EDUCATIONAL ENROLLMENT: OPTIMAL PRICING POLICY

by

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ABSTRACT

Optimal registration fees for students of diverse income classes is considered. A numerical example suggests that the efficiency of the University of Nairobi, as measured by enrollment of qualified students, may be significantly increased given a change in admissions policy.

1. Introduction

Educational institutions are often faced with an excess of qualified candidates who are frustrated in their attempts to secure limited enrollment space. In developing countries where the existing tax base is gerally insuffient to underwrite expenditures to eliminate this excess demand, what pricing policy will insure that available resources will enable the greatest number of students to be educated?

This paper examines optimal pricing policy when candidates can be differentiated into separate demand groups, e.g., regional origin, income classes. The objective here is to maximize enrollment subject to the constraining demand groups and an overall budget subsidy applicable to the educational institution. Section II treats the problem mathematically while Section III presents a numerical example. Section IV interprets the results in light of actual behavior of educational institutions.

2. Mathematical Formulation

The problem facing the educational institution is to set prices P_i for enrollment for $i=1, \ldots, n$ demand groups whose demand functions are given by

(2.1) $$P_i = d_i(X_i) \quad \text{for all } i=1,\ldots,n .$$

In equation (2.1) X_i for all $i=1,\ldots,n$ is the amount of people who will enter the educational institution when charged P_i. Clearly $X_i \geq 0$. It is assumed that

(2.2) $$\frac{\partial d_i}{\partial X_i} < 0 \text{ and}$$

d_i is convex for all $i=1,\ldots,n$. Equation (2.2) is guaranteed if the demand functions are constructed by rank ordering according to willingness to pay.

Moreover the educational institution operates under an overall fiscal deficit allocation S and a total cost curve of educational expenditures $C(\sum_{i=1}^{n} X_i)$ which is also convex; the marginal cost of educating student is not a decreasing function of the number of students in the relevant range.

The educational institutions problem is therefore to

(2.3) Maximise $\sum_{i=1}^{n} X_i$

Subject to

$$P_i = d_i(X_i) \qquad \text{for all } i=1, \ldots,n.$$

$$S + \sum_{i=1}^{n} P_i X_i \geq C(\sum_{i=1}^{n} X_i)$$

$$X_i \geq 0 \qquad \text{for all } i=1,\ldots,n.$$

The Lagrangian of (2.3) is therefore:

(2.4) $\pounds(X_1,\ldots,X_n,P_1,\ldots,P_n,\lambda_1,\ldots,\lambda_n,\lambda_S)$

$$= \sum_{i=1}^{n} X_i + \sum_{i=1}^{n} \lambda_i [d_i(X_i) - P_i]$$

$$+ \lambda_S [S + \sum_{i=1}^{n} P_i X_i - C(\sum_{i=1}^{n} X_i)].$$

Assuming an interior solution ($X_i > 0$ for all $i=1,\ldots,n$; this is necessary to insure optimal pricing under relevant demand groups) the necessary first order conditions are:

(2.5) $$\frac{\partial \pounds}{\partial X_i} = 1 + \lambda_i \frac{\partial d_i}{\partial X_i} + \lambda_S P_i - \frac{\partial C}{\partial X_i} = 0 \text{ for all } i=1,\ldots,n.$$

(2.6) $$\frac{\partial \pounds}{\partial P_i} = -\lambda_i + \lambda_S X_i = 0 \quad \text{for all } i=1,\ldots,n.$$

(2.7) $$\frac{\partial \pounds}{\partial \lambda_i} = d_i(X_i) - P_i = 0 \quad \text{for all } i=1,\ldots,n.$$

(2.8) $$\frac{\partial \pounds}{\partial \lambda_S} = S + \sum_{i=1}^{n} P_i X_i - C(\sum_{i=1}^{n} X_i) = 0.$$

Equation (2.8) states that it is optimal to spend all resources, both subsidy and revenues obtained from enrollment. Equations (2.7) state that it is optimal to charge each demand group according to its demand curve. Specifically it is never optimal to have excessive demand or excessive price.[2] λ_S is the marginal value of educational services which ideally should be set equal to the marginal value society receives from educating the student. In a developing country the tax; base may be insuffient to achieve the desired value of λ_S.

