SYSTEMATIC MODELING OF WHITENOISE WITH FINANCIAL TIME SERIES IN DECISION MAKING

A THESIS SUBMITTED TO THE UNIVERSITY OF NAIROBI FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN MATHEMATICAL STATISTICS IN THE SCHOOL OF MATHEMATICS

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DECLARATION AND APPROVAL

Declaration

I, the undersigned declare that this thesis contains my own work. To the best of my knowledge, no portion of this work has been submitted in support of an application for another degree or qualification of this or any other university or other institution of learning.

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This thesis has been Under our supervision and has our approval for submission.

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DEDICATION

This thesis is dedicated to the Holy Trinity,
my loving mother Regina Ibasha Shileche,
and my family members.

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ABSTRACT

In this study real non - diversifiable (systematic) risk is derived. This risk together with diversifiable (non - systematic) risk is weighted against expected returns of certain assets to determine maximum returns of these assets at minimum risk. A Real Risk Weighted Pricing Model (RRWPM) is thus developed which is able to postulate expected returns and risks of assets in the past and in the near future. This enables discounting rates and costs of capital to be accurately determined. Decisions can also be made based on the point estimators determined for example expected returns and total risks of assets obtained.

Finite investment decision making using real market risk (Non-diversifiable risk) is then undertaken. The Non-diversifiable risks estimates of a portfolio of stocks as determined by the RRWPM are used as initial data. The variance of non-diversifiable risk is estimated as a random variable referred to as random error (white noise). The estimator is used to calculate estimates of white noise. A curve estimation of the white noise is made using Kernel density estimation. This is used to derive probability estimates of the non-diversifiable risks of the various stocks. This enable comparison among the portfolio of stocks and propagates good decision making.

Actual future market risks (systematic or non-diversifiable) of investment portfolios are then determined. Future returns are forecasted using past returns and GARCH (Generalized Autoregressive Conditional Heteroskedastic) models. RRWPM is used to estimate future systematic risk among other point estimators and determines the

future costs of the portfolios. Forecasted random error is then calculated as a random variable and used to determine probability density estimates of systematic risk. This enables future actual market risks of portfolio investments to be derived hence facilitating proper future investment decision making.

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COMMON SYMBOLS

t - Time index

r_i - Security return i

P - Share price

∞ - Infinity

R_n - Sample space of returns

Ω - Parametric mean

μ - Population mean

σ - Population standard deviation

E - Expected value

var - Variance

Pr - Probability

cov - Covariance

w_i - Weight of security return i

D_m - Diversifiable risk

N_q - Non - diversifiable risk

$E[R_i]$ - Weighted expected return of security i

VaR - Value at risk

wn - White noise

h_{sj} - Sheather - Jones plug - in bandwidth

ϵ - Model's prediction error

MA - Moving average

AR - Autoregression

h_{AMISE} - A bandwidth that minimises the absolute minimum integrated square error

r^2 - Coefficient of determination

S_{ey} - The standard error for the y estimate

$E(r_i)$ - The Capital Asset Pricing Model Cost of capital for the firm

ARCH - Autoregressive Conditional Heteroskedasticity

GARCH -Generalized Autoregressive Conditional Heteroskedasticity

Chapter 1

INTRODUCTION

1.1 Background

Decision making is a broad subject encompassing most of our day to day activities. It forms the basis of most actions. In most financial institutions investment decision making is an important financial management function. Financial management is that management activity which is concerned with the planning and controlling of the firm's financial resources. Financial functions include, Investment decision making, Capital - mix decision making, profit allocation and liquidity decision making. Financial accounting is the process of identifying, measuring and communicating economic information to permit informed judgement and decisions by users of the information. The goal of financial managers is the maximization of owners' economic welfare. To this end financial managers use the information prepared by financial accountants in making decisions concerning the use of limited resources, including the identification of crucial decision areas and determination of objectives and goals.

Continuous technological development and competition requires that past investment decisions are reanalyzed and updated and current ones are monitored to ensure that they are within the required expectations. These can be classified into short term, intermediate and long term investment decision making. Good investment decision making is essential for the sustainability and growth of financial institutions. It is the good investment strategists' that experience success in financial environments. This necessitates employment of suitable investment decision making methods.

In the past two decades, analysts have observed increasing integration of international financial markets. Barriers to international investment among developed economies have slowly but steadily diminished. Hence, global risk factors are increasingly important for portfolio selection and asset pricing. Recent empirical evidence indicates, specifically, that global occurrences affect the pricing of stocks in industrialized countries. This study investigates global risk factors with a better pricing model. It in fact investigates the cost of capital of randomly selected companies on the New York stock exchange at the height of the global credit crunch of 2008 to 2009.

Cost of capital estimation is becoming increasingly important. First introduced during the 1970s in regulatory proceedings, the application of reserving and other types of financial decision making has grown rapidly over the past two decades. The use of an incorrect cost of capital in capital budgeting, pricing, and other applications can have serious consequences, with firms losing market value if the cost of capital is underestimated and losing market share if the cost of capital is over estimated. Essentially using incorrect cost of capital estimates can lead to the firms' investing in negative net present value projects that destroy firm value.

Most pricing models use betas or standard deviation to represent systematic risk. These are determined as asset covariance or error terms. The actual definition of non - diversifiable risk i.e. the risk that still exists in all well diversified portfolio postulates that it is one which cannot be diversified thus has an element of independence. Using an asset covariance or error terms to represent this risk goes against this phenomenon since these incorporate the dependency factor. Also the weight in the other models is a fraction of the total returns.

Probability is important in investment decision making process since it helps address the problem of uncertainty. Many of the investment decision making methods have incorporated the expectation and risk of an event in making investment decisions. Most of the models that use risk account for diversifiable risk only, limiting the accuracy of these investment methods since total risk are not properly accounted for. A few of these methods incorporate uncertainty. These include Value at Risk method which uses covariance matrices as total risk and the binning system which always assumes normal distribution and thus does not take care of discrete cases.

This study determines probability estimates of various entities in comparison with one another and by incorporating total risk thus making a strong case for good decision making. Recent years have seen a surge of interest in econometric models of changing conditional variance. Probably the most widely used but by no means the only such models, are the family of ARCH (Autoregressive Conditional Heteroskedasticity) models introduced by Engle (1982). This methodology together with the GARCH (1, 1) has been successfully applied in asset pricing models. This study uses a RRWPM that avoids the explosion of conditional moments of GARCH (1, 1). With this model the relationship between the actual and estimated values with GARCH forecasted time series data is almost perfect.

1.2 Statement of the problem

In most of the estimators of risk and expected returns covariances are used to determine market risk or non - diversifiable risk. From the definition of non - diversifiable risk we know that this is the risk that cannot be diversified no matter the number of portfolios used. This has tended to affect the estimates of risk and expected returns being made thus leading to wrong financial or investment decisions being made. For example a wrong estimate of expected returns of a particular portfolio may attract large sums of investments in the business portfolios only to lead to losses. It may even be worse if the risk estimates were not accurate too, as is bound to happen incase the expected returns are not, since the two parameters are interrelated. This study has determined non-diversifiable risk from its basic definition with the aid of well calculated infinit weights. This has also aided the determination of probabilities of these estimates and thus quantified their uncertainities.

1.3 Objectives

The overall objective of the study was to estimate factors which influence investment decision making processes for example real total risk and expected returns. These are determined by weighing expected returns against total risk to determine the weight that will maximize expected returns and minimize total risk. Random error of the non - diversifiable risk is also generated by assuming that these errors are random variables which are independent and thus will have independent parameters. This facilitates the determination of probability distribution functions of the random error with the Kernel density estimation, their parameters and probability estimates. Non - diversifiable risk is then used for the quantification of uncertainty as a basis for comparison among various investment entities.

The specific objectives were as follows:

i) Derivation of models that are used to determine variables which influence investment decision making. For example pricing and risk models.

ii) Deriving estimators of the models.

iii) Show statistical properties of the estimators.

iv) Testing the models in ii) with organised data to determine their plausibility.

v) Using the derived estimators in ii) to model random error.

vi) Deriving probability density estimators (pde) for discrete cases using kernel density estimation

vii) Deriving probability density estimators for continuous cases with GARCH models and kernel density estimators.

viii) Using the pde in vi) and vii) to determine probability density estimates and hence make current and future investment decisions.

1.4 Significance of the study

Investment appraisal as an inclusive function encompassing relevant factors such as real total risk and expected returns and the quantification of their uncertainty will have a great impact on the financial environment.

i) Non - diversifiable and diversifiable risk is considered not as a risk premium but in deriving the expected cash flows thus correct expected cash flows are determined.

ii) A Real Risk Weighted Pricing Model (RRWPM) is determined which gives us almost perfect values of cost of capital and total risk thus ensuring proper capital allocation is made.

iii) Random error estimated as a random variable which enables probability density estimation of the non - diversifiable parameters in part ii) be determined. This ensures that the exact value of this parameter is made.

iv) The probability element determined ensures proper comparability among different portfolios. Investors therefore make an informed choice of portfolios to invest in.

v) Long term investors are aided in their decision making with the results of forecasted returns by GARCH models going through processes iii) and iv).

Chapter 2 gives a brief look at past studies in this area and how they are made better in this paper. Chapter 3 determines the RRWPM by first estimating non- diversifiable risk then weighing it against diversifiable risk and expected returns. Section

3.1 carries out a survey of other past models and compares them with the RRWPM using data from the New York Stock Exchange (NYSE). The variance (white noise) of non-diversifiable risk is estimated and its probability density function modeled in Chapter 4. Probability estimates of the estimated non-diversifiable risk are then used in decision making. Chapter 5 outlines how returns of a portfolio of stocks are forecasted using the GARCH (1, 1) model. Section 5.1 uses forecasted returns determined above and RRWPM modeled by Anyika *et al* (2011) to determine forecasted future cost and total risk. Section 5.2 calculates estimates of white noise using an estimator derived by Anyika *et al* (2010) and determines probability density estimates of the portfolio systematic risk using the Gaussian kernel. Probability estimates of future portfolio risks are then estimated as well as actual market risk. Chapter 6 illustrates the data used and the surveys carried out on the data to give the results presented in the same chapter. Finally Chapter 7 summarises what has been done and concluded based on the results and recommendation for future research.

Chapter 2

LITERATURE REVIEW

Substantial research has been done in the area of determination of expected returns in relation to systematic and non - systematic risk. Early researchers determined risk using standard deviation and covariance function. Later, these were determined by using an expected returns function with a weight of one over the total number of time series.

In the recent past Chu Sheng (2003) has used the conditional version of international Capital Asset Pricing Model (ICAPM) in the absence of Purchasing Power Parity (PPP) to control economic fundamentals. The empirical results of this control indicate that there are no mean spillovers among futures markets unless there is a crisis or conditional volatility influenced by the negative volatility shocks from other assets. Thus there is a close relationship between the Capital Asset Pricing Model (CAPM) and ICAPM since CAPM uses local variables while ICAPM uses similar international variables like foreign exchange and foreign markets. This was experienced in the year 2008 when the tumbling of the markets in the United States of America affected markets which have close trading with them including Japan's, Britain's, France's, German's, but not countries in Africa and South America. This model although justified could not detect the anomalies beforehand hence solutions to such crisis are not timely. It also has defects similar to those of CAPM.

The CAPM by Sharp (1964) and Fama and French (1992) are frequently used in determining the cost of capital in practical application as illustrated by Graham and Harvey (2001) and have been extensively tested in the academic literature. The Fama-French three -factor model (FF3F) of Fama and French (1993) was developed in response to the criticism that the CAPM systematic market risk factor alone does not provide an adequate explanation of the cross-sectional variation of average stock returns. The FF3F as studied by Fama and French (1996) model achieves significantly better explanatory power by adding risk factors to capture the effects of firm size (total market capitalization) and the ratio of the book value of equity (BE) to the market value of equity (ME). David and Richard (2005) uses Full Information Beta (FIB) technique to explain the factors of the FF3F model. The FF3F model results of Fama and French (1997) have been a source of controversy as some researchers question whether the mimicking portfolios truly capture non - diversifiable risk and therefore macroeconomic variables as risk factors have failed to explain a significant fraction of the variation in these returns.

Martin and Sydney (2001) demonstrates that an asset's covariance with scaled consumption growth can go a long way towards accounting for the value premium thereby lending support to the view that the reward for holding high-book to-market stocks arises at least partly as a consequence of true systematic risk elements. This study determines non - diversifiable risk (systematic risk) according to its real definition and weights it against diversifiable risk and expected returns to determine the maximum returns at minimum risk. A Real Risk Weighed Pricing Model is thus developed with an almost perfect correlation between the estimated and actual values. We also use the parameters so derived to make decisions about the asset portfolios analyzed. Most prior research in this area focuses on industries that are exposed to one major source of financial risk. For instance, Schrand (1999) examines whether derivatives activities lower, savings and loans interest rates exposure, measured by the sensitivity of stock prices to movements in interest rates. Similarly Rajgopal (1999) finds that information about derivatives presented in a tabular and sensitivity analysis format is related to the share price exposure of oil and gas firms to oil and gas prices. Wong (2000) examines the foreign exchange rate exposure for a sample of United States manufacturing firms and finds that derivatives notional partially explained exposure. In contrast, a variety of risk factors affect large banks trading portfolios. This study uses non-diversifiable risk which is determined from its true definition of that risk that cannot be diversified.

Jorion (2000) determines the Value at Risk (VaR) measure as the forecasted volatility, S_t multiplied by standard normal deviate, α for the selected confidence level (e.g, $\alpha = 2.33$ for a one-tailed confidence level of 99 percent). The portfolio variance then becomes $S_t^2 = w_t' \Sigma_t w_t$ where Σ_t is the forecasted covariance matrix for the market risk factors as of the close day t of trading. Hence we have, $VaR_t = \alpha S_t$. Although this research takes care of all the other shortcomings of previous researches, the portfolio variance is determined as a covariance which goes against the definition of market risk as that which cannot be diversified. This has been clearly addressed in this research by using non-diversifiable risk which is determined without covariance. Brian *et al* (2006) in estimating density dependence process noise and observation error offers a statistical approach for jointly estimating density dependence, process error and observation error. Although this model is relatively easy for ecologists to use and is applicable in many population systems, the process noise has a normal distribution with mean μ and variance $\sigma^2 (E_t \sim N(0, \sigma^2))$. This research looked at a case of no assumption of normality for the noise process and normality using GARCH models. White noise is determined as a random variable on the precincts of Sklar (1996) where he says no common probability space can be found for a given set of random variables, but such common probability spaces exists for arbitrary proper subsets of the given set. In this study the subsets were the portfolios of different companies used giving a common probability space that is estimated. The results of Wu (1980) show that for finite parameters the consistency of the least squares estimator is equivalent to the existence of a consistent estimator thus the estimator of the white noise derived in this study is an unbiased estimator.

Researchers have successfully applied the new ARCH methodology in asset pricing models. For example, Engle *et al* (1986) used GARCH (1, 1) to model the risk premium on the foreign exchange market and Bollerslev *et al* (1988) extended GARCH

(1, 1) to a multivariate context to test a conditional CAPM (Capital Asset Pricing Model) with time varying covariance. However their results show that shocks may persist in one norm and die out in another, so the conditional moments of GARCH (1, 1) may explode even when the process itself is strictly stationary and ergodic as researched by Nelson (1990). Achia *et al* (2008) revealed that the GARCH (1, 1) model provided a good explanation of the dynamics of the market returns but failed to obey the efficient market principle indicating that there is market risk. This research used a RRWPM as determined by Anyika *et al* (2011) that avoids the explosion of conditional moments(spiked) of GARCH (1, 1). With this model the relationship between the actual and estimated values with GARCH forecasted time series data is a perfect fit. With the determination of total forecasted risk using the RRWPM the assumption of an efficient market need not be upheld.

Chapter 3

REAL RISK WEIGHTED PRICING MODEL

3.1 Realistic Pricing Model

Many models exist in the financial environment. These range from simple investment decision models such as return on investment, Payback period, Net present value, Internal rate of return to the more detailed models such as Scholes (1973) options pricing model to the Capital Asset Pricing Model and Value at risk among others. The common factor about these models are the assumptions made in order for them to be functional. For example the cornerstone equilibrium and efficiency assumptions of the theory of finance are inconsistent with empirical observations. In particular Shiller (1989) argues that while some of the implications of efficient markets hypothesis (that speculative prices always present the best information about the true economic value) are substantiated by data, investor attitudes are of great importance in determining the course of prices of speculative assets. In fact, prices change in substantial measure because the investing public en masse capriciously changes its mind. Shiller (1989) further emphasizes the profound practical implications of his findings. That is price change for no good reason is of great importance for many purposes. For instance, when an asset is under priced incentives are created to neglect or abuse it. When it is overpriced incentives are created to invest too many resources in it.

On the other hand Peters (1991) pursues a not dissimilar theme using chaos theory as one of his main investigative tools and concludes that current theories are inadequate since there is evidence that the capital markets are non - linear systems and that current capital market theory does not take these effects into account. Due to these and some other unrealistic assumptions we have encountered like the CAPM and VaR models in chapter two there is need for proper determination of pricing models incorporating realistic, current and practical assumptions. The sections below determine an asset pricing model governed by these principles.

