A Hybrid Finite Element/Moment Method for Solving Electromagnetic Radiation Problem of Arbitrarily-Shaped Apertures in a Thick Conducting Screen

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***Abstract* – In this paper a hybrid numerical technique is presented for characterization of the transmission properties of a three-dimensional slot in a thick conducting plane. The slot is of arbitrary shape and is excited by an electromagnetic source far away from its plane. The analysis of the problem is based on the "generalized network formulation" for aperture problems. The problem is solved using the method of moments(MOM) and the finite element method(FEM) in a hybrid format. The finite element method is applicable to inhomogeneously filled slots of arbitrary shape while the method of moments is used for solving the electromagnetic fields in unbounded regions of the slot. The cavity region has been subdivided into tetrahedral elements resulting in triangular elements on the surfaces of the apertures. Validation results for rectangular slots are presented. Close agreement between our data and published results is observed. Thereafter, new data has been generated for cross-shaped, H-shaped and circular apertures.**

***Index Terms* – Finite–Element Method, Method of Moments, Perfect Electric Conductor(PEC)**

I. INTRODUCTION

THE problem of electromagnetic transmission through apertures has been extensively investigated[1-6]. Many specific applications, such as apertures in a conducting screen, waveguide-fed apertures, cavity-fed apertures, waveguide-to-waveguide coupling, waveguide-to-cavity coupling, and cavity-to-cavity coupling have been investigated in the literature.

Both intentional and inadvertent apertures of various shapes are encountered in the many applications afforded by the advances in technology and increased scale of utilization of microwave and millimeter-wave bands for communications, radar, industrial and domestic applications. Examples of undesirable coupling are leakage from microwave ovens and printed circuit boards, electromagnetic penetration into vehicles, and electromagnetic pulse interaction with shielded electronic equipment. These lead to problems of Electromagnetic Compatibility (EMC) and Electromagnetic Interference (EMI) which should be minimized or eliminated. Desirable coupling exist in slotted antenna arrays, microstrip-patch antennas, directional couplers, cavity resonators, etc.

For the purposes of either minimizing or enhancing electromagnetic coupling through apertures, it is desirable to quantify the electromagnetic penetration through apertures. The past seventeen years have witnessed an increasing reliance on computational methods for the characterization of electromagnetic problems. Although traditional integral-equation methods continue to be used for many applications, one can safely state that in recent times the greatest progress in computational electromagnetics has been in the development and application of hybrid techniques, such as FEM/MOM and (finite-difference time-domain)FDTM/MOM .

The FDTM requires a fine subdivision of the computational domain for good resolution and so is computationally intensive just like the MOM. The method of moments is an integral equation method which handles unbounded problems very effectively but becomes computationally intensive when complex inhomogeneities are present. In contrast, inhomogeneities are easily handled by the finite element method, which requires less computer time and storage because of its sparse and banded matrix. The matrix filling time is also negligible when simple basis functions are used [7]. However, it is most suitable for boundary value problems. Since the investigation to be carried out encompasses unbounded problems with complex inhomogeneities, a procedure combining the finite element method and the method of moments would be effective.

It is the objective of this paper to develop and demonstrate the validity and accuracy of a new hybrid FEM/MOM method. Apertures having various shapes, such as rectangular, circular, cross and H have been studied.

II. GENERAL FORMULATION OF THE PROBLEM

Consider the 3–dimensional (aperture–cavity–aperture) structure illustrated in Fig.1.

