STOCHASTIC INTEREST RATES MODEL

And

CONTINGENT CLAIM PRICING

BY

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A research project submitted to the University of Nairobi in partial fulfillment of the requirement for the award of Masters of Science degree in Actuarial Science

DECLARATION

Declaration by the candidate ………………………………………………………………………………………….

The project is my original work and has never been presented for a degree in any other university for examination purposes

Sign: …………………………………………….   Date: ………………………………………………..

Name: **AMBROSE J. SEWE**   Reg: **I56/60952/2010**

Declaration by the supervisor…………………………………………………………………………………………..

This project has been submitted for examination with my approval as university supervisor

Sign: …………………………………………….   Date: ………………………………………………..

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DEDICATION

This project is dedicated to my lovely wife Christine A. Sewe; you are the foundation and the inspiration behind this paper. You have always believed that I should only go for the best and be the best that you believe strongly I am

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I feel greatly indebted and would like with humility to appreciate and acknowledge people without whom I would not have dreamt to be an Actuary. Prof. Weke, may I say you are the best, I pray God to grant me the valuable blessing to be only a quarter of who and what you are, I appreciate your brotherly support, your encouragement have mentored me, you have made a mark in my life and in my heart of Hearts I say thank you

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ABSTRACT

This project designs and formulates prices and the inherent factors used in contingent securities. Participating contingent contracts are most popular in most financial jurisdictions. They present many different covenants and depend on sector regulations. This work tries to design the new participatory contract although structurally unchanged from the traditional contracts, the stochastic nature of the interest rates are taken into consideration in the design of this new contract this research envisages. After an in-depth analysis of the factors stochastic or otherwise but with a guaranteed rate matching the rate of interest in Kenyan government bonds, we prove that this new type of contract can be valued in closed form when interest rates are stochastic and the company can default.

The stochastic interest rate model used here borrows heavily from Schwartz and Gibson's work (1989) as it is used to capture the empirical properties of the financial time series. Most of these applications are made on the assumptions that the conditional distribution of interest rates given that the log distribution of volatilities is normal. This research project aims to analyze the other side of the standard Black- Scholes and GARCH- Models and re evaluate the parameters as used in BSM Model using Stochastic Volatility models (SV) and applying the estimated rate of the interest in a two factor stochastic model to price a contingent security. The traditional BSM pricing assumption of interest rate is looked upon as continuous time processes and the re evaluation is done using the continuous time model of SV. These models are derived and applied on the two factor security pricing formulae. The standard SV Model is examined and applied in statistical sense- linear model. The revised stochastic interest rate model is then applied to the pricing of contingent claims using the Nairobi stock exchange prices as the underlying security. Emphasis is laid on the estimation of the parameter interest rates that is looked upon as a stochastic random variable depending on time and other factors the motivation thus is the inherent failures of the traditional option pricing Models as BSM Model. This is due to the realization that most of parameters used in the standard Black-Scholes and assumed constant and are in real sense are time dependent variables and should be looked upon as such given the complex business environment that requires effective pricing that reflect this modern challenges and factors. The study therefore aims to go beyond the norm by doing in depth analysis into the Black-Scholes pricing formulae and the time proven time series model- GARCH Model and concentrating on the synergy between the two and proposing a more robust model for security pricing

CHAPTER ONE

1.1 Brief introduction and background of study

The behavior of interest rates is essential to the profitability and to the solvency of financial institutions and insurance companies. A great deal of financial literature deals with the term structure of interest rates where interest rates used for valuation and pricing are pre determined and assumed deterministic over time, and many models have been written to fit the various theories. These models usually form the basis of a much larger model that depends on the interest rate scenarios, such as option pricing or asset liability studies. Models that are used for option pricing (hedging choices) require an interest rate generator that reproduces the current asset prices (Tilley 1992).When interest rate scenarios are used for other purposes, interest rate generators that match current asset prices are not appropriate. This research work to shift the focus generally on long term behavior of the interest rates with particular interest on time dependency going forward on the long term. In this paper the consideration are time and resource costs involved in running models based on stochastic interest rate scenarios, especially when the scenarios are a small part of a much larger asset/liability model. Thus it is necessary to balance the desirability of the knowledge gained from running a large number of scenarios with the costs of running them. So many studies have been done on this subject of interest rate and efforts to limit the number of scenarios have been done with paths created by the binomial tree method. This paper concentrates all the efforts on the Stochastic volatility Model and its architectures to try to generate a large volume of time dependant interest rates, akin to avoiding the inherent challenges that the traditional asset pricing models such as BSM models have been known to have, here pricing factors are at best deterministic, as such the resultant assets are either overpriced or underpriced. This study endeavors to use the financial interest rates series to generate future time dependent interest rates and use them in a two factor model to price assets It is generally acknowledged that the volatility of many financial interest rate series is not constant over time. This paper examines main classes of models that have been developed over the last two decades to capture the time-varying autocorrelated volatility process: the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) and the Stochastic Volatility (SV) model. GARCH models define the time-varying variance as a deterministic function of past squared innovations and lagged conditional variances whereas SV model defines volatility as a logarithmic first order autoregressive process. Although SV models are more sophisticated than GARCH models, their empirical application has been limited under the assumption that the conditional distribution of rates is normal, given the latent volatility process. This SV-normal model is not able to capture the empirical regularities of financial interest rates series: first, volatility clustering is often observed. That is, large changes tend to be followed by large changes and small changes tent to be followed by small changes; second, financial time series often exhibit leptokurtosis, meaning that their distribution is symmetrical in shape, similar to a normal distribution, but the center peak is much higher, so it has a fat tail; third, squared returns exhibit serial correlation whereas little or no serial dependence can be detected in the return series itself. In the papers of Ruiz (1994),0 Harvey, Ruix and Shephard (1994),Sandmann and Koopman (1998) and Chib, Nardari and Shephard (1998) the SV model is extended to allow the conditional distribution of the returns to be more heavy-tailed distribution than the normal distribution by using Student t-distribution for the standardized residual.

The purpose of this paper is to examine the ability of the SV model to capture the properties of financial return series mentioned earlier under the assumptions based on a conditional normal distribution for log volatility and a conditional Student t-distribution for the returns.

It will finally be shown that such assumptions regarding the conditional distribution of the interest rates systematically affects the estimates of the parameters in the latent interest rates volatility process.

This paper will also take into consideration the salient reasons for the limited empirical applications of the SV model such as (1.1a) the difficulty to evaluate the likelihood function directly. The marginal likelihood function of the SV model is given by a high dimensional integral, which cannot be calculated by standard maximum likelihood method (ML). This paper will therefore use the simulated maximum likelihood (SML) approach as developed by Danielsson and Richard (1993) and is employed to calculate the model integral. This estimation method allows us to demonstrate the statistical inference with the standard instruments of inference for the ML method.

1.3 PROBLEM STATEMENT

The option and stock markets have grown over the years to complex machines of trade and investment in the financial services sector. The pricing and the prices of option, futures and related stocks requires meticulous considerations and the Black- Scholes models have been used over the years for this purpose, as like any other model this is never expected to be the reality but should capture the objective to which it intended but the business environment over the years have metermophosised, the parameters used though assumed constant in the traditional BSM Model are very critical and should reflect the volatile trading environment. The pricing of this assets using the traditional BSM Model though fair, doesn't reflect the reality. This project therefore seeks to bridge the pricing gap that exists when the pricing parameters are assumed constant and a more robust approach when the parameters are adjusted using a generalized time series model- GARCH Model and Stochastic volatility models to reflect the stochastic nature of the parameters and the investment environment. Of particular interest is the interest rate used the pricing process but which chameleons over time and a pricing model that does not reflect this is somewhat overly unrealistic and returns calculated on this are barely optimal either very optimistic to make imaginarily unfair profits or very conservative to make imaginary loses which could easily be avoided by the right pricing mechanism

1.4 PROPOSED SOLUTION

This paper seeks to establish the behavior of the interest rates, study and analyse their patterns and volatilities with a view to estimating future time dependent interest rates in an SVL model that is then used in a more robust two factor model to price the contingent claim. The paper focuses primarily on the interest rate used the pricing process but which chameleons over time and setting up a pricing model that reflects this so as to make the process acceptably realistic and returns calculated based on this is made optimal neither to be very optimistic to make imaginarily unfair profits nor very conservative to make imaginary loses. This therefore provides the right pricing mechanism.

