 percent of the students who were in grade III had dropped out of the system. Again, they were the students who had not attained maximum qualification. On the other hand, two years later, by the year 2006, 87.81 percent of the students had dropped out of the system as was expected, since most of the students who were
in grade III graduated two years later. Eventually, the repeaters will continue dropping out of the system until by the year 2010, all the grade III students will have dropped out of the system.

Finally, 87.88 percent of the students who were in grade IV had dropped out of the system one year later. They comprised of both of the students who graduated from the system and students who were discontinued from the system. Repeaters continued dropping out of the system such that within five years, by the year 2009, it is expected that all grade IV students will have dropped out of the system.

3.4.2 COMPLETION RATES UNDER DOUBLE ABSORBING STATES

Considering the double absorbing states model, students from grade IV were grouped into those who dropped out of the system before attaining the maximum qualification and those who actually graduated from the system. The completion rate is the \((i, k)^{th}\) element of 
\[(I + Q + Q^2 + \ldots + Q^{r-1})R_2\]

a summary of which is displayed in Table 3.3 overleaf. Below each grade, are two columns; one for the proportion of students who dropped out of the system and the other for the proportion of students who graduated from the system.
Table 3.3 Completion rates within x years using a Double absorbing states model.

<table>
<thead>
<tr>
<th>Years(x)</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>D</td>
<td>G</td>
<td>D</td>
<td>G</td>
</tr>
<tr>
<td>1</td>
<td>0.0800</td>
<td>0.0000</td>
<td>0.0824</td>
<td>0.0000</td>
</tr>
<tr>
<td>2</td>
<td>0.1557</td>
<td>0.0000</td>
<td>0.1942</td>
<td>0.0000</td>
</tr>
<tr>
<td>3</td>
<td>0.2577</td>
<td>0.0000</td>
<td>0.2214</td>
<td>0.6587</td>
</tr>
<tr>
<td>4</td>
<td>0.2847</td>
<td>0.5884</td>
<td>0.2251</td>
<td>0.7598</td>
</tr>
<tr>
<td>5</td>
<td>0.2887</td>
<td>0.6944</td>
<td>0.2256</td>
<td>0.7726</td>
</tr>
<tr>
<td>6</td>
<td>0.2892</td>
<td>0.7087</td>
<td>0.2257</td>
<td>0.7741</td>
</tr>
<tr>
<td>7</td>
<td>0.2893</td>
<td>0.7104</td>
<td>0.2257</td>
<td>0.7743</td>
</tr>
<tr>
<td>8</td>
<td>0.2893</td>
<td>0.7107</td>
<td>0.2257</td>
<td>0.7743</td>
</tr>
<tr>
<td>9</td>
<td>0.2893</td>
<td>0.7107</td>
<td>0.2257</td>
<td>0.7743</td>
</tr>
<tr>
<td>10</td>
<td>0.2893</td>
<td>0.7107</td>
<td>0.2257</td>
<td>0.7743</td>
</tr>
</tbody>
</table>

Key: D represents Dropouts and G represents Graduates.
Making inferences from Table 3.3, within two years, by the year 2006, 15.57 percent of the students who were in grade I had dropped out of the system before attaining the maximum qualification. It is expected that by the year 2011, a maximum of 28.93 percent of the same students will fall in this category. On the other hand, there were no graduates by the year 2006, neither will there be any by the year 2007, as is expected. However, in the year 2008, 58.84 percent of the students are expected to graduate from the system. This figure is expected to build up to a maximum 71.07 percent by the year 2012.

Considering students who were in grade II, it is expected that by the year 2010, a maximum of 22.57 percent will drop out of the system without attaining the maximum qualification. In contrast, within three years, by the year 2007, 65.87 percent of the students are expected to graduate from the system. Eventually, students who repeated various grades are expected to continue graduating from the system until by the year 2011, it is expected that a maximum of 77.43 percent of the students will have graduated from the system.