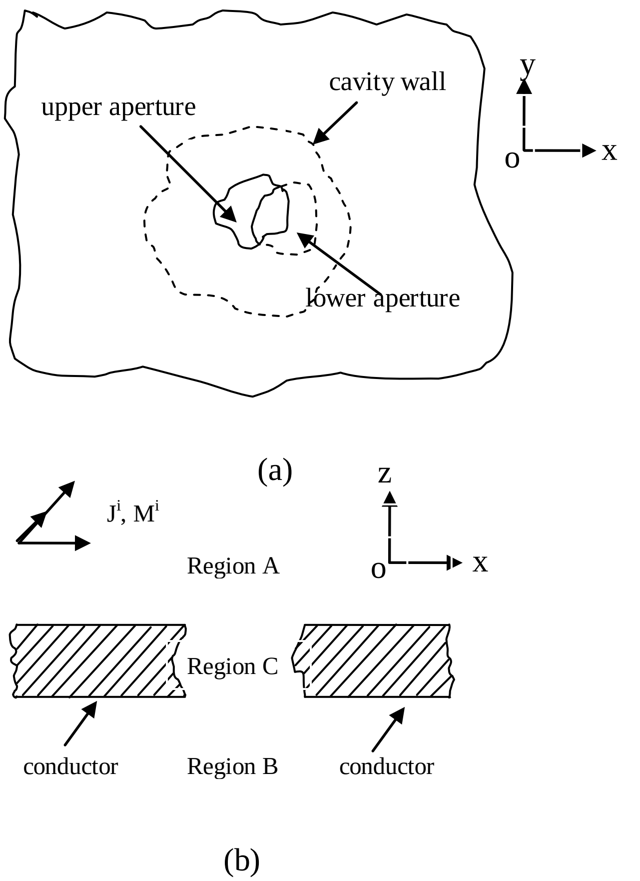


Fig. 1. Problem geometry: (a) Top view (b) Cross-sectional view

The specific configuration consists of a cavity in a thick conducting plane having an aperture at the top surface and another one at the bottom surface. The free space region above the plane (z > 0) is denoted as region A and that below the plane (z < -d) as region B. Also, the volume occupied by the cavity (-d < z < 0) will be referred to as region C. It is assumed that the cavity is filled with an inhomogeneous material having a relative permittivity $\varepsilon_c(r)$ and relative permeability $\mu_c(r)$.

Using an approach similar to that presented in [8], one can replace the tangential electric fields in the aperture planes with equivalent magnetic currents M_1 in the $z=0^+$ aperture plane and $-M_2$ in the $z=-d^-$ aperture plane. In the interior region, the equivalent magnetic currents $-M_1$ and M_2 in the $z=0^-$ and the $z=-d^+$ aperture planes, respectively. Perfect conductors can be placed in the $z=0\,(S_1)$ and the $z=-d\,(S_2)$ planes, as illustrated in Fig. 2, without altering the fields. The problem geometry is then decomposed into 3 regions as illustrated in Fig. 3.

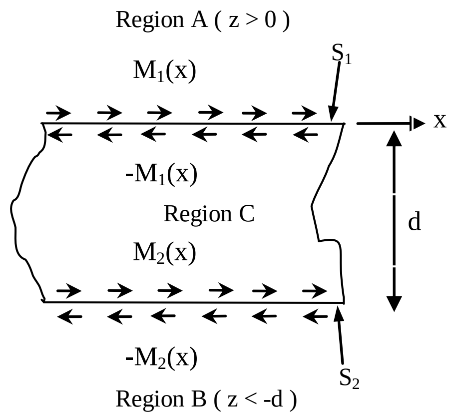


Fig. 2: Equivalent currents introduced in the aperture planes.

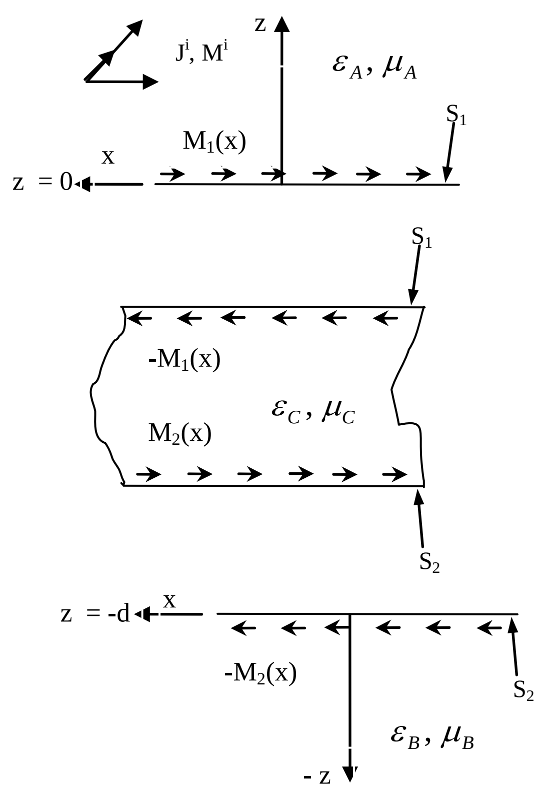


Fig. 3: Problem geometry decomposed into three regions; A, B, and C.

The total magnetic field in regions A and B can be expressed as a superposition of the short-circuited magnetic field $\vec{H}^{sc}$ (i.e., the incident magnetic field plus the specular magnetic field $\vec{H}^{scat}$, produced by the equivalent surface currents. The tangential components of the total magnetic field in the aperture planes are then expressed as

$$\hat{n}\times\vec{H}^{tot} = \hat{n}\times\vec{H}^{sc} + \hat{n}\times\vec{H}^{scat} \tag{1}$$

