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Modeling of Wholesale Prices for Selected Vegetables Using Time Series Models in Kenya

By

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Declaration

I, the undersigned, declare that this project is my original work and to the best of my knowledge has not been presented for the award of the degree in any other University.

Signature:………………………….. Date:……………………………….

(I56/79482/2012)

This project has been submitted with my approval as University supervisor.

Signature:………………………….. Date:……………………………….

Dr. Ivivi Mwaniki

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Abstract

Price forecasting is more sensitive with vegetable crops due to their highly nature of perishability and seasonality and is often used to make better-informed decisions and to manage price risk. Further, to improve domestic market potential for smallholder producers, who are the biggest suppliers in the market and in line with the government's Agriculture Sector Development Strategy (ASDS). Three autoregressive models are used to predict and model the wholesale prices for selected vegetables in Kenya shillings per kilogram. The models are; Autoregressive Moving Average (ARMA), Vector Autoregressive (VAR), Generalized Autoregressive Condition Heterostadicity (GARCH) and the mixed model of ARMA and GARCH. This time series data for tomato, potato, cabbages, kales and onions for markets in Nairobi, Mombasa, Kisumu, Eldoret and Nakuru wholesale markets are considered as the classical national average. The result indicates the models are valid in predicting. Based on the model selection criterion the best forecasting models in ARIMA are; Potato ARIMA (1,1,0), Cabbages ARIMA (2,1,2), tomato ARIMA (3,0,1), onions ARIMA (1,0,0), Kales ARIMA (1,1,0) . Further, the mixed model of ARMA (1, 1) and GARCH (1, 1) model is also identified best model in forecasting.

Key words; Tomato, Potato, Cabbages, Kales and Onions

Dedication

This project work is dedicated to; my wife Joyce Kamau, my daughters (Maryann Wanjiku and Terryann Wangui) and my parents; Mr. Peter Gathondu and Mrs. Jane Gathondu for their outstanding support, prayers, encouragement and availing conducive environment during my study.

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Abbreviations

RMSE	Root Mean Square Error
MAE	Mean Absolute Error
FPE	Final Prediction Error
MA	Moving Average
AR	Autoregressive
ARIMA	Autoregressive Integrated Moving Average
ARMA	Autoregressive Moving Average
VAR	Vector Autoregressive
GARCH	Generalized Autoregressive Condition Heterostadicity
AIC	Akaike Information Criterion
ARCH	Autoregressive Conditional Heteroscedastic
SARIMA	Seasonal Autoregressive Integrated Moving Average
ACF	Autocorrelation Function
PACF	Partial Auto Correlation Function
HCDA	Horticulture Crop Development Authority
KHCP	Kenya Horticulture Competitiveness Project
USAID	United States Agency for International Development

Ksh	Kenya shilling
Kg	Kilogram
ASDS	Agriculture Sector Development Strategy
OLS	Ordinary Least Squares
ADF	Augmented Dickey-Fuller
FARIMA	Fractional Autoregressive Integrated Moving Averages
Ha	Hectares
WN	White Noise
ESS	Error Sum of Squares
HQIC	Hannan Quinn Information Criterion
SIC	Schwarz Information criterion
CV	Coefficient of Variation
BIC	Bayesian Information Criterion
MLE	Maximum Likelihood Estimator
DF	Degrees of freedom
STD	Standard deviation

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1.0 Introduction

A time series is the collection of observations made sequentially over time and methods of analyzing this data of time series constitute an important area of statistics (Chatfield, C., 2005). Several objectives for a time series data analysis are classified as explanation, description, control and prediction.

1.1 Back ground

In Kenya, the agriculture sector is the mainstay in the Kenyan economy, contributing 30% of the GDP and accounting for 80% of employment. The total domestic value in the horticulture sector in 2012 amounted to Ksh 217 billion occupying an area of 662,835 ha with a total production quantity of 12.6 million tons. As compared to 2011, the total value, area and production increased by 6%, 9% and 38% respectively (HCDA, 2012). Vegetables contributed 38% to the domestic value of horticulture with 287,000 ha under production and producing 5.3 million tons valued at Ksh 91.3 billion. Production increased by 13% while there was a slight reduction in value by 4% from 2011 levels. The increased production is occasioned by favorable weather conditions that resulting to high yield, thus reducing the value of vegetables. However, there was a drop in prices for commodities like cabbages, tomatoes, kales and carrots thereby reducing the overall value for the year. Vegetable production and consumption is becoming more and more popular due to health concerns and the search for alternative economic opportunities. The fast growing industry consists of a wide array of crops and products including potatoes, tomatoes, onions, cabbage, etc. Together with fruits, vegetables account for about 73% of horticultural retail trades at most retail outlets sales today (USAID-KHCP, June 2012).

1.2 Statement of the problem

In Kenya, there is stiff competition for vegetables between the formal and informal market outlets, yet many smallholder producers sell vegetables at low prices, not competitive enough to ensure positive returns. It is suggested that market information flow between vegetable producers and market actors' impact on the price that farmers receive for their vegetables. Access and use of market information could explain why some are able to sell competitively while others are not. Knowledge of the flow and use of market information on prices would be useful in; designing effective price information, dissemination strategies to help farmers sell their vegetables at profitable prices and realize positive returns. The domestic market faces numerous challenges such as: Inadequate market information for smallholders, insufficient data on market flows, and lack of awareness on the use of available data by actors among others (USAID-KHCP, 2012).

1.3 Overall objectives

To inform on proper planning mechanism of the identified vegetables in stabilizing food security and improved marketing

1.3.1 Study objectives

1. To identify trends relationship and inform on proper planning
2. To compare the different time series models in modeling wholesale prices
3. To analyze seasonal price variation and develop a model to forecast the weekly prices at wholesale level
4. To investigate the performance of parametric models in modeling selected vegetable wholesale prices

1.4 Research questions

1. Does the inequality coefficient used be the selection criteria determine the best model?
2. Does a positive linear trend influence the time series data?
3. Does the seasonal variation have effect on the prices
4. Does the time series data became stationary after the first order differencing?
5. Does the Autocorrelation function (ACF) and the Augmented Dickey-Fuller (ADF) tests show that the time series data was stationary or not stationary?

1.5 Justification:

These crops were selected partly because of their relative importance in the horticulture industry; they constitute 71% of the vegetables produced in Kenya and partly due to data availability. Further, to improve domestic market potential for smallholder producers, who are the biggest suppliers in the market and in line with the government's Agriculture Sector Development Strategy (ASDS), these smallholder producers need to develop a business approach to maintain and grow their market share. The main purpose of agricultural commodity price forecasting is to allow producers to make better-informed decisions and to manage price risk (Ticlavilca et al., (2010)). Price forecasting is more sensitive with vegetable crops due to their highly nature of perish-ability and seasonality (Fenyves et al., 2008).

2.0 Literature Review

In Kenya the increase in the demand for vegetables is driven mainly by increasing wealth and the need of consumers for fiber, low cholesterol, low fat and high vitamins A and C (Love, 1991). On the supply side, the desire for more profits will motivate producers to expand the supply of vegetables. Vegetable prices play a major role in coordinating the supply and demand of these products. Hence, modeling of their prices will be useful to producers, consumers, processors, rural development planners and other people involved in the vegetable market. Fildes & Lusk (1984) considers a range of methods and analyze their comparative performance over a random selection of series. Four different models of univariate time series are compared by Assis. et al,. (2010), namely the exponential smoothing, autoregressive integrated moving average (ARIMA), generalized autoregressive conditional heteroskedasticity (GARCH) and the mixed ARMA and GARCH models.

Dieng, A., (2008) investigates the performance of parametric models in forecasting selected vegetables prices in Senegal and makes recommendations to potential users. In his case two forecasting approaches are used and evaluated using both qualitative and quantitative methods, these consist of parametric and non-parametric models. The study finding suggested that, among the parametric models ARIMA model is a worthy technique to use in generating price models for both producers and consumers. However, the study recommended additional research to test the forecasting accuracy of parametric against non-parametric models with respect to other crops.

Moghaddasi,R and Badr, B.R (2008) in their economic model paper aimed to assess the statistical accuracy of alternative wheat prices forecast over the last 40 years. The model performance was evaluated and compared using common criteria’s such as; root mean square error (RMSE), mean absolute error (MAE), mean absolute percentage error (MAPE) and their inequality coefficient. The study findings revealed the supremacy of time series models (unit root and ARIMA (3, 2, 5)) in predicting wheat prices.

Structure models emphasize the importance of explanatory variables describing the peculiar characteristics of the commodity market. Bourke (1979) study expanded the analysis of Beef forecasting model by applying both Box-Jenkins methodology and structural techniques in generating prediction. He indicated that for the period of 1966 to 1975, more accurate quarterly and monthly price projection can be obtained using the Box-Jenkins approach than those derived from econometric structure methods. The absolute and relative accuracy evaluation procedures have been the two major approaches used in forecast evaluation by Moghaddasi, R and Badr, B.R (2008). Absolute accuracy of forecast is evaluated using statistical measures describing the differences between the predicted and actual realized values. Empirical results on structural models are estimated using ordinary least squares (OLS).

By performing unit root tests and rejection of the null hypothesis of a unit root under both the ADF test and the Phillips-Perron (PP) test is taken as evidence of stationary. ARIMA time series models outperformed the structural models in prediction when using historical information than developing an econometric structure. Assis et al,.(2010) compared different univariate time series models in forecasting monthly average cocoa beans prices namely the exponential smoothing, ARIMA, GARCH and the mixed ARIMA & GARCH models. RMSE, MAPE, MAE and their inequality coefficient were used as the selection criteria to determine the best model. The study revealed the time series data was influenced by a positive linear trend factor whereas a regression test showed non-existence of seasonal factors.

The wholesale prices for vegetables are characterized by large seasonal variation the degree and timing of the changes are different (Fenyves et al., 2008). Due to the price fluctuations, vegetable producers normally have large losses, therefore the adaption of production to seasons, market research and technological development should all be improved. Among the seasonal decomposition models of forecasting, Seasonal Autoregressive Integrated Moving average (SARIMA) method could enable producers achieve better market positions by adopting the practice. The SARIMA model is an extension of the ARIMA model into capturing both seasonal and non-seasonal behaviour of a time series data (Sampson et al., 2013).

2.1 Types of time series modeling methods

a) **Univariate modeling method:** This is a type of modeling which generally uses only time as an input variable with no other outside explanatory variable.(Celia et al., 2003).Some of the few employed methods are exponential smoothing, ARIMA and Autoregressive Conditional Heteroscedastic (ARCH) (Elham et al., 2010).

b) **Multivariate modeling method:** When two or more variables are used to measure a person, place or thing. Variables may or may not be dependent on each other.

Fatimah and Roslan (1986) confirmed the suitability of univariate models in agriculture prices forecasting whereas Mad Nasir (1992) noted, the ARIMA models have the advantage of relatively low research cost when compared with econometric/structural models. ARCH is one of the initial time series models allowing for heteroscedasticity as introduced by Engle (1982). The idea was extended by Bollerslev (1986) into GARCH which gives more careful results than ARCH models.

Zhou et al., (2006) proposed a new traffic network prediction model based on non-linear time series mixed model of ARIMA and GARCH. They established that, the proposed mixed models outperformed the existing Fractional Autoregressive Integrated Moving Averages (FARIMA) model in terms of prediction and accuracy. Therefore the objective of this research is to compare the modeling performance of the different time series methods in modeling wholesale prices for Potato, cabbages, onions, tomatoes and kales in Kenya.

3.0 Methodology

3.1 Data Overview

The wholesale price data is gathered from the Ministry of Agriculture, Livestock and Fisheries (MALF) in the agribusiness department which was collected by extension officers in the various wholesale markets. The data was available on weekly prices and covered the four year period from 2010 to 2013. Under this study, the average wholesale prices for five Nairobi, Mombasa, Kisumu, Eldoret and Nakuru markets are considered as the classical national average. The time series data is measured in Kenya shillings per Kilograms (Ksh/Kg) and the data ranged from week 1 in January 2010 until week 208 in December 2013.

3.2 Methods

The study follows the Box-Jenkins (1970) methodology for modeling, generally known as ARIMA model by PadhanPurna Chandra (2012) in the Journal of Agriculture and Social Science. Let $\boldsymbol{Y}_t$ be a discrete time series variable which takes different variable over a period of time. The corresponding AR (p) model of Y_t series, which is the generalizations of the autoregressive model, is expressed as;

$$\text{AR(p); } Y_t = \theta_0 + \theta_1 Y_{t-1} + \theta_2 Y_{t-2} + \ldots\ldots \theta_p Y_{t-p} + \varepsilon_t \tag{3.1}$$

Where Y_t is the response variable at time t, $Y_{t-1}, Y_{t-2}, \ldots\ldots Y_{t-p}$ are the respective variables at different time lags; $\theta_0, \theta_1, \ldots\ldots \theta_p$ are the coefficients and ε_t is the error factor. Similarly, the MA (q) model which is the generalization of the moving average model is specified as;

$$\text{MA (q); } Y_t = \mu_t + \varepsilon_t + \sigma_1 \varepsilon_{t-1} + \ldots\ldots\ldots \sigma_q \varepsilon_{t-q} \quad , \varepsilon_t \sim WN(0, \sigma_t^2) \tag{3.2}$$

Where, μ_t is the constant mean of the series, $\sigma_1, \sigma_2, \ldots\ldots, \sigma_q$, the coefficients of the estimated error term and εt is the error term.

