# DETERMINING THE OPTIMAL PORTFOLIO SIZE ON THE NAIROBI SECURITIES EXCHANGE

BY

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# DECLARATION

This research project is my original work and has not been presented to any other university for the award of a degree. All information sources have been acknowledged.

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| Joseph Aloise Mbithi | __________________________ | _____________________ |
| D61/75773/2012 | Signature | Date |

This research project report has been submitted for examination with my approval as the University of Nairobi supervisor.

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| Dr Sifunjo E. Kisaka | __________________________ | _________________ |
| Supervisor | Signature | Date |

## ABSTRACT

There is consensus that diversification results in risk reduction. However there is no consensus on the number of securities required for maximum risk diversification. Studies done on different capital markets have yielded differing results. This study aimed to determine the optimal portfolio size for investors on the Nairobi Securities Exchange in Kenya. The study used mean variance optimization model and secondary data consisting of monthly security returns over a five year period from January 2009 to December 2013. Forty three of the sixty listed firms had complete information on monthly security returns and were used in the study. Portfolios of different sizes were formed by random selection of securities. The study found that portfolio risk reduced as the number of securities in the portfolio increased and that the optimal portfolio size in the Nairobi Securities Exchange was between 18 and 22 securities.

# DEDICATION

This research work is dedicated to my dear wife Maggy and sons Amos and Austin

# ACKNOWLEDGEMENTS

I thank the Almighty God for his grace. I wish to sincerely express my gratitude and appreciation to the people who have contributed and inspired me to complete the study. First my supervisor Dr Sifunjo E. Kisaka for his guidance, encouragement and support throughout the study. To my family and friends who have been very understanding, supportive and encouraging. Thank you all.

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# ABBREVIATIONS

ATS – Automated Trading System

BSE – Bombay Stock Exchange

CBK – Central Bank of Kenya

CMA – Capital Markets Authority

ESOPS – Employee Share Ownership Plans

FTSE – Financial Times Stock Exchange

GOK – Government of Kenya

IRA – Insurance Regulatory Authority

KLSE – Kuala Lumpur Stock Exchange

MPT – Modern Portfolio Theory

NASI – NSE All Shares Index

NSE – Nairobi Securities Exchange

NSE-20 – Nairobi Securities Exchange 20 share Index

NYSE – New York Stock Exchange

OLS – Ordinary Least Squares

RBA – Retirements Benefits Authority

TSE – Toronto Stock Exchange

U.K – United Kingdom

U.S. – United States of America

WAN – Wide Area Network

# CHAPTER ONE

# INTRODUCTION

## 1.1 Background to the Study

Financial theory assumes utility maximization. Investors endeavor to maximize their expected utility. Return from a portfolio of financial assets is of utmost importance to all investors (Elton, Gruber, Brown & Goetzmann, 2010). Equally important is the number of securities to invest in and their combination. Asset allocation is the first step in portfolio management. Strategic asset allocation is an important determinant of investment returns and success. It identifies the best allocation of investment funds among various categories of securities for an investor whose saving and consumption pattern is predictable over a given investment period such as pension and insurance funds (Brennan, Schwartz & Lagnado, 1997). But how many securities are optimal?

Markowitz (1952) developed the basic portfolio model that incorporated diversification benefits into portfolio asset allocation. However his model was limited to one holding period. Merton (1971) considered the multi-period holding portfolio strategy which is more realistic in practice. Sharpe (1985) simplified the methodology. Modern portfolio theory is based on portfolio mean-variance optimization model, where the complete investment opportunity set (all assets) are considered simultaneously. This differs from practice where investors consider the asset classes separately in their allocation models (Reilley & Brown, 2012).

Investors continually deal with the trade-off between risk and return. They strive to maximize their growth potential with the minimum possible risk. The conflicting objectives of maximizing expected return and minimizing uncertainty or risk must be balanced against each other. Markowitz (1952) suggested that there is a portfolio which will give the investor maximum expected return and minimum variance. He called this the optimal portfolio (Elton et al, 2010). This study used the mean-variance analysis Model to determine the optimal portfolio size in the Nairobi Securities Exchange.

### 1.1.1 Financial Assets Return and Risk

Financial asset return refers to earnings generated from invested capital (assets). Financial asset returns are related to future economic activity. Investors spend money at present with the expectation of earning more money in the future, the utility maximization assumption. Financial asset returns come in two ways: dividend or interest payments and capital gains. The total return is given by the ratio of the sum of capital gain and dividend or interest payments to the initial investment. The concept of return provides an investor with a convenient way of expressing the financial performance of an investment. The value of a financial asset is the value of all expected future cash flows discounted to the present (Elton et al, 2010).

The expected return of a portfolio is represented by the mean of the expected returns of the constituent assets. It is represented as

$$E(R_p) = \sum_{i=1}^{n} w_i E(R_i) \qquad (1)$$

Where $R_p$ is the return on the portfolio, $R_i$ is the return on asset i and $w_i$ is the proportion of asset i in the portfolio.

Risk refers to the chance of unfavourable events. Investors normally buy stocks in anticipation of a particular return but fluctuations in stock prices result in fluctuating returns. Finance theory defines risk as the probability that the actual returns will deviate from the expected returns. There are two types of risks namely unsystematic and systematic risk. Unsystematic risk is also referred as diversifiable, is the risk that can be diversified away by holding the investment in a suitably wide portfolio. This type of risk is usually firm specific. Systematic (non-diversifiable) risk, is the risk inherent in the market as a whole and is attributable to market wide factors. This risk is not diversifiable and must be accepted by investor who chooses to hold the asset (Elton et al, 2010).

The risk of a portfolio is represented by the variance of return and is expressed as

$$\sigma_p^2 = \sum_i w_i^2 \sigma_i^2 + \sum_i \sum_{j \neq i} w_i w_j \sigma_i \sigma_j \rho_{ij} \qquad (2)$$

Where $\rho_{ij}$ is the correlation coefficient between the returns on assets i and j (Reilley & Brown, 2012).The volatility of portfolio return is the standard deviation of return, $\sigma_p$

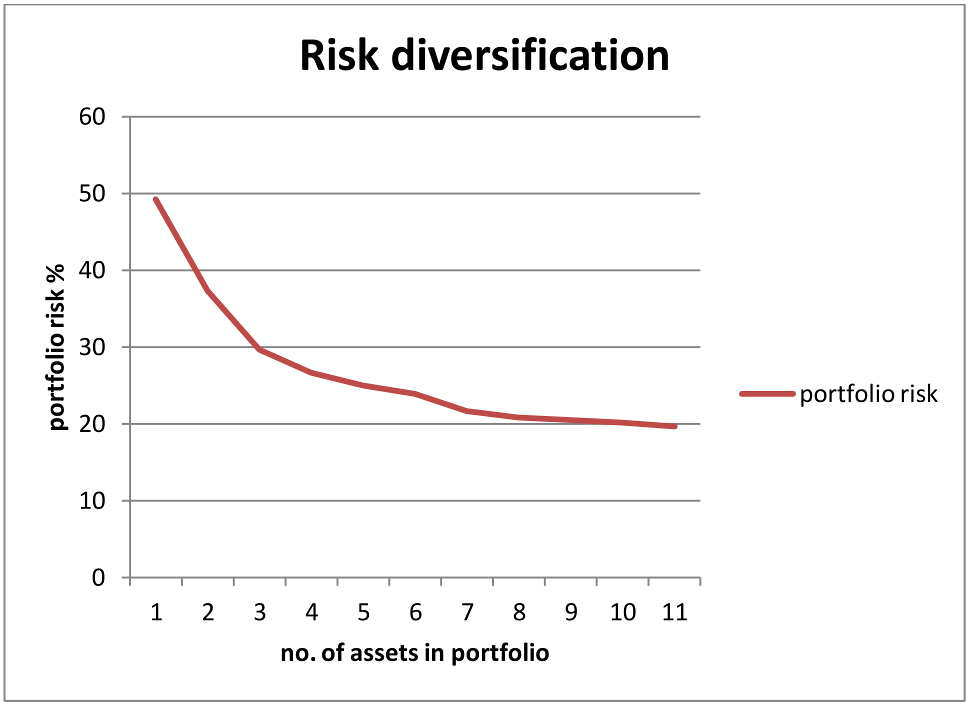
$$\sigma_p = \sqrt{\sigma_p^2} \qquad (3)$$

### 1.1.2 Diversification and Optimal Portfolio size

Investors strive to maximize their expected return with the minimum possible risk. These are two conflicting objectives that must be balanced against each other. Portfolios that satisfy this requirement are called efficient portfolios. In constructing efficient portfolios investors are assumed to be risk averse and when presented with two efficient portfolios with same expected return they will prefer the less risky one. The optimal portfolio is the efficient portfolio which is most preferred by the investor. Modern portfolio theory postulates that as the number of securities in a portfolio increases, the portfolio risk decreases (Elton et al, 2010). Markowitz (1952) suggested that there is a portfolio which will give the investor maximum expected return with minimum variance and he called this the optimal portfolio. Increasing the number of securities in a portfolio beyond this optimal size depends on the marginal benefits of risk reduction derived from diversification against the marginal cost of maintaining the portfolio in terms of operational costs (Evans & Archer, 1968). Beyond the optimal portfolio risk reduction benefits becomes insignificant.

Investment managers use diversification as one of the main concepts to eliminate as much risk as possible from their portfolios. Diversification eliminates or lessens firm- or company- specific risk factors. However investment managers can do nothing to eliminate exposure to market-wide (systematic) risk factors. Generally a diversified portfolio has the potential to earn much higher risk-adjusted return than undiversified one (Treynor & Black, 1973). Incorporating different securities in a portfolio, the investment manager optimizes diversification, earning a target return with the least risk. However too much diversification increases operating costs and reduces the returns thus decreasing the portfolio efficiency. This is why it is essential to determine the optimal portfolio. Investors reduce their risk by holding a combination of assets that are not perfectly positively correlated i.e. correlation coefficient greater or equal to -1 but less than 1 (Reilley & Brown, 2012).

Figure 1: Risk Diversification




Source: Plot by author using Microsoft Excel for illustration

### 1.1.3 The Kenyan Stock Market

The Kenyan capital markets are regulated and supervised by the Capital Markets Authority (CMA) through legislative power of CMA Act of 1989. The act came into effect in 1990. The Authority supervises and regulates the activities of market intermediaries including the stock exchange, central depository and settlement system and all other persons licensed under Capital Markets Act. The capital market is part of the financial markets that provides funds for long-term development. It facilitates mobilization and allocation of capital resources to finance long-term productive investments.

According to CMA (2013) the sector consisted of 5 approved institutions, 10 investment banks, 11 stock brokers, 21 fund managers, 17 investment advisors, 15 authorized depositories, 16 approved collective investment schemes and 10 approved employee share ownership plans (ESOPS). The NSE is among the approved institutions. Several funds are run under the collective investment schemes including money market fund, equity fund, balanced fund and fixed income (bond) fund.

The Nairobi Securities Exchange was established in 1954. It has a computerized delivery and settlement system and an Automated Trading System (ATS) which enables trading in equities and immobilized corporate and treasury bonds. In 2007 the NSE established a Wide Area Network (WAN) platform to enable brokers conduct trading from their offices. The NSE-20 share index acts as the gauge for market activity while the NASI acts as an alternative index. There are 60 companies listed on the Main Investment Market Segment (MIMS) of the NSE. Trading at the exchange is on the equities of these listed companies and immobilized corporate and government bonds (NSE, 2014).

Thus financial instruments available for investment in the Kenyan capital markets include equities, bonds and the collective investment schemes' funds. This study utilized monthly returns of equities of listed firms to determine the optimal portfolio size in the Nairobi Securities Exchange over a five year period from January 2009 to December 2013.

## 1.2 Research Problem

The Nairobi Securities Exchange has continued to develop over the last ten years. This has seen an increase in available securities through more listing of firms at the NSE. Trading in the NSE has been automated and the number of investors participating in the market has increased both individual/institutional and local/foreign. With the increased choices investors have to make a decision on number of securities to include in their portfolio in order to maximize return and minimize risk. Portfolio return is very important to all investors. Diversification reduces risk without compromising on expected return (Reilley & Brown, 2012). However the number of securities to invest in, their combination in a portfolio and the risk involved are equally important considerations. Kenyan investors will be exposed to reduced risk if they diversified their portfolios. Information on the optimal portfolio size in Kenya is necessary to help investors in selecting stocks to invest in and reduce their exposure to diversifiable risk.

Investors make decisions to invest with expectation of a return for a given level of risk. Total risk of an investment asset consists of non-systematic and systematic risks. Non-systematic risk is caused by firm specific random factors and can be diversified away (eliminated) by holding investment in an optimal portfolio. According to Modern Portfolio Theory portfolio risk is negatively related to portfolio size. Portfolio variance decreases as the number of securities in the portfolio increases (Markowitz, 1952; Evans & Archer, 1968; Reilley & Brown, 2012).

Research has shown that most of the risk reduction benefits of diversification can be gained by forming portfolios containing 8-20 randomly selected securities (Newbould & Poon, 1993; Tang, 2004; Solnik, 1990). Treynor and Black (1973) showed that portfolio performance can be improved by optimally weighting a fund manager's stocks selection. Studies by Evans and Archer (1968), Fisher and Lorie (1970) and Tole (1982) indicated that the major benefits of diversification are achieved rather quickly, with about 90% of maximum benefit of diversification derived from portfolios of 12 to 18 stocks. Statman (1987) considered the trade-off between diversification benefits and transaction costs involved in increasing the size of the portfolio. He concluded that a well diversified portfolio must contain at least 30 to 40 stocks. Frahm and Wiechers (2011) using monthly return data for equally weighted 40-assets portfolios found that diversification effect among different assets contributed to portfolio performance.

Gupta, Khoon and Shahnon (2001) found that 27 securities were required for a well diversified portfolio in the Malaysia stock market while Zulkifli et al (2008) after examination of the same market concluded that the benefit of diversification can be fully achieved by investing in a portfolio of 15 stocks. Tsui, Low and Kwok (1983) found that a well diversified portfolio in Singapore stock market consisted of 40 randomly selected securities. Nyariji (2001) evaluated the risk reduction benefits of portfolio diversification at the NSE and established that the risk minimizing portfolio was 13 securities. There is therefore no consensus on the optimal portfolio size.

The NSE is an emerging capital market. Very few studies on optimal portfolio size have been done for emerging capital markets and the NSE in particular. Nyariji (2001) found that the risk minimizing portfolio for equities listed at the NSE was 13 securities. Kamanda (2001) concluded that the equity portfolios held by Kenyan insurance sector are poorly diversified and performed worse than the NSE. Mwangangi (2006) found that over 60% of fund managers considered mean-variance model in their allocation criteria.

The NSE has continued to develop as evidenced by automated trading and listing of new firms. The success of these listings has increased the number of investors in the stock market. However, most investors want to maximize returns without risk consideration. This is attributed mainly to herd mentality. Determining the optimal portfolio size is important to help decision making for investors on the NSE. There is no consensus on optimal size as many of the studies done have provided varied results (Solnik, 1974). This study aimed at contributing to this debate by using the mean-variance model to determine the optimal portfolio size on the NSE over a five year period starting January 2009 to December 2013. The study sought to answer the following question: what is the optimal portfolio size for an investor on the Nairobi Securities Exchange?

## 1.3 Objectives of the Study

The objective of this study was to determine the optimal portfolio size for an investor in the Nairobi Securities Exchange.

## 1.4 Importance of the Study

This study contributes to the empirical evidence on the determination of optimal portfolio size for investors within Kenyan and other emerging capital markets. This study has important implications for investors in making portfolio securities selection and funds allocation decisions in Kenya.

This study can inform future review of policy and regulatory guidelines for regulated institutional investors in Kenya. The study is of interest to researchers and financial analysts of the Kenyan economy. The study is of interest to portfolio and investment managers of insurance companies and retirement benefits schemes.

# CHAPTER TWO
# LITERATURE REVIEW

## 2.1 Introduction

This chapter discusses the theories on investment risk, return and diversification. Section 2.2 discusses theoretical literature. Section 2.3 presents the empirical literature. Section 2.4 presents studies on optimal portfolio size done in Kenya. Section 2.5 summarizes the literature reviewed and support for the knowledge gap identified.