In the interior region C, the closed cavity is perturbed by the equivalent currents $-M_1$ and M_2, as illustrated by Fig. 3. The total tangential magnetic field that is produced on S_1 is expressed as the superposition of the fields produced by each current

$$\hat{n}\times\left[\vec{H}_1^{tot}\left(-\vec{M}_1\right)+\vec{H}_1^{tot}\left(+\vec{M}_2\right)\right] \tag{2}$$

with a similar expression on S_2. To ensure the uniqueness of the solution, the tangential magnetic field must be continuous across the aperture planes. To this end, continuity of the total tangential magnetic fields across the aperture planes is enforced. Therefore, on S_1

$$\hat{n}\times\vec{H}_1^{sc} = \hat{n}\times\left[\vec{H}_1^{tot}\left(-\vec{M}_1\right)+\vec{H}_1^{tot}\left(+\vec{M}_2\right)-\vec{H}_1^{scat}\left(\vec{M}_1\right)\right] \tag{3}$$

At this point, the equivalent currents are still unknown. Their approximate solution is derived using the method of moments. To this end, the equivalent magnetic currents are expanded into a series of known basis functions weighed by unknown coefficients.

$$\vec{M}_1(x,y,z=0) = \sum_{n=1}^{N} V_{1n}\vec{M}_{1n}(x,y) \tag{4}$$

$$\vec{M}_2(x,y,z=-d) = \sum_{n=1}^{P} V_{2n}\vec{M}_{2n}(x,y) \tag{5}$$

The approximate equivalent currents are substituted into equation (3). Two sets of vector testing functions are also introduced : $\vec{W}_{1m}(x,z=0)$ and $\vec{W}_{2m}(x,z=-d)$ in the S_1 and S_2 planes, respectively. Taking the inner product of equation (3) with $\vec{W}_{1m}(x,z=0)$ and taking advantage of the linearity of the operators results in

$$\begin{aligned}\left\langle\vec{W}_{1m},\vec{H}_{1t}^{sc}\right\rangle &= -\sum_{n=1}^{N}V_{1n}\left\langle\vec{W}_{1m},\vec{H}_{1t}^{tot}\left(\vec{M}_{1n}\right)\right\rangle+\sum_{n=1}^{P}V_{2n}\left\langle\vec{W}_{1m},\vec{H}_{1t}^{tot}\left(\vec{M}_{2n}\right)\right\rangle\\ &-\sum_{n=1}^{N}V_{1n}\left\langle\vec{W}_{1m},\vec{H}_{1t}^{scat}\left(\vec{M}_{1n}\right)\right\rangle\end{aligned} \tag{6}$$

where, a similar expression is derived on S_2

$$\begin{aligned}\left\langle\vec{W}_{2m},\vec{H}_{2t}^{sc}\right\rangle &= \sum_{n=1}^{P}V_{2n}\left\langle\vec{W}_{2m},\vec{H}_{2t}^{tot}\left(\vec{M}_{2n}\right)\right\rangle-\sum_{n=1}^{N}V_{1n}\left\langle\vec{W}_{2m},\vec{H}_{2t}^{tot}\left(\vec{M}_{1n}\right)\right\rangle\\ &+\sum_{n=1}^{P}V_{2n}\left\langle\vec{W}_{2m},\vec{H}_{2t}^{scat}\left(\vec{M}_{2n}\right)\right\rangle\end{aligned} \tag{7}$$

where the subscript t denotes the tangential fields. By introducing $N+P$ vector testing functions, equations (6) and (7) provide $N+P$ linear equations from which the unknown coefficients V_{1n} and V_{2n} can be obtained. The first two inner products of equations (6) and (7) are referred to as the aperture admittance matrices for region C [10]. These matrices relate the tangential magnetic fields in the aperture to the equivalent currents, which are equivalent to the tangential electric fields. The last inner products appearing in equations (6) and (7) are referred to as the interior aperture admittance matrix for regions A and B respectively. In terms of the aperture admittance matrix, the coupled equations (6) and (7) are expressed as

$$\begin{bmatrix}\vec{H}_1^{sc}\\ \vec{H}_2^{sc}\end{bmatrix} = \begin{bmatrix}\left[Y_{11}^C\right] & \left[Y_{12}^C\right]\\ \left[Y_{21}^C\right] & \left[Y_{22}^C\right]\end{bmatrix}\begin{bmatrix}V_{1n}\\ V_{2n}\end{bmatrix} + \begin{bmatrix}\left[Y^A\right] & 0\\ 0 & \left[Y^B\right]\end{bmatrix}\begin{bmatrix}V_{1n}\\ V_{2n}\end{bmatrix} \tag{8}$$

The advantage of using this function is that the admittance matrices of the interior and the exterior regions can be constructed independently. The aperture admittance matrices $\left[Y^A\right]$ and $\left[Y^B\right]$ are computed by solving the problem of the equivalent magnetic currents radiating into a half-space. The aperture admittance matrix $\left[Y^C\right]$ is computed by solving the interior cavity problem for each equivalent magnetic current basis function to compute in the aperture planes, and then taking the inner products with the appropriate testing functions. The following section describes the solution of this interior problem using the **FEM**.

