"FORECASTING STANDARD ONE ENROLMENT USING MARKOV CHAIN PROCESS A CASE OF NAKURU MUNICIPALITY"

BY

LUCY AKUMU OJODE

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FACULTY OF COMMERCE UNIVERSITY OF NAIROBI NAIROBI, KENYA

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THIS MANAGEMENT PROJECT IS MY ORIGINAL WORK AND HAS NOT BEEN PRESENTED FOR A DEGREE IN ANY OTHER UNIVERSITY

LUCY AKUMU OJODE

THIS MANAGEMENT PROJECT HAS BEEN SUBMITTED FOR EXAMINATION WITH MY APPROVAL AS UNIVERSITY SUPERVISOR

TOM KONGERÈ

LECTURER
DEPARTMENT OF MANAGEMENT SCIENCE
FACULTY OF COMMERCE
UNIVERSITY OF NAIROBI
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DEDICATION

TO
Sam
and
Titi
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Special appreciation goes to my beloved husband and friend, Sam for his patience, prayers and love during the trying periods of separation. Finally, I wish to acknowledge my daughter Titi for her patience and love at such a tender age.

To all who contributed in one way or another, I say: Thank you and God bless you!
ABSTRACT

Standard one enrolment present a recurring problem to both the parents and the authorities in the urban centres in Kenya. The study detailed in this report sought to represent the standard one enrolment in Nakuru Municipality as a Markovian Model and to use the Model to predict the demand for standard one for the future.

Data on standard one enrolment from 1969 to 1988 and the demand for standard one places for 1989 were collected from the Municipal council records.

Part of the data collected was used to build a Model of the system and part was used to validate the said Model. The model was then used with the 1989 demand to predict the demand for 1990, 1991 and 1992. Similar analysis was performed for the boys only, the girls only and both boys and girls data combined.

The results of the study indicate that the demand for standard one will be greatest in the central zone, followed by the western and the eastern zones respectively. The boys demand for standard one places on the other hand will be greatest in the eastern zone followed by the central and then the western zones. The girls demand for standard one places will be greatest in the western zone followed
by the central zone and the eastern zone respectively.

With such results, it is recommended in this report for the Municipal council to concentrate their resources like administrative effort, time, money and personnel in the central zone, for administering in the area where demand will be greatest. It would also be appropriate to use the results given above as a resource allocation measure, so that resources are distributed equitably among the zones. Finally, the Model may be used to allocate most of the girls' exclusive facilities (like girls only schools) to the West and most of those exclusive to the boys (like boys only schools) to the East.
SECTION ONE

INTRODUCTION

1.1 BACKGROUND

From as early as 1965, Kenya had recognised certain constraints which educational planners had to face, viz; population growth, economic pressures, political pressures, teacher supply, manpower demand and need to maintain quality. However, a quarter of a century has not been enough to meet most of these hurdles and planning for educational activity is becoming more complex, intricate and involving with the passage of time. In present day Kenya, the complexity in planning has resulted from such factors as; economic planning restructuring, that is, from the centralized form of planning at the central Government level to the decentralised form at the district level (The District Focus), population increase, rural-urban migration, restructuring of the education system from the "7-4-2-3" to the "8-4-4" system of education and other development related factors. Usually, the major manifestations of rapid change, and all the problems that change brings are found in the urban centres. The degree of urban complexity tends to be a function of size and scale of operations. As the population increases in size, the larger the scale of modern sector activities, the more complex the urban society becomes and the more difficult it becomes for the Government to plan and respond to the problems created by changes.
In the developing countries, education and economics are closely interdependent for it is necessary to ensure that there are enough persons with all the skills needed for economic growth, and also enough jobs to take advantage of their skills - a balancing act that requires continuous adjustment and hence continuous planning. For appropriate manpower planning, therefore, education is necessary and is a basic foundation for national development. Primary school education, particularly, is important as this is the "mouldable" age, the kind of education exposure received determine to a great extent the direction of the future personal development opportunities.

A common feature of most developing countries like Kenya is a "pyramidal" population structure, that is a population where the majority are young people (school age), this implies that by virtue of their number the youth deserve a lot of the country's resources. These resources must, therefore, be planned for adequately.

It is in this light that education constitutes one of the most important national development priorities and consumes a large proportion (38%) of the Kenya National budget.

The importance vested on primary education is not a new phenomenon. For, as early as 1969, Municipal and County Councils were already vested with the responsibility of financing and developing of primary education.

It is not until the Local Government (Transfer of Functions) Act, enacted in 1969, that the overall responsibility for primary education in rural areas
transferred to the Central Government. In urban areas, Municipalities have retained responsibility for financing and operating primary schools and this has resulted into greater standaziation within the town and better (smaller) planning units.

An outstanding feature of most developing countries like Kenya is a population growth rate (3.64%) above the annual economic development rate (3.5%) This means that whereas there is growth in the economy, the population rate of increase outstrips this growth, thus there is a general rate of decline within the general economy.

This feature signifies a major problem to planners as resources keep diminishing in the face of a largely young populace. With an estimated annual net growth rate (1980-1990) of 3.54% and an estimated population of 24.9 million for 1990, there is a greater need for planning than ever before. The pressure of this population on resources is felt most in the provision of basic needs like food, shelter, health services and education.

The Rural-Urban drift, a feature of countries like Kenya, occurs when people move from rural to the Urban areas in search of a better means of livelihood. This drift makes the population density
higher in the urban areas than the rural ones and the impact of such a population is thus higher in the urban areas than in the rural areas. With a higher population density than the National average (this is diluted by rural areas), the urban areas also have a higher population rate of growth far above the National average. Whereas the whole country is facing a projected rate of growth of 3.64% for the population, Nairobi and Nakuru both face a population growth rate of 4.71%, about one and a third \( \left( 1 \frac{1}{3} \right) \) times the national rate. The provision of school amenities is, thus, a major problem to urban authorities, and the planning for primary school enrolment alone is a very difficult and complex process. With a projected population of 952,000 people by the year 2000, the primary school age children (5-14 years) shall constitute 291,023 or 31% in Nakuru district alone. In Nairobi for instance, parents miss schools for their children because the schools are not enough. Indeed, cases abound where parents stay in the cold as they queue for their children to enrol in standard one at the beginning of each year (notwithstanding the fact the "place" was reserved three years earlier) and Nakuru has not been spared this pain! As a control measure, the Nakuru Municipal insists that children must attend pre-unit (preprimary) centres for about three years until they attain the minimum age of 6 years before they can enrol in standard one. A child whose age is five years and nine months
would not enrol and even where a child has attained 6 years, their older counterparts would be given priority.

Such problems as above are brought about by inadequate planning as well as slow growth in the provision of the amenities in the face of an escalating population. To arrest such a situation, more aggressive planning is necessary especially in the urban areas of developing countries like Kenya. Intensive research is necessary if developing countries are to have a chance of making sufficiently rapid progress in resolving their problems. The painful alternative is to have their education too much patterned on the advanced countries, unsuited to their own needs, and far beyond their financial resources.

1.2 BACKGROUND OF NAKURU MUNICIPALITY

The origin of Nakuru town, just like many other towns in Kenya, is closely linked with the development of the Kenya-Uganda Railway. In 1900 when the first shop opened its doors, apart from the indigenous people, Nakuru residents consisted mainly of railway employees. The town grew so fast that on January 28, 1904, the then British Commissioner in Kenya declared Nakuru a township in the Official Gazette. The town was then within only a mile's radius of the entrance of Nakuru Railway station. In 1905, Nakuru township was just a collection of the railway station, a few houses for the railway staff and two shops. The following year the first hotel Nakuru Hotel (now, Midlands) opened its doors, followed by
the National Bank of India (The Kenya Commercial Bank today).

The town's status was bolstered when in 1912, following the completion of the railway survey of the line from Nakuru to Eldoret, it was decided that it should be the main junction for the line running to Kisumu as well as Eldoret. This new development necessitated the boundaries of the town to be expanded in 1913. In 1929, the town had become so important that a Municipal Board was formed thereby giving it Municipality status. Although the depression of 1930s and later the Second World War affected the town by reducing its rate of development, progress resumed after these which enabled the Governor of Kenya to proclaim the elevation of the Municipal Board to the status of a Municipal Council in 1952.

Nakuru Municipality consist of the town center and its suburbs (see Map), and, it is run by the Nakuru Municipal Council although the town also houses the headquarters of Nakuru County Council as well as the Provincial Commissioner's and District Commissioner's headquarters. The town is situated on the bed of the geographical wonder, the Great Rift Valley. It is the fourth largest town in Kenya and is surrounded by the Lion Hills, Lake Nakuru, the extinct volcanic hill, Menengai, and Honeymoon Hill to the Southern part. With an area of some 78 square kilometres, the town lies 36 degrees East and 0.4 degrees South of the Equator. Nakuru is some 97 miles from Kenya's capital city, Nairobi and 451 miles from the port town of Mombasa, at an attitude of 6,070 feet above sea level.
Nakuru is known as the capital of the country's rich agricultural highlands while administratively, it is the headquarters of the Rift Valley Province. His Excellency President Daniel Arap Moi's Permanent home is barely 25 kilometres from the town at Kabarak, and his frequent presence at the Nakuru State House has enhanced the town's political status.

With a population of some 200,000 people, Nakuru's appeal to people emanates from its healthy climate, which is usually temperate to warm, varying with seasonal rains between March and May while dry weather prevails during Mid-November to the end of February. It receives an average annual rainfall of 103.8 mm. Nakuru is famous worldwide for its over 400 species of birds found at the Lake Nakuru National Park.

The smooth running of Nakuru Municipal council is made possible by a number of standing and special committees. Viz:-

(i) Finance and General Purposes Committee.
This has the superintendence, management and control of the Town Clerk's and the Municipal Treasurer's Departments. The committee also; procures and examines estimates of expenditure, from the several other committees and prepares the general estimates of receipts and expenditure; considers and reports to the council upon the financial effect of any scheme or work proposed to be carried out by the council; directs and superintends the keeping of accounts of all the departments of the council, examines and passes accounts for the salaries and wages
of the various officers, clerks and workmen of the council, and, for the money payable to contractors of the council; negotiates all loans of money which may be ordered by the council; deals with the powers and duties of the council in relation to matters of valuation of hereditaments within the Municipality; hears and determines applications to be excused payment of any rates and carries out the general instructions of the committee in relation to fulfilling all duties not delegated to any other committee of the council.

(ii) Town Planning and Works Committee.
This is involved with the implementation of both short and long term plan to accommodate the town's ever growing population. The Town planning department works very closely with the Ministry concerned with urban development and housing's physical planning, while the work's department looks after the water supply, roads, trees and gardens, sewers and sewage disposal, housing schemes, construction of schools, street lighting and council's workshop.

(iii) Housing and social services Committee.
Carries out the duties of providing houses for the council staff and the general public within the municipality and provides markets and trading centres in the Municipality. The committee, through the social services department looks after day nursery schools, destitutes, libraries and information services, cinema shows, the stadium, youth club activities and adult literacy and evening continuation classes.
(iv) Public Health and Education Committee

This has superintendence and control over the management of public health department while taking into consideration all questions affecting the health of the inhabitants of the Municipality over which the council may have jurisdiction. The Committee also looks after the education vote, plans any development of education in the Municipality, and co-ordinates the work of school committees with that of their own local school committees. This Committee was split up into two separate committees, Education and Public Health, operating under two different Chairmen in 1988.

Currently, Nakuru Municipality has about twenty secondary schools (four of them being council maintained), forty primary schools and fifty Nursery Schools. The number of pupils attending primary schools within the Municipality has risen from 17,050 in 1979 to 31,700 in 1989, while the demand for standard one places rose from 3,170 to 3,550 in the same period. In 1980, the council was maintaining only about thirty primary schools, which number has risen to thirty-eight. This rise has been necessitated by increase in student population. The nursery schools have also increased from thirty one to fifty (though the Municipality only maintains six of these), thus increasing demand for standard one places.

The education department is manned by about thirty staff members headed by the Municipal Education Officer (M.E.O) and his deputy. This task force is responsible to the committee for the administration of education.
system within the Municipality. Due to the large number of schools involved, three educational zones were established in 1986 as follows: Central Zone, with eleven schools, Western Zone - twelve schools and Eastern zone-twelve schools. The zones are manned by Zonal Inspectors; thus as well as facilitating administration, the establishment of the zones was meant to intensify easier schools' inspection. In 1987 an extra zone, the Southern Zone, was added and in 1988 three more schools were started while some old ones were expanded to accommodate increased enrolment. The Zones were however reduced back to three in early 1989, when one of the inspectors was promoted to the post of Assistant Municipal Education Officer. The map of Nakuru Municipality clearly shows the distribution of the schools within the zones.

1.3 STATEMENT OF THE PROBLEM

The provision of Educational amenities in Kenya do not match the needs particularly within the urban areas. This mismatch is due to the problems of high population growth and density in the urban areas. The rural-urban migration also makes the population
growth rate within the urban areas high and in the face of limited resources, the local authorities cannot match the resources to the needs. This has been evidenced in Nairobi and other towns in Kenya where parents miss primary school places. The major causes of the mismatch are the inability to make accurate projections of needs (brought about by the above factors) and lack of funds to meet the needs.

