TESTING THE VALIDITY OF CAPITAL ASSEST PRICING MODEL IN NAIROBI SECURITIES EXCHANGE

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DECLARATION

I declare that this proposal is my original work and to the best of my knowledge it has not been presented for registration in any other learning institution.

Sign:………………………………………..Date………………………..

Calvin Bitange Maina

Supervisor

This research proposal has been submitted with my approval as the University supervisor.

Sign:………………………………………..Date………………………..

Mr. Cyrus Iraya

Supervisor

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DEDICATION

To my loving wife Edwinah Onyancha and my yet to be born twins.

ABSTRACT

There have been various innovation of the capital asset pricing model (CAPM). These innovation are meant to redress the assumption of the CAPM in view of capturing the variability of the asset returns. Many attempts have been made to capture this variability by extending the factors affecting the individual assets. These factors include the economical, fundamental and statistical factors. The time varying aspect of returns has also been addressed by using time series models like the ARCH models. These attempts have however not been satisfactory. This emerges from the fact that returns have been assumed to be normally distributed. High frequency (daily or weekly) financial data exhibit non-normal characteristics. The returns are usually skewed, more peaked and experience fat tails than the normal distribution. The class of generalized hyperbolic distribution is a class of normal variance-mean mixtures with generalized inverse Gaussian distribution as the mixing distribution. The distribution captures skewness, peakedness of data and fat tailed empirical data depicted by high frequency financial data. The class nest other distributions as special or limiting cases.

In this dissertation, the distribution has been used to model weekly returns of the NSE20 index, Safaricom company and Mumias Sugar company. The marginal distributions results are satisfactory and realible compared to the normal distribution. Correspondingly, the marginal distribution have been used to construct copula functions and bivariate distribution to capture the dependence between returns. The estimation difficulty is overcome using the EM algorithm which is easily programmable and surely converges to give precision result with few iteration compared to other optimization technique. The suggested model does not underestimate the risk measured by the beta of the company. The beta of the company measures in part the covariance of the returns between company and the market. This bivariate data is modeled using the bivatiate normal inverse Gaussian distribution. Unlike the bivariate normal distribution, the bivariate NIG does not underestimate the risk of the company. For the NSE20 index, Safaricom company and Mumias Sugar company weekly follow the normal inverse Gaussian distribution which is a special case of generalized hyperbolic distributions. The distribution fits well to the empirical weekly returns than the normal distribution.

The exploratory data analysis using the QQ-plots show that the NIG distribution is better compared to the normal distribution. The bivariate NIG model fits well to the bivariate weekly returns of NSE20 index and safaricom weekly returns and NSE20 index and Mumias Sugar weekly returns. The bivariate returns is underestimated in using bivariate normal distribution. In particular, the beta of Safaricom company is estimated by the bivariate distribution to be 1.512042 and normal distribution to be 1.10226. The beta of Mumias Sugar company is estimated by the bivariate distribution to be 1.012042 and normal distribution to be 0.9226. Clearly, the normal distribution underestimate the systematic risk of the company. The required returns computed using the beta of the bivariate NIG distribution are 21.87% for the Safaricom company and 18.42% for the Mumias Sugar company. These returns better describe the trade-off between risk and returns in non-normal settings.This study ensure that the assumption of normal distribution of high frequency returns as depicted in theory of CAPM is replaced by the generalized hyperbolic distribution which are more competent. The practical value of study is the fact that the systematic risk of the company as measured by the beta is not underestimated or overestimated. This ensures policy making, especially in risk management, is accurately done and observed.

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LIST OF ABBREVIATIONS AND ACRONYMS

GHD..Generalized hyperbolic distribution

var...variance

cov..covariance

c (u,v)..copulas function

E(R_i)…………………………………………expectation of return to asset *i*.

I_{t-1}………………………………………..…information set

R_i…………………………………………… return to asset *i*.

EM algorithm………………………………….Expectation maximization algorithm

NIG……………………………………………Normal Inverse Gaussian

VG……………………………………………Variance gamma distribution

AIC……………………………………………Alkaike Information Creteria

CHAPTER ONE: INTRODUCTION

1.1 Background of the Study

The single-period capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965) postulates a simple linear relationship between the expected return and the market risk of a security.

While the results of direct tests have been inconclusive, recent evidence suggests the existence of additional factors which are relevant for asset pricing. Litzenberger and Ramaswamy (1979) showed a significant positive relationship between dividend yield and return of common stocks for the 1936-1977 periods. Basu (1977) finds that price earnings ratios and risk adjusted returns are related. He chooses to interpret his findings as evidence of market inefficiency but as Ball (1978) points out, market efficiency tests are often joint tests of the efficient market hypothesis and a particular equilibrium relationship. Thus, some of the anomalies that have been attributed to a lack of market efficiency might well be the result of a misspecification of the pricing model.

In summary, research has given a number of factors that influence the return of the asset which should be included in determining its return. The factors fall mainly under four categories. These are: macroeconomic factors, fundamental factors and statistical factors.

Macroeconomic factors include observable time series as the factors. They include factors such as the annual rate of inflation and economic growth, short term interest rate, the yield on long term government bonds, and the yield marginal on corporate bonds over government bonds. These are typically expected to influence the future cashflows of an asset and the discounted rate used to value them.

Fundamental factors include factors that are company-specific variables. Examples are: the level of gearing, the price earnings ratio, the level of R&D and the industry group to which the company belongs. These factors once incorporated in a model are used for risk control by comparing the sensitivity of the asset to one of the factors with the sensitivity of a benchmark portfolio.

Statistical factors include factors a researcher considers imperative but not in the above categories.

Innovation of the CAPM has addressed the imperfection of the traditional CAPM by incorporating the stochastic nature of the markets. Most of early test of CAPM have employed the methodology of first estimating betas using time series regression and then running a cross section of regression using the estimated betas as explanatory variables to test the hypothesis implied by the CAPM. The first tests of CAPM on individual stock in the excess return form have been conducted by Lintner (1965) and Douglas (1968). They have found that the intercept has value much larger than R_f the coefficient of beta is statistically significant but has a lower value and residual risk has effect on security returns. Their results seem to be a contradiction to the CAPM model. But both the Douglas and Lintner studies appear to suffer from various statistical weaknesses that might explain their anomalies results. The measurement error has incurred in estimating individual stock betas, the fact that estimated betas and unsystematic risk are highly correlated and also due to skewness present in the distribution of observed stock returns. Thus Lintner's results have seemed to be in contradiction to the CAPM.

The multifactor asset pricing model generalized the result of Sharpe-Lintner-Black (SLB) model. In these models, the return generating models involve multiple factors and the cross section of expected returns is explained by the cross section of factor loadings or sensitivities. One approach suggested by Ross (1976) arbitrage pricing theory (APT) uses factor analysis to extract the common factors and then tests whether expected returns are explained by the cross section of the loading of asset returns on the factor Roll and Ross (1980), Chen (1983) and Lehmann and Modest (1988) have tested this approach in detail. The factor analysis approach to test of the APT leads to unreasonable conflict about the number of common factors and what these factors are. The factor analysis approach is limited, but it confirms that there is more than one common factor in explaining expected returns. However, it’s impractical to account for all the factors affecting the returns of assets. In addition, the processes can be very costly for any meaning economic benefit.

Modeling the dependence returns of the company using the non-gaussian distributional approach will not be adequate to address the challenges of determining all factors affecting the returns of the CAPM model. A class of distribution that fits dependence financial data well is the multivariate generalized hyperbolic distribution.

1.1.1 Capital Asset Pricing Model (CAPM)

A single investor given his own estimate of security returns, variances and covariance can apply portfolio theory. The CAPM developed by Sharpe (1964) and Lintner (1965) introduces assumptions regarding the market and the behavior of other investors to allow the construction of an equilibrium model of price in the whole market. The assumption include: All investors have the same one-period horizon, all investors can borrow or lend unlimited amount at the same risk-free rate, market for risky asset are perfect. Information and instantly available to all investors and no investor believes that they can affect the price of the security line by their own actions, investors have the same estimates of the expected returns, standard deviations and covariance of securities over the one period horizon, investors measure in the same "currency" eg pounds or dollars or in "real" or money "terms".

The formula is given by

$R_i = R_f + \beta_i (R_m - R_f)$

Where

R is the risk free interest rate

R_i is the required return of the asset

β_i is the beta of the asset given by $(cov(R_i, R_m))/var(R_m)$

R_m is the expected return of the market

1.1.2 Test of CAPM

Numerous studies have examined the effectiveness of the original Capital Asset Pricing Model of Sharpe (1964) and Lintner (1965) and most have found that for developing country markets this is subject to considerable ambiguity. More recently, additional factors have been proposed to provide a more reliable explanation of the cross section of average returns. These include firm size, the book to market equity ratio, the price earnings ratio, the cash flow to price ratio and the performance of the firm in terms of sales growth. A major innovation proposed by Fama and French (1993) by introducing a number of changes to previous research, including the set of asset returns, the variables in the model and the estimation approach, choosing a time series cross section method rather than simply a cross section dimension.

Tests of the CAPM on markets other than those in OECD countries are somewhat limited. Shum and Tang (2006) test common risk factors in assessing returns in Asian stock markets, using a sample of asset listed on the Hong Kong, Singapore and Taiwan Stock Exchanges. They found that augmented model that includes size and book-to-market ratios reports no significant improvement over the traditional CAPM. Only with past values of these variables is there any enhanced accuracy of asset pricing in these markets.

1.1.3 CAPM and Asset Pricing

There are three models covered in this study of Capital Asset Pricing Model: Sharpe-Lintner Version, Black version conditional version. Sharp-Lintner model is static. The parameters are considered to be time invariant which is inconsistence with market dynamics. In addition, the risk-free rate may be discordant with the model.

Black version of CAPM fills the gap of CAPM when the riskless asset is absent. He suggest to use zero beta portfolio. The conditional version of CAPM incorporate the time varying aspect by using time series models such as the ARCH models.

