Note on the effect of cross-sensitivity in the determination of stress

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The concentric Mohr circles of modified apparent strain, true strain and stress permit quick visualisation and calculation of true strains and stresses from readings of gauges with known cross-sensitivities.

Expressions for the principal stresses determined from readings of bonded wire resistance strain gauges after correction for cross sensitivity of the gauges have been derived analytically in an earlier article in Strain1. These expressions can also be derived without much effort by consideration of the three concentric circles for modified apparent strain, for true strain and for stress, which can be drawn from the gauge readings in three arbitrary directions in a bi-axial stress field.

Meir and Mehaffey2 have proved that a circle representing modified readings of strain gauges can be drawn concentric with the true strain circle for a point in a bi-axial stress field. The following proof, somewhat different from that of Meir and Mehaffey, is here presented for the drawing of the two strain circles from gauge readings obtained by means of a strain indicator and of the constants \( \mu_o \) and \( K \) given for individual gauges by the manufacturer.

The strain circles

The true strain \( \varepsilon_t \) along the axis of the gauge and the apparent axial strain \( \varepsilon_a \) read by a strain indicator connected to the gauge are related by

\[
\varepsilon_a = 1 + \frac{\varepsilon_o \cdot K}{1 - \mu_o K} \cdot \varepsilon_t \quad \text{[1]}
\]

At a point in a bi-axial stress field, the true strain \( \varepsilon_t \) along an arbitrary direction inclined at an angle \( \alpha \) to the algebraically larger principal strain axis through the point can be expressed as

\[
\varepsilon_t = A + B \cos 2\alpha \quad \text{[2]}
\]

where \( A \) and \( B \) are functions of strain with respect to a pair of orthogonal axes through the point.

Similarly

\[
\varepsilon_a = A - B \cos 2\alpha \quad \text{[3]}
\]

From [1], [2] and [3]

\[
\varepsilon_a \left( \frac{1 - \mu_o K}{1 + K} \right) = A + B' \cos 2\alpha
\]

\[\text{Notations}\]
\[
\begin{align*}
\varepsilon_a & \quad \text{Apparent strain along the axis of the grid of a resistance strain gauge, as read by a strain indicator} \\
\varepsilon_{ma} & \quad \text{Modified apparent strain reading of the gauge} \\
\varepsilon_t & \quad \text{True strain along the axis of the gauge} \\
\varepsilon_{t2} & \quad \text{True strain transverse to the axis of the gauge} \\
\mu_o & \quad \text{Poisson's ratio of the material on which the manufacturer has calibrated the gauge (supplied by the manufacturer)} \\
K & \quad \text{Cross-sensitivity coefficient of the gauge (supplied by the manufacturer)} \\
\mu & \quad \text{Poisson's ratio of the material to which the gauge is fixed} \\
E & \quad \text{Young’s modulus of the material to which the gauge is fixed} \\
E_{max}, E_{min} & \quad \text{Maximum and minimum principal stresses}
\end{align*}
\]

Notations:

\[
\begin{align*}
\text{where } B' &= B \left( \frac{1 - K}{1 + K} \right) \quad \text{[4]} \\
\varepsilon_{ma} &= \varepsilon_a \left( \frac{1 - \mu_o K}{1 + K} \right) = A + B' \cos 2\alpha \quad \text{[5]}
\end{align*}
\]

[2] and [5] represent concentric circles of radii \( B \) and \( B' \) related as in [4].

Thus from values of \( \varepsilon_{ma} \) in three directions, as obtained from a rosette gauge, the modified apparent strain circle can be drawn, and from it the true strain circle.

The stress circle

A stress circle concentric with the true strain circle can be drawn by recalling the well known relations

\[
\text{Radius of stress circle} = \frac{1 - \mu}{1 + \mu} \text{ times radius of true strain circle.} \quad \text{[6]}
\]

\[
\text{Stress scale} = \frac{1}{1 - \mu} \text{ times strain scale} \quad \text{[7]}
\]

The readings for a rosette yield
\[
\varepsilon_{ma1} = \varepsilon_{a1} \left( \frac{1 - \mu_o K}{1 + K} \right), \varepsilon_{ma2} = \varepsilon_{a2} \left( \frac{1 - \mu_o K}{1 + K} \right), \varepsilon_{ma3} = \varepsilon_{a3} \left( \frac{1 - \mu_o K}{1 + K} \right) \quad \text{[8]}
\]

and, in the case of a rectangular rosette as shown in Fig. 1,

\[
\frac{\gamma_m}{2} = \frac{\varepsilon_{ma1} + \varepsilon_{ma3} - \varepsilon_{ma2}}{2} \quad \text{[9]}
\]

"Strain", April, 1971
where \( \gamma_m \) is the modified shear strain on the axes 011 and 013 in Fig. 2. Also in Fig. 2

\[
00_1 = \frac{e_{m1} + e_{m3}}{2} \quad \ldots [10]
\]

With the aid of the abscissa relation [10] for the centre, the relation [9] and the radii relations [4] and [6], the modified apparent strain circle, the true strain circle and the circle of stress can be drawn as shown in Fig. 2.

Analytically we obtain with [7] the expressions shown in equations [11] and [12].

It is obvious from Fig. 2 that the inclination \( \beta \) of the maximum principal stress to any given direction (e.g. that of the axis of gauge 1) is not affected by the cross-sensitivity of the gauges and can be evaluated analytically from

\[
\tan 2\beta = \frac{e_{m1} + e_{m3} - e_{m2}}{e_{m1} - (e_{m1} + e_{m3})/2} \quad \ldots [13]
\]

\[
\tan 2\beta = \frac{2e_{a2} - (e_{a1} + e_{a3})}{e_{a1} - e_{a3}} \quad \ldots [14]
\]

**Conclusion**

The circles, once sketched correctly, can greatly help in visualising the strains and stresses in any direction; analytical expressions like [11] and [12] can easily be written down using the data on the circles and their numerical values can be calculated to any desired arithmetical accuracy consistent with the experimental accuracy.

**References**