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The Effects of Vegetation Roots on Stability of Slopes

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Doctor of Philosophy in Civil Engineering

By

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Declaration

This Thesis is my original work and has not been presented for a Degree in any other University

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This Thesis has been submitted to the University for award of the Degree of Doctor of Philosophy in Civil Engineering with our approval as University Supervisors

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Dedication

This work is dedicated to my parents
Mr. Stephen Osano and Mrs. Belliah Kemunto.
Their special place in me made it happen.

Abstract

The main objective of this investigation was to provide information about the contribution of plant roots to soil shear strength. Reconnaissance of the case study areas was undertaken and geology of the areas established, rainfall patterns collected, type of vegetation obtained, theoretical research on stability of slopes studied, soil samples were collected and laboratory soil tests were conducted. Laboratory tests were conducted at soils laboratories in the University of Nairobi and University of Portsmouth, UK.

It was established that unvegetated soils are weaker than vegetated slopes in shear. Tests conducted at Sasumua backslope and Murang'a landslide sites indicated that shear values reached 16 kPa for unvegetated soils while vegetated soil had maximum values of 80 kPa for saltbush, 90 kPa grass and 120 kPa for tree fern. Rooted soils were thus stronger in shear as compared to fallow soils. Shear stress increases at the end of testing for the rooted samples. Observations of the roots after test indicates roots elongated. This elongation can be related to the root biomass density to explain why varying strengths were obtained for different samples of a same species.

The perpendicular model of Wu *et al.* (1979) was used to calculate soil reinforcement by action of roots. The tensile strength – diameter curves (T-D) relationships indicate that root tensile strength decreases with increasing root diameter, and follows a power law equation $f(x) = ax^k$, where a and k are parameters obtained from *T-D* curves. Strong roots have high *a*-values and low *k*-values and vice versa. Shrubs generally have high *a*-values, but a great variation is noted within individual plant species.

It was observed that smaller diameter roots have higher tensile strengths. Tensile strengths for shrubs decreased from 34 N/mm^2 to 4 N/mm^2, grasses from 45 N/mm^2 to 5 N/mm^2 and ferns from 30 N/mm^2 to 10 N/mm^2 for root diameters ranging from 1.5 mm to 6 mm. Maximum root area ratio (RAR) values, defined as the ratio of the sum of the root areas to the area of soil profile they intersect, were located within 0.1 m for all the species, with maximum rooting depth of 0.7 m for fern tree. Shrubs species showed high RAR values between 0.1 – 0.3 m depth. In general, vegetations growing in the Sasumua backslope have shallow roots (maximum root depth 0.7 m) therefore unable to reinforce the soils to stop the landslides occurring at 1 m depths. The results strongly imply that shrubs species have prominent root mechanical properties and it is anticipated that these particular plants have the necessary features to be outstanding slope plants.

Overall correlation and regression analysis show that the pull-out resistances of plants have a positive, either weak or strong, linear relationships with all the morphological properties. Bigger plants can resist pull-out force better than the smaller plants. The increase in plant size will normally generate high pull-out resistance. Taller plants will resist uprooting better than the shorter ones. The increase in pull-out resistance of plants that have root systems with extensive number of lateral root is due to the fact that the stronger soil-anchorage is developed by the lateral roots

The safety factor decreases with increasing slope angle until the slope fails at a critical angle of about 45° and follows a power law equation of the form $f(x) = ax^k$. Moisture content of 50% accelerates failure of slopes.

Design charts and graphs have been developed. If θ (the angle of shear distortion) and ϕ (the soil friction angle) are known, using the relevant chart, K-Values will be obtained, which will be multiplied by t_R-Values (the total mobilized tensile stress of roots fibers per unit area of soil) also obtained from charts. The contribution of roots to shear strength is thus deduced.

Tests conducted using a variable tensile test machine indicate that vegetation roots increase their tensile strength at high strain rates. This is the case during a landslide spell, indicating that roots could induce high resistance to the forces impacted and thus offering to the stability of slopes.

Several recommendations have been drawn. Deeper rooted vegetation (> 1 m) with high RAR values are recommended for slopes to reduce the potential of shallow landslides from occurring. Vegetation with high surcharge (weight / unit area) should be avoided. Vegetation should be spaced 1m apart in order to influence full mobilization of shear strength as the roots integrate with the soil mass. Population living in slopes greater than 30° should be resettled in gentle areas as safety factor for slopes beyond 30° is less than 1.5, and failure can be triggered during a rainy season. Proper drainage system should be designed and constructed over slopes to dissipate surface runoff immediately it occurs in order to avoid premature failure of slopes as moisture content of more than 50% reduces the shear strength significantly. The easy to use design charts developed under this research should be applied for root reinforced soils design process.

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Notation

α_1, α_2, α_3, α_4: angles

A_R: root cross-sectional area

T_R: average tensile strength of the roots,

α: slope angle

w_t: vegetation surcharge (weight / unit area),

Δc: increased cohesion due to tree roots

A: reference area of soil occupied by roots

γ_{sat}: saturated unit weight of soil

γ_w: unit weight of water

ϕ': internal angle of friction

θ: angle of shear distortion in the shear zone

ΔS_R: shear strength increase resulting from root displacement,

ΔS_R: increase in shear resistance

a, b: empirical constants

c': soil cohesion

C_r: increase in shear strength due to the presence of roots

C_u undrained shear strength, and

D: diameter

F_{max}: maximum force (N) needed to break roots

K: coefficient of subgrade reaction, dimensionless,

l_a, l_b and l_c: lengths of root sections a, b and c respectively

n_r: residual soil-root adhesion

P_y: ultimate soil reaction

R: concentration or density of roots in the soil

RAR: root area ratio

s: soil shear strength without roots

s_r: shear strength of the soil with roots

t_R: total mobilized tensile stress of roots fibers per unit area of soil

UTS: Ultimate Tensile Strength

YS: Yield Strength

θ: angle of shear distortion in the shear zone

Chapter one

Introduction

1.1 General

For many years man has realized the potential of vegetation in controlling surface erosion. Shrubs and grasses help deter erosion on slopes by serving as a blanket against wind and rainwater impact and as a sieve removing soil particles from surface water runoff. Though man has for some time used vegetation to protect the earth's surface against the elements, our concern has only recently focused on the contribution of vegetation in deterring mass movement.

Past experience shows that slopes under vegetation are more resistant against mass movements and water erosion. However, vegetation cover can be very limited in semi-arid environments and is often damaged by surface fire, overgrazing, drought or flooding. Even following vegetation removal, an increase in water erosion phenomena or shallow mass movements usually appears after a lag period attributed to the time required for roots of the removed vegetation to decay. The effects of roots in protecting the soil from being eroded can therefore not be neglected (Gray and Sotir, 1996, Osano and Mwea, 2008).

Soils covered by vegetation run less risk of erosion from both water and land movement (Burroughs and Thomas, 1977; Ziemer, 1981; Greenway, 1987; Gray and Sotir, 1996). The role roots play in slope stabilization has been recognized for many years (e.g. Gray and Sotir, 1996;), whereas interest in bio-mechanical tests on roots (of Mediterranean species in particular) has arisen only in more recent years (De Baets et al., 2008). De Baets et al. (2008) showed how some typical Mediterranean plants increase topsoil resistance to erosion and shallow landslides from runoff and superficial flow.

As seen in Table 1.1, some Mediterranean species were subjected to root tensile strength, shear stress and/or pull-out tests, and also the architecture of their rooting system grown on slopes was studied. Spanish Broom (*Spartium junceum L.*) has been studied by Chiatante *et al.*, (2001, 2003a, b) with regard to the architecture of the Spanish Broom root system when grown on slopes: and it was observed that its orientation and root density undergo modification. Its root growth is asymmetric and follows the orientation of the slope, concentrating mainly on the uphill direction. This is a characteristic that guarantees the stability of the plant (Chiatante et al., 2001, 2003a, b.)

Table 1.1: Mediterranean species subjected to root tensile strength, shear stress and/or pull-out tests

Authors	*Studied species*
Operstein and Frydman (2000)	*Medicago sativa, Rosmarimus officinalis, Pistacia lentiscus e Cistus(all dicotyledonous shrub species)*
Gallotta et al. (2000, 2003)	*Cupressus, Crataegus, Juglans, Prunus, Pyrus, morus, tamarix*
Amato et al. (1997, 2000)	*Citrus sinensis, Prunus avium, Ailanthus altissima, Castanea sativa, Ficus carica, Pinus, Quercus pebescens, Prunus, Arundo, Festuca, Poa, Dactylis, Trifolium, Cyclamen, Brassica and Rubus frutticosus*
Mattia et al. (2005)	*Lygeum spartum L. (herb), Atriplex halimus L. and Pistacia lentiscus L.* (shrub)
De Baets et al. (2008)	*Atriplex halimus* (shrub), *Salsola genistoides (shrub), Brachypodium retusum* (grass), *Thymelaeahirsuta (shrub), Phragmites australis (reed), Limonium supinum* (herb), *Tamarix canariensis* (tree),
Chiatante et al. (2001, 2003a, b)	*Architecture of the Spartium j unceum L. rooting system grown on slopes*

The concern regarding the effect of vegetation on slope stability occurs mainly from problems encountered in logging. The methods used to remove and transport timber from a logging site also removes a large amount of groundcover. Upon removal of this vegetation, deterioration of the remaining root structure causes a decrease in the stability of the slope. The denuded slope is subjected to soil erosion, and with the decrease in slope stability caused by root deterioration, it is more susceptible to mass movement.

Roots affect properties of the soil, such as infiltration rate, aggregate stability, moisture content, shear strength and organic matter content, all of which control soil erosion rates to various degrees. One of the important mechanical characteristics of roots is that they are strong in tension. Soils, on the other hand, are strong in compression and weak in tension. A combined effect of soil and roots results in a reinforced soil. When shearing the soil, roots mobilize their tensile strength whereby shear stresses that develop in the soil matrix are transferred to the soil fibers via interface friction along the root length or via the tensile resistance of the roots. The magnitude of root reinforcement depends on;

(i) morphological characteristics of the root system (e.g. root distribution with depth, root distribution over different root diameter classes),

(ii) root tensile strengths,

(iii) root tensile modulus values,

(iv) the interface friction between roots; and

(v) the soil and the orientation of roots to the principal direction of strain (Greenway, 1987, Osano and Mwea, 2011).

Landslides are a common form of erosion in steep hills (Ekanayake & Phillips, 1999). Widespread shallow translational landslides (soil slips or debris flows) have been associated with severe storms, and with the removal of indigenous forest and conversion to grassland or pastoral farming. Comprehensive surveys and analyses of landslides after a high-intensity rainstorm have shown that few landslides take place in forested regions as compared to bear slopes (Ekanayake and Phillips, 1999; Ekanayake et al., 1997; Marden and Rowan, 1993). Vegetation is able to provide stability to such slopes due to a variety of physical processes. Most commonly, vegetation is applied for surface erosion control, as in the use of turf grass for protection of slopes and swales against the action of rain, wind, and frost. Natural plant communities are also understood to play a significant role in the stability of many slopes, riverbanks, and shorelines where deeper-seated instability problems, waves, or concentrated flow create larger engineering challenges. Various designers have incorporated the geotechnical as well as ecological benefits of vegetation into water resource and slope stabilization projects, with comprehensive documentation dating back to the early part of the 20th century (Schiectl and Stern, 1997). The documentation provides an excellent review of design objectives and typical treatments for waterways bioengineering projects. Greenway (1987) reviews geomorphic factors influencing bioengineering design and discusses mechanisms and analysis tools related to reinforcement of soil by roots with a focus on slope and embankment sites.

1.2 Problem statement

The use of vegetation to stabilize soil slopes is becoming an increasingly used environmentally friendly alternative to traditional soil improvement methods. However, at present, widespread use of vegetation to stabilize soil has been inhibited by a lack of verified methods to predict the reinforcing effect of various plant types (Bransby, 2004). Thus a research is required to investigate the link between root systems, root mechanical properties and soil-slope stability.

While many practitioners discuss selecting appropriate tree, shrub, and herb species which are adapted to site conditions, little quantitative research has been conducted into the structural properties or strengthening influences of herbaceous plants. While many woody plants are known to have long, strong roots, many herbaceous species have highly fibrous roots that often form dense mats within the upper 0.3-1 meter of soil, or deeper. In many natural conditions, herbaceous species are better adapted than trees and shrubs to zones where soil saturation and inundation are common. In other settings, woody and herbaceous plants can coexist and provide complementary functions. Due to the significant differences in rooting characteristics between woody and herbaceous plants, it is important to understand how roots and soils interact from an engineering standpoint. Additional targeted information about the qualitative and quantitative effects of roots in slope stability analyses in vegetated landfills, riverbanks, shorelines, embankments, cut slopes and retaining walls, and other reclamation applications where the mechanical contribution of root reinforcement is important to predicting soil behavior could guide the design and management of vegetative stabilization systems. In practice, the scarcity of data and systematic analytical approaches to applying vegetation for soil reinforcing purposes leads to a consistent tendency to favor structural measures, like stem size and plant height, which have little to no ecological values, unlike their vegetative counterparts.

1.3 Other past researches

In the Department of Geology and Geological Engineering at the University of Mississipi, research has been carried out on the effects of riparian vegetation on bank stability (Ekanayake *et al*, 1997). Modelling results indicated that there is a contrast between root-reinforced and unreinforced soil. When root reinforcement existed, slope failed marginally.

Ekanayake *et al*, (1997) have provided a detailed analysis of the contribution of root to soil strength. In their paper 'Tree roots and slope stability: a comparison between *pinus radiata* and kānuka', they compared the contribution of live roots of a naturally regenerating indigenous species (kānuka) with that of planted and managed exotic species *(pinus radiata)* to soil reinforcement using *in situ* direct shear tests. Results suggested that for individual trees the contribution from the roots to soil strength was independent of species for the two tree species tested. There were, however, significant differences in stand density. A simple model was developed using the relationship between the shear strength of the soil-root system, the specific root cross-section area, and slope angle to determine safety factors for typical stand densities of naturally regenerating kānuka for comparison with different *pinus radiata* management regimes at equivalent stages of growth. The model predicted that safety factors for stands of *pinus radiata* in the first 8 years after establishment would be lower than for equivalent-aged stands of fully-stocked regenerating kānuka under similar conditions. However, after 16 years the safety factor for a stand of kānuka would be lower than that for *pinus radiata* at final stocking densities typical of framing and biomass regimes. They concluded that in areas where vegetation plays a major role in soil conservation and erosion control, the model could be used to compare the stability of slopes forested with different species.

1.4 Background information: The Kenyan perspective

The problem of slope instability has been encountered for centuries. Many cases of slopes failures (landslides) have been reported in many parts of the world, Kenya being one of them. In the past, many parts of Kenya have experienced many cases of slope instability leading to very destructive landslides that have destroyed properties and caused loss of life. Slope instability is very common in Central Kenya and parts of the Rift valley. A physical map of Kenya is shown in Fig. 1.1, while the topographical map of Kenya is in Fig. 1.2. The figures clearly indicate that the most affected places are the highlands where slopes are steep. Appendix 1 presents the floods and landslides affected areas in Kenya which occurred in 2008.