1.1.4 Nairobi Securities Exchange

Iraya and Musyoka (2013) give detailed description of the Nairobi Securities Exchange (NSE), formerly Nairobi Stock Exchange, as the principal stock exchange of Kenya. It began in 1954 as an overseas stock exchange while Kenya was still a British colony with permission of the London Stock Exchange. The NSE is a member of the African Securities Exchanges Association. It is Africa's fourth largest stock exchange in terms of trading volumes, and fifth in terms of market capitalization as a percentage of GDP. The Exchange works in cooperation with the Uganda Securities Exchange and the Dar es Salaam Stock Exchange, including the cross listing of various equities. NSE is reorganized into ten independent market sectors including: Agricultural, Commercial and Services, Telecommunication and Technology, Manufacturing and Allied, Banking, Automobiles and Accessories, Insurance, Energy and Petroleum, Construction and Allied and Investment. Two indices are popularly used to measure performance. The NSE 20-Share Index has been in use since 1964 and measures the performance of 20 blue-chip companies with strong fundamentals and which have consistently returned positive financial results. The other index is the NSE All Share Index (NASI) which was introduced as an alternative index. Its measure is an overall indicator of market performance. The Index incorporates all the traded shares of the day (NSE, 2013).

1.2 Research Problem

Different theories have been proposed in the application of CAPM. The variance-mean portfolio theory proposes that investors make decision based on the mean and covariance of the return of the asset only. That for a give risk, the investors prefer maximum returns from their investment. Empirical studies have established that efficient frontiers cannot be based only on mean and variance of return.

The arbitrage pricing theory proposed by Ross (1976) is a general theory of asset pricing that holds that the expected return of a financial asset can be modeled as a linear function of various macro-economic factors or theoretical market indices, where sensitivity to changes in each factor is represented by a factor-specific beta coefficient. The model-derived rate of return will then be used to price the asset correctly - the asset price should equal the expected end of period price discounted at the rate implied by the model. If the price diverges, arbitrage should bring it back into line. The major limitation of the theory is that is assumes normality of high frequency returns.

Numerous studies have examined the effectiveness of the original Capital Asset Pricing Model of Sharpe (1963, 1964), Lintner (1965), Mossin (1966) and most have found that for developing country markets this is subject to considerable ambiguity. More recently, additional factors have been proposed to provide a more reliable explanation of the cross section of average returns. These include firm size, the book to market equity ratio, the price earnings ratio, the cash flow to price ratio and the performance of the firm in terms of sales growth (Shum and Tang, 2005). A major innovation to asset pricing was proposed by Fama and French (1993) by introducing a number of changes to previous research, including the set of asset returns, the variables in the model and the estimation approach, choosing a time series cross section method rather than simply a cross section dimension.

Most economic and financial phenomena are modeled using probability distributions with finite variance. In particular, the CAPM and its innovations have been developed by many authors in a gaussian framework. However, the assumption of gaussianity is in general not verified empirically. It has long been known that high frequency (daily or weekly) financial returns from financial market variables such as exchange rates, equity prices, and interest rates used in financial applications are characterized by non-normality. The empirical distribution of such returns is more peaked and has fatter tails than the normal distribution, which implies that changes in return occur with a higher frequency than under normality. In addition it is often skewed towards the left tail and has a kurtosis greater than three.

Mandelbrot (1963) and Fama (1965) proved that empirical distribution of asset returns such as stocks, foreign currencies, exchange rates and interest rates conform better to stable distributions than to normal distribution. These distributions are heavy tailed, skewed and peaked. However, the asymmetric stable Paretian, which is theoretically well motivated, has the drawback that the variance does not exist, an assumption which might be deemed too extreme for certain applications (Paolella, 2007).

The problem therefore in dependence modeling using copulas approach is to fit non-gaussian distribution to the high frequency financial returns. We propose the class of generalized hyperbolic distributions introduced by Barndorff-Nielsen (1977) which is a mixture of the normal variance-mean model with the generalized inverse gaussian distribution as the mixing distribution. The following are the stylized facts of the distribution: the Generalized Hyperbolic Distribution is very flexible in the sense that it nests several other distributions, namely: Variance-Gamma distribution, Normal inverse

Gaussian distribution, scaled and shifted student t-distribution, hyperbolic Distribution, etc. The Generalized Hyperbolic Distribution features both fat tails and skewness. These properties account for some of the frequently reported stylized facts of financial returns and also financial returns volatility.

In recent years, risk management analysis has become increasingly important to financial institutions due to the rapid globalization and increased trading volumes with the associated potential risk. Regulators are beginning to design new regulations around it, such as bank capital standards for market risk and the reporting requirements for the risks associated with assets used by corporations. One of the fundamental issues in the financial risk management is to fully characterize the distribution of the returns. In other words, a good approximation for the unconditional distribution of the returns is very important for a further risk construction. Thus, to reflect this aspect in the CAPM is important so as not to underestimate the risk associated with the company's returns.

Therefore, in this study the research question is: does the applicability of the CAPM in NSE justifiable?

1.3 Objective of the study

To test the validity of CAPM in the Nairobi Securities Exchange.

1.4 Value of Study

It has long been Known that high frequency (daily or weekly) financial returns from financial market variables such as exchange rates, equity prices, and interest rates used in financial applications are characterized by non-normality. The empirical distribution of such returns is more peaked and has fatter tails than the normal distribution, which implies that changes in return occur with a higher frequency than under normality. In addition it is often skewed towards the left tail and has a kurtosis greater than 3. A promising non-normal distribution with mentioned stylized facts is the generalized hyperbolic (GH) distribution. This is a Normal Variance-Mean (NVM) model with generalized Inverse gaussian (GIG) distribution as the mixing distribution. The Generalized Hyperbolic Distribution is very flexible in the sense that it nests several other distributions, namely: Variance-Gamma distribution, Normal inverse Gaussian distribution, scaled and shifted student t-distribution, hyperbolic Distribution, etc. Its parameter estimation using the maximum likehood estimation via the EM-algorithm (expectation maximization algorithm) introduced by Liu and Rubin (1977) is easily programmable.

In practice, this class of distribution, when used in the framework of CAPM, it substitutes the assumption of normality which is not supported in empirical studies. GH distributions are heavy tailed. The normal distribution underestimates the risks facing the company. Thus, using GHD implies that changes in return occur with a higher frequency than under normality. This feature in financial modeling ensures that risk in not underestimated which leads to making informed decisions.

CHAPTER TWO: LITERATURE REVIEW

2.1 Introduction

In this chapter, we start by reviewing the theoretical framework of CAPM. It includes theories on testing of CAPM. The tradition CAPM is reviewed and its weakness briefly discussed. In the empirical literature, different innovation proposed are mentioned and their weakness illustrated. Attempts to address the weakness are sequentially discussed. We then finish by making the summary of the chapter.

2.2 Theoretical Framework

Most economic and financial phenomena are modeled using probability distributions with finite variance. In particular, the CAPM and its innovations has been developed by many authors in a gaussian framework. However, the assumption of gaussianity is in general not verified empirically. It has long been Known that high frequency (daily or weekly) financial returns from financial market variables such as exchange rates, equity prices, and interest rates used in financial applications are characterized by non-normality.

The study includes the theoretical derivation of equilibrium model, usually referred to as capital asset pricing model (CAPM). This model was developed almost simultaneously by Sharpe (1964), Treynor (1961), while Lintner (1965), Mossin (1966) and Black (1972) have extended and clarified it further. The variation through time in expected returns is common in securities and in related in plausible ways to business conditions. Therefore modified version of the asset-pricing model, known as conditional capital asset pricing model (CCAPM) is derived from static CAPM. An alternative equilibrium asset-pricing model, called the arbitrage asset pricing theory (APT) was developed by Ross (1976).

The fundamental principles underlying the arbitrage prong theory are also discussed the empirical literature is reviewed and the critical analysis of empirical and theoretical model are provided. The success of CAPM is inherent in the accurate computation of the beta of the asset. We compute the beta using the copulas approach while using the GHD as the marginal laws.

2.2.1 The variance-mean portfolio theory

Different theories have been proposed in the application of CAPM. The variance-mean portfolio theory proposes that investors make decision based on the mean and covariance of the return of the asset only. That for a give risk, the investors prefer maximum returns from their investment. Empirical studies have established that efficient frontiers cannot be based only on mean and variance of return.

2.2.2 The arbitrage pricing theory

The arbitrage pricing theory proposed by Ross (1976) is a general theory of asset pricing that holds that the expected return of a financial asset can be modeled as a linear function of various macro-economic factors or theoretical market indices, where sensitivity to changes in each factor is represented by a factor-specific beta coefficient. The model-derived rate of return will then be used to price the asset correctly - the asset price should equal the expected end of period price discounted at the rate implied by the model. If the price diverges, arbitrage should bring it back into line. The major limitation of the theory is that is assumes normality of high frequency returns.

2.3 Empirical Literature

Numerous studies have examined the effectiveness of the original Capital Asset Pricing Model of Sharpe (1964) and Lintner (1965) and most have found that for developing country markets this is subject to considerable ambiguity. More recently, additional factors have been proposed to provide a more reliable explanation of the cross section of average returns. These include firm size, the book to market equity ratio, the price earnings ratio, the cash flow to price ratio and the performance of the firm in terms of sales growth as explained by (Shum and Tang, 2005). These innovations capture the factors affecting the returns of a company beyond the market return in the traditional CAPM. However, Shum and Tang (2005) for the variables in the model and the estimation approach, use simply a cross section dimension which does not take into consideration the changes over time.

The Sharpe-Lintner model is the extension of one period mean-variance portfolio models of Markowitz (1959) and Tobin (1958), which in turn are built on the expected utility model of von Nuemann and Morgenstern (1953). The Markowitz mean variance analysis are concerned with how the consumer investor should allocate his wealth among the various assets available in the market, given that he is one-period utility maximize. The Sharpe-Lintner asset pricing model then uses the characteristics the consumer wealth allocation decision to derive the equilibrium relationship between risk and expected return for assets and portfolios. In the development of capital asset pricing model simplifying assumption about the real world are used in order to define the relationship between risk and return that determines security prices. These assumptions are, (a) all investors are risk-averse individuals, who maximize the expected utility of their end of period wealth, (b) the investors are price takers and have homogenous expectations about asset returns that have joint normal distribution, (c) there exist a risk-free asset such that investor may borrow or lend unlimited amounts at the risk-free rate, (d) the quantities of asset are fixed, also all assets are marketable and perfectly divisible, (e) asset markets are frictionless and information is costless and simultaneously available to all investors, and (f) there are no market imperfections such as taxes, regulations, or restrictions on other sellings. Sharpe and Lintner thus making a number of assumptions extended Markowitz's mean variance framework to develop a relation for expected return.