## 2.2 Theoretical Literature

This section reviews the Modern Portfolio Theory, Efficient frontier, Utility theory, indifference curves and the Capital Asset Pricing Model (CAPM).

### 2.2.1 Modern Portfolio Theory

Modern Portfolio Theory (MPT) was pioneered by Markowitz in 1952. MPT is a finance theory that attempts to maximize portfolio expected return for a given amount of portfolio risk by carefully choosing the proportions of various assets. Expected returns must consider uncertainty due to market imperfections. The theory is a mathematical formulation of the concept of diversification, with the aim of selecting a collection of assets that has collectively lower risk than any individual asset (Markowitz, 1952).

The MPT is based on the mean-variance analysis model developed by Markowitz (1952). He derived the expected rate of return for a portfolio of assets and an expected risk measure. Markowitz showed that variance of the rate of return was a meaningful measure of portfolio risk under a reasonable set of assumptions (Reilley & Brown, 2012). He derived the formula for computing the variance of a portfolio which indicated the importance of investment diversification to reduce total risk and how to diversify effectively.

The Markowitz's model is based on the following assumptions: First, investors care for risk over a single holding period. Second, investors maximize one-period utility and their curves indicate diminishing marginal utility for wealth. Third, investors estimate portfolio risk on the basis of the variability of the expected returns. Fourth, investors base decisions solely on expected return and expected risk. Fifth, for a given level of risk investors prefer higher returns to lower returns or for a given level of expected return they prefer less risk to more risk. He showed that when investors consider the mean–variance criteria, they choose a combination of market portfolio and a risk-free asset when constructing their portfolio structure (Reilley & Brown, 2012).

However, Markowitz's analysis was limited by the assumption of a single holding period. Many real financial problems do not fit that assumption e.g. pension or insurance problems. Merton (1971) considered multi-period planning and portfolio strategy by an investor. However his solutions required an investor to have log utility consumption with constant relative risk aversion equal to one. Diversification eliminates unsystematic risk leaving systematic risk which is market-wide. Different assets often change in value in opposite directions e.g. stock markets and bond markets. MPT models asset returns as normally distributed function, defines risk as standard deviation and models a portfolio as a weighted combination of the assets returns (Elton et al, 2010).

Portfolio return is expressed as the mean of expected returns of component assets while risk is expressed as variance of the asset returns. MPT seeks to reduce portfolio variance of returns. It assumes that investors are rational and that markets are efficient. The MPT was developed in the 1950s through early 1970s. According to Elton et al (2010) the contribution of covariance between different pairs of individual assets to the portfolio variance increases with the number of assets. The portfolio variance gradually approaches average covariance which is the minimum for the portfolio (Jorion & Khoury, 1996).

The expected return for a portfolio (E ($R_p$)) is the weighted average of the expected rates of returns of the individual investments in the portfolio. The weights are the proportion of the total value for the individual assets (Reilley & Brown, 2012).

$$E\,(R_p) = \sum_{i=1}^{n} w_i R_i \qquad (4)$$

Where $w_i$ = Weight of individual asset in the portfolio

$R_i$ = Expected rate of return of asset i

There are two concepts which are important in defining the variance of returns of a portfolio namely covariance of returns and correlation coefficient. Covariance measures the to which two variables move together relative to their individual mean values over time. The covariance of the rates of return of portfolio components is important. A positive covariance indicates the rates of return for a pair of assets move in the same direction relative to their individual means during the same time period while a negative covariance indicates movement in different directions (Reilley & Brown, 2012).

For two assets:

$$\text{Co}v_{ij} = \{(R_i - E(R_i))(R_j - E(R_j))\} \quad (5)$$

Covariance is affected by the variability of the individual returns indices thus it is standardized by the product of the individual assets standard deviations to yield Correlation coefficient ($r_{ij}$) which varies in the range of -1 to +1.

$$r_{ij} = \frac{Cov_{ij}}{\sigma_i \sigma_j} \quad (6)$$

A value of +1 indicates a perfect positive linear relationship, meaning returns of the two assets move together in a completely linear manner. A value of -1 indicates a perfect negative relation , meaning when one of the assets rate of return is above its mean the other assets rate of return will be below its mean by a comparable amount.

Markowitz developed the general formula for standard deviation of a portfolio as follows.

$$\sigma_p = \sqrt{} \{\sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j cov_{ij} \} \quad i \neq j \quad (7)$$

The formula indicates that the standard deviation of a portfolio is a function of the weighted average of the individual variances and the weighted covariance between all the assets of the portfolio. In a portfolio with large number of securities the formula reduces to the sum of the weighted covariances (Reilley & Brown, 2012).

The concept of the efficient frontier was introduced by Markowitz (1952) and others. A portfolio of assets is said to be efficient if it has the best possible expected return for its level of risk. Every possible combination of assets, excluding risk-free asset, can be plotted in a risk-expected return space. The upward slopped part of the left boundary of this region is referred to as the efficient frontier. Thus the efficient frontier is the portion of the opportunity set that offers the highest expected return for a given level of risk (Sharpe & Alexander, 1990).

According to the efficient set theorem, an investor will choose an optimal portfolio from a set of for varying levels of risk and minimum risk for varying levels of expected return. The set of portfolios meeting these two conditions are referred as the efficient set or the efficient frontier. The set of all attainable portfolio combinations with their expected returns and standard deviations make the feasible or opportunity set for the investor (Sharpe & Alexander, 1990).

The Modern Portfolio Theory is important to this study because it provides a mathematical linkage between the concept of risk diversification and the selection of a portfolio of assets. The model links the expected rate of return of portfolio to the expected risk and indicates the importance of diversification to reduce the total risk of a portfolio of investments.

### 2.2.2 Utility Theory

Utility concept represents the satisfaction experienced by the consumer of a good. Utility theory is used to explain risk and return and it assumes that investors are rational and consistent i.e. individuals make rational choices among alternative investments and they expect to get utility through a combination of risk and return. Investors who are risk indifferent attach the same value to a nominal wealth gain as an equal nominal loss in wealth (derive equal utility). Investors who are risk averse will lose more utility from a nominal lose in wealth than the gain from the same amount of additional wealth. This satisfies the diminishing marginal utility of wealth and they will require higher returns for risky investments. Risk seekers will attach greater weight to a nominal gain in wealth than the loss of an equal amount in wealth. An increase in wealth will cause a higher utility (Sharpe & Alexander, 1990).

Any investor can have various combinations which will give equal utility irrespective of their attitude towards risk. The risk and return trade-off for an investor can be depicted through the indifference curves. The general shape of the curve for a risk-averse investor means that, for any additional risk taken, the investor will require increased return at an accelerating (increasing) rate. The higher the indifference curves the higher the satisfaction the individual will derive i.e. get higher returns with lesser risk. The steepness if the indifference curve depicts the degree of risk aversion. This concept is important in determining the optimal portfolio for an investor (Elton et al, 2010).

Each indifference curve indicates a distinct level of utility and the curves cannot intersect. The construction makes the assumptions of non-satiation and risk aversion. The indifference curves of risk-averse investors are convex and upward slopping, those for risk-neutral investors are parallel to the standard deviation axis and those for risk-seekers are downward slopping. Combining the efficient set with the investor's indifference curves will give the optimal portfolio which the point where the highest possible indifference curve is tangent to the efficient frontier (Sharpe & Alexander, 1990).

Utility theory is important to this study because investors endeavor to maximize their expected utility. The theory explains the attitude of investors when making choices among alternative investment assets. The utility theory explains indifference curves for investors which depict the risk and return trade off for investors. The concept is thus important in determining the optimal portfolio size for investors.

### 2.2.3 Capital Asset Pricing Model (CAPM)

This model was developed by William Sharpe (1964) following the foundation laid by Markowitz (1952). It gives a precise prediction about the relationship between an asset and its expected return. It provides a benchmark return for evaluating possible investments. CAPM is also used to predict the expected return on assets that have not been traded. The model assumes that: First, all investors are focused on a single holding period and they seek to maximize the expected utility. Second, all investors can borrow and lend unlimited amounts at a given risk-free rate. Third, all investors have identical estimates of variance and covariances among all assets (homogenous expectations). Fourth, there are no transaction costs, no taxes and investors are price takers and fifth, the quantities of all assets are given and fixed (Reilley & Brown, 2012).

The model gives a framework to enable prediction of asset prices and returns under equilibrium conditions. CAPM links together non-diversifiable risk and return for all assets. The model is concerned with how systematic risk is measured and how it affects required returns and security values. CAPM theory includes the following propositions: First, investors require return in excess of the risk-free rate to compensate them for systematic risk. Second, investors require no premium for bearing non-systematic risk since it can be diversified away and third, because systematic risk varies between companies, investors will require higher return for investments where systematic risk is greater (Sharpe, 1964).

The return on individual assets or portfolios is expressed as follows:

$$R_i = R_f + \beta_i ( R_m - R_f ) \qquad (8)$$

Where $R_i$ is the expected return from asset i, $R_f$ is the risk-free rate of return, $R_m$ is return from the market as a whole, $\beta_i$ is the beta factor of asset i and ($R_m$ - $R_f$) is the market premium.

Sharpe established the relationship between an individual asset and the return of an efficient portfolio which contains the asset. He formulated that assets which were more responsive to the efficient portfolio should have higher expected returns. Thus in equilibrium asset prices adjusts linearly with assets responsiveness to systematic risk of the efficient portfolio and the expected returns of the assets (Reilley & Brown, 2012).

CAPM is important to this study because it links together non-diversifiable risk and return for all assets. The model is concerned with how systematic risk is measured and how it affects required returns and security values. The higher the risk the higher the premium investors will require to be induced to hold the asset. The model thus links security return to its risk.

## 2.3 Empirical Literature

Evans and Archer (1968) examined the relationship between diversification and the level of variation of returns for randomly selected portfolios. They used 470 securities listed in the Standard and Poor's Index for the period January 1958 to July 1967. They used ordinary least squares (OLS) regression to analyze the data. T-tests and F-tests were then performed to test for significance and convergence respectively. The results of the analysis indicated a relatively stable and predictable relationship exists between number of securities included in a portfolio and the level of portfolio dispersion. They concluded that a portfolio consisting of 10 different stocks was sufficiently diversified and that the results of their study raised doubt on the economic justification of raising portfolio sizes beyond 10 securities.

Fisher and Lorie (1970) examined frequency distributions and dispersions of wealth ratios of investments in different sized portfolios of stocks listed in the NYSE. Using mean variance model, they used data from1926 to 1965 with equal initial investments made in each stock in a portfolio. The study found out that reduction of dispersion by increasing the number of stocks in the portfolio is rapidly exhausted with 40% achievable reduction obtained by holding two stocks, 80% with eight stocks, 90% with 16, 95% with 32 and 99% with 128 stocks respectively.

Solnik (1974) examined the additional portfolio risk reduction that could be achieved by diversifying internationally. He studied the weekly stock returns of 8 countries over the period 1966 to 1971. He used data on more than 300 stocks from the U.S. and seven major European markets of U.K., France, Germany, Switzerland, Belgium, Italy and Netherlands. Using the mean variance model, he found out that, whether hedged against exchange rate risk or not, an internationally diversified portfolio is likely to carry a much smaller risk than a domestic portfolio. Another finding of the study was that well diversified stock portfolios from most of the European markets had higher proportions of systemic risk than those for U.S. stocks.

Tsui, Low and Kwok (1983) employed monthly data of 40 common stocks listed on the Securities Exchange of Singapore to analyse systematic and unsystematic risks. The period of study was June 1973 to December 1981 and used mean variance model. They found that 40 randomly selected securities in a portfolio gave a well diversified portfolio for the Singapore stock market.

Statman (1987) investigated how many stocks made a diversified portfolio. He used data over five years 1979 to 1984 and different sized portfolios of randomly selected stocks listed in the Standard & Poor's Index. He used a 500-stock benchmark portfolio to compare with other less diversified portfolios using mean variance optimization model and the security market line to allow for borrowing and lending. The study showed that a well diversified portfolio of randomly chosen stocks must include at least 30 stocks for a borrowing investor and 40 stocks for a lending investor. This is contrary to widely held notion that most of the diversification benefits are exhausted with a portfolio of 10 stocks. He suggested that diversification should be increased as long as the marginal benefits of risk reduction exceed the marginal costs of increasing portfolio size.

Cleary and Copp (1999) evaluated diversification with Canadian stocks using the mean variance model. They used data on mean rates of return and monthly standard deviations of those returns for a randomly selected sample of 222 stocks listed in the Toronto Stock Exchange (TSE) over thirteen year period from January 1985 to December 1997. The results of the study indicated that 30 to 50 Canadian stocks are required to capture most of the diversification benefits. However substantial benefits occur by diversifying across as few as 10 stocks.

Byrne and Lee (2000) tested the empirical relationship between asset sizes, the level of diversification of UK property portfolios. They used a sample of 136 property funds and data over 11 years from 1989 to 1999. The study used multiple regression analysis of both systematic and specific risk against size and a series of variables describing the portfolio investment structure of the funds. Results of the study showed that size is negatively related to specific risk but positively related to systematic risk. This runs counter to portfolio theory which predicts that only specific risk is affected by portfolio size. They concluded that there was significant positive correlation between size and risk.

Gupta, Khoon and Shahnon (2001) examined the relationship between portfolio risk and the number of stocks in the portfolio for a given return in the Malaysian stock market for the period September 1988 to June 1997. They used 213 stocks traded on the Kuala Lumpur Stock Exchange (KLSE) and applied the random diversification approach based on Statman (1987) technique. The study found out that, on average a well diversified portfolio of stocks contain at least 27 randomly selected securities. The study extended to determine optimal portfolio for borrowing and lending investors. They concluded that 30 securities give well diversified portfolio for borrowing investors while for lending investors 50 securities were required.

Ahuja (2011) evaluated portfolio diversification in the Karachi Stock Exchange using mean variance model. He used data on daily returns for 15 randomly selected securities over three year period 2007 to 2009. From the results he concluded that diversification theory is applicable in the Karachi Stock Exchange and a reduction of 52.25% of risk was achieved. The results indicate that 10 securities can diversify away significant amount of risk.

Rani (2013) used mean variance optimization model to investigate the relationship between portfolio size and risk in the Indian stock market. He used a random sample of 225 securities listed in the Bombay Stock Exchange (BSE-500). The study period was 11 years from 2001 to 2011 and used secondary data to calculate daily security returns. He applied regression method to test the hypothesized relationship between portfolio size and risk. The results of the study showed that portfolio risk decreased as the number of securities increased. The study doubted the 20-30 securities range as the minimum number of securities for a well diversified portfolio suggested by prior studies.

The empirical studies reviewed indicated varied optimal portfolio sizes. This varied from 10 to 50 securities. There is also no consensus as to whether or not diversification benefits are dependent on the market involved. Most of the studies have been done in developed markets. This necessitates a study to determine the optimal portfolio size in the Kenyan capital markets.

## 2.4 Research on Optimal Portfolio in Kenya

Nyaraji (2001) evaluated the risk reduction benefits of portfolio diversification at the NSE. The study used mean-variance analysis model and the period of study was 1996 to 2000. He used a census of 49 companies listed on the NSE. The study used weekly returns computed from secondary data on share prices and dividend distributions of the quoted securities. The study indicated a significant risk reduction at the NSE as the portfolio grew in size up to 13 securities after which risk reduction becomes insignificant. He concluded that 13 securities were the risk minimizing portfolio size at the NSE. The study applied correlation empirical model and was done over twelve years ago when few firms were listed and few investors participating. The current study will apply regression empirical model to determine the optimal portfolio size for investors in Kenyan stock market and to contribute in bridging the knowledge gap that exists.

Kamanda (2001) evaluated quoted equity portfolios held by Kenyan insurance companies and the extent of their diversification. He determined the relationship between different equity portfolios of respective insurance companies and the NSE-20 share index. The study used both primary and secondary data to generate portfolio returns. Regression analysis was used to derive the beta. Four models: Sharpe, Treynor, Jenssen and coefficient of variation were used to determine the relative performance and the extent of diversification. From the study he concluded that quoted equity portfolios held by Kenyan insurance companies were poorly diversified and the insurance industry portfolio performed much worse than the market portfolio. If the optimal portfolio size at the NSE is determined, it will help insurance managers in their decision making and improve performance.