Fig. 1.1: Physical map of Kenya

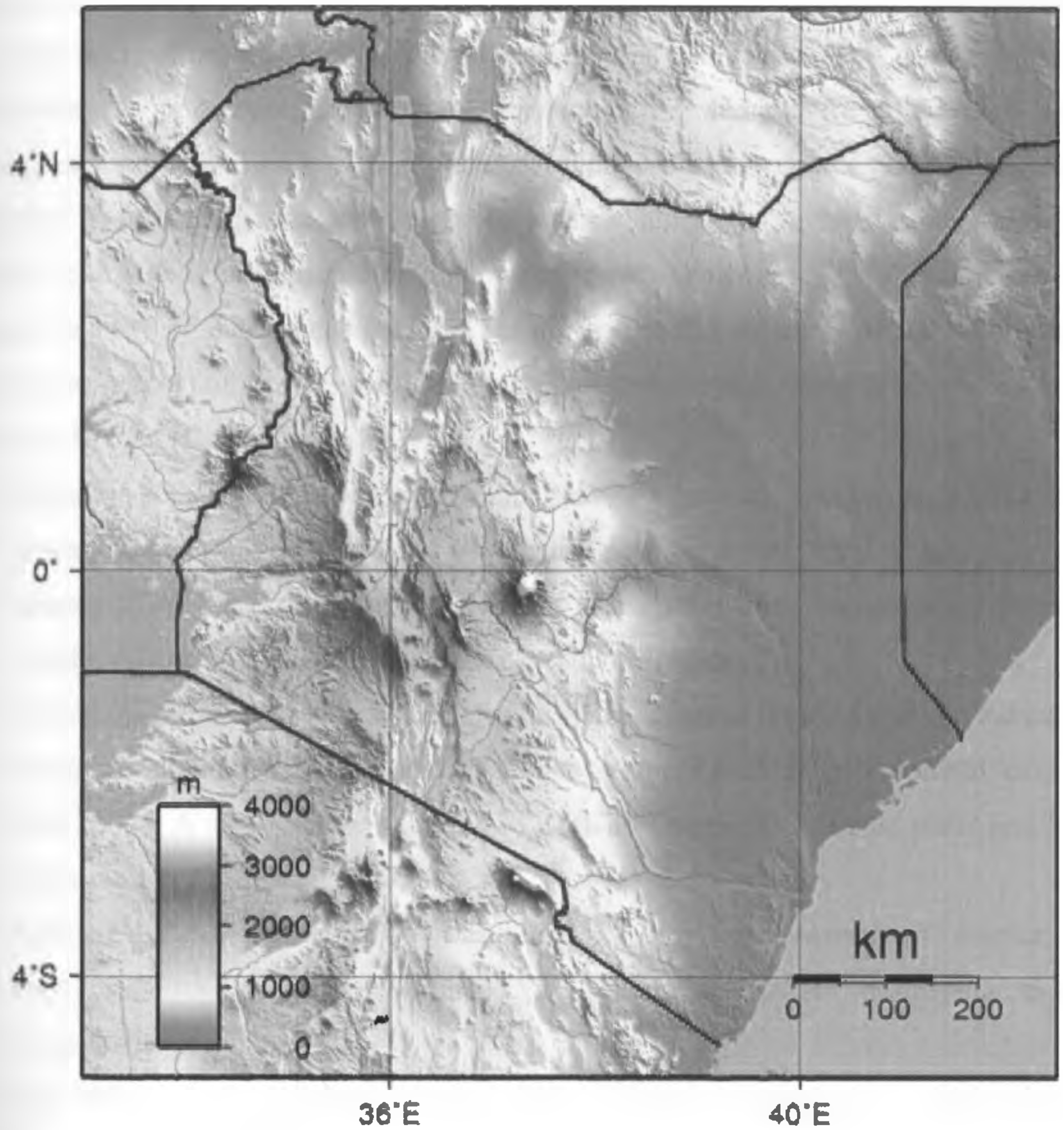


Fig. 1.2: Topographical map of Kenya

The volcanic processes that took place millions of years ago left the present landforms in Central Kenya that is highly susceptible to slope failures. The Rift Valley regions experience slope instability due to seismic effects within the earth interior. A major impediment to the effective disaster prevention from mass movement is the lack of sufficient knowledge on the part of local communities of the main factors responsible for the failure and the best methods to minimize the instability. In many cases a slope failure disaster is simply taken as an act of God. Thus few bother to seek the expert opinion on the possible causes and mechanisms to put in place to avoid a repeat of the same. To attest to this fact is the lack of records of past occurrences. Since the last decade, tremendous increases in cases of mass earth movements in Kenya have been witnessed. The resulting damages have also increased. A major reason for this is the increase in population resulting in a corresponding demand for land which has led to increasing cutting and filling of earth masses. This is also accompanied by environmental degradation. Thus some potentially dangerous areas which were previously forest reserves have now been cultivated for agriculture and even human settlement. This results not only in decreased stability of these areas but also an increase in the elements of risk. An attempt has been made to gather information on past incidences and the following illustrate the instability instances in Kenya, (Ocha Kenya, 2009);

1 February 2007 - landslide occurred in Enkararo area in Transmara District in which several homes were damaged and people displaced from their homesteads.

2 January 2004 - A family lost their home when a landslide completely destroyed it in Mithiu Village in Kalama Division in Machakos district.

3 March / April 2003 - Slopes failures in Murang`a and Meru Central Districts buried several people in their sleep and displaced many more besides their material losses.

4 June 2002 - A slope failure occurred in Kericho District and caused panic and hundreds of people were displaced from their homesteads.

5 April 2001 - A family of four was buried in their sleep in Meru North district.

6 May 1999 - A failure occurred in Murang`a District destroying crops and homesteads.

7 May 1998 - in Kijabe area, two slope failures occurred, one which blocked the Nairobi- Nakuru railway line and disrupted train services for 9 days and resulting in ksh. 90 million losses. (Daily Nation, Friday June 28th 1998). The other blocked the Kijabe-Githigo road and disrupted traffic for 10 days before repairs were completed.

8 The heavy El-nino rains triggered a huge landslide at Mutonga River along the busy Embu –Meru road where over 100m of the road was either swept away or blocked by the displaced land mass. Transport operations were completely cut off for a couple of days.

9 February 1998 - 30 houses were destroyed by earth movement in Mbooni-Makueni District, displacing 400 people.

10 December 1997 - A landslide damaged a commercial building and cash crop at Kigarutara in Gathanga division Murang`a district. The other failures occurred along the Nairobi –Murang`a road in Sabasaba and at Kangema.

11 May 1997 - Rock fill in Chesikaki sub-location of Mt. Elgon district destroyed dwelling places and crops (maize and coffee). A similar event is reported to have taken place in the area in 1961.

12 May 1997 - eleven members of one family were buried alive by a 2.00 a.m landslide in Murang`a district, 7 houses were completely destroyed.

13 January 1993 - A failure of a railway line bridge embankment at Ngai Ndethya area of Makueni district caused a train accident resulting to 65 people losing their lives.

14 May 1993 – A landslide in Rugaiti area of Murang`a district buried 8 people of the same family.

15 Late 1982 – A landslide in Mikindani area along Nairobi – Mombasa road (Gichaga, 1984)

Majority of landslides occurring in Kenya have only been reported without conducting any investigation on their causes. Gichaga (1984) report on the Mikindani landslide which affected Nairobi – Mombasa road and the railway line, with the permission and co-operation of the Ministry of Transport & Communication of Kenya, was among the landslides that were investigated, and provided several recommendations. This landslide, crossing an erosion valley at the edge of a large plateau, was 12m thick with 20m elevation difference between the ground levels at the top of the plateau and the bottom of the valley (Fig. 1.3). The embankment was constructed in 1956 whereas the A109 road was built in 1970.

A culvert, carrying surface water drainage from the marshalling yard and surrounding areas, was laid at the bottom of the valley. The natural valley slope varied from 1:8 to 1:5 while the embankment slope was 1:2. The original soil, grey clay, underlying the embankment was decomposed Changamwe shale from the upper Jurassic period. The embankment material consisted of red/brown/grey clay with variable contents of silt and sand. It originated from Magarini sands which formed the upper layer of the plateau. The section of the road had shown considerable deformations which had been corrected by resealing during periodic routine maintenance of the road. However, the deformation continued to be a major source of concern to the Provincial Roads Engineer, Coast Province. By late 1982, the deformation led to a 525mm diameter water pipe burst accompanied by severe erosion of the embankment.

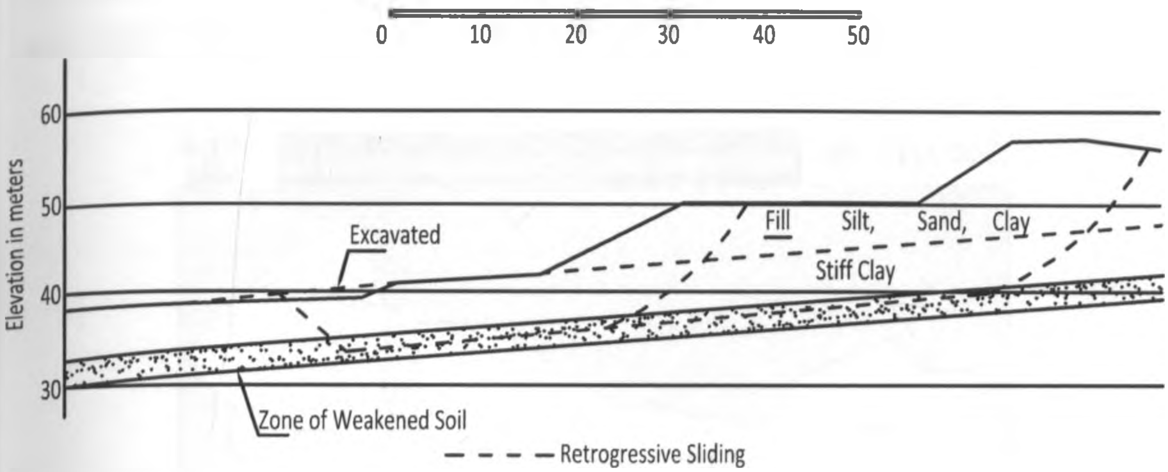


Fig. 1.3: Schematic representation of the embankment at the Mombasa landslide at Mikindani (After Gichaga (1984))

A factory at the bottom of the embankment underwent considerable distortion with heaving floor and bent columns and beams. As the embankment showed progressively increasing vertical and horizontal movements together with severe vertical cracks, the authorities were forced to close the road and abandon one line of railway. Field investigations consisted of seven boreholes to recover undisturbed samples, installation of 10 piezometers to monitor the ground water levels and ground survey works.

Gichaga (1984) acknowledged that the landslide occurred as a retrogressive creep failure in the layer of soft clay, in the original ground surface. He recommended to the then Ministry of Transport and Communication several remedial measures which included increasing the overburden pressure by constructing a counterbeam at the foot of the slope, reducing the pore pressure by constructing sand drains down to the fine sand layer and reconstructing the surface water drainage system including culverts through the embankment.

These are just but a few cases of slope failures in Kenya and the magnitude of their occurrence; hence slope instability is the main focus of this research. Landslide prone areas have been highlighted in the Kenya Map shown in Fig. 1.4.

Fig. 1.4: Landslide areas in Kenya

1.5 Scope and objectives

As the use of vegetation to stabilize soil slopes has become an increasingly used environmentally friendly alternative to traditional soil improvement methods, a better understanding on its application is important. At present, widespread use of vegetation to stabilize soil has been inhibited by a lack of verified methods to predict the reinforcing effect of various plant types (Bransby, 2004). Thus a research is required to investigate the link between root systems, root mechanical properties and soil-slope stability.

The main objective of this investigation was to provide information about the contribution of plant roots to soil shear strength. This information will help designers and land managers to;

(i) Make qualitative assessment, and

(ii) Perform quantitative slope stability analyses in vegetated landfills, riverbanks, shorelines, embankments, cut slopes and retaining walls, and other reclamation applications where the mechanical contribution of root reinforcement is important to predicting soil behavior.

To obtain the objectives, the following investigations were conducted;

(i) Investigations into the behavior of an individual root located across a shearing zone. Testing was performed to determine the behavior of the soil and root below the shearing zone and what effect this behavior has on the soil being sheared.

(ii) Investigations about individual root tensile strength of typical plant species and their contribution to soil shear strength. Root area ratio (RAR) values were obtained of the individual species to determine their distribution in the soil in a typical site where shallow landslide problems are rampant.

(iii) Investigations into root pull-out strengths were conducted.

(iv) Investigations into the effect of strain rate and specimen length on stress-strain relationships of typical vegetation roots used in slope stability problems were conducted. By varying the strain rate, the behavior of these roots during sudden movements, as in landslides, when the strain rates were increased, was established. This investigation yielded information that lead to establishment of a standardized mechanism of testing of roots and the establishment of a generalized design criteria using design charts.

(v) Determination of critical slope angles for most slopes was conducted.

The methodology and findings of this study will support the work of a broad set of planners, designers, and regulatory reviewers who are often faced with difficult decision regarding the selection and development of sites that could benefit from more rigorous evaluation of the soil strength contribution of plants. Ideally, this investigation and perhaps follow-on studies can help to establish both the merit and a practical approach to incorporating vegetation for its physical functions in land stabilization, including situations where it is currently routinely disregarded.

The questions that were answered in this research were:

- Do the roots contribute to the stability of slopes?
- Which kind of tree species should be introduced in a typical slope?
- What is the critical slope angle for any root-reinforced soil?

In order to achieve this, the following was undertaken:

1 Reconnaissance of the case study areas so as to establish geology of the areas, rainfall patterns, type of vegetation, types of soil and human activities which generally contribute to slope instability.

2 Theoretical research on stability of slopes, methods of slope stability analysis, types of slope failure modes, and slope stability mitigation measures.

3 The role of vegetation roots in slope stability.

4 Data collection from the case study area which included soil samples and other documented information.

5 Conducting laboratory soil tests.

This research presentation is broken down into the following sections;

(i) Chapter one presents a general introduction and objectives of the research

(ii) Chapter two presents literature review where background on slope stability using vegetation is analyzed together with existing theoretical models for predicting the effect of root reinforcement on the shear strength of a soil.

(iii) Chapter three presents the methodology used to achieve the objectives which included a series of tests designed to investigate the effect of a single root on the shear strength of a soil. These tests investigate the anchoring effect of roots penetrating below the shear zone. Specific attention was paid to the soil's resistance to root displacement and the effect of root's pull-out resistance to the normal load on the shear surface.

(iv) Results and analysis is presented in Chapter four together with conclusions drawn from the tests presented in Chapter three.

(v) Chapter five deals with design charts and graphs for root reinforced soils.

(v) Chapter six presents conclusions, recommendations and further research.

