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AN APPROXIMATION OF FISHER'S INFORMATION FOR THE NEGATIVE BINOMIAL PARAMETER k

by

I. C. Kipchirchir

Pushpa Publishing House
Vijaya Niwas, 198 Mumfordganj
Allahabad 211002, INDIA
http://pphmj.com/journals/fjts.htm
fjts@pphmj.com & arun@pphmj.com

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I. C. KIPCHIRCHIR

School of Mathematics
University of Nairobi
P. O. Box 30197-00100, Nairobi, Kenya
e-mail: kipchirchir@uonbi.ac.ke

Abstract

The negative binomial distribution is a versatile distribution in describing dispersion. The negative binomial parameter k is considered as a measure of dispersion. The aim of this paper is to present an approximation of Fisher's information for the parameter k which is used in successive approximation to the maximum likelihood estimate of k. The approximation utilized a relationship between a convergent series and a convergent improper integral.

1. Introduction

Dispersion is the description of the pattern of distribution of organisms in a space (Southwood [6]), often referred to as the spatial distribution. It is a characteristic ecological property. In order to describe dispersion, organisms are assumed to be confined to discrete habitable sites called units (sampling units). Probability distribution models are used to quantify and classify the dispersion of organisms. They have been widely used in entomological research to describe the dispersion of insects.

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Keywords and phrases: dispersion, negative binomial, measure of dispersion, Fisher's information, maximum likelihood estimate.

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Let a random variable X represent the number of individuals that a unit may contain and let $p_x = P(X = x)$ be the probability distribution of X. If variance (v) is larger than the mean (m), the spatial distribution is said to be *contagious* and the population pattern is said to be *clumped* or *patchy* or *aggregated* or *clustered* or *overdispersed*. If the mean and variance are equal, then both the spatial distribution and the population pattern are said to be *random*. If variance is less than the mean, then the spatial distribution is said to be *regular* and the population pattern is said to be *underdispersed* (*uniform*).

Many overdispersed pest populations that have been studied can adequately be described by the negative binomial distribution

$$p_x = \binom{k + x - 1}{x}\left(\frac{p}{1+p}\right)^x \left(\frac{1}{1+p}\right)^k, \quad x = 0, 1, 2, \ldots; k > 0,\ p > 0 \tag{1}$$

so that $v = m(1 + p) > m = kp$ implying it describes a contagious distribution, however, for fixed $m = kp$,

$$\lim_{k \to \infty} p_x = \frac{m^x e^{-m}}{x!}, \quad x = 0, 1, 2, \ldots; m > 0 \tag{2}$$

which is the Poisson distribution with parameter m and describes a random distribution $(v = m)$.

The positive exponent, k, is considered as a dispersion parameter so that the negative binomial distribution describes contagion $(k \to 0)$ and randomness $(k \to \infty)$ (Anscombe [1]). The foregoing discussion calls for tractable methods of estimating the parameter k.

2. Maximum Likelihood Estimate of k

Since a maximum likelihood estimator is a function of a sufficient statistic, a maximum likelihood estimate supplies essentially all the information in the sample about the unknown parameter. Method of moments estimator may not be a function of sufficient statistic. Generally, maximum likelihood estimator is superior to method of moments estimator.

Let n be the number of units and n_x be the number of units containing x counts. Then according to Bliss and Fisher [2], the reduced maximum likelihood equation

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e-mail: kipchirchir@uonbi.ac.ke

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1. Introduction

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for estimating k is

$$\frac{\partial \ln L(k)}{\partial k} = \sum_{x=0}^{\infty} \frac{N_x}{k+x} - n \ln\left(1 + \frac{\overline{x}}{k}\right) = 0, \tag{3}$$

where $N_x = n_{x+1} + n_{x+2} + n_{x+3} + \cdots$, $x = 0, 1, 2, \ldots$ is the sum of units containing more than x counts and $\overline{x}$ is the mean number of counts per unit. That $\hat{k} = \infty$ is a permissible solution. The maximum likelihood estimate cannot be written down explicitly. Starting with the method of moments estimate, (3) is solved by trial and error, limiting its practical use. A more tractable method is that of successive approximation.

