# NURSE SCHEDULING BY SHIFT: <br> THE CASE OF INPATIENT CARE IN THE MEDICINE UNIT AT KENYATTA NATIONAL HOSPITAL 

## BY

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This management paper is my original work and has not been presented for a degree in any other university in Kenya

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## DEDICATION

To God, All the Glory.
To my late father, Paul Mutua for encouraging me while young and to my mother for her patience, understanding and perseverance.
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## ABSTRACT

Literature indicates that the use of Management Science techniques in health service management is less frequent than expected. Yet, health service management is faced with many challenging problems such as allocating scarce health personnel efficiently and effectively. Management Science techniques can play a major role in developing solutions to these problems.

This study presents the results of a shift assignment problem using one of the Management Science techniques-goal programming- to assign nurses to shifts in ward 28 of the medicine unit of Kenyatta National Hospital(KNH). Goal programming application is suitable for situations where there are multiple objectives or goals to be achieved which many times are conflicting. Goal programming output provides a satisficing solution.

In this study, four goals were considered in assigning the nurses. The goals were prioritized as follows:

1) A nurse can only be on duty on only one shift a day;
2) A required number of nurses should be on duty per shift each day.
3) A Registered Nurse (RN) should be on duty in shift A (from 7.30a.m. to 4.30p.m.)
4) Individual nurse preferences for a shift assignment should be considered.

The fourth goal will usually conflict with the hospital policies and the results indicated that while the other three goals were fully met, this one was partially met on Mondays, Saturdays and Sundays. Also, a high percentage of nurses (approximately 67\%) were assigned at least a shift they prefered. As a result of participating in the scheduling decision and the assignment being fair, nurses are likely to be more motivated and therefore provide better patient care. This will benefit both the hospital and the patient.

## SECTION ONE

### 1.0 INTRODUCTION

### 1.1 Background

Provision of adequate health services is a major objective for a developing nation. The importance given to this sector of development may be gauged from the amount of public funds set aside annually for health care. For example, total Kenya government expenditure is expected to rise by $79.4 \%$ between $1987 / 88$ and 1990/91. Major shares in the total are accounted for by health among others [1]. Table 1 below gives details of expenditure incurred by the government on various services.

Table 1
Central Government Expenditure on Main Services 1990/91


* Provisional

Source: Adapted from Economic Survey. 1991: 68.
Table 1 indicates that the government spends $4.43 \%$ of its recurrent account on health and $5.42 \%$ of its development account on the same. In total, health accounts for $4.68 \%$ of the total
government service's expenditure, being the fourth in the expenditure hierarchy in 1990/91.

The health of a population is affected by many factors of which medical services form an important component. Good health at the individual , community and national level is a prerequisite for productivity and creativity. (Kanani et al, 1984). Maintaining high standards of health care require efficient and effective health delivery systems. Kenyatta National Hospital is an important subsystem in the Kenya's health delivery system. Deficiencies in Kenya's health services have been a widely debated issue in our local press.[2] Problems identified include drug shortages , poor maintenance of hospital equipment and facilities, patient-medical personnel relationship, long queues and so on. Many of the problems are management related and Management Science can play a big role in helping solve some of these problems.

Health personnel involved in delivery of health services include physicians, nurses, dentists, pharmacists, sanitary engineers and a host of various categories of paramedical staff. Among these, physicians and nurses carry out important leadership and management fuctions and decision making roles in delivery of medical services. Table 2 details the number of registered medical personnel including those in training institution such as Kenya Medical Training colleges and the College of Health Sciences of the University of Nairobi.

Table 2 Registered Medical Personnel 1989-1990

|  | 1989 |  | 1990 |  | IN TRAINING |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | No. per |  | No. per | \| 1989-90 | \| 1990-91 |
|  | \| Number |1 | 100,000 | \| Number | 100,000 | \| Number | \| Number |
| Type of Personnel |  | Population |  | Population |  |  |
| Doctors | 3,266\| | 14 | 3,357\| | 13.8 | 913 | 1,072 |
| Dentists | 561\| | 2 | 596\| | 2.4 | 131 | 150 |
| Pharmacists | 413\| | 2 | 4431 | 1.8 | 194 | 234 |
| Pharmaceutical |  |  |  |  |  |  |
| Technologists | 5591 | 2 | 6041 | 2.5 | 147 | 145 |
| Registered Nurses | 4,712\| | 24 | 5.441 | 22.4 | 1,169 | 1,174 |
| Enrolled Nurses | \| 15,200| | 65 | \|17,734| | 72.4 | 7,604 | 7,639 |
| Clinical Officers | 2,534\| | 11 | 2,630\| | 10.8 | 470 | 530 |
| Public Health Officers | 5501 | 2 | 5851 | 2.4 | 124 | 122 |
| Public health |  |  | 1 \| |  |  |  |
| Technicians | 2,3931 | 10 | 2,528 | 10.4 | 668 | 670 |

* Provisional

Source: Economic Survey, 1991.
An increase is observed in all cadre of personnel trained between 1989 and 1990. This increase in the number of doctors and nurses is also attributable to establishment of a medical school at Moi University and a school of Nursing in Kabarnet. Enrollment of doctors, pharmacists and clinical officers in training institutions grew by over $10 \%[3]$. This is encouraging taking into account the rate of population growth.

Nurses constitute the largest single group of health professionals. This is because nursing service is a most, if not the most critical component in fulfilling hospital objectives for patient care.

### 1.2 Nursing and Health Care

Nursing is a valued service in health care. Its special concern is the individual's need for self-care action and the provision and management of it in a continous basis in order to sustain life and health, recover from diseases or injury and cope with their effects. Therefore, the physical and mental health of individuals depends in part on the efficiency and effectiveness
of the nursing service. Nurses are the only professional workers providing a continuous and direct caring services in hospitals.

Care is a concept implying a measure of constancy and continuity. These two aspects, reinforced by communication, coordination, explanation, education and empathy, are some of the main components in the nurse's contribution to care. (Hockey,1976)

However, nurses are limited by external and internal factors in what they can do to meet nursing requirements. Limiting factors include; time, prevailing conditions (since health itself is a subjective value-laden concept liable to change over time and relative not only to the individual but to the value system of the society), interests, values, and abilities of the nurses. Effective organisation of nursing services is essential both in provision of nursing and the maintenance of satisfaction among nurses.

Nurses have a responsibility for the quality of the services they provide. Therefore, there is need to ensure their job satisfaction, motivation and retention. Job satisfaction in this case has been defined as the extent to which an employee expresses a positive orientation toward a job. (Hinnings et al, 1980)

One of the most pervasive problems facing hospital
administrators is nurse staffing or scheduling[4]. This includes the size, mix, utilization and practice profile of various cadre of nursing personnel. Assigning nurses to specific shifts can become a very time-consuming and difficult task for the nurse in charge particularly to meet the ever-increasing patient demand and to cater for individual nurse's preferences on shifts.

Kenyatta National Hospital (KNH)
Kenyatta National Hospital is the largest government hospital. It is a 1900-bed hospital built in several phases between 1901 and 1981.

The functions of KNH are:
a) To receive patients for health care,
b) As the national referral hospital to participate in national health planning,
c) As a teaching hospital for the university of Nairobi to provide facilities for medical education and research either directly or indirectly through other co-operating health institutions,
d) To provide facilities for nursing and other paramedical education and training,
e) To assist in the prevention and promotion of health care in Kenya.

Due to the increased cost of providing health care, KNH has recently stopped providing free medical services and has introduced a nominal fee charged to all consumers not working in the civil service. It is also undergoing a reorganisation plan with the help of the World Bank in order to meet the proposed changes in the running of the organisation. For example, KNH is to receive Ksh 700 million from the World Bank to help fund a government programme to raise health care standards at the hospital and to restore "its reputation as the biggest and best in east and central Africa"[5].

Appendix 1 shows the proposed organisational structure for KNH with emphasis on clinical administration and the existing structure for nursing.

### 1.3 Scheduling of Nurses

Scheduling is generally defined as the allocation of resources over time to perform a collection of tasks. (Grant, 1986: 49 ). This includes both material and human resources.

The nursing service of a hospital has responsibilities for quality delivery of patient care and accounts for a major proportion of hospital expenditure. In order for nurses to provide proper patient care, many complex decisions are to be made. There must be adequate coverage for each shift, sufficient time off between shifts, hospital policies must be observed and within the context of shortage of nursing personnel, try to consider the individual nurse's wants and needs so as to maintain job satisfaction and the necessary staffing level. Therefore, the problem of scheduling nurses is not an easy one and has been defined as:
> "How to generate a configuration of nurse schedules fulfilling the hospital staffing requirements while simultaneously satisfying the individual nurse's preferences for various scheduling pattern characteristics." [6]

The scheduling decision process must resolve the conflicting viewpoints of the hospital on the one hand, and nursing personnel on the other. Apart from providing proper patient care, the hospital has other interests in scheduling decisions such as minimizing overtime, maintaining continuity of care, maintaining morale and minimizing turnover and absenteeism.

Warner (1976) identified some characteristics of scheduling systems as being important. These are summarised in Table 3 below:

| Characteristic | Explanation |
| :---: | :---: |
| Coverage | The number of nurses (by skill class) assigned to be on duty in relation to to some minimum requirements |
| Quality | A measure of a schedule's desirability as judged by the nurse who will have to work it. This measure includes weekends off, work stretches, single days on, split days off, and certain rotation patterns in addition to how a schedule conforms to her requests for days off for a particular scheduling period. |
| Stability | A measure of the extent to which nurses know their future days off and on duty and the extent to which they feel that their schedules are generated consistent with a set of stable policies ( e.g weekend policy, rotation policy). |
| Flexibility | The ability of a scheduling system to handle changes, such as rotation to working only one shift, and special requirements- requests for day off, leaves of absence etc. |
| Fairness | A measure of the extent to which each nurse perceives that she exerts the same amount of influence upon the scheduting system as other nurses. |
| $\operatorname{cost}$ | The resources consumed in making the scheduling decision. |

The scheduling decision can be confined to the nursing personnel of a single unit since, unlike the allocation decision, there are few interdependencies among units. This is why this study was designed to meet most of the scheduling characteristics mentioned above for the scheduling decision of the medicine unit at KNH.

### 1.4 The statement of the Problem

## The Medicine Unit at KNH

The unit deals with examination and treatment of internal ailments such as diseases of the stomach, kidneys, heart, metabolic, heamatological hormonal disorders, etc for patients
aged over thirteen years. It consists of 10 wards under a matron whose work is administrative in nature. She also takes ward rounds to ensure that patients are receiving the required care. Each ward has a ward in charge and her deputy, both of who are registered nurses. Their work is to assign nurses to shifts, make sure that all nursing activities are carried out, doctors have come for rounds and that all supplies needed are ordered and delivered. In general, they are ward managers and report directly to the matron. Registered nurses deal with general nursing management whereas enrolled nurses are responsible for practical bed-side nursing.

One of the most important decision situations for the nurse in-charge is assigning nursing personnel to shifts in order to meet the ever-increasing demand of patient care in her ward. This can be very time-consuming. The unit is allocated 14 to 16 nurses to be scheduled within 4 shifts every week. The four shift are arranged such that some nurses will work from $7.30 \mathrm{a} . \mathrm{m}$ to $4.30 \mathrm{p} . \mathrm{m}$, others from $7.30 \mathrm{a} . \mathrm{m}$ to $1.00 \mathrm{p} . \mathrm{m}$, others will come in at $1.00 \mathrm{p} . \mathrm{m}$ and leave at $8.00 \mathrm{p} . \mathrm{m}$ and then others take over from $8.00 \mathrm{p} . \mathrm{m}$ to $8.00 \mathrm{a} . \mathrm{m}$. In addition, other factors as presented in Figure 1 can increase the size and complexity of the scheduling problem. Fortunately, much of the scheduling problem is simplified to some extent by the hospital staffing policy. Yet for the nurse in-charge, the remaining or residual nurse assignments might require several hours of tedious "pencil and paper" allocation effort.