Chapter two

Literature review

2.1 Introduction

Roots are equal in importance to leaves as the life support system for plants and thus for all life in terrestrial ecosystems (Arora, 1991). Fig. 2.1 shows the root structure. The figure is rotated into upside-down position to demonstrate the importance that roots offer just as leaves do. Fig. 2.2(a - f) demonstrates different types of root systems found in the universe.

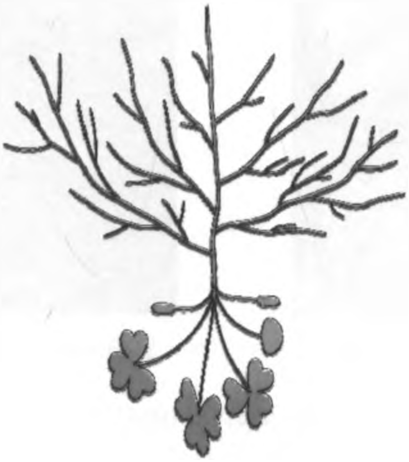
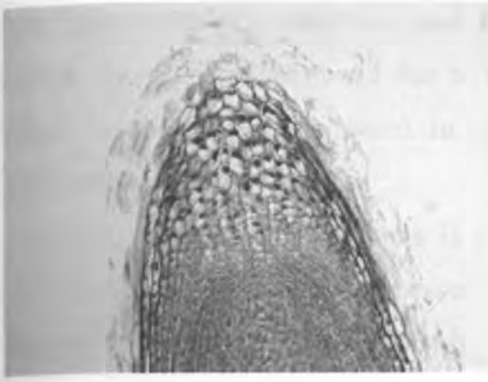


Fig. 2.1: Root structure

(a)Primary and secondary roots in a cotton plant

(b) Aerating roots of a mangrove



(c) The growing tip of a fine root

(d) The stilt roots of Socratea exorrhiza

(e) Cross section of a mango tree root system

(f) Roots on onion bulbs

Fig. 2.2(a - f): Different types of root systems

In vascular plants, the root is the organ of a plant that typically lies below the surface of the soil. This is not always the case, however, since a root can also be aerial (growing above the ground) or aerating (growing up above the ground or especially above water). Furthermore, a stem normally occurring below ground is not exceptional either. Early root growth is one of the functions of the apical meristem located near the tip of the root. The meristem cells more or less continuously divide, producing more meristem, root cap cells and undifferentiated root cells. The latter become the primary tissues of the root, first undergoing elongation, a process that pushes the root tip forward in the growing medium. Gradually these cells differentiate and mature into specialized cells of the root tissues (Chen *et. al.*, 1999).

Roots will generally grow in any direction where the correct environment of air, mineral nutrients and water exists to meet the plant's needs. Roots will not grow in dry soil. Over time, given the right conditions, roots can crack foundations, snap water lines, and lift sidewalks. At germination, roots grow downward due to gravitropism, the growth mechanism of plants that also causes the shoot to grow upward. In some plants such as ivy, the "root" actually clings to walls and structures (Coutts, 1987).

Growth from apical meristems is known as primary growth, which encompasses all elongation. Secondary growth encompasses all growth in diameter, a major component of woody plant tissues and many nonwoody plants. For example, storage roots of sweet potato have secondary growth but are not woody. Secondary growth occurs at the lateral meristems, namely the vascular cambium and cork cambium. The former forms secondary xylem and secondary phloem, while the latter forms the periderm (Raven and Edwards, 2001).

In plants with secondary growth, the vascular cambium, originating between the xylem and the phloem, forms a cylinder of tissue along the stem and root. The vascular cambium forms new cells on both the inside and outside of the cambium cylinder, with those on the inside forming secondary xylem cells, and those on the outside forming secondary phloem cells. As secondary xylem accumulates, the "girth" (lateral dimensions) of the stem and root increases. As a result, tissues beyond the secondary phloem (including the epidermis and cortex, in many cases) tend to be pushed outward and are eventually "sloughed off" (shed). At this point, the cork cambium begins to form the periderm, consisting of protective cork cells containing suberin. In roots, the cork cambium originates in the pericycle, a component of the vascular cylinder. The vascular cambium produces new layers of secondary xylem annually. The xylem vessels are dead at maturity but are responsible for most water transport through the vascular tissue in stems and roots (Raven and Edwards, 2001).

2.2 Specialized roots

The roots, or parts of roots, of many plant species are specialized to serve adaptive purposes.

- **Adventitious roots** arise out-of-sequence from the more usual root formation of branches of a primary root, and instead originate from the stem, branches, leaves, or old woody roots. They commonly occur in monocots and pteridophytes, but also in many dicots, such as clover (*Trifolium*), ivy (*Hedera*), strawberry (*Fragaria*) and willow (*Salix*). Most aerial roots and stilt roots are adventitious. In some conifers adventitious roots can form the largest part of the root system.
- **Aerating roots** (or **knee root** or **knee** or **pneumatophores** or **cypress knee**): Roots rising above the ground, especially above water such as in some mangrove genera (*Avicennia, Sonneratia*). In some plants like *Avicennia* the erect roots have a large number of breathing pores for exchange of gases.
- **Aerial roots**: Roots entirely above the ground, such as in ivy (*Hedera*) or in epiphytic orchids. They function as prop roots, as in maize or anchor roots or as the trunk in strangler fig.
- **Contractile roots**: They pull bulbs or corms of monocots, such as hyacinth and lily, and some taproots, such as dandelion, deeper in the soil through expanding radially and contracting longitudinally. They have a wrinkled surface.
- **Coarse roots**: Roots that have undergone secondary thickening and have a woody structure. These roots have some ability to absorb water and nutrients, but their main function is transport and to provide a structure to connect the smaller diameter, fine roots to the rest of the plant.
- **Fine roots**: Primary roots usually <2 mm diameter that have the function of water and nutrient uptake. They are often heavily branched and support mycorrhizas. These roots may be short lived, but are replaced by the plant in an ongoing process of root 'turnover'.
- **Haustorial roots**: Roots of parasitic plants that can absorb water and nutrients from another plant, such as in mistletoe (*Viscum album*) and dodder.
- **Propagative roots**: Roots that form adventitious buds that develop into aboveground shoots, termed suckers, which form new plants, as in Canada thistle, cherry and many others.

- **Proteoid roots** or cluster roots: These are dense clusters of rootlets of limited growth that develop under low phosphate or low iron conditions in Proteaceae and some plants from the following families Betulaceae, Casuarinaceae, Elaeagnaceae, Moraceae, Fabaceae and Myricaceae.
- **Stilt roots**: These are adventitious support roots, common among mangroves. They grow down from lateral branches, branching in the soil.
- **Storage roots**: These roots are modified for storage of food or water, such as carrots and beets. They include some taproots and tuberous roots.
- **Structural roots**: Large roots that have undergone considerable secondary thickening and provide mechanical support to woody plants and trees.
- **Surface roots**: These proliferate close below the soil surface, exploiting water and easily available nutrients. Where conditions are close to optimum in the surface layers of soil, the growth of surface roots is encouraged and they commonly become the dominant roots.
- **Tuberous roots**: A portion of a root swells for food or water storage, e.g. sweet potato. A type of storage root distinct from taproot (Phillips, 1963).

2.3 Rooting depths

The distribution of vascular plant roots within soil depends on plant form, the spatial and temporal availability of water and nutrients, and the physical properties of the soil. The deepest roots are generally found in deserts and temperate coniferous forests. The deepest observed living root, at least 60 m below the ground surface, was observed during the excavation of an open-pit mine in Arizona, USA. Some roots can grow as deep as the height of the tree. The majority of roots on most plants are however found relatively close to the surface where nutrient availability and aeration are more favorable for growth. Rooting depth may be physically restricted by rock or compacted soil close below the surface, or by anaerobic soil conditions (Phillips, 1963). Table 2.1 shows typical depths of roots.

Table 2.1: Rooting depth records

Species	*Location*	*Maximum rooting depth (m)*	*References*
Boscia albitrunca	Kalahari desert	*68*	Jennings (1974)
Juniperus monosperma	Colorado Plateau	*61*	Cannon (1960)
Eucalyptus sp.	Australian forest	*61*	Jennings (1971)
Acacia erioloba	Kalahari desert	*60*	Jennings (1971)
Prosopis juliflora	Arizona desert	*54*	Phillips (1963)

Roots are:

- Carbon pumps that feed soil organisms and contribute to soil organic matter
- Storage organs
- Chemical factories that may change soil pH, poison competitors, filter out toxins, concentrate rare elements, etc.
- A sensor network that helps regulate plant growth
- Absorptive network for limiting soil resources of water and nutrients
- Mechanical structures that support plants, strengthen soil, construct channels, break rocks, etc.
- Hydraulic conduits that redistribute soil water and nutrients
- Habitats for mycorrhizal fungi, rhizosphere and rhizoplane organisms (Bougher & Syme, 1998).

2.4 Root systems and architecture

The pattern of development of a root system is termed as *root architecture*, and is important in providing a plant with a secure supply of nutrients and water as well as anchorage and support. The architecture of a root system can be considered in a similar way to above-ground architecture of a plant—i.e. in terms of the size, branching and distribution of the component parts. In roots, the architecture of fine roots and coarse roots can both be described by variation in topology and distribution of biomass within and between roots. Having a balanced architecture allows fine roots to exploit soil efficiently around a plant, but the plastic nature of root growth allows the plant to then concentrate its resources where nutrients and water are more easily available. Balanced coarse root architecture, with roots distributed relatively evenly around the stem base, is necessary to provide support to larger plants and trees (Sutton and Tinus, 1983).

Tree roots normally grow outward to about three times the branch spread. Only half of a tree's root system occurs between the trunk and the circumference of its canopy. Roots on one side of a tree normally supply the foliage on that same side of the tree. Thus when roots on one side of a tree are injured, the branches and leaves on that same side of the tree may die or wilt. For some trees however, such as the maple family, the effect of a root injury may show itself anywhere in the tree canopy (Phillips, 1963).

The recognition of different types of roots is important because these can have different functions. Most plants produce one or more orders of lateral root branches. Different orders of roots vary in their thickness, branching patterns, growth rates, capacity for secondary growth, lifespans, structural features, etc. These variations will influence their capacities to obtain water and nutrients, support mycorrhizal associations and survive adverse conditions. Higher order lateral roots are generally thinner, shorter and don't live as long as those of lower (Brundrett, 1991).

Types of roots

- Seminal root - from a seed
- Adventitious root - from a stem

- First order lateral root - from a seminal or adventitious root
- Second order laterals, etc. - from first order laterals, which in turn produce third order laterals, and so on.
- Feeder roots - fine, relatively short-lived roots that acquire nutrients and water in the topsoil
- Primary roots - from primary growth by the apical meristem
- Secondary roots - mature, thicker "woody" roots with bark and additional vascular tissue
- Coarse roots - may live for a long time and have roles in transport and mechanical support (Brundrett *et al.*, 1985).

Fig. 2.3 shows the orders in a typical root assembly

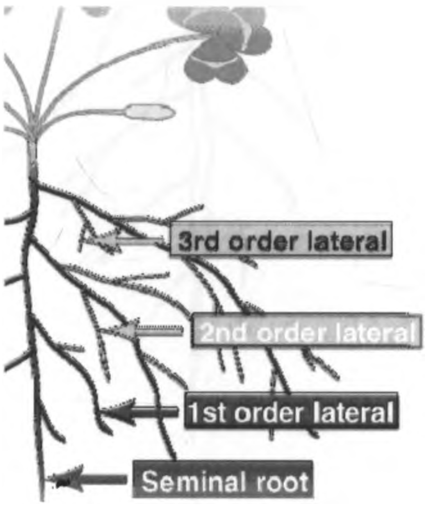


Fig. 2.3: Root orders

2.5 Fibrous and taproot systems

A taproot system (Fig. 2.4) derives its nutrients directly from the first root that emerge from a seed (the radicle or primary root) that enlarges and forms a prominent central root that is called the taproot. The taproot is larger in diameter than the lateral roots. Lateral roots branch off from the taproot, and subsequent lateral roots can branch off other lateral roots. Taproots generally grow more deeply into the soil than do fibrous roots. It often becomes a modified storage organ for food reserves such as carbohydrate or for reaching water deep in the ground. A taproot system, generally found in dicotyledons and conifers. Most trees begin life with a taproot, but after one to a few years change to a wide-spreading fibrous root system with mainly horizontal surface roots and only a few vertical, deep anchoring roots. A typical mature tree 30-50 m tall has a root system that extends horizontally in all directions as far as the tree is tall or more, but well over 95% of the roots are in the top 50 cm depth of soil.

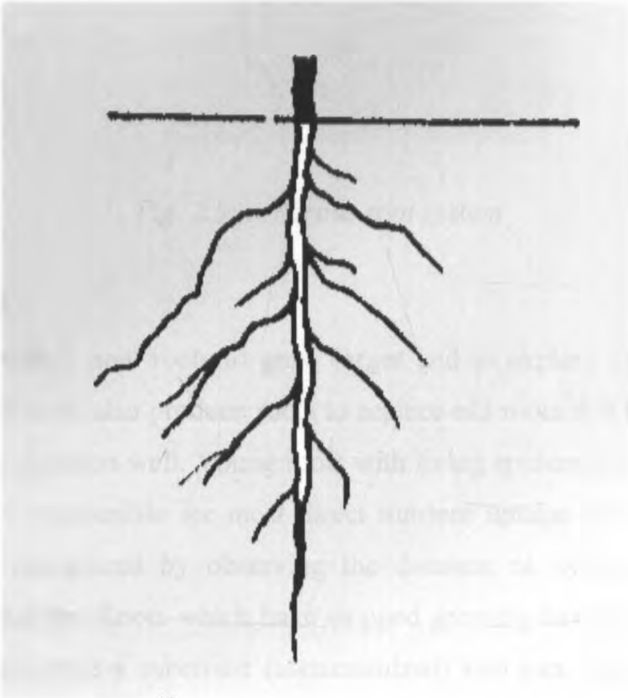


Fig. 2.4: A tap root system

A fibrous (Also diffuse or fasciculate) root system (Fig. 2.5) is a kind of root system in which both primary and lateral roots are finely divided and have approximately equal diameters, without evident thickening or an enlarged central root. Most monocots have a fibrous root system consisting of an extensive mass of similarly sized roots. In these plants, the radicle is short lived and is replaced by a mass of adventitious roots which are roots that form organs other than roots; for example the root of the grass.