Mwangangi (2006) surveyed the applicability of Markowitz's portfolio optimization model in overall asset allocation decisions by pension fund managers in Kenya. He used a questionnaire and secondary data from RBA on funds allocation for three years from 2003 to 2005. The results of the study showed that 60% of the fund managers applied the Markowitz's optimization model in their allocation criteria. From the survey he concluded that most fund managers considered the model in their allocation criteria and the key challenge faced was client investment constraints. Determining the optimal portfolio size in the Kenya is essential to fund managers in their allocation decisions and improvement of performance of funds under their management.

## 2.5 Summary

Kenyan investors have a wide range of assets to choose from. The Kenyan capital markets have continued to develop with more listings and increased investor participation in the NSE. Without information investors may be tempted to invest all available stocks which is costly due to increased maintenance costs or invest in very few or what is new (e.g. new listings) and miss on the benefits of diversification. Determining the optimal portfolio size will help them in making decisions in the selection of assets to invest in based on trade off between risk and return. Few studies have been done in Kenya to determine the optimal portfolio size. There is thus need for more studies in this area to support investor decisions.

Most of the empirical studies reviewed have indicated varied optimal sizes. This has ranged from 10 to over 50 securities. Studies in the same market have yielded different results for example Malaysian stock market (Gupta et al, 2001; Zulkifli et al, 2008). Nyariji (2001) did a study and concluded that the risk minimizing portfolio for stocks listed at the NSE was 13. This study determined the optimal portfolio size at the NSE to help bridge the knowledge gap and contribute to the debate.

# CHAPTER THREE
# RESEARCH METHODOLOGY

## 3.1 Introduction

This chapter discusses research methodology. Section 3.2 presents and justifies the research design. Section 3.3 discuses the study population, the sampling method and the period covered by the study. Section 3.4 discusses the type of data for the study, its measurement and the data collection instruments. Section 3.5 discusses data analysis, presents conceptual and analytical models and relevant tests of statistical significance.

## 3.2 Research Design

This was an empirical quantitative study based on secondary data to determine the optimal number of securities to hold for an investor in the Kenyan capital markets. Due to the historical nature of securities prices data collected and analyzed were treated as secondary sources. Statistical measures were used to analyze the relationship between portfolio variance (as a measure of risk) and the portfolio size. Most of the earlier studies on optimal portfolio size have employed this design hence this ensures consistency and ease of comparability of the results against earlier studies.

The study used monthly returns of listed equities over a five year period from January 2009 to December 2013 to calculate portfolio variance and standard deviation as a measure of risk. Portfolios of increasing number of securities were constructed by random selection of the assets. For each portfolio size, several samples were drawn and their mean and variance averaged to obtain a representative portfolio risk.

## 3.3 Population and Sample

The population for the study comprised all 60 (sixty) firms listed in the Nairobi Securities Exchange. The study used a census of all securities in the population which had complete information on prices for all the months over the study period January 2009 to December 2013 (i.e. not delisted or suspended from trading over the study period).

Portfolios of increasing number of securities were constructed by random selection of assets. For each portfolio size several samples were randomly drawn and their mean and variance of returns averaged to give a representative portfolio risk. According to the law of large numbers, this avoided undue bias of one or few securities on the results.

## 3.4 Data and Data Collection Instruments

The study used secondary data of monthly opening and closing security prices and dividend distributions over the study period January 2009 to December 2013. The data was obtained from Nairobi Securities Exchange, Capital Markets Authority websites, published company annual reports, Newspapers and periodicals on capital markets.

From the monthly opening and closing prices for each security and dividend distribution, expected return, variance and standard deviation of returns were computed.

## 3.5 Data Analysis

Statistical measures of arithmetic mean, variance and standard deviation of returns were used. Regression analysis was used to test the variables relationship.

### 3.5.1 Conceptual Model

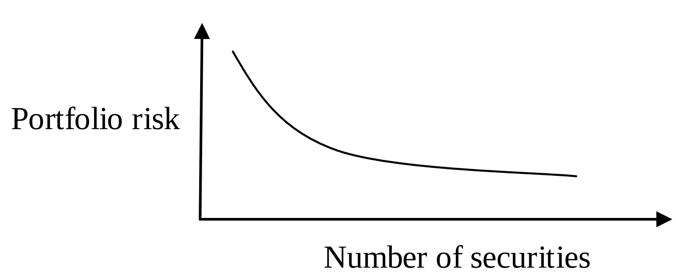
Portfolio Variance (or Standard deviation) was used as the measure of risk. The number of securities combined constituted portfolio size. Portfolio variance is computed from dispersion of security rates of return from their mean. Using equally weighted portfolios, the time series standard deviation is expected to decline to an asymptote as the number of securities in the portfolio increases (Evans & archer, 1968).

Portfolio risk is negatively related to portfolio size. As the number of securities in a portfolio (diversification) increases, portfolio risk decreases. The model below represents that relationship.

$$Y = \beta\left(\frac{1}{X}\right) + A \qquad (9)$$

Where Y is portfolio standard deviation (risk), X is portfolio size, β is a parameter of the model and A is a constant

Figure 2: graphic conceptual model



### 3.5.2 Analytical Model

The study applied the mean variance model (Evans & Archer, 1968; Cleary & Copp, 1999). According to Brealey and Myers (1991) return and risk calculation for each security is as follows $R_t = \frac{(P_t - P_{t-1}) + D}{P_{t-1}}$ (10)

where $P_t$ is the price at the end of period, $P_{t-1}$ is the price at the beginning of the period, $R_t$ is the security return for the period, t is the period of return, D is the dividend paid over the period and n is the total number of periods considered.

The mean security return $\bar{R} = \frac{1}{n}\sum_{t=1}^{n} R_t$ (11)

Security Variance $(\sigma^2) = \frac{1}{n}\sum_{t=1}^{n}(R_t - \bar{R})^2$ (12)

Security standard deviation (σ) = √variance

Total market Return = sum of all securities returns for the five years

Average market return ( $\overline{R_m}$ ) = Total market return / Number of periods considered

Security Covariance with Market

Covariance $(R_m R_i) = \frac{1}{n}\sum_{t=1}^{n}(R_m - \overline{R}_m)(R_i - \overline{R_i})$ (13)

According to Pandey (2005) Portfolio Risk for naive N-security portfolio is given by

$\sigma_p^2 = N\left(\frac{1}{N}\right)^2 \times \text{Average Variance} + (N^2 - N)\left(\frac{1}{N}\right)^2 \times \text{Average Covariance}$ (14)

$\sigma_p^2 = \frac{1}{N} \times \text{Average Variance} + \left(1 - \frac{1}{N}\right) \times \text{Average Covariance}$ (15)

Where N is the number of securities in the portfolio

Portfolio standard deviation is the square root of portfolio variance

$\sigma_p = \sqrt{\sigma_p^2}$ (16)

Portfolio standard deviation (risk) was plotted against number of securities and the point at which the curve became asymptotic was the optimal portfolio size.

Regression analysis was used to measure strength of variables relationship at 95% confidence level ($p<0.05$). The estimated regression model equation (9) was used. Portfolio standard deviation was regressed on the inverse of portfolio size.

# CHAPTER FOUR
# DATA ANALYSIS, RESULTS AND DISCUSSION

## 4.1 Introduction

This chapter presents data analysis and results together with their discussion. Section 4.2 presents descriptive statistical measures of securities and portfolios mean monthly rates of return and their variances. Section 4.3 presents the inferential statistical results indicating the strength of the relationship between portfolio size and portfolio risk. Section 4.4 discusses the results. A summary of the chapter will be given in section 4.5.

## 4.2 Summary Statistics

A total of forty three (43) securities had complete information on returns for the study period January 2009 to December 2013. The forty three were the securities used the study and are provided as appendix A. Table 1 indicates the securities mean monthly rates of return, variances from expected rates of monthly returns and the securities covariances with total market monthly rates of return. Seven securities had negative mean monthly rates of return while thirty six had positive returns. The mean monthly rates of returns ranged from -1.44% to 4.79%. The average monthly rate of return was 1.26% over the 60 month study period.

Thirty random samples were drawn to determine the average variance of securities for each portfolio size from a portfolio of one security to a thirty security portfolio. Appendix 3 shows the results of the sampling.

The portfolio risk was then determined using the naive N-security formula

$$\sigma_P^2 = \frac{1}{N} * \text{Average variance} + \frac{N-1}{N} * \text{Average covariance of all securities} \qquad (17)$$

Where N is the number of securities in the portfolio

$\sigma = \sqrt{\text{variance}}$

Table 2 presents the portfolio variances and standard deviations calculated.

Table 1: Securities mean monthly rates of returns, variances and covariances

| security | mean return | mean variance | covariance with market |
|---|---|---|---|
| ATHI RIVER MINING | 0.0300927 | 0.0069056 | 0.0027656 |
| BAMBURI CEMENT | 0.0099881 | 0.0047271 | 0.0019527 |
| BRITISH AMERICAN TOBACCO | 0.0346510 | 0.0036429 | 0.0011855 |
| BARCLAYS BANK | 0.0023454 | 0.0193697 | 0.0041521 |
| CROWN BERGER | 0.0373895 | 0.0127001 | 0.0034285 |
| CAR & GENERAL | 0.0032534 | 0.0115842 | 0.0022979 |
| EAST AFRICAN CABLES | 0.0022105 | 0.0089671 | 0.0027889 |
| CFC STANBIC BANK | 0.0117935 | 0.0106387 | 0.0019639 |
| COOPERATIVE BANK | 0.0157726 | 0.0104212 | 0.0032228 |
| DIAMOND TRUST BANK | 0.0227795 | 0.0062131 | 0.0026819 |
| EAST AFRICA BREWARIES | 0.0176268 | 0.0070210 | 0.0022678 |
| EGAADS | -0.0004196 | 0.0227453 | 0.0025106 |
| EQUITY BANK | 0.0178805 | 0.0141698 | 0.0032754 |
| EVEREADY | -0.0000283 | 0.0107833 | 0.0027386 |
| FIRESTONE (SAMEER) | 0.0090704 | 0.0147443 | 0.0038900 |
| HOUSING FINANCE COMPANY | 0.0184541 | 0.0100815 | 0.0032893 |
| CENTUM (ICDC) | 0.0170803 | 0.0155293 | 0.0051036 |
| JUBILEE HOLDING | 0.0205481 | 0.0105957 | 0.0037945 |
| KAPCHORUA TEA | 0.0196883 | 0.0113911 | 0.0019266 |
| KENYA COMMERCIAL BANK | 0.0206165 | 0.0084935 | 0.0027758 |
| KENYA ELECTRICITY GENERATING CO | 0.0027796 | 0.0110904 | 0.0031790 |
| KENOL KOBIL | 0.0021448 | 0.0240514 | 0.0021050 |
| KENYA REINSURANCE | 0.0107198 | 0.0131810 | 0.0031589 |
| KENYA POWER | 0.0058499 | 0.0068963 | 0.0021287 |
| KENYA AIRWAYS | -0.0046012 | 0.0158096 | 0.0031019 |
| KUKUZI LTD | 0.0366498 | 0.0140353 | 0.0032871 |
| MUMIAS SUGAR | -0.0014082 | 0.0166430 | 0.0031137 |
| NATIONAL BANK | 0.0008246 | 0.0141948 | 0.0038332 |
| NIC BANK | 0.0134152 | 0.0077038 | 0.0025482 |
| NATION MEDIA GROUP | 0.0205914 | 0.0067824 | 0.0023885 |
| OLYMPIA CAPITAL HOLDINGS | -0.0075113 | 0.0115182 | 0.0018991 |
| PANAFRICA INSURANCE | 0.0195247 | 0.0161309 | 0.0033227 |
| EAST AFRICA PORTLAND CEMENT | 0.0037174 | 0.0150603 | 0.0018549 |
| SASINI TEA | 0.0214933 | 0.0140396 | 0.0036407 |
| SCAN GROUP | 0.0168622 | 0.0127470 | 0.0025758 |
| STANDARD CHARTERED BANK | 0.0164396 | 0.0037851 | 0.0017734 |
| SAFARICOM | 0.0265137 | 0.0077673 | 0.0024229 |
| STANDARD GROUP | -0.0068123 | 0.0064529 | 0.0019321 |
| TOTAL KENYA | 0.0031593 | 0.0079059 | 0.0012494 |
| TPS SERENA EA | 0.0044667 | 0.0114645 | 0.0038993 |
| UNGA GROUP | 0.0140529 | 0.0090025 | 0.0025507 |
| WILLIAMSON TEA | 0.0478993 | 0.0417524 | 0.0047585 |
| EXPRESS KENYA | -0.0143762 | 0.0069786 | 0.0020676 |

Source: Calculation by author

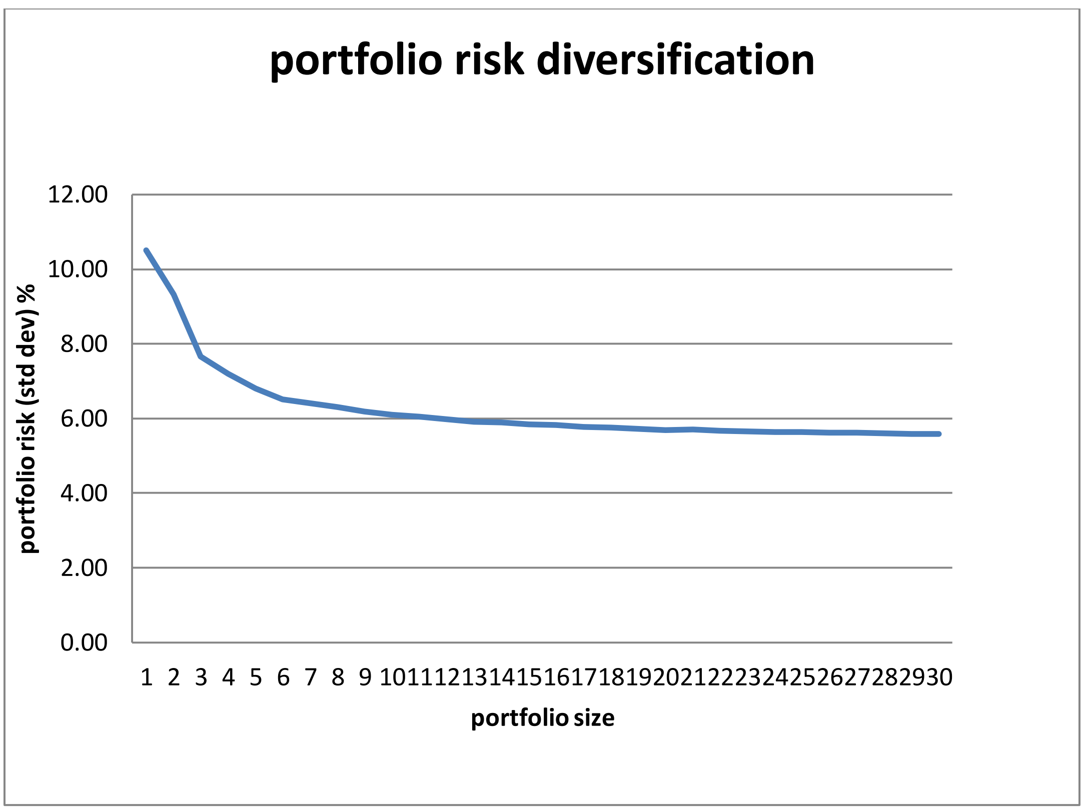
Table 2: Portfolio size, variance and standard deviation of monthly returns

| portfolio Size | portfolio variance | portfolio risk (std dev) % | risk reduction % |
|---|---|---|---|
| 1 | 0.011035 | 10.50 | 0 |
| 2 | 0.008677692 | 9.32 | 11.32 |
| 3 | 0.005859589 | 7.65 | 27.13 |
| 4 | 0.005183787 | 7.20 | 31.46 |
| 5 | 0.004637106 | 6.81 | 35.18 |
| 6 | 0.004231986 | 6.51 | 38.07 |
| 7 | 0.004117614 | 6.42 | 38.91 |
| 8 | 0.003969835 | 6.30 | 40.02 |
| 9 | 0.003826452 | 6.19 | 41.11 |
| 10 | 0.003731245 | 6.11 | 41.85 |
| 11 | 0.003653803 | 6.04 | 42.46 |
| 12 | 0.003565934 | 5.97 | 43.15 |
| 13 | 0.003502815 | 5.92 | 43.66 |
| 14 | 0.003469141 | 5.89 | 43.93 |
| 15 | 0.003420024 | 5.85 | 44.33 |
| 16 | 0.003386797 | 5.82 | 44.60 |
| 17 | 0.003338243 | 5.78 | 45.00 |
| 18 | 0.003322528 | 5.76 | 45.13 |
| 19 | 0.003278047 | 5.73 | 45.50 |
| 20 | 0.003238464 | 5.69 | 45.83 |
| 21 | 0.00326427 | 5.71 | 45.61 |
| 22 | 0.003215502 | 5.67 | 46.02 |
| 23 | 0.003197366 | 5.65 | 46.17 |
| 24 | 0.003183325 | 5.64 | 46.29 |
| 25 | 0.003173008 | 5.63 | 46.38 |
| 26 | 0.003155984 | 5.62 | 46.52 |
| 27 | 0.003154628 | 5.62 | 46.53 |
| 28 | 0.003140155 | 5.60 | 46.66 |
| 29 | 0.003121646 | 5.59 | 46.81 |
| 30 | 0.00311957 | 5.59 | 46.83 |

Source: Calculation by author

The results indicate that portfolio risk decreases as the number of securities in the portfolio (diversification) increases. The rate of risk reduction is high initially with 40% of diversifiable risk being eliminated by holding a portfolio of eight securities. The rate of risk reduction then slows with only an additional 7% risk reduction being eliminated with the increase of portfolio size from 8 to 30 securities. This is supported by the plot in figure 3 showing that portfolio risk reduces to an asymptote as the number of securities in the portfolio is increased. This occurs at a portfolio size of between 18 and 22 securities portfolios.