Another consideration in the assignment process, and perhaps one of the most important, is the nurses' personal preferences on shifts and off-duty days. The present manual system used cannot cater for the personal preferences of all the nurses at the same time, in an objective manner.

The nurse in-charge of assigning has to ensure each week that each nurse is fairly assigned her duties and day offs. Balancing all these requirements by pencil and paper is not easy due to the many conflicting objectives that have to be met. An efficient and more reliable method is therefore required.

Figure $1 \quad$| Factors that increase and decrease the size and |
| :--- |
| complexity of the nurse assignment problem |



Factors that decrease assignment problem size and complexity
 preference

Residual assignment problem

Unassigned nurses on shift
1.5 objective of Study

The objective of this study is to establish an effective and efficient method for weekly scheduling of nurses by shift in the medicine unit of KNH in order to
(1) - meet staff preference on shifts
(2) - meet minimum requirement on hospital working policies in terms of working hours and type of staff.

Due to the multi-objective nature of the problem and the conflicting goals that are exhibited, goal programming will be used. The objective is to obtain satisfactory levels of goal attainment that represents the best possible combinations of goal achievements.

### 1.6 Importance and Benefits of the Study

(1) The study will be of value to hospital administrators. Its application can help improve the management efficiency and increase employee satisfaction and consequently the quality of health care.
(2) The study can also serve as a prototype for personnel scheduling in other units of the hospital.

## SECTION TWO

### 2.0 LITERATURE REVIEW

Management science models have been widely utilized in helping administrators or managers in making objective decisions in various fields such as government, defence, health and education.

The application of management science techniques to health care systems has increased substantially in recent years. (Taylor III, B. W. et al, 1980) This is attributed to a desire on the part of health administrators to increase the operational efficiency of health facilities, in terms of reduced costs and improved services. It is also a result of perceived opportunities on the part of the researchers to apply management science techniques in a beneficial manner to practical problems.

In service oriented operations where demand fluctuates,e.g nursing care, problems of staff requirements and allocation arises Further, staffing and scheduling decisions are complicated by varying worker requirements and assorted work rules must be made frequently as they operate seven days a week.

Several management science models have been used to deal with such problems as will be seen in the literature reviewed below.

### 2.1 Management Science in Health Services

Health care is a very important sector in the country. Modern industrial methods in quality and productivity could have great impact in health care sector, but such methods are least known in this sector. [7]

Improving quality and availability of care and containing health care costs are two of the most pressing problems in health care. However, a panel of experienced analysts in health
care recently agreed, " The health care industry is further back than manufacturing was in the 1920s " [8] in adopting and utilizing modern quality and productivity measurement and management.

William Pierskalla, former ORSA president lists scheduling, forecasting , market research analysis, staffing and facility expansion and contraction, disease diagonistics and error checking, and service system design as the high impact areas for Operations Research or Management science in health care. He noted that congestion is associated with quality problems and that it is a good opportunity for Management Science to help. (Samuelson , 1991: 34 )

In a sector as important and as rapidly growing as health care , there are many opportunities for Management Science analysts to contribute in a beneficial manner. That is, the health administrators are liable to benefit from application of management science techniques by increasing the operational efficiency of health care facilities in terms of reduced costs and improved services.

### 2.2 Studies on Scheduling

Over the years there has been an increasing awareness for the need for better productivity. The productivity problem exists in both manufacturing and service organisations. Infact, the productivity issue is particularly acute in the service sector.

In service industries , attempts have been made to use technology to deliver the service. However, services are still labour intensive due to technology costs to provide certain type of services. With such conditions present, one way of improving productivity is the better management of the workforce, by better scheduling, work station design, motivation and so on.

Several management science models have been used in scheduling of workers or human resource planning in general. Cavalier and Chandra (1986) suggested that mathematical programming models such as linear programming, non-linear programming , integer programming , goal programming and others can be used to solve optimum allocation problems (scheduling ) efficiently.

Since the mid 1970s, there has been a number of methods developed to solve the nurse scheduling problem. cyclical approaches have been made by Megeath (1978) , Marchionno (1987) and Rosenbloom and Goertzen (1987). Computational solutions have been presented by Warner (1976) , Miller et al (1976) and Smith and Wiggins (1977). Kostreva and Jennings (1991) presented a computer package using FORTRAN.

Megeath's (1978) method came from a management science class who solved the scheduling problem in an hospital. The class made schedules equal in length to the number of nurses being scheduled. Off-shifts were scheduled first with the remaining filled in with day-shifts.

Machionno (1987) gave a step-by-step guide in developing her cyclical schedules. She explained how to determine the number of necessary personnel and the number of off shifts. She described how to set up the schedules using two week periods, assigning weekends off, then off shifts, days off, and finally day shifts.

Rosenbloom and Goertzen (1987) presented an algorithm with three stages:(1) generate a set of possible schedules;(2) formulate and solve an integer programming problem and (3)implement the solution. The set of schedules are 7 -tuples of $0 s$ and $1 s$ depending on whether the day is off or worked. The 128 possible schedules for a week are then reduced using dynamic elimination which corresponds
to the work restrictions. The resulting integer programming problem representing the work constraints, determines how many nurses should work two schedules successively. The same two week period is then repeated for the schedule.

Warner (1976) used a block pivoting method in his two phase algorithm. Phase 1 searched for feasible schedules, for each unit given the policy decisions of coverage, weekends off, rotation schemes and nurse aversion to certain properties of schedules. The aversion and rotation information defined the objective function which is improved throughout phase 2.

Miller et al (1976) generated schedules using a cyclic coordinate descent method with an objective function that minimized a combination of the hospital staffing costs and individual nurse dissatisfaction costs. Each hospital defined constraints for two classes : constraints to define feasible nurse schedules and non binding constraints which incur a penalty cost in the objective function when violated.

Smith and Wiggins (1977) devised a heuristic approach to the scheduling problem. The heuristic was designed to operate on a non-interactive computer with three phases: (1) summarize weekly staffing status for each unit;(2) generate tentative shift schedules indicating shortages and overages in each unit;(3) and processing manual adjustments and generate final schedules. The heuristic allowed for various staff levels, shifts and days off preferences, and part time employees.

Kostreva and Jennings (1991) developed a computer package. It had two main modules. The survey module was designed to query the nurses and the supervisors to obtain information concerning nurse preferences and unit requirements, and to produce the input file for the scheduler. The scheduler module, created and solved (approximately) an optimization problem and hence
generated the desired shift schedules for the individual nurses.
Other scheduling studies include that of Emmons and Burns (1991) on off day scheduling when considering a workforce with hierarchical worker category. The authors suggest that though integer programs can be used, it is often possible to determine the necessary workforce size directly using simple formulas , and to make a feasible schedule of those workers in one pass. They applied the direct combinatory approach to a problem involving workers of different types , with more qualified staff being able to substitute for the less qualified.

Agnihothri and Taylor (1991) used a queuing model to find the optimal staffing level in a centralized appointment scheduling department in a hospital in order to handle the variation in call arrivals within a day. The problem was solved by just rearranging the work shifts , without adding any staff.

### 2.3 Scheduling and Goal Programming

Goal programming is a useful technique in situations where several objectives are to be met simultaneously in a satisfactory manner. Trivedi (1981: 1019-1034) came up with a mixed integer goal programming model for expense budgeting in a hospital nursing department. The model incorporated several different objectives based upon such considerations as cost containment and providing appropriate nursing hours for delivering quality nursing care.

Also considered were possible trade-offs among full-time, part-time and overtime nurses on weekdays as well as weekends. The budget included vacation, sick leave, holiday and seniority policies of a hospital and various constraints on a hospital nursing services imposed by nursing unions. The results were based on data from a study hospital and indicated that the model
is practical for budgeting in a hospital nursing department.
In another study, Baker et al (1989: 423-432) developed an integer, non-linear goal programming model to allocate emergency medical service (EMS) ambulances to sectors within a county in order to meet a government-mandated response-time criterion. In addition, the model also reflected criteria for budget and work-load. Since ambulance response is best described within the context of a queuing system, several of the model system constraints were based on queuing formulations adapted to a mathematical programming format. The solution results provided ambulance allocations to sectors within the county, the probability of an ambulance exceeding a pre-specified response time, and the utilization factor of an ambulance per sector.

Scheniederjans and Kim ( 1987 ) utilized a goal programming model to optimize departmental preferences in course assignment.

From the literature, several modeling techniques have been utilized in scheduling problems. However, due to the conflicting objectives inherent in many scheduling problems, especially those that consider personal preferences, goal programming has been successfully used.

Goal programming is also an especially useful technique for organisations such as government agencies, hospitals, schools and charitable organisations. (Taylor, 1990). It allows the decision maker to set the priority structure for the goals in the goal programming model formulation and to reformulate the model and systematically compare solutions in terms of their achievement of multiple conflicting objectives.

## SECTION THREE

### 3.0 RESEARCH METHODOLOGY

### 3.1 Data collection

Both primary and secondary data was used in the study. Secondary data was obtained from the matron in charge of the medicine unit. This included the skill levels of the nurse, number of nurses required per shift as per their skills and other hospital policies. Primary data included the individual nurse preferences of shifts for each day of a week obtained from the nurses themselves.

Prior to obtaining data, an approval had to be obtained from the Ethical and Research Committee of KNH. The period between handing in the letter of request and obtaining the approval took approximately two and a half weeks. Appendix IA shows the letter of request and the approval letter.

### 3.2 Modelling Technique

Many public organisations , such as government hospitals , have no profit objective at all but a myriad of social objectives. In other words, they have multiple objectives.

Management models are intended to help the decision maker to derive a better solution to managerial problems. Thus, a growing number of studies attempt to describe the decision making behaviour in organisations. Although some such studies are conceptual in nature, the 'satisficing' approach based on the concept of bounded rationality as suggested by simon (1955) has emerged as a pragmatic methodology of decision making. In this approach, a set of tangible multiple aspiration criteria replaces an abstract, global optimization criterion. Thus, multiple criteria decision making (MCDM) or multiple objective decision making (MODM) has been fairly well established as a practical approach to seek a cognitive ability of the decision maker (Lee et al, 1985). One such approach that exhibits such characteristics is goal programming and was therefore chosen as a suitable technique for this study.

### 3.2.1 Goal Programming

Goal programming is one of the most powerful and widely applied techniques for modeling modern decision making problems , which reflect Simon's (1955) theory of 'satisficing '. It is a relatively new approach which began with the work of Charnes and Cooper (1961) and has been refined and extended by Lee in the 1970s.

It has found application in many real world problems in business firms, government agencies and public institutions characterized by one or more of the following :

- multiple , possibly conflicting goals
- priorities of individuals , groups or governing policy that must be respected ( including those dictated by physical laws )
- hard constraints that must be satisfied in order for 'solution' to be deemed feasible
- soft constraints that must be satisfied to some approximate degree. Such an approximate degree may be expressed in terms of a penalty for deviating from a target value, or in terms of a relational hierarchy and so forth.

Goal programming is very similar to linear programming models. However, its implicit within this system is that the objectives may be incommensurable - they may not be based on the same unit of measure.

Goal programming can be employed for decision problems with a single goal and multiple subgoals, as well as having multiple goals and subgoals. Although it may not be possible to optimize every goal, goal programming attempts to obtain satisfactory levels of goal attainment that represents the best possible combinations of goal achievements.This necessitates the establishment of a weighting
system for the goals such that lower ranked or weighted goals are considered only after higher ranked goals have been satisfied or have reached the point beyond which no further improvement is desirable. These weights can either be cardinal or ordinal in nature.