III. COMPUTING THE INTERIOR ADMITTANCE MATRIX BY THE **FEM**

A. SOLUTION OF THE INTERIOR PROBLEM

Within the cavity region, the magnetic fields must satisfy the vector Helmholtz equation

$$\therefore \nabla\times\frac{1}{\varepsilon_r}\nabla\times\vec{H} - k^2\mu_r\vec{H} = 0 \tag{9}$$

where $\vec{H}$ is the total magnetic field. The cavity region is defined as a closed space confined by PEC walls, which will be referred to as the domain V. A functional described by the inner product of the vector wave equation and the vectors testing function, $\vec{H}$, is expressed as

$$F\left(\vec{H}\right) = \iiint_V\left[\vec{H}^*.\nabla\times\frac{1}{\varepsilon_r}\nabla\times\vec{H} - k^2\mu_r\vec{H}^*.\vec{H}\right]dV \tag{10}$$

is defined. The solution of the boundary-value problem is then found variationally, by solving for the magnetic field at a stationary point of the functional via the first variation

$$\delta F\left(\vec{H}\right) = 0 \tag{11}$$

The functional has a stationary point (which occurs at an extremum) and equations (9) and (10) can be used to derive a unique solution for the magnetic field.

For an arbitrary V the solution of the above variational expression cannot be found analytically and is derived via the FEM. To this end, the domain V is discretized into a finite number of subdomains, or element domains, V_e (Fig. 4). It is assumed that each element domain V_e has a finite volume and that the material profile within this volume is constant. Within each element domain the vector magnetic field is expressed as $\vec{H}_e$. If there are N_e element domains within V, the functional can be expressed in terms of the approximate magnetic fields as

$$F(\vec{H})=\sum_{e=1}^{N_e}\left\{\iiint_{V_e}\left[\vec{H}_e^*.\nabla\times\frac{1}{\varepsilon_r}\nabla\times\vec{H}_e-k_0^2\mu_r\vec{H}_e^*.\vec{H}_e\right]dV_e\right\}$$

$$F(\vec{H})=\sum_{e=1}^{N_e}\iiint_{V_e}\left[\vec{H}_e^*.\nabla\times\frac{1}{\varepsilon_r}\nabla\times\vec{H}_e-k_0^2\mu_r\vec{H}_e^*.\vec{H}_e\right]dV_e \tag{12}$$

Applying the vector identity

$$\vec{H}_e^*.\nabla\times\frac{1}{\varepsilon_r}\nabla\times\vec{H}_e=-\nabla.\left[\vec{H}_e^*\times\left(\frac{1}{\varepsilon_r}\nabla\times\vec{H}_e\right)\right]+\left(\frac{1}{\varepsilon_r}\nabla\times\vec{H}_e\right).\left(\nabla\times\vec{H}_e^*\right) \tag{13}$$

and the divergence theorem, (12) can be written as

$$\therefore F(\vec{H})=\sum_{e=1}^{N_e}\left\{\begin{array}{l}\iiint_{V_e}\left[\frac{1}{\varepsilon_r}\left(\nabla\times\vec{H}_e\right)^*.\left(\nabla\times\vec{H}_e\right)-k_0^2\mu_r\vec{H}_e^*.\vec{H}_e\right]dV_e\\ +\iint_{S_e}\left[\frac{1}{\varepsilon_r}\vec{H}_e^*.\hat{n}\times\nabla\times\vec{H}_e\right]dS_e\end{array}\right\} \tag{14}$$

Also

$$\frac{1}{\varepsilon_r}\vec{H}_e^*.\hat{n}\times\nabla\times\vec{H}_e=j\frac{k_0}{\eta_0}\vec{H}_e^*.\vec{M} \tag{15}$$

implying contribution from only regions of implied magnetic current sources, i.e, at the apertures only. $\vec{M}$ = surface magnetic current source.