Further analysis of Table 3.3 indicates that by the year 2006, 72.72 percent of the students who were in grade III graduated from the system. On the contrary, it is expected that a maximum of 15.52 percent of the same students will drop out of the system without attaining the maximum qualification by the year 2008.
Finally, considering students who were in grade IV, 84.48 percent actually graduated from the system in the next year, 2005. More so, it is expected that by the year 2009, a maximum of 96.54 percent of the students will graduate from the system. However, it is also expected that a maximum 3.46 of the same students will drop out of the system without attaining the maximum qualification within four years, that is by the year 2008.

3.4.3 COMPLETION RATES UNDER MULTIPLE ABSORBING STATES

In this case, consideration is given to students who dropped out of the system after successful completion of particular grades, students who were discontinued from particular grades and students who graduated from the system. A student who is in a particular grade can only graduate with final education level of the same grade, if not of a higher grade. The completion rate is the \((i, k)\)th element of \((I + Q + Q^2 + \ldots + Q^{x-1})R_s\). Hence, summary results for students who were in various grades against the various possible final grade qualifications are tabulated separately as shown in the pages that follow in Tables 3.4(i), 3.4(ii), 3.4(iii) and 3.4(iv).
Table 3.4(i) Completion rates within $x$ years using a Multiple absorbing states model.

Student Grade: I

<table>
<thead>
<tr>
<th>Years(x)</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0533</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0267</td>
</tr>
<tr>
<td>2</td>
<td>0.0547</td>
<td>0.0421</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0589</td>
</tr>
<tr>
<td>3</td>
<td>0.0548</td>
<td>0.0437</td>
<td>0.0660</td>
<td>-0.0000</td>
<td>0.0932</td>
</tr>
<tr>
<td>4</td>
<td>0.0548</td>
<td>0.0437</td>
<td>0.0699</td>
<td>0.5884</td>
<td>0.1163</td>
</tr>
<tr>
<td>5</td>
<td>0.0548</td>
<td>0.0437</td>
<td>0.0701</td>
<td>0.6944</td>
<td>0.1201</td>
</tr>
<tr>
<td>6</td>
<td>0.0548</td>
<td>0.0437</td>
<td>0.0701</td>
<td>0.7087</td>
<td>0.1207</td>
</tr>
<tr>
<td>7</td>
<td>0.0548</td>
<td>0.0437</td>
<td>0.0701</td>
<td>0.7104</td>
<td>0.1207</td>
</tr>
<tr>
<td>8</td>
<td>0.0548</td>
<td>0.0437</td>
<td>0.0701</td>
<td>0.7107</td>
<td>0.1207</td>
</tr>
<tr>
<td>9</td>
<td>0.0548</td>
<td>0.0437</td>
<td>0.0701</td>
<td>0.7107</td>
<td>0.1207</td>
</tr>
<tr>
<td>10</td>
<td>0.0548</td>
<td>0.0437</td>
<td>0.0701</td>
<td>0.7107</td>
<td>0.1207</td>
</tr>
</tbody>
</table>
As per the information in Table 3.4(i), which refers to students who were in grade I only, within one year, 5.33 percent dropped out of the system with final education of grade I. It is expected that a maximum 5.48 percent of the students will drop out of the system with final education of grade I by the year 2007. Again, by the year 2006, 4.21 percent of the students dropped out of the system with final education of grade II and this figure is expected to build up to a maximum 4.37 percent within three years, that is by 2007. Three years down the line, by 2007, 6.6 percent of the students are expected to drop out with final education of grade III, the maximum percentage of this category of students supposedly being 7.01 by the year 2009. Again, within four years of being in the system, 58.84 percent of the students are expected to graduate by the year 2008 after which repeaters who are expected to continue graduating will make this figure attain a maximum value of 71.07 percent of the students, by the year 2012.

On the other hand, 2.67 percent of the students were discontinued from the system by the year 2005. This figure is expected to gradually build up to a maximum 12.07 percent of the students within six years, that is by 2010.

Summary results for students who were in grade II are available in Table 3.4(ii) as follows.
Table 3.4(ii) Completion rates within x years using a Multiple absorbing states model.