When (Y_t) in the data is replaced with $(\Delta Y_t = Y_t - Y_{t-1})$, then the ARMA models become the **ARIMA (p,d,q)** models, where **p** is order of autocorrelation (Indicates weighted moving average over past observations), **d** is order of integration (differencing) and **q** is order of moving averaging. By combining the models in (3.1) and (3.2), this is referred as ARMA model, which have the general form of;

$$Y_t = \theta_0 + \theta_1 Y_{t-1} + \theta_2 Y_{t-2} + \ldots\ldots + \theta_p Y_{t-p} + \varepsilon_t + \sigma_1 \varepsilon_{t-1} + \ldots + \sigma_q \varepsilon_{t-q} \tag{3.3}$$

If Y_t is stationary at level or I(0) or at first difference I(1) then this determines the order of integration. To identify the order of p and q the ACF and PCF is applied.

3.3 Stationary versus non-stationary

ARIMA model is generally applied for stationary time series data. The stationary and non-stationary properties are checked by applying ADF test and the results are estimated with the first difference. The ADF statistic is a negative number and the more negative it is, the stronger the rejection of the hypothesis that there is a unit root at some level of confidence.

ARIMA forecasting model involves four different but interrelated steps as;

a) **Identification**: The first step of applying the model is to identify appropriate order of ARIMA (p,d,q) model. Identification of ARIMA model involves selection of order of AR(p), MA(q) and I(d). The order of d is estimated through I(1) or I(0) process.

The model specification and selection of order p and q involves plotting of ACF and partial PACF or correlogram of variables at different lag length.

The significance level of individual coefficients is measured by Box-Pierce Q statistics and jointly together by Ljung-Box LB statistics. The Box-Pierce Q statistics is defined as;

$$Q = \sum_{k=1}^{m} \tilde{\rho}_k^2 \sim \chi_m^2 \qquad (3.4)$$

And Ljung Box (LB) Statistics is defined as

$$LB = n(n+2)\sum_{k=1}^{m} \frac{\tilde{\rho}_k^2}{n-k} \sim \chi_m^2 \qquad (3.5)$$

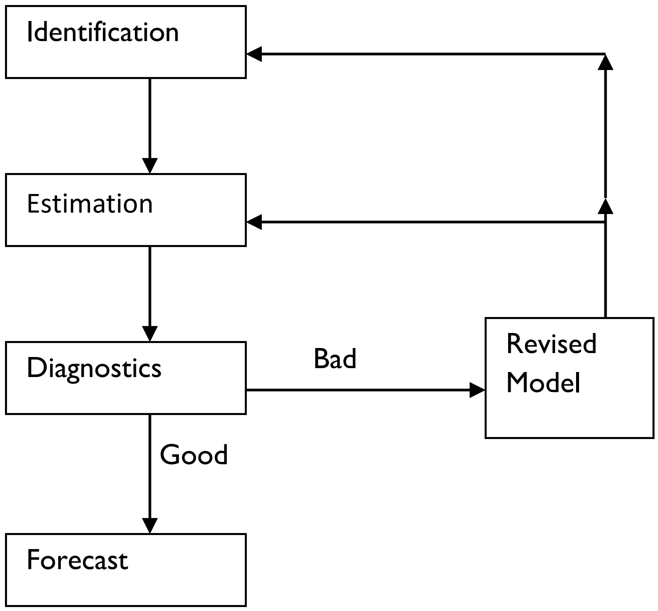
Where n=sample size and m is lag length.

b) **Model estimation**: Once the order of p, d and q are identified, next step is to specify appropriate regression model and estimate. With the help of R software various order of ARIMA model has been estimated to arrive at the optimal model. For instance by ARIMA (2,1,1) it means the series is stationary at first difference and follows AR (2) and MA (1) process.

c) **Diagnostic checking**: According to PadhanPurna Chandra (2012) this is done by checking on the residual term obtained from ARIMA model by applying ACF and PACF functions.

d) **Forecasting**: Forecasted values are obtained by estimating appropriate model

3.4 ARIMA model frame work



The coefficient of variation (V) is used to measure the index of instability of the time series data as defined by

$$V = \frac{\sigma}{\overline{Y}} \tag{3.6}$$

Where σ is the standard deviation, and

$$\overline{Y} = \frac{1}{n}\sum_{t=1}^{n} Y_t \quad \text{, is the mean change in price} \tag{3.7}$$

A complete stable data has **V=1**, but unstable data is characterized by **V<1** (Telesca et al., (2008)). Regression analysis is used to test whether the trends and seasonal factors exist in the time series data.

3.5 Regression model

The existence of linear trend factors was tested through the regression equation.

$$Y_t = \beta_0 + \beta_1 t + \varepsilon_t \quad \text{, where } \varepsilon_t \sim WN(0, \sigma^2) \tag{3.8}$$

Where, Y_t is the various time series data of the study, t is the linear trend factor of time, β_0 & β_1 are the parameters and ε_t is the error of the model with an assumption of white noise (WN).

The hypotheses of the model are

$H_0 : \beta_1 = 0$ (Non-existence of linear trend factor)

$H_1 : \beta_1 \neq 0$ (Linear trend factor exists)

3.6 SARIMA Model

SARIMA model takes into account the seasonal characters of the time series data (Fenyves et al., 2008). The model is used in the analysis of stochastic but not stationary time series and complements ARIMA models. The model is useful when the time series data exhibit seasonality-periodic fluctuations that recur with about the same intensity each year (Martinez,et al., 2011).The SARIMA model is denoted by;

$\mathrm{ARIMA(p, d, q)} \times \mathrm{(P, D, Q)S}$, as written in lag form by (Halim and Bisono, 2008)

$$\begin{aligned}
&\phi(B)\Phi(B^s)(1-\mathrm{B})^d(1-B^s)^D Y_t = \theta(B)\Theta(B^s)\varepsilon_t \\
&\phi(B) = 1-\phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p \\
&\phi(B^s) = 1-\phi_1 B^s - \phi_2 B^{2s} - \ldots - \phi_p B^{ps} \\
&\theta(B) = 1-\theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q \\
&\Theta(B^S) = 1-\theta_1 B^S - \theta_2 B^{2S} - \ldots - \theta_p B^{QS}
\end{aligned} \tag{3.9}$$

With p= non-seasonal AR order, d= non-seasonal differencing, q=non-seasonal MA order, P=seasonal AR order, D=seasonal differencing, Q= seasonal MA order, and S= time span of repeating seasonal pattern (in a weekly data s=52).

Y_t , represents the time series data at period t, B , represent backward shift operator

ε_t , represents white noise error at period t

To avoid fitting an over parameterized model, the Akaike Information criterion (AIC) is employed in selecting the best model (Sampson et al., 2013). The model with the minimum values of AIC is considered as the best. In addition, RMSE, MAE and MAPE are employed for comparison of the best models selected.

3.7 Vector Auto-Regression (VAR)

The vector auto-regression (VAR) model proposed as proposed by Sims (1980) is one of the successful, flexible, and easy to use models in the analysis of multivariate time series. The model often provides superior models to those from univariate time series models and is based on elaborate theories of simultaneous equations. VAR model extends the univariate autoregressive (AR) model to dynamic multivariate time series by allowing more than one evolving variable (Zhang Haonan 2013).All variables in a VAR model are treated symmetrically in a structural sense as each variable has an equation explaining its evolution based on its own lags and the lags of the other model variables (Walter, 2003).

Let $Y_t = (y_{1t}, y_{2t}, ... y_{nt})'$ denote an ($n \times 1$) vector of time series variables then a VAR model with p lags is then expressed as follows:

$$Y_t = c + \pi_1 Y_{t-1} + \pi_2 Y_{t-2} + \pi_3 Y_{t-3} + ... + \pi_p Y_{t-p} + \varepsilon_t, \; t = 1, 2, ..., \mathrm{p} \qquad (3.10)$$

Where π_i is a $(n \times n)$ coefficient matrix, ε_t is an $(n \times 1)$ unobservable zero mean white noise vector process i.e. $((E(\varepsilon_t = 0))$, and c is an $(n \times 1)$ vector of constants (intercepts)

3.8 GARCH Model

The natural frequency of data to feed a GARCH estimator is daily data, weekly or monthly data may be used. There is volatility throughout the day and highly depends on the particular market where the trading happens, and possibly on the specific asset. The estimation of the model is mostly about estimating how fast the decay happens. The standard structure of a GARCH (p, q) models is specified by the three equations as stated by Assis et al,.(2010);

$$Y_t = x_t \gamma + \varepsilon_t \tag{3.11}$$

$$\varepsilon_t = V_t \sqrt{\sigma_t^2}, \quad \text{where } V_t \text{ is i.i.d. N (0, 1)} \tag{3.12}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \tag{3.13}$$

With $\alpha_0 > 0, \alpha_1, \beta_1 \geq 0; and \alpha_1 + \beta_1 < 1$

Where; p is the order of GARCH term, q is the order of ARCH term, and $\sigma_t^2 = 1$. Equations (3.11) and (3.13) respectively are called mean equation and conditional variance equation. The mean equation is indicated as a function of exogenous variables (x_t) with an error term (μ_t). The variance equation is a function of mean (ω), ARCH (μ_{t-i}^2) and GARCH term (σ_{t-j}^2), where (ω, α, β) are the parameters of the process.

3.9 The mixed model ARIMA and GARCH

The combination of ARIMA (p,d,q) and GARCH (p,q) is written as;

$$(\Delta Y_t)^d = \sum_{i=1}^{p} \phi_i (\Delta Y_{t-i})^d + \varepsilon_t + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j}, \qquad (3.14)$$

$$\sigma_t^2 = \omega + \sum_{j=1}^{q} \beta_j \varepsilon_{t-j}^2 + \sum_{i=1}^{p} \alpha_i \sigma_{t-i}^2 \qquad (3.15)$$

The multivariate forecasting approach is used to generate samples of forecast models as suggested by Ramanathan (2002) where the model selection criterion are used to choose the best models.

3.10 Model selection criterion

Model selection criteria allow the best model to be fit the data by striking a balance and finding a model that neither under-fits nor over-fits the data. This is the motivation for model selection criteria (Burnham & Anderson, 2002).

A. Akaike's Information Criterion (AIC)

The intent of AIC is to measure the mathematical distance between the true population and the fitted model, by using the so called Kullback-Leibler discrepancy. To differentiate between models with different numbers of parameters, AIC adds two times the number of model parameters to the estimated Kullback-Leibler discrepancy. Thus, when two models of differing complexity fit a data set equally well; AIC chooses the simpler model by penalizing the complex model for having more model parameters. The rule of parsimony advice a researcher to choose the simplest model that adequately describes the behavior of the population. Use of AIC generally supports this rule (Gosky, R and Gosky, S, 2009).

The AIC function can be defined as follows;

$$AIC = \left(\frac{ESS}{n}\right) e^{2f/n} \tag{3.16}$$

Where;

n=Number of observations

f=Number of parameters and

ESS=Error sum of square.

Or; $AIC = -2\log(L) + 2(\mathrm{p}+\mathrm{q})$ (3.17)

Where (L) indicates the likelihood of the data with a certain model, p and q indicate the lag orders of AR and MA term (Zhang Haonan, 2013).

B. **Final Prediction Error(FPE)** is defined as;

$$FPE = \left(\frac{ESS}{n}\right)\frac{n+f}{n-f} \tag{3.18}$$

C. **Generalized Cross Validation(GCV)** defined by;

$$GCV = \left(\frac{ESS}{n}\right)\left[1-\left(\frac{f}{n}\right)\right]^{-2} \tag{3.19}$$

GCV estimates prediction error, but does not control the probability of selecting irrelevant predictors of the target variable

D. **Hannan–Quinn Information Criterion (HQIC)** stated as;

$$HQ = \left(\frac{ESS}{n}\right)(\ln n)^{2f/n} \tag{3.20}$$

E. **Schwarz Information Criterion [Schwarz, 1978] - (SIC)** stated as;

$$SIC = \left(\frac{ESS}{n}\right) n^{f/n} \tag{3.21}$$

3.11 Model forecast Accuracy Criteria

A. **Root mean square error** (RMSE) defined as;

$$RMSE = \sqrt{\frac{ESS}{n}} \tag{3.22}$$

This measures the "mean prediction error", "Perfect" fit (RMSE=0). The Root Mean Square Error (RMSE) (also called the root mean square deviation, RMSD) is frequently used to measure the difference between values predicted by a model and the values actually observed from the environment that is being modeled. These individual differences are also called residuals.

