"A COMPARATIVE STUDY OF THE EFFECTIVENESS OF PROGRAMMED INSTRUCTION, PROGRAMMED INSTRUCTION COMBINED WITH TEACHER-INSTRUCTION IN SMALL GROUPS AND CONVENTIONAL CLASSROOM TEACHING"

FRANCIS OBUNGA-OKAMBI

THIS THESIS HAS BEEN ACCEPTED FOR
THE DEGREE OF M.Ed. 1978
AND A COPY MAY BE PLACED IN THE
UNIVERSITY LIBRARY

"A thesis submitted to the University of Nairobi in partial fulfilment for the Degree of Masters of Education.

1978

DECLARATION

"This thesis is my original work and has not been submitted for a degree in any other University."

FRANCIS OBUNGA-OKAMBI

"This thesis has been submitted for examination with my approval as University supervisor.

DR. G. S. ESHIWANI,

SENIOR LECTURER,

MATHEMATICS EDUCATION.

ACKNOWLEDGEMENTS

In the life of this study several debts have been accumulated. First and foremost, the investigator is indebted to Dr. G. S. Eshiwani of the University of Nairobi who has rendered him invaluable service throughout the duration of the study. Without his untiring and effective supervision, this study would not have been a success. The investigator is also happy to acknowledge the help rendered to him by Dr. Andrew Young also of the University of Nairobi, who helped in the interpretation of data. Dr. Peackock of Kamwenja Teachers College also deserves mention. He read the manuscript and offered very constructive criticisms and suggestions.

Acknowledgements are also due to my colleagues in the M.Ed. Mathematics Education class, Mr. Oyor Otieno, Mr. Okiya Toka and Mrs. Hellen Kithiinji, with whom I had useful discussions on the subject of my research.

Mention should also be made of Mr. J. D. Owuor-Onyango of the Forest Headquarters. He kindly lent me a powerful dest calculator without which data analysis would have been an uphill task, since by the time of data analysis there was hardly any money left for computer use.

I would also like to thank the kind ladies who typed the manuscript. Those were Mrs. Mary Oteto, Mrs. Margaret Abondo and Miss Selpha Wakuze, all of the Survey of Kenya, Nairobi.
Special mention should bemade of Mrs. Peresa Gaya who typed the learning materials i.e. the program and the lesson and the measuring instruments.

This list of acknowledgements would be incomplete without mentioning the research assistants who worked very hard to make the investigation at least successful. Thanks go to Mr. Ochieng, Adipo, Mr. Oyier and Mr. Atieno of Lake Primary School, Mr. Angwenyi, Mr. Olando and Mrs.Obunga of Manyatta Primary School; and to Mr. Ogoti Mr. Sijenyi and Mr. Odero of Kisumu Union School.

TABLE OF CONTENTS

Page

LIST OF TABLES

ABSTRACT

Since the introduction of free primary education in Kenya in 1974, the primary schools have witnessed an influx of pupils. Total enrolments rose from 74% in 1973 to 107% in 1974. In 1975 enrolments were 2881155 and in 1976 enrolments stood at 2894617. (Social Perspectives: Vol. 2 No. 6 Nov. 1977 and Vol. 2 No. 3 August 1977). This number was matched by 87,076 teachers out of whom 63% were qualified. This gives a minimum of 52 pupils per teacher. Now that the primary school fees are gradually being phased out, one can assume that the rate of drop out in upper classes will be minimized. This means that the number of pupils will continue to rise. If this trend continues then the problem of finding suitably qualified teachers will be aggravated. It is hypothesized in this study that this problem can be alleviated by finding out a suitable teaching method which would help alleviate the shortage of teachers in vital subjects, such as mathematics. One such method is programmed learning.

Comparisons between traditional teaching and programmed learning in Kenya have been made in the secondary schools. Among those known to the present researcher who have done work in this field are Eshiwani (11,1974) and Parkar (12,1974). The present study deviates from this trend and looks at the suitability of programmed learning in the primary schools in Kenya.

The major purpose of the study was to find out whether three modes of instruction - programmed instruction (PI), programmed instruction supplemented by teacher instruction in small groups (IPI), and the conventional classroom instruction (CI) - would produce different levels of attainment and retention. In addition, the study sought to investigate possible relationships between attainment and predictor variables (maths ability, reading ability, attitude towards mathematics and toward the method by which the subject is presented), and between attainment and retention.

The subjects for the study comprised 353 pupils from three schools randomly drawn from all low cost primary schools with treble standard six classes in Kisumu Town. In each school, the three streams were randomly assigned to the three treatments.

After the administration of pre-tests the groups were subjected to the three different treatments followed by a post-test. Eight weeks later, a retention test was administered.

In most respects, a two-way analysis of covariance showed no significant treatment effects on post-test achievement on knowledge, comprehension and application subtasks. Significant treatment differences, however, existed on the analysis subtask. No significant treatment effects existed in retention on all levels of cognition.

It was found that no wide differences existed among the three treatment groups with regard to the variables examined for the prediction of post-test achievement. Mathematical ability and reading ability were found to be significant predictors of achievement for the IPI group. Attitude towards mathematics was a good predictor of achievement for the subjects in the IPI and CI groups. No significant correlations were found between attitude towards the program and post-test.

The post-test scores were a better predictor of retention than pre-test scores for total scores and for each cognitive level. The initial pre-test scores significantly predicted retention for the PI group while post-test scores predicted retention for IPI and CI groups.

The findings of this study have revealed that the IPI and CI methods are powerful teaching aids for higher cognitive abilities. Programmed learning, however, scores well over traditional learning in saving student time. The time element coupled with its effectiveness as a learning tool argue well for the establishment of programmed workshops in a country such as Kenya where the population of primary school children cannot match the output of trained teachers.

CHAPTER ONE

1.0 INTRODUCTION

Recent reforms that have taken place in both the primary and secondary schools in Kenya have usually come from outside the schools. Changes have normally been introduced without finding out what pupils and teachers feel about them. For example, the introduction of the Kenya Primary Mathematics books was done without having carried out proper research to determine their suitability for our primary school children, and without having given teachers sufficient training to enable them to handle the new mathematics courses with relative ease.

Kenya is not alone in experiencing such problems like shortage of properly trained teachers, overcrowdedness of the schools and inadequate facilities and equipment in most of the primary schools. The number of pupils in the primary schools increases every year outnumbering the output of trained teachers thereby giving rise to a larger pupil-teacher ratio. This brings about the problem of maintaining the quality of teaching.

In the light of these problems that we face today in our schools it is necessary to search for new approaches to teaching that would help solve such problems. One such approach is programmed learning. Programmed learning has been introduced in many countries, notably, the western countries to

(1) help alleviate such educational problems like shortage of suitably qualified teachers and the overcrowdedness of the schools and,

(2) to help the children meet the educational objectives of their countries.

Although several researches carried out in the west have attested to the general effectiveness of the program as a method of instruction and thus have pointed out the obvious potentiality of programmed materials in the schools, such a potentiality has not been demonstrated in Kenyan primary schools. Apart from Eshiwani's study (1974) which compared three modes of instruction - programmed instruction (PI), integrated programmed instruction (IPI) and the conventional classroom approach (CCA) in Kenyan secondary schools, and Parker's study (1974) which sought to find out whether programmed workcards can be of significant value as a method of instruction as compared to the formal classroom method in the secondary schools, no study known to the present investigator has been conducted in Kenyan primary schools to establish which method (s) could be more suitable. This fact has encouraged the investigator of this study to investigate which of the three methods: programmed instruction (PI), programmed instruction supplemented by teacher-supervision in small groups (IPI) and the conventional mode of instruction (CI) can be a more effective learning instrument for our primary school children.

The origins of programmed learning go back to the work of Sidney Pressy (in the 1920's), Professor B.F. Skinner and Dr. Norman Crowder (in the mid fifties). Their work on the experiments with teaching machines came about as a result of the dissatisfactions with traditional method of learning, shortage of skilled teachers and by the competition of Russian technological advances.

The inauguration of a new era of programmed learning came about with Professor Skinner's suggestion in 1954 that the experimental analysis of behaviour could be applied in the construction of a teaching machine which would present a carefully sequenced set of material to a student and reinforce his responses to direct behavioural capabilities (1,1973). Skinner's work on instrumental conditioning saw the development of linear programs. His theory centres around rewards, which is a development and an expansion of Dr. Thorndike's work. Thomas (1,1963) sees rewards as a means of ensuring that a particular response is likely to be repeated. He has cited the following characteristics of programmed learning that render it different from the conventional method.

(1) Programmed learning is an individual learning process in which the student accepts a far wider measure of responsibility for his own learning and proceeds at his own rate.

1
Thomas, C. A., et al: Programmed Learning in perspective, London Publicity Services, 1963.

Ibid.

(2) Programmed learning requires an active response from the student and provides immediate confirmation of results.

(3) Programmed learning ensures that the student is more often successful, and is therefore strongly motivated.

(4) The subject matter is programmed in such a way that the student's learning (behaviour) is shaped in a particular way.

Most psychologists believe that more learning and retention take place when the learner makes responses and have them immediately confirmed. Programmed learning has one important characteristic in that it permits the learner to progress at his own pace. This removes the boredom that is sometimes experienced by slow learners when they have to work at the same pace with faster learners in a conventionally taught classroom.

1.1 THE STATEMENT OF THE PROBLEM

The problem of finding a suitable teaching method that would enable children in our primary schools to meet the educational objectives set by the Kenya Institute of Education (a body in charge of curriculum development in Kenya) continues to occupy the minds of many educators in Kenya today. For educators to recommend a method of instruction to be adopted by schools, its effectiveness needs to be ascertained through research. The present study, therefore seeks to find out whether programmed learning can be a more effective learning instrument for our primary school children than the conventional method. Specifically, the problem was to investigate whether Kenyan primary school children learn and retain better when they receive individualised programmed instruction, (PI), when they receive programmed instruction supplemented by teacher-supervision in small groups, hereinafter called the Integrated Programmed Instruction (IPI), or when they learn through the conventional mode of instruction (CI).

1.2 PURPOSE OF THE STUDY

The major purposes of the study were:

(i) To investigate whether there would be any achievement differences in a unit on probability among the programmed instruction (PI), the programmed instruction combined with teacher supervised small groups (IPI), and the conventional instruction groups at each of the following cognitive levels.

(a) knowledge of specific facts,
(b) comprehension,
(c) application, and
(d) analysis.

(ii) To investigate whether performance in mathematics is related to sex.

The subordinate objectives of the study were as follows:

1. To find out possible differences in reading ability among the PI, IPI and the CI groups.
2. To investigate possible differences in attitude towards mathematics among the three treatment groups.
3. To investigate any differences in attitude towards the program between the PI and the IPI groups.
4. To investigate possible differences in retention of probability concepts among the three treatment groups.

5. To investigate which of the following variables would be valid predictors of achievement in probability:- reading ability, attitude towards mathematics, attitude towards the program, and mathematical reasoning ability.

6. To investigate which of the following would be a valid predictor of retention:- pretest achievement or posttest achievement.

7. To investigate possible differences in mathematical ability.

1.3 THE STATEMENT OF THE HYPOTHESES

The following hypotheses, stated in the null form were tested:-

1. HO_1 : There is no achievement differences in test scores as measured by probability post test.

   (a) among the three treatment groups (PI, IPI and CI);

   (b) between the two sex groups at each of the following cognitive levels:

      (i) knowledge of specific facts,

      (ii) comprehension,

      (iii) application and

      (iv) analysis.

2. HO_2: There is no differences in attitude towards mathematics.
   (i) among the PI, IPI and the CI groups,
   (ii) between the two sex groups in the study.

3. HO_3: There is no difference in attitude towards the program between
   (i) the PI and the IPI groups,
   (ii) the two sex groups in the study.

4. HO_4: There is no differences in reading ability as determined by Schonnell's Reading Ability Test A.
   (i) among the three treatment groups,
   (ii) between the two sex groups in the study.

5. HO_5: There is no difference in mathematical scores
   (a) among the three instructional groups,
   (b) between the two sex groups in the study.

6. HO_6: There is no differences in retention
   (i) among the three treatment groups,
   (ii) between male and female pupils.

7. HO_7: There is no correlation between pupils'
   (a) reading ability,
   (b) attitudes towards the program,

(c) attitudes towards mathematics,

(d) mathematical reasoning ability,

and their achievement in probability post-test scores.

8. HO_8 : There is no correlation between pupils'

(i) Pre-test achievement scores,

(ii) Post-test achievement scores

and their retention scores.

9. HO_9 : There is no achievement difference in test scores as measured by probability pre-test

(a) among the three treatment groups

(b) between the two sex groups at each of the following cognitive levels:

(i) knowledge

(ii) comprehension

(iii) application and

(iv) analysis.

1.4 LIMITATIONS OF THE STUDY

Some of the limitations of the study are:-

1. It was not possible to conduct a countrywide study due to the nature of the problem and to the time available for the study. Hence the subjects for this study were limited to standard six pupils in three treble-streamed

schools in Kisumu town.

2. The pupils who did not sit for all the tests were eliminated from final analysis.
3. It was not possible to employ the services of teachers of the same grade in all the sample schools. Nor was it possible to have a single teacher for all the three instructional groups in each school.
4. It was not possible to control for teacher enthusiasm and competence towards any particular method.
5. It was difficult to control for mental or emotional state of each child.
6. It was originally proposed to measure time taken by each child to complete a program. However, some instructors did not comply with this instruction in the first two days of the investigation. It was therefore decided to leave out the time variable.
7. It was not ascertained whether the groups in each sample school were intellectually comparable.
8. Pupils had more experience with the traditional method than the other two at the start of the research study. This was beyond control.

9. It was difficult to ascertain the degree to which a teacher was able to motivate his pupils, especially when they came to section three of the program and the lesson which needed abstract thinking.

1.5. ASSUMPTIONS

1. Since the task learned involved some experimental activity by all pupils in each of the three groups, it was assumed that there would be no differences in the motivation of pupils of all treatments. In view of the fact that all children were involved in some experimental activity, it was further assumed that a teacher's age, grade or experience would not significantly affect the performance of his pupils.

2. The programmed materials in probability covered sufficient material to be learned by the standard six pupils.

3. The tests used were valid and reliable.

4. The subjects in the study were assumed to be at the same level of understanding before the investigation began since none of them had prior exposure to probability.

5. The sample schools were assumed to be comparable in teacher distribution and material supply.

1.6 DEFINITION OF TERMS USED

PROGRAMMED INSTRUCTION

This is a technique whereby students study individually from sequentially arranged materials. Programmed instruction is usually characterised by self-instructional, self-paced sequences of short questions and answers which are presented in teaching machines or as programmed textbooks (2,1973).

2. Integrated Programmed Instruction (IPI)

This is a technique whereby teacher instruction supplements the programme.

3. Conventional Instruction (CI)

This is a method of instruction characterised by teacher lectures, demonstrations and homework.

---

2 Roebuck, M; Frames from Ibadan: Programmed Learning in a West Nigerian Context. Bulletin of Programmed Learning, Research Unit Dept. of Education, August 1973.

4. Discovery Learning:- This refers to those teaching situations in which the student achieves the instructional objective with limited or no help from the teacher. If the learner completes the task with little or no guidance, he is said to have learned by discovery (3,1973). Sometimes learning by discovery occurs when children are led step by step/by the teacher appropriate questioning and activities using concrete materials to discover concepts for themselves (4,1966).

---

[3] Kersh, B. Y.: Learning By Discovery: What is learned? Arithmetic Teacher. Vol. II, 1974.

[4] Glaser, R.: "Variables in Discovery Learning" in L. S. Shulman and E. R. Keislar (Eds). Learning by Discovery: A Critical Appraisal. Rand MacNally and Co., Chicago, 1966.

5. Reinforcement:-

According to Peel (5,1963) reinforcement means the strengthening of any "on-going" behaviour by a consequent event contingent upon the behaviour. He lists what he calls the components of the "simplest reinforcing state" as:-

1. a person carrying on observable behaviour i.e. the learner;

2. the learner's condition of need or want.

3. a strengthening event, i.e. reward.

---

5 Peel, E. A.: Some Psychological Principles underlying Programmed Learning. Educational Research. Vol. 5, No. 3, 1963.

CHAPTER TWO

REVIEW OF RELATED LITERATURE

The aim of this chapter is to review some of the research findings in areas of programmed learning that are related to the present study. Among the numerous investigations that have been made relating to programmed learning only ten most related studies have been cited by the investigator of the present study. Four of these are investigations carried out in Africa while the remaining six have been carried out in the west.

A good number of researches done in the field of programmed learning have compared programmed learning with conventional learning. Some of these researches have reported the superiority of programmed instruction over the conventional mode of instruction; others have found the conventional instruction to be superior to the programmed instruction while others have reported no differences between the two instructional modes.

Daniel and Murdoch (6,1968) compared learning from a programmed text with learning from a conventional text covering the same material. The major purpose of the study was to determine which of the two methods of teaching would lead to a better performance on a content examination.

577 students enrolled in an introductory psychology at Chapel Hill comprised the subjects for the study. Two of these students were dropped for being suspected of cheating in an examination. This left 575 students for data analysis. The subjects were assigned to the conventional and programmed sections, with each section having between 18 and 26 students. 12 instructors, 5 female and 7 male graduate students took part in the experiment. Each instructor taught two sections - one programmed instruction section and one conventional instruction section. Two texts were used: one by Holland and Skinner and the other by Skinner. The programmed and conventional texts were written by the same author.

---

6 Daniel, W. J. and Murdoch, P: Effectiveness of learning from a Programmed Text compared with a Conventional Text covering the same material. Journal of Educational Psychology ..., Vol. 59, No.6, 1968 pp. 425 - 431.

At the end of the course the subjects were administered a 100 - item test on operant psychology. The items were taken from a large pool of items that had been contributed by the teaching assistants. The items were categorized into objective and essay type items. The objective type items were subdivided into six categories, viz: multiple - choice format (MC); knowledge of specific content (A); responding to new concepts and principles (B); responding to new materials (C); free - recall format (FR); and application to everyday life (D). Multivariate F - ratios computed for the six objective item types revealed no evidence of a sex effect ($P > .300$). The between - instructor - within - sex effect was statistically significant ($P < .010$). Murdoch (note that William J. Daniel died before data analyses were completed or the report written. He, however, initiated the research) attributes this significance to the fact that each instructor taught both the programmed and conventional sections. This, in his view, increased the sensitivity of the experiment. The textbook effect was also found to be statistically significant ($P < .002$), an indication that as a set, the objective measures differed according to the kind of textbook studied. The results of the multivariate F-tests for the equality of the mean vectors for the essay items were similar to those for the six objective item types. There was no statistically significant sex effect, ($P > .422$ in the first analysis and $P > .250$ in the second analysis). However, the between - instructor - within - sex effect was statistically significant ($P < .001$).

The hypothesis that learning from the programmed text is greater than learning from the conventional text covering the same material was supported by the univariate F-tests. In this study, the programmed group on the average obtained a 10% higher score on each multiple - choice item type and a 7% higher score on each essay type item.

It should be noted that the greater learning shown by the programmed instruction group may be attributed to the method of instruction. The contents covered by the two books were assumed to be comparable in style, difficulty, and content, etc, since they were written by the same author for the same purpose. Further, the teaching assistants were very familiar with both books. After their ratings after responding to a questionnaire given to them, the instructors were of the opinion that the texts were comparable and the examination administered to the research subjects did not favour any one text to the exclusion of the other. Since the texts were found to be comparable and the examination was not biased in favour of any one text, the author, therefore, concluded that the experiment provided a fair comparison of the practical usefulness of the two texts by Skinner, and that the programmed text was more effective for teaching operant psychology than the conventional text.

It is not clear from the report to what extent the programmed and the conventional texts covered the same instructional ground, though the report indicates that the two texts were comparable in style, difficulty and content since they were both written by the same author, i.e. Skinner, for the same purpose. The report does not make it clear whether the level of motivation was the same for all instructional groups.

In the 1961 - 62 school year, Banghart,

et al (7,1963) carried out an experimental study of programmed versus traditional method of instruction. The purpose of the study was to compare programmed materials with non-programmed materials in elementary school mathematics. Specifically, the study aimed at finding out possible differences in total arithmetic scores, problem solving and comprehension between the two instructional groups (programmed and conventional) and between the two sex groups.

The study was conducted in the Norfolk, Virginia, public school system. The subjects for the study consisted of 195 control and experimental fourth-grade children representing an acceptable cross-section of fourth graders in intelligence, achievement, and socio-economic status. The subjects were in a relatively superior school in terms of facilities and personnel.

The experimental class learned through the program while the control class learned through

---

7 Banghart, F. W. et al: An experimental study of Programmed versus Traditional Elementary School Mathematics. Arithmetic Teacher, Vol. No. 4, 1963, pp. 199 - 207.

the regular materials. The programmed materials consisted of specially programmed text books which included the usual sequence of fourth-grade arithmetic, skills and content, language of sets, number lines, and simple equations involving one unknown. The experimental materials were constructed by the author after making an extensive survey of the content of leading elementary school arithmetic text books and the arithmetic curriculum organization in several large school systems to assure that all the major skills normally taught at the fourth-grade level are properly treated in the experiment.

The control materials for the control subjects consisted of the standard text and supplemental materials used as a regular part of the normal instruction in fourth-grade arithmetic in the Norfolk, Virginia, public school system.

The length of the normal class period for both the experimental and control classes was 30 - 40 minutes a day. The author met occasionally with the experimental teachers to discuss their observations in the classroom.

A t-test was computed to compare the performance of the experimental and control subjects. The differences between the experimental and the control groups for total scores and comprehension scores (total, boys, and girls) are significant.

The differences between the two groups for problem solving (total), problem solving (boys) and problem solving (girls) are not statistically significant ($P > .05$). The author attributes the non-significance between the groups for problem solving (boys) to the large amount of variability between the groups. No reason can be advanced for the difference between means for problem solving (girls). The author finds it interesting to note that the mean score for experimental problem solving (girls) is consistent with the other experimental means.

The general observation by the experimental teachers was that the children in the experimental groups showed high enthusiasm for the programmed materials. The experimental teachers felt that learning through individual self-paced programs was a very effective means for teaching elementary school arithmetic. The teachers also noted certain disadvantages associated with individualized learning. They noticed that when working with programmed materials, pupils quickly covered a wide range of content which threatened to increase as pupils advanced through their programmed materials. They also noted that the pupils did not work at constant rates, rather the rates at which some pupils worked was erratic. This made it necessary for the teachers to keep a daily record of the progress and speed of each child. Infact the lack of constancy in pupil work constitutes one of the advantages of individualized programmed instruction for the slow learners who would otherwise be bored if they had to learn at the same rate with the fast learners, and for fast learners who would be relieved from the frustrations of being detained by the slow learners. Teachers noticed children forming voluntary groups to discuss the program.

The author observed that if programmed materials are well designed and well-tested and if competent and interested teachers are employed to supervise programmed learning, then one can expect a significant achievement in favour of programmed materials over conventional materials. Another useful observation made by the author is that programs should be integrated with the teacher. He contends that programmed materials are most effective when used to supplement the classroom teacher.

Jamieson (8,1969) investigated the relative effectiveness of two methods of instruction - programmed and guided discovery methods and the effects of the same upon people of different age groups. (This research is related to the present study in-sofar as the comparison between the programmed and the guided discovery methods is concerned: otherwise it is not, since the present study is not investigating the effects of different instructional modes upon different age groups).

---

8 Jamieson, G. H.: Learning by Programmed and Guided Discovery Methods at Different Age Levels. Programmed Learning and Educational Technology, Vol. 6, 1969, pp. 26 - 30.

The subjects for this study consisted of 80 females categorized on the basis of their ages as follows:-

1. 20 pupils drawn from a state primary school mean age 11, range 10 years, 1 month to 11 years 8 months. (This was the youngest group).

2. 20 students from a college of occupational therapy, mean age 21, range 20 years 8 months to 22 years 11 months.

3. 20 students from a college of education for mature students, mean age 40 years 6 months, range 34-47 years.

4. 20 members of the Liverpool Medical Research Council's voluntary panel, mean age 57 years 5 months, range 51 - 66 years.

These subjects were randomly assigned, in equal numbers, within their age groups to the two different modes of learning i.e. programmed and guided discovery. The programmed group went through a 154 - frame linear program on binary number, presented on small manually operated teaching machines. The subjects worked independently, but in the presence of others in their group. No time limit was specified, but a record of individual time to completion was covertly kept.

In the guided discovery method group each subject was provided with a binary light indicator and binary/denary conversion scale specially designed for the experiment. The subjects used these learning aids to discover the base of the binary system, and to make numerical conversion between the binary and denary systems. The content taught was the same for the two instructional groups, i.e. the programmed and the guided discovery groups. The speed of the lesson depended on the subjects' response and the feedback needed to clarify any points at issue.

Before the beginning of each course, Vernon's graded-arithmetic mathematics test was administered to all the subjects in the study. At the end of each course all the subjects were administered a written test in binary number. The test sampled all the work in number which had been taught.

A five-point scale attitude questionnaire was administered to the subjects to sample the subjects' responses to the learning aids and to elicit a comparison between the programmed and the guided discovery methods.

The data obtained were subjected to the Mann-Whitney "U" tests and to rank-difference (Rho) correlations. The results of the "U" - tests showed that:-

Subjects in the youngest and oldest groups, i.e. groups 1 and 4 who learned by the discovery method performed significantly better in binary criterion test scores ($P < 0.02$ in both cases) than the other subjects in the same age groups who learned by the programmed group (2). The programmed method subjects in the same age groups 2 and 4 were significantly quicker than the subjects in the same age groups learning by the guided discovery method ($P < 0.002$ in both cases).

Rank difference correlation coefficients were computed for arithmetic scores and binary criterion scores; for arithmetic test scores and learning time (binary); for age and learning time (binary); and for age and binary criterion scores.

The results showed a significant correlation between arithmetic ability and scores on the binary criterion test ($P<0.01$ for the programmed group and a significant positive relationship between age and time ($P<0.05$ for the programmed group). No significant relationships were found between age and binary criterion scores for both the programmed and the guided discovery groups.

The foregoing discussion of the results show that the youngest and oldest groups using the guided discovery method performed better in binary criterion scores than members of the same groups learning by the other method. The author advances three reasons for the better performance shown by these two groups under the discovery method:-

1. the role of the teacher. The author argues that the supportive role of the teacher may have benefitted the arithmetically less able subjects.

2. The principles of the number system could have been more readily grasped under the guided discovery method than under the programmed instruction method.

3. Subjects in these groups had not developed an independent learning style to assist them cope with the teaching machines.

The first reason advanced by the author does not seem to be convincing since the report does not indicate anywhere that members of groups one and four learning by the discovery method were arithmetically less able subjects. The reason for this surprise superior performance is therefore to be found elsewhere. It could be that the guided discovery method groups were more motivated than the programmed groups of the same age groups.

An interesting finding was the stronger association between arithmetic ability scores and post programmed learning scores on the criterion, than between arithmetical ability scores and post guided discovery learning. This suggests that transfer was greater for those learning by program.

Studies on programmed learning have not only involved comparisons between programmed instruction and conventional instruction, but have also gone further to include a new element, that is, the teacher and the program. In his comparative study of programmed and traditional elementary mathematics, Banghart (7,1963) observed that

---

&
7 Ibid.

"allowing programmed materials to become the sole source of instruction to the exclusion of the teacher does not make most effective use of the programmed material nor of the teacher. Programmed materials are most effective when used to supplement the classroom teacher."

Meadowcroft (9,1965) conducted a comparative study of the textbook method and the programmed combine with teacher instruction.

The subjects consisted of 294 students of both sexes in the seventh grade of Wilkinsburg Junior High School in the year 1962 - 63. In the previous year, i.e. in the sixth grade the subjects learned by the conventional method. The experiment was extended into the 8th grade when the subjects learned again by the conventional method. During this time the sample size had reduced from original 294 in the 7th grade to 249 in the 8th grade.

---

9 Meadowcroft, B.A.: The effects on conventionally taught eighth-grade math following seventh-grade math. The Arithmetic Teacher, Vol. 12, No. 8, 1965, pp.614 - 616.

The purpose of the study was twofold:

1. to investigate which of the two methods-programmed or conventional - was more effective;

2. to find out whether the utilization of programmed materials in the seventh grade had any adverse effects on eighth-grade achievement when subjects again learned by the traditional textbook method.

The experimental group, constituting one-half of the students in the seventh grade learned arithmetic by means of the program supplemented by teacher instruction while the other half, the control class learned by means of teacher-textbook method. The experimental group used the programmed materials 70% of the class time and received teacher instruction 30% of the class time. The experimental group learned at their own pace and were individually tested. The pupils in the control group were instructed by means of assignments, lectures and recitutions.

The t-tests computed revealed no significant difference between the means of the two instructional groups (total). But when the t-tests were computed for the accelerated, above average, average and slow subgroups, the difference between the means of the experimental, and the control groups in the accelerated section was found to be significant (t = 3.33, ($P < .01$), with the mean for the accelerated section of the control group (10.3) being higher than the mean for the accelerated section of the experimental group. All the means for the other sections of the two instructional groups were not significantly different. But on a special achievement test average section of the experimental group had significantly higher mean than the average of the control group.

The results of this study indicate that programmed instruction was not superior to the textbook method as far as arithmetic achievement was concerned. However, programmed Learning was found to be more efficient in saving student time than the conventional method of learning.

When separate sections of the experimental and control groups were considered, the accelerated section of the control group was found to have a significantly higher mean than its counterpart in the experimental section.

The author attributes this difference to the use of materials but not to the programmed materials themselves. This indicates that the accelerated section of the experimental group did not make proper use of the programmed materials.

The author puts forward a strong case for using programmed materials when he considered total advance by the students in terms of achievement; the experimental group advanced 1.3 years while the control group advanced only 1.1 years.

An interesting point to note in this investigation is that educators can insert programmed materials in one grade without ~~necessarily~~ following it up in the next grade without anticipating dire results. This follows from the fact that use of the programmed materials in the seventh grade did not very much affect achievement in the 8th grade when the experimental group no longer learned by the programme.

It is not clear from this investigation whether the test used to measure retention in the eighth grade was the same test administered to the subjects in the seventh grade. It is further not clear whether the two groups in the study were taught by the same teacher.

Another study on integrated programmed learning was by Holmberg (10,1966). He compared the conventional classroom method with a combination of programmed instruction and teacher supervised small group instruction in the seventh grade.

The subjects were 19 boys and 17 girls from two classes at the School of Education in Malmo, Sweden. Each class had 18 pupils, subdivided on the basis of their mathematical and intellectual abilities into a high, a middle and a low group, with each group comprising six pupils.

One class received programmed instruction supplemented by teacher-supervision in small groups, ranging from two to six pupils per group. Time for group instructions ranged from 10 to 20 minutes. The control class received conventional instruction where the teacher prepared new items on the blackboard and the pupils given exercises to be done

---

10
Holmberg, I.: A combination of programmed instruction and teacher - supervised small group instruction compared with conventional classroom method. Didakometry (Malmo, Swe den: School of Education), 1966 No. 10.

in class and at home. Teacher made tests were given every month to test for achievement. The two instructional groups were taught by one teacher.

Before the start of the experiment subjects were tested with the Cattell Culture Fair Scale 2A to find out whether they were intellectually comparable. The results showed no difference between the programmed instruction group (mean of 26.83 with a standard deviation of 6.81) and the conventional instruction group (mean 28.44, standard deviation, 4.58 , $P > 0.20$). To control for the arithmetic ability of the pupils they were administered a 10 - item arithmetic test. The results were not statistically significant (PI class, $X = 2.72$, $S = 0.96$ and CI class: $X = 2.83$, $S = 0.92$, $P > .20$).

The pupils were divided into high, middle and low groups following their scores on intelligence tests and arithmetic tests.

Changes in arithmetic, reading ability, classroom behaviour, play and passivity, disturbing interactions, working habits, preference for arithmetic, were analysed. An analysis of variance revealed no significant differences in arithmetic achievement between the two instructional groups ($P > 0.05$). But variation between high, middle and low groups was found to be significant, with the high group getting the best results and the low group the poorest ($P < 0.001$). The results of reading ability, investigated once a semester with Diagnostic reading tests designed by Dr. Esse L'ovgren at the School of Education in Stockholm, Sweden, revealed some changes in reading technique. Pupils decreased in reading speed but gained in comprehension. The experimental class showed significant superiority in reading instructions and tended to be more independent in their laboratory work than the control class. There was a significant difference between the two instructional groups in the variable "disturbing interaction", the variable being frequently noted in the control class ($P < 0.001$). Classroom behaviour was found to be rather similar in both classes in spite of the different teaching methods being employed.

Though the study shows no evidence of the superiority of either method with regards to arithmetic achievement, the integrated programmed instruction gains over the conventional instruction in reading ability test. The integrated programmed learning group is shown to be more independent in the laboratory work than the conventional group.

The pupils' attitude towards the program was not very promising. The pupils found programmed instruction more tiring than the conventional instruction. But their general opinion was that they learned more from programmed materials than from conventional materials.

The two factors mentioned above, i.e. that the pupils of the programmed instruction group showed more independence in their work than the pupils of the conventional instruction group and that the pupils of the programmed instruction expressed the opinion that they learned more from programmed materials argue well for programmed learning to be introduced in a country such as ours where suitably qualified teachers are in short supply.

Comparative studies involving programmed learning and conventional instruction have also been carried out in Africa. Among the researchers who have done work in this field in Africa are

Eshiwani (11,1974), Parkar (12,1974), Okunrotifa (13,1968) and Roebuck (14,1968).

Eshiwani (11,1974) carried out a study involving three methods of teaching: programmed instruction (PI), integrated programmed instruction (IPI), and the conventional classroom approach (CCA). The major purpose of the study was to find out whether boys' superiority in mathematics as reported by researches from the west is true with Kenyan children.

The subordinate purpose of the study was to investigate whether attitude towards mathematics, mathematical reasoning, vocabulary of mathematical terms, vocabulary of scientific terms and computation are valid predictors of achievement in mathematics for Kenya boys and girls.

354 form two students from two boys' and two girls' high schools in Nairobi constituted the sample for the study. Three classes within each of the four selected schools were randomly assigned to each of the following treatments:

---

11 Eshiwani, G. S. : Sex Differences in the Learning of Mathematics among Kenyan High School Students. Mathematics Education Research Report, No. 3 - August 1974.

programmed instruction, conventional classroom approach and integrated programmed instruction.

At the beginning of the experiment the following pre-tests were administered to all the subjects in the study:-

1. Attitude towards mathematics scale.
2. Five Dots - measuring mathematical ability.
3. Fractions - measuring ability to compute.
4. Arithmetic reasoning - measuring mathematical reasoning ability.
5. Probability pre-test.
6. Comprehension of mathematical vocabulary test.
7. Comprehension of science vocabulary.

After the administration of the pre-tests the subjects underwent an instructional course in probability. The programmed instructional group learned through the program, edited by the investigator, the conventional classroom approach group learned through the teacher-talk method while the integrated programmed instruction group learned through the program supplemented by the teacher.

The first and the second achievement tests, designed by the investigator were administered to all the subjects in the study half-way through the instruction and at the end of the session which lasted two weeks. A retention test was administered to all the subjects in the study six weeks after the instruction. The students were not informed of the impending retention test. They had reverted to their normal class routine after the post-test achievement.

The results of the pre-tests show that boys in the PI and CCA groups scored higher than girls in attitude toward mathematics, five dots, computation (Fractions), Arithmetic Reasoning, Comprehension of Mathematical and Scientific terms, while girls performed better on probability pre-test. The IPI girls performed better than boys in all the pre-tests except on comprehension of science terms.

The results of the achievement tests reveal the following:

1. In the first achievement test, boys in the PI and IPI groups had a higher mean than girls, while girls in the CCA group had a higher mean than boys of the same group.

2. In the second achievement test, girls in the PI and IPI groups performed better than boys of the same group while boys in the CCA group had a slightly better mean than girls in the same group.

A t-test was used to test for possible differences between boys and girls in the pre-tests, post-test and retention test. Girls in the PI and IPI groups performed better than boys in the same groups on the retention test. Boys in the CCA group performed significantly better than girls of the same group on the retention test.

When total scores for boys and girls were analysed, it was found that

1. Girls performed significantly better than boys ($t = 2.89$; $P < 0.05$; on the arithmetic reasoning test.
2. Girls performed better than boys on the probability pre-test and on the second achievement test ($t = 4.09$; and $t = 3.13$ respectively, $p < 0.05$).

3. There were no significant differences between boys and girls on attitude toward mathematics, five dots, fractions (computation), comprehension of mathematics, and science terms, first achievement test and retention test.

The results of the stepwise regression analysis computed to determine the relationship between the pre-test variables and first achievement test were as follows:-

1. For boys, five dots and arithmetic reasoning were significant predictors of probability achievement ($p < 0.05$).

2. Comprehension of mathematical terms, arithmetic reasoning and computational ability (Fractions) were valid predictors of achievement at the 0.05 level of significance.

The study reveals that sex differences in mathematics do exist among Kenyan high school children. This sex differences, however, cannot be attributed to the students' attitudes towards mathematics. If positive attitude towards the subject were to go with higher achievement in a mathematics test, then boys in this study would have scored much higher than the girls on the achievement and on the retention test. If the hypothesis that girls are better readers than boys is anything to go by, then one would expect girls to do better from programs than boys. But this was not the case in this study. Boys gained more from the programs while girls gained more from the human teacher. The investigator did not consider reading ability as one of the variables, hence it is difficult to say whether boys were better readers or girls.

It is not indicated in the study whether the content taught to the CCA group covered the same information ground as the program.

The results of the pre-tests show that girls were superior to boys in pre-test probability. This is a clear indication that subjects were not initially comparable. For this reason, the groups should have been statistically equated by the use of analysis of covariance.

Another related study on programmed learning in Africa was carried out by Parkar (12,1974).

12

Parkar, K. D.: The Impact of the Programmed workcards on the Quality of Teaching Mathematics in the secondary schools of Kenya. M.Ed. Thesis,1974.

The purpose of the study was to find out whether there would be any difference in achievement between students taught by a textbook - lecture method and those taught by programmed workcards. Subsidiary purposes of the study was to examine the students' attitudes towards mathematics and how these attitudes change during the course of learning and to examine their attitudes towards the program as a method of instruction.

219 Form 1 students from a boys' and a girls' school in Nairobi comprised the subjects for this stud y. The subjects were distributed among six classes with three classes forming the control group and the other three, the experimental group. The classes used were intact Form 1 classes (i.e. there was no re-organisation of the classes for the purposes of the experiment).

In order to control for certain extraneous factors like motivation, method of instruction, teaching aids, length of class period, time of day, size of class, assignments, etc. a single teacher was assigned to teach at least one experimental class and one control class. Both the experimental and the control groups learned the same material with similar vocabulary.

symbolism and problems. The investigator contended that all errors would not be removed by the controls exercised. He, however, expressed the hope that such errors would be eliminated by the process of randomization.

At the beginning of the study all the subjects were administered Dutton's attitude scale, revised by the investigator to suit the needs of his study. The purpose of administering the attitude questionnaire was to learn how Kenyan high school students felt about mathematics. Three attitude scales were used for this purpose:

1. Attitude towards mathematics as a process.
2. Attitude about difficulty of learning mathematics.
3. Attitude towards the place of mathematics in society.

Reliability data for these scales were not calculated. The same questionnaire was administered to all the subjects in the study at the end of the year to see if any changes towards mathematics had been made during the course of the study.

The results of the pre-test attitude towards mathematics revealed no significant differences between the experimental and control groups (total) and between boys and girls in the study at the 0.05 level of significance.

The results of the attitudes scale administered at the end of the study were as follows:

1. There were no significant differences between the experimental group and the control group in their attitude toward mathematics as a process and in their attitude towards the place of mathematics in society.

2. No significant difference between the two sexes was shown for the three attitude scales.

3. There was a significant difference towards the difficulty of learning mathematics between the control boys and control girls ($t = 2.25$; $P < 0.05$) with the control boys having a higher mean attitude score than the control girls.

On attitude changes, the control boys showed slight improvements in their attitude towards mathematics during the second term compared to first term. There were no significant differences between pre-test and post-test attitude towards mathematics.

Boys tended to favour the program more than the girls. Boys had a significantly higher mean of 40.75 than the girls' mean of 35 ($t = 2.19$; $P < 0.05$).

Achievement tests covering what had been taught were given at the end of every term. At the end of the third term, a 50 - multiple choice item covering the material learned for the whole year was administered.

The results indicate that the experimental girls performed significantly better than the experimental boys in the first, and second achievement tests ($t = 4.21$ and $t = 5.04$; respectively $P < 0.05$) while boys did better in the third achievement test ($t = 4.25$; $P < 0.05$). The control girls performed better in the first achievement test ($t = 5.55$; $P < 0.05$) while the boys of the same instructional group did better in the second and third achievement tests ($t = 1.15$ and $t = 7.29$ respectively; $P < 0.05$).

The results show that girls performed significantly better in the first achievement test while boys' performance was significant in the third achievement test. This would indicate that boys had a higher retentive power than girls if the subjects were not informed at the end of the year that the final test would include work done in the first and second terms.

Parkar's study has provided a useful information on the general effectiveness of the programme and its retentive effect when used with Kenya's high school children.

Okunrotifa (13,1968) compared programmed learning with the conventional method of instruction.

The purpose of the study was threefold:

1. To find whether there would be any difference in attitude to programmed materials between those who learned by the program and those who learned by conventional method of instruction.

---

[13] Okunrotifa, P. O.: Attitude of Nigeria Secondary School children to programmed instruction in geography. Educational Research Journal, Vol. 17, No. 2, Feb. 1975, pp. 110 - 114.

2. To find whether the two instructional groups exhibited any attitude differences towards geography as a subject.

3. To find which of the two instructional methods under investigation would be superior.

The subjects for the study consisted of 200 second formers - 100 boys and 100 girls, randomly drawn from four schools representing urban boys', urban girls' rural boys' and rural girls' schools in the North Central State of Nigeria. The mean age of the subjects was 14 years. None of the subjects had prior exposure to programmed materials.

Before the study commenced the subjects were administered a pre-test geography achievement, verbal aptitude and quantitative aptitude tests.

After these tests, all the subjects in the study went through a linear program in civics. The instruction went on for three sessions. (3 days) the end of the third session the subjects were divided into experimental and control groups on the basis of their pre-test geography achievement, verbal aptitude and quantitative aptitude scores.

The subjects were then administered a

pre-test attitude towards geography and a pre-test attitude towards the program (Likert - type).

After the administration of the pre-tests the subjects underwent an instructional course in map reading in geography. The experimental group learned through the programs while the control group learned from the conventional texts. The experimental group was presented with five geography programmed texts and the conventional group with five conventional texts in geography. The conventional texts covered the same span of information as the programs. An American map reading program, with a version adapted to Nigeria geographical conditions was used in the study.

The investigator hoped to control for possible methodological errors by making the students aware that they were involved in an experiment concerned with their learning in geography and by warning them against any leakage as it was thought that leakage would destroy the experimental test of the independent variables.

The subjects were not aware that they were divided into two different treatment groups. The investigator hoped that the "Rosenthal Effect" would cancel out since both the instructors and students were not aware of his hypotheses to be tested. He further hoped that by employing instructors who were not normal teachers in the experimental schools and by testing all the subjects in the study, he would level out the "Hawthorne Effect."

After the instruction, a post-test achievement and a post-test attitude towards geography and the program were administered to all the subjects in the study.

The results of the pre-tests showed no significant differences in verbal aptitude, quantitative aptitude, pre-test geography achievement and age. There were significant differences in pre-test geography attitude and pre-test program attitude, the control group showing more positive attitude in both cases. The significant differences in the pre-tests indicated a need for statistically equating the two groups.