There is a problem of lack of accurate projections of school enrolment, particularly, in standard one in the urban areas. If accurate projections can be obtained, the resources can be mobilised to meet the needs from one year to the next. For instance, if standard one enrolment can be projected with some measure of accuracy, say, for the next five years then, funds can be sought to prepare for such now. To evolve an efficient system of education, the activities need to be co-ordinated, for which an abstract realization of real system may be necessary. An effective projection or forecasting model is necessary to enable adequate planning and co-ordination. The forecasting of primary school enrolment, particularly, in standard one would go along way towards helping to match the resources to the anticipated needs.
The purpose of this study is to use a stochastic model to forecast standard one enrolment in any year. The stochastic model is appropriate because the situation under study is dynamic. For instance the standard 1 enrolment in one year may stay constant for a certain period, fall below the previous period or rise above it. A stochastic model will accommodate all these possibilities.

1.4 OBJECTIVE OF THE STUDY

The objective of this study is to apply a stochastic model, specifically the Markov chain process in the prediction of standard one enrolment in the different zones in Nakuru Municipality for any given year.

1.5 IMPORTANCE OF THE STUDY

1. (a) The study will be of importance to educational planners in urban planning of education and for projecting enrolment. This will help them prepare for such by recruiting enough staff, building enough classrooms, acquiring equipment and other necessary facilities.

(b) Educationists could also use the study to analyse the educational characteristics like dropout and staying ratio of boys and girls, in different classes and in the different regions of the country.

2. The study may also be used to evaluate the new system of education against the old system with respect to the enrolment proportions of boys and girls.
3. Finally, it is intended to stimulate interest and research in stochastic model applications, in areas other than education like health planning, agriculture, and manufacturing in both business and non-business organizations.
Research in education is needed to answer such central problems as: what kind of formal and informal education should be developed in a country that cannot afford full primary and secondary education for all children? Will education differ not in degree, but in concept, from that of an advanced country? How can it be planned so as to contribute to social and economic development? How can it be afforded? What forms of education are suitable for a predominantly agricultural country? Are there appropriate models in advanced countries which can be studied? For this reason, therefore, several varieties of quantitative models for educational analyses have been developed. Most of them focus on the flows of students through a system as Steel (1979) found out in his analysis. This interest in the flows of students may be from the broad perspective of a national planner evaluating the long term implications of present educational policies in terms of future availabilities of trained adults, or from the narrow perspective of a departmental chairman deciding on classroom assignments for the next semester.

As the only policy-making body with powers for co-ordinating overall educational programs in Kenya, the Ministry of Education has not laid emphasis on models which link the total educational system to forecast of future requirements. This is not only true of Kenya, but Steel (1979) found out that even in Britain the primary focus
has been on the shorter term problems of local educational institutions like universities and other educational institutions. In Europe for instance, only a few countries like Sweden and the U.S.S.R have used intensive research in formulating their educational policies. In most, education has changed more slowly in response to varied pressures, with little research to influence policy. Such a situation is not accidental, the reason being that when looked at in totality, the educational systems are generally very wide and planners tend to lose sight of the objectives. Most effective quantitative models for such analysis would, therefore, be those which focus on the flows of students through a system; particularly the stochastic models. Stochastic models in particular are suitable because of their inherent ability to encompass the variability and dynamism underlying "real life" systems behaviour.

In Kenya, Stochastic model applications have not been widely researched on. A study by Owino J. (1982) used Stochastic (Markovian) model for educational planning which is not easy to apply in reality as the study covered the whole country of Kenya which in itself is so varied. The results of the study, the forecasted enrolment for 1983 and the number of new classrooms forecasted for each class are total figures for the whole country. In planning, one need to decide in advance what to do, when to do it and how to do it. The knowledge of such "block" figures would just highlight on
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what should be done and when it should be done but not how, thus may not be of much help for planning purposes. If these totals are broken down for the different regions, then distribution of the necessary resources can be done according to the peculiar needs of each region. For instance, the forecasts may be broken down to the smallest planning unit possible, say the locational or sub-locational level. The planning for standard one enrolment may be more urgent in the urban areas than in the rural ones and such factors need to be taken into consideration when aggregating.

Anklesaria (1986) developed a stochastic model for analysing time-series nominal data. This general purpose model explicitly considers the observed time-relationship as well as how previous behavior appears to affect later behavior, and presented this information as a finite state Markov chain. Markov analysis is used to predict a system's behavior over time and the models have been used extensively to analyse system behavior, see Ashby (1956), Bartholomew (1967), Hillier and Lieberman (1967). Most of these models, however, are special purpose, they do not take time relationships into explicit consideration and, or ignore the effect of previous behavior or system's memory. The study overcame these deficiencies and also allows the aggregation and testing of several processes with one another thereby allowing the determination of similarities and differences.

Gani (1963) proposed one of the early Markovian models for educational planning in Australian universities when he used the model to forecast enrolment and degree
awards. Later on, Thonstad (1969) made an extensive use of stochastic models in his educational planning manual. Nearer home, Uche (1978) applied Markovian models to Nigerian education system. One common feature in these studies is that the system under study is formulated as a Markovian process and the Markovian model then used to forecast or predict the future state of the system.

Stochastic models have also been used to study consumer behavior. Kuehn and Day (1964) applied stochastic models to study consumer behaviour by emphasising on the principle marketing variables (the four p's of product, price, place and promotion) rather than on sales forecasting. They used the idea that purchase probabilities for an individual consumer can be expected to change over time, thus, the dynamic feature necessary under stochastic process. They also emphasised the fact that a dynamic model of consumer brand choice behavior must provide a method of revising the individual's set of purchase probabilities to show changes induced by the passage of time, new purchase experiences, and exposures to merchandising influences. Their findings were that although they had gained considerable attention and were conceptually appealing, simple Markov chain models were of limited value as the basis of a brand choice model. Acting on this strong evidence "introduced" which suggests that brand choice behavior is substantially stochastic, Bass (1974) presented a general theory of stochastic preference and tested it ten years later. He used the theory of stochastic preference under dynamic conditions to show
from the empirical evidence from empirical studies of individual consumer choice that this behavior is substantially stochastic. This suggests stochastic modeling in consumer behavior where the deterministic tradition in theory seems to have been the mode. From these two studies, it can be concluded that even though much remains to be done before a model can be developed which can adequately treat all the marketing variables under the assumption of a stochastic consumer behavior, a modified Markovian process offers an encouraging start. As Howard points out though, there are some major problems in using markovian analysis under such circumstances. These can be classified into three categories: irregular patterns of purchase or random interpurchase time, difficulties of aggregation, and difficulties in revising transition matrices to reflect new information.

Another area where stochastic models applications have been tried is in personal selling strategies. Magee (1952) tried as early as 1952 to apply stochastic models in personal selling, by arranging dealer customers according to average order size, which provided the basis for allocation of the sales effort to consumers. Having allocated sales effort to control unit, the sales manager then considers the call strategies to be used. The result was not very practical until 1960's and after when Shuchman proposed a modified Markov chain approach to the allocation decision.12
Personnel supply in an organization can also be forecasted using Markov chains to model the flow of people through various states (usually skill or position levels and sometimes years of service). The majority of Markovian manpower applications noted in the literature comes from military, government and public agencies rather than business. Moreover, much of the new works in this area have been published in volumes of conference proceedings, including Wilson (1969), Bartholomew and Smith (1977), and Bartholomew (1967). White's (1970) work on chain of opportunity is a detailed study of job mobility based on the idea of modeling the flow of vacancies through the system.

Although statistical tests for validating Markov chain models have been known for more than twenty nine years, they have been used only in few applications, especially, in the analysis of system behavior. In demography, for example, Keyfitz has provided a basic source of methodology for mathematical analysis of population. Chatfield and Goodhardt (1969) have also modelled buying behavior as a stochastic process. Hagerstrand's work on diffusion of innovations by use of stochastic methods has triggered quantitative geographers' interest in this area.

In business, Cyert and Thompson (1967) have used a stochastic model for selecting a portfolio of credit risks. It is a common practice with credit agencies to classify each credit applicant with several risk categories. This is usually done by a crude form of multiple regression model using "scoring functions" of relevant independent
variables. Suppose there are $C$ risk categories and $m+2$ stages of the account; out of which 2 refers to the "paid up" and the "bad debt" states of an account. Assuming that the credit applicant passes through the states of the account based on a transition probability matrix, the management can determine the elements of $C$ transition probability matrices one for each risk category. This seems to be a more realistic assumption than the one based on a single transition probability matrix for all credit applicants. The end objective of this procedure is to enable the firm to arrive at a portfolio of accounts receivables with different proportions of customers in the various risk categories. From the standpoint of total risk, the firm then should be able to afford more people in higher-risk categories when it has relatively more in the low-risk categories. The criterion for this policy proposed by Cyert and Thompson is based on the coefficient of variation of the total expected receipts and acceptance of customers as long as this coefficient remains below a preset value.

The term structure problem can also be analysed using a stochastic model. The problem of term structure may be considered to be a special case of the problem of determining capital values under uncertainty. The payments stream may be taken as deterministic, but future interest rates are stochastic. In his study Pye (1966) accounted for the stochastic nature of
interest rates, by assuming that there are finite number of interest rates $P_i$ ($i=1, 2, \ldots, m$), and from period to period the interest rates vary as a Markov Chain. Pye also discusses the properties of the yield curves and the behavior of the expected future one-period interest rates based on the properties of the transition probability matrix.

Stochastic models have also been used in the study of the spread of rumours and epidemics, see Becker (1968), Doley and Kendall (1965).

2.2. FORECASTING MODELS

Man has always striven to account for things that he observed happening in the universe about him. One reason for this is obvious: if man "understands" a phenomenon, he may be able to control it in some way, or will at least be able to predict future behaviour from a knowledge of current behaviour. The formation of theories (Models) which account for observed behavior is not a new phenomenon. The historical development of probability models for instance, has been long and interesting. The Mathematic giants P. Fermat (1601-1665) and B. Pascal (1623-1662) worked on such Models (in connection with gambling problems). The present approach to the axiomatic system commonly called "probability theory" was proposed by A. Kolmogorov in 1933. One striking difference between the earlier models and the recent ones is associated with assessment of the likelihood of various alternative outcomes in an experiment.
Forecasting models can be seen as either causal or extrapolative. The various extrapolative models available can be subdivided further into: Trend curve analysis, Smoothing and Box-Jenkins methods, Bayesian forecasting (under which falls the Markov Chain analysis) and adaptive models. Although by custom trend curve analysis has more often been used for longer term forecasting while the remainder are usually perceived as suitable for the shorter term, they are all strictly comparable in terms of the criteria of cost effectiveness for the forecast user.

Statistical forecasting is generally based on the assumption of constancy, this has emphasised the traditional role of statistical forecasting as extrapolative. That is some model based on past data has been developed, which subsequently has been used to 'project' the past patterns and relationships beyond the sample date, thus providing forecasts for the future. This usually works well as long as the established patterns and relations do not change. However, if changes do occur, statistical forecasting cannot deal with this situation because the assumption of constancy will not hold. The resulting errors do not have to follow previous patterns: they can be non-random, their variance can be wider, and or they can be non-symmetric.
Major Functions in Forecasting

![Diagram showing Major Functions in Forecasting]

If an enlarged role of forecasting is accepted (Fig. 2.2.1) then the problem becomes how to continue forecasting when systematic changes from established patterns and relationships are involved. Forecasters and end users however, must accept the inevitable, this being the inability to forecast statistically when the assumption of constancy does not hold.

After using the regression models based on anticipated workload, sales or economic indicators and the Delphi procedures, to forecast personnel demand; Hopes (1973) also used Markov Chains Model to forecast personnel supply and these have been used with some measure of success for manpower planning.

Several studies have been reported in which multiple-objective approaches are used to select an appropriate forecast model by comparing forecast statistics generated by each method. Reeves G.R and Lawrence (1982) studied combination of forecasting methods based on multiple objectives. Bunn (1975) also studied linear combination of forecasts while Carbone et al (1983) compared for different time series methods, the value of technical
expertise, individualised analysis and judgemental adjustments needed in forecasting. Dickinson (1975) also made some observations on the combination of forecasts.

The benefits of combining forecasts using goal programming (G.P) lies in the ability of goal programming to reflect the preference of the decision-maker or planner. A Model may then be selected to yield forecasts which are implicitly "weighted" by either the preferences of the planner or a linear approximation of a preference structure.

Two questions usually dominate any assessment of a forecasting model:
(1) Is the model statistically satisfactory?
(2) Will the model once developed be used and perform cost effectively?

While much of research in statistics and operations research has concentrated on the former question, it is the latter that is often of more concern in practical forecasting. Little (1979) suggests that as well as accuracy, a good model should be:
(i) Easy to understand for the user
(ii) Easy to control, that is, the user should be able to make the model behave in the way he wants it to, because of its comprehensive structure.
(iii) Adaptive, that is, should be able to accept fresh information and use it to update the values of the parameters.
(iv) Complete on important issues: It should include those factors perceived to be important by the user.
(v) Easy to communicate with: The user should be able to change user specified input information easily and obtain model output in a simple form with equal facility.

(vi) Robust: It should be specified so that it gives sensible answers to plausible input information.

