MODEL FOR THE ESTIMATION OF INITIAL CONDITIONS IN A CONFLICT ENVIRONMENT

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Abstract

Conflict can be described as a condition in which actions of one person prevent or compel some outcome at the resistance of the other. Quite often this can be seen as "two or more competing, often incompatible, responses to same event". Model-based predictions and formulations of trends are becoming more commonly used by researchers in the field of conflicts. These models to a great extent rely on fundamental or empirical models that are frequently described by systems of differential equations. In this paper, we have developed the dynamic time varying model for estimating control variables (initial conditions) which play a significant role in the success of conflict resolution estimated using a logistic probability model. A real conflict data set, from International Peace Institute, Oslo (PRIO), was used to test on the workability of the model.

Key words: *Bayesian theory, Conflict, dynamic state, Initial conditions, Logit model, Ultimatum game*

1.0. Introduction

The application of formal models and quantitative analysis techniques have come a long way towards explaining how strategic actors bargain in a variety of conflict settings. For instance, in the political setting or international relations, bargaining plays a central role in understanding and solving any conflict and thus the mastery of the concept of bargaining is very important, [1, 2, 3, 4, 5 and 6].

The basics of logic of bargaining in the face of conflicting interests have been explained by Game theory. Political scientist have employed for instance, bargaining models based on the Game theory to analyze effects of open and closed rules on the distributive politics of legislative appropriation to the study of war initiation and termination, [7, 8 and 9].

Understanding the interplaying factors in a conflict is very important in solving the conflict. In the likelihood that the factors are not known, a reliable model can be used to predict them. Most conflicts are generally triggered by the differences in opinions and interpretation of an idea, [6]. It is therefore, important that these differences are understood in terms of their magnitude in a conflict. In this paper, we categorize these factors into two broad distinct variables, that is, control variables and state variables. Control variables are the most critical factors to any conflict. According to [6], the control variables are represented as reservation values. In general, it is important to understand the effects of these substantive variables (control variables) on the bargaining process.

The control and state variables can be represented in a model that describes a conflict situation using statistical and numerical models of the system dynamics. The fundamental or empirical models that are frequently described by systems of ordinary differential equations (ODEs) can also be used to describe a conflict situation, [10, 11]. The systems of ordinary differential equations can be used to predict the future behaviour/dynamics of the conflict, provided that the initial states of the conflict are known. An account on the modelling of a conflict from the perspective of social welfare theory and social choice theory has been given by [12]. A complete data defining all of the states of a conflict system at a specific time are, however, rarely available. This challenge can however, be handled using missing data analysis techniques, [13, 14 and 15].

In a conflict, for instance, there are some underlying issues that can be described to be private and as such may not be available. Moreover, both the models and the available initial data/information contain inaccuracies and random noise that can lead to significant differences between the predicted states of the system and the actual states of the system. In this case, observations of the system over time can be incorporated into the model equations to derive improved estimates of the states and also to provide information about the uncertainty in the estimates.

The popularity of model-based algorithms in a number of systems and situations like control and process optimization has consequently led to an increased interest in developing fundamental models with precise parameter estimates [16, 17]. It is therefore, crucial for researchers to develop models in a dynamic conflict system that responds to these needs reliably and efficiently.

In this paper, we present a model for the estimation of initial conditions of a conflict situation based on the state dynamics using ordinary differential equations (ODEs). The initial conditions estimated are to be integrated in the linear and exponential dynamic models to predict on the future trends of a conflict. A method for estimation of the initial conditions (initial control conditions) in a dynamic state system is given in section (3).

A brief description of the linear and exponential dynamic system models is provided where the estimated initial conditions can be used to form a predictor model.