Since goal programming is a form of a linear programming , goal programming models must be formulated under the same limitations , assumptions and conditions as linear programming. (Ignizio, 1989)

General Goal Programming Model Formulation

Minimize $\quad Z=E P_{i}\left(d_{i}{ }^{-}, d_{i}{ }^{+}\right)$
Subject to :

$$
\begin{equation*}
\mathrm{h}_{\mathrm{i}}\left(\mathrm{x}_{\mathrm{i}}\right)<=\mathrm{b}_{\mathrm{i}} \tag{2}
\end{equation*}
$$

$$
\begin{align*}
& f_{i}\left(x_{i}\right)+d_{i}^{-}-d_{i}^{+}=r_{i}  \tag{3}\\
& x_{i}, d_{i}^{-}, d_{i}^{+}>=0 \tag{4}
\end{align*}
$$

Where :
$-X_{i}$ are the structural variables ( $i=1,2, \ldots . . . . n$ );
$-d_{i}^{-}$and $d_{i}^{+}$are the negative and positive goal
deviation variables respectively;

- $\mathrm{b}_{\mathrm{i}}$ are the right hand side of the rigid constraints ;
- $r_{i}$ are the right hand sides of the soft goals
i.e. the goal aspiration levels );
- relationship (1) is the Archimedian Goal achievement function, a weighted linear function of the unwanted goal deviation variables;
- the relationships in (2) are the rigid constraints ;
- the relationships in (3) are the soft goals ;
- and (4) denotes the necessary non - negativity restrictions on the goal deviation variables and when appropriate, non negativity restrictions on the structural variables.

The key thing is that the goal programming (GP) achievement function consists solely of unwanted goal - deviation variables. This is summarized in Table 6.

```
Table 6 Deviations to be minimized in the GP achievement function
```

| Original goal formulation | GP format | ```Unwanted deviation variable(s) ( i.e. to minimize)``` |
| :---: | :---: | :---: |
| $f(x)<=b$ | $f(x)+d^{-}-d^{+}$ | $d^{+}$ |
| $f(x)>=b$ | $f(x)+d^{-}-d^{+}$ | ${ }^{-}$ |
| $f(x)=b$ | $f(x)+d^{-}-d^{+}$ | $d^{-}+d^{+}$ |

## Goal Programming Solution Procedure

A modified simplex procedure is used to solve goal programming problems. The simplex method is an algorithmic method that employes an interative process of obtaining the optimal solution through progressive operations. The simplex solution procedure for goal programming is very similar to that of linear programming though with some modifications. Thus, the simplex based method for goal programming is often refered to as modified simplex method. Figure 2 gives the flowchart of the solution process of goal programming. In this study the LINDO package was used to solve the GP model.

## SECTION FOUR

### 4.0 MODEL APPLICATION AND FINDINGS

4.1 Goal Programming Model Construction for the Medicine Unit

The model seeks to satisfy the Kenyatta Hospital's medicine unit's goals on the number and type of nurses required, the nursing workload requirements as well as their preferences on shifts.

The definitions of the goal programming model's variables , some constants , constraints and objective function are presented below :

Model Variables

$$
\begin{aligned}
& x_{i j}= \\
& \begin{array}{l}
\text { If } 1 \text {, then the } i^{\text {th }} \text { shift is } \\
\text { assigned to the jth nurse i } \\
\text { If o, then the ith shift is } \\
\text { not assigned to the } j \text { nurse. } \\
\text { n no }
\end{array} \\
& \mathbf{d r}_{\mathbf{u}}{ }^{-}=\text {negative deviation from the shifts assigned the } \\
& \text { same } u^{\text {th }} \text { preference level } \\
& \mathbf{a}^{\mathbf{r}} \mathbf{u}^{+}=\text {positive deviation from the shifts assigned } \\
& \mathbf{d}^{\mathbf{s}}{ }_{\mathbf{i}}{ }^{-}=\text {negative deviation from the } i^{\text {th }} \\
& \text { shift } \\
& \mathbf{d}^{\mathbf{s}}{ }_{\mathbf{i}}{ }^{\mathbf{+}}=\text { positive deviation from the } i^{\text {th }} \\
& \text { shift } \\
& \mathrm{d}^{\mathrm{P}}{ }_{\mathrm{j}}{ }^{+}=\text {positive deviation from overassigning } \\
& \text { (assigning more than one shift per day } \\
& \text { to the jth nurse) } \\
& \mathbf{d}_{\mathbf{i}} \mathbf{f -}^{\mathbf{-}} \text { negative deviation from assigning the required } \\
& \text { number of RNs for shift A. } \\
& \mathbf{d}_{i}{ }^{\mathbf{f}+}=\text { positive deviation from assigning the required } \\
& \text { number of RNs for shift } A \text {. }
\end{aligned}
$$

## Model Constants

$\mathrm{n}=$ total number of nurses to assign to shifts
$\mathrm{m}=$ total number of shifts to be assigned
q = total number of ranks used by nurses to define their shift preferences
$s_{i}=$ number of nurses required for each $i^{\text {th }}$ shift
$r_{\mathbf{u}}=$ number of shifts permitted within the same $u^{t h}$ rank ( within the week)
$\mathbf{w}_{\mathbf{u}}=$ ranked weight given by nurses on the preferences to be assigned to work in a specific shift
$\mathbf{p}_{\mathbf{j}}=$ number of shifts a nurse should work per day.

## Constraints

The first category of constraints represents a set of goals to ensure that a nurse is not assigned more than one shift per day.

$$
E_{i=1} \text { to } m x_{i j}+d_{j} p^{-}-d_{j} p^{+}=p_{j}
$$

$$
\begin{array}{r}
\text { For each } k=1,2, \ldots, 7 \text {. } \\
\text { (for i=1, } 2, \ldots \ldots, m \text { ) }-\cdots-(1)
\end{array}
$$

The second category represents a set of goals that need to be satisfied to ensure that the required number of nurses per shift a assigned per day.

$$
\begin{align*}
& E_{i=1} \text { to } m x_{i j k}+d_{i} s--d_{i}{ }^{s+}=s_{i} \\
& \text { For each } k=1,2, \ldots, 7 . \\
&\text { (for } i=1,2, \ldots \ldots m) \tag{2}
\end{align*}
$$

The third category represents a set of goals to ensure that the required number of RNs per shift are met.

$$
E_{i=1} \text { to } 2 x_{i j k}+d_{i}^{f-}-d_{i}^{f+}=1
$$

```
For each k= 1, 2,...,7. ----(3)
```

Nurses' preferences on shifts are built into the model by the following equation

$$
\begin{aligned}
& E_{i=1} \text { to } m^{x_{i j k}+d_{u}} r^{-}-d_{u}{ }^{r+}= \\
& \text { For each } k=1,2, \ldots 7 \text { and } j=1,2,3,4 . \\
&(f \circ r u=1,2, \ldots . q)--(4)
\end{aligned}
$$

The $r_{u}$ right-hand side values seeks to restrict the assignment of shifts to those chosen by the nurses. This is accomplished by structuring the $q^{\text {th }}$ goal constraints in the order of the nurses' ranking of shift preferences and using a weighted system in the objective function.

## Objective Function

The objective function is

$$
\begin{aligned}
\text { Minimize } z & =P_{1} E_{j=1} \text { to m } d_{j}{ }^{p+}+P_{2} E_{i=1} \text { to } n^{\left(d_{i}{ }^{s-}+d_{i}^{s+}\right)} \\
& +P_{3} E_{i=1} \text { to } n\left(d_{i}{ }^{f-}+d_{i}{ }^{f+}\right)+P_{4} E_{u=1} \text { to } q^{W}{ }_{k}\left(d_{u}^{r-}+d_{u}^{r+}\right)--6-
\end{aligned}
$$

The first priority, $P_{1}$ in equation $\sigma$ is designed to satisfy shift assignment such that a nurse is on duty on only one shift per day. As formulated, the model seeks to minimize overassignment (assigning more than one shift to a nurse is physically impossible).

The second priority, $\mathrm{P}_{2}$ represents the set of required number of nurses to be assigned per day per shift. The model seeks the exact number of nurses because the hospital has fewer number of nurses than desired.

The requirement that at least an $R N$ should be assigned to shift $A$ is placed as a third priority. An $R N$ is required to assist the nurse in charge or her deputy since there is more administrative work during that shift.

The nurse's shift assignment preference goal constraints are placed at the fourth priority, $P_{4}$ to logically position them as subordinate goals to those of the hospital. The number of ranks defines $q$ and $w_{u}$. Each of the $q$ goal constraints includes only
the decision variables that were attached to the specific ranks. The mathematical weights $w_{u}$ used to differentiate the deviational variables in the objective function in equation 6 are the reversed order of the nurses' rankings. For example, the decision variables placed in the goal constraint with a rank of one received the most desirable mathematical weight of $q$ for that goal constraint's deviational variables. The variables with the rank of two would receive a weight of $q-1$ and so on. These are simply the reverse ordering of the four ranks.

The variables in the above objective function reflect the deviations between goals and what actually is achieved within a given set of constraints rather than the decision variables themselves.

### 4.2 Model Input

Each nurse gave her preference for the four shifts namely, A, $B, C$ and $D$ as shown under the data collection form(Appendix II). The nurses were at first reluctant to state their preference because they are not familiar with research studies that require their participation and were thus suspicious that the administration might use the information for some other purposes.

Therefore it took three weeks to obtain the nurse's preferences data. The ward in charge had to convince them that the data was strictly for research purpose. Table 4 indicates the nurses preference rankings (1= most prefered to $4=$ least prefered).


The total number of nurses allocated to this ward is fifteen. Table 5 indicates their number by skill levels.

| Table 5 Nurses number by skill levels |
| :--- |
| Nursing skill levels |
| RNs |
| Total |

Out of the six RNs, two are the ward in-charge and her deputy. One of them has to be on duty on shift A since they play a major role in the ward management. From the remaining $R N s$, at least one RN is required to assist the ward in-charge or her deputy.

For each shift, a certain number of nurses are required. Table 6 illustrates this.
A
B
C
D

Formulation was done on daily basis. The formulation consisted of 52 decision variables and 26 contraints. Formulation details are provided in Appendix III.

In the formulation, the ward in-charge and her deputy were not included due to the fact that it is simpler to decide between themselves on who would work on a certain day. Also, their working hours are fixed-i.e from 7.30 a.m. to $4.30 \mathrm{p} . \mathrm{m}$. Therefore, only 13 nurses (4RNs and 9 ENs) were considered in the formulation. They were numbered 1 to 13 consecutively. (Apendix III)

The formulations were done in such a way that the formulation for the next day depended on the output of the previous day. This was done to avoid "impossible" assignments such as assigning nurse 1 to a night shift the next day after being on duty on shift $A$ the previous day.

The formulations for each day are presented in the Appendixes IV,V, VI, VII, VIII, X and XI for the days Monday to Sunday respectively. As noted from the formulations, the number of nurses required decreased as we move towards sunday. This is because once a nurse has covered more than 42 hours she is dropped out of the formulation to cater for the requirement that a nurse should work a maximum of fourty two hours a week. However some nurses are allowed to work more hours but be compensated on a later date due to the shortage of nursing personnel. Also, as can be seen from the formulations, a nurse who is assigned night
shift on Monday is expected to be on duty on the same shift for the consecutive two or three days. This is a hospital policy requirement.

The LINDO package was used to solve the models as a 0-1 integer problem.

### 4.3 Model Output

In goal programming, instead of trying to maximize or minimize the objective criteria directly as in linear programming, deviations between goals and what can be achieved within a given set of constraints are minimized. Such a structure of the objective function implies that in a goal programming problem, the actual value of an objective function is not of much significance -the important question is to what extent various goals are satisfied under a given set of constraints on the problem. ( Trivedi, 1981)

In our formulations, four priorities were considered for each day in the following order:
$P_{1}$ : Ensure that a nurse is assigned only one shift;
$\mathbf{P}_{2}$ : To meet the required number of nurses per shift ( those available);
$P_{3}$ : To ensure that at least one $R N$ is assigned shift $A$;
$\mathbf{P}_{\mathbf{4}}$ : To meet nurses' preferences on shifts.
Table 8 below shows the solution for this application. The solution assignment is indicated by an asterix next to the nurses' preferences for the shifts.


Days 1 to 7 indicates llonday to Sunday

The summarized goal deviation values are presented by priority level in Table 9.