1.5 OBJECTIVES OF THE STUDY

1.5.1 General Objectives.

In the evaluation and review of the traditional option pricing models and adjusting the technical mindset on say BSM pricing models, this paper intents to do an in depth analysis of the behavior and structure of the time dependent interest rates in pricing options and other contingent claims. To do this, this paper employs SVL models and a two factor stochastic models for option pricing

1.5.2 Specific Objectives

- Approximate stochastic time dependent interest rate variables using the Stochastic Volatility Model and OLS with volatility as regression factor.
- Evaluate the prices of a contingent claim specifically using the estimated time dependent stochastic interest rate factor on a two factor model
- Model a robust contingent claim using two factor models based on Nairobi stock exchange stocks as the underlying securities.

1.6 SIGNIFICANCE OF THE STUDY

Participating contingent contracts are most popular in most financial jurisdictions. They present many different covenants and depend on sector regulations many of which are priced using a deterministic rate of interest, it's in this view that this paper looks at this inadequacy as most factors used in most traditional options pricing models ought to be continuous and time dependent. This paper tries to design the new participatory contract. After an in-depth analysis of the factors stochastic or otherwise but with a guaranteed rate matching the rate of interest in Kenyan government bonds, we prove that this new type of contract can be valued in closed form when interest rates are stochastic and the company can default. This in its intent reduces the possibility of either an over priced or under priced Asset as the paper envisages happens in most contingent contracts. It's to be shown that a robust pricing and factor modeling mechanism works in synergy with the financial services industry which has grown to be sophisticated and dynamic. This paper therefore envisages pricing Model that reflects this reality and accommodate the factors therein and their determinants.

The stochastic interest rate model used here borrows heavily from Taylor (1986) as it is used to capture the empirical properties of the financial time series. Most of these applications are made on the assumptions that the conditional distribution of interest rates given that the long distribution of volatilities is normal. This research project aims to analyze and look at the other side of the standard Black- Scholes and GARCH- Models and re evaluate the parameter interest rate as used in BSM pricing Model using Stochastic Volatility models (SV) and applying the estimated SV interest rate in a two factor stochastic model to price a contingent security. The traditional BSM pricing assumption of interest rate is looked upon as continuous time processes and the re evaluation is done using the continuous time model of SV. These models are derived and applied on the two factor security pricing formulae. The standard SV Model is examined and applied in statistical sense- linear model. The revised stochastic model is then applied to the pricing of Nairobi stock share prices and the variation observed between this and the recursive kallman filter method. This s is due to the realization that most of parameters used in the standard Black- Scholes and assumed constant are in real sense, time dependent variables are should be looked upon as such given the complex business environment that requires effective pricing that reflect this modern challenges and factors. The study therefore aims to go beyond the norm by doing in depth analysis into the Black-Scholes pricing formulae and the time proven time series model- GARCH Model and concentrating on the synergy between the two and proposing a more robust model for security. The use of Kallman filter process is thus taken purely as a tie breaker in the two factor estimation mechanisms of course due to its historical use and over time improvement thus some reliability

1.7 SCOPE OF THE STUDY

This research paper examines the stochastic interest rate by using the SV model and its ability to capture the properties of financial time series mentioned fore mentioned under the assumptions based on the conditional normal distribution for log volatility and conditional student t distribution for the interest rates. We then endeavor to show that such assumptions regarding conditional distributions of the rates of interest systematically affects the estimates of the parameters in the latent volatility process, with this modeled stochastic interest rate from the SV process, we try to use to factor model to price contingent claims

Finally this work develops and tests a two factor model for pricing financial assets. The factors are stock spot prices and the stochastic interest rates. The parameters of the model are estimated using weekly stock prices from Nairobi Stock exchange and the model performance is assessed by valuing futures contracts over a defined period. The model is then applied to determine the prices of a financial Asset.

CHAPTER TWO

2.1 LITERATURE

2.1.1 General Review

INTRODUCTION

It's an accepted fact that the volatility of many financial rates of interest time series is not constant over time. Over time two main classes of models have been developed that captures the time varying autocorrelated volatility process. The Generalised autoregressive conditional heteroscedasticity(GARCH) model and the stochastic volatility model. GARCH models define the time varying variance as a deterministic function of past innovations and lagged conditional variances. SV Models defines volatility as a logarithmic first order autoregressive process. SV models are more sophisticated than GARCH models but their empirical applications have been limited under the assumptions that the conditional distribution of returns is normal given the volatility process.

The beauty of the sophisticated nature of the SV model informs this work, attempts will be made to assume the shortcomings of this model and the fact that the SV –normal model is not able to capture the empirical regularities of financial time series is given a blackout. The shortcomings includes (1.1a) volatility clustering is often observed i.e. large changes tend to be followed by large changes and small changes tend to be followed by small changes short circuiting the mean reversion theorists. (1.1b) financial time series often exhibit leptokurtosis i.e. the distribution is symmetritised in shape, similar to a normal distribution but centre peak is higher and a fat tail, (1.1c) squared returns exhibit serial correlation whereas little or no serial dependence can be detected in the return series itself, Ruiz (1994), Harvey, Ruiz and Shepard(1994), sadman and koopman(1998). The SV model is therefore extended to allow the conditional distribution of returns to be more heavy tailed distribution than the normal distribution by using the t-distribution for the standardised residual.

Another reason for the limited application of the SV model is(1.1d) the difficulty to evaluate the likelihood function directly. The marginal likelihood function of the SV model is given by high dimension which cannot be calculated by standard maximum likelihood method (ML). In this paper, the simulated maximum likelihood (SML) approach developed by Danielson and Richard (1993) is used to calculate the integral. This helps us to demonstrate the statistical inference for the ML method. The paper is organized in steps such that we first describe the volatility models and more details of SV model, secondly use the interest rate Data from the central bureau of statistics publications to underscore the study variables and some statistics of the rates of interest and the squared rates, after which it describes SML (simulated maximum likelihood) and the Accelerated Gaussian Importance Sampling (AGIS) Methods, provide the parameter estimation and modest evaluation of the model. Then lastly look at the two factor pricing model as used in security valuation.

2.1 Introduction to pricing mechanisms

Ideally spot price process and the interest rate should be estimated simultaneously from the time series & cross section of future prices and discount bond prices. To simplify the estimation, we first estimate the parameters of the spot price and interest rate process with an assumption that the parameters of the interest rate process are not affected.

Stocks or Shares as most corporate organizations know them are a sure way of raising working capital for organizations and stocks are also used as underlying assets to many financial instruments such as futures options on futures as well as other unit linked Assets. While most financial instruments cover the short to medium term maturity range, we want to try and price our options using the contingent claim framework viz a viz the stochastic interest rate regime. Most options valuations models aimed at valuing natural resources have been based on the assumption that there is a single source of uncertainty related to the price of the underlying commodity.

We therefore look at it from a more general perspective which can easily be applied to the pricing of real and financial contingent claims. We assume that stock price is fundamental but not unique determinant of option price. Additionally we allow a stochastic interest rate in order to develop a two factor option pricing model.

Interest rate here will be viewed as dividend yield accruing to the owner of a physical commodity and it derives the relationship between futures and stock prices of many commodities. Using Gibsons and Schwartz (1989) mean reverting assumption as well as the variability of its changes requires a stochastic representation in order to price stock related securities accurately. The research work attempts to derive a more realistic two factor pricing model and subsequently analyze its performance in valuing short and long term contracts.