A maximum likelihood estimate $\hat{k}$ of the parameter k can be found by successive approximation by expressing (3) as a Taylor series expansion about a value in the neighbourhood of the estimate (Kendall and Stuart [4]). Thus, expanding about k_0 and evaluating at $\hat{k}$, we have

$$\left.\frac{\partial \ln L(k)}{\partial k}\right|_{\hat{k}} \approx \left.\frac{\partial \ln L(k)}{\partial k}\right|_{k_0} + (\hat{k} - k_0)\left.\frac{\partial^2 \ln L(k)}{\partial k^2}\right|_{k'} = 0, \tag{4}$$

where k' lies between $\hat{k}$ and k_0 so that

$$\hat{k} \approx k_0 - \frac{\left.\frac{\partial \ln L(k)}{\partial k}\right|_{k_0}}{\left.\frac{\partial^2 \ln L(k)}{\partial k^2}\right|_{k'}}. \tag{5}$$

If we can choose k_0 so that it is likely to be in the neighbourhood of $\hat{k}$, we can replace k' in (5) by k_0 and obtain

$$\hat{k} \approx k_0 - \frac{\left.\frac{\partial \ln L(k)}{\partial k}\right|_{k_0}}{\left.\frac{\partial^2 \ln L(k)}{\partial k^2}\right|_{k_0}} \tag{6}$$

which gives a closer approximation to $\hat{k}$. The process can be repeated until no further correction is achieved to the desired degree of accuracy. The choice of k_0 is taken as the value of some (preferably, simply calculated) consistent estimate of k. The two consistent estimates k_0 and $\hat{k}$ converge to k^* and consequently k' also converges to k^*.

The three variables

$$\left.\frac{\partial^2 \ln L(k)}{\partial k^2}\right|_{k'}, \quad \left.\frac{\partial^2 \ln L(k)}{\partial k^2}\right|_{k_0}, \quad \left.E\left(\frac{\partial^2 \ln L(k)}{\partial k^2}\right)\right|_{k_0} \tag{7}$$

all converge to

$$\left.E\left(\frac{\partial^2 \ln L(k)}{\partial k^2}\right)\right|_{k^*}. \tag{8}$$

Moreover, using the third variable instead of the second variable, (6) yields the alternative iterative procedure

$$\hat{k} \approx k_0 + \left.\left(\frac{1}{nI(k)} \frac{\partial \ln L(k)}{\partial k}\right)\right|_{k_0}, \tag{9}$$

where

$$I(k) = -E\left(\frac{\partial^2 \ln p_x}{\partial k^2}\right) \tag{10}$$

is the Fisher's information for k which is the amount of information in X about k (Hogg and Craig [3]) and consequently, $nI(k)$ is the amount of information supplied by n units about k.

The information in X is a weighted mean of the derivative $-\dfrac{\partial^2 \ln p_x}{\partial k^2}$, where the weights are given by p_x. The greater this derivative on the average, the more information we get about k.

From (9) and (3), an iterative procedure for finding $\hat{k}$ is

$$k_{i+1} = k_i + \left[\frac{1}{nI(k_i)}\left(\sum_{x=0}^{\infty} \frac{N_x}{k_i + x} - n \ln\left(1 + \frac{\bar{x}}{k_i}\right)\right)\right], \quad i = 0, 1, 2, \ldots, \tag{11}$$

where k_0 is the method of moments estimate.

3. Fisher's Information $I(k)$

On rewriting (1), we obtain

$$p_x = \frac{(k+x-1)!}{x!(k-1)!}\frac{p^x}{(1+p)^{k+x}} \tag{12}$$

so that

$$\ln p_x = \ln(k+x-1)! - \ln(k-1)! - \ln x! + x\ln p - (k+x)\ln(1+p). \tag{13}$$

Differentiating with respect to k, we obtain

$$\frac{\partial \ln p_x}{\partial k} = F(k+x-1) - F(k-1) - \ln(1+p), \tag{14}$$

where

$$F(z) = \frac{d\ln z!}{dz} \tag{15}$$

so that

$$F(z) - F(z-1) = \frac{d}{dz}\ln\left(\frac{z!}{(z-1)!}\right) = \frac{d}{dz}\ln z = \frac{1}{z}. \tag{16}$$