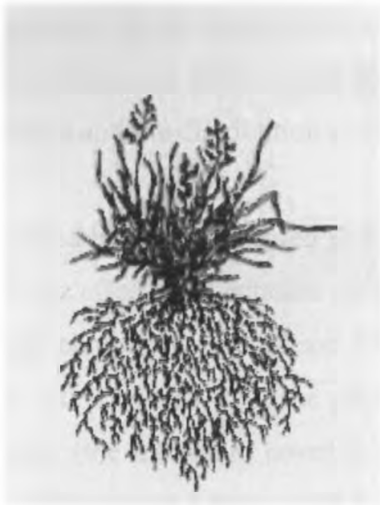


Fig. 2.5: A fibrous root system

2.6 Root growth

Plants must produce new roots to grow larger and to explore new volumes of soil to acquire nutrients. They must also produce roots to replace old roots that have died, were lost to predation, or no longer function well. Young roots with living epidermal cells and root hairs, are often considered to be responsible for most direct nutrient uptake (Lyr & Hoffmann, 1967). Young roots can be recognized by observing the distance of xylem and endodermis cell maturation from the root tip. Roots which have stopped growing have mature xylem vessels at their apex and may also have a suberised (metacutinized) root cap. These features are readily apparent after roots have been cleared and stained (O'Brien & McCully, 1981).

Root tissues are produced by cell division in the root apex and cell expansion in subapical regions. Cell division at the apical meristem produces new root cap cells in an outward direction and new root cells in an inward direction. Root tissues progressively mature at greater distances from the root tip and may develop specialized features of their cell walls or cytoplasm. Most of what we know about plants comes from scientific studies of crops selected from weedy ancestors for rapid growth in highly fertile soils. However, this information may not be relevant to plants in natural ecosystems. Crop plants typically have roots which elongate 1 cm or more in a day (Russell, 1977), while roots of plants from a natural ecosystem may grow 1 mm or less a day (Brundrett, 1991).

2.7 Roots as slope reinforcement material

The additional strength created by the roots is defined as the growing cohesion, which increases with vertical stress and is occupied by the roots. The main effect of vegetation on slope stability is generally considered to be mechanical stabilization due to the response of the roots. This effect is applied through the strength and the distribution of roots within the soil (Osano and Mwea, 2011).

Stresses acting on an infinite hillslope are represented in Fig. 2.6. According to Wu *et al.* (1979) and Osano and Mwea, (2011), the effect of vegetation roots on soil shear strength can be taken as part of the cohesive strength component of the soil-root system. Assuming that the phreatic surface is at the soil surface and the location of the potential shear plane is z distance below the soil surface, the safety factor (the minimum possible shear strength / the maximum possible shear stress) for a vegetated infinite slope is given by Equation 2.1;

$$SF = \frac{\left[c' + \Delta c + \left(z\cos^2(\alpha)(\gamma_{sat} - \gamma_w) + w_t\cos\alpha\right)\tan\phi'\right]}{\left[z\gamma_{sat}\cos\alpha\sin\alpha + w_t\sin\alpha\right]}$$

Equation 2.1

Where;

(i) c' and ϕ' are the effective soil strength parameters,

(ii) Δc is the increased cohesion due to tree roots,

(iii) α is the slope angle,

(iv) w_t is the vegetation surcharge (weight / unit area),

(v) γ_{sat} is the saturated unit weight of soil,

(vi) γ_w is the unit weight of water.

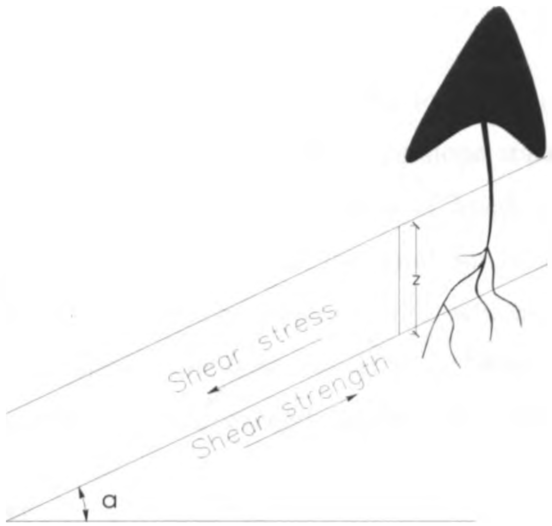


Fig. 2.6: Schematic of infinite slope stability analysis for a planar failure

In conventional limit equilibrium slope stability analysis, the shear strength ($c'+\Delta c+\left(z\cos^2(\alpha)(\gamma_{sat}-\gamma_w)+w_t\cos\alpha\right)\tan\phi'$) is usually estimated using the soil strength parameters c' and ϕ' with the estimated effective normal stress on the potential shear plane under the given pore-water pressure condition. In order to predict the landslide threshold conditions, these soil strength parameters are estimated from the Mohr-Coulomb failure envelope derived from the peak values of a series of shear stress-displacement curves (Ekanayake *et al.,* 1997).

Vegetation and slope stability are interrelated by the ability of the plant life growing on slopes to both promote and hinder the stability of the slope. The relationship is a complex combination of the type of soil, the rainfall regime, the plant species present, the slope aspect, and the steepness of the slope. Knowledge of the underlying slope stability as a function of the soil type, its age, horizon development, compaction, and other impacts is a major underlying aspect of understanding how vegetation can alter the stability of the slope (Mattia *et al.* 2005).

There are four major ways in which vegetation influences slope stability: wind throwing, the removal of water, mass of vegetation (surcharge), and mechanical reinforcement of roots.

2.7.1 Wind throwing

Wind throw is the toppling of a tree due to the force of the wind, this exposes the root plate and adjacent soil beneath the tree and influences slope stability. Wind throw is a factor when considering one tree on a slope, however it is of lesser importance when considering general slope stability for a body of trees as the wind forces involved represent a smaller percentage of the potential disturbing forces and the trees which are in the centre of the group will be sheltered by those on the outside (Greenwood et al. 2004). The roots anchor the plant in position thereby increasing the soil shear strength and prevents against tree toppling

2.7.2 Removal of water

Vegetation influences slope stability by removing water through transpiration. Transpiration is the vaporization of liquid water contained in plant tissue and the vapour removal to the air (Food and Agriculture Organization of the United Nations, 2007). Water is drawn up from the roots and transported through the plant up to the leaves .

The major effect of transpiration is the reduction of soil pore water pressures which counteracts the loss of strength which occurs through wetting. This is most readily seen as a loss of moisture around trees. However it is not easy to rely on tree and shrub roots to remove water from slopes and consequently help ensure slope stability. The ability to transpire in wet conditions is severely reduced and therefore any increase in soil strength previously gained in evaporation and transpiration will be lost or significantly reduced. Consequently the effects of transpiration cannot be taken into account at these times. However it can be assumed that the chance of slope failure following saturation by storm event or periods of extended rainfall will be lessened as a result of transpiration. Moreover, although changes in moisture content will affect the undrained shear strength, the effective shear stress parameters as commonly used in routine slope stability analysis are not directly influenced by changing moisture content,, although the water pressures (suctions) used in the analysis will change (Food and Agriculture Organization of the United Nations, 2007; Greenwood et al., 2004). This is because shear stress in soil structure can only be resisted by the skeleton of solid particles by means of forces developed at the inter-particle forces.

It is important to note that desiccation cracks can potentially be extended by vegetation in dry weather promoting the deeper penetration of water to a potential slip plane and increased water pressure into the soil during the wet periods. Nevertheless these cracks will be filled by roots growing deeper into the soil as they follow the path of least resistance (Greenwood et al., 2004).

Studies in Malaysia have shown that there is a significant relationship between root length density, soil water content and ultimately slope stability. Slopes that had high root density (due to dense vegetation on the surface) were less likely to undergo slope failure. This is because a high root length density results in low soil water content which in turn results in an increase in shear strength and a decrease in soil permeability. It is suggested that root length density and soil water level could be used as indicators of slope stability and possibly could be used to predict future slope failure (Perry et al. 2003).

Transpiration is accentuated when the vegetation has an extensive root system and rapid transpiration continues throughout winter (Perry et al. 2003).

The removal of water is also affected by the shading provided by vegetation. Shading helps prevent the desiccation of the soils which results in shrinkage and cracking allowing the deep penetration of rain water. Plants need to have a high leaf to root ratio and have the ability to persist through hot summer months in order to provide effective shading of the soils (Perry et al. 2003).

2.7.3 The mass of vegetation

The mass of vegetation is only likely to have an influence on slope stability when larger trees are growing on the slope. A tree of 30 -50m height is likely to have a loading of approximately 100 -150kN/m^2. The larger trees should be planted at the toe of the slope with a potential rotational failure as this could increase the factor of safety by 10%. However if the tree is planted at the top of the slope this could reduce the factor of safety by 10%.

Each slope stability situation should be considered independently for the vegetation involved. It is important to remember that transpiration will reduce the weight of the slope as moisture is lost. This can be significant on slopes of marginal stability (Greenwood et al. 2004).

If larger trees are removed from the toe area of a slope there will be both a reduction in soil strength due to the loss of evapo-transpiration effects and a reduction in applied loading which may result in temporary suctions in clay soils which could lead to softening as the available water is drawn in to compensate for the suction forces. This is similar to the recognized softening of over-consolidated clays due to the relaxation of overburden pressures when placed in the top layers of an embankment from deep cutting (Greenwood et al. 2004).

2.7.4 Mechanical reinforcement of roots

Roots reinforce the soil through growing across failure planes, root columns acting as piles, and through limiting surface erosion.

(a) Root growth across failure planes

When roots grow across the plane of potential failure there is an increase in shear strength by binding particles. The roots anchor the unstable soil into the deeper stable layers or bedrock (Mattia et al., 2005). This most readily occurs when there is rapid deep growth (1.5m deep) of roots which last for more than two years. However it is important to note that the strength exerted by roots generally only extends down to 1m while most failures occur between 1.0 – 1.5m soil depth (Perry *et al.* 2003).

(b) Root reinforcement model

The root reinforced earth root model is the result of the root elongation across a potential slip plane which produces a tensile root force which is transferred to the soil by cohesive and frictional contacts between the root and the soil (van Beek *et al.* 2005).

(c) Tensile root strength contribution and pull out resistance

The pull-out resistance of a root is the measured resistance of root structure to be pulled out of the ground. The measured tensile strength of the root is found to be more than the pull-out strength. In the cases where there is no pull out data available, the tensile strength data can be used as a rough guide to the maximum pull out resistance available (Greenwood *et al.* 2004, Osano and Mwea, 2011).

The tensile root strength of a range of diameters over a range of species has been tested in the laboratory and has been found to vary between 5 and 60MN/m^2. In order for the root to actually enhance slope stability the root must have sufficient embedment and adhesion with the soil. The way that roots interact with the soil is intricate but for engineering purposes the available force contributions may be measured with *in situ* pull out tests (Greenwood et al. 2004).

(d) Root morphology and modes of failure

The root length and the type of root branching affects the way root failure occurs (Greenwood *et al.* 2004; Norris 2005). Three different modes of failure have been identified in hawthorn roots which relate to the root soil relationship which is shown in the shape of the roots and the shape of the failure curve. Roots which have no branches tend to fail in tension and pull straight out of the ground with minimal resistance. Roots which have multiple branches generally fail in stages as each branch breaks inside the soil. These roots can then separated into two different groups;

1) those that initially reach their maximum peak force and then maintain a high force that progressively decreases as the root branches fail after significant strain and

2) those that break with increasingly applied force.

In a number of tests, considerable adhesion between a segment of the root and the soil can be measured prior to the root eventually slipping out of the soil mass (Greenwood et al. 2004).

(e) Type A failure

Roots that do not have branches generally fail in tension and pull straight out of the ground with only minimal resistance. The root reaches its maximum pullout resistance then rapidly fails at a weak point. The roots easily slip out of the soil due to the gradual tapering (progressive decrease in root diameter along its length) which means that as the root is pulled out it is moving through a space that is larger than its diameter which consequently has no further bonds or interaction with the surrounding soil (Norris, 2005).

(f) Type B failure

Type B failure occurs when branched roots initially reach their maximum peak resistance, and then sustain a high resistance which slowly reduces as the branches of the roots fail after significant strain. In some tests considerable adhesion between a section of the root and the soil mass can be measured before the root eventually slips out. Forked roots require a greater force to be pulled out as the cavity above the fork is thinner than the root which is trying to move through the cavity, this can then result in deformation of the soil as the root moves through the soil (Norris, 2005).

(g) Type C failure

Roots that have multiple branches or forked branches also can undergo tensile failure but predominantly fail in stages as each branch breaks within the soil. These roots break with increasingly applied force in stages in the form of stepped peaks corresponding to the progressive breaking of roots of greater diameters. The root progressively releases its bonds with the soil until final tensile failure (Norris, 2005).

In some cases when the root has a sinusoidal shape with many small rootlets along its length the root reaches its maximum pull out resistance on straightening and then breaks at the weakest point, however at this point the root is not pulled out of the soil as it adheres and interacts with the soil producing a residual strength. If pulling was stopped at this point, the root would give increased strength to the soil. However if the root is completely pulled out of the ground then there is no further interaction with soil and therefore no increase in soil strength is provided (Norris, 2005).

2.8 Factors which affect root pull-out resistance

Studies by Norris (2005) have shown that the pull out resistance of hawthorn and oak roots are affected by intra species differences, inter-species variations and root size (diameter) in a similar as way as root tensile strength varies (as measured in the laboratory). In the pull out test the applied force acting on the root acts across a larger root area, which involves multiple branches, longer lengths) than the short (approximately 150mm) length of root used in tensile strength tests. In pull out test the root is likely to fail at weak points such as branching points, nodes or damaged areas.

Norris (2005) also showed that there is a positive correlation between maximum root pull out resistance and root diameter for hawthorn and oat root. Smaller diameter roots had a lower pull out resistance or breaking force than the larger diameter roots.

2.9 Root columns acting as piles

Trees and root columns can prevent shallow mass movement through acting as piles when there is buttressing and soil arching through a woody deep root system which has multiple sinker roots with embedded stems and laterals (Perry et al. 2003). Section 2.5 presents the literature on this theory.

2.10 Limitation of surface erosion

Vegetation can be used to control water erosion by limiting surface processes such as sheet wash and overland flow. Vegetation can provide a considerable contribution to the stability of slope through enhancing soil cohesion. This cohesion is dependent upon the morphological characteristics of root systems and the tensile strength of single roots (Mattia et al. 2005).

There is considerable evidence of fine roots resisting surface erosion. The role of fine roots in general slope stability is not fully understood. It is thought that the fine roots help keep the surface soil together and prevent surface erosion. The fine root network may have an apparent enhanced cohesion which is comparable to geosynthetic mesh elements. The limitation of surface erosion processes is particularly apparent in areas of shrub and grass where the fine root distribution is consistent and clearly defined, however cohesion is generally limited to the top 1m of soil (Greenwood et al. 2004).

2.11 Soil bioengineering systems & root geometry

Bayfield et al. (1992) explained that the surfaces of slope from the direct impact of raindrops is protected by plants, and they help trap waterborne sediment, reduce the velocity of surface runoff and strengthen the soil by the binding action of their roots. Similarly, vegetation has been indicated to have the tendency to reduce the moisture content of soil through interception of rain and transpiration. These usually increase the stability of slope; however, they also increase infiltration through root penetration, and may in some circumstances (such as where there is heavy surface runoff) decrease stability.