The idea is to empirically show that the model can be used to value short term options and try it on a long term contract. It also attempts to explain the intrinsic difference in price volatility between stocks prices and futures contracts as well as its decreasing maturity pattern.

Proposition 2a: Diffusion Process

We assume that it price depends only upon the stock price S , the instantaneous rate of interest σ and the time to maturity τ(T-t) The stock price and interest rates follow a joint diffusion process specified

$$\frac{ds}{s} = udt + \delta_1 dz_1$$
$$d\delta = u(\alpha - \delta)dt + \delta_2 dz_2.$$

Study this again

Where dZ_2 and dZ_1 are correlated increments to standard Brownian process and dZ_1 and dZ_2 $= \rho dt$ where ρ is the correlation coefficient between the two Brownian motions.

The form (2.1) Is based on the assumption that the spot stocks prices has a log-stationary distribution and provide the time series properties of the spot prices

The form (2.2) Is motivated by Gibson's and Schwartz's (1989) study of the time series properties of the forward yields and applied to the behavior of the convenience yields of crude oil where they find strong empirical evidence in favour of the mean reverting pattern.

Assuming that the price B (S, δ, τ) of the stock contingent claim is twice continuously differentiable functions of S and δ and we shall use the Ito's lemma to define its instantaneous price as follows $dB = B_s + B_\delta d_\delta - B_\tau d_\tau + \frac{1}{2}B_{ss}(ds)^2 + \frac{1}{2}B_{\delta\delta}(d\delta)^2 + B_{s\delta}d_s$

$$d_\delta dB = \left[-B - \frac{1}{2}B_{ss}\delta_1^2 s^2 + B_{ss}S_p\delta_1\delta_2 + \frac{1}{2}B_{\delta\delta}\delta_2^2 + B_s uS + B_\delta\big(K(\alpha - \delta)\big)\right]dt$$

$$+\delta_1 s B_s d_{z_1} + \delta_2 B_\delta d_{z_2}$$

(2.3)

This is abstracted from the fact that interest rates are uncertain and that the standard perfect market assumptions will hold. The no arbitrage price is assumed and that the equation therefore satisfies the following equation

$$B_{ss}\delta_1^2 s^2 + \frac{1}{2}B_{\delta\delta}\delta_2^2 + B_{ss}S_p\delta_1\delta_2 + B_s S(r - \delta) + B_\delta(K(\alpha - \delta) - \lambda\delta_2) - B_\tau - rB = 0$$

(2.4)

Proposition 2b:

Consider the stock price at time t,s_t and what happens if Δlns_t depends on lns_t on the following manner: Δ$lns_t = \gamma[lns_t - ut] + uh + \delta\sqrt{h\varepsilon_{h+h}}$ where$\gamma < 0$, in this case if lns_t departs from the trend ut , then the expected change in the next time period will pull $lnS(t+h)$ Back to the trend by an amount determined by the factor γ

Proposition: Ornstein-uhlenbeck process

We examine carefully whether Ornstein-uhlenbeck process meaningfully describes the evolution of the convenience yield of yield of oil. In particular we discuss its relevance in the light of the conclusion of the theory of storage. The derivation of the equation below relies on the same methodology as the one underlying the two factor partial equilibrium bond pricing model developed by Brenan and Schwartz (1979). The no arbitrage condition leads to the following relationship between the total (expected) return of the claim and its risk exposure

In the forward equation, when $a(y,t) = -\beta(x), b(y,t) = \delta^2$, $f(y,s)$ satisfies the partial differential equation $dB = B_s + B_\delta d_\delta - B_t d_t + \frac{1}{2}B_{ss}(ds)^2 + \frac{1}{2}B_{\delta\delta}(d\delta)^2 + B_{s\delta}d_s d_\delta$ and solving it using the initial boundary conditions

This meaningfully describes the evolution of the instantaneous interest; in particular it describes its relevance in light of Mean reversion and the conclusion to the theory of storage The derivation of the general pricing equation (2.4) relies on the Sample methodology as the one underlying the two factor partial equilibrium band pricing modeled by Brennan and Schwartz (1979) looked at extensively as follows

The no arbitrage condition leads to the following relationship between total (expected) return of the claim u_B and its risk exposure.

$U_b = r + \frac{\lambda' s B_s \delta_1}{B} + \frac{\lambda B_s \delta_2}{B}$ The option price must satisfy (i) and that the total expected return u_s to the owner of the option contract derives from two sources namely the time dependent interest rate δ and expected stock price change u, we define the market price per unit of stock price risk λ' by solving the partial differential equation for S. hence

$\lambda' = \frac{u_s - r}{\delta_1} = \frac{(\mu + \delta) - r}{\delta_1}$ From which (2.4) is derived for a contingent claim.

Brennan (1986) derives a two factor model for pricing of commodities in which λ is a constant, for this to hold the investor has to have a logarithmic utility function and a covariance between return on aggregate wealth and the instantaneous rate of interest is said to be proportional to δ_2

$$\frac{1}{2}F_{ss}s^2\delta_1^2 + \frac{1}{2}F_{\delta\delta}\delta_2^2 + F_{\delta s}s_p\delta_1\delta_2 + F_s S(r-\delta) + F_\delta(K(\alpha-\delta) - \lambda\delta_2) - F_r = 0$$

(2.5)

Subject to the boundary condition $F(S, \delta, 0) = S$

(2.6)

Any other on stock under this framework satisfies equation (2.4) subject to relevant boundary condition. For example using equation (2.4) to price a European option entitling its owner to by one unit of stock at time T at an exercise price of K, the initial condition is as follows

$C(S, \delta, 0) = \max[0, S - K]$
(2.7)

$C(S, \delta, 0)$ Denotes the price of the European call option at maturity under the pretext that there is no analytical solution to the partial differential equation 4 and 5, therefore use a numerical technique to compute the present value factor $B(S, \delta, \tau)$ and the future prices $F(S, \delta, \tau)$
To apply this model, we will estimate the market prices of the stochastic interest rate risk λ together with parameters $K, \alpha, \delta_1, \rho_2$ *and* δ of the joint stochastic process and followed by stock price and interest rates

Proposition 2D: 'The Kalman Filter
Kalman filter is a recursive procedure for computing the optional estimation of the state vector at time t, based on the information available at time t. this enables the estimate of the state vector to be continuously updated as new information becomes available.

When distribution & initial state vector are normally distributed, the Kalman filter enables the likelihood function to be calculated this allows for the estimation of any unknown parameter of the model and provides the basis for statistical testing & model specification (Kalman filter-Harvy 1989)

Measurement equation can be written $y_t = d_t + Z_t X_t + \varepsilon_t, \; t = 1, 2, 3, \ldots \ldots . . N$ where y_t **is** the estimated convenience yield and the above equation forms a simple regression and equation with the sensitivity factors that informs it.