Using (16),

$$F(k+x-1) - F(k-1) = \frac{1}{k+x-1} + \frac{1}{k+x-2} + \cdots + \frac{1}{k+1} + \frac{1}{k} \tag{17}$$

for $x = 1, 2, 3, \ldots$ and (14) becomes

$$\frac{\partial \ln p_x}{\partial k} = \frac{1}{k+x-1} + \frac{1}{k+x-2} + \cdots + \frac{1}{k+1} + \frac{1}{k} - \ln(1+p). \tag{18}$$

Noting that $\frac{\partial^2 \ln p_0}{\partial k^2} = 0$, we have from (18),

$$I(k) = -\sum_{x=0}^{\infty} p_x\left(\frac{\partial^2 \ln p_x}{\partial k^2}\right)$$

$$= \sum_{x=1}^{\infty}\left(\frac{p_x}{k^2} + \frac{p_x}{(k+1)^2} + \cdots + \frac{p_x}{(k+x-2)^2} + \frac{p_x}{(k+x-1)^2}\right)$$

$$= \frac{p_1 + p_2 + p_3 + \ldots}{k^2} + \frac{p_2 + p_3 + p_4 + \ldots}{(k+1)^2} + \frac{p_3 + p_4 + p_5 + \cdots}{(k+2)^2} + \ldots$$

$$= \frac{1 - p_0}{k^2} + \frac{1 - (p_0 + p_1)}{(k+1)^2} + \frac{1 - (p_0 + p_1 + p_2)}{(k+2)^2} + \cdots < \sum_{x=k}^{\infty} \frac{1}{x^2}, \tag{19}$$

that is, $I(k)$ is dominated by a convergent series (by integral test), and hence $I(k)$ converges by comparison test.

Now, the negative binomial probabilities are generated by the recurrence formula

$$p_x = \frac{k + x - 1}{x} \beta p_{x-1}, \quad x = 1, 2, 3, \ldots;$$

$$p_0 = (1 - \beta)^k, \quad \beta = \frac{p}{1+p}, \tag{20}$$

and using (20) in (19), we obtain

$$I(k) = \frac{1}{k^2}(1 - p_0) + \frac{1}{(k+1)^2}(1 - (1 + k\beta) p_0)$$

$$+ \frac{1}{(k+2)^2}\left(1 - \left(1 + k\beta + \frac{(k+1)k\beta^2}{2!}\right) p_0\right)$$

$$+ \frac{1}{(k+3)^2}\left(1 - \left(1 + k\beta + \frac{(k+1)k\beta^2}{2!} + \frac{(k+2)(k+1)k\beta^3}{3!}\right) p_0\right) + \ldots$$

$$= (1 - p_0)\sum_{x=0}^{\infty} \frac{1}{(k+x)^2} - \frac{k\beta p_0}{1!}\sum_{x=1}^{\infty} \frac{1}{(k+x)^2} - \frac{(k+1)k\beta^2 p_0}{2!}\sum_{x=2}^{\infty} \frac{1}{(k+x)^2}$$

$$- \frac{(k+2)(k+1)k\beta^3 p_0}{3!}\sum_{x=3}^{\infty} \frac{1}{(k+x)^2} - \ldots . \tag{21}$$

4. Approximation for $I(k)$

To approximate (21), consider the convergent series

$$\sum_{x=1}^{\infty} \frac{1}{x^2} = \frac{\pi^2}{6} \tag{22}$$

and the convergent improper integral

$$\int_1^{\infty} \frac{1}{x^2} dx = 1 \tag{23}$$

which are related by

$$\sum_{x=1}^{\infty} \frac{1}{x^2} = \frac{\pi^2}{6} \int_1^{\infty} \frac{1}{x^2} dx. \tag{24}$$

In view of (24), formulate the approximation

$$\sum_{x=a}^{\infty} \frac{1}{(k+x)^2} \approx \frac{\pi^2}{6} \int_a^{\infty} \frac{1}{(k+x)^2} dx = \frac{\pi^2}{6(k+a)} \tag{25}$$

for $k > 0$ and a non-negative integer a. This formulation corresponding to $a = 0$ and $k = 1$ is exact and is equivalent to (24). The sum of the series from a to 1000 and its approximation are graphed as shown in Figure 1.