So that

$$\therefore F(\vec{H})=\sum_{e=1}^{N_e}\left\{\iiint_{V_e}\left[\frac{1}{\varepsilon_r}\left(\nabla\times\vec{H}_e\right)^*.\left(\nabla\times\vec{H}_e\right)-k_0^2\mu_r\vec{H}_e^*.\vec{H}_e\right]dV_e\right\}$$

$$+\sum_{b=1}^{N_b}\iint_S j\frac{k_0}{\eta_0}\vec{H}_b^*.\vec{M}_n dS \tag{16}$$

The approximate vector field in each element domain is represented as

$$\vec{H}=\sum_{j=1}^{m}\alpha_{ej}\vec{W}_{ej} \tag{17}$$

with the spacial vector $\vec{W}_{ej}$ being the Whitney basis function defined by

$$\vec{W}_{ij}=\lambda_i\nabla\lambda_j-\lambda_j\nabla\lambda_i \tag{18}$$

and λ_i is the barycentric function of node i expressed as

$$\lambda_i(\vec{r})=\frac{1}{4}+\frac{(\vec{r}-\vec{r}_b).\vec{A}_i}{3V_e} \tag{19}$$

where $\vec{A}_i$ is the inwardly directed vectorial area of the tetrahedron face opposite to node i, V_e the element volume, and $\vec{r}$ the position vector. Also, $\vec{r}_b$ is the position vector of the barycenter of the tetrahedron defined as

$$\vec{r}_b=\frac{\left(\vec{r}_0+\vec{r}_1+\vec{r}_2+\vec{r}_3\right)}{4} \tag{20}$$

in which $\vec{r}_i$ is the position vector of node i.

Using equations (17) in (16) leads to:

$$F(\vec{H}_e)=\sum_{e=1}^{N_e}\left\{\sum_{j=1}^{m}\alpha_{ej}^*\sum_{i=1}^{m}\alpha_{ei}\iiint_{V_e}\left[\frac{1}{\varepsilon_r}\left(\nabla\times\vec{W}_{ej}\right).\left(\nabla\times\vec{W}_{ei}\right)-k^2\vec{W}_{ej}.\vec{W}_{ei}\right]dV_e\right\}$$

$$-\sum_{b=1}^{N_b}\alpha_b^*\left\{j\frac{k_0}{\eta_0}\iint_S\vec{W}_b.\vec{M}_n dS\right\} \tag{21}$$

Letting

$$N_{ji}=\iiint_{V_e}\left[\frac{1}{\varepsilon_r}\left(\nabla\times\vec{W}_{ej}\right).\left(\nabla\times\vec{W}_{ei}\right)-k^2\vec{W}_{ej}.\vec{W}_{ei}\right]dV_e \tag{22}$$

and

$$P_{nb}=j\frac{k_0}{\eta_0}\iint_S\vec{W}_b.\vec{M}_n dS \tag{23}$$

equation (21) becomes

$$F(\vec{H}_e)=\sum_{e=1}^{N_e}\left\{\sum_{j=1}^{m}\alpha_{ej}^*\sum_{i=1}^{m}\alpha_{ei}N_{ji}\right\}-\sum_{b=1}^{N_b}\alpha_b^*P_{nb} \tag{24}$$

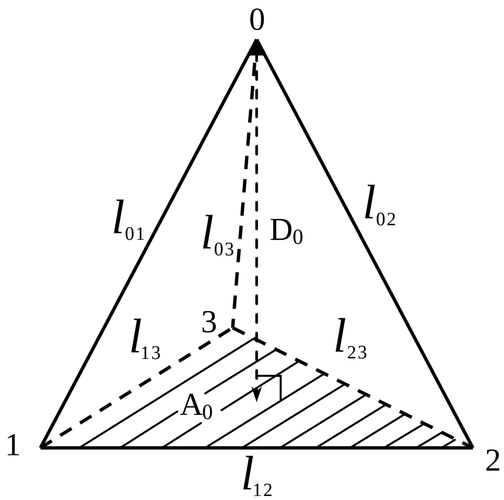
By enforcing the continuity of the magnetic field, a global number scheme can be introduced. The following symmetric and highly sparse matrix results

$$\begin{bmatrix}N_{ee} & N_{eb}\\ N_{be} & N_{bb}\end{bmatrix}\begin{bmatrix}\alpha_e\\ \alpha_b\end{bmatrix}=\begin{bmatrix}0\\ P_{nb}\end{bmatrix} \tag{25}$$

where : α_b are coefficients weighting the edge elements lying in the aperture plane S_1 and S_2. α_e are coefficients weighting the remaining edge elements in V.

B. EVALUATION OF N_{ji}

Consider the tetrahedron in Fig ( ) :



Fig(4): Tetrahedron simplex 3-dimensional finite element

$$\therefore P_{ib} = j\frac{k_0}{\eta_0}\iint_S \vec{W}_{pq}.\vec{M}_{i1}dS$$ **,** for the i^{th} common non-boundary edge whose end nodes are p and q.