And $y_t = [lnFT_i] \; i = 1, 2, 3, 4, \ldots . . N, N * 1$ Vector of observations

$d_t = \left[(1 - e^{-KTi})\alpha^* + \frac{\delta^2}{4k}(1 - e^{-2KTi})\right] i = 1, 2, 3 \ldots . N, N * 1$ Vector

$Z_t = [e^{-KT_i}] \; i = 1, 2, 3 \ldots \ldots . N, N * 1$ Vector

$\varepsilon_t, \quad t = 1, 2, 3, \ldots \ldots N, N * 1$ Vector of serially uncorrelated distribution with

$E(\varepsilon_t) = 0$ and $Var(\varepsilon_t) = H$

The transition equation can be written as

$X_t = C_t + \emptyset X_t + \square_{t-1}, \; t = 1, 2, 3\ 4 \ldots \ldots \ldots \ldots NT$ **.** In this paper the factor y_t forms the central theme of the study as will be compared later to the interest rate factor from the SV-model. NT will the total observational period under the interest rate regime under investigation

Where $C_t = k\alpha\Delta_t$ and $\emptyset_t = 1 - k\Delta t$

$\square_t$ is a set of serially uncorrelated disturbance with $E(\square_1 t)$ = 0 and $Var\square_t = \delta^2\Delta_t$

Alternatively this can be remodeled into a measurement equation written as

$$y_t = d_t + Z_t[X_t\delta_t]' + \varepsilon_t,\ t = 1, 2, 3, \ldots\ldots..NT$$

The exact transition equation is

$X_t = \alpha(1 - e^{-K\Delta t}) + e^{-K\Delta t}X_{t-1} \cap_t$ Where

$y_t = [lnFT_i]\ \ i = 1, 2, 3, 4, \ldots..N, N * 1$ Vector of variables

$d_t = [A(T_i)]\ \ i = 1, 2, 3\ \ \ldots\ldots.N, N * 1$ Vector

$Z_t = \left[1 - \left(1 - \frac{e^{-KTi}}{K}\right)\right]\ \ i = 1, 2, 3, \ldots\ldots..N,$ $N * 2$ Matrix

$$[X_1t\ \ \delta_1(t)] = C_1(t) + \emptyset_1 t\ [X_1(t-1)\ \delta_1 t\ ]^{1\prime} + \cap_1(t\ \ \ )\ t = 1, 2, 3, \ldots\ldots\ldots..NT$$

Where

$C_t = \left[\left(\mu - \frac{1}{2}\delta^2\right)\Delta_t, k\alpha\Delta_t\right]'$ $2 * 1\ vector$ And

$$\emptyset_t = \begin{bmatrix} 1 & -\Delta_t \\ 0 & 1 - k\Delta_t \end{bmatrix}$$

$\cap_t$ is a set of serially uncorrelated disturbances with $E(\cap_t) = 0$ **and**

$$Var(\cap_t) = \begin{pmatrix} \delta^2\Delta_t & \rho\Delta_1\delta_2\Delta_t \\ \rho\delta_1\delta_2\Delta_t & \delta_1^2\Delta_t \end{pmatrix}$$

and lastly we look at the model where the spot price process, the convenience yield process and the interest rate process are estimated simultaneously from a time series and a cross section of future prices and discount bond prices. To simplify this we first estimate the parameters of the interest rate process and use this last model to estimate the parameters of the spot prices and convenience yield process. We essentially assume that the parameters of the interest rate process are not affected by the commodity prices.

Once we have estimated the interest rate, we only have to estimate the parameters and the state variables from spot price and the convenience yield process. From the above equations we $y_t = d_t + Z_t[X_t \delta_t]' + \varepsilon_{t,}\ t = 1, 2, 3, \dots \dots . . NT$

$y_t = [lnFT_i]\ \ i = 1, 2, 3, 4, \dots . . N, N * 1$ Vector of variables

$〖d〗_{\downarrow t} = [((r_1 i\ (1 - e^{\uparrow}(-\alpha T)\ ))/\alpha + C(T_1 i\ )\ ]\quad i = 1, 2, 3\ \dots \dots . N, N * 1$ Vector

$Z_t = \left[1 - \left(1 - \frac{e^{-KTi}}{K}\right)\right]\quad i = 1, 2, 3, \dots \dots . . N,\qquad N * 2$ Matrix

2.1.2 SPECIFIC REVIEW

2.1.3 Introduction

2.1. 4 VOLATILITY MODEL.

Financial series is either called conditionally heteroscedastic if the conditional variance depends on time or unconditional if the variance is constant. Models like those of volatility are called the conditional heteroscedastic model since volatility evolves over time, modeling interest rates plays important roles in product pricing and particularly contingent claim pricing and risk management. It also improves the efficiency in parameter estimation and the accuracy in interval forecast.

Three main Univariate models are disclosed by this work here. The autoregressive conditional heteroscedasticity (ARCH) Model of Engle (1982). The Bollerslev's Generalised ARCH (GARCH) and the stochastic volatility model introduced by Taylor (1986)

2.1.5 THE ARCH AND GARCH MODELS

Engle's paper (1982) introduced the ARCH model to express the conditional variance of today's rates of interest as function previous observations. The basic idea of the ARCH model is that the mean-correlated asset returns are serially uncorrelated but dependent of past observation. The ARCH (q) model is defined by

$r_t = \sqrt{\lambda_t u_t}$ u_t , Is independent and identically distributed standard normal random variable {$N(0,1)$} and $\sum_{i=1}^{q} \alpha_i < 1$ is the necessary and sufficient condition for the rates of interest to be weakly stationary. The order q of the process determines the volatility persistence which increases with the value q

$\lambda_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i r_{t-i}^2$ Where r_t is the rate of interest on day t and u_t is the white noise process

The generalized ARCH (GARCH) model is an extension of Engel's work by Bollerslev's (1986). This work allows the conditional variance to depend on the previous conditional variances and squares of previous rates of interest. The possibility that the estimated parameters in ARCH models do not satisfy the stationary conditions which increases with lag. The GARCH model is an alternative to ARCH model. The GARCH (p , q) model is defined by

$r_t = \sqrt{\lambda_t \mu_t}$ Where $\mu_t \approx iid\, N(0,1)$ and $\lambda_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i r_{t-i}^2 + \sum_{j=1}^{p} \beta_j \lambda_{t-1}$

λ_t Is the conditional variance of the rates given by $R_{t-1} = r_{t-1}.r_{t-1}$ ... and the parameters given $(\alpha_1 \alpha_2 \dots . \alpha_q . \beta_1 \beta_2 \dots . \beta_p)$ are restricted for any t, $\lambda_t > 0$

2.1.6 THE SV MODEL

The standard version of the stochastic volatility (SV) is given by

$$r_t = \exp\left(\lambda_t / 2\right) \tag{2.2.8}$$

$$\lambda_t = \alpha + \beta \lambda_{t-1} \tag{2.2.9}$$

Where

r_t Is the observed rate of interest at time t

λ_t Is the log volatility is $\lambda_t = 2 ln \delta_t$

δ_t Is the interest rate volatility.

Both u_t *and* v_t are identically and independently distributed random variables with zero mean and unit variance.

However v_t is normally distributed.

These error processes are stochastically independent and unobservable. The unobservable volatility process λ_t is assumed a Gaussian AR (1) process with a persistent parameter β for $|\beta| < 1$, the latent process is stationary. The parameter γ^2 represents the variance of volatility shocks and is assumed to be positive. Most cases u_t are assumed to be normal but with a heavy tail and with a leptokurtic distribution for u_t better captures the empirical regularity of the financial time series. Now we examine the statistical properties of two SV models with a normally distributed u_t (SV-model) and the heavy tailed SV model, u_t (SV-t}

2.1.7 Properties of the SV-Normal Model

For $|\beta| < 1$ the latent volatility process $\lambda_t \approx N(\mu, \delta^2)$ with $u = \dfrac{\alpha}{1-\beta}$ and $\delta^2 = \dfrac{\gamma^2}{1-\beta^2}$ ie the unconditional mean and variance of λ_t respectively. Let $\delta_t^2 = \exp(\lambda_t)$ and assuming $E(u_t^4) < \infty$ the moments of $E(r_t^2)$ and $E(r_t^4)$ and defined as follows

$$E(r_t^2) = E(\delta_t^2) = \exp\left(u + \frac{\delta^2}{t}\right) \qquad (2.2.10)$$

$$E(r_t^4) = E(\delta_t^2) exp(\delta^2) E(u_t^4) = \exp\{2u + 2\delta^2\} E(u_t^4) \qquad (2.2.11)$$