1.1. Linear model:

In the dynamic system continuous time formulation, it is assumed that the absolute change with respect to time of the series is equal to a constant. That is, the average growth is constant during the period. Hence, the dynamics can be represented by

$$\frac{dy}{dt} = \phi, \quad y(0) = \theta, \quad \text{.......................(1)}$$

where θ is the initial condition of the series. This is equivalent to assuming that $\frac{d^2y}{dt^2} = 0$ and $\frac{dy(0)}{dt} = \phi$ are the initial conditions.

The solution to this equation is given by

$$y_t = \theta + \phi t, \text{ ....(2)}$$

which is the linear model in time. Thus, we can view the estimation of the parameters in (1) as fitting the solution (2) to a discrete data set.

1.2. Exponential model:

The dynamics in this case can be modelled by

$$\frac{dy}{dt} = \phi y, \quad y(0) = y_0 = e^{\theta}, \text{ ....(3)}$$

That is, the percent growth rate is equal to a constant or that the absolute change is proportional to the current value of the series. We denote by y_0 the initial condition for the problem with a solution given by,

$$y_t = e^{(\theta + \phi t)}, \text{ ....(4)}$$

This is the exponential model of time used in estimating trends and growth rates in dynamic environments like economic setting. Its advantage is that the estimated coefficient is the average growth rate.

The linear and exponential functions of time are often used for forecasting for instance in economics, business, and finance.

2.0. Dynamic representation of the system models

Purely linear and exponential functions of time can be used for trend estimation as a solution to their corresponding time dynamics equations, that is, equations used to describe how systems change or evolve over time. This is important because understanding the relationships can be very useful to researchers in a conflict situation. It is often the case that reality necessitates the relaxation of the linearity assumptions in a number of situations like conflict and economic environments giving rise to nonlinear dynamic systems. Analytical solutions of these systems are in general unattainable for some relatively more complicated dynamics and the only method of estimation may be the dynamic approach.

In a static state, the initial data point is used as the initial condition of the differential equation, while in the dynamic option; the initial condition(s) is estimated as an additional parameter. The nice thing about this procedure is that the dynamics are written as they occur in the model equations. It is very important to understand the difference between the static and dynamic options when fitting dynamic models to data.

The model developed in this paper, can be used for the estimation of the initial conditions for both static and dynamic systems.

3.0. Model for the estimation of initial conditions

In modelling any conflict, control variables play a crucial role. Control variables can be any private information that is relevant to the party's decision making in a conflict environment. In the context of this paper, the control variable is modelled to constitute the following components:

1. Demand to the other parties.
2. Demands from the other parties.

The two components are the conditioning variable to a probability of one another.

Suppose we have a conflict control variable ℓ_i, it can be defined by a Bayes probability distribution which is drawn independently and identically distributed (i.i.d) from a logistic distribution function $F_i(.)$ with a corresponding everywhere positive density $f_i(.)$, mean $\mu_i = 0$ variance $\sigma_i^2 < \infty$, and assuming that f_i's are continuously differentiable.

It is also assumed that a conflict is most likely to arise if the demands from one party are not met by the other party. These demands are usually private information/reservation. Understanding these demands is critical for success of a bargaining game towards the resolution of a conflict, since they constitute a greater part of the control variables of a conflict.

The private reservations R_i usually will lead to a conflict in opinion. As a consequence of conflict of opinion, a conflict state described by these divergent opinions is developed. This conflict state is characterised by initial conditions (control variables) which must be understood and quantified to successfully model and solve any conflict. In most cases, the initial conditions might be known or unknown and therefore a good model for the estimation of these conditions is paramount. The estimation of the initial conditions in a conflict situation can be compared to the estimation of types in a Bayesian game theory.

Suppose the initial conditions are the state set, θ, (current state), they can be represented by:

$$\theta = X_{i \in N} \theta_i, \quad \text{.........................} (5)$$

where N is a set of parties to a conflict, X is the state vector, θ_i is the state of the system at i.