$$\vec{M}_{i1}(\vec{r}) = \begin{cases} \dfrac{l_i}{2A_i^+}\bar{\rho}_{i1}^{\,+}, & \bar{r} \ in \ T_i^{\,+} \\ \dfrac{l_i}{2A_i^-}\bar{\rho}_{i1}^{\,-}, & \bar{r} \ in \ T_i^{\,-} \\ 0 \quad , & otherwise \end{cases}$$

$$\vec{W}_{pq} = \lambda_p \nabla\lambda_q - \lambda_q \nabla\lambda_p$$

$$\lambda_p(\vec{r}) = \frac{1}{4} + \frac{(\vec{r}-\vec{r}_b).\vec{A}_p}{3V_e}$$

$$[D] = \begin{bmatrix} K_1^2+M_1^2+N_1^2 & K_1K_2+M_1M_2+N_1N_2 & K_1K_3+M_1M_3+N_1N_3 & K_1K_4+M_1M_4+N_1N_4 & K_1K_5+M_1M_5+N_1N_5 & K_1K_6+M_1M_6+N_1N_6 \\ K_2K_1+M_2M_1+N_2N_1 & K_2^2+M_2^2+N_2^2 & K_2K_3+M_2M_3+N_2N_3 & K_2K_4+M_2M_4+N_2N_4 & K_2K_5+M_2M_5+N_2N_5 & K_2K_6+M_2M_6+N_2N_6 \\ K_3K_1+M_3M_1+N_3N_1 & K_3K_2+M_3M_2+N_3N_2 & K_3^2+M_3^2+N_3^2 & K_3K_4+M_3M_4+N_3N_4 & K_3K_5+M_3M_5+N_3N_5 & K_3K_6+M_3M_6+N_3N_6 \\ K_4K_1+M_4M_1+N_4N_1 & K_4K_2+M_4M_2+N_4N_2 & K_4K_3+M_4M_3+N_4N_3 & K_4^2+M_4^2+N_4^2 & K_4K_5+M_4M_5+N_4N_5 & K_4K_6+M_4M_6+N_4N_6 \\ K_5K_1+M_5M_1+N_5N_1 & K_5K_2+M_5M_2+N_5N_2 & K_5K_3+M_5M_3+N_5N_3 & K_5K_4+M_5M_4+N_5N_4 & K_5^2+M_5^2+N_5^2 & K_5K_6+M_5M_6+N_5N_6 \\ K_6K_1+M_6M_1+N_6N_1 & K_6K_2+M_6M_2+N_6N_2 & K_6K_3+M_6M_3+N_6N_3 & K_6K_4+M_6M_4+N_6N_4 & K_6K_5+M_6M_5+N_6N_5 & K_6^2+M_6^2+N_6^2 \end{bmatrix}$$

$$[K] = \begin{bmatrix} R_1^2+Q_1^2+P_1^2 & R_1R_2+Q_1Q_2+P_1P_2 & R_1R_3+Q_1Q_3+P_1P_3 & R_1R_4+Q_1Q_4+P_1P_4 & R_1R_5+Q_1Q_5+P_1P_5 & R_1R_6+Q_1Q_6+P_1P_6 \\ R_2R_1+Q_2Q_1+P_2P_1 & R_2^2+Q_2^2+P_2^2 & R_2R_3+Q_2Q_3+P_2P_3 & R_2R_4+Q_2Q_4+P_2P_4 & R_2R_5+Q_2Q_5+P_2P_5 & R_2R_6+Q_2Q_6+P_2P_6 \\ R_3R_1+Q_3Q_1+P_3P_1 & R_3R_2+Q_3Q_2+P_3P_2 & R_3^2+Q_3^2+P_3^2 & R_3R_4+Q_3Q_4+P_3P_4 & R_3R_5+Q_3Q_5+P_3P_5 & R_3R_6+Q_3Q_6+P_3P_6 \\ R_4R_1+Q_4Q_1+P_4P_1 & R_4R_2+Q_4Q_2+P_4P_2 & R_4R_3+Q_4Q_3+P_4P_3 & R_4^2+Q_4^2+P_4^2 & R_4R_5+Q_4Q_5+P_4P_5 & R_4R_6+Q_4Q_6+P_4P_6 \\ R_5R_1+Q_5Q_1+P_5P_1 & R_5R_2+Q_5Q_2+P_5P_2 & R_5R_3+Q_5Q_3+P_5P_3 & R_5R_4+Q_5Q_4+P_5P_4 & R_5^2+Q_5^2+P_5^2 & R_5R_6+Q_5Q_6+P_5P_6 \\ R_6R_1+Q_6Q_1+P_6P_1 & R_6R_2+Q_6Q_2+P_6P_2 & R_6R_3+Q_6Q_3+P_6P_3 & R_6R_4+Q_6Q_4+P_6P_4 & R_6R_5+Q_6Q_5+P_6P_5 & R_6^2+Q_6^2+P_6^2 \end{bmatrix}$$

