"FINITE ELEMENT METHOD APPLIED TO SOME BOUNDARY VALUE PROBLEMS"

BY

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DECLARATION

This dissertation is my own work and has not been presented for a degree in any other University.

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This dissertation has been submitted for examination with my approval as University Supervisor.

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CHAPTER ONE

1.1 GENERAL INTRODUCTION

The Finite Element Method (FEM) is one of the numerical analysis techniques for obtaining approximate solutions to a wide variety of engineering problems. Others are finite difference and boundary element methods. Suitability of any method depends on the nature of the problem to be solved.

The basic premise of the FEM is that a solution region can be analytically modeled or approximated by replacing it with an assemblage of discrete elements. Since these elements can be put together in a variety of ways, they can be used to represent exceedingly complex shapes.

In a continuum problem of any dimension the field variable possesses infinitely many values because it is a function of each generic point in the body or solution region. Thus the problem is one with an infinite number of unknowns. The finite element discretization procedures reduce the problem to one of a finite number of unknowns by dividing the solution region into elements and by expressing the unknown field variable in terms of assumed approximating function within each element. The approximating functions are defined in terms of the values of the field variables at specified points called nodes or nodal points. Nodes usually lie on the element boundaries where adjacent elements are considered to be connected. In addition to boundary nodes, an element may also have a few interior nodes. The nodal values of the field variable
and the interpolation functions for the elements completely define the behaviour of the field variable within the elements. For the finite element representation of a problem, the nodal values of the field variable become the new unknowns. Once these unknowns are found, the interpolation functions define the field variable throughout the assemblage of elements.

Clearly, the nature of the solution and degree of approximation depend not only on the size and number of the elements used but also on the interpolation functions selected. Often functions are chosen so that the field variable or its derivatives are continuous across adjoining element boundaries.

An important feature of the FEM which sets it apart from other approximate numerical methods is the ability to formulate solutions for individual elements before putting them together to represent the entire problem. In essence a complex problem reduces to considerably simplified problems.

FEM has also a variety of ways in which one can formulate the properties of individual elements. These are basically four approaches:

(i) Direct approach.
(ii) Variational approach
(iii) Weighted residuals approach
(iv) Energy balance approach.

Regardless of the approach used to find the element properties, the solution of a continuum problem by the FEM always follows an orderly step-by-step process.
The summary of how the FEM works is:

(i) Discretize the solution region into a finite number of elements

(ii) Select interpolation functions

(iii) Find element properties

(iv) Perform coordinate transformation if need be

(v) Assemble the element properties to obtain the system equations

(vi) Solve the system equations

(vii) Make additional computations if desired.