Most tests of the asset pricing models have been performed by estimating the cross sectional relation between average return on assets, and their betas over some time interval and comparing the estimated relationship implied by CAPM. The time series estimation approach is also used in the literature.

In the absence of riskless asset Black (1972) has suggested to use zero beta portfolio R_z that is $(cov(R_{z,}\ R_m))= 0$, as a proxy for riskless asset In this case CAPM depends upon two factors; zero beta and non zero beta portfolios, and it is refereed as two factor CAPM, which may be represented as,

$$E(R_i) = E(R_z)+\beta_i(E(R_m)-E(R_f))$$

In excess return form

$$E(R_i)-E(R_z)=\beta_i(E(R_m)-E(R_f))$$

The zero-beta model specifies the equilibrium expected return on asset to be a function of market factor defined by the return on market portfolio R_m and a beta factor defined by the return on zero-beta portfolio-that is minimum variance portfolio which is uncorrelated with market portfolio. The zero-beta portfolio plays the role equivalent to risk free rate of return in Sharpe-Lintner model. The intercept term is zero implies that CAPM holds. Gibbons (1982), Stambaugh (1982) and Shanken (1985) have tested CAPM by first assuming that market model is true, that is the return as the ith asset is a linear function of a market portfolio proxy.

$R_i = \alpha_i + \beta_i R_m - \mu_t$

Black (1972) two-factor version requires the intercept term $E(R_z)$ to be the same for all assets. Gibbons (1982) points out that the Black's two factor CAPM requires the constraint on the intercept of the market model.

$\alpha_i = E(R_z)(1-\beta_i)$

for all the assets during the same time interval. When the above restriction is violated the CAPM must be rejected. Stambaugh (1982) has estimated the market model and using the Lagrange multiplier test has found evidence in support of Black's version of CAPM. Gibbons (1982) has used a similar method as used by Stambaugh but employed likelihood ratio test (LRT), MacBeth (1975) has used Hostelling T^2 statistics to test the validity of CAPM.

The traditional CAPM, which describes stock return solely on measure, is based on the assumption that all market participants share identical subjective expectations of mean and variance of return distribution, and portfolio decision is exclusively based on these moments. But empirical evidence from literature suggests a deviation of the model from its formal theory. It has been observed that return distribution varies over time Engle (1982) and Bollerslev (1986). In other words, the stock return distribution is time variant in nature and hence, the subjective expectation of moment differs from one period to another. This implies that the investor expectations of moments behave like random variables rather than constant as assumed in the traditional CAPM for stock returns. The main proposition while taking care of time varying moments in CAPM is that, the investors still share identical subjective expectations of moments but these moments are conditional on the information at the time t. In symbols the conditional version of Sharpe-Lintner CAPM hereafter referred as conditional CAPM from Equation (1) can be written as

$R_{it} = R_{ft} + \lambda cov(u_{it}-u_{mt} \mid I_{t-1})+u_{it}$

Where:

u_{it} is $(R_{it} – E (R_{it} \mid I_{t-1}))$

u_{mt} is $((R_{mt} – R_{ft}) + \lambda var(R_{mt} \mid I_{t-1}))$

I_{t-1} is the information.

In earlier research works the presence of time varying moments in return distribution has been in the form of clustering large shocks of dependent variable and thereby exhibiting a large positive or negative value of the error term by Mandelbrot (1963) and Fama (1965). A formal specification was ultimately proposed by Engle (1982) in the form of Autoregressive Conditional Hetroscedastic (ARCH) process. Some of latter studies have attempted to improve upon Engle's ARCH specification Engle and Bollerslev (1986). The approaches which are helpful in specifying functional form of error term in the test of

CCAPM include the approaches given by Engle and Bollerslev (1986), (Bollerslev et al., 1992) and (Ng et al., 1992) in case of family of ARCH model.

A major innovation to asset pricing was proposed by Fama and French (1993) by introducing a number of changes to previous research, including the set of asset returns, the variables in the model and the estimation approach, choosing a time series cross section method rather than simply a cross section dimension. However, their results confirm that the augmented model that includes size and book-to-market ratios reports no significant improvement over the traditional CAPM. Only with past values of these variables is there any enhanced accuracy of asset pricing in these markets.

Gathoni (2002) did a study on forecasting ability of valuation ratios (Nairobi Stock Exchange). She did predictive regression model on a small sample of fourteen organizations with a financial year end of 31st Dec, over a period of five years (1996 to 2000). The ratios were then lagged for one quarter in order to see what impact this had on the predictive ability of the valuation model. She concluded that price earnings ratio explains future stock returns. She also concluded that price earnings ratio have predictive ability in majority of samples observed and are again determinant of future stock returns. However, she did not address the time series aspect which affects the returns of any asset.

Kiweu (1991) did a study to determine the behavior of share prices in the Nairobi Stock Exchange. He did examine the behavior of ordinary share price of ten selected "blue chip" companies in the Nairobi Stock Exchange. He investigated the behavior of bid price change over five years from January 1986 to December 1990. He concluded that weekly returns of shares traded in the Nairobi Stock Exchange are serially independent (random). The evidence presented suggested that no important dependencies could be identified in the stock market. He did not however model the distribution of the weekly returns or suggest whether they are normally distributed.

Asiemwa (1992) did an empirical study to identify the relationship between investments ratios and share performance of companies quoted on the NSE. She did multiple regression analysis of establish the relationship between investment ratios and share price and concluded that earnings per share, dividend per share, price earnings and dividend yield have a significant effect on share prices. She concluded that a significant association between share prices and investment ratios exists. She however assumed that the empirical data is normally distributed which empirically is incorrect.

Mwangi (1997) did a study to analyze the price movement for selected stocks in Nairobi Stock Exchange. He developed a model using a PC (version) software package and using this model; he computed and compared the prices from the month of Jan, 1992 to April, 1997 with the actual ones. He did t-test to determine whether the two prices were significantly different from one another. He concluded that it is not always possible to develop models that are only as good as being proxy for the investor's decision process and are limited by the inaccuracies in estimating future earnings of the company. At best they are only a framework for analyses which is useful for structuring the way an investor can conceptualize share valuation. He however used the pooled variance approach which assumed that sampled data comes from population with equal variance which is not necessarily correct. The Welch approach is a better alternative.

(Bruce and Jenifer, 2007) incorporate some aspects of the Fama and French method, that is, the time series approach, and the inclusion of a firm size factor. They remark that they are the first to incorporate a measure of illiquidity, following Liu (2006), in the specific context of emerging markets. Liquidity is a major factor in explaining asset returns and a number of measures have been suggested. These include the quantity of trades (Datar et al. 1998), the speed of trades Liu (2006) and the costs of trading Amihud and Mendelson (1986) or by the impact that a trade has on price Amihud (2002), Pastor and Stambaugh (2003). However, many of these aspects are difficult to capture in emerging markets. Bruce and Jenifer (2007) focus on the price effect. The market wide illiquidity factor is constructed following the work of Amihud (2002) and is based on intraday trading volumes and order flow that impacts stock prices. The countries in the study are all in southern and eastern Africa and, with the exception of South Africa, represent some of the most illiquid financial markets in the world. However, they are also countries that have attracted some interest from international investors and some multinationals, particularly those in the mining sector, for example, Anglo American, Anglo Gold, and Anglo Ashanti and the financial sector, such as Old Mutual, Standard Bank, Standard Chartered, Barclays, Société General, and BNP Paribas. These companies dominate the domestic markets and create a very uneven degree of liquidity. They conclude that the differences between the periods for South Africa and Kenya reflect political and economic events that influence markets. However, the use of such models for Swaziland and Mozambique when analyzed separately are less positive. The illiquidity in these markets is too extreme that any form of CAPM fails to predict excess returns with any degree of confidence. This evidence suggests that while the firm size factor is as important in pricing assets as in developed markets the major risk component in emerging markets is illiquidity. However, they assume that returns are normally distributed.

The assumption of gaussianity is in general not verified empirically. It has long been Known that high frequency (daily or weekly) financial returns from financial market variables such as exchange rates, equity prices, and interest rates used in financial applications are characterized by non-normality. Barndorff-Nielsen (1977) constructed the class of GHD which is a mixture of normal-variance mean model with the GIG distribution as the mixing distribution. The GHD was first used in finance by Eberlein and Keller (1992) to model financial stock returns. However the systematic risk to the company was not addressed explicitly through multivariate distribution as captured by the beta of the asset (company).

2.4 Summary of Literature Review

The CAPM and its innovation success depend on the factors used to capture the randomness in asset returns. These factors are quite many and attempts to make a choice between them is rather difficulty. The gap exist in identifying the factors explaining the variability of returns under the usual assumption of normality of returns. In addition, is impractical to know all the factors that may exist for all asset of interest. The most powerful and attractive way of addressing this problem, is modeling the return of the asset and the market using hyperbolic distribution. Using these distributions, and the correct choice of copula, then calculate the beta of the company. Therefore, modeling the returns of assets (companies) accurately suffice using many factors under normal distribution to address indirectly the assumption of normality.

The failure of the CAPM to capture the time varying aspect of the returns, fails to adequately explain the time varying aspect of returns. Extending the model to incorporate the time series model, address the problem.

However, the time series models assume normality distribution of returns which is contrary empirical studies. This problem can be addressed by using non-normal distribution such as the class of hyperbolic distribution.

The distribution of returns may fit to different distributions in the class of GHDs. To capture this aspect in modeling, the copula functions are used to simultaneously model the joint distribution.

CHAPTER THREE: RESEARCH METHODOLOGY

3.1 Introduction

In this chapter, the research design used is presented and justified. The population of study, sampling methods, data collection techniques and data analysis methods used are presented.

3.2 Research design

In this study descriptive survey was used. In the study, the CAPM assumption underlying the model was tested in the NSE to evaluate its validity. The rapid globalization and increased trading volumes with the associated potential risk in the NSE have complicated the market dynamics for the assumptions of CAPM to hold. In particular, the assumption of normality of asset returns. The goal was to describe relevant aspects of the CAPM in NSE in testing its validity which makes descriptive survey valid in the study.

3.3 Population of Study

The target population of study was all the listed companies in the Nairobi Securities Exchange. The market returns for these companies can all be tested for normality assumption irrespective of the sector the company is in.