Putting 2.2.10 and 2.2.11 into the definition of kurtosis, the kurtosis for the unconditional distribution of the rates is given by

$$K = {E(r_t^2)}/{E(r_t^2)^2} = E(r_t^2) exp\{\delta^2\} \qquad (2.2.12)$$

Equation (2.2. 12) has two components, 1st is the baseline kurtosis due to the term $E(u_t^4)$ that represents the kurtosis of the standardised residuals, the 2nd is the kurtosis due to the variation in the volatility process λ_t, under the SV model with a conditional normal distribution for the rates of interest (SV- Normal) the baseline kurtosis $E(u_t^4)$ is equal to three and so unconditional kurtosis of the rates is greater than three

The autocorrelated function (ACF) of the squared rates is defined as

$$p(h) = {cov(r_t^2, r_{t-1}^2)}/{var(r_t^2)} \qquad h = 1, 2, \ldots \ldots .. \qquad (2.2.13)$$

Provided $|\beta| < 1$ the autocovariance of the squared value given by

$cov(r_t^2, r_{t-h}^2) = cov(\delta_t^2, \delta_{t-h}^2)$ By independence of u_t (2.2.14)

= {exp ($\delta^{\uparrow}2\ \beta^{\uparrow}h) - 1\} E\ [(\delta_{\downarrow}t^{\uparrow}2)]^{\uparrow}2$ and additional assumption $E(u_t^4) < \infty$ its variance is

$$var(r_t^2) = E((\delta_t^2)^2) - E(h_t u_t^2)^2$$

$= \{(\delta_t^2)\} E(u_t^2) - E(\delta_t^2)^2$ By independence

$$= \left(var(\delta_t^2) + E(\delta_t^2)^2\right)E(u_t^4) - E(\delta_t^2)^2$$

$$= E(\delta_t^2)^2 \left\{\left(\frac{var(\delta_t^2)}{E(\delta_t^2)^2} + 1\right)E(u_t^4) - 1\right\}$$

$$= E(h_t)^2 \left\{\exp(\delta^2)\left(u_i^4\right) - 1\right\} \qquad \textbf{(2.2.15)}$$

Putting 2.2.14 and 2.2.15 into 2.2.13, then the ACF of the squared rates of interest r_t^2 is given by

$$p(h) = \frac{\exp\{\delta^2 \beta^h\} - 1}{E(u_t^4)\exp\{\delta^2\} - 1} \qquad h = 1, 2, 3, 4 \ldots\ldots\ldots\ldots\ldots.$$

The SV-Normal model predicts a positive autocorrelation on the second squared interest rates which is exponentially decaying out. The rate is determined by the parameter β and hence persistence of volatility shocks depend on β

The characteristics of the SV_{-t} Model

The thick tailed distribution for the error u_t means that the fourth moment of is u_t greater than three. This implies that the kurtosis is a conditional heavy tailed distribution for the rates is larger than that of conditional normal rates and that the level of ACF of the squared rates of interest declines as verified by equation 2.2.12 and 2.2.15. A widely known leptokurtic distribution is t-distribution. The density function of a t-distributed random variable u_t with mean zero and unit variance is given by

$$f(u_t) = [\pi(w-2)]^{-\frac{1}{2}} \frac{\Gamma\left(\left(w+1\right)/2\right)}{\Gamma\left(w/2\right)} \left[1 + \frac{w_i^2}{w-2}\right]^{\frac{-(w+1)}{2}} \qquad w > 4$$

(2.2.15)

Is the Gamma function and the parameter w denotes the degree of freedom. The kurtosis of the t-distribution is given by, as long as $w > 4$

$K = \exp\{\delta^2\} E(u_t^4)$ And defines the level of peakedness of volatility

$= \exp\{\delta^2\}\left[3\frac{(w-2)}{(w-4)}\right]$ If $w > \infty$, the kurtosis is greater than 3 and does not exist if $w = 4$, the t-distribution approaches a normal distribution as w goes to infinity

$$p(1) = \frac{\exp\{\delta^2\beta\} - 1}{E(u_i^4 \exp(\delta^2) - 1)} \; where\ h = 1\ and\ K\ can\ be\ considered\ as\ a\ function\ of\ p(1)$$

Liesenfield and Jung (2000) shows using an empirical study that SV_t model can capture the relationship between K and $p(1)$ better than the SV normal model

It therefore can be said that the financial interest rates series that a leptokurtic error distribution such as a student t-distribution helps to capture the empirical properties of the financial time series, a high Kurtosis of the returns and a low but decaying ACF of the returns.

CHAPTER THREE

3.0 METHODOLOGY

3.0.1 Outliers

An outlier is an observation that is far removed from the general pattern of the data, in the case of this paper, the obviously ambiguous interest rates shocks in the market resulting either from the financial turmoil in 2008 or the inflation rates influences due to food prices. The determination of an outlier depends on the model to be used in this case SVL Models. An outlier with respect to one model may not be an outlier in the next. But for the case of this study the outlier can be determined as follows

First we graphically view the data and given the small sample size used here, it's easy to see which points may be considered as outliers. Then the next step, you can do none of the following in trying to clean your data

Use statistical tests such as the six-sigma rule to determine the outliers. This rule states that if a point is within three standard deviations of the mean, it is not considered an outlier. Statistical computing package routinely produces diagnostic measures for determining the outliers

Remove the point from the data if you are sure, based on your knowledge of interest rates pattern and history, that the point is an outlier. Care is taken here to avoid just removing data elements without in depth considerations

3.1 DATA REQUIRMENTS

Hypothetical half yearly interest rates of the interest series are used in this paper. They are taken from the central Bank of Kenya reports at the national statistical Bureau reports. The total number of monthly rates that is biased and predetermined for the sake of this paper and the half year rates are conveniently transformed into the log rates of interest, using the formulae given below

$$r_t = ln\frac{p_t}{p_{t-1}}$$

Where p_t represents the yearly averages of interest rates taken as the average base lending rates and that $t = 2006, 2007, 2008, 2009, 2010, 2011$. The year 2006 rates are taken as the anchor year for the process.

3.2 ESTIMATION METHOD

The likelihood function associated with the known observation $R = \{r_t\}_{t=1}^{T}$ and the vector of the latent variable

$\square = \{\lambda_t\}_{t=1}^{T}$ is given by $f\left({}^{R}/_{\theta}\right) = \int_{R^T} f\left(R, \square/_{\theta}\right) d\square$
(3.2.10)

Note that $T = 2011$ the end period time and the beginning period time is 2006

Where $\theta = (\alpha, \beta, \gamma)$ denotes the vector of parameter to be estimated and T is the number of observations. The latent process λ_t in the SV model makes the direct calculation of the integral in (3.2.10) difficult in this paper; the simulated maximum likelihood (SML) approach is employed as introduced by Danielson and Richard (1995) to estimate the parameters in the model. It depends heavily on the Monte Carlo (MC) integration in evaluating the likelihood (3.0)

Infinite sample space, the SML methods performs almost identical to MC (Danielson1994) the standard instruments for instruments can be used even if the number of MC iterations is very large. More so the applications to be SV models can not only be obtained easily once SML process is implemented but extended to the multivariate case of the latent process λ_t.

