Note on the effect of cross-sensitivity in the determination of stress

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The concentric Mohr circles of modified apparent strain, true strain and stress permit quick visualisation and calculation of true strains and stresses from readings of gauges with known cross-sensitivities.

Expressions for the principal stresses determined from readings of bonded wire resistance strain gauges after correction for cross sensitivity of the gauges have been derived analytically in an earlier article in *Strain*[1]. These expressions can also be derived without much effort by consideration of the three concentric circles for modified apparent strain, for true strain and for stress, which can be drawn from the gauge readings in three arbitrary directions in a bi-axial stress field.

Meir and Mehaffey[2] have proved that a circle representing modified readings of strain gauges can be drawn concentric with the true strain circle for a point in a bi-axial stress field. The following proof, somewhat different from that of Meir and Mehaffey, is here presented for the drawing of the two strain circles from gauge readings obtained by means of a strain indicator and of the constants μ_o and K given for individual gauges by the manufacturer.

Notations

ε_a	Apparent strain along the axis of the grid of a resistance strain gauge, as read by a strain indicator
ε_{ma}	Modified apparent strain reading of the gauge
ε_t	True strain along the axis of the gauge
ε_n	True strain transverse to the axis of the gauge
μ_o	Poisson's ratio of the material on which the manufacturer has calibrated the gauge (supplied by the manufacturer)
K	Cross-sensitivity coefficient of the gauge (supplied by the manufacturer)
E	Young's modulus of the material to which the gauge is fixed
μ	Poisson's ratio of the material to which the gauge is fixed
$\sigma_{max}, \sigma_{min}$	Maximum and minimum principal stresses

The strain circles

The true strain ε_t along the axis of the gauge and the apparent axial strain εa read by a strain indicator connected to the gauge are related by[3]

$$\varepsilon_a = \frac{1 + \frac{\varepsilon_n}{\varepsilon_t} . K}{1-\mu_o K} . \varepsilon_t \qquad \ldots.[1]$$

At a point in a bi-axial stress field, the true strain ε_t along an arbitrary direction inclined at an angle α to the algebraically larger principal strain axis through the point can be expressed as

$$\varepsilon_t = A + B \cos 2\alpha \qquad \ldots\ldots[2]$$

where A and B are functions of strain with respect to a pair of orthogonal axes through the point. Similarly

$$\varepsilon_n = A - B \cos 2\,\alpha \qquad \ldots\ldots[3]$$

From [1], [2] and [3]

$$\varepsilon_a \left(\frac{1 - \mu_o K}{1 + K}\right) = A + B' \cos 2\alpha$$

$$\text{where } B' = B\left(\frac{1-K}{1+K}\right) \qquad \ldots.[4]$$

$$\varepsilon_{ma} = \varepsilon_a \left(\frac{1 - \mu_o K}{1 + K}\right) = A + B' \cos 2\alpha \qquad \ldots.[5]$$

[2] and [5] represent concentric circles of radii B and B' related as in [4].

Thus from values of ε_{ma} in three directions, as obtained from a rosette gauge, the modified apparent strain circle can be drawn, and from it the true strain circle.

The stress circle

A stress circle concentric with the true strain circle can be drawn by recalling the well known relations[3]

Radius of stress circle $= \dfrac{1-\mu}{1+\mu}$ times radius of true strain circle. ....[6]

Stress scale $= \dfrac{E}{1-\mu}$ times strain scale ....[7]

The readings for a rosette yield

$$\varepsilon_{ma1} = \varepsilon_{a1}\left(\frac{1-\mu_o K}{1+K}\right),\ \varepsilon_{ma2} = \varepsilon_{a2}\left(\frac{1-\mu_o K}{1+K}\right),$$

$$\varepsilon_{ma3} = \varepsilon_{a3}\left(\frac{1-\mu_o K}{1+K}\right) \qquad \ldots.[8]$$

and, in the case of a rectangular rosette as shown in Fig. 1,

$$\frac{\gamma_m}{2} = \frac{\varepsilon_{ma1} + \varepsilon_{ma3}}{2} - \varepsilon_{ma2} \qquad \ldots.[9]$$

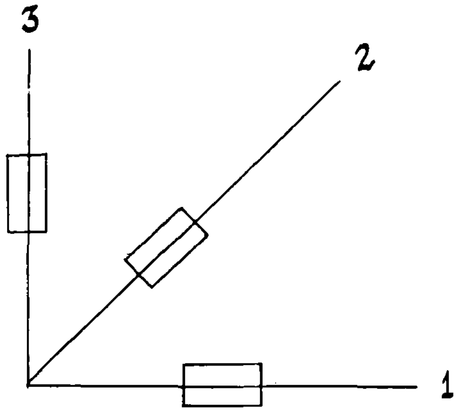

Fig. 1. Orientation of gauges in a rectangular rosette.