A three-way analysis of covariance was computed to compare geography achievement, attitude towards mathematics and attitude toward the program. Teaching methods, sex and school environment were used as the main effects while pre-test scores were used as covariate and post-test scores as criterion. An F-ratio of 23.56 showed that there were significant differences in teaching methods ($P < 0.01$), the programmed group performing better than the control group. No significant differences were found among other variances. The groups' learning times scores were compared by a 2x2x2 factorial analysis of variance. There were no significant main effects for methods, sex and school environment. Neither were there any significant interactions involving sex, methods and school environment variables. On the basis of these results, it was concluded that programmed instruction was more efficient than the conventional instruction in contributing to pupils' achievement.

A 2x2x2 analysis of covariance computed to compare the subjects' attitude towards map reading revealed significant differences in method variances ($F = 16.19$; $P < 0.01$). The programmed instruction group showed a more significant favourable attitude towards map reading than the conventional text group. The other variances were not significant. The author attributed the more favourable attitude shown by the experimental group to two factors:

1. The programs were well validated;
2. Programmed instruction usually emphasizes immediate confirmation of results, active response, constant evaluation, appropriate practice and graduated sequence.

The author contends that these two factors might have made map reading easier and more satisfying and therefore more liked by the programmed group.

A 2x2x2 analysis of covariance was also computed to compare the subjects' attitude towards the program. Methods ($F = 43.12$; $P < .01$) and sex ($F = 3.99$; $P < .05$) variances were found to be significant. The programmed group had a more favourable attitude towards the program than the conventional text group. Boys were found to be more inclined to programmed instruction than girls. Initially, the control group exhibited a more positive attitude towards the program than the experimental group. After instruction, the trend changed, the experimental group now showing a more positive attitude than the control group. The author suggests the reason for this to be the length of time both groups were in contact with the programs. The control group were in contact with the programs for only three sessions at the beginning of the study while the experimental group were in contact with the program throughout the learning session

A correlation of r = 0.142 between post-test geography achievement and post-test attitude towards the programs in the experimental group suggests that a positive attitude towards the program does not necessarily result in high achievement.

The results of this study have confirmed some of the earlier researches reviewed in this thesis that learning is greater from programmed materials than from the conventional materials covering the same span of information as the program. The finding that boys tend to like the program more than the girls indicate that boys like to show more independence in their work than the girls who like the supportive role of the teacher. This fact is confirmed by Eshiwani's study (11, 1974) which reported that boys learned more through the programs while girls gained more from the human teacher.

Another comparative study involving programmed and conventional instruction in Africa was carried out by Roebuck (14,1968).

The investigation was done with fourth-year students in a secondary grammar school in West Nigeria. The programmed and the conventional groups were stated to be of equal ability in physics. The groups were arranged on the basis of their end-of-year examination taken in December, 1967.

A pre-test (Kuder-Richardson reliability: 0.58) was administered to both groups on Feb. 2, 1968. The 17 - item objective test

---

11 Op. cit.

14 Roebuck, M.: A. definite conclusion in a comparison between conventional and programmed instruction. Programmed Learning and Educational Technology, Vol. 7, 1970.

contained pre-requisite and pre-knowledge items.

Between Feb. 2 and Feb.19, 1968 both groups underwent an instructional course on mass, weight and density during normal lessons. Care was taken not to disrupt the syllabus arrangements for the term. The programmed group worked through a programmed text supplemented by the standard school practical experiments supervised by the teacher while the conventional group followed the normal school syallabus but carried out the same standard experiments as the programmed instruction group.

At the end of the instruction, on Feb. 19, 1968, a 19 - item post-test (Kuder-Richardson reliability: 0.85) was administered to both groups in the study. No retention test was administered due to what the writer calls "political and administrative difficulties." The tests administered were based on the content of the program and on those supplied by the writer of the program.

A t-test was used to compare the means for the two groups in the pre-test and post-test achievement.

The results show that the non-programmed group performed significantly better than the programmed group in the pre-test achievement ($t = 2.964$; $P < .01$). In the post-test, the programmed group showed superior performance to the conventional group ($t = 2.635$; $P < .05$).

On the basis of the initial differences shown in the pre-test analysis, the author decided to equate the groups by using an analysis of covariance. This analysis revealed that there was a significant difference between the regression coefficients ($F = 31.67$; $P < .001$). According to the author, this significance shows that the pre-test/post-test relationship for the two groups was not of the same form. The two groups may have learned different aspects of the subject matter.

The author concludes that though the programmed group showed superiority over the conventional group, the results show that the two methods emphasized different concepts and hence the observed differences in attainment were a function of the testing procedures used.

Studies on programmed learning have not been limited to comparisons between programmed instruction and conventional instruction. Some studies have examined the effect of the program on retention. Dick (15,1965) compared the immediate delayed performance of students who worked in pairs with the performance of students who worked alone.

The major purpose of the study was to determine if the paired use of programmed materials, which involved verbal interaction between two students resulted in superior retention when compared to a group of students who worked alone.

The subjects were students who enrolled in mathematics at the Pennsylvania State University in the Winter Term, 1962. The subjects were randomly assigned to two groups, one group consisting of students who worked in pairs and the other group, consisting of students who worked alone. Both groups were tested for verbal and quantitative ability with the School and College Ability Test (SCAT). The paired group had the program placed between two students. The students

---

15 Dick, W.: Individual use of programmed instruction. The Mathematics Teacher, Vol. 58, No. 7, 1965, pp. 649 - 654.

discussed the material in the program with which they had difficulty.

Daily tests, mid-term tests and final examination were administered. An analysis of covariance using total SCAT scores as the control variable showed no significant difference between the two groups on their total daily test points, midterm and final examinations, or the test of transfer. No significant difference was shown in the subjects' attitude toward the course in general or toward the program.

One year later, during the latter weeks of the Winter Term, 1963, 80% of the students were retested. An analysis of covariance was used to test for the significance of the difference in retention of paired and individual groups. The post-test scores were used as the control variable while the retention test scores as the criterion. The results show a significant difference ($F = 3.77$; $0.05 < P < 0.07$) in favour of the paired group.

When ability measure (SCAT) and post-test scores were correlated with the retention test to determine which variable was a better predictor of retention, it was found that the post-test scores was a significantly better predictor of retention. The correlation between the final examination and the retention test was r = 0.89 for the paired group. The correlation between total SCAT and the retention test scores was r = 0.43 for the same group. The difference between the correlations was significant (t = 4.95; P∠ 0.01). For the individual treatment group the correlations were r = 0.77 between the final examination and retention test, and r = 0.50, between SCAT scores and retention test scores. The difference between these correlations was also significant (t = 1.96; P∠ 0.06).

The results of this study led the investigator to conclude that the benefits of paired learning are found in the retention of the material and not in the immediate performance of the material. The correlations between the final examination and retention test for both groups show that the best predictor of retention is the post-test achievement and not a general ability measure.

Though the researches reviewed here have conflicting results regarding the superiority of the program over the conventional method, the general view conveyed is that the program teaches better than the conventional method and that the program encourages the students to be more independent in their work.

The superiority of the program over the conventional mode of instruction was reported by Murdoch (6, 1968), Banghart, et al (7, 1963), Parker (12, 1974), Okunrotifa (13, 1968) and Roebuck (14, 1968).

Eshiwani (11, 1974) found the program to be more effective with boys but girls find themselves more at home when they learn through the human teacher. Although Holmberg (10, 1966) found no significant difference in arithmetic achievement, his experimental group, i.e., the group that learned through the programs showed more independence in their work. Pupils' general opinion was that they learned more from materials than from conventional materials. Dick's study (15, 1965) reports no difference when the performance of children who learned by the program in pairs was compared with the performance of children who learned individually also by the program when measured by a post-test achievement.

But when a delayed post-test achievement was given, the paired group retained more material than the group working individually. This shows that the program is more useful for the students learning in pairs than for those learning individually.

Meadowcroft's findings (9, 1965) are somewhat contrary to the findings of researches reviewed here. He found no significant differences between the programmed instruction group and the conventional instructional group when total scores were considered. But when scores for different ability groups were considered, significant differences were found between the accelerated sections, with the accelerated section of the control group performing better than the same section in the experimental group. Meadowcroft put forward one useful point that should be considered carefully by future researchers, namely that the success of any program remains with the design of the materials and equipment and the utilisation of the instructional devices. Banghart's (7, 1963) suggestion reinforces this view. Banghart suggests that for any program to be useful, it must be used in combination with the teacher. The teachers involved with the program must show competence in handling the program and they must also show interest in the program.

Jamieson's study (8, 1969) considered such variables like mathematical ability and age. He found that it is mathematical ability and not age that affect achievement. The study further revealed that older people are just as enthusiastic to new materials as the younger ones.

The findings from the researches reviewed in this chapter have guided the investigator of the present study to plan his work. Specifically, the present study has been planned and designed along the lines of some of the researches so far reviewed.

CHAPTER THREE

PROCEDURE AND DESIGN OF THE STUDY

3.0 INTRODUCTION

This chapter reports the procedure and design of the present study. The chapter begins by describing the construction of the learning materials and measuring instruments, then moves on to describe the conduct of the pilot study. Finally, the chapter dwels at length on the main study.

3.1.0 CONSTRUCTION AND TRY-OUT OF LEARNING MATERIALS

The Program

In January 1977, the investigator of this study set out to write a program which would constitute the learning materials for the subjects in his main study. The writing and the try-out of the programmed materials went on simultaneously.

The investigator secured permission from the Headmaster of Nairobi Primary School to test his programmed materials with the pupils of standard six. The purpose of the try-out was to enable the writer to reconstruct the frames so as to enable the pupils to go through the program with minimum difficulty.

The subjects for the try-out consisted of all the pupils in the three streams in class six in Nairobi Primary School. The headmaster felt that making use of only one class would put that class at an advantage over the other classes when they come to do the topic now under investigation (probability). The investigator was granted three lessons a week of 35 minutes each in each class.

The topic chosen for investigation was to be learned during the third term according to the syllabus arrangements. The material was presented to the children by means of an overhead projector which was borrowed from the Resource Section of the Faculty of Education, Nairobi University.

Each pupil read each frame projected on one of the walls of the classroom. Pupils wrote down their responses to the questions in each frame on pieces of paper supplied by the investigator. At the end of each lesson the investigator collected the papers for marking. The frames so far presented to the pupils were then revised on the basis of the pupils' responses. If a frame was correctly responded to by 80% or more of all the pupils, such a frame was thought to be good and was therefore not revised. The revised frames were again presented to the pupils and again revised on the basis of their responses to the frames. The process continued for a period of one month. By the end of the fourth week, all the 175 frames had been revised at least three times. The revised frames were then typed and stapled into small booklets, with answers given at the back of each page. Three booklets comprised the whole program. The first booklet was on ideas about chance events. Here are two examples.

Example 00

3. Some things are more likely to happen than others.

(a) Which is more likely, that one of the pupils in this class will be absent or that the mathematics teacher in this class will be absent? ______________

(b) Which is more likely, that you will have ugali for breakfast or that you will have ugali for lunch? ________

(Ugali is a stiff porridge made of maize,millet or cassava flour)

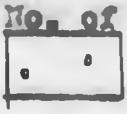
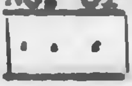
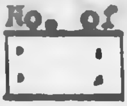
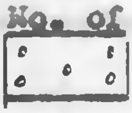
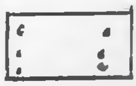
Example 01

16. You are to play a game with your friend. The game is "Toss a die once and see who wins" (The die is cubic)
In this game you win if 1 shows up. The other player wins if 3 shows up. In order to decide whether the game is fair or unfair we first list all the possible outcomes _____, _____, _____, ______, and ______, _______.

The second booklet was about Experiments in Probability. An example follows.

Example 02

43. Mama tossed a die 20 times and recorded her outcomes in the following table:

	No. of 	No. of 	No. of 	No. of 	No. of 	
Tally	//	////	///	//	///	~~////~~ /
Total	2	4	3	2	3	6

You now toss a die 60 times and make a record of the number of dots on the top face. Record your results in a table such as the one shown above.

44. Use the results of frame 43 to answer the following questions:-

(a) How many 1's did you get? ___________

(b) How many 3's did you get? ___________

(c) Did you get each outcome about the same number of times? ___________

The third booklet was on "Finding Probabilities." Here is an example:-

Example 03

108. When tossing one die, we have six outcomes. We write the 6 under the bar of a fraction:

$$\frac{\ }{6}$$

Getting the outcome 3 is just as likely as any of the others, so we expect it about $\frac{1}{6}$ of the time. We say, "the probability of 3 is ________ we write p(3) = ____________

The investigator found it expedient to divide the program into the three sections because he believes that pupils should first involve themselves with experiments before they come to find probabilities of events. It was hoped that the first and the second sections of the program would enable the pupils to distinguish between the expected and experimental outcomes. After the pupils have familiarized themselves with the two sections, they would then be able to find the probability of events by employing some abstract reasoning.

The program was constructed by the investigator from the following books:-

1. Kenya Primary Mathematics Book 6 (Pupil's and Teacher's Books).
2. Kenya Primary Mathematics Book 7 (Pupil's and Teacher's Books).
4. Secondary School Mathematics (Special Edition, Student's Text and Teacher's Commentary) by the School Mathematics Study Group.

During the try-out of the program, the pupils were not exposed to the answers to the frames, so there was no immediate confirmation of results. The answers to the frames were read out to the pupils by the investigator after he had marked their scripts.

At the end of the try-out, a 20-item achievement test and a 5 - item attitude questionnaire towards the program were administered to all the subjects in the try-out study. Item analyses for these tests were not carried out)

The purpose of administering the achievement test was to find out whether the program, now believed to be in its final form could teach. The mean performance of the pupils was 8.5, with a standard deviation of 3.3. marks. (The test was marked out of 20).

The attitude towards the program questionnaire was administered to find out whether the pupils liked the program or not. It was clear from the pupils lively behaviour in the class that they were highly enthusiastic to programmed materials. One item enquired whether pupils would like to use programmed materials everyday. 70% of the pupils said that they would like to use programmed materials everyday and 20% said they would not while 10% were not sure. 75% of the pupils positively responded to an item which required them to state whether they preferred programmed instruction to their usual mode of instruction. The positive response indicated that they preferred the program to the mode of instruction to which they had been accustomed. In general, the pupils of the try-out study expressed favourable attitude to programmed materials.

Since the program was tried-out with pupils of a high cost school in Nairobi (the high cost school pupils are generally assumed to be superior in academic performance to the pupils from low-cost schools), and since the actual study was to be conducted in Kisumu with pupils from low-cost schools, the investigator thought it fitting to try-out the "finished" program with a sample of pupils from a low-cost primary school in Kisumu Town. One single-streamed school was randomly selected from all single-streamed schools in Kisumu Town. In this school, all the 50 standard six pupils were subjected to the try-out study. The study was conducted in mid February, 1977. The linear programs were handed to the pupils section by section. They went through the program individually and at their own pace. When a pupil completed a booklet he collected another one from the investigator. The investigator allowed the pupils to take the programs home so that the study could be completed in the shortest time possible. This was done under the assumption that the pupils would continue to read the programs at home.

Answers to each frame were provided at the back of each page. At the end of each section, there was a self-test with answers following the test. The pupils were required to do the test, then confirm their responses from the answers. The major difference in learning between the try-out subjects in Kisumu and the try-out subjects in Nairobi is that the try-out subjects in Kisumu were provided with immediate confirmation of results. This is to say that the program, now considered to be in its final form had answers to the frames at the back of each page.

At the end of the third week of instruction, the subjects were administered a test on probability similar to the one administered to the pupils in the try-out study in Nairobi Primary School. The mean and the standard deviation for the scores for 50 pupils were computed. The mean was found to be 7.6 with a standard deviation of 3.5 marks. The results here were found to be somehow comparable to the results of the subjects in Nairobi (mean 8.5, standard deviation 3.3). The investigator was then of the opinion that the program could teach, and did not therefore require any further revision. He then set out to conduct a pilot study.

3.1.1 THE LESSON

The lessons used by the control group were constructed from the programs. This was to ensure that the content covered in the lessons was comparable in style and difficulty to the content covered in the program. Like the program, the lesson was also divided into three sections: section one contained various "Ideas about chance," section two was about "Experiments in Probability" and the third section dealt with "Finding Probabilities."

3.2.0 CONSTRUCTION OF THE TESTS

Before the pilot study commenced, pre-test achievement and post-test achievement to be used in the main study were constructed. The tests were constructed from the programmed and the lesson notes. This was to ensure that the items in the test did not favour any one instructional method.

3.2.1 PRE-TEST ACHIEVEMENT

This consisted of 15 multiple-choice items designed to measure the pupils' initial knowledge of probability. The test sampled the information to be covered in the program and in the lesson. These questions were a revised version of the questions originally administered to the try-out subjects. A final revision of items was done in the pilot study before administration in the main study. Some of the items were of a general nature, designed to measure a child's ability to reason intuitively.

The items were categorized according to Bloom's Specifications (16, 1971). Bloom's categories of cognitive levels into which the items were divided include knowledge of specific facts, comprehension, application and analysis. The subjects in the study were not aware of

---

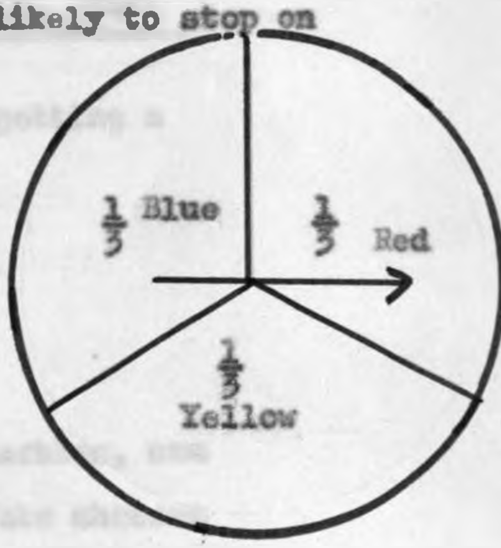
16 Bloom, B.S., et al: Handbook of formative and summative evaluation of student learning. McGraw - Hill Book Co., 1971, pp. 271 - 273.

this specification.

After careful screening of the pre-test, only three items remained to test children's knowledge of specific facts. As it has been stated earlier in this section, knowledge items were rather general, mainly designed to test a child's intuitive reasoning. Examples of knowledge, comprehension, application and analysis items follow below:-

<u>Example 003</u> (Knowledge)

Think of spinning the pointer of the spinner on the right. The pointer is likely to stop on red



(a) ½ of the time.
(b) 1/3 of the time
(c) 0 of the time
(d) all of the time.

<u>Example 004</u> (Comprehension)

Ataka spins the pointer of a spinner 100 times and gets 25 red, 25 blue, and 50 yellow.

Which of the following statements is true?

(a) The dial of the spinner is ½ yellow.
(b) The dial of the spinner is $\frac{1}{3}$ green.

(c) The dial of the spinner is $\frac{1}{4}$ blue.

(d) The dial of the spinner is all red.

<u>Example 005</u> (Application)

The table below shows all the possible outcomes when two coins are tossed.

		Second Coin	
		Head	Tail
First	Head	Head, Head	Head, Tail
Coin	Tail	Tail, Head	Tail, Tail

What is the probability of getting a head and a tail?

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{8}$ (d) 1.

<u>Example 006</u> (Analysis)

Tabu's bag contains three marbles, one red, one white and one blue. If Tabu chooses one marble without looking, what is the probability that the marble Tabu chooses is red?

(a) $\frac{2}{3}$ (b) 1 (c) $\frac{1}{3}$ (d) 0.

3.2.2. POST-TEST ACHIEVEMENT

This consisted of 20 objective type items covering the program and the lesson. The items in this test were also categorized according to Bloom's specification (16, 1971). Examples of such items are given below.

Example 007 (Knowledge)

1) If an event is certain to occur, its probability is:

(a) 0 (b) ½ (c) 1 (d) Greater than 1.

Example 008 (Comprehension)

5) The probability of throwing exactly four heads and one tail in a toss of five coins is $\frac{5}{32}$. What is the probability of not throwing four heads and one tail?

(a) $\frac{5}{32}$ (b) 1 (c) $\frac{27}{32}$ (d) 0.

---

16 Op. cit.

Example 009 (Application)

11) In a gambling game where one coin is to be tossed a player wins if he scores two heads and one tail. How many times must he toss the coin?

(a) 8 times (b) once (c) three times

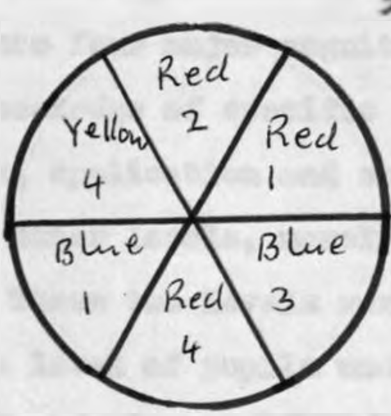
(d) twice.

Example 010 (Analysis)

18) Two dice are tossed together. What is the probability of getting a sum of 6 or a sum of 7 ?

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{11}{36}$ (d) $\frac{5}{36}$

20)



The spinner above is divided into six equal regions. Use it to find the probability of either Red or 1.

(a) $\frac{2}{3}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) ½

Like the pre-test items, the post-test items received careful scrutiny, being revised twice before the investigation.

The items were arranged in the following order: items one to four were knowledge items, items 5 to 8 were comprehension items; number 9 to 15 were application items and items number 16 to 20 comprised analysis items. The pupils were not aware of such an arrangement.

3.2.3. DESCRIPTION OF THE COGNITIVE LEVELS

As has been mentioned in sections 3.2.1 and 3.2.2, the pre-test and post-test were categorized according to Bloom's Taxononomy of Educational objectives into four major cognitive levels. These are: knowledge of specific facts, comprehension, application and analysis. Bloom includes two other levels, namely synthesis and evaluation. These two levels were considered to be beyond the level of pupils under investigation. The study, therefore, was limited to the first four levels of cognition. A description of each of these four levels follows below.

Knowledge of Specific Facts

Bloom defines knowledge as recall of specifics and universals, the recall of methods and processes, or the recall of a pattern, structure or setting. In this level the pupil was required to recall the material learned earlier. For example, if he is able to recall that if an event is certain to occur, then its probability is 1, then he has displayed his knowledge of that fact. This fact will be remembered from the fact that $0 \leq P \leq 1$.

Comprehension

This represents the lowest level of understanding. Here, an individual is supposed to know what is being communicated and should be able to make use of the material or idea being communicated without necessarily relating it to other material or seeing its fullest implication. An example of an item included in this category has been given in example 008. Here the child was told that the probability of obtaining exactly four heads and one tail in a toss of five coins was $\frac{5}{32}$. He was then asked to state the probability of not obtaining four heads and one tail? This item required the child to know that the probability of a sure thing is 1 and then use this knowledge to find the complement of obtaining four heads and one tail.

Application

In this level the child is required to apply the material already learned and comprehended to new situations. Application is defined by Bloom as the use of abstractions in particular and concrete situations. The abstractions may be for general ideas, rules of procedures, generalized methods, technical principles and theories. These should be remembered and applied in new situations.

Analysis

In this level a pupil is required to break a given problem into its constituent parts so that he clearly understands the relations between ideas expressed. One example of this level is given in Example 010, item no. 18. In this example the child is told that two dice are tossed. He is then asked to find the probability of getting either a sum of 6 or a sum of 7. The child is required to break down the information as follows:

(a) He draws up a table.

(b) He lists the outcomes when one dice is tossed, on the top of the table.

(c) He lists the outcomes when the second dice is tossed, on the left of the table.

(d) He then writes down the elements in an ordered pair in each cell of the table when the two dice are tossed together.

(e) Finally, he adds up each of the ordered pairs to see which ordered pair gives him a sum of 6 or a sum of 7. Such a table is given below.

No. on Second die

		1	2	3	4	5	6
	1	1,1	1,2	1,3	1,4	1,5	1,6
No. of	2	2,1	2,2	2,3	2,4	2,5	2,6
First die	3	3,1	3,2	3,3	3,4	3,5	3,6
	4	4,1	4,2	4,3	4,4	4,5	4,6
	5	5,1	5,2	5,3	5,4	5,5	5,6
	6	6,1	6,2	6,3	6,4	6,5	6,6

From the table the child will see that five cases will give him a sum of 6 and 6 pairs will give him a sum of 7. The total is 11. Hence the probability of obtaining a sum of 6 or a sum of 7 when two dice are tossed is $\frac{11}{36}$. Thus before the child finally settles to an answer of $\frac{11}{36}$, he would have reasoned through six stages. The sixth stage involves counting the number of cells to get the sample space.

3.2.A READING ABILITY

One of the variables commonly investigated in a programmed learning experiment is children's reading ability. One question often asked is: "Do good readers always perform better in an achievement test than poor readers?" In order to answer this question a 17-item reading ability test was given to the pupils in the study. The test was constructed by J. F. Schonnell. It was administered to the subjects without any revision made as an analysis of the pilot study revealed that the test had a high reliability coefficient of r = 0.89.

In this test children were required to read short sentences silently then respond to a question at the end of the sentence.

3.2.5. MATHEMATICAL ABILITY

An average of three tests, namely, Diagnostic Test in Vulgar Fractions, Arithmetic Reasoning and working with Numbers constituted the mathematical ability measure. The Diagnostic Test in Vulgar Fractions was a 24 - item Test constructed by J. F. Schommel. The remaining two tests, Arithmetic Reasoning and Working with Numbers, each comprising 10 items were obtained from the supervisor, Dr. G. S. Eshiwani. Reliability coefficients for these three tests were as follows:- Fractions (KR_{20} = 0.93); Arithmetic Reasoning ($r_{\frac{1}{2}\frac{1}{2}}$ = 0.64) and Working with Numbers ($r_{\frac{1}{2}\frac{1}{2}}$ = 0.55).
($r_{\frac{1}{2}\frac{1}{2}}$ = split-half reliability coefficient)

3.2.6.0 ATTITUDE QUESTIONNAIRE

Two attitude questionnaires were administered to the subjects in the main study. These were pupils' attitude towards mathematics and pupils' attitude towards the program. These two scales were obtained from the supervisor Dr. G.S. Eshiwani. The major purpose for administering these two attitude scales was to see whether a pupil's attitude towards a subject or towards the method by which that subject is learned is a good predictor of achievement in the criterion test.

3.2.6.1. ATTITUDE TOWARDS MATHEMATICS

The questionnaire comprised 22 items. This was a four-point Likert-type attitude scale ranging from Strongly Agree to Strongly Disagree. The scales used to quantify the items were strongly agree, agree, disagree and strongly disagree. The pupils were asked to tell how they felt about each statement by circling one of the categories. The categories were respectively given differential scores of +2, +1, -1, -2 for strongly agree, agree, disagree and strongly disagree. The attitudes expressed by the subjects were scored in the same direction, with agreement with a positive statement scoring the same as disagreement with a negative statement.

All the three instructional groups were administered this attitude questionnaire. The attitude inventory was not extended to teachers as it was thought that the teachers involved in the study were too few to give a representative opinion of all teachers. However, the teaching assistants expressed very useful ideas.

3.2.6.2 ATTITUDE TOWARDS THE PROGRAM

This was a 12 - item Likert - type attitude scale. Five categories were used to quantify the items. These were strongly agree, agree, undecided, disagree, and strongly disagree. The categories were given scores of +2, +1, 0, -1, and -2, respectively. The pupils were asked to indicate the strength of their preferences by circling one of the categories. It was assumed that the pupils' choices would represent a true picture of their opinion.

The items contained six positive and six negative items randomly mixed within the test. As in the attitude towards mathematics questionnaire, the attitude towards the program were scored in the same direction, with agreement with a positive statement scoring the same as disagreement with a negative statement.

3.3. EXPERIMENTAL MATERIALS

The experimental materials consisted of glass marbles, dice, spinners of various sides and coins.

Dice were made out of a 1 in. x1 in. x1 in. pieces of wood by the investigator at the Kenya Institute of Education workshop. The faces of the cubes were marked in such a way that the sum of the opposite faces was 7. Glass marbles were bought from the shops. The teachers were asked to construct spinners with their classes and have them ready before the start of the experiment. Each pupil involved in the study was asked to provide himself with a 5 - cent, a 10 - cent and a 50 - cent coin.

All these experimental materials were ready before the main study commenced. The investigator provided all the materials needed for the pilot study save the coins which were provided by every child involved in the pilot study.

3.4.0 THE PILOT STUDY

At the close of the try-out study, in the second week of March when all the measuring instruments and learning tasks had been constructed, the investigator set to conduct a pilot study.

3.4.1 PURPOSE OF THE PILOT STUDY

The purpose of the pilot study was twofold:-

1. To find out whether the revised program could be successful in teaching, and
2. To find out the suitability of the tests to be administered in the main study.

3.4.2. THE SAMPLE FOR THE PILOT STUDY

One single-streamed school was randomly selected from all single-streamed schools in Kisumu Town. The school selected was one of the low-cost primary schools not involved in the try-out study. All the standard six pupils in the selected school constituted the subjects for the pilot study.

3.4.3. THE MEASURING INSTRUMENTS

After the pilot study school had been selected all the standard six pupils in the school were administered the following tests:

1. A 15 - item probability pre-test designed to measure the pupil's initial knowledge of probability. The pre-test was a revised form of the test administered to the try-out subjects.

2. A test on children's reading ability taken from J. F. Schonnell's "Diagnostic and

Attainment Testing" Test A.

3. A test of children's general mathematical ability. This included arithmetic reasoning, fractions and working with numbers.

4. A post-test achievement. This test was administered to all the pupils present immediately after instruction.

No retention test and attitude questionnaires were administered to the pilot study group.

3.4.4. THE LEARNING TASK

After the administration of the pre-tests, a course of instruction in probability, supervised by the investigator was given to all the pupils in the class. The learning task consisted of a 175 - frame linear programme presented in three sections. As noted in section 3.1.0, section one of the program was about "Thinking" about chance." The purpose of this section was to stimulate pupils to think more objectively about chance events. Pupils would have an opportunity to test their intuition through participation and discussion. This section was supposed to encourage children to make guesses, estimates and predictions about chance events.

Section two dealt with "Experiments in Probability." This section was designed to help pupils clarify their concepts of chance and uncertainty. By performing experiments with dice, marbles, spinners and coins, and tabulating their results, pupils were supposed to discover possible patterns among chance events and to use these patterns to estimate future outcomes. An estimation of future outcomes would enable the pupils to distinguish between experimental and expected occurences.

Frame 99 of section two requires children to interpret a bar chart. This is to reinforce their ideas relating to probability.

Finally, when the children come to section three of the program, an assumption is made that they have gathered enough data from their activities in sections one and two and can summarize these data in tables. In section three, pupils are supposed to use the ideas gained from the previous two sections to calculate the probabilities of events. Here, they are introduced to the use of rational numbers as a measure of probability. At this stage of learning probability children are assumed to be capable of abstract thinking.

3.4.5. PROCEDURE

A linear program was handed section by section to each pupil in the class. A pupil could discuss the contents of the program with a friend if need be. Pupils were allowed to take the program home. Problems encountered by the pupils when going through the program were explained by the investigator. The questions asked by the pupils were generally concerned with language problems. The pupils were informed that they were involved in an experiment.

Five periods a week was allocated for this exercise. The instruction lasted 3½ weeks, at the end of which a post-test in probability was given to all the subjects present.

3.4.6. PROBLEMS ENCOUNTERED IN THE PILOT STUDY

At the beginning of the pilot study, all seemed to be going on very well. The regular mathematics teacher who was also the headmaster of the school initially showed a very positive attitude towards the whole programme. He indicated to the investigator that he would be present during the experiment to gather some ideas on probability as he himself was not sure of the subject.

One week later, the regular mathematics teacher changed his attitude. He now asked the investigator to wind off his work since the class was behind and he wanted to complete Book Six in time - that is before the end of the year. The investigator told the regular teacher that the experiment had just began and that he still had three weeks before he could wind off his work. He reminded the teacher that permission to conduct the study in the school had been obtained from the Municipal Education Officer and that the results of the investigation would be beneficial to the whole country. Further, the experimental materials and the programmed booklets would remain in the school. The investigator felt that choosing another school for the pilot study would be expensive both in terms of time and money, as he had scheduled the main study to start in May immediately the schools open for second term business and all the pre-test and section one of the program had been given to the subjects in the school. After two days, the regular class teacher changed his mind and allowed the investigator to continue. As has been stated earlier, the investigator was allowed five periods a week for his experiment while the regular mathematics teacher in the class used the remaining five mathematics periods.

It should be noted that lack of co-operation was only limited to the teacher. The pupils on the other hand, showed much enthusiasm throughout the experiment. Even though the pupils were enthusiastic to the programmed materials, the investigator strongly feels the attitude of the headmaster to the whole exercise contributed greatly to the pupils' low achievement in the post-test.

3.4.7. ITEM ANALYSIS FOR THE PILOT STUDY RESULTS

Item analysis was carried out for all the tests used in the pilot study. This was to find out the appropriateness of the tests to be used in the main study. The mean and standard deviation for each test were computed. Besides, the facility value for each item, item discrimination index, the reliability coefficient and the variance of the proportions for each test were computed.

TABLE I

FACILITY VALUES AND DISCRIMINATION INDICES FOR THE PILOT STUDY

ITEM NUMBERS

Variable	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Fractions																								
Facility:	.64	.50	.17	.14	.33	.36	.43	.57	.79	.26	0	.33	.21	.83	.74	.07	.36	.24	.29	.07	.02	0	.39	.10
Discrimination:	.25	.35	.3	.2	.25	.4	.2	.3	.25	.3	-.05	.15	.2	.2	.25	.15	.4	.2	.05	.05	.05	0	.3	.1
Arithmetic Reasoning:																								
Facility:	.63	.49	.80	.34	.56	.51	.32	.41	.39	.39														
Discrimination:	.14	0	.27	.45	.23	.36	.36	.36	-.05	.18														
Silent Reading:																								
Facility:	.97	.95	.89	.84	.95	.76	.55	.89	.74	.74	.74	.55	.21	.29	.50	.58	.18							
Discrimination:	.05	.1	.15	.15	.05	.3	.15	.05	.4	.1	.35	.15	.2	.2	.2	.35	.2							
Working with Numbers																								
Facility:	.19	.57	.74	.46	.33	.45	.4	.28	.26	.19														
Discrimination:	.18	.09	0	.23	.09	.36	.23	.09	.23	.09														
Probability Post-test																								
Facility:	.4	.71	.31	.66	.23	.34	.46	.14	.40	.69	.69	.23	.40	.20	.43	.20	.09	.37	.06	.17				
Discrimination:	.39	.49	.33	.69	.28	.39	.56	.17	.5	.62	.73	.28	.39	.11	.39	.17	.11	.17	.06	.28				

As has been mentioned items were revised on the basis of the facility and discrimination indices obtained. For fractions items 3, 4, 11, 13, 16, 20, 21, 22 and 24 were revised. But the values obtained did not differ much from the original indices. But there was slight improvement. The facilitity values obtained after revising the items were .28, .39, .27, .35, .25, .34, .28, .27 respectively.

The items on the Arithmetic reasoning, silent reading and working with numbers were not revised though there were some items which did not discriminate well between the top 27% and the lower 27%. For post-test items, items 5, 8, 12, 14, 16, were revised. The revised facility values were as follows: .25, .32, .35, .45 respectively. Items 17 - 20 were left to challenge the bright students. The above table reveals that many items needed revision. Due to shortage of time, this could not be done and the investigator went ahead to administer the tests. The selection of each item was done on the basis of the facility values and discrimination indices obtained. Items with facility values less than 0.25 and discrimination indices lower than 0.2 were rejected as poor items. Similarly, items with facility values higher than 0.85 were regarded as of little use as part of a measuring instrument. Note that items with high discriminating power are good items since they discriminate well between the top 27% and the lower 27%. If an item was rejected, another one was constructed and then re-tested. Finally, the items thought to constitute good measures had facility values spread from a lower limit of 0.25 to an upper limit of 0.85 (i.e., $0.25 \leq F \leq 0.85$). The table above presents a summary of the facility values, discrimination indices before the items were revised.

TABLE 2

MEANS, STANDARD DEVIATIONS, VARIANCE OF PROPORTIONS AND RELIABILITY COEFFICIENTS FOR THE PILOT STUDY

Test	No. of Subjects N	No. of items in the test n	Mean	SD	Var(P)	Reliability Coefficient
Probability Pre-test	44	15	3.82	1.77	0.13	0.53 (K.R.)
Working with Numbers	42	10	3.79	1.72	0.21	0.55 (S.H.)
Fractions	42	24	7.74	3.62	0.16	0.93 (K.R.)
Arithmetic Reasoning	41	10	4.76	1.89	0.23	0.64 (S.H.)
Silent Reading	38	17	11.37	2.52	0.16	0.89 (K.R.)
Probability Post-test	35	20	7.2	1.86	0.19	0.81 (K.R.)

S. H. = Split - half

K. R. = Kuder-Richardson

Kuder - Richardson formula 20 (KR_{20}) was used to calculate the reliability coefficient of the following tests: probability pre-test achievement, probability post-test achievement, Fractions and Silent Reading. The remaining tests, namely, Working with Numbers and Arithmetic Reasoning were subjected to the Spearman - Brown formula (split-half method) of calculating reliability. It was found that Kuder-Richardson formula 20 yielded unusually high reliability coefficients for these two tests and hence it could not be applied.

Table two above shows the average score, the variance of the scores and the variance of the proportion of correct answers for each item. From these figures, KR20 was calculated as an index of the extent to which the variation of the subjects' raw scores was a true indication of their variation (17, 1971). Kuder-Richardson formula 20 is also an index of the extent to

---

17
Keats, J. A.: An Introduction to Quantitative Psychology. John Wiley and Sons Australasia pty Ltd., 1971.

which scores on the second testing would reproduce scores on the first testing. It is also an index of the extent to which the items can be used to rank the subjects in the same order.

In order to test whether or not the observed discrimination between subjects was likely to have risen by chance responses, the following formular was used:

$$x^2 (N - 1) = \frac{n(n - 1)}{n(1 - R) + R}$$

where n = number of items in the test

R = Kuder-Richardson formula 20

N = Number of subjects.

The values of the chi-square obtained for the tests that were subjected to Kuder-Richardson formula 20 were significant at the .01 level of significance (Table 3, page 101).

This means that the observed discrimination between subjects did not occur by chance but could have been due to some other factor, possibly intelligence.

TABLE 3

χ^2 VALUES FOR THE TESTS SUBJECTED TO KUDER-RICHARDSON TEST

Test	N	df	χ^2	P
Probability Pre-test	44	43	110	$< .01$
Fractions	42	41	377	$< .01$
Silent Reading	38	37	227.9	$< .01$
Probability Post-test	35	34	147.5	$< .01$

Table 3 above gives the χ^2 - values for the four tests.

3.5.0 THE MAIN STUDY

3.5.1 The Sample for the main study

In February 1977 a list of all primary schools in Kisumu was obtained from the Kisumu Municipal Education Officer for the purpose of selecting research subjects. Since the investigation to be carried out involved three treatments and in order not to disturb the existing classroom arrangements in the schools, a decision was made to select schools with three streams of standard six for the research. Accordingly, a random sample of three schools with three streams in standard six was drawn using random digit numbers. After the three schools had been randomly drawn, the three standard six classes in each school were randomly assigned to each of the following treatments: programmed instruction, (PI), programed instruction with teacher instruction in small groups, (IPI), or conventional classroom instruction (CI).

Originally, the sample consisted of 447 pupils, with 250 boys and 197 girls. But for reasons not known to the investigator nor to the teaching assistants, a number of pupils did not do all the tests administered. These were finally dropped at the analysis stage, leaving only 353 pupils -

192 boys and 161 girls.

Table 4 shows how the subjects were distributed per treatment in each school before the dropout. A basic 2 x 3, sex x treatment factorial design was adopted for the experiment.

TABLE 4:

DISTRIBUTION OF SUBJECTS BEFORE DROP-OUT

		TREATMENTS			
		PI	IPI	CI	TOTALS
Sex	Boys	30, 29	28, 30	29, 30	
		27	19	28	
		(86)	(77)	(87)	250
	Girls	21, 28	22, 19	16, 19	
		25	28	19	197
		(74)	(69)	(54)	
	Totals	160	146	141	447

The uncircled numbers in each cell represent the number of subjects by sex in each of the three sample schools. The circled numbers represents the total number of subjects in each cell.

The table below shows the distribution of subjects per school per treatment after some pupils had been eliminated at the analysis stage.

TABLE 5:

DISTRIBUTION OF SUBJECTS AFTER DROP-OUT

		TREATMENTS			
		PI	IPI	CI	TOTALS
	Boys	21, 28 19	22, 29 15	24, 20 14	
Sex		(58)	(66)	(58)	192
	Girls	11, 20 24	19, 25 14	16, 20 12	
		(55)	(58)	(48)	161
	Totals	123	124	106	353

The uncircled numbers in each cell represent the number of subjects by sex in each of the three sample schools. The total number of pupils for each cell is shown circled.

3.5.2. THE TEACHERS

The regular classroom teachers in each sample school were used in this study. It was not possible to enlist the services of a single teacher in all the three classes as it was found that this would interfere with the existing arrangements in each school.

Before the instruction commenced the investigator thoroughly trained all the teachers involved in the study. All the three programmed booklets were given to each teacher. The investigator used these booklets as a basis for instruction. The training lasted four days. During the experimental period the investigator visited each school at least twice a week meeting the teachers in each school and reviewing the progress made by the pupils. During this time the investigater also reviewed the program. The investigater was satisfied that the teachers involved in the study now understood probability which was the subject to be taught.

In addition to teaching the teaching assistants the content of the subject to be taught, the investigator also briefed the teachers on how to conduct programmed and conventional sessions.

At the end of the teacher training, the teaching assistants were handed the following pre-tests to administer to the subjects under their control:

a) A 15 - item achievement pre-test in probability;
b) a test on childrens' reading ability;
c) a test on children's general mathematical ability;
d) pupils' attitude towards mathematics.

After the pre-test administration, a course of instruction in probability was given to the three treatment groups. A description of each of the treatment groups follows:

3.5.3. THE INSTRUCTIONAL METHODS

Three instructional methods - individualised programmed instruction (PI), integrated programmed instruction (IPI), and the conventional method of instruction (CI) were used in the study.

(a) The Individualised Programmed Instruction (PI)

As has been mentioned in section 3.1.0, the program in probability was a linear one, of the Skinner type with 175 frames written by the investigator of the present study.

The pupils in this instructional group went through the program individually and at their own pace. The teacher passed out the first booklet to each pupil. The pupil read a frame then constructed his own answer to that frame on a separate answer sheet. The answer was then compared with the answer given at the back of each page. If the answer was correct, the pupil continued on. If the answer was incorrect the pupil reread the frame until he understood it. After an individual had completed a booklet he collected another one from the teacher. At the end of each booklet there was a self-test intended for self-evaluation. The answers were given following the test. The purpose was to enable the pupil to grade himself. No teacher-made tests were given to this group.

The pupils in this treatment group were not allowed to take the programs home with them. This was to ensure that the subjects of all the three instructional groups were in contact with the learning materials for approximately the same amount of time. The teacher provided help only to those pupils who sought help, otherwise he remained effectively passive in the class.

He was in class throughout the learning session to ensure that the subjects did not cheat by working from the answers.