3.4 Sample and Sampling methods

The study adopted a random survey approach since the distribution of weekly empirical returns of the listed companies are all affected by the market dynamics. All public listed company returns can be computed to access their distribution. As such, random sampling of any company is suitable for study since the market dynamics affect all companies.

The historical cross sectional correlational survey design that will be used relies on the population of all the companies that have been quoted in the equity securities market of the Nairobi Securities Exchange for the four year period between January 2008 and December 2013. This time period is considered because it coincides with the time the Nairobi All Share Index (NASI) has been operational at the NSE. NASI is a price index that shows the daily performance of all the equity securities quoted at the NSE.

3.5 Data Collection techniques

The research relied upon secondary data obtained from Nairobi Securities Exchange. High frequency data (weekly) was obtained for the company share price. Correspondingly, the returns were obtained.

3.6 Data Analysis techniques

Exploratory data analysis was first performed to the data to test whether its normally distributed. Here, QQ-plots and histogram were used. Further, statistical test for normality like Shapiro test and Bartellet test were also used to ascertain.

The excess weekly returns of the risk-free rate for the company and the market was obtained. The data was then analyzed using the R program, specifically to estimate the beta of the company using the simple regression approach. The result was compared to the proposed model and interpreted so as to make a conclusion. To compute the returns, we let (P_t) for $t \geq 0$ denote the price process of a security, in particular of a stock. In order to allow comparison of investments in different securities we shall investigate the rates of return defined by ($X_t = \log P_t - \log P_{t-1}$). The reason for this is that the return over n periods, for example n days, is then just the sum of

$X_t + X_{t+1} + X_{t+2} + \ldots + X_{t+n-1} = \log P_{t+n-1} - \log P_{t-1}$

Which does not hold for Y_t defined by?

$Y_t = (P_t - P_{t-1}) / P_{t-1}$

The Generalized Normal Variance-Mean Mixture Mechanics. The generalized hyperbolic distribution proposed by Barndorff-Nielsen (1977) and first used in finance by Eberlein and Keller (1992) is a normal variance-mean mixture where the mixing distribution is generalized inverse Gaussian. Thus if the conditional distribution of X given W is normal distribution with parameters (μ+w,w), and that W is a random variable following the generalized inverse gaussian distribution, the distribution of X is generalized hyperbolic distribution.

Put mathematically,

$X|W=w \sim \text{normal}(\mu+w, w)$

And

$W \sim GIG(\lambda, \delta, \gamma)$

Then the marginal distribution of X will be generalized hyperbolic distribution, i.e,

$X \sim GHD(\lambda, \alpha, \beta, \delta, \mu)$

Estimation of parameters is done via the Estimation Maximization (EM) algorithm which is a powerful method for calibration and has been used for some time in various contexts. Dempster, Laird and Rubin (1977) showed that the EM algorithm can be used to find maximum likelihood estimates for data containing missing values or being considered as containing missing values. Karlis (2005) states that this formulation is particularly suitable for distributions arising as mixture since the mixing operation can be considered as producing missing data. He further states that an important feature of the EM algorithm is that it is not merely a numerical technique but it offers statistical insight.

We used QRM package installed in R statistical software that impliment the EM algorithm for parameter estimation.

The theory of copulas dates back to Sklar (1959), but its application in financial modeling is far more recent and dates back to the late 1990s. The strongest argument for using copula approach is that one can separate the dependence structure from marginal distributions completely. In finance modeling, it is a big advantage to have this property of separation. With this separation, the choice of dependence of modeling is independent from the choice of modeling of the marginal. This adds great flexibility to the modeling of financial products that depend on the joint law.

CHAPTER FOUR: DATA ANALYSIS

4.1 Introduction.

In this chapter the validity of CAPM will be considered. Weekly returns of the dataset will be computed. Exploratory data analysis, in particular the QQplots, will be performed to test whether the returns are normally distributed.

Generalized hyperbolic distributions will then be fitted to the returns. Alkaike Information Criteria (AIC) will be used to select the best model. The joint distribution between the specific company and NSE20 index is modeled using the copula function. The variance of the NSE20 index and covariance between the companies returns and NSE20 index returns will be computed. The beta of the company will then be calculated and the result compared with that of regression and approximation methods.

4.2 Model Choice

4.2.1 NSE20 index weekly returns

Choice of model is based on AIC.

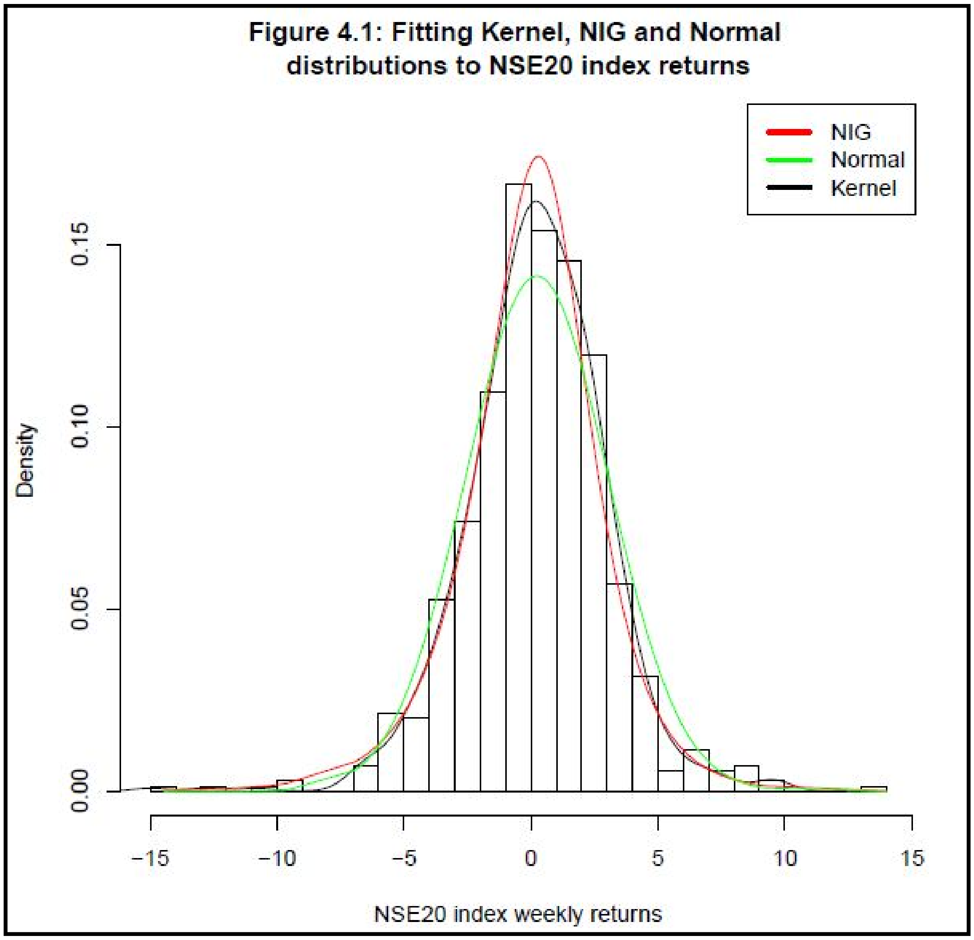
Table 4.1: Model choice of NSE20 returns based on AIC

Model	Likehood	AIC
NIG	-3120.760	6249.520
t	-3121.047	6250.093
ghyp	-3120.626	6251.253
hyp	-3121.886	6251.772
VG	-3122.992	6253.984
Normal	-3163.906	6331.812

Source: Author’s computation

From the table, the Normal Inverse Gaussian distribution is the best model to model weekly returns of the NSE20 index. This is because based on the alkaike information criteria, the NIG has the minimum AIC.

Fitting the NIG to weekly returns gives the figure below:



Source: Based on author's computation

The Kernel distribution depicts the empirical distribution, which is presented by the histogram, of the returns based on the computed quantiles. Therefore the best distribution will imitate the kernel. From the figure above, the normal distribution (green) fails to model empirical returns precisely. The normal inverse Gaussian (NIG) distribution (red) closely imitates the kernel distribution compared to the normal. By far, the NIG is the best model.

The parameter estimates of the NIG are:

Table 2: parameter estimate of the NIG distribution for the Safaricom weekly returns.

Parameter	Value
β	-0.006709572
α	0.04492583
μ	3.493509
δ	20.80091
λ	0.500000
γ	0.04442198

Source: Author's computation

The table gives parameter estimate for the NIG distribution for the NSE20 index weekly returns. The λ is the index parameter, δ is the scale parameter, β models the skewness of the returns and μ is the location parameter.

Which then means that the expected return of the NSE20 index will be:

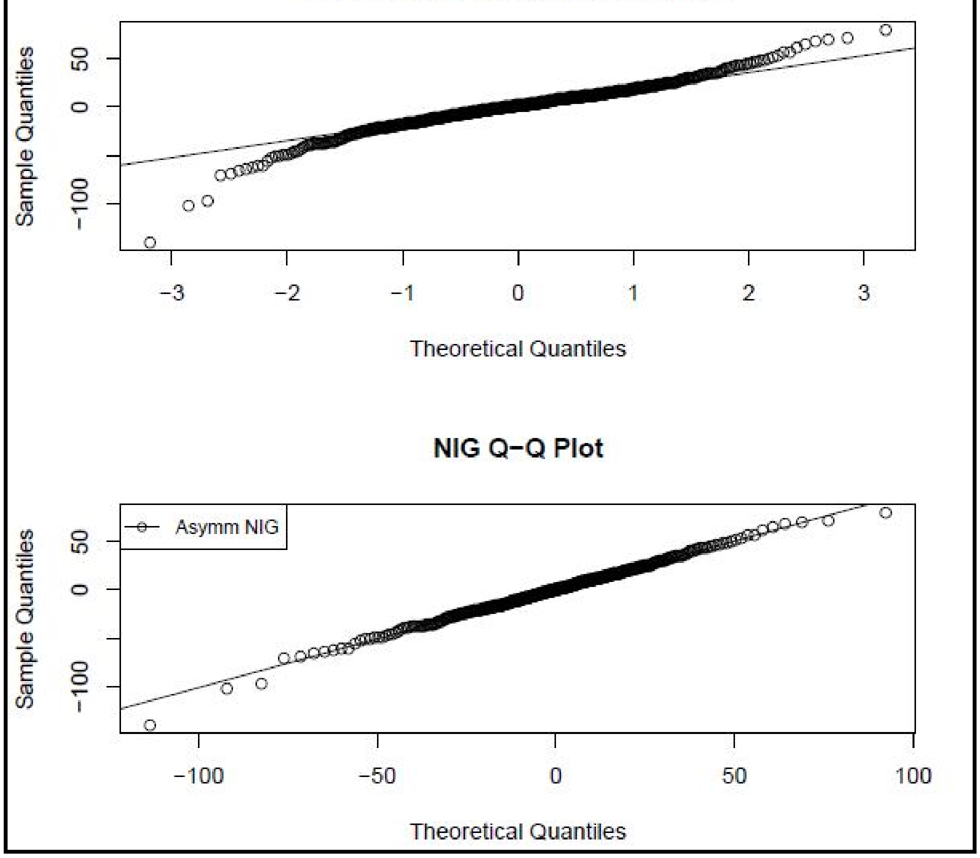
$$E(r_m) = \mu + \frac{\beta\delta}{\gamma} = 0.3517048$$