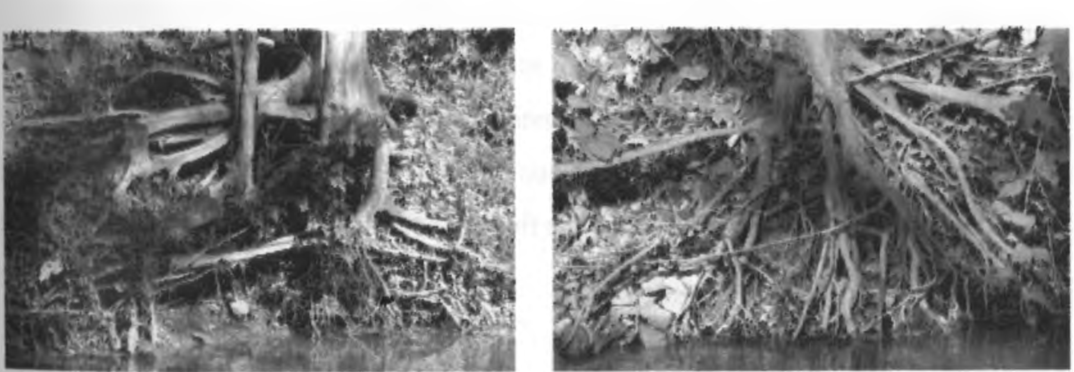
Grasses are the most widely used vegetation on slopes as their roots concentrate usually in the top of 30 to 50 centimeters of soils, but they can also penetrate up to about one meter. Shrubs and trees, on the other hand, provide deeper slope reinforcement, with roots which penetrate to three meters and more, but are mainly concentrated in top one to two meters. Specialized methods have been developed to establish vegetation on slopes. In these methods, un-rooted cuttings, which are cut from live plants, are used, imbedded and arranged in the ground, in special patterns and configurations. These embedded cuttings take root, become established on the slope and act as barriers to earth movement, soil reinforcement, moisture wicks and hydraulic drains.

Two commonly used systems are extended to sufficient depth to serve as reinforcement in shallow slides. Brush layers consist of live branches which are placed in trenches or between layers of compacted fill (Fig. 2.7). Live stakes or live poles are stems cut from live trees and installed vertically or in a direction perpendicular to the slope (Wu, 2007). Live poles (Steele et al., 2004) consisting of willow stems, with diameter of 4-10cm and length of about 2m, were used to stabilize shallow slides.

Fig. 2.7: Brush layering

Wu (2007) showed that, root geometry denotes all the properties which are necessary to define the positions and dimensions of the roots in the system.

Plate-shaped root systems are composed mainly of lateral roots. The diameter of lateral roots decreases rapidly with the distance from the root crown. The mass which contains most of lateral roots is sometimes called the root mat (Fig. 2.8 a and b)



(a) (b)

Fig. 2.8 (a) and (b): Tangled root mat

More detailed correlations between different dimensions of a root system can be developed and used in computer simulations so as to generate a distribution of root diameters at various distances from the stem (e.g. Wu & Watson, 1998).

2.12 Root reinforcement theories

2.12.1 Introduction

Root reinforcement theory has basically been developed along two avenues. The first method originated with the effects to quantify the effects of deforestation and precipitation on the stability of slopes, and entailed a description of root soil interaction within a shear band through force equilibrium. The formulations were proposed by Waldon (1977) and Wu *et al.* (1979). Subsequent advances to these approaches mainly comprised refinements (for instance) in the form of explicit definitions of reinforcement element orientation (Gray & Ohashi, 1983) and improved description of load transfer from soil to reinforcement elements (Juran *et al.*, 1988). These advances were, however, increasingly based on fiber reinforced soil behavior with root reinforcement, owing its origin to the description of the behavior of composite materials. This method considers the macroscopic properties of composites, with the distinct characteristics of fibers and matrix having been homogenized or averaged with roots. Within this context of fiber reinforcement, root reinforcement is clearly identified as a specific case.

2.12.2 The Wu *et al.* (1979) Model

The model of Wu *et al.* (1979) is used to estimate the increase in soil shear strength due to presence of roots. Their model assumes that roots grow vertically and act as loaded piles, so tension is exerted to them as the soil is sheared. This model was also used by De Baets et al. (2008) where they tested root tensile strength and root distribution of typical Mediterranean plant species. If the soil is rooted, the increased soil shear strength can be expressed as an additional cohesion in Equation 2.2;

$$s_r = s + Cr$$ **Equation 2. 2**

where;

(i) s is soil shear strength without roots (kPa),

(ii) s_r (kPa) is the shear strength of the soil with roots, and

(iii) C_r (kPa) is the increase in shear strength due to the presence of roots.

When shear forces occur, the root fiber deforms. This deformation causes the fiber to stretch, provided there is sufficient interface friction, confining stress and anchorage length to lock the fiber in place and to prevent slippage or pullout. The fiber elongation mobilizes the tensile resistance in the fiber (Gray and Sotir, 1996). The tension developed in the roots is resolved with a tangential component resisting shear and a normal component increasing the confining pressure on the shear plane. The most critical assumption of this model implies that all roots attain ultimate tensile strength simultaneously during soil shearing.

From Wu et al. (1979 model, the predicted shear strength increase from a full mobilization of root tensile strength is given by Equation 2.3;

$$Cr = t_R(\sin\theta + \cos\theta \tan\phi) \qquad \textbf{Equation 2.3}$$

Where;

(i) θ is the angle of shear distortion in the shear zone, which is the angle the root makes across a shear zone in a soil profile when the root develops tension (Fig. 2.9); the component of this tension tangential to the shear zone directly resists shear, while the normal component increases the confining pressure on the shear zone.

(ii) ϕ is the soil friction angle (°), and

(iii) t_R is the total mobilized tensile stress of roots fibers per unit area of soil.

Generally, the model simplifies the actual behavior of the sinker roots (Fig. 2.10) and assumes that a flexible, elastic root penetrates vertically across a shear zone. The root is initially oriented perpendicularly to the shear zone. A tensile force develops in the root as the soil is sheared (Fig. 2.11). This tensile force can be resolved into components acting normally and tangentially to the shear zone. The tangential component directly resists shear while the normal component contributes to the confining stress on the shear zone.

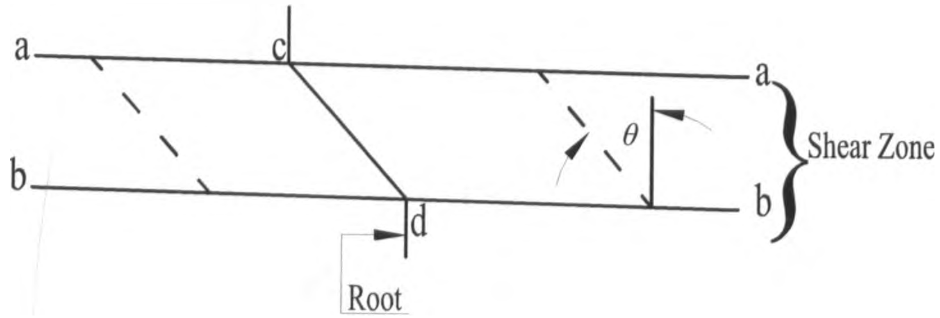


Fig. 2.9: Tensile force development during shearing

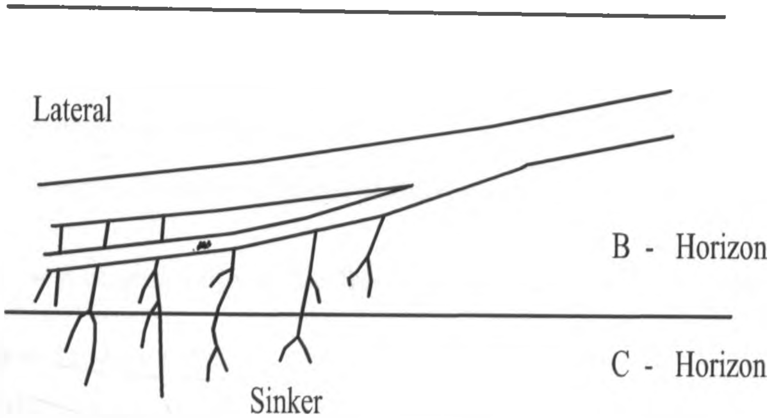


Fig. 2.10: General sinker roots development in soil

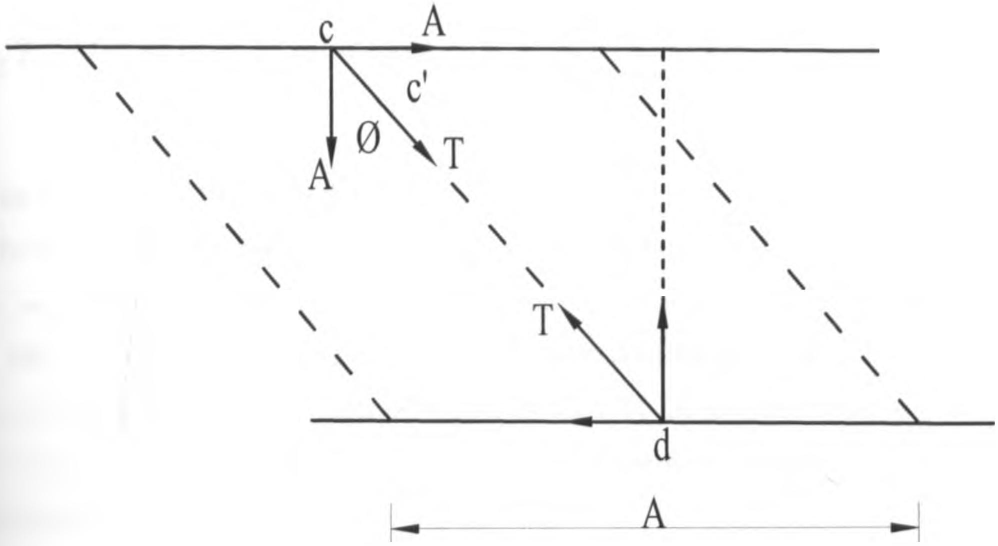


Fig. 2.11: Tensile force resolving in the shear zone

The mobilization of the root's tensile resistance increases the shear strength of the soil by an amount as in Equation 2.4.

$$\Delta S_R = T_R \frac{A_R}{A}(\sin\theta + \cos\theta \tan\phi)$$ **Equation 2.4**

Where,

(i) ΔS_R = shear strength increase resulting from root displacement,

(ii) θ = angle of shear distortion,

(iii) ϕ = angle of internal friction,

(iv) T_R = average tensile strength of the roots,

(v) $\frac{A_R}{A}$ = root area ratio or the fraction of the soil cross-sectional area by roots, which is dimensionless.

Wu *et al.* (1979) pointed out that the bracketed term in Equation 2.3 is relatively insensitive to any change in the expected value of either the shear distortion angle, θ, or the internal friction angle, Φ, of the soil. An average value of 1.2 was determined for the bracketed term after considering all possible combinations of θ and Φ. The maximum shear strength increase resulting from root reinforcement is now approximated as shown in Equation 2.5;

$$\Delta S_R = 1.2\, T_R \frac{A_R}{A}$$ **Equation 2.5**

2.12.3 The McOmber (1981) Model

McOmber (1981) presented a model for soil interaction which analyzed the mechanism of failure as the soil-root adhesion is exceeded and the root pulls out of the soil. His model assumed that soil-root interaction resulted from the adhesion between the soil and the root, and from the resistance of the soil to lateral motion of the root. In his simplified soil-root system, shown in Fig. 2.12 – 2.14, McOmber (1981) assumed that roots behave as perfectly flexible fibers resisting forces in pure tension.

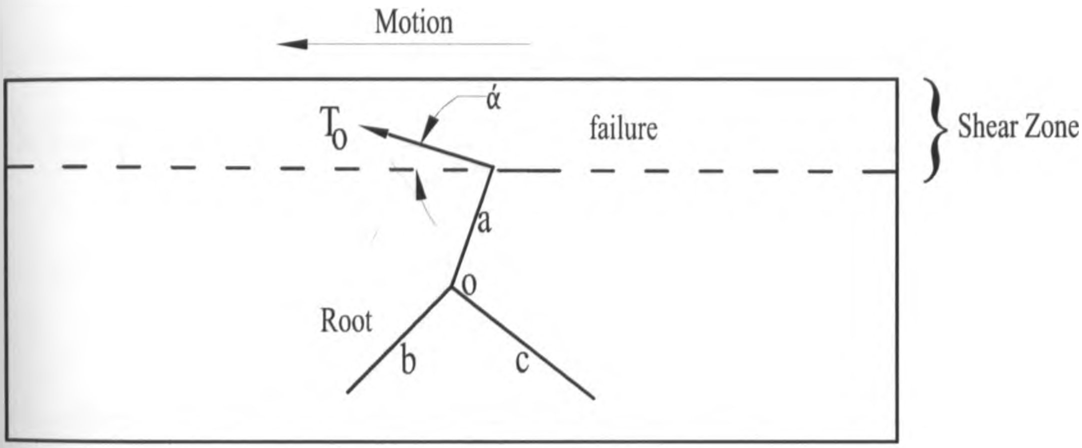


Fig. 2.12: Idealized soil-root system - tensile root failure (after McOmber (1981))

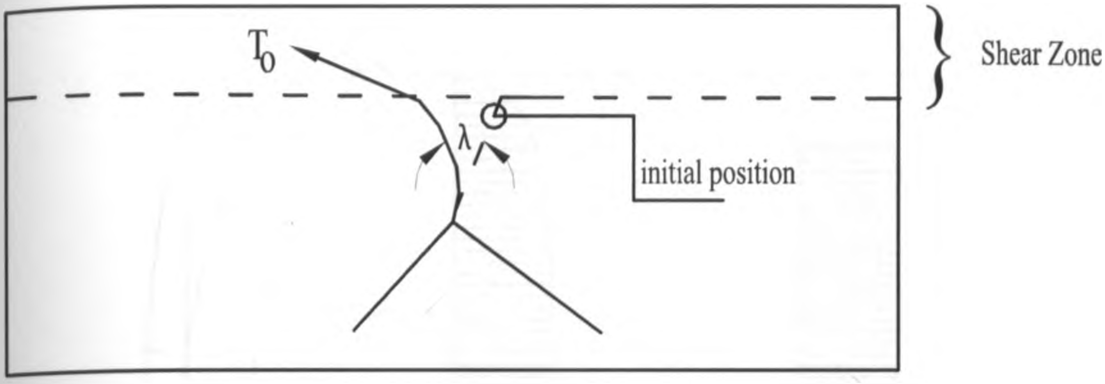


Fig. 2.13: Idealized soil-root system - deflection of root section a (after McOmber (1981))

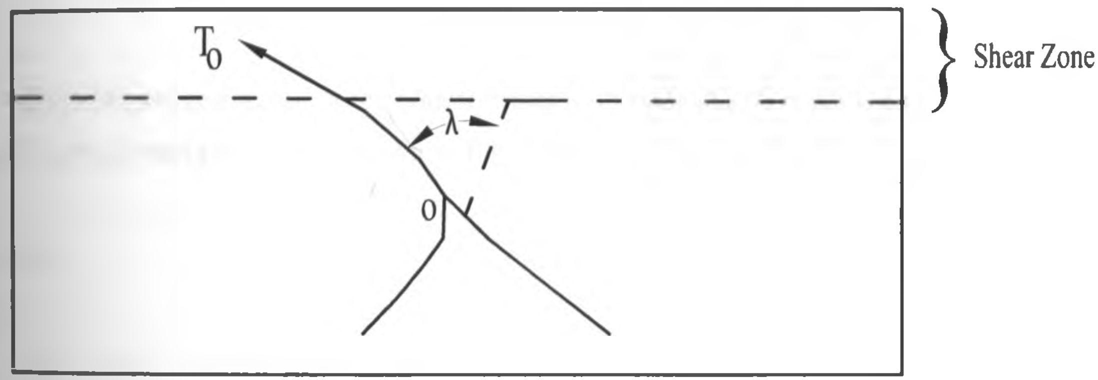


Fig. 2.14: Idealized soil-root system - initiation of movement in root section b and c (after McOmber (1981))

Therefore, the roots could only fail in tension and never in shear. This assumption is consistent with the assumption made in the model developed by Wu *et al.* (1979).