Each teacher involved with this group was asked to covertly record the time taken by each pupil to complete each section of the program and the number of times the teacher offered assistance to a subject. This instruction was not strictly adhered to by some teachers, who after keeping the record for two days felt that the exercise was laborious and hence abandoned it.

b). The Integrated Programmed Instruction Group (IPI)

The subjects in this group also used a linear program similar to the one used by the individualized programmed instruction group. In addition to the program, this group also received teacher instruction. The pupils formed voluntary groups between two and four pupils in each group The program was placed between two pupils in each group. They read a frame and constructed their answers individually on separate answer sheets. The answers were then compared with the answers given at the back of the page. If the pupils in the same group had the same answer, they continued on. But if one of them had an incorrect response to a frame, they discussed the material in the frame until they all understood it. If, however, after the discussion some members of the group had not understood the material in the frame, they consulted the teacher who helped them overcome their difficulties.

The teacher of this instructional group visited each subgroup from time to time during the learning session discussing with the members of each group the difficulties they encountered in the frames. He also offered them hints for further discussion on their own. Exceptions were observed in one or two schools where the teachers concerned left the pupils to go through the programme alone without offering them help. The investigator drew their attention to the possible effects of their behaviour. But this did not change the situation very much in one school where the teaching assistant got involved in sporting activities during the last week of investigation. The investigator considered it too late to recruit another teacher.

As in the programmed instruction group, no teacher-made tests were given to this group.

c). <u>The Conventional Instruction Group (CI)</u>

The pupils in this group received conventional instruction. (This method, recommended to be used in the primary schools of Kenya, is usually referred to as the Guided Discovery Method).

A booklet in probability containing the same material as the program was constructed by the investigator and issued to each teacher involved with the CI group. The teacher prepared his daily lessons from the lesson booklets. The pupils worked in groups of four throughout the learning session. The teacher posed questions to the pupils to help them discover mathematical relationships. Unlike the pupils in the PI or IPI groups, pupils in this group were given exercises to be done in class and at home. The teacher marked these class exercises and homework assignments and then discussed the assignments in class. At the end of each section the teacher administered a short test to the members of this group. The test was constructed and marked by the teacher.

It should be noted that the pupils in this group were not given probability booklets. Rather, the teacher wrote the problems for discussion on the blackboard. The experimental materials were made available to them just as in the other instructional groups.

3.5.4 TESTING PROCEDURES AND TEST SCORING

At the close of the instruction which lasted four weeks, a 20 - item probability test was administered to all the subjects in the study. A 12 - item attitude towards the program questionnaire was also administered to the subjects in the programmed instruction and the integrated programmed instruction groups. Eight weeks later, a retention test was administered to all the subjects in the study.

The normal mathematics teachers administered the tests during double mathematics lessons. The investigator collected the answer scripts from the schools.

The tests were marked manually by the investigator. For the tests, one mark was awarded for each correct answer, an 8 for more than two responses to an item, and a 9 for a missed our response. In the analysis of data, only a correct response and an incorrect response were considered. As has been mentioned earlier, the attitude questionnaire items were given differential scores of +2, +1, 0, -1 and -2 according to whether an item had a positive or negative connotation.

A desk calculator was used for data analysis.

3.5.5. CONTROL

In a study such as the present one, it is often difficult to hold all variables constant except the experimental variable which we wish to manipulate through our experimental treatment. In this study an attempt has been made to control for some errors that would have been extraneous to the purposes of this investigation. Such errors and how they were possibly controlled for are listed below:

1. To control for the contents to be studied, the investigator prepared lesson materials from the programmed materials. The program and the lesson were therefore comparable in style and difficulty.

2. Children's natural interest in games provides a high level of motivation for the study of probability. This study capitalised on that interest by using experimental activities to introduce some of the basic ideas of probability.

3. It was considered that the conventional method employing the same experimental materials would have similar novelty value as the programmed instruction or the integrated programmed instruction methods, and that this would help to balance any "Hawthorne Effects."

4. The subjects were aware that they were involved in an experiment, as this was relayed to them by their mathematics teachers. They however, were not aware that they were being subjected to different treatments. Further, the investigator did not inform the teaching assistants of his hypotheses. It was hoped that this would help to cancel out the "Rosenthal Effect."

5. The methodological errors due to different teachers were controlled for by thoroughly training the teachers in the subject to be taught. Before training, the teachers had little knowledge of probability as was

exemplified by their responses to the questions posed by the investigator before commencing the training programme. After training and subsequent reviews of the topic during the course of the experiment, it was hoped that teachers were on the same level of understanding of probability.

6. The investigator did not visit the classrooms frequently. Rather, he conferred with teachers in each school at least twice a week. During these meetings the teachers reported what they had observed in their classes and the problems they had encountered when handling probability.

7. Before the start of the investigation, all subjects were assumed to be initially comparable. However, after the pre-test probability scores had been subjected to a one-way analysis of variance, the assumption of the initial equality of all the subjects in the study was not supported. Hence a two-way analysis of covariance was used to control for any factor that might have been responsible for the differences among the subjects.

3.5.6. GENERAL OBSERVATIONS

As has been mentioned elsewhere in this chapter, teachers involved in this study met frequently with the investigator to report their observations in the classrooms. In these meetings many useful ideas were discussed. Some of the programmed instruction teachers complained that the program was too long and that if they continued using it, they would not be able to cover the syllabus by the end of the year.

Teachers of all instructional groups reported that pupils were highly enthusiastic, especially when using experimental materials like dice, marbles, etc. Teachers of the PI groups reported seeing voluntary groups being formed in the classrooms, an indication that pupils cannot read the program wholly on their own without seeking the assistance of other pupils. This confirms Banghart's (7, 1963)) contention that proper use of programmed materials can be made if they are combined with teacher instruction.

---

[7] Ibid.

The PI teachers also reported seeing pupils at first hurry through the program as if it was a test. But this slowed down after some time with students settling down to a more serious work. Most of the pupils in the PI group were seen working from the answers. This was, however stopped by their teachers.

CHAPTER FOUR

FINDINGS

4.0 INTRODUCTION

This chapter outlines the techniques of analysis used and describes the findings pertinent to each hypothesis as given on page 8 chapter one.

The initial probability achievement test was subjected to a one-way analysis of variance. This is described in section 4.1.1. Following a significant analysis of variance, a scheffe' test for post-hoc comparisons was computed to determine which of the means were significantly different. This statistic is described in section 4.1.3.

For testing the null hypothesis of no differences among the treatment groups at each of the four cognitive levels with respect to the criterion variable, a two-way analysis of covariance was computed. A two-way analysis of covariance was also used to find out which of the treatment groups under investigation had a higher retentive power at each of the four cognitive levels. A description of the assumptions underlying the analysis of covariance is given in section 4.1.4.

Following a rejection of no differences among the means of the treatment groups, a special t-test was computed to determine which pair of means significantly differed. This statistic is described in section 4.1.5.

The ordinary t-test was computed to test for significant differences between the means of boys and girls in the pre-test probability achievement, mathematical reasoning, reading ability and attitude inventories. The assumptions underlying the t-statistic are reported in section 4.1.2.

Pearson product-moment correlation coefficients were computed between predictor variables: mathematical ability, silent reading ability, attitude towards mathematics, attitude towards the program, and probability post-test scores, to find out which of the predictor variables significantly predicted achievement. The pre-test and post-test probability scores were each correlated with probability retention scores to determine which of the two predicted retention.

4.1.0. DESCRIPTION OF STATISTICAL TESTS USED FOR DATA ANALYSIS

Before the start of the investigation, a pre-test probability achievement, mathematics ability test, silent reading ability test, and attitude towards mathematics questionnaire were administered to all the subjects in the study. All these tests were subjected to a one-way analysis of variance to test for differences among treatment means. The assumptions underlying this statistic are listed below.

4.1.1. ASSUMPTIONS UNDERLYING THE F-TEST

1. All the treatment groups are drawn randomly from normally distributed parent population.
2. The variance of each treatment population is the same.
3. Observations represent random samples from populations.

4.1.2. ASSUMPTION OF THE t-test

1. The scores in each of the two populations from which the groups are randomly selected are normally distributed (assumption of normality of the distribution).

2. The variances of the scores in the two populations are equal (the assumption of homogeneity of variance).

3. The two groups are independently selected.

4.1.3 SUPPLEMENTARY ANALYSIS FOLLOWING ANALYSIS OF VARIANCE

Following a significant F in the context of analysis of variance, a Scheffe' test for post-hoc comparisons was computed. The various t-tests following analysis of variance are mutually interdependent and hence would not be ideal in this case. Two points are advanced about the Scheffe' procedure.

1. The method is exceedingly general in that it may be applied regardless of the number of means under study and regardless of the number of cases in each group.

2. The Scheffe' test is very conservative, thus leading to relatively few significant results.

The principal reason for the conservation of the Scheffe' test is that the associated significance level (e.g. .05) applies simultaneously to all possible Scheffe' comparisons.

4.1.4. ASSUMPTIONS UNDERLYING THE ANALYSIS OF COVARIANCE.

1. The criterion scores in each group must be regarded as a random sample from a population of possible scores.

2. The covariate measures are unaffected by the treatments.

3. The regression of the Y-scores (the criterion measures), the measure forming the basis of the adjustment is the same for all the populations.

4. The adjusted scores in each of the populations are normally distributed and have the same variance.

5. The mean of the adjusted scores is the same for all treatment populations.

4.1.5 SUPPLEMENTARY ANALYSIS FOLLOWING THE ANALYSIS OF COVARIANCE

Following a rejection of the null hypothesis of no difference between the means of the samples on the basis of an analysis of covariance, it was necessary to determine which pairs of means differed significantly. To do this, a special t-test was computed. The following formula gives the error variance of the difference between two adjusted criterion means ($\bar{Y}_i^1 - \bar{Y}_j^1$).

$$\sigma^2_{Y_i' - Y_j'} = \left[\frac{1}{ni} + \frac{1}{nj} + \frac{(\bar{x}_i - \bar{x}_j)^2}{SS_W X} \right] MS'Y_W$$

and

$$t = \frac{Y_i' - Y_j'}{\sigma_{Y_i' - Y_j'}},$$

where,

n_i = sample size for group i,

n_j = sample size for group j

$\bar{X}_i$ = mean for the x-measures for group i

$\bar{x}_j$ = mean for the x-measures for group j

$\bar{Y}_i'$ = adjusted criterion mean for group i

$\bar{Y}_j'$ = adjusted criterion mean for group j

$SS_W X$ = within-groups sums of squares for the x-measures

$SS^1_W Y$ = adjusted within-groups mean squares for the criterion measures

$\sigma^2_{Y_i' - Y_j'}$ = error variance of the difference between two adjusted criterion means

4.1.6. CORRELATIONAL STUDIES

The purpose of performing a correlational analysis was to study the relation between the independent variables such as attitude towards mathematics, attitudes towards the program, mathematical reasoning, reading ability, and probability post-test. Specifically, the study aimed at finding out which of the independent variables was a good predictor of achievement in a probability post-test.

Correlation coefficients were also computed to determine which of the two: pre-test achievement or post-test achivement, was a better predictor of retention.

4.2.0. FINDINGS OF THE INVESTIGATION

4.2.1. PRE-TEST PROBABILITY ACHIEVEMENT

As has been stated earlier in the introductory part of this chapter, pre-test probability achievement scores were subjected to a one-way analysis of variance. The assumptions concerning this statistic have been given in section 4.1.1. The means and standard deviations for each instruction group have been computed. Table 6 gives a summary of the results.

TABLE 6:

MEANS AND STANDARD DEVIATION FOR THE PRE-TEST PROBABILITY

Cognitive Level and Sex	Programmed Instruction (PI)			Integrated Programmed Instruction (IPI)			Conventional Instruction (CI)		
Knowledge	N	$\bar{X}$	SD	N	$\bar{X}$	SD	N	$\bar{X}$	SD
Total Scores	123	53.0	22.09	124	47.6	26.8	106	48.8	27.0
Male	68	55.5	24.37	66	49.2	27.7	58	55.2	27.6
Female	55	50.0	18.65	58	45.7	25.7	48	41.1	24.5
Comprehension									
Total Scores	123	34.3	26.47	124	36.5	27.3	106	30.0	24.98
Male	68	36.0	26.06	66	39.4	27.8	58	31.9	28.41
Female	55	32.3	27.07	58	33.2	26.7	48	27.6	20.1
Application									
Total Scores	123	16.6	14.22	124	17.5	12.8	106	15.1	11.5
Male	68	15.6	13.37	66	19.1	13.3	58	14.0	11.0
Female	55	17.9	15.23	58	15.7	12.1	48	16.4	11.98
Analysis									
Total Scores	123	26.0	14.92	124	19.0	15.8	106	20.2	15.2
Male	68	26.8	14.91	66	20.0	16.5	58	21.4	16.7
Female	55	25.1	15.02	58	17.9	15.1	48	18.8	13.3

Table 7 presents a summary of a one-way analysis of variance for probability pre-test achievement.

TABLE 7:

ANALYSIS OF VARIANCE FOR PRE-TEST PROBABILITY

Source of Variation / ...gnitive level	Between Groups			Within Groups			
	SS	df	MS	SS	df	MS	F
...nowledge	2009.43	2	1004.715	224283.57	350	640.8102	157
...omprehension	2504.49	2	1252.245	240670.38	350	687.63	1.82
...pplication	341.349	2	170.67	58504.791	350	167.16	1.021
...nalysis	3411.6	2	1705.8	82228.07	350	234.9	7.26*

* $P < .01$

An examination of the above table reveals no significant differences among the means of the PI, IPI and CI groups in the knowledge, comprehension and application levels. Significant differences however, existed among the means of the three instructional groups in the analysis level ($F = 7.26$, $P < .01$).

Since a significant F-ratio was obtained in the analysis level, a Scheffe' test for post-hoc comparisons was computed to determine which pairs of means were significantly different. Table 8 shows a comparison of different pairs of means in the three treatment groups in the Analysis level.

TABLE 8:

SCHEFFE' TEST FOR COMPARISON OF THE MEANS OF THE THREE INSTRUCTIONAL GROUPS IN ANALYSIS LEVEL

Treatment Groups	Difference between Means	95% CI	99% CI
PI Vs IPI	7.0	2.2 to 11.8*	1.03 to 12.97+
PI Vs CI	5.8	0.8 to 10.8*	-0.42 to 12.0
IPI Vs CI	1.2	-3.79 to 6.19	-5.0 to 7.4

*Significant at the 95% Confidence Interval.

+Significant at the 99% Confidence Interval.

The results presented in the above table show that a significant difference existed between the programmed and integrated programmed instruction groups at both the 95% and 99% confidence intervals, with the programmed instruction group ($\bar{X}$ = 26.0; SD = 14.92) performing significantly better than the integrated programmed instruction group ($\bar{X}$ = 19.0; SD = 15.79). Since the null hypothesis that the difference between the means of the two instructional groups is zero is not included in the two confidence intervals, we are relatively confident that the means for the pupils in the IPI group was from 2.2. to 11.8 marks lower than the mean for the PI pupils. In the 99% confidence interval, the mean for the IPI pupils was from 1.03 to 12.97 lower than the mean for the PI pupils. There was a significant difference between the mean of the programmed instruction group and the mean of the conventional instruction group at the 95% confidence interval, the programmed instruction group ($\bar{X}$ = 26.0; SD = 14.92) performing much better than the conventional instruction group ($\bar{X}$ = 20.2; SD = 15.42). The mean for the CI pupils was from 0.8 to 10.8 points lower than the mean for the PI pupils. The means of the IPI and the CI groups are statistically insignificant (d = 1.2, $P > .05$), where d refers to the differences between means.

In order to throw more light on the overal performance of boys and girls in pre-test probability in the four cognitive levels, a t-test was computed. The results of this statistic are summarized in table 9. There was no significant difference between the means of the two sex groups in the analysis cognitive level, ($t = 1.33$; $P > .05$). Nor were there significant differences in the Application cognitive level ($t = 0.289$; $P > .05$). Significant differences, however, existed on the knowledge ($t = 2.79$ $P < .05$) and on the Comprehension ($t = 1.67$; $P < .05$ - one - tailed) cognitive levels. The significant difference was in favour of boys in both cases.

TABLE 9:

COMPARISON OF THE PERFORMANCE OF BOYS AND GIRLS

Variable/Sex	N	$\bar{X}$	s	t	P
Knowledge					
Boys:	192	53.3	26.57	2.79	< .05
Girls:	161	45.8	23.6		
Comprehension					
Boys:	192	39.9	27.39	1.67	< .05 (one-tailed)
Girls:	161	31.2	25.00		
Application					
Boys:	192	16.3	12.78	0.289	(NS)
Girls:	161	16.7	13.14		
Analysis					
Boys:	192	22.8	16.2	1.33	(NS)
Girls:	161	20.6	14.82		

4.2.2.0 ANALYSIS OF POST-TEST PROBABILITY ACHIEVEMENT

The assumption of the initial equality of the groups was tested by a one-way analysis of variance. The results have been summarized in tables 7 and 8. There are no significant differences in knowledge, comprehension and application levels in the context of anova. Significant differences, however, existed in Analysis level. The significant differences indicated a need for statistically equating the groups. Subsequently, a two-way analysis of covariance was computed for this level. In order to increase the precision of the experiment for the other three cognitive levels, the two-way analysis of covariance was computed, even though no significant differences had been found among the means of the three treatment groups in these cognitive levels in the context of anova.

As has been mentioned in the foregoing paragraph, treatment groups were compared for probability achievement by a two-way analysis of covariance. Teaching methods and sex of pupil served as the main effects with the pre-test probability scores as the covariate and the post-test scores as the criterion. Before computing the analysis of covariance, the means and standard deviations for the treatment groups and sex groups were computed, for each cognitive level. Table 10 presents a summary of the means and standard deviations while table 11 gives a summary of the analysis of covariance for all cognitive levels, and table 12, page 136 gives the adjusted means for the subjects in the three treatment groups.

TABLE 10:

MEANS AND STANDARD DEVIATIONS FOR PROBABILITY POST-TEST SCORES

Cognitive Level and Sex	Programmed Instruction			Integrated Programmed Instruction			Conventional Instruction		
	N	Y	S	N	Y	S	N	Y	S
Knowledge									
Total Scores	123	59.8	21.14	124	60.1	25.69	106	62.7	22.69
Male	68	60.7	21.74	66	63.6	25.64	58	60.8	21.52
Female	55	58.6	20.53	58	56.0	25.36	48	65.1	24.04
Comprehension									
Total Scores	123	51.6	23.9	124	53.0	21.41	106	55.2	23.07
Male	68	46.3	21.27	66	55.3	19.37	58	57.8	22.56
Female	55	58.2	25.49	58	50.4	23.41	48	52.1	23.54
Application									
Total Scores	123	44.1	16.16	124	46.6	15.26	106	48.1	17.74
Male	68	49.1	16.03	66	46.2	14.14	58	48.0	18.93
Female	55	37.8	14.08	58	47.1	16.56	48	48.2	16.38
Analysis									
Total Scores	123	36.7	16.86	124	40.5	18.12	106	42.6	15.63
Male	68	36.2	18.69	66	39.7	18.73	58	41.4	14.92
Female	55	37.5	14.43	58	41.4	17.51	48	44.2	16.48

TABLE 11:

ANALYSIS OF COVARIANCE

Source of Variation / Cognitive level	Teaching Methods				Sex				Interaction (Method X Sex)				
	SS	df	MS	F	SS	df	MS	F	SS	df	MS	F	SS
Knowledge	697.09	2	348.54	0.651	155.52	1	155.52	0.291	2373.23	2	1186.615	2.22	185175.
Comprehension	654.13	2	327.07	0.644	24.2	1	24.2	0.048	5848.45	2	2924.225	5.76•	175736.
Application	1068.88	2	534.44	1.101	1140.92	1	1140.92	4.427+	3146.43	2	1573.21	6.19•	87976.
Analysis	2386.73	2	1193.37	4.13+	366.08	1	366.08	1.266	20.9	2	10.45	0.04	100061.

+ P∠.05

• P∠.01

TABLE 12:

TREATMENT GROUP MEANS POST-TEST SCORES

ADJUSTED FOR PRE-TEST SCORES

Cognitive Level	Treatment Group	N	X-Means	Y-Means	Adjusted Y-Means
Knowledge	PI				
	Boys —	68	55.5	60.7	60.14
	Girls —	55	50.5	58.6	58.59
	IPI				
	Boys —	66	49.2	63.6	63.67
	Girls —	58	45.7	56.0	56.42
	CI				
	Boys —	58	55.2	60.8	60.27
	Girls —	48	41.1	65.1	64.22
Comprehension	PI				
	Boys —	68	36.0	46.3	46.38
	Girls —	55	32.3	58.2	58.14
	IPI				
	Boys —	66	39.4	55.3	55.51
	Girls —	58	33.2	50.4	50.38
	CI				
	Boys —	58	31.9	57.8	57.73
	Girls —	48	27.6	52.1	51.87
Application	PI				
	Boys —	68	15.6	49.2	49.35
	Girls —	55	17.9	37.8	37.56
	IPI				
	Boys —	66	19.1	46.2	45.76
	Girls —	58	15.7	47.1	47.23
	CI				
	Boys —	58	14.0	48.0	48.41
	Girls —	48	16.4	48.2	48.21
Analysis	PI				
	Boys —	66	26.8	36.2	35.9
	Girls —	55	25.1	37.5	37.3
	IPI				
	Boys —	66	20.0	39.7	39.8
	Girls —	58	17.9	41.4	41.6
	CI				
	Boys —	58	21.4	41.4	41.4
	Girls —	48	18.8	44.2	44.4

4.2.2.1 ANALYSIS OF POST-TEST PROBABILITY SCORES ON THE KNOWLEDGE COGNITIVE LEVEL

Table 11 shows no significant F-ratios for sex, teaching methods or method by sex interaction on the knowledge cognitive level. Though no significant differences are revealed on this level, an examination of table 12 shows that the programmed instruction boys (adjusted mean 60.14) performed better than the girls of the same instructional group (adjusted mean 58.59). The integrated programmed instruction boys (mean 63.67) did better than the girls of the same group. The conventional instruction boys, on the other hand, showed a relative inferiority in their performance to that of the girls in the same group (adjusted mean for boys: 60.27 and adjusted mean for girls 64.22). On the whole, the girls who went through the conventional instruction performed slightly better than the boys and the girls of the other instructional groups. It can be implied from the foregoing discussion of results that girls learn specific facts better when they are taught by the human teacher than when they are taught by either the program or the program supplemented by the human teacher. Boys who learned from the program supplemented by the teacher were slightly superior in performance to those who were taught by the program or by the human teacher.

4.2.2.2. ANALYSIS OF PROBABILITY POST-TEST SCORES ON COMPREHENSION COGNITIVE LEVEL

An examination of table 11 reveals that there were no significant differences with regards to teaching methods and sex. However, significant differences existed in sex by methods interaction ($F = 5.757$, $P < .01$). To gain further insight into the nature of these differences, a special t-test following a significant analysis of covariance was computed. The results of this statistic are presented on table 13 below.

TABLE 13:

SPECIAL t-test FOLLOWING ANALYSIS OF COVARIANCE ON COMPREHENSION LEVEL

Treatment Group	Sex	t	P	
PI and IPI	Boys	2.34	< .05; P < .01	(one-tailed)
	Girls	1.831	< .01	(one-tailed)
PI and CI	Boys	2.815	< .01	
	Girls	1.407	> .05	
IPI and CI	Boys	0.545	> .05	
	Girls	0.339	> .05	
PI Boys and PI Girls		2.875	< .01	
IPI Boys and IPI Girls		1.262	> .05	
CI Boys and CI Girls		1.33	> .05	
PI Boys and IPI Girls		0.991	> .05	
PI Boys and CI Girls		1.286	> .05	
PI Girls and IPI Boys		0.638	> .05	
PI Girls and CI Boys		0.0978	> .05	
IPI Girls and CI Boys		1.757	< .05	(one-tailed)
IPI Girls and CI Girls		0.34	> .05	

The results of the special t-test revealed the following:

1. (a) There was a significant difference between the boys of the PI group and the boys of the IPI group ($\bar{Y}^{1}PI_{b} = 46.35$, $\bar{Y}^{1}IPI_{b} = 55.51$; $t = 2.34$, $P < .05$ and $P < .01$, one-tailed), the difference being in favour of the IPI boys.

   (b) For the girls of the two instructional groups, the PI girls performed better than the IPI girls ($\bar{Y}^{1}PI_{g} = 58.14$, $\bar{Y}^{1}IPI_{g} = 50.38$, $t = 1.831$, $P < .05$, one-tailed).

2. (a) There was a significant difference between the boys of PI and those of the CI groups, with the CI boys' performance being significantly superior to that of the PI boys ($\bar{Y}^{1}CI_{b} = 57.73$, $\bar{Y}^{1}PI_{b} = 46.38$, $t = 2.815$, $P < .01$).

   (b) the girls of the same instructional groups did not differ significantly in their performance ($\bar{Y}^{1}CI_{g} = 51.67$, $\bar{Y}^{1}PI_{q} = 58.14$, $t = 1.407$, $P > .05$).

3. (a) There were no significant treatment effects for the boys in the IPI group and those in the CI group ($\bar{Y}^{1}_{IPI_b} = 55.51$, $\bar{Y}^{1}_{CI_b} = 57.73$, $t = 0.545$, $P > .05$) although the CI group boys performed relatively better than the IPI group boys as evidenced by their mean scores,

(b) The girls of the same treatment groups did not exhibit any significant differences in their performance ($t = 0.339$) $P > .05$). The performance of the girls in these two instructional groups was almost comparable. The adjusted mean for the IPI girls was 50.35 while that for the girls of the CI group was 51.87.

4. The boys and the girls undergoing programmed instruction differed significantly in their performance ($\bar{Y}^{1}PI_b = 46.38$, $\bar{Y}^{1}PI_g = 58.14$, $t = 2.875$, $P < .01$), with PI girls scoring higher than PI boys.

5. There was no significant difference between IPI boys and IPI girls ($t = 1.262$, $P > .05$) though from the mean scores, boys in this instruction group ($\bar{Y}^{1} = 55.51$) performed better than girls of the same group ($\bar{Y}^{1} = 50.38$).

6. The performance of boys and girls undergoing the conventional mode of instruction was not statistically significant ($t = 1.33$, $P > .05$). Judging from the mean scores, boys who learned through this mode of instruction ($Y^1 = 57.73$) did better in the probability achievement post-test than girls who learned through the same method ($Y^1 = 51.87$).

7. No difference between PI boys' and IPI girls' performance was observed ($t = 0.991$, $P > .05$). However, the IPI girls ($Y^1 = 50.38$) did better than the PI boys ($Y^1 = 46.38$).

8. Although the difference between the adjusted means of the PI boys and CI girls was not statistically significant ($t = 1.286$, $P > .05$), the CI girls' performance ($Y^1 = 51.87$) was somewhat superior to the PI boys' performance ($Y^1 = 46.38$).

9. No significant difference was found between PI girls and IPI boys ($t = 0.638$, $P > .05$). But the PI girls ($Y^1 = 58.14$) did slightly better than the IPI boys ($Y^1 = 55.51$), judging from their mean performance.

10. No significant difference was observed between the PI girls and the CI boys ($t = 0.0978$, $P > .05$). The means for the two sex groups were quite close ($\bar{Y}^1$ PI girls = 58.14) and ($\bar{Y}^1$ IPI boys = 57.73).

11. The girls who learned from the program supplemented by the human teacher i.e. the IPI girls, and the boys who learned from the human teacher alone i.e. CI boys showed significant differences in their achievement scores ($t = 1.757$, $P < .05$, one-tailed). The significant performance was in favour of the CI boys ($\bar{Y}^1 = 57.73$). The mean for the IPI girls was $\bar{Y}^1 = 50.38$.

12. The girls of the IPI group and the girls in the CI group did not exhibit any significant differences in their performance ($t = 0.34$, $P > .05$), though the mean performance for the CI girls was slightly higher than the mean performance for the IPI girls.

From the forgoing discussion it is clear that boys do relatively better in comprehension tasks when they are taught by the human teacher while girls do well when they learn through self-instructional materials. It should further be observed that the mean performance of the girls who learned through the program was relatively higher than the mean performance of the other subjects. The finding that girls seem to learn better through self-instructional materials is contrary to the findings on the knowledge level where girls were found to perform better on knowledge tasks when taught by the human teacher.

4.2.2.3. ANALYSIS OF POST-TEST PROBABILITY SCORES ON APPLICATION COGNITIVE LEVEL

The results of the analysis of covariance used to test for the significance of the difference in probability achievement in the instructional groups, i.e. the PI, IPI and the CI groups is presented on table 11, Page 135. Table 12, page 136 give the adjusted means for the subjects in the three treatment groups.

There were significant main effects for sex (F = 4.487, P $<$ .05) and for sex by methods interaction (F = 6.187, P $<$ .01). There were no significant differences for teaching methods. The significant interaction revealed by the two-way analysis of covariance called for further analysis to determine which group interacted with which method. For this reason a special t-test was computed. Table 14 presents a summary of the results.

TABLE 14:

SPECIAL t-test FOLLOWING ANALYSIS OF COVARIANCE ON APPLICATION COGNITIVE LEVEL

Treatment Group	t	P
PI boys and IPI boys	1.298	> .05
PI boys and CI boys	0.329	> .05
IPI girls and CI girls	0.315	> .05
PI girls and CI girls	3.38	< .01
IPI boys and CI boys	0.917	> .05
PI girls and IPI girls	3.218	< .01
PI boys and PI girls	4.072	< .01
PI boys and IPI girls	0.333	> .05
PI girls and CI girls	0.379	> .05
IPI boys and IPI girls	0.511	> .05
IPI boys and CI girls	0.809	> .05
CI boys and IPI girls	0.398	> .05
CI boys and CI girls	0.064	> .05
CI boys and PI girls	3.602	< .01
IPI boys and PI girls	2.816	< .01

The table reveals that only five pairs of means were significantly different. The remaining ten pairs were not significantly different; at the 0.5 level of significance.

1. There was a significant difference between the performance of the PI girls and the performance of the CI girls ($t = 3.38$, $P < .01$), with the CI girls ($Y^1 = 48.21$) performing significantly better than the PI girls ($Y^1 = 37.56$).

2. The girls undergoing programmed instruction supplemented with teacher instruction ($Y^1 = 47.23$) did significantly better than the girls who learned through the program only ($t = 3.218$, $P < .01$).

3. The mean performance of the PI boys ($Y^1 = 49.35$ and the mean performance of the PI girls ($Y^1 = 37.56$) were significantly different ($t = 4.072$, $P < .01$) the difference being in favour of boys.

4. The performance of the boys who received teacher instruction ($Y^1 = 48.41$) was significantly superior to the performance of girls who went through programmed materials individually ($Y^1 = 37.56$, $t = 3.602$, $P < .01$).

5. When a comparison of the performance between IPI boys and PI girls was made, it was found that the performance by the IPI boys ($\overset{1}{Y} = 45.76$) was significantly superior to that of the PI girls ($\overset{1}{Y} = 37.56$) $t = 2.816$, $P < .01$).

From the forgoing discussion, it seems appropriate to conclude that girls performed significantly better in an achievement test when they received teacher instruction than when left to study on their own through the program on the application cognitive level. This is evidenced by the relatively low mean of 37.56 for the girls who received individualized programmed instruction.

It is interesting to note that boys who went through the program on their own performed relatively better than those who received programmed instruction supplemented by teacher instruction in small groups. Another interesting observation comes to light when one considers the performance of the pupils who received conventional instruction. The difference between the means of boys and girls in this group was statistically insignificant (t = 0.0642, $P > .05$) and negligible. This can be interpreted to mean that boys and girls who received this instruction learned equally well.

When the performance of all the boys and all the girls in the study was considered boys' performance was somewhat better (mean: 47.8, SD= 16.33) than that of girls (mean 44.2, SD= 16.30). On the whole, the CI group ($\bar{Y}^1$ = 48.33) performed better than either the IPI ($\bar{Y}^1$ = 46.43) or the PI ($\bar{Y}^1$ = 44.08) groups.

4.2.2.4. ANALYSIS OF PROBABILITY POST-TEST SCORES ON ANALYSIS COGNITIVE LEVEL

The results of the subjects' performance on this cognitive level are summarised on tables 11 and 12. Table 11 gives a summary of the analysis of covariance for post-test probability scores adjusted for pre-test probability scores while table 12 presents a summary of the treatment group means for post-test probability adjusted for pre-test probability scores. There was a significant treatment effects (F = 4.126, $P < .05$). Following this significant F in the context of ancova, a special t-test was again computed to find out which pair of means were significantly different. Table 15 gives the results of the special t-test.

TABLE 15:

A SPECIAL t-test ON ANALYSIS LEVEL

Treatment Group	difference between means (d)	t
PI and IPI	4.24	2.50+
PI and CI	6.3	2.76*
IPI and CI	2.02	0.90(NS)

+ P <.01 (one-tailed)
* P <.01 (two-tailed)

The significant differences favoured the IPI and the CI groups over the PI group. There was no significant difference between the IPI and the CI groups. Judging from group mean performance, the CI group was on the whole more favoured than the other two instructional groups. The adjusted mean for the CI group was 42.7 while that for the IPI group was 40.68 and that for the PI group was 36.43.

4.2.3.0. ANALYSIS OF PREDICTOR VARIABLES

The subjects' performance on the variables for predicting achievement in probability post-test - Reading Ability, Attitude towards Mathematics and Attitude towards the Program - were also examined in this study. Table 16, page 152 presents the means and standard deviations of the subjects' scores on the four predictor variables. Table 17, page 153 presents a summary of a one-way analysis of variance for the four variables and Table 18, page 154 gives a summary of the t-values for the comparison of boys and girls in the study.

4.2.3.1. ANALYSIS OF DIFFERENCES IN READING ABILITY.

An F-ratio of 1.49 revealed no significant differences among the three instructional groups, at the .05 level of significance (table 17). Nor was there a significant difference between boys and girls in the study ($t = 0.43$, $P > .05$). (table 18).

An examination of table 16 reveals the following:

1. The programmed instruction group had a slightly higher mean ($\overline{X} = 59.8$) than either the integrated programmed instruction group ($\overline{X} = 55.5$) or the conventional instruction group ($\overline{X} = 57.1$). It should be noted that the IPI mean was the lowest.

TABLE 16:

MEANS AND STANDARD DEVIATIONS

FOR PREDICTOR VARIABLES

Variable	Programmed Instruction			Integrated Programmed Instruction			Conventional Instruction		
	N	$\bar{X}$	SD	N	$\bar{X}$	SD	N	$\bar{X}$	SD
Reading Ability									
Total	123	59.8	17.78	124	55.5	20.71	106	57.1	20.34
Boys	68	60.2	17.57	66	55.9	20.5	58	57.4	20.9
Girls	55	59.3	18.3	58	55.1	21.2	48	56.6	19.8
Mathematics Ability									
Total	123	34.1	11.54	124	34.7	14.56	106	33.5	13.52
Boys	68	34.3	11.65	66	36.6	15.41	58	36.6	15.03
Girls	55	33.9	11.51	58	32.5	13.3	48	29.7	10.34
Att. Toward Maths									
Total	123	8.5	9.56	124	7.4	9.18	106	8.8	10.39
Boys	68	7.8	8.96	66	8.5	9.86	58	8.4	10.20
Girls	55	9.0	10.25	58	6.2	8.25	48	9.3	10.71
Att. Toward the Program									
Total	123	3.6	6.33	124	2.7	5.85	t = 1.14, P > .05		
Boys	68	4.5	6.03	66	2.8	6.08			
Girls	55	2.5	6.59	58	2.5	5.62			
	t = 1.76 *P < .05			t=0.28, P > .05					

TABLE 17:

ONE-WAY ANALYSIS OF VARIANCE FOR READING ABILITY, MATHEMATICS ABILITY AND ATTITUDE TOWARDS MATHEMATICS

Source of Variation / Variables	Between Groups			Within Groups				
	SS	df	MS	SS	df	MS	F	P
Reading Ability	1148.8	2	574.4	134765.2	350	385.04	1.49	>.05
Mathematics Ability	82.8	2	41.4	61491.21	350	175.72	0.236	>.05
Attitude Towards Mathematics	132.474	2	66.237	32836.48	350	93.82	0.71	>.05

TABLE 18:

TOTAL SCORES FOR BOYS AND GIRLS ON THE FOUR PREDICTOR VARIABLES

Variable	Sex	n	$\bar{x}$	s	t	P
Reading Ability:	Male	192	57.9	19.6	0.43	> .05
	Female	161	57.0	19.8		
Mathematics Ability:	Male	192	35.8	14.06	2.64	< .01
	Female	161	32.1	11.91		
Attitudes Towards Maths:	Males	192	8.2	9.61	0.097	> .05
	Female	161	8.3	9.78		
Attitude Towards The Program:	Male	134	3.7	6.09	1.54	> .05
	Female	113	2.5	6.08		

2. The mean for the boys in the PI group was higher than any for the subjects in the other instructional groups. It was also higher than that of the girls in the same instructional group.

3. The girls of the PI group had a higher mean than their counterparts in the other instructional groups.

4.2.3.2. ANALYSIS OF DIFFERENCES IN MATHEMATICS ABILITY

The results of the subjects' performance in mathematics ability are summarized on tables 16 - 18. There were no significant differences among the three treatment groups ($F = 0.236$, $P > .05$). The t-test computed to compare the overal performance of boys and girls in the study revealed a significant difference in mathematical ability in favour of boys ($t = 2.64$, $P < .01$).

When table 16 is examined, the following come to light:

1. The boys in the IPI group had the same mean with their counterpart in the CI group $\bar{X} = 36.6$). This mean was however, higher than that for the boys in the PI group

($\bar{X}$ = 34.3).

2. The CI girls had the lowest mean ($\bar{X}$ = 29.7).

3. Significant differences existed between PI boys and CI girls (t = 2.69, $P < .01$) the difference favouring the PI boys.

4. There were significant differences between the CI boys and the CI girls (t = 2.19; $P < .05$), CI boys performing significantly better than the CI girls.

4.2.3.3. ANALYSIS OF DIFFERENCES IN ATTITUDES TOWARDS MATHEMATICS

The information on pupils' attitudes towards mathematics was collected from all pupils in the study. The data was subjected to a one-way analysis of variance for the comparison of the PI, IPI and CI groups and to a t-test for the comparison of total boys' and total girls' scores in the study. The results have been summarized in tables 16 - 18.

There were no significant differences among the three instructional groups as is revealed by an F-ratio of 0.71, $P > .05$. Table 18 shows no significant differences between boys' (total) and girls' (total) attitude scores towards mathematics (t = 0.097, $P > .05$). However, table

16 reveals that girls of the PI and the CI groups slightly favoured mathematics more than the boys in the same treatment groups and also more than the subjects in the integrated programmed instruction group. The lowest preference for mathematics was exhibited by the girls of the IPI group (mean 6.2, SD = 8.25).

4.2.3.4. ANALYSIS OF ATTITUDES TOWARDS THE PROGRAM

Only the attitudes of the subjects who learned by the program, that is, those who received individual programmed instruction and those who were taught by the program and the teacher, were investigated. The attitude scores for the two instructional groups were compared by a t-test. The results of this statistic are presented in table 16 page 152. A t-value of 1.14 revealed no significant differences in attitudes towards the program between the PI and IPI groups, at the 5% level of significance, though the PI group had a slightly higher mean ($\bar{X}$ = 3.6) than the IPI group ($\bar{X}$ = 2.7). This can be interpreted to mean that those pupils who learned individually by the program found the programmed materials more interesting than those who learned through program and the teacher. A significant difference in attitude towards the program existed between boys and girls undergoing the individualized programmed instruction ($t = 1.76$, $P < .05$ – one-tailed) with the boys scoring higher ($\bar{X} = 4.5$, $S = 6.03$) than the girls ($\bar{X} = 2.5$, $S = 6.59$). The boys and the girls of the IPI group did not show any significant difference in attitude ($t = 0.28$, $P > .05$).

When total attitude scores for boys and girls were compared, no significant differences were found ($t = 1.54$, $P > .05$). This t-value, however, was almost significant in favour of boys at the 5% level (one-tailed). On the whole, boys favoured the program more than the girls.

4.2.4.0. <u>ANALYSIS OF RETENTION TEST SCORES</u>

It has been mentioned in chapter three section 3.5.4 that a retention test was administered to all the subjects in the study eight weeks after the post-test administration. The items in the retention test were the same as the post-test items. The subjects were not informed of the impending delayed post-test. The results of the retention test scores were subjected to a two-way analysis of covariance in each of the four cognitive levels-knowledge comprehension, application and analysis. The post-test scores were used as the covariate while the retention test scores served as the criterion. Tables 19 and 20 pages 163-164 summarize the results of data analysis.

4.2.4.1. ANALYSIS OF RETENTION TEST-SCORES ON THE KNOWLEDGE COGNITIVE LEVEL

There were no significant differences in teaching methods, sex or in methods x sex interaction. But an F-ratio of 3.34 for sex was nearly significant at the .05 level (df = 1,346).

The following observations are made from table 20, page 164.

1. Boys undergoing individualised programmed instruction significantly dropped in their retention scores whereas the mean retention score for girls of the same instructional group was higher than their post-test mean. There was a significant difference in retention between the PI boys and PI girls ($t = 2.50$, $P < .05$), the PI girls retaining more materials than the PI boys.

2. Boys who learned through the program and the teacher did not show a significant drop in their retention scores, the mean difference between the post-test and retention test scores being only 2.28.

o On the other hand, the girls' retention scores were higher than their post-test scores, the unadjusted mean difference being 5.64.

3. Both boys and girls undergoing the conventional mode of instruction showed a drop in their retention scores. The difference between the mean scores for the boys was 3.9 while that for girls was 8.3, an indication that girls had the highest drop.

Further analyses were carried out by t-tests to provide a more concise picture of the statistical group differences on retention. The girls learning through the individualized programmed instruction performed significantly better than the girls learning by the conventional method ($M_1 - M_2 = 8.8$, df = 101, t = 1.99 $P < .05$). The girls of the PI group also retained significantly more material than the boys of the CI group ($M_1 - M_2 = 7.4$, t = 1.86, $P < .05$- one-tailed).

Retention by girls learning by the program was again significantly higher than that exhibited by boys learning by the same method. This is shown by a t-value of 2.50, $P < .01$ for the unadjusted means. The mean retention scores for the other groups were not statistically significant.

The foregoing discussion reveals that girls who learned by the program individually retained more material than either the boys who received the same instruction or the boys and girls in the other instructional groups. This may mean that after the instruction and the post-test, the girls in the PI group continued to study the programs at home. This reason may similarly apply to boys and girls who were taught by the program and the teacher. The subjects in this treatment group also took the programs home with them and may have studied them after instruction and subsequent post-testing. No reason can be advanced for the poor retention by the boys of the programmed instruction group. Their performance in probability post-test on this cognitive level was a bit better than that for the girls of the same instructional group, the mean for the PI boys being 60.14 and that for the PI girls, 58.59. The poor retention by the CI group can be attributed to the fact that they had no programs and were not exposed to lesson transcripts after instruction. On the knowledge cognitive level, therefore, those learning by program, whether individually or with teacher guidance seem to retain more material than those being taught by the teacher.