Danielson and Richard (1993) introduced the SML approach using an Important Sampling Method and an accelerated Gaussian Importance Sampling(AGIS), SML method can also be looked after with two sampling technique

3.2.1 Importance Sampling (IS)

To obtain the MC estimate of $f\left({}^{R}/_{\theta}\right)$, the joint density function of $f\left(R, \square/_{\theta}\right)$ is factorized into an important sampling function (IF) $\psi\left(\square/_{R}\right)$ and a remainder function (RF)Φ(□,R) such that

$$f\left(R, \square/_{\theta}\right) = \Phi(\square, R)\psi\left(\square/_{R}\right) \qquad (3.2.11)$$

An initial factorization of $f\left(R, \square^{\square}/_{\theta}\right)$ is derived from 12 as follows

$$\psi_0\left(\square^{\square}/_{R}\right) = \prod_{t=1}^{T} f\left({}^{\lambda_t}/_{\lambda_{t-1}}\right)$$

(3.2.12)

$$\Phi_0(\square, R) = \prod_{t=1}^{T} f\left({}^{r_t}/_{\lambda_t}\right)$$

(3.2.13)

is the conditional density of λ_t given λ_{t-1} satisfying condition (3.2.11) which is a normal distribution and $f\left({}^{r_t}/_{\lambda_t}\right)$ is the density of the t –th half year rate of interest conditional to λ_t. In a SV-normal model $f\left({}^{r_t}/_{\lambda_t}\right)$ is given by

$$f\left({}^{r_t}/_{\lambda_t}\right) = \frac{1}{\sqrt{2\pi \exp(\lambda_t)}} \exp\left\{-\frac{r_t^2}{2\exp(\lambda_t)}\right\}$$

and in a SV-t distribution model, $f\left({}^{r_t}/_{\lambda_t}\right)$ has a form

Since the expected value of the RF is given by

$$E_{\psi}[\Phi(\square, R)] = \int_{R^T} \Phi(\square, R)\, \psi\left(\square^{\square}/_{R}\right) d\square = \int_{R^T} f\left(R, \square^{\square}/_{\theta}\right) d\square = f\left({}^{R}/_{\theta}\right)$$

(3.2.14)

The MC sampling mean is given by

$$\int_N^{\square}\left({}^{R}/_{\theta}\right) = \sum_{n=1}^{N} \Phi_0(\square_n, R)$$

(3.2.15)

Where $\{\square_n\}_{n=1}^{N}$ denotes N number of the simulated sample from the probability distribution $\psi\left(\square^{\square}/_{R}\right)$. Thus the ML estimate of θ obtained by maximizing $\ln\left[f\left({}^{R}/_{\theta}\right)\right]$

3.2.2 Accelerated Gaussian Importance Sampling (AGIS)

It often occurs that the initial, IF (3.2.12) and RF (3.2.13) in (3.2.14) technique leads to inefficiency in the variance remarkably increases with this dimension of the integral T. The AGIS method is proposed by Danielson and Richard (1993) can solve this inefficiency problem. The

AGIS method is based on minimizing the variance of the remaining function $\Phi(\square, R)$ while the condition (3.2.11) and (3.2.14) holds. The process based on AGIS solves the minimizing problem

$$Min_{\psi} Var_{\psi}[\Phi(\square, R)]$$

(3.2.16)

Subject to the constraints $f\left(R, {}^{X}/_{\theta}\right) = \Phi(\square, R)\ \psi\left(\square\ {}^{\square}/_{R}\right)$ and $E_{\psi}[\Phi(\square, R)] = f\left({}^{R}/_{\theta}\right)$

(3.2.17)

Where the variance of $\Phi(\square, R)$ evaluated over an importance function $\psi\left(\square\ {}^{\square}/_{R}\right)$ is given by

$$\llbracket Var \rrbracket \downarrow\psi\, [\Phi(\square, R)\,] = \int \downarrow (R^{\uparrow}T)\ \llbracket [\Phi(\square, R)\ \llbracket - f(R\,/\,\theta)] \rrbracket\ \uparrow] \rrbracket\ \uparrow 2\ \ \psi(\square\,/\,R)\, d\square$$

The variance reduction function $\xi(\square, \Omega)$ is such that a new IF and a new RF are given by

$$\psi_1\left(\square\ {}^{\square}/_{R}\right) = \frac{\left[\psi_0\left(\square\ {}^{\square}/_{R}\right)\xi(\square, \Omega)\right]}{k(\Omega)}$$

(3.2.18)

$$\Phi_1(\square, R) = \frac{\left[\ \Phi_0(\square, R)\,k(\Omega)\right]}{\square}\,\xi(\square, \Phi)$$

(3.2.19)

Where $k(\Omega)$ is the constant which makes the new IF $\Phi_1(\square, R)$ a probability density function is given by

$$k(\Omega) = \int_{R^T} \psi_0\left(\square\ {}^{\square}/_{R}\right)\xi(\square, \Omega)\, d\square \qquad (3.2.20)$$

These transformation for the new RF and IF are suggested to change the variance of the remainder function while keeping the constraints (3.7) and $\xi(\square, \Omega)$ defined by

$$\xi(\square, \Omega) = \prod_{t=1}^{T} \xi(\lambda_t, \eta_t) \qquad (3.2.21)$$

With $\xi(\lambda_t, \eta_t) = \exp\left\{-\frac{1}{2}\eta_t' \eta_t \Omega_t\right\}$

Where $\eta'_t = (\lambda_t, \lambda_{t-1}, 1)$ this reduces the computational burden for calculating a new IF and RF. To obtain $\Omega = \{\Omega\}_{t=1}^{T}$ and hence $\xi(\square, \Omega_t)$ the iterations needed are as follows

We generate a set of N independent random vectors $\{\lambda_{0,t}\}$ for $t = 1, 2, 3, \ldots . T$ satisfying the following equations when $t = 1$, $\lambda_{0,t}/_{\theta} \sim N\left(\alpha_0, \gamma_0^2\right)$ where α_0 and γ_0 are the initial values of α *and* γ respectively where t is greater than one

$$\lambda_{0,t}/_{\lambda_{t-1,\theta}} \sim N(\alpha + \beta\lambda_{t-1}\gamma^2)$$

Construct a matrix $\Omega = \{\Omega\}_{t=1}^{T}$ with OLS estimate of the coefficients which is given by

$$\widehat{\Omega}_{1,t} = \begin{bmatrix} -2\hat{a}_{1,t} & 0 & -\hat{b}_{1,t} \\ 0 & 0 & 0 \\ -\hat{b}_{1,t} & 0 & -2\hat{c}_{1,t} \end{bmatrix}$$ For t =1, 2, 3 …T

Construct a first new IF $\psi_1\left(\square/_R\right) = \dfrac{\psi_0\left(\square/_R\right)\xi\left(\widehat{\Omega}_1\right)}{K\left(\widehat{\Omega}_1\right)}$ with $\widehat{\Omega}_1 = \{\Omega\}_{t=1}^{T}$ this implies that a new random variable

$$N\left(\frac{\alpha_0 + \beta_0\lambda_{0,t-1} + (\sqrt{2}\hat{a}_{1,t})^{-1}\gamma_0\hat{b}_{1,t}}{(1-\sqrt{2}\hat{a}_{1,t})\gamma_0}, \hat{\gamma}_0{}^2\left(1-\gamma_0\sqrt{2}\hat{a}_{1,t}\right)^2\right)$$

Construct the i^{th} matrix $\Omega_i = \{\Omega_{i,t}\}_{t=1}^{T}$ with the coefficients obtained by regressing $\ln \square\, \emptyset_0(\lambda_{t-1,n,t})$ on $\lambda_{i-1,n,t}$ and $\lambda^2{}_{i-1,n,t}$

Determine a i^{th} step IF $\psi_i\left(\square/_R\right) = \psi_{i-1}\left(\square/_R\right) = \dfrac{\psi_{i-1}\left(\square/_R\right)\xi\left(\widehat{\Omega}_{i-1}\right)}{k\left(\widehat{\Omega}_{i-1}\right)}$

The iteration algorithm is repeated until $\widehat{\Omega}_i$ is sufficiently close to $\widehat{\Omega}_{i-1}$ usually the number of iterations is less than 5then the MC sample mean is given by $\int_N\left(\frac{R}{\theta}\right) = \frac{1}{N}\sum_{n=1}^{N}\square$

3.2.3 Pricing Data.

After estimating the interest rates volatilities and using OLS to run and find the future interest rate patterns and predictions, the estimated interest rates are used in a Schwartz (1997) Two factor model to try and price a contingent claim

The data used to test the model consists of weekly observation of futures prices for two commercial commodities KCB and one precious metal, SASINI in every case five future contracts (n=5) were used in the estimation. For different commodities and different time periods, however, different specific futures contracts had to be used since they vary across commodities through time for a particular commodity. The data of interest are yield on 3 months treasury bills. These data must only be used in models requiring variable interest rates

Schwartz (1997) two factor technical model.