Predicted errors/residue= Actual value-Predicted value

B. MAE-mean absolute error

$$MAE = \frac{\sum_{t=1}^{n} \left| Y_t - \hat{Y}_t \right|}{n} \tag{3.23}$$

C. MAPE-mean absolute Percentage Error

$$MAPE = \frac{\sum_{t=1}^{n} \left| \frac{Y_t - \hat{Y}_t}{Y_t} \right|}{n} \times 100\% \tag{3.24}$$

Where,

Y_t =the actual observation value at time t

$\hat{Y}_t$ = the forecast observation value at time t

n = the total number of observations and

ESS = the error sum of squares

4.0 Data Analysis and Results

The wholesale price data used in the analysis is on weekly basis and covered the four year period for potato, cabbages, tomato, onions and kales. The average price for Nairobi, Mombasa, Kisumu, Eldoret and Nakuru markets is considered as the classical national average for each of the vegetables.

4.1 Testing for normality of the data -QQ-Norm

From the overall data results only potato data seems to be normal

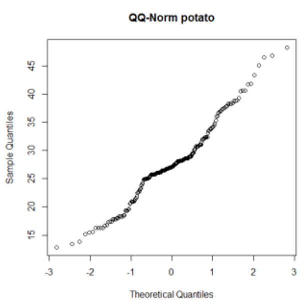


Figure 1: QQ-Norm Potato prices

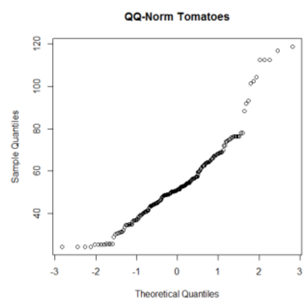


Figure 3: QQ-Norm Tomato prices

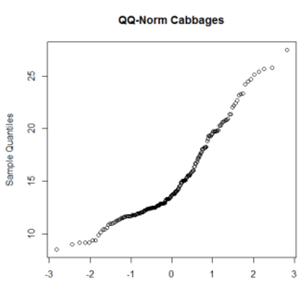


Figure 2: QQ-Norm Cabbage prices

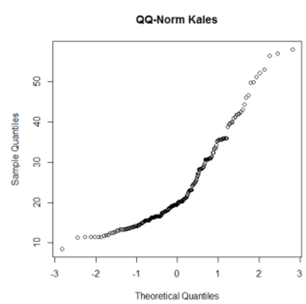


Figure 4: QQ-Kale prices

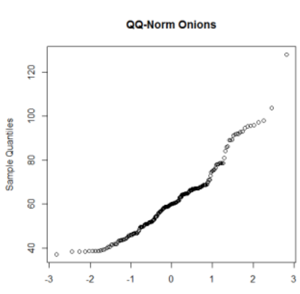
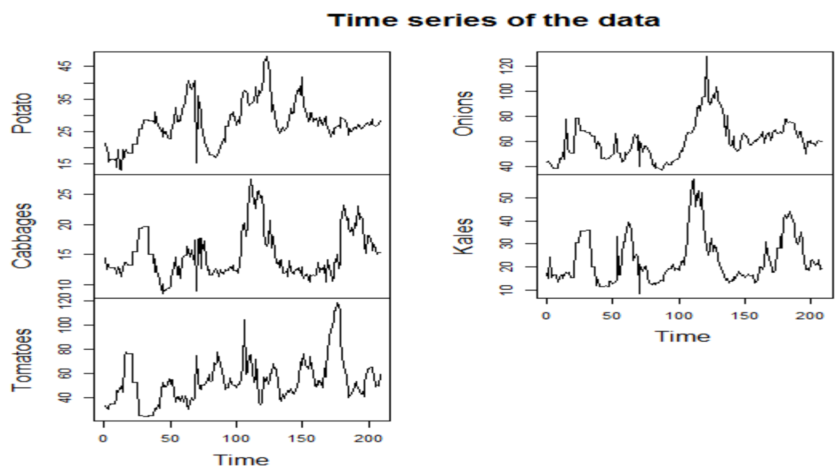


Figure 5**:** QQ-Norm onions prices

4.2 Plotting of the time series- Overall observations



From the plotted results, the time series data is volatile and not stable

Figure 6: Time series for the data

4.3 Testing normality Transformed data

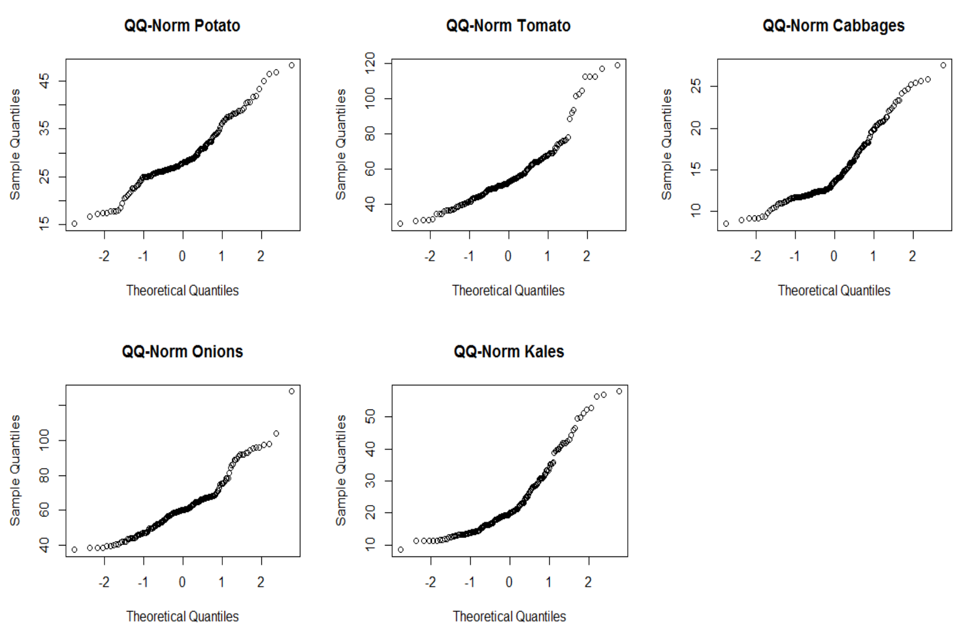


Figure 7: Testing normality of the data

After transforming the data through truncation of the first thirty seven (37) observations only potato data looked to be normal.

4.4 Regression model

The existence of linear trend factors was tested by fitting a Linear Regression

a. Potato

The model equation was presented as; Price (Ksh/Kg) = 24.018+0.035 time (weeks). This means for every one unit increase in time (week), an average prices increase by Ksh 0.035 and from the results of adjusted R-squared only 9.2% of the data is explained by the model thus regression model is not the best fit.

b. Cabbages

The model equation was presented as; Price (Ksh/Kg) = 13.373+0.016 time (weeks). This means for every one unit increase in time (week), an average prices increase by Ksh 0.016,from the results of adjusted R-squared only 5.2% of the data is explained by the model thus regression model is not the best fit.

c. Tomato

The model equation was presented as; Price (Ksh/Kg) = 40.735+0.124 time (weeks). This means for every one unit increase in time (week), an average prices increase by Ksh 0.124, from the results of adjusted R-squared, only 16.85% of the data is explained by the model thus regression model is not the best fit.

d. Onions

The model equation was presented as; Price (Ksh/Kg) = 51.863+0.089 time (weeks). This means for every one unit increase in time (week), an average prices increase by Ksh 0.089, and from the results of adjusted R-squared, only 11.97% of the data is explained by the model thus regression model is not the best fit.

e. Kales

The model equation was presented as; Price (Ksh/Kg) = 20.021+0.033 time (weeks). This means for every one unit increase in time (week), an average prices increase by Ksh 0.033 and from the results of the adjusted R-squared, only 3.11% of the data is explained by the model thus regression model is not the best fit.

4.5 Data summary and descriptive analysis

Table 1: Overall Summary-208 weeks

Variable	**Potato**	**Cabbages**	**Tomatoes**	**Onions**	**Kales**
Min	12.86	8.53	24.28	37.15	8.47
1st Qu	24.90	12.21	43.27	50.60	15.90
Median	27.14	13.71	51.22	59.95	19.59
Mean	27.69	15.00	53.69	61.14	23.48
3rd Qu	30.93	17.41	63.50	67.42	29.97
Max	48.20	27.49	118.68	128.00	58.00
variance	46.07	15.37	322.92	230.54	110.58
S.t.d	6.79	3.92	17.97	15.18	10.52
C.V	0.25	0.26	0.33	0.25	0.45
Model	ARIMA(2,1,0), ARIMA(1,1,1)	ARIMA (2,1,1)	ARIMA (1,1,6)	ARIMA (2,1,3)	ARIMA (1,1,13)

For all the five vegetables in the data, stationary of the data was achieved on the first differencing, potato data generates two models i.e ARIMA (2,1,0), ARIMA (1,1,1) whereas for Kales the model generated is ARIMA (1,1,13) of which the researcher beliefs that 13 variables are too many, thus the desire to transform the data.

Table 2: Overall Summary-Transformed data

Variable	Potato	Cabbages	Tomato	Onions	Kales
Min	15.18	8.53	28.72	37.15	8.47
1st Qu	25.84	12.00	45.28	51.63	15.85
Median	27.82	13.73	52.48	60.15	20.00
Mean	29.15	15.02	56.09	62.03	23.67
3rd Qu	32.28	17.54	63.80	67.46	28.84
Max	48.20	27.49	118.68	128.00	58.0
Var	38.67	17.19	287.78	237.66	119.56
S.t.d	6.23	4.15	16.96	15.42	10.93
C.V	0.21	0.28	0.30	0.25	0.46
Model	ARIMA (1,1,0)	ARIMA (2,1,2)	ARIMA (1,1,6)	ARIMA (2,1,3)	ARIMA (1,1,0)

Data transformation is achieved by truncating the first 37 observations that were missing most of the first observations. Based on ARIMA models and comparing the two data sets, the transformed data seems to give the best models estimate where kales model was identified as ARIMA (1, 1, 0). Using the coefficient of variation (V) to test the stability of the time series data, the results showed that the V value were less than one (V<1), thus this study concludes the time series data is not stable (Telesca et al., 2008).

Table 3: Estimated ARIMA model forecast by Model criterion

Vegetable	**AIC**	**RMSE**	**MAE**	**MAPE**
Potato	1,1,0	3,1,6	3,2,6	3,2,6
Cabbages	2,1,2	2,0,5	2,0,4	2,0,4
Tomato	1,1,6	3,0,1	3,1,6	3,1,6
Onions	2,1,3	3,1,6	1,0,0	1,0,0
Kales	1,1,0	3.0.3	3,2,1	3,1,5

Based on the three model selection criterion (AIC, RMSE, MAE and MAPE), the best fit models are potato ARIMA (1,1,0), cabbages ARIMA (2,1,2), tomato ARIMA (3,0,1), onions ARIMA (1,0,0) and kales ARIMA (1,1,0).

4.6 Testing for stationary of the time series data

The ADF test was conducted to test for the stationary of the data on the selected vegetables using the transformed data and was tabulated as;

Table 4: Testing for stationary of the time series data

Vegetable	**ADF-test**	**Lag order**	**p-value**	**hypothesis(H_1)**	**Decision**
Potato	-3.2247	5	0.086	Stationary	Fail to reject H_0
Cabbages	-2.3719	5	0.421	Stationary	Fail to reject H_0
Tomatoes	-4.4473	5	0.01	Stationary	Fail to reject H_0
Onions	-2.0523	5	0.5544	stationary	Fail to reject H_0
Kales	-2.9501	5	0.1796	stationary	Fail to reject H_0

From the ADF test results, the time series data was not stationary. As the more negative it is, the stronger the rejection of the hypothesis that there is a unit roots at some level of confidence, but after first order differencing was carried out, the data became stationary.

Table 5: Testing for stationary of the time series data after first differencing

Crop	**ADF test**	**Lag order**	**p-value**	**Hypothesis (H_1)**	**Decision**
Potato	-5.2525	5	0.01	stationary	reject H_0
Cabbages	-5.5792	5	0.01	stationary	reject H_0
Tomato	-4.9373	5	0.01	stationary	reject H_0
Onions	-5.3517	5	0.01	stationary	reject H_0
Kales	-4.6186	5	0.01	stationary	reject H_0

4.7 Box-Ljung test & Box-Pierce test

Table 6: Test of independent versus dependent

Crop	Chi-square	df	p-value	Null hypothesis	Decision
Potato	127.786	1	< 2.2e-16	Independent	Reject H_0
Cabbages	138.533	1	< 2.2e-16	Independent	Reject H_0
Tomato	130.577	1	< 2.2e-16	Independent	Reject H_0
Onions	145.809	1	< 2.2e-16	Independent	Reject H_0
Kales	149.302	1	< 2.2e-16	Independent	Reject H_0

Notice, from the results the p-values for the Box-Ljung tests are all below 0.05 indicating significance for the coefficients.