Figure 3: Portfolio risk diversification on the NSE




Source: Plot by author from study findings

## 4.3 Empirical Model

The results in Table 1 and the plot on figure 3 suggest an inverse relationship between portfolio risk and portfolio size. This was tested using the following regression model

$$Y = \beta \left(\frac{1}{X}\right) + A \quad (18)$$

Where X is the portfolio size, β is a parameter of the model, Y is the mean portfolio risk and A is a constant. Portfolio risk was regressed against the inverse of portfolio size. The regression results are presented in tables 3, 4 and 5.

Table 3: Regression Model Goodness of fit results

| Model | Multiple R | $R^2$ | Adjusted $R^2$ | Standard error |
|---|---|---|---|---|
| $Y = \beta \left(\frac{1}{X}\right) + A$ | 0.976032 | 0.952639 | 0.950948 | 0.248088 |

Source: Research findings

The model yielded a good fit as evidenced by a coefficient of determination ( $R^2$ ) of 0.952639. This indicated that 95.26% of portfolio reduction in risk can be explained by increase in the number of securities in the portfolio.

Table 4: Analysis of Variance (ANOVA)

| | df | SS | MS | F | Sigf. F |
|---|---|---|---|---|---|
| Regression | 1 | 34.66388 | 34.66388 | 563.2048 | 4.37E-20 |
| Residual | 28 | 1.723332 | 0.061548 | | |
| Total | 29 | 36.38721 | | | |

Source: Research findings

The results indicate that portfolio size is significant in determining the level of portfolio risk. This is supported by a significance level of less than 0.05 ($p<0.05$)

Table 5: Regression Model coefficients

| | Coefficients | Standard error | t Stat | P-Value |
|---|---|---|---|---|
| Intercept (A) | 5.504822 | 0.055336 | 99.48059 | 3.03E-37 |
| X Variable 1(β) | 5.664949 | 0.238706 | 23.73194 | 4.37E-20 |

Source: Research findings

Fitting the coefficients in table 5 into the regression model, results into the following equation. This is at a significance level less than 0.05 ($p < 0.05$)

$$Y = 5.664949*\left(\frac{1}{X}\right) + 5.504822 \quad (19)$$

As the number of securities in a portfolio increases the first component of the right hand side of the model decreases and this yields a lower figure for portfolio risk Y. This is because the coefficient 5.664949 is being divided by larger figures as the number of securities in the portfolio (X) increases.

## 4.4 Discussion

The study sought to determine the optimal number of securities an investor in the Nairobi Securities Exchange should hold to minimize exposure to diversifiable risk. Naive portfolios were constructed and their variance and standard deviations of monthly returns determined. The results of regression of portfolio risk (std. deviation) against the reciprocal of portfolio size indicated a strong inverse relationship as indicated by a high coefficient of determination of 0.952639. This indicates that 95.26% of the decrease in portfolio risk can be explained by increase in the number of securities in the portfolio. The results show a very strong relationship of the variables and a very good fit of the model to the study. This supports portfolio theory and the diversification principle (Markowitz, 1952).

Investors aim to maximize returns and minimize risk. The results of the study show that 40% of risk reduction is achieved by holding eight securities. Addition of 8 more securities eliminates a further 4% of risk while addition of the next fourteen securities yields only a further 3% of diversifiable risk reduction. This supports earlier studies that indicated that most diversification benefits are achieved rather quickly and are gained by forming portfolios containing 8-20securities (Newbould & Poon, 1993; Evans & Archer, 1968). This study shows that increasing portfolio size from 8 to 30 securities achieves only 7% further reduction in risk. The results thus indicate investors in the NSE will gain maximum diversification benefits by holding portfolios containing 18-22 stocks.

## 4.5 Summary

The results of the study show that portfolio risk reduced with increase in portfolio size. This supports the diversification concept and modern portfolio theory. Initially the risk reduction is rapid with 40% risk reduction with 8 securities but after portfolio size of 8 rate of reduction becomes minimal. The regression results show that there is a strong inverse relationship between portfolio risk and portfolio size evidenced by a high coefficient of determination of 0.952639.

The risk reduced by 40% with 8-securities portfolio, 46% with 20-securities portfolio and 47% with 30-ssecurities portfolio. The risk reduction achieved with 8-securities portfolio represents 85% of the risk reduction achievable with a 30-security portfolio. This indicates substantial benefits of diversification. This finding is especially beneficial to retail investors who may not have the capacity to invest in larger portfolios compared to institutional investors.

# CHAPTER FIVE
# SUMMARY AND CONCLUSION

## 5.1 Introduction

This chapter gives a summary of the study and the conclusions drawn from the study. Section 5.2 provides a summary of the study. Section 5.3 draws conclusions on the study while section 5.4 highlights the limitations of the study. Section 5.5 provides recommendations for further research.

## 5.2 Summary of the Study

Investors continually deal with the trade-off between risk and return. This creates conflicting objectives of maximizing expected return and minimizing risk which have to be balanced against each other. Diversification is one of the tools to minimize risk. This study aimed to contribute to the existing information gap on the optimal number of securities to invest in and help Kenyan investors achieve maximum diversification benefits.

The study applied the mean variance model to determine the optimal portfolio size in the NSE. The population of the study comprised securities of all firma listed at the NSE with complete return information over the study period of sixty months. Forty three securities met the criteria over the period January 2009 to December 2013. Monthly closing prices were used to determine mean monthly returns and their variances for the securities. The average market return and the covariances of the securities with market return were then determined.

Portfolios consisting of one to thirty securities were constructed by random selection of securities with 30 samples drawn for each portfolio size. This was to ensure that the resulting portfolios were representative. The variance and standard deviation of returns were determined for each portfolio size. This enabled the determination of the extent of risk reduction increasing number of securities in a portfolio. From the results of the study 40% risk reduction in the NSE is achieved with a portfolio of 8 securities. Further increase in portfolio size results in minimal risk reduction. This is supported by a graph of portfolio risk against portfolio size which becomes asymptotic at between 8-12 securities. Regression of portfolio risk against the inverse of portfolio size yielded a high coefficient of determination 0.952639 suggesting 95.26% of portfolio risk reduction can be explained by increase in the number of securities in the portfolio.

## 5.3 Conclusions

This study sought to determine the relationship between portfolio risk and portfolio size, the optimal portfolio size in the Nairobi Securities Exchange and the extent of risk reduction achieved by diversification. The results of the study indicate that diversification results in risk reduction benefits. Portfolio risk decreased as the number of securities in the portfolio increased. This was supported by a plot of portfolio standard deviation against portfolio size which reduced to an asymptote. The results of regression of portfolio risk against the inverse of portfolio size, further supports the strong inverse relationship as indicated by a high coefficient of determination ($R^2$) of 0.952639. This implies that 95.26% of the portfolio risk reduction can be explained by the increase in the number of securities in the portfolio. The model coefficients are significant at $p<0.05$.

The results of the study indicate that portfolio risk at the NSE reduced from 10.5% standard deviation for one-security portfolio to 5.59% standard deviation with 30-security portfolio. Risk reduction is initially rapid with risk reduction of 40% is achieved with a portfolio size of 8 securities. Adding 8 more securities achieves a further 5% reduction in portfolio risk while the next 14 additional securities achieves only a further 2% risk reduction. The curve of the plot of portfolio risk against portfolio size becomes asymptotic at 18 securities. Using this together with the results of risk reduction calculation it can be concluded that the optimal portfolio size for investors in the Nairobi Securities Exchange is between 18 and 22 securities. This information will help investors avoid under- or over- diversification. Investors will thus be exposed to reduced unsystematic risk if they diversified their investments to hold such a portfolio size. The results support Modern Portfolio theory that portfolio risk and portfolio size are inversely related thus portfolio risk decreases as the number of securities in the portfolio increases and the applicability of the diversification concept in the Nairobi Securities Exchange. However increasing portfolio size results in additional operational costs of maintaining the portfolio. Holding portfolios higher than the optimal size must be justified by conducting cost benefit analysis (Evans & Archer, 1968).

## 5.4 Limitations of the Study

Many investors consider multi-period holding of securities. However the mean-variance optimization model assumes single holding period. Distribution of dividends is done periodically either semi-annually or annually. In calculating the monthly security returns the dividends are distributed evenly over the 12 months which may affect the results. Historical data for security prices from the NSE is not readily available but has to be purchased. This limits the amount of data that can be obtained. There was time constraint in conducting the study.

The study does not address the issue of how much money an investor requires to enable the application of the optimal portfolio size diversification strategy. This can be a challenge to retail investors. The study does not address the issue of transaction and monitoring costs associated with diversification. These can be substantial and limit the benefits of diversification. The study considers a specific period January 2009 to December 2013. Due to variation in economic conditions of different time periods compared to the period of this study different result may be obtained.

## 5.5 Recommendations for Further Research

This study used naive (equally-weighted) portfolios. This is different from practice were investors allocate different amounts of investment funds to different securities. Further research can be conducted using differently-weighted portfolios by capitalization.

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# APPENDICES

Appendix A Firms Listed at the NSE used in study

| | Code | Security Name |
|---|---|---|
| 1 | ARM | Athi River Mining |
| 2 | BAMB | Bamburi Cement |
| 3 | BAT | British American Tobacco |
| 4 | BBK | Barclays Bank of Kenya |
| 5 | BERG | Crown Berger |
| 6 | C&G | Car & General |
| 7 | CABL | East African Cables |
| 8 | CFC | CFC Stanbic Bank |
| 9 | COOP | Cooperative Bank |
| 10 | DTK | Diamond Trust Bank |
| 11 | EABL | East African Breweries |
| 12 | EGAD | Eaagads |
| 13 | EQTY | Equity Bank |
| 14 | EVRD | Eveready ltd |
| 15 | FIRE | Sameer E.A. ltd |
| 16 | HFCK | Housing Finance company |
| 17 | ICDC | Centum Investment |
| 18 | JUB | Jubilee Holdings |
| 19 | KAPL | Kapchorua Tea |
| 20 | KCB | Kenya Commercial Bank |
| 21 | KEGN | Kenya Electricity Generating company |
| 22 | KENO | KenolKobil |
| 23 | KNRE | Kenya Reinsurance |
| 24 | KPLC | Kenya Power |
| 25 | KQ | Kenya Airways |
| 26 | KUKZ | Kakuzi Ltd |
| 27 | MSC | Mumias Sugar Company |
| 28 | NBK | National Bank of Kenya |
| 29 | NIC | NIC Bank |
| 30 | NMG | Nation Media Group |
| 31 | OCH | Olympia Capital Holdings |
| 32 | PAFR | Pan Africa Insurance |
| 33 | PORT | East Africa Portland Cement |
| 34 | SASN | Sasini Tea |
| 35 | SCAN | ScanGroup Ltd |
| 36 | SCBK | standard Chartered Bank |
| 37 | SCOM | Safaricom |
| 38 | SGL | Standard Group Ltd |
| 39 | TOTL | Total Kenya |
| 40 | TPSE | TPS EA (serena) |
| 41 | UNGA | Unga Group |
| 42 | WTK | Williamson Tea |
| 43 | XPRS | Express Ltd |