TABLE 19

ANALYSIS OF COVARIANCE FOR RETENTION SCORES

Source of Variation	Teaching Methods				Sex				Interaction (Method X Sex				Error		
Cognitive-Level	SS	df	MS	F	SS	df	MS	F	SS	df	MS	F	SS	df	MS
Knowledge	1840.89	2	920.45	1.83	1681.2	1	168.2	3.34	2170.19	2	1085.10	2.16	174159.5	346	503.35
Comprehension	375.53	2	187.77	0.41	650.6	1	650.6	1.43	99.94	2	49.97	0.11	156891.5	346	453.44
Application	491.92	2	245.96	0.71	16.21	1	16.2	0.05	284.06	2	142.03	0.41	120305.3	346	347.7
Analysis	960.56	2	480.28	1.81	503.0	1	503.0	1.89	288.01	2	144.01	0.54	92010.9	346	266.79
Total scores	600.24	2	300.12	2.71	62.51		62.5	0.57	251.53	2	125.76	1.14	38305.1	346	110.71

TABLE 20:

TREATMENT MEANS ADJUSTED FOR POST-TEST SCORES

Cognitive Level		Treatment Group	N	X - Means	Y - Means	Adjusted Y-Means
	PI	Boys	68	60.7	53.31	53.4
		Girls	55	56.6	63.6	64.2
Knowledge	IPI	Boys	66	63.6	61.4	60.4
		Girls	58	56.0	61.6	63.3
	CI	Boys	58	60.8	59.9	56.8
		Girls	48	63.1	56.8	53.4
	PI	Boys	68	46.3	51.8	52.3
		Girls	55	58.2	50.0	49.7
Comprehension	IPI	Boys	66	55.3	54.2	54.0
		Girls	58	50.4	52.2	52.3
	CI	Boys	58	57.8	55.6	55.3
		Girls	48	52.1	51.0	51.1
	PI	Boys	68	49.2	43.1	42.7
		Girls	55	40.6	43.5	44.5
Application	IPI	Boys	66.	46.2	46.6	46.6
		Girls	58	47.1	44.2	44.1
	CI	Boys	58	48.0	46.8	46.8
		Girls	48	48.2	46.2	46.0
	PI	Boys	68	36.2	35.9	36.3
		Girls	55	37.5	33.5	33.8
Analysis	IPI	Boys	66	39.7	38.5	38.6
		Girel	58	41.4	38.3	38.1
	CI	Boys	58	41.4	36.9	36.7
		Girls	48	44.2	32.5	32.0
Total	PI	Boys	68	47.7	45.3	45.8
		Girls	55	46.7	46.4	47.1
Scores Without Considering the cognitive Level	IPI	Boys	66	49.6	50.2	49.9
		Girls	58	47.8	47.7	48.1
	CI	Boys	58	51.1	48.1	47.3
		Girls	48	51.4	45.8	45.0

4.2.4.2 ANALYSIS OF RETENTION TEST-SCORES ON THE COMPREHENSION COGNITIVE LEVEL

A two-way analysis of covariance computed to find out whether there existed any differences in retention of probability material among the subjects of the three treatment groups revealed no significant differences. The results of the analysis of data are summarized in tables 19 and 20.

On an examination of table 20 page 162 it is clear that:

1. the boys of the PI group had an increase of 5.5 marks in their retention scores over their post-test scores while the girls of the same instructional group dropped by 8.2 points.
2. the subjects in the IPI group show that they have retained the material learned, the retention scores and post-test scores not being markedly different. The means for the CI group show a similar trend.

All the t-values computed to compare the means of different groups were not statistically significant. However, the mean retention score for boys of the CI group nearly differed significantly from the mean retention score for the PI girls ($t = 1.43$, $P > .05$), $M_1 - M_2 = 5.6$). The CI boys had a higher retention mean than the PI girls. On the whole the mean retention scores for the boys of the CI group was higher than that for any single sex group in the study (adjusted mean, 55.3, Table 20).

4.2.4.3. ANALYSIS OF RETENTION TEST-SCORES ON APPLICATION COGNITIVE LEVEL

The F-ratios computed to compare differences in retention for sex, teaching methods and sex by method interaction were not significant. Table 19 page 163 presents a summary of the results.

A look at table 20 page 162 shows that there was a drop in retention for the boys of the PI group, the girls of the IPI group and the girls of the CI group. Some small increase in retention scores was shown by the PI girls, the IPI boys and the CI boys. On the whole, the CI boys' retention scores were slightly higher than for any single sex group in the study. The mean retention scores for the PI boys was the lowest. There were no significant group differences in retention.

4.2.4.4. ANALYSIS OF RETENTION TEST-SCORES ON THE ANALYSIS COGNITIVE LEVEL

There were no significant differences among the three treatment groups, sex or sex x method interaction. Table 19 gives the F-ratios for this cognitive level.

All the subjects in each of the three treatment groups had a drop in their retention scores (see table 20). The girls receiving conventional instruction had the greatest drop - 11.7 points while the boys receiving programmed instruction had the least drop - 0.3. The results indicate that boys who learned probability individually through programmed materials and boys who were taught by the program and the teacher retained greater material than the other subjects. Boys in each treatment group scored relatively higher points in the retention test than girls in a similar treatment group. On the post-test, girls in each treatment group scored higher than boys in a similar group on the retention test, the situation was reversed. This may be interpreted to mean that girls are not capable of retaining material at a higher cognitive level.

4.2.4.5. ANALYSIS OF TOTAL RETENTION SCORES

The total probability retention scores were also subjected to an analysis of covariance, with the probability post-test scores used as the covariate and the retention scores as the criterion. The purpose was to find out whether there were any differences in retention among the subjects of the three treatment groups when total scores were considered. All the variances were statistically insignificant. The variance for teaching methods approached significance at the .05 level. Table 19 summarizes the results of the analysis of covariance.

An examination of table 20 reveals the following:

1. Boys in the IPI group increased their scores in the retention test.
2. All the other subjects dropped in their retention test scores. The girls in the CI group dropped by 5.6 points followed by boys in the CI group by 3.0 points. The boys in the PI group dropped by 2.4 points. The lowest drop was observed in the PI girls (0.3) and in the IPI girls (0.1).
3. Girls who went through programmed materials

individually scored higher than boys who received the same instruction.

4. In the IPI and the CI groups, boys scored higher than girls of the same group in the retention test.

The foregoing discussion reveals that the PI girls,the IPI boys and IPI girls retained more material than the boys of the PI group, the boys of CI group and the girls of the CI group.

4.2.5.0. CORRELATIONAL STUDIES

Variables for the prediction of probability post-test achievement and retention test scores were also investigated in this study. For the prediction of probability post-test achievement, mathematical ability, reading ability attitude towards mathematics and attitude towards the program, were each correlated with the post-test achievement scores by the Pearson product-moment correlation coefficients. The Pearson product-moment correlation coefficients were also computed for pre-test scores and retention test scores, for post-test scores and retention test scores to determine which variable, pretest or post-test was a better predictor of retention at each of the four cognitive levels-knowledge, comprehension application and analysis.

4.2.5.1. PREDICTION OF POST-TEST ACHIEVEMENT

Table 21 indicates the product-moment correlation coefficients between each of the predictor variables and the post-test scores.

TABLE 21:

PRODUCT-MOMENT CORRELATIONS BETWEEN PREDICTOR VARIABLES AND POST-TEST SCORES

Treatment Group	Mathematics Ability		Reading Ability		Attitude Towards Mathematics		Attitude Towards the Program	
	r	z	r	z	r	z	r	z
PI:Total	0.131	1.452	-0.035	-0.381	-0.096	-1.063	-0.118	-1.3
Boys	0.188	1.554	-0.0236	-0.193	-0.031	-0.252	-0.[illegible]	-0.609
Girls	0.0514	0.378	0.318	-0.384	-0.051	-0.374	-0.[illegible]	-1.499
IPI:Total	0.351	3.89*	0.273	3.023*	0.1904	2.1124+	-0.01	-0.07
Boys	0.424	3.41*	0.290	2.338+	0.0954	0.769	-0.0[illegible]	-0.264
Girls	0.24	1.841≠	0.254	1.915≠	0.306	2.31+	0.[illegible]	0.169
CI:Total	0.205	2.099+	0.161	1.647≠	0.255	2.612+		
Boys	0.290	2.192+	0.111	0.84	0.263	1.989+		
Girls	0.0959	0.657	0.223	1.531	0.245	1.679≠		

*P < .01 +P < .05 ≠P < .05 (one-tailed)

The above table reveals that mathematics ability was a good predictor of achievement for the integrated programmed instruction group, where the correlation coefficients were all significant. For the conventional instruction group. the correlation coefficients of 0.205 for total scores and 0.290 for boys' scores were significantly greater than zero at the .05 level. There was no significant correlation between the girls' scores on mathematics ability and their scores on probability post-test in the conventional instruction group. The low correlation coefficients of the PI group are not significant. But a correlation of 0.188, z = 1.554 for the PI boys approached significance at the .05 level of significance.

The correlation coefficients between boys' scores on mathematics ability and their scores on probability post-test are relatively higher than those for girls. On this basis, one can conclude that mathematics ability is a better predictor of achievement for boys than for girls.

Reading ability seems to be a good predictor of achievement for the subjects in the integrated programmed instruction group. The correlation coefficients between probability achievement and reading ability for the other two instructional groups are fairly low. An interesting case is seen in the low negative correlations for the programmed instruction group, an indication that for this group reading ability is not a good predictor of achievement. It was observed from table 16 that the mean performance in reading ability test for the programmed instruction group was the highest ($\overline{X}$ = 59.8) which according to earlier findings should mean that this instructional group should have performed better in an achievement test. From the correlational results therefore, it seems appropriate to assert that better reading cannot be attributed to higher achievement in probability.

There were low negative correlations between attitude towards mathematics and probability achievement scores for the subjects in the programmed instruction group. An examination of table 16 page 152 reveals that girls of the PI group favoured mathematics more than any other sex group in the study. This is shown by a relatively higher mean attitude score of 9.50 for the PI girls. One would expect a high attitude score to be positively and highly related to higher achievement. But as it stands here, this was not the case. The low negative correlations exhibited by the subjects in this treatment group may imply that a high positive attitude cannot be attributed to higher achievement in probability.

For the IPI group, it was found that significant correlations existed between attitude towards mathematics scores and probability post-test scores when the total scores were considered ($r = 0.1904$, $P < .05$). The correlation for boys was not significant ($r = 0.0954$, $z = 0.769$, $P > .05$). The correlation for girls of the same group was significant ($r = 0.306$; $z = 2.31$, $P < .05$).

It was also found that significant correlations existed for the subjects in the conventional instruction group at the .05 level of significance. It can therefore, be interpreted that with the exception of boys in the IPI group, attitude towards mathematics is a good predictor of achievement for the subjects in the IPI and the CI groups. Hence, for these groups, Murdoch's (6, 1968) contention that if liking for a subject is great, then learning is enhanced was supported.

All the correlation coefficients between attitude towards the program and probability achievement scores are low and insignificant (see table 21). It may mean that the subjects did not understand the items in the attitude towards the program questionnaire, although each item in the attitude inventory was thoroughly explained to them by their regular teachers. Possibly, a percentage count of the subjects response to each item would have provided a more precise picture of the nature of the subjects' response to each item.

4.2.5.2. PREDICTION OF RETENTION

Table 22 presents the Pearson product-moment correlation coefficients between pre-test probability achievement, post-test probability achievement and retention test scores, on each of the four cognitive levels - knowledge, comprehension, application and analysis.

---

6 Ibid.

TABLE 22:

PRODUCT-MOMENT CORRELATIONS BETWEEN PRE-TEST, POST-TEST AND RETENTION TEST ON THE FOUR COGNITIVE LEVELS

Cognitive Level	Pre-test and Retention test Scores	Post-test and Retention test Scores
Total Scores	0.176 ; $P < .05$	0.329 ; $P < .05$
Knowledge	0.1298 ; $P < .05$	-0.628 ; $P < .01$
Comprehension	0.0459 ; $P > .05$	0.0768 ; $P > .05$
Application	0.0623 ; $P > .05$	0.138 ; $P < .05$
Analysis	-0.0766 ; $P > .05$	0.119 ; $P > .05$

There exists a significant positive relationship between pre-test scores and retention scores ($r = 0.176$, $P < .05$) and a significant positive relationship between post-test scores and retention test scores ($r = 0.329$, $P < .05$) when total scores are considered. The relationship between post-test scores and retention test scores is stronger than that between pre-test scores and retention test scores.

On the knowledge cognitive level, there is a strong negative relation between post-test and retention test scores ($r = 0.628$, $P < .01$), while the relation between pre-test scores and retention scores is positively low ($r = 0.1298$).

The correlation coefficients on the comprehension level are very low with that between post-test and retention test being relatively higher than that between pre-test and retention test scores. This trend also obtains for the application and analysis cognitive levels, namely, the correlation between post-test and retention test is greater than that between pre-test and retention test scores. From this discussion it is clear that the post-test scores are a better predictor of retention than the initial pre-test scores.

Further product-moment correlation coefficients were computed to provide more information on the predictive nature of the variables under consideration, namely, pre-test scores and post test scores for the sex groups in each of the three instructional groups. Table 23 shows the correlations for the instructional groups.

TABLE 23:

PRODUCT-MOMENT CORRELATIONS BETWEEN PRE-TEST, POST-TEST AND RETENTION TEST FOR TOTAL SCORES

Method of Learning		Pre-test and Retention test	Post-test and Retention test
PI:	Total	0.235; $P < .05$	0.126; $P > .05$
	Boys	0.210; $P < .05$ (one-tailed)	0.192; $P > .05$
	Girls	0.269; $P < .05$	0.0317; $P > .05$
IPI:	Total	0.145; $P > .05$	0.464; $P < .01$
	Boys	0.123; $P > .05$	0.49; $P < .01$
	Girls	0.137; $P > .05$	0.424; $P < .01$
CI:	Total	0.225; $P < .05$ (one-tailed)	0.361; $P < .01$
	Boys	0.145; $P > .05$	0.394; $P < .01$
	Girls	0.203; $P < .05$ (one-tailed)	0.322; $P < .05$

An examination of table 23 reveals the following:

1. Significant positive correlations existed between pre-test scores and retention test scores at the .05 level for the PI group. The correlations between post-test scores and retention test scores for this same group were not significant at the .05 level of significance. This means that for the individualized programmed instruction group pre-test scores were a better predictor for retention than post-test scores.

2. The results for the integrated programmed instruction group show that the correlations between post-test scores and retention scores were positively higher than correlations between pre-test and retention test scores. Further, the correlations between post-test scores and retention test scores were significant at the 1% level while correlations between pre-test and retention test scores were not significant at the 5% level. The results for this instructional group indicated that post-test scores are a more valid predictive variable for retention than pre-test scores.

3. The correlations between post-test and retention test scores are higher than correlations between pre-test and retention test scores for the subjects in the conventional instruction group, an indication that post-test scores are a better predictor of retention than pre-test scores.

CHAPTER FIVE

SUMMARY, INTERPRETATIONS, RECOMMENDATIONS AND CONCLUSIONS

5.0 SUMMARY

This study has described the design procedure and analysis of the investigation conducted to determine the relative effectiveness of three modes of instruction: - Programmed Instruction, Integrated Programmed Instruction and the Conventional mode of instruction and has assessed the relative effectiveness of the three modes of instruction upon different sex groups.

Differences among the three instructional groups in variables such as reading ability, mathematical ability, attitude toward mathematics and attitude toward the program have also been examined. Further examinations have been made on the relationship between these predictor variables and achievement in probability post-test; and between pre-test achievement, post-test achievement and retention test.

For data analysis,[8] one-way analysis of variance, a two-way analysis of covariance, t-tests and the Scheffe' test have been used. Analysis was done by means of a desk calculator.

A one-way analysis of variance was used to test for differences among the treatment groups in the following variables:

1. probability pre-test achievement;
2. reading ability test;
3. mathematical ability test; and
4. attitude toward mathematics.

A two-way analysis of covariance was used to compare subjects' scores in probability post-test and retention test. The t-testswere used to compare the performance between boys and girls in the study. The Scheffe' test and the special t-test were computed following significant analyses of variance and covariance respectively to find out which pairs of means differed significantly. The Pearson product-moment correlation coefficients were computed to find out which variables were good predictors of achievement and retention.

5.1.0. SUMMARY OF FINDINGS

This section gives a summary of the findings pertinent to the hypotheses stated on page 8 chapter one on each of the four cognitive levels, and for the two sex-groups.

5.1.1. SUMMARY OF FINDINGS OF POST-TEST ACHIEVEMENT ON KNOWLEDGE SUB-TASKS

The hypothesis of no difference among the three instructional groups and between the two sex groups was accepted. But an inspection of the subjects' mean performance revealed that:

1. Girls who learned by the conventional method performed better on the knowledge sub-tests than boys and girls in the other instructional groups,

2. Boys learned better when they received programmed instruction supplemented by teacher instruction.

5.1.2 SUMMARY OF FINDINGS OF POST-TEST ACHIEVEMENT ON THE COMPREHENSION SUB-TASKS

The hypothesis of no differences among the treatment groups and between the two sex groups was supported. But different sex groups were found to interact significantly with different teaching methods. The analysis of results showed that boys performed relatively better in comprehension tasks when they were taught by the human teacher than when they received other instructions. Girls on the other hand learn better when they go through self - instructional materials.

5.1.3. SUMMARY OF FINDINGS OF POST-TEST ACHIEVEMENT ON THE APPLICATION COGNITIVE LEVEL

The hypothesis of no differences among the three instructional groups was again accepted. But that of no difference between the two sex groups was rejected. A significant interaction between the two sex groups and the instructional methods was also found.

Girls were found to perform well when they received teacher instruction than when left to study on their own through the program. The boys who learned through the program were relatively superior in performance to the boys who were taught by the program and the teacher. In the conventional instruction group, the two sex groups were comparable. When the overall performance of boys and girls was considered on this level, boys were found to have performed significantly better than girls.

5.1.4. SUMMARY OF FINDINGS OF POST-TEST ACHIEVEMENT ON ANALYSIS COGNITIVE LEVEL

The hypothesis of no difference among the treatment groups was rejected. The integrated programmed instruction and the conventional instruction groups performed better on analysis subtasks than the programmed instruction group. On the whole, the performance by the conventional instruction group was superior to that of the other two instructional groups.

In summary, the results of this study have shown that in an achievement test, there were no wide differences among the three instructional groups on knowledge, comprehension and application subtasks; but that treatment differences did exist on analysis subtasks.

On knowledge/and application subtasks, girls were found to perform well in the achievement test when they were taught by the teacher, while on the comprehension subtasks they learned better through self-instructional materials.

Pupils who received teacher instruction performed well on analysis subtasks. The performance of those pupils who received individualized programmed instruction was the poorest.

It can be said that on higher cognitive levels, pupils who received individualized instruction performed poorly while those who had the support of the teacher performed well. This argues well for the program to be supplemented by teacher instruction. And Banghart's (1968) contention that programmed materials are most effective when used to supplement the classroom teacher was supported by the findings of this study.

5.2. SUMMARY OF ANALYSIS OF PREDICTOR VARIABLES

The hypothesis of no difference among the three instructional groups in the variables for the prediction of achievement - reading ability, mathematical ability, attitude toward mathematics and attitude toward the program were all accepted.

The following were, however, revealed from the subjects' mean scores:

1. On reading ability, the programmed instruction group performed better than the other two instructional groups with the programmed instruction boys being relatively better readers than all the other subjects in the study. The programmed instruction girls were better readers than their counterparts in the other instructional groups. The integrated programmed instruction group's reading performance was the poorest.

2. When the overall performance was compared for boys and girls in the mathematical ability test, boys were found to be superior to girls.

   (a) The IPI boys' mathematical ability scores were comparable to the CI boys' scores.

   (b) the IPI boys and CI boys performed better than the CI girls. The poor performance by the CI girls can be interpreted to mean that the CI girls were unable to perform mathematical computation and reasoning tasks.

3. On the attitude toward mathematics scores

   (a) Girls of the PI and CI groups slightly favoured mathematics more than the boys in the same treatment groups and also more than the subjects in the IPI group.

   (b) The lowest preference for mathematics was exhibited by the girls of the IPI group.

4. Pupils who learned individually by the program found the programmed materials more interesting than those who learned through the program and the teacher. In the PI group boys favoured the program more than the girls. In the IPI group both boys and girls equally favoured the program.

On the whole, boys were found to favour the program more than the girls.

5.3. SUMMARY OF ANALYSIS OF RETENTION TEST.

The hypothesis of no difference in retention among the three instructional groups was supported in each of the four cognitive levels.

Though there were no differences in retention among the three treatment groups, an examination of the mean scores revealed the following:

1. On the knowledge subtasks, the PI girls retained more material than either the PI boys, the IPI or the CI groups.

2. On the comprehension subtasks, the PI boys increased in their retention scores while girls of a similar group dropped in their retention scores.

   On the whole, the mean retention scores for boys of the CI group was higher than for any single sex group.

3. There was a drop in retention scores by the boys of the IPI, PI and CI groups. A small increase in retention was shown by the girls of similar treatment groups.

   On the whole, the CI girls' retention scores were slightly higher than those of the other subjects. The mean retention score for the boys was the lowest.

4. On the analysis subtests, all the subjects dropped in their retention scores. The CI girls had the greatest drop and the PI boys the lowest drop. The results showed that PI boys and IPI boys retained greater material than the other subjects in the study. Boys in each treatment group scored

relatively higher than girls of similar treatment groups.

5. When the subjects were compared for their total scores it was found that the IPI boys increased in their retention scores while the other subjects dropped in their retention scores. The PI girls and the IPI girls had the lowest drop. The boys of the IPI and the CI groups scored higher than girls. The PI girls scored higher than the PI boys.

On the whole, it can be said that the PI girls, the IPI boys and girls showed a greater retentive power than boys of the PI and CI groups and the CI girls.

It seems appropriate to assert here that though the subjects did not show wide differences in the retention test, some small differences did, however, exist. The girls of the PI group seem to have shown a superior retentive power on those tasks that required the recall of specifics. The CI group's superiority was evident on comprehension subtasks. It is interesting to note that the scores for the girls of the three treatment groups were relatively higher than those of the boys in similar groups.

When the total scores were considered the retention die rolled in favour of the PI girls, IPI boys and IPI girls. In other words, with the exception of the PI boys, the two programmed groups showed relative superiority in retention over the CI group. One possible reason that can be advanced for the superiority shown by the programmed group is that these two groups took the programs home with them at the end of the investigation. It is suspected that they may have continued to read the program. No reason can be found for the poor retention by the PI boys, particularly when their attitude to programmed materials is considered. The boys of this treatment group significantly favoured the program more than the girls of a similar group. More would possibly have been gained if attitude scores had been correlated with retention test scores.

5.4. SUMMARY OF CORRELATIONAL ANALYSIS

5.4.1. PREDICTION OF POST-TEST ACHIEVEMENT

The results of correlational analysis show that mathematical ability was a good predictor of achievement for the boys and girls who received programmed instruction supplemented with teacher instruction in small groups. When total scores were considered, it was found that mathematical ability was a better predictive variable for boys than it was for girls.

Reading ability was also found to be a good predictor of achievement for the IPI group. It is interesting to note that the correlation coefficients for the PI group were all negative and low. This may indicate that reading ability is not a good predictor of achievement for the subjects who learned through this method.

It was observed from table 16 page 152 that the mean performance in reading ability test for the PI group was the highest, which, according to research literature should imply high performance in the achievement test.

From the correlational results, therefore, it seems appropriate to assert that for this group the ability to read cannot be attributed to higher achievement in probability. It may also mean that comprehension of the reading ability test items may not necessarily be associated with comprehension of programmed materials.

Low correlations were recorded between attitude toward mathematics scores and post-test achievement scores for the PI subjects. Inspection of table 16 page 152 revealed that the girls favoured mathematics more than the boys. This was shown by a relatively higher mean attitude score of 9.50 for the PI girls. One would expect a high attitude score to be positively and highly related to higher achievement. But this was not the case. Hence it may be said that a high positive attitude towards a subject cannot be attributed to higher achievement in probability.

With the exception of boys in the IPI group attitude toward mathematics was a good predictor of achievement for the subjects in the IPI and the CI groups. For these two groups, Murdoch's (1968) contention that if liking for a subject is great, then learning is enhanced, was supported.

All the correlation coefficients between attitude toward the program scores and probability achievement scores were low and insignificant. No suitable reason can be advanced for this surprise results. The queer results may be interpreted to mean that the attitude toward the program was not a good predictor of achievement for all the three instructional groups and for the two sex groups.

The boys of the PI group favoured the program more than the girls of the same group. This favourable attitude, as suggested by Murdoch should have been followed by high achievement.

5.4.2. PREDICTION OF RETENTION

Predictor variables for retention, namely, pre-test achievement and post-test achievement were examined for total scores and for each of the four cognitive levels.

There was a positive relationship between pre-test and retention scores and a significant positive relationship between post-test and retention test scores, the relationship between post-test and retention test scores being stronger than that between pre-test and retention test scores, for total scores.

On the knowledge cognitive level, the relationship between post-test and retention test was strongly negative, and significant while the relation between pre-test and retention test scores was positively low.

On the comprehension, application and analysis levels, all the correlations were very low. The correlation between post-test and retention test were relatively higher than those between pre-test and retention test scores.

In summary, it can be said that post-test scores are a better predictor of retention than the initial pre-test scores for total scores and for each cognitive level.

Correlation coefficients were also computed between pre-test, post-test and retention test for total scores, for boys and girls for the three instructional groups.

For the PI group the correlation between pre-test and retention for total scores, for boys and girls were all significant, while those between post-test and retention test were not significant - an indication that pre-test scores were a better predictor of retention than post-test scores.

For the IPI and the CI groups, all the correlations between post-test scores and retention test scores were significantly higher than those between pre-test and retention test scores. This means that post-test scores predict retention more than the initial pre-test scores.

5.4.3. INTERPRETATION AND RECOMMENDATIONS

The present study has found that the treatments had no effect on the subjects' performance for knowledge, comprehension and application cognitive levels. For the analysis cognitive level the treatments affected the pupils' scores. The IPI and the CI methods were superior to the PI method and the CI method was superior to the IPI method. This means that the CI method is best suited for higher cognitive processes. The findings here have contradicted some of the research findings from the west which have attested to the general effectiveness of the program as a method of instruction. The findings from the west that programmed instruction combined with teacher instruction produce better results than individualized programmed learning have been supported by the results of this study.

The present investigator agrees with Banghart's observation that if programmed materials are well designed and well tested and teachers trained to competently supervise programmed learning, then one can expect a significant achievement in favour of programmed materials. The teachers involved in the present study had a brief training period (4 days) in the use of programs and in the topic taught, i.e. probability. The training period was, however too short to enable the teachers to competently handle the programs. It should also be noted that this was the first time such teachers were exposed to programs. This factor, i.e. lack of competence to handle programs may have contributed to the poor performance by the pupils of the programmed instruction group. It is therefore recommended that future researchers of programmed learning train teachers for a longer time in the use of programs to ensure efficiency in utilizing programmed materials.

It was mentioned in section 1.4 that it

was not possible to employ the services of teachers of the same grade and to have a single teacher responsible for the three instructional groups. It was however hoped that the training of the teachers in probability would help to remove some teacher variability with regards to the content of the subject. But the training of teachers in the content to be taught would not remove teacher interest in probability or teacher competence to handle the topic. Both variables, namely teacher interest and teacher competence in the topic could have affected the performance by the subjects in the programmed instruction and programmed instruction combined with teacher instruction groups. It is therefore recommended that in any future investigation into the effectiveness of programmed instruction, teacher interest in the subject and teacher competence to handle the subject should be carefully looked into.

On occasional visits to experimental schools during the course of instruction, the investigator observed that some IPI teachers did not actually integrate with the program. Instead their classes reverted to individualised programmed instruction. This is likely to have contributed to the relatively poor performance by the IPI group in comparison to the performance by the CI group. One would expect the IPI group's performance to be much better than either the PI group's or the CI group's performance if the IPI teachers had really integrated with the program. Future researchers should ensure that IPI teachers really integrate with the program to produce better results as suggested by Baghart and Jamieson who emphasize the supportive role of the teacher as being a significant factor in programmed learning.

Instruction was conducted during the normal mathematics periods and all the three instructional groups had the same amount of learning time. In order to ensure that the subjects of the three treatment groups were in contact with learning materials for approximately the same amount of time, the programs from the PI and IPI groups were collected at the end of every lesson. It was observed that the programmed instruction group took a relatively shorter time to complete the program. This observation agrees with Meadowcroft's observation. The results of his study showed that although programmed instruction was not superior to the textbook method, it was more efficient in saving student time.

It is not known how much time the PI group took to complete the whole program as this was not measured by the teachers of the PI group as was required. Time taken by each individual to complete a program is an important factor and should therefore be measured and correlated with post-test achievement in future investigations. It is also necessary to know how much time a teacher spends with each subgroup in the IPI group as the intensity of teacher interaction with each subgroup is likely to affect the performance of the whole class. Hence it is suggested that future researches look into this variable.

On pages 108, 109 and 111 it was mentioned that the pupils of the PI and the IPI groups did self-tests at the end of each booklet and that the pupils in the CI group were given class exercises and homework. This means that the CI group had extra work which the other two groups did not have. This may have contributed to the superior performance by the CI group.

The exercises for the CI group and the self-tests for the PI and the IPI groups were comparable. The post-test sampled the information taught in the program and the lesson. The post-test, the self-test and the exercises were closely related in most cases. For this reason it can be said that the criterion test did not favour any one method to the exclusion of the others. It is therefore the opinion of the investigator that the observed differences were not due to the test procedures as found in Roebuck's study.

In this study, some PI teachers' apparent negative attitude towards the program was noted on two occasions, one, when they failed to record time taken by each pupil to complete a program and two, when they raised complaints that the program was too long and that if they continued using it, they would not finish the syllabus in time. This suggests that these teachers did not properly utilize the programmed materials and as has been noted elsewhere,many have contributed to the relatively poor performance by the PI group in comparison to the CI group. Teachers' attitude both towards the subject and the method by which that subject is presented is an important variable that can affect students' performance. It is unlikely that a teacher with low value attachment to mathematics will motivate his pupils into liking the subject. Similarly, a teacher who is not sure of a particular method is not likely to efficiently guide children to learn by that method. It is therefore necessary for future researchers of programmed learning to examine teachers' attitude towards mathematics and towards the program as a method of instruction, and whether instruction would produce attitude changes.

One of the most important variables in programmed learning is a child's reading ability. It is hypothesized that if a child can perform well in a reading ability test then he can read and understand programmed materials on his own. The results of this study do not support this hypothesis. The correlation coefficients for the PI group between reading ability test scores and post-test scores are negative and low, suggesting inverse relationship. This may imply that the reading ability test used in this study did not use the same terminology used in the program and can therefore be regarded as not being valid with regards to modern mathematics programs like probability.

The advantages of programmed instruction are to be found in the retention test. Retention scores are more important than immediate post-test scores. Hence a teaching method that produces greater retention power is most ideal. In this study, the three teaching methods, in general, seem to have produced equal retentive power. But a close examination of total scores reveals that the IPI group retained more material than the other two groups. This helps to boost the power of the IPI method as a powerful learning tool and supports Dick's contention that the benefits of paired learning are found in the retention of the material and not in the immediate performance.

In this study, item analysis on the attitude questionnaire was not undertaken. This should be done in future researches to establish the reliability and validity of the questionnaire.

CONCLUSION

The results of this study are limited to the group for which the study was undertaken.

However, it is hoped that these results will be of value to curriculum developers, administrators, primary school teachers, mathematics tutors in the teacher training colleges, and above all, to researchers in primary mathematics education in Kenya. Further, the findings of this study should open the way for further research in other areas of programmed learning which have been mentioned earlier in this section.

If the educational problems mentioned in the introductory section of chapter one are to be overcome, then programmed workshops should be established in the country where teachers interested in the use of programs can be trained in the construction and execution of the programmes. This will ensure widespread use of the programmes in Kenya and will subsequently give them a place in mathematics education.

During the investigation, some teachers expressed their fears of possible replacement by the programs, should they be found effective. The programs should not be seen as an attempt to replace the human teacher, but should be viewed as a powerful and effective aid for the teacher. As suggested by Lawless, the use of programs will release the teacher from the rigid pattern of class teaching.

REFERENCES

1. Thomas, C. A.,et al: Programmed Learning in Perspective. London Publicity Services, 1963.

2. Roebuck, M.: Frames from Ibadan: programmed learning in a West Nigerian Context. Bulletin of Programmed Learning. Reasearch Unit, Dept. of Education, August, 1973.

3. Kersh, B. Y.: Learning by Discovery: What is Learned? Arithmetic Teacher. Vol. II, 1974.

4. Glaser, R.: "Variables in Discovery Learning:" In L. S. Shulman and E. R. Keislar (Eds). Learning by Discovery: A Critical Appraisal. Rand MacNally and Co., Chicago, 1966.

5. Peel, E. A.: Some Psychological Principles underlying Programmed Learning. Educational Research. Vol. 5, No. 3, 1963.

6. Daniel, W. L. and Murdoch, P.: Effectiveness of Learning from a Programmed Text compared with a conventional Text Covering the same material. Journal of Educational Psychology: Vol. 59, No. 6, 1968 pp. 425-431.

7. Banghart, F. W., et al: An experimental study of Programmed Versus Traditional Elementary School Mathematics. Arithmetic Teacher. Vol. 10, No. 4, 1963, pp. 199 -207.

8. Jamieson, G. H.: Learning by Programmed and Guided Discovery Methods at Different Age Levels. Programmed Learning and Educational Technology. Vol. 6, 1969, pp. 26 - 30.

9. Meadowcroft, B. A.: The effects on conventionally taught eighth-grade math following seventh-grade math. The Arithmetic Teacher. Vol. 12 and 8, 1965, pp. 614 - 616.

10. Holmberg, I.: A Combination of Programmed Instruction and Teacher-supervised small group Instruction compared with Conventional Classroom Method. Didacometry No. 10 (Malmo, Sweeden: School of Education), 1966.

11. Eshiwani, G. S.: Sex differences in the learning of mathematics among Kenyan High School Students. Mathematics Education Research Report. No. 3, August, 1974.

12. Parkar, K. D.: The Impact of the Programmed Workcards on the Quality of Teaching Mathematics in the Secondary Schools of Kenya, M.Ed. Thesis, 1974.

13. Okunrotifa, P.O.: Attitudes of Nigerian Secondary School Children to Programmed Instruction in Geography. Educational Research Journal, Vol. 17, No. 2, Feb., 1975 pp. 110 - 114.

14. Roebuck, M.: A Definite Conclusion in a Comparison Between Conventional and Programmed Instruction. Programmed Learning and Educational Technology, Vol. 7, 1970.

15. Dick, W.: Individual use of Programmed Instruction. The Mathematics Teacher, Vol. 58, No. 7, 1965, pp. 649 - 654.

16. Bloom, B. S., et al: Handbook of Formative and Summative Evaluation of Student Learning. McGraw-Hill Book Co. 1971 pp. 271-273.

17. Keats, J. A.: An Introduction to Quantitative Psychology. John Wiley and Sons Australasia PTY Ltd., 1971.

GENERAL REFERENCES

18. Lawless, C. J.: Programmed Learning in the Developing Countries of Africa. Programmed Learning and Educational Technology. Vol. 6 No. 3. pp. 189-196.

19. Jacob, P. I.: Development, Use and Self Instructional Programs in Israel. Journal of Experimental Education. Vol. 36 No. 3 Spring 1968.

20. Keislar, E. R.: The Development of Understanding in Arithmetic by a Teaching Machine. The Journal of Educational Psychology. Vol. 50 Dec. 1959, pp. 247 - 253.

21. Flournoy, F.: Meeting Individual Differences in Arithmetic. The Arithmetic Teacher. Vol. 7, Feb., 1960, pp. 80 - 86.

22. Pixas, A.: Comparison Between Traditional and Programmed Learning as a Function of Passive Performance and Active Application and Time till Application. Programmed Learning and Educational Technology. Vol. 6, 1969.

23. Lambert, P.: Mathematical Ability and Masculinity. The Arithmetic Teacher, Vol. 7, No. 1 Jan., 1960.

24. Lipson, J. I.: Hidden Strengths of Conventional Instruction. The Arithmetic Teacher, Vol. 23, No. 1, Jan., 1976.

25. Roberts. F: Attitudes of College Freshmen Towards Mathematics. The Mathematics Teacher, Vol. 62, No. 1, Jan., 1969.

26. Schunen, H. L.: Self-Paced Mathematics Instruction: How effective it has been? The Arithmetic Teacher, Vol. 23, No. 2, Feb., 1976.

27. De Cecco, J. P.: Psychology of Learning and Instruction, Educational Psychology; Prentice-Hall Inc. Englewood Cliffs, New Jersey, 1968. pp. 483 - 95.

28. Leith, G. O. M., et al: A Handbook of Programmed Learning. University of Birmingham. 1964.

29. Fund for the Advancement of Education. Four case studies of Programmed Instructions, Georgian Lithographers, Inc. June. 1964.

30. Borg, W. R.: Educational Research: An Introduction, David Mckay Co., Inc., New York, 1963.

31. Lindquist, E. F. : Design and Analysis of Experiments in Psychology and Education, Honghton Mifflin Co., Boston, 1953.

32. Lewis, D. G.: Experimental Design in Education, University of London Press Ltd., 1968.

33. Yamane, T.: Statistics, An Introductory Analysis,Harper International Edition, 1973.

34. Schonnel, F. J.: Diagnostic and Attainment Testing, Oliver and Boyd Edinburgh, Great Britain, 1970.

35. K.I.E.: Kenya Primary Mathematics (Standard 6 and 7 pupil's books and Teachers copies): Jomo Kenyatta Foundation, 1976.

36. School Mathematics Study Group: Probability for Intermediate Grades (Students Text and Teachers Commentary). The Board of Trustees of the Leland Stanford Junior University, 1965 and 1966.

31. School Mathematics Study Group. Secondary School Mathematics (Special Edition. Student's Text and Teacher's Commentary): The Board of Trustees of the Leland Stanford Junior University, 1970.

APPENDIX A

PROGRAMMED INSTRUCTION

PROBABILITY

INTRODUCTION

The material presented to you here is in the form of a program for self-instruction. The subject matter covered in this program has been broken into items or frames which permit you to learn efficiently by studying and answering each step or frame separately.

The most effective way to study a program for self-instruction is to read and study each frame carefully, you should study definitions and formulas thoroughly as you go along so that you will be able to acquire new information step by step.

After you have studied a question frame, write out your answer fully on a separate piece of paper, then compare your answer with the answer given at the back of the page.

The material will be arranged in this manner:-

---

1. Look in your multiplication table.

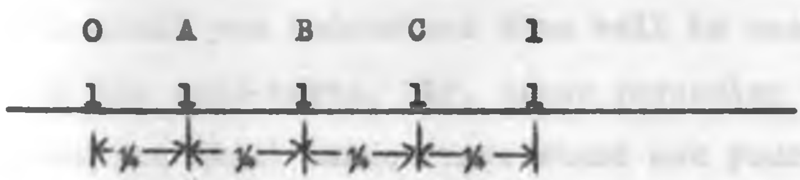
   Can you find an "answer" to

   $\frac{5?}{2}$ ______; $\frac{6?}{4}$ ______; $\frac{4?}{3}$ ______;

   $\frac{8?}{5}$ ______; $\frac{17?}{9}$ ______; $\frac{9?}{7}$ ______.

---

2. Look at the number line below.



   Into how many smaller segments have we divided the unit segment (the length from O to 1)? ____________

---

3. Refer to frame 2.

   What rational number is shown by the length of the segment from.

   (a) O to A? __________
   (b) O to B? __________
   (c) O to C? __________
   (d) O to 1? __________

Answers.

1. $2\frac{1}{2}$; $1\frac{1}{2}$; $1\frac{1}{3}$; $1\frac{3}{5}$; $1\frac{8}{9}$; $1\frac{2}{7}$.

2. 4 smaller segments

3. (a) ¼
   (b) ½ (c) ¾ (d) 1.

You are to write down your answer in the blank of each frame. Then compare your answer with the answer given at the bottom of each page. If you do not get a question at the end of a frame correct, read again the frame corresponding to the question.

At the end of each section there is a short test intended as a self-test. The answers are given following this test. If you do not get the questions in the self-test correct, read the frames again until you understand them well to enable you to do the self-tests. If, after rereading the frames you still cannot understand ask your teacher to help you.

Why study probability

Probability, is an important branch of mathematics. It is used in making decisions in military operations, scientific research, design and quality control of manufactured products, insurace. calculations, governmental operations, etc. It is also important in all games of chance.

When learning about probability, you are learning about a very important branch of mathematics.

This unit is divided into three sections. Section one deals with ideas about chance, section two is on Experiments in Probability and section three is about Finding Probability.

Francis Obunga-Okambi

1. Thinking about chance.

Materials.

Materials needed for this unit include dice, coins - 5-cent piece, 10-cent piece and 50-cent piece; marbles and spinners.

Terms to be learned.

Likely, unlikely, chance, probably, certain, uncertain, probability, fair, unfair.

Purpose.

To stimulate pupils to think more objectively about chance events. Through participation, discussion, and sometimes, demonstrations by the teacher, pupils will have opportunities to test their intuition regarding the results of some activities involving chance and to make guesses, estimates, and predictions about such results.

Suggested time:- 5 to 6 lessons.

Introduction:

You probably have heard or made statements like:

1. It is more likely that I shall go to see my uncle during the holidays.
2. Chances are good that my father will buy me a shirt at the end of this month.

3. Kamau and Barasa have equal chances to win.
4. I am almost certain that I can come to your house after school.

These sentences are alike in one way. They have words and ideas which are used in mathematics. These words and ideas are used in a part of mathematics called probability. In probability we are interested in things which happen by chance. By using mathematics we can often estimate quite accurately what will probably happen.

---

1. Answer the following questions:
   a) Which footbal club will win the East and Central Africa Club Championships cup next year? ______________
   b) Will all the members of your class be in school next Monday? ______________

---

2. The questions in frame 1 are chance events. Can you be certain of their answers?

---

3. Some things are more likely to happen than others,
   a) Which is more likely, that one of the pupils in this class will be absent or that the mathematics teacher in this class will be absent? ____________________

b) Which is more likely, that you will have ugali for breakfast or that you will have ugali for lunch? ____________

---

4. Some things are more likely to happen than not.

a) In Kisumu in July, is it more likely than not that it willxrain at noon? ________

b) Is it more likely than not that you can find the sum of 324 and 465? __________

---

5. Some things are certain and some things are impossible. Write C if an event is certain or I if the event is impossible.

i) A man can live without water for three months___

ii) Barasa's dog can write his first and last names in swahili ____________

iii) All new cars from China this year will use water for fuel.______________

iv) Tomorrow, today, will be yesterday.________

Answers:

4 (a) Not likely, (b) More likely, 324 + 465 = 789

5. (i) I (ii) I (iii) I (iv) I

6. Our ideas about chance might be classified as <u>certain</u>, <u>uncertain</u>, or <u>impossible</u>.

In the following sentences, write C, U, or I for <u>certain</u>, <u>uncertain</u>, or <u>impossible</u>.

(a) It is ____ that the sun will set in the east.

(b) It is ____ that a river flows downhill.

(c) It is _____ that we will see the sun tomorrow.

(d) It is ____ that a river flows uphill.

(e) It is ____ that I will not sleep at all this week

(f) It is ____ that a river is deep today than yesterday.

---

7. When we say a teacher gives a test on Friday, it does not mean we are sure he is going to give one this Friday. We can use numbers to tell how likely it is that he will give a test this Friday.

Mrs. Obunga gave a test on 3 Fridays out of every 4 last year.

Mr. Ogoti gave a test on 7 Fridays out of every 8 last year.

Mrs. Okiya gave a test on 2 out of every 3 Fridays last year.

Mrs. Oyor gave a test on 20 out of every 21 Fridays last year.

Who is the most likely to give a test on Friday?____

Who is the least likely to give a test on Friday?___

Answers:

6. (a) I (b) C (c) U (d) I (e) I (f) U.

7. Mrs. Oyor, Mr. Ogot.

8. For Mrs. Obunga's 3 out 4, we write $\frac{3}{4}$.

For Mrs. Ogoti's 7 out of 8, we write $\frac{7}{8}$.

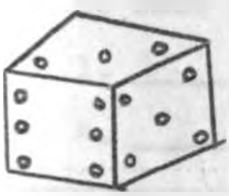
For Mrs. Okiya's 2 out of 3, we write ____

For Mrs. Oyor's 20 out of 21, we write ____

---

9. Write the outcomes in frame 8 as equivalent fractions. ______, ______, ______, ______.

10.