Student Grade: II

<table>
<thead>
<tr>
<th>Years(x)</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0000</td>
<td>0.0471</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0353</td>
</tr>
<tr>
<td>2</td>
<td>0.0000</td>
<td>0.0477</td>
<td>0.0739</td>
<td>0.0000</td>
<td>0.0727</td>
</tr>
<tr>
<td>3</td>
<td>0.0000</td>
<td>0.0477</td>
<td>0.0763</td>
<td>0.6587</td>
<td>0.0975</td>
</tr>
<tr>
<td>4</td>
<td>0.0000</td>
<td>0.0477</td>
<td>0.0764</td>
<td>0.7598</td>
<td>0.1011</td>
</tr>
<tr>
<td>5</td>
<td>0.0000</td>
<td>0.0477</td>
<td>0.0764</td>
<td>0.7726</td>
<td>0.1016</td>
</tr>
<tr>
<td>6</td>
<td>0.0000</td>
<td>0.0477</td>
<td>0.0764</td>
<td>0.7741</td>
<td>0.1016</td>
</tr>
<tr>
<td>7</td>
<td>0.0000</td>
<td>0.0477</td>
<td>0.0764</td>
<td>0.7743</td>
<td>0.1016</td>
</tr>
<tr>
<td>8</td>
<td>0.0000</td>
<td>0.0477</td>
<td>0.0764</td>
<td>0.7743</td>
<td>0.1016</td>
</tr>
<tr>
<td>9</td>
<td>0.0000</td>
<td>0.0477</td>
<td>0.0764</td>
<td>0.7743</td>
<td>0.1016</td>
</tr>
<tr>
<td>10</td>
<td>0.0000</td>
<td>0.0477</td>
<td>0.0764</td>
<td>0.7743</td>
<td>0.1016</td>
</tr>
</tbody>
</table>
In Table 3.4(ii), only students who were in grade II are considered. One year later, 4.71 percent of the students dropped out of the system with final education of grade II and by the year 2006, a maximum figure of 4.77 percent was attained for this category of students. Within two years, by the year 2006, 7.39 percent of the students dropped out of the system with final education of grade III and by the year 2008, it is expected that a maximum 7.64 percent of the students will fall in the same category of students. Since most of the students are expected to graduate three years later, the table indicates that by the year 2007, 65.87 percent of the students are expected to graduate from the system, with continual graduation of repeaters bringing this figure to a maximum 77.43 percent, by the year 2011.

On the other hand, within one year, 3.53 percent of the students were discontinued from the system. It is expected that a maximum 10.16 percent of the students will be discontinued from the system by the year 2009.

Summary results for students who were in grade III are available in Table 3.4(iii) as follows.
Table 3.4(iii) Completion rates within x years using a Multiple absorbing states model.

Student Grade: III

<table>
<thead>
<tr>
<th>Years (x)</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0816</td>
<td>0.0000</td>
<td>0.0408</td>
</tr>
<tr>
<td>2</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0833</td>
<td>0.7272</td>
<td>0.0677</td>
</tr>
<tr>
<td>3</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0833</td>
<td>0.8302</td>
<td>0.0714</td>
</tr>
<tr>
<td>4</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0833</td>
<td>0.8430</td>
<td>0.0719</td>
</tr>
<tr>
<td>5</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0833</td>
<td>0.8446</td>
<td>0.0719</td>
</tr>
<tr>
<td>6</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0833</td>
<td>0.8447</td>
<td>0.0719</td>
</tr>
<tr>
<td>7</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0833</td>
<td>0.8448</td>
<td>0.0719</td>
</tr>
<tr>
<td>8</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0833</td>
<td>0.8448</td>
<td>0.0719</td>
</tr>
<tr>
<td>9</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0833</td>
<td>0.8448</td>
<td>0.0719</td>
</tr>
<tr>
<td>10</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0833</td>
<td>0.8448</td>
<td>0.0719</td>
</tr>
</tbody>
</table>
Table 3.4(iii) displays the completion rates for students who were in grade III. Within one year, 8.16 percent of the students dropped out of the system with final education of grade III. A maximum value of 8.33 percent was attained for this category of students by the year 2006. Most of the students who were in grade III are expected to graduate two years later therefore by the year 2006, 72.72 percent of the students graduated from the system. It is expected that the students who had repeated various grades will continue graduating from the system until within seven years, by the year 2011, a maximum of 84.48 percent of the students are expected to graduate from the system.

However, 4.08 percent of the students were discontinued from the system by the year 2005. It is expected that a total of 7.19 percent of the students will be discontinued from the system by the year 2008.