# Appendix B Monthly Securities and Market Return

| | | ARM | BAMB | BAT | BBK | BERG | C&G | CABL | CFC | COOP |
|---|---|---|---|---|---|---|---|---|---|---|
| 2009 | january | -0.07473 | -0.06922 | 0.045469 | -0.03899 | -0.08787 | -0.05751 | -0.09669 | -0.03851 | -0.14707 |
| | february | -0.20241 | -0.18276 | 0.036544 | -0.25415 | -0.27541 | 0.001396 | -0.35508 | -0.09043 | -0.14679 |
| | march | -0.03761 | -0.0089 | 0.005708 | 0.121384 | -0.16604 | -0.1611 | 0.217409 | -0.09173 | -0.13084 |
| | april | 0.232692 | -0.02301 | 0.01547 | 0.085197 | 0.247188 | -0.08789 | 0.130675 | 0.016303 | -0.01973 |
| | may | -0.02344 | 0.042385 | 0.107183 | 0.04391 | 0.411863 | 0.083798 | -0.021 | 0.152083 | 0.146754 |
| | june | 0.19391 | 0.215972 | 0.127319 | 0.237784 | 0.138251 | 0.274419 | 0.159516 | 0.202532 | 0.48164 |
| | july | -0.02823 | -0.05575 | 0.037506 | -0.06755 | 0.052133 | 0.032996 | -0.00762 | -0.03504 | -0.10671 |
| | august | 0.010139 | 0.109681 | 0.006905 | -0.10695 | 0.076282 | -0.00633 | -0.08402 | -0.09225 | -0.02422 |
| | september | 0.032129 | 0.072778 | -0.01557 | -0.03353 | -0.0519 | -0.02196 | -0.04913 | -0.04206 | -0.03378 |
| | october | 0.049393 | 0.019542 | 0.012811 | 0.011041 | -0.03727 | 0.048948 | -0.06714 | -0.10394 | -0.01193 |
| | november | -0.0152 | -0.01001 | 0.007024 | 0.03209 | -0.05013 | -0.15919 | 0.026893 | -0.08 | 0.061671 |
| | december | 0.147369 | -0.01724 | 0.032738 | -0.00769 | 0.050728 | -0.05101 | 0.024791 | -0.02174 | -0.00258 |
| 2010 | january | -0.04202 | -0.0145 | 0.008712 | 0.096039 | 0.075868 | 0.102572 | 0.159671 | -0.04664 | 0.084533 |
| | february | -0.03742 | 0.104957 | 0.100446 | 0.027313 | 0.041993 | -0.025 | -0.03276 | -0.02387 | 0.005464 |
| | march | 0.127736 | 0.11303 | 0.068599 | 0.021477 | 0.078889 | 0.031373 | -0.01981 | -0.08554 | 0.026926 |
| | april | 0.021129 | 0.021564 | -0.02573 | 0.125889 | 0.328958 | 0.06619 | 0.02422 | 0.255985 | 0.189144 |
| | may | 0.113762 | 0.042641 | 0.05994 | 0.038784 | -0.07751 | 0.303803 | -0.13114 | 0.35334 | 0.06515 |
| | june | 0.107872 | 0.003542 | 0.027576 | 0.045101 | -0.05219 | 0.001375 | 0.055213 | 0.084648 | 0.192969 |
| | july | 0.08547 | 0.004792 | 0.185886 | 0.090158 | 0.113526 | 0.001375 | -0.05417 | 0.141715 | 0.000888 |
| | august | 0.079211 | 0.007283 | 0.053796 | -0.00523 | 0.079394 | 0.104467 | -0.01438 | 0.028427 | 0.10696 |
| | september | 0.099119 | 0.05825 | 0.040338 | 0.038655 | -0.01604 | -0.12025 | -0.00035 | 0.015709 | 0.106481 |
| | october | -0.01632 | -0.02496 | 0.038465 | 0.001991 | 0.024453 | 0.065248 | -0.03016 | 0.013326 | 0.092036 |
| | november | -0.0337 | -0.04268 | 0.006231 | -0.07332 | -0.12646 | -0.03867 | -0.13653 | -0.09835 | -0.04497 |
| | december | 0.073539 | -0.04474 | -0.02622 | 0.014659 | 0.135026 | -0.01424 | 0.05092 | -0.05976 | 0.003332 |
| 2011 | january | 0.050926 | 0.062281 | 0.007371 | -0.0132 | 0.061313 | 0.19881 | -0.01397 | 0.058123 | 0.056416 |
| | february | -0.05203 | -0.02262 | 0.066444 | 0.096511 | -0.0323 | -0.0909 | 0.182431 | -0.02264 | 0.003588 |
| | march | -0.05594 | -0.12328 | -0.06687 | -0.15961 | -0.02929 | -0.2415 | -0.22934 | -0.04392 | -0.18685 |
| | april | -0.01127 | -0.0049 | -0.02553 | 0.145239 | 0.023855 | -0.07211 | 0.026195 | -0.21488 | 0.059166 |
| | may | 0.070625 | 0.012801 | -0.04048 | -0.73175 | 0.016416 | -0.07408 | -0.08507 | -0.09781 | -0.00493 |
| | june | 0.021343 | 0.055794 | 0.072029 | -0.01617 | -0.04684 | -0.03374 | -0.04473 | -0.04897 | -0.06227 |
| | july | -0.01925 | -0.01769 | 0.007535 | -0.0921 | -0.01822 | -0.06304 | -0.00991 | 0.026926 | -0.05858 |
| | august | -0.06327 | -0.0381 | -0.10505 | -0.15645 | -0.08782 | -0.05876 | -0.13837 | -0.05455 | -0.07084 |
| | september | -0.06636 | -0.0995 | -0.00499 | -0.16255 | -0.01141 | -0.09006 | 0.006533 | -0.09919 | -0.01172 |
| | october | 0.167224 | 0.005556 | 0.038979 | 0.391325 | 0.017954 | 0.032155 | 0.015606 | -0.07124 | 0.033761 |
| | november | -0.14233 | -0.05444 | -0.01046 | -0.18154 | -0.16264 | -0.17467 | -0.06184 | -0.01307 | -0.13532 |
| | december | 0.019271 | -0.10757 | 0.080616 | 0.101419 | 0.012728 | 0.089325 | 0.018429 | -0.00858 | -0.01016 |
| 2012 | january | -0.03795 | 0.08483 | 0.063596 | -0.05009 | 0.240936 | 0.100916 | 0.124228 | 0.004771 | 0.029536 |
| | february | 0.027693 | 0.080556 | 0.260224 | 0.111021 | -0.0108 | 0.051833 | -0.04254 | 0.048114 | -0.09142 |
| | march | 0.045582 | 0.040517 | -0.11461 | -0.10737 | 0.048284 | -0.08397 | -0.00705 | -0.04022 | 0.087868 |
| | april | 0.149853 | -0.0075 | 0.141959 | 0.063859 | 0.075737 | 0.210243 | 0.049798 | 0.021636 | 0.074713 |
| | may | 0.032407 | -0.01057 | 0.123806 | 0.015885 | 0.144333 | -0.0329 | -0.02927 | 0.073084 | -0.03996 |
| | june | 0.042154 | 0.125 | 0.040856 | 0.026574 | 0.129973 | -0.00265 | -0.04713 | -0.00998 | -0.13715 |
| | july | 0.018213 | 0.115392 | 0.040963 | 0.076125 | -0.0605 | -0.14326 | 0.038714 | -0.0255 | 0.02869 |
| | august | 0.015049 | -0.04997 | 0.069119 | 0.049173 | 0.129349 | 0.05058 | -0.0226 | 0.135385 | 0.004614 |
| | september | 0.07194 | 0.005764 | 0.059521 | -0.00249 | -0.01264 | -0.01297 | 0.037767 | -0.1654 | 0.032239 |
| | october | 0.041854 | 0.042895 | 0.090647 | 0.087274 | 0.126307 | 0.016883 | 0.046252 | -0.05748 | 0.053132 |
| | november | -0.05245 | -0.00189 | -0.01551 | -0.0733 | 0.020666 | -0.03817 | 0.082335 | 0.11745 | -0.03009 |
| | december | 0.018083 | 0.05461 | 0.087543 | 0.097203 | 0.101774 | 0.00191 | 0.026015 | 0.001448 | 0.03826 |
| 2013 | january | 0.045464 | 0.085667 | 0.037887 | 0.032419 | 0.017557 | -0.03733 | 0.13173 | 0.035402 | 0.01112 |
| | february | 0.198466 | -0.03464 | 0.033349 | 0.030895 | 0.003431 | 0.002778 | 0.145404 | 0.066117 | 0.067887 |
| | march | 0.102322 | 0.105036 | 0.024724 | 0.03792 | 0.137549 | -0.03889 | 0.063268 | 0.312042 | 0.19544 |
| | april | -0.08969 | -0.0632 | 0.00571 | 0.028491 | 0.185598 | 0.133333 | 0.036356 | -0.00927 | 0.000102 |
| | may | 0.108838 | 0.098354 | 0.027191 | 0.026655 | -0.00042 | 0.041026 | -0.07237 | 0.068458 | 0.030238 |
| | june | -0.075 | -0.0239 | -0.00335 | -0.12134 | -0.01204 | -0.1179 | -0.01711 | 0.030842 | -0.08488 |
| | july | 0.048221 | -0.00406 | 0.055634 | 0.105993 | 0.020461 | -0.02877 | 0.113461 | 0.030003 | 0.047322 |
| | august | 0.061532 | -0.05894 | 0.005372 | -0.01162 | 0.103436 | 0.078986 | -0.04413 | 0.04238 | 0.006382 |
| | september | 0.010393 | 0.076233 | 0.007114 | -0.00244 | 0.022244 | -0.12862 | 0.050086 | 0.061587 | 0.013863 |
| | october | 0.130477 | -0.02992 | -0.02386 | 0.096429 | 0.039779 | 0.009147 | 0.038024 | 0.077821 | 0.102762 |
| | november | 0.12263 | 0.018921 | 0.039203 | -0.05017 | 0.141805 | 0.266605 | 0.021809 | 0.124286 | 0.036069 |
| | december | -0.01185 | -0.04087 | 0.045205 | 0.008438 | -0.00693 | 0.100134 | -0.00571 | 0.000218 | -0.03797 |

| | | DTK | EABL | EGAD | EQTY | EVRD | FIRE | HFCK | ICDC | JUB |
|---|---|---|---|---|---|---|---|---|---|---|
| 2009 | january | 0.001832 | -0.05697 | 0.003087 | -0.16448 | -0.09065 | 0.036876 | -0.11661 | -0.22902 | 0.077479 |
| | february | -0.28583 | -0.25568 | -0.10815 | -0.28195 | -0.09657 | -0.1943 | -0.19917 | -0.32923 | -0.27404 |
| | march | 0.138773 | 0.147569 | 0.001615 | 0.543253 | 0.048276 | 0.042726 | 0.121892 | 0.047959 | -0.02261 |
| | april | 0.083056 | 0.048595 | 0.047769 | -0.21085 | -0.0625 | 0.162227 | -0.00669 | -0.02434 | 0.107923 |
| | may | 0.095495 | 0.004758 | -0.08669 | 0.020696 | -0.09825 | -0.03345 | 0.057807 | 0.292415 | 0.135693 |
| | june | 0.096384 | 0.250528 | -0.2725 | 0.171688 | 0.035019 | -0.02932 | 0.102322 | 0.240154 | 0.140339 |
| | july | -0.05652 | 0.002954 | -0.01989 | 0.042536 | 0.007519 | -0.01122 | -0.01826 | -0.05106 | -0.07404 |
| | august | 0.009053 | 0.006583 | -0.09534 | -0.15134 | 0.041045 | -0.02908 | -0.03497 | -0.15551 | -0.08746 |
| | september | -0.01958 | -0.04915 | 0.010202 | -0.02066 | -0.05018 | 0.041073 | -0.03633 | -0.10179 | 0.038899 |
| | october | -0.01271 | 0.007932 | 0.002625 | 0.033024 | -0.03774 | -0.05293 | -0.05675 | -0.0891 | -0.11173 |
| | november | -0.00549 | -0.01417 | 0.002625 | -0.06409 | 0.14902 | 0.044468 | 0.077948 | 0.115859 | 0.13375 |
| | december | 0.061629 | 0.041855 | 0.002625 | 0.06395 | -0.00341 | 0.059162 | 0.166236 | -0.0383 | 0.021018 |
| 2010 | january | 0.102051 | 0.038181 | 0 | 0.123371 | 0.303448 | 0.11068 | 0.007157 | 0.164758 | 0.199473 |
| | february | -0.01758 | 0.008038 | 0.276667 | -0.01893 | -0.05291 | 0.145105 | -0.02009 | -0.05749 | -0.03161 |
| | march | 0.128703 | 0.075271 | 0.658676 | 0.013206 | 0.318436 | 0.454962 | -0.02683 | 0.248796 | 0.270062 |
| | april | 0.085612 | 0.047104 | -0.30839 | 0.185349 | 0.019068 | 0.018888 | 0.375269 | 0.150386 | 0.042154 |
| | may | -0.01865 | 0.053433 | -0.04821 | 0.215371 | -0.10603 | -0.15242 | -0.11807 | 0.041341 | -0.02201 |
| | june | 0.071553 | 0.02569 | 0.130435 | 0.062684 | -0.06047 | 0.055893 | 0.027629 | 0.200644 | -0.01685 |
| | july | 0.070885 | 0.01307 | -0.03846 | 0.003201 | -0.05198 | 0 | 0.126107 | -0.02189 | 0.010167 |
| | august | 0.123993 | -0.01214 | 0 | 0.04911 | 0.049608 | 0.03107 | 0.013443 | 0.054363 | 0.141988 |
| | september | 0.018403 | 0.053235 | -0.04 | 0.074458 | -0.04478 | -0.03683 | 0.125661 | 0.101386 | -0.02225 |
| | october | 0.214955 | 0.123347 | 0.087708 | -0.01315 | -0.04687 | -0.06837 | 0.073571 | -0.05193 | 0.000674 |
| | november | -0.01283 | 0.029408 | -0.09021 | -0.02583 | -0.2623 | -0.25124 | -0.11946 | -0.02199 | 0.000675 |
| | december | 0.020984 | -0.07786 | 0.052632 | 0.043839 | 0.125926 | 0.27907 | 0.052087 | -0.0297 | -0.02748 |
| 2011 | january | 0.041595 | -0.10333 | 0.091232 | 0.072384 | -0.11672 | -0.17097 | 0.034275 | 0.023296 | 0.022884 |
| | february | -0.00979 | -0.00882 | -0.12098 | -0.01215 | -0.09643 | -0.03646 | 0.078216 | -0.06197 | -0.04631 |
| | march | -0.03962 | -0.0023 | -0.11792 | -0.12019 | -0.05534 | -0.13062 | -0.12415 | -0.03596 | -0.06079 |
| | april | 0.028973 | 0.129714 | 0.036458 | 0.078626 | -0.09623 | 0.203877 | 0.046784 | 0.068998 | 0.202807 |
| | may | -0.13074 | 0.030542 | -0.00145 | -0.07184 | -0.00926 | -0.08802 | -0.0557 | -0.01744 | -0.09037 |
| | june | -0.03996 | -0.06631 | -0.08771 | 0.030618 | -0.05607 | -0.12085 | -0.06439 | 0.015979 | -0.0618 |
| | july | -0.07897 | -0.06139 | -0.17929 | -0.08599 | 0 | -0.07908 | -0.12543 | -0.17125 | -0.01619 |
| | august | -0.10824 | -0.03662 | 0.040123 | -0.17406 | -0.05941 | -0.12037 | -0.15735 | -0.10912 | -0.10081 |
| | september | 0.000223 | -0.06605 | -0.0256 | -0.07992 | 0.042105 | 0.182114 | -0.03511 | -0.09172 | 0.00422 |
| | october | -0.0529 | -0.04363 | -0.01164 | 0.114501 | -0.06566 | -0.01932 | -0.0183 | -0.00391 | 0.002934 |
| | november | -0.01973 | 0.038727 | 0.107587 | -0.1019 | -0.09189 | -0.16384 | -0.14572 | -0.14977 | 0.027262 |
| | december | 0.012167 | 0.083026 | -0.09178 | -0.06045 | 0.035714 | 0.131467 | -0.06268 | 0.044615 | -0.02839 |
| 2012 | january | 0.026455 | -0.04712 | 0.003255 | 0.066597 | -0.07263 | -0.07091 | 0.105331 | 0.033259 | -0.0171 |
| | february | 0.046241 | 0.140042 | 0.003255 | 0.075207 | -0.12048 | -0.03453 | 0.019594 | 0.035765 | 0.071985 |
| | march | -0.08007 | 0.071193 | 0.003255 | 0.019724 | 0.054795 | -0.03079 | -0.00864 | -0.08978 | 0.113021 |
| | april | 0.116333 | 0.047636 | 0.003255 | 0.084936 | 0.071429 | 0.254915 | 0.053894 | 0.174507 | 0.023005 |
| | may | 0.059942 | 0.039906 | 0.00638 | 0.017533 | 0.157576 | -0.04489 | 0.0604 | -0.0491 | -0.1128 |
| | june | 0.044372 | 0.041034 | 0.00013 | 0.018268 | -0.10471 | -0.05399 | -0.06753 | -0.15285 | 0.022396 |
| | july | -0.10706 | -0.01472 | 0.222005 | 0.02272 | 0.087719 | -0.04536 | 0.085141 | -0.00561 | 0.006646 |
| | august | 0.055997 | 0.010064 | 0.412927 | 0.049063 | 0.016129 | -0.05029 | -0.0366 | -0.04839 | -0.00762 |
| | september | 0.109703 | 0.043898 | -0.0872 | 0.024025 | -0.06349 | 0.016266 | -0.03014 | 0.005085 | 0.025443 |
| | october | 0.065659 | 0.034987 | -0.18954 | 0.063411 | -0.06215 | -0.03682 | 0.101531 | 0.10118 | -0.02643 |
| | november | -0.0089 | 0.031402 | -0.42755 | -0.03255 | 0.13253 | 0.141114 | -0.02543 | -0.05283 | 0.052002 |
| | december | -0.0103 | 0.078578 | 0.087702 | 0.020984 | 0.095745 | 0.017234 | 0.046569 | -0.00566 | -0.01141 |
| 2013 | january | 0.121334 | 0.129806 | 0.010526 | 0.107831 | -0.03922 | 0.085616 | 0.154677 | 0.094245 | 0.073438 |
| | february | 0.071986 | -0.06295 | -0.04167 | 0.080327 | -0.03571 | -0.0666 | 0.12256 | 0.089939 | -0.00046 |
| | march | 0.057954 | 0.107028 | 0.121304 | 0.180968 | 0.126984 | 0.183371 | 0.244685 | 0.383916 | 0.37269 |
| | april | 0.05931 | -0.02691 | 0.008143 | -0.05837 | 0.276995 | 0.00677 | 0.029931 | 0.00758 | 0.006361 |
| | may | 0.100475 | 0.221181 | 0.076923 | 0.15728 | 0.073529 | 0.022201 | 0.050175 | 0.165998 | 0.022064 |
| | june | 0.027068 | -0.08139 | -0.02679 | -0.12056 | -0.03082 | -0.07685 | -0.01173 | -0.08559 | -0.13669 |
| | july | -0.07184 | -0.00881 | -0.00917 | 0.052423 | 0.053004 | 0.056818 | 0.053997 | 0.078551 | 0.239437 |
| | august | 0.097719 | -0.14712 | -0.03704 | -0.00831 | -0.07718 | -0.06974 | -0.08327 | 0.067597 | -0.03488 |
| | september | 0.027443 | 0.15605 | -0.13462 | 0.047554 | -0.03636 | 0.05414 | 0.041324 | 0.096405 | -0.04581 |
| | october | 0.020017 | -0.03045 | 0.044444 | 0.044975 | 0.022642 | 0.120445 | 0.072851 | 0.134501 | 0.092707 |
| | november | 0.066896 | 0.019117 | 0 | 0.002954 | 0.0369 | 0.013612 | 0.168681 | 0.040722 | 0.063345 |
| | december | -0.02714 | -0.09999 | 0.117021 | -0.12824 | -0.02847 | -0.05845 | 0.014892 | 0.03692 | -0.04412 |