3.2 Asset Returns

Real Risk Weighed Pricing Model focuses on the joint distribution of returns at a single time index t the distribution of $r_1(t), \cdots$ for asset returns, $r_i(t); i = 1, 2, 3, \cdots,$ and time $t = 2, 3, 4, \cdots, n$, n being the sample size. It is customary to treat asset returns as continuous random variables especially for index return of stocks calculated at a low frequency, and use their probability functions. In this case, we can write the probability density function, as,

$$f(r_i(t), \cdots, r_i(n)|\Theta) = f(r_i(t)|\Theta) \prod_{t=2}^{n} f(r_i(t)/r_i(t-1), \cdots, r_i(1)|\Theta)$$

For higher frequency asset returns, discreteness becomes an issue. For example, stock prices change in multiples of a tick size (frequency). Therefore, the tick-by-tick returns of an individual stock listed on many stock exchanges is not continuous. The rate of return on asset i at time t as used in this study is the simple return defined as:

$$r_i(t) = \frac{\mathrm{P_i(t) - P_i(t-1)}}{\mathrm{P_i(t-1)}} \tag{3.1}$$

where: $P_i(t)$ is the share price of asset i at time t.

3.3 Estimation of Non - diversifiable risk

Assume that

- $r_1(t), r_2(t), \cdots$ are, normally distributed with mean zero (0) variance σ^2
- Sample space is $R_n = \{(r_1(t), r_2(t), \cdots, r_n(t)) : -\infty < r_i(t) < \infty\}$
- Parametric space is $\Omega = \{(\mu, \sigma) : -\infty < \mu, \sigma \geq 0\}$
- Autocovariance function $=Cov[r_i(t), r_i(t+h)]$
- $r_m(t), r_i(t), r_j(t)$ are returns of asset m, i and j respectively.

Thus if $r_m(t) = r_i(t) + r_j(t)$

then $Var(r_m(t)) = Var(r_i(t)) + Var(r_j(t)) + 2Cov(r_i(t), r_j(t))$, also expanded to include infinit weights with a range of $-\infty < w_i < \infty$

$$Var\left[\sum_{i=1}^{\infty} w_i r_i(t)\right] = \left[\sum_{i=1}^{\infty} w_i^2 \sigma_i^2 + 2 \sum_{i \neq j} \sum w_i w_j \sigma_{ij}\right] \tag{3.2}$$

Taking the square root of equation 3.2 gives

$$\left[Var \sum_{i=1}^{\infty} w_i r_i(t)\right]^{\frac{1}{2}} = \left[\sum_{i=1}^{\infty} w_i^2 \sigma_i^2 + 2 \sum_{i \neq j}\sum w_i w_j \sigma_{ij}\right]^{\frac{1}{2}} \tag{3.3}$$

which is known as diversifiable risk (D_m) experienced in most organisations and can be controlled by these organisations. Diversifiable risk can also be reduced by a well constituted portfolio of investments as noted by Stulz (1999), Harper (2003) and Andrew *et al* (2005):

Let non - diversifiable risk which is that risk that is not reduced due to diversification be

$$N_g = \left[\sum_{i=1}^{\infty} w_i^2 \sigma_i^2\right]^{\frac{1}{2}} \tag{3.4}$$

where,

$Cov[r_i(t), r_j(t)] = 0$

This is the risk that organisations have no control of and thus it cannot be diversified by a well constituted portfolio. Total risk will thus be equal to diversifiable risk plus non - diversifiable risk since risk can either be diversifiable or non - diversifiable.

That is

$$\left[\sum_{i=1}^{\infty} w_i^2 \sigma_i^2 + 2 \sum_{i \neq j}\sum w_i w_j \sigma_{ij}\right]^{\frac{1}{2}} + \left[\sum_{i=1}^{\infty} w_i^2 \sigma_i^2\right]^{\frac{1}{2}} \tag{3.5}$$

Proposition 3.1: Non-diversifiable risk $(w_i^2 \sigma_i^2)^{\frac{1}{2}}$ of a given investment i remains unchanged for $i = 1, 2, 3, \cdots, \infty$ investments of a given portfolio.

Proof:

Assuming that $r_i(t) \sim N(\mu, \sigma^2)$, let n be a sample of investments.

Total sample risk of these investments will be given by

$$\left[\sum_{i=1}^{n} w_i^2 s_i^2 + 2 \sum^{n} \sum^{n}_{i \neq j} w_i w_j s_{ij}\right]^{\frac{1}{2}} + \left[\sum_{i=1}^{n} w_i^2 s_i^2\right]^{\frac{1}{2}} \tag{3.6}$$

and total sample variance represented by portfolio variance (D_m^2) plus non- portfolio variance (N_g^2),

$$\sum_{i=1}^{n} w_i^2 s_i^2 + 2 \sum \sum_{i \neq j} w_i w_j s_{ij} + \sum_{i=1}^{n} w_i^2 s_i^2 \tag{3.7}$$

For a two investment portfolio total variance is equal to

$$w_1^2 s_1^2 + w_2^2 s_2^2 + 2 w_1 w_2 s_{12} + w_1^2 s_1^2 + w_2^2 s_2^2$$

$$= 2 w_1^2 s_1^2 + 2 w_2^2 s_2^2 + 2 w_1 w_2 s_{12}$$

Thus the value of w_1 that minimizes risk for a two investment portfolios is given by,

$$\frac{\partial(\mathrm{D_m^2} + \mathrm{N_g^2})}{\partial_{\mathrm{w_1}}} = 4 w_1 s_1^2 + 2 w_2 s_{12} \tag{3.8}$$

$$4 w_1 s_1^2 + 2 w_2 s_{12} = 0$$

$$4 w_1 s_1^2 = -2 w_2 s_{12}$$

$$\frac{4\mathrm{w_1 s_1^2}}{4\mathrm{s_1^2}} = \frac{-2\mathrm{w_2 s_{12}}}{4\mathrm{s_1^2}}$$

$$w_1 = -\frac{1}{2}\left(\frac{\mathrm{w_2 s_{12}}}{\mathrm{s_1^2}}\right)$$

For three investments the value of w_1 that minimizes risk is given by,

$$= \frac{\partial(\mathrm{D_m^2} + \mathrm{N_g^2})}{\partial_{\mathrm{w_1}}} = 4w_1 s_1^2 + 2w_2 s_{21} + 2w_3 s_{31} \tag{3.9}$$

which gives

$$4w_1 s_1^2 = 2w_2 s_{21} - 2w_3 s_{31}$$

so that

$$w_1 = -\frac{1}{2}(\frac{\mathrm{w_2 s_{21}}}{\mathrm{s_1^2}}) - \frac{1}{2}(\frac{\mathrm{w_3 s_{31}}}{\mathrm{s_1^2}})$$

For four investments the value that minimizes risk is given by,

$$w_1 = -\frac{1}{2}(\frac{\mathrm{w_2 s_{21}}}{\mathrm{s_1^2}}) - \frac{1}{2}(\frac{\mathrm{w_3 s_{31}}}{\mathrm{s_1^2}}) - \frac{1}{2}(\frac{\mathrm{w_4 s_{41}}}{\mathrm{s_1^2}}) \tag{3.10}$$

continuing in the same manner we see that for n investments the value of w_1 that minimizes risk is given by,

$$w_1 = -\frac{1}{2}(\frac{\mathrm{w_2 s_{21}}}{\mathrm{s_1^2}}) - \frac{1}{2}(\frac{\mathrm{w_3 s_{31}}}{\mathrm{s_1^2}}) - \frac{1}{2}(\frac{\mathrm{w_4 s_{41}}}{\mathrm{s_1^2}}), \cdots, \frac{1}{2}(\frac{\mathrm{w_n s_n}}{\mathrm{s_1^2}}) \tag{3.11}$$

Similarly $w_2, w_3, ..., w_n$ can be obtained.

Taking expectations of equations 3.8, 3.9, and considering that the investments are not correlated for non-diversifiability we obtain

$$E(w_1 s_1^2) = E(-\frac{1}{2} w_2 s_{12})$$

$$w_1 \sigma_1^2 (\frac{\mathrm{n}-1}{\mathrm{n}}) = (-\frac{1}{2} w_2(0))$$

as $n \to \infty$

$$\begin{aligned} w_1\sigma_1^2((\frac{\mathrm{n}}{\mathrm{n}}) - (\frac{1}{\mathrm{n}})) &= 0 \\ w_1\sigma_1^2((1) - (\frac{1}{\infty})) &= 0 \\ w_1\sigma_1^2(\frac{\mathrm{n}}{\mathrm{n}}) - 0 &= 0 \\ w_1\sigma_1^2 &= 0 \qquad (3.12) \end{aligned}$$

Multiplying Equation 3.12 by w_1 results in $w_1^2\sigma_1^2 = 0$ Where n is the sample size, $n > 1$ and $s_1^2 = \sigma_1^2(\frac{\mathrm{n}-1}{\mathrm{n}})$ as shown by Tobago (2010). NOTE: $E(s_{ij}) = 0$ for non-diversifiability.

From equation 3.9,

$$\begin{aligned} E(w_1 s_1^2) &= E(-\frac{1}{2}w_2 s_{21}) + E(-\frac{1}{2}w_3 s_{31}) \\ w_1\sigma_1^2(\frac{\mathrm{n}-1}{\mathrm{n}}) &= (-\frac{1}{2}w_2(0) - \frac{1}{2}w_3(0)) \end{aligned}$$

as $n \to \infty$

$$w_1\sigma_1^2 = 0 \qquad (3.13)$$

Multiplyng both sides of Equation 3.13 by w_1 results in $w_1^2\sigma_1^2 = 0$ Similarly for investments $4, 5, 6, ..., \infty$. $w_1^2\sigma_i^2 = 0$.

Since for any number of investments, $w_1^2\sigma_1^2 = 0$, then non - diversifiable risk $N_g = (w_i^2\sigma_i^2)^{\frac{1}{2}}$ of investment $i = 1, 2, ..., \infty$ for given portfolio remains unchanged that is it is not affected by the number of portfolio you use, thus not diversified.

Weights of asset of portfolios which minimize total risk and maximize returns as determined by Anyika *et al* (2005) are given by

$$\begin{pmatrix} w_1 & w_2 & w_3, \cdots, w_\infty \end{pmatrix} \begin{pmatrix} 3\sigma_1^2 & 2\sigma_{12} & \cdots & 2\sigma_{1m} \\ 2\sigma_{21} & 3\sigma_2^2 & \cdots & 2\sigma_{2m} \\ \vdots & \ddots & \vdots & \vdots \\ 2\sigma_{\infty 1} & \cdots & \cdots & 3\sigma_m^2 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} E(r_1)(t) \\ E(r_2)(t) \\ \vdots \\ E(r_\infty)(t) \end{pmatrix} \tag{3.14}$$

Analysis of returns of stocks of three portfolios Kenya Commercial Bank, East African Breweries Limited and Standard chartered Bank undertaken by Anyika *et al* (2005) and substituting the values obtained for the unknowns in equation 3.14 gave the following results

$$\begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$$

$$= 0.15 \begin{pmatrix} -1.1 \times 10^{-18} & 2.8 \times 10^{-20} & -9.1 \times 10^{-15} \\ -2.8 \times 10^{-20} & -9.1 \times 10^{-15} & 6.8 \times 10^{-21} \\ -9.11 \times 10^{-15} & 6.8 \times 10^{-21} & -4.8 \times 10^{-20} \end{pmatrix}$$

$$\begin{pmatrix} -6.8 \times 10^{-08} \\ -1.5 \times 10^{-07} \\ 3.3 \times 10^{-07} \end{pmatrix}$$

$$(3.15)$$

where the right hand side is the product of the reciprocal of determinant of variance covariance matrix, inverse of variance covariance matrix and values representing returns of the three investment returns.

Thus their weights are:

$$\begin{aligned} w_1 &= -3.48 \\ w_2 &= 1.58 \\ w_3 &= 0.72 \end{aligned}$$

Substituting these weights in equations 3.3 and 3.4 gives the diversifiable risk of a portfolio of Kenya Commercial Bank, East African Breweries Limited and Standard chartered Bank as $2.54382\,X10^{-06}\,\%$ and its non - diversifiable risk as $8.93518\,X10^{-06}\,\%$. For a two investment portfolio of Kenya Commercial Bank and East African Breweries Limited non - diversifiable risk for East African Breweries Limitedis $0.01755\,\%$ and that of East African Breweries Limited is $0.15460\,\%$. For an investment portfolio of Kenya Commercial Bank, East African Breweries Limited and Standard chartered Bank, non - diversifiable risk for Kenya Commercial Bank is $2.15374X10^{-09}\,\%$, East African Breweries Limited is $8.71066\,X10^{-08}\,\%$ and that of Standard chartered Bank is $9.14787\,X10^{-11}\,\%$.

Clearly portfolio risk has been diversified but non - diversifiable risk for Kenya Commercial Bank and East African Breweries Limited is not the same as that one for two investments as per the definition of non - diversifiable risk and the findings of proposition 3.1. This is also true for turnover of a three and four investment portfolio. Thus there is some white noise in the data analyzed. We therefore adjust our estimator for non - diversifiable risk to include the random error. This is done by adding $\sum_{i=1}^{n}(s_{ei}^2)^{\frac{1}{2}}$ to the non - diversifiable risk estimator. We thus denote this Non - diversifiable risk estimator as,

$$N_G = (\sum_{i=1}^{n}(w_i^2 s_i^2))^{\frac{1}{2}} + (\sum_{i=1}^{n}(s_{ei}^2))^{\frac{1}{2}} \tag{3.16}$$

where s_{ei} is an independent random variable for investment i with mean zero and variance $\frac{\sigma_{ei}^2}{n-1}$ as shown by Stephen (2008). This estimator is found to be asymtotically unbiased by the proof of proposition 3.2 below.

Proposition 3.2: Non-diversifiable risk estimator N_G is a consistent estimator of non-diversifiable risk N_g.

Proof:

Let:

i) s_{ei} be an independent random variable with mean zero i.e. $\mu = 0$ and variance $\frac{\sigma_{ei}^2}{n-1}$

ii) Variance of investments $i = 1, 2, 3, ..., \infty$

By estimation we note that

$$N_G = (\sum_{i=1}^{n} w_i^2 s_i^2)^{\frac{1}{2}} + (\sum_{i=1}^{n} s_{ei}^2)^{\frac{1}{2}}$$

Thus

$$N_G^2 = \sum_{i=1}^{n} w_i^2 s_i^2 + 2(\sum_{i=1}^{n}(w_i^2 s_i^2)^{\frac{1}{2}} \sum_{i=1}^{n}(s_{ei}^2)^{\frac{1}{2}}) + \sum_{i=1}^{n} s_{ei}^2 \tag{3.17}$$

The expectation of equation 3.17 is

$$E\Big(N_G^2\Big) = E\Big(\sum_{i=1}^{n} w_i^2 s_i^2\Big) + E\Big[2\sum_{i=1}^{n}\Big(w_i s_i\Big)\Big(s_{ei}\Big)\Big] + E\Big(\sum_{i=1}^{n} s_{ei}^2\Big) \tag{3.18}$$

Since i) s_{ei} is an independent random variable the covariance of s_{ei} and s_i is equal to zero.

Then

$$\begin{aligned} E\Big(N_G^2\Big) &= \frac{n-1}{n}\Big(\sum_{i=1}^{n} w_i^2\sigma_i^2\Big) \\ &= (1-\frac{1}{\text{n}})\Big(\sum_{i=1}^{n} w_i^2\sigma_i^2\Big) \end{aligned}$$

as $n \to \infty$

$$E\Big(N_G^2\Big) = (1-0)\Big(\sum_{i=1}^{\infty} w_i^2\sigma_i^2\Big)$$

Hence

$$E(N_G) = \Big(\sum_{i=1}^{\infty} w_i^2\sigma_i^2\Big)^{\frac{1}{2}} = N_g \tag{3.19}$$

since the square root of a second moment is the standard deviation (Bainlee and Euglehardt, 1992)

Proving that N_G is an asymtotically unbiased estimator of N_g.