Therefore the annual rate of return will be:

$$0.1758519 \times 52.12 = 18.33085$$

The QQ-plots of weekly returns are as follows:

Figure 4.2: The comparison between the boxplot of the normal and NIG distributions



Source: Based on Author's Competition

The figure shows that the normal distribution is far from explaining the variability of the NSE20 weekly returns. The NIG distribution fits well and this support the choice of model. This is supported by the fact that most data point are close to the QQ-line for the NIG distribution compared to the normal distribution. In addition, the QQ-line is almost the diagonal of the plot for the NIG model compared to the normal model a fact that is required for the chosen distribution.

4.2.2 Safaricom company

Table 4.3: The choice of model based on AIC for safaricom company

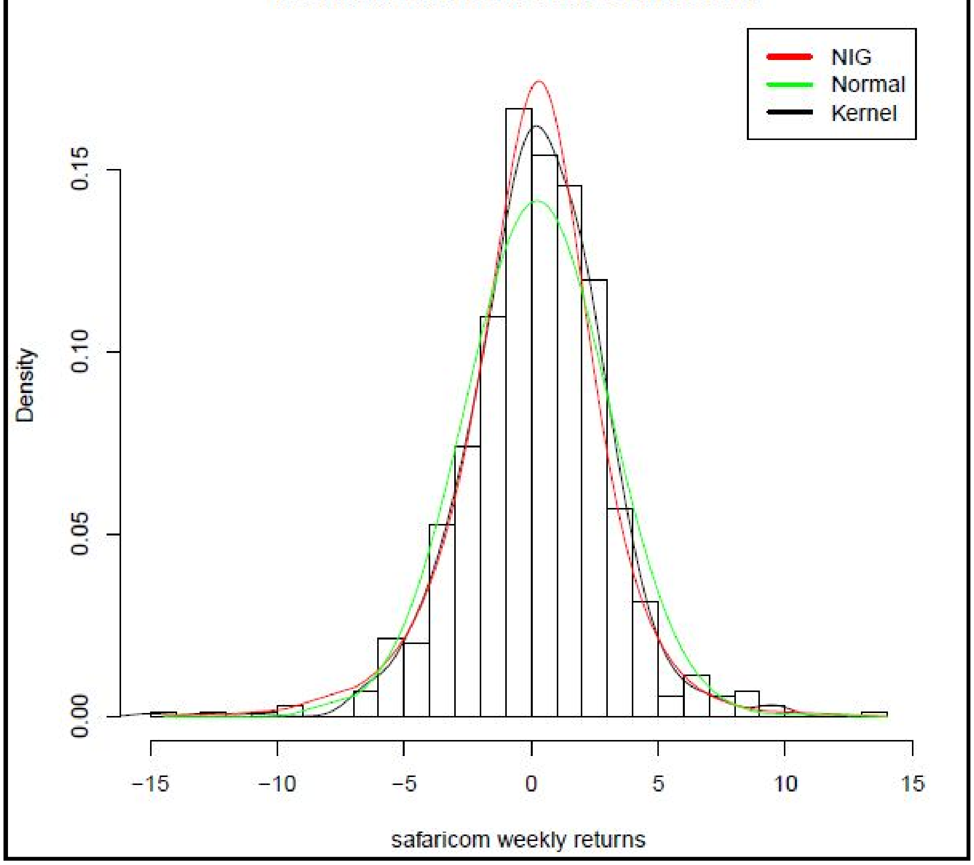
Model	Likehood	AIC
t	-2284.410	4576.819
NIG	-2285.220	4578.440
hyp	-2284.410	4578.820
ghyp	-2285.743	4579.485
VG	-2286.408	4580.816
Normal	-2316.426	4580.816

Source: Author’s computation

From the table, the t distribution is the best model to model weekly returns of the Safaricom returns. This is because based on the alkaike information criteria, it has the minimum AIC. The second best model for the Safaricom weekly returns is NIG distribution.

Fitting the distribution to weekly returns we obtain:

Figure 4.3: Fitting the Kernel, NIG, and Normal distributions for the Safaricom returns



Source: Derived from author's estimation of parameters

Here the Kernel distribution depict the empirical distribution, which is presented by the histogram, of the returns based on the computed quantiles. Therefore the best distribution will imitate the kernel. From the figure above, the normal distribution (green) fails to model empirical returns precisely. The normal inverse Gaussian (NIG) distribution (red) closely imitates the kernel distribution compared to the normal. By far, the NIG is the best model.

Here, the parameters of the Normal inverse Gaussian distribution is:

Table 4.4: Parameter estimate for the NIG distribution for the Safaricom Company

Parameter	Value
β	-0.05531936
α	0.226518
μ	2.640184
δ	8.688738
λ	0.500000
γ	0.2196592

Source: Author's computation

The table gives parameter estimate for the NIG distribution for the Safaricom weekly returns. The λ is the index parameter, δ is the scale parameter, β models the skewness of the returns and μ is the location parameter.

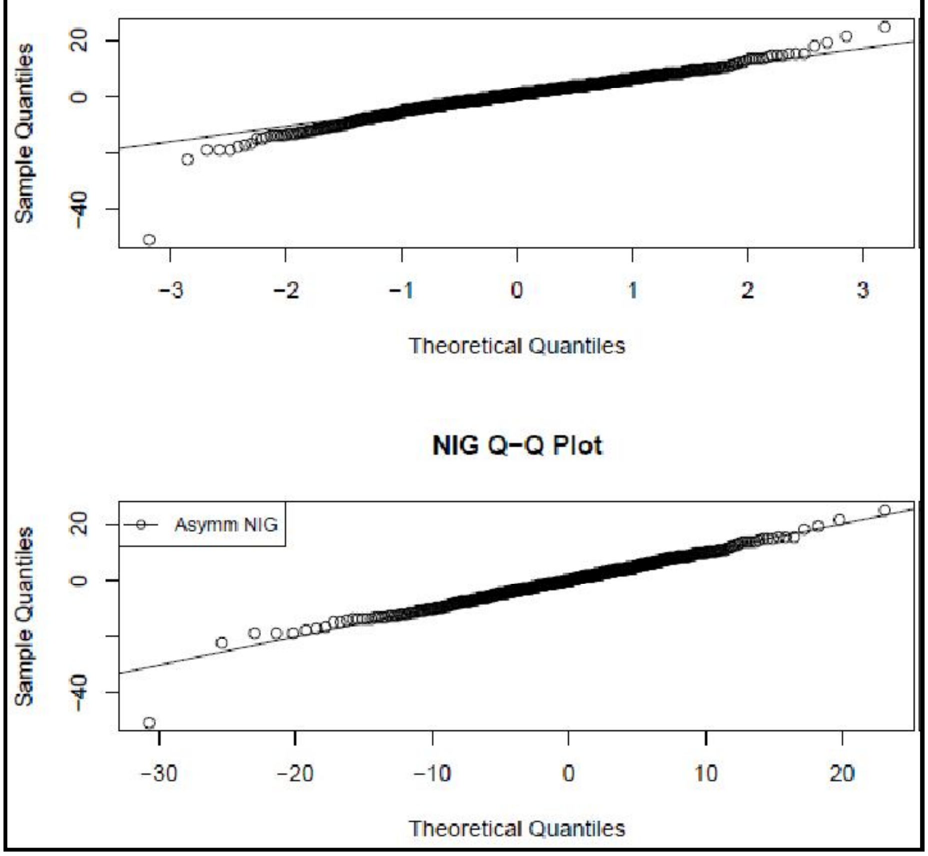
Which then means that the expected return of the Safaricom company will be:

$$E(r_s) = \mu + \frac{\beta\delta}{\gamma} = 0.4519965$$

Therefore the annual rate of return will be:

$$0.4519965 \times 52.12 = 23.55806$$

Figure 4.4: The comparison between the boxplot of the normal and NIG distributions



Source: Author's computation

The figure shows that the normal distribution is far from explaining the variability of the Safaricom weekly returns. The NIG distribution fits well and this support the choice of model. This is supported by the fact that most data point are close to the QQ-line for the NIG distribution compared to the normal distribution. In addition, the QQ-line is almost the diagonal of the plot for the NIG model compared to the normal model a fact that is required for the chosen distribution.

Parameters:

alpha.bar =1.429526

mu:

(2.050576, 4.750866)

gamma:

(2.050576, 4.750866)

sigma:

	ratem	ratense
ratem	40.81847	81.88455
ratense	81.88455	447.81578

Optimization information:

log-Likelihood = -5228.778

AIC = 10473.56

Fitted parameters: alpha.bar, mu, sigma, gamma; (Number: 8)

Number of iterations: 55

Converged: TRUE

The variance covariance matrix for the model is:

Table 4.5: The variance-covariance matrix for the NSE20 index and Safaricom returns

	ratem	ratense
Ratem	214.8861	707.0016
Ratense	707.0016	467.58064

Source: Author's computation

The table illustrates the variance-covariance matrix between the NSE20 index and safaricom weekly returns. These have been computed using the R-Gui statistical package. The corresponding matrix assuming that the returns are normally distributed is different.