| | | KAPL | KCB | KEGN | KENO | KNRE | KPLC | KQ | KUKZ | MSC |
|---|---|---|---|---|---|---|---|---|---|---|
| 2009 | january | 0.105511 | -0.1349 | 0.017951 | 0.137159 | 0.047309 | 0.063748 | -0.03238 | 0.074052 | 0.023659 |
| | february | 0.007966 | -0.23958 | -0.13229 | -0.16256 | -0.08737 | -0.10312 | -0.02942 | -0.03442 | -0.26121 |
| | march | 0.007966 | 0.258446 | -0.25682 | -0.19508 | -0.27649 | -0.19317 | -0.28965 | -0.12689 | -0.27822 |
| | april | -0.06556 | 0.043964 | 0.221314 | 0.028883 | 0.4286 | 0.157342 | 0.006768 | 0.16886 | 0.30455 |
| | may | 0.008598 | -0.05385 | -0.05121 | 0.303796 | 0.093182 | 0.014225 | 0.152574 | 0.032197 | -0.09606 |
| | june | 0.056217 | 0.190075 | -0.02158 | -0.01934 | -0.0191 | 0.011765 | -0.02719 | 0.087037 | 0.007994 |
| | july | 0.269571 | -0.01937 | 0.252071 | 0.060365 | 0.000131 | 0.251437 | 0.093873 | 0.245704 | 0.444444 |
| | august | 0.027528 | 0.000151 | -0.04038 | -0.08342 | -0.04486 | -0.01269 | 0.051475 | 0.173611 | -0.01948 |
| | september | 0.006373 | -0.07323 | -0.10175 | -0.03533 | -0.1458 | -0.15023 | -0.10262 | -0.09405 | 0.20605 |
| | october | -0.05245 | 0.017449 | -0.09762 | -0.06454 | -0.03493 | 0.021389 | -0.10319 | 0.054233 | -0.01236 |
| | november | 0.078396 | -0.03151 | -0.05689 | 0.005527 | 0.023496 | 0.190867 | 0.220615 | 0.021465 | 0.012095 |
| | december | 0.009468 | 0.02695 | 0.070367 | -0.01488 | 0.083748 | 0.022607 | 0.37283 | -0.0496 | -0.08453 |
| 2010 | january | -0.08444 | 0.087203 | 0.102357 | 0.224667 | 0.073496 | 0.058229 | 0.40449 | 0.06901 | 0.30106 |
| | february | 0.157219 | -0.07027 | -0.06981 | 0.038459 | 0.011116 | 0.096331 | -0.0176 | 0.401422 | 0.112969 |
| | march | 0.480008 | 0.079793 | 0.128331 | 0.210131 | 0.060108 | 0.0952 | 0.215004 | 0.669653 | 0.055611 |
| | april | -0.04438 | 0.047441 | 0.053731 | 0.095226 | -0.02252 | 0.00718 | -0.04915 | -0.05432 | 0.248647 |
| | may | -0.00345 | -0.1118 | 0.070897 | 0.189881 | -0.09983 | 0.112007 | -0.02559 | -0.00391 | -0.01431 |
| | june | 0.083484 | -0.0851 | 0.017413 | -0.90326 | 0.024715 | 0.005067 | -0.15469 | 0.064167 | 0.00104 |
| | july | -0.05691 | 0.035456 | 0.023039 | 0.057459 | 0.032539 | -0.01914 | 0.034097 | 0.046707 | 0.06901 |
| | august | -0.10342 | 0.001272 | 0.000672 | -0.04799 | -0.01711 | 0.047225 | -0.02336 | 0.016809 | -0.13162 |
| | september | 0.084167 | 0.100643 | 0.005293 | 0.04093 | -0.01994 | 0.149953 | -0.0263 | 0.012484 | 0.040045 |
| | october | -0.03318 | 0.087074 | 0.021973 | 0.082664 | -0.00176 | -0.04736 | -0.02071 | 0.048075 | -0.12208 |
| | november | -0.07292 | -0.04709 | -0.08009 | -0.06114 | -0.02558 | -0.02439 | 0.028947 | -0.05615 | -0.10407 |
| | december | -0.16233 | 0.024655 | 0.019795 | -0.00066 | -0.06394 | 0.066873 | 0.003995 | -0.01792 | 0.001392 |
| 2011 | january | 0.1875 | 0.054611 | -0.02183 | 0.001325 | 0.022986 | -0.05995 | -0.02248 | -0.0023 | -0.0797 |
| | february | -0.03365 | 0.018795 | -0.07827 | -0.05372 | -0.05161 | -0.07422 | -0.13014 | -0.01775 | -0.07909 |
| | march | -0.075 | 0.012582 | -0.05276 | 0.030238 | -0.05935 | -0.04626 | -0.1778 | -0.09599 | -0.10733 |
| | april | -0.03804 | 0.096266 | 0.069396 | 0.048415 | -0.01028 | -0.00012 | 0.120769 | -0.02828 | 0.067084 |
| | may | -0.05795 | -0.00686 | 0.009296 | -0.01374 | 0.019542 | 0.00884 | 0.129339 | 0.078297 | -0.01815 |
| | june | 0.316748 | -0.05043 | -0.12571 | 0.18431 | -0.15354 | 0.01346 | -0.06594 | -0.05099 | -0.028 |
| | july | 0.00463 | -0.03502 | -0.14348 | 0.035256 | -0.01215 | -0.08896 | -0.13995 | -0.01696 | -0.02341 |
| | august | -0.19537 | -0.13006 | -0.11642 | -0.12217 | -0.06865 | -0.01896 | -0.02776 | 0.02646 | -0.07856 |
| | september | -0.07292 | -0.16283 | -0.07653 | -0.03678 | -0.00143 | -0.10772 | -0.17279 | -0.04554 | 0.014345 |
| | october | 0.15201 | 0.115639 | 0.146443 | 0.095433 | 0.058245 | 0.064653 | -0.08253 | 0.132519 | 0.093427 |
| | november | 0.101974 | -0.16478 | -0.19908 | -0.10693 | -0.10577 | -0.0864 | -0.14481 | -0.0625 | -0.28405 |
| | december | 0.005 | 0.125111 | 0.006151 | 0.065179 | 0.022481 | 0.054099 | 0.040923 | 0.002036 | 0.078806 |
| 2012 | january | -0.03131 | 0.146854 | -0.0907 | 0.018293 | 0.020637 | -0.10987 | -0.05402 | 0.031429 | -0.03946 |
| | february | 0.014254 | 0.086047 | -0.07026 | 0.156687 | -0.01015 | -0.0657 | -0.10426 | 0.104464 | -0.02154 |
| | march | 0.057609 | 0.091049 | -0.00951 | 0.032787 | -0.01165 | -0.01304 | -0.19465 | 0.007305 | -0.02835 |
| | april | -0.04442 | 0.01567 | 0.128453 | 0.0401 | 0.335155 | 0.08738 | 0.06763 | 0.059061 | 0.117376 |
| | may | 0.157609 | 0.027134 | -0.01232 | 0.005622 | 0.271389 | -0.05375 | 0.016622 | -0.01457 | 0.100128 |
| | june | 0.023585 | 0.014794 | 0.087829 | 0.147764 | -0.14791 | 0.033127 | -0.12558 | 0.003906 | 0.084651 |
| | july | 0.007074 | 0.043312 | -0.0232 | 0.119694 | -0.06546 | 0.023476 | 0.075324 | -0.05859 | 0.137303 |
| | august | -0.03477 | 0.077787 | -0.00358 | -0.08515 | 0.10986 | 0.123719 | -0.06932 | 0.0051 | -0.07912 |
| | september | -0.03365 | 0.076918 | 0.069964 | 0.027174 | 0.078939 | 0.008849 | -0.07717 | -0.03673 | 0.025499 |
| | october | 0.005 | 0.090085 | 0.031746 | -0.0377 | 0.054457 | 0.066184 | 0.049287 | 0.00434 | -0.06476 |
| | november | 0.005 | -0.05997 | 0.096133 | 0.015808 | -0.08558 | -0.10182 | -0.03637 | -0.05122 | -0.15806 |
| | december | -0.051 | 0.075201 | -0.10334 | -0.08119 | -0.05979 | 0.043289 | -0.03041 | 0.063419 | -0.05356 |
| 2013 | january | -0.04308 | 0.118065 | 0.347032 | 0.005036 | 0.117647 | 0.024075 | -0.04643 | 0.056481 | 0.026477 |
| | february | 0.025926 | 0.145322 | 0.044255 | -0.01626 | 0.045183 | -0.00172 | -0.00375 | 0.088908 | -0.15675 |
| | march | 0.047101 | 0.082306 | 0.191489 | -0.24415 | 0.27846 | 0.091327 | 0.029135 | 0.043019 | 0.058824 |
| | april | 0.045139 | 0.021827 | 0.017229 | -0.04265 | -0.00386 | -0.05421 | 0.000913 | 0.078906 | 0.051111 |
| | may | 0.047333 | -0.00343 | 0.026513 | 0.083506 | 0.108091 | -0.07012 | 0.027372 | 0.050145 | -0.0444 |
| | june | 0.037676 | -0.10972 | 0.006645 | -0.15474 | -0.10545 | -0.12747 | -0.11545 | -0.10764 | -0.07301 |
| | july | -0.07099 | 0.159507 | 0.06755 | -0.00358 | 0.083443 | -0.04047 | -0.06124 | 0.066406 | 0.023866 |
| | august | -0.02067 | -0.01397 | -0.00933 | -0.0297 | -0.08577 | 0 | 0.006417 | 0.0625 | -0.12587 |
| | september | 0.003415 | 0.113055 | 0.036547 | -0.04938 | -0.04873 | 0.025018 | 0.03932 | -0.04653 | -0.008 |
| | october | -0.01298 | 0.050068 | 0.035976 | 0.109536 | -0.00282 | -0.01185 | 0.067485 | 0.10307 | -0.06452 |
| | november | 0.057639 | -0.00833 | -0.06257 | 0.047646 | 0.160879 | 0.05434 | 0.302682 | -0.02593 | -0.01724 |
| | december | -0.00856 | -0.00947 | -0.14972 | 0.04977 | -0.07348 | -0.05957 | -0.04926 | 0.044521 | -0.03509 |

| | | NBK | NIC | NMG | OCH | PAFR | PORT | SASN | SCAN | SCBK |
|---|---|---|---|---|---|---|---|---|---|---|
| 2009 | january | 0.090717 | 0.084666 | -0.02188 | -0.05938 | -0.13844 | 0.05072 | -0.19613 | -0.05464 | 0 |
| | february | -0.08505 | -0.16502 | -0.1852 | -0.11406 | -0.07668 | 0.001275 | -0.21397 | -0.27981 | -0.10881 |
| | march | -0.29357 | -0.10955 | 0.118164 | 0.1725 | -0.09864 | -0.11637 | 0.157153 | 0.087937 | -0.00171 |
| | april | 0.023664 | 0.055537 | 0.035629 | -0.13859 | 0.026515 | 0.001444 | 0.072304 | 0.140543 | -0.02352 |
| | may | 0.177933 | 0.109098 | -0.03681 | 0.163366 | 0.003704 | 0.001444 | 0.055747 | 0.038519 | 0.014721 |
| | june | 0.007434 | 0.120389 | 0.137467 | -0.1266 | 0.075926 | -0.05189 | 0.015326 | 0.12297 | 0.059925 |
| | july | 0.152597 | -0.06427 | -0.09057 | -0.14129 | 0.039724 | -0.01256 | 0.047696 | -0.00878 | -0.05247 |
| | august | 0.011524 | -0.13022 | -0.03756 | 0.009929 | 0.003333 | 0.001548 | -0.01352 | 0.084402 | 0.026333 |
| | september | -0.11165 | -0.0037 | -0.05339 | -0.1573 | -0.00667 | 0.115833 | -0.01855 | -0.08357 | 0.020894 |
| | october | -0.00256 | -0.07924 | 0.016596 | 0.091667 | -0.08754 | 0.02703 | 0.138979 | -0.03524 | 0.019464 |
| | november | -0.07971 | -0.00302 | 0.038731 | -0.04733 | 0.137037 | 0.001354 | 0.121169 | 0.024915 | 0.051157 |
| | december | 0.125737 | 0.080154 | -0.03648 | 0.041667 | -0.11438 | 0.001354 | -0.04848 | 0.036691 | 0.066191 |
| 2010 | january | -0.00349 | 0.110212 | 0.013502 | 0.193548 | 0.072222 | 0 | 0.116915 | 0.020224 | 0.052357 |
| | february | 0.021892 | 0.028963 | 0.041841 | -0.00676 | -0.05729 | 0.0625 | 0.078348 | 0.026766 | 0.079121 |
| | march | 0.442096 | -0.10497 | 0.121188 | 0.14966 | 0.138889 | 0.137294 | 0.64069 | 0.020888 | 0.071752 |
| | april | -0.24033 | 0.202296 | 0.051282 | 0.123077 | 0.102941 | 0.241337 | 0.009852 | 0.001775 | 0.049523 |
| | may | -0.06997 | -0.06846 | -0.01972 | -0.10643 | 0.075893 | -0.04167 | -0.07723 | 0.210078 | 0.102155 |
| | june | -0.01903 | 0.085657 | -0.00017 | -0.04953 | 0.035833 | 0.004348 | 0.060936 | 0.208283 | 0.034333 |
| | july | -0.00955 | 0.016757 | 0.013476 | -0.09181 | 0.102585 | 0.064935 | 0.010054 | -0.07516 | 0.144643 |
| | august | -0.01198 | 0.159789 | 0.123967 | 0.096995 | 0.128676 | -0.02439 | -0.05556 | 0.479889 | -0.00765 |
| | september | 0.001128 | 0.00091 | 0.058248 | -0.15318 | 0.003268 | 0.033333 | 0.001622 | 0.141613 | 0.10503 |
| | october | 0.040994 | 0.057716 | -0.01208 | 0.027941 | 0.009804 | -0.07258 | 0.096077 | 0.102143 | -0.06369 |
| | november | -0.04005 | -0.00741 | -0.02515 | -0.02718 | -0.07468 | -0.13043 | -0.1158 | -0.15706 | 0.045892 |
| | december | 0.001658 | -0.04395 | 0.050064 | -0.12059 | -0.07394 | -0.2 | 0.025032 | 0.077848 | -0.03391 |
| 2011 | january | 0.133616 | 0.080941 | 0.050228 | -0.01675 | -0.11574 | 0.488021 | -0.02068 | 0.034359 | 0.068197 |
| | february | 0.035261 | 0.007484 | 0.028599 | -0.12947 | 0.267664 | -0.14881 | -0.09964 | -0.07513 | 0.020408 |
| | march | -0.19955 | -0.07978 | 0.012961 | -0.03914 | 0.148097 | -0.10082 | -0.1989 | -0.07257 | -0.0573 |
| | april | 0.034645 | -0.01933 | 0.058554 | 0.099796 | 0.021254 | -0.06361 | 0.106507 | 0.162872 | -0.02739 |
| | may | -0.0948 | -0.04127 | -0.00339 | -0.1037 | -0.5539 | -0.00151 | 0.249669 | -0.17689 | -0.01855 |
| | june | -0.02237 | 0.016964 | -0.08616 | -0.10124 | -0.0008 | -0.05833 | -0.09045 | 0.022642 | -0.04084 |
| | july | -0.18797 | -0.12753 | -0.09237 | -0.06207 | -0.249 | -0.06198 | -0.0559 | -0.1446 | -0.03996 |
| | august | -0.12084 | -0.15916 | -0.05889 | 0.041667 | -0.12527 | -0.17278 | 0.039197 | -0.06 | -0.08296 |
| | september | -0.06078 | -0.04173 | 0.002752 | -0.02353 | -0.07545 | -0.01739 | 0.130844 | -0.14624 | -0.1238 |
| | october | 0.072944 | -0.04198 | 0.003185 | -0.06024 | 0.016835 | -0.07126 | 0.121997 | 0.075446 | 0.002961 |
| | november | -0.17562 | -0.07658 | -0.0131 | -0.01026 | -0.09333 | -0.03466 | -0.06116 | 0.004092 | -0.09557 |
| | december | -0.00231 | -0.1067 | 0.02303 | -0.15803 | -0.07659 | 0.028287 | 0.036232 | 0.027541 | -0.00817 |
| 2012 | january | 0.065654 | 0.129092 | 0.068897 | -0.00641 | -0.02651 | 0 | -0.04874 | -0.05179 | -0.02184 |
| | february | -0.00254 | 0.09914 | 0.072761 | 0.166667 | 0.237594 | 0 | -0.01838 | 0.04557 | 0.08587 |
| | march | -0.07936 | -0.10719 | 0.106469 | -0.13889 | 0.015139 | 0 | -0.08251 | 0.228606 | 0.026155 |
| | april | -0.00069 | 0.150655 | 0.015562 | 0.041294 | 0.175081 | 0 | 0.042838 | 0.024491 | -0.04315 |
| | may | 0.090736 | 0.149266 | -0.00666 | 0.117337 | 0.083537 | 0.066667 | 0.1191 | -0.02451 | 0.093055 |
| | june | -0.07452 | 0.052139 | 0.07119 | 0.002148 | 0.097561 | -0.0625 | -0.05867 | 0.053861 | 0.127851 |
| | july | 0.007045 | 0.031643 | -0.00388 | -0.03651 | 0.022388 | -0.06667 | 0.030386 | -0.00583 | -0.0216 |
| | august | -0.05806 | -0.07407 | 0.162301 | -0.04602 | 0.106765 | 0 | -0.05131 | 0.074607 | 0.04247 |
| | september | 0.001452 | 0.00721 | 0.030924 | 0.044601 | -0.00348 | -0.07018 | -0.02665 | 0.02381 | 0.03022 |
| | october | 0.036815 | 0.116288 | 0.079997 | -0.01126 | 0.060811 | -0.38295 | 0.024396 | 0.026379 | 0.097205 |
| | november | -0.10049 | 0.017827 | 0.014292 | -0.0032 | 0.032051 | 0.338313 | 0.031854 | 0.082549 | 0.048369 |
| | december | 0.010325 | 8.69E-05 | -0.03681 | -0.06107 | 0.0125 | -0.09302 | 0.014769 | 0.04212 | -0.03875 |
| 2013 | january | 0.008501 | 0.090382 | 0.178407 | -0.02703 | 0.067702 | 0.076563 | -0.00496 | -0.01239 | 0.115273 |
| | february | 0.033629 | 0.072087 | 0.032909 | 0.013889 | 0.195892 | 0.148663 | -0.07076 | 0.079625 | 0.024671 |
| | march | 0.199529 | 0.18011 | 0.337541 | 0.10137 | 0.172335 | 0.075664 | 0.092803 | -0.00305 | 0.132864 |
| | april | -0.04965 | -0.03908 | -0.21178 | 0.039801 | 0.043032 | 0.024764 | 0.085069 | -0.06534 | -0.07298 |
| | may | 0.081341 | 0.109666 | 0.128476 | 0.239234 | -0.06591 | 0.051843 | 0.09391 | 0.010935 | 0.078737 |
| | june | -0.05567 | -0.06416 | -0.04181 | -0.11197 | -0.08461 | -0.01645 | -0.0436 | -0.10273 | -0.02952 |
| | july | 0.047294 | 0.075844 | 0.03664 | -0.02391 | 0.075243 | -0.04353 | 0.033985 | 0.089145 | 0.050194 |
| | august | -0.06239 | 0.015656 | 0.003897 | -0.17595 | 0.106818 | 0.010514 | -0.00494 | -0.03474 | -0.02222 |
| | september | -0.00061 | 0.033263 | -0.00429 | 0.256757 | 0.022727 | 0.140046 | -0.03804 | -0.04173 | 0.020732 |
| | october | -0.00845 | 0.007001 | 0.026889 | -0.03226 | 0.063008 | 0.15874 | 0.045721 | -0.03345 | 0.016602 |
| | november | 0.36999 | 0.024648 | 0.006193 | -0.13333 | 0.271154 | 0.114642 | 0.014397 | -0.04184 | 0.046913 |
| | december | 0.046909 | -0.01624 | -0.05917 | 0.166667 | 0.098784 | -0.1291 | 0.042522 | -0.13532 | -0.0495 |

