This Thesis is my original work and has not been presented for a degree in any other University.

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This Thesis has been submitted for examination with my approval as University Supervisor.

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OPERATOR EQUATIONS IN HILBERT SPACES.

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By

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ABSTRACT

The development of operator theory in Hilbert spaces, owes its origin to complex analysis. Thus, given a bounded linear operator A, we would like to view it as a generalised complex number viz:

$$A = B + iC.$$

with B and C as real and imaginary parts respectively. However, it turns out that it is not always true that BC = CB as in the case of complex numbers. This in itself has been a drawback, which necessitated the classification of operators. Hence, the operator A for which BC = CB is said to be normal because it behaves like a complex number.

It is therefore, quite natural that given an operator equation like

$$AB + BA^* = A^*B + BA = I,$$

one would first address oneself to the problem of finding normal solutions. Similarly, we note that if A, H and K are complex numbers and AH = KA, one would not require any condition on A in order to conclude that H = K. However, this is not always the case if A, H and K are bounded linear operators. It is therefore motivating enough to try and find conditions under which H = K. In the light of the remarks above, we address ourselves in this thesis to the following task

in the form of chapters:

(i) On the operator equation $AH = KA$, in which we find sufficient conditions under which $H = K$ and its consequences.

(ii) The operator equation $AB + BA^* = A^*B + BA = I$, in which we find necessary and sufficient conditions for existence of A or B.

(iii) We revist the operator equation $AB + BA^* = A^*B + BA = I$, in which we apply some of the results of chapter one in order to find sufficient conditions for normality of A or B. It is infact shown that some of the already published sufficient conditions under which A is normal, can easily be derived as corollaries to our main results in this chapter.

(iv) The operator equation $TST^* = S$, in which we deduce unitary solutions.

Finally, while the study of each of these three operator equations may appear to be done in isolation, the three equations are shown infact to be interrelated. We would also like to note that the question of applicability of these operator equations is not of our primary concern in this thesis. Here, we concern ourselves with the abstract theory of these operator equations in which there is also sufficient merit.