From (3.17), observe that

$$Var\Big(N_G^2\Big) = Var\Big(\sum_{i=1}^{n} w_i^2 s_i^2\Big) + Var\Big[\sum_{i=1}^{n}\Big(w_i^2 s_i^2\Big)^{\frac{1}{2}}\Big(s_{ei}^2\Big)^{\frac{1}{2}}\Big] + Var\Big(\sum_{i=1}^{n} s_{ei}^2\Big)$$

$$= \Big(\frac{2}{n-1}\sum_{i=1}^{n} w_i^4\sigma_i^4\Big) + \Big(\frac{2}{n-1}\Big)^2\Big(\sum_{i=1}^{n} w_i^4\sigma_i^4\Big)\Big(\sum_{i=1}^{n}\sigma_{ei}^4\Big)Var\Big[\Big(\sum_{i=1}^{n} w_i^2\sigma_i^2\Big)^{-\frac{1}{2}}\Big(\sigma_{ei}^2\Big)^{-\frac{1}{2}}\Big] + \Big(\frac{2}{n-1}\Big)\Big(\sum_{i=1}^{n}\sigma_{ei}^4\Big)$$

$$= \frac{2}{n-1}\left[\left(\sum_{i=1}^{n} w_i^4\sigma_i^4\right)+\left(\frac{2}{n-1}\right)\left(\sum_{i=1}^{n} w_i^4\sigma_i^4\right)\left(\sum_{i=1}^{n} \sigma_{ei}^4\right)Var\left[\left(\sum_{i=1}^{n} w_i^2\sigma_i^2\right)^{-\frac{1}{2}}\left(\sigma_{ei}^2\right)^{-\frac{1}{2}}\right]+\left(\sum_{i=1}^{n} \sigma_{ei}^4\right)\right] \tag{3.20}$$

Thus as $n \to \infty$

$$Var(N_G^2) = \frac{2}{\infty}\left[\left(\sum_{i=1}^{\infty} w_i^4\sigma_i^4\right)+\left(\frac{2}{n-1}\right)\left(\sum_{i=1}^{\infty} w_i^4\sigma_i^4\right)\left(\sum_{i=1}^{\infty} \sigma_{ei}^4\right)Var\left[\left(\sum_{i=1}^{\infty} w_i^2\sigma_i^2\right)^{-\frac{1}{2}}\left(\sigma_{ei}^2\right)^{-\frac{1}{2}}\right]+\left(\sum_{i=1}^{\infty} \sigma_{ei}^4\right)\right]$$

$$= 0\left[\left(\sum_{i=1}^{\infty} w_i^4\sigma_i^4\right)+\left(\frac{2}{n-1}\right)\left(\sum_{i=1}^{\infty} w_i^4\sigma_i^4\right)\left(\sum_{i=1}^{\infty} \sigma_{ei}^4\right)Var\left[\left(\sum_{i=1}^{\infty} w_i^2\sigma_i^2\right)^{-\frac{1}{2}}\left(\sigma_{ei}^2\right)^{-\frac{1}{2}}\right]+\left(\sum_{i=1}^{\infty} \sigma_{ei}^4\right)\right]$$

Hence $Var(N_G) \to 0$

Note: In equation (3.20) $(w_i^2\sigma_i^2)^{\frac{1}{2}}$ is rewritten as $(w_i^2\sigma_i^2)(w_i^2\sigma_i^2)^{\frac{-1}{2}}$ and $(\sigma_{ei}^2)^{\frac{1}{2}}$ as $(\sigma_{ei}^2)(\sigma_{ei}^2)^{\frac{-1}{2}}$ to assist in the simplification of the equation. Also $Var\ (s^2) = \frac{2}{n-1}\sigma^4$ as proved by Tobago (2010)

Recalling that as $n \to \infty$, $E(N_G) = N_g$, it follows from this result (i.e. $Var(N_G) \to 0$) that N_G is a consistent estimator of Non - diversifiable risk.

3.4 Determination of Real Risk Weighted Pricing Model

To determine the RRWPM we let the weighted expected returns be

$$E(R(t)_{lw}) = a_{lw} + b_{lw}E(R(t)_{mw}) \tag{3.21}$$

Where, $a_{lw} = \sum_{i=1}^{n} w_i a_i$, $b_{lw} = \sum_{i=1}^{n} w_i b_i$, w_i is the weight of security i, a_i is the constant return unique to security i, b_i is a measure of the sensitivity of the return of security i to the return on the market index, $E(R(t)_{lw})$ is the weighted expected return of security i, $E(R(t)_{mw})$ is the weighted expected return of the market index. Then take weighted diversifiable risk estimator as seen in section 3.2 to be

$$D_{mw} = (c_{lw} + d_{lw})^{\frac{1}{2}} \tag{3.22}$$

and weighted non - diversifiable risk estimator determined in section 3.2 as

$$N_{Gw} = (c_{lw} + e_{lw})^{\frac{1}{2}} \tag{3.23}$$

where

$$c_{lw} = \sum_{i=1}^{n} w_i^2 s_i^2, d_{lw} = 2 \sum^{n} \sum^{n}_{i \neq j} w_i w_j s_{ij} \ and \ e_{lw} = \sum_{i=1}^{n} w_i^2 s_{ie}^2.$$

s_j^2 is the sample variance of security j, s_i^2 is the sample variance of security i, s_{ei}^2 the sample variance of random error of security i and s_{ij} the sample covariance of security i and j.

To find the weight of investment i that will maximize expected returns and minimize total variance we apply the classical optimization method with no constraints as given by Rao (1994). We thus differentiate the expression;

$$E(R(t)_{lw}) - c_{lw} - d_{lw} = a_{lw} + b_{lw} E(R(t)_{mw}) - c_{lw} - d_{lw} \tag{3.24}$$

With respect to w_i, and differentiate

$$c_{lw} - d_{lw} + e_l \tag{3.25}$$

With respect to w_i, where $E(R(t)_{lw}) = a_{lw} + b_{lw} E(R(t)_{mw})$ and $E(R(t)_{lw}) - c_{lw} - d_{lw}$ are maximum returns (derived by subtracting diversifiable portfolio variance from portfolio expected returns), and $2c_{lw} + d_{lw} + e_l$ is the total variance (derived by adding portfolio variance to non-diversifiable variance) and is determined by adding Equation 3.22 to 3.23. Note: The second derivative of the differential of Equation 3.24 is equal to $-2\sum_{i=1}^{n} s_i^2$ implying that w_i obtained will always maximize returns and that of Equation 3.25 is equal to $4\sum_{i=1}^{n} s_i^2$ implying that w_i obtained will always minimize risk. Equate the differentials of Equation 3.24 to Equation 3.25 to get the value of w_i,

$a_{lw} + b_{lw}E(R(t)_{mw}) - 2c_{lw} - 2d_{lw} = 2c_{lw} + 2d_{lw} + 2c_{lw}$

$$-6c_{lw} = 4d_{lw} - a_{lw} - b_{lw}E(R(t) \tag{3.26}$$

w_j is similarly derived.

Thus

$$w_j = \frac{-2\sum_{i\neq j}^{n}\sum^{n} \mathrm{w_i s_{ij}}}{3\sum_{j=1}^{n} \mathrm{s_j^2}} \tag{3.27}$$

Replacing w_j in equation 3.22 gives the value of

$$w_i = \frac{3\sum_{i\neq j}^{n}\sum^{n} \mathrm{s_j(a_i + b_i E(R(t)_m)}}{\sum_{i\neq j}^{n}\sum^{n} \mathrm{(18 s_i^2 s_j^2 - 8 s_{ij}^2)}} \tag{3.28}$$

For $i = 1$ $j = 1$

$$w_1 = \frac{\mathrm{3s_2(a_1 + b_1 E(R(t)_m))}}{\mathrm{18 s_1^2 s_2^2 - 8 s_{12}^2}}$$

Once these weights are determined, they are substituted in Equation 3.21 to give maximum returns and in both Equations 3.22 and 3.23 to give minimum total risk. The costs of capital are also determined which enable future predictions. RRWPM is tested with data and compared with CAPM to determine which model has better predictive characteristics. The non - diversifiable risk determined by substituting w_i and w_j is used to model random error as an independent random variable. w_i and w_j are determined as optimal values of total risk and expected returns and not as one divided by the number of time series as in most risk and expected returns models.

Chapter 4

DENSITY ESTIMATION USING KERNELS WITH WHITE NOISE IN DECISION MAKING

4.1 Risk of Non-diversifiable Risk

In the past few years there has been evidence of collapse of well established business entities. This has been attributed to lack of appropriate methods of preventing or measuring risk and uncertainty as opposed to lack of the same methods. Many companies on Wall Street in 2008 went under despite having extensive measures of mitigating risk such as futures and forwards. An investigation into some of these methods revealed the lack of properly estimated market risk measures. It is an obvious fact now that it was the external reactions that brought down the companies on Wall Street. Once the markets got a hint of the internal financial and investment affairs of the companies this spread so rapidly and in a matter of hours these companies had collapsed. A good example is the Lehman Brothers Holdings limited, Merrill Lynch and companies, and American Investment group as explained by Lucchetti *et al* (2008), which indicate that market environments are so critical in the existence of business entities such that variables affecting the business entities from the market environments should be estimated appropriately. This study has determined total risk in Chapter three which clearly has both the systematic and non- systematic components. It should be noted that non - systematic risk is internal in nature, and in most cases well known and relatively less difficult to estimate.

This chapter determines the risk factor of systemic risk a phenomenon lacking in many risk models. Since we have seen from most examples that market risk is the precursor of most companies down fall, it is hoped that investors and companies will be able to easily estimate probability of the risk measures thus enabling them make informed decisions.

4.2 Determination of White Noise of Non-diversifiable Risk

White noise is a purely random process whose random variables are a sequence which is mutually independent, and identically distributed. Thus it describes an event and is a function with a domain that makes some real number correspond to each outcome of the experiment. In this study white noise is taken as the random error of non-diversifiable risk N_{Gwi} of an investment i. Proposition 4.1 below seeks to show that white noise is a random variable.

Proposition 4.1: Let V_i be the white noise of the non-diversifiable risk N_{Gwi}, then $V_i(\cdot)$ is a random variable.

Proof:

Given N_{Gwi} and

i) The domain $\Omega = V_{i;1 \leq i \leq n}, V_{\substack{j;-\infty \leq j \leq \infty \\ i \neq j}}$,

ii) Counter domain r is such that $0 \leq r \leq 1$, and

iii) The range of returns i is $-\infty \leq i \leq \infty$,

then $V_i(\cdot)$ is an event. That is $V_i(\cdot)$ is such that the subset $w_r = \{s : V_i(s) \leq r\}$, where s is a subset of the domain. This is true since $0 \leq V_i(s) \leq 1$.

If w_r belongs to W for every real number r, where W is the set of all outcomes of event $V_i(\cdot)$, then the probability of $V_i(\cdot)$, $P[V_i(\cdot)]$ is a set function having domain $V_i(\cdot)$ and counter domain the interval $[0, 1]$. Therefore W has a probability space (Ω, W, P).

Also W consists of four subsets; ϕ, $V_{i;1 \leq i \leq n}, V_{\substack{j;-\infty \leq j \leq \infty \\ i \neq j}}$ and Ω.

such that if:

i) $r < 0$, then $s : V_i(s) \leq r = \phi$

ii) $0 \leq r < 1$, then $s : V_i(s) \leq r$, where $V_{\substack{-\infty \leq j \leq \infty \\ i \neq j}}$ and

iii) $r \geq 0$, then $s : V_i(s) \leq r = \Omega = V_{i;1 \leq i \leq n}, V_{\substack{j;-\infty \leq j \leq \infty \\ i \neq j}}$.

Since $V_i(\cdot)$ has a probability space, and W consists of the four subsets above, then, for each r the set $\{s : V_i(s) \leq r\}$ belongs to W, thus $V_i(\cdot)$ is a random variable.

Since $V_i(\cdot)$ is a random variable it is independent.

Therefore the parameters of white noise for example its mean and variance as well as its unique probability distribution can be determined. The probability estimates of non - diversifiable risk for investment decisions are then estimated as shown in the following subsections.

4.2.1 Determination of random error

The non-diversifiable variance estimator

$$N_{Gwi}^2 = \sum_{i=1}^{n} w_i^2 s_i^2 + \sum_{i=1}^{n} w_i^2 s_{ei}^2 \tag{4.1}$$

derived from the non-diversifiable risk estimated in section 3.2 indicates the presence of random error in the risk estimator. This error is taken to be white noise (wn) thus it can be said to be a random variable $V_1, V_2, V_3, ..., V_\infty$ which is independent and identically distributed. This is estimated from a sample of data by first varying the variance of individual return values of r_i resulting in

$$w\hat{n}_i = T \sum_{i=1}^{z} s_{ri}^2 s_{Gw}^2 \tag{4.2}$$

where $T = \frac{\text{z}-2}{(\text{z}-1)^2}$, z being the total number of returns and (4.2) is the predicted random error.

From (4.2) the actual value of $w\hat{n}_i$ is given by

$$wn_i = \sum_{i=1}^{z} w_i^2 s_{ri}^2 C + L \tag{4.3}$$

Where C and L are values representing the scale (variance) and location (mean) parameters. These parameters are determined such that the bias and variance of the actual and predicted values of wn are minimized as follows;

Let the variance between actual and sample white noise be

$$\operatorname{var}(w\hat{n}_i, wn_i) = \frac{2}{z-1} w\hat{n}_i^{\ 2} - \left(\frac{2}{z-1}\right)^2 w\hat{n}_i wn_i + \frac{2}{z-1} wn_i^{\ 2} \tag{4.4}$$

The values of C and L which will minimize variance are given by the partial derivatives of C and L , f_C and f_L respectively. After several iterations;

$$f_C = 2wn_i - \frac{2}{\mathrm{z}-1}w\hat{n}_i = L$$

$$f_L = \frac{2}{\mathrm{z}-1}w\hat{n}_i - 2wn_i = L$$

Thus the value of

$$w\hat{n}_i = \frac{3(\mathrm{z}-1)}{4}wn_i \tag{4.5}$$

Proposition 4.2:

$w\hat{n}_i$ is an asymtotically unbiased estimator of wn_i

Let s^2_{gw} be independent and identically distributed with mean 0 and variance $\frac{(\mathrm{z}-2)}{(\mathrm{z}-1)^2}\sigma^2_{ri}$

Proof: From equation (4.2)

$$w\hat{n}_i = T\sum_{i=1}^{z}(s^2_{ri}s^2_{gw})$$

$$E(w\hat{n}_i) = \frac{\mathrm{z}-2}{(\mathrm{z}-1)^2}\sum_{i=1}^{z}E(V_i^2 - \bar{V}_r^2)$$

$$= \frac{\mathrm{z}-2}{(\mathrm{z}-1)^2}\sum_{i=1}^{z}\left(E(V_i^2) - E(\bar{V}_r^2)\right)$$

$$= \frac{\mathrm{z}-2}{(\mathrm{z}-1)^2}\left(\sum_{i=1}^{z}(\mu^2 + wn_i) - z \times \frac{1}{\mathrm{z}}\left(\sum_{i=1}^{z}\bar{V}_i^2 + \sum_{i<j}^{z}\bar{V}_i\bar{V}_j\right)\right)$$

$$\frac{\mathrm{z}-2}{(\mathrm{z}-1)^2}\left(z\mu^2 + zwn_i - \frac{1}{\mathrm{z}}(z\mu^2 + zwn_i) - z(z-1)\mu^2\right)$$

$$= \frac{\mathrm{z}-2}{(\mathrm{z}-1)^2}\left(z\mu^2 + zwn_i - \mu^2 - wn_i - (z-1)\mu^2\right)$$

$$= \frac{\mathrm{z}-2}{(\mathrm{z}-1)^2}\left(z\mu^2 + zwn_i - \mu^2 - wn_i - z\mu^2 + \mu^2\right)$$

$$= \frac{\mathrm{z}-2}{(\mathrm{z}-1)^2}(z-1)wn_i$$

$$= \frac{\mathrm{z}-2}{(\mathrm{z}-1)}wn_i \tag{4.6}$$

where wn_i and μ are the actual variance and mean of non - diversifiable risk respectively.

Dividing equation 4.6 by z results in

$$= \frac{\left(1-\frac{2}{\mathrm{z}}\right)wn_i}{1-\frac{1}{\mathrm{z}}}$$

lim as $z \to \infty$

$$= \frac{\left(1-0\right)wn_i}{1-0}$$

$$= wn_i$$

Thus $w\hat{n}_i$ is asymtotically an unbiased estimator of wn_i

From the results of Wu (1980), equation 4.6, and proposition 4.2, $w\hat{n}_i$ is a consistent estimator of wn_i

4.3 Derivation of a Probability Density Function for Random Error of Non - Diversifiable Risk

4.3.1 Bandwidth selection for kernel density estimation of the wn of non-diversifiable risk

It is generally known that the value of the bandwidth is of critical importance while the shape of the kernel function has little practical impact. Thus we estimate bandwidth and use a given kernel function to get the density estimation of the white noise of non-diversifiable risk. Assuming that the underlying density is sufficiently smooth and that the kernel has fourth moment using the Taylor series

$$Bias \quad \left\{\hat{f}_h(v)\right\} = \frac{h^2}{2}\mu_2(k) f''(v) + (h^2) \tag{4.7}$$

$$Var\{\hat{f}_n(v)\} = \frac{1}{\mathrm{nh}} R(k) f(v) + 0(\frac{1}{\mathrm{nh}}) \tag{4.8}$$

where $R(k) = \int k^2(v)dv$ (Wand and Jones(1994)). Adding the leading variance and squared bias terms produces the asymptotic mean squared error (AMSE)

$$\left\{\hat{f}_n(v)\right\} = \frac{1}{nh} R(k) f(v) + \frac{h^4}{4}\mu_2(k)^2 \left[f''(v)\right]^2 \tag{4.9}$$

The overall measure of the discrepancy between $\hat{f}$ and f is the mean integrated squared error (MISE) which is given by

$$MISE \ \left(\hat{f}_n\right) = E\left\{\int \left(\hat{f}_h(y) - f(y)\right) dy\right\}$$

$$\int Bias(\hat{f}_h(v)^2) \ dv + \int Var(\hat{f}_h(v)) \ dv \tag{4.10}$$

Under an integrability assumption on f, integrating the expression for AMSE gives the expression for the asymptotic mean integrated squared error (AMISE), that is,

$$AMISE\left(\hat{f}_h\right) = \frac{1}{nh} R(k) + \frac{h^4}{4} \mu_2(k)^2 R(f'') \tag{4.11}$$

Where

$$R(f'') = \int [f''(v)^2] dv. \tag{4.12}$$

The value of the bandwidth that minimizes the AMISE is given by

$$h_{AMISE} = \left[R(k) / \mu_2(k)^2 R(f'') \right]^{1/5} n^{-1/5} \tag{4.13}$$

Using the rule of thumb method a global bandwidth h is based on replacing $R(f'')$ the unknown part of h_{AMISE}, by its value for a parametric family expressed as a multiple of a scale parameter, which is then estimated from the data. The method dates back to Deheuvels (1977) and Scott (1979). It has been popularized for kernel estimates by Silverman (1986). The plug - in method is used to estimate h_{AMISE} in this study. Here the unknown quantity $R(f'')$ in the expression for h_{AMISE} is replaced by an estimate. The "solve - the - equation" plug - in approach developed by Sheather and Jones (1991) is based on deriving, the pilot bandwidth for the estimate $R(f'')$, as a function of h, namely

$$g(h) = C(k) \left[R(f'') / R(f''') \right]^{1/7} h^{5/7} \tag{4.14}$$

The unknown functionals of f are estimated using kernel density estimation with bandwidth based on normal rules of thumb resulting in

$$h_{sj} = \left[R(k) / \mu_2(k)^2 R(\hat{f}'' g_{(h)}) \right]^{1/5} n^{-1/5} \tag{4.15}$$

where h_{SJ} is known as the Sheather-Jones plug-in bandwidth. Under smoothness assumption on the underlying density, $n^{\frac{5}{14}}(\frac{h_{SJ}}{h_{AMISE-1}})$ has an asymptotic $N(0, \sigma^2_{SJ})$ distribution. Thus, the Sheather-Jones plug-in bandwidth has a relative convergence rate of order $n^{\frac{-5}{14}}$, which is much higher than that of Biased Cross-Validation (BCV).