Initially, state is assigned a prior belief $P(\theta)$ which reflects existing knowledge about the conflict state environment. As the system evolves, some new information and data say D will become available. To estimate these new outcomes, the available beliefs can be updated using the Baye's rule.

$$posterior = p(\theta / D) = \frac{p(D/\theta)p(\theta)}{\int_N p(D/\theta)p(\theta)d\theta} = \frac{likelihood \times prior}{nomalizer} \quad \text{...........(6)}$$

Equation (6) gives a new set of initial conditions.

3.1. Conflict and Ultimatum game

Suppose in a conflict the first party has made an offer y based on the state set θ given by (5), then the second party will chose between the offer and her reservation value given by $R_2 + \ell_2$. Equilibrium and hence settlement of a conflict can be achieved if the second party will play the cut-point strategy given by:

$$s_2(y, \ell_2) = \begin{cases} accept & if \;\; y \geq R_2 + \ell_2 \\ reject & if \;\; y < R_2 + \ell_2 \end{cases} \quad \text{...........(7)}$$

From a negotiation stand point, the first party does not observe ℓ_2, but must assess the probability that the second party will accept or reject his offer, where;

$$\begin{aligned} \Pr(accept \;/\; y) &= \Pr(y \geq R_2 + \ell_2) \\ &= \Pr(\ell_2 \leq y - R_2) \\ &= F_{\ell_2}(y - R_2) \end{aligned} \quad \text{................(8)}$$

Considering the optimization problem for the first party, given the second part's strategy (8), then the expected utility for the first party is:

$$Eu_1(y/Q^*) = F_{\ell_2}(y - R_2).(Q^* - y) + (1 - F_{\ell_2}(y - R_2)).(R_1 + \ell_1); \quad \text{..........(9)}$$

By the first order condition (F.O.C) and the log-concavity of f_{ℓ_2}, the first party's optimal offer is the unique y* that implicitly solves

$$y^* = Q^* - R_1 - \ell_1 - \frac{F_{\ell_2}(y^* - R_2)}{f_{\ell_2}(y^* - R_2)}, \quad \text{..........(10)}$$

However, $0 \leq y^* \leq Q^*$ and sometimes y^* will be outside the feasible set. We can then show that an end-point (0 or Q^*) is optimal and in any perfect Bayesian equilibrium (PBE), the first party will have the strategy:

$$s_1(\ell_1 / R_1, R_2, Q^*, F_{\ell_2}(.)) = \begin{cases} Q^*, & \ell_1 \leq -R_1 \frac{F_{\ell_2}(Q^* - R_2)}{f_{\ell_2}(Q^* - R_2)} \\ y^*, & -R_1 - \frac{F_{\ell_2}(Q^* - R_2)}{f_{\ell_2}(Q^* - R_2)} < \ell_1 < Q^* - R_1 - \frac{F_{\ell_2}(-R_2)}{f_{\ell_2}(-R_2)} \\ 0, & \ell_1 \geq Q^* - R_1 \frac{F_{\ell_2}(-R_2)}{f_{\ell_2}(-R_2)} \end{cases} \quad \text{..........(11)}$$

Taking variables δ_k, $k \in \{0, y, 1\}$ such that $\delta_0 = 1$, if y = 0, $\delta_y = 1$, if 0 < y < Q^* and $\delta_1 = 1$, if y = Q^* that is, a censored model with a "latent" best offer in the constraint set. Otherwise, there is the best feasible offer, at, a boundary point.

Taking the second party's acceptance as $\delta_{accept} = 1$, if she accepted the offer and $\delta_{accept} = 0$, if she rejected the offer and assuming we have data on both parties actions (i.e., y and δ_{accept}) from the state set, θ, then the likelihood would be

$$L = \prod_{i=1}^{n} \Pr(y^* < 0)^{\delta_0} . \Pr(y^* = y)^{\delta_y} . (1 - \Pr(y^* < Q^*))^{\delta_1} \Pr(accep)^{\delta_{accept}} . \Pr(rejec)^{1-\delta_{accept}} \quad \text{..........(12)}$$

Equation (12) is based on the existing control variables in θ. It gives the log-likelihood function for our data in terms of distributions already derived, which are functions of regressors.