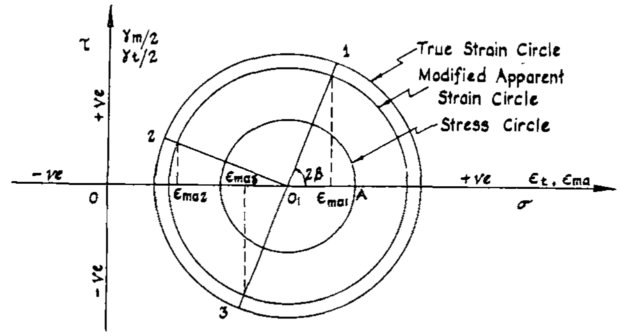

Fig. 2. Circles of modified apparent strain, true strain and stress

where γ_m is the modified shear strain on the axes $0_1 1$ and $0_1 3$ in Fig. 2. Also in Fig. 2

$$00_1 = \frac{\varepsilon_{ma1}+\varepsilon_{ma3}}{2} \qquad \ldots.[10]$$

With the aid of the absicissa relation [10] for the centre, the relation [9] and the radii relations [4] and [6], the modified apparent strain circle, the true strain circle and the circle of stress can be drawn as shown in Fig. 2.

Analytically we obtain with [7] the expressions shown in equations [11] and [12].

It is obvious from Fig. 2 that the inclination β of the maximum principal stress to any given direction (e.g. that of the axis of gauge 1) is not affected by the cross-sensitivity of the gauges and can be evaluated analytically from

$$\tan 2\beta = \frac{\frac{\varepsilon_{ma1}+\varepsilon_{ma3}}{2} - \varepsilon_{ma2}}{\varepsilon_{ma1} - \frac{\varepsilon_{ma1}+\varepsilon_{ma3}}{2}} \qquad \ldots.[13]$$

$$\tan 2\beta = \frac{2\varepsilon_{a2} - (\varepsilon_{a1} + \varepsilon_{a3})}{\varepsilon_{a1} - \varepsilon_{a3}} \qquad \ldots.[14]$$

Conclusion

The circles, once sketched correctly, can greatly help in visualising the strains and stresses in any direction; analytical expressions like [11] and [12] can easily be written down using the data on the circles and their numerical values can be calculated to any desired arithmetical accuracy consistent with the experimental accuracy.

References

1. E. M. Beaney, "The effect of cross-sensitivity on the determination of stresses by strain gauges'. *Strain*, 1970, **6**, 2.
2. J. H. Meir and W. R. Mehaffey, 'Electronic computing apparatus for rectangular and equiangular rosettes. *S.E.S.A.* II 1, Addison Wesley Press, Cambridge, Mass., 1944.
3. C. C. Perry and H. R. Lissner, 'The strain gauge primer', McGraw Hill Book Co., New York, 1962.

$$\sigma_{max} = \frac{E}{1-\mu} OA = \frac{E}{1-\mu}\left[\frac{\varepsilon_{ma1}+\varepsilon_{ma3}}{2} + \frac{1-\mu}{1+\mu}\left\{\frac{1+K}{1-K}\sqrt{\left(\frac{\varepsilon_{ma1}-\varepsilon_{ma3}}{2}\right)^2+\left(\frac{\varepsilon_{ma1}+\varepsilon_{ma3}}{2}-\varepsilon_{ma2}\right)^2}\right\}\right]$$

$$\sigma_{max} = \frac{E(1-\mu_o K)}{2}\left[\frac{\varepsilon_{a1}+\varepsilon_{a3}}{(1-\mu)(1+K)} + \frac{1}{(1+\mu)(1-K)}\sqrt{(\varepsilon_{a1}-\varepsilon_{a3})^2 + (\varepsilon_{a1}+\varepsilon_{a3}-2\varepsilon_{a2})_2}\right] \qquad \ldots\ldots[11]$$

$$\sigma_{min} = \frac{E(1-\mu_o K)}{2}\left[\frac{\varepsilon_{a1}+\varepsilon_{a3}}{(1-\mu)(1+K)} - \frac{1}{(1+\mu)(1-K)}\sqrt{(\varepsilon_{a1}-\varepsilon_{a3})^2 + (\varepsilon_{a1}+\varepsilon_{a3}-2\varepsilon_{a2})^2}\right] \qquad \ldots\ldots[12]$$