Table 9 Goal Deviation Values

|  | Priority | Goal | Total Weighted Deviations from Goal | Goal <br> Achievement |
| :---: | :---: | :---: | :---: | :---: |
| Monday |  |  |  |  |
|  | $P_{1}$ | Assigning one shift only | 0 | Full |
|  | $\mathrm{P}_{2}$ | Required number of nurses | 0 | Full |
|  | $P_{3}$ | Required number of RNs in $A$ | 0 | Full |
|  | $P_{4}$ | Nurses' preferences | 4 | Partial |
| Tuesday |  |  |  |  |
|  | $P_{1}$ | Assigning one shift only | 0 | Full |
|  | $\mathrm{P}_{2}$ | Required number of nurses | 0 | Full |
|  | $P_{3}$ | Required number of RNs in $A$ | 0 | Full |
|  | $\mathrm{P}_{4}$ | Nurses' preferences | 0 | Full |
| Wednesday |  |  |  |  |
|  | $P_{1}$ | Assigning one shift only | 0 | Full |
|  | $P_{2}$ | Required number of nurses | 0 | Futl |
|  | $P_{3}$ | Required number of RNs in $A$ | 0 | Full |
|  | $\mathrm{P}_{4}$ | Nurses' preferences | 0 | Full |
| Thursday |  |  |  |  |
|  | $P_{1}$ | Assigning one shift only | 0 | Full |
|  | $P_{2}$ | Required number of nurses | 0 | full |
|  | $P_{3}$ | Required number of RNs in $A$ | 0 | Full |
|  | $P_{4}$ | Nurses' preferences | 0 | Full |
| Friday |  |  |  |  |
|  | $P_{1}$ | Assigning one shift only | 0 | Full |
|  | $P_{2}$ | Required number of nurses | 0 | Full |
|  | $\mathrm{P}_{3}$ | Required number of RNs in $A$ | 0 | Full |
|  | $\mathrm{P}_{4}$ | Nurses' preferences | 0 | Full |
| Saturday |  |  |  |  |
|  | $P_{1}$ | Assigning one shift only | 0 | Full |
|  | $P_{2}$ | Required number of nurses | 0 | Full |
|  | $\mathrm{P}_{3}$ | Required number of RNs in $A$ | 0 | Full |
|  | $P_{4}$ | Nurses' preferences | 7 | Partial |
| Sunday |  |  |  |  |
|  | $P_{1}$ | Assigning one shift only | 0 | Full |
|  | $P_{2}$ | Required number of nurses | 0 | Full |
|  | $\mathrm{P}_{3}$ | Required number of RNs in $A$ | 0 | Full |
|  | $\mathrm{P}_{4}$ | Nurses' preferences | 5 | Partial |

It took roughly about 5 minutes of execution time on a compaq PC for each of the assignment problems. Table 10 below illustrates the number of iterations taken to arrive at an optimal solution for each day's formulation.

Table 10 Total number of iterations required for each assignment problem

| Day | Number of iterations |
| :---: | :---: |
| Monday | 43 |
| Tuesday | 17 |
| Wednesday | 35 |
| Thursday | 32 |
| Friday | 45 |
| Saturday | 16 |
| Sunday | 28 |

### 4.4 Results and Discussion

In the solution, while the first priority $\left(P_{1}\right)$ was fully achieved, nurse number two was not assigned any shift that day and therefore has an off-day on Monday. Table 11 indicates the nurses who were not assigned a shift on a particular day and thus have an off-day.

Table 11 Nurses assigned off-days

| Off-Day | Nurse Number |  |
| :---: | :---: | :---: |
| Monday | 2 |  |
| Tuesday | 7 |  |
| Wednesday | $6 \& 13$ |  |
| Thursday | 9 |  |
| Friday | 10 |  |
| Saturday | $5,8,9 \& 10$ |  |
| Sunday | $5,8,9 \& 10$ |  |

The reason for having not been assigned shifts on days Friday to Sunday was that the nurses would have exceeded the maximum 42 hours of working hours per week as required by the Ministry of Health.

The second priority $\left(\mathrm{P}_{2}\right)$ also was fully achieved and so was the third priority $\left(\mathrm{P}_{3}\right)$. The weighted nurses' preferences $\left(P_{4}\right)$ for shift assignment were minimized on days Tuesday to Friday. Their preferences for shifts were fully met. Only Monday, Saturday and Sunday nurses' shift preferences were not fully met as observed previously in Table 9. This was because the $\mathrm{P}_{4}$ rankings by their very nature are conflicting and thus this goal
will usually not be fully achieved.
The resulting solution on Table 8 indicates that the solution is fair in the light that out of the thirteen nurses, at least twelve have been assigned a shift they mostly desired or desired most of the days -i.e shifts they had given the highest ranks (1 and 2). Table 12 indicates the percentage of nurses who were given preferences 1 or 2 on a given day.

Table 12 Percentage of Nurses Given Preference 1 or 2

| Days | $\%$ of nurses |
| :--- | :--- |
| Monday | 67 |
| Tuesday | 67 |
| Wednesday | 72 |
| Thursday | 67 |
| Friday | 58 |
| Soturday | 55 |
| Sunday | 67 |

In the view of the above, it is more likely that the nurses will be satisfied with the resulting assignment and thus minimize nurses' job dissatifaction. In addition, since data (individual nurse's preferences on shifts) can be collected at least two weeks before and it only took one to two days to come up with the assignment result, the nurse's in charge time in coming up with a nurse assignment list will be substantially reduced.

## SECTION FIVE

### 5.0 DISCUSSION, SUMMARY AND LIMITATION

### 5.1 Discussion

There are many advantages to be gained from using such a technique as goal programming for scheduling nurses. The procedure allows for assignment of nurses by skill to shifts in accordance to some minimum hospital policy. This ensures that the number of nurses required per shift are assigned in order to meet the required workload.

The scheduling technique is desirable in the sense that a nurse is assigned to a shift she prefers. In a similar way, offduty follows from the unassigned shifts and in this case mostly the least prefered shifts.

The schedule program is stable enough. This is because the nurse can know well in advance their off-duty and on-duty. It is possible to generate schedules many weeks in advance.

Changes can easily be incorporated in the scheduling system such as requests for off-days by simply not obtaining or including a nurse's shift preference for that day. The scheduling system can thus be flexible. This is more desirable as it makes it easier for the scheduler to implement changes without having to start the whole process of formulation. Thus time and resources are saved.

The system is fair to the extent that a nurse feels a participant and exerts the same influence upon the scheduling system as other nurses by giving her own shift preferences on which the schedules are based on.

The resources consumed in making the schedule are minimum in the sense that the ward in-charge's time is saved as compared to the manually designed schedules and thus she is able to concentrate on her nursing duties. It also takes a minimum of one
day to come up with the schedule using a microcomputer which is available and a goal programming package which is relatively cheap as it can be used for many other problem solutions.

Thus the scheduling fairness, advanced knowledge of schedules and involvement in decision making process should lead to increased job satisfaction and motivation, and consequently to better patient care.

### 5.2 Summary

The goal programming model presented can be useful in assigning nurses to shifts in any ward in the medicine unit. This can also be applied in any other unit of the hospital or personnel assignment problems.

The modeling enables the user to participate in its initial stages by setting up priorities that helshe would like to meet in order of importance. The priorities set are met in that order in the final result of the model solution. The models dealt with in this study recognized nurse assignment requirements while at the same time included the nurses' preferences on shifts per day. The resulting solution was a more superior assignment to currently used subjective method of "pencil and paper' assignment.

Only thirteen out of the fifteen nurses were incorporated in the models. This made the models sizes smaller and therefore it was possible to solve the formulations using the available package (i.e LINDO). The other two nurses were not incorporated in the model because they work on only shift $A$ and when one is on duty, the other is off duty. Therefore it is simpler for the two to decide on their off-duty days.

The scheduling approach presented in this study is not an assignment problem for the entire hospital, but it is a wardapproach which may be applied throughout the hospital and be individualized for each unit and each ward. To ease acceptance,
the approach allows the individual nurse to become involved in the schedule making process by indicating her preference for a shift. Such individual involvement leads to schedules the nurse prefers. Therefore, the use of goal programming approach in this study may potentially increase motivation, job satisfaction and morale which benefit the nurse, the hospital and ultimately the patient.

Since the basic structure of goal programming model is quite general, it is possible to use the scheduling model in other hospital units such as surgery, radiology etc.

In conclusion, various issues are suggested for successful implementation of the method covered in this study. The method requires that nurses give their shifts preferences well in advance to enable the scheduler to come out with the schedule at least a week before reporting for duty. Therefore this requires their cooperation. One way of encouraging nurses to become cooperative is to encourage nursing research to be carried out often enough by either the nurses themselves or by other researchers interested in the areas of nursing services. Their superiors should teach and advise them on importance of research. Although the method is based on the hospital policies and nurses' preferences, it can allow for incorporating users'judgement. The method can also incorporate other policies that the administration deems important.

It is also possible to perform sensitivity analyses to answer
"what if' questions for management such as changes in staffing levels, nurse mix and any policy change. This requires coorporation understanding and close interaction with the decision maker.

The proposed method requires the use of a Personal computer (PC) and a goal programming package. Due to the unavailability of computer in the unit, this can be done in other departments where there
are PCs as this takes a short time, basically a day to complete the assignment problem. A goal programming package can be bought. Though it may be relatively expensive to have the package, it may be worthw in the long run. This is because goal programming can be used in various fields such as nurse service budgetting (Trivedi, 1981), assigning of ambulances to emergencies etc. Due to the nature of decision making situations in health services -i.e it is characterized with diverse and often conflicting objectives- goal programming may turn out to be a very useful technique to help in decision making. As observed the method is relatively cheap to implement.

### 5.3 Limitations of Study

The major limitation in the study was lack of a goal programming package that could handle at least 420 variables. As observed from the study, we had to formulate on daily basis instead of weekly and therefore some subjectivity was introduced as a result. Also, a priority that was originally intended to be met was left out due to the package limitation. Thus, nurses work load in terms of working hours per week was not inputed as a goal to achieve but as a cumulative sum of nurse hours after assignment.

Therefore, with the availability of a goal programming package that could handle a large data set, a better assignment solution may be obtained. Problem formulation and solution would also take a much shorter period.

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# UNIVERSITY OF NAIROBI <br> FACULTY OF COMMERCE <br> DEPARTMENT OF MANAGEMENT SCIENCE 

P.O. Box 30197

Nairobi, Kenya

```
28th April, 1992.
Chairman,
Research and Ethics Committee,
Kenyatta National Hospital,
Nairobi.
Dear Sir,
This is to certify that miss J. Mutua's research project proposal
"Nurse Scheduling by Shift: The Case of the Medicine Unit at KNH"
has been approved for study.
I will supervise her research project. Please assist her in
obtaining any relevant information.
Thank you.


\section*{UNIVERSITY OF NAIROBI}

\section*{COLLEGE OF HEALTH SCIENCES}

Miss J.M. Mutur, 14th May, 1992. Department of Commerce, Lower Kabete Campus, University of Nairobi.

Re: Research Proposal entitled "Nurse scheduling by shift. The case of in-patient care in the medicine unit at Kenyatta National Hospital" (P250/5/92)

I am pleased to inform you that at its May, 1992 meeting, the KNH-ERC reviewed, discussed and approved your above cited study.

On behalf of the Committee, I wish you best of luck in carrying out your study and look forward to receiving a summary of your research findings and recommendations if any. This information will assist the Committee to build a data base, that can be used as reference in guiding future studies in the KNH to avoid undue duplication of studies.

By a copy of this letter, I am requesting all the relevant sections to give you the necessary support.

Thank you.


Dr. Anastasia N. Guantai
SECRETARY - KNH-ERC
```