Using equation (12), the Likelihood, $P(D/\theta)$, which is a measure of the probability of seeing particular realization of the state θ, can therefore be estimated, where y = ultimatum offer from the first party to the conflict, Q^*=upper bound of the contested prize, δ_i = actions, $\sigma_i : \ell_i \rightarrow A^i$, $i = \{1, 2\}$, where A^i defines the action set for the i^{th} party.

Since, party 1 is making the ultimatum offer, $A^1 = \{y: y \in [0; Q^*]\}$, the second party is then left to accept or reject the offer, so $A^2 = \{accept; reject\}$.

$$\Pr(accept / y) = \wedge(y - Z\gamma), \qquad (13)$$

Suppose the public portion of the parties' reservation values are $R_1 = \beta X$ and $R_2 = \gamma Z$, where X and Z are sets of substantive regressors.

Then, for party 1, logistic distribution of y^* implies that

$$y^* = Q^* - \beta X - \ell_1 - \frac{\wedge(y^* - \gamma Z)}{\lambda(y^* - \gamma Z)}, \qquad (14)$$

which is the optimal offer, where $\wedge(.)$ is the logit cumulative distribution function (c.d.f) and $\lambda(.)$ is the logit p.d.f.. Solving for y^* gives

$$y^* = Q^* - \beta X - \ell_1 - 1 - \omega\left(e^{(Q^* - \beta X - \gamma Z - \ell_1 - 1)}\right), \qquad (15)$$

where ω is Lambert's ω, which solves transcendental functions of the form $z = \omega e^w$ for ω. Lambert's ω is useful here because it is known to have nice properties. First, Lambert's ω is single valued on R_+. Second, ω's first and second derivatives exist and are well behaved, making it easy to show that y^* is a monotonic function of ℓ_1 and allowing for the derivation of the probability density function for equilibrium offers.

From (6), the new initial conditions estimates of θ in a dynamic system, estimated as posteriors can then be given by:

$$\hat{\theta} = \frac{LP(\theta)}{\int_N p(D/\theta)p(\theta)d\theta}, \qquad (16)$$

where $\int_N p(D/\theta)p(\theta)d\theta$ is used to ensure that the values of $P(D/\theta)$ sum up to one and thus define a proper probability distribution.

3.2. Application of the model to an armed conflict

We examine the application of the model in the estimation of initial conditions in an armed conflict situation. Modelling the initial conditions in this situation can be compared to the modelling of the risk related to the previous conflict, [18].

It is believed that countries that have experienced an armed conflict are more prone to another conflict in the future and thus their risk levels of an armed conflict are high. We have developed a model that estimates the initial conditions which can act as the pointer to the current risk levels using the past and current state control variables. The estimates of the initial conditions can be used to make predictions for the future trends of a conflict in a dynamic state system.

Assuming that all countries in the world are a universal set Ψ and the countries that are likely to be in a conflict are its subset denoted by Q^*. Our concern is on the subset which can be described as the "prize". A country becomes an element (member) of Q^* if it has experienced an armed conflict at any time in the period of interest. The set Q^* is described as a semi-open space since it allows individuals to become members but does not allow them to get out.

We can therefore define an indicator variable X_{tc}, such that

$$X_{tc} = \begin{cases} 0 & \text{if } c \text{ is not in conflict in year } t \\ 1 & \text{if } c \text{ is is in conflict in year } t \end{cases} \qquad (17)$$

Thus,

The total number of countries in a conflict in year t is:

$$s_t = \sum_{c=1}^{n} X_{tc} \qquad (18)$$

The number of countries that are at conflict in year t and have experienced at least one armed conflict in the past is:

$$m_t = \sum_{c=1}^{n} X_{tc} \quad \text{if } X_{tc} = 1 \text{ and } \ni y < t / X_{yc} = 1 \qquad (19)$$