Which implies that the beta of the company is:

$$\beta_s = \frac{707.0016}{467.58064} = 1.512042$$

Using the approximation method with assumption of normality:

$$\beta_s = \frac{531.0666}{481.7981} = 1.10226$$

Clearly the assumption on normality underestimates the systematic risk of the company.

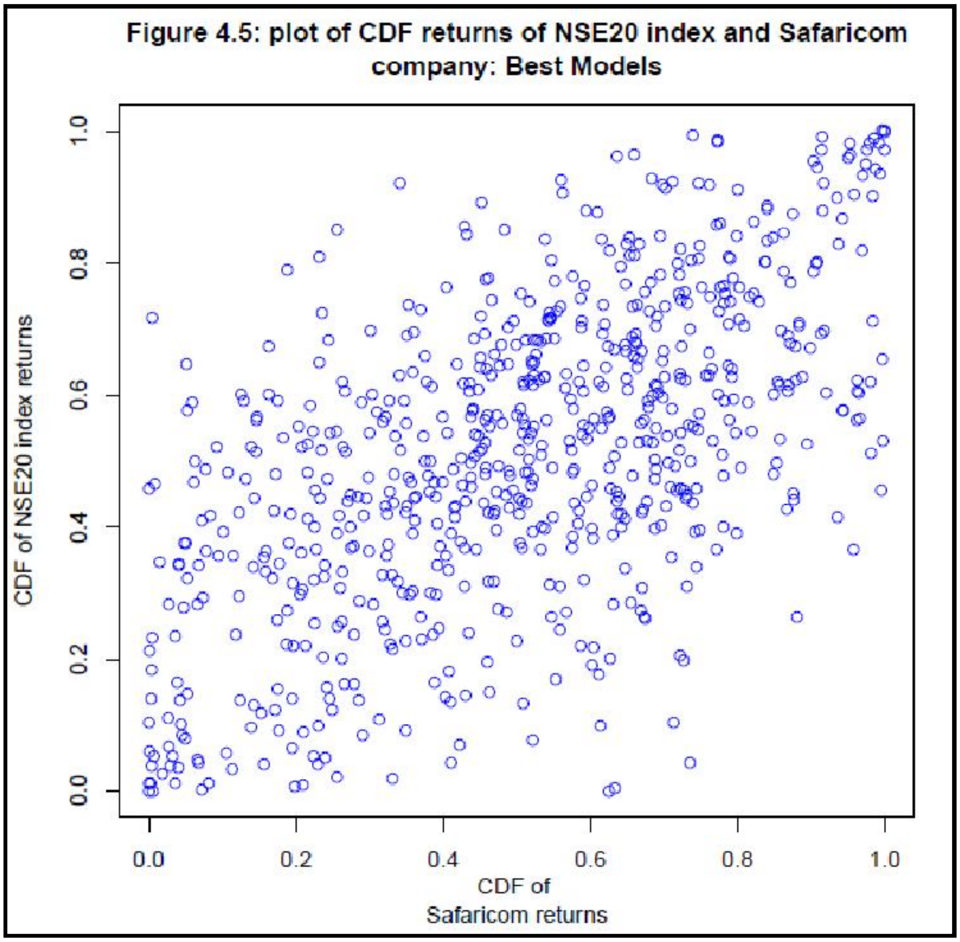
The expected return of the company based on the bivariate NIG distribution is:

$$E(r_m) = 11.4 + 1.512042(18.3305 - 11.4) = 21.87921$$

While that based on the normality assumption is:

$$E(r_m) = 11.4 + 1.10226(18.3305 - 11.4) = 19.03921$$

The scatter plot between the cumulative returns is:



Source: Author’s computation

The joint distribution of the returns of the NSE20 index and Safaricom is based on the scattet plot. It illustrates the dependence structure of the returns and guides the choice of copula. The figures represents tail dependence structure which points that the t copula which is the only elliptical copula with tail dependence is a favorable candidate for constructing joint distribution.

The copula parameter estimates are as follows:

Table 4.6: copula parameter estimates

copula	Parameter(s) estimate
gaussian	0.6026715
T	(0.6013155, 3.815338)
clayton	1.062018
Gumbel	1.66119
Frank	4.35389
Joe	1.836395

Source: Author's computation

The best copula choice based on the AIC for the bivariate data considered here is the t copula with parameters: 0.6013155 and 3.815338.

4.2.3 Mumias Sugar Company

Table 4.7:The model choice based on AIC

Model	Likehood	AIC
NIG	-1701.047	3410.093
t	-1701.120	3410.240
hyp	-1701.294	3410.588
VG	-1701.816	3411.632
ghyp	-1700.982	3411.963
Normal	-1718.622	3441.245

Source: Author's computation

From the table, the NIG distribution is the best model to model weekly returns of the Mumias Sugar returns. This is because based on the alkaike information criteria, it has the minimum AIC. The second best model for the Safaricom weekly returns is t distribution and so on.

Fitting the model to the returns we obtain:

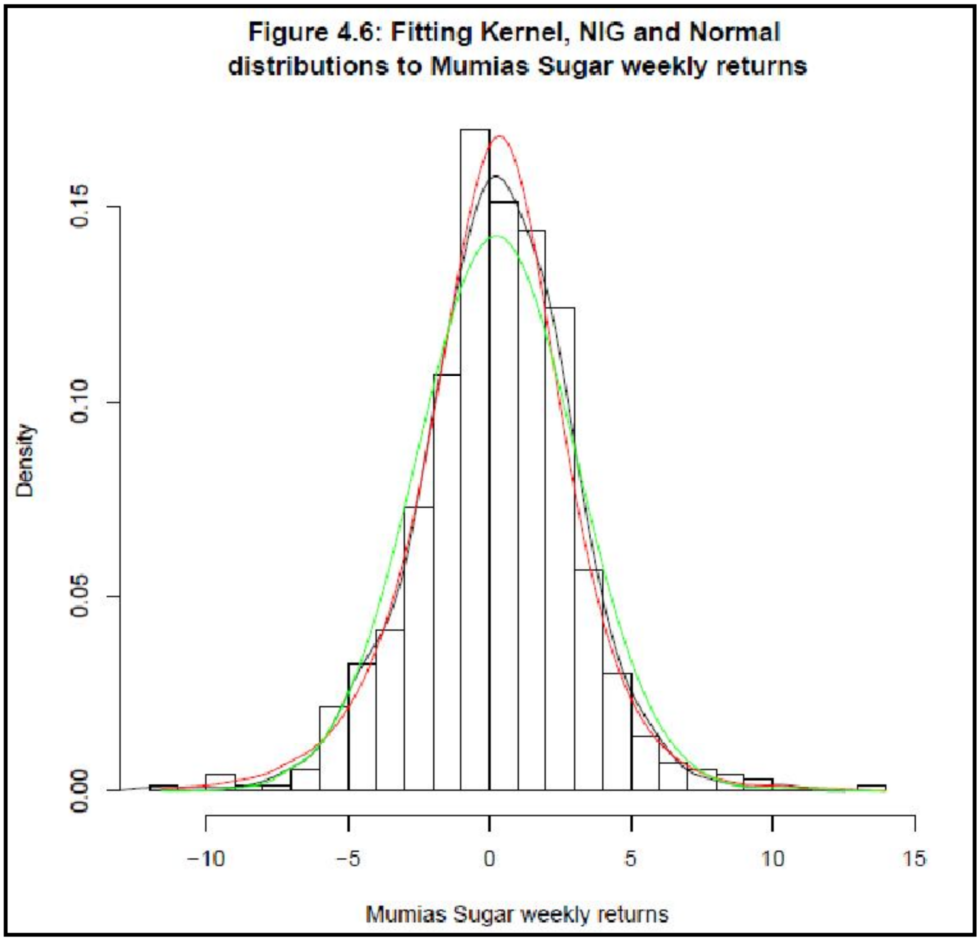


Figure 4.6: Fitting Kernel, NIG and Normal distributions to Mumias Sugar weekly returns

Source: Based on author's parameter estimation values

The Kernel distribution depict the empirical distribution, which is presented by the histogram, of the returns based on the computed quantiles. Therefore the best distribution will imitate the kernel. From the figure above, the normal distribution (green) fails to model empirical returns precisely. The normal inverse Gaussian (NIG) distribution (red) closely emitates the kernel distribution compared to the normal. By far, the NIG is the best model.

The parameter estimates of the NIG distribution are:

Table 4.8: Parameter estimation values for Mumias Sugar weekly returns

Parameter	Value
β	-0.01384742
α	0. 4661607
μ	0.3516722
δ	3.652304
λ	0.500000
γ	0.465955

Source: Author's Computation

The table gives parameter estimate for the NIG distribution for the Safaricom weekly returns. The λ is the index parameter, δ is the scale parameter, β models the skewness of the returns and μ is the location parameter.

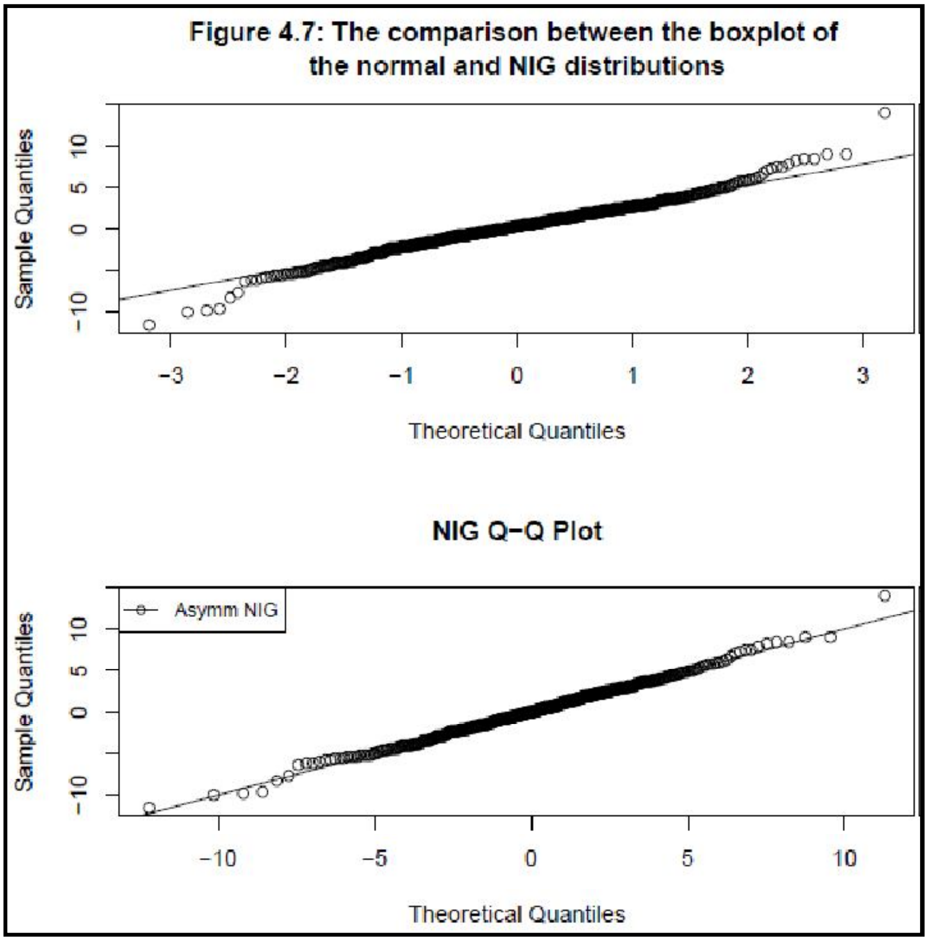
Which then means that the expected return of the Mumias company will be:

$$E(r_s) = \mu + \frac{\beta\delta}{\gamma} = 0.2431317 \times 1.49$$

Therefore the annual rate of return will be:

$$0.2431317 \times 52.12 \times 1.49 = 18.88132$$

Which give the following QQ-plots:



Source: Based on author's computation

The figure shows that the normal distribution is far from explaining the variability of the Safaricom weekly returns. The NIG distribution fits well and this support the choice of model. This is supported by the fact that most data point are close to the QQ-line for the NIG distribution compared to the normal distribution. In addition, the QQ-line is almost the diagonal of the plot for the NIG model compared to the normal model a fact that is required for the chosen distribution.

Therefore, fitting the bivariate (NIG) distribution to the weekly returns of Mumias Sugar Company and NSE20 index we get:

Parameters:

alpha.bar =1.383942

mu:

(0.4458086, 0.2529323)

gamma:

(-0.202703,1 -0.2353775)

sigma:

	ratem	ratense
ratem	40.81847	81.88455
ratense	81.88455	447.81578

Optimization information:

log-Likelihood = -2636.577

AIC = 5289.154

Fitted parameters: alpha.bar, mu, sigma, gamma; (Number: 8)

Number of iterations: 55

Converged: TRUE

The variance covariance matrix for the model is:

Table 4.9: variance covariance matrix for Mumias Sugar and NSE20 index returns

	ratem	ratense
Ratem	234.8762	487.5999
Ratense	487.5999	481.7981

Source: Author's computation

Which implies that the beta of the company is:

$$\beta_{mumias} = \frac{487.5999}{481.7981} = 1.012042$$

Using the approximation method with assumption of normality:

$$\beta_{mumias} = \frac{426.0549}{461.7981} = 0.9226$$

Clearly the assumption on normality underestimates the systematic risk of the company.

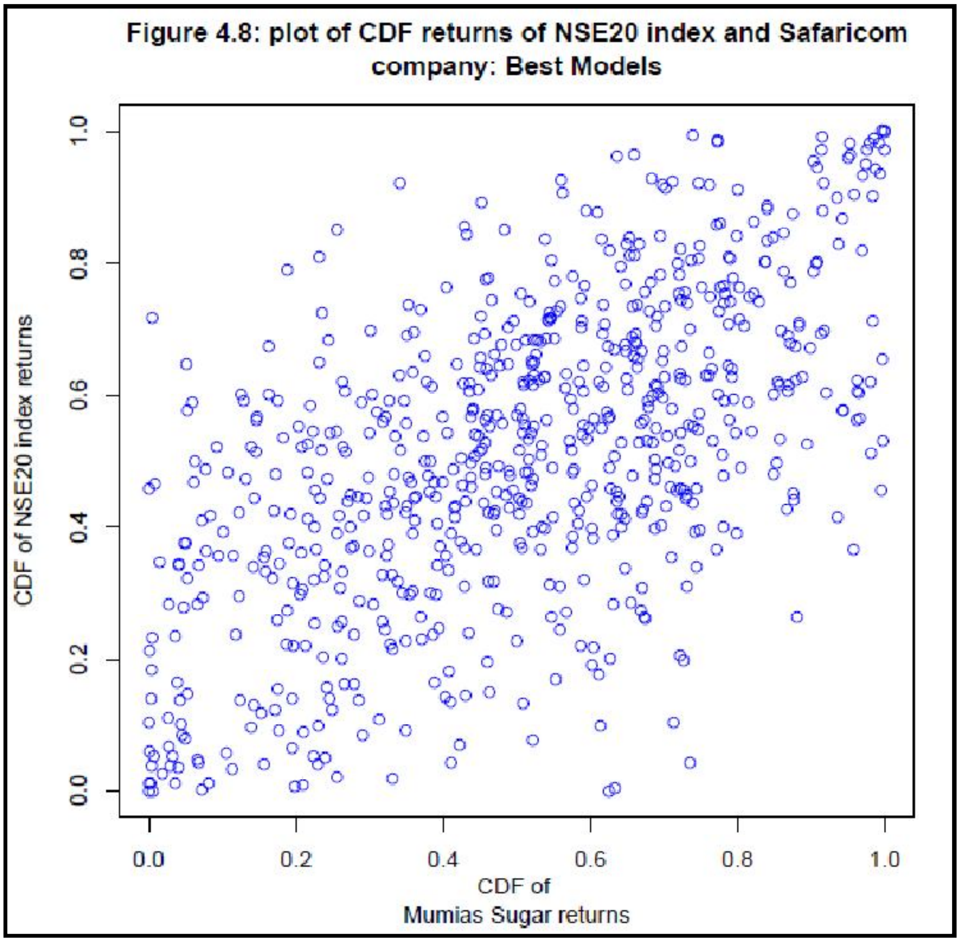
The expected return of the company based on the bivariate NIG distribution is:

$$E(r_m) = 11.4 + 1.012042(18.3305 - 11.4) = 18.41396$$

While that based on the normality assumption is:

$$E(r_m) = 11.4 + 0.9226(18.3305 - 11.4) = 17.79408$$

The scatter plot between the cumulative returns is:



Source: based on author's computation

The joint distribution of the returns of the NSE20 index and Safaricom is based on the scattet plot. It illustrates the dependence structure of the returns and guides the choice of copula. The figures represents tail dependence structure which points that the t copula which is the only elliptical copula with tail dependence is a favorable candidate for constructing joint distribution.

The copula parameter estimates are as follows:

Table 4.10: copula parameter estimate

copula	Parameter(s) estimate
gaussian	0.4209198
t	(0.4250304, 4.961417)
clayton	0.6258091
Gumbel	1.352555
Frank	2.703336
Joe	1.417481

Source: Author's computation

The best copula choice based on the AIC for the bivariate data considered here is the t copula with parameters: 0.4250304 and 4.961417.

CHAPTER FIVE: SUMMARY, CONCLUSION AND RECOMMENDATION

5.1 Introduction

In this chapter we consider the results in the previous chapter and make conclusion and recommendation as it regards the validity of CAPM.

5.2 Summary

The original CAPM and its innovation are based on the assumption of normality of empirical returns. However, the high frequency (daily and weekly) returns exhibit non-normal characteristics. The returns are usually skewed, heavy tailed and more peaked than the normal distribution. The dataset considered: NSE20 index, safaricom and Mumias Sugar weekly returns exhibit these properties.

The generalized hyperbolic distributions capture these properties. However, the difficulty of estimating the parameter of the distributions: NIG, variance-gamma, t, hyperbolic and generalized hyperbolic distributions come on the way of applying them. To overcome this difficulty, the Expectation maximization algorithm is used which is easily programmable.

The normal distribution underestimates the risk inherent in the company compared with the normal inverse Gaussian. Correspondingly, the beta of the company which measures the systematic risk of the company is underestimated.

The marginal distributions of the all the financial data considered follow the normal inverse Gaussian law. This ensures that the bivariate distribution considered have NIG distribution as their marginal laws.

The joint distribution of returns is modeled using the copula functions. copulae are of interest in finance because of two reasons: First, as a way of studying the dependence structure of an asset portfolio irrespective of its marginal asset-return distributions; and second, as a starting point for constructing multidimensional distributions for asset portfolios, with a view to simulation

As long as the underlying distribution of financial data is not accurately measured, the precision of using the CAPM is not guaranteed. Attempts to capture the variability of returns using various factors under the assumption of normality is incorrect and weak especially for high frequency financial data. The systematic risk as measured by the beta of the company under the generalized hyperbolic distribution depict the company's situation more accurately compared to the normal distribution of the financial data.

The boxplot figures shows that the normal distribution is far from explaining the variability of the Safaricom weekly returns. The NIG distribution fits well and this support the choice of model. This is supported by the fact that most data point are close to the QQ-line for the NIG distribution compared to the normal distribution. In addition, the QQ-line is almost the diagonal of the plot for the NIG model compared to the normal model a fact that is required for the chosen distribution.

The joint distribution of the returns of the NSE20 index and Safaricom is based on the scatter plot. It illustrates the dependence structure of the returns and guides the choice of copula. The figures represents tail dependence structure which points that the t copula which is the only elliptical copula with tail dependence is a favorable.

5.3 Conclusions

The assumption of normality of high frequency (daily or weekly) financial data does not hold. The weekly returns of Safaricom and Mumias companies fit well with the normal inverse Gaussian distribution.

The normal distribution underestimates the risk inherent in the company compared with the normal inverse Gaussian. Correspondingly, the beta of the company which measures the systematic risk of the company is underestimated.

The marginal distributions of the all the financial data considered follow the normal inverse Gaussian law. This ensures that the bivariate distribution considered have NIG distribution as their marginal laws.

The EM algorithm considered overcomes the difficulty of estimation of parameters. It ensures that the algorithm converges and can be easily programmed. Compared to other algorithm, the algorithm has the least number of iterations.

The copula functions models the bivariate distribution using the marginal distribution of the company and the market returns.

The CAPM formula is realistic when generalized hyperbolic distributions substitute the assumption of normality of financial data. As long as the underlying distribution of financial data is not accurately measured, the precision of using the CAPM is not guaranteed. Attempts to capture the variability of returns using various factors under the assumption of normality is incorrect and weak especially for high frequency financial data.