4.3.2 Kernel density estimation of white noise of non-diversifiable risk

The triangle kernel is used for smoothing.

This is given by

$$K_{tri}(t) = \begin{cases} 1 - |t|/c, |t| \leq 1/c \\ 0, \qquad\quad |t| > 1/c \end{cases} \tag{4.16}$$

Where c is the constant used to scale the resulting kernel so that the upper and lower quartiles occur at $\pm$ 0.25.

Let $V_1, V_2, ..., V_n$ denote a sample of size n from the random variable $V_i(\cdot)$ of white noise, V_i with density f. The kernel density estimates of f at the point V_i is given by

$$\hat{f}_n(V_i) = \frac{1}{\mathrm{nh}} \sum_{i=1}^{n} k[\frac{\mathrm{V_i} - \bar{\mathrm{V}}_\mathrm{i}}{\mathrm{h}}] \tag{4.17}$$

where the kernel k satisfies $\int_{-\infty}^{\infty} k(v)dv = 1$, the smoothing parameter h is known as the bandwidth, and $\bar{V_i}$ is the mean of V_i.

From equation 4.16 and 4.17, $\hat{f}_n(V_i) = \frac{1}{\mathrm{nh}} \sum_{i=1}^{n} [1 - \frac{\mathrm{V_i} - \bar{\mathrm{V}}_\mathrm{i}}{\mathrm{hc}}]$

Proposition 4.3: $\sum_{i=1}^{n} V_i$ is a minimum sufficient statistics of $\bar{V}_i$.

Proof: Given the function $\hat{f}_n(V_i) = \frac{1}{\mathrm{nh}} \sum_{i=1}^{n} [1 - \frac{\mathrm{V_i} - \bar{\mathrm{V}}_\mathrm{i}}{\mathrm{c}}]$, then the likelihood (L) of V_i is

$$L(\underline{\mathrm{V}}_i : \bar{V}_i) = \frac{\partial}{\partial \underline{\mathrm{V}}_\mathrm{i}} \ln \frac{1}{\mathrm{hn}} \sum_{i=1}^{n} [1 - \frac{(\sum_{\mathrm{i=1}}^{\mathrm{n}} \mathrm{V_i} - \bar{\mathrm{V}}_\mathrm{i})}{\mathrm{hc}}] \tag{4.18}$$

Since, $\sum V_i = \bar{V}_i$, then $\bar{V}_i - \bar{V}_i = 0$

Let $V_i \cdot = (V_1, V_2, ..., V_n)$be a point in $\underline{V_i} : V_i = (V_1, V_2, ..., V_n)$

Then

$$\frac{\mathrm{L}(\underline{\mathrm{V}}_\mathrm{i} : \bar{\mathrm{V}}_\mathrm{i})}{\mathrm{L}(\underline{\mathrm{V}}_\mathrm{i} \cdot : \bar{\mathrm{V}} \cdot_\mathrm{i})} = \frac{\frac{\partial}{\partial \underline{\mathrm{V}}_\mathrm{i}} \ln \frac{1}{\mathrm{hn}} \sum_{i=1}^{n} 1}{\frac{\partial}{\partial \underline{\mathrm{V}} \cdot_\mathrm{i}} \ln \frac{1}{\mathrm{hn}} \sum_{i=1}^{n} 1} \tag{4.19}$$

and therefore $\frac{\mathrm{L}(\underline{\mathrm{V}}_\mathrm{i} : \bar{\mathrm{V}}_\mathrm{i})}{\mathrm{L}(\underline{\mathrm{V}}_\mathrm{i} \cdot : \bar{\mathrm{V}} \cdot_\mathrm{i})} = 1$ Meaning that it is independent of $\bar{V}_i$ and thus, $\sum_{i=1}^{n} v_i$ is a minimum sufficient statistics of $\bar{V}_i$. Lehmann and scheffe (1950) remarks that if the sample space is discrete or a finite dimensional Euclidean space then a minimal sufficient statistic will always exists. Since a minimum sufficient statistic exists then the sample space is discrete and the probability density function given by equation 4.17 exists. This probability density function together with the random error estimate given by equation (4.5) have been estimated with all the good properties of estimation for example unbiased, consistency and sufficiency lending credence to these estimators. These are used to generate probabilities of non - diversifiable risks of given portfolios thus ensuring that actual systematic risk is determined. This is clearly shown in chapter six section 6.2.4.

Chapter 5

INVESTMENT DECISION MAKING WITH GARCH MODELS AND REAL MARKET RISK

5.1 Decision Making

The essence of business success is using current and past data in order to make informed decisions. These could be financial or investment in nature. Budgeting for example is an important planning and control tool. Its success depends on appropriate prediction of the future by using either the past when using traditional budgeting methods or the present using Zero based budgeting where all budgetary estimates are made a fresh without looking at the past figures but at the expectations of the future. Both methods of budgeting require appropriate estimates of total risk since the future is not known, thus providing generally risky or uncertain as well as credible forecasting environments that will give an appropriate estimation of future variables.

Investment is the cornerstone of sustainability of businesses. These decisions are normally strategic since they are long term. Therefore they involve large amounts of sunk in assets and relatively long periods of waiting before returns on investments are realized. This necessitates well researched and well tested methods to be applied in the determination of the choice of projects to invest in. Otherwise it could mean the thin line between large future sustainable profits or heavy instant loses, as well as bankruptcy.

At the core of this research is the decision making process. Therefore this chapter explicitly embraces the two previously well determined variables that is total risk and expected return with their statistical properties from chapter three and the determined probability estimates of non - diversifiable risk done in chapter four as well as good forecasting models to aid proper future investment decision making. This is done in the sections below.

5.2 Forecasting Returns Using Garch (1, 1)

ARCH models make the conditional variance at time t prediction error a function of time system parameters, exogenous and lagged endogenous variables, and past prediction errors. For each integer t, let ξ_t be a model's (scalar) prediction error, b a vector of parameters, x_t a vector of predetermined variables, and σ_t^2 the variance of ξ_t given past information at time t.

A univariate ARCH model based on Engle (1982) sets

$$\xi_t = \sigma_t Z_t \tag{5.1}$$

Where, $Z_t \sim i.i.d$, with $E(Z_t) = 0$, $\mathrm{Var}(Z_t) = 1$,

$$\sigma_t^2 = \sigma^2(\xi_{t-1}, \xi_{t-2}, \cdots, t, x_t, b) = \sigma^2(\sigma_{t-1}Z_{t-1}, \sigma_{t-2}Z_{t-2}, \cdots, t, x_t, b) \qquad (5.2)$$

Equations (5.1) and (5.2) can be given a multivariate interpretation as suggested by Brooks *et al* (2003), in which case Z_t is an $n \times 1$ vector and σ_t^2 is an $n \times n$ matrix. We refer to any model of the form of equations (5.1) and (5.2) whether univariate or multivariate, as an ARCH model. The most widely used specification of equation (5.2) are the linear ARCH and GARCH models introduced by Engle (1982), which make σ_t^2 linear in lagged values of

$\xi_t^2 = \sigma_t^2 z_t^2$, by defining

$$\sigma_t{}^2 = \omega + \sum_{j=1}^{p} \alpha_j Z_{t-j}^2 \sigma_{t-j}^2 \qquad (5.3)$$

$$\sigma_t{}^2 = \omega + \sum_{i=1}^{n} \beta_i \, \sigma_{t-1}^2 + \sum_{j=1}^{p} \alpha_j Z_{t-j}{}^2 \sigma_{t-j}^2 \qquad (5.4)$$

respectively, where ω, α_j and β_i are non negative. Since equation (5.3) is a special case of equation (5.4) we refer to both equations (5.3) and (5.4) as GARCH models, to distinguish them as special cases of equation (5.2). The GARCH - M (the multivariate model) model of Engle and Bollerslev (1986) adds another equation

$$R_t = a + b\sigma_t{}^2 + \xi_t \qquad (5.5)$$

in which σ_t^2, the conditional variance of R_t, enters the conditional mean of R_t as well. For example if R_t is the return on a portfolio at time t, its required rate of return may be linear in its risks as measured by σ_t^2.

The RRWPM as illustrated in chapter 3 is used to determine the future cost of equity and future real risk using forecasted returns determined in this section. The RRWPM can be tested only as a sub model, and it is natural to search for this nesting model in the class of GARCH models. Since we do not have an established link between a structural formulation on which RRWPM is based and a descriptive specification, the GARCH - M model is used for forecasting in this research. The GARCH - M model is a two successive application model, the first one on the process itself and the second on the squared innovations. A two step estimation procedure that takes this particular structure into account is considered.

Let us consider a regression model with ARCH errors:

$$\begin{cases} R_t = a + b\sigma_t^2 + \epsilon_t \\ V(\epsilon_t/\epsilon_{t-1}) = c + a_1\epsilon_{t-1}^2 + ... + a_p\epsilon_{t-p}^2 \end{cases}$$

A consistent estimator of b is the ordinary least square (OLS) estimator from the regression of R_t on σ_t, denoted by $\tilde{b}_T$. From this estimation, we obtain as residuals

$$\epsilon_t = R_t - \hat{a}_T - \hat{b}_T\sigma_t^2$$

The other parameters $c, a_1, ..., a_p$ appearing in the variance may also be estimated in a consistent way by regressing $\tilde{\epsilon}_t^2$ on $1, \tilde{\epsilon}_{t-1}^2, ..., \tilde{\epsilon}_{t-p}^2$. They are denoted by $\tilde{c}_T, \tilde{a}_{1T}, ..., \tilde{a}_{pT}$. These two successive regressions are estimated by OLS, that is without accounting for the conditional heteroscedasticity. The estimator of b is improved in the second step by applying the quasi-generalised least squares to a regression of R or σ. Due to the volume of data GARCH models available as procedure in the econometric package Mat lab is used for forecasting in this research. In estimating a GARCH model in Mat lab the true GARCH (A,B) model parameters of the time series are entered. These parameters correspond to a given GARCH (A,B) model for the conditional variance F(t) and Y(t) innovation, sequences:

$$F(t) = L(t) + S(1)^{\star}F(t-1) + S(2) \star F(t-2) + ... + S(p) \star F(t-p) +$$

$$W(1)Y^2(t-1) + W(2)Y^2(t-2) + ... + W(Q)Y^2(t-Q)$$

for time steps t = 1, 2, ... N

where:

S is the order of AutoRegression, W, the order of the Moving Average, A and B are the model orders determined by the number of elements of S and W.

where S represents the number of partial autocorrelation spikes while W represents the number of autocorrelation spikes and t, the current time index. $Y(t) = \sqrt{F(t)L(t)}$ and L(t) is an independent sequence N (0,1) while Y and F are related.

5.3 Determination of Forecasted White Noise

White noise of the real risk is given by the equation (4.1) in section 4.2.1 as determined by Anyika *et al* (2010) where

$$w\hat{n}_i = T \sum_{i=1}^{\infty} s^2_{r_f i} s^2_{gw} \tag{5.6}$$

The only differences are the returns r_i which are forecasted thus represented by r_{fi} where T is still represented by $T = \frac{\mathrm{z}-2}{(\mathrm{z}-1)^2}$

Similarly as in section 4.2.1 the forecasted white noise is estimated as a random error from its definition of being mutually independent identically distributed with constant mean and variance and $\operatorname{cov}(wn_i, wn_{i+l}) = 0 \quad \text{for} \quad l = \pm 1, \pm 2, \ldots$

The probability distribution of a continuous-valued random variable X is conventionally described in terms of its probability density function (pdf), $f(x)$, from which probabilities associated with X can be determined using the relationship

$$P(a \leq x \leq b) = \int_a^b f(x)dx$$

The objective of many investigations is to estimate $f(x)$ from a sample of observations $x_1, x_2, \cdots, x_n$. The parametric approach for estimating $f(x)$ is to assume that $f(x)$ is a member of some parametric family of distributions, e.g. $N(\mu, \sigma^2)$, and then to estimate the parameters of the assumed distribution from the data. For example, fitting a normal distribution leads to the estimator

$$\hat{f}(x) = \frac{1}{\sqrt{(2\pi\hat{\sigma})}} e^{\frac{(\mathrm{x}-\hat{\mu})^2}{2\hat{\sigma}^2}}, x\varepsilon\Re$$

where $\hat{\mu} = \frac{1}{\mathrm{n}} \sum_{i=1}^{n} x_i$ and $\hat{\sigma}^2 = \frac{1}{\mathrm{n}-1} \sum_{i=1}^{n} (x_i - \hat{\mu})^2$

This approach has advantages as long as the distributional assumption is correct, or at least is not seriously wrong as written by Kotz *et al* (2006), that is, it is easy to apply and yields relatively stable estimates. The main disadvantage of the parametric approach is lack of flexibility. Each parametric family of distributions imposes restrictions on the shapes that $f(x)$ can have. For example the density function of the normal distribution is symmetrical and bellshaped, and therefore is unsuitable for representing skewed densities or bimodal densities. Another alternative weighting function is the Gaussian kernel:

$$f(t,h) = \frac{1}{\sqrt{(2\pi \mathrm{h})}} e^{\frac{(-\mathrm{t}^2)}{2\mathrm{h}^2}}, -\infty < t < \infty \tag{5.7}$$

In this function the fluctuations in $f(x)$ decrease with increasing h thus is a better model.

The function is determined from the definition of the probability density function $f(x)$ such that

$$P(x-h < X < x+h) = \int_{x-h}^{x+h} f(t)dt \approx 2hf(x)$$

The probability above can be estimated by a relative frequency in the sample, hence

$$\hat{f}(x) = \frac{1}{2\mathrm{h}} \frac{\text{number of observations in } (\mathrm{x}-\mathrm{h}, \mathrm{x}+\mathrm{h})}{\mathrm{n}}$$

An alternative way to represent $\hat{f}(x)$ is

$$\hat{f}(x) = \frac{1}{\mathrm{n}} \sum_{i=1}^{n} w(x - x_i, h)$$

Where $x_1, x_2, \cdots, x_n$ are the observed values and $w(t,h)$ is the Gaussian kernel.

Due to the advantages and the ease of application this research uses the Gaussian kernal for estimating non- diversifiable risk from forecasted returns. Here it is generally known that the value of the bandwidth is of critical importance while the shape of the kernel function has little practical impact. The value of the bandwidth that minimizes the AMISE is given by

$$h_{AMISE} = \left[R(k) / \mu_2 \left(k\right)^2 R\left(f''\right) \right]^{1/5} n^{-1/5}$$

The Gaussian kernel by Sheather and Jones (1991) is used to determine the probability estimates.

This is done using forecasted stock returns of 20 stock from the New York stock exchange. These enable the mitigation of the element of riskiness of the future non-diversifiable risks of the portfolios as a basis for deciding on the portfolio to invest in.

Chapter 6

DATA ANALYSIS AND RESULTS

6.1 Preliminary Data

A twenty stock portfolio is picked randomly from the New York Stock Exchange. The New York Share Index (NYSE) is used as the market share index and the long term Treasury bond as the risk free asset. The monthly returns of the twenty stocks, the NYSE and Treasury bond since September 1998 - September 2008 are calculated using equation 3.1 and tabulated in the appendix.