In effect what these criteria demand, is a model as flexible as the forecaster himself and yet formalized so that statistical testing can be carried out. Any such model would be expensive and in most forecasting situations it would appear that a model is selected for its simplicity rather than with regard to the other five criteria. However, Little's somewhat idealistic criteria can serve as benchmark for assessing a model's acceptability to the user but note that points (iv)-(vi) are of more concern in causal modelling than extrapolative models.

Makridaki's et al (1982) summarises the criteria of selecting forecasting methods as depending on:

1. The pattern of the data
2. The type of series
3. The time horizon
4. The ease of application
5. The cost
6. The accuracy of the forecast.

2.3. THE MARKOV CHAIN PROCESS: DEFINATIONS AND PROPERTIES

Markov Chain analysis is attributed to A. Markov, a Russian Mathematician who developed the technique in 1907. As a descriptive (none normative) tool, the major objective of Markov Chain analysis is the prediction of the future behavior of managerial systems. Such prediction can be achieved, in some cases, by other tools such as decision trees (coupled
with complete enumeration) or simulation. The advantage of Markov chain analysis is that, the computational work is relatively uncomplicated and can be carried out very rapidly, and with the relevant computer packages in a few hours.

The analysis applies in a dynamic, probabilistic environment where most other management science tools fail; yet most management situations in reality are of such nature. Therefore, it is considered a good tool for providing information which can be used as a basis for making decisions by either complete enumeration of all alternatives or through additional optimization models. The relationship between Markov analysis and the managerial analysis is shown below:

Managerial analysis using Markov Chains

PROBLEM → Prediction of system's Behavior → Decision making via complete enumeration or an optimization model

Fig. 2.3.1 MARKOV ANALYSIS → MANAGERIAL ANALYSIS

Stochastic means probabilistic, that is, with an exhaustive set of probabilities or chances of outcomes.

Some Common Stochastic Processes are:

(i) The Bernoulli Process, that is a process with discrete state and parameter space. For instance, a series of independent repeated trials with two outcomes, say "success" and "failure" whose probabilities are p and q respectively at each trial; and (p+q) = 1. Sn implies the number of successes in n trials, thus (Sn) is a stochastic process with state space
The probability distribution of $S_n$ for a given $n$ is:

$$P(S_n = K) = \binom{n}{k} p^k q^{n-k}, \quad K = 0, 1, 2 \ldots$$

The process $(S_n)$ is a Bernoulli process. This process is a simple discrete time renewal counting process; it is also a Markov chain with discrete state space.

(ii) The Poisson process, a process with discrete state space and continuous parameter space. Let $x(t)$ be the number of events occurring in time $(0,t)$; the process $(x(t))$ is stochastic with state space $(s: 0, 1, 2 \ldots)$ and parameter space $(t \geq 0)$. For a given $t$, $x(t)$ has the Poisson distribution with mean $\mu$:

$$P[x(t) = K] = \frac{e^{-\mu} \mu^K}{K!}, \quad K = 0, 1, 2 \ldots$$

The process $[x(t)]$ is called the Poisson process. The time interval between consecutive occurrences of events in a Poisson process are independent random variables identically distributed with probability density function $f(x)$ given by: $f(x) = \lambda e^{-\lambda t}$. Therefore the Poisson Process is also a renewal counting process, thus has the Markov property, consequently, the process is also a Markov process.

(iii) The Gaussian Process: This is a process with continuous state and parameter space. Consider a stochastic process $[x(t)]$ with the property that for an arbitrary set of $n$ time points $(t_1, t_2, \ldots, t_n)$ the joint distribution of $x(t_r), (r=1, 2, \ldots, n)$ is $n$-variable normal. Then the process is called Gaussian.
(iv) The Wiener Process (Brownian Motion Process).

This is a process with continuous state and parameter spaces. Consider a stochastic process \( (x(t)) \) with the following properties:

(a) The process \( [x(t), t \geq 0] \) has stationery independent increments. This means that for \( t_1, t_2 \in T \) (parameter space) and \( t_1 < t_2 \), the distribution of \( x(t_2) - x(t_1) \) is the same as \( x(t_2 + h) - x(t_1 + h) \) for \( h > 0 \), and for any non-overlapping time intervals \( (t_1, t_2) \) and \( (t_3, t_4) \) with \( t_1 < t_2 < t_3 < t_4 \), the random variables \( x(t_2) - x(t_1) \) and \( x(t_4) - x(t_3) \) are independent.

(b) For any given time interval \( (t_1, t_2) \), \( x(t_2) - x(t_1) \) is normally distributed with mean zero and variance \( \sigma^2(t_2 - t_1) \). Then the process \( [x(t)] \) is called the Wiener process.

A Markovian Process for which all realization or sample function \( [X_t, t \in (0, \infty)] \) are continuous functions, is called a diffusion process. The Poisson process is a continuous time Markov Chain and Brownian Motion is a diffusion process.

Stochastic Processes occurring in most real-life situations are such that for discrete set of parameters \( t_1, t_2, \ldots, t_n \in T \), the random variable \( x(t_1), x(t_2), \ldots, x(t_n) \) exhibit some sort of dependence. For instance, the state of the standard one after \( n \) periods \( (X_n) \) may conceivably depend on the state of the same during the preceding \( n-1 \) periods. The analysis of the process gets complicated as the dependence structure becomes complex. The simplest type of dependence is the first-order dependence underlying a stochastic process. This dependence
is termed Markov-dependence and may be defined as follows:

Consider a finite (or countably infinite) set of points \( t_0, (t_1, \ldots, t_n, t), \) where \( t_0 < t_1 < t_2 < \cdots < t_n < t \) and \( t, t_r \in T \) (\( r = 0, 1, \ldots, n \)) where \( T \) is the parameter space of the process \( X(t) \). The dependence exhibited by the process \( (X(t), t \in T) \) is called Markov-dependence if the conditional distribution of \( X(t) \) for given values of \( x(t_1), x(t_2), \ldots, x(t_n) \) depends only on \( x(t_n) \) which is the most recent known value of the process, that is:

\[
P[X(t) \leq x | X(t_n) = x_n, \ldots, x(t_1)] = P[X(t) = x | X(t_n) = x_n]
\]

The stochastic process exhibiting this property is called a Markov process. A Markov process is therefore, a stochastic process whose probability of being in any state depends only on its previous state and the transition matrix. In a Markov process, therefore, if the state is known for any specific value of the time parameter \( t \), that information is sufficient to predict the behavior of the process beyond that point.

Depending on the nature of the state space and the parameter space, we can divide Markov Processes into four different classes (see table). Whenever the parameter space is discrete, we shall call such Markov process Markov chains, that is a Markov process with constant transition probabilities (chances of a system moving from one state to another). In this
Contino s arkov process wit discrete state space

Table 2.3.1

<table>
<thead>
<tr>
<th>Parameter space</th>
<th>state space</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discrete Markov chain with discrete state space</td>
<td>Markov chain with continuous state space</td>
</tr>
<tr>
<td>Continous Markov process with discrete state space</td>
<td>Markov Process with continuous state space</td>
</tr>
</tbody>
</table>

study, the emphasis is on Markov chains.

A Markovian Decision Process (MDP) is the stochastic process that describes the evolution of a dynamic system controlled by a sequence of decisions. Such a process exist in a continuous state space and more sophisticated tools for its analysis are needed.

Symbolically, a Markovian process can be defined in terms of the probability of being a given state $S$ at time $T$ as follows:

$$\text{Prob} (S_{1}=K_{1} \mid S_{t-1}=K_{t-1}, S_{t-2}, S_{t-3}, \ldots, S_{1}, S_{0})$$

Thus, the probability of being in a given state at time $T$ depends only on the previous state the system was in.

The Markov chain process describes the movement of a system from a certain condition (or state) in the current stage (time period) to one of $n$ possible states in the next stage. The system moves in an uncertain environment. All that is known is the probability associated with any possible move (transition). This concept of the transition probability is the key to Markov analysis.

The transition probability at time $t$ for a Markov chain is defined by:

$$P_{ij}(t)=\text{Prob}(S_{t}=K_{j} \mid S_{t-1}=K_{i}) \ldots (2.3.2)$$

and gives the probability that the system is in state $K_{j}$ at the time $t$ given that it was in state $K_{i}$.
previously (time t-1). Alternatively, it is the probability of moving from state Ki to state Kj in one time period (t). If the transition probabilities do not depend on t, that is the transition probabilities of moving from one state to the next remain constant over time, then it is said that the Markov chain is homogenous. In which case:

$$P_{ij} = \text{Prob}(S_t = K_j \mid S_{t-1} = K_i) \quad \cdots \quad (2.3.3)$$

If the state space of the Markov chain is finite then it is discrete Markov chain, otherwise it is a continuous Markov chain. In this paper the concern is with homogenous finite Markov chains.

Given a set of states i=1, ..., M, then $P_{ij}$ is the percentage of time that state j is the outcome if the system starts in state i. The set of transition probabilities across any row (current state) is called a probability vector and represent all possibilities of moving from one state in the current period to one of the n states in the next period.

Given the transition Matrix $P$:

$$P = \begin{bmatrix}
S_1 & S_2 & \cdots & S_j & \cdots & S_n \\
\begin{bmatrix}
P_{11} & P_{12} & \cdots & P_{1j} & \cdots & P_{1n} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
P_{i1} & P_{i2} & \cdots & P_{ij} & \cdots & P_{in} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
P_{n1} & P_{n2} & \cdots & P_{nj} & \cdots & P_{nn}
\end{bmatrix}
\end{bmatrix}$$
The state of the system after \( n \) moves is described by numbers \( P_{ij}(n) \), that is the percentage of the time that \( j \) is the outcome after \( n \) moves if the system starts in state \( i \).

As \( n \) becomes large \( P_{ij}(n) \) approaches \( \pi_j \): that is, all the numbers in a column become identical. The \( \pi_j \)'s may be computed from \( \pi_j = \sum_{i=1}^{M} \pi_i P_{ij} \), where \( \sum_{j=1}^{M} \pi_j = 1 \),

\[
\pi_j > 0 \quad . \quad . \quad . \quad (2.3.4)
\]

**Transient behavior and State Probabilities**

Let the probability that the system will occupy a particular state \( i \), at period \( k \), be denoted by \( q_i(k) \). This probability is the state probability. As the system must occupy only one of the \( n \) possible states at any given period, including period \( k \), then the sum of all \( q \) values must equal 1. Thus:

\[
q_1(k) + q_2(k) + \ldots + q_n(k) = 1 \quad \text{for every} \quad k
\]

That is:

\[
\sum_{i=1}^{n} q_i(k) = 1 \quad . \quad . \quad . \quad . \quad (2.3.5)
\]

where: \( n = \) number of states

\( k = \) number of transitions = 0, 1, 2, ....

In general therefore, the state probability distribution \( Q(k) = [q_1(k), q_2(k), \ldots, q_n(k)] \). . . (2.3.6)

Thus the probability of \( q_1(1) \) together with \( q_2(1) \) and \( q_3(1) \) form the components of \( Q(1) \).

But in Matrix notation: \( Q(1) \) is the product of \( Q(0) \) and \( P \).

Thus: \( Q(1) = q_1(1), q_2(1), \ldots, q_n(1) = Q(0)P \ldots (2.3.7) \)
where $P$ is the transition probability matrix of the system.

Similarly: $Q(2) = Q(1)P \ldots (2.3.8)$

Thus: $Q(2) = Q(1)P = Q(o)P^2$

In general therefore:

$$Q(k) = Q(k-1)P = Q(k-2)P^2 = \ldots = Q(o)P^k \ldots (2.3.9)$$

A major property of Markov chains is that in the long run, the process usually stabilizes: that is the system exist in a steady state or in equilibrium. This phenomenon of equilibrium probabilities is expressed as:

$$Q(k) = Q(k-1) = Q(k)P \ldots \ldots (2.3.10)$$

But $Q = QP \ldots \ldots (2.3.11)$

which implies the state probabilities at equilibrium remains the same from period to period.

$$Q = \begin{pmatrix} q_1 & q_2 & \ldots & q_n \end{pmatrix} \begin{pmatrix} P_{11} & P_{12} & \cdots & P_{1n} \\ P_{21} & P_{22} & \cdots & P_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ P_{n1} & P_{n2} & \cdots & P_{nn} \end{pmatrix} \ldots (2.3.12)$$

Thus: $Q = (q_1, q_2, \ldots, q_n)$

The key feature of a Markovian Model is its ability to capture probabilistic dependencies between successive events. Determining the steady-state behavior of a Markovian system involves solving a set of linear equations. Consequently the Matrix multiplication in equation 2.3.12 above, then result in a system of $n$ simultaneous linear equations as follows:

$$q_1 = P_{11}q_1 + P_{21}q_2 + \cdots + P_{n1}q_n$$

$$\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$$

$$q_n = P_{1n}q_1 + P_{2n}q_2 + \cdots + P_{nn}q_n$$
The Limitations of this model are:
(a) For accurate $\pi_j$'s (equation 2.3.4), the $P_{ij}$'s must be well determined.
(b) The size of $n$ for $P_{ij}(n)$ to be close to $\pi_j$ may be quite large; the $\pi_j$'s are useful only if the requisite size of $n$ is reasonable for the problem at hand.
(c) Difficulties usually arise in the application of Markov chain analysis regarding the development of transition matrix. Historical data can serve this purpose in some cases, while in others, the subjective beliefs of management may be used.

A major property of Markov chains process is thus: "The future, given the present, is independent of the past".