On the left is a picture of a die.

(a) How many faces has the die? ____

(b) List the number of dots on the remaining faces ____________

---

11. If the die on frame 10 is tossed once, there are six possible outcomes ____, ____, ____, ____, ____, ____.

Answers:

8. $\frac{2}{3}$ $\frac{20}{21}$

9. $\frac{126}{168}$, $\frac{147}{168}$, $\frac{112}{168}$, $\frac{160}{168}$.

10. (a) 6 (b) 1, 2, 4.

11. 1, 2, 3, 4, 5, 6.

12. If a die is tossed, the face that is on top is the one that counts. For example, in frame 10, the face with 3 dots shows up. So this is the face that we consider. If we were playing a game with this die, we would consider the result as a score of 3.

---

13. If the die in frame 10 is tossed once, how many times are the following numbers likely to show on the top face?

1. ________ 2. ________ 3. ________
4. ________ 5. ________ 6. ________

---

14. If we toss a die once, are there equal chances that a number on any of the six faces will show up? ______________

---

15. If events have equal chances of occuring, we say that they are equally likely. If you were playing a game with a friend and each one of you had an equal chance of winning, we would say that the game was <u>fair.</u> But if one of you had more chances of winning, we would say that the game was ______

Answers:

12. No answer is required
13. Once; once; once; once; once; once;
14. Yes.
15. Unfair.

16. You are to play a game with your friend. The game is "Toss a die once and see who wins."

In this game you win if I shows up. The other wins if 3 shows up. In order to decide whether the game is fair or unfair, we first list all the possible outcomes of the game. These outcomes are

_____, _______, ______, ______, _______, _______.

---

17. After we have listed all the possible outcomes of a single toss of a die, we then find the number of times I is likely to show out of the six possible occurrences. We also find the number of times 3 is likely to show on the top face of the die.

We see that I is likely to show on the top face once and 3 is also likely to show up on the face once We say that these events, I showing up and 3 showing up are _____ likely. And the two players have equal chances of winning the game. Therefore the game is ______________

Answers:

16. 1, 2, 3, 4, 5, 6.

17. Equally; fair.

18. In the game "Toss one die and see who wins", you win if an odd number shows up. The other player wins if an even number shows up. Write down the set of odd numbers and the set of even numbers that are likely to show up.

(a) (odd numbers) = (___, ___, ___).

(b) (even numbers) = (___, ___, ___).

(c) Are these events equally likely? ____

(d) Is the game fair or unfair? ________

---

19. In the game "Toss one die and see who wins", you win if 3 is up. The other player wins if a number greater than 3 is up. List the outcomes for each player.

(a) Outcomes for first player ________

(b) Outcomes for second player ____, ____, ____.

(c) Are these outcomes equally likely? _______

(d) Is the game fair or unfair ________.

Answers:

18. (a) (odd numbers) = (1, 3, 5.)
(b) (even numbers)= (2, 4, 6.)
(c) Yes (d) Fair.

19. (a) 3 (b) 4, 5, 6. (c) No
(d) Unfair.

20. If one die is tossed, there are 6 possible outcomes. If two dice are tossed, there are 36 possible outcomes. These are

(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)
(_,_), (_,_), (_,_), (_,_), (_,_), (_,_)
(_,_), (_,_), (_,_), (_,_), (_,_), (_,_)
(_,_), (_,_), (_,_), (_,_), (_,_), (_,_)

---

21. In frame 20 the first number in the ordered pair refers to the outcome on the first die, while the second number refers to the outcome on the second die. Thus in the outcome (1,3), 1 is the number that shows up on the first die and 3 is the number that shows up on the second die.

---

22. You are to play a game with your friend. The game is, "Toss two dice together". One die is white, the other die is green. In this game, you will win if 1 is on each die, that is you win if the outcome is (1,1).

Answers:

20. (4,1), (4,2), (4,3), (4,4), (4,5), (4,6)
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)

21. No answer is required.

The other player wins if 5 is on each die.

That is he wins if the outcome is (5,5).

(a) Outcome for first player ______

(b) Outcome for second player ______

(c) Are these outcomes equally likely? ______

(d) Is the game fair or unfair? ______

---

23. In the game of frame 22, you win if there is an even number on the white die. The other player wins otherwise.

(a) Outcomes for the first player are

(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)

(4,1), (_,_), (_,_), (_,_), (_,_), (_,_)

(_,_), (_,_), (_,_), (_,_), (_,_), (_,_)

(b) Outcomes for the second player are:

(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)

(3,1), (_,_), (_,_), (_,_), (_,_), (_,_)

(_,_), (_,_), (_,_), (_,_), (_,_), (_,_)

Answers:

22. (5,5)

(a) (1,1), (b) (5,5) (c) Yes

(d) fair, because each player has one chance out of 36 possible chances.

23. (a) (4,2), (4,3), (4,4), (4,5), (4,6)
(6,1), (6,2), (6,4), (6,5), (6,6)

(b) (3,2), (3,3), (3,4), (3,5), (3,6)

(5,1), (5,2), (5,3), (5,4), (5,5), (5,6).

24.(a) Are the outcomes in frame 23 equally likely?___

(b) Is the game in frame 20 fair or unfair? _______

---

25. In the game in frame 22, you win if 6 is on the white die, and he wins if 4 is on the green die.

(a) Outcomes for the first player are (6,1), (_,_), (_,_), (_,_), (_,_).

(b) Outcomes for the second player are (1,4), (,), (,), (,), (,)

(c) When will the two players tie?

____

---

26. (a) Are the outcomes for the players in frame 25 equally likely? ________

(b) Is the game in frame 25 fair or unfair?

____

Answers:

24. Yes Because each player has 18 chances out of 36 possible outcomes.

(b) The game is fair.

25. (a) (6,1) (6,2), (6,3), (6,5), (6,6)

(b) (1,4), (2,4), (3,4), (4,4), (4,5)

(c) There will be a tie if the outcome for each is (6,4).

26. (a) Yes, since each player has 5 chances of winning.
(b) The game is fair.

27. Refer to the game in frame 22.

You win if I is on each die. He wins if one die has 1 and the other die has 2.

(a) Outcome(s) for the first player (_,_)

(b) Outcomes for the second player (_,_), (_,_)

(c) Are these outcomes equally likely? ________

(d) Is this game fair or unfair? ____________

---

28. Refer to frame 22.

You win if the number on the white die is greater than the number on the green die. He wins otherwise.

(a) Outcomes for first player are

(2,1), (3,1), (_,_), (_,_), (_,_), (_,_),
(_,_), (_,_), (_,_), (_,_), (_,_), (_,_)
(_,_), (_,_), (_,_), (_,_), (_,_), (_,_)

Answers:

27. (a) (1,1), (b) (1,2), (2,1)
(c) No (d) Unfair

28. (a) (2,1), (3,1), (3,2), (4,1), (4,2), (4,3)
(5,1), (5,2), (5,3), (5,4)
(6,1), (6,2), (6,3), (6,4), (6,5).

28. (b) Outcomes for second player are:

(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)

(2,2), (2,3), (_,_), (_,_), (_,_)

(3,3), (_,_), (_,_), (_,_)

(_,_), (_,_)

(_,_)

---

29. (a) Are the outcomes of frame 28 equally likely?

____

(b) Is the game fair or unfair? __________

---

30. In this game one die is tossed twice. You win if the number the second time is greater than the number the first time. Otherwise he wins.

Answers:

28. (b) (1,1), (1,2), (1,3), (1,4), (1,5), (1,6)

(2,2), (2,3), (2,4), (2,5), (2,6)

(3,3), (3,4), (3,5), (3,6)

(4,4), (4,5), (4,6)

(5,5), (6,6)

29. (a) No. Player 1 has 15 out of 36 chances of winning. Player 2 has 21 chances of winning. The outcomes are therefore, not equally likely.

(b) The game is unfair.

30. (i) List the outcomes for the first player.

(1,2), (1,3), (1,4), (1,5), (1,6)

(2,3), (___), (___), (___),

(___), (___), '___)

(___), (___)

(___)

(ii) List the outcomes for the second player.

(1,1),

(2,1) (___)

(___), (___), (___),

(___), (___), (___), (___)

(___), (___), (___), (___), (___),

(___), (___), (___), (___), (___), (___).

Answer:

30. (i) (1,2), (1,3), (1,4), (1,5), (1,6)

(2,3), (2,4), (2,5), (2,6)

(3,4), (3,5), (3,6)

(4,5), (4,6)

(5,6)

(ii) (1,1)

(2,1), (2,2)

(3,1), (3,2), (3,3)

(4,1), (4,2), (4,3), (4,4)

(5,1), (5,2), (5,3), (5,4), (5,5)

(6,1), (6,2), (6,3), (6,4), (6,5), (6,6).

31. (i) Are the outcomes in frame 30 equally likely? __________

(ii) Is the game in frame 30 fair or unfair?

____

---

32. For the game in frame 30 you win if the number each time is even. He wins if the number each time is odd.

(i) List the outcomes for the first player.

(2 -), (2,4), (___), (___), (___)

(___), (___), (___), (___)

(ii) List the outcomes for the second player.

(___), (___), (___), (___), (___), (___)

(___), (___), (___)

---

33. (i) Are the outcomes in frame 32 equally likely?

____

(ii) Is the game in frame 32 fair or unfair? ____

Answers:

31. (i) No. Player 1 has 15 chances while player 2 has 21 chances.

(ii) The game is unfair.

32. (i) (2,2), (2,4), (2,6)
(4,2), (4,4), (4,6)
(6,2), (6,4), (6,6)

(ii) (1,1), (1,3), (1,5)
(3,1), (3,3), (3,5)
(5,1), (5,3), (5,5)

33. (i) Yes

(ii) The game is fair.

34. The word outcomes is often used in talking about probability. People often ask, "How did the football game come out?" or they might say "what was the outcome of the football game?".

In probability, when we talk about the outcomes of an activity, we mean all the things that can happen (all the possibilities). For a football game, for example, there are three possiblities or outcomes. Your team will win; your team will _____ or there will be a ____

---

35. In the game, "Toss a die once and see who wins," the first player won if an odd number showed up and the second player won if an even number showed up. We can make a list of outcomes and see whether or not the outcomes are equally likely. For instance for the first player, the outcomes were, (1, __, ___). For the second player the outcomes were (__, __, __). Since there are ____ elements in each set, we say that the outcomes are ___ likely.

Answers:

34. Lose
a draw or a tie.

35. Outcomes for first player: 1, 3, 5.
Outcomes for second player: 2, 4, 6.
3 elements
equally.

36. You are to play a game with your friend. The game is Toss a coin once and see who wins." You win if a tail shows up. Your friend wins if a head shows up.

(a) How many outcomes are there for the game ________

(b) List the outcomes ________

---

37. Refer to frame 36.
Write T if the following statement is true, if it is false, write F.

(a) My friend is more likely to win _____

(b) I stand a better chance of winning ____

(c) We are both equally likely to win _____

---

38. (a) What are the outcomes when you toss a die. Remember a die has six faces, and any one of these faces may be up. The outcomes are 1, 2, 3, __, __, __.

(b) Are these outcomes equally likely? _____

(c) Are there just six outcomes when you toss two dice?

(d) How many outcomes are there when you toss two dice?

Answers:

36. (a) 2 outcomes altogether
    (b) Head; Tail

37. (a) F (b) F (c) T.

38. (a) 1, 2, 3, 4, 5, 6.
    (b) Yes
    (c) No (d) 36 outcomes.

39. To make a list of the outcomes, you make a table. The left side of the table shows the number of dots on the white die. The top of the table shows the number of dots on the green die.

Use a number pair for each outcome. The first number is the outcome on the white die, and the second number is the outcome on the green die. If the white die has 1 up, the green die might have 1 also. This is shown as (1,1).

Finish the table below. Write a number pair for each outcome.

Green die

		1	2	3	4	5	6
White die	1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
	2	-	-	-	-	-	-
	3						
	4						
	5						
	6						

(a) How many different number pairs are shown in the table ________

(b) How many outcomes are there for tossing two dice ____________

(c) Are all these outcomes equally likely? ____

Answer:

39.

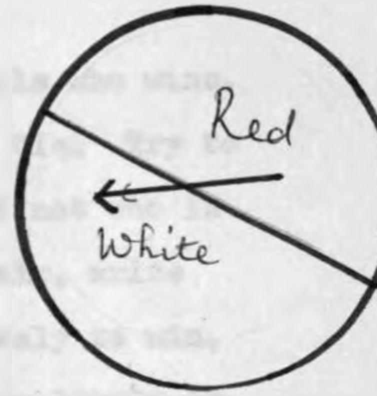
White die	Green die 1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

(a) 36 different number pairs shown on the table.

(b) There are 36 outcomes for tossing two dice.

(c) Yes.

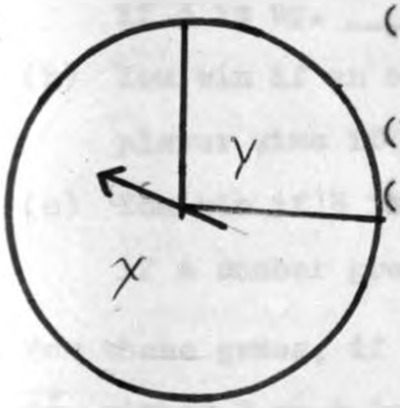
40. The spinner on the right is half white and half red. If you spin the pointer, what are the outcomes? (Assume that the pointer does not stop on the boundary. If the pointer stops on the boundary, do not count it as a spin.



(a) The outcomes are ______ and ________.

(b) Are these outcomes equally likely? ______.

41.



(a) What are the outcomes in the spinner below? __ and _____

(b) Are they equally likely?___

(c) The rule is <u>you</u> win if the pointer stops on Y; you lose if it stops on x. Do you want to play? ________
Why? ____________________

Answers:

40. (a) The outcomes are Red and White.

(b) Yes.

41. (a) X and Y.

(b) No. The pointer of the spinner is likely to stop on X most of the time (in fact it will stop on X $\frac{3}{4}$ of the time and on Y $\frac{1}{4}$ of the time).

(c) No.

(d) Because I may lose most of the time.

SELF-TEST I

For each game a rule is given that tells who wins. If neither player wins, the game is a tie. Try to tell whether each game is fair and, if not who is more likely to win. If the game is fair, write "F" in the blank. If you are more likely to win, write "Y". If the other player is more likely to win, write "O".

1. Use one die.
   (a) You win if 1 is up. The other player wins if 3 is up. ____________
   (b) You win if an odd number is up. The other player wins if an even number is up. ______
   (c) You win if 3 is up. The other player wins if a number greater than 3 is up. _______

2. For these games, if 1 is up, call it Result 1. If either 2 or 4 is up, call it Result 2. If 3, 5, or 6 is up, call it Result 3.
   (a) You win on Result 3. The other player wins on Result 1. ________
   (b) You win on Result 3, and he wins on any Result less than 3. ________
   (c) You win on an even numbered Result, and he wins otherwise. __________

Answers to self-test 1:

1. (a) F (b) F (c) O
2. (a) Y (b) F (c) O.

3. Use two dice, one white and one green.

Toss them together.

(a) You win if 1 is on each die. The other player wins if 5 is on each die. ______

(b) You win if there is an even number on the white die. The other player wins otherwise. ______________

(c) You win if 6 is on the white die, and he wins if 4 is on the green die. _________

(d) You win if 1 is on each die. He wins if one die has 1 and the other has 2. ___________

(e) You win if the number on the white die is greater than the number on the green die. He wins otherwise. _____________

4. Use one die, and throw it two times for each game.

(a) You win if the number the second time is greater than the number the first time. Otherwise, he wins. ________

(b) You win if the number each time is even. He wins if the number each time is odd. _______

5. What are the outcomes when you toss a die? It has six faces, and any one of these faces may be up. The outcomes are 1, 2, 3, __, __, ___.

Answers:

3. (a) F (b) F (c) F (It is also possible for both or neither to win).

(d) O (e) O

4. (a) O (b) F

5. 1, 2, 3, 4, 5, 6.

SECTION II

EXPERIMENTS IN PROBABILITY

Objective: To help the pupils with the techniques for gathering, tabulating, graphing and interpreting data which they generate by tossing a coin, tossing a die and drawing marbles.

The ideas gained from activities should sharpen children's intuition about chance events by analyzing the results of a large number of trials.

Vocabulary:- Tabulate, horizontal, vertical, tally.

Materials:- Spinner, coin: 5-cent piece and 50-cent piece; dice, marbles.

Suggested Time:- 6 to 8 lessons.

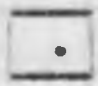
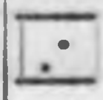
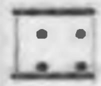
A. Tossing a Die

42. If you toss a die once, you have six outcomes. 1, 2, 3, 4, 5, 6. If you toss the die once you may get any of these outcomes. If you toss the die six times, do you think you will get each of the outcomes exactly once? ___________

Answer:

42. No.

43. Maua tossed a die 20 times and recorded her outcomes in the following table.

	No. of 	No. of 	No. of 	No. of 	No. of 	No. of 
Tally	11	1111	111	11	111	𝍸 1
Total	2	4	3	2	3	6

You now toss a die 60 times and make a record of the number of dots on the top face. Record your results in a table such as the one shown above.

---

44. Use your results of frame 43 to answer the following questions:-

(a) How many 1's did you get? __________

(b) How many 3's did you get? __________

(c) Did you get each outcome about the same number of times? __________

---

45. Toss a die 100 times. Keep a record of your results in the table below.

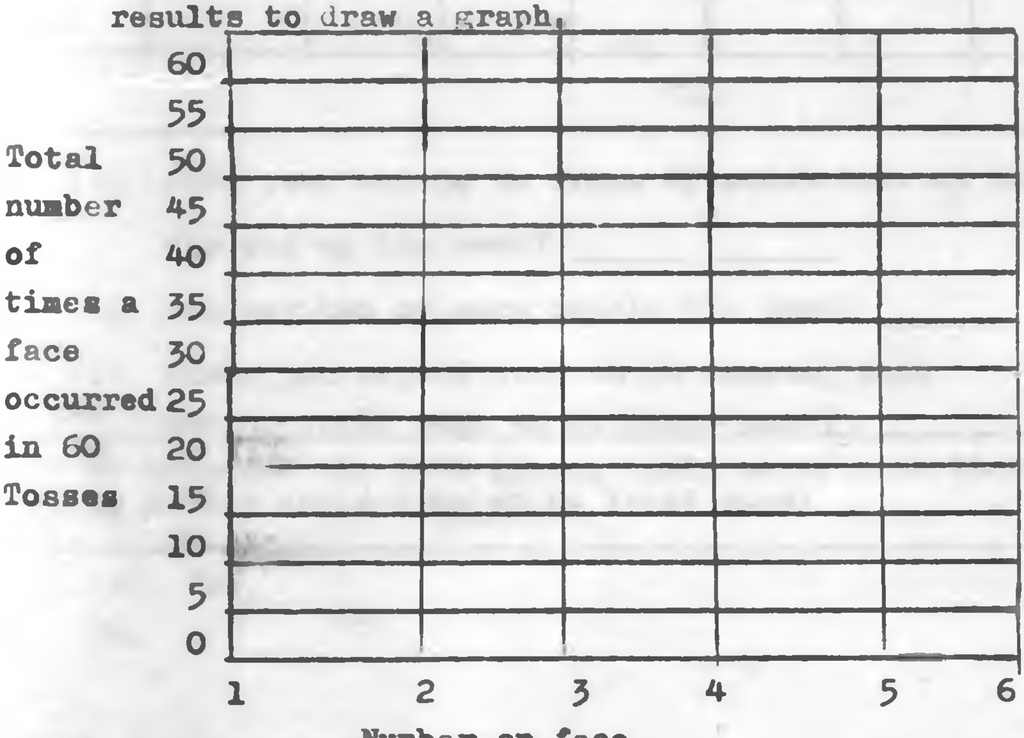
	No.of 1'S	No.of 2'S	No.of 3'S	No.of 4'S	No.of 5'S	No.of 6'S
Tally						
Total						

(a) Did you get each outcome about the same number of times? ________________

(b) Does your experiment make you think that in the long run you are likely to get each outcome 1 time in 6? ____________

---

46. In frame 43, you tossed a die 60 times and recorded your results in a table. Use these results to draw a graph.



---

45 . .

47. Toss a die 10 times and record your results in the following table.

	No. of ⚀	No. of ⚁	No. of ⚂	No. of ⚃	No. of ⚄	No. of ⚅	Total No. of Tosses
1st Toss							
2nd Toss							
3rd Toss							
4th Toss							
5th Toss							
6th Toss							
7th Toss							
8th Toss							
9th Toss							
10th Toss							
Total							

---

48. (a) From your totals in frame 47 which face of the die was up the most? ________

(b) Are any two or more totals the same? ______

(c) Would you expect that on 10 tosses, each number would come up at least once? ________

---

49. If we tossed a die 1000 times, could we be sure that every number would come up at least once? ________

---

48. (c) Yes

49. No.

50. If two dice are tossed at the same time each of the number 1, 2, 3, 5, 6 are equally likely to show up on the top face of the first die and each of the number 1, 2, 3, 5, 6 are equally likely to show up on the second die.

We can write the scores on the top faces of two dice using ordered pairs as follows:-

(1,1), (1,2), (1,3), (2,6), etc.

The first number represents a number on the top face of the first die and the second on the second die.

(a) List all the possible outcomes in a single throw of two dice.

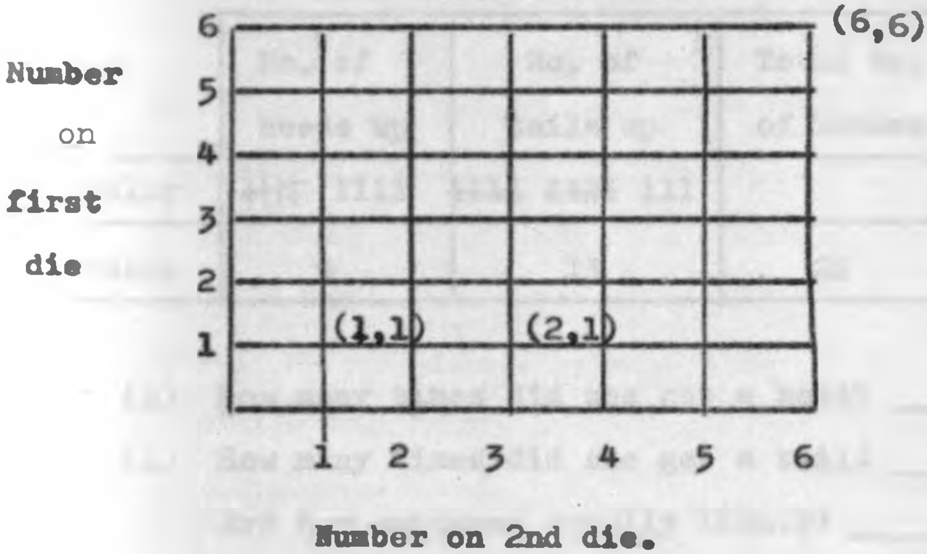
(b) How many possible outcomes are there? ______

Answer:

50. (a) (1,1), (1,2), (1,3), (1,4), (1,5) (1,6)
(2,1), (2,2), (2,3), (2,4), (2,5) (2,6)
(3,1), (3,2), (3,3), (3,4), (3,5),(3,6)
(4,1), (4,2), (4,3), (4,4), (4,5),(4,6)
(5,1), (5,2), (5,3), (5,4), (5,5),(5,6)
(6,1), (6,2), (6,3), (6,4), (6,5),(6,6).

(b) 36.

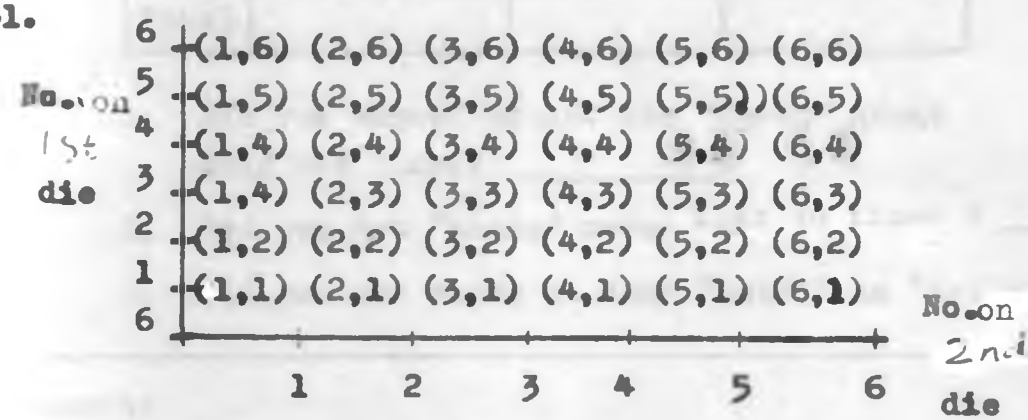
51. The best way to try and answer the question in frame 50 is to use a graph. Thus



Complete this graph.

Answer:

51.



B. Tossing a Coin

52. Maria tossed a coin 22 times and recorded her results in a table below:-

	No. of heads up	No. of tails up	Total No. of Tosses
Tally	~~1111~~ 1111	~~1111~~ ~~1111~~ 111	
Total	9	13	22

(a) How many times did she get a head? ______

(b) How many times did she get a tail? _____

Are her outcomes equally likely? ______

(b) How many times did she expect to get a head? ______ a tail? ________

---

53. Toss a coin 100 times. Keep a record of the results in table below:-

	No. of heads	No. of tails	Total No. of Tosses
Tally			
Total			

1. Did you expect to get the "heads" about half the times? __________
2. Did you get "heads" more than 40 times ? ______
3. Did you get about as many "heads" as "tails"?__

Answers:

52. (a) 9 times; 13 times; No. (b) 11 times; 11 tim

53. (i) Yes.

54. If you toss a coin once,

(a) How many times would you expect a head to show up? ____________

(b) How many times would you expect a tail to show up? ____________

(c) Are these outcomes equally likely? _____

---

55. If you toss a coin <u>once</u>, you would expect a head to come down <u>once</u> and a tail to come down once. If you toss a coin 10 times

(a) How many times would you expect a head to come down? ____________

(b) How many times would you expect a tail to come down? _________

---

56. Toss a coin 200 times and keep a record in a table such as the one shown on frame 52.

(a) How many times did you get a head? _____

(b) How many times did you get a tail? _____

(c) How many times do you expect to get a head? ____________

(d) How many times do you expect to get a tail? ________

Answers:

54. (a) once (b) Once (c) Yes.
55. (a) 5 times (b) 5 times
56. (c) 100 times (d) 100 times

57. Refer to frame 56.

Is the number of times the "heads" showed up when you tossed a coin 200 times closer to the number of times you would expect "heads" to show up when a coin is tossed 200 times? ____

---

58. Take two coins, a 10-cent coin and a 5-cent coin. Toss them together. Record your result in a table below:-

	No.of heads up	No.of tails up	Total No. of Tosses
10-cent coin			
5-cent coin			

Repeat this experiment 40 times.

(a) How many times do you get two heads? _____

(b) How many times do you get two tails? _____

(c) How many times do you get a head and tail?__

(d) Do you think you would get a head and a tail almost 2 times as you would get 2 heads or 2 tails?

Answers:

57. Yes.

58. Yes.

59. When two coins are tossed, a 10-cent piece and a 5-cent piece, they can fall in one of the the following ways shown in the table below:-

Ten-cent piece	Five-cent piece
Head	Head
Head	Tail
____	____
____	____

Complete this table.

---

60. The table in frame 59 can be drawn as:

		10 - cent piece	
		H	T
5 - cent piece	H	HH	
	T		

Complete the table.

Answers:

59.

Ten-cent piece	Five-cent piece
Head	Head
Head	Tail
Tail	Head
Tail	Tail

60.

	H	T
H	HH	HT
T	TH	TT

61. Toss 3 coins, a 5-cent coin, a 10-cent coin and a 50-cent coin. Record your results in a table below:-

	No.of heads up	No.of tails up	Total No. of Tosses
5-cent coin			
10-cent coin			
50-cent coin			

How many times do you get

(a) 3 heads ________

(b) 3 tails ________

(c) a head and a tail.

---

62. If you toss 3 coins they can fall in 8 different ways. Complete the table below.

50-cent piece	10-cent piece	5-cent piece
Head	Head	Head
Head	Head	Tail
-	-	-
-	-	-
-	-	-
-	-	-
-	-	-
-	-	-

Answers:

62.

50-cent piece	10-cent piece	5-cent piece
Head	Head	Head
Head	Head	Tail
Head	Tail	Head
Head	Tail	Tail
Tail	Head	Head
Tail	Head	Tail
Tail	Tail	Head
Tail	Tail	Tail

63. All the outcomes from a toss of three coins can also be shown in a table such as this:

50-cent coin

5-cent coin and 10-cent coin.	H	T
HH	HHH	HHT
HT		
TH		
TT		TTT

Complete this table.

(a) How many times are we likely to get 3 heads?___

(b) 3 tails ______________

(b) How many times are we likely to get two heads and one tail? _________

(c) How many times are we likely to get one head and 2 tails?

---

64. If you toss one coin it can fall in one of two different ways. If you toss two coins they can fall in one of four different ways. If you toss three coins they can fall in one of _____ different ways.

Complete the above pa ttern and use it to decide in how many different ways you think four coins fall. _______

63.

50-cent coin

	H	T
HH	HHH	HHT
HT	HTH	HTT
TH	THH	THT
TT	TTH	TTT

(a) One time; one time

(b) 3 times

(c) 3 times

64. 8 different ways; that is $2 \times 2 \times 2 = 2^3 = 8$
4 coins can fall in 16 different ways. That is $2^4 = 2 \times 2 \times 2 \times 2 = 16$.

65. If we toss 2 coins we may record the outcomes as is shown in the table below:-

	2H,OT	IH,IT	OH,2T
	HH	HT	TT
No. of outcomes	1	-	1

---

66. Make a table as the one in frame 65 for 3 tosses of a coin.

Answers:

65.

	2H,OT	IH,IT	OH,2T
	HH	HT	TT
		TH	
No. of outcomes	1	2	1

66.

	3H,OT	2H,IT	IH,2T	OH,3T
	HHH	HHT	HTT	TTT
		HTH	THT	
		THH	TTH	
No. of outcomes	1	3	3	1

67. Complete the table below for 4 tosses of a coin.

	4H,OT	3H,IT	2H,2T	IH,3T	OH,4T
	HHHH	HHHT	HHTT		TTTT
No. of outcomes	1		6		1

Answer:

67.

	4H,OT	3H,IT	2H,2T	IH,3T	OH,4T
	HHHH	HHHT	HHTT	HTTT	TTTT
		HHTH	HTHT	TTHT	
		HTHH	HTTH	TTTH	
		THHH	THHT	THTT	
			THTH		
			TTHH		
No. of Outcomes	1	4	6	4	1

68. We can organize our results from 65,66 and 67 in a triangular form. This triangle is called Pascal's triangle.

First Toss 1 1
1H,OT Oh,IT

1 2 1
2H,OT 1H,IT OH,2T

1 3 3 1
3H,OT 2H,IT 1H,2T OH,3T

1 4 6 - -
4H,OT

Complete the row for the 4th toss.

---

69. Look at the pattern in the triangle on frame 68. Complete the 5th and the 6th rows (5 and 6 tosses of a coin) without making a table.

---

70. I tossed a coin 40 times and counted 18 heads. Is this less or more than you would have expected?

Answers:

68. 4th toss:
1 4H,OT 4 3H,IT 6 2H,2T 4 1H,3T 1 OH,3T

69. 5th row:
1 5H,OT; 5 4H,IT; 10 3H,2T; 10 2H,3T; 5 1H,4T; 1 OH,5T

6th row:
1 6H,OT 6 5H,IT; 15 4H,2T; 20 3H,3T; 15 2H,4T; 6 1H,5T 1 OH,6T

70. Less than you would expect. You would expect to get 20 heads when you tossed a coin 40 times.

C. Drawing Marbles

71. Put three marbles, one red one green and one yellow, into a box you can't see through, without looking take out one marble, keep a record of the colour in this table.

Red	Green	Yellow

Put the marble back in the box, mix the marbles and draw again. Do this 50 times.

(a) Did you get about the same number of each colour? ___________

(b) What are the outcomes of this activity? ____

(c) Did you get each outcome about $\frac{1}{3}$ of the time? _________

---

72. Put three marbles, two white and one blue into the box. Do as you did in frame 71. Mix, draw, keep a record and put the marble back in the box.

White	Blue

Do this experiment 50 times.

(a) What are the outcomes of this activity?_____
(b) Did you get the outcome, blue, about $\frac{1}{3}$ of the time?_
(c) Did you expect to get blue as often as you got white

---

71. (b) Outcomes of the activity are, Red; Green; Yellow.
72. (a) Outcomes are white and Blue.
(c) No.

73. Put six marbles in a box. Three marbles are red, two are blue and one is green. Without looking, take out one marble. Keep a record of the colour in this table.

Red	Blue	Green

What are the outcomes of this activity? ______, ______, ______.

---

74. Suppose the experiment in frame 73 was repeated 100 times, about how many times would you expect to draw out a red marble? ______

---

75. Kamau and Omungu, each has one white and one green marble, Kamau picks one of his marbles without looking and then Omungu picks one of his, also without looking. The four possible outcomes are listed in the table on the next page, complete the table on the right to show the outcomes in a shorter way.

Answers:

73. Outcomes are Red, Blue and Green.

74. About 50 times.

75.

	Kamau's Marble	Omungu's Marble
1	White	White
2	White	Green
3	Green	White
4	Green	Green

	Kamau's Marble	Omungu's Marbles
1.	W	W
2.	W	--
3.	G	--
4.	--	--

76. There are two bags in Okiya's house. In the first bag, there are two marbles, Red and Blue. In the second bag, there are again two marbles, Red and blue.

Sometimes we can use a table such as the one shown below to help find the possible outcomes.

		Second Bag	
		Red	Blue
First Bag.	Red	Red, Red	
	Blue	Blue, Red	

Complete the table.

How many possible outcomes are there?

Answers:

75.

Kamau's marble	Omungu's marble
W	W
W	G
G	W
G	G

76.

		Second Bag	
		Red	Blue
First Bag.	Red	Red, Red	Red, Blue
	Blue	Blue, Red	Blue, Blue

There are four possible outcomes

77. In the table in frame 76 the left side shows the colour of the marble taken from the first bag. This colour is shown first in the row. The top of the table shows the colour from the _____ bag.

---

78. Okiya now has three bags in his house. In the first bag, there are one red and one blue marbles; in the second bag, there are one red and one blue marbles and the third bag contains one red and one blue marbles.
How many outcomes are there for the three bags?___

Answers:

77. Second Bag.

78. Eight outcomes for the three bags.

79. The outcomes in frame 78 can be put in a table such as the one below. The outcomes for the two bags are on the left. The top of the table shows the outcome from the third bag. Complete this table.

Outcomes from the third bag.

Outcomes from the first and Second bags.		R_3	B_3
	R_1R_2	$R_1R_2R_3$	
	R_1B_2		
	B_1R_2	$B_1R_2R_3$	
	B_1B_2		

<u>Note:</u> R_1-red marble drawn from bag 1; B_1-blue marble from bag 1. R_2-red marble from bag 2; B_2-blue marble from bag 2; R_3-red marble from bag 3; B_3-blue marble from bag 3.

---

79. Outcome from the third bag

Outcome from 1st and 2nd bags		R_3	B_3
	R_1R_2	$R_1R_2R_3$	$R_1R_2B_3$
	R_1B_2	$R_1B_2R_3$	$R_1B_2B_3$
	B_1R_2	$B_1R_2R_3$	$B_1R_2B_3$
	B_1B_2	$B_1B_2R_3$	$B_1B_2B_3$

80. Each time we add a bag, we double (multiply by 2) the number of outcomes. For instance, with one bag there were 2 outcomes (Red, Blue); with 2 bags there were 4 outcomes (Red; Red; Red; Blue; Blue; Red; Blue; Blue) and with 3 bags, there are _____ outcomes. List these outcomes. RRR RRB RBR RBB.

____, ____, ____, ____.

---

81. The number of outcomes increases by powers of 2. The number of outcomes for one bag is 2. The number of outcomes for two bags is $2^2 = 2 \times 2 = 4$.

The number of out comes for 3 bags is _____ = _____ = _____.

---

80. 8 outcomes.

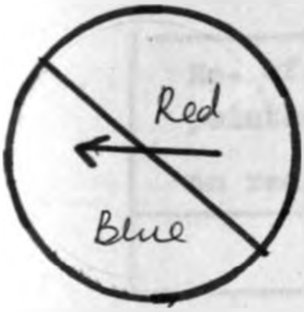
R R R, R R B, R B R, R B B, B R R,

B R B, B B R, B B B.

81. $2^3 = 2 \times 2 \times 2 = 8$.

D. Spinning the Pointer of a Spinner.

82.



On the left is a picture of a spinner. It is divided into two equal parts. One part is red, the other part is blue.

If you spin the pointer of this spinner 100 times

(a) How many times is it likely to stop on red? ______

(b) How many times is it likely to stop on blue? ________

---

83. If you were to play a game with the spinner in frame 82, you would win if the pointer stopped on red, your friend would win if the pointer stopped on blue.

Write T if you think the statement below is true. If it is false, write F.

(a) I would be more likely to win the game since the pointer would stop on red most of the time__________

(b) My friend would win most of the time ______

(c) Both of us would have equal chances to win this game since the pointer would stop on red about the same number of times it would stop on blue. ______

---

82. (a) 50 times (b) 50 times.

83. (a) F (b) F (c) T.

84. Spin the pointer of the spinner shown in frame 82, 20 times. Keep a record of your results in a table below.-

No. of times pointer stops on red.	No.of times it stops on blue.

(a) How many times does the pointer stop on red? __________

(b) How many times does the pointer stop on blue? _________

(c) Would you expect the pointer to stop on red the same number of times as it would stop on blue? ________

---

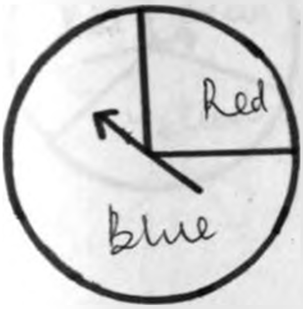
85. About how many time would you expect the pointer of frame 82 to stop on red if you spun the pointer 400 times? ___________

---

84. (c) Yes

85. About 200 times.

86. The pointer on the left is divided into two sections. The red section is $\frac{1}{4}$ of the whole and the blue section is $\frac{3}{4}$ of the whole. Spin the pointer of this spinner 20 times and keep a record of your results in a table below.



No. of times pointer stops on red	No. of times pointer stops on blue

---

87. Refer to frame 86.

(a) How many times did the pointer stop on red? ____

(b) How many times did the pointer stop on blue? ____

(c) Is the pointer equally likely to stop on red as on blue? ____________

---

88. About how many times would you expect the pointer to stop on blue if the pointer of the spinner in frame 86 were spun 400 times ________

---

87. (c) No.

88. About 300 times.

89. 

The spinner on the left is divided into 3 equal parts. Each part is $\frac{1}{3}$ of the whole.

Spin the pointer of this spinner 20 times and keep track of the outcomes in a table such as this.

No. of times pointer falls on blue	No.of times pointer falls on red	No.of times pointer falls on yellow

---

90. Refer to frame 89.

(a) How many times did the pointer stop on red? ____

on blue? ________

(b) Is each of these colours equally likely? _____

---

91. If you were to spin the pointer of the spinner in frame 89 900 times, about how many times would you expect it to fall on?

(a) Red? ________

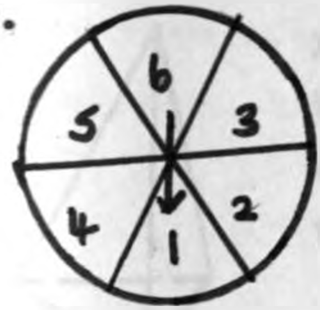
(b) Blue? ________

(c) Yellow? ________

---

90. (b) Yes.

91. 300 times; 300 times; 300 times.

92. 

The spinner on the left is divided into six equal parts.

Maua spun the pointer of this spinner 20 times and kept a record of her results in a table such as this:

	No. of times pointer stops on 1	No. of times pointer steps on 2	No. of times pointer stops on 3	No. of times pointer stops on 4	No. of times pointer stops on 5	No. of times pointer stops on 6
Tally	1111	HHT	1	11	111	HHT
Total	4	5	1	2	3	5

(a) How many times did her pointer stop on 1? ______

(b) How many times did it stop on 2? ______

(c) How many times did it stop on 3? ______

(d) How many times did it stop on 4? ______

(e) How many times did it stop on 5? ______

(f) How many times did it stop on 6? ______

---

93. Refer to frame 92.

(a) Is each of the numbers equally likely? ______

(b) If Maua spun the pointer of the spinner 600 times, about how many times would she expect it to stop on 4? ______

---

92. (a) 4 times (b) 5 times (c) 1 time
(d) 2 times (e) 3 times (f) 5 times.

93. (a) Yes (b) About 100 times.

94. This tetrahedron has one of its faces coloured red, one blue, another yellow, and the last green.

Toss the tetrahedron 20 times and note the face that is down. Keep track of the outcomes in a table such as this:

	Red	Blue	Green	Yellow
Tally				
Total				

How many times did the tetrahedron fall on Red? ____________ Blue ? __________

Green ? ____________ Yellow? __________

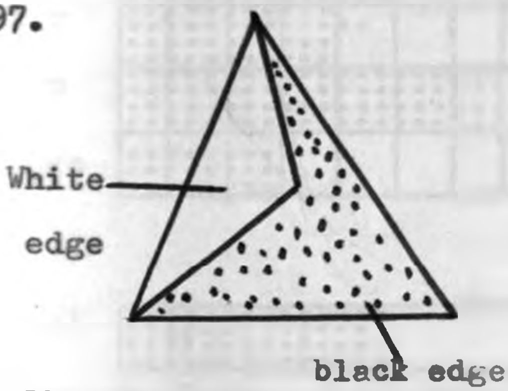
95. (i) Add the number of times it fell on red and on Blue ____________

(ii) Add the number of times it fell on green and yellow ____________

(iii) Is each of these sums about ½ of the total number of tosses, or about ¼ of the total number of tosses? About ____________

96. If you were to throw the tetrahedron in frame 94 1,000 times, about how many times would you expect it to fall on red? ________

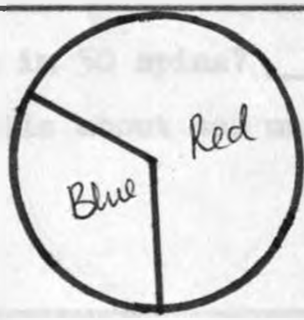
97.



Look at this 3-sided spinner which is divided into 3 equal parts. If you spin it 42 times, how many times would you expect it to fall on the black edge? __

98. If after 60 spins of the spinner in frame 97 you had recorded 23 times for the red edge, would this be less or more than you would expect? _____

99. Four girls, Maua, Betty, Carro and Anyango, each used the spinner shown on the right.

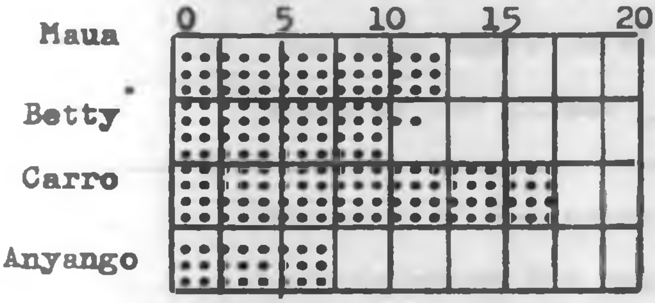


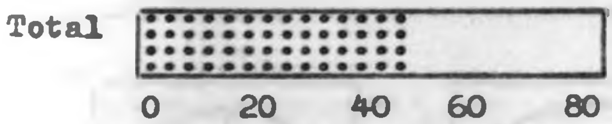
ANSWERS

96. About 250 times.
97. 14 times
98. More.

99. They drew a bar graph shown below.

No.of Blues in 50 spins





Number of Blues in 200 spins.