Table 7: Box-Pierce test

Crop	Chi-square	df	p-value	Null hypothesis	Decision
Potato	125.570	1	< 2.2e-16	Independent	Reject H_0
Cabbages	136.130	1	< 2.2e-16	Independent	Reject H_0
Tomatoes	128.312	1	< 2.2e-16	Independent	Reject H_0
Onions	143.280	1	< 2.2e-16	Independent	Reject H_0
Kales	146.713	1	< 2.2e-16	Independent	Reject H_0

Still, from the results, the p-values for the Box-Pierce tests are all well below 0.05 indicating significance for the coefficients

4.8 Fitting the mixed model of ARMA and GARCH

Shifting gears in an attempt to find some sort of pattern in the data by the use of different models to assess the price volatility for accurate measurement and to give reliable forecast. We started to model the conditional volatility as GARCH (1, 1) process since previously has been shown to be parsimonious representation of conditional variances that adequately fits many high-frequency time series (Bollerslev (1987) and Engle (1993)). Four GARCH (p,q) models are selected and compared, namely GARCH(1,1), GARCH(1,2), GARCH(2,1) and GARCH(2,2). Using the model selection criteria suggested by Ramanathan (2002), the GARCH (1, 1) model was selected as the best model in most of the selected vegetables among the other models.

4.9 Estimating the parameters-GARCH Model

1. The mixed model of ARMA (1, 1) & GARCH (1, 1) -Potato

This analysis uses the potato wholesale prices weekly data, on the mixed model of ARMA (1, 1) & GARCH (1, 1). The model is fitted to the time series data using R's garchFit function after installing the fGarch package. The mixed model is fitted, assuming t-distribution errors and the conditional distribution equals the standard deviation ("std") and the results are;

Table 8: Standard errors analysis based on Hessian matrix

Coefficient(s)	Estimate	Std. Error	t-value	Pr(>|t|)
μ	0.766	0.763	1.004	0.315
ϕ	0.976	0.028	35.093	0.000
MA(1)	0.004	0.072	0.050	0.960
ω	2.137	1.194	1.790	0.074
α_1	0.310	0.222	1.396	0.163
β_1	0.468	0.140	3.351	0.001
γ	2.998	0.719	4.169	0.000

In the output, ϕ is denoted by AR1, the mean is mean, and ω is called omega. ϕ =0.976 and is st atistically significant, implying there is a small amount of positive autocorrelation. α_1 =0.310 and is not significant, whereas β_1=0.468 and is highly significant which implies some volatility.

Table 9: Standardized Residuals Tests-Potato

Standardized Residuals Tests		Statistic	p-Value
Jarque-Bera Test	Chi^2	17,315.57	0.000
Shapiro-WilkTest	W	0.634	0.000
Ljung-Box Test	Q(10)	3.872	0.953
Ljung-Box Test	Q(15)	7.433	0.944
Ljung-Box Test	Q(20)	11.502	0.932
Ljung-Box Test for squared residuals	Q(10)	0.209	0.999
Ljung-Box Test for squared residuals	Q(15)	0.352	1.000
Ljung-Box Test for squared residuals	Q(20)	0.457	1.000
LM Arch Test	TR^2	0.250	1.000

In the output garchfit, the normalized log-likelihood is the log-likelihood divided by n, where n=171. The AIC and BIC values have also been normalized by dividing by n. The Jarque-Bera test of normality strongly rejects the null hypothesis that the white noise innovation process { ε_t } is Gaussian thus the t-model is the suitable model. The Ljung-Box test with an R in the second column are applied to the residuals(R=residuals), while the Ljung-Box tests with squared residuals (R^2) are applied to the squared residuals. None of the tests is significant, which indicates that the model fits the data well. The LM ARCH test indicates the same non-significance results.

Table 10: Model information criterion ARMA (p, q) & GARCH (p, q)-Potato

ARMA	GARCH	AIC	BIC	SIC	HQIC
1,1	1,1	4.438	4.567	4.435	4.490
1,2	1,2	4.391	4.557	4.386	4.458
2,1	2,1	4.432	4.597	4.427	4.499
2,2	2,2	4.414	4.616	4.406	4.496

Based on the summarized information criterion statistics for Potatoes, the mixed model of ARMA (1, 2) and GARCH (1, 2) is identified as the best model since they have the lowest results based on AIC, BIC. SIC and HQIC.

2. The mixed model of ARMA (1, 1) & GARCH (1, 1) -Cabbages

Table 11: Standard Errors analysis based on Hessian matrix-Cabbages

Coefficient(s)	Estimate	Std. Error	t-value	Pr(>|t|)
μ	0.780	0.327	2.387	0.017
ϕ	0.939	0.025	38.220	0.000
MA(1)	(0.253)	0.087	(2.892)	0.004
ω	0.311	0.208	1.495	0.135
α_1	0.594	0.272	2.181	0.029
β_1	0.506	0.119	4.259	0.000
γ	3.835	1.216	3.154	0.002

Similarly in the mixed model of ARMA (1, 1) and GARCH (1, 1) model output for cabbages, ϕ is denoted by AR1, the mean is mean, and ω is called omega. $\hat{\phi}$ =0.939 and is statistically significant, implying there is a small amount of positive autocorrelation.

Table 12: Standardized Residuals Tests-Cabbages

Standardized Residuals Tests		Statistic	p-Value
Jarque-Bera Test	Chi^2	63.385	0.000
Shapiro-WilkTest	W	0.956	0.000
Ljung-Box Test	Q(10)	15.600	0.112
Ljung-Box Test	Q(15)	27.284	0.027
Ljung-Box Test	Q(20)	34.052	0.026
Ljung-Box Test for squared residuals	Q(10)	4.075	0.944
Ljung-Box Test for squared residuals	Q(15)	10.726	0.772
Ljung-Box Test for squared residuals	Q(20)	18.815	0.534
LM Arch Test	TR^2	5.212	0.951

Both α_1 and β_1 are statistically significant, where α_1 =0.594, β_1=0.506 and implies persistent volatility clustering. The Jarque-Bera test of normality strongly rejects the null hypothesis that the white noise innovation process { ε_t } is Gaussian thus the t-model is best fitting.

The Ljung-Box test applied to the R^2 residuals indicates none of the tests is significant. Thus the model fits the data well as similarly indicated by LM ARCH test non-significance results.

Table 13: Model information criterion ARMA (p, q) & GARCH (p, q) -Cabbages

ARMA	GARCH	AIC	BIC	SIC	HQIC
1,1	1,1	3.770	3.898	3.767	3.822
1,2	1,2	3.777	3.943	3.772	3.845
2,1	2,1	3.773	3.939	3.768	3.840
2,2	2,2	3.796	3.998	3.788	3.878

Based on the summarized information criterion statistics for cabbages, the mixed model of ARMA (1, 1) and GARCH (1, 1) is identified as the best model as has the lowest results based on AIC, BIC, SIC and HQIC model selection criterion.

Prediction of the mixed ARMA (1, 1) and GARCH (1, 1) models-Cabbages

Table 14: Prediction of mixed ARMA (1, 1) and GARCH (1, 1) models-Cabbages

Time (Week)	Mean Forecast	Mean Error	Standard Deviation
1	15.188	0.927	0.927
2	15.041	1.289	1.121
3	14.903	1.625	1.301
4	14.773	1.951	1.474
5	14.652	2.273	1.644

Based on the prediction results, the model is only able to predict one week ahead with minimum variations.

3. Mixed model of ARMA (1, 1) and GARCH (1, 1) -Onions

Table 15: Standard Errors analysis based on Hessian matrix-Onions

Coefficient(s)	Estimate	Std. Error	t value	Pr(>|t|)
μ	2.786	1.012	2.754	0.006
ϕ	0.957	0.017	55.399	0.000
MA(1)	0.050	0.074	0.676	0.499
ω	13.595	12.701	1.070	0.284
α_1	1.000	0.819	1.222	0.222
β_1	0.454	0.198	2.286	0.022
γ	2.229	0.212	10.512	0.000

In the mixed model of ARMA (1, 1) and GARCH (1, 1) model output for onions, $\widehat{\phi}$ =0.957 & is statistically significant. Implying small amount of positive autocorrelation, α_1 is not statistically significant though β_1 is statistically significant indicating some volatility, α_1 =1.00 and β_1 =0.454.

Table 16: Standardized Residuals Tests-Onions

Standardized Residuals Tests		Statistic	p-Value
Jarque-Bera Test	Chi^2	2,070.39	0
Shapiro-WilkTest	W	0.764	0
Ljung-Box Test	Q(10)	9.785	0.46
Ljung-Box Test	Q(15)	10.946	0.756
Ljung-Box Test	Q(20)	20.923	0.402
Ljung-Box Test for squared residuals	Q(10)	3.425	0.97
Ljung-Box Test for squared residuals	Q(15)	4.27	0.997
Ljung-Box Test for squared residuals	Q(20)	7.394	0.995
LM Arch Test	TR^2	3.524	0.991

The Jarque-Bera test of normality strongly rejects the null hypothesis that the white noise innovation process $\{\varepsilon_t\}$ is Gaussian, thus the t-model is the best fitting. The Ljung-Box test applied to the squared residuals (R^2) indicates none of the tests is significant, which implies the model fits the data well.

Table 17: Model information criterion ARMA (p, q) & GARCH (p, q) -Onions

ARMA	GARCH	AIC	BIC	SIC	HQIC
1,1	1,1	5.750	5.879	5.747	5.803
1,2	1,2	5.741	5.907	5.736	5.808
2,1	2,1	5.752	5.917	5.747	5.819
2,2	2,2	5.762	5.965	5.755	5.844

Based on the summarized information criterion statistics for onions, the mixed model of ARMA (1, 1) and GARCH (1, 1) is identified as the best model since has the lowest results based on AIC, BIC, SIC and HQIC.

Prediction of mixed ARMA (1, 1) and GARCH (1, 1) model- Onions

Table 18: Prediction results of mixed ARMA (1, 1) & GARCH (1, 1)-onions

Time (weeks)	Mean Forecast	Mean Error	Standard Deviation
1	60.111	5.096	5.096
2	60.339	8.814	7.166
3	60.556	12.825	9.393
4	60.765	17.357	11.911
5	60.964	22.579	14.826

Based on the prediction results, the model is only able to predict one week ahead with minimum variation in wholesale price for onions.

4. Mixed model of ARMA (1, 1) and GARCH (1, 1) -Tomatoes

Table 19: Standard Errors analysis based on Hessian matrix-Tomatoes

Coefficient(s)	Estimate	Std. Error	t-value	Pr(>|t|)
μ	4.803	2.391	2.009	0.045
ϕ	0.908	0.047	19.426	0.000
MA (1)	0.043	0.068	0.633	0.527
ω	11.397	11.589	0.983	0.325
α_1	0.162	0.157	1.035	0.301
β_1	0.770	0.184	4.177	0.000
γ	2.686	0.762	3.526	0.000

In the mixed ARMA (1, 1) and GARCH (1, 1) model output for tomatoes, $\hat{\phi}$ =0.908 and is statist ically significant, implying a small amount of positive autocorrelation. α_1 is not statistically sign ificant, β_1 is statistically significant indicating some volatility, with α_1 =0.162 and β_1 =0.770 .

Table 20: Standardized Residuals Tests-Tomatoes

Standardized Residuals Tests		Statistic	p-Value
Jarque-Bera Test	Chi^2	892.143	0.000
Shapiro-WilkTest	W	0.863	0.000
Ljung-Box Test	Q(10)	9.156	0.517
Ljung-Box Test	Q(15)	14.530	0.486
Ljung-Box Test	Q(20)	19.797	0.471
Ljung-Box Test for squared residuals	Q(10)	3.208	0.976
Ljung-Box Test for squared residuals	Q(15)	3.870	0.998
Ljung-Box Test for squared residuals	Q(20)	4.411	1.000
LM Arch Test	TR^2	3.551	0.990

The Jarque-Bera test of normality strongly rejects the null hypothesis that the white noise innovation process $\{\varepsilon_t\}$ is Gaussian thus the t-model is best fit. The Ljung-Box test applied to the squared residuals (R^2) indicates none of the tests is significant, which indicates the model fits the data well.

Table 21: Model information criterion ARMA (1, 1) and GARCH (1, 1) - Tomatoes

ARMA	GARCH	AIC	BIC	SIC	HQIC
1,1	1,1	6.784	6.913	6.781	6.836
1,2	1,2	6.759	6.924	6.754	6.826
2,1	2,1	6.793	6.958	6.788	6.860
2,2	2,2	6.782	6.984	6.774	6.864

Based on the summarized information criterion statistics for tomatoes, the mixed model of ARMA (1,2) and GARCH (1, 2) is identified as the best fit model since has the lowest results based on table 21.

Prediction of mixed model ARMA (1, 1) and GARCH (1, 1)-Tomatoes

Table 22: Prediction results of ARMA (1, 1) and GARCH (1, 1)-Tomatoes

Time (weeks)	Mean Forecast	Mean Error	Standard Deviation
1	59.231	8.540	8.540
2	58.604	12.059	8.910
3	58.034	14.552	9.241
4	57.517	16.510	9.539
5	57.046	18.122	9.809

From the results, the mixed model of ARMA (1, 1) and GARCH (1, 1) gives higher deviation in the mean price forecast compared to ARMA (1,2) and GARCH (1, 2) mean prediction which have smaller week to week price deviation.