In analyzing the soil's resistance to root motion, McOmber compared the soil reaction to root displacement to the soil reaction of a laterally loaded pile. McOmber determined the horizontal and vertical soil reaction by using the method proposed by Broms (1964a) to solve the pile deflection problem. Fig. 2.15 illustrates a pile subjected to a lateral load and the resulting soil reaction.

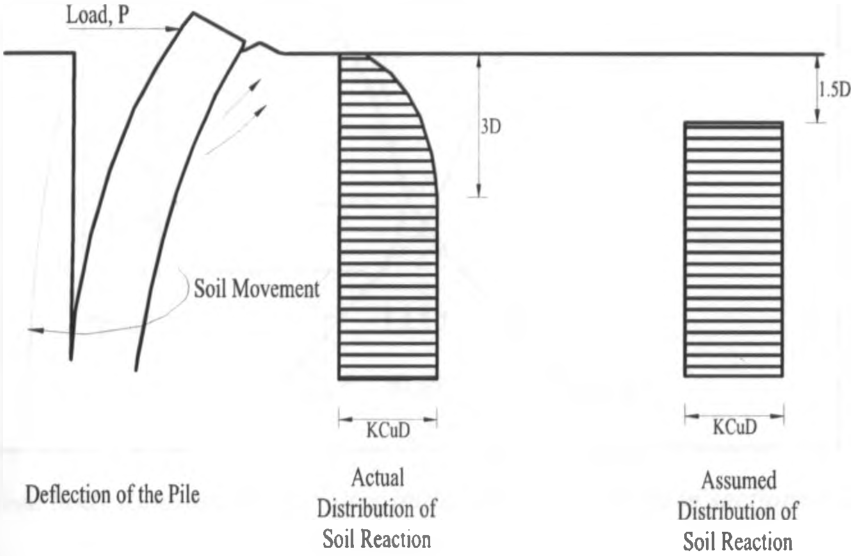


Fig. 2.15: Distribution of lateral soil reaction (after Broms (1964a))

The ultimate resistance of the cohesive soil to the lateral pile motion is approximated by a uniformly distributed pressure as shown in Fig. 2.15.

$p_y = KC_uD$ **Equation 2.6**

Where,

(i) P_y = the ultimate soil reaction,

(ii) K = the coefficient of subgrade reaction, dimensionless,

(iii) C_u = the undrained shear strength, and

(iv) D = the diameter of the pile.

Fig. 2.16 is a free body representation of the root model during the latter stages of loading.

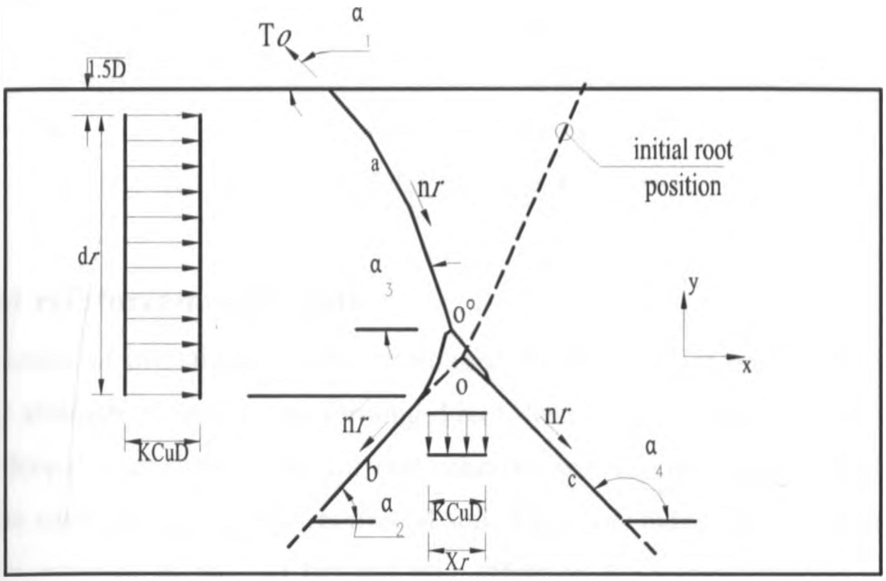


Fig. 2.16: Free body diagram of root after initiation of movement in sections b and c (after McOmber (1981))

At this stage, the soil-root adhesion in sections b and c has exceeded. Point O has moved to point O', section a has deflected horizontally and section b and c have deflected both horizontally and vertically. Therefore, the ultimate soil resistance must be overcome both in the horizontal and vertical directions. Summing the forces in the x and y directions yields Equation 2.7 and 2.8;

$\Sigma F_y = 0$

$$T_o \sin\alpha_1 = KC_u D x_r + n_r D\left[l_b \sin\left(\frac{\alpha_3}{2}\right) + l_c\left(\frac{\alpha_4}{2}\right) + l_a \sin(\alpha_1 + \alpha_2)\right]$$

Equation 2.7

and

$\Sigma F_x = 0$

$$T_o \cos\alpha_1 = KC_u D d_r + n_r D\left[-l_b \cos\left(\frac{\alpha_3}{2}\right) + l_c\left(\frac{\alpha_4}{2}\right) + l_a \sin\left(\frac{\alpha_1 + \alpha_2}{2}\right)\right]$$

Equation 2.8

where,

(i) l_a, l_b and l_c = lengths of root sections a, b and c respectively,

(ii) n_r = residual soil-root adhesion, and

(iii) α_1, α_2, α_3, and α_4 = angles shown in Fig. 2.16.

2.13 Root reinforcement testing

A number of investigators have determined the influence of root or fiber reinforcement on the shear strength of soils by performing direct shear tests on laboratory and field samples. Kassif and Kopelovitz (1968) tested a non-cohesive soil that contained synthetic fibers and compared the shear strength of the reinforced soil. They concluded that the fiber reinforcement increased the cohesion of the soil but had little effect on the internal friction angle. Cohesion increased with increases in the surface area, bulk density, and fixity of the fibers. Kassif and Kopelovitz suggested deformation and failure of a reinforced soil occurs in a number of stages. The first stage is the elastic deformation of both the soil and reinforcement. Next, the soil undergoes plastic deformation while the reinforcement continues to deform elastically. During this stage or the following stage the shear strength of the reinforced soil surpasses the maximum shear strength of the non-reinforced soil. Eventually both the soil and reinforcement exhibit plastic behavior.

Endo and Tsurata (1969) determined the reinforcing effect of tree roots on soil shear strength by performing *in-situ* direct shear tests on soil blocks containing live tree roots. Fig. 2.17 – 2.19 illustrates Endo and Tsuruta's testing procedure.

The shear strength of the reinforced soil increased directly with the bulk weight of roots per unit volume of soil. Endo and Tsurata (1969) summarized their results in the empirical Equation 2.9

$$\Delta S_R = a(R + b)$$ **Equation 2.9**

Where;

(i) ΔS_R = increase in shear resistance

(ii) R = concentration or density of roots in the soil, and

(iv) a and b= empirical constants (a = 0.93x10, b = 53 g/m^3)

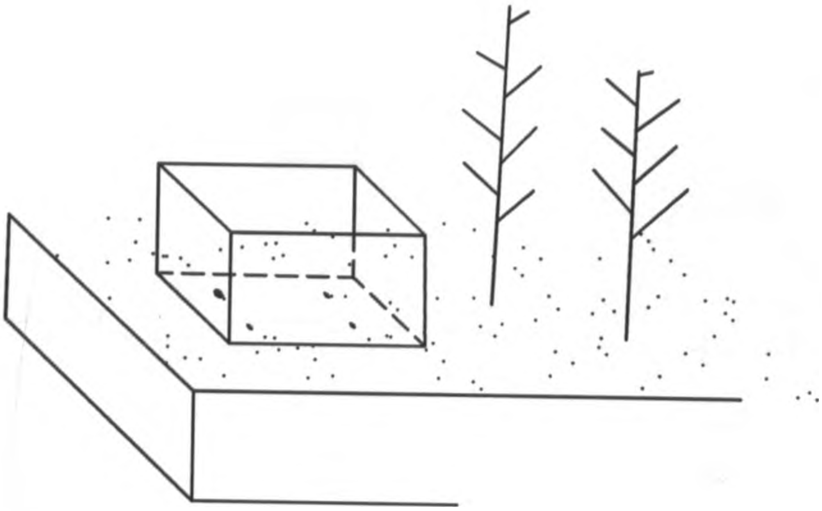


Fig. 2.17: In-situ shear test: soil pedestal guide box in place (after Endo and Tsuruta (1969))

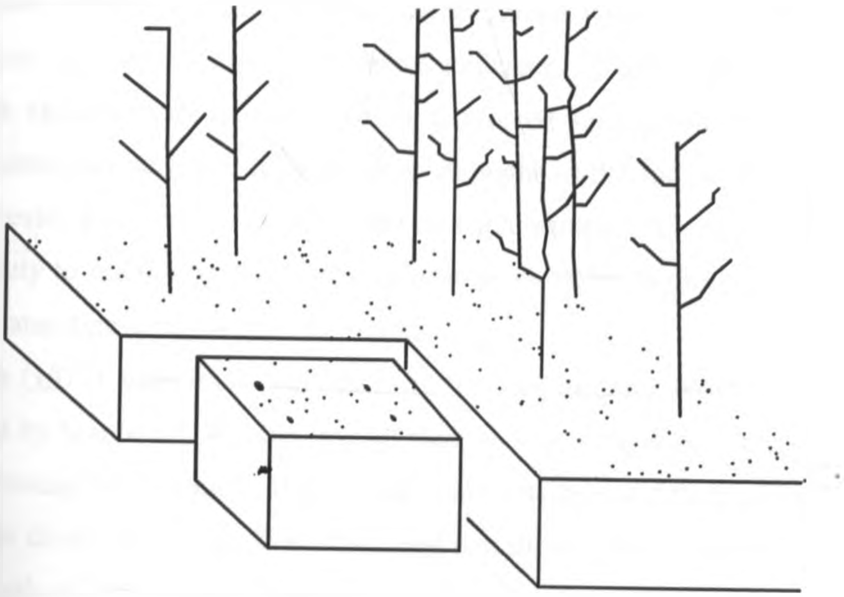


Fig. 2.18: In-situ shear test: soil pedestal excavated and exposed (after Endo and Tsuruta (1969))

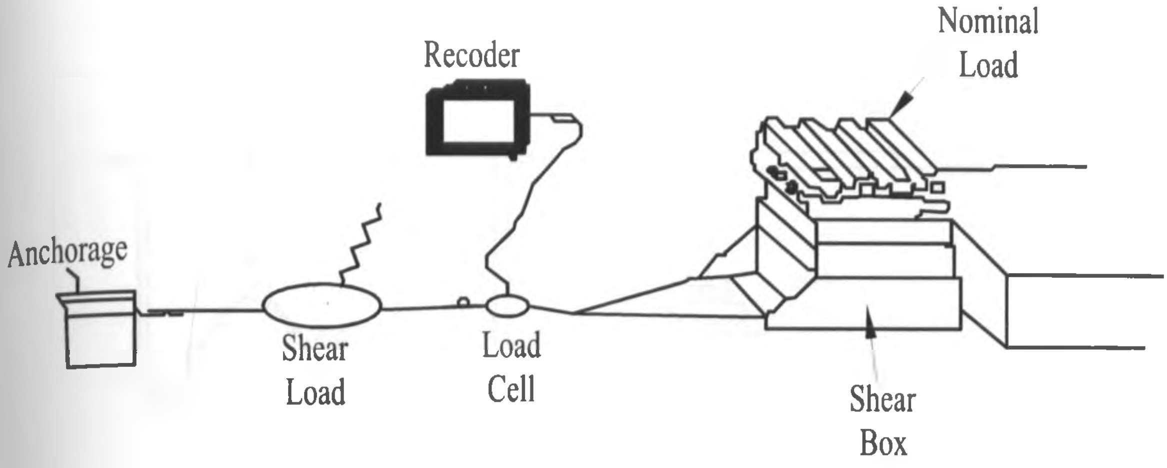


Fig. 2.19: In-situ shear test: shear box emplaced over pedestal (after Endo and Tsuruta (1969))

Manbeian (1973) sheared soil columns reinforced with barley, sunflower, and alfalfa roots. The plants were grown in large diameter containers of homogeneously packed silty clay loam. He then sheared both fallow and root reinforced samples in a direct shear machine. Manbeian reported that peak and residual shear strengths of the root reinforced samples were 2 and 4 times greater than shear strength of the root free samples. These strength increases were attributed entirely to the mechanical reinforcement provided by the roots since soil suction was eliminated by saturating samples prior to testing.

Waldon (1977) and Osano and Mwea (2011) conducted a series of tests similar to the tests performed by Manbeian. Waldon investigated the soil reinforcing effects of alfalafa, barley and pine by growing these plants in large cylindrical containers and then subjecting the saturated soil cylinders to direct shear tests. Waldron found that the presence of each plant type increased the shear strength of the soil. He also attributed this increase to the mechanical reinforcement provided by the roots.

McOmber performed tests to measure the pullout resistance of roots in a cohesive soil. McOmber conducted these tests using a simplified model root (Fig. 2.20) constructed of 1/8 inch diameter steel cable.