(a) Who had the smallest number of blues in 50 spins? _____

(b) Who had the largest number of blues in 50 spins? _____

---

100. Refer to frame 99.

(a) How many reds did Betty get in 50 spins? ________

(b) Which of these fractions tells about how much of the dial is blue?

½, ⅕, ⅛ __________

---

99. (a) Anyango

(b) Carro.

100. (a) 40 reds

(b) ⅕.

101. Refer to frame 99.

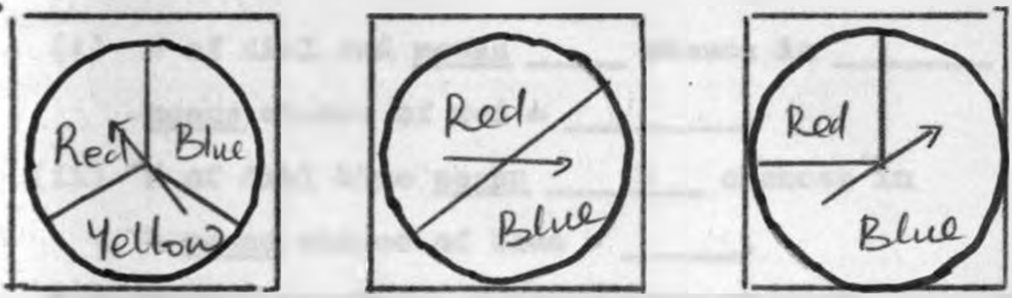
(a) Did any girl get 25 or more blue? ______

(b) How many times in all was the spinner spun by the girls? ______________

---

102. (a) How many of the spins ended on blue? ______
is this about the number of blues you would expect on 200 spins? __________

103.



Look at these spinners. You can use fractions to compare the chances of different results.

Complete the following.

½ of dial red means 1 chance in 2 means chance of red = ½.

½ dial blue means ___ chance in 2 means chance of blue= ___

---

101. (a) No girl got 25 or more blues in 50 spins.
(b) 200 times.

102. 48.
No. We would expect about 50 spins.

103. 1 chance in 2 means chance of blue = ½.

104. Look at the spinners in frame 103.

(i) $\frac{1}{3}$ of dial red means 1 chance in ___

means chance of red = ______

(ii) $\frac{1}{3}$ of dial blue means _____ chance in 3

means chance of blue = ________

(iii) $\frac{1}{3}$ of dial yellow means ½ chance in _____

means chance of yellow = $\frac{1}{3}$ ________

---

105. Again look at the spinners in frame 103.

(i) ¾ of dial red means ____ chance in _______

means chance of red = ________.

(ii) ¾ of dial blue means _______ chances in

4 means chance of blue = _____.

---

106. Refer to frame 103.

(i) All of dial red means red is certain means

chance of red = _________

(ii) ______ of dial red means red is impossible

means chance of red =

---

104. (i) 1 chance in 3 means chance of red = $\frac{1}{3}$
(ii) 1 chance in 3 means chance of blue = $\frac{1}{3}$
(iii) 3.

105 3 chances in 4.
chance of red = ¾

106. (i) Chance of red = 1 (ii) None.

107. Ateka was told that she would get Shs.20/= if she could get one of the following outcomes:-

1. Blue on a spinner whose dial is ½red and ½ blue.
2. A 2 on one toss of a die.

Which one would she choose? ________

---

107. She would choose blue on a spinner whose dial is ½ red and ½ blue, because she would have half the chance of getting Shs. 20/=.

If she chose a 2 on one toss of a die, she would have only $\frac{1}{6}$ chance of getting the Shs.20/=.

SELF-TEST 2

1. Awinja spins the pointer of a spinner 100 times and gets 35 reds. Which of the following statement is most likely to be true?

   (a) The dial of the spinner is all red.

   (b) The dial of the spinner is one-half- blue.

   (c) The dial of the spinner is one-sigth red.

   (d) The dial of the spinner is one-third red.

2. Omutsimi spins the pointer of a spinner 100 times and gets 25 red, 25 blue and 50 yellow. Which of the following statements cannot be true?

   (a) The dial of the spinner is one-fourth yellow.

   (b) The dial of the spinner is One-third green.

   (c) The dial of the spinner is one-fourth blue.

   (d) The dial of the spinner is all red.

3. A spinner has a dial that is one-third red, one-half white, and one-sixth blue. Which of the following cannot result from exactly 100 spins.

   (a) 30 reds, 50 whites and 20 blues.

   (b) 40 reds, 40 whites and 20 blues.

   (c) 50 reds, 5 whites and 10 blues.

   (d) 60 reds, 40 whites and 0 blues.

Answers:

1. (d) The dial of the spinner is $\frac{1}{3}$ red.
2 (a), (b); (c).
3. (b); (c), (d).

4. You wish to get exactly 5 reds and 5 blues in 15 spins. Which of the following dials could not give this result?

   (a) One-half red and one-half blue.

   (b) One-third red, one-third blue and one-third yellow.

   (c) One-fourth red, one-fourth blue and one-half yellow.

   (d) One-fifth red, two-fifths blue and two- fifths yellow

5. In which of the following is the chance of red equal to ¼?

   (a) One chance in two of red.

   (b) Two chances in four of red.

   (c) One chance in five of red.

   (d) Two chances in eight of red.

6. Which of the following spinners is likely to give about the same number of reds and yellows?

   (a) One-half red, one-fourth yellow, one-fourth blue.

   (b) One-third red, two-thirds yellow.

   (c) One-third red, one-third yellow, one-third blue.

   (d) Four-fifths yellow, one-fifth red.

---

4. (a); (c); (d).

5. (d)

6. (c).

7. If the dial of a spinner is all red, we say the chance of red is equal to:

   (a) any other chance.

   (b) one chance in two.

   (c) one-half.

   (d) one.

8. If the dial of a spinner is all blue, we say the chance of red is equal to:

   (a) one

   (b) zero

   (c) one chance in one

   (d) one-half.

9. The dial of spinner is one-third red, one-third yellow, and one-third blue. Which of the following statements are true?

   (a) Red, yellow, and blue are equally likely to occur.

   (b) The chance of getting red is equal to $\frac{1}{3}$.

   (c) One spin must result in either red or yellow or blue.

   (d) The chance of getting green is equal to zero.

---

7. (d)

8. (b)

9. (a); (b); (c).

10. If the chance of red on a spinner is equal to zero, which of the following statements could be true?

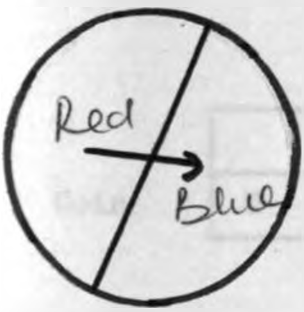
(a) The dial is all red.

(b) The dial is all blue.

(c) The dial has at least two colours.

(d) The dial has at least three colours.

11. The spinner shown on the left is divided into two equal parts. One part is painted red, the other part is painted blue. Tabu spins this spinner twice. Complete the table below to show the possible outcomes.



		Second Spin	
		Red	Blue
First spin.	Red		

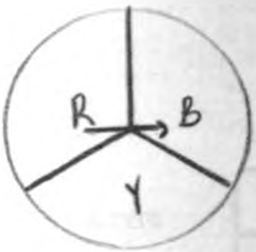
---

10. (b); (c).

11.

		Second Spin	
		Red	Blue
First Spin.	Red	Red, Red	Red, Blue
	Blue	Blue, Red	Blue, Blue

12. A coin is tossed once and the spinner shown below is spun once. Complete the table below to show all the possible outcomes. The dial of the spinner is divided into three equal parts.



Spinner

		R		Y
Coin	T			

(a) How many possible outcomes are there? _____

(b) How many of these outcomes give a head and red? _____

12.

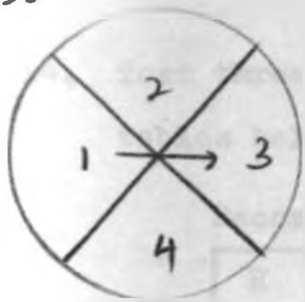
Spinner

		R	B	Y
Coin	H	HR	HB	HY
	T	TR	TB	TY

(a) There are 6 possible outcomes.

(b) One outcome gives a red and a head.

13.



The dial of the spinner shown on the left is divided into four equal regions. The pointer of this spinner is spun twice.

Complete the table below to show all the possible outcomes.

Second Spin

First Spin

	1			4
1	1,1		1,3	
4				

What is the total number of outcomes?_______

13.

Second Spin

First Spin

	1	2	3	4
1	1,1	1,2	1,3	1,4
2	2,1	2,2	2,3	2,4
3	3,1	3,2	3,3	3,4
4	4,1	4,2	4,3	4,4

There are 16 outcomes.

14. Toss three coins together. Fill in the tables below to show all possible outcomes.

Second Coin

		H	T
First Coin	H		
	T		

Third Coin

		H	T
First and Second Coins	HH		
	HT		
	TH		
	TT		

(a) What is the total number of outcomes when three coins are tossed? ________

(b) How many of these outcomes include three heads? ________

(c) How many of these outcomes include two heads and one tail? ________

14.

Second Coin

		H	H
First Coin	H	HH	HT
	T	TH	TT

Third Coin

		H	T
First and Second Coin	HH	HHH	HHT
	HT	HTH	HTT
	TH	THH	THT
	TT	TTH	TTT

(a) There are 8 outcomes when 3 coins are tossed.

(b) One outcome includes 3 heads.

(c) 3 outcomes includes 2 heads and 1 tail.

15. Complete this table to show all the possible outcomes for two tosses of a coin.

	2H,0T	1H,1T	0H,2T
	HH	HT	TT
		TH	
No. of Outcomes	1	2	1

16. Complete the table for 3 tosses of a coin.

	3H,0T	2H,1T	1H,2T	0H,3T
	HHH	HHT		
No. of Outcomes	1	3	3	1

15.

	2H,0T	1H,1T	0H,2T
	HH	HT	TT
		TH	
No. of Outcomes	1	2	1

16.

	3H,0T	2H,1T	1H,2T	0H,3T
	HHH	HHT	HTT	TTT
		HTH	THT	
		THH	TTH	
No. of Outcomes	1	3	3	1

SECTION III - Finding Probabilities

Introduction:-

When we talk about the probability of a particular outcome, we tell how likely it is that the outcome is the one we get. We use a number that tells what part of the total outcomes we expect a particular one to happen. This means that probabilities can be written as fractions.

108. When tossing one die, we have six outcomes.

We write the 6 under the bar of a fraction:

$\overline{6}$

Getting the outcome 3 is just as likely as any others, so we expect it about $\frac{1}{6}$ of the time.

We say, "The probability of 3 is ________.
We write P(3) = ________

---

109. In the experiment, "Tossing a coin once", there are 2 outcomes. Since the outcomes are equally likely, we can say that

P(heads) = $\overline{2}$

P(tails) = $\overline{2}$

---

110. Sometimes we give probabilities for things that can't possibly happen. In tossing one die, there is no chance at all of getting the outcome "7".

Answers:

108. $\frac{1}{6}$. (P(3) = $\frac{1}{6}$

109. P(heads) = ½ P(tails) = ½

The number of times you would get 7 in tossing one die is _____. We could write $P(7) = \frac{}{6}$ or P(7) = _____.

---

111. We can also give a probability for a "sure thing." (That is, a thing which must happen). If we ask, "What is the probability of getting a number less than seven when we toss one die?" There are six ways to get a number less than seven. All six of the six outcomes are less than 7, so we write: P(number less than 7) = $\frac{}{6}$ = _______.

---

112. In the experiment, "Tossing one die", what is the probability of the outcome 5? P(5) = _____.

b) What is P(2) = _____

P(1) = _____; P(4)= _____ ; P(6) = _____.

---

113. In the experiment, "Tossing Two Dice", there are _____ outcomes. Since these outcomes are equally likely, we can say

i) P(4,3) = _____

ii) P(6,6) = _____

iii) P(7,1) = _____

Answers:

110. 0; $P(7) = \frac{0}{6}$; P(7) = 0

111. P(number less than 7) = $\frac{6}{6}$ = 1.

112. $P(5) = \frac{1}{6}$; $P(2) = \frac{1}{6}$; $P(1) = \frac{1}{6}$; $P(4) = \frac{1}{6}$; $P(6) = \frac{1}{6}$.

113. 36 outcomes. (i) $P(4,3,) = \frac{1}{36}$; (ii) $P(6,6) = \frac{1}{36}$ (iii) $P(7,1) = \frac{0}{36} = 0$.

114. In the experiment of drawing marbles of frame 71, you used one red, one green, and one yellow marble. Since drawing a red marble is one of the three equally likely outcomes, we can say,

P(red) = ________ P(blue) = ________

P(green) = ________ P(yellow) = ________

P(not blue) = ______

---

115. In the experiment, "Tossing Two Dice", find the probability of the first die showing 3 and the second die showing a 5; that is, find P(3,5).

P(3,5) = ________.

---

114. P(red) $= \frac{1}{3}$; P(blue) $= \frac{0}{3}$

P(green) $= \frac{1}{3}$; P(yellow) $= \frac{1}{3}$

P(not blue) $= \frac{3}{3} = 1.$

115. P(3,5) $= \frac{1}{36}$.

116. If we toss two dice, we can show all the possible sums of the dots on the two dice in a table such as the one below.

Number on Second Die

No. on First Die	1	2	3	4	5	6
1	2	3				
2						
3		5				
4				8		
5					10	
6						12

Complete the table to show all the possible sums of the dots on two dice.

---

117. One way to get a sum of 7 is to get a 1 on the first die and a 6 on the second die. We write this as (1,6). There are five more ways to get a sum of 7. These are (,), (,), (,), (,), (,).

116.

Number on Second Die

No. on First Die	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

117. (2,5), (3,4), (4,3), (5,2), (6,1).

118. (a) How many entries are there in the table? ____

(b) How many possible entries are there when you toss two dice? ________

---

119. (a) Of the entries in the table of frame 116, how many are 6's? ________

(b) What is the probability of getting a sum of 6 when two dice are tossed? ________

---

120. (i) How many of the entries in the table of frame 116 are odd numbers? ________

(ii) What is the probability of getting the sum that is an odd number? ________

---

121. (i) How many of the sums in the table of frame 116 are either 5 or 9? ________

(ii) What is the probability that the sum will be either 5 or 9? ________

---

118. (a) 36 entries

(b) 36 possible entries.

119. (a) 5

(b) $P(\text{sum} = 6) = \frac{5}{36}$

120. (i) 18 odd numbers

(ii) $P(\text{sum is odd number}) = \frac{18}{36} = ½$

121. (i) 8

(ii) $P(\text{sum either 5 or 9}) = \frac{8}{36} = \frac{2}{9}$

122. Refer to frame 116 to answer the following.

a). P(sum = 3) = ________

b). P(sum = 8) = ________

c). P(sum = 12)= ________

d). P(sum = 2) = ________

e). P(sum = 11)= ________

---

123. Refer to frame 116 to answer the following.

a). P(sum = 2 or sum = 12) = ______

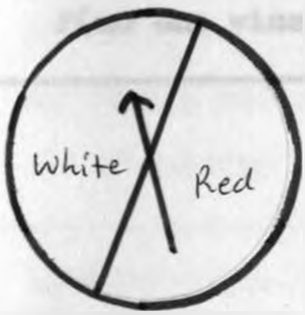
b). P(sum = 6 or sum = 8) = ______

c). P(sum = 5 or sum = 9) = ______

d). P(sum ≠ 7) = ______

e). P(sum > 9) = ______

124.



In the spinner on the left, there are _____ equally likely outcomes

(i) P(red) = _____

(ii) P(white) = ____

(iii) P(yellow) = ____

---

122. (a) P(sum = 3) = $\frac{2}{36} = \frac{1}{18}$

(b) P(sum = 8) = $\frac{5}{36}$

(c) P(sum =12) = $\frac{1}{36}$

(d) P(sum = 2) = $\frac{1}{36}$ (e) P(sum = 11) = $\frac{2}{36} = \frac{1}{18}$

123. (a) P(sum = 2 or sum = 12) = $\frac{2}{36} = \frac{1}{18}$

(b) P(sum)= 6 or sum = 8) = $\frac{10}{36} = \frac{5}{18}$

(c) P(sum = 5 or sum 9) = $\frac{8}{36} = \frac{2}{9}$

(d) P(sum ≠ 7) = $\frac{30}{36} = \frac{5}{6}$ (e) P(sum > 7) = $\frac{15}{36} = \frac{5}{12}$

124. 2 equally likely outcomes.

(i) P(red) = ½ (ii) P(white = ½ P(yellow = $\frac{0}{2}$ = 0.

125. You played some games at the beginning. You were asked to decide whether or not the games were fair. You found that some games were fair and some were not. You saw that the game was fair if your winning outcome was just as likely as the other players. P(you win) = P(   )?

---

126. In the game, "Toss one die", the rule was:- "You win if 1 is up, the other player wins if 3 is up." Since P(1) = $\frac{1}{6}$ and P(3) = ___ we say that the game was _____. Each one of you had an equal chance of winning.

---

127. In the game of frame 126, how many outcomes out of the 6 let neither of you win? _______

P(no one wins) = _______

---

125. P(you win) = P(other player wins)

126. P(3) = $\frac{1}{6}$; Fair.

127. 4 outcomes

P(no one wins) = $\frac{4}{6}$ = $\frac{2}{3}$

128. In another game, the rule was: "You win if an odd number is up; the other player wins if an even number is up." To find whether or not you and your friend had equal chances to win, you found out how many of the outcomes were odd and how many were even.

a) How many outcomes out of the 6 are odd? ____

b) How many outcomes are even? ______

c) P(odd) = $\frac{\ }{6}$ = ______

d) P(even) = $\frac{\ }{6}$ ________

---

129. The rule for one game using one die is: "You win if 3 is up; the other player wins if a number greater than 3 is up." (See frame 19).

a) P(3) = ______

b) Which outcomes are greater than 3? ______

c) P(outcome greater than 3) = ___________

---

130. In frame 129, who has a better chance to win you or the other player? ________

Answers:

128. (a) 3 outcomes are odd.
(b) 3 outcomes are even.
(c) P(odd) = $\frac{3}{6}$ = ½
(d) P(even) = $\frac{3}{6}$ = ½

129. (a) P(3) = $\frac{1}{6}$
(b) 4, 5, 6.
(c) P(outcome greater than 3) = $\frac{3}{6}$ = ½.

130. The other player.

131. For the game, Toss one die, call the Result 1 if 1 shows up; call the Result 2 if either 2 or 4 shows up; call the Result 3 if 3, 5, or 6 is up.

(a) P(Result 1) = ________

(b) How many outcomes give Result 2? ______

(c) P(Result 2) = ________

---

132. (a) How many outcomes give result 3? ______

(b) P(Result 3) = ________

---

133. Write L if you are more likely to win and H if the other player is more likely to win for each rule. Write E if both are equally likely to win.

a) You win on Result 3. He wins on Result 1______

b) You win on Result 3. He wins on any Result less than 3. ______

c) You win on an even-numbered result and he wins otherwise. ________

Answers:

131. (a) P(Result 1) = $\frac{1}{6}$

(b) 2 outcomes give Result 2.

(c) P(Result 2) = $\frac{2}{6}$ = $\frac{1}{3}$

132. (a) 3 outcomes give result 3.

(b) P(Result 3) = $\frac{3}{6}$ = ½

133. (a) L (b) E (c) H.

134. When you toss a green die and a wh...
together, how many outcomes are there? ____
What is the probability of any of these outcomes? __

---

135. Refer to frame 134.
The rule is: "You win if 1 is on each die; the other player wins if 5 is on each die."
(a) For how many outcomes do you win? ______
(b) P(1 on each die) = ______

---

136. (a) For how many outcomes does the other player win? ________
(b) P(5 on each die) = ________
(c) For how many outcomes does nobody win? ______
(d) P(no one wins) = ____________

Answers:

134. 36 outcomes
P(any of the outcomes) = $\frac{1}{36}$

135. (a) I win for one outcome. That is, I win if I have (1,1).
(b) P(1 on each die) = $\frac{1}{36}$

136. (a) The other player wins for one outcome.
(b) P(5 on each die) = $\frac{1}{36}$
(c) No one wins for 34 outcomes
(d) P(no one wins) = $\frac{34}{36} = \frac{17}{18}$

137. The rule is: "You win if there is an even number on the white die; the other player wins otherwise."

(a) Does it matter what the outcome on the green die is? ________

(b) P(even) = __________

(c) P(odd) = __________

(d) P(no one wins) = ________

(e) P(both win) = __________

---

138. The rule is: "You win if 6 is on the white die, and the other player wins if 4 is on the green die."

(a) P(6 on white) = ________

(b) P(4 on green) = ________

---

139. In frame 138, can you get 6 on the white die and 4 on the green die at the same time? ________

P(both win) = ______

P(no one wins) = _______

Answers:

137. (a) No

(b) P(even) = $\frac{18}{36}$ = ½

(c) P(odd) = $\frac{18}{36}$ = ½

(d) P(no one wins) = $\frac{0}{36}$ = 0

(e) P(both win) = $\frac{0}{36}$ = 0

140. The rule is: "You win if 1 is on each die; the other player wins if 1 is on one die and 2 is on the other die."

a) P(you win) = ________

b) For how many outcomes does the other player win? ________

c) P(other wins) = _____

---

141. In frame 140,

a) Who has a better chance to win, you or the other player? ________

b) P(both win) = ________

c) P(no one wins) = _____

Answers:

140. (a) P(you win) $= \frac{1}{36}$

(b) Other player wins on 2 outcomes.

(c) P(other wins) $= \frac{2}{36} = \frac{1}{18}$

141. (a) Other player.

(b) P(both win) $= \frac{0}{36} = 0.$

(c) P(no one wins) $= \frac{33}{36} = \frac{11}{12}$.

142. The rule is: "You win if the number on the white die is greater than the number on the green die; the other player wins otherwise."

(i) P(you win) = _____

(ii) P(other wins) = _____

(iii) P(no one wins) = _____

iv) P(both win) = _____

---

143. The part of the rule which says, "He wins otherwise" is now changed to "He wins if the number on the white die is less than the number on the green die."

a) Does this change your chance to win? _____

b) With this change, P(other wins) = _____ and P(no one wins) = _____

---

144. In frame 140, you found the probability of "1 on one die and 2 on the other". You found that the pair would be <u>either</u> (1,2) <u>or</u> (2,1). You probably counted those outcomes and found 2 out of 36 outcomes, so the probability is _____

Answers:

142. (i) P(you win) = $\frac{15}{36} = \frac{5}{12}$

(ii) P(other wins) = $\frac{21}{36} = \frac{7}{12}$

(iii) P(no one wins) = $\frac{0}{36}$ = 0 (iv) P(both win) = 0

143. (a) No. (b) P(other wins) = $\frac{15}{36} = \frac{5}{12}$

(c) P(no one wins) = $\frac{6}{36}$

144. Probability $\frac{2}{36}$ or $\frac{1}{18}$.

145. Sometimes you cannot find the probability of <u>either</u> this event <u>or</u> that event by counting. Look at this spinner. There are 2 outcomes

just as there are 2 outcomes for the spinner in frame 82. The spinner in frame 82 has two equally likely outcomes. So P(white) = P(red) = ______

---

146. The spinner in frame 145 has two outcomes, but these outcomes are not equally likely. If the spinner is honest (if the pointer does not stop on a line each time it is spun), We would expect the pointer to stop on red about one out of four times, so P(red) = $\frac{1}{4}$ and P(white)=____

---

147. By looking at the spinner of frame 145 you know that it is certain the outcome will be either white or red. so P(either white or red) = ______
P(red) + P(white) = $\frac{1}{4}$ + ______ = ______

Answers:

145. P(white) = P(red) = $\frac{1}{2}$

146. P(white) = $\frac{3}{4}$

147. 1
P(red) + P(white) = $\frac{1}{4}$ + $\frac{3}{4}$ = 1.

148\. 

This spinner is $\frac{1}{4}$ white, $\frac{1}{4}$ red, $\frac{1}{8}$ blue and $\frac{1}{8}$ yellow.

(a) Are these outcomes equally likely? ________

(b) P(white) = $\frac{1}{4}$, P(blue) = ____

P(yellow) = ____; P(red) = ____

149\. To find the probability of <u>either</u> white <u>or</u> blue, we add P(white) and P(blue).

P(either white or blue) = $\frac{1}{4} + \frac{1}{8}$

so P(either white or blue) = $\frac{4}{8} + \frac{1}{8}$

P(either white or blue) = $\frac{4 + 1}{8}$

P(either white or blue) = $\frac{\quad}{8}$

150\. To find P(either blue or yellow) we add P(blue) and P(yellow):

P(either blue or yellow) = ________.

151\. To find P(either red or blue), we add P(red) and P(blue)

So P(either red or blue) = ________.

Answers

148\. (a) No. (b) P(blue) = $\frac{1}{8}$
P(yellow) = $\frac{1}{8}$; P(red) = $\frac{1}{4}$

149\. P(either white or blue)
P(white) + P(blue) = $\frac{1}{4} + \frac{1}{8} = \frac{4}{8} + \frac{1}{8} = \frac{5}{8}$.

150\. P(either blue or yellow) = P(blue) + P(yellow)
$\frac{1}{8} + \frac{1}{8} = \frac{2}{8} = \frac{1}{4}$.

151\. P(either red or blue) = P(red) + P(blue)

152. You can't always find either - or probabilities just by adding. Look at the spinner below. It is divided into six equal parts, all the same size. P(red) = ____

P(1) = ______ $\frac{1}{3} + \frac{1}{3} =$ ________

---

153. You can see that 3 of the 6 parts of the spinner are neither red nor 1. So P(either red or 1) ≠ ____

---

154. Count the parts of the spinner that are red ______

Count the parts of the spinner that have 1 ______

Put an X on each part of the spinner that is either red or 1. How many X's do you have? ____

Answers:

152. P(red) = $\frac{2}{6}$ or $\frac{1}{3}$; P(1) = $\frac{2}{6}$ or $\frac{1}{3}$

$\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

You can't just add P(red) and P(1) to find P(either red or 1) because one section has both red and 1.

153. P(either red or 1) ≠ $\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$.

154. 2 parts are red.
2 parts have 1.
1 part of the spinner is either red or 1.
There is one X.

155. You can find P(either red or 1) by adding the probabilities of each one and then <u>subtracting</u> the probability of that part of the spinner that has a 1 and is red

that P(either red or 1) = P(red) + P(1) - P(red and 1)

$$= \frac{1}{3} + \frac{1}{3} - \frac{1}{6}$$

$$= \frac{\ }{6} + \frac{\ }{6} - \frac{1}{6}$$

$$= \frac{\ + \ - 1}{6}$$

$$= \_\_\_\_\_\_.$$

---

155. P(either red or 1) = P(red) + P(1) - P(red and 1)

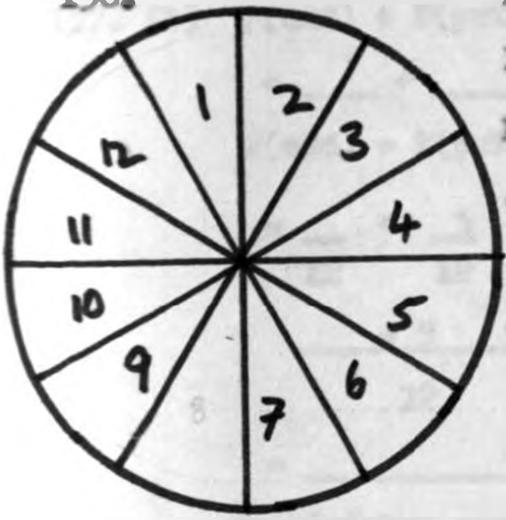
$$= \frac{1}{3} + \frac{1}{3} - \frac{1}{6}$$

$$= \frac{2}{6} + \frac{2}{6} - \frac{1}{6}$$

$$= \frac{4 - 1}{6}$$

$$= \frac{3}{6} = ½.$$

156.



Look at the spinner on the left. It is divided into 12 parts, all the same size.

(a) Which outcomes are odd? ____

P(odd) = __________

(b) Which outcomes are prime? ____

P(prime) = ________

(c) Which outcomes are both odd and prime? ________

---

157. (a) What is the probability of getting either 3 or 5 or 7 or 11?

Answers:

156. (a) Odd outcomes are 1, 3, 5, 7, 9, 11.

$P(\text{odd}) = \frac{6}{12} = \frac{1}{2}$

(b) Prime outcomes are 2, 3, 5, 7, 11.

$P(\text{prime}) = \frac{5}{12}$.

(c) Outcomes that are both odd and prime are 3, 5, 7, 11.

157. (a) P(either 3 or 5 or 7 or 11)

$= P(3) + P(5) + P(7) + P(11)$

$= \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} = \frac{4}{12}$

157. (b) P(odd) + P(prime) - P(either 3 or 5 or 7 or 11)

= ____ + ____ - ____

P(odd) + P(prime) - P(either 3 or 5 or 7 or 11)

$= \frac{\_\_}{12} + \frac{5}{12} - \frac{\_\_}{12}$

$\frac{\_\_ + 5 - \_\_}{12}$

= ____________

So P(either odd or prime) = ________

Check your answer by counting.

---

158. When one coin is tossed, there are 2 outcomes, head or tail. The probability of a head showing up is ½ and the probability of a tail showing up is also ½.

Suppose you toss two coins, say a 10 cent piece and a 5 cent piece do you think the probability of both heads showing up will be ½? ________

---

159. Here is a table for tossing two coins. See frame 60.

		10-cent piece	
		H	T
5-cent piece	H	H H	
	T		

Answers:

157. (b) P(odd) + P(prime) - P(either 3 or 5 or 7 or 11)

$= \frac{6}{12} + \frac{5}{12} - \frac{4}{12} = \frac{11 - 4}{12} = \frac{7}{12}$

158. No.

159. (a) Complete the table.

(b) Since there are four outcomes, all equally likely, the probability for any one of the outcomes is ¼.

So P(both heads) = _____ and

P (both tails) = _____

but P(1 head, 1 tail) = _____.

---

160. Here is a table for tossing 3 coins. See frame 63.

5 cent coin and 10 cent coin.

	50-cent piece H	50-cent piece T
HH	HHH	HHT
HT		
TH		
TT		TTT

(a) Complete the table.

(b) Since there are ____ outcomes, all equally likely, the probability of any one of the outcomes is ____.

Answers:

159.

5 cent piece

	10-cent piece H	10-cent piece T
H	HH	HT
T	TH	TT

P(both heads) = ¼ and P(both tails) = ¼

P(1 head, 1 tail) = $\frac{2}{4}$ = ½

160.

5 cent coin and 10 cent coin.

	50 cent piece H	50 cent piece T
HH	HHH	HHT
HT	HTH	HTT
TH	THH	THT
TT	TTH	TTT

(b) 8 outcomes
P(any one out outcome) = $\frac{1}{8}$

161. In frame 160. Find.

(a) P(3 heads) = ________

(b) P(3 tails) = ___________

(c) P(2 heads, 1 tail) = ______

(d) P(1 head, 2 tails) = ______

---

162. Two white marbles and two green marbles are put into a box. If you take out one marble without looking, what is the probability that it is white? _______

(b) If you take out 2 marbles, do you think the probability that they will both be white is still ½? ______

---

163. We can think of the problem in frame 162 like this: Uhuru and Ponto, each draw one of the marbles. The possible outcomes are shown in a table below.

		Uhuru's draws	
		White	Green
Ponto's draws	White	WW	WG
	Green	GW	GG

Answers:

161. (a) P(3 heads) = $\frac{1}{8}$
(b) P(3 tails) = $\frac{1}{8}$
(c) P(2 heads, 1 tail) = $\frac{3}{8}$
(d) P(1 head, 2 tails) = $\frac{3}{8}$

162. (a) P(white) = $\frac{2}{4}$ = ½
(b) No.

163. Uhuru may get a white marble, and Ponto may get any of the three others.

How many outcomes are there? ________

Are these outcomes equally likely? _____

---

164. WG and GW are two different outcomes.

Uhuru can draw white and Ponto can draw green, or Ponto can draw white and Uhuru can draw green.

But there is only one way they can draw white <u>and</u> there is only one way they can draw green.

P(WW) = _____ ; P(GG)= ________

P(WG) = _____ P(GW) = _______

---

165. A bag contains several marbles. Some are red, some white, and the rest blue. If you pick one marble without looking, the probability of red is $\frac{1}{3}$ and the probability of white is aslo $\frac{1}{3}$.

What is the probability of blue? _______

Answers:

163. There are 4 outcomes.

Yes. The outcomes are equally likely.

164. P(WW) = $\frac{1}{4}$; P(GG) = $\frac{1}{4}$

P(WG) = $\frac{1}{4}$; P(GW) = $\frac{1}{4}$.

165. The probability of blue is $\frac{1}{3}$

166. A bag contains one red marble, two white marbles, and three blue marbles. If you pick one marble without looking, what is the probability that it will be red? ______

---

167. In frame 166. Find

(a) The probability that the marble drawn will will be white? ______

(b) P(marble blue) = ______

(c) How many white marbles must we add to the bag to make the probability of white equal to ½? ______

---

168. Use the table of frame 75 to find the probability that

(a) Kamau picks a white marble ______

(b) Omungu picks a white marble ______

(c) Both Kamau and Omungu pick white marbles ______

Answers:

166. P(red) = $\frac{1}{6}$

167. (a) P(white marble) = $\frac{2}{6}$ = $\frac{1}{3}$

(b) P(marble blue) = $\frac{3}{6}$ = ½

(c) We must add 2 white marbles to the bag to make the probability of white equal to ½.

168. (a) P(Kamau picks white marble) = $\frac{2}{4}$ = ½

(b) P(Omungu picks white marble) = $\frac{2}{4}$ = ½

(c) P(both pick white marbles) = ¼.

169. Refer to frame 75. Find.

(a) P(both Kamau and Omungu pick green marbles) ________

(b) P(the boys pick a marble of the same colour) ________

---

170. Sometimes tree diagrams are used to show possible outcomes of an experiment and hence to calculate probabilities of those outcomes. When a coin is tossed once, we get two outcomes, a head and a tail. We can represent these outcomes by a "tree" as follows:

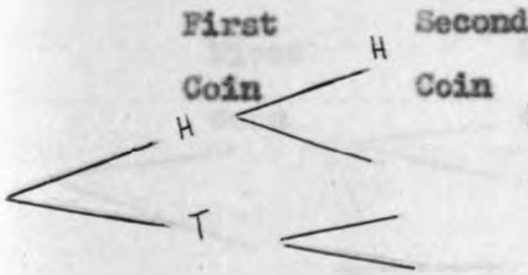
Answers:

169. (a) P(both Kamau and Omungu pick green marbles) = ¼

(b) P(the boys pick a marble of the same colour) = ½

170. No answer is required for this frame.

171. Fill in the tree diagram and the table to show all the outcomes when two coins are tossed.

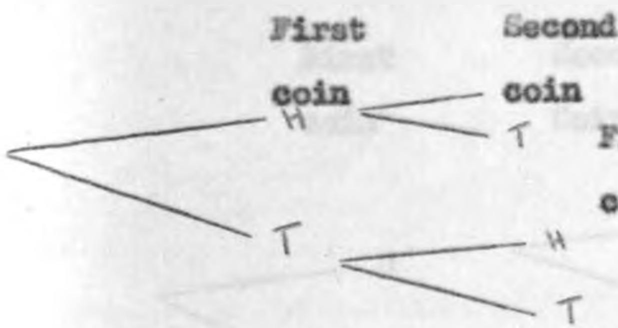


		Second Coin	
		H	T
First Coin	H		
	T		

List all the possible outcomes when 2 coins are tossed. HH , ____, ____, ____.

The table of the right should help you to read the tree diagram.

Answer:

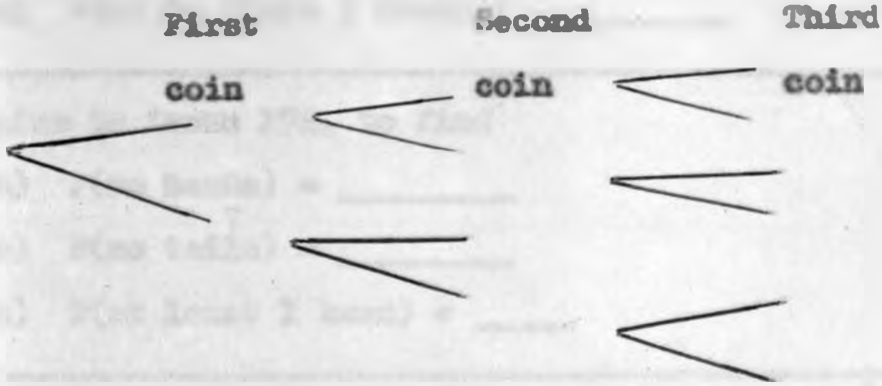


		Second coin	
		H	T
First coin	H	HH	HT
	T	TH	TT

Outcomes when 2 coins are tossed are HH, HT, TH, TT.

The tree diagram is read from left to right and from top to bottom. For example on the top branch we have HH and HT. On the bottom branch we have TH and TT.

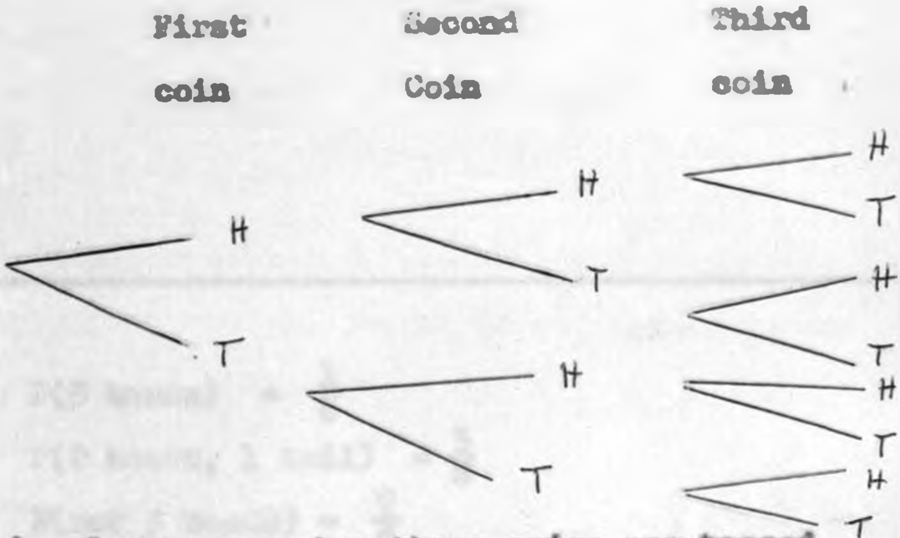
172. Fill in the tree diagram to show all the outcomes when three coins are tossed.



(a) What is the total number of outcomes when three coins are tossed? ______

(b) How many of these outcomes include 3 heads? __________

Answer:



(a) 8 outcomes when three coins are tossed.

(b) One outcome includes three heads.

173. (a) From the tree diagram of frame 172, find the probability of getting 3 heads. _____

(b) What is P(2 heads, 1 tail)? ________

(c) What is P(not 3 heads)? __________

---

174. Refer to frame 172, to find

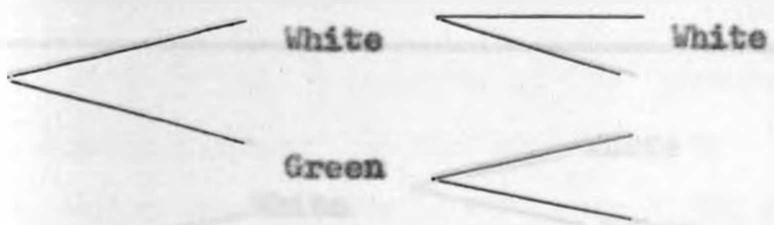
(a) P(no heads) = _________

(b) P(no tails) = _________

(c) P(at least 1 head) = _____

---

175. Fill in the following tree diagram for the marbles of frame 75.



Answers:

173. (a) P(3 heads) = $\frac{1}{8}$

(b) P(2 heads, 1 tail) = $\frac{3}{8}$

(c) P(not 3 heads) = $\frac{7}{8}$

174. (a) P(no heads) = $\frac{1}{8}$

(b) P(no tails) = $\frac{1}{8}$

(c) P(at least 1 head) = $\frac{7}{8}$

175. The possible outcomes are read from the tree, going from left to right.

List these outcomes ______, ______, ______, ______.

What is the probability of getting a white marble from each boy?

---

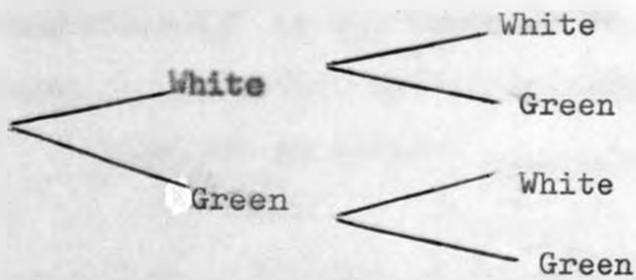
176. Refer to frame 175 to find

(a) The probability of getting both a white marble and a gree marble P(WG) = ______

(b) P(GW) = ______.

Answers:

175.



Outcomes are white; white; White; green green; white; green; white; groen, green.

P(WW) = ¼

176. (P(WG) = ¼

P(GW) = ¼.

SELF - TEST 3

1. Look at the spinner below.

The probabilities for each colour are:

	Red	Blue	Yellow	Green
Probability	$\frac{1}{6}$	¼	¼	$\frac{1}{3}$
Renamed Probabilities	$\frac{\quad}{12}$	$\frac{\quad}{12}$	$\frac{\quad}{12}$	$\frac{\quad}{12}$

(a) Fill in the numerators of the "renamed probabilities" in the table above.

(b) Write P(yellow or blue) as an addition problem:

P(yellow or blue) = _____ + ______

= _____

(c) Write P(green or red) as an addition problem:

P(green or red) = ______ + ______

= _____

(a)

	Red	Blue	Yellow	Green
Probability	$\frac{1}{6}$	¼	¼	$\frac{1}{3}$
Renamed Probabilities	$\frac{2}{12}$	$\frac{3}{12}$	$\frac{3}{12}$	$\frac{4}{12}$

(b) P(yellow or blue) = P(yellow) + P(blue)

$= \frac{3}{12} + \frac{3}{12} = \frac{6}{12} = ½.$

(c) P(green or red) = P(green) + P(red)

= 4 + 2 = 6 = ½

1. (d) Write P(red or blue) as an addition problem:

P(red or blue) = ____ + ____

= ____

(e) Write P(green or blue) as an addition problem:

P(green or blue) = ____ + ____

= ____

(f) Write P(yellow or red) as an addition problem:

P(yellow or red) = ____ + ____

= ____

(d) P(red or blue) = P(red) + P(blue)

$$= \frac{2}{12} + \frac{3}{12} = \frac{5}{12}$$

(e) P(green or blue) = P(green) + P(blue)

$$= \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$$

(f) P(yellow or red) = P(yellow) + P(red)

$$= \frac{3}{12} + \frac{2}{12} = \frac{5}{12}$$

2. Look at the spinner below. It is divided into 12 parts, all the same size.

(a) Give the following probabilities.