6.2 Data Analysis and Results

6.2.1 CAPM Cost of Capital Estimation

For comparison purposes with RRWPM, CAPM cost of capital estimation is conducted as follows: Returns on specific stocks in the sample are regressed on the market risk factor to obtain the beta coefficient for each firm as given by the following formula:

$$E(r_i) = r_f + \beta_{mi}\left[E\left(r_m\right) - r_f\right], \tag{6.1}$$

where

$E(r_i)$ = the CAPM cost of capital for firm i,

r_f = the expected return on a default risk-free asset, also known as the alpha

$E(r_m)$ = the expected return on the market portfolio, and

β_{mi} = firm i's beta coefficient for systematic market risk

$= \frac{Cov(r_i, r_m)}{Var(r_m)}$.

The beta coefficients are inserted into equation 6.1 along with the estimated market risk-premium to obtain the cost of capital estimate for each firm. Results of twenty stocks are tabulated in Table 6.1 below.

TABLE 6.1 Cost of Capital Estimates for Portfolio of 20 stock

COMP	BETA	ALPHA	r^2	s_{ey}	$E(r_i)$
TM	0.00249	0.86871	0.25702	0.06905	0.004929
HMC	0.00562	0.73927	0.20272	0.06854	0.004928
PARD	0.03197	1.88501	0.05442	0.36735	0.004927
VICL	0.00463	1.68021	0.14661	0.18952	0.004928
DWCH	0.03137	0.12547	0.00024	0.3759	0.004927
BP	0.00562	0.75481	0.30408	0.05339	0.004928
STI	-0.0006	0.98887	0.24018	0.08223	0.004929
PNC	0.00416	1.06745	0.3324	0.07073	0.004929
AIG	-0.00005	2.15503	0.14884	0.24093	0.004929
F	0.00145	1.81696	0.27366	0.13839	0.004929
AMR	0.00238	2.21958	0.2375	0.18593	0.004929
BPH	0.012171	1.12014	0.30141	0.07973	0.004928
CTL	0.00619	0.86327	0.202	0.08033	0.004928
PFE	0.00302	0.71805	0.21677	0.06381	0.004929
RTI	0.00226	1.28053	0.18034	0.12764	0.004929
GSK	0.00416	0.55442	0.19376	0.05287	0.004929
BCE	0.01387	0.9863	0.2971	0.07093	0.004928
SBGI	-0.00655	1.62275	0.22271	0.14173	0.004929
YAH	0.008903	1.60829	0.37162	0.09778	0.004929
TIF	0.02780	2.02885	0.18776	0.19728	0.004928

Values for the survey of CAPM Where; COMP = Company, TM = Toyota Motors, HMC = Honda motors, PARD = Ponard pharmaceuticals, VICL = Vical, DWCH = Data watch, BP = British power, STI = Sun Trust Bank, PNC = PNC Finance services, AIG = American International group, F = Ford, AMR = Amr company BHP = BHP Billiton, CTL = CENTURY TEL, PFE = Pfizer, RTI = RTI Intl Metals, GSK = GlaxoSmithKline, BCE = BCE Company, SBGI = Sinclair Broadcast Group, YAH = Yahoo group, TF = Tiffany. r^2 = The coefficient of determination. S_{ey} = The standard error for the y estimate. Although there is a correlation between the variables in the CAPM model the average of the r^2 value for the 20 stock is only approximately 0.25. This indicates a very weak relationship between the variables. Thus the predictors of expected returns of the stock are not reliable.

6.2.2 Cost of Capital for RRWPM

The cost of capital for Real Risk Weighed Pricing Model is determined as follows: Returns on specific stocks are regressed on the market risk factor to obtain the initial estimates of beta and alpha values. These are substituted in equations 3.27 and 3.28 to get the weights. The weights are plugged into equation 3.21 and the regression repeated to get the real beta and alpha coefficients. Other values for the stocks are calculated as indicated by the various equations in section 3.4 and tabulated in Table 6.2 below.

TABLE 6.2 A table of results of the evaluation of 20 stock using RRWPM

COMP	BETA	ALPHA	r^2	s_{ey}	$E(r_i)$	N_{Gw}	w_i	w_j
TM	-0.028	1.001	0.999	1.951	1.006	538.1	-0.013	0.0049
HMC	0.0245	1.016	0.999	0.56	1.021	28.23	407.3	-0.0003
PARD	-0.171	1.021	0.999	2.424	1.026	27.97	75.78	-0.0002
VICL	-0.044	1.068	0.921	0.484	1.073	0.5611	2.936	0
DWCH	-0.069	1.031	0.998	0.807	1.036	25.58	67.89	0.0002
BP	-0.093	1.004	0.999	2.617	1.009	105.2	1932	0.0014
STI	-0.014	1.008	0.999	2.057	1.013	108.3	1325	0.00004
PNC	0.1797	0.984	0.999	1.474	0.99	7.876	111.7	0
AIG	0.3043	1.002	0.999	13.24	1.006	7163	99180.0	-0.1047
F	0.2971	0.996	0.999	3.076	1.001	1897	-13291	0.0337
AMR	0.3706	0	1	0	0.005	25.17	-130.1	0.0005
BPH	0.0534	1.000	0.996	0.593	1.005	1.752	-9.057	0.00003
CTL	-0.112	0.98	0.999	1.379	0.985	5.547	68.96	0
PFE	0.034	0.986	0.999	1.80	0.991	46.58	581.14	0.00024
RTI	-0.041	1.011	0.999	1.018	1.016	9.054	139.8	0.00012
GSK	-0.133	1.002	0.999	3.259	1.007	35.81	280.8	-0.0002
BCE	-0.007	1.001	1	1.51	1.007	209.1	-3700	0.0051
SBGI	-0.022	0.982	0.998	1.636	0.988	15.91	221.6	0
YAH	0.8847	0.901	0.988	14.45	0.906	29.33	-143.46	0.0005
TIF	0.0218	0.988	0.999	1.621	0.993	63.5	-607.72	0.00114

Values for the survey of RRWPM

With the RRWPM there is also a correlation between the variables which are, the weighted expected return of the security i and the weighted expected return of the market index. The average value of r^2 for the twenty stock is 0.99. This is an indication of a strong correlation between the two variables thus the predictors of the expected return of the security i can be greatly relied upon. When we compare CAPM and RRWPM we find a very big difference between their values of r^2. Thus RRWPM is a much more improved model than CAPM.

6.2.3 Value at Risk

VaR of the 20 companies from NYSE is investigated for comparison purposes with actual non-diversifiable risk. Value at Risk is determined as a measure of the forecasted volatility as illustrated by Jorion (2000), S_t (portfolio risk) is multiplied by standard normal deviate, α, for the selected confidence level (e.g, $\alpha = 2.33$ for a one-tailed confidence level of 99 percent). The portfolio variance then $S_t^2 = w_t' \sum_t w_t$ where $\sum_t$ is the forecasted covariance matrix for the market risk factors as of the close day t. Hence we have, $VaR_t = \alpha S_t$. Using the Matlab software with a safety factor (scaling factor) of 3 the VaR for the 20 portfolio from NYSE are determined in the table below.

TABLE 6.3 Expected return, Risk and VaR at different α for the 20 portfolio from NYSE

COMPANY	EXPECTED RETURN	RISK	VaRα = 0.01	VaRα = 0.05	VaRα = 0.10
Yahoo	0.0377	0.2182	0.4699	0.3212	0.2419
Tiffany	0.0168	0.1229	0.2693	0.1855	0.1408
Toyota	0.0067	0.0799	0.1790	0.1246	0.0956
HM	0.0092	0.0765	0.1688	0.1166	0.0888
Ponardph	0.0412	0.3766	0.8349	0.5844	0.4483
Vical	0.0128	0.2045	0.4629	0.3235	0.2493
DataWatch	0.032	0.3747	0.8398	0.5844	0.4483
Bp	0.0093	0.0638	0.1391	0.0956	0.0724
Suntrust	0.0042	0.094	0.2145	0.1505	0.1163
Pnc	0.0094	0.0863	0.1914	0.1326	0.1012
AIG	0.0105	0.2603	0.5951	0.4177	0.3232
Ford	0.0103	0.1619	0.3662	0.2559	0.1971
Amr	0.0132	0.2123	0.4806	0.3359	0.2588
Bph	0.0176	0.0951	0.2036	0.1388	0.1042
Ctl	0.0104	0.0895	0.1978	0.1368	0.1043
pfe	0.0065	0.0719	0.1607	0.1117	0.0856
rti	0.0085	0.1405	0.3184	0.2226	0.1796
gsk	0.0068	0.0587	0.1297	0.0897	0.0684
bce	0.0186	0.0843	0.1775	0.12	0.0894
sbg	0.0014	0.1603	0.3714	0.2622	0.2040

Value at Risk for the 20 Portfolio from NYSE

VaR returns the maximum potential loss in the value of a portfolio over one period of time, given the loss probability level risk threshhold. It is an N ports by 1 vector of the estimated maximum loss in the portfolio predicted with a confidence probability of 1 risk threshold. A value of 0 indicates no losses. Although this is a popular method of estimating risk the market risk is not conventionally calculated since covariances are used yet market risk is not by definition diversifiable.

6.2.4 Calculating Actual non-diversifiable risk

With the proof that non - diversifiable risk is a random sample in Proposition 4.1 the sample white noise is estimated by varying the variance of individual return values r_i resulting in the expression given by equation (4.2). The non-diversifiable risk estimates in table 6.4 below are substituted into equation (4.3) which is determined such that the bias and variance of the actual and predicted values of white noise are minimized to give equation (4.5).

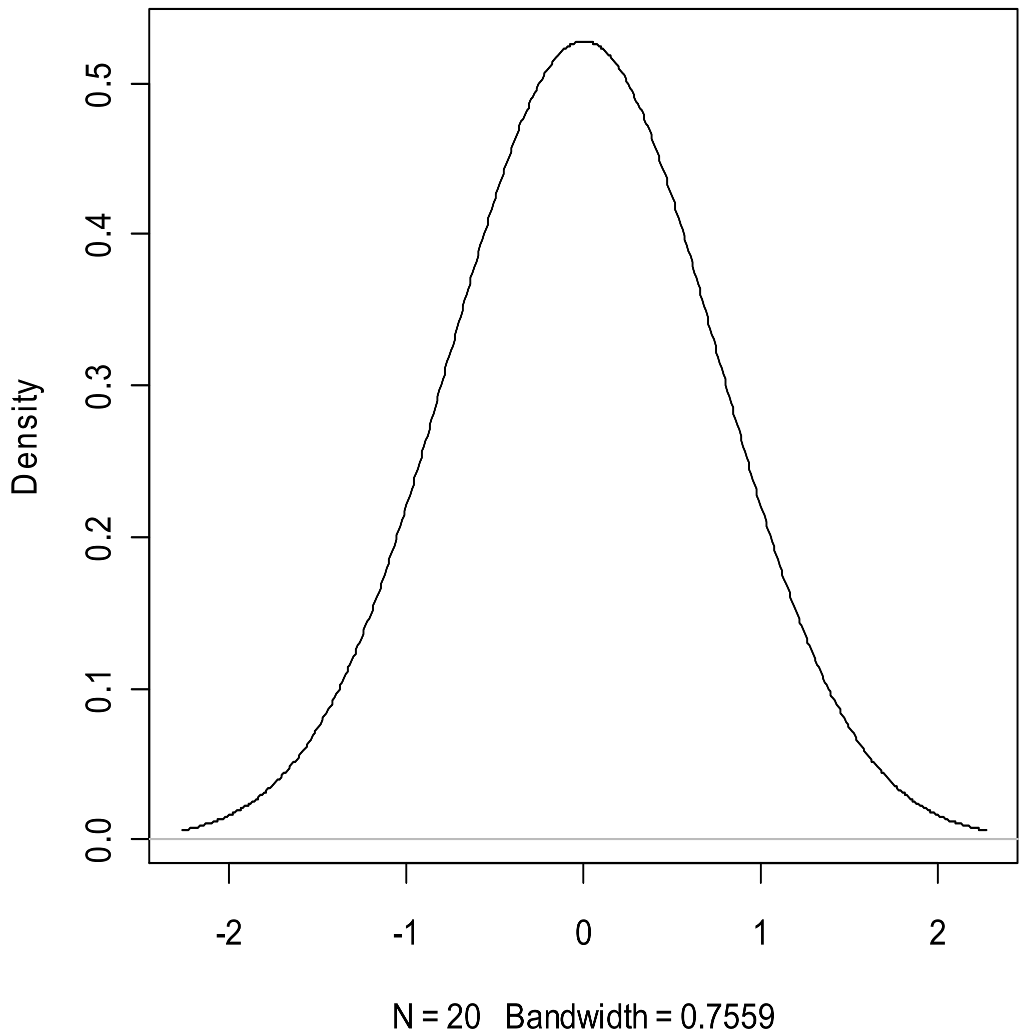
TABLE 6.4 The non-diversifiable risks of 20 stocks used to determine White noise

COMP	N_{Gw}
YH	29.33
TIF	63.5
TM	538.1
HM	28.23
PONARD	27.97
VIC	0.561
DAWT	25.58
BP	105.2
SUNTB	108.3
PNC	7.876
AIG	7164
FORD	1898
AMR	25.17
BPH	1.752
CTL	5.547
PFE	46.58
RTI	9.054
GSK	35.81

The non-diversifiable risks of 20 stocks from NYSE

The sample White Noise estimates are then substituted into equation (4.5) to determine the actual white noise. R statistical software is used to calculate Sheather Jones (sj) bandwidth and hence the density estimates of actual wn as plotted in figure 6.1.

FIGURE 6.1The density estimates of actual white noise



A plot of the density estimates of actual white noise resulting from

Call: Density (x=x, bw=0.7559, xlim=c(-2,2) Data: x (20 obs); Bandwidth 'bw'=0.7559)

as illustrated by section 4.3

A summary of statistics resulting from Sheather Jones density estimation in Table 6.5 below enables us apportion densities of the different quartile ranges.

TABLE 6.5 A summary of statistics resulting from Sheather Jones density estimation

Value	X	Y
Min	-2.2676128	0.005892
1st Qu	1.133396	0.041992
Median	0.00081	0.170879
Mean	0.0008191	0.219614
3rd Qu	1.135035	0.397272
Max	2.269251	0.527674

A summary of the results of a kernel density estimation of a portfolio white noise

F values are calculated as follows:

$F = \frac{v_i - \bar{v}}{\sigma_v}$

where v_i = white noise of portfolio i,

$\bar{v}$ = mean of white noise of all the portfolios, and

σ_v = standard deviation of all the portfolios.

The probability density estimate of a portfolio is determined by comparing the F values with the apportioned densities of the different quartile ranges and the maximum value.

An F score of positive 0.827722 has its density calculated as follows:

0.527674+(1-0.827722)0.130402 = 0.63556

Where:

0.527674 is the maximum value,

0.130402 is the value apportioned to the first quartile and

1- 0.827722 represents the fraction occupied in the first quartile. Final results of the survey are tabulated in Table 6.6 below.

TABLE 6.6 A table of Actual white noise and its determinants

Company	wn	F	Probabilities	Actual N_{Gw}
YH	0.000729	0.827722	0.63566	18.64391
TIF	0.00027	-0.27869	0.491	31.1785
TM	0.00011	-0.66196	0.4414	237.5173
HM	0.000128	-0.62098	0.4467	12.61034
PONARD	0.001551	2.809139	0.979	27.38263
VIC	0.00046	0.179302	0.5511	0.309222
DAWT	0.001456	2.580143	0.9388	24.0145
BP	0.000113	-0.65714	0.442	46.4984
SUNTB	0.00011	-0.66437	0.441	47.7603
PNC	0.000142	-0.58723	0.4511	3.552864
AIG	0.000657	0.654167	0.613	4390.919
FORD	0.000308	-0.18709	0.5033	954.7601
AMR	0.000491	0.254027	0.521	13.11357
BPH	0.000227	-0.38234	0.4778	0.837106
CTL	0.0000884	-0.71655	0.4342	2.408507
PFE	0.0000988	-0.69146	0.4375	20.37875
RTI	0.000238	-0.35582	0.4813	4.35769
GSK	0.0000872	-0.71933	0.4339	15.53796
BCE	0.00022	-0.39921	0.4756	99.44796
STGI	0.000227	-0.38234	0.4778	7.601798

Final results of white noise and kernel density estimation of portfolios of stocks It is interesting to note that the portfolios with the highest actual non-diversifiable risks were AIG with 4390.919%, FORD; 954.7661%, and TM; 237.5173%. These are corporates which experienced financial difficulties during the credit crunch in the

United States of America in 2008. AIG and TM had to be given some financial rescue packages to stay afloat until the financial crump was reversed.

6.2.5 Wilcoxon Signed Rank Test

Wilcoxon signed rank test of hypothesis is used to compare the VaR method of determining risk and Kernel white noise method. Here we test the hypothesis that risks obtained by Kernel White noise are a reflection of actual risks than those obtained by VaR.