**Absorbing Markov chains**

A system is said to be in an "absorbing" state if, once there, it cannot exit to some other state. Similarly a state is absorbing if once entered, cannot be left. Practical examples of absorbing states are:
A bankrupt business, a river or lake irreversibly destroyed by pollution and sediment, and a building destroyed by fire.

A Markov chain is absorbing if, it has at least one absorbing state or it is possible to reach an absorbing state from any non-absorbing states.
Given a transition Matrix \( T \) (where transition probabilities are assumed constant over time)

\[
T = \begin{bmatrix}
S_1 & S_2 & S_3 & S_4 \\
S_1 & P_{11} & P_{12} & P_{13} & P_{14} \\
S_2 & P_{21} & P_{22} & P_{23} & P_{24} \\
S_3 & P_{31} & P_{32} & P_{33} & P_{34} \\
S_4 & P_{41} & P_{42} & P_{43} & P_{44}
\end{bmatrix}
\]

Assume that states \( S_1 \) and \( S_3 \) are absorbing states, thus all the elements in rows \( S_1 \) and \( S_3 \) are zero save for the one corresponding to the same state which has the value of one.

Thus: \( P_{11} = 1, \ P_{lj} = 0 \) for \( j = 2, 3, 4 \) and \( P_{3j} = 0 \) save for \( j = 3 \)

The transition Matrix then appears in the following form:

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
P_{21} & P_{22} & P_{23} & P_{24} \\
0 & 0 & 1 & 0 \\
P_{41} & P_{42} & P_{43} & P_{44}
\end{bmatrix}
\]

The Fundamental Matrix Formulation:

Cross off the rows with the absorbing states (\( S_1 \) and \( S_3 \) in the above case), then re-arrange the remaining rows and columns into Absorbing (A) and Nonabsorbing (N).

\[
A = \begin{bmatrix}
S_1 & S_3 \\
S_2 \\
S_4
\end{bmatrix}, \quad
N = \begin{bmatrix}
S_2 & S_4 \\
S_2 \\
S_4
\end{bmatrix}
\]

Define the Fundamental Matrix (F), where

\[
F = (I - N)^{-1}, \quad \ldots \ldots \ldots \quad (2.3.15)
\]

the inverse of Identity Matrix less Matrix N.
Thus:

\[
F = \begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix} - \begin{pmatrix}
P_{22} & P_{24} \\
P_{42} & P_{44}
\end{pmatrix}^{-1}
\]

The elements of the Fundamental matrix represent the average number of periods the system will be in each non-absorbing state until it gets absorbed. The sum of the elements in each row of matrix \( F \) provide the average number of periods to absorption.

The probabilities of moving from any nonabsorbing state to each absorbing state are given by matrix \( M \).

\[
M = F A \quad \ldots \ldots \quad (2.3.16)
\]

That is:

\[
M = \begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix} - \begin{pmatrix}
P_{22} & P_{24} \\
P_{42} & P_{44}
\end{pmatrix}^{-1} \begin{pmatrix}
P_{21} & P_{23} \\
P_{41} & P_{43}
\end{pmatrix}
\]

Analysis of absorbing Markov chains can provide Management with answers to at least three important questions:

(i) What is the average number of periods that the system will be in each non-absorbing state before it is absorbed?

(ii) How long is the system expected to stay in non-absorbing state starting from each non-absorbing state?

Such information can be of practical value for management decisions in such areas as: replacement of equipment, marketing, and maintenance. Other relevant areas of application of such information range from queueing theory to production planning, brand loyalty or market share analysis, investment evaluation, new product development, Pest control and ecology, dynamic programming and organization theory among others.
3.1. THE POPULATION

The population of study consisted of all the standard one enrolments in Nakuru Municipal primary schools from 1969 to 1989 (beginning of year). This period of twenty years was chosen because of the major educational policies and developments which took place, and whose effects are felt on enrolment in standard one, in Kenya's urban areas. In 1969 for instance the local (Transfer of Functions) Act, was enacted which transferred the overall responsibility for primary education in the rural areas to the Central Government, while leaving the same in urban areas to the Municipalities. This policy greatly influenced the municipal school systems, and with the unflux of people to the towns, the municipalities had to engage in proper planning and administration of the various resources to meet their local needs, as the central government was no longer responsible directly for the running of primary schools within these areas.

In 1974, there as was a presidential decree which removed school fees for lower primary school (standard one to four). This encouraged many people to send their children to schools consequently enrolment in these classes, especially in standard one soared so high that in some places (like Nakuru) it more than doubled. Such other factors like the gesture by the President in 1979 which introduced milk to primary schools also occurred within this period.
In 1985 a new syllabus was introduced to primary schools which changed the whole of the education system from the 7-4-2-3 to 8-4-4 system. Such factors may have influenced enrolment in one way or the other and the last two decades covered by the study ensures that all these effects are taken into consideration.

The 1969 to 1989 period gives data for two decades. The behavior of the system in the last two decades provide a good indication of the expected behavior for the coming decade. This period thus provide enough data for standard one enrolment behavior, which can therefore be used to predict enrolment for the coming decade. Finally the 1989 (beginning of year enrolment) gives the most current data, whose inclusion in the model made the projections from it more realistic. This is necessary, because like most forecasting or projection models, the further away the period to be predicted from the prediction point, the less realistic the predictions and the nearer the period to be predicted from the prediction point the more realistic the predictions are likely to be.

3.2 DATA COLLECTION METHOD

The data was collected from a secondary source, the Nakuru Municipal Council records. These records are compiled for council school administration purposes. Monthly returns are sent by the schools to the Municipal Education office, showing among other things the pupil attendance over the month for each class. The pupils may not attend classes due to several reasons:
Non-payment of school fees, (when standard one pupils paid school fees), sicknesses, death, transfers out of town, transfers within town (to different schools within the same zones or in different zones). In cases where pupils were absent due to non-payment of school fees or sicknesses, they were treated as present, because these are temporary absences. In cases where pupils were transferred out of town (Municipality) or cases of death or drop-outs, they were treated as "absorbed" or as if they entered a non-return state. In cases where pupils shifted between schools in the same zone, such enrolments are really within the same zone and their effects are not felt outside the zone, as such these cases are ignored. Movement out of each zone and into a zone were considered as transitions.

The data collection sheets (Appendices 3, 4 & 5) show how the relevant data was captured. All the data from 1969 to 1988 represent actual enrolment; this is due to the fact that no comprehensive list of the applicants (demand) exists for these years. Furthermore the municipality claims that all the applicants in each of those years were enrolled and where these over-numbered the facilities available, temporary measures were taken like converting other rooms for classrooms, employing untrained teachers, or simply doing without some facilities. In other words, the situation was dealt with as it arose. If this is true, then, there was insignificant difference between the applicants (demand) and actual enrolment.
In 1989, however, the application forms sent to the various nursery schools for standard one prospects from the municipality was far below the actual applicants, this was evidenced by the actual requests sent to the Municipal office. Such information while available for 1989 was not available for the previous years as such it was possible to obtain the applicants figures for 1989 that was used to estimate the demand for standard one places in 1990, 1991 and 1992.

The demand estimates are much more useful for planning purposes than the estimated enrolment, because it provides what should be the target for proper allocation of resources rather than what would be the case with the actual enrolment. The three years forecasts were chosen because the shortest planning period (for local government) is three years. Thus it is possible to plan for the three years as part of the usual planning programs rather than for a year.

3.3 DATA ANALYSIS

The data was analysed by building a Markov chain Model of the enrolment, and employing this model in the prediction of future enrolment. The actual process follows.

3.3.1 MODEL DEVELOPMENT:

3.3.1.1 Introduction

Data for actual movement between the zones during any year were not readily available, indeed the documented evidence showed that the movement,
particularly for standard one pupils is negligible. If such information is used to build the model, it would not result in a realistic model. It became necessary therefore, to come up with an appropriate transition matrix which would be realistic, this is due to the fact that for a Markov chain Model, the transition Matrix is the core.

The enrolment for each zone for each year were arranged in a tabular form (appendix 2). The proportion of the enrolment for each zone were then calculated for every year, from 1974 to 1979 and from 1984 to 1988. The 1974 was chosen as the base year to start studying the behavior of the system for the following reason:-

In 1974 there was a presidential decree which abolished school fee payment for standard one to standard four. This decree affected enrolment so much that it increased from 2,154 in 1973 to 4,359 in 1974 for standard one alone; an increase of 102%. Indeed the enrolment has never been that high again! From 1974 therefore a new era started in the primary school system in which parents paid no school fees to enrol their children in standard one. It therefore follows that all schools were the same and parents could therefore enrol their children only in the most convenient regions. Before 1974, Price discrimination (differential school fees in different
schools) kept pupils only in schools their parents could afford. Indeed some schools were reserved for the very rich, whites, Indians. Such arrangements constrained enrolment to specific regions, and it is possible that parents would send their children to different zones from where they lived. It can therefore be concluded that the distribution of standard one enrolment (behavior of the system) before this period was constrained or artificial. When this barrier was removed, it is expected that the demand for standard one places shifted from the affordable (for the majority of Nakuru resident) to the most convenient regions. The system therefore assumed its own behavior which can safely be represented by a Stochastic Model.

The 1980, 1981, 1982 and 1983 data was left out when building the Model because they were to be used for the Model validation. 1989 data was also left out because this demand is to be used for forecasting for the next decade once the model is validated.

This process was followed for the total enrolments in standard one (boys and girls), for boys only and for girls only. In effect, three different transition Matrices were obtained showing the transition between the zones for boys and girls separately and for the two together.

The assumptions underlying the Markov Chain Model are covered under section two of this study and shall only be referred to, in this section, where applicable.
3.3.1.2 The Model

One of the major properties of Markov chains is that, in the long run, the process usually stabilizes (equilibrium). This phenomenon is expressed in equations 2.3.7, 2.3.9 and 2.3.10; these imply that the state probabilities at equilibrium remain the same from period to period. Therefore at equilibrium, if the current state $Q(o)$ is known and the transition matrix $T$ is known, then the future state $Q(k)$ can be predicted.

Since the transition matrix in our case is not known, the transition from state to state is studied for the nine years, in the following manner:

Given that:

$$Q(k) = Q(o)T,$$

where:

- $Q$ is the future state.
- $q_E$ is the proportion of standard one enrolled in the Eastern zone currently.
- $q_C$ is the proportion of standard one enrolled in the Central zone currently.
- $q_W$ is the proportion of standard one enrolled in the Western zone currently.
- $P_{ij}$ for $i,j =$ Eastern, Central and Western, represent the probability of moving from the $i$ zone to the $j$ zone.
It is necessary to add here that, \( q_E + q_C + q_W = 1 \), that is, the enrolment within the Municipality can only occur in either of the three zones. Also, the row vector of the transition Matrix, \( T \), add up to one, thus in every transition, a zone retains a certain proportion while the rest it shifts to either of the other two zones. A shift from the municipality and shifts into the Municipality are taken care of by the increases or decreases within the zones. For instance, if there is an influx of people to the Municipality, these will distribute themselves into the various zones as they find convenient, as such the proportions in the zones will change. If more people move to the Eastern zone for instance, then the proportion enroled in the Eastern zone shall rise, similarly if there is a major move from the Municipality from one particular zone, say, the western zone, then the proportions in this zone shall fall accordingly. Otherwise the moves into or out of the Municipality should be distributed equally between the zones if there is no preference of one zone over the others; in which case the proportions remain unchanged.

In most instances the current state, \( Q(0) \) and the transition Matrix (\( T \)) are known; but \( Q(k) \) is unknown. In our case the \( T \) is unknown, but the transition from state to state for the nine years (1974-1979, 1984-1988) were used to arrive at an estimate of the best transition Matrix to be used.
Wolfe and Dantzig (1962) showed that there is an iteration procedure which when applied result in the choice of the best Transition Probability Matrix. Since a Markov chain (system) can be in any one of the \( M(< \infty) \) states and associated with each state \( i(=1,2,...,m) \), let \( P_i \) be the set of available alternate probability vectors \( \{^kP_i1, ^kP_i2, ..., ^kP_im\} \), \( K \in D_i \). The idea is to select one vector out of \( P_i \), in such a way that when it is used for the one-step transition probabilities out of state \( i \) in the Markov Chain, the ultimate gain is maximised, assuming that the system stays in operation for a sufficiently long time. Repeating the iteration procedure for all the states, \( i=1,2,...,m \), then a transition probability Matrix can be assembled for the Markov Chain which can be considered to be the best under the given circumstances. The iteration method used in achieving this result has been shown to be a special extension of the simplex method of Linear programming allowing for Multiple substitution (see Ghellink, 1960).