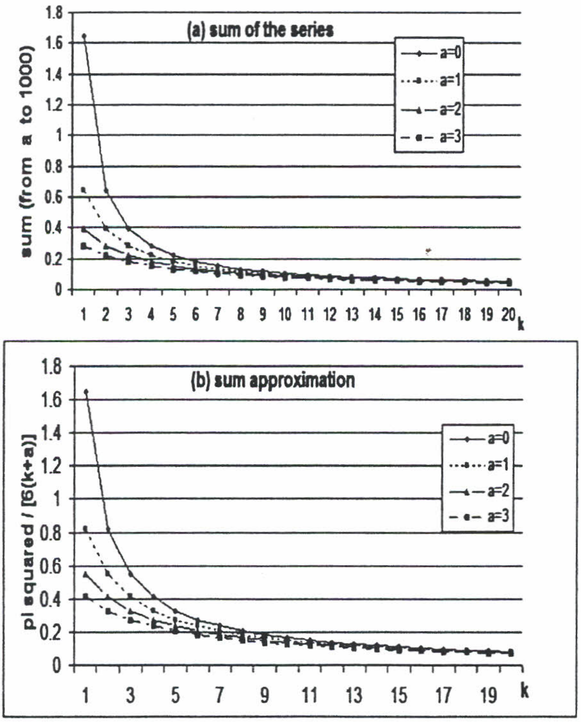

Figure 1. (a) Sum of the series (b) sum approximation.

From (21) and (25),

$$I(k) \approx \frac{\pi^2}{6}\left(\frac{1-p_0}{k} - p_0\left(\frac{k\beta}{(k+1)1!} + \frac{(k+1)k\beta^2}{(k+2)2!} + \frac{(k+2)(k+1)k\beta^3}{(k+3)3!} + \dots\right)\right)$$

$$= \frac{\pi^2}{6}\left(\frac{1-p_0}{k} - p_0\left(\sum_{x=1}^{\infty}\frac{((k+x-1)(k+x-2)\dots k)\beta^x}{(k+x)x!}\right)\right)$$

$$= \frac{\pi^2}{6}\left(\frac{1-p_0}{k} - p_0\sum_{x=1}^{\infty}\frac{1}{k+x}\binom{k+x-1}{x}\beta^x\right)$$

$$= \frac{\pi^2}{6}\left(\frac{1}{k} - \frac{p_0}{k}\sum_{x=0}^{\infty}\frac{k}{k+x}\binom{k+x-1}{x}\beta^x\right)$$

$$= \frac{\pi^2}{6}\left(\frac{1}{k} - \frac{p_0}{k}\sum_{x=0}^{\infty}\left(1-\frac{x}{k+x}\right)\binom{k+x-1}{x}\beta^x\right)$$

$$= \frac{\pi^2}{6}\left(\frac{1}{k} - \frac{p_0}{k}\left(\sum_{x=0}^{\infty}\binom{k+x-1}{x}\beta^x - \sum_{x=1}^{\infty}\frac{x}{k+x}\binom{k+x-1}{x}\beta^x\right)\right). \tag{26}$$

Finally, make the approximation

$$\frac{x}{k+x}\binom{k+x-1}{x} \approx \frac{x}{k+x-1}\binom{k+x-1}{x} = \binom{k+x-2}{x-1}. \tag{27}$$

Substituting in (26) and using (20), $I(k)$ is approximated by

$$I(k) \approx \frac{\pi^2}{6k}\left(1 - p_0\left(\sum_{x=0}^{\infty}\binom{k+x-1}{x}\beta^x - \beta\sum_{x=1}^{\infty}\binom{k+x-2}{x-1}\beta^{x-1}\right)\right)$$

$$= \frac{\pi^2}{6k}(1 - p_0((1-\beta)^{-k} - \beta(1-\beta)^{-k}))$$

$$= \frac{\beta\pi^2}{6k}$$

$$= \frac{m\pi^2}{6k(m+k)}, \tag{28}$$

where $m = kp = E(X)$ which is estimated by $\hat{m} = \overline{x}$.