cc. Prof. F.E. Onyango - Chairman - KNH-ERC
Dept. of Paediatrics, CHS.
Director - KNH
Danny Fernades - Dept, of Management Science
Faculty of Commerce, UoN.

```


\[
\begin{array}{ll}
\text { N.C.N.O. } & -2 \\
\text { S.A.O. } & -7 \\
\text { N.O.I } & -50 \\
\text { N.O.LI } & -364 \\
\text { N.O.III } & -96 \\
\operatorname{ENLI} & -4 \\
\operatorname{LH} L I & -111 \\
\operatorname{EN} L I I & -307
\end{array}
\]

\section*{APPENDIX II}

\section*{Data Collection Form}

\section*{INDIVIDUAL NURSE PREFERENCE TO SHIFTS}

UNIT: MEDICINE UNIT
Name: \(\qquad\)

Please indicate in the table below the shifts labelled \(A, B\), \(C\), and \(D\) you would prefer to work in. You indicate this by putting down against each shift for each day, numbers 1 to 4, whereby:

1 indicates the shift most prefered,
2 indicates the shift prefered,
3 indicates the shift moderately prefered, and
4 indicates the shift least prefered.
Table to indicate your preferences on shifts

SHIFTS


Thanks for your cooperation.
Mutua J. Mutheu
MBA student,
Department of Management Science, U. O. N.

\section*{APPENDIX III}

For the sake of confidentiality, initials will be used instead of the nurse's full name.

NURSE NUMBERING
\begin{tabular}{cll}
\hline Numbers & Name Initials & Skill Levels \\
\hline 1 & S.W. . & RN \\
2 & E. S. & RN \\
3 & J. B. & RN \\
4 & S. A. & RN \\
5 & A. M. & EN \\
6 & B. K. & EN \\
7 & J. & EN \\
8 & K. & EN \\
9 & OK. & EN \\
10 & Ob. & EN \\
11 & E.S. M. & EN \\
12 & R.O. & EN \\
13 & Obo. & EN
\end{tabular}

\section*{APPENDIX IV}
\(X_{i j}=\) If 1 , then assign the \(i^{\text {th }}\) shift to the \(j^{\text {th }}\) nurse
If 0 , then the \(i^{\text {th }}\) shift is not assigned to the \(j^{\text {th }}\) nurse
\(d_{i}\) are the decision variables.

\section*{Monday Formulation}

\begin{abstract}
MIN D1B+D2b+D3B+D4B+D5B+D6B+D7B+D8B+D9B+D10B+D11B+D12B+D13B+D14A+D14B \(+D 15 A+D 15 B+D 16 A+D 16 B+D 17 A+D 17 B+D 18 A+D 18 B+4 D 19 A+4 D 19 B+3 D 20 A+3 D 20 B\) \(+2 D 21 A+2 D 21 B+D 22 A+D 22 B\)
\end{abstract}

SUBJECT TO
2) \(X 1+X 2+X 3+X 4+D 1 A-D 1 B=1\)
3) \(\mathrm{X} 5+\mathrm{X} 6+\mathrm{X} 7+\mathrm{X} 8+\mathrm{D} 2 \mathrm{~A}-\mathrm{D} 2 \mathrm{~B}=1\)
4) \(\mathrm{X} 9+\mathrm{X} 10+\mathrm{X} 11+\mathrm{X} 12+\mathrm{D} 3 \mathrm{~A}-\mathrm{D} 3 \mathrm{~B}=1\)
5) \(\mathrm{X} 13+\mathrm{X} 14+\mathrm{X} 15+\mathrm{X} 16+\mathrm{D} 4 \mathrm{~A}-\mathrm{D} 4 \mathrm{~B}=1\)
6) \(\mathrm{X} 17+\mathrm{X} 18+\mathrm{X} 19+\mathrm{X} 20+\mathrm{D} 5 \mathrm{~A}-\mathrm{D} 5 \mathrm{~B}=1\)
7) \(\mathrm{X} 21+\mathrm{X} 22+\mathrm{X} 23+\mathrm{X} 24+\mathrm{D} 6 \mathrm{~A}-\mathrm{D} 6 \mathrm{~B}=1\)
8) \(X 25+X 26+X 27+X 28+D 7 A-D 7 B=1\)
9) \(\mathrm{X} 29+\mathrm{X} 30+\mathrm{X} 31+\mathrm{X} 32+\mathrm{D} 8 \mathrm{~A}-\mathrm{D} 8 \mathrm{~B}=1\)
10) \(\mathrm{X} 33+\mathrm{X} 34+\mathrm{X} 35+\mathrm{X} 36+\mathrm{D} 9 \mathrm{~A}-\mathrm{D} 9 \mathrm{~B}=1\)
11) \(\mathrm{X} 37+\mathrm{X} 38+\mathrm{X} 39+\mathrm{X} 40+\mathrm{D} 10 \mathrm{~A}-\mathrm{D} 10 \mathrm{~B}=1\)
12) \(\mathrm{X} 41+\mathrm{X} 42+\mathrm{X} 43+\mathrm{X} 44+\mathrm{D} 11 \mathrm{~A}-\mathrm{D} 11 \mathrm{~B}=1\)
13) \(\mathrm{X} 45+\mathrm{X} 46+\mathrm{X} 47+\mathrm{X} 48+\mathrm{D} 12 \mathrm{~A}-\mathrm{D} 12 \mathrm{~B}=1\)
14) \(\mathrm{X} 49+\mathrm{X} 50+\mathrm{X} 51+\mathrm{X} 52+\mathrm{D} 13 \mathrm{~A}-\mathrm{D} 13 \mathrm{~B}=1\)
15) \(\mathrm{X} 1+\mathrm{X} 5+\mathrm{X} 9+\mathrm{X} 13+\mathrm{X} 17+\mathrm{X} 21+\mathrm{X} 25+\mathrm{X} 29+\mathrm{X} 33+\mathrm{X} 37+\mathrm{X} 41+\mathrm{X} 45+\mathrm{X} 49+\mathrm{D} 14 \mathrm{~A}-\mathrm{D} 14 \mathrm{~B}=5\)
16) \(\mathrm{X} 2+\mathrm{X} 6+\mathrm{X} 10+\mathrm{X} 14+\mathrm{X} 18+\mathrm{X} 22+\mathrm{X} 26+\mathrm{X} 30+\mathrm{X} 34+\mathrm{X} 38+\mathrm{X} 42+\mathrm{X} 46+\mathrm{X} 50+\mathrm{D} 15 \mathrm{~A}-\mathrm{D} 15 \mathrm{~B}=2\)
17) \(\mathrm{X} 3+\mathrm{X} 7+\mathrm{X} 11+\mathrm{X} 15+\mathrm{X} 19+\mathrm{X} 23+\mathrm{X} 27+\mathrm{X} 31+\mathrm{X} 35+\mathrm{X} 39+\mathrm{X} 43+\mathrm{X} 47+\mathrm{X} 51+\mathrm{D} 16 \mathrm{~A}-\mathrm{D} 16 \mathrm{~B}=3\)
18) \(\mathrm{X} 4+\mathrm{X} 8+\mathrm{X} 12+\mathrm{X} 16+\mathrm{X} 20+\mathrm{X} 24+\mathrm{X} 28+\mathrm{X} 32+\mathrm{X} 36+\mathrm{X} 40+\mathrm{X} 44+\mathrm{X} 48+\mathrm{X} 52+\mathrm{D} 17 \mathrm{~A}-\mathrm{D} 17 \mathrm{~B}=2\)
19) \(\mathrm{X} 1+\mathrm{X} 5+\mathrm{X} 9+\mathrm{X} 13+\mathrm{D} 18 \mathrm{~A}-\mathrm{D} 18 \mathrm{~B}=1\)
20) \(\mathrm{X} 2+\mathrm{X} 7+\mathrm{X} 10+\mathrm{X} 15+\mathrm{X} 18+\mathrm{X} 22+\mathrm{X} 25+\mathrm{X} 30+\mathrm{X} 36+\mathrm{X} 38+\mathrm{X} 42+\mathrm{X} 45+\mathrm{X} 50+\mathrm{D} 19 \mathrm{~A}-\mathrm{D} 19 \mathrm{~B}=4\)
21) \(\mathrm{X} 3+\mathrm{X} 5+\mathrm{X} 11+\mathrm{X} 14+\mathrm{X} 19+\mathrm{X} 23+\mathrm{X} 26+\mathrm{X} 31+\mathrm{X} 34+\mathrm{X} 40+\mathrm{X} 41+\mathrm{X} 51+\mathrm{D} 20 \mathrm{~A}-\mathrm{D} 20 \mathrm{~B}=4\)
22) \(\mathrm{X} 1+\mathrm{X} 6+\mathrm{X} 9+\mathrm{X} 16+\mathrm{X} 17+\mathrm{X} 24+\mathrm{X} 27+\mathrm{X} 35+\mathrm{X} 39+\mathrm{X} 43+\mathrm{X} 46+\mathrm{X} 49+\mathrm{D} 21 \mathrm{~A}-\mathrm{D} 21 \mathrm{~B}=4\)
23) \(\mathrm{X} 4+\mathrm{X} 8+\mathrm{X} 12+\mathrm{X} 13+\mathrm{X} 20+\mathrm{X} 21+\mathrm{X} 28+\mathrm{X} 29+\mathrm{X} 32+\mathrm{X} 33+\mathrm{X} 37+\mathrm{X} 44+\mathrm{X} 48+\mathrm{X} 52+\mathrm{D} 22 \mathrm{~A}\) \(-D 22 B=4\)
24) \(\mathrm{D} 1 \mathrm{~B}+\mathrm{D} 2 \mathrm{~B}+\mathrm{D} 3 \mathrm{~B}+\mathrm{D} 4 \mathrm{~B}+\mathrm{D} 5 \mathrm{~B}+\mathrm{D} 6 \mathrm{~B}+\mathrm{D} 7 \mathrm{~B}+\mathrm{D} 8 \mathrm{~B}+\mathrm{D} 9 \mathrm{~B}+\mathrm{D} 10 \mathrm{~B}+\mathrm{D} 11 \mathrm{~B}+\mathrm{D} 12 \mathrm{~B}+\mathrm{D} 13 \mathrm{~B}=0\)
25) \(\mathrm{D} 14 \mathrm{~A}+\mathrm{D} 14 \mathrm{~B}+\mathrm{D} 15 \mathrm{~A}+\mathrm{D} 15 \mathrm{~B}+\mathrm{D} 16 \mathrm{~A}+\mathrm{D} 16 \mathrm{~B}+\mathrm{D} 17 \mathrm{~A}+\mathrm{D} 17 \mathrm{~B}=0\)
26) \(\mathrm{D} 18 \mathrm{~A}+\mathrm{D} 18 \mathrm{~B}=0\)

END.

\section*{APPENDIX V}

\section*{Tuesday Formulation}

MIN \(D 1 B+D 2 B+D 3 B+D 4 B+D 5 B+D 6 B+D 7 B+D 8 B+D 9 B+D 10 B+D 11 B+D 12 A+D 12 N B+D 13 A+\) \(D 13 B+D 14 A+D 14 B+D 15 A+D 15 B+4 D 16 A+4 D 16 B+3 D 17 A+3 D 17 B+2 D 18 A+2 D 18 B\)

\section*{SUBJECT TO}