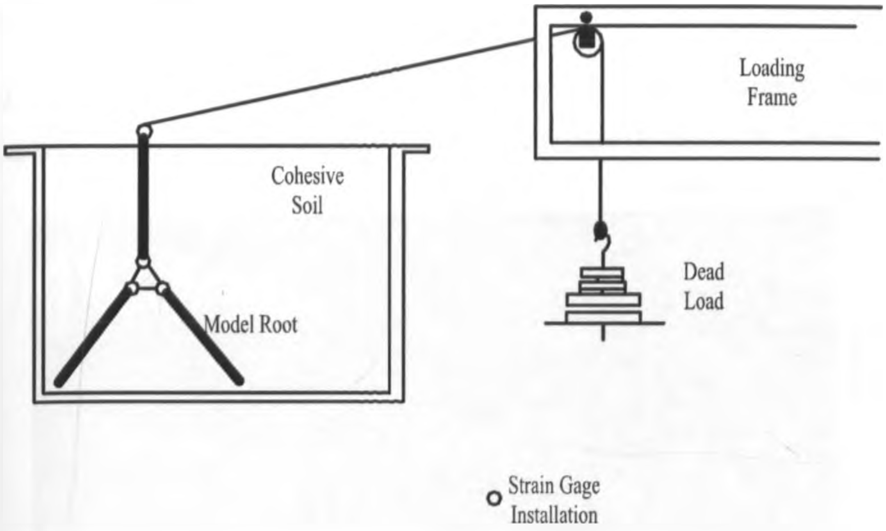


Fig. 2.20: McOmber's testing apparatus and model root

Cable sections a, b, and c were connected by brass connectors. Strain gauges were mounted on these connectors to measure the strain, and in turn the stress, at these points.

The root was buried in a sandy clay soil and then the soil was compacted under a 716 kN/m^2 load. At various stages of loading, x-rays were taken to determine the relative position of the buried root. Knowing the load applied to the top of the root, the loads measured at the connections of the root, and the relative position of the root, McOmber provided information regarding the pullout resistance of the model root in a cohesive soil. McOmber analyzed the soil-root interaction using the concept of subgrade reaction. Analysis of McOmber's test data provided an average value for the coefficient of subgrade reaction, k, of approximately 6.0.

2.14 Catastrophic landslide occurrences around the world

(a) Rock Avalanche Dammed Lake Gojal, Pakistan

On January 4th, 2010, a 50 M m^3 rockslide formed a dam approximately 1200 m long, 350 m wide, and 125 m high. The initial mass movement killed at least 19 people, dammed the Hunza River at Attabad (northern Pakistan), and formed a large impounded lake (Lake Gojal). Fig. 2.21 shows the satellite image showing the location and extent of the slide. The lake began to overtop the rockslide debris through a spillway constructed by Pakistani authorities. At maximum height, the lake was 22 km long, covered and area of 12 km^2, and contained a volume of 585 M m^3 of water. Currently, the dam is still stable, with no credible estimates of catastrophic failure (Evans *et al*, 2009).

Fig. 2.21: ASTER satellite image showing the location and extent of Lake Gojal (Source: Evans et al, 2009).

(b) The 1949 Khait Rockslide-loess Flow

In July 1949, Khait earthquake, which was having a magnitude of 7.4 on the ritcher scale, triggered many hundreds of landslides in a mountainous region near the southern limit of the Tien Shan Mountains, central Tajikistan. These landslides involved widespread rock-slope failure as well as large numbers of flowslides in loess that mantles the steep slopes of the region. The Khait landslide involved a transformation of an earthquake triggered rockslide into a very rapid flow by the entrainment of saturated loess into its movement. The moving mass travelled a horizontal distance of 7.41 km, while descending vertically 1421 m. The total volume of mass was 75 M m^3, an order of magnitude lower than previously published estimates. It was estimated that the total loss of life due to landslide inundation in the Khait epicentral region was approximately 7,200 people (Evans *et al*, O. 2009). The image showing the location and extent of the 1949 Khait rockslide is shown in Fig. 2.22.

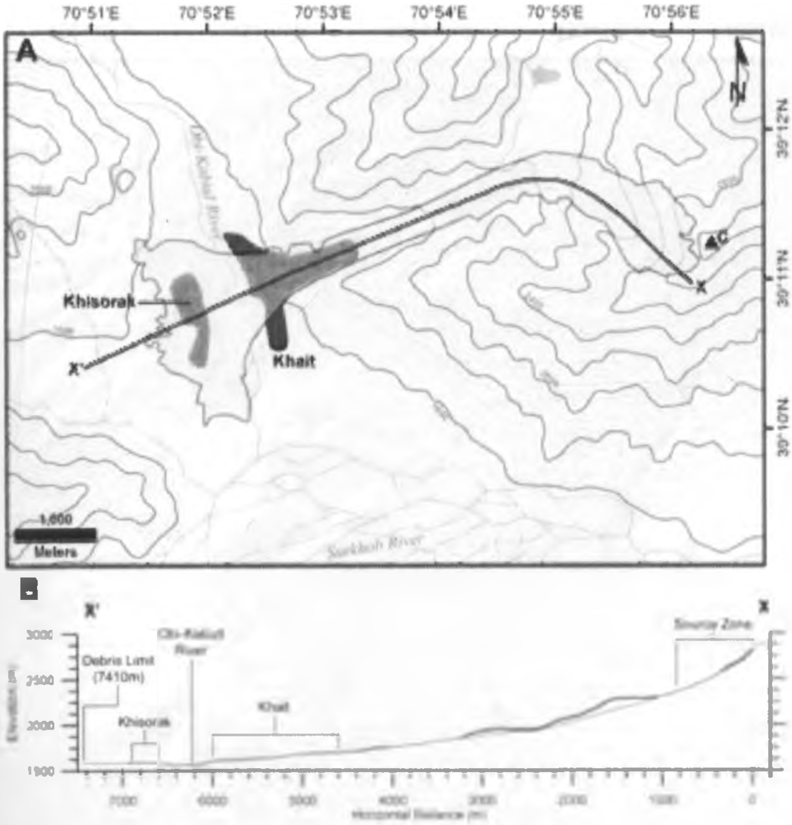


Fig. 2.22: Image showing the location and extent of the 1949 Khait rockslide-loess flow from Chokrak mountain, created from SRTM data (NASA) and ASTER GDEM data (METI and NASA) (Source: Evans et al, 2009).

(c) 1772 Catastrophic Flank Collapse, Papandayan Volcano, Indonesia

The Papandayan stratovolano complex is located in the North West region of the Island of Java, Indonesia (07° 19' S / 107° 44' E). A satellite image of the location is shown in Fig. 2.23. During the catastrophic 1772 eruption the north east flank collapsed, creating a massive debris avalanche. The failed mass travelled downslope to the NE over 10 kilometres, covering a total area of over 20 km^2. This event destroyed 40 villages and killed almost 3000 people.

This event is part of an on-going study researching global catastrophic landslides (Evans *et al.* 2009).

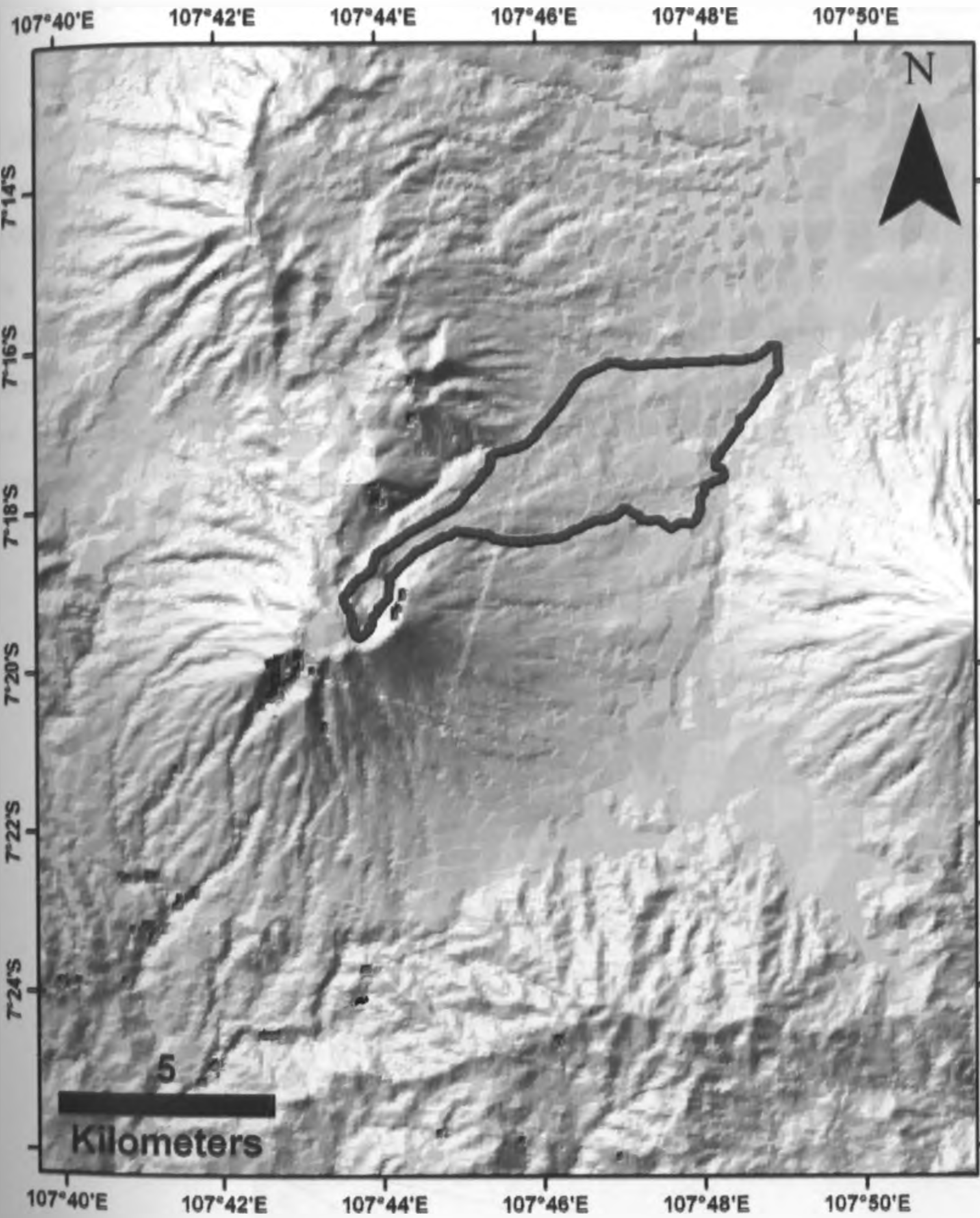


Fig. 2.23: Image showing the location and extent of the 1772 flank collapse of Papandayan created from NASA's SRTM data and geological mapping. (Source: Asmoro et al, 1989)

(d) Sea To Sky Highway (Olympic Corridor) Rockfall, British Columbia, Canada

The site of the July 31st 2008 Sea to Sky rockfall/rockslide was examined in 1997. Fig. 2.24 shows the site at that time. The photo at the lower right was taken soon after the 2008 event by Canadian Press. The slope consists of resistant Coast Plutonic Complex granite but as seen on the 1997 photograph the rock mass is characterized by more-or-less planar stress relief (sheeting) joints that dip west (downslope) towards Howe Sound. Undercutting of the slope during construction of highway in 1958 resulted in the sheeting joints daylighting in the rock slope. The location of the 2008 rockfall is known as Porteau Bluffs, they run for just under 1 km along Howe Sound (49° 33' 52.15"N; 123° 14' 01.44"W), and the rock slopes along this section of highway exhibit similar rock mass characteristics to those involved in the July 2008 event (Evans *et al.* 2009).

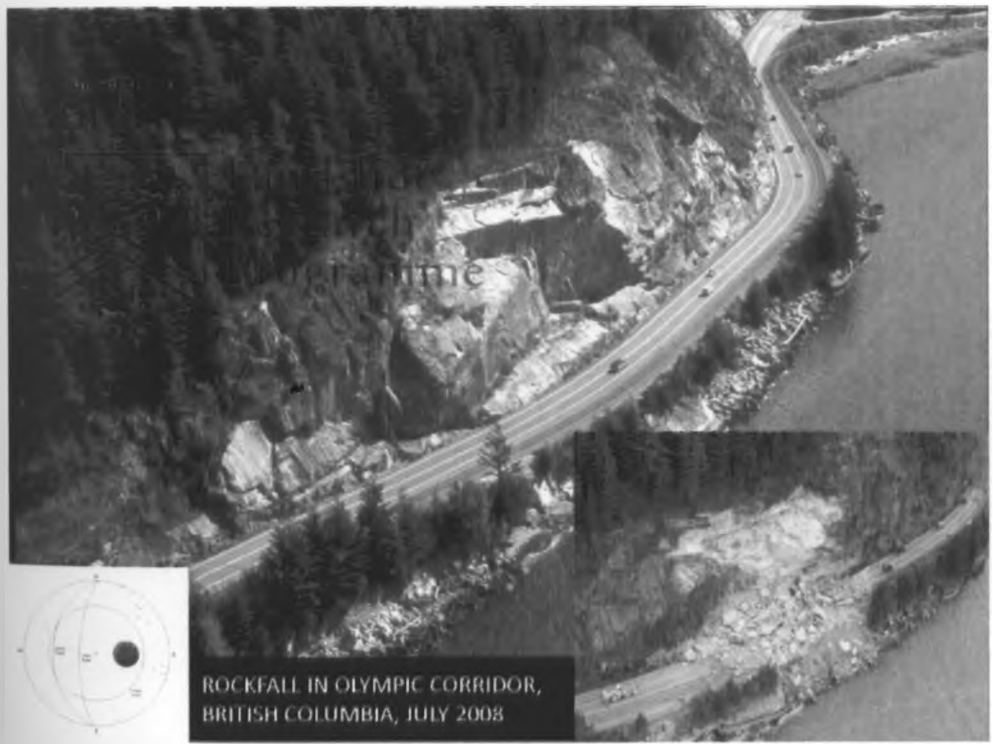


Fig. 2.24: Image showing the exact location of the failed cliff in 1997 and the subsequent failure in July, 2008 (LRP, 2008; inset Canadian Press, 2008)

(e) Rock and Ice Avalanches: Mount Steele, Yukon, Canada

A large rock and ice avalanche occurred on the north face of Mount Steele, southwest Yukon Territory, Canada, on July 24, 2007 as shown in Fig. 2.25. In the days and weeks preceding the landslide, several smaller avalanches initiated from the same slope. The ice and rock debris traveled a maximum horizontal distance 5.76 km with a maximum vertical descent of 2,160 m, leaving a deposit 3.66 km^2 in area on Steele Glacier. The seismic magnitude estimated from long period surface waves (Ms) is 5.2. Modeling of the waveforms suggests an estimated duration of approximately 100 s and an average velocity of between 35 and 65 m/s. This landslide is one of 18 large rock avalanches known to have occurred since 1899 on slopes adjacent to glaciers in western Canada. Lipovsky *et al.*, 2008 described the setting, reconstructed the event chronology and presented a preliminary characterization of the Mount Steele ice and rock avalanches based on field reconnaissance, analysis of seismic records and an airborne LiDAR survey. They also presented the results of a successful dynamic simulation for the July 24 event.