H_o: The population difference are centered at zero

H_a : The population differences are centered at a value less than zero

Based on a significance level of $\alpha = 0.01$, the proper test is to reject H_o if $|Z| > Z\alpha$

Determining Z and Z_α

$Z =$ test statistic $= \frac{\bar{x}_t - \mu_t}{\sigma_t}$

Where $\mu_t = \frac{n(n+1)}{4}$, $\sigma_t = \sqrt{\frac{n(n+1)(2n+1)}{24}}$, and $\sum t = \frac{n}{2}\left(2a + (n-1)d\right)$

TABLE 6.7 A table of the ranks of the difference between risks from kernel density from table 6.4 and VaR from sub section 6.2.3

COMPANY	Actual N_{Gw} (i)	VaR at $\alpha = 0.01$ (ii)	i)- ii)	Rank
Yahoo	18.64	0.4699	18.17	10
Tiffany	31.18	0.2693	30.91	14
Toyota	237.52	0.1790	237.34	18
HM	12.61	0.1688	12.44	7
Ponardph	27.38	0.8349	26.55	13
Vical Inc	0.31	0.4629	-0.15	1
Data Watch	24.01	0.8398	23.17	12
Bp	46.50	0.1391	46.36	15
Suntrust	47.76	0.2145	47.55	16
Pnc	3.55	0.1914	3.36	4
AIG	4390.92	0.5951	4390.32	20
Ford	954.76	0.3662	954.39	19
amr	13.11	0.4806	12.63	8
Bph	0.84	0.2036	0.63	2
ctl	2.41	0.1978	2.211	3
pfe	20.38	0.1607	20.22	11
rti	4.358	0.3184	4.039	5
gsk	15.54	0.1297	15.41	9
bce	99.45	0.1775	99.27	17
sbg	7.602	0.3714	7.230	6

Wilcoxon signed Rank Test for Large samples paired

Using normal tables $-Z_\alpha$ = - 1.645, using the difference in risks and their ranks in table 6.4, Z = - 3.88. Since, $Z < -Z_\alpha$ we reject the null hypothesis, so there is sufficient evidence to conclude that the kernel white noise risks are better estimates of risks as compared to those obtained by VaR.

6.2.6 Determining Forecasted Actual Non-diversifiable Risk

The monthly returns of the twenty stocks, the NYSE and Treasury bond from July 1998 to September 2009 are forecasted and their returns calculated. A sample of the forecasted parameters and returns of Toyota using Matlab forecasting software is given below.

TABLE 6.8 Parameters of Forecasted Actual Non - Diversifiable Risk

PARAMETER	Standard Value	T Error	Statistic
C	0.007386	0.0067064	1.1013
MA	-0.05125	0.1055	-0.4858
K	0.00019654	0.00028766	0.6833
GARCH	0.90643	0.063142	14.3555
ARCH	0.034543	0.027396	1.2609
Leverage	0.084469	0.063075	1.3392

Conditional Probability Distribution: Gaussian Parameter.

Where: Parameter refers to the Standard value, the T error and the Statistic value.

Standard value = the determined values of the unknowns,

T Error = the error values in determining the standard values,

Statistic = the ratio of Standard value to T Error,

C and K are the constant values used in estimating the MA (1), GARCH (1) and ARCH (1),

Leverage (1) = the value that compares the actual value and estimated value.

TABLE 6.9 18 month forecasts of Toyota returns

Month 1-6	Month 7-12	Month 13-18
0.0851	0.0877	0.0899
0.0856	0.0881	0.0902
0.0861	0.0885	0.0906
0.0865	0.0888	0.0909
0.0869	0.0892	0.0912
0.0873	0.0895	0.0915

The forecasts in table 6.9 are determine using the equation:

$\sigma_t^2 = 0.00019654 + 0.90643\sigma_{t-1} + 0.034543\varepsilon_{t-1}^2$

Where:

$0.00019654 = \mathrm{k}$

$0.90643\sigma_{t-1}$ = GARCH (1)

$0.034543\varepsilon_{t-1}^2$ =ARCH

as given in table 6.8. The forecasted returns are substituted into equation 3.21 to give the forecasted real risk weighed expected returns, cost of equity, and in equation 3.22 and 3.23 to get the forecasted total real risk as shown in the table below

TABLE 6.10 Parameters for the survey of RRWPM with forecasted returns

COMPANY	BETA	ALPHA	r^2	s_{ey}	$E(r_i)$	N_{Gw}	w_i
TM	-9.54292	1.00134	0.99999	86.6696	1.152211	41.2738	210876
HMC	3.431529	1.01662	0.99994	30.2805	0.88325	17.2158	17913.27
PARD	-8.24994	0.99774	0.99741	75.8045	3.63853	24.3518	5274.66
VICL	-9.29791	4.56046	0.99969	114.899	2.5129	14.2396	1303.08
DWCH	-3.13457	1.02443	0.9968	38.65	1.98147	13.9194	2926.67
BP	2.041693	1.00334	0.99999	110.8701	0.90452	36.738	90836.6
STI	17.49379	1.00795	0.99999	172.977	2.10794	69.8255	185721
PNC	8.196005	1.00763	0.99001	180.9757	1.54059	29.1505	9829.81
AIG	115.6898	1.00203	0.99999	871.5171	4.29225	316.416	1538914
F	-3.34379	0.99638	1.00000	177.2241	2.39588	118.724	1378801
AMR	1.4558	0.96866	0.99979	41.4946	2.89675	25.7771	11355.9
BPH	-0.53771	1.000191	0.99999	0.007131	1.42972	0.537676	20.4607
CTL	-1.75989	0.98106	0.99935	85.5529	1.0979	31.5394	34748.1
PFE	2.22284	1.02424	0.99984	32.7984	0.987518	18.4911	15985.3
RTI	-1.97823	0.99658	0.99896	132.1698	1.74984	29.7557	33276.7
GSK	8.56784	1.00197	0.99999	76.18854	0.72527	32.6397	176875
BCE	-0.18985	0.99658	0.99454	58.8545	1.36037	23.6328	10082.1
SBGI	3.819447	1.01005	0.99999	74.32407	2.851	44.0047	77721.6
YAH	-6.02508	0.94946	0.99423	50.99261	2.24331	21	5798.92
TIF	-0.00427	0.73454	0.99898	0.093718	2.2033	0.033178	0.579569

A table of values for the survey of RRWPM with forecasted returns

where:

r^2 = The coefficient of determination,

S_{ey} = The standard error for the y estimate and $E(r_i)$= Cost of equity.

Non - diversifiable risk estimated above is used to calculate white noise as an independent random variable as given by equation 4.5 and tabled below.

Table 6.11 Forecasted returns of estimated white noise

COMPANY	TM	HMC	PARD	VICL	DWCH	BP	
wn_i	0.000068	0.000052	0.001436	0.000436	0.001299	0.00004	
COMPANY	STI	PNC	AIG	F	AMR	BPH	CTL
wn_i	0.000217	0.00101	0.000108	0.000282	0.000458	0.000096	0.000076
COMPANY	PFE	RTI	GSK	BCE	SBGI	YAH	TIF
wn_i	0.000055	0.000197	0.000033	0.000084	0.000408	0.000022	0.000195

A table of estimated white noise of forecasted returns.

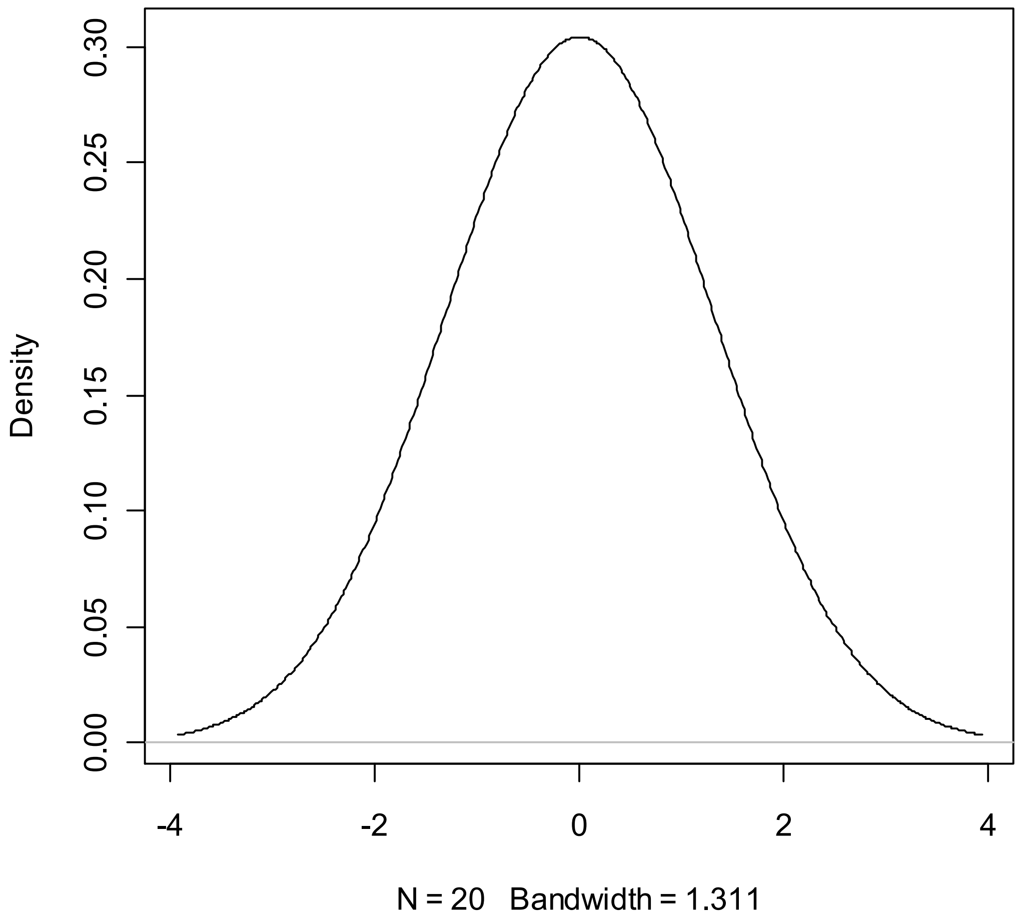
Gaussian kernel is used to determine the probability estimates of non - diversifiable risk using white noise as an independent random variable and thus actual non - diversifiable risk is calculate as tabulate below.

Table 6.12 A summary of statistics resulting from Sheather Jones density estimation of the forecasts

Value	X	Y
Min	-3.9316733	0.003405
1st Qu	-1.9654271	0.024243
Median	0.0008191	0.098588
Mean	0.0008191	0.126682
3rd Qu	1.9670653	0.229129
Max	3.9333115	0.304343

A summary of the results of a Gaussian kernel density estimation of forecasted white noise. A maximum value of 0.304343 represents the probability at 3.9333115 deviations from the mean. The probability apportioned to the first quartile being determined by taking 0.098588-0.024243, second quartile 0.126682-0.098588 and third quartile 0.229129-0.126682

FIGURE 6.2 The density estimates of a Gaussian Kernel density estimation of forecasted whitenoise



A Plot of the density estimates of a Gaussian kernel density estimation of forecasted white noise

The Z values range from -4 to +4 with most of the data being within a density of 0.6

TABLE 6.13 A Table of the Actual Forecasted N_{Gw}

Company	F	Probability estimates	N_{Gw}
TM	-0.62972	0.253574	10.46597
HMC	-0.66799	0.250696	4.315931
PARD	2.675201	0.560296	13.64422
VICL	0.259601	0.323869	4.611757
DWCH	2.344264	0.560296	7.456496
BP	-0.69795	0.323869	9.127267
STI	-0.26942	0.535691	19.59819
PNC	1.646155	0.248442	13.52311
AIG	-0.53272	0.280674	82.54386
F	-0.1124	0.463907	34.72481
AMR	0.312744	0.260872	8.451437
BPH	-0.56243	0.292484	0.139062
CTL	-0.61065	0.327866	8.04281
PFE	-0.66142	0.258635	4.644776
RTI	-0.31773	0.255009	8.243527
GSK	-0.71329	0.25119	8.070741
BCE	-0.58966	0.27704	6.063864
SBGI	0.191005	0.247267	14.0247
YAH	-0.74105	0.245201	5.149221
TIF	-0.32256	0.276677	0.00918

Final results of a Gaussian kernel density estimation of forecasted white noise

The value of F in table 6.13 is determined in the same way as that for Kernel density estimation on page 56. If the market risk 12 months after the credit crunch in the United States of America's (US) economy as shown in table 6.7 above is compared with that at the height of the crunch as shown in table 6.6 on page 58 it is seen that 12 months later the risks are much lower as it was true with the US economy right then.

TABLE 6.14 A Table of forecast parameter for the survey of CAPM

COMPANY	α_i	β_i	r^2	S_{ey}	$E(r_i)$
TM	0.0119	-1.0427	0.6582	0.0529	- 0.0008
HMC	0.00734	0.8256	0.2768	0.067	0.0110
PARD	0.0734	3.3783	0.1577	0.3917	0.025
VICL	0.0268	2.4439	0.2748	0.1992	0.0199
DWCH	0.0636	1.7913	0.0487	0.3972	0.025
BP	0.0073	0.8793	0.4099	0.0529	0.0113
STI	0.0234	1.9342	0.3598	0.1294	0.0171
PNC	0.0149	1.4848	0.4589	0.0809	0.0146
AIG	0.0533	3.9906	0.2956	0.3090	0.0283
F	0.0196	2.3596	0.3993	0.1459	0.0194
AMR	0.0243	2.8686	0.3601	0.1918	0.0222
BPH	0.0196	1.4113	0.4277	0.0819	0.0142
CTL	0.0105	1.0525	0.3031	0.0801	0.0127
PFE	0.0074	0.8987	0.3211	0.0656	0.0114
RTI	0.0144	1.6717	0.2901	0.1317	0.0157
GSK	0.0052	0.6571	0.2848	0.0522	0.0101
BCE	0.0216	1.3111	0.4264	0.0761	0.0137
SBGI	0.0238	2.677	0.3583	0.1797	0.0211
YAH	0.0386	2.2973	0.2663	0.1913	0.0191
TIF	0.0252	- 2.2086	0.5088	0.1090	- 0.0056

A table of values for the survey of CAPM with forecasted returns

The value of r^2 averages 0.34432 which is synonymous with other errors from CAPM survey.

Chapter 7

CONCLUSION AND RECOMMENDATION

7.1 Conclusion

Most of the objectives set out for this research have been achieved. Basic functions that are used to determine variables that influence investment decision making for example expected returns, diversifiable risk and non - diversifiable risk and their estimators have been derived in chapter three. Extensive analysis of the estimators and comparison of their results with other estimators has been done in chapter five. Random error has been modeled as an independent random variable and supported with proved statistical properties. Both discrete and continuous probability density functions and probability estimates using kernel estimation have been determined which have then been used to calculate respective probability estimates. Thus actual estimates of non - diversifiable risks have been made which have reflected the true scenario of the actual happenings of the various stocks and organisations in the World.

The Coefficient of determination value for RRWPM averages 0.999 for the twenty stocks indicating that it is almost a perfect estimator of cost of equity. This is in comparison to the Fama and French (1995) which averaged 0.93, the full-information industry beta method which averaged 0.75 and the CAPM model which averaged 0.25. The real risks are also determined as given by equations 3.22 and 3.23 thus aiding decision making since we have more than one risk to consider. The other models take the Non-diversifiable risk as S_{ey} and the diversifiable risk as one minus the non - diversifiable risks with no justifiable evidence for their accuracy. A cost of capital of 1.031 for Sun Trust Bank in table 6.2 means it costs 3.1 % that is the excess of 100 by 1.031 (103.1) to receive the given returns of the Sun Trust Bank stock. It is important to note that the expected return given by equation 3.22 can also be used to measure the cost of equity.

An estimate of random error is made with the least bias and variance. Probability estimates of the asset parameters are made thus boosting the level of surerity. These are made in comparison i.e. by analysing given portfolios one is able to make a decision among a variety of them. Methods like VaR use generated variances to give probability estimates using extreme values which lacks the comparability factor and uses covariances to determine market risk. They also use covariance parameters as market risks thus going against its definition. From the analysis of the results it is clear that there is a relationship between the determined actual non-diversifiable and the actual market risk on the ground over the past two years. These research findings can aid investors make solid investment decisions as well as the different corporate cution themselves against any financial stress currently and in future.

The r^2 value for RRWPM averages 0.999 for the twenty stock companies forecasted indicating that it is almost a perfect estimator of cost of equity. This is in comparison to the CAPM model which averages 0.25. This shows that a RRWPM avoids the explosion of conditional moments of GARCH (1) since this has not deterred the RRWPM from being a perfect estimator of cost of equity. The actual non-diversifiable risk determined using derived white noise enables one make future predictions on the various portfolios. The study also indicates that the companies which needed financial assistance to stay afloat 12 months before AIG, TM and FORD, had market risks of, 4390.919, 237.5173 and 954.7601 respectively and 12 months later they had market risks of 82.54386, 10.46597 and 34.72481 respectively. Also of importance is the flexibility that RRWPM has where it varies risk measures between $-\infty$ to $+\infty$ unlike other risk measures which are limited to 0 to 100 %. This makes it easy to compare the risk change as seen in the comparisons above where the difference in the risk measures are so apparent. Therefore this research is a true reflection of the actual market risks. Also the least risky stocks twelve months later included BPH, TIF, AMC and VICL which was also true on the NewYork Stock Exchange then.