2) \(\mathrm{X} 1+\mathrm{X} 2+\mathrm{X} 3+\mathrm{D} 1 \mathrm{~A}-\mathrm{D} 1 \mathrm{~B}=1\)
3) \(\mathrm{X} 5+\mathrm{X} 6+\mathrm{X} 7+\mathrm{D} 2 \mathrm{~A}-\mathrm{D} 2 \mathrm{~B}=1\)
4) \(\mathrm{X} 9+\mathrm{X} 10+\mathrm{X} 11+\mathrm{D} 3 \mathrm{~A}-\mathrm{D} 3 \mathrm{~B}=1\)
5) \(\mathrm{X} 13+\mathrm{X} 14+\mathrm{X} 15+\mathrm{D} 4 \mathrm{~A}-\mathrm{D} 4 \mathrm{~B}=1\)
6) \(\mathrm{X} 17+\mathrm{X} 18+\mathrm{X} 19+\mathrm{D} 5 \mathrm{~A}-\mathrm{D} 5 \mathrm{~B}=1\)
7) \(\mathrm{X} 21+\mathrm{X} 22+\mathrm{X} 23+\mathrm{D} 6 \mathrm{~A}-\mathrm{D} 6 \mathrm{~B}=1\)
8) \(X 25+X 26+X 27+D 7 A-D 7 B=1\)
9) \(\mathrm{X} 29+\mathrm{X} 30+\mathrm{X} 31+\mathrm{D} 8 \mathrm{~A}-\mathrm{D} 8 \mathrm{~B}=1\)
10) \(\mathrm{X} 41+\mathrm{X} 42+\mathrm{X} 43+\mathrm{D} 9 \mathrm{~A}-\mathrm{D} 9 \mathrm{~B}=1\)
11) \(X 45+X 46+X 47+D 10 A-D 10 B=1\)
12) \(\mathrm{X} 50+\mathrm{X} 51+\mathrm{X} 52+\mathrm{D} 11 \mathrm{~A}-\mathrm{D} 11 \mathrm{~B}=1\)
13) \(\mathrm{X} 1+\mathrm{X} 5+\mathrm{X} 9+\mathrm{X} 13+\mathrm{X} 17+\mathrm{X} 21+\mathrm{X} 25+\mathrm{X} 29+\mathrm{X} 41+\mathrm{X} 45+\mathrm{X} 49+\mathrm{D} 12 \mathrm{~A}-\mathrm{D} 12 \mathrm{~B}=5\)
14) \(\mathrm{X} 2+\mathrm{X} 6+\mathrm{X} 10+\mathrm{X} 14+\mathrm{X} 18+\mathrm{X} 22+\mathrm{X} 26+\mathrm{X} 30+\mathrm{X} 42+\mathrm{X} 46+\mathrm{X} 50+\mathrm{D} 13 \mathrm{~A}-\mathrm{D} 13 \mathrm{~B}=2\)
15) \(\mathrm{X} 3+\mathrm{X} 7+\mathrm{X} 11+\mathrm{X} 15+\mathrm{X} 19+\mathrm{X} 23+\mathrm{X} 27+\mathrm{X} 31+\mathrm{X} 43+\mathrm{X} 47+\mathrm{X} 51+\mathrm{D} 14 \mathrm{~A}-\mathrm{D} 14 \mathrm{~B}=3\)
16) \(\mathrm{X} 1+\mathrm{X} 5+\mathrm{X} 9+\mathrm{X} 13+\mathrm{D} 15 \mathrm{~A}-\mathrm{D} 15 \mathrm{~B}=1\)
17) \(\mathrm{X} 3+\mathrm{X} 5+\mathrm{X} 6+\mathrm{X} 11+\mathrm{X} 13+\mathrm{X} 15+\mathrm{X} 19+\mathrm{X} 22+\mathrm{X} 25+\mathrm{X} 30+\mathrm{X} 43+\mathrm{X} 47+\mathrm{X} 51+\mathrm{D} 16 \mathrm{~A}-\mathrm{D} 16 \mathrm{~B}=3\)
18) \(\mathrm{X} 2+\mathrm{X} 7+\mathrm{X} 10+\mathrm{X} 14+\mathrm{X} 18+\mathrm{X} 23+\mathrm{X} 26+\mathrm{X} 31+\mathrm{X} 42+\mathrm{X} 45+\mathrm{X} 50+\mathrm{D} 17 \mathrm{~A}-\mathrm{D} 17 \mathrm{~B}=3\)
19) \(\mathrm{X} 1+\mathrm{X} 9+\mathrm{X} 17+\mathrm{X} 27+\mathrm{X} 41+\mathrm{X} 46+\mathrm{X} 49+\mathrm{D} 18-\mathrm{D} 18 \mathrm{~B}=3\)
20) \(\mathrm{D} 1 \mathrm{~B}+\mathrm{D} 2 \mathrm{~B}+\mathrm{D} 3 \mathrm{~B}+\mathrm{D} 4 \mathrm{~B}+\mathrm{D} 5 \mathrm{~B}+\mathrm{D} 6 \mathrm{~B}+\mathrm{D} 7 \mathrm{~B}+\mathrm{D} 8 \mathrm{~B}+\mathrm{D} 9 \mathrm{~B}+\mathrm{D} 10 \mathrm{~B}+\mathrm{D} 11 \mathrm{~B}=0\)
21) \(\mathrm{D} 12 \mathrm{~A}+\mathrm{D} 12 \mathrm{~B}+\mathrm{D} 13 \mathrm{~A}+\mathrm{D} 13 \mathrm{~B}+\mathrm{D} 14 \mathrm{~A}+\mathrm{D} 14 \mathrm{~B}=0\)
22) \(\mathrm{D} 15 \mathrm{~A}+\mathrm{D} 15 \mathrm{~B}=0\)

END
INTEGER VARIABLES= 33

\section*{Wednesday Formulation}

MIN D1B+D2B+D3B+D4B+D5B+D6B+D7B+D8B+D9B+D10B+D11B+D12A+D12B+D13A+ \(D 13 B+D 14 A+D 14 B+D 15 A+D 15 B+4 D 16 A+4 D 16 B+3 D 17 A+3 D 17 B+2 D 18 A+2 D 18 B\)

\section*{SUBJECT TO}
2) \(X 1+X 2+X 3+D 1 A-D 1 B=1\)
3) \(\mathrm{X} 5+\mathrm{X} 6+\mathrm{X} 7+\mathrm{D} 2 \mathrm{~A}-\mathrm{D} 2 \mathrm{~B}=1\)
4) \(\mathrm{X} 9+\mathrm{X} 10+\mathrm{X} 11+\mathrm{D} 3 \mathrm{~A}-\mathrm{D} 3 \mathrm{~B}=1\)
5) \(X 13+X 14+X 15+D 4 A-D 4 B=1\)
6) \(\mathrm{X} 17+\mathrm{X} 18+\mathrm{X} 19+\mathrm{D} 5 \mathrm{~A}-\mathrm{D} 5 \mathrm{~B}=1\)
7) \(\mathrm{X} 21+\mathrm{X} 22+\mathrm{X} 23+\mathrm{D} 6 \mathrm{~A}-\mathrm{D} 6 \mathrm{~B}=1\)
8) \(\mathrm{X} 25+\mathrm{X} 26+\mathrm{X} 27+\mathrm{D} 7 \mathrm{~A}-\mathrm{D} 7 \mathrm{~B}=1\)
9) \(\mathrm{X} 29+\mathrm{X} 30+\mathrm{X} 31+\mathrm{D} 8 \mathrm{~A}-\mathrm{D} 8 \mathrm{~B}=1\)
10) \(\mathrm{X} 41+\mathrm{X} 42+\mathrm{X} 43+\mathrm{D} 9 \mathrm{~A}-\mathrm{D} 9 \mathrm{~B}=1\)
11) \(\mathrm{X} 45+\mathrm{X} 46+\mathrm{X} 47+\mathrm{D} 10 \mathrm{~A}-\mathrm{D} 10 \mathrm{~B}=1\)
12) \(\mathrm{X} 49+\mathrm{X} 50+\mathrm{X} 51+\mathrm{D} 11 \mathrm{~A}+\mathrm{D} 11 \mathrm{~B}=1\)
13) \(\mathrm{X} 1+\mathrm{X} 5+\mathrm{X} 9+\mathrm{X} 13+\mathrm{X} 17+\mathrm{X} 21+\mathrm{X} 25+\mathrm{X} 29+\mathrm{X} 41+\mathrm{X} 45+\mathrm{X} 49+\mathrm{D} 12 \mathrm{~A}-\mathrm{D} 12 \mathrm{~B}=4\)
14) \(\mathrm{X} 2+\mathrm{X} 6+\mathrm{X} 10+\mathrm{X} 14+\mathrm{X} 18+\mathrm{X} 22+\mathrm{X} 26+\mathrm{X} 30+\mathrm{X} 42+\mathrm{X} 46+\mathrm{X} 50+\mathrm{D} 13 \mathrm{~A}-\mathrm{D} 13 \mathrm{~B}=2\)
15) \(\mathrm{X} 3+\mathrm{X} 7+\mathrm{X} 11+\mathrm{X} 15+\mathrm{X} 19+\mathrm{X} 23+\mathrm{X} 27+\mathrm{X} 31+\mathrm{X} 43+\mathrm{X} 47+\mathrm{X} 51+\mathrm{D} 14 \mathrm{~A}-\mathrm{D} 14 \mathrm{~B}=3\)
16) \(\mathrm{X} 1+\mathrm{X} 5+\mathrm{X} 9+\mathrm{X} 13+\mathrm{D} 15 \mathrm{~A}-\mathrm{D} 15 \mathrm{~B}=1\)
17) \(\mathrm{X} 2+\mathrm{X} 10+\mathrm{X} 15+\mathrm{X} 18+\mathrm{X} 22+\mathrm{X} 25+\mathrm{X} 30+\mathrm{X} 42+\mathrm{X} 47+\mathrm{X} 50+\mathrm{D} 16 \mathrm{~A}-\mathrm{D} 16 \mathrm{~B}=3\)
18) \(\mathrm{X} 1+\mathrm{X} 5+\mathrm{X} 7+\mathrm{X} 13+\mathrm{X} 14+\mathrm{X} 19+\mathrm{X} 23+\mathrm{X} 26+\mathrm{X} 31+\mathrm{X} 41+\mathrm{X} 46+\mathrm{X} 51+\mathrm{D} 17 \mathrm{~A}-\mathrm{D} 17 \mathrm{~B}=3\)
19) \(\mathrm{X} 3+\mathrm{X} 6+\mathrm{X} 9+\mathrm{X} 11+\mathrm{X} 17+\mathrm{X} 27+\mathrm{X} 45+\mathrm{X} 49+\mathrm{D} 18 \mathrm{~A}-\mathrm{D} 18 \mathrm{~B}=3\)
20) \(\mathrm{D} 1 \mathrm{~B}+\mathrm{D} 2 \mathrm{~B}+\mathrm{D} 3 \mathrm{~B}+\mathrm{D} 4 \mathrm{~B}+\mathrm{D} 5 \mathrm{~B}+\mathrm{D} 6 \mathrm{~B}+\mathrm{D} 7 \mathrm{~B}+\mathrm{D} 8 \mathrm{~B}+\mathrm{D} 9 \mathrm{~B}+\mathrm{D} 10 \mathrm{~B}+\mathrm{D} 11 \mathrm{~B}=0\)
21) \(\mathrm{D} 12 \mathrm{~A}+\mathrm{D} 12 \mathrm{~B}+\mathrm{D} 13 \mathrm{~A}+\mathrm{D} 13 \mathrm{~B}+\mathrm{D} 14 \mathrm{~A}+\mathrm{D} 14 \mathrm{~B}=0\)
22) \(\mathrm{D} 15 \mathrm{~A}+\mathrm{D} 15 \mathrm{~B}=0\)

END
INTEGER VARIABLES= 33

\section*{APPENDIX VII}

\section*{Thursday Formulation}

MIN \(\mathrm{D} 1 \mathrm{~B}+\mathrm{D} 2 \mathrm{~B}+\mathrm{D} 3 \mathrm{~B}+\mathrm{D} 4 \mathrm{~B}+\mathrm{D} 5 \mathrm{~B}+\mathrm{D} 6 \mathrm{~B}+\mathrm{D} 7 \mathrm{~B}+\mathrm{D} 8 \mathrm{~B}+\mathrm{D} 9 \mathrm{~B}+\mathrm{D} 10 \mathrm{~B}+\mathrm{D} 11 \mathrm{~B}+\mathrm{D} 12 \mathrm{~A}+\mathrm{D} 12 \mathrm{~B}+\mathrm{D} 13 \mathrm{~A}+\) \(D 13 B+D 14 A+D 14 B+D 15 A+D 15 B+D 16 A+D 16 B+4 D 17 A+4 D 17 B+3 D 18 A+3 D 18 B+\) \(2 D 19 A+2 D 19 B+D 20 A+D 20 B\)