3.3.1.3 The Total enrolment Model

The transition between 1974 and 1975

\[
Q(1974) \begin{bmatrix}
P_{11} & P_{12} & P_{13} \\
P_{21} & P_{22} & P_{23} \\
P_{31} & P_{32} & P_{33}
\end{bmatrix} = Q(1975)
\]

Thus:

\[
\begin{bmatrix}
.378 & .362 & .259 \\
.383 & .356 & .261
\end{bmatrix} = \begin{bmatrix}
P_{11} & P_{12} & P_{13} \\
P_{12} & P_{22} & P_{23} \\
P_{13} & P_{32} & P_{33}
\end{bmatrix}
\]
Similarly the transition between 1975 and 1976

\[
\begin{bmatrix}
0.383 & 0.356 & 0.261 \\
0.21 & 0.22 & 0.23 \\
0.31 & 0.32 & 0.33
\end{bmatrix}
= \begin{bmatrix}
0.374 & 0.346 & 0.280 \\
\end{bmatrix}
\]

This process is done up to the transition between 1987 and 1988. The result is a set of nine simultaneous equations with nine unknowns; which were thus solved to obtain the initial estimate of the transition matrix. For the total enrolment the initial matrix obtained was:

\[
T = \begin{bmatrix}
0.52 & 0.37 & 0.11 \\
0.12 & 0.78 & 0.10 \\
0.10 & 0.25 & 0.65
\end{bmatrix}
\]

This matrix reached equilibrium at the ninth transition, that is \( T^9 = T^{10} = T^{11} = \ldots = T^n \) for a very large \( n \).

The equilibrium transition matrix for the total enrolment is thus:

\[
T_e = \begin{bmatrix}
0.095 & 0.590 & 0.315 \\
0.095 & 0.590 & 0.315 \\
0.095 & 0.590 & 0.315
\end{bmatrix}
\]

The transition probability matrix gives the probability of moving from state \( K_i \) to state \( K_j \) in one time period \( t \). Since the transition probabilities above do not depend on \( t \), that is the transition probabilities of moving from one state to the next remain constant over time, the Markov Chain is homogeneous and takes the form of equation 2.3.3. The state space of the Markov Chain above is finite (three) as such it is a discrete Markov Chain.
Pij from the transition Matrix above represent the percentage of time that state j is the outcome if the system starts in state i. The state of the system after n moves is described by Pij(n), that is, the percentage of the time that j is the outcome after n moves if the system starts in state i. As n becomes large (nine in our case) Pij(9) approaches \( \pi_j \); that is all the numbers in a column become identical. In column one, all the numbers are .095, while in Column two, they are .590 and in column three they are .315; also \[ \sum_{j=1}^{3} \pi_j = 1, \pi_j \geq 0 \] (equation 2.3.4).

3.3.1.4 The Boys enrolment Model

A similar procedure to the one used for the total enrolment is used for the boys enrolment.

With the known states and an unknown transition Matrix, the transition is built from 1974 to 1975, from 1975 to 1976, 1976 to 1977 up to from 1978 to 1979. Also from 1984 to 1985 and so on up to 1988.

\[
Q(1974) = \begin{bmatrix}
P_{11} & P_{12} & P_{13} \\
P_{21} & P_{22} & P_{23} \\
P_{31} & P_{32} & P_{33}
\end{bmatrix} = Q(1975)
\]

Thus: \((.375 .351 .274) \begin{bmatrix}
P_{11} & P_{12} & P_{13} \\
P_{21} & P_{22} & P_{23} \\
P_{31} & P_{32} & P_{33}
\end{bmatrix} = (.383 .348 .278)\)

for 1974-1975
similarly for the 1975-1976

\[
\begin{pmatrix}
0.383 & 0.348 & 0.278 \\
0.382 & 0.315 & 0.303
\end{pmatrix}
\]

\[
\begin{pmatrix}
P_{11} & P_{12} & P_{13} \\
P_{21} & P_{22} & P_{23} \\
P_{31} & P_{32} & P_{33}
\end{pmatrix}
\]

and so on up to 1987-1988 period.

The result is a set of nine simultaneous equations with nine unknowns. These are then solved using the iteration procedure as above to obtain the initial transition matrix

\[
T_b = 
\begin{pmatrix}
0.690 & 0.174 & 0.136 \\
0.220 & 0.568 & 0.212 \\
0.279 & 0.249 & 0.492
\end{pmatrix}
\]

The matrix \( T_b \) reached a steady state (equilibrium) at the tenth power, so that \( T_b^{10} = T_b^{11} = T_b^{12} = \ldots = T_b^n \) for a large \( n \). The equilibrium transition matrix for the boys enrolment is thus :-

\[
T_b_e = 
\begin{pmatrix}
0.510 & 0.309 & 0.181 \\
0.510 & 0.309 & 0.181 \\
0.510 & 0.309 & 0.181
\end{pmatrix}
\]

To conform with the transition probability matrix property that as \( n \) becomes large (ten in this case) \( P_{ij}^{(10)} \) approaches \( \pi_j \); thus all the numbers in a column become identical. In the first column, in this case, the number is 0.510, the second column has 0.309 while the third column has 0.181.
3.3.1.5 The Girls enrolment Model

A similar procedure to the ones already used above is used for the girls data. The iterations from 1974 to 1975, 1975 to 1976 up to 1978-1979, and also from 1984 to 1985, 1985 to 1986 up to 1988 are followed:

So that:

\[ Q(1974) \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix} = Q(1975) \]

Thus:

From 1974 to 1975

\[ (1974) \begin{bmatrix} .382 & .374 & .244 \\ .384 & .364 & .256 \end{bmatrix} \]

This procedure, if followed up to the 1987-1988 period result in nine simultaneous equations with nine unknowns. These when solved provide the initial matrix, \( T_g \).

\[ T_g = \begin{bmatrix} .406 & .566 & .028 \\ .110 & .547 & .343 \\ .108 & .030 & .862 \end{bmatrix} \]

Raising \( T_g \) to higher powers provided a steady state transition probability matrix at the thirteenth iteration thus \( T_g \) became steady at the fourteenth power. Thus, \( T_g^{14} = T_g^{15} = T_g^{16} = \ldots T_g^n \), for a very large \( n \). The equilibrium transition probability matrix for the girls is thus:

\[ T_{ge} = \begin{bmatrix} .154 & .234 & .612 \\ .154 & .234 & .612 \\ .154 & .234 & .612 \end{bmatrix} \]

This also conforms to the transition probability matrix property that as \( n \) becomes large (fourteen in this case), \( P_{ij}(14) \) approaches \( j \); thus all the numbers in a column become identical. In this case, the first column number is .154, the second column number is .234 while the third column number is .612.
### 3.3.2 Model Validation

#### 3.3.2.1 Introduction

In validating the model, actual enrolment for 1980 (in proportions) are premultiplied with the transition matrix $T$, to provide an estimate for 1981. 1981 estimated proportions are also premultiplied with the transition matrix again to provide estimates for 1982, which is also premultiplied by the transition matrix to provide estimates for 1983 and similarly for 1983 and 1984.

A statistical test is then done to compare the estimated enrolment against the actual enrolment first at 0.10 significance level (90% confidence level) and then at 0.05 significance level (95% confidence level).

This procedure is followed for all the three transition matrices (The totals, boys only and girls only).

#### 3.3.2.2 The Total Enrolment Model

$$
\begin{bmatrix}
Q(1980) \\
E & C & W
\end{bmatrix}
\begin{bmatrix}
.52 & .37 & .11 \\
.12 & .78 & .10 \\
.10 & .25 & .65
\end{bmatrix}
= 
\begin{bmatrix}
Qe(1981) \\
E & C & W
\end{bmatrix}
$$

$$
\begin{bmatrix}
340 & 320 & .340 \\
.12 & .78 & .10 \\
.10 & .25 & .65
\end{bmatrix}
= 
\begin{bmatrix}
.249 & .460 & .291
\end{bmatrix}
$$
A comparison of the estimated proportions against the actual proportions at a 90% level of confidence. This is testing a hypothesis about the difference between two population proportions.

Hypotheses:

\[
\begin{align*}
H_0: & \quad \text{PE}_e = \text{PE}_a, \quad \text{PC}_e = \text{PC}_a, \quad \text{PWe} = \text{PW}_a \\
H_1: & \quad \text{PE}_e \neq \text{PE}_a, \quad \text{PC}_e \neq \text{PC}_a, \quad \text{PWe} \neq \text{PW}_a
\end{align*}
\]
That is the hypothesis that the estimated proportions are the same as the actual proportions in other words, that the model predict the proportions with some measure of accuracy.

Test Statistics:

\[ Z = \frac{(P_a - P_e) - 0}{S(P_a - P_e)} \]

Where \( P_a \) are the actual proportions

\( P_e \) are the estimated proportions

\( S(P_a - P_e) \) are the standard error of the difference between the two proportions.

This is given by

\[ S(P_a - P_e) = P(1 - P) + P(1 - P) \]

\[ \frac{na}{na + ne} \]

Where \( P \) is the pooled estimate of the hypothesized common proportions, \( na \) is a sample size for the actual proportions and \( ne \) is a sample size taken for the estimated proportions. \( P \) is given by

\[ \frac{X_a + X_e}{na + ne} \]

Where \( X_a \) represent the actual numbers from the sample while \( X_e \) represent the estimated number from the sample, for instance if a sample of fifty (50) is chosen at random then \( X_a \) is the actual number from this fifty enrolled in a given zone while \( X_e \) is the estimate (by the model) number enrolled in a given zone.

In which case \( na = ne = 50 \).

**Significance Level:** \( \alpha = 0.10 \)
Decision Rule: If the computed value of the test statistics, $Z$, is greater than or equal to $1.645$ or less than or equal to $-1.645$ we reject the null hypothesis (Ho).

Calculations:

Given that: $n_a = 50$ (50 is chosen arbitrarily, but since it is a large number, that is, greater than thirty (30), the central limit theorem applies).

Then for the different zones:

<table>
<thead>
<tr>
<th>Eastern</th>
<th>Central</th>
<th>Western</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_a$</td>
<td>$X_e$</td>
<td>$P$</td>
</tr>
<tr>
<td>$0.34 \times 50 = 17$</td>
<td>$0.39 \times 50 = 19.50$</td>
<td>$0.27 \times 50 = 13.5$</td>
</tr>
<tr>
<td>$0.249 \times 50 = 12.45$</td>
<td>$0.46 \times 50 = 23$</td>
<td>$0.291 \times 50 = 14.55$</td>
</tr>
<tr>
<td>$P = \frac{17 + 12.45}{50} = 0.2945$</td>
<td>$P = \frac{19.5 + 23}{100} = 0.425$</td>
<td>$P = \frac{13.5 + 14.55}{1000} = 0.2805$</td>
</tr>
</tbody>
</table>

$SP_a - P_e = \frac{P(1-P)}{n_a}$ + $\frac{P(1-P)}{n_e} = \frac{2P(1-P)}{50}$

$SP_a - P_e = \frac{2 \times 0.2945 \times 0.7055}{50} = 0.091$ \[= \frac{2 \times 0.425 \times 0.575}{50} = 0.099\] \[= \frac{2 \times 0.2805 \times 0.7195}{50} = 0.090\]

$Z = \frac{(0.34 - 0.249) - 0}{0.091} = 1.00$ \[= \frac{(0.39 - 0.46) - 0}{0.099} = -0.707\] \[= \frac{(0.27 - 0.291) - 0}{0.09} = -0.233\]
Statistical Decision:

Since \(1.00 \leq 1.645\), \(-.707\) and \(-.233\) are greater than \(-1.645\), we do not reject the null hypothesis.

If unpooled estimate of the standard error is used, then

\[
Z = \frac{(P_a - P_e) - 0}{\sqrt{\frac{P_a(1-P_a) + P_e(1-P_e)}{n_a + n_e}}}
\]

\[
Z = \frac{-0.91}{0.0907} = 1.003 = \frac{-0.07}{0.099} = -.707 = \frac{-0.021}{0.090} = -.233
\]

The two methods give the same answer because \(n_a = n_e\), thus pooling does not affect the result.

Administrative Decision

Based on the data from above, we conclude that the two proportions may be equal. These data do not allow us to accept the alternative hypothesis. The model predictions for 1981 are not so far off from the actual proportions; as such the model may be accepted for one transition or for one year estimation.

In testing the same hypothesis at the 95% level of confidence, the test statistics remain the same, however the significance level changes to \(\alpha = 0.05\) and the decision rule also changes.
Decision rule

If the computed value of the test statistics, $Z$, is greater than or equal to $+1.96$ or less than or equal to $-1.96$, we reject the null hypothesis (Ho).

Statistical Decision

Since the calculated values, 1.00, -0.707, -0.233 all fall within the given range, the decision of not rejecting the null hypothesis is upheld.

Administrative decision remains the same.

Hypotheses:

$H_0$: $P_{Ee} = P_{Ea}$, $P_{Ce} = P_{Ca}$, $P_{We} = P_{Wa}$

$H_1$: $P_{Ee} \neq P_{Ea}$, $P_{Ce} \neq P_{Ca}$, $P_{We} \neq P_{Wa}$

Test statistics: $Z = \frac{(P_a - P_e) - 0}{\sqrt{\frac{P_a(1-P_a) + P_e(1-P_e)}{n}}}$

Significance level: $\alpha = .10$

Decision rule: If the computed value of the test statistics, $Z$, is greater than or equal to $+1.645$ or less than or
equal to -1.645, we reject the null hypothesis (Ho).