7.2 Recommendation

Expected returns determined by the RRWPM which is also used as an estimate of cost of capital is a factor considered when chosing among a portfolio of assets for investments. Market risk determined in this research leading to the computation of total risk is also a factor considered in determining the choise of portfolio to invest in. Thus these should be used in finace and in investment decision making since they have been found to be better when compared to CAPM as well as VaR. The random error model enabling the generation of probability density functions and estimates should be taken up since this ascertains the non - diversifiable risk estimates thus making these risks actual. It also enables comparison among the portfolios making performance rankings possible. Wilcoxon signed Rank test gives sufficient evidence to conclude that the kernel white noise risks are a reflection of actual risks as compared to those obtained by VaR thus we should use kernel white noise risks to evaluate performance of various portfolios and thus decide on the better portfolio.

The Garch forecasts of Non - diversifiable risk as investigated by Anyika *et al* (2011) give a true picture of the changes that were experienced in NYSE stocks thus forecasting does not reduce the estimation properties of the various estimators but add to their credibility. Forecasting in Garch assumes continuous distribution. Random error estimator is able to model continuous situations thus seen to be cutting across all distributions and able to facilitate easy comparison enabling sound decision making. Research is encouraged in the area of observation error and its effect on investment decision making since this has not been done in this research. During the credit crunch a lot of financial measures were taken to save the financial markets and the economy as a whole. These measures and their effect on the crisis and the eventual solutions should be investigated.

A global risk mitigating model could be a start so that every continent's risk measure is weighed as the model is derived so that we do not experience spillover effects that facilitate persistent financial crises even as they are properly dealt with in their countries of origin.

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APPENDIX

Returns of Stocks as Calculated by Equation 3.1 for 20 NYSE Stock, the NYSE Index, and the Treasury Bond from Year 1998 -2008

Returns for the selected 20 stocks for the year 1998

$C \setminus M$	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
TM	0.06*	0.14*	0.07	0.08	0.04	0.05	0.01*	0.00	0.10	0.00	0.01*	0.19
HMC	0.04	0.09*	0.09*	0.01	0.18	0.08*	0.11	0.02	0.18	0.01*	0.07*	0.06
VICL	0.32*	0.33*	0.44	0.18	0.26	0.13*	0.02	0.12*	0.20*	0.11	0.05	0.01
DWCH	0.21*	0.23*	0.10	0.09*	0.23	0.10*	0.09*	0.23	0.06	0.17*	0.07	0.04
BP	0.09*	0.08*	0.19	0.01	0.05	0.01*	0.11*	0.05	0.20	0.12	0.05*	0.01
STI	0.10*	0.23*	0.11	0.12	0.01	0.10	0.08*	0.03*	0.08*	0.15	0.05*	0.03
PNC	0.19*	0.34*	0.28*	0.21*	0.00	0.10*	0.63	0.29*	0.02*	0.16*	0.12	0.06
AIG	0.03	0.23*	0.01	0.09	0.10	0.03	0.07	0.11	0.06	0.03*	0.02*	0.03
F	0.03*	0.22*	0.05	0.17	0.01	0.07	0.05	0.03*	0.04*	0.14	0.11*	0.01*
AMR	0.14*	0.24*	0.02	0.21	0.02*	0.01*	0.01*	0.06*	0.06	0.19	0.07*	0.05
BPH	0.06*	0.14*	0.04	0.19	0.06*	0.09*	0.03	0.02	0.15	0.30	0.06*	0.14
CTL	0.08	0.09*	0.04	0.20	0.00	0.19	0.01	0.09*	0.14	0.14*	0.05*	0.04
PFE	0.01	0.15*	0.14	0.01	0.05	0.12	0.03	0.03	0.05	0.17*	0.07*	0.02
RTI	0.08*	0.15*	0.13	0.26*	0.01	0.07*	0.12*	0.14*	0.06*	0.31	0.01	0.10
GSK	0.02	0.08*	0.03	0.09	0.02	0.09	0.02*	0.06*	0.06	0.13*	0.03*	0.01
BCE	0.06*	0.20*	0.11*	0.22	0.04	0.09	0.18	0.09*	0.11	0.03	0.01	0.09
SBGI	0.09*	0.39*	0.01	0.20*	0.05*	0.59	0.04*	0.22*	0.02*	0.04*	0.01*	0.18
YAH	0.16	0.24*	0.88	0.01	0.47	0.23	0.49	0.13*	0.10	0.04	0.15*	0.16
TIF	0.11*	0.13*	0.15*	0.03	0.35	0.19	0.11	0.01*	0.31	0.12	0.01*	0.17
NYSE	0.01*	0.15*	0.06	0.08	0.06	0.06	0.04	0.03*	0.04	0.04	0.02*	0.05
BOND	0.00	0.02	0.02	0.01*	0.01	0.00	0.01	0.02*	0.01	0.00	0.01*	0.00

Returns for the selected 20 stocks for the year 1999

$C \setminus M$	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
TM	0.09	0.10*	0.00	0.10	0.01*	0.43	0.10*	0.07*	0.28	0.04*	0.09*	0.02
HMC	0.01*	0.06*	0.01	0.03	0.02*	0.07*	0.14*	0.03	0.21	0.09	0.23*	0.01
VICL	0.22	0.09*	0.03	0.03	0.52	0.38	0.29	0.53	0.43*	0.46*	0.06	0.01
DWCH	0.50*	0.25	0.07	0.25*	2.62	0.25*	0.39	1.31	0.43*	0.32*	0.20*	0.06
BP	0.07	0.03*	0.01*	0.04	0.06	0.03*	0.09*	0.12*	0.13	0.04*	0.08	0.04
STI	0.07*	0.01	0.02	0.11	0.04*	0.02*	0.13*	0.13*	0.14	0.12*	0.18	0.24*
PNC	0.20	0.07	0.14*	0.22*	0.48	1.10	0.35	2.23	0.08	0.23*	0.04*	0.33
AIG	0.01*	0.00	0.06*	0.18	0.01	0.04	0.03*	0.15*	0.24	0.00	0.03	0.04
F	0.13*	0.07	0.04*	0.10	0.08*	0.06	0.06*	0.16*	0.10	0.20	0.11*	0.08*
AMR	0.05*	0.10*	0.07*	0.17	0.04*	0.10	0.02*	0.02*	0.42	0.07	0.16*	0.07*
BPH	0.07*	0.02*	0.08	0.09*	0.05	0.21	0.11*	0.16*	0.09	0.03*	0.04*	0.19
CTL	0.08	0.08*	0.04	0.00	0.14	0.03	0.18*	0.13*	0.11	0.34*	0.10	0.06
PFE	0.07*	0.12	0.05*	0.11	0.08*	0.11*	0.12	0.11*	0.14	0.15	0.06	0.08
RTI	0.12*	0.28*	0.07	0.28*	0.09*	0.13	0.02*	0.04*	0.25	0.20	0.23	0.12*
GSK	0.08*	0.03	0.02*	0.15	0.01*	0.06*	0.06*	0.08*	0.19	0.10	0.10*	0.03
BCE	0.01	0.06*	0.08	0.21	0.12	0.34	0.13	0.08	0.15	0.08*	0.14*	0.05
SBGI	0.16	0.04*	0.44*	0.10	0.15	0.06	0.34*	0.19	0.06*	0.12*	0.05	0.33
YAH	0.21*	0.08	0.22	0.00	0.19	1.03	0.26*	0.01*	0.07	0.24*	0.13*	0.10
TIF	0.04	0.06	0.12	0.01*	0.30	0.15	0.17*	0.14*	0.31	0.13*	0.16*	0.11
NYSE	0.03*	0.01*	0.03*	0.06	0.02	0.06	0.05*	0.02*	0.10	0.03*	0.02*	0.02
BOND	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.02

Returns for the selected 20 stocks for the year 2000

$C \setminus M$	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
TM	0.09*	0.03	0.10*	0.02	0.11*	0.12*	0.08	0.02	0.02	0.04*	0.05	0.01*
HMC	0.07	0.01*	0.01	0.05*	0.00	0.07	0.04	0.03	0.04	0.00	0.02	0.05
VICL	0.16*	0.51	0.06	0.18*	0.31*	0.25	0.02	0.26*	0.32*	0.52	0.03*	0.00
DWCH	0.13*	0.02*	0.35*	0.31*	0.39*	0.31*	2.10	0.23*	0.11*	0.31*	0.13	0.02*
BP	0.08*	0.06	0.04*	0.04*	0.06*	0.01	0.08	0.03*	0.00	0.09	0.01*	0.07*
STI	0.05	0.04	0.01	0.02*	0.05	0.24	0.06	0.01*	0.01*	0.03*	0.02*	0.05
PNC	0.13*	0.15	0.29	0.34*	0.58*	0.23*	1.01	0.36*	0.38*	0.62	0.26*	0.40*
AIG	0.12	0.02	0.07	0.02	0.01*	0.02	0.14*	0.04*	0.02*	0.02	0.01*	0.05
F	0.09	0.09*	0.05	0.04	0.13*	0.03	0.22	0.01*	0.01	0.06	0.17*	0.01
AMR	0.25	0.01*	0.00	0.00	0.02	0.17	0.00	0.15*	0.06	0.09	0.02	0.07*
BPH	0.11*	0.05	0.07*	0.02*	0.03	0.04	0.01*	0.06	0.12*	0.14	0.03	0.02*
CTL	0.02	0.02*	0.05*	0.41	0.08*	0.02	0.12*	0.08*	0.00	0.06*	0.05	0.07
PFE	0.01*	0.00	0.04	0.04*	0.03	0.04	0.02*	0.00	0.09*	0.06	0.01*	0.07*
RTI	0.23	0.00	0.03	0.00	0.10*	0.10	0.09	0.03	0.16*	0.05	0.02	0.06
GSK	0.01*	0.02	0.05	0.04*	0.02*	0.01*	0.06*	0.05	0.05*	0.02	0.03	0.03
BCE	0.04*	0.02*	0.06	0.16	0.01	0.07	0.01*	0.07*	0.15*	0.11	0.02	0.05
SBGI	0.00	0.10	0.09*	0.16*	0.05*	0.15	0.11	0.19*	0.20*	0.13	0.10	0.15
YAH	0.04	0.06*	0.25*	0.36*	0.32*	0.24*	0.24	0.36*	0.34*	0.28	0.10*	0.10
TIF	0.01	0.22	0.07*	0.11	0.20*	0.07*	0.18	0.17*	0.12*	0.19	0.07	0.05
NYSE	0.02*	0.06	0.05*	0.00	0.08*	0.00	0.03	0.09*	0.06*	0.08	0.01	0.03*
BOND	0.01	0.02	0.01	0.01	0.02	0.02	0.01	0.01	0.01	0.00	0.01	0.00

Returns for the selected 20 stocks for the year 2001

$C \setminus M$	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
TM	0.06*	0.08*	0.15*	0.05*	0.06	0.02*	0.03	0.02*	0.13	0.06*	0.01	0.04*
HMC	0.04	0.21*	0.12*	0.13	0.05	0.07	0.01*	0.02*	0.07	0.07	0.04*	0.05*
VICL	0.12*	0.06*	0.12*	0.26	0.08*	0.03	0.16*	0.18*	0.11	0.02	0.22*	0.28*
DWCH	0.31*	0.18*	0.2*	0.79	0.2*	0.16*	0.12	0.04*	0.89	0.10	0.16*	0.48
BP	0.01*	0.04	0.03*	0.02*	0.08*	0.05	0.00	0.07	0.07	0.04*	0.01	0.01*
STI	0.07	0.01*	0.02*	0.10*	0.06	0.01*	0.02*	0.03	0.06	0.02	0.01	0.01*
PNC	0.13	0.13*	0.16*	0.14	0.81	0.12	0.19*	0.15*	0.13*	0.28*	0.06	0.55*
AIG	0	0.02*	0.06*	0.00	0.01	0.05	0.04*	0.07*	0.00	0.02*	0.04*	0.03*
F	0.04	0.21*	0.13*	0.07*	0.18	0.17*	0.02*	0.03*	0.11	0.03*	0.11	0.09*
AMR	0.03*	0.09*	0.40*	0.05*	0.17	0.04	0.12	0.05	0.01	0.19*	0.02*	0.20*
BPH	0.06*	0.02	0.14*	0.06	0.18	0.01	0.08	0.05	0.00	0.06*	0.07	0.04*
CTL	0.02	0.13	0.04*	0.06*	0.07	0.03*	0.06*	0.08	0.03	0.19*	0.12	0.05*
PFE	0.03	0.07*	0.05	0.05	0.04	0.08*	0.05	0.01*	0.03*	0.09*	0.04*	0.01
RTI	0.16*	0.13*	0.25*	0.18	0.09*	0.11	0.01	0.02*	0.17	0.14	0.14*	0.07
GSK	0.03	0.08*	0.06	0.04*	0.05*	0.02*	0.03*	0.03	0.04*	0.02	0.14*	0.06
BCE	0.02	0.07*	0.11*	0.00	0.04	0.00	0.04*	0.05*	0.14*	0.01*	0.06	0.04*
SBGI	0.05*	0.01	0.19*	0.08*	0.07	0.19	0.06	0.15	0.18	0.01*	0.11	0.02*
YAH	0.12*	0.33*	0.26*	0.23	0.43	0.14	0.03*	0.16*	0.28	0.20*	0.09	0.08*
TIF	0.03*	0.12*	0.30*	0.08	0.23	0.09	0.13	0.08*	0.08	0.12	0.06*	0.06*
NYSE	0.01*	0.06*	0.08*	0.02	0.08	0.01	0.02*	0.02*	0.04	0.06*	0.01*	0.07*
BOND	0.02	0.01	0.01	0.02	0.01*	0.01*	0.01	0.01	0.02*	0.02	0.01	0.00

Returns for the selected 20 stocks for the year 2002

$C \setminus M$	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
TM	0.09*	0.04	0.03	0.05*	0.09	0.00	0.10*	0.01*	0.05*	0.01	0.05	0.09
HMC	0.01	0.01	0.06*	0.09*	0.05	0.05*	0.08*	0.10	0.09*	0.00	0.08	0.06
VICL	0.23	0.16*	0.58*	0.47	0.14	0.10*	0.11*	0.18*	0.03	0.06	0.53	0.07
DWCH	0.01*	0.19*	0.37	0.23*	0.05*	0.07*	0.07*	0.01*	0.01*	0.03*	0.02	0.00
BP	0.08*	0.02	0.15*	0.04*	0.03	0.04	0.04*	0.01*	0.01	0.00	0.10	0.00
STI	0.03*	0.03*	0.09*	0.01*	0.03*	0.03*	0.00	0.00	0.06*	0.09	0.04	0.00
PNC	0.26*	0.36*	0.32*	0.67	0.06	0.38*	0.09	0.04	0.55	2.04	0.48	0.01*
AIG	0.06*	0.02*	0.13*	0.14	0.04	0.11*	0.06*	0.09*	0.00	0.17	0.00	0.05*
F	0.15*	0.13*	0.17*	0.13*	0.35	0.18*	0.01*	0.09*	0.10*	0.38	0.02	0.05
AMR	0.34*	0.09*	0.59*	0.13*	0.64*	0.15*	0.56*	0.19*	0.10*	1.13	0.42	0.74
BPH	0.08	0.03*	0.03*	0.09*	0.10*	0.00	0.08*	0.06	0.01*	0.00	0.03	0.02
CTL	0.10*	0.02	0.17*	0.26	0.09	0.05*	0.03	0.10*	0.01	0.07	0.14	0.04
PFE	0.08*	0.03	0.12*	0.09	0.00	0.03*	0.00	0.02*	0.04	0.01*	0.01	0.10
RTI	0.21*	0.06	0.04	0.01*	0.11	0.12*	0.02	0.11*	0.07	0.03	0.01	0.06
GSK	0.08*	0.04*	0.01	0.01*	0.01	0.01*	0.03	0.08*	0.00	0.15	0.01*	0.01
BCE	0.05*	0.10	0.01*	0.02*	0.06	0.00	0.05	0.01*	0.01*	0.08	0.11	0.06
SBGI	0.19*	0.08	0.08	0.14*	0.17	0.16*	0.03*	0.23*	0.01*	0.35	0.15	0.04*
YAH	0.11*	0.22*	0.07*	0.56*	0.23	0.11*	0.11	0.15	0.15	0.03	0.20	0.10
TIF	0.30*	0.01	0.13*	0.22	0.08	0.16*	0.03*	0.03	0.04	0.11	0.18	0.00
NYSE	0.08*	0.00	0.11*	0.09	0.06	0.06*	0.03*	0.02*	0.01	0.08	0.05	0.01
BOND	0.01	0.02	0.01	0.01*	0.00	0.02	0.00	0.01	0.00	0.01	0.02	0.00