Summary results for students who were in grade IV are available in Table 3.4(iv) as follows.
Table 3.4(iv) Completion rates within x years using a Multiple absorbing states model.

Student Grade: IV

<table>
<thead>
<tr>
<th>Years(x)</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.8484</td>
<td>0.0304</td>
</tr>
<tr>
<td>2</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.9512</td>
<td>0.0341</td>
</tr>
<tr>
<td>3</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.9637</td>
<td>0.0345</td>
</tr>
<tr>
<td>4</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.9652</td>
<td>0.0346</td>
</tr>
<tr>
<td>5</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.9654</td>
<td>0.0346</td>
</tr>
<tr>
<td>6</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.9654</td>
<td>0.0346</td>
</tr>
<tr>
<td>7</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.9654</td>
<td>0.0346</td>
</tr>
<tr>
<td>8</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.9654</td>
<td>0.0346</td>
</tr>
<tr>
<td>9</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.9654</td>
<td>0.0346</td>
</tr>
<tr>
<td>10</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.9654</td>
<td>0.0346</td>
</tr>
</tbody>
</table>
Information in Table 3.4(iv) refers to students who were in grade IV. One year later by the year 2005, 84.84 percent of the students graduated after attaining the maximum qualification. It is expected that by the year 2009, 96.54 percent of the students will graduate from the system after attaining the maximum qualification. Within one year, by the year 2005, 3.04 percent of the students were discontinued from the system and by the year 2008, it is expected that a maximum 3.46 percent of the students will be discontinued from the system.

3.5 ABSORBING RATES

If students were allowed to remain in the system indefinitely, then the absorbing rate is given by

\[
\begin{align*}
\lim_{n \to \infty} r_{ik}^{(n)} &= \sum_{n=1}^{\infty} r_{ik}^{(n)} \\
&= (I + Q + Q^2 + ...) R
\end{align*}
\]  

(3.11)  

(3.12)

Actually, \( r_{ik}^{(\infty)} \) is the \((i, k)^{th}\) element of the matrix product \((I - Q)^{-1} R\) since

\[(I + Q + Q^2 + ...) R = (I - Q)^{-1} R \]

(3.13)

The absorbing rate under single absorbing state is

\[
\begin{pmatrix}
1.0000 \\
1.0000 \\
1.0000 \\
1.0000
\end{pmatrix}
\]
The absorbing rate under double absorbing states is

\[
\begin{pmatrix}
0.2893 & 0.7107 \\
0.2257 & 0.7743 \\
0.1552 & 0.8448 \\
0.0346 & 0.9654
\end{pmatrix}
\]

The absorbing rate under multiple absorbing states is

\[
\begin{pmatrix}
0.0548 & 0.0437 & 0.0701 & 0.7107 & 0.1207 \\
0.0000 & 0.0477 & 0.0764 & 0.7743 & 0.1016 \\
0.0000 & 0.0000 & 0.0833 & 0.8448 & 0.0719 \\
0.0000 & 0.0000 & 0.0000 & 0.9654 & 0.0346
\end{pmatrix}
\]

Note:

In Table 3.2, we considered proportions of students who dropped out of the system with time. Most of the students who were in grade I dropped out within four years, most of the students who were in grade II dropped out within three years, most of the students who were in grade III dropped out within two years and most of the students who were in grade IV dropped out within one year. After these time periods, students who repeated continued dropping out. In the long run, all the students are expected to drop out of the system, thus justifying the eventual invariant entry of 1 with time. The absorbing rate gives us the eventual long run scenario thus under single absorbing state, all the entries are 1.

In Table 3.3, we considered proportions of students who dropped out of the system without
attaining the maximum qualification and proportions of those who graduated from the system, with time. Considering students who were in grade I, in the long run, 28.93 percent dropped out without attaining the maximum qualification whereas 71.07 percent graduated from the system. Considering students who were in grade II, in the long run, 22.57 percent dropped out without attaining maximum qualification whereas 77.43 percent graduated from the system. With respect to students who were in grade III, in the long run, 15.52 percent dropped out without attaining the maximum qualification whereas 84.48 percent graduated from the system. Again, considering students who were in grade IV, in the long run, 3.46 percent dropped out without attaining the maximum qualification whereas 96.54 percent graduated from the system. These proportions are the same as the entries in the absorbing rate under double absorbing states.