P(1) = ____ P(2) = ____ P(3) = ____

(b) P(either 1 or 2) = ____

(c) List the prime number outcomes: ____, ____, ____, ____, ____.

(d) P(prime number) = ____.

(e) List the outcomes that are factors of 12: ____, ____, ____, ____.

(a) $P(1) = \frac{1}{12}$; $P(2) = \frac{1}{12}$; $P(3) = \frac{1}{12}$

(b) $P(\text{either 1 or 2}) = P(1) + P(2)$

$$= \frac{1}{12} + \frac{1}{12}$$

$$= \frac{2}{12} = \frac{1}{6}$$

(c) Prime number outcomes are: 2, 3, 5, 7, 11.

(d) $P(\text{prime numbers}) = \frac{5}{12}$

(e) Outcomes that are factors of 12 are:

1, 2, 3, 4, 6, 12.

2. (f) P(factor of 12 = ______

(g) List the outcomes that are either 4 or odd: ___, ___, ___, ___, ___, ___, ___

(h) P(either 4 or odd) = ______.

(i) P(n $>$ 0) = ______

(j) P(not 5) = ______

(k) List the outcomes that are neither 5 nor 6.
___, ___, ___, ___, ___, ___, ___.

((l) P(neither 5 nor 6) = ____

(m) P(even) = _____

(n) P(odd) = _____

(p) P(factor of 13) = _______

(f) P(factor of 12) = $\frac{6}{12}$ = ½

(g) Outcomes that are either 4 or odd:-

4, 1, 3, 5, 7, 9, 11.

(h) P(either 4 or odd) = P(4) + P(odd)

= $\frac{7}{12}$.

(i) P(n $>$ 0) = $\frac{12}{12}$ = 1.

(j) P(not 5) = $\frac{11}{12}$.

(k) Outcomes that are neither 5 nor 6

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.

(l) P(neither 5 nor 6) = $\frac{10}{12}$ = $\frac{5}{6}$

(m) P(even) = $\frac{6}{12}$ = ½

(n) P(odd) = $\frac{6}{12}$ = ½

(p) P(factor of 13) = $\frac{1}{12}$

APPENDIX B

CONVENTIONAL INSTRUCTION

PROBABILITY

INTRODUCTION

Probability is an important branch of mathematics. It is used in making decisions in military operations, scientific research, design and quality control of manufactured products, insurance calculations, governmental operations, etc. It is also important in all games of chance.

When learning about probability therefore, you are learning about a very important branch of mathematics.

This unit is divided into three sections: Section One deals with "Ideas about Chance;" Section Two is on "Experiments in Probability" and Section Three is about "Finding Probabilities."

SECTION ONE

IDEAS ABOUT CHANCE

Purpose:

To stimulate pupils to think more

objectively about chance events. Through participation, discussion and demonstrations by the teachers: Pupils are expected to have opportunities to test their intuition regarding the results of some activities involving chance, and to make guesses, estimates, and predictions about such results.

Objectives: Throughout this Section,

1. pupils will be able to think objectively about chance events;
2. pupils will be able to distinguish between expected and experimental outcomes of events.

Lesson 1

Purpose:- To introduce ideas about chance.

Materials needed: None

Mathematical words to be learned: Chance; probability; certain; uncertain; probably; likely; unlikely.

Introduction:

Today we are going to learn about chance. Some of you have heard statements that talk about chance. For example, you might have heard or made the following statements:

1. It is more likely that I shall go to see my uncle during the holidays.
2. Chances are good that my father will buy me a shirt at the end of this month.
3. Kamau and Barasa have equal chances to win.
4. I am almost certain that I can come to your house after school.

These sentences are alike in one way. They have words and ideas which are used in a part of mathematics called probability. In probability, we are interested in things which happen by chance. By using mathematics we can often estimate quite accurately what will probably happen.

The pupils should discuss the implications of statements 1 to 4 above.

1. Now try to answer the following questions.
   a) Which Football Club will win the East and Central African Club Championships next year?
   b) Will all the members of your class be in school next Monday?

2. Some things are more likely to happen than others.

   a) Which is more likely, that one of the pupils in this class will be absent or that the mathematics teacher in this class will be absent?

   b) Which is more likely, that you will have ugali for breakfast or that you will have ugali for lunch?

3. Some things are more likely to happen than not.

   a) In Kisumu in July, is it more likely than not that it will rain at noon?

   b) Is it more likely than not that you can find the sum of 324 and 465?

4. Some things are certain and some things are impossible. Which of the following events are certain or impossible?

   a) A man can live without water for three months.

   b) Barasa's dog can write his first and last names in Swahili.

   c) All new cars from China this year will use water for fuel.

   d) Tomorrow, today, will be yesterday.

5. Our ideas about chance might be classified as certain, uncertain, or impossible. In the following sentences write C, U or I for certain, uncertain or impossible.

a) ______ The sun will set in the east.

b) ______ A river flows downhill.

c) ______ We will see the sun tomorrow.

d) ______ A river flows uphill.

e) ______ I will not sleep at all this week.

f) ______ A river is deep today than yesterday.

6. When we say a teacher gives a test on Friday, it does not mean we are sure he is going to give one this Friday. We can use numbers to tell how likely it is that he will give a test this Friday.

Mrs. Obunga gave a test on 3 Fridays out of every 4 last year.

Mr. Ogoti gave a test on 7 Fridays out of every 8 last year.

Mrs. Okiya gave a test on 2 out of every 3 Fridays last year.

Mrs. Oyor gave a test on 20 out of every 21 Fridays last year.

(a) Who is the most likely to give a test on Friday?
(b) Who is the least likely to give a test on Friday?

If you know a teacher usually gives a test on Friday, you decide to study a little more on Thursday night.

Lesson 2.

Purpose: To extend the children's understanding of the ideas about chance.

Materials needed: Dice.

Mathematical Words: Outcome, possible, fair, unfair.

Revise: Chance, Certain, likely.

Introduction:

Go quickly through the last lessons by asking children questions.

The Lesson:-

Class Discussion Exercises.

Show the children a die.

1. How many faces has this die?
2. Can you name the number of dots on each face?
3. If I toss this die once, how many possible outcomes are there? Which are these outcomes?
4. If a die is tossed, the face that is on top is the one that counts.
   Draw a die on the board to show the face that counts.

5. Look at the die on the board, which face shows up?

   How many times are each of the numbers on the die likely to show on the top face if we toss the die once?

6. If we toss a die are there equal chances that a number on any of the six faces will show up?

7. If events have equal chances of occuring, we say that they are equally likely. If you were playing a game with a friend and each one of you had an equal chance of winning we would say that the game was fair. But if one of you had more chances of winning, we would say that the game was unfair.

8. Imagine that you are playing a game with your friend. For each game a rule is given that tells who wins. If neither player wins, the game is a tie. Try to tell whether each game is fair and, if not, who is more likely to win, you or the other player?

   1. Use one die.

      a) You win if 1 is up. The other player wins if 3 is up.

b) You win if an odd number is up. The other player wins if an even number is up.
First list the set of odd numbers and the set of even numbers.
Are these outcomes equally likely?

c) You win if 3 is up. The other player wins if a number greater than 3 is up.

2. For these games, if 1 is up, call it Result 1. If either 2 or 4 is up, call it Result 2. If 3, 5, 6 is up, call it Result 3.

a) You win on Result 3. The other player wins on Result 1. __________

b) You win on Result 3, and he wins on any Result less than 3. __________

c) You win on an even-numbered Result, and he wins, otherwise. __________

3. Use two different colours, perhaps one white and one green. Toss them together.
List all the 36 possible outcomes for a toss of two dice.

a) You win if 1 is on each die. The other player win if 5 is on each die. __________

b) You win if there is an even number on the white die. The other player wins otherwise. ______

c) You win if 6 is on the white die, and he wins if 4 is on the green die. ________

d) You win if 1 is on each die. He wins if one die has 1 and the other has 2. _____

e) You win if the number on the white die is greater than the number on the green die. He wins otherwise. ____________

4. Use one die, and throw it two times for each game.

a) You win if the number the second time is greater than the number the first time. Otherwise, he wins. _______________

b) You win if the number each time is even. He wins if the number each time is odd.

Lesson 3.

Purpose:- To extend the children's understanding of the ideas about chance.

Materials needed:- dice, spinners.

Words to be learned:- Outcome.

The word outcome is often used in talking about probability. People often ask, "How did the football game come out?" or they might say "what was the outcome of the football game?"

In probability when we talk about the outcomes of an activity, we mean all the things that can happen (all the possibilities). For a football game, for example, there are three possibilities or outcomes: Your team will win, your team will lose, or it will be a tie. If all the outcomes are equally likely, their probabilities are equal.

In order to see whether or not the outcomes for any game are equally likely it is better to make a list of outcomes.

A. CLASS DISCUSSION

1. What are the outcomes when one die is tossed?
2. Are these outcomes equally likely?
3. How many outcomes are there when you toss two dice.

   To make a list of the outcomes when two dice are tossed you can make a table such as the one below:

Green die

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2						
3						
4						
5						
6						

White die

Complete the above table then answer the following questions:

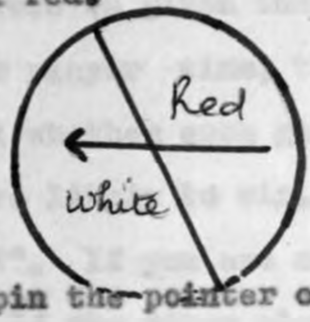
(i) How many different number pairs are shown in the table?

(ii) How many outcomes are there for tossing two dice?

(iii) Are all these outcomes equally likely?

Note that the first number is the outcome on the white die, and the second number is the outcome on the green die.

B. Draw this spinner on the board. It is half white and half red.

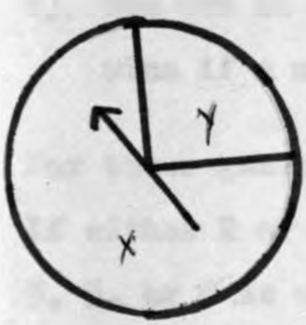


Suppose you spin the pointer of this spinner once and it does not stop on the boundary.

a) How many possible outcomes are there? List these outcomes.

b) Are these outcomes equally likely?

C. Look at the spinner on the left.



a) What are the outcomes if the pointer of this spinner is spun once?

b) Are the outcomes equally likely?

c) The rule is <u>you</u> win if the pointer stops on Y, you lose if it stops on X. Do you want to play?

EXERCISES

For each game a rule is given that tells who wins. If neither player wins, the game is a tie. Try to tell whether each game is fair and, if not who is more likely to win. If the game is fair, write "F". If you are more likely to win, write "Y". If the other player is more likely to win, write "O".

1. Use one die.
   a) You win if 1 is up. The other player wins if 3 is up. ________
   b) You win if an odd number is up. The other player wins if an even number is up. ____
   c) You win if 3 is up. The other player wins if a number greater than 3 is up. ___

2. For these games, if 1 is up, call it Result 1. If either 2 or 4 is up, call it Result 2. If 3, 5, or 6 is up, call it Result 3.
   a) You win on Result 3. The other player wins on Result 1. ________
   b) You win on Result 3, and he wins on any Result less than 3. ________
   c) You win on an even-numbered Result, and he wins otherwise. ________

3. Use two dice, one white and one green. Toss them together.

   a) You win if 1 is on each die. The other player wins if 5 is on each die. ______

   b) You win if there is an even number on the white die. The other player wins otherwise ________

   c) You win if 6 is on the white die, and he wins if 4 is on the green die. ________

   d) You win if 1 is on each die. He wins if one die has 1 and the other has 2. _____

   e) You win if the number on the white die is greater than the number on the green die. He wins otherwise. ________

4. Use one die and throw it two times for each game.

   a) You win if the number the second time is greater than the number the first time. Otherwise, he wins ____________.

   b) You win if the number each time is even. He wins if the number each time is odd.

   ____

5. a) What are the outcomes when you toss one die? List the outcomes.

b) What are the outcomes when you toss two dice? List these outcomes.

SECTION II

EXPERIMENTS IN PROBABILITY

Objective:

To help the pupils with the techniques for gathering, tabulating, graphing and interpreting data which they generate by tossing a coin, tossing a die, drawing marbles and spinning the pointer of a spinner.

The ideas gained from activities should sharpen children's intuition about chance events by analysing the results of a large number of trials.

Vocabulary:- Tabulate, horizontal vertical, tally.

Material needed:- Spinner, coins: 5 - cent piece, 10 - cent piece and 50 - cent piece, dice, marbles.

Lesson 4 and 5:- Tossing a die.

Purpose:- To extend children's ideas about chance events by involving them in experimental work.

Materials:- Iken tossed a die 20 times and recorded her outcomes in the following table:-

	No.of ⚀	No.of ⚁	No.of ⚂	No.of ⚃	No.of ⚄	No.of ⚅
Tally	//	////	///	//	///	~~////~~/
Total	2	4	3	2	3	6

Study the table carefully and see how many times each face of the die showed up.

1. You now toss a die 60 times and make a record of the number of dots on the top face. Record your results in a table such as the one shown above.
   Now answer the following questions:
   a) How many 1's did you get?
   b) How many 3's did you get?
   c) Did you get each outcome about the same number of times?
   d) How many times did you expect to get each outcome?

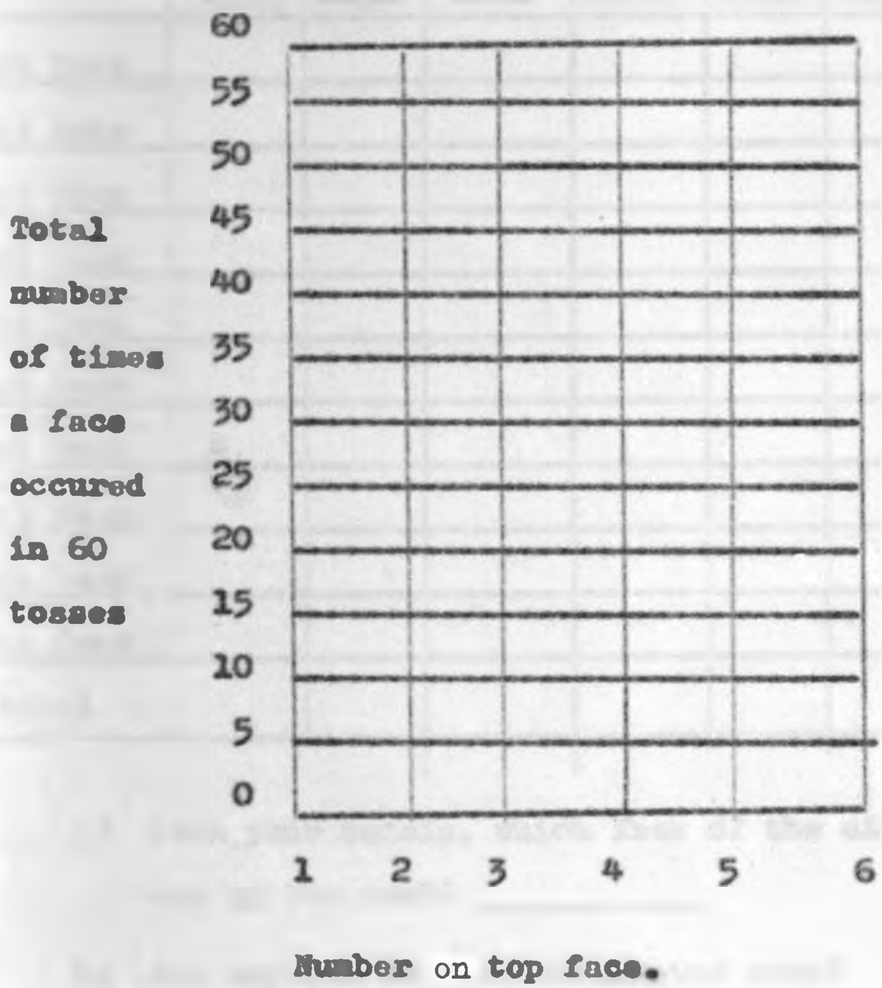
2. Toss a die 100 times. Keep a record of your results in the table below.

	No. of 1's	No. of 2's	No. of 3's	No. of 4's	No. of 5's	No. of 6's
Tally						
Total						

a) Did you get each outcome about the same number of times?

b) Does your experiment make you think that in the long run you are likely to get each outcome 1 time in 6?

3. In question 1 you tossed a die 60 times and recorded your results in a table. Use these results to draw a graph.



4. Toss a die 10 times and record your results in the following table.

	No. of ⚀	No. of ⚁	No. of ⚂	No. of ⚃	No. of ⚄	No. of ⚅
1st Toss						
2nd Toss						
3rd Toss						
4th Toss						
5th Toss						
6th Toss						
7th Toss						
8th Toss						
9th Toss						
10th Toss						
Total						

a) From your totals, which face of the die was up the most? ____________

b) Are any two or more totals the same? ________ Which? ___________

c) Would you expect that on 10 tosses, each number would come up at least once? __________

d) (i) If we tossed a die 1000 times, could we be sure that every number would come up at least once? ___________

(ii) How many times would you expect every number to come up when you toss a die 1000 times?

5. If two dice are tossed at the same time there are 36 possible outcomes. We can represent these outcomes in a table such as this.

Number on second die						
6						
5						
4	(1,4)					
3	(1,3)					(6,3)
2	(1,2)					
1	(1,1)	(2,1)				
	1	2	3	4	5	6

Number on first die

Complete the above table to show all the possible outcomes in a throw of two dice.

In the table, the first number in the ordered pair, say (1,2), refers to the outcome on the first die while the second number refers to the outcome on the second die.

Lesson 6 and 7: Tossing a coin

Purpose: To extend children's ideas about chance events.

Materials: Coins - 5 - cents piece; 10 - cent piece and 50 - cent piece.

Words to learn:- expect; actual, about.

A coin has two faces: a head and a tail (the side of the coin with the coat of arms is referred to as "tail").

If you toss a coin once there is one chance out of two possiblities of getting a head.

(i) How many times would you expect a head to show up?

(ii) How many times would you expect a tail to show up?.

1. Maria tossed a coin 22 times and recorded her results in a table below:-

	No.of heads up	No.of tails up	Total No. of Tosses
Tally	~~////~~ ////	~~/////~~ ~~////~~ ///	
Total	9	13	22

a) (i) How many times did she get a head?

(ii) How many times did she get a tail?

(iii) Number of times she got a head plus number of times she got a tail is equal to ________.

(iv) Are her outcomes equally likely?

b) (i) How many times did she expect to get a head?

(ii) How many times did she expect to get a tail?

2. Toss a coin 100 times. Keep a record of the results in a table such as this.

	No. of heads up	No. of tails up	Total No. of Tosses
Tally			
Total			

a) How many times did you expect to get the "heads"?

b) How many times did you actually get the "heads?

c) Did you get the "heads" more than 40 times?

d) Did you get about as many heads as tails?

3. Toss a coin 200 times and keep a record in a table such as the one shown in question 1.

   a) How many times did you get a head?

   b) How many times did you get a tail?

   c) How many times would you expect to get a head?

   d) How many times would you expect to get a tail?

   e) Is the number of times the "heads" showed up closer to the number of times you would expect "heads" to show up?

Note:- If a coin is tossed a large number of times, the number of times a head actually shows up would be closer to the number of times we would expect a head to show up.

<u>Lessons 8 - 10</u>

<u>Purpose:-</u> To extend children's knowledge about ideas about expected and experimental outcomes of an activity.

<u>Revise:</u> actual outcomes and expected outcomes. Throughout these lessons stress the difference between <u>actual</u> outcomes and <u>expected</u> outcomes. Assuming that the coin is a balanced one, i.e. if it does not stand on its edge when it is

tossed, the actual number of times it lands "heads" up would approximately approach the number of times we would expect it to land "heads" up.

1. Take two coins, a 10 - cent piece and a 5 - cent piece. Toss them together 40 times. Record your results in a table below:-

	No. of heads up	No. of tails up	Total No. of Tosses
5 - cent coin			
10 - cent coin			

a) How many times do you get two heads?

b) How many times do you get two tails?

c) How many times do you get a head and a tail?

d) Do you think you would get a head and a tail almost 2 times as you would get 2 heads or 2 tails?

2. When two coins are tossed, say a 5 - cent piece and a 10 - cent piece, they can fall in one of the following ways shown in the table below.

5 - cent piece	10 - cent piece
Head	Head
Head	Tail
-	-
-	-

Complete this table.

3. We can also draw a table to show the outcomes when two coins are tossed.
Complete the table.

		10 - cent piece	
		H	T
5 - cent piece	H	H	
	T		

4. Toss 3 coins, a 5 - cent coin, a 10 - cent coin and a 50 - cent coin. Record your results in a table below.

	No. of times heads shows	No. of times tails shows	Total No. of Tosses
5 - cent piece			
10 - cent piece			
50 - cent piece			

How many times do you get

a) 3 heads?

b) 3 tails?

c) a head and a tail?

5. If you toss 3 coins they can fall in one of 8 different ways. Complete the table below.

5 - cent piece	10 - cent piece	50 - cent piece
Head	Head	Head
Head	Head	Tail
———	———	———
———	———	———
———	———	———
———	———	———
———	———	———

6. All the outcomes from a toss of three coins can also be shown in a table such as this:

5 - cent coin and 10 - cent coin	50 cent coin H	50 cent coin T
H H	H H H	H H T
H T		
T H		
T T		T T T

Complete this table then answer the following questions:-

a) How many times are we likely to get 3 heads?

b) How many times are we likely to get 3 tails?

c) How many times are we likely to get two heads and one tail?

d) How many times are we likely to get one head and two tails?

7. If we toss 2 coins we may record the outcomes as is shown in the table below:-

	2H, OT	1H, 1T	OH, 2T
	HH	HT	TT
No. of Outcomes	1	-	1

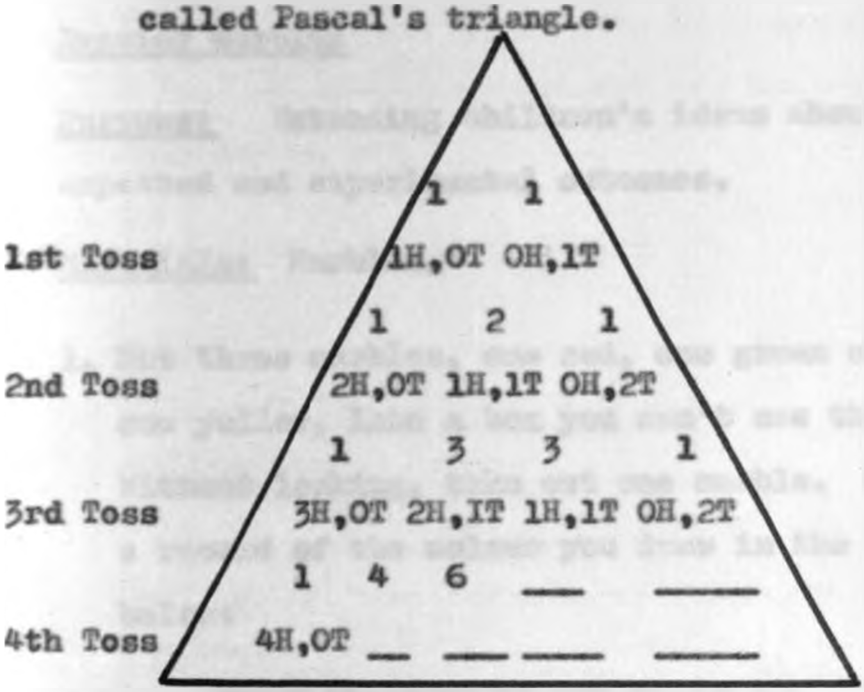
Complete the table.

8. Make a table like the one in question 7 for three tosses of a coin.

9. **Complete the table below for 4 tosses of a coin.**

	4H,0T	3H,1T	2H, 2T	1H,3T	0H,4T
	HHHH	HHH	HH		TTTT
No. of outcomes	1		6		1

10. We can organise the results of questions 7, 8 and 9 in a triangular form. This is called Pascal's triangle.



Complete the row for the 4th toss.

11. Look at the pattern in the triangle on question 10. Complete the 5th and the 6th rows (5 and 6 tosses of a coin) <u>without</u> making a table.

12. I tossed a coin 40 times and counted 18 heads. Is this more or less than you would have expected?

Lesson 11 - 12.

Drawing marbles

Purpose: Extending children's ideas about expected and experimental outcomes.

Materials: Marbles.

1. Put three marbles, one red, one green and one yellow, into a box you can't see through. Without looking, take out one marble. Keep a record of the colour you draw in the table below:

Red	Green	Yellow

Put the marble back in the box, mix the marbles and draw again. Do this 50 times.

a). Did you get about the same number of each colour?

b) What are the outcomes of this activity?

c) Did you get each outcome about $\frac{1}{3}$ of the time?

2. Put three marbles, two white and one blue into the box. Do as you did in question 1: mix, draw, keep a record and put the marble back in the box.

Do this experiment 50 times.

White	Blue

a) What are the outcomes of this activity?

__________, __________, __________.

b) Suppose this experiment was repeated 100 times, about how many times would you expect to draw out a white marble?

3. Kamau and Omangu, each has one white and one green marble. Kamau picks one of his marbles without looking and then Omangu picks one of his, also without looking. The four possible outcomes are listed in the table below. Complete the table on the right to show the outcomes in a shorter way.

	Kamau's marble	Omangu's marble
1	White	White
2	White	Green
3	Green	White
4	Green	Green

	Kamau's marble	Omangu's marble
1	W	W
2	W	___
3	6	___
4	___	___

5. There are two bags in Okiya's house. In the first bag, there are two marbles, red and blue. In the second bag, there are again two marbles, red and blue.

Sometimes we can use a table such as the one shown below to help find the possible outcomes.

		Second Bag	
		Red	Blue
First Bag	Red	Red, Red	
	Blue	Blue, Red	

Complete the table.

In the above table, the left side shows the colour of the marble taken from the first bag. This colour is shown first in the row. The top of the table shows the colour from the second bag.

6. Okiya now has three bags in his house. In the first bag, there are one red and one blue marbles; in the second bag, there are one red and one blue marbles and the third bag contains one red and one blue marbles. If Okiya draws a marble from each bag, his outcomes can be put in a table such as the one below.

The outcomes for the first two bags are on the left. The top of the table shows the outcome from the third bag. Complete the table.

		Outcomes from the third bag	
		R_3	B_3
Outcomes from the first and second bag.	R_1R_2	$R_1R_2R_3$	
	R_1B_2		
	B_1R_2		
	B_1B_2		

7. Each time we add a bag, we double (multiply by 2) the number of outcomes. For instance, with one bag there were 2 outcomes - Red and Blue. With two bags there were 4 outcomes - (Red, Red); Red, Blue); (Blue, Red) and (Blue, Blue).

   a) How many outcomes are there when there are three bags?

   b) List these outcomes.

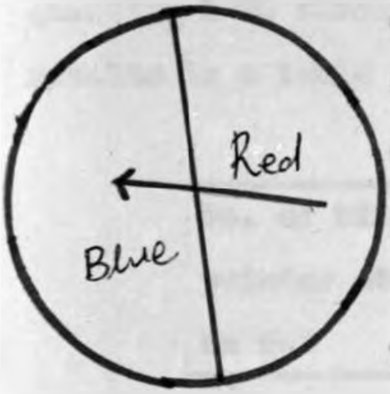
8. The number of outcomes increases by powers of 2. The number of outcomes for one bag is 2. The number for two bags is $2^2 = 2 \times 2 = 4$. In the same way, write down the number of outcomes for 3 bags.

<u>Lesson 13 - 16:</u>

<u>SPINNING THE POINTER OF SPINNER</u>

<u>Purpose:</u> To extend children's knowledge about <u>expected</u> and <u>actual</u> outcomes and to enable them to distinguish between these words (expected and actual outcomes).

<u>Material:</u> Spinners.

1. 

The spinner on the left is divided into two equal parts. Suppose you spin the pointer of this spinner 100 times.

a) How many times is it likely to stop on red?

b) How many times is it likely to stop on blue?

2. If you were to play a game with the spinner in question 1, you would win if the pointer stopped on red, your friend would win if the pointer stopped on blue.

Write T if you think the statement below is true. If it is false, write F.

a) I would be more likely to win the game since the pointer would stop on red most of the time ____________.

b) My friend would win most of the time ______.

c) Both of us would have equal chances to win this game since the pointer would stop on red about the same number of times it would stop on blue_________.

3. Spin the pointer of the spinner shown in question 1 20 times. Keep a record of your results in a table below:

No. of times pointer stops on red	No. of times pointer stops on blue

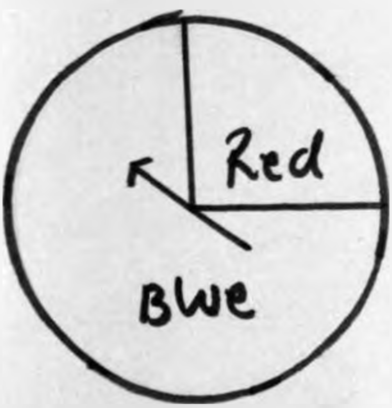
a) How many times does the pointer stop on red?

b) How many times does the pointer stop on blue?

c) Would you expect the pointer to stop on red the same number of times as it would stop on blue?

d) About how many times would you expect the pointer of this spinner to stop on red if you spun it 400 times?

4.



The spinner on the left is divided into two sections. The red section is $\frac{1}{4}$ of the whole and the blue section is $\frac{3}{4}$ of the whole. Spin the pointer of this spinner 20 times and keep a record of your results in a table below.

No. of times pointer stops on red	No. of times pointer stops on blue

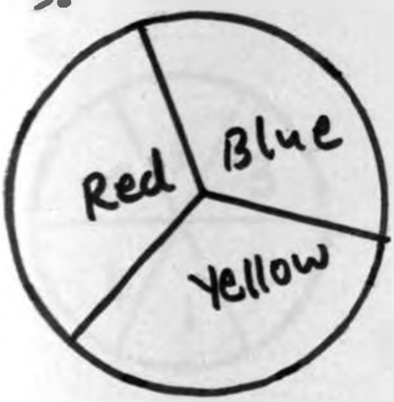
a) How many times did the pointer stop in red?

b) How many times did the pointer stop on blue?

c) Is the pointer equally likely to stop on red as on blue?

d) About how many times would you expect the pointer to stop on blue if the pointer of this spinner were spun 400 times?

5.



The spinner on the left is divided into 3 equal parts. Each part is $\frac{1}{3}$ of the whole.

Spin the pointer of this spinner 20 times and keep track of the outcomes in a table such as this.

No. of times pointer falls on blue	No. of times pointer stops on red	No. of times pointer stops on yellow

a) How many times did the pointer stop on red? ________; on blue? ________: on yellow ____.

b) Is each of these colours equally likely?

c) If you were to spin the pointer of the spinner shown above 900 times, about how many times would you expect it to fall on

a) Red? (b) Blue? (c) yellow?

6.

The spinner on the left is divided into six equal parts. Nana spun the pointer of this spinner 20 times and kept a record of her results in a table such as this.

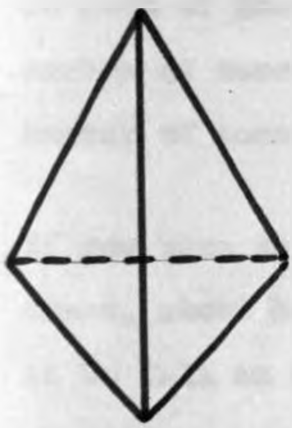
	No. of times pointer stops on 1	No. of times pointer stops on 2	No. of times pointer stops on 3	No. of times pointer stops on 4	No. of times pointer stops on 5	No. of times pointer stops on 6
Tally	////	~~////~~	/	//	///	~~////~~
Total	4	5	1	2	3	5

(i) How many times did Maua's pointer stop on

a) 1? b) 2? c) 3? d) 4? e) 5? f) 6?

(ii) Is each of the numbers equally likely?

(iii) If Maua spun the pointer of the spinner 600 times, about how many times would she expect it to stop on 4?

7. 

This tetrahedron has one of its faces coloured red, one blue, another yellow, and the last green.

Toss the tetrahedron 20 times and note the face that is <u>down</u>. Keep track of the outcomes in a table such as this.

	Red	Blue	Green	Yellow
Tally				
Total				

a) How many times did the tetrahedron fall on Red? ____ on Blue? ____ on Green ____ on Yellow? ________.

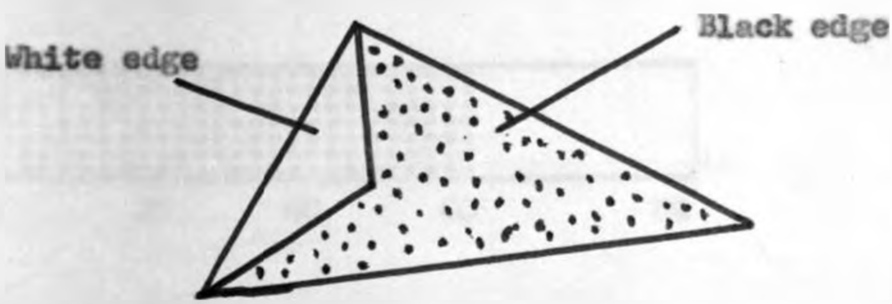
b) Add the number of times it fell on red to the number of times it fell on Blue ________.

c) Add the number of times it fell on green and yellow ________.

d) Is each of these sums about half of the total number of tosses, or about ¼ of the total number of tosses? About ________.

e) If you were to throw the tetrahedron 1,000 times, about how many times would you expect it to fall on red?

8.

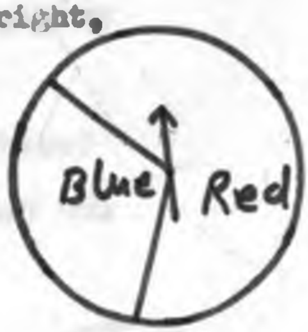


This 3 - sided spinner is divided into three equal parts.

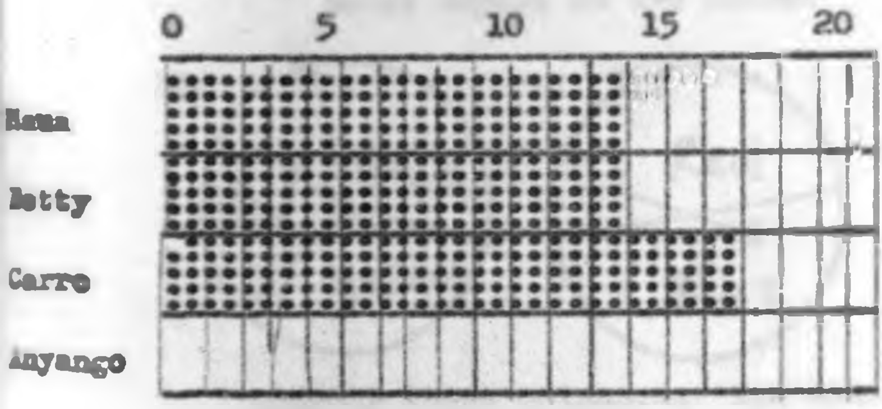
a) If you spin it 42 times how many times would you expect it to fall on the black edge?

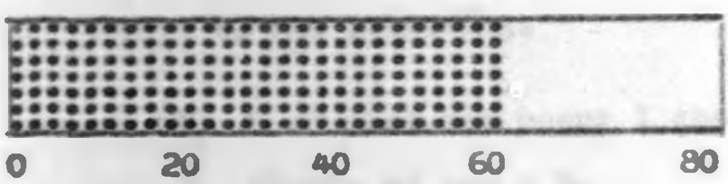
b) If after 60 spins of this spinner you had recorded 23 times for the red edge. Is this less or more than you would expect?

c) Four girls, Maua, Betty, Carro and Anyango, each used the spinner shown on the right, they drew a bar graph shown below:-



No. of blues in 50 spins





No. of blues in 200 spins.

a) Who had the smallest number of blues in 50 spins?

b) Who had the largest number of blues in 50 spins?

c) How many reds did Betty get in 50 spins?

d) Which of these fractions tells about how much of the dial is blue?

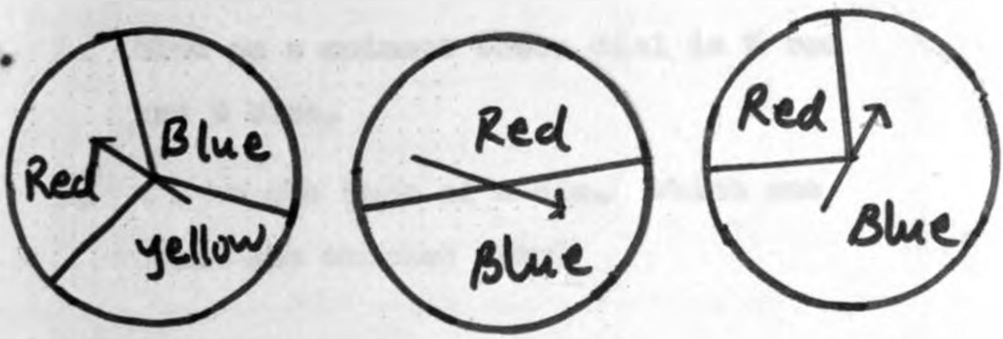
½, ⅓, ¼.

e) Did any girl get 25 or more blues?

f) How many times in all was the spinner spun by the girls?

g) How many of the spins ended on blue? Is this about the number of blues you would expect in 200 spins?

10.



Look at these spinners. You can use fractions to compare the chances of different results.

Complete the following.

a) (i) ½ of dial red <u>means</u> 1 chance in 2 <u>means</u> chance of red = ½.

(ii) ½ of dial blue means ____ chance in 2 <u>means</u> chance of blue = ____

b) (i) $\frac{1}{3}$ of dial red <u>means</u> 1 chance in _____

means chance of red = ________.

(ii) $\frac{1}{3}$ of dial blue <u>means</u> ____ chance in

3 means chance of blue = _______.

(iii) $\frac{1}{3}$ of dial yellow <u>means</u> $\frac{1}{3}$ chance in

___ <u>means</u> chance of yellow = $\frac{1}{3}$.

c) (i) ½ of dial red <u>means</u> _____ chance in

____ <u>means</u> chance of red = _____.

(ii) ½ of dial blue means red is impossible

<u>means</u> chance of red = 0.

11. Ateka was told that she would get Shs. 20/= if she could get one of the following outcomes:-

1. Blue on a spinner whose dial is ½ red and ½ blue.

2. A 2 on one toss of a die. Which one would she choose? Why?

SECTION 3:

Finding Probabilities

Introduction:- When we talk about the probability of a particular outcome, we tell how likely it is that the outcome is the one we get. We use a number that tells what of the total outcomes we expect a particular one to happen. This means that probabilities can be written as fractions.

Lesson 17 and 18.

Purpose:- To introduce probability.

Mathematical words: Revise chance, certain. Teach probability. Revise chance events in the last two sections. These sections contain elementary ideas of probability. Some experiments performed earlier will now be used to draw conclusions about their outcome.

Throughout the last section a distinction was drawn between expected and experimental results. Expected results are used to calculate probabilities.

1. When tossing one die, we have six outcomes. We write the 6 under the bar of a fraction.

$$\frac{\quad}{6}$$

Getting the outcome 3 is just as likely as any of the others, so we expect it about $\frac{1}{6}$ of the time. We say, "The probability of getting a 3 in a single toss of a die is $\frac{1}{6}$.

2. In the experiment, "Tossing a coin once", there are 2 outcomes.

   a) What is the probability of obtaining a head? P (H) = ½

   b) What is the probability of obtaining a tail? P (T) = ½.

3. Sometimes we give probabilities for things that can't possibly happen. In tossing a die, there is no chance at all of getting the outcome "7".

   The number of times you would get 7 in tossing one die is 0 so $P(7) = \frac{0}{6} = 0$

4. We can also give a probability for a "sure thing" (that is, a thing which must happen). For example, what is the probability of getting a number less than 7 when one die is tossed?

There are six ways to get a number less than 7. So the probability of getting a number less than 7 is $\frac{6}{6}$ = 1. We write this as

P(number less than 7) = $\frac{6}{6}$ = 1.

Classs Discussion.

1. In the experiment, "Tossing one Die", what is the probability of the outcome 5?

   P(5) = ______ P(2) = ______ P(1) = ______

   P(4) = ______ P(6) = ______

2. a) How many outcomes are there when two dice are tossed? Find

   (i) P(4, 3) (ii) P(6, 6)

   (ii) P(7, 1) (iv) P(3, 5).

3. In the experiment of "Drawing marbles" you used one red, one green, and one yellow marble. What is

   a) P(red marble drawn)
   b) P(drawing blue marble);
   c) P(green)
   d) P(yellow)
   e) P(not blue).

Note: The outcomes in the experiment were equally likely.

4. If we toss two dice we can show all the possible sums of the dots on the two dice in table such as the one below:-

Number on Second Die

Number on First Die		1	2	3	4	5	6
	1	2	3				
	3		5				
	4				8		
	5					10	
	6						12

Complete the above table and use it to answer questions (a) to (f) below:-

a) One way to get a sum of 7 is to get a 1 on the first die and a 6 on the second die. We write this as (1, 6). There are five more ways to get a sum of 7. Write down the five ways.

b) (i) How many entries are there in the table?

(ii) How many possible entries are there when you toss two dice?

c) (i) Of the entries in the table, how many are 6's?

(ii) What is the probability of getting a sum of 6 when two dice are tossed?

d) (i) How many entries are odd numbers?

(ii) What is the probability of getting the sum that is an odd number?

e) (i) How many of the sums are either 5 or 9?

(ii) What is the possibility that the sum will be either 5 or 9?

f) (i) P(sum = 3) (ii) P(sum = 8)

(iii) P(sum =12) (iv) P(sum = 2)

(v) P(sum = 11) (vi) P(sum = 2 or sum = 12)

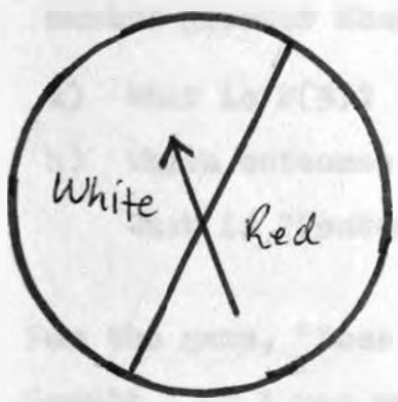
(vii) P(sum = 6 or sum = 8)

(viii) P(sum = 5 or sum = 9)

(ix) P(sum = 7)

(x) P(sum > 9).

5.



The spinner on the left is divided into two equal parts. Find

(a) P(red)

(b) P(white)

(c) P(yellow).

Lesson 19:

Purpose:- To extend the children's understanding of probability.

Mathematical words:- Revise probability, equally likely outcomes, impossible outcomes, outcomes which are certain to occur.

You played some games at the beginning. You were asked to decide whether or not the games were fair. You found that some games were fair and some were not. You saw that the game was fair if your winning outcome was just as likely as the other player's. We say that the game is fair if P(you win) = P(your friend wins)

EXERCISES

1. The rule for a game using one die is: "You win if 3 is up; the other player wins if a number greater than 3 is up."
   a) What is P(3)?
   b) Which outcomes are greater than 3? What is P(outcome greater than 3)?

2. For the game, "Toss one Die" you called the Result 1 if 1 was up; you called the Result 2 if either 2 or 4 was up; you called the Result 3 if 3, 5, or 6 was up.

a) What is P(Result 1)?

b) How many outcomes give Result 2?

What is P(Result 2)?

c) How many outcomes give Result 3?

What is P(Result 3)?