To give formulae and relations needed to understand the Schwartz two factor commodity model {Schwartz 1997}. This includes parameter estimates using the Kalman filter, pricing of European option as well as computation of risk measures

Schwartz two factor model is essential in understanding the R package Schwartz (1997).

Two factors are spot prices of a commodity together with instantaneous convenience yield was first done by Gibson & Schwartz (1990) and re done again by Schwartz (1997) for pricing futures contracts. Miltersen and Schwartz (1998) and Hillard & Reis (1998) presented equations for arbitrage free prices of European option on commodities. The derivation of the transition density of the two state variables

MODEL

The spot price of the commodity and instantaneous convenience yield are assumed to follow the joint stochastic process

$$ds_t = (u - \delta_t)s_t\, d_t + \delta_s\, s_t\, d_{w_s} \qquad \textbf{Factor one}$$

$$d\delta_t = K(\alpha - \delta_t)dt + \delta_\varepsilon d_{w_\varepsilon} \qquad \text{Factor two}$$

With Brownian motions w_s and w_ε under the objective measure

ψ $and\ correlation\ dw_s dw_\varepsilon = \rho dt$

Under the pricing measure Q the dynamics are of the form

$$ds_t = (r - \delta_t)s_t\, d_t \; + \delta_s\, s_t\, d_{\widehat{w}_s}$$

$$d\delta_t = [K(\alpha - \delta_t) - \lambda]dt \; + \delta_\varepsilon d_{\widehat{w}_\varepsilon}$$

$\lambda = \textbf{Market price of convenience yield risk}$
And $\widehat{\mathbf{w}}_s$ & $\widehat{\mathbf{w}}_\varepsilon$ **are Q – brownian motion**

A new mean level for the convenience yield process Q is also introduced

$\widehat{\alpha} = \alpha - \lambda/_k$ Leading to the dynamics

$$dd\delta_t = K(\alpha - \delta_t)dt \; + \delta_\varepsilon d_{w_\varepsilon} = K(\widehat{\alpha} \; - \delta_t)dt \; + \delta_\varepsilon d_{w_\varepsilon}$$

Distribution

Joint Distribution of state variables density is $\begin{pmatrix} X_t \\ \delta_t \end{pmatrix} \cong N\left[\begin{pmatrix} \mu_{x(t)} \\ \mu_{\delta(t)} \end{pmatrix}, \begin{pmatrix} \delta^2{}_{x(t)} & \delta_{x\delta(t)} \\ \delta_x\delta(t) & \delta^2{}_{\delta(t)} \end{pmatrix}\right]$

With parameters

$$\mu_{x(t)} = X_0 + \left(\mu - \frac{1}{2}\delta_s^2 \mid \alpha\right)t + (\alpha - \delta_o)\frac{1 - e^{-Kt}}{K}$$

$$\mu_{\delta(t)} = e^{-Kt}\delta_0 \; + \alpha\left(1 - e^{-rt}\right)$$

$$\delta^2{}_{x(t)} = \frac{\delta_\varepsilon^2}{k^2}\left[\left(\frac{1}{2k}\left(1 - e^{-2kt}\right)\right) - \frac{2}{k}\left(1 - e^{-kt}\right) + t\right] + \frac{\rho 2\delta_s\delta_\varepsilon}{k}\left(\frac{\left(1 - e^{-kt}\right)}{k} - t\right) + \delta_s^2$$

$$\delta^2{}_{\delta(t)} = \frac{\delta_\varepsilon^2}{2k}\left(1 - e^{-2kt}\right)$$

$$\delta_x\delta(t) = \frac{1}{k}\left\{(\delta_s\delta_\varepsilon\rho)\left(1 - e^{-kt}\right) + \frac{\delta_\varepsilon^2}{2k}\left(1 - e^{-2kt}\right)\right\}$$

Mean parameters3. 8 and 3.9 refer to the P Dynamics

To obtain parameters under Q simply replace μ by r and α by $\widehat{\alpha}$ defined in (3.5)

The futures Price

The futures and the forward prices coincide in this research work by the interest rate applicable is time dependent and any other statement made about the futures contract holds also for the forward contracts.

The parameter estimation

Attempts are made in this work to elegantly estimate the Schwartz two factor models. That's the estimation of the model parameters using Kalman filter as in Schwartz(1997) and express this two factor model into a state space form. Once in this form the likelihood can be computed and numerically maximized

4.0 CHAPTER FOUR

4.1 DATA ANALYSIS

4.1.0 Introduction

This chapter is broken into two parts with different data requirement. Part 1 involves the use of interest rates prevailing on the Government Bonds to test the SV models used to estimate the stochastic interest rates over time. Specifically, the Data used here are of the average half yearly rates of interest from the year 2006 as obtained from the Central Bureau of Statistics publications other determining factors as the rates of inflation are ignored

4.1.1 Estimation of the joint stochastic interest rate process

Let's define two processes for $S \text{ and } \delta$ since neither of this two state variables are easily observed using the concept of Gibson and Schwartz (1989) that assumes that there is no true spot market price for stock, we identify this variable with the settlement price of the closed maturity stock futures contract.

It can be shown that the log the rates of interest series are serially uncorrelated but dependent as usual however in 2007 and 2008 series; the first log autocorrelation appears to be large. This is because the limits are very narrow due to the large number of observations

Table 1: Summery of Statistics of Data

Statistics	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**
Sample Size	12	12	12	12	12	12
Mean	0.11	0.13	0.15	0.16	0.16	0.13
Standard deviation	0.25	0.015	0.019	0.025	0.024	0.049
Minimum	-0.17	-0.08	-0.17	0.15	-0.15	0.16
Maximum	0.179	0.08	0.1	0.1	0.13	0.25

Parameter estimation results

The SV model estimation results are summarized along with statistical standard error of the parameter estimate and the MC sampling standard deviation of the parameter estimates. The small MC standard deviations indicate that SML estimates are precise. The estimated β are highly significant in all cases, however 2006 and 2007 data, the β are slightly lower than the rest.

Table 2: SML estimation of the SV-Normal Model

This table summarises the SV normal results along with standard normal errors of the parameter estimates and the MC sampling standard deviations of the parameter estimates. The small MC standard deviations and

Parameter	2006	2007	2008	2009	2010	2011
α	0.003	-0.034	-0.009	0.002	-0.02	0.003
MC std dev	0.005	0.001	0.0004	0.002	0.001	0.003
Std error	0.012	0.002	0.043	0.04	0.001	0.02
β	0.9	0.9	0.9	0.98	0.95	0.95
MC std dev	0.01	0.0002	0.001	0.0004	0.001	0.0004
Std error	0.01	0.05	0.06	0.04	0.01	0.05
γ	0.14	0.1	0.2	0.3	0.11	0.12
MC std dev	0.001	0.001	0.001	0.001	0.001	0.004
Std error	0.06	0.05	0.05	0.05	0.06	0.05

The results for the rates of interest with student t distribution with $\omega = 10$ are displayed below. The MC standard deviations and standard errors for all the parameter estimates are quite smaller than the ones from the SV-normal model. The results may be already expected from the fact that the unconditional variance of the latent process is equal to $\delta^2 = \gamma^2 / (1-\beta^2)$ which appears to be smaller for the SV-t than the SV- normal model in all the series. The β estimates are all around 0.95 which are reasonable. Furthermore, the standard error of the parameter estimates in most cases are quite small, but the estimates of β and γ for 2009 are somewhat high than the estimated
Volatility Estimated Result the following tables, the MC estimate of the sequence of volatilities $E\left(\delta_t / R, \theta\right)$ resulting from the SV-normal and the SV-t model are presented along with log returns, all the log rates seems to be stationary compared to the actual rates of

interests, the volatility of the estimates from the SV-normal model does not seem to reflect the movement of the rates of interests compared to the one from SV-t model, its estimated that the GM shows that estimated volatility from SV-normal model is slightly stable than the SV-t model and both estimates doesn't reflect the movement of the interest rates series. In the rest of the cases, the estimated volatilities from the SV-t model exhibit smoother movements than the ones from the SV-normal mod