In Tables 3.4(i) upto 3.4(iv), we considered proportions of students who dropped out of the system without attaining the maximum qualification, proportions of students who graduated from the system and proportions of students who were discontinued from the system, with reference to time. Considering students who were in grade I, in the long run, 5.48 percent will drop out with final education grade I, 4.37 percent will drop out with final education grade II, 7.01 percent will drop out with final education grade III and 71.07 percent will drop out with final education grade IV. On the other hand, 12.07 percent will be discontinued. Considering students who were in grade II, in the long run, 4.77 percent will drop out with final education grade II, 7.64 percent will drop out with final education grade III,
77.43 percent will drop out with final education grade IV and 10.16 percent will be discontinued. Considering students who were in grade III, in the long run, 8.33 percent will drop out with final education grade III, 84.48 percent will drop out with final education grade IV and 7.19 will be discontinued. Considering students who were in grade IV, in the long run, 96.54 percent will drop out with final education grade IV and 3.46 percent will be discontinued.

These proportions are the same as the entries in the absorbing rate under multiple absorbing states. The absorbing rate is thus a powerful tool for determining the relevant proportions in the long run.
Chapter 4

CONCLUSION AND RECOMMENDATION

4.1 COMPARATIVE DISCUSSIONS

Single versus Double Absorbing States

Considering the single absorbing state case, students who left the system due to various reasons like discontinuation, health or lack of fees were grouped together with those who graduated from the system. It was therefore, not possible to ascertain whether a greater percentage of the students graduated from the system or dropped out of the system before attaining the maximum qualification. On the other hand, in the double absorbing states case, students who left the system due to various reasons were separated from those who graduated from the system. Hence, the proportions of students who graduated from the system and those who left the system due to various reasons were ascertained.
Single versus Multiple Absorbing States

In the single absorbing state case, all the students who were discontinued, who dropped out with particular final grades and those who graduated from the system were grouped together. It was impossible to ascertain the proportions of students who were grouped in each category. Considering the multiple absorbing states case, all the three categories of students were distinctly separated. Hence, the proportions of students who graduated from the system, who were discontinued from the system and who left the system due to other reasons were obtained.

Double versus Multiple Absorbing States

In the double absorbing states case, students were divided into those who graduated from the system and those who dropped out from the system, whereas in the multiple absorbing states case, not only were students divided into graduates and dropouts, but the dropouts were further separated into those who were discontinued and those who attained particular education grades. Therefore, if our major interest is on the graduates only, then the double absorbing states model is sufficient, otherwise, the multiple absorbing states model serves the necessary purpose.
4.2 MERITS AND DEMERITS

Computations using the three different absorbing states have given us information with varying degrees of precision. In the single absorbing state case, computations were simple but all cadres of students were lumped together, so it was not possible to determine whether most of the students graduated from the system, as would be the main objective of the education system or not. In contrast, computations for the double absorbing states case were more laborious but the graduates were distinctly separated from students who dropped out of the system. So, the proportions of graduates were determined. Considering the multiple absorbing states case, the computations were quite laborious but the proportions of students who graduated from the system, who dropped out due to discontinuation and who dropped out with particular final grades were determined.

4.3 CONCLUSION

In conclusion, the double absorbing states model is an improvement on the single absorbing state model since the proportions of students who graduated from the system and those who dropped out of the system were precisely determined. The multiple absorbing states model further gave us the proportions of students who not only graduated from the system, but also of those who dropped out due to discontinuation as well as those who dropped out with particular final grades. Therefore, it was an improvement on the double absorbing states model.
Finally, in an education system, students either graduate from the system or dropout of the system before attaining the maximum qualification. In this study, two categories of dropouts were considered; students who dropped out due to discontinuation and students who dropped out with specific final qualifications. The latter category dropped out from particular grades despite successful completion of the same, due to various reasons. Further research can be carried out on this category of students to ascertain the various reasons for dropping out with a view to formulating ways of curbing such dropouts, where possible.
REFERENCES