N_{ji} are the elements of matrix $[N]=[D]-[K]$

C. EVALUATION OF P_{nb}

$$P_{nb} = j\frac{k_0}{\eta_0}\iint_S \vec{W}_b.\vec{M}_n dS$$

where $\vec{r}_b$ is the position vector of the tetrahedron barycenter.

$$P_{ib} = j\frac{k_0}{\eta_0}\frac{l_i}{2A_i^{\pm}}\iint_S \vec{W}_{pq}.\bar{\rho}_{i1}^{\pm} dS$$

Eliminating surface integration over each triangle by approximating $\vec{W}_{pq}$ by its value at the centroid of each triangle :

$$P_{ib} = j\frac{k_0}{\eta_0}\left\{A_i^+\left[\vec{W}_{pq}\left(\bar{r}_i^{c+}\right)\left(\frac{l_i}{2A_i^+}\bar{\rho}_{i1}^{c+}\right)\right] + A_i^-\left[\vec{W}_{pq}\left(\bar{r}_i^{c-}\right)\left(\frac{l_i}{2A_i^-}\bar{\rho}_{i1}^{c-}\right)\right]\right\}$$

$$= j\frac{l_i k_0}{2\eta_0}\left\{\vec{W}_{pq}\left(\bar{r}_i^{c+}\right)\bar{\rho}_{i1}^{c+} + \vec{W}_{pq}\left(\bar{r}_i^{c-}\right)\bar{\rho}_{i1}^{c-}\right\}$$

D. COMPUTING $\left[Y^C\right]$

The computation of the nth column of $\left[Y^C\right]$ in ( ) and ( ) is performed by perturbing the cavity with an equivalent current basis function M_n. Then, with the use of ( ) or ( ), the interior tangential magnetic fields in the aperture planes S_1 and S_2 are computed. The first N row elements of the nth column of $\left[Y^C\right]$ are then computed by taking the inner product of the aperture field on S_1 successively with the N testing functions $\vec{T}_m$. The next P row elements of the nth column are computed by taking the inner product of the aperture field on S_2 with with the P testing functions $\vec{T}_q$. Similarly, the remaining P columns of $\left[Y^C\right]$ can be computed by perturbing the cavity with equivalent current basis functions M_p.

IV. COMPUTING THE EXTERIOR ADMITTANCE MATRICES BY THE MOM

$$[Y^A] = [<\vec{T}_m, \vec{H}_{1t}^{scat}\left(\vec{M}_n\right)>]_{NxN}$$

$$[Y^B] = [<\vec{T}_q, \vec{H}_{2t}^{scat}\left(\vec{M}_p\right)>]_{PxP}$$

The discretization of the MOM computational domain is based on triangular patch modeling scheme as proposed by Rao et al [12] and as implemented by Konditi and Sinha [paper u carried].

A. EVALUATION OF MATRIX ELEMENTS

As explained in [paper u carr], following Galerkin procedure ($\vec{T}_m = \overline{M}_m$), a typical matrix element for the r[th] region is given by

$$Y_{mn}^r = 2 <M_m, H_t^r(\overline{M}_n)>$$

$$= 2\left\{\iint_{T_m^+} \overline{M}_m \bullet \overline{H}_t^r(\overline{M}_n)\,ds + \iint_{T_m^-} \overline{M}_m \bullet \overline{H}_t^r(\overline{M}_n)\,ds\right\}$$

$$= 2\iint_{T_m^{\pm}} \overline{M}_m \bullet \overline{H}_t^r(\overline{M}_n)\,ds$$

where the notation $\iint_{T_m^+} (\quad)ds$ has been introduced for compactness.

In terms of the electric vector potential $\overline{F}(\bar{r})$ and the magnetic scalar potential $\phi(\bar{r})$, the magnetic field $\overline{H}^r(\overline{M}_n)$ can be written as

$$\overline{H}^r(\overline{M}_n) = -j\omega\overline{F}_n(\bar{r}) - \nabla\phi_n(\bar{r})$$

where $\overline{F}_n(\bar{r}) = \varepsilon^r \iint_T \overline{\overline{G}}(\bar{r}/\bar{r}')\bullet\overline{M}_n(\bar{r}')ds'$

$$\phi_n(\bar{r}) = \frac{\nabla\bullet\overline{F}_n(\bar{r})}{-j\omega\mu^r\varepsilon^r}$$