Consequently, from (11) and (28), an iterative procedure for finding $\hat{k}$ is

$$k_{i+1} = k_i + \frac{6k_i}{n\pi^2}\left(1 + \frac{k_i}{\bar{x}}\right)\left(\sum_{x=0}^{\infty} \frac{N_x}{k_i + x} - n \ln\left(1 + \frac{\bar{x}}{k_i}\right)\right), \quad i = 0, 1, 2, \ldots, \tag{29}$$

where k_0 is the method of moments estimate.

5. Illustration

We shall use the data in Table 1 generated by a mixture of Poisson distribution with parameter λ and Pearson Type III distribution with parameters $k = 10$ and $p = \frac{1}{2}$.

Table 1. Mixture of Poisson and Pearson Type III Data.

x	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
n_x	0	3	8	10	2	11	4	4	0	1	2	4	0	1	0	0	0

Source: Maritz and Lwin [5]

The negative binomial distribution can be obtained as a continuous mixture of Poisson and Pearson Type III distributions. The Poisson distribution is the kernel and the Pearson Type III distribution is the mixing distribution. More precisely, for a given λ, let the number of individuals per unit X have a Poisson distribution with parameter λ, that is,

$$P(X = x/\lambda) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad x = 0, 1, 2, \ldots; \lambda > 0. \tag{30}$$

Then the population pattern would be random (units are equally receptive or attractive to individuals). Further, suppose that some units provide more favourable environment than others (units are not equally receptive or attractive to individuals), then the expected number λ of individuals in a unit, varies from unit to unit. In particular, let λ be a realization of a random variable Λ having the Pearson Type III distribution

$$g(\lambda) = \frac{\lambda^{k-1}}{\Gamma(k) p^k} e^{-\lambda/p}, \quad \lambda, k, p > 0 \tag{31}$$

then the marginal distribution of X (mixture distribution) is

$$P(X = x) = \int_0^{\infty} P(X = x/\lambda) g(\lambda) d\lambda$$

$$= \frac{\Gamma(k + x)}{x!\Gamma(k)} \left(\frac{p}{1 + p}\right)^x \left(\frac{1}{1 + p}\right)^k, \quad x = 0, 1, 2, \ldots; k, p > 0 \tag{32}$$

which is the negative binomial distribution with parameters k and p.

Thus, the data in Table 1 correspond to the negative binomial distribution with parameters $k = 10$ and $p = \frac{1}{2}$. The negative binomial probabilities are generated by (20) and as per the following seven categories:

$$x = \overbrace{0, 1, 2,}^{1} \overbrace{3,}^{2} \overbrace{4,}^{3} \overbrace{5,}^{4} \overbrace{6,}^{5} \overbrace{7, 8,}^{6} \overbrace{9, 10, 11, 12, 13, 14, 15, 16}^{7};$$

these data fit negative binomial distribution with parameters $k = 10$ and $p = \frac{1}{2}$ at 5% level of significance.

For the data in Table 1, $n = 50$, $\bar{x} = 5$ and the method of moments estimate is $k_0 = 6.068$. Noting that $N_x = n_{x+1} + n_{x+2} + n_{x+3} \ldots$, $x = 0, 1, 2, \ldots$ and using tolerance of $k_{\text{new}} - k_{\text{old}} = 10^{-6}$ with S-PLUS, (29) yielded $\hat{k} = 6.548893$ (447 iterations) as the maximum likelihood estimate of k as it satisfies (3).

References

[1] F. J. Anscombe, The statistical analysis of insect counts based on the negative binomial distribution, Biometrics 5 (1949), 165-174.

[2] C. I. Bliss and R. A. Fisher, Fitting the negative binomial distribution to biological data and a note on efficient fitting of the negative binomial distribution, Biometrics 9 (1953), 176-200.

[3] R. V. Hogg and A. T. Craig, Introduction to Mathematical Statistics, Prentice-Hall Inc., 1995.

[4] M. G. Kendall and A. Stuart, Inference and Relationship, Charles Griffin and Company Ltd., 1969.

[5] J. S. Maritz and T. Lwin, Empirical Bayes Methods, Chapman and Hall, 1989.

[6] T. R. E. Southwood, Ecological Methods, Methuen and Co. Ltd., 1966.

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