Table 3: SML estimates of the SV-t distribution model with $\omega = 10$

parameter	2006	2007	2008	2009	2010	2011
α	0.002	-0.03	-0.01	0.001	-0.02	0.003
MC std dev	0.0001	0.0003	0.0003	0.001	0.001	0.0004
Std error	0.01	0.01	0.01	0.03	0.03	0.01
β	0.9	0.96	0.989	0.98	0.99	0.98
MC std dev	0.002	0.0001	0.004	0.001	0.0002	0.0001
Std error	0.01	0.05	0.03	0.1	0.04	0.04
γ	0.09	0.09	0.13	0.114	0.09	0.08
MC std dev	0.006	0.0003	0.0002	0.0002	0.0004	0.0007
Std error	0.05	0.03	0.04	0.08	0.03	0.05
K	4.24	4.48	8.19	5.82	6.67	4.89

Conclusion

This paper tries to use the SV model with a normal error distribution and a t-distribution. The Simulated Maximum Likelihood (SML) method, proposed

By Danielsson and Richard (1993) is applied. The results from the empirical hypothetical figures can be details by the following elaborations as follows. First, the Simulated Maximum Likelihood (SML) approach with the Accelerated Importance Sampling (AGIS) technique provides a reasonably accurate of parameter estimation. Second, we find that the SV-t model captures the properties of high kurtosis and slowly decaying ACF of squared returns usually seen in the financial time series. Finally, the SV-t model is a reasonable choice for the financial interest rates series in our example.

3.1.2 Model Verification

It's imperative it do some model validation to justify the appropriateness of this model. Define the standardised error from the equation (1)

$\hat{u}_t \cong \exp\left\{\frac{\hat{\lambda}_t}{2}\right\}$ and t =1, 2, 3, 4,....1000. the $\hat{\delta}_t's$ are the estimated rates of interest by the estimated volatilities and $r_t's$ are the observed rates of interest. If the model is appropriate for our data, then there should be no autocorrelations if the standardized error $\hat{u}_t$ and the squared standardized error u_t^2

Figures in the appendix exhibit the ACF of u_t^2 *and* $\hat{u}_t$. Most plots show no significant serial autocorrelations at certain lags. For example RBC and CIBC seem to have large autocorrelation at lag one, but this may be due to narrow limits resulting from a large sample size. The standard deviations of $\widehat{u}_t$ are summarized in table 4. This result is expected from the original assumption of $u_t \approx t(0,1)$

Table4. The Standard Deviations of the error process u_t *and* v_t

Parameter	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**
SD of u_t	0.784	0.624	0.674	0.735	0.53	0.68
SD of v_t	0.95	0.99	0.98	0.92	1.04	0.97

We examine the error process $\{v_t^*\}$ where

$v_t^* = \gamma u_t$ in (2) with the same criterion. $\hat{v}_t$ Is generalized from $\hat{\lambda}_t = \widehat{\alpha} + \hat{\beta}\hat{\lambda}_{t-1} + \hat{\gamma}\,\hat{u}_t$

$t = 1, 2, 3, \ldots 1000$, $\widehat{\alpha}$, $\hat{\beta}$ and $\hat{\gamma}$ are the ACF of the estimated error v_t^*

The standard errors in table 4 are all closer to one, which is the standard deviation of the error process v_t.

Model Testing and Examination

Lastly we re examine the applicability and the practicality of the model given the modest data to justify the model. Let's begin by re defining the standardised error from the equation

$r_t = \exp\left({\lambda_t}/{2}\right)\mu_t$ And $\dot{u}_t = {r_t}/{\hat{\delta}_t}$ where $\hat{\delta}_t = \exp\left\{{\hat{\lambda}_t}/{2}\right\}$

and $T = 2006, 2007, 2008, 2009, 2010$ *and* 2011 conveniently for our notations, we label the time periods as $t = 1, 2, 3, 4, \ldots\ldots\ldots\ldots\ldots$ The $\hat{\delta}_t's$ are the estimated volatility by the estimation method discussed above and r_t are the model observed rates of interest. If the model is appropriate for our data then there should be no autocorrelation of the standardised error u_t and the squares standard error $\hat{\mu}_t^2$

Further checking of the model, we examine the error process $\hat{v}_t$ where $\hat{v}_t = \gamma v_t$ as

$\lambda_t = \alpha + \beta\lambda_{t-1} + \gamma v_t$ As the same criterion where $\widehat{\alpha}, \widehat{\beta}$ and $\hat{\gamma}$ are the SML Parameter estimates and $\widehat{\lambda}_t$ and $\widehat{\lambda}_{t-1}$ are the estimated volatilities .

CHAPTER FIVE

5.0 CONCLUSION AND STUDY LIMITATION

5.1. Limitation of the study

Analysis of behavior of the interest rates is important in pricing and valuation of assets. Any model intended to do this must be robust enough to be able to capture the ever changing business environment. This project paper has endeavored to use the SV model to estimate the volatility of the interest model, regress the future rates and use the fairly stochastic estimated rate of interest to price a contingent claim using a two factor Model. However due to limitation of time the paper has to be re looked at going forward to ascertain the exact and salient Model weaknesses.

Its normally a known fact within financial practitioners that there exists a gap between the factor model and other pricing models. Most factor models are time-homogeneous models and as such they cannot be consistent with the current yield curve it would have been more promising to price this claims and compare the yield curves. Most factor models do not have closed-form formulas for the zero-coupon bond prices. Thus for practical implementation of the factor models some form of recombining binomial trees or partial differential equations may not be applicable and bushy trees or Monte Carlo methods are required. It is necessary to keep track of the entire yield curve.

This paper would have provided the bridge for the gap between the factor model and the HJM model and derives a class of efficient factor models that can be readily extended to fit the current yield curve and have closed-form formulas for pricing default-free zero-coupon bonds. The volatility of interest rates is a fundamental determinant of the values of interest rate derivatives.

We, therefore, generalize the model to incorporate explicitly the stochastic volatility of the short rate.

The following are some of the observed Limitations

5.2: STUDY LIMITATIONS

There is a silent observed threat that the projected interest rate in this SV model could turn negative in this model is undesirable business wise and presents a pricing challenge. The study proposes a further examination of this estimate going forward including the proposing of exponentialising the final estimated parameter

There is unfulfilled attempt of resolve the simple most observed weakness of the SV model that is the volatility clustering. This contradicts the use mean reverting nature of interest rate regimes. As such the SV model is unable to capture the empirical regularities of the financial time series

The hypothetical interest rates captured in this alludes to the dirt and may not be accurate but desirably represents the best estimate of the interest rates

5.3: STUDY CONCLUSIONS:

Going forward there is the need to build on the positive of the model used to develop a more robust pricing Model that will reflect the vulgarism of time and other asset pricing factors. However these are the key aspects to build on:

This paper has analyzed the SV model with a normal error distribution and a leptokurtic error distribution (t-distribution). The Simulated Maximum Likelihood (SML) method, proposed by Danielsson and Richard (1993) is applied.
The results from the empirical example can be summarized as follows. To begin with, the SML approach with the Accelerated Importance Sampling (AGIS) technique produces a high accuracy of parameter estimation. Secondly, it's observed that the SV-t model captures the properties of high kurtosis and slowly decaying ACF of squared returns usually seen in the financial time series. Hence we can conclude that the SV-t model is reasonable choice for the financial interest rates series as has been observed. Given that the volatilities estimated can be trusted accurately to model future rates of interest

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