**Calculations**

<table>
<thead>
<tr>
<th></th>
<th>Eastern</th>
<th>Central</th>
<th>Western</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pa</td>
<td>.352</td>
<td>.338</td>
<td>.309</td>
</tr>
<tr>
<td>Pe</td>
<td>.214</td>
<td>.523</td>
<td>.263</td>
</tr>
<tr>
<td>n</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
</tbody>
</table>

Eastern

\[
Z = \frac{(0.352 - 0.214) - 0}{\sqrt{0.352(0.648) + 0.214(0.786)}} = \frac{0.138}{0.089} = 1.55
\]

Central

\[
Z = \frac{(0.338 - 0.523) - 0}{\sqrt{0.338(0.662) + 0.523(0.477)}} = \frac{0.185}{0.098} = -1.89
\]

Western

\[
Z = \frac{(0.309 - 0.263) - 0}{\sqrt{0.309(0.691) + 0.263(0.737)}} = \frac{0.046}{0.090} = 0.511
\]

**Statistical Decision**

Since one of the calculated values, -1.89 is less than the critical -1.645, we reject the null hypothesis at the significance level \( \alpha = 0.10 \).

**Administrative Decision:**

Based on the above data, we conclude that the two
proportions may not be equal, however further tests are to be carried out below to check on this decision at a higher level of confidence.

At a 95% level of confidence, the test statistics remain the same, but the significance level changes to $\alpha = 0.05$ and the decision rule also changes.

**Decision rule:**

If the computed value of the test statistics, $Z$, is greater than or equal to $+1.96$ or less than or equal to $-1.96$, we reject the null hypothesis.

**Statistical decision**

Since the calculated values, $1.55$, $-1.89$, $0.511$ all fall within the non-rejection range, that is between $\pm 1.96$, we do not reject the null hypothesis at the significance level $\alpha = 0.05$.

**Administrative Decision**

Based on the above data, we conclude that the two proportions may be equal. The data do not allow us to accept the alternative hypothesis. The model predictions for 1982 are not so far off from the actual proportions observed at the confidence level of 95% as such the model may be accepted for two transitions or for two year forecast at 95% confidence level.
Hypotheses:

Ho: \( P_{Ee} = P_{Ea} \), \( P_{Ce} = P_{Ca} \), \( P_{We} = P_{Wa} \)
H1: \( P_{Ee} \neq P_{Ea} \), \( P_{Ce} \neq P_{Ca} \), \( P_{We} \neq P_{Wa} \)

Test Statistics

\[
Z = \frac{(P_e - P_a) - 0}{\sqrt{\frac{Pa(1-Pa) + Pe(1-Pe)}{n}}}
\]

Significance level: \( \alpha = .10 \)

Decision Rule:

If the computed value of the test statistics, \( Z \), is greater than or equal to +1.645 or less than or equal to -1.645, we reject the null hypothesis at the .10 significance level.

Calculations

<table>
<thead>
<tr>
<th></th>
<th>Eastern</th>
<th>Central</th>
<th>Western</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_e )</td>
<td>.338</td>
<td>.361</td>
<td>.301</td>
</tr>
<tr>
<td>( P_a )</td>
<td>.200</td>
<td>.553</td>
<td>.247</td>
</tr>
<tr>
<td>( n )</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
</tbody>
</table>
Statistical Decision

Since one of the computed values of $Z$, $-1.956$ is less than the critical $-1.645$, we reject the null hypothesis at the significance level $\alpha = 0.10$.

Administrative Decision

Based on the above data, we conclude that the two proportions may not be equal, however further tests are carried out below to check on this decision at a higher level of confidence.

At a 95% level of confidence the test statistics remain the same, but the significance level changes to $\alpha = .05$ and thus the decision rule also changes.

Decision Rule:

If the computed value of the test statistics, $Z$, is
greater than or equal to +1.96 or less than or equal to -1.96, we reject the null hypothesis.

Statistical Decision

Since the calculated values of $Z$, 1.568, -1.959 and .607, all fall within the non rejection, range, We fail to reject the null hypothesis at the $\alpha=0.05$ level of significance.

Administrative Decision

From the above data, we conclude that the two sets of proportions may be equal. These data do not allow us to accept the alternative hypothesis. The model predictions for 1983 are not significantly different from the actual observed proportions, the model may be thus used for prediction of the third year, or the third transition.

1984

Hypotheses:

$H_0$: $P_{ee} = P_{ea}$, $P_{ce} = P_{ca}$, $P_{we} = P_{wa}$

$H_1$: $P_{ee} \neq P_{ea}$, $P_{ce} \neq P_{ca}$, $P_{wa} \neq P_{we}$

Test Statistics: $Z = \frac{(P_a - P_e) - 0}{\sqrt{P_a(1-P_a)+P_e(1-P_e)}}$
Significance Level: $\alpha = .10$.

Decision Rule:

If the computed value of the test statistics, $Z$, is greater than or equal to $+1.645$ or less than or equal to $-1.645$, we reject the null hypothesis, $(H_0)$ at significance level of $\alpha = .10$.

Calculations

<table>
<thead>
<tr>
<th></th>
<th>Eastern</th>
<th>Central</th>
<th>Western</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_a$</td>
<td>.340</td>
<td>.362</td>
<td>.298</td>
</tr>
<tr>
<td>$P_e$</td>
<td>.195</td>
<td>.567</td>
<td>.238</td>
</tr>
<tr>
<td>$n$</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
</tbody>
</table>

$Z = \frac{(.340-.195)-0}{\sqrt{.34(.66)+.195(.805)}} = \frac{.145}{.87} = 1.667$

$Z = \frac{(.362-.567)-0}{\sqrt{.362(.638)+.567(.433)}} = \frac{-.205}{.098} = -2.092$

$Z = \frac{(.298-.238)-0}{\sqrt{.299(.702)+.238(.762)}} = \frac{.060}{.088} = .682$
Statistical Decision

Since two of the computed values of $Z$, 1.667 and -2.092 are greater than 1.645 and less than -1.645 respectively, we reject the null hypothesis at the significance level of $\alpha = .10$.

Administrative Decision

Based on the above data we conclude that the two sets of proportions may not be equal, however, further tests are carried out below to check on this decision at a higher level of confidence.

At 95% level of confidence, the test statistics remain the same, but the significance level changes to $\alpha = .05$, and thus the decision rule also changes.

Decision Rule:

If the computed value of the test statistics, $Z$, is greater than or equal to +1.96 or less than or equal to -1.96, we reject the null hypothesis.

Statistical Decision

Since one of the calculated values of $Z$, -2.092 is less than -1.96, we reject the null hypothesis at the significance level $\alpha = .05$. 
From the above data, we conclude that the two sets of proportions may not be equal. This implies that the model for 1984 or for four transitions has weaker prediction power.

This confirms the suspicion that as the powers keep rising or the further away the predicted point from the starting point, the larger the difference between the predicted and the actual observation.

**General Conclusion**

The general conclusion for the total enrolment model is that the model should be used with caution especially for long period forecasts. The model works well for the first three transitions and starts to fail at the fourth transition. It is for this reason that the model is used to forecast for only three years. Such forecasts though short for long term planning are long enough for the short term planning purposes.

### 3.3.2.3 The Boys Enrolment Model

\[
\begin{align*}
Q(1980) & \quad Tb & \quad Qe(1981) \\
E & C & W & = & E & C & W \\
(.302 & .364 & .334) & \begin{bmatrix} .690 & .174 & .136 \\ .220 & .568 & .212 \\ .279 & .249 & .472 \end{bmatrix} & = & (.382 & .342 & .276)
\end{align*}
\]
\[ Q_e(1981) \]
\[
\begin{pmatrix}
.690 & .174 & .136 \\
.220 & .568 & .212 \\
.279 & .249 & .472
\end{pmatrix}
\]
\[ Q_e(1982) \]
\[
\begin{pmatrix}
.690 & .174 & .136 \\
.220 & .568 & .212 \\
.279 & .249 & .472
\end{pmatrix}
\]
\[ Q_e(1983) \]
\[
\begin{pmatrix}
.690 & .174 & .136 \\
.220 & .568 & .212 \\
.279 & .249 & .472
\end{pmatrix}
\]
\[ Q_e(1984) \]
\[
\begin{pmatrix}
.690 & .174 & .136 \\
.220 & .568 & .212 \\
.279 & .249 & .472
\end{pmatrix}
\]

Year | Actual Proportions | Estimated Proportions
--- | --- | ---
1981 | .341 | .386 | .273 | .382 | .342 | .276
1983 | .342 | .339 | .320 | .430 | .323 | .247
1984 | .356 | .334 | .310 | .437 | .370 | .343

A comparison of the actual proportions against the estimated proportions first at 90% confidence level then at 95% confidence level. This is a test of the hypothesis about the difference between two sets of population proportions, the actual and the estimated.
Hypotheses

\[ H_0: \; P_e = P_a, \; P_C = P_C, \; P_W = P_W \]
\[ H_1: \; P_e \neq P_a, \; P_C \neq P_C, \; P_W \neq P_W \]

Test Statistics:

\[ Z = \frac{(P_a - P_e) - 0}{\sqrt{\frac{P_a(1-P_a) + P_e(1-P_e)}{n_a + n_e}}} \]

Significance Level: \( \alpha = 0.10 \)

Decision Rule:

If the computed value of the test statistics, \( Z \), is greater than or equal to 1.645 or less than or equal to -1.645, we reject the null hypothesis.

Calculations

<table>
<thead>
<tr>
<th></th>
<th>Eastern</th>
<th>Central</th>
<th>Western</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_a )</td>
<td>0.341</td>
<td>0.386</td>
<td>0.273</td>
</tr>
<tr>
<td>( P_e )</td>
<td>0.382</td>
<td>0.342</td>
<td>0.255</td>
</tr>
<tr>
<td>( n_a = n_e = n )</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
</tbody>
</table>

\[ Z = \frac{(0.341 - 0.382) - 0}{\sqrt{\frac{0.341(0.659) + 0.382(0.618)}{50}}} \]
\[ = -0.041 = -0.427 \]

\[ \frac{0.096}{0.096} \]

\[ \frac{0.096}{0.096} \]

\[ \frac{0.096}{0.096} \]
Since all calculated values fall within the non-rejection region (± 1.645), we do not reject the null hypothesis.

At the significance level $\alpha = .05$: The decision rule changes to greater than or equal to +1.96 or less than or equal to -1.96. Thus even at this level of significance, we do not reject the null hypothesis.

Administrative Decision

Based on the data from above, we conclude that the two proportions may be equal. These data do not allow us to accept the alternative hypothesis. The model predictions for 1981 are not so far off from the actual proportions observed, as such the model is acceptable for one transition or one year prediction.

**Hypotheses**

$H_0$: $\text{PE}_a = \text{PC}_e$, $\text{PC}_e = \text{PC}_a$, $\text{PW}_a = \text{PW}_e$

$H_1$: $\text{PE}_a \neq \text{PC}_e$, $\text{PC}_e \neq \text{PC}_a$, $\text{PW}_a \neq \text{PW}_e$
Test Statistics: \[ Z = \frac{(P_a - P_e) - 0}{\sqrt{P_a(1-P_a) + P_e(1-P_e)/n}} \]

Significance Level: \( \alpha = 0.10 \).

Decision Rule:

If the computed values of the test statistics, \( Z \), is greater than or equal to +1.645 or less than or equal to -1.645, we reject the null hypothesis.

Calculations

<table>
<thead>
<tr>
<th>Eastern</th>
<th>Central</th>
<th>Western</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_a )</td>
<td>.364</td>
<td>.319</td>
</tr>
<tr>
<td>( P_e )</td>
<td>.416</td>
<td>.329</td>
</tr>
<tr>
<td>( n )</td>
<td>50</td>
<td>50</td>
</tr>
</tbody>
</table>

\[ Z = \frac{(.364 - .416) - 0}{\sqrt{.364(1-.636) + .416(1-.584)/50}} = \frac{-0.052}{0.097} = -0.536 \]

\[ Z = \frac{(.319 - .329) - 0}{\sqrt{.319(1-.681) + .329(1-.671)/50}} = \frac{-0.01}{0.094} = -0.106 \]

\[ Z = \frac{(.317 - .255) - 0}{\sqrt{.317(1-.683) + .255(1-.745)/50}} = \frac{0.062}{0.090} = .689 \]
Statistical Decision

Since all the calculated values fall within the non-rejection region (±1.645) we do not reject the null hypothesis (Ho).

At the significance level $\alpha = .05$, the decision rule changes to greater than or equal to +1.96 or less than or equal to -1.96, thus even at this level of significance (95% confidence level), we do not reject the null Hypothesis.

Administrative Decision

Based on the data from above, we conclude that the two proportions may be equal. These data do not allow us to accept the alternative hypothesis. The model predictions for 1982 are not so far off from the actual proportions observed, as such the model is acceptable for two transitions or for two years’ predictions.

1983

Hypotheses remain the same as above.

Test statistics remain the same as above.

Significance Level: $\alpha = .10$
Decision Rule:

If the computed values of the test statistics, Z, is greater than or equal to +1.645 or less than or equal to -1.645, we reject the null hypothesis.

Calculations

<table>
<thead>
<tr>
<th></th>
<th>Eastern</th>
<th>Central</th>
<th>Western</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pa</td>
<td>.342</td>
<td>.339</td>
<td>.320</td>
</tr>
<tr>
<td>Pe</td>
<td>.430</td>
<td>.326</td>
<td>.247</td>
</tr>
<tr>
<td>n</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
</tbody>
</table>

Eastern

\[ Z = \frac{(.342 - .430) - 0}{\sqrt{.342(.658) + .430(.570)}} = \frac{-.088}{.097} = -.907 \]

Central

\[ Z = \frac{(.339 - .326) - 0}{\sqrt{.339(.661) + .326(.674)}} = \frac{.013}{.090} = .138 \]

Western

\[ Z = \frac{(.320 - .247) - 0}{\sqrt{.320(.680) + .247(.753)}} = \frac{.073}{.090} = .811 \]

Statistical Decision

This is upheld as in the above case (1982).