Prediction of mixed models ARMA (1, 2) and GARCH (1, 2)-Tomatoes

Table 23: Prediction results of ARMA (1, 2) and GARCH (1, 2)-Tomatoes

Time (weeks)	Mean Forecast	Mean Error	Standard Deviation
1	59.402	8.175	8.175
2	60.012	11.355	8.382
3	59.079	14.277	8.900
4	58.280	16.318	9.191
5	57.596	17.951	9.556

5. Mixed model of ARMA (1, 1) and GARCH (1, 1) -Kales

Table 24: Standard Errors analysis based on Hessian matrix-Kales

Coefficient(s)	Estimate	Std. Error	t-value	Pr(>|t|)
μ	1.392	0.485	2.868	0.004
ϕ	0.918	0.028	33.245	0.000
MA (1)	0.062	0.078	0.799	0.424
ω	1.806	1.768	1.021	0.307
α_1	1.000	0.854	1.171	0.241
β_1	0.653	0.100	6.532	0.000
γ	2.238	0.240	9.316	0.000

In the mixed ARMA (1, 1) and GARCH (1, 1) model output for kales, $\widehat{\phi}$ =0.918 and is statistically significant, implying an amount of positive autocorrelation. α_1, is not statistically significant though β_1 is highly significant indicating some volatility, α_1 =1.000 and β_1 =0.653.

Table 25: Standardized Residuals Tests-Kales

Standardized Residuals Tests		Statistic	p-Value
Jarque-Bera Test	Chi^2	7537.387	0.000
Shapiro-WilkTest	W	0.679	0.000
Ljung-Box Test	Q(10)	4.485	0.923
Ljung-Box Test	Q(15)	11.441	0.721
Ljung-Box Test	Q(20)	19.946	0.461
Ljung-Box Test for squared residuals	Q(10)	0.547	1.000
Ljung-Box Test for squared residuals	Q(15)	0.755	1.000
Ljung-Box Test for squared residuals	Q(20)	5.297	1.000
LM Arch Test	TR^2	0.691	1.000

The Jarque-Bera test of normality strongly rejects the null hypothesis that the white noise innovation process $\{\varepsilon_t\}$ is Gaussian thus the t-model is best fitting. The Ljung-Box test applied to the squared residuals (R^2) indicates none of the tests is significant, which indicates that the model fits the data well.

Table 26: Model information criterion for ARMA (p, q) and GARCH (p, q) - Kales

ARMA	GARCH	AIC	BIC	SIC	HQIC
1,1	1,1	4.995	5.123	4.992	5.047
1,2	1,2	4.969	5.134	4.964	5.036
2,1	2,1	4.996	5.162	4.991	5.063
2,2	2,2	4.985	5.187	4.977	5.067

Based on the summarized model information criterion, the mixed model of ARMA (1, 2) and GARCH (1, 2) is identified as the best model fit since has the lowest results based on AIC, SIC and HQIC. However, the mixed model of ARMA (1, 1) and GARCH (1, 1) is identified through BIC, but this study recommends the mixed model of ARMA (1, 2) and GARCH (1, 2) as the best fit in price prediction.

Table 27: Prediction results of ARMA (1, 2) and GARCH (1, 2)-Kales

Time (weeks)	Mean Forecast	Mean Error	Standard Deviation
1	18.873	2.796	2.796
2	18.744	5.149	4.304
3	18.590	7.118	4.909
4	18.447	9.142	5.854
5	18.315	11.254	6.844

From the results, the mixed model of ARMA (1, 2) and GARCH (1, 2) means forecast indicates minimal week to week price deviation.

4.10 Estimating of parameters and model diagnostics-VAR

Although the structure of the VAR model looks very complex, the estimation of the parameters is not difficult and the most common methods used in estimation are; the maximum likelihood estimator (MLE) and ordinary least square estimator (OLS) (Yang & Yuan 1991). Similar to ARIMA a Q test is applied to test whether the residuals of the VAR models are white noise.

Table 28: Estimates of AR models coefficients using OLS for VAR (1)

Variable	dPotato=A	dCabbages=B	dTomatoes=C	dOnions=D	dKales=E
dPotato=A	0.846	(0.097)	(0.003)	(0.004)	0.053
dCabbages=B	(0.022)	0.677	0.007	(0.008)	0.105
dTomatoes=C	0.225	(0.164)	0.857	(0.041)	(0.061)
dOnions=D	0.121	0.132	0.045	0.828	0.087
dKales=E	0.002	0.002	0.060	(0.031)	0.946

In the matrix given on (table 28), across the row we get the coefficients for the vector variables and on the diagonal we have the variances for the vector variables.

The intercepts of the equations are given under one intercept per variable.

Table 29: intercepts for AR models coefficients in VAR (1)

dPotato171=A	dCabbages171=B	dTomatoes171=C	dOnions171=D	dKales171=E
0.006	(0.007)	0.055	(0.079)	(0.001)

In this study, the OLS method where each variable is considered as a linear function of the lag 1 value for each of the five variables in the set is applied. As a regression predictors for each variable and using notation A = de-trended Potato, B = de-trended cabbages, C= de-trended tomato, D= de-trended onions, and C= de-trended kales, the equation for de-trended VAR (1) model becomes;

$\hat{A}_t$ =0.06499+0.846A_ {t-1}-0.097B_{t-1}-0.003C_{t-1}-0.004D_{t-1}+0.053E_{t-1}

$\hat{B}_t$ =-0.007-0.022A_{t-1}+0.677B_{t-1}+0.007C_{t-1}-0.008D_{t-1}+0.105E_{t-1}

$\hat{C}_t$ =-0.055+0.025A_{t-1}-0.164B_{t-1}+0.857C_{t-1}-0.041D_{t-1}-0.061E_{t-1}

$\hat{D}_t$ =-0.079+0.121A_{t-1}+0.132B_{t-1}+0.045C_{t-1}+0.828D_{t-1}+0.087E_{t-1}

$\hat{E}_t$ =-0.001+0.002A_{t-1}+0.002B_{t-1}+0.060C_{t-1}-0.031D_{t-1}+0.946E_{t-1}

Table 30: The variance-covariance matrix of the residuals from the VAR (1)

Variable	dPotato=A	dCabbages=B	dTomatoes=C	dOnions=D	dKales=E
dPotato=A	10.162	2.549	(6.610)	7.818	5.862
dCabbages=B	2.549	3.055	(1.788)	2.835	3.710
dTomatoes=C	(6.610)	(1.788)	65.664	(3.114)	(5.048)
dOnions=D	7.818	2.835	(3.114)	35.520	4.909
dKales=E	5.862	3.710	(5.048)	4.909	15.006

The matrix under (table 30) gives the variance-covariance matrix of the residuals from the VAR (1) for the five variables. The variances are the diagonals of the matrix and could be used to compare this model to higher order VAR. The determinant of the matrix is used to compute the BIC statistic that could be used to compare the fitted model to other models.

Table 31: The standard errors of the AR coefficients VAR (1)

AR coefficients	dPotato=A	dCabbages=B	dTomatoes=C	dOnions=D	dKales=E
dPotato=A	0.050	0.123	0.016	0.022	0.045
dCabbages=B	0.027	0.067	0.009	0.012	0.025
dTomatoes=C	0.127	0.312	0.041	0.056	0.115
dOnions=D	0.094	0.229	0.030	0.041	0.084
dKales=E	0.061	0.149	0.020	0.027	0.055

As with the coefficients, read across rows. The first row gives the standard errors of the coefficients for the lag 1 variables that predict de-trended time potato. The second row gives the standard errors for the coefficients that predict de-trended cabbages. The final row gives the standard errors for the coefficients that predict de-trended kales.

5.0 Conclusions and Recommendations

Using the coefficient of variation (V) to test the stability of the time series data, the results showed that the V values were less than 1 ($V<1$). Thus this study concludes the time series data is not stable.

Based on the model selection criterion the best forecasting model on ARIMA are; Potato ARIMA (1,1,0), Cabbages ARIMA (2,1,2), tomato ARIMA (3,0,1), onions ARIMA (1,0,0), Kales ARIMA (1,1,0) since the models have the list variables.

Of the four GARCH (p, q) models used and compared, GARCH (1, 1) model is selected as the best fitting model when conditioned on t-distribution.

The mixed model of ARMA (1, 1) and GARCH (1, 1) model fitted on the data when conditioned on t-distribution errors is identified as the best fit and further research should be considered in relation to other conditions as required for accurate measures and reliable forecast.

The auto regressive models could be used to model and predict wholesale prices and better assist users and policy makers. Other VAR models need to be explored further on wholesale price modeling and forecasting on the selected vegetables.

Predictions for vegetable prices can be enhanced by combining predictions from different models and would lead to better price information about the commodities in the wholesale markets.

References

Adanacioglu, H. & Yercan, M. (2012). An analysis of Tomato Prices at Wholesale Level in Turkey: an application of SARIMA model. *Custosea Gronegocio Online,* Vol(8), No (4),52-75.

Armstrong, J.S. (1994). The Fertile Field of Meta-Analysis: Cumulative Progress in Agricultural Forecasting. *International Journal of Forecasting, University of Pennsylvania,* Vol(10),147-140.

Assis,K., Amran, A., Remali, Y. &Affendy, H. (2010). A comparison of Univariate Time Series Methods for Forecasting Coco Bean Prices. *Trends Agriculture Economics*, 3(4), 207-215.

Assis, K., Amran, A. & Remali, Y. (2010). Forecasting Coco Bean Prices Using Univariate Time Series Models. *Journal of arts science & commerce,* Vol (1), 71-79.

Cooray, T.M.J.A. (2006). Statistical Analysis and Forecasting of Main Agriculture Output of Sri Lanka: Rule-based Approach.10th *International Symposium*, *University of Moratuwa*, 221.

Dieng, A. (2008). Alternative Forecasting Techniques for Vegetable Prices in Senegal.*Institutsénégalais de recherchesagricoles (ISRA),* Vol (1), No.3, 5-9.

Fahimifard, S. M., Salarpour, M., Sabouhi, M. & Shirzady, S. (2009). Application of ANFIS to Agricultural Economic Variables Forecasting Caste Study: Poultry Retail Price. *Journal of Artificial intelligence*, 2(2):65-72.

Fenyves et al. (2008). Evaluation of Different Predicting Methods in Forecasting Hungarian, Italian and Greek Lamb Prices. *International Conference on Applied Economics* (ICOAE), 315-319.

Huberty, C.J. & Morris, D.J. (1989). Multivariate Analysis Versus Multiple Univariate Analysis. *Psychological Bulletin*, Vol (105), No (2), 302-308.

Kunst,R.M. (2012). Econometric Forecasting. *University of Vienna & Institute for Advanced Studies Vienna*, 41-75.

Lehecka, G.V. (2013). Forecasting Agricultural Commodity Prices Using Combined Forecasts. *Institute for Sustainable Economic Development, University of Natural Resources and Life Sciences,* Vienna, 1-33.

Maindonald, J. H. (2004). Using R for Data Analysis and Graphics Introduction, Code and Commentary. *Centre for Bioinformation Science, Australian National University*, 9-90.

McGuire,M. (2012). Forecasting Class iii Milk Prices in a Volatile Market. *Master Thesis, Colorado State University*

Moghaddasi, R.& Bita. R. B. (2008). An Econometric Model for Wheat Price Forecasting in Iran. *International Conference on Applied Economics (ICOAE),* 671-677.

Nakajima, J. (2011). Time-varying Parameter VAR Model with Stochastic Volatility: An Overview of Methodology and Empirical Applications. *Economist Institute for Monetary and Economic Studies, Bank of Japan*, 107-138.

Padhan, P.C. (2012). Application of ARIMA Model for Forecasting Agricultural Productivity in India. *Journal of Agriculture &Social Sciences, 8: 50-56.*

Pecican, E. S. (2010). Forecasting Based on Open VAR model. *Romanian Journal of Economic Forecasting,* Vol (1), 59-68.

Pfaff,B. (2008).VAR,SVAR and SVEC models: Implementation within R packages vars. *Journal of Statistical Software*,27(4):1-29.

Ramanathan, R. (2002). Introductory to Econometric with Applications, 5th edition. *Thomson learning*, South Carolina, 688.

Ramirez, O. A. & Mohamadou, F. (2003). Forecasting Agricultural Commodity Prices with Asymmetric- Error GARCH Models. *Journal of Agricultural and Resource Economics*, 28(1):71-85.

Ruppert.D. (2011). GARCH models, Statistics and Data Analysis for Financial Engineering, *Springer Texts in Statistics*, 477-499.

Sampson et al. (2013). Proposed Seasonal Autoregressive Integrated Moving Average Model (SARIMA) for Forecasting Rainfall Pattern in the Navrongo Municipality, Ghana. *Journal of Environment and Earth Science, the International Institute for Science, Technology and Education (IISTE*), Vol(3), No(12), 80-86.

Shukla,M. & Sanjay,J. (2013).Applicability of ARIMA Models in Wholesale Vegetable Market;An Investigation. *International Journal of Information Systems and Supply Chain Management*, 6(3), 105-119.

Sims, C. A. (1973). Seasonality in Regression, *Center for Economic Research, Department of Economics, University of Minnesota*, Vol (23), 2-30.

Sims, C.A., Stock, J.H. & Watson, M.W. (1990). Inference in Linear Time Series Models With Some Unit Roots. *Econometrica*, 58(1), 113-144.