\section*{SUBJECT TO}
2) \(\mathrm{X} 1+\mathrm{X} 2+\mathrm{X} 3+\mathrm{D} 1 \mathrm{~A}-\mathrm{D} 1 \mathrm{~B}=1\)
3) \(\mathrm{X} 5+\mathrm{X} 6+\mathrm{X} 7+\mathrm{D} 2 \mathrm{~A}-\mathrm{D} 2 \mathrm{~B}=1\)
4) \(\mathrm{X} 9+\mathrm{X} 10+\mathrm{X} 11+\mathrm{D} 3 \mathrm{~A}-\mathrm{D} 3 \mathrm{~B}=1\)
5) \(\mathrm{X} 13+\mathrm{X} 14+\mathrm{X} 15+\mathrm{D} 4 \mathrm{~A}-\mathrm{D} 4 \mathrm{~B}=1\)
6) \(\mathrm{X} 17+\mathrm{X} 18+\mathrm{X} 19+\mathrm{D} 5 \mathrm{~A}-\mathrm{D} 5 \mathrm{~B}=1\)
7) \(\mathrm{X} 21+\mathrm{X} 22+\mathrm{X} 23+\mathrm{D} 6 \mathrm{~A}-\mathrm{D} 6 \mathrm{~B}=1\)
8) \(\mathrm{X} 25+\mathrm{X} 26+\mathrm{X} 27+\mathrm{D} 7 \mathrm{~A}-\mathrm{D} 7 \mathrm{~B}=1\)
9) \(\mathrm{X} 29+\mathrm{X} 30+\mathrm{X} 31+\mathrm{D} 8 \mathrm{~A}-\mathrm{D} 8 \mathrm{~B}=1\)
10) \(\mathrm{X} 41+\mathrm{X} 42+\mathrm{X} 43+\mathrm{D} 9 \mathrm{~A}-\mathrm{D} 9 \mathrm{~B}=1\)
11) \(\mathrm{X} 45+\mathrm{X} 46+\mathrm{X} 47+\mathrm{D} 10 \mathrm{~A}-\mathrm{D} 10 \mathrm{~B}=1\)
12) \(\mathrm{X} 49+\mathrm{X} 50+\mathrm{X} 51+\mathrm{D} 11 \mathrm{~A}-\mathrm{D} 11 \mathrm{~B}=1\)
13) \(\mathrm{X} 1+\mathrm{X} 5+\mathrm{X} 9+\mathrm{X} 13+\mathrm{X} 17+\mathrm{X} 21+\mathrm{X} 25+\mathrm{X} 29+\mathrm{X} 41+\mathrm{X} 45+\mathrm{X} 49+\mathrm{D} 12 \mathrm{~A}-\mathrm{D} 12 \mathrm{~B}=5\)
14) \(\mathrm{X} 2+\mathrm{X} 6+\mathrm{X} 10+\mathrm{X} 14+\mathrm{X} 18+\mathrm{X} 22+\mathrm{X} 26+\mathrm{X} 30+\mathrm{X} 42+\mathrm{X} 46+\mathrm{X} 50+\mathrm{D} 13 \mathrm{~A}-\mathrm{D} 13 \mathrm{~B}=2\)
15) \(\mathrm{X} 3+\mathrm{X} 7+\mathrm{X} 11+\mathrm{X} 15+\mathrm{X} 19+\mathrm{X} 23+\mathrm{X} 27+\mathrm{X} 31+\mathrm{X} 43+\mathrm{X} 47+\mathrm{X} 51+\mathrm{D} 14 \mathrm{~A}-\mathrm{D} 14 \mathrm{~B}=3\)
16) \(\mathrm{X} 4+\mathrm{X} 8+\mathrm{X} 12+\mathrm{X} 16+\mathrm{X} 20+\mathrm{X} 24+\mathrm{X} 28+\mathrm{X} 32+\mathrm{X} 44+\mathrm{X} 48+\mathrm{X} 52+\mathrm{D} 15 \mathrm{~A}-\mathrm{D} 15 \mathrm{~B}=1\)
17) \(\mathrm{X} 1+\mathrm{X} 5+\mathrm{X} 9+\mathrm{X} 13+\mathrm{D} 16 \mathrm{~A}-\mathrm{D} 16 \mathrm{~B}=1\)
18) \(\mathrm{X} 1+\mathrm{X} 5+\mathrm{X} 6+\mathrm{X} 11+\mathrm{X} 15+\mathrm{X} 19+\mathrm{X} 22+\mathrm{X} 25+\mathrm{X} 30+\mathrm{X} 41+\mathrm{X} 48+\mathrm{X} 51+\mathrm{D} 17 \mathrm{~A}-\mathrm{D} 17 \mathrm{~B}=4\)
19) \(\mathrm{X} 3+\mathrm{X} 7+\mathrm{X} 10+\mathrm{X} 13+\mathrm{X} 14+\mathrm{X} 18+\mathrm{X} 23+\mathrm{X} 26+\mathrm{X} 31+\mathrm{X} 42+\mathrm{X} 45+\mathrm{X} 50+\mathrm{D} 18 \mathrm{~A}-\mathrm{D} 18 \mathrm{~B}=4\)
20) \(\mathrm{X} 2+\mathrm{X} 9+\mathrm{X} 16+\mathrm{X} 17+\mathrm{X} 24+\mathrm{X} 27+\mathrm{X} 32+\mathrm{X} 43+\mathrm{X} 47+\mathrm{X} 49+\mathrm{D} 19 \mathrm{~A}-\mathrm{D} 19 \mathrm{~B}=4\)
21) \(\mathrm{X} 4+\mathrm{X} 8=\mathrm{X} 12+\mathrm{X} 20+\mathrm{X} 21+\mathrm{X} 28+\mathrm{X} 29+\mathrm{X} 44+\mathrm{X} 46+\mathrm{X} 52+\mathrm{D} 20 \mathrm{~A}-\mathrm{D} 20 \mathrm{~B}=4\)
22) \(\mathrm{D} 1 \mathrm{~B}+\mathrm{D} 2 \mathrm{~B}+\mathrm{D} 3 \mathrm{~B}+\mathrm{D} 4 \mathrm{~B}+\mathrm{D} 5 \mathrm{~B}+\mathrm{D} 6 \mathrm{~B}+\mathrm{D} 7 \mathrm{~B}+\mathrm{D} 8 \mathrm{~B}+\mathrm{D} 9 \mathrm{~B}+\mathrm{D} 10 \mathrm{~B}+\mathrm{D} 11 \mathrm{~B}=0\)
23) \(\mathrm{D} 12 \mathrm{~A}+\mathrm{D} 12 \mathrm{~B}+\mathrm{D} 13 \mathrm{~A}+\mathrm{D} 13 \mathrm{~B}+\mathrm{D} 14 \mathrm{~A}+\mathrm{D} 14 \mathrm{~B}+\mathrm{D} 15 \mathrm{~A}+\mathrm{D} 15 \mathrm{~B}=0\)
24) \(\mathrm{D} 16 \mathrm{~A}+\mathrm{D} 16 \mathrm{~B}=0\)

END
INTEGER VARIABLES= 44

\section*{APPENDIX VIII}

\section*{Friday Formulation}

MIN D1B+D2B+D3B+D4B+D5B+D6B+D7B+D8B+D9B+D10B+D11A+D11B+D12A+D12B+ \(D 13 A+D 13 B+D 14 A+D 14 B+4 D 15 A+4 D 15 B+3 D 16 A+3 D 16 B+2 D 17 A+2 D 17 B\)

SUBJECT TO
2) \(\mathrm{X} 1+\mathrm{X} 2+\mathrm{X} 3+\mathrm{D} 1 \mathrm{~A}-\mathrm{D} 1 \mathrm{~B}=1\)
3) \(\mathrm{X} 5+\mathrm{X} 6+\mathrm{X} 7+\mathrm{D} 2 \mathrm{~A}-\mathrm{D} 2 \mathrm{~B}=1\)
4) \(\mathrm{X} 9+\mathrm{X} 10+\mathrm{X} 11+\mathrm{D} 3 \mathrm{~A}-\mathrm{D} 3 \mathrm{~B}=1\)
5) \(\mathrm{X} 13+\mathrm{X} 14+\mathrm{X} 15+\mathrm{D} 4 \mathrm{~A}-\mathrm{D} 4 \mathrm{~B}=1\)
6) \(\mathrm{X} 17+\mathrm{X} 18+\mathrm{X} 19+\mathrm{D} 5 \mathrm{~A}-\mathrm{D} 5 \mathrm{~B}=1\)
7) \(\mathrm{X} 21+\mathrm{X} 22+\mathrm{X} 23+\mathrm{D} 6 \mathrm{~A}-\mathrm{D} 6 \mathrm{~B}=1\)
8) \(\mathrm{X} 25+\mathrm{X} 26+\mathrm{X} 27+\mathrm{D} 7 \mathrm{~A}-\mathrm{D} 7 \mathrm{~B}=1\)
9) \(\mathrm{X} 41+\mathrm{X} 42+\mathrm{X} 43+\mathrm{D} 8 \mathrm{~A}-\mathrm{D} 8 \mathrm{~B}=1\)
10) \(X 45+X 46+X 47+D 9 A-D 9 B=1\)
11) \(\mathrm{X} 49+\mathrm{X} 50+\mathrm{X} 51+\mathrm{D} 10 \mathrm{~A}-\mathrm{D} 10 \mathrm{~B}=1\)
12) \(\mathrm{X} 1+\mathrm{X} 5+\mathrm{X} 9+\mathrm{X} 13+\mathrm{X} 17+\mathrm{X} 21+\mathrm{X} 25+\mathrm{X} 41+\mathrm{X} 45+\mathrm{X} 49+\mathrm{D} 11 \mathrm{~A}-\mathrm{D} 11 \mathrm{~B}=5\)
13) \(\mathrm{X} 2+\mathrm{X} 6+\mathrm{X} 10+\mathrm{X} 14+\mathrm{X} 18+\mathrm{X} 22+\mathrm{X} 26+\mathrm{X} 42+\mathrm{X} 46+\mathrm{X} 50+\mathrm{D} 12 \mathrm{~A}-\mathrm{D} 12 \mathrm{~B}=2\)
14) \(\mathrm{X} 3+\mathrm{X} 7+\mathrm{X} 11+\mathrm{X} 15+\mathrm{X} 19+\mathrm{X} 23+\mathrm{X} 27+\mathrm{X} 43+\mathrm{X} 47+\mathrm{X} 51+\mathrm{D} 13 \mathrm{~A}-\mathrm{D} 13 \mathrm{~B}=3\)
15) \(\mathrm{X} 1+\mathrm{X} 5+\mathrm{X} 9+\mathrm{X} 13+\mathrm{D} 14 \mathrm{~A}-\mathrm{D} 14 \mathrm{~B}=1\)
16) \(\mathrm{X} 3+\mathrm{X} 10+\mathrm{X} 14+\mathrm{X} 18+\mathrm{X} 22+\mathrm{X} 25+\mathrm{X} 43+\mathrm{X} 50+\mathrm{X} 6+\mathrm{X} 48+\mathrm{D} 15 \mathrm{~A}-\mathrm{D} 15 \mathrm{~B}=3\)
17) \(\mathrm{X} 2+\mathrm{X} 5+\mathrm{X} 7+\mathrm{X} 11+\mathrm{X} 19+\mathrm{X} 23+\mathrm{X} 26+\mathrm{X} 42+\mathrm{X} 46+\mathrm{X} 51+\mathrm{D} 16 \mathrm{~A}-\mathrm{D} 16 \mathrm{~B}=3\)
18) \(\mathrm{X} 1+\mathrm{X} 9+\mathrm{X} 15+\mathrm{X} 17+\mathrm{X} 27+\mathrm{X} 41+\mathrm{X} 45+\mathrm{X} 49+\mathrm{X} 24+\mathrm{D} 17 \mathrm{~A}-\mathrm{D} 17 \mathrm{~B}=3\)
19) \(\mathrm{X} 4=0\)
20) \(\mathrm{X} 20=0\)
21) \(X 24=0\)
22) \(\mathrm{X} 28=0\)
23) \(\mathrm{X} 48=0\)