Administrative Decision

The model may be used to forecast for the three years or three transitions.
Hypotheses remain the same as above.

Test statistics remain the same as above.

Significance Level: \( \alpha = .10 \)

Decision rule remains the same as above.

Calculations

<table>
<thead>
<tr>
<th></th>
<th>Eastern</th>
<th></th>
<th>Central</th>
<th></th>
<th>Western</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pa</td>
<td>.356</td>
<td>Pe</td>
<td>.437</td>
<td></td>
<td>.334</td>
</tr>
<tr>
<td>Pe</td>
<td>.356</td>
<td></td>
<td>.437</td>
<td></td>
<td>.320</td>
</tr>
<tr>
<td>n</td>
<td>50</td>
<td></td>
<td>50</td>
<td></td>
<td>50</td>
</tr>
</tbody>
</table>

\[
Z = \frac{(0.356-0.437)-0}{\sqrt{0.356(0.644)+0.437(0.563)}} = -0.081 = -0.835
\]

\[
= \frac{(0.334-0.320)-0}{\sqrt{0.334(0.666)+0.320(0.680)}} = 0.014 = 0.149
\]

Statistical decision is upheld for 1984 as for 1983.

From the data above, the model may be used to predict for the fourth year with reasonable measure of accuracy with 95% confidence. That is, the projected proportions are not
so different from the observed proportions.

3.3.2.4 The Girls Enrolment Model

\[
\begin{align*}
Q(1980) &\quad T_g &\quad Qe(1981) \\
E & C & W & 0.566 & 0.406 & 0.028 & E & C & W & 0.238 & 0.399 & 0.363 \\
(282, 0.415, 0.303) & & 0.110 & 0.547 & 0.343 & & 0.108 & 0.190 & 0.702 \\
Qe(1981) &\quad T_g &\quad Qe(1982) \\
& 0.566 & 0.406 & 0.028 & & 0.218 & 0.384 & 0.398 \\
(0.238, 0.399, 0.363) & & 0.110 & 0.547 & 0.343 & & 0.108 & 0.190 & 0.702 \\
Qe(1982) &\quad T_g &\quad Qe(1983) \\
& 0.566 & 0.406 & 0.028 & & 0.209 & 0.374 & 0.417 \\
(0.218, 0.384, 0.398) & & 0.110 & 0.547 & 0.343 & & 0.108 & 0.190 & 0.702 \\
Qe(1983) &\quad T_g &\quad Qe(1984) \\
& 0.566 & 0.406 & 0.028 & & 0.204 & 0.369 & 0.427 \\
(0.209, 0.374, 0.417) & & 0.110 & 0.547 & 0.343 & & 0.108 & 0.190 & 0.702 \\
\end{align*}
\]

<table>
<thead>
<tr>
<th>Year</th>
<th>Actual Proportions</th>
<th>Estimated Proportions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ea</td>
<td>Ca</td>
</tr>
<tr>
<td>1981</td>
<td>0.340</td>
<td>0.395</td>
</tr>
<tr>
<td>1982</td>
<td>0.340</td>
<td>0.360</td>
</tr>
<tr>
<td>1983</td>
<td>0.334</td>
<td>0.383</td>
</tr>
<tr>
<td>1984</td>
<td>0.323</td>
<td>0.391</td>
</tr>
</tbody>
</table>

A comparison of the actual proportions against the
estimated proportions, first at 90% confidence level then at 95% confidence level. This is a test of the hypothesis about the difference between two sets of population proportions, the actual and the estimated.

1981

Hypotheses:

Ho: \( P_{Ea} = P_{Ee}, \quad PC_{a} = PC_{e}, \quad PW_{a} = PW_{e} \)

Hi: \( P_{Ea} \neq P_{Ee}, \quad PC_{a} \neq PC_{e}, \quad PW_{a} \neq PW_{e} \)

Test Statistics: \[ Z = \frac{(P_{a} - P_{e}) - 0}{\sqrt{\frac{P_{a}(1 - P_{a}) + P_{e}(1 - P_{e})}{n}}} \]

Significance Level: \( \alpha = 0.10 \)

Decision Rule:

If the computed value of the test statistics, \( Z \), is greater than or equal to 1.645 or less than or equal to -1.645, we reject the null hypothesis (about equality, between the actual and the model predicted proportions).

Calculations

<table>
<thead>
<tr>
<th></th>
<th>Eastern</th>
<th>Central</th>
<th>Western</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{a} )</td>
<td>0.340</td>
<td>0.395</td>
<td>0.265</td>
</tr>
<tr>
<td>( P_{e} )</td>
<td>0.238</td>
<td>0.399</td>
<td>0.363</td>
</tr>
<tr>
<td>( n )</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
</tbody>
</table>
\[ Z = \frac{(.340 - .238) - 0}{\sqrt{\frac{.340(.660) + .238(.762)}{50}}} = \frac{.102}{.090} = 1.133 \]

\[ (.395 - .399) - 0 \]
\[ \sqrt{\frac{.395(.605) + .399(.601)}{50}} = \frac{.004}{.098} = -.041 \]

Western
\[ (.265 - .363) - 0 \]
\[ \sqrt{\frac{.265(.735) + .363(.633)}{50}} = \frac{-.098}{.092} = -1.065 \]

Statistical Decision:

Since the calculated values above fall within the non-rejection region (+1.645), we do not reject the null hypothesis at the 0.10 significance level.

At the higher significance level \( \alpha = .05 \), the decision rule changes to greater than or equal to +1.96 or less than or equal to -1.96. The decision above is thus upheld even at this significance level.

Administrative Decision:

Based on the data above, we conclude that the two proportions may be equal. These data do not allow us to accept the alternative hypothesis. The model predictions for 1981 are not so far off from the actual proportions observed, as such the model is acceptable for one transition or for one year predictions.
Hypotheses remain the same as above (for 1981)

Test statistics also remain the same as for 1981.

**Significance Level:** $\alpha = .10$, $\alpha = .05$

**Decision Rule:**
If the computed value of the test statistics, $Z$, is greater than or equal to $+1.645$ or less than or equal to $-1.645$ at the 0.10 significance level then we reject the null hypothesis. If the value of $Z$ is greater than or equal to $+1.96$ or less than or equal to $-1.96$ at the 0.05 significance level, then we reject the null hypothesis at this significance level.

### Calculations

<table>
<thead>
<tr>
<th></th>
<th>Eastern</th>
<th>Central</th>
<th>Western</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pa</td>
<td>.340</td>
<td>.360</td>
<td>.300</td>
</tr>
<tr>
<td>Pe</td>
<td>.218</td>
<td>.384</td>
<td>.398</td>
</tr>
<tr>
<td>$n$</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
</tbody>
</table>

\[
Z = \frac{(\cdot340 - \cdot218) - 0}{\sqrt{\cdot340(\cdot660) + \cdot218(\cdot782)}}
\]

\[
\cdot122 = 1.37
\]

\[
\cdot089
\]

\[
Z = \frac{(\cdot360 - \cdot384) - 0}{\sqrt{\cdot360(\cdot640) + \cdot384(\cdot616)}}
\]

\[
\cdot024 = -\cdot247
\]

\[
\cdot097
\]
\[\sqrt{\frac{0.300(0.700) + 0.398(0.602)}{50}} = -0.098 = -1.032\]

**Statistical decision**

Since the calculated values fall within the non-rejection region (±1.645) at the significance level \(\alpha = 0.10\) and also at the significance level \(\alpha = 0.05\) that is ±1.96, we do not reject the null hypothesis at both levels.

**Administrative Decision**

Based on the data above, we conclude that the two proportions may be equal. The model predictions for 1982 are therefore not so far off from the actual observed proportions, as such the model is acceptable for two year transition or for two year projections.

1983

Hypotheses remain the same as above (1982). Test statistics also remain the same.

Significance Level: \(\alpha = 0.10, \alpha = 0.05\)

Decision rules remain the same as above for \(\alpha = 0.10\) and also for \(\alpha = 0.05\).
### Calculations:

<table>
<thead>
<tr>
<th></th>
<th>Eastern</th>
<th>Central</th>
<th>Western</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_a )</td>
<td>.334</td>
<td>.383</td>
<td>.283</td>
</tr>
<tr>
<td>( P_e )</td>
<td>.209</td>
<td>.374</td>
<td>.417</td>
</tr>
<tr>
<td>( n )</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
</tbody>
</table>

**Eastern**

\[
Z = \frac{(.334-.209) - 0}{\sqrt{.334(.666)+.209(.791)}} = \frac{.125}{.088} = 1.42
\]

**Central**

\[
Z = \frac{(.383-.374) - 0}{\sqrt{.383(.617)+.374(.626)}} = \frac{.009}{.097} = .093
\]

**Western**

\[
Z = \frac{(.283-.417) - 0}{\sqrt{.283(.717)+.417(.583)}} = \frac{-.134}{.094} = -1.426
\]

### Statistical Decision

Since the computed values of \( Z \) fall within the non-rejection ranges (± 1.96), we do not reject the null hypothesis.

### Administrative Decision:

Based on the data above, we conclude that the two proportions may be equal. These data do not allow us to accept the alternative hypothesis. The model predictions for 1983 are not so far off from the actual observation. The model is thus acceptable for three years predictions.
or for three transitions.

Hypotheses remain the same as above.
Test statistics also remain the same.

Significance Level: \( \alpha = .10, \ \alpha = .05 \)

**Decision Rule:**

If the computed values of the test statistics, \( Z \), is greater than or equal to +1.645 or less than or equal to -1.645 at the 0.10 significance level then we reject the null hypothesis. If the value of \( Z \) is greater than or equal to +1.96 or less than or equal to -1.96 at the 0.05 significance level, then we reject the null hypothesis at this level of significance.

**Calculations**

<table>
<thead>
<tr>
<th></th>
<th>Eastern</th>
<th>Central</th>
<th>Western</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pa</td>
<td>.323</td>
<td>.391</td>
<td>.286</td>
</tr>
<tr>
<td>Pe</td>
<td>.204</td>
<td>.369</td>
<td>.427</td>
</tr>
<tr>
<td>n</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
</tbody>
</table>

\[
Z = \frac{(.323 - .204) - 0}{\sqrt{.323(.677) + .204(.796)}} = \frac{.119 = .368}{.087} = 1.368
\]

\[
(\frac{.391 - .369) - 0}{\sqrt{.391(.609) + .369(.631)}} = \frac{.022 = .227}{.097}
\]
Western
\[
\frac{(0.286 - 0.427) - 0}{\sqrt{\frac{0.286(0.714) + 0.427(0.573)}{50}} = -1.141 = -1.484}
\]

**Statistical Decision**

Since the calculated values of \( Z \), fall within the relevant ranges (± 1.645 and ± 1.96), we do not reject the null hypothesis at 0.10 level of significance and at 0.05 level of significance respectively.

**Administrative Decision**

From the above data the model is acceptable for the prediction of the fourth year or up to the fourth transition, because the two proportions (actual and predicted) may not be different.

3.3.2.5 **Conclusion**

The three models built in this section for the total enrolment, the boys enrolment and the girls enrolment are validated above. The validation showed that for the latter two, the model may be used to predict up to the fourth year from the prediction point and there would be no statistical difference between the actual and the estimated proportions. For the total enrolment, however, the model can only be used to predict up to the third year from the prediction point.
without any statistical difference between the actual and the estimated difference.

It is necessary to note that the tests performed on the model are only statistical and hence have the inherent statistical tests assumptions and limitations.
SECTION FOUR

RESULTS OF THE APPLICATION OF THE MODEL

The Model developed and validated in the last section is used with the demand proportions for 1989 to predict demand proportions for 1990, 1991 and 1992.

4.1 TOTAL STANDARD ONE DEMAND

<table>
<thead>
<tr>
<th>Demand proportions for 1989:</th>
<th>Transition Matrix</th>
<th>Forecasted Demand Proportions for 1990</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eastern Central Western</td>
<td>T</td>
<td>Eastern Central Western</td>
</tr>
<tr>
<td>(.302 .387 .311)</td>
<td>.52 .37 .11</td>
<td>(.235 .491 .274)</td>
</tr>
<tr>
<td>Forecasted Proportions for 1990:</td>
<td>T</td>
<td>Forecasted Demand Proportions for 1991</td>
</tr>
<tr>
<td>(.235 .491 .274)</td>
<td>.52 .37 .11</td>
<td>(.209 .538 .253)</td>
</tr>
<tr>
<td>Forecasted Proportions for 1991:</td>
<td>T</td>
<td>Forecasted Demand Proportions for 1992</td>
</tr>
<tr>
<td>(.209 .538 .235)</td>
<td>.52 .37 .11</td>
<td>(.199 .560 .241)</td>
</tr>
</tbody>
</table>

The results of the Model application shows that in 1990 24% of the total standard one demand in Nakuru municipality will be in the Eastern Zone. In the same period, 49% will be in the Central Zone while 27% will be in the Western Zone.
In 1991, 21% of the standard one demand will be in the Eastern zone, 54% in the Central zone and 25% in the Western zone.