The systematic risk as measured by the beta of the company under the generalized hyperbolic distribution depict the company's situation more accurately compared to the normal distribution of the financial data.

5.4 Recommendation

We recommend using other normal mixture models with finite mixture of generalized inverse gaussian distribution as mixing distributions and making a comparison with the generalized hyperbolic distribution. In addition other models such as the stable paratian distribution and the skew normal mixtures should be used.

We also recommend assessing the significant of addition of factors under the assumption of normal mixtures to explain the variability of returns. The parameter estimation technique involves the EM algorithm. To enhance convergence, we recommend the extensions of the EM algorithms.

We recommend the use of mixtures of copula to model the complicated dependence structure. This is done by making the parameter of the standard copula random following a cetain distriburion.

We also recommend finite mixture of the GIG to be used as mixing distribution in the construction of the generalized hyperbolic distribution to make the model more competitive.

5.5 Limitation of the Study

The Study involved collection of financial data of Safaricom and Mumias Sugar companies over a long period which we had to purchase. Only daily share prices are quoted and free. This limited the number of companies studied.

The use of special function in the construction of the generalized hyperbolic distribution also made the study challenging and the reference materials were difficult to get. This made the study time consuming and limited the choice of mixing distribution.

The special functions in the distribution made the usual computation for parameter estimate difficult and this lead us to developing algorithms to overcome the difficulty. The algorithm could only cover two dimensional case only.

The only copula function considered are the standard copulas which are installed in the R statistical packages. This limited the use of other copulas relevant in the study.

The other limitation is the conditional distribution used in the study. The parameterization considered the case when the variance is variance in such a way that it influences the mean. The other cases are not considered such as the varying of the variance without influencing the mean as is the case of scale normal mixtures.

5.6 Suggestion for further studies

We suggest studying more companies in the NSE market to quantify the study. This should be done with regard to different industries to assess the distribution and sensitivity of high frequency returns.

We suggest the use of other mixing distribution in constructing the normal mixtures to be used. This will capture the different market dynamics that keep on changing over time. Such mixing distributions include the finite mixture of the generalized inverse Gaussian distribution.

We suggest the use of manual algorithm to compute the copulas not incorporated in statistical packages. This will increase the choice of copula function used in the study.

We also suggest the use of generalized representation of the conditional normal distribution to capture all the possible cases of the variation of the parameters.

REFERENCES

Dempster, A. P., Laird, N. M., and D. Rubin (1977). Maximum likelihood from incomplete data using the EM algorithm. *Journal Royal Statistical Society*, 39, 1-38.

Asiemwa T. K. J. (1992), The relationship between investment ratios and share performance of companies quoted on the Nairobi Stock Exchange. Unpublished MBA project, University of Nairobi.

Ball, R. (1978). Anomalies in relationships between securities and yield surrogates, *Journal of Financial Economics*, 6, 103-126.

Basu, S. (1977). Investment performance of common stocks and relation to their price-earnings ratios. A test of market efficiency, *Journal of Finance,* 32, 663-682.

Black, F. (1972). Capital Market Equilibrium with Restricted Borrowing. Journal of Business, 45, 444-455.

Bollerslev, T. (1986). Generalised Autoregressive Conditional Hetroscedasticity. *Journal of Econometrics,* 31, 307-327.

Bollerslev, T., Chou, R. Y., and Kroner K. F. (1992). ARCH Modeling in Finance: A Review of the Theory and Empirical Evidence. *Journal of Econometrics,* 55, 5-59.

Chen, N. (1983). Some Empirical Tests of the Theory of Arbitrage Pricing. *Journal of Finance*, 38(5), 1393-1414.

Iraya, C. and Musyoki, L. N. (2013). Performance of Socially Screened Portfolio at the Nairobi Securities Exchange. *International Journal of Humanities and Social Science3*(6)[Special Issue]

Karlis, D. (2002). An EM type algorithm for maximum likelihood estimation of the normal-inverse Gaussian distribution. *Statistics & Probability Letters*, 57,43-52.

Douglus, G. W. (1968). Risk in the Equity Market: Application of Market Efficiency. *Yale Economic Essays*, 9, 3-45.

Engle, R. F. (1982). Autoregressive Conditional Hetroscedasticity with Estimates of UK Inflation. *Journal of Econometrica,* 50, 987-1007.

Engle, R. F., and Bollerslev, N. (1986). Modelling the Persistence of Conditional Variance. *Econometric Review,* 5(1), 1-50.

Fama, E. F. (1965). The Behaviour of Stock Market Prices. *Journal of Business,* 38, 34-50.

Gibbons, M. R. (1982) Multivariate Test of Financial Models: A New Approach. *Journal of Financial Economics,* 10, 3-27.

Kiweu J.M., (1991). The behavior of share prices in the Nairobi stock Exchange: An empirical Investigation. Unpublished MBA project, University of Nairobi

Lehmann, B. N., and David M. M. (1988). The Empirical Foundation of the Arbitrage Pricing Theory. *Journal of Financial Economics,* 21, 213-254.

Lintner, J. (1965) The Valuation of Risk Assets and Selection of Risky Investments in Stock Portfolio and Capital Budgets. *Review of Economics and Statistics,* 47, 13-47.

Litzenberger, R. H. and Krishna R. (1979). The effect of personal taxes and dividends on capital asset prices: Theory and empirical evidence, *Journal of Financial Economics,* 7, 163-195.

MacBeth, J. D. (1975). Tests of Two Parameters Models of Capital Market Equilibrium. PhD. Dissertation. Graduate School of Business, University of Chicago, Chicago IL.

Macharia G. (2002). Forecasting ability of valuation ratios (Nairobi Stock Exchange). Unpublished MBA project, University of Nairobi.

Mandelbrot, B. (1963). New Methods in Statistical Economics. *Journal of Political Economy,* 71, 421-440.

Markowitz, H. (1959). Portfolio Selection: Efficient Diversification of Investments. New York: Wiley.

Mwangi N. M. (1997). An analysis of CAPM. Unpublished MBA project, university of Nairobi.

Ng, V. K., Engle R. F., and M. R. (1992). A Multi Dynamic Sector Model for Stock Returns. *Journal of Econometrics,* 52, 245-266.

Barndorff-Nielsen O. (1977). Exponentially decreasing distributions for the logarithm of particle size. *Proceding of Royal Society of London* , 353, 409-419.

Roll, R. W., and Stephen A. R. (1980). An Empirical Investigation of Arbitrage Pricing Theory. *Journal of Finance,* 35(5), 1073-1103.

Ross, S. A. (1976). An Arbitrage Theory of Capital Asset Pricing. *Journal of Economic Theory,* 13, 341-360.

Sharpe, W. F. (1964). Capital Asset Pricing Theory of Market Equilibrium under Conditions of Risk. *Journal of Finance,* 19, 425-442.

Stambaugh, R. F. (1982). On the Exclusion of Assets from Tests of the Two Parameter Model. *Journal of Financial Economics,* 10, 235-268.

Tobin, J. (1958). Liquidity Preferences as Behaviour Towards Risk. *Review of Economic Studies* 25, 65-86.

Treynor, J. L. (1961). Towards a Theory of Market Value of Risky Assets. (Unpublished Manuscript).

Von N., John, and Margenstern O. (1953). Theory of Games and Economic Behaviour. 3rd (ed.) Princeton: Princeton University Press.

Appendix 1: The extract of the NSE20 index weekly returns

Safaricom returns
8.27398268
-20.36127552
-9.72954692
7.37720118
-10.9866326
-10.2946386
19.54858418
11.34271584
19.23717306
16.43872896
-20.42191426
21.63292558
-9.73671934
7.5187259
-2.33530268
5.84591436
20.53306086
1.63765908
-2.63776918
1.17004416
3.00084824
6.46866012
10.85580788
8.9988858
3.6286331
42.66237702
-18.70087356
11.1346367
-3.03001042

The returns are in percentage.

Appendix 2: The extract of the Safaricom weekly returns

Safaricom Returns
-1.094786764
-2.484726041
-2.583647853
-1.891130028
-4.789592685
3.591307437
2.114805169
-0.467722238
6.233236533
7.732096853
-7.547155916
4.48584421
-2.422497397
-4.327393527
2.745621569
2.049170301
3.128954213
1.772603404
1.902782485
0.405743648
-1.637703134
0.576622665
1.836000526
6.799398179
2.435348962
8.227034663
-6.642736943
3.824600662
1.707451366

The returns are in percentage.

Appendix 3: The extract of the Mumias Sugar weekly returns

Mumias returns
0.012358658
2.492707232
-1.757828659
0.619034778
-1.186757784
0.35481646
0.200356332
1.110814515
0.276942184
-0.990832456
-2.079810762
-0.12703018
-1.146019406
0.165425222
-0.728287615
1.902586649
0.887703762
3.63881057
0.908823725
-1.194707661
2.988243871
-0.651786323
0.300258665
2.798217281
-0.553824809
2.339633742
-1.285569879
1.208530041
-1.894091668

Appendix 4.The iteration table for parameter estimation

	gamma	delta	beta	alpha	mu	Loglik
1	1	1	1	1	1	1
2	0.043627	20.19263	-0.00883	0.044511	4.437467	-3121.19
3	0.043772	20.29279	-0.00813	0.04452	4.119533	-3119.73
4	0.043871	20.37749	-0.00765	0.044533	3.906202	-3119.19
5	0.043948	20.45117	-0.00733	0.044555	3.762884	-3118.74
6	0.044017	20.51647	-0.00711	0.044588	3.666639	-3118.37
7	0.044083	20.57512	-0.00696	0.044629	3.602155	-3118.09
8	0.044149	20.62831	-0.00686	0.044679	3.559167	-3117.88
9	0.044216	20.6769	-0.0068	0.044735	3.530769	-3117.74
10	0.044284	20.72154	-0.00675	0.044796	3.512297	-3117.66
11	0.044353	20.76274	-0.00673	0.04486	3.500593	-3117.63
12	0.044422	20.80091	-0.00671	0.044926	3.493509	-3117.65

The variable gamma, delta, beta, alpha, mu are the parameters of the NIG distribution and Loglik the log likelihood of the distribution. It illustrate the convergence of the algorithm.