| | | SCOM | SGL | TOTL | TPSE | UNGA | WTK | XPRS | mkt return | mkt deviation |
|---|---|---|---|---|---|---|---|---|---|---|
| 2009 | january | -0.11372 | -0.08417 | -0.06958 | -0.1144 | -0.31245 | 0.044872 | -0.11538 | -0.04211 | -0.054741 |
| | february | -0.17552 | -0.00827 | -0.05075 | -0.42371 | -0.20111 | -0.08642 | -0.04348 | -0.15704 | -0.169669 |
| | march | 0.151457 | -0.00636 | 0.187663 | 0.127457 | 0.005563 | -0.03401 | -0.09091 | -0.0064 | -0.019037 |
| | april | -0.06015 | -0.07685 | 0.031341 | 0.160901 | 0.106501 | 0.04539 | -0.113 | 0.054057 | 0.041425 |
| | may | -0.01119 | 0.093293 | -0.14939 | 0.11967 | 0.035 | -0.05055 | 0.014656 | 0.05383 | 0.041198 |
| | june | 0.139187 | -0.00636 | 0.075846 | 0.166387 | 0.217391 | 0.137681 | 0.017778 | 0.096295 | 0.083663 |
| | july | 0.182439 | -0.03241 | -0.02203 | -0.09391 | 0.095238 | 0.717949 | 0.077511 | 0.04737 | 0.034737 |
| | august | -0.02986 | -0.10433 | -0.05515 | 0.007233 | -0.06703 | -0.09457 | -0.00405 | -0.02204 | -0.034675 |
| | september | 0.035451 | -0.06419 | 0.022113 | 0.059084 | -0.27184 | 0.018858 | -0.03357 | -0.03147 | -0.044105 |
| | october | 0.069073 | -0.03679 | 0.019299 | -0.05249 | 0.053333 | 0.084776 | -0.03368 | -0.01105 | -0.023685 |
| | november | 0.230159 | 0.086905 | -0.00865 | -0.01705 | 0.098734 | 0.929924 | -0.10675 | 0.050574 | 0.037942 |
| | december | -0.06973 | -0.00864 | 0.018866 | 0.125604 | 0.099078 | -0.11898 | 0.003659 | 0.027195 | 0.014563 |
| 2010 | january | 0.172137 | 0.030315 | 0.021373 | 0.064465 | 0.003476 | -0.03318 | 0.241379 | 0.088021 | 0.075388 |
| | february | 0.006866 | -0.05668 | 0.034699 | -0.06909 | 0.059524 | 0.038083 | 0 | 0.031185 | 0.018552 |
| | march | 0.038557 | 0.071117 | -0.01985 | 0.426961 | 0.09809 | 0.685047 | 0.281111 | 0.178984 | 0.166352 |
| | april | 0.055255 | 0.082096 | -0.00108 | -0.00799 | 0.056349 | -0.15699 | -0.11101 | 0.054954 | 0.042322 |
| | may | -0.04509 | -0.02309 | -0.00741 | -0.04427 | 0.073454 | 0.055373 | -0.15512 | 0.013626 | 0.000994 |
| | june | 0.047962 | -0.13477 | -0.03373 | -0.05276 | 0.037366 | 0.092604 | 0.039261 | 0.008669 | -0.003964 |
| | july | 0.016638 | 0.076333 | 0.066277 | 0.006969 | 0.021413 | -0.08477 | 0.058889 | 0.028853 | 0.016220 |
| | august | -0.17204 | 0.13398 | -0.01979 | 0.002712 | -0.00228 | -0.04261 | 0.037775 | 0.033984 | 0.021352 |
| | september | -0.08505 | 0.056572 | 0.067479 | 0.077929 | 0.044597 | -0.04289 | 0.011122 | 0.026982 | 0.014350 |
| | october | 0.1076 | 0.054259 | -0.03576 | 0.125333 | -0.06664 | 0.023112 | -0.05 | 0.019603 | 0.006971 |
| | november | -0.06817 | -0.04448 | -0.04015 | -0.01854 | -0.06831 | -0.09124 | -0.09474 | -0.06377 | -0.076404 |
| | december | 0.041116 | 0.006446 | -0.0018 | 0.00555 | -0.02233 | 0.106926 | -0.09302 | 0.00393 | -0.008703 |
| 2011 | january | -0.04326 | -0.0663 | -0.00043 | -0.0455 | 0.034748 | 0.086712 | 0.034839 | 0.026648 | 0.014016 |
| | february | -0.09896 | -0.05325 | -0.0123 | -0.01037 | -0.02879 | -0.01479 | -0.05611 | -0.02451 | -0.037146 |
| | march | -0.04312 | -0.0265 | -0.09403 | 0.03671 | -0.07113 | -0.05081 | -0.18098 | -0.07935 | -0.091982 |
| | april | 0.040905 | -0.02414 | 0.046623 | -0.00637 | -0.01806 | 0.084009 | -0.05323 | 0.032419 | 0.019787 |
| | may | -0.02099 | -0.09421 | -0.02757 | 0.008196 | 0.015902 | -0.06621 | -0.18228 | -0.05422 | -0.066852 |
| | june | 0.019811 | 0.016851 | -0.0832 | -0.01016 | 0.027118 | 0.15758 | -0.0625 | -0.01882 | -0.031452 |
| | july | -0.08482 | -0.15714 | 0.008397 | -0.06355 | -0.02404 | -0.0462 | 0 | -0.0663 | -0.078928 |
| | august | -0.14898 | 0.022712 | -0.14992 | -0.06815 | -0.07907 | -0.08128 | -0.07333 | -0.0839 | -0.096529 |
| | september | -0.0077 | -0.07193 | -0.11913 | -0.00637 | 0.129281 | 0.005631 | -0.0048 | -0.03957 | -0.052202 |
| | october | 0.015608 | -0.01786 | -0.05833 | 0.03157 | 0.096897 | 0.628441 | -0.03614 | 0.048588 | 0.035956 |
| | november | -0.0574 | 0.023636 | -0.03243 | -0.11313 | 0.034748 | -0.04723 | 0 | -0.07525 | -0.087886 |
| | december | 0.058893 | -0.1119 | -0.05142 | -0.05302 | -0.17838 | -0.00466 | -0.0125 | 0.001611 | -0.011021 |
| 2012 | january | 0.090348 | 0.045657 | 0.108453 | -0.0422 | -0.03553 | -0.06478 | 0 | 0.016049 | 0.003416 |
| | february | -0.0471 | -0.16924 | 0.115226 | -0.05859 | 0.026648 | 0.139706 | 0.0325 | 0.034584 | 0.021952 |
| | march | 0.05847 | -0.02558 | -0.21736 | 0.012258 | 0.04445 | -0.20474 | 0.004843 | -0.01177 | -0.024398 |
| | april | 0.043094 | 0.002387 | 0.129459 | 0.002355 | 0.084372 | 0.050543 | -0.03614 | 0.072064 | 0.059432 |
| | may | -0.01552 | 0.080952 | -0.01905 | -0.06286 | 0.26421 | 0.135373 | 0 | 0.046076 | 0.033444 |
| | june | 0.070041 | 0.048899 | -0.0047 | 0.025775 | -0.04039 | 0.02793 | -0.055 | 0.007408 | -0.005224 |
| | july | 0.106148 | 0.049979 | 0.04943 | -0.05549 | -0.00862 | -0.11205 | -0.00794 | 0.013434 | 0.000802 |
| | august | 0.010035 | -0.0968 | -0.00267 | -0.07459 | 0.029472 | -0.09022 | 0.021333 | 0.018133 | 0.005501 |
| | september | 0.06467 | 0.129318 | -0.07058 | 0.028976 | 0.100198 | -0.01944 | 0.078329 | 0.012709 | 0.000077 |
| | october | 0.088043 | 0 | -0.07137 | -0.02399 | -0.04764 | -0.08807 | -0.0799 | 0.012163 | -0.000470 |
| | november | 0.133409 | -0.06863 | -0.0411 | 0.014092 | 0.051414 | 0.000625 | -0.09211 | -0.00228 | -0.014910 |
| | december | 0.017738 | -0.08421 | 0.040918 | 0.002804 | -0.04291 | 0.005639 | 0.014493 | 0.008221 | -0.004411 |
| 2013 | january | 0.080545 | -0.02704 | 0.000722 | 0.076242 | 0.052299 | 0.002917 | 0.008646 | 0.055587 | 0.042955 |
| | february | 0.064067 | 0.115408 | 0.047757 | -0.03009 | 0.068215 | 0.037917 | -0.02571 | 0.038381 | 0.025749 |
| | march | 0.051939 | 0.281203 | -0.09217 | 0.243497 | -0.03284 | 0.11876 | 0.01173 | 0.120165 | 0.107533 |
| | april | 0.14438 | -0.04299 | 0.261303 | 0.023388 | 0.111607 | -0.0018 | 0.217391 | 0.026017 | 0.013385 |
| | may | 0.064112 | 0.045434 | 0.045704 | 0.015156 | -0.00371 | 0.162971 | -0.0119 | 0.057879 | 0.045247 |
| | june | -0.09115 | -0.15352 | -0.03857 | -0.13131 | -0.00894 | -0.1548 | -0.03614 | -0.06703 | -0.079659 |
| | july | 0.11743 | 0.044635 | -0.04573 | 0.0625 | 0.028491 | 0.047037 | -0.0825 | 0.03651 | 0.023878 |
| | august | 0.061699 | -0.01601 | 0.101282 | 0.022325 | 0.077974 | 0.045035 | 0.008174 | -0.00437 | -0.017007 |
| | september | 0.107824 | -0.02261 | 0.081728 | -0.04485 | 0.023503 | -0.0466 | 0.062162 | 0.021659 | 0.009026 |
| | october | 0.121364 | 0.013016 | 0.334957 | -0.10875 | 0.062977 | -0.07046 | 0.005089 | 0.042762 | 0.030130 |
| | november | 0.146691 | 0.072003 | 0.031275 | 0.10512 | 0.033952 | 0.081404 | -0.04304 | 0.063783 | 0.051151 |
| | december | 0.009165 | -0.07489 | -0.01144 | -0.01568 | -0.02838 | 0.075465 | 0.031746 | -0.01177 | -0.024406 |