Equations (2.6) imply

(2.9) $$\lambda_i = \lambda_S X_i \quad \text{for all } i=1,\ldots,n.$$

Substituting into equations (2.5):

$$(2.10) \qquad 1 + \lambda_S X_i \frac{\partial d_i}{\partial X_i} + \lambda_S P_i - \frac{\partial C}{\partial X_i} = 0 \text{ for all } i=1,\ldots n.$$

Rearranging

$$(2.11) \qquad X_i \frac{\partial d_i}{\partial X_i} + P_i = \frac{\frac{\partial C}{\partial X_i} - 1}{\lambda_S} \qquad \text{for all } i=1,\ldots,n.$$

From equations (2.11) we conclude that optimal pricing policy implies:

$$(2.12) \qquad X_i \frac{\partial d_i}{\partial X_i} + P_i = X_j \frac{\partial d_j}{\partial X_j} + P_j \text{ for all } i,j=1,\ldots,n;\ i \neq j.$$

As a direct result of equations (2.12) a best pricing policy is a single constant price iff

$$(2.13) \qquad X_i = \frac{\frac{\partial d_j}{\partial X_j} X_j}{\frac{\partial d_i}{\partial X_i}} \qquad \text{for all } i,j=1,\ldots,n; i \neq j.$$

A sufficient condition for this to be true is that all demand curves be horizontal translations; this being empirically unlikely. Normally an equal pricing policy will be suboptimal. The more distinct demand groups the more divergent an optimal pricing policy will be from a uniform price and the greater the enrollment.

3. Numerical Example

Consider the possibility of price differentiation when students can be divided into two broad income classification; the rich and the not-

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2.) Thus it is never optimal to have an excess of qualified candidates who are frustrated in securing enrollment space.

so-rich.[2] The rich have a demand function which can be approximated in the relevant range by

(3.1) $$P_1 = -X_1 + 1400,$$

whereas the not-so-rich have a demand function which can be approximated in the relevant range by

(3.2) $$P_2 = -600.$$

The cost function is linear and given by

(3.3) $$C(\sum_{i=1}^{2} X_i) = 100 + 500(\sum_{i=1}^{2} X_i).$$

Overall subsidy given by the government is given by

(3.4) $$S = 2,300,100.$$

The maximization problem facing the educational institution is therefore to:

(3.5) $$\text{Maximize } X_1 + X_2$$

subject to

$$P_1 = -X_1 + 1400$$

$$P_2 = -600$$

$$2,300,100 + P_1X_1 + P_2X_2 \geq 100 + 500(X_1 + X_2).$$

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2. Numbers chosen for this example are roughly representative of the situation at the University of Nairobi. As such numerical quantities should be interpreted as monthly allocations in terms of shillings. The objective function of the maximization problem should be interpreted as enrollment of new freshmen students.

The necessary first order conditions are:

$$(3.6) \qquad X_1 \frac{\partial d_1}{\partial X_1} + P_1 = X_2 \frac{\partial d_2}{\partial X_2} + P_2$$

$$(3.7) \qquad P_2 = -600.$$

$$(3.8) \qquad P_1 = -X_1 + 1400.$$

$$(3.9) \qquad 2{,}300{,}100 + P_1X_1 + P_2X_2 = 100 + 500(X_1 + X_2).$$

Solving we obtain:

$$(3.10) \qquad P_1 = 400.$$

$$(3.11) \qquad X_1 = 1000.$$

$$(3.12) \qquad P_2 = -600.$$

$$(3.13) \qquad X_2 = 2000.$$

with total output X* given by

$$(3.14) \qquad X^* = X_1 + X_2 = 3000 \text{ students}$$

This optimal enrollment level should be compared with a single price enrollment scheme. Suppose that the single price is a per student subsidy of K.shs. 600 per month. The enrollment can be calculated directly from the budget constraint:

$$(3.15) \qquad S - 600X = 100 + 500X.$$

This implies

$X \simeq 2091$ students.[3]

3. This corresponds to an average annual expenditure of about K.Shs. 13,200 per student; not an unrealistic estimate in the University of Nairobi case.

If this numerical example can be viewed as representative then even differentiation into but two distinct income groups of students can increase the efficiency of the educational institution; in the example given almost a fifty per cent gain is realized. If students are divided into even finer classifications (as, for example, by parental income tax bracket) even greater gains might be obtained.

4. Conclusions

As shown in Section 3 the gains from price differentiation in relation to one of an educational institution's objectives is likely to be substantial. This gain can in part be utilized to explain the behavior of many educational institutions where the parents of candidates for scholarships must file a confidential statement of their assets before acceptance to a university. Such behavior is much closer to optimal than a single pricing mechanism. This analysis introduces the specific optimality conditions which should be met under such systems.

In developing countries, such as Kenya, where educational resources are less than desired arising from the existence of a small industrial sector and therefore a small tax base, among other reasons, a single pricing policy can be viewed as a definite detriment to long run growth and progress.