Sujjaviriyasup, T. & Pitiruek, K. (2013).Hybrid ARIMA-Support Vector Machine Model for Agricultural Production Planning. *Applied Mathematical Sciences*, Vol (7), No.57, 2833 – 2840.

Taylor, J.W. (2008).Exponentially Weighted Information Criteria for Selecting Among Forecasting Models. *International Journal of Forecasting*, Vol (24):513-524.

Yang, H. & Hu, J. (2013). Forecasting of Fresh Agricultural Products Demand Based on the ARIMA Model. *Advanced Journal of Food Science and Technology*, 5(7), 855-858.

Zhang,H. (2013).Modeling and Forecasting Regional GDP in Sweden Using Autoregressive Models. *Master thesis in micro-data analysis, Dalarna University*, 2-35.

Appendix 1: Linear Regression results in R

Potato

fit=lm(Potato~Time);fit

summary(fit)

Model summary-potato

Coefficients:	Estimate	Std. Error	t-value	Pr(>|t|)
(Intercept)	24.01834	0.899858	26.691	<2.00E-16
Time	0.035133	0.007466	4.706	4.64E-06

Residual standard error: 6.466 on 206 degrees of freedom

Multiple R-squared: 0.09705,   Adjusted R-squared: 0.09267

F-statistic: 22.14 on 1 and 206 DF, P-value: 4.64e-06

####Cabbages

fit=lm(Cabbages~Time);fit

summary(fit)

Model summary-cabbages

Coefficients:	Estimate	Std.Error	t-value	Pr(>|t|)
(Intercept) (Ksh)	13.37277	0.531139	25.178	<2.00E-16
Time (Wks)	0.015539	0.004407	3.526	0.00052

Residual standard error: 3.816 on 206 degrees of freedom

Multiple R-squared: 0.05692,   Adjusted R-squared: 0.05234

F-statistic: 12.43 on 1 and 206 DF, P-value: 0.0005198

####Tomatoes

fit=lm(Tomatoes~Time);fit

summary(fit)

Model summary-Tomatoes

Coefficients:	Estimate	Std.Error	t value	Pr(>|t|)
(Intercept)-Ksh	40.73457	2.28061	17.861	<2.00E-16
Time (Wk)	0.124	0.01892	6.553	4.44E-10

Residual standard error: 16.39 on 206 degrees of freedom

Multiple R-squared: 0.1725,   Adjusted R-squared: 0.1685

F-statistic: 42.94 on 1 and 206 DF, P-value: 4.439e-10

####Onions

fit =lm(Onions ~ Time);fit

summary(fit)

Model summary-Onions

Coefficients:	**Estimate**	**Std.Error**	**t value**	**Pr(>|t|)**
(Intercept)-Ksh	51.86252	1.98269	26.158	<2.00E-16
Time (week)	0.08881	0.01645	5.399	1.84E-07

Residual standard error: 14.25 on 206 degrees of freedom

Multiple R-squared: 0.124, **Adjusted R-squared:** 0.1197

F-statistic: 29.15 on 1 and 206 DF, P-value: 1.839e-07

Kales regression model

fit=lm(Kales ~ Time)

summary(fit)

Model summary-Kales

Coefficients	**Estimate**	**Std. Error**	**t value**	**Pr(>|t|)**
(Intercept)-Ksh	20.02094	1.44059	13.898	<2e-16
Time (Weeks)	0.03305	0.01195	2.765	0.0062

Residual standard error: 10.35 on 206 degrees of freedom

Multiple R-squared: 0.03579, **Adjusted R-squared:** 0.03111

F-statistic: 7.647 on 1 and 206 DF, P-value: 0.006203

Appendix 2: Results of mixed models ARMA (p, q) and GARCH (p, q)-Potato in R

######## Potato

ARMA(1, 1) + GARCH(1, 1)

Call:garchFit(formula =~arma(1, 1)+garch(1, 1), data = Potato171,cond.dist= "std")
Mean and Variance Equation:data~arma(1, 1) + garch(1, 1)[data = Potato171]
Conditional Distribution:std
Coefficient(s):
muar1 ma1 omega alpha1 beta1 shape
0.7662861 0.9762901 0.0035909 2.1365760 0.3099668 0.4683308 2.9979295
Std. Errors: based on Hessian
Error Analysis:
Estimate Std. Error t value Pr(>|t|)
mu0.766286 0.763285 1.004 0.315412
ar1 0.976290 0.027820 35.093 < 2e-16 ***
ma1 0.003591 0.071616 0.050 0.960010
omega2.136576 1.193892 1.790 0.073520 .
alpha1 0.309967 0.222080 1.396 0.162791

beta1   0.468331   0.139739   3.351 0.000804 ***
shape2.997930   0.719055   4.169 3.06e-05 ***
Signif.codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Log Likelihood:
-372.4515   normalized: -2.178079
Standardized Residuals Tests:
Statistic p-Value
Jarque-Bera Test   R   Chi^2 17315.57 0
Shapiro-WilkTest R   W   0.6337871 0
Ljung-Box Test   R   Q(10) 3.87224   0.9529264
Ljung-Box Test   R   Q(15) 7.433125 0.9444985
Ljung-Box Test   R   Q(20) 11.50183 0.9321566
Ljung-Box Test   R^2 Q(10) 0.2091511 0.9999999
Ljung-Box Test   R^2 Q(15) 0.3518208 1
Ljung-Box Test   R^2 Q(20) 0.4574722 1
LM Arch Test   R   TR^2   0.2502313 1
Information Criterion Statistics:
AIC   BIC   SIC   HQIC
4.438029 4.566635 4.434850 4.490212

################

ARMA(1, 2) + GARCH(1, 2) -Potato

Call:garchFit(formula = ~arma(1, 2) + garch(1, 2), data = Potato171,
cond.dist = "std")
Mean and Variance Equation:data ~ arma(1, 2) + garch(1, 2)[data = Potato171]
Conditional Distribution: std
Coefficient(s):
Muar1   ma1   ma2   omega
1.04080164 0.96749289 0.01581170 0.16995907 3.77977437
alpha1   beta1   beta2   shape
0.36880443 0.00000001 0.28200713 2.67379286
Std. Errors:based on Hessian
Error Analysis:
Estimate Std. Error t value Pr(>|t|)
mu   1.041e+00   8.335e-01   1.249 0.21176
ar1 9.675e-01 3.007e-02   32.174 < 2e-16 ***
ma1   1.581e-02   7.680e-02   0.206 0.83687
ma2   1.700e-01   5.171e-02   3.287 0.00101 **
omega 3.780e+00   2.213e+00   1.708 0.08763 .
alpha1 3.688e-01   2.336e-01   1.579 0.11434
beta1 1.000e-08   NA   NANA
beta2 2.820e-01   NA   NANA
shape 2.674e+00   6.482e-01   4.125 3.71e-05 ***
Signif.codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Log Likelihood:
-366.4559   normalized: -2.143017
Standardised Residuals Tests:
Statistic p-Value
Jarque-Bera Test   R   Chi^2 16237.640
Shapiro-WilkTest R   W   0.6388235 0
Ljung-Box Test   R   Q(10) 4.736877 0.9080477
Ljung-Box Test   R   Q(15) 8.508542 0.9017824
Ljung-Box Test   R   Q(20) 11.98111 0.9167248
Ljung-Box Test   R^2 Q(10) 0.1970646 0.9999999
Ljung-Box Test   R^2 Q(15) 0.3352245 1
Ljung-Box Test   R^2 Q(20) 0.4710161 1
LM Arch Test   R   TR^2   0.2462037 1

Information Criterion Statistics:
AIC BIC SIC HQIC
4.391297 4.556648 4.386118 4.458389
################

ARMA(2, 1) + GARCH(2, 1) -Potato

Call:garchFit(formula = ~arma(2, 1) + garch(2, 1), data = Potato171, cond.dist = "std")
Mean and Variance Equation:
data ~ arma(2, 1) + garch(2, 1)[data = Potato171]
Conditional Distribution:std
Coefficient(s):

mu	ar1	ar2	ma1	omega
0.97112621	0.55179096	0.41805741	0.38250482	2.35070106

alpha1	alpha2	beta1	shape
0.31710291	0.00000001	0.48581711	2.79264513

Std. Errors:based on Hessian
Error Analysis:

	Estimate	Std. Error	t value	Pr(>|t|)	
mu	9.711e-01	1.064e+00	0.913	0.3613	
ar1	5.518e-01	2.344e-01	2.354	0.0186	*
ar2	4.181e-01	2.304e-01	1.814	0.0696	.
ma1	3.825e-01	2.181e-01	1.754	0.0794	.
omega	2.351e+00	1.539e+00	1.528	0.1265	
alpha1	3.171e-01	2.602e-01	1.219	0.2230	
alpha2	1.000e-08	2.468e-01	0.000	1.0000	
beta1	4.858e-01	1.899e-01	2.558	0.0105	*
shape	2.793e+00	7.062e-01	3.954	7.67e-05	***

Signif.codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
Log Likelihood:
-369.9286 normalized: -2.163325
Standardised Residuals Tests:

			Statistic	p-Value
Jarque-Bera Test	R	Chi^2	17222.86	0
Shapiro-WilkTest	R	W	0.634233	0
Ljung-Box Test	R	Q(10)	3.1134	0.9786279
Ljung-Box Test	R	Q(15)	7.0216	0.9570476
Ljung-Box Test	R	Q(20)	11.12929	0.9428004
Ljung-Box Test	R^2	Q(10)	0.2081988	0.9999999
Ljung-Box Test	R^2	Q(15)	0.3532266	1
Ljung-Box Test	R^2	Q(20)	0.4545337	1
LM Arch Test	R	TR^2	0.2523629	1

Information Criterion Statistics:
AIC BIC SIC HQIC
4.431914 4.597264 4.426734 4.499006
################

ARMA(2, 2) + GARCH(2, 2) -Potato

Call:garchFit(formula = ~arma(2, 2) + garch(2, 2), data = Potato171, cond.dist = "std")
Mean and Variance Equation:
data ~ arma(2, 2) + garch(2, 2)[data = Potato171]
Conditional Distribution: std
Coefficient(s):

mu	ar1	ar2	ma1	ma2
1.05883989	0.99999999	-0.03324893	-0.01620395	0.17433963

omega	alpha1	alpha2	beta1	beta2
3.81639632	0.36647021	0.00000001	0.00000001	0.28099856

shape
2.66806007
Std. Errors:based on Hessian
Error Analysis:
Estimate Std. Error t value Pr(>|t|)
mu1.059e+00 8.165e-01 1.297 0.19472
ar1 1.000e+00 4.139e-01 2.416 0.01569 *
ar2 -3.325e-02 4.081e-01 -0.081 0.93507
ma1 -1.620e-02 3.959e-01 -0.041 0.96735
ma2 1.743e-01 7.999e-02 2.179 0.02930 *
omega 3.816e+00 1.305e+00 2.923 0.00346 **
alpha1 3.665e-01 NA NANA
alpha2 1.000e-08 NA NANA
beta1 1.000e-08 NA NANA
beta2 2.810e-01 NA NANA
shape 2.668e+00 3.355e-01 7.953 1.78e-15 ***
Signif.codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
Log Likelihood:
-366.4075 normalized: -2.142734
Standardised Residuals Tests:
Statistic p-Value
Jarque-Bera Test R Chi^2 16120.85 0
Shapiro-WilkTest R W 0.6397832 0
Ljung-Box Test R Q(10) 4.794177 0.9044962
Ljung-Box Test R Q(15) 8.562336 0.8992682
Ljung-Box Test R Q(20) 11.99955 0.9160914
Ljung-Box Test R^2 Q(10) 0.1964181 0.9999999
Ljung-Box Test R^2 Q(15) 0.3347906 1
Ljung-Box Test R^2 Q(20) 0.4727199 1
LM Arch Test R TR^2 0.2454739 1
Information Criterion Statistics:
AIC BIC SIC HQIC
4.414123 4.616218 4.406495 4.496125