In 1992, only about 20% of the demand will be in the Eastern zone, 56% in the Central zone while the Western zone will have 24% of the demand.

4.2 THE BOYS DEMAND FOR STANDARD ONE

<table>
<thead>
<tr>
<th>Demand Proportions for 1989</th>
<th>Transition Matrix</th>
<th>Forecasted Demand Proportions for 1990</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eastern Central Western</td>
<td>( T_b )</td>
<td>Eastern Central Western</td>
</tr>
<tr>
<td>(.323 .340 .337)</td>
<td>( \begin{bmatrix} 0.690 &amp; 0.174 &amp; 0.136 \ 0.220 &amp; 0.568 &amp; 0.212 \ 0.279 &amp; 0.249 &amp; 0.472 \end{bmatrix} )</td>
<td>(.392 .333 .275)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Forecasted Proportions for 1990</th>
<th>( T_b )</th>
<th>Forecasted Demand Proportions for 1991</th>
</tr>
</thead>
<tbody>
<tr>
<td>E C W</td>
<td>(.392 .333 .275)</td>
<td>E C W</td>
</tr>
<tr>
<td>(.690 .174 .136)</td>
<td>(.220 .568 .212)</td>
<td>(.420 .326 .254)</td>
</tr>
<tr>
<td>(.279 .249 .472)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Forecasted Proportions for 1991</th>
<th>( T_b )</th>
<th>Forecasted Demand Proportions for 1992</th>
</tr>
</thead>
<tbody>
<tr>
<td>E C W</td>
<td>(.420 .326 .254)</td>
<td>E C W</td>
</tr>
<tr>
<td>(.690 .174 .136)</td>
<td>(.220 .568 .212)</td>
<td>(.432 .322 .246)</td>
</tr>
<tr>
<td>(.279 .249 .472)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The results of the model application shows that in 1990, 39% of the boys will demand to be enrolled in Eastern zone, 33% in the Central zone and about 28% in the Western zone. In 1991, 42% of the boys will demand to be enrolled in the Eastern zone, 33% in the Central zone and 25% in the Western zone. In 1992, 43% of the boys will demand to be enrolled in the Eastern zone, 32% in the Central zone and 25% in the Western zone.
4.3 THE GIRLS DEMAND FOR STANDARD ONE

Demand Proportions for 1989

Transition Matrix

Forecasted Demand Proportions for 1990:

Eastern Central Western

\[
\begin{bmatrix}
.566 & .406 & .028 \\
.110 & .547 & .343 \\
.108 & .190 & .702
\end{bmatrix}
\]

Eastern Central Western

\[
\begin{bmatrix}
.239 & .404 & .357 \\
.218 & .386 & .396 \\
.209 & .375 & .416
\end{bmatrix}
\]

The results of the model application shows that in 1990, 24% of the girls will demand to be enrolled in the Eastern zone, 40% in the Central zone and 36% in the Western zone. In 1991, 22% will demand to be enrolled in the Eastern zone, 39% in the Central zone and 39% in the Western zone. In 1992, 21% will demand to be enrolled in the Eastern Zone, 37% in the Central zone and about 42% in the Western zone.
# SUMMARY OF RESULTS

## Table 4.1: Total Demand for Standard One

<table>
<thead>
<tr>
<th>Zone</th>
<th>Expected Demand in 1990</th>
<th>1991</th>
<th>1992</th>
<th>Expected long run Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eastern</td>
<td>24%</td>
<td>21%</td>
<td>20%</td>
<td>9%</td>
</tr>
<tr>
<td>Central</td>
<td>49%</td>
<td>54%</td>
<td>56%</td>
<td>59%</td>
</tr>
<tr>
<td>Western</td>
<td>27%</td>
<td>25%</td>
<td>24%</td>
<td>32%</td>
</tr>
<tr>
<td>Total</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
</tbody>
</table>

## Table 4.2: Boys Demand for Standard One

<table>
<thead>
<tr>
<th>Zone</th>
<th>Expected Demand in 1990</th>
<th>1991</th>
<th>1992</th>
<th>Expected long run Demand</th>
</tr>
</thead>
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## Table 4.3: Girls Demand for Standard One

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<th>1991</th>
<th>1992</th>
<th>Expected long run Demand</th>
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<tbody>
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<td>Western</td>
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SECTION FIVE

SUMMARY, RECOMMENDATIONS AND CONCLUSION

5.1 SUMMARY

The study set out to represent the standard one enrolment in Nakuru Municipality as a Markovian process and to use the Markov chain process to forecast the demand for standard one for the years: 1990, 1991 and 1992. The forecasts are needed for municipal school administration and planning purposes.

By treating the movement between the different zones of the municipality as transitions, a Markovian model of the system was developed which was used to estimate the future enrolment proportions within the zones. The results of the study show that for 1990, about a half of the total standard one demand will be in the Central zone, while the Eastern and the Western zones shall share the rest, with the Western zone demand slightly higher than the eastern zone demand. More or less the same conditions shall prevail in the 1991 and 1992, with each successive year's demand rising in the central zone while the demand for the other zones fall accordingly. In the long run, when the system will have reached a stable state, about 60% of standard one demand will be for the central zone, 10% for the eastern zone and 30% for the western zone.
In 1990, about 39% of the boys will demand to be enrolled in the Eastern Zone, 33% in central zone and 28% in the Western zone. These proportions will increase to 42%, remain the same at 33% and increase to 25% in 1991 for the eastern, central and western zone respectively. In 1992, the demand of the boys for the eastern zone will be 43%, 32% for the central zone and 25% for the western zone. Most of the boys demand for standard one will therefore be concentrated in the eastern zone followed by the central zone; which would demand about a third of such places. In the long run, when the system will have reached a stable state, about a half of the boys demand would be for the eastern zone, a third for the central zone and the rest for the western zone.

The demand of the girls for standard one will be 40% in the central zone, 24% in the eastern zone and 36% in the Western zone for 1990. The proportions will change by 1991, so that about 39% of the girls will prefer the western zone, while 39% will prefer the central zone and 22% the eastern zone. By 1992, the western zone percentage will have increased by 42% while the central zone will have dropped to 37% and eastern to 21%. In the long run therefore, when the system will have reached a stable state, most of the girls (61%) or about two-thirds (2/3) of the girls would demand to be enrolled in standard one in the western zone; followed by the central zone which would be demanded by about 20% or a fifth of the girls.
An interesting observation is that while most of the total demand for standard one places will be concentrated in the central zone, the boys demand for standard one places will be concentrated mainly in the eastern zone while the girls demand will be concentrated mainly in the western zone. The available information does not seem to explain this phenomenon, consequently further research to find why this phenomenon occurs is recommended.
5.2 RECOMMENDATIONS

From the above observations, most of standard one demand will be for the central zone. It has to be noted that the central zone includes the town centre and the busiest parts of Nakuru town. With the observation above, the Municipality has to act fast to be able to meet the standard one demand for this region. One of the possible ways of doing this is to expand the existing schools in this area by, say, erecting storied classrooms, removing structures near such institutions to create room for sportsground and such type of actions that would ensure that adequate resources, about a half needed for standard one, are concentrated in the central zone.

The administrative effort of the Municipal council of Nakuru education department should be concentrated in the central zone, as this is where demand for standard one and possibly the other classes is greatest. Other resources like time, personnel, equipment and money should be apportioned such that the central zone receives more than the other two zones.

For the short planning period (1990-1992), about 50% of the resources should be in the central zone, 30% in the western zone and 20% in the Eastern zone. These ratios should be changed in the long-run to about 60% in the Central zone, 30% in the western zone and 10% in the Eastern zone.
One of the possible reasons why the demand for standard one is concentrated near the town centre is that most offices and workplaces are in the town centre itself, parents therefore find it convenient to drop their young children (six or seven year olds) to schools near their offices. The Municipality can discourage this indirectly by allocating no more land in the town centre for offices, so that the town expands outwards. The Municipal council can also charge higher rates and rents for such areas than the other areas and discourage the coming up of new office structures. Some residential estates within the central zone like the Langalanga area are densely populated than most residential estates within the Western and the Eastern zones. To correct this imbalance, as far as standard one enrolment is concerned, the Municipal can provide safe school transport from such estates to the eastern and western zones, so that fear of transport problems does not force parents to enrol their young children only in schools near their homes.

The recommendations above can be useful both for the short planning period (as the one used in forecasting with the above Model) or the long planning period. A simple definition of planning is: "deciding in advance what to do, when to do it, how to do it, where to do it and who to do it". Taking this simple definition in relation to the Municipal education department, the "what" may be known, say expand
classrooms for standard one, when?, say for the coming three years or five years planning period, how? say by allocating a certain proportion of their budget to this purpose and by who?, say, calling tenders from various constructors and whoever qualifies to get the tender. A difficult question to answer, usually, is, where? This is not accidental but is due to the fact that the Municipality is so large and each area or even each school needs more resources. Usually each school sends requests for what it needs. When going through the Municipal records, one finds so many requests for each month, some of which are repeated month after month, implying that they are never answered in time (if at all). One therefore realises they are dealing with a typical situation of limited resources. Like most other towns, Nakuru faces the problem of allocation of limited resources among the rivaling needs of its residence, like education, health, security and housing. The Model above can help the Municipality in answering this universal economic question of resource allocation. By indicating the proportions likely to be enroled in the different areas, the Municipality can apply similar proportions to allocate the resources.

The long run (steady state) proportions show what the position would be in a very long time to come, if the system is not interfered with. It shows a possible gross imbalance in the demand
for standard one within the different zones. This knowledge can help the Municipality to look for ways in which to redirect this imbalance before it actually occurs. New attractions may be set up in the Eastern zone for instance to encourage people to move into this area. Such actions may help spread out of people more uniformly and thus spread out development in the long run. By trying to identify reasons why the demand for standard one would shift to the central zone, the Municipality can then make the Eastern and western zones also attractive to lure demand into these areas.

The Model can also be used to allocate more of those facilities peculiar to girls in the Western zone and more of those peculiar to boys in the Eastern zone, thus ensuring that they are allocated where the need is likely to be felt most.

LIMITATIONS OF THE STUDY

The study was limited to standard one enrolment and the results may not apply for the other classes. Certain data for this study were not easy to obtain; indeed the demand for standard one in the Municipality has never been recorded in the past. The only data kept from 1969 to 1988 were the actual enrolment within the different zones. The model was therefore built with the actual enrolment rather than the demand. This limits the study to the extent that
the demand is assumed to behave in a similar way to the actual enrolment.

Time allocated to the study was too short to allow for more intensive analysis of the data, and this contrained the study to some extent.

SUGGESTIONS FOR FURTHER RESEARCH

Any further research could address itself to:

(a) Finding out why the standard one girls and boys in Nakuru Municipality tend to demand the Western and Eastern zones respectively.

(b) Finding out how the Municipality can ease the unequal demand facing the different schools within the same zones.

(c) Similar studies for the other primary school classes within the Municipality and even for the other urban areas like Nairobi.

(d) The whole of education system within a Municipality for instance, by taking enrolment within a particular year, say, 1980, until they graduate in 1988, to find out:

(i) actual movements from zone to zone

(ii) dropout ratios among the girls, boys and the two combined.

(iii) staying ratios, that is the probability that a pupil will graduate from the system and such other enrolment related factors.
### APPENDIX 1

#### Schools in the different zones

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<thead>
<tr>
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<th>Central</th>
<th>Western</th>
</tr>
</thead>
<tbody>
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<td>Baharini</td>
<td>Freehold</td>
</tr>
<tr>
<td>Crater</td>
<td>Flamingo</td>
<td>Harambee Khalsa</td>
</tr>
<tr>
<td>Gilani*</td>
<td>Kaloleni*</td>
<td>Heshima</td>
</tr>
<tr>
<td>Hyrax</td>
<td>Kimathi</td>
<td>Hill special*</td>
</tr>
<tr>
<td>Jamhuri</td>
<td>Langalanga</td>
<td>Kaptembwa</td>
</tr>
<tr>
<td>Kisulisuli</td>
<td>Moi</td>
<td>Kariba Road*</td>
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<td>Ngala special*</td>
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* Gilani was closed down in 1984 because the area where it was situated was not suitable.

* Lions is a school run by the Lion's club of Nakuru and does not fall under the Municipal Council administration directly.

* Kaloleni was started in 1989

* Prisons was started in 1989

* Ngala is a special school for the deaf

* Hill (or Nakuru Hill) is a special school for the mentally handicaped.

* Kariba Road was started in 1989
### APPENDIX 2

Standard one enrolment between 1969 to 1988

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B - Boys  
G - Girls  
T - Totals
### APPENDIX 3

Proportion (of the Totals) enrolled within the zones from 1969 to 1988

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APPENDIX 4

Proportion of the boys enroled within
the zones from 1969 to 1988

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## APPENDIX 5

Proportion of the girls enroled within the zones from 1969 to 1988

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Rejection Region at the 90% confidence level.

Rejection Region at the 95% confidence level.
APPENDIX 7

Demand for standard one in the different zones in 1989

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FOOTNOTES

SECTION ONE


6. See footnote no.4

7. 
SECTION TWO


Bunn, D.W.: "A Bayesian Approach to the Linea Combination of forecasts". Operational Research Quarterly 26, pp 325-329, 1975


Magee, J.F. "The Effect of Promotional Effort on Sales", JORS, Vol.1 pp 64-74 February, 1953