24) \(\mathrm{D} 1 \mathrm{~B}+\mathrm{D} 2 \mathrm{~B}+\mathrm{D} 3 \mathrm{~B}+\mathrm{D} 4 \mathrm{~B}+\mathrm{D} 5 \mathrm{~B}+\mathrm{D} 6 \mathrm{~B}+\mathrm{D} 7 \mathrm{~B}+\mathrm{D} 8 \mathrm{~B}+\mathrm{D} 9 \mathrm{~B}+\mathrm{D} 10 \mathrm{~B}=0\)
25) \(\mathrm{D} 11 \mathrm{~A}+\mathrm{D} 11 \mathrm{~B}+\mathrm{D} 12 \mathrm{~A}+\mathrm{D} 12 \mathrm{~B}+\mathrm{D} 13 \mathrm{~A}+\mathrm{D} 13 \mathrm{~B}=0\)
26) \(\mathrm{D} 14 \mathrm{~A}+\mathrm{D} 14 \mathrm{~B}=0\)

END
INTEGER VARIABLES= 30

\section*{APPENDIX X}

\section*{Saturday Formulation}

MIN D1B+D2B+D3B+D4B+D5B+D6B+D7B+D8B+D9B+D10A+D10B+D11A+D11B+D12A+D12B \(+D 13 A+D 13 B+D 14 A+D 14 B+4 D 15 A+4 D 15 B+3 D 16 A+3 D 16 B+2 D 17 A+2 D 17 B+D 18 A+D 18 B\)

SUBJECT TO
2) \(X 1+X 2+X 3+X 4+D 1 A-D 1 B=1\)
3) \(X 5+X 6+X 7+X 8+D 2 A-D 2 B=1\)
4) \(\mathrm{X} 9+\mathrm{X} 10+\mathrm{X} 11+\mathrm{X} 12+\mathrm{D} 3 \mathrm{~A}-\mathrm{D} 3 \mathrm{~B}=1\)
5) \(\mathrm{X} 13+\mathrm{X} 14+\mathrm{X} 15+\mathrm{X} 16+\mathrm{D} 4 \mathrm{~A}-\mathrm{D} 4 \mathrm{~B}=1\)
6) \(\mathrm{X} 21+\mathrm{X} 22+\mathrm{X} 23+\mathrm{X} 24+\mathrm{D} 5 \mathrm{~A}-\mathrm{D} 5 \mathrm{~B}=1\)
7) \(\mathrm{X} 25+\mathrm{X} 26+\mathrm{X} 27+\mathrm{X} 28+\mathrm{D} 6 \mathrm{~A}-\mathrm{D} 6 \mathrm{~B}=1\)
8) \(\mathrm{X} 41+\mathrm{X} 42+\mathrm{X} 43+\mathrm{X} 44+\mathrm{D} 7 \mathrm{~A}-\mathrm{D} 7 \mathrm{~B}=1\)
9) \(\mathrm{X} 45+\mathrm{X} 46+\mathrm{X} 47+\mathrm{X} 48+\mathrm{D} 8 \mathrm{~A}-\mathrm{D} 8 \mathrm{~B}=1\)
10) \(\mathrm{X} 49+\mathrm{X} 50+\mathrm{X} 51+\mathrm{X} 52+\mathrm{D} 9 \mathrm{~A}-\mathrm{D} 9 \mathrm{~B}=1\)
11) \(\mathrm{X} 1+\mathrm{X} 5+\mathrm{X} 9+\mathrm{X} 13+\mathrm{X} 21+\mathrm{X} 25+\mathrm{X} 41+\mathrm{X} 45+\mathrm{X} 49+\mathrm{D} 10 \mathrm{~A}-\mathrm{D} 10 \mathrm{~B}=3\)
12) \(\mathrm{X} 2+\mathrm{X} 6+\mathrm{X} 10+\mathrm{X} 14+\mathrm{X} 22+\mathrm{X} 26+\mathrm{X} 42+\mathrm{X} 46+\mathrm{X} 50+\mathrm{D} 11 \mathrm{~A}-\mathrm{D} 11 \mathrm{~B}=2\)
13) \(\mathrm{X} 3+\mathrm{X} 7+\mathrm{X} 11+\mathrm{X} 15+\mathrm{X} 23+\mathrm{X} 27+\mathrm{X} 43+\mathrm{X} 47+\mathrm{X} 51+\mathrm{D} 12 \mathrm{~A}-\mathrm{D} 12 \mathrm{~B}=2\)
14) \(\mathrm{X} 4+\mathrm{X} 8+\mathrm{X} 12+\mathrm{X} 16+\mathrm{X} 24+\mathrm{X} 28+\mathrm{X} 44+\mathrm{X} 48+\mathrm{X} 52+\mathrm{D} 13 \mathrm{~A}-\mathrm{S} 13 \mathrm{~B}=2\)
15) \(\mathrm{X} 1+\mathrm{X} 5+\mathrm{X} 9+\mathrm{X} 13+\mathrm{D} 4 \mathrm{~A}-\mathrm{D} 14 \mathrm{~B}=1\)
16) \(\mathrm{X} 2+\mathrm{X} 5+\mathrm{X} 6+\mathrm{X} 11+\mathrm{X} 14+\mathrm{X} 22+\mathrm{X} 26+\mathrm{X} 42+\mathrm{X} 46+\mathrm{X} 51+\mathrm{D} 15 \mathrm{~A}-\mathrm{D} 15 \mathrm{~B}=4\)
17) \(\mathrm{X} 3+\mathrm{X} 10+\mathrm{X} 23+\mathrm{X} 27+\mathrm{X} 43+\mathrm{X} 47+\mathrm{X} 50+\mathrm{D} 16 \mathrm{~A}-\mathrm{D} 16 \mathrm{~B}=4\)
18) \(\mathrm{X} 1+\mathrm{X} 9+\mathrm{X} 15+\mathrm{X} 24+\mathrm{X} 28+\mathrm{X} 41+\mathrm{X} 45+\mathrm{X} 49+\mathrm{D} 17 \mathrm{~A}-\mathrm{D} 17 \mathrm{~B}=4\)
19) \(\mathrm{X} 4+\mathrm{X} 7+\mathrm{X} 8+\mathrm{X} 12+\mathrm{X} 13+\mathrm{X} 16+\mathrm{X} 21+\mathrm{X} 25+\mathrm{X} 44+\mathrm{X} 48+\mathrm{X} 52+\mathrm{D} 18 \mathrm{~A}-\mathrm{D} 18 \mathrm{~B}=4\)
20) \(\mathrm{X} 16=0\)
21) \(\mathrm{X} 28=0\)
22) \(\mathrm{X} 48=0\)
23) \(\mathrm{X} 52=0\)
24) \(\mathrm{D} 1 \mathrm{~B}+\mathrm{D} 2 \mathrm{~B}+\mathrm{D} 3 \mathrm{~B}+\mathrm{D} 4 \mathrm{~B}+\mathrm{D} 5 \mathrm{~B}+\mathrm{D} 6 \mathrm{~B}+\mathrm{D} 7 \mathrm{~B}+\mathrm{D} 8 \mathrm{~B}+\mathrm{D} 9 \mathrm{~B}=0\)
25) \(\mathrm{D} 10 \mathrm{~A}+\mathrm{D} 10 \mathrm{~B}+\mathrm{D} 11 \mathrm{~A}+\mathrm{D} 11 \mathrm{~B}+\mathrm{D} 12 \mathrm{~A}+\mathrm{D} 12 \mathrm{~B}+\mathrm{D} 13 \mathrm{~A}+\mathrm{D} 13 \mathrm{~B}=0\)
26) \(\mathrm{D} 14 \mathrm{~A}+\mathrm{D} 14 \mathrm{~B}=0\)

END

\section*{APPENDIX XI}

\section*{Sunday Formulation}

MIN D1B+D2B+D3B+D4B+D5B+D6B+D7B+D8A+D8B+D9A+D9B+D10A+D10B+D11A+D11B \(+4 D 12 A+4 D 12 B+3 D 13 A+3 D 13 B+2 D 14 A+214 B+D 15 A+D 15 B\)

SUBJECT TO
2) \(\mathrm{X} 1+\mathrm{X} 2+\mathrm{X} 3+\mathrm{D} 1 \mathrm{~A}-\mathrm{D} 1 \mathrm{~B}=1\)
3) \(\mathrm{X} 5+\mathrm{X} 6+\mathrm{X} 7+\mathrm{D} 2 \mathrm{~A}-\mathrm{D} 2 \mathrm{~B}=1\)
4) \(\mathrm{X} 9+\mathrm{X} 10+\mathrm{X} 11+\mathrm{D} 3 \mathrm{~A}-\mathrm{D} 3 \mathrm{~B}=1\)
5) \(\mathrm{X} 13+\mathrm{X} 14+\mathrm{X} 15+\mathrm{D} 4 \mathrm{~A}-\mathrm{D} 4 \mathrm{~B}=1\)
6) \(\mathrm{X} 25+\mathrm{X} 26+\mathrm{X} 27+\mathrm{D} 5 \mathrm{~A}-\mathrm{D} 5 \mathrm{~B}=1\)
7) \(\mathrm{X} 45+\mathrm{X} 46+\mathrm{X} 47+\mathrm{D} 6 \mathrm{~A}-\mathrm{D} 6 \mathrm{~B}=1\)
8) \(\mathrm{X} 49+\mathrm{X} 50+\mathrm{X} 51+\mathrm{D} 7 \mathrm{~A}-\mathrm{D} 7 \mathrm{~B}=1\)
9) \(\mathrm{X} 1+\mathrm{X} 5+\mathrm{X} 9+\mathrm{X} 13+\mathrm{X} 25+\mathrm{X} 45+\mathrm{X} 49+\mathrm{D} 8 \mathrm{~A}-\mathrm{D} 8 \mathrm{~B}=3\)
10) \(\mathrm{X} 2+\mathrm{X} 6+\mathrm{X} 10+\mathrm{X} 14+\mathrm{X} 26+\mathrm{X} 46+\mathrm{X} 50+\mathrm{D} 9 \mathrm{~A}-\mathrm{D} 9 \mathrm{~B}=2\)
11) \(\mathrm{X} 3+\mathrm{X} 7+\mathrm{X} 11+\mathrm{X} 15+\mathrm{X} 27+\mathrm{X} 47+\mathrm{X} 51+\mathrm{D} 10 \mathrm{~A}-\mathrm{D} 10 \mathrm{~B}=2\)
12) \(\mathrm{X} 1+\mathrm{X} 5+\mathrm{X} 9+\mathrm{X} 13+\mathrm{D} 11 \mathrm{~A}-\mathrm{D} 11 \mathrm{~B}=1\)
13) \(\mathrm{X} 2+\mathrm{X} 5+\mathrm{X} 9+\mathrm{X} 14+\mathrm{X} 27+\mathrm{X} 47+\mathrm{X} 50+\mathrm{D} 12 \mathrm{~A}-\mathrm{D} 12 \mathrm{~B}=3\)
14) \(\mathrm{X} 3+\mathrm{X} 10+\mathrm{X} 45+\mathrm{X} 51+\mathrm{D} 13 \mathrm{~A}-\mathrm{D} 13 \mathrm{~B}=3\)
15) \(\mathrm{X} 1+\mathrm{X} 7+\mathrm{X} 11+\mathrm{X} 15+\mathrm{X} 26+\mathrm{X} 46+\mathrm{X} 49+\mathrm{D} 14 \mathrm{~A}-\mathrm{D} 14 \mathrm{~B}=3\)
16) \(\mathrm{X} 6+\mathrm{X} 13+\mathrm{X} 25+\mathrm{D} 15 \mathrm{~A}-\mathrm{D} 15 \mathrm{~B}=3\)
17) \(\mathrm{D} 1 \mathrm{~B}+\mathrm{D} 2 \mathrm{~B}+\mathrm{D} 3 \mathrm{~B}+\mathrm{D} 4 \mathrm{~B}+\mathrm{D} 5 \mathrm{~B}+\mathrm{D} 6 \mathrm{~B}+\mathrm{D} 7 \mathrm{~B}=0\)
18) \(\mathrm{D} 8 \mathrm{~A}+\mathrm{D} 8 \mathrm{~B}+\mathrm{D} 9 \mathrm{~A}+\mathrm{D} 9 \mathrm{~B}+\mathrm{D} 10 \mathrm{~A}+\mathrm{D} 10 \mathrm{~B}=0\)
19) \(\mathrm{D} 11 \mathrm{~A}+\mathrm{D} 11 \mathrm{~B}=0\)

\footnotetext{
END
INTEGER VARIABLES= 21
}```