## Appendix C Portfolios Construction and Average Variance

| port. Size | sample 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.010596 | 0.01417 | 0.004727 | 0.003785 | 0.003785 | 0.014195 | 0.006979 | 0.007906 | 0.014744 |
| 2 | 0.017818 | 0.008994 | 0.011777 | 0.027974 | 0.010602 | 0.007588 | 0.007554 | 0.012911 | 0.019931 |
| 3 | 0.012903 | 0.010435 | 0.014896 | 0.011277 | 0.02438 | 0.009619 | 0.009784 | 0.010681 | 0.011729 |
| 4 | 0.021179 | 0.012855 | 0.010577 | 0.012439 | 0.013081 | 0.009654 | 0.009714 | 0.011323 | 0.01099 |
| 5 | 0.009485 | 0.009692 | 0.012244 | 0.010843 | 0.016957 | 0.015598 | 0.008335 | 0.019898 | 0.010837 |
| 6 | 0.011358 | 0.010551 | 0.01228 | 0.01054 | 0.018731 | 0.007702 | 0.012462 | 0.009837 | 0.011893 |
| 7 | 0.012472 | 0.018687 | 0.010777 | 0.011595 | 0.011017 | 0.012703 | 0.010431 | 0.00847 | 0.011034 |
| 8 | 0.012245 | 0.011305 | 0.010603 | 0.011899 | 0.011541 | 0.014721 | 0.008945 | 0.01117 | 0.01212 |
| 9 | 0.011918 | 0.0169 | 0.008541 | 0.011729 | 0.010902 | 0.012086 | 0.01388 | 0.008025 | 0.014283 |
| 10 | 0.012121 | 0.010469 | 0.014773 | 0.012135 | 0.013188 | 0.012095 | 0.014706 | 0.013027 | 0.010395 |
| 11 | 0.015606 | 0.014142 | 0.010318 | 0.012 | 0.012549 | 0.013766 | 0.010654 | 0.014567 | 0.014483 |
| 12 | 0.011667 | 0.015066 | 0.008925 | 0.012822 | 0.013436 | 0.014462 | 0.012262 | 0.011464 | 0.017176 |
| 13 | 0.010456 | 0.016268 | 0.013525 | 0.014489 | 0.012092 | 0.010514 | 0.010769 | 0.013168 | 0.010868 |
| 14 | 0.009645 | 0.009277 | 0.012785 | 0.011091 | 0.016139 | 0.011503 | 0.01326 | 0.014795 | 0.011148 |
| 15 | 0.010863 | 0.012462 | 0.014044 | 0.010055 | 0.013852 | 0.013062 | 0.011392 | 0.009367 | 0.011452 |
| 16 | 0.010995 | 0.013125 | 0.01091 | 0.012421 | 0.010929 | 0.010935 | 0.015085 | 0.012255 | 0.01086 |
| 17 | 0.009805 | 0.011687 | 0.01366 | 0.012931 | 0.012841 | 0.013028 | 0.01192 | 0.01081 | 0.011494 |
| 18 | 0.008464 | 0.014075 | 0.011506 | 0.011727 | 0.009498 | 0.010781 | 0.010678 | 0.012708 | 0.013782 |
| 19 | 0.011376 | 0.011206 | 0.012331 | 0.015448 | 0.013036 | 0.01095 | 0.010615 | 0.013737 | 0.013141 |
| 20 | 0.013049 | 0.013113 | 0.010689 | 0.010545 | 0.011526 | 0.012498 | 0.009088 | 0.015288 | 0.012057 |
| 21 | 0.012742 | 0.015175 | 0.011053 | 0.014182 | 0.011807 | 0.015158 | 0.012937 | 0.010971 | 0.012364 |
| 22 | 0.012669 | 0.011099 | 0.010205 | 0.012312 | 0.013193 | 0.00997 | 0.011461 | 0.011707 | 0.012919 |
| 23 | 0.011333 | 0.010881 | 0.010583 | 0.010861 | 0.012215 | 0.011402 | 0.010723 | 0.009753 | 0.01162 |
| 24 | 0.011713 | 0.012872 | 0.010226 | 0.010612 | 0.015284 | 0.012271 | 0.012932 | 0.012506 | 0.011525 |
| 25 | 0.013648 | 0.010479 | 0.011696 | 0.011875 | 0.015119 | 0.011901 | 0.010645 | 0.012871 | 0.011958 |
| 26 | 0.010477 | 0.012998 | 0.012201 | 0.012513 | 0.011339 | 0.01545 | 0.014181 | 0.011467 | 0.01305 |
| 27 | 0.013477 | 0.009844 | 0.011089 | 0.011144 | 0.011963 | 0.011203 | 0.012081 | 0.011574 | 0.012827 |
| 28 | 0.011884 | 0.012074 | 0.012849 | 0.011643 | 0.011791 | 0.012115 | 0.011093 | 0.013927 | 0.010817 |
| 29 | 0.009813 | 0.010566 | 0.010075 | 0.010591 | 0.014641 | 0.011766 | 0.012458 | 0.01306 | 0.011063 |
| 30 | 0.010299 | 0.010948 | 0.011315 | 0.011471 | 0.013893 | 0.011486 | 0.01139 | 0.009174 | 0.014396 |

| port. Size | sample10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.012747 | 0.007767 | 0.013181 | 0.011391 | 0.007021 | 0.0127 | 0.003785 | 0.014195 | 0.022745 |
| 2 | 0.008994 | 0.013675 | 0.007517 | 0.016527 | 0.016387 | 0.011325 | 0.012964 | 0.009958 | 0.00924 |
| 3 | 0.012947 | 0.013752 | 0.010971 | 0.01985 | 0.013002 | 0.015582 | 0.009968 | 0.010102 | 0.008012 |
| 4 | 0.007922 | 0.017872 | 0.008692 | 0.010915 | 0.012774 | 0.008785 | 0.013178 | 0.012237 | 0.018448 |
| 5 | 0.012594 | 0.013874 | 0.011733 | 0.009116 | 0.015149 | 0.008484 | 0.009595 | 0.009498 | 0.010511 |
| 6 | 0.010317 | 0.009202 | 0.009768 | 0.008983 | 0.010952 | 0.011254 | 0.009628 | 0.009184 | 0.009157 |
| 7 | 0.010316 | 0.010326 | 0.014773 | 0.011622 | 0.010744 | 0.008669 | 0.01081 | 0.012881 | 0.012292 |
| 8 | 0.010805 | 0.011875 | 0.010077 | 0.009595 | 0.011444 | 0.015954 | 0.014913 | 0.011974 | 0.010601 |
| 9 | 0.013683 | 0.010386 | 0.009532 | 0.018439 | 0.012988 | 0.008561 | 0.011203 | 0.010098 | 0.014497 |
| 10 | 0.009919 | 0.01036 | 0.011249 | 0.013964 | 0.011694 | 0.013949 | 0.010174 | 0.01141 | 0.011149 |
| 11 | 0.009799 | 0.016177 | 0.013095 | 0.011054 | 0.00973 | 0.011669 | 0.009787 | 0.012626 | 0.010315 |
| 12 | 0.011505 | 0.008514 | 0.010082 | 0.011205 | 0.010162 | 0.011262 | 0.011185 | 0.00974 | 0.012421 |
| 13 | 0.011854 | 0.01084 | 0.01246 | 0.010422 | 0.009155 | 0.009662 | 0.011481 | 0.012926 | 0.011529 |
| 14 | 0.010329 | 0.01468 | 0.010282 | 0.010848 | 0.012154 | 0.012929 | 0.013757 | 0.0129 | 0.013145 |
| 15 | 0.018614 | 0.010315 | 0.01285 | 0.011583 | 0.011268 | 0.011528 | 0.012096 | 0.013547 | 0.010313 |
| 16 | 0.014644 | 0.015006 | 0.010757 | 0.010054 | 0.009988 | 0.013359 | 0.010349 | 0.012054 | 0.011325 |
| 17 | 0.012317 | 0.010433 | 0.011266 | 0.01273 | 0.013454 | 0.013387 | 0.013057 | 0.009997 | 0.011492 |
| 18 | 0.010184 | 0.011903 | 0.01078 | 0.011294 | 0.011893 | 0.014702 | 0.014267 | 0.010481 | 0.013561 |
| 19 | 0.010635 | 0.014124 | 0.010379 | 0.013735 | 0.010761 | 0.010426 | 0.011364 | 0.013313 | 0.010565 |
| 20 | 0.010462 | 0.012741 | 0.011782 | 0.011042 | 0.010197 | 0.011863 | 0.01249 | 0.010168 | 0.009513 |
| 21 | 0.011944 | 0.010908 | 0.011637 | 0.012489 | 0.010552 | 0.01464 | 0.012794 | 0.010421 | 0.011784 |
| 22 | 0.011865 | 0.01347 | 0.010113 | 0.009579 | 0.013194 | 0.010604 | 0.011842 | 0.010925 | 0.012188 |
| 23 | 0.011395 | 0.012774 | 0.01134 | 0.010444 | 0.012266 | 0.011957 | 0.010035 | 0.012717 | 0.013527 |
| 24 | 0.010311 | 0.010623 | 0.010178 | 0.011188 | 0.012046 | 0.011128 | 0.011251 | 0.011783 | 0.010427 |
| 25 | 0.011267 | 0.011871 | 0.01372 | 0.011201 | 0.010686 | 0.010759 | 0.012896 | 0.011425 | 0.01171 |
| 26 | 0.011197 | 0.012886 | 0.009314 | 0.010252 | 0.011333 | 0.010641 | 0.013296 | 0.011369 | 0.012741 |
| 27 | 0.011103 | 0.013189 | 0.013158 | 0.013571 | 0.012894 | 0.01105 | 0.013198 | 0.010042 | 0.014447 |
| 28 | 0.011737 | 0.012287 | 0.012267 | 0.010962 | 0.011094 | 0.013988 | 0.012176 | 0.011888 | 0.015857 |
| 29 | 0.012339 | 0.011288 | 0.010928 | 0.011609 | 0.010313 | 0.012669 | 0.012899 | 0.011604 | 0.012042 |
| 30 | 0.011356 | 0.012678 | 0.012385 | 0.011015 | 0.012734 | 0.012043 | 0.012612 | 0.012984 | 0.01337 |

| port. Size | sample19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.015529 | 0.01506 | 0.011464 | 0.011464 | 0.009003 | 0.010596 | 0.007704 | 0.011464 | 0.016643 |
| 2 | 0.009676 | 0.032902 | 0.024103 | 0.024366 | 0.013363 | 0.028248 | 0.011718 | 0.008808 | 0.007362 |
| 3 | 0.010523 | 0.012241 | 0.011628 | 0.011043 | 0.008652 | 0.008457 | 0.011747 | 0.008431 | 0.008988 |
| 4 | 0.011437 | 0.014438 | 0.007762 | 0.010538 | 0.010571 | 0.007413 | 0.014351 | 0.017095 | 0.015414 |
| 5 | 0.012909 | 0.0106 | 0.013387 | 0.012001 | 0.010083 | 0.010822 | 0.015433 | 0.012363 | 0.008221 |
| 6 | 0.011519 | 0.011355 | 0.016017 | 0.010072 | 0.017989 | 0.008691 | 0.011928 | 0.011292 | 0.010114 |
| 7 | 0.012109 | 0.009061 | 0.009492 | 0.01247 | 0.010337 | 0.010143 | 0.012743 | 0.017858 | 0.011593 |
| 8 | 0.010942 | 0.012793 | 0.017725 | 0.01184 | 0.011586 | 0.012128 | 0.010137 | 0.016066 | 0.011242 |
| 9 | 0.011919 | 0.014938 | 0.011726 | 0.012839 | 0.012269 | 0.01189 | 0.011358 | 0.01361 | 0.009753 |
| 10 | 0.011692 | 0.011398 | 0.010713 | 0.012149 | 0.01084 | 0.01329 | 0.012483 | 0.01298 | 0.017386 |
| 11 | 0.010549 | 0.015433 | 0.012502 | 0.010901 | 0.01149 | 0.012001 | 0.009857 | 0.010814 | 0.010137 |
| 12 | 0.011383 | 0.011771 | 0.013796 | 0.013862 | 0.01594 | 0.015608 | 0.010871 | 0.010624 | 0.009815 |
| 13 | 0.012065 | 0.011874 | 0.015263 | 0.012938 | 0.011705 | 0.009998 | 0.011394 | 0.012186 | 0.012302 |
| 14 | 0.011855 | 0.01487 | 0.012765 | 0.010105 | 0.012208 | 0.011997 | 0.010814 | 0.011616 | 0.011883 |
| 15 | 0.014741 | 0.009683 | 0.010122 | 0.010517 | 0.012708 | 0.011333 | 0.013482 | 0.009335 | 0.011163 |
| 16 | 0.013223 | 0.012996 | 0.01196 | 0.013544 | 0.011573 | 0.013784 | 0.011591 | 0.01141 | 0.011334 |
| 17 | 0.012175 | 0.010076 | 0.012706 | 0.009635 | 0.011352 | 0.011505 | 0.01085 | 0.010892 | 0.013928 |
| 18 | 0.011909 | 0.014255 | 0.011065 | 0.011942 | 0.013042 | 0.013376 | 0.014179 | 0.009997 | 0.012317 |
| 19 | 0.011597 | 0.011688 | 0.009885 | 0.011167 | 0.010055 | 0.011118 | 0.011002 | 0.011245 | 0.013381 |
| 20 | 0.014284 | 0.010364 | 0.011528 | 0.011004 | 0.013799 | 0.01046 | 0.008878 | 0.010045 | 0.009984 |
| 21 | 0.012123 | 0.009951 | 0.014765 | 0.01212 | 0.012824 | 0.011992 | 0.013613 | 0.014147 | 0.012066 |
| 22 | 0.011733 | 0.015715 | 0.010631 | 0.012274 | 0.010811 | 0.012717 | 0.010918 | 0.0135 | 0.011713 |
| 23 | 0.011863 | 0.010873 | 0.014473 | 0.012769 | 0.011619 | 0.01012 | 0.015324 | 0.01285 | 0.012446 |
| 24 | 0.012034 | 0.01099 | 0.010679 | 0.012338 | 0.012704 | 0.013988 | 0.01132 | 0.011918 | 0.011261 |
| 25 | 0.012181 | 0.008941 | 0.013054 | 0.011062 | 0.011384 | 0.011226 | 0.012069 | 0.010285 | 0.014493 |
| 26 | 0.010602 | 0.012305 | 0.010746 | 0.011104 | 0.013774 | 0.011598 | 0.011827 | 0.010954 | 0.011311 |
| 27 | 0.010194 | 0.011777 | 0.012448 | 0.012726 | 0.011002 | 0.012923 | 0.012536 | 0.013599 | 0.012236 |
| 28 | 0.013468 | 0.010751 | 0.010589 | 0.012011 | 0.013519 | 0.011207 | 0.011067 | 0.012062 | 0.011102 |
| 29 | 0.013077 | 0.011397 | 0.01229 | 0.013037 | 0.011032 | 0.012067 | 0.012888 | 0.010854 | 0.012083 |
| 30 | 0.012998 | 0.012706 | 0.011598 | 0.010701 | 0.012945 | 0.010628 | 0.013721 | 0.015184 | 0.011826 |

| port. Size | sample28 | 29 | 30 | average variance |
|---|---|---|---|---|
| 1 | 0.013181 | 0.014035 | 0.008493 | 0.011035 |
| 2 | 0.013608 | 0.015479 | 0.015002 | 0.014546 |
| 3 | 0.0106 | 0.01524 | 0.011367 | 0.01196 |
| 4 | 0.017129 | 0.010756 | 0.009679 | 0.012307 |
| 5 | 0.0107 | 0.015417 | 0.012062 | 0.011948 |
| 6 | 0.012166 | 0.011735 | 0.013664 | 0.011345 |
| 7 | 0.018311 | 0.014578 | 0.010694 | 0.011967 |
| 8 | 0.016404 | 0.011096 | 0.00905 | 0.012093 |
| 9 | 0.010758 | 0.009349 | 0.010839 | 0.011963 |
| 10 | 0.012389 | 0.00965 | 0.00909 | 0.012028 |
| 11 | 0.010801 | 0.013174 | 0.012953 | 0.012098 |
| 12 | 0.010608 | 0.008646 | 0.010351 | 0.011888 |
| 13 | 0.012351 | 0.009784 | 0.01038 | 0.011824 |
| 14 | 0.010629 | 0.012974 | 0.008987 | 0.012046 |
| 15 | 0.013492 | 0.010984 | 0.012855 | 0.011969 |
| 16 | 0.011472 | 0.011068 | 0.012428 | 0.012048 |
| 17 | 0.010526 | 0.01046 | 0.013588 | 0.0118 |
| 18 | 0.012126 | 0.012871 | 0.012027 | 0.012046 |
| 19 | 0.009902 | 0.01083 | 0.012423 | 0.011714 |
| 20 | 0.011382 | 0.011489 | 0.010403 | 0.011391 |
| 21 | 0.011711 | 0.011781 | 0.010205 | 0.012362 |
| 22 | 0.010199 | 0.01117 | 0.011622 | 0.011744 |
| 23 | 0.011085 | 0.012085 | 0.010656 | 0.011733 |
| 24 | 0.012012 | 0.011931 | 0.013458 | 0.011784 |
| 25 | 0.011888 | 0.012158 | 0.01253 | 0.0119 |
| 26 | 0.010073 | 0.012082 | 0.011535 | 0.011821 |
| 27 | 0.011875 | 0.011614 | 0.013153 | 0.012131 |
| 28 | 0.010306 | 0.014097 | 0.011506 | 0.012071 |
| 29 | 0.013685 | 0.012324 | 0.011502 | 0.011865 |
| 30 | 0.013371 | 0.01257 | 0.009659 | 0.012115 |