The number of countries that have experienced an armed conflict before year t, they are not at conflict in year t, but are reported to have experienced another conflict later is:

$$z_t = \sum_{c=1}^{n} X_{tc} + 1 \quad \text{if } X_{tc} = 0 \text{ and } \ni y < t,\ j > t / X_{yc} = 1,\ X_{jc} = 1 \qquad (20)$$

The number of countries at conflict in year t that are reported to be still at conflict at any later period is:

$$r_t = \sum_{c=1}^{n} X_{tc} \quad \text{if } X_{tc} = 1 \text{ and } \ni y > t / X_{yc} = 1 \qquad (21)$$

The total of armed conflicts in a country which is subset of Q^* is:

$$a_c = \sum_{c=1}^{n} X_c \quad \text{.........(22)}$$

The probability, $P(D/\theta))$, given by equation(12) that an armed conflict is likely to occur given that a country is a member of Q^* in t can be estimated by:

$$P(D/\theta)) \equiv L = \frac{m_t r_t}{m_t r_t + s_t z_t} \quad \text{.........(23)}$$

And the prior belief $P(\theta)$ can be obtained as:

$$P(\theta) = \frac{a_c}{s_t} \quad \text{.........(24)}$$

Using the PRIO/Uppsala Conflict Data Project, data set that can be obtained from *http://www.prio.no/cwp/ArmedConflict* and estimated values by equation (23) and (24), our estimated initial condition $\hat{\theta}$, for the various conflict situations in the various countries in the year 2000, 2003 and 2004 can be estimated using equation (16). These estimates are shown in table 1 below:

Table1: Estimated initial conditions as posterior.

Country	2000		Country	2003		Country	2004	
	$\hat{\theta}$	No. of conflicts		$\hat{\theta}$	No. of conflicts		$\hat{\theta}$	No. of conflicts
India	0.68	8	India	0.69	7	India	0.74	6
Nepal	0.60	1	Nepal	0.60	1	Nepal	0.67	1
DRC	0.50	1	DRC	0.34	-	DRC	0.56	-
Colombia	0.68	1	Colombia	0.68	1	Colombia	0.74	1
Peru	0.14	0	Peru	0.14	0	Peru	0.18	0
Pakistan	0.49	1	Pakistan	0.49	1	Pakistan	0.56	2
Ethiopia	0.68	3	Ethiopia	0.69	2	Ethiopia	0.74	2
Turkey	0.68	1	Turkey	0.69	1	Turkey	0.74	1
Indonesia	0.55	1	Indonesia	0.55	1	Indonesia	0.52	1
Mali	0.25	0	Mali	0.25	0	Mali	0.31	0
Nigeria	0.14	0	Nigeria	0.14	0	Nigeria	0.18	1
Niger	0.37	0	Niger	0.25	0	Niger	0.31	0
Thailand	0.55	0	Thailand	0.55	1	Thailand	0.62	1

The estimated initial conditions for the various countries based on the past armed conflicts and the current state conditions are given in table 1 as $\hat{\theta}$. The estimates reflect the risk level predictions of an occurrence of an armed conflict and can give a pointer to the future trends of the existing conflict. The initial conditions which estimate the likelihood of an occurrence of a conflict have been compared with the actual occurrence of a conflict for various countries. For DRC, there was no data available to indicate any new conflict and thus it was indicated as a dash. From the table, there is a direct relation between the initial conditions and the actual occurrence of a conflict for most countries.

4.0. Conclusion

The model gives initial conditions based on the previous and available conditions for the country in conflict. The estimated initial conditions gives the probability of the occurrence of an armed conflict and can thus form the basis for further investigation and prediction of the trend that a conflict is likely to take as other new interplaying factors come into play. The model is dynamic in the sense it can be adjusted over the time under investigation. The threshold of the initial conditions upon which a conflict must occur needs to be investigated.

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