In Eqn. (14), $\overline{\overline{G}}(\bar{r}/\bar{r}')$ denotes the dyadic Green's function of the half space.
Substitution of Eqn. (13) in Eqn. (12) and use of two-dimensional divergence theorem leads to

$$Y_{mn}^r = -2j\omega\iint_{T_m^{\pm}} \overline{M}_m \bullet \overline{F}_n\,ds + 2\iint_{T_m^{\pm}} m_m\,\phi_n\,ds \tag{2.19}$$

where $$m_m = \frac{\nabla\bullet\overline{M}_m}{-j\omega} \tag{2.20}$$

Eqn. (16) contains quadruple integrals; a double integral over the field triangles $T_m^{\pm}$ and a double integral over the source triangles $T_n^{\pm}$ involved in the computation of $\overline{F}_n(\bar{r})$ and $\phi_n(\bar{r})$. In order to reduce the numerical computations, the integrals over $T_m^{\pm}$ can be approximated by the values of integrals at the centroids of the triangles. This procedure yields

$$Y_{mn}^{r} = -2l_m \begin{Bmatrix} j\omega\left[\overline{F}_n(\overline{r}_m^{c+}) \bullet \dfrac{\overline{\rho}_m^{c+}}{2} + \overline{F}_n(\overline{r}_m^{c-}) \bullet \dfrac{\overline{\rho}_m^{c-}}{2}\right] \\ + \phi_n(\overline{r}_m^{c-}) - \phi_n(\overline{r}_m^{c+}) \end{Bmatrix} \quad (18)$$

where $\overline{F}_n(\overline{r}_m^{c\pm}) = \varepsilon^r \iint_{T_n^{\pm}} \overline{\overline{G}}(\overline{r}^{c\pm}|\overline{r}') \bullet \overline{M}_n(\overline{r}')\, ds'$

(19)

$$\phi_n(\overline{r}_m^{c\pm}) = \frac{-1}{j\omega\mu^r} \iint_{T_n^{\pm}} \left\{\nabla \bullet \overline{\overline{G}}(\overline{r}^{c\pm} | \overline{r}')\right\} \bullet \overline{M}_n(\overline{r}')\, ds'$$

(20)

In (18), $\overline{\rho}_m^{c\pm}$ are the local position vectors to the centroids of $T_m^{\pm}$ and $\overline{r}_m^{c\pm} = (\overline{r}_m^{1\pm} + \overline{r}_m^{2\pm} + \overline{r}_m^{3\pm})/3$ are the position vectors of centroids of $T_m^{\pm}$ with respect to the global coordinate system.

Similarly, as explained in[paper u carried], using the centroid approximation in Eqn. (2.10), an element of excitation vector can be written as

$$I_m^i = -l_m \left\{ \overline{H}_t^i(\overline{r}_m^{c+}) \bullet \frac{\overline{\rho}_m^{c+}}{2} + \overline{H}_t^i(\overline{r}_m^{c-}) \bullet \frac{\overline{\rho}_m^{c-}}{2} \right\}$$

(21)

Following the procedure as outlined in [paper u carried], transmission coefficient and transmission cross-section are evaluated for the aperture problem.

V. RESULTS AND DISCUSSION

REFERENCES

1. Collin, R.E.: Field Theory of Guided Waves, New York: McGraw-Hill, 1960
2. Harrington, R.F.: Time-harmonic Electromagnetic Fields, New York : McGraw-Hill, 1961
3. N. Amitay. V. Galindo. and C. P. Wu, Theory and Analysis of Phased Array Antennas. New York: Wiley-Interscience, 1972
4. W. F. Croswell. "Antennas and wave propagation," Electronics Engineers' Handbook, D. G. Fink and A. A. McKenzie, Eds. New York: McGraw-Hill, 1975, pp. 18-26.
5. R.F. Harrington, J.R. Mautz, and D.T. Auckland: "Electromagnetic coupling through apertures," Dept. Elec. Comput. Eng., Syracuse Univ., Syracuse NY, Rep. TR-81-4, Aug. 1981.
6. C. M. Butler. Y. Rahmat-Samii. and R. Mittra. "Electromagnetic penetration through apertures in conducting surfaces," IEEE Trans. Electromagn. . vol. EMC-20. pp. 82-93, Feb. 1978.
7. Xingchao Yuan, "Three-dimensional electromagnetic scattering from inhomogeneous objects by the hybrid moment and the finite element method", IEEE Trans Microwave Theory Tech., Vol. MTT-38, pp. 1053-1058, 1990.
8. Harrington, R.F. and Mautz, J.R. : "A generalized network formulation for aperture problems," IEEE Trans. Antennas Propagat., Vol. AP-24, pp. 870-873, 1976
9. Harrington, R.F. and Mautz, J.R. : "A generalized network formulation for aperture problems," IEEE Trans. Antennas Propagat., Vol. AP-24, pp. 870-873, 1976
10. S. K. Jeng, "Scattering from a cavity-backed slit in a ground plane TE case," IEEE Tram. Antennas Propagat., vol. 38, pp. 1523-1529, 9, Oct. 1990.
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