Returns for the selected 20 stocks for the year 2003

$C \setminus M$	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
TM	0.01*	0.08	0.08	0.0*	0.03	0.15	0.0*	0.04	0.08	0.02*	0.01*	0.13
HMC	0.04	0.03	0.01*	0.01*	0.01	0.11	0.05*	0.02	0.07	0.13*	0.07	0.13
VICL	0.21	0.03*	0.10	0.02	0.16*	0.02*	0.30	0.12	0.14*	0.14*	0.05	0.05
DWCH	1.21	0.02	0.23	0.01*	0.26*	0.02*	0.29	0.04	0.60	0.28*	0.01*	0.07
BP	0.01*	0.01	0.01	0.01	0.02	0.16	0.04*	0.04	0.04	0.03	0.01	0.01
STI	0.03	0.01	0.01*	0.11	0.07	0.01	0.01	0.01	0.04*	0.02*	0.04*	0.00
PNC	0.18*	0.01*	1.20	0.15*	0.18*	0.18*	0.14	0.05*	0.02*	0.44*	0.38	0.26*
AIG	0.16	0.07*	0.03*	0.05	0.05*	0.15	0.05	0.07	0.03*	0.00	0.02	0.03*
F	0.02	0.05	0.07*	0.14	0.09	0.21	0.09*	0.05*	0.01*	0.14	0.03*	0.05
AMR	0.15*	0.18	0.04	0.16	0.03*	0.01	0.27	0.07*	0.16*	0.11*	0.01	0.05
BPH	0.12	0.10	0.01*	0.16	0.01	0.11	0.06*	0.12	0.02*	0.12*	0.05	0.02
CTL	0.02*	0.01	0.02*	0.06	0.08*	0.00	0.19*	0.08	0.04*	0.05	0.03	0.01
PFE	0.02*	0.10*	0.02	0.04	0.07	0.05	0.04	0.01	0.04*	0.02	0.01*	0.03*
RTI	0.07*	0.04	0.00	0.14	0.13	0.25	0.12*	0.14	0.07*	0.07*	0.01*	0.10
GSK	0.05*	0.01	0.09	0.03	0.06	0.02	0.06*	0.02*	0.06*	0.05	0.02	0.02*
BCE	0.04*	0.02*	0.02	0.04	0.01*	0.02	0.00	0.02*	0.03*	0.05*	0.00	0.02
SBGI	0.11*	0.07	0.08*	0.14	0.00	0.29	0.09*	0.08*	0.08*	0.08*	0.10*	0.07*
YAH	0.05*	0.07	0.06	0.24	0.02*	0.05	0.04	0.06*	0.09	0.04	0.21	0.19
TIF	0.05	0.13	0.04*	0.27	0.04*	0.00	0.12*	0.06	0.09*	0.02	0.09*	0.04
NYSE	0.02	0.02	0.01*	0.05	0.01	0.05	0.02	0.01	0.02*	0.02*	0.01	0.02
BOND	0.03*	0.01	0.03	0.01*	0.00	0.01	0.00	0.01	0.01	0.03*	0.00	0.01

Returns for the selected 20 stocks for the year 2004

$C \setminus M$	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
TM	0.02*	0.01*	0.03*	0.02	0.04*	0.10	0.04*	0.01*	0.04*	0.02*	0.01*	0.00
HMC	0.00	0.03	0.02*	0.00	0.01*	0.09	0.01	0.02	0.06*	0.04*	0.03	0.00
VICL	0.21*	0.07	0.01*	0.06	0.16*	0.10	0.02	0.20	0.30*	0.06*	0.01	0.28
DWCH	0.28*	0.00	0.22	0.11*	0.11	0.36	0.11*	0.11*	0.21	0.21*	0.05	0.08*
BP	0.05	0.04*	0.07	0.01	0.06	0.05*	0.02	0.10	0.04*	0.02*	0.00	0.04
STI	0.01	0.04	0.03	0.00	0.02	0.04	0.03*	0.01	0.01*	0.01	0.02	0.02*
PNC	0.18*	0.07	0.22*	0.08	0.04*	0.19	0.01*	0.23*	0.38*	0.36*	0.10*	0.05
AIG	0.01*	0.01	0.04*	0.11*	0.04	0.04	0.01	0.01	0.17*	0.08*	0.09	0.05
F	0.05*	0.04*	0.00	0.06*	0.09	0.03	0.09*	0.04*	0.01*	0.19*	0.10	0.03
AMR	0.30*	0.06	0.18*	0.05	0.17	0.21	0.21*	0.01*	0.26	0.02*	0.23	0.06*
BPH	0.05	0.01	0.12	0.00	0.15	0.01	0.06	0.21	0.08*	0.10*	0.01*	0.09
CTL	0.03	0.04	0.07	0.06*	0.03	0.08	0.08*	0.03	0.02*	0.07*	0.07	0.06
PFE	0.07*	0.03	0.06*	0.05*	0.03*	0.03*	0.10*	0.10	0.00	0.03	0.03	0.01*
RTI	0.06*	0.01*	0.30	0.03	0.08	0.05*	0.19	0.11	0.14*	0.04*	0.22	0.15
GSK	0.01*	0.01	0.06	0.03*	0.01	0.11	0.06*	0.09	0.05*	0.10	0.01*	0.02*
BCE	0.05	0.01*	0.05	0.07	0.04	0.01	0.01*	0.02*	0.07	0.04*	0.04*	0.04
SBGI	0.04*	0.19*	0.09*	0.04*	0.03	0.28	0.11*	0.06*	0.05	0.05*	0.15	0.04
YAH	0.15*	0.07*	0.19	0.07	0.04	0.00	0.07*	0.08*	0.05	0.02	0.08	0.07*
TIF	0.03*	0.13*	0.00	0.05*	0.04	0.05	0.02*	0.04*	0.15	0.13*	0.03	0.06
NYSE	0.03*	0.00	0.01	0.01	0.04	0.03	0.03*	0.02	0.02*	0.02*	0.03	0.00
BOND	0.01	0.02	0.00	0.01	0.01*	0.01	0.01	0.01*	0.01*	0.01	0.01	0.01

Returns for the selected 20 stocks for the year 2005

$C \setminus M$	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
TM	0.06	0.08	0.14	0.00	0.04	0.08	0.01*	0.03	0.02	0.08	0.08*	0.03*
HMC	0.05	0.04	0.06	0.02*	0.01	0.04	0.02*	0.03	0.06	0.14	0.07*	0.04*
VICL	0.10	0.11*	0.03	0.04	0.01	0.18*	0.08	0.04	0.31	0.13*	0.20	0.14*
DWCH	0.05	0.07*	0.06*	0.09*	0.40	0.09*	0.12*	0.01*	0.08	0.02*	0.08*	0.04
BP	0.06	0.05	0.04	0.06*	0.00	0.02*	0.13	0.01*	0.04	0.07	0.03*	0.02*
STI	0.01	0.03*	0.01*	0.04	0.01	0.00	0.02*	0.02	0.01	0.06	0.01*	0.01
PNC	0.1	0.53	0.09*	0.01*	0.11	0.26*	0.07	0.68	0.01*	0.05*	-0	0.01*
AIG	0.04	0.01*	0.05	0.05	0.04	0.02	0.04*	0.01	0.00	0.01*	0.07*	0.03*
F	0.06	0.07*	0.01*	0.15*	0.02*	0.05*	0.12	0.07*	0.00	0.11*	0.03	0.03*
AMR	0.16	0.10*	0.11*	0.21	0.25	0.32	0.02	0.11	0.08	0.09*	0.00	0.03
BPH	0.09	0.06	0.09	0.09*	0.04	0.04	0.18	0.09*	0.11	0.14	0.05*	0.00
CTL	0.01*	0.05	0.03*	0.06*	0.01	0.00	0.00	0.08	0.09	0.04*	0.05*	0.04
PFE	0.04*	0.13*	0.02*	0.01*	0.02*	0.10	0.10	0.03	0.05*	0.02	0.06*	0.01*
RTI	0.10	0.01	0.13	0.15*	0.11	0.02	0.19	0.07*	0.30	0.10	0.00	0.07*
GSK	0.02*	0.03	0.05	0.01	0.04*	0.02	0.02	0.00	0.03	0.09	0.02*	0.01
BCE	0.02	0.08	0.06	0.10*	0.04*	0.02	0.01	0.00	0.00	0.03	0.02*	0.01*
SBGI	0.01*	0.04	0.05*	0.07*	0.16	0.03*	0.14*	0.01*	0.15	0.04*	0.08	0.02
YAH	0.04*	0.00	0.02	0.09	0.09	0.03*	0.12*	0.07*	0.01	0.02	0.04*	0.04
TIF	0.04	0.10	0.07	0.01*	0.03	0.06*	0.02*	0.02*	0.01	0.07*	0.02*	0.03*
NYSE	0.04	0.01*	0.01	0.02*	0.04	0.00	0.03	0.00	0.01	0.01	0.03*	0.00
BOND	0.01*	0.01	0.01*	0.01*	0.00	0.01	0.00	0.00	0.01*	0.00	0.00	0.00

Returns for the selected 20 stocks for the year 2006

$C \setminus M$	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
TM	0.01	0.03	0.01	0.08	0.02	0.12	0.02*	0.01	0.04*	0.05*	0.01*	0.04
HMC	0.04	0.03	0.00	0.05	0.00	0.12	0.01*	0.06*	0.06*	0.01*	0.03	0.03
VICL	0.14*	0.19	0.11*	0.27	0.01	0.00	0.03*	0.16*	0.08*	0.02	0.00	0.06
DWCH	0.0*	0.07*	0.13*	0.07*	0.17	0.07*	0.1	0.06	0.04*	0.52	0.11	0.11
BP	0.04	0.05*	0.04*	0.02	0.02	0.01*	0.05*	0.02*	0.05	0.04	0.00	0.08
STI	0.03	0.02*	0.01	0.02	0.04	0.03	0.02*	0.02	0.02*	0.02	0.07	0.04*
PNC	0.35*	0.00	0.07*	0.29	0.55	0.29*	0.14	0.04	0.04*	0.22	0.17	0.16*
AIG	0.03	0.05	0.04	0.01	0.05	0.02	0.04*	0.02*	0.00	0.04	0.04	0.03*
F	0.03*	0.25	0.03*	0.02	0.02*	0.08*	0.08	0.03*	0.00	0.02	0.04	0.13
AMR	0.13*	0.06*	0.12	0.22	0.13	0.05*	0.23	0.08*	0.11*	0.14*	0.09	0.07*
BPH	0.02*	0.00	0.09*	0.12	0.03*	0.04*	0.03	0.06	0.13	0.01	0.08	0.13
CTL	0.04	0.03	0.00	0.01	0.06	0.03	0.03	0.00	0.01	0.02	0.07	0.01*
PFE	0.11	0.07	0.03	0.06*	0.04	0.06*	0.01	0.04*	0.01	0.05	0.05	0.07*
RTI	0.17*	0.06*	0.00	0.41	0.23	0.03	0.05	0.06	0.05	0.04	0.06*	0.15*
GSK	0.01*	0.03	0.06*	0.00	0.01	0.01*	0.03	0.05	0.02*	0.05	0.09*	0.00
BCE	0.03*	0.09	0.10	0.04	0.13*	0.11	0.03*	0.00	0.09	0.19	0.09	0.03
SBGI	0.02*	0.08*	0.03	0.15	0.10	0.07	0.12	0.21	0.09	0.06	0.06*	0.06*
YAH	0.18*	0.06	0.12*	0.04	0.03	0.05*	0.11	0.09	0.01	0.10*	0.02	0.05*
TIF	0.04*	0.00	0.05	0.08	0.08	0.02	0.00	0.11	0.05	0.05	0.10	0.01
NYSE	0.01	0.02	0.02	0.03	0.02	0.01	0.01	0.02*	0.01	0.04	0.03	0.02*
BOND	0.01	0.02	0.01	0.01	0.01	0.01*	0.00	0.02	0.00	0.01	0.01*	0.00

Returns for the selected 20 stocks for the year 2007

$C \setminus M$	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
TM	0.04*	0.04*	0.01	0.02*	0.02*	0.06*	0.02	0.00	0.07*	0.01	0.01	0.08*
HMC	0.01*	0.09*	0.01	0.12	0.04*	0.03*	0.05*	0.03*	0.06*	0.10	0.05	0.02
VICL	0.02*	0.04*	0.01	0.05	0.11*	0.08*	0.14*	0.04	0.08*	0.00	0.09*	0.06
DWCH	0.04*	0.12*	0.04*	0.10	0.42	0.15*	0.27*	0.20*	0.01	0.00	0.24*	0.11*
BP	0.04*	0.02*	0.03	0.12	0.06*	0.01	0.13*	0.03	0.06*	0.20	0.01	0.04*
STI	0.09*	0.02	0.04*	0.04*	0.02*	0.11*	0.10	0.15*	0.05*	0.01	0.05*	0.31*
PNC	0.15*	0.05	0.07*	0.18*	0.03*	-0	0.18	0.21*	0.18*	0.10	0.08	0.07
AIG	0.08	0.03	0.03	0.07*	0.08*	0.01	0.06*	0.15*	0.07*	0.07	0.22*	0.26*
F	0.10*	0.08*	0.09	0.04	0.15*	0.10*	0.01*	0.02*	0.12*	0.44	0.18*	0.29*
AMR	0.06*	0.01*	0.09*	0.08	0.12*	0.34*	0.01*	0.08*	0.30*	0.03*	0.18*	0.29*
BPH	0.07	0.01*	0.26	0.11	0.13*	0.08*	0.04*	0.09	0.10*	0.22	0.05	0.01
CTL	0.06*	0.05	0.04*	0.05*	0.03*	0.03*	0.11*	0.02*	0.08*	0.02*	0.09	0.01
PFE	0.08*	0.07	0.02*	0.01	0.02*	0.04*	0.03	0.03*	0.06*	0.04*	0.02*	0.01*
RTI	0.05	0.12*	0.14	0.01*	0.06*	0.06*	0.02*	0.01*	0.18*	0.09*	0.05	0.17*
GSK	0.02*	0.03	0.02	0.03*	0.03	0.04*	0.06*	0.06*	0.03*	0.05	0.01	0.01*
BCE	0.00	0.01	0.06	0.09	0.01*	0.02	0.12*	0.04	0.06*	0.08	0.04*	0.01*
SBGI	0.08*	0.04*	0.02*	0.00	0.14*	0.19*	0.10	0.03	0.01*	0.01*	0.02	0.13*
YAH	0.14*	0.02*	0.18	0.16	0.14*	0.13*	0.18*	0.45	0.04	0.05*	0.02*	0.23*
TIF	0.09*	0.06	0.02	0.03	0.14*	0.01*	0.14*	0.05*	0.12	0.04	0.13	0.17*
NYSE	0.03*	0.01	0.04	0.01	0.04*	0.01*	0.06*	0.03*	0.01*	0.05	0.01	0.09*
BOND	0.01	0.01	0.01	0.01	0.01	0.00	0.02	0.00	0.00	0.00	0.01*	0.00

Returns for the selected 20 stocks for the year 2008

$C \setminus M$	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
TM	0.08*	0.04	0.04*	0.11*	0.17*	0.04	0.03*	0.01*	0.00	0.25	0.01	0.03*
HMC	0.06*	0.02	0.08*	0.18*	0.11*	0.03*	0.06	0.04	0.00	0.23	0.00	0.06*
VICL	0.00	0.05*	0.33*	0.23*	0.33*	0.27	0.37	0.22*	0.29	0.32	0.16*	0.27
DWCH	0.00	0.19*	0.07*	0.24*	0.05*	0.06*	0.42	0.32*	0.10	0.28	0.05	0.04
BP	0.12*	0.05*	0.13*	0.01*	0.00	0.04*	0.09*	0.08*	0.05	0.06	0.19	0.04*
STI	0.13	0.04	0.07	0.11*	0.19*	0.07*	0.59*	0.01*	0.02*	0.23	0.08*	0.25
PNC	0.09*	0.25	0.01*	0.27*	0.05*	0.35*	0.24	0.17*	0.07	0.49	0.41	0.33
AIG	0.02*	0.18*	0.84*	0.43*	0.05*	0.22*	0.18*	0.67*	1.38	0.38	0.22	0.31*
F	0.00	0.07*	0.17	0.58*	0.23	0.15*	0.18*	0.07	0.32	1.27	0.04*	0.06
AMR	0.76	0.14	0.05*	0.04	0.14*	0.22	0.44*	0.31*	0.22*	0.49	0.07*	0.10*
BPH	0.12*	0.06*	0.25*	0.25*	0.03	0.07	0.12*	0.01*	0.22	0.08	0.17	0.03*
CTL	0.05	0.04	0.03*	0.31*	0.09	0.03	0.01*	0.03*	0.10	0.03*	0.14	0.02
PFE	0.07	0.04	0.04*	0.04*	0.06*	0.08	0.18*	0.14*	0.11	0.02*	0.15	0.01*
RTI	0.23*	0.24	0.42*	0.19*	0.24*	0.19	0.07*	0.19*	0.08	0.11	0.10	0.23
GSK	0.06	0.01	0.07*	0.10*	0.11*	0.08	0.05*	0.13*	0.03	0.00	0.10	0.05
BCE	0.09	0.00	0.08*	0.16*	0.32*	0.05	0.00	0.04*	0.03	0.08	0.07	0.08*
SBGI	0.00	0.07*	0.26*	0.36*	0.02*	0.05	0.40*	0.39*	0.08*	0.08	0.59	0.10
YAH	0.04*	0.03*	0.11*	0.26*	0.10*	0.06	0.04*	0.13	0.03*	0.12	0.11	0.01*
TIF	0.07*	0.17	0.19*	0.23*	0.28*	0.20	0.12*	0.08*	0.14	0.34	0.02*	0.10*
NYSE	0.01*	0.01	0.09*	0.17*	0.07*	0.01	0.09*	0.11*	0.09	0.09	0.05	0.00

Where:

$*$:represents cells whose values are negatives,

C :Company

M :Month