3. Write L if you are more likely to win and H if the other player is more likely to win for each rule. Write E if both are equally likely to win.

a) You win on Result 3. He wins on Result 1.

b) You win on Result 3. He wins on any result less than 3.

c) You win on an even-numbered result and he wins otherwise.

4. When you toss a green die and a white die together, how many outcomes are there?

What is the probability of any of these outcomes?

5. The rule is: "You win if there is an even number on the white die; the other player wins otherwise. Find

a) P(even number) (b) P(odd number)

c) P(no one wins) (d) P(both win).

6. The rule is: "You win if 6 is on the white die, and the other player wins if 4 is on the green die." Find

a) P(6 on white die) (b) P(4 on green die)

c) P(both win) (d) P(no one wins)

7. The rule is: "You win if 1 is on each die; the other player wins if 1 is on one die and 2 is on the other die." Find

a) P(you win) (b) P(other wins)

c) P(both win) (d) P(no one wins)

8. The rule is: "You win if the number on the white die is greater than the number on the green die; the other player wins otherwise." What is

a) P(you win) (b) P(other wins)

c) P(no one wins) (d) P(both win)

9. The part of the rule which says, "He wins otherwise" is now changed to "He wins if the number on the white die is less than the number on the green die."

a) Does this change your chance to win?

b) With this change, what is P(other wins); P(no one wins).

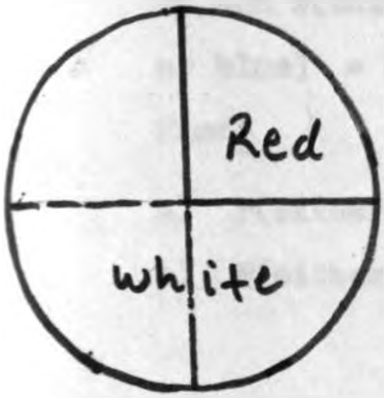
Lesson 20:

Purpose: To introduce "Either - or " Probabilities.

Mathematical words: Teach or. You can mention And but do not teach it. In connection with set language or means union ($\cup$) and And means intersection ($\cap$).

1. In question 7 above, you found the probability of "1 on one die and 2 on the other." You found that the pair would be either (1, 2) or (2, 1). You probably counted these outcomes and found 2 out of 36 outcomes, so you found the probability to be $\frac{2}{36}$ or $\frac{1}{18}$.

Sometimes you cannot find the probability of either this event or that event by counting. Look at this spinner.



There are two outcomes, red and white. But these outcomes are not equally likely. If the spinner is honest, i.e. if the pointer does not stop on a line each time it is spun, we would expect the pointer to stop on red about one out of four times.

What is

a) P(red)? (b) P(white)?

By looking at this spinner, you know that it is <u>certain</u> the outcome will be either white or red.

What is

a) P(either white or red)?

b) P(red) + P(white)?

2. 

The spinner on the left is ½ white, ¼ red, $\frac{1}{8}$ blue and $\frac{1}{8}$ yellow. These outcomes are not equally likely.

P(white) = ½, P(blue) = $\frac{1}{8}$

P(red) = ¼ P(yellow) = $\frac{1}{8}$

To find the probability of <u>either</u> white <u>or</u> blue, we add P(white) and P(blue). P(either white or blue) = ½ + $\frac{1}{8}$ = $\frac{5}{8}$

Find

a) P(either blue or yellow)

b) P(either red or blue).

EXERCISES

1. Look at the spinner below.

The probabilities for each colour are:

	Red	Blue	Yellow	Green
Probability	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{3}$
Renamed Probabilities	$\frac{}{12}$	$\frac{}{12}$	$\frac{}{12}$	$\frac{}{12}$

a) Fill in the numerators of the "renamed probabilities" in the table above.

b) Write P(yellow or blue) as an addition problem.

c) Write P(yellow or red) as an addition problem.

d) What is P(red or blue)?

e) What is P(green or blue)?

f) What is P(yellow or red)?

2. Look at the spinner below. It is divided into 12 parts, all the same size.

What is

a) P(1)? P(2)? P(3)?

b) P(either 1 or 2)?

c) i) List the prime number outcomes.

ii) What is P(prime number)?

d) i) List the outcomes that are factors of 12.

ii) P(factor of 12) = ?

e) i) List the outcomes that are either 4 or odd.

ii) P(either 4 or odd) = ?

f) P(number greater than zero) = ?

g) i) List the outcomes that are neither 5 nor 6.

ii) P(neither 5 nor 6) = ?

h) P(even) = ?

i) P(odd) = ?

j) P($n > 7$) = ?

k) P(factor of 13) = ?

l) P($n < 0$) = ?

Lesson 21:

Purpose: To extend children's ideas about either - or probabilities.

Mathematical words: Revise or; Either.

Briefly teach the Mathematical meaning of And.

You can't always find either - or probabilities just by adding. Look at the spinner below. It is divided into six parts, all the same size.

P(red) = ________ P(1) = ________.

P(red) + P(1) = ________.

You can see that the probability of either red or 1 is not $\frac{2}{3}$, because three of the six parts of the spinner (½ of it) are neither red nor 1. You can't just add P(red) and P(1) to find P(either red or 1). Why.

a) How many parts of the spinner are red?

b) How many parts of the spinner have 1?

c) Put an X on each part of the spinner that is either red or 1. How many X's do you have?

You can find P(either red or 1) by adding the probabilities of each one and then <u>subtracting</u> the probability of that part of the spinner that has a 1 <u>and</u> is red.

$$P(\text{either red or } 1) = P(\text{red}) + P(1) - P(\text{red and} 1)$$

$$P(\text{either red or } 1) = \frac{1}{3} + \frac{1}{3} - \frac{1}{6}$$

$$= \frac{2}{3} - \frac{1}{6}$$

$$= \frac{4}{6} - \frac{1}{6} = ?$$

EXERCISES

Look at the spinner below. It is divided into 12 parts, all the same size.

a) i) Which outcomes are odd?

ii) P(odd) = ?

b) i) Which outcomes are prime?

ii) P(prime) = ?

c) i) Which outcomes are odd and prime?

ii) P(odd and prime) = ?

iii) P(either odd or prime) = P(___)+P(____)-P(___).

Follow the arguement of the example above and then check your answer by counting.

Lesson 22:

Purpose: To extend children's knowledge about probability by using two or more coins, spinners, marbles and dice.

Mathematical words:- Teach both.

When one coin is tossed, there are 2 outcomes, head or tail. The probability of a head showing up when one coin is tossed is ½ and the probability of a tail showing up is also ½, provided the coin is "honest."

The outcome on one toss doesn't have anything to do with the outcome on the next toss. Even if you get heads five times in a row or ten times in a row, the probability of heads on the next toss is still ½.

Suppose you toss two coins together, say a 10 - cent piece and a 5 - cent piece, do you think the probability of both heads showing up will still be ½?

1. Here is a table for tossing two coins.

		10 - cent piece	
		H	T
5 - cent piece	H	HH	
	T		

a) Complete the table. How many outcomes are there when two coins are tossed?

b) What is

i) P(both heads) = ?

ii) P(both tails) = ?

iii) P(1 head, 1 tail) = ?

2. Here is a table for tossing 3 coins.

		50 - cent piece	
		H	T
5 - cent coin and 10 - cent coin	HH	HHH	HHT
	HT		
	TH		
	TT		TTT

a) Complete the table. How many outcomes are there when you toss 3 coins?

b) What is

i) P(3 heads)? =

ii) P(3 tails)? =

iii) P(2 heads, 1 tail)? =

iv) P(1 head, 2 tails)? =

v) P(at least 1 head) = ?

vi) P(at least 2 tails) = ?

3. Two white marbles and two green marbles are put into a box. If you take out one marble without looking, what is the probability that it is white?

b) If you take out 2 marbles, what is the probability that the marbles drawn are white?

4. We can think of the problem of question 2 like this: Uhuru and Ponto, each draws one of the marbles. The possible outcomes are shown in a table below:

		Uhuru's draws	
		White	Green
Ponto's draws	White	WW	WG.
	Green		

a) Complete the table. How many outcomes are there?

b) What is the probability that both boys drew a white marble ?

c) What is the probability that the boys drew a white marble and a green marble?

d) What is the probability that the boys drew a green marble?

5. i) A bag contains several marbles. Some are red, some white, and the rest blue. If you pick one marble without looking, the probability of red is $\frac{1}{3}$, and the probability of white is also $\frac{1}{3}$. What is the probability of blue?

6. A bag contains one red marble, two white marble, and three blue marbles. If you pick one marble without looking,

a) What is the probability that it will be red?

b) What is the probability that the marble drawn will be white?

c) P(marble blue) = ?

d) How many white marbles must we add to the bag to make the probability of white equal to ½?

Lesson 23:

Purpose:- To use probability trees to calculate probabilities.

Mathematical words:- Teach probability tree.

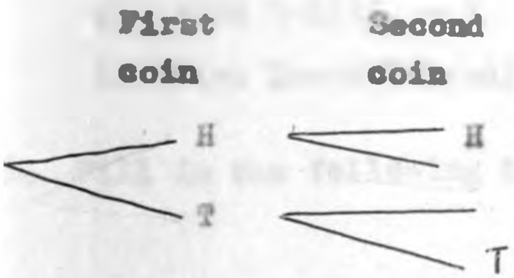
Sometimes tree diagrams are used to show possible outcomes of an experiment and hence to calculate probabilities of those outcomes.

When a coin is tossed once, we get two outcomes, a head and a tail. We can represent these outcomes by a "tree" as follows:

H

T

1. Fill in the tree diagram and the table to show all the outcomes when two coins are tossed.



		Second coin	
		H	T
First coin	H		
	T		

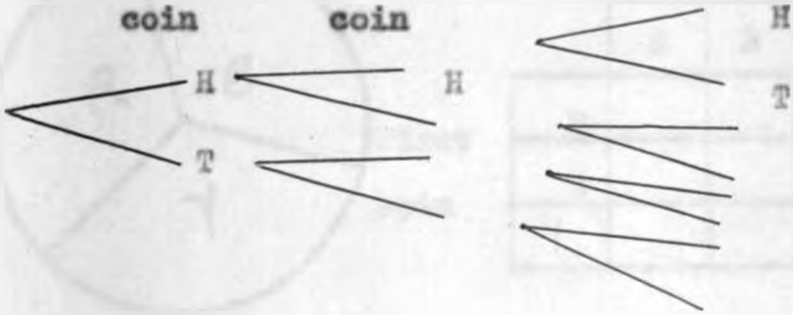
Use the tree diagram to calculate

a) P(both heads) (b) P(both tails)

c) P(1 head, 1 tail).

2. Fill the tree diagram to show all the outcomes when three coins are tossed.

First coin   Second coin



a) What is the total number of outcomes when three coins are tossed?

b) What is P(3 heads)?

c) What is P(2 heads, 1 tail)?

d) What is P(1 head, 2 tails)?

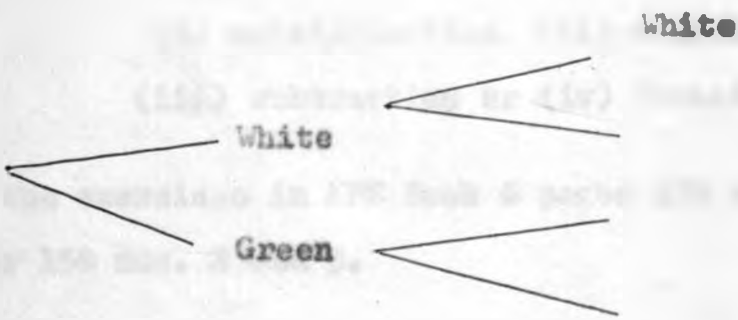
e) What is P(not 3 heads)?

f) P(no heads) = ?

g) P(no tails) = ?

i) P(at least 1 head) = ?

3. Fill in the following tree diagram.



Find

a) P(W W)   (b) P(G G)   (c) P(G W) = ?

4. Finish the table below to show the outcomes for two spins of the spinner.

First spin

	Second spin		
	R	B	Y
R			
B			
Y			

a) For the first spin what is

a) P(R) = ? (b) P(B) = ? (c) P(X) = ?

b) For the second spin

i) P(R) = ? (ii) P(B) = ? (iii) P(Y) = ?

c) For 2 spins

i) P(RR) = ? (ii) P(RB) = ? (iii) P(RY)=?

iv) P(YY) = ? (v) P(BB) = ?

vi) What mathematical operation connects the results of the first spin to the results of the second spin to yield the results of both spins: Is it (i) multiplication (ii) addition (iii) subtraction or (iv) division?

5. Do the exercises in KPE Book 6 pages 156 and 197. Page 158 nos. 2 and 3.

APPENDIX CI

PROBABILITY

PRE-TEST EXERCISES TIME: 40 MIN.

NAME ____________________ SCHOOL ____________

BOY OR GIRL ____________ AGE ______________

READ THE FOLLOWING INSTRUCTIONS AND EXAMPLES CAREFULLY.

1. The questions presented to you are designed to find out how much knowledge of probability you already have. They are like the problems that will be on the lesson.

2. Read carefully and think hard before you attempt any question. Attempt all questions.

3. Write your answers on the question paper

4. You should work out the examples given with your teacher.

EXAMPLES:

1. Think of spinning the pointer of the spinner on the right.

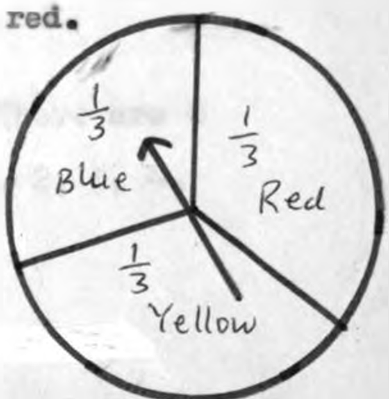
   The pointer is likely to stop on red.

   a) ½ of the time

   (b) $\frac{1}{3}$ of the time

   c) 0 of the time

   d) all the time.



Circle the letter of the correct answer.

Answer. Since the spinner is divided into three equal parts, if the pointer is fair it is likely to stop on red the same number of times it will stop on blue or on yellow. Hence it will stop on red $\frac{1}{3}$ of the time. So we circle choice (b).

2. For the spinner in question 1 we say that the probability that the pointer will stop on red is

a) ½ (b) $\frac{1}{3}$ (c) 0 (d) 1.

Answer:- The correct answer is $\frac{1}{3}$. So we circle choice (b).

3. Think of tossing a ten-cent coin once. There are two possible outcomes. Either head shows up or a tail shows up.

What is the probability of getting a head?

(a) P(H) = ½ (b) P(H) = ⅓ (c) P(H) = 0

(d) P(H) = 1.

Answer:- There are two possible outcomes and one of the possible outcomes may be a head. Hence the probability that a head will show up is ½. We therefore circle choice (a).

4. Think of tossing a di e once. There are 6 outcomes. These outcomes are 1, 2, 3, 4,

5 and 6. The outcomes are equally likely. What is the probability that the face that shows up will be one with 4 dots?

(a) $\frac{1}{12}$ (b) $\frac{4}{6}$ (c) $\frac{1}{6}$

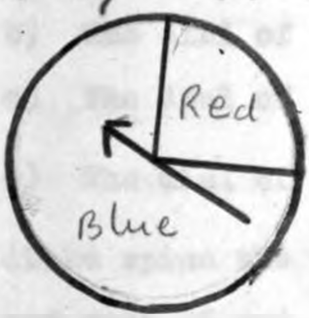
(d) $\frac{6}{6}$.

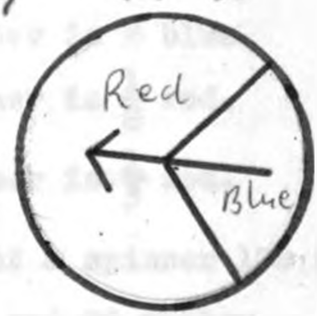
Answer: Since there are 6 outcomes, all equally likely, the probability that the face that shows up will be one with 4 dots is $\frac{1}{6}$. So we put a circle around choice (c).

---

1. Write $\frac{9}{7}$ as a mixed number.

a) $1\frac{2}{7}$ (b) 2 (c) $\frac{9}{7}$ (d) 9.





2. Study the two spinners above. Suppose a pirate captain said to you "I will give you just one chance on a spinner. If the pointer stops on blue, I will push you into the sea. If it stops on red, you may go free." Which spinner would you choose?

Which spinner would you choose?

(a) Spinner A

(b) Spinner B

(c) None of the spinners.

3. Lumumba has 3 green marbles and 2 blue mr marbles in his pocket. How many marbles must he remove to be sure of getting a blue marble?

a) one (b) at least 2 (c) more.

4. Omungu spins the pointer of a spinner 100 times and gets 35 reds. Which of the following statements is likely to be true? Put a circle around the letter that is likely to be true.

a) The dial of the spinner is all red.

b) The dial of the spinner is ¼ blue.

c) The dial of the spinner is $\frac{1}{8}$ red.

d) The dial of the spinner is $\frac{1}{3}$ red.

5. Ateka spins the pointer of a spinner 100 times and gets 25 red, 25 blue and 50 yellow.

Which of the following statements is true?

a) The dial of the spinner is ½ yellow.

b) The dial of the spinner is $\frac{1}{3}$ green.

c) The dial of the spinner is ¼ blue.

d) The dial of the spinner is all red.

6. You wish to get exactly 5 reds and 5 blues in 15 spins of a spinner. Which of the following dials could give this result?

   a) One-half red and one-half blue.
   b) One-third red, one-third blue and one-third yellow.
   c) One-fourth red, one-fourth blue and one-half yellow.
   d) One-fifth red, two-fifths blue and two-fifths yellow.

7. In which of the following is the chance of red equal to ¼?

   a) One chance in two of red.
   b) Two chances in four of red.
   c) One chance in five of red.
   d) Two chances in eight of red.

8. Which of the following spinners is likely to give about the same number of reds as yellow?

   a) One-half red, one-fourth yellow, one-fourth blue.
   b) One-third red, two-thirds yellow.
   c) One-third red, one-third yellow, one-third blue.
   d) Four-fifths yellow, one-fifth red.

9. If the dial of a spinner is all red, we say the chance of red is equal to:

   a) Any other chance.

   b) One chance in two.

   c) One-half.

   d) One.

10. If the dial of a spinner is all blue, we say the chance of red is equal to:

    a) One (b) zero (c) one chance in one

    d) one-half.

11. Awinja and Omutsimi each have one white and one green marble. Awinja picks one of her marbles without looking and then Omutsimi picks one of hers. The four possible outcomes are listed in a table below:

Awinja's marble	Omutsimi's marble
White	White
White	Green
Green	White
Green	Green

If the two girls pick the marbles at the same time, their chance of picking a white marble is equal to:

a) Any other chance.

b) One chance in two.

c) One chance out of one.

12. Tabu's bag contains three marbles, one-red, one white and one blue. If Tabu chooses one marble without looking, what is the probability that the marble Tabu chooses is red?

(a) $\frac{2}{3}$ (b) 1 (c) $\frac{1}{3}$ (d) 0.

13. The dial of the spinner shown on the right is divided into six equal regions. Pento spins the pointer of this spinner once. What is the probability that the pointer will stop on 3.

a) $\frac{1}{3}$ (b) $\frac{1}{6}$ (c) ¼ (d) 1.

14. The table below shows the possible outcomes if the pointer of the spinner shown on the left is spun twice. The spinner is divided into two equal parts.

		Second spin	
		Red	Red
First spin	Red	Red, Red	Red, Blue
	Blue	Blue, Red	Blue, Blue

What is the probability that the pointer will stop on red the first time and on blue the second time, i.e. what is P(RB)?

a) $\frac{1}{6}$ (b) ¼ (b) ½ (d) 0.

15. The table below shows all the possible outcomes when two coins are tossed.

		Second Coin	
		Head	Tail
First Coin	Head	Head, Head	Head, Tail
	Tail	Tail, Head	Tail, Tail

What is the probability of getting a head and a tail?

a) ½ (b) ¼ (c) $\frac{1}{8}$ (d) 1.

APPENDIX C2

PROBABILITY

POST-TEST

TIME: 1 HR. 30 MIN.

NAME ____________________ SCHOOL ____________________

BOY OR GIRL ____________ AGE ____________________

CLASS ____________________

Read the following instructions carefully before you begin the test.

1. Attempt all questions.
2. Write your answers on the question paper, you should circle the letter of the correct answer of your choice.
3. Do your rough work on a separate sheet of paper, then write your answer on the question paper

Example

1. Think of tossing a 10-cent coin once. There are two possible outcomes. Either a head shows up or a tail shows up. What is the probability that a head will show up?

(a) $P(H) = \frac{1}{2}$ (b) $P(H) = \frac{1}{4}$

(c) $P(H) = 0$ (d) $P(H) = 1$.

<u>Answer:-</u> There are two possible outcomes and one of the 2 possible outcomes may be a head. Hence the probability that a head will show up is ½. We therefore circle choice (a).

---

1. If an event is certain to occur, its probability is

   a) 0 (b) ½ (c) 1 (d) Greater than 1.

2. If an event can never happen, its probability is

   a) 0 (b) ½ (d) 1 (d) Greater than 1.

3. A box of marbles contains 7 blue marbles and 11 white marbles. Betty draws one marble from the box without looking. What is the probability that the marble drawn is white?

   a) $\frac{7}{11}$ (b) $\frac{7}{18}$ (c) $\frac{11}{18}$ (d) $\frac{11}{7}$

4. If the set of possible outcomes for an experiment consists of four equally likely outcomes, then the probability of each outcome is

   a) $\frac{4}{4}$ (b) ¼ (c) 0 (d) Greater than one.

5. The probability of throwing exactly four heads and one tail in a toss of five coins is $\frac{5}{32}$. What is the probability of <u>not</u> throwing four heads and one tail?

a) $\frac{5}{32}$ (b) 1 (c) $\frac{27}{32}$ (d) 0.

6. Think of spinning this spinner once. Each small section is $\frac{1}{10}$ of it.

P(Blue) = ____________

a) $\frac{1}{10}$ (b) ½ (c) $\frac{9}{10}$ (d) 5.

7. If you spin the pointer of a spinner 100 times and get 25 red, 25 blue and 50 yellow, which of the following statements is true?

a) The dial of the spinner is ¼ yellow.

b) The dial of the spinner is $\frac{1}{3}$ green.

c) The dial of the spinner is ¼ blue.

d) The dial of the spinner is all red.

8. When a die is tossed once, what is the probability of the outcome "greater than 2"?

a) $\frac{5}{6}$ (b) $\frac{2}{3}$ (c) $\frac{1}{6}$ (d) 1.

9. If you get exactly 5 reds and 5 blues in 15 spins of a spinner, which of the following dials could give this result?

a) ½ red and ½ blue.

b) $\frac{1}{3}$ red, $\frac{1}{3}$ blue and $\frac{1}{3}$ yellow.

c) ¼ red, ¼ blue and ½ yellow.

d) $\frac{1}{5}$ red, $\frac{2}{5}$ blue and $\frac{2}{5}$ yellow.

10. Ponto spins the pointer of a spinner whose dial is divided into six equal regions numbered 1, 2, 3, 4, 5, and 6. What is the probability that the pointer will stop on 3?

a) $\frac{1}{3}$ (b) $\frac{1}{6}$ (c) ¼ (d) 1.

11. In a gambling game where one coin is to be tossed, a player wins if he scores two heads and one tail. How many times must he toss the coin?

a) 8 times (b) once (c) three times

d) twice.

12. In the game "Toss two dice", you win if each die has 1 on the top face; the other player wins if one die has 1, and the other die has 2.
What is the probability that the other player wins?

a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{18}$ (d) $\frac{1}{36}$

13. The dial of a spinner is ½ white, ¼ red, $\frac{1}{8}$ blue and $\frac{1}{8}$ yellow. What is the probability that the pointer of the spinner will stop on either white or blue if the spinner is spun once?

a) ½ (b) $\frac{1}{8}$ (c) ¼ (d) $\frac{5}{8}$

14. Okiya has three bags in his house. In the first bag, there are one red and one blue marbles; in the second bag, there are one red and one blue marbles and the bag contains one red and one blue marbles. Okiya picks a marble from each bag without looking. What is the probability that he will draw at <u>least</u> a red marble?

a) 1 (b) $\frac{7}{8}$ (c) $\frac{3}{8}$ (d) 0.

15. A spinner whose dial is $\frac{1}{3}$ red, $\frac{1}{3}$ blue and $\frac{1}{3}$ yellow is spun twice. What is the probability of getting both red in two spins of the spinner?

a) $\frac{1}{9}$ (b) $\frac{2}{9}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

16. A spinner whose dial is $\frac{3}{8}$ red and $\frac{5}{8}$ blue and a spinner whose dial is ¼ blue and $\frac{3}{4}$ red are spun at the same time.
What would you do to find the probability of both red in two spins?

a) add P(Red) on first spinner to P(Red) on second spinner.
b) subtract P(Red) on first spinner from P(Red) on second spinner.
c) P(Red) on first spinner multiplied by P(Red) on second spinner.
d) Divide P(Red) on second spinner by P(Red) on first spinner.

17. A coin is tossed once, and a spinner whose dial is coloured Red, Blue and Yellow is spun once. What is the probability of <u>not</u> getting a head on a coin and a blue on a spinner

a) $\frac{1}{3}$ (b) ½ (c) $\frac{5}{6}$ (d) $\frac{1}{6}$.

18. Two dice are tossed together, what is the probability of getting either a sum of 6 or a sum of 7?

a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{11}{36}$ (d) $\frac{5}{36}$.

19. There are three boys and three girls in a house. Their mother wishes to take any two of them at a time for a walk in town. In how many different ways can she select two children to go with her?

a) 6 (b) 12 (c) 15 (d) 21.

20.

The spinner above is divided into six equal regions. Use it to find the probability of either Red or 1.

a) $\frac{2}{3}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$.

APPENDIX DI

NAME ____________________ SCHOOL ____________

BOY OR GIRL ____________ DATE ____________

AGE ____________________

ARITHMETIC REASONING

Instruction

This section consists of problems in arithmetic. However, you do not have to find the answer to each problem. You only have to tell how the answer could be found.

Example O.

Jane's father was 26 years old when she was born. Jane is 8 years old. How old is her father now?

(A) subtract
(B) divide
(C) add
(D) multiply.

Jane's father is now 34 years old. But, you are not asked to find this. You are asked how to find this. Since his age is found by adding 26 and 8, choice (C) should be circled.

Example 00

Desks are priced at Shs. 40/= each. If bought in lots of 4, the total price is reduced by Shs. 20/=. How much would 4 desks cost?

(A) divide and add

(B) multiply and multiply

(C) subtract and divide

(D) multiply and subtract.

One way to solve the problem would be to multiply Shs 40/= by 4 and then subtract 20 from the product. So you should circle choice (D).

Although some problems may be worked in more than one way, only one of the ways will be given among the answer choices.

You should only guess if you can rule out some of the choices. DO NOT guess wildly.

You will have 40 minutes for this section. If you finish before time is called, check your work.

DO NOT TURN THIS PAGE UNTIL ASKED TO DO SO.

1. There are 4 quarts in a gallon and 4 cups in a quart. How many cups are there in a gallon?

   (A) add

   (B) subtract

   (C) multiply

   (D) divide.

2. An electric planer is set to remove .02 of an inch each time a piece of wood is passed through it. If a board is put through 7 times, how much will have been removed?

   (A) multiply

   (B) subtract

   (C) divide

   (D) add.

3. There are 54 children at a small holiday camp. If there are 33 boys attending the camp, how many campers are girls?

   (A) add

   (B) multiply

   (C) subtract

   (D) divide.

4. A man wants to seed a lawn around his new home. His plot is 120 feet by 90 feet (10,800 sq. feet). His house is centered on the lot and occupies 2,785 sq. feet. How many square feet of ground may be put into lawn?

(A) add

(B) divide

(C) multiply

(D) subtract.

5. A wholesale fruit dealer sells oranges at 72 cents per pound and lemons at 31 cents per pound. One day he sold 79 pounds of each type of fruit. How much money was taken in?

(A) add and divide

(B) add and multiply

(C) multiply and subtract

(D) divide and divide.

6. A cyclist in an international bicycle race has covered an average of 9 miles every 20 minutes. If he can maintain the same average speed, how long will it take him to cycle the remaining 84 miles of the race?

(A) divide and multiply

(B) subtract and divide

(C) add and subtract

(D) divide and add

7. A grocer sells oranges for 59 cents a dozen. The oranges cost him 33 cents a dozen. How much profit is there on each orange?

(A) subtract and multiply

(B) divide and subtract

(C) add and divide

(D) subtract and divide

8. A boy works in a shop after school for a total of 10 hours a week. He also works 8 hours on Saturdays. How much is he being paid per hour, if he makes Shs. 20/70 per week?

(A) multiply and subtract

(B) add and divide

(C) divide and subtract

(D) add and multiply

9. A housewife took a job which pays Shs.65/00 per week. After paying taxes she is left with 76% of her salary, and each week she spends a total of Shs. 56/00 on lunches and bus fares. How much does her job increase the family income?

(A) divide and subtract

(B) subtract and multiply

(C) add and divide

(D) multiply and subtract

10. A rectangular underground reservoir is 15 feet deep and contains 2,000,000 gallons of water, when it is full. The short rains filled the reservoir, but a drought in January caused the water level to drop 8 feet. Approximately how many gallons of water were consumed during the drought?

(A) subtract and divide

(B) add and subtract

(C) divide and multiply

(D) subtract and multiply.

APPENDIX D2

DIAGNOSTIC TEST IN VULGAR FRACTIONS

NAME ____________________ SCHOOL ____________

BOY OR GIRL ______________ CLASS ____________

AGE _____________________ DATE _____________

READ CAREFULLY

a) This test is designed to help your teacher discover the difficulties you experience when working with fractions.

b) You should work quickly and carefully.

c) Where possible reduce fractions in the answers to lowest terms.

d) You are allowed 40 minutes to answer these questions.

1\.a) $\frac{3}{7} + \frac{1}{7} =$ (b) $\frac{5}{9} + \frac{1}{9} =$

c) $7 + \frac{2}{3} =$ (d) $2\frac{5}{6} + 5 =$

2\.a) $1\frac{5}{8} + \frac{1}{8} =$ (b) $3\frac{4}{9} + 1\frac{2}{9} =$

c) $3\frac{7}{10} + 1\frac{3}{10} =$

3\.a) $3\frac{3}{4} + 2\frac{5}{8} =$ (b) $\frac{4}{9} + \frac{5}{9} =$

c) $3\frac{7}{8} + 11\frac{1}{3} =$

4. a) $2\frac{1}{2} + 1\frac{4}{5} + 3\frac{7}{10} =$ (b) $2\frac{1}{2} + 3\frac{1}{4} + 1\frac{2}{5} =$

   c) $3\frac{7}{10} + 4\frac{8}{15} + 2\frac{5}{6} =$

5. a) $\frac{7}{9} - \frac{2}{9} =$ (b) $\frac{9}{10} - \frac{3}{10} =$

   c) $6\frac{3}{8} - 5\frac{5}{8} =$

6. a) $5\frac{8}{9} - 2\frac{1}{2} =$ (b) $5\frac{1}{2} - 3 =$

   c) $6 - 5\frac{5}{8} =$

7. a) $5\frac{3}{10} - \frac{7}{10} =$ (b) $6\frac{5}{12} - 3\frac{11}{15} =$

   c) $4\frac{7}{15} - \frac{7}{9} =$

8. a) $\frac{1}{4} - \frac{1}{8} =$ (b) $1\frac{2}{5} - 1\frac{1}{4} =$

APPENDIX D3

WORKING WITH NUMBERS

NAME ______________________ SCHOOL ____________

BOY OR GIRL ______________ CLASS ____________

AGE ______________________ DATE ____________

READ CAREFULLY

There are 10 questions about working with numbers. Each question has five answer choices. You should circle the letter in front of the answer you choose.

Here is an example of how you should mark your answers.

Example

Subtract 918 from 1,725.

a) 819 (b) 807 (c) 928

d) 1,018 (e) 1,622.

The answer is 807, so (b) has been circled.

You are to do as many questions as you can. Do not spend too much time on any one question.

You are allowed 40 minutes to work out these problems.

The first two questions are about a ringtoss game. In ringtoss each player gets three rings to toss. Rings on the peg win 23 points each. Rings off the peg lose 10 points each.

1. Bernina has two on and one off. How many points does she get?

   a) 5 (b) 15 (c) 36 (d) 40

   e) 60.

2. Otieno has one on and two off. How many points does he get?

   a) 3 (b) 20 (c) 25 (d) 40

   e) 45.

3. What number does S stand for if

   3 X 4 X S = 12 X S is a true statement?

   a) 20 (b) 0 (c) 3 (d) 4

   e) 5.

4. Which formula would you use to find how many stamps each person should get if 31 people share equally a package of 2325 stamps?

   a) 31 ÷ 2325 = n

(b) $2325 \div 31 = n$

(c) $2325 - 31 = n$

(d) $31 \times 2325 = n$

(e) $n - 2325 = 31$.

5. Suppose we decide to write fractions in a different way. For example, instead of $\frac{2}{3}$ we would write (2, 3) and instead of $\frac{7}{5}$ we would write (7,5). What would be the sum of (1,5) and (3,5)?

(a) (5,5) (b) (4,5) (c) (3,10)

(d) (4,10) (d) 3,25).

6. $1 \times 1 = 0$, $2 \times 2 = 3$, $5 \times 6 = 29$,

$7 \times 2 = 13$, $4 \times 4 = 15$, $9 \times 2 = 17$.

What does 6×3 equal?

(a) 6 (b) 3 (c) 9 (d) 17 (e) 18.

7. Which of the following will always produce an odd number?

i) The sum of two odd numbers

ii) The sum of any 3 even numbers.

iii) The sum of any 3 odd numbers.

(a) i) only (b) (ii) only (c) (iii) only

(d) i) and (ii) only (e) i) and (iii) only.

8. The sum of the odd numbers less than 4 and the even numbers less than 9 is

(a) 11 (b) 13 (c) 24 (d) 42

(e) 45

9. If you multiply a number less than 1,000 by one less than 100, the greatest possible answer you could get is

a) 98,901 (b) 100,000 (c) 1,000,000

d) 999,901 (e) 99,999.

10. How many pieces of wood will you have if you cut across a long board 17 times with a saw?

a) 16 (b) 17 (c) 18 (d) 19

e) none of these.

APPENDIX E

SILENT READING

NAME ____________________ SCHOOL ____________

BOY OR GIRL ____________ CLASS ____________

AGE ____________________ DATE ____________

(Time - 30 Minutes)

Read carefully but quickly each paragraph and the question at the end of it. Write the answers to the questions on the space provided. Write only one word answers whenever possible.

Paragraphs (a) and (b) are examples to be done by the teacher and the class.

(a) I have a cat. It is black and white. It is one year old. It sleeps in a box. It likes to play with a ball of wool.

Where does the cat sleep?

____

(b) Every now and then along the roads we see low wooden houses with tightly shut windows and little gardens stocked with flowers.

Choose the word below that tells about the windows, and write it on your answer paper.

half-open; open; closed apart

---

1. I am a wild bird. My home is in a tree. I can fly high in the air. I can sing a song.

   Question: Where is the bird's home?

   Answer: ______________________________

2. We have a baby. When we speak to him he waves his little hand. He has ten teeth. He sleeps in a cot most of the day.

   Question: How many teeth has the baby?

   Answer: ______________________________

3. It was getting so dark that Alice thought there must be a storm coming on. "What a thick black cloud that is!" She cried. "And how fast it comes! Why, I do believe it's got wings."

   Question: Do you think the sun was shining?

   Yes

   No

   Cannot tell.

Put a circle around the answer that is appropriate.

4. Otieno picked up a small bag full of money and went off with a light heart. His eyes sparkled for joy and he said to himself, "I must have been born in a lucky hour; everything that I wish for comes to me of itself."

Question: Was Otieno happy or unhappy?

Answer: ____________________

5. In some cities coloured lights are used to direct the cars at cross streets. A red light means "Stop" an orange light means "Get Ready," and a green light means "Go".

Questions: What light is used for "Get Ready?"

Answer: ____________________

6. Last Monday we went to the Zoo. We spent much time in front of an iron cage which held seven monkeys. They made us laugh when they put out their paws for nuts.

Question: What was the monkey's cage made of?

Answer: ____________________

7. There was once a shoemaker who worked very hard and was very honest, but still he could not earn enough to live on, and at last he

all he had in the world was gone except enough leather for one pair of shoes.

Choose the word below that tells what the shoemaker was and put a circle round it

lazy dishonest hardworking

proud idle

8. When a duck wants to come to rest on water it draws its head backward, tilts its body upward, thrusts its feet forward and spreads its tails outward. Choose the word below telling how the duck places its head. Put a circle around it.

upward forward backward

downward

9. I can skip, I go to school everyday, I wear a pretty dress, I have a long hair.

What am I?

____

10. Long ago there lived on the sea coast of Japan a young man named Yaina, a kindly fellow and clever with his rod and line.

In the space provided below write the word Yaina. If you think he was a fisherman put a line under his name; if you think he was not put a cross under his name.

11. The daylight is dying,
Away in the West,
The wild birds are flying,
In silence to rest.

Do these lines tell about evening or morning?

____

12. Over the grazing field,
In the reeds on the shore,
Lived a mother water-rat,
And her little water-rats four.

How many water-rats altogether lived in the reeds?

____

13. A sailor dropped the captain's silver tea-pot into the sea. The captain went to the sailor and said to him, "You let my tea-pot fall into the sea, did you not? It is lost." "No, no," said the sailor, I know where it is. It is at the ________ of the sea."

Write the word that has been left out.

14. If you are waiting on shore for a ship to come in the first thing you see is the smoke, later the funnels and masts come in sight, and lastly the hull of the ship itself is seen.

Suppose you were watching a ship leaving the land. Choose the word below that tells you the last thing you would see. Put a circle around the word you have chosen.

people	masts	smoke
funnels	hull	

15. Behind the little house were orange trees, a mango tree and two or three paw paw trees. Then came a stretch of rough grass and a stone wall with a gate leading into the pasture.

Was the stone wall in front, or at the side of the house?

____

16. A field mouse had a friend who lived in a house in town. Now the town mouse was asked by the field mouse to dine with him, so out he went and sat down to a meal of wheat.

Where did they dine? At the field mouse's home, or at the town mouse's home?

____

17. Upon a mountain height,
Far from the sea,
I found a shell,
And to my listening ear the lonely thing,
Ever a song of ocean seemed to sing,
Ever a tale of ocean seemed to tell.

Which seemed to sing a song? The mountain, The shell, or the ocean?

____

APPENDIX FI

NAME ______________ SCHOOL ______________

BOY OR GIRL ______________ CLASS ______________

AGE ______________ DATE ______________

ATTITUDE TOWARD MATHEMATICS

This is not a test. There are no "right" or "wrong" answers. Just answer the questions as honestly as you can.

In each question you are asked to tell how you feel about each statement by selecting one of the ways given beneath the statement.

Here is a practice sample:

Example 00

It is more fun to play hockey than to dance.

(A) Strongly agree
(B) Agree
(C) Disagree
(D) Strongly disagree

Which one of the four ways tells best how you feel about the statement?

(A) or (B) or (C) or
(D) Put an x on the letter of the answer you chooce.

Now do the same for the following statement. Work carefully and quickly. Do not spend a long time on one question. Please answer all the items and give only one answer to each.

All answers must be on the question papers

1. I hate Mathematics and avoid using it.

   (A) Strongly Agree

   (B) Agree

   (C) Disagree

   (D) Strongly Disagree

2. I have never liked Mathematics.

   (A) Strongly Agree

   (B) Agree

   (C) Strongly Disagree

   (D) Disagree.

3. I am afraid of doing word problems.

   (A) Strongly Agree

   (B) Agree

   (C) Strongly Disagree

   (D) Disagree

4. I have always been afraid of Mathematics.

   (A) Strongly Agree

   (B) Agree

   (C) Disagree

   (D) Strongly Disagree.

5. I can't see much value in Mathematics.

(A) Strongly Agree

(B) Agree

(C) Disagree

(D) Strongly Disagree

6. I avoid Mathematics because I am not very good with numbers.

(A) Strongly Agree

(B) Agree

(C) Disagree

(D) Strongly Disagree.

7. Mathematics is something you have to enjoy even though it is not enjoyable.

(A) Strongly Agree

(B) Agree

(C) Disagree

(D) Strongly Disagree

8. I do not feel sure of myself in Mathematics.

(A) Strongly Agree

(B) Agree

(C) Disagree

(D) Strongly Disagree.

9. I do not think Mathematics is fun, but I always want to do well in it.

(A) Strongly Agree

(B) Agree

(C) Disagree

(D) Strongly Disagree

10. I am not enthusiastic about Mathematics, but I have no real dislike for it either.

(A) Strongly Agree

(B) Agree

(C) Disagree

(D) Strongly Disagree

11. I like Mathematics, but I like other subjects just as well.

(A) Strongly Agree

(B) Agree

(C) Disagree

(D) Strongly Disagree

12. Mathematics is important as any other subject.

(A) Strongly Agree

(B) Agree

(C) Strongly Disagree

(D) Disagree

13. I enjoy doing problems when I know how to work them well.

(A) Strongly Agree

(B) Agree

(C) Disagree

(D) Strongly Disagree

14. Sometimes I enjoy the challenge presented by Maths problem.
    (A) Strongly Agree
    (B) Agree
    (C) Disagree
    (D) Strongly Disagree

15. I like Maths because it is practical.
    (A) Strongly Agree
    (B) Agree
    (C) Disagree
    (D) Strongly Disagree

16. Maths is very interesting.
    (A) Strongly Agree
    (B) Agree
    (C) Disagree
    (D) Strongly Disagree

17. I enjoy seeing how rapidly and accurately. I can work Maths problems.
    (A) Strongly Agree
    (B) Agree
    (C) Disagree
    (D) Strongly Disagree

18. I would like to spend more time in school working Maths.
    (A) Strongly Agree
    (B) Agree
    (C) Disagree
    (D) Strongly Disagree

19. I think about Maths problems outside school and like to work them out.

(A) Strongly Agree

(B) Agree

(C) Disagree

(D) Strongly Disagree

20. I never get tired of working with numbers.

(A) Strongly Agree

(B) Agree

(C) Disagree

(D) Strongly Disagree

21. I think that Mathematics is the most enjoyable subject.

(A) Strongly Agree

(B) Agree

(C) Disagree

(D) Strongly Disagree

22. Mathematics thrills me, and I like it better than any other subject.

(A) Strongly Agree

(B) Agree

(C) Disagree

(D) Strongly Disagree

APPENDIX F2

ATTITUDE INVENTORY

The statements below represent varying attitudes towards the use of programmed materials or teaching machines as a means of studying a subject. Read each statement and indicate the extent to which you agree or disagree by circling SA(Strongly Agree), A(Agree), U(Undecided or Neutral), D(Disagree), or SD(Strongly Disagree).

1. Classes in which programmed materials are used are dull and uninteresting.

   SA A U D SD

2. I feel that using programmed materials is the most effective method of studying that I have ever used.

   SA A U D SD

3. I am glad that I am not using programmed materials in more classes that I am at present.

   SA A U D SD

4. I do not like to work with programmed materials.

   SA A U D SD

5. School would be more interesting if programmed materials were used in more classes.

   SA A U D SD

6. I wish that I could study programmed materials in my other classes.

SA A U D SD

7. Using programmed materials results in too much wasted time.

SA A U D SD

8. Using programmed materials is interesting because you have to keep thinking.

SA A U D SD

9. I would rather be working with a group of classmates than working alone with a programmed textbook.

SA A U D SD

10. When I use programmed materials I can keep interested in my work.

SA A U D SD

11. When I use programmed materials I understand everything that I study.

SA A U D SD

12. I would rather have a teacher explain the subject than be left on my own with a programmed text.

SA A U D SD