Appendix 3: Results of mixed models ARMA (p, q) and GARCH (p, q)--Cabbages in R

> GARCH Modelling- cabbages (1,2)
Call:garchFit(formula = ~arma(1, 2) + garch(1, 2), data = Cabbages171,
cond.dist = "std")
Mean and Variance Equation:
data ~ arma(1, 2) + garch(1, 2)[data = Cabbages171]
Conditional Distribution:std
Coefficient(s):
mu ar1 ma1 ma2 omega alpha1
0.888493 0.930356 -0.254105 0.087661 0.342328 0.608498
beta1 beta2 shape
0.443189 0.054024 3.584554
Std. Errors:based on Hessian
Error Analysis:
Estimate Std. Error t value Pr(>|t|)
mu 0.88849 0.38330 2.318 0.02045 *
ar1 0.93036 0.02865 32.478 < 2e-16 ***
ma1 -0.25410 0.09574 -2.654 0.00795 **
ma2 0.08766 0.08552 1.025 0.30533
omega 0.34233 0.24712 1.385 0.16597

```
alpha1   0.60850     0.31884    1.908  0.05633 .
beta1    0.44319     0.38008    1.166  0.24360
beta2    0.05402     0.25889    0.209  0.83470
shape    3.58455     1.10235    3.252  0.00115 **
Signif.codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
Log Likelihood:
 -313.9703    normalized:  -1.836084
Standardised Residuals Tests:
                                Statistic p-Value
Jarque-Bera Test   R    Chi^2  83.05831  0
 Shapiro-WilkTest  R    W      0.949983  9.454504e-06
Ljung-Box Test     R    Q(10)  14.41095  0.1550584
Ljung-Box Test     R    Q(15)  25.97785  0.03825616
Ljung-Box Test     R    Q(20)  31.6296   0.04740863
Ljung-Box Test     R^2  Q(10)  4.385457  0.9282878
Ljung-Box Test     R^2  Q(15)  10.32114  0.7990679
Ljung-Box Test     R^2  Q(20)  18.29202  0.5681784
LM Arch Test       R    TR^2   5.241904  0.9494072
Information Criterion Statistics:
     AIC      BIC      SIC     HQIC
3.777431 3.942782 3.772251 3.844523
################
summary(fit9.42) cabbages (2,1)
Title: GARCH Modelling
Call:garchFit(formula = ~arma(2, 1) + garch(2, 1), data = Cabbages171,
cond.dist = "std")
Mean and Variance Equation:
data ~ arma(2, 1) + garch(2, 1)[data = Cabbages171]
Conditional Distribution:std
Coefficient(s):
mu         ar1         ar2         ma1       omega
1.07474033  0.64705450  0.26849449  0.03367772  0.33956286
    alpha1      alpha2       beta1       shape
0.59830010  0.00000001  0.51010462  3.57183759
Std. Errors:based on Hessian
Error Analysis:
Estimate  Std. Error  t value Pr(>|t|)
mu      1.075e+00   4.844e-01    2.219 0.026506 *
ar1    6.471e-01   1.905e-01    3.396 0.000683 ***
ar2    2.685e-01   1.730e-01    1.552 0.120734
ma1     3.368e-02   2.137e-01    0.158 0.874788
omega  3.396e-01   2.574e-01    1.319 0.187031
alpha1 5.983e-01   3.096e-01    1.932 0.053316 .
alpha2 1.000e-08   2.904e-01    0.000 1.000000
beta1  5.101e-01   1.783e-01    2.861 0.004225 **
shape  3.572e+00   1.093e+00    3.267 0.001087 **
Signif.codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
Log Likelihood:
 -313.611    normalized:  -1.833983
Standardised Residuals Tests:
                                Statistic p-Value
Jarque-Bera Test   R    Chi^2  82.14891  0
 Shapiro-WilkTest  R    W      0.9495606 8.672119e-06
Ljung-Box Test     R    Q(10)  14.81506  0.1389514
Ljung-Box Test     R    Q(15)  25.93842  0.03867493
Ljung-Box Test     R    Q(20)  31.56531  0.04815618
```

```
Ljung-Box Test     R^2  Q(10)  4.091188  0.9431396
Ljung-Box Test     R^2  Q(15)  10.11748  0.8122859
Ljung-Box Test     R^2  Q(20)  18.4934   0.5549401
 LM Arch Test       R    TR^2   5.173797  0.9519205
Information Criterion Statistics:
    AIC      BIC      SIC     HQIC
3.773228 3.938579 3.768049 3.840321
################
>summary(fit9.43) cabbages (2,2)
Title: GARCH Modelling
Call:garchFit(formula=~arma(2,2)+garch(2,2),data=Cabbages171,cond.dist="std")
Mean and Variance Equation: data~arma(2,2)+garch(2,2)[data=Cabbages171]
Conditional Distribution:std
Coefficient(s):
mu          ar1          ar2          ma1          ma2
1.04880827   0.61209487   0.30500099  0.06589968  -0.02541346
omegaalpha1       alpha2       beta1        beta2  shape
0.35885789   0.65095063   0.00000001  0.39052375   0.08076401 3.56474435
Std. Errors:based on Hessian
Error Analysis:
Estimate  Std. Error  t value Pr(>|t|)
mu      1.049e+00   4.779e-01    2.195  0.02818 *
ar1     6.121e-01   3.148e-01    1.944  0.05187 .
ar2     3.050e-01   3.022e-01    1.009  0.31279
ma1      6.590e-02   3.421e-01    0.193  0.84724
ma2    -2.541e-02   1.714e-01   -0.148  0.88211
omega   3.589e-01   4.228e-01    0.849  0.39598
alpha1  6.510e-01   3.459e-01    1.882  0.05985 .
alpha2  1.000e-08   6.627e-01    0.000  1.00000
beta1   3.905e-01   9.481e-01    0.412  0.68040
beta2   8.076e-02   4.696e-01    0.172  0.86346
shape   3.565e+00   1.089e+00    3.273  0.00106 **
Signif.codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Log Likelihood:
 -313.5469    normalized:  -1.833608
Standardised Residuals Tests:
                              Statistic p-Value
Jarque-Bera Test   R    Chi^2  95.20974  0
 Shapiro-WilkTest  R    W      0.9466308 4.811676e-06
Ljung-Box Test     R    Q(10)  15.10953  0.1281192
Ljung-Box Test     R    Q(15)  26.35437  0.03445883
Ljung-Box Test     R    Q(20)  32.09486  0.04229791
Ljung-Box Test     R^2  Q(10)  4.231646  0.9362925
Ljung-Box Test     R^2  Q(15)  10.84993  0.763158
Ljung-Box Test     R^2  Q(20)  19.50121  0.4894939
 LM Arch Test       R    TR^2   5.047417  0.9563764
Information Criterion Statistics:
    AIC      BIC      SIC     HQIC
3.795870 3.997966 3.788242 3.877872
```

Appendix 4: Results of mixed models ARMA (p, q) and GARCH (p, q)-Onions in R

```
>summary(fit9.51) onions (1,2)
Title: GARCH Modelling
```

```
Call:garchFit(formula = ~arma(1, 2) + garch(1, 2), data = Onions171,
cond.dist = "std")
Mean and Variance Equation:data ~ arma(1, 2) + garch(1, 2) [data= Onions171]
Conditional Distribution:std
Coefficient(s):
mu        ar1        ma1        ma2       omega     alpha1
 2.740132   0.958275   0.048469   0.009164  14.854573   1.000000
   beta1      beta2      shape
 0.076549   0.390087   2.183692
Std. Errors:based on Hessian
Error Analysis:
Estimate  Std. Error  t value Pr(>|t|)
mu      2.740132    1.180281    2.322   0.0203 *
ar1     0.958275    0.020982   45.670  < 2e-16 ***
ma1     0.048469    0.081631    0.594   0.5527
ma2     0.009164    0.054211    0.169   0.8658
omega  14.854573   14.607790    1.017   0.3092
alpha1  1.000000    0.912285    1.096   0.2730
beta1   0.076549    0.220153    0.348   0.7281
beta2   0.390087    0.624951    0.624   0.5325
shape   2.183692    0.271347    8.048 8.88e-16 ***
Signif.codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
Log Likelihood:
 -481.8708    normalized:  -2.817958
Standardised Residuals Tests:
                              Statistic p-Value
Jarque-Bera Test   R    Chi^2  1703.451  0
 Shapiro-WilkTest  R    W      0.7675186 3.502733e-15
Ljung-Box Test     R    Q(10)  8.951468  0.5367156
Ljung-Box Test     R    Q(15)  9.540549  0.8476031
Ljung-Box Test     R    Q(20)  20.12803  0.449947
Ljung-Box Test     R^2  Q(10)  2.838104  0.9849813
Ljung-Box Test     R^2  Q(15)  3.658927  0.9986576
Ljung-Box Test     R^2  Q(20)  9.285991  0.9793518
 LM Arch Test      R    TR^2   3.216009 0.9938173
Information Criterion Statistics:
     AIC      BIC      SIC     HQIC
5.741179 5.906530 5.736000 5.808272
################
>summary(fit9.52) onions (2,1)
Title: GARCH Modelling
Call:garchFit(formula = ~arma(2, 1) + garch(2, 1), data = Onions171,
cond.dist = "std")
Mean and Variance Equation:data ~ arma(2, 1) + garch(2, 1)[data = Onions171]
Conditional Distribution:std
Coefficient(s):
mu         ar1         ar2         ma1        omega
3.1380e+00  8.0513e-01  1.4700e-01  1.9999e-01  1.6615e+01
   alpha1      alpha2       beta1       shape
1.0000e+00  1.0000e-08  5.1929e-01  2.1598e+00
Std. Errors:based on Hessian
Error Analysis:
Estimate  Std. Error  t value Pr(>|t|)
mu     3.138e+00   1.394e+00    2.251  0.02440 *
ar1    8.051e-01   3.431e-01    2.347  0.01894 *
ar2    1.470e-01   3.311e-01    0.444  0.65704
```

```
ma1    2.000e-01   3.306e-01    0.605  0.54526
omega  1.662e+01          NA       NANA
alpha1 1.000e+00   3.985e-01    2.509  0.01209 *
alpha2 1.000e-08          NA       NANA
beta1  5.193e-01   1.909e-01    2.721  0.00652 **
shape  2.160e+00   9.773e-02   22.098  < 2e-16 ***
Signif.codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Log Likelihood:
 -482.782    normalized:  -2.823287
Standardised Residuals Tests:
                              Statistic p-Value
Jarque-Bera Test   R    Chi^2  1786.278  0
 Shapiro-WilkTest  R    W      0.7717845 4.889029e-15
Ljung-Box Test     R    Q(10)  9.821575  0.4562843
Ljung-Box Test     R    Q(15)  10.70952  0.7729011
Ljung-Box Test     R    Q(20)  20.48866  0.4277587
Ljung-Box Test     R^2  Q(10)  3.296978  0.9735466
Ljung-Box Test     R^2  Q(15)  4.131147  0.9972754
Ljung-Box Test     R^2  Q(20)  7.774266  0.9932589
 LM Arch Test      R    TR^2   3.488845  0.9909983
Information Criterion Statistics:
     AIC      BIC      SIC     HQIC
5.751836 5.917187 5.746657 5.818929
################
>summary(fit9.53) onions 2,1
Title: GARCH Modelling
Call:garchFit(formula = ~arma(2, 2) + garch(2, 2),data = Onions171,
cond.dist = "std")
Mean and Variance Equation: data ~ arma(2, 2) + garch(2, 2)[data = Onions171]
Conditional Distribution:std
Coefficient(s):
mu          ar1         ar2         ma1         ma2
3.2169e+00  7.5303e-01  1.9863e-01  2.5115e-01  1.7888e-02
omega       alpha1      alpha2      beta1       beta2
1.5344e+01  1.0000e+00  1.0000e-08  7.1975e-02  4.5374e-01
shape
2.1499e+00
Std. Errors:based on Hessian
Error Analysis:
Estimate  Std. Error  t value Pr(>|t|)
mu     3.217e+00   1.556e+00    2.067   0.0387 *
ar1    7.530e-01   3.308e-01    2.277   0.0228 *
ar2    1.986e-01   3.185e-01    0.624   0.5329
ma1    2.512e-01   3.396e-01    0.740   0.4595
ma2    1.789e-02   6.007e-02    0.298   0.7659
omega  1.534e+01   1.960e+01    0.783   0.4336
alpha1 1.000e+00   9.210e-01    1.086   0.2776
alpha2 1.000e-08   8.664e-01    0.000   1.0000
beta1  7.198e-02   6.200e-01    0.116   0.9076
beta2  4.537e-01   4.797e-01    0.946   0.3442
shape  2.150e+00   2.013e-01   10.681   <2e-16 ***
Signif.codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Log Likelihood:
 -481.6879    normalized:  -2.816888
Standardised Residuals Tests:
                              Statistic p-Value
```

```
Jarque-Bera Test   R    Chi^2  1575.937  0
 Shapiro-WilkTest  R    W      0.7724919 5.169191e-15
Ljung-Box Test     R    Q(10)  9.186316  0.5145176
Ljung-Box Test     R    Q(15)  9.808871  0.8315871
Ljung-Box Test     R    Q(20)  20.67289  0.4166059
Ljung-Box Test     R^2  Q(10)  2.831535  0.9851152
Ljung-Box Test     R^2  Q(15)  3.689002  0.9985909
Ljung-Box Test     R^2  Q(20)  9.66053   0.9739181
 LM Arch Test      R    TR^2   3.260902 0.9934049
Information Criterion Statistics:
     AIC      BIC      SIC     HQIC
5.762432 5.964527 5.754803 5.844433
################
```

Appendix 5: Results of mixed models ARMA (p, q) and GARCH (p, q)-Tomatoes in R

```
>summary(fit9.6) tomatoes 1,2
Title: GARCH Modelling
Call:garchFit(formula = ~arma(1, 2) + garch(1, 2), data = Tomatoes171,
cond.dist = "std")
Mean and Variance Equation:d ata ~arma(1,2)+garch(1, 2)[data = Tomatoes171]
Conditional Distribution:std
Coefficient(s):
mu         ar1         ma1         ma2        omega
7.7377e+00  8.5551e-01  8.1527e-02  1.6888e-01  1.6344e+01
    alpha1       beta1       beta2       shape
3.8339e-01  1.0000e-08  5.3764e-01  2.8658e+00
Std. Errors:based on Hessian
Error Analysis:
Estimate  Std. Error  t value Pr(>|t|)
mu     7.738e+00   3.043e+00    2.542  0.01101 *
ar1    8.555e-01   5.914e-02   14.465  < 2e-16 ***
ma1    8.153e-02   1.108e-01    0.736  0.46167
ma2    1.689e-01   9.400e-02    1.797  0.07240 .
omega  1.634e+01   1.331e+01    1.228  0.21946
alpha1 3.834e-01   3.915e-01    0.979  0.32742
beta1  1.000e-08   8.310e-01    0.000  1.00000
beta2  5.376e-01   6.654e-01    0.808  0.41906
shape  2.866e+00   9.061e-01    3.163  0.00156 **
Signif.codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Log Likelihood:
 -568.8917    normalized:  -3.326852
Standardised Residuals Tests:
                              Statistic p-Value
Jarque-Bera Test   R    Chi^2  1443.759  0
 Shapiro-WilkTest  R    W      0.8371223 1.55974e-12
Ljung-Box Test     R    Q(10)  6.106461  0.8062409
Ljung-Box Test     R    Q(15)  10.28313  0.8015625
Ljung-Box Test     R    Q(20)  16.32854  0.6960441
Ljung-Box Test     R^2  Q(10)  1.032247  0.9998009
Ljung-Box Test     R^2  Q(15)  1.703678  0.9999899
Ljung-Box Test     R^2  Q(20)  2.217776  0.9999997
LM Arch Test       R    TR^2   1.57954   0.9998279
Information Criterion Statistics:
     AIC      BIC      SIC     HQIC
```

6.758967 6.924318 6.753787 6.826059
################
>summary(fit9.6) tomatoes 2,1
Title: GARCH Modelling
Call:garchFit(formula =~arma(2,1) +garch(2,1),data=Tomatoes171,cond.dist= "std")
Mean and Variance Equation: data ~ arma(2, 1) + garch(2, 1)[data=Tomatoes171]
Conditional Distribution:std
Coefficient(s):

```
mu          ar1          ar2          ma1          omega
 4.35353030   0.99999999  -0.08095849  -0.02822254  92.75556348
     alpha1       alpha2        beta1        shape
 0.99999999   0.00000001   0.85947451   2.03608725
```

Std. Errors:based on Hessian
Error Analysis:

```
Estimate  Std. Error  t value Pr(>|t|)
mu      4.354e+00   3.111e+00    1.399    0.162
ar1     1.000e+00   1.683e+00    0.594    0.552
ar2    -8.096e-02   1.632e+00   -0.050    0.960
ma1    -2.822e-02   1.515e+00   -0.019    0.985
omega   9.276e+01          NA       NANA
alpha1  1.000e+00   1.221e+00    0.819    0.413
alpha2  1.000e-08          NA       NANA
beta1   8.595e-01   9.621e-02    8.934   <2e-16 ***
shape   2.036e+00          NA       NANA
```

Signif.codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
Log Likelihood:
 -571.7982   normalized: -3.343849
Standardised Residuals Tests:

```
                              Statistic p-Value
Jarque-Bera Test   R    Chi^2  657.4075  0
 Shapiro-WilkTest  R    W      0.8736461 8.150674e-11
Ljung-Box Test     R    Q(10)  10.61608  0.3881999
Ljung-Box Test     R    Q(15)  16.22059  0.3675449
Ljung-Box Test     R    Q(20)  20.6511   0.417918
Ljung-Box Test     R^2  Q(10)  9.030082  0.5292511
Ljung-Box Test     R^2  Q(15)  9.706804  0.8377662
Ljung-Box Test     R^2  Q(20)  10.39066  0.9605227
LM Arch Test       R    TR^2   9.739483  0.6388037
```

Information Criterion Statistics:

```
     AIC      BIC      SIC     HQIC
6.792961 6.958312 6.787782 6.860054
```

################
>summary(fit9.6) tomatoes 2,2
Title:GARCH Modelling
Call:garchFit(formula = ~arma(2, 2) + garch(2, 2), data = Tomatoes171,
cond.dist = "std")
Mean and Variance Equation:data ~ arma(2, 2) + garch(2, 2)[data=Tomatoes171]
Conditional Distribution:std
Coefficient(s):

```
mu          ar1          ar2          ma1          ma2
 6.86534929   0.99999999  -0.12795777  -0.06000199   0.15496351
omega        alpha1       alpha2       beta1        beta2
16.79905147   0.38374003   0.00000001   0.00000001   0.54822103
shape
 2.79942355
```

Std. Errors:based on Hessian

Error Analysis:
Estimate Std. Error t value Pr(>|t|)
mu 6.865e+00 NA NANA
ar1 1.000e+00 NA NANA
ar2 -1.280e-01 NA NANA
ma1 -6.000e-02 NA NANA
ma2 1.550e-01 9.836e-03 15.754 < 2e-16 ***
omega 1.680e+01 1.367e+01 1.229 0.219
alpha1 3.837e-01 3.087e-01 1.243 0.214
alpha2 1.000e-08 NA NANA
beta1 1.000e-08 NA NANA
beta2 5.482e-01 4.733e-02 11.584 < 2e-16 ***
shape 2.799e+00 7.129e-01 3.927 8.61e-05 ***
Signif.codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Log Likelihood:
-568.859 normalized: -3.326661
Standardised Residuals Tests:
Statistic p-Value
Jarque-Bera Test R Chi^2 1397.49 0
Shapiro-WilkTest R W 0.8377639 1.662852e-12
Ljung-Box Test R Q(10) 6.279508 0.791259
Ljung-Box Test R Q(15) 10.31024 0.7997845
Ljung-Box Test R Q(20) 16.36855 0.6935128
Ljung-Box Test R^2 Q(10) 1.041371 0.9997928
Ljung-Box Test R^2 Q(15) 1.718001 0.9999893
Ljung-Box Test R^2 Q(20) 2.237926 0.9999997
LM Arch Test R TR^2 1.589153 0.9998222
Information Criterion Statistics:
AIC BIC SIC HQIC
6.781976 6.984072 6.774348 6.863978
################

Appendix 6: Results of mixed models ARMA (p, q) and GARCH (p, q)-Kales in R

>summary(fit9.71) Kales 1,2
Title: GARCH Modelling
Call:
garchFit(formula = ~arma(1, 2) + garch(1, 2),data = Kales171,cond.dist="std")
Mean and Variance Equation:
data ~ arma(1, 2) + garch(1, 2)[data = Kales171]
Conditional Distribution:std
Coefficient(s):
mu ar1 ma1 ma2 omega
1.19868109 0.92784079 0.08306114 0.05041848 2.21967105
alpha1 beta1 beta2 shape
0.99999999 0.00000001 0.42946098 2.41462669
Std. Errors:ased on Hessian
Error Analysis:
Estimate Std. Error t value Pr(>|t|)
mu 1.199e+00 2.827e-01 4.240 2.23e-05 ***
ar1 9.278e-01 1.485e-02 62.499 < 2e-16 ***
ma1 8.306e-02 7.468e-02 1.112 0.2660
ma2 5.042e-02 5.281e-02 0.955 0.3397
omega 2.220e+00 1.227e+00 1.809 0.0705 .
alpha1 1.000e+00 4.981e-01 2.008 0.0447 *

```
beta1  1.000e-08         NA       NANA
beta2  4.295e-01   1.054e-01    4.073 4.64e-05 ***
shape  2.415e+00   2.507e-01    9.633  < 2e-16 ***
Signif.codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Log Likelihood:
 -415.8432    normalized:  -2.431831
Standardised Residuals Tests:
                              Statistic p-Value
Jarque-Bera Test   R    Chi^2  7493.937  0
 Shapiro-WilkTest  R    W      0.6842309 0
Ljung-Box Test     R    Q(10)  4.831528  0.9021441
Ljung-Box Test     R    Q(15)  12.92612  0.6080046
Ljung-Box Test     R    Q(20)  20.3347   0.4371757
Ljung-Box Test     R^2  Q(10)  0.5262494 0.9999916
Ljung-Box Test     R^2  Q(15)  0.7807039 1
Ljung-Box Test     R^2  Q(20)  4.657443  0.9998412
 LM Arch Test      R    TR^2  0.6733808 0.9999985
Information Criterion Statistics:
     AIC      BIC      SIC     HQIC
4.968926 5.134276 4.963746 5.036018
#################
summary(fit9.72) kales 2,1
Title: GARCH Modelling
Call: garchFit(formula =~arma(2, 1)+garch(2,1),data=Kales171,cond.dist = "std")
Mean and Variance Equation: data~arma(2, 1)+garch(2,1)[data = Kales171]
Conditional Distribution:std
Coefficient(s):
muar1         ar2         ma1       omega
1.74586814  0.48763594  0.40779702  0.47135990  1.76872486
    alpha1      alpha2       beta1       shape
0.99999999  0.00000001  0.67870800  2.20866570
Std. Errors:based on Hessian
Error Analysis:
Estimate  Std. Error  t value Pr(>|t|)
mu     1.746e+00   7.050e-01    2.476   0.0133 *
ar1    4.876e-01   2.810e-01    1.735   0.0827 .
ar2    4.078e-01   2.630e-01    1.550   0.1210
ma1    4.714e-01   2.623e-01    1.797   0.0723 .
omega  1.769e+00   1.697e+00    1.042   0.2972
alpha1 1.000e+00   7.404e-01    1.351   0.1768
alpha2 1.000e-08   6.768e-01    0.000   1.0000
beta1  6.787e-01   1.148e-01    5.912 3.39e-09 ***
shape  2.209e+00   1.967e-01   11.228  < 2e-16 ***
Signif.codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Log Likelihood:
 -418.1915    normalized:  -2.445564
Standardised Residuals Tests:
                             Statistic p-Value
Jarque-Bera Test   R    Chi^2  8194.806  0
 Shapiro-WilkTest  R    W      0.6740251 0
Ljung-Box Test     R    Q(10)  4.164881  0.9396025
Ljung-Box Test     R    Q(15)  9.660394  0.840541
Ljung-Box Test     R    Q(20)  17.41176  0.626097
Ljung-Box Test     R^2  Q(10)  0.4956424 0.9999937
Ljung-Box Test     R^2  Q(15)  0.757358  1
Ljung-Box Test     R^2  Q(20)  4.346875  0.9999085
```

LM Arch Test R TR^2 0.6326093 0.9999989
Information Criterion Statistics:

AIC	BIC	SIC	HQIC
4.996392	5.161742	4.991212	5.063484

################
summary(fit9.73) Kales 2,2
Title: GARCH Modelling
Call: garchFit(formula = ~arma(2, 2) + garch(2, 2), data = Kales171,cond.dist = "std")
Mean and Variance Equation:
data ~ arma(2, 2) + garch(2, 2)[data = Kales171] Conditional Distribution:std
Coefficient(s):

mu	ar1	ar2	ma1	ma2
1.60027903	0.44106448	0.46205412	0.57053429	0.05453350

omega	alpha1	alpha2	beta1	beta2
2.09007914	0.99999999	0.00000001	0.00000001	0.43621483

shape
2.41467957
Std. Errors: based on Hessian
Error Analysis:

	Estimate	Std. Error	t value	Pr(>|t|)	
mu	1.600e+00	9.484e-01	1.687	0.0915	.
ar1	4.411e-01	2.712e-01	1.626	0.1039	
ar2	4.621e-01	2.510e-01	1.841	0.0657	.
ma1	5.705e-01	2.761e-01	2.066	0.0388	*
ma2	5.453e-02	6.070e-02	0.898	0.3690	
omega	2.090e+00	1.657e+00	1.261	0.2072	
alpha1	1.000e+00	5.862e-01	1.706	0.0880	.
alpha2	1.000e-08	2.810e-01	0.000	1.0000	
beta1	1.000e-08	2.048e-01	0.000	1.0000	
beta2	4.362e-01	1.116e-01	3.909	9.29e-05	***
shape	2.415e+00	3.295e-01	7.329	2.32e-13	***

Signif.codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
Log Likelihood:
-415.2163 normalized: -2.428166
Standardised Residuals Tests:

			Statistic	p-Value
Jarque-Bera Test	R	Chi^2	7967.098	0
Shapiro-WilkTest	R	W	0.6775692	0
Ljung-Box Test	R	Q(10)	5.041154	0.888412
Ljung-Box Test	R	Q(15)	12.21494	0.6626924
Ljung-Box Test	R	Q(20)	19.63058	0.4812416
Ljung-Box Test	R^2	Q(10)	0.5087623	0.9999928
Ljung-Box Test	R^2	Q(15)	0.7882664	1
Ljung-Box Test	R^2	Q(20)	4.448923	0.9998897
LM Arch Test	R	TR^2	0.6555249	0.9999987

Information Criterion Statistics:

AIC	BIC	SIC	HQIC
4.984986	5.187081	4.977358	5.066988