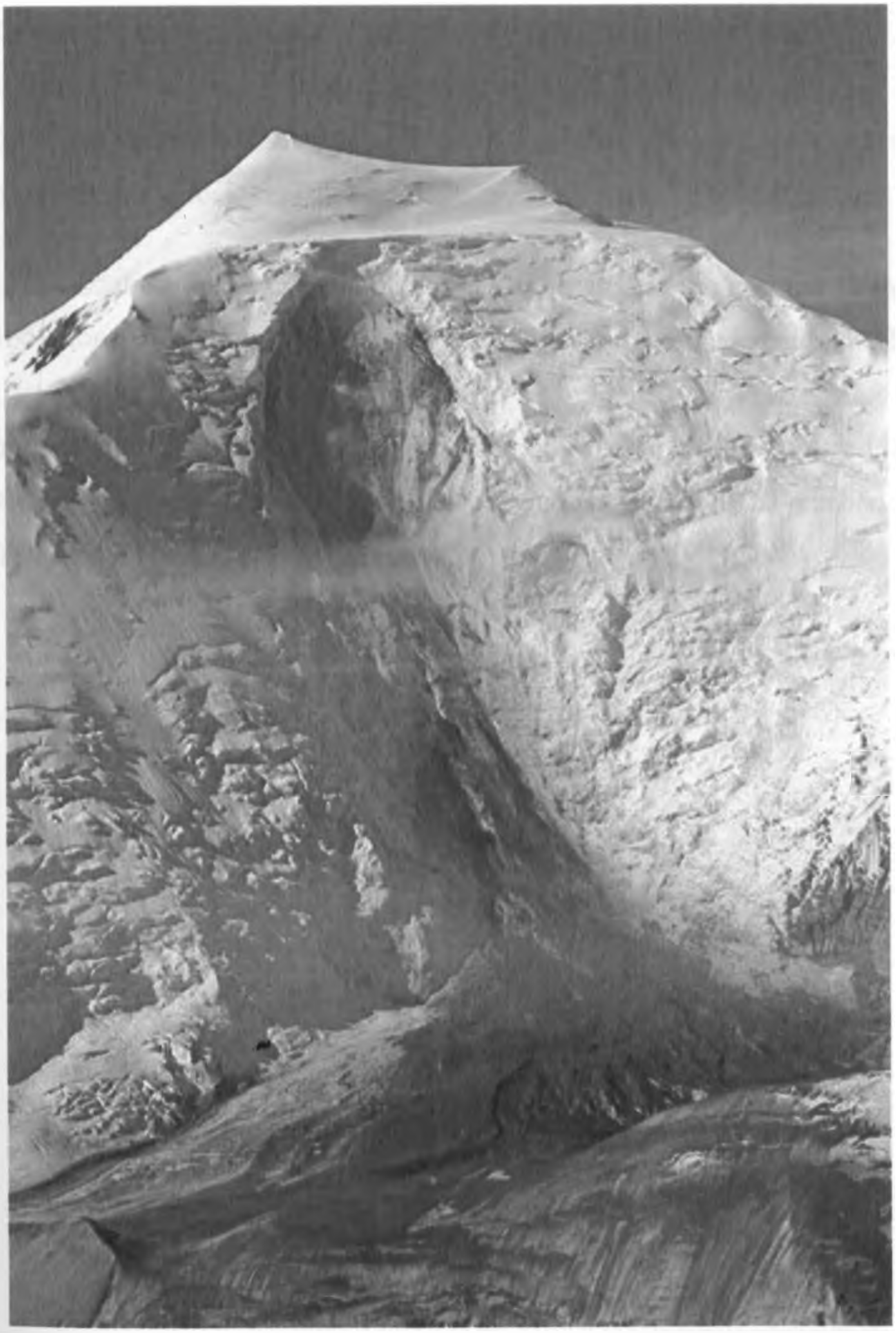


Fig. 2.25: Oblique image showing the failure on the North Face of Mount Steele (Source: Yukon Geological Survey, 2007), (after Lipovsky et al., 2008)

(f) Rock Avalanche and Debris Flows, Nevado Huascaran, Peru

The 1962 and 1970 Huascaran mass movements originated as rock/ice falls from the mountain's North Peak, transformed into higher-volume high-velocity mud-rich debris flows by incorporation of snow from the surface of a glacier below Huascaran and the substantial entrainment of morainic and colluvial material from slopes below the glacier terminus. Fig. 2.26 is the aerial photograph of the rock avalanche.

Water for fluidization of the entrained material originated in the melting of incorporated snow and the liberation of soil moisture contained within the entrained materials. Eyewitness reports indicate very high mean velocities for the events; 17-35 m/s (1962) and 50-85 m/s (1970). The runout distances and velocity profiles of both events were simulated. Both mass movements continued downstream as debris floods (aluviones), that in 1970 reached the Pacific at a distance of 180 km (Evans *et al.* 2009).

Fig. 2.26: Aerial photo of the major rock avalanche and debris flows from Nevado Huascaran, Peru, 1970 (NASA). (Evans et al. 2009).

2.15 Database of plants suitable for slope stability

Plants suitable for erosions control and slope stability have been categorized by various organizations and researchers. These plants have been summarized in Appendix 2. When selecting plants for erosion control or slope stabilization, it is important to use fast growing species that have root systems that will "hold" the soil in place. Heavy plants are not recommended for slope control because their dense foliage can actually contribute to slope erosion. For example, ice plants (*Carpobrotus sp., Drosanthemum sp.*, etc.) are a poor choice for slope control because their shallow root systems cannot hold their heavy leaves. All of the listed species, however, are lightweight, have fibrous root systems and provide great coverage for slopes. A database similar to Appendix 2 showing plants suitable for erosion control and slope stability in Kenya will help designers and land managers to make qualitative assessment and perform quantitative slope stability analyses.

2.16 Conclusion

It has been shown that roots contribute to the shear strength of a soil. The root's influence on the shear strength of the soil is dependent on the bulk density of roots in the soil, the orientation of roots with respect to the shear zone, and the tensile strength of the roots. Research investigating the effect of roots on soil shear strength has determined that the roots tend to increase the cohesive component of shear strength while showing little influence on the internal friction angle of the soil. The soil-root interaction model proposed by Waldon and *wu et al.* provides insight into the behavior of a root located across a shear zone and its contribution to soil shear strength.

As the use of vegetation to stabilize soil slopes has become an increasingly used environmentally friendly alternative to traditional soil improvement methods, a better understanding on its application in important. At present, widespread use of vegetation to stabilize soil has been inhibited by a lack of verified methods to predict the reinforcing effect of various plant types (Bransby, 2004). This research investigates the link between root systems, root mechanical properties and soil-slope stability to shed more light on this subject.

Chapter three

Methodology

3.1 Introduction

This chapter presents the testing procedure designed to analyze the strength of individual roots, the root's effect on soil strength, root's pull-out characteristics, the determination of the critical angle of slopes and the effect of strain rate on stress-strain relationships of individual roots for different species. By performing a large direct shear test on a soil containing roots together with a control test, it was possible to determine the root's contribution to the shear strength of the soil.

For pull-out tests, real roots rather than model roots as used by McOmber (1981) were tested. A series of tests using actual roots obtained from landslide susceptible sites (Sasumua and Murang'a) on remoulded soil samples were performed. Extreme care was observed in order to replicate the soil conditions and characteristics during the remoulding process at the laboratory by performing various physical tests at site and laboratory. The testing procedure used by McOmber (1981) was modified by pulling-out roots axially and repeating the tests to obtain additional information regarding pullout resistance of the actual roots.

Direct shear tests were performed on soil containing the roots and those without. Specific attention was given to the soil's resistance to root motion and the effect of the pullout resistance of the root on the normal force on the shear plane.

3.2 Description of study sites

3.2.1 Sasumua site

Root sampling of different plant species, each plant species comprising of 3-5 individual samples, took place in the Sasumua Water Treatment Backslope, situated in Njambini Division, Nyandarua District of Central Province in Kenya. Fig. 3.1 shows the geographical map of the research site.

Sasumua is situated at a latitude of 0° -47' 0 S and a longitude 36° 42' 0 E. The location is situated 245 kilometres south west (216°) of the approximate centre of Kenya and 57 kilometres north (347°) of the capital Nairobi.

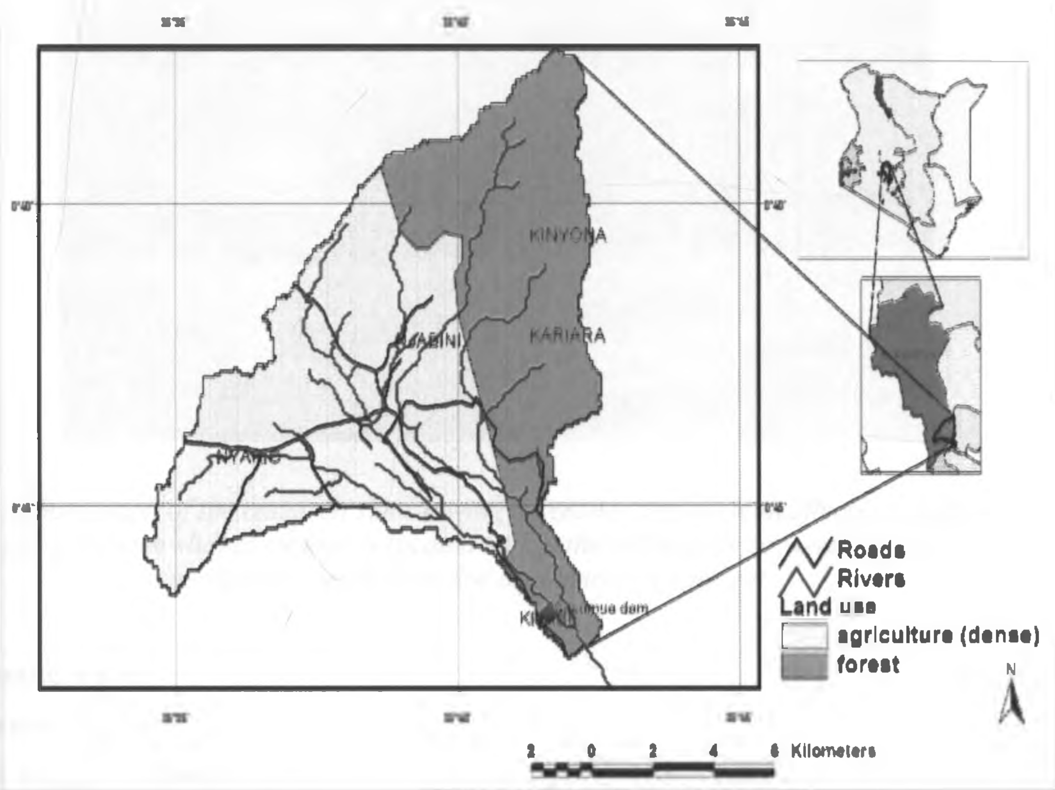


Fig. 3.1: Geographical map of Nyandarua District indicating the location of Sasumua

The Sasumua backslope had steep slopes, ranging from 15° to 55°. The length of entire slope is about 180m. Soils comprise of hard murram, fine grained gravels to red coffee soils. During the rainy season, the slope experiences seepage from underground water which tends to reduce its stability. Fig. 3.2 is the photograph showing the extent of the shallow landslide of within 1m depths and rampant gully erosion.

Fig. 3.2: Photograph of the research site showing an eroded terrain with effects of shallow landslides in which the sliding surface is located within the soil mantle or weathered bedrock (typically to a depth from few decimetres to 1 metre)

3.2.2 Murang'a site

(a) Site description

The typical landslide comprehensively investigated under this research occurred in Mung'aria Village situated in Murang'a North District, Kenya. A major landslide occurred on 6th November 2008. Interviews with the local residents revealed that it had been raining heavily for the whole week preceding the landslide occurrence. The landslide destroyed a house and collapsed onto a girl who was sleeping, killing her instantly.

Field investigations were conducted at the landslide site five days subsequent to its occurrence. The terrain was particularly steep and varied from steep dissected slopes (25 – 30^0) on relatively hard, fine grained gravel soils to very steep slopes (40 -65^0) on red soils. Vegetation consisted of tea and coffee plantations on the uphill to unvegitated ground at midlands to nappier grass plantation downhill sides.

The depth of landslide failure was largely a function of the thickness of the soil and colluvium overlying mudstone lithologies.

The soils on the site were found to be shallow (<3 m). At the landslide occurrence, soils were brown and well drained. This is typical of land that is being eroded. The abrupt loss of strength of the soil during wetting was found to be a distinct feature of the soil in the area. Fig. 3.3 and 3.4 show the landslide site. The portion that underwent landsliding measured 7m long by 5m wide by 1m deep.

Figure 3.3: Muranga location in Kenya

Figure 3.4: Murang'a County map

(b) The Landslide

The landslide measured 7m high by 5 m long as shown in Fig. 3.5 from left side view. Fig. 3.6 is right side view and Fig. 3.7 presents the aerial view of Mung'aria Village landslide site

Figure 3.5: Left Side View of Mung'aria Village Landslide

Figure 3.6: Right Side View of Mung'aria Village Landslide

Figure 3.7: Aerial View of Mung'aria Village Landslide Site

(c) Weather pattern during the Month of October-November-December (OND) 2008 – *Courtesy of Kenya Meteorological Department*

The October-November-December (OND) 2008 "Short-rains" seasonal rainfall was characterized by very poor temporal distribution. Most parts of the Central Kenya experienced heavy and continuous rainfall during the month of October and the first half of November (Fig. 3.8 – 3.11). This rainfall resulted into flash floods and landslides/mudslides leading to loss of life and property as well as destruction of infrastructure in several areas.

However, the entire country remained generally sunny and dry throughout the second half of November and the entire month of December. During the month of December, for example, most stations in the country recorded monthly totals not exceeding 10mm.

Stations at Nyeri recorded a highest seasonal rainfall total of 566.6mm (105%).

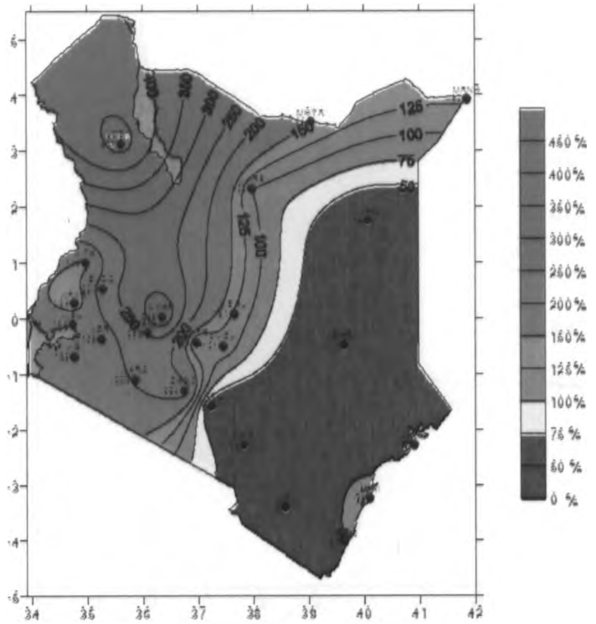


Fig 3.8: October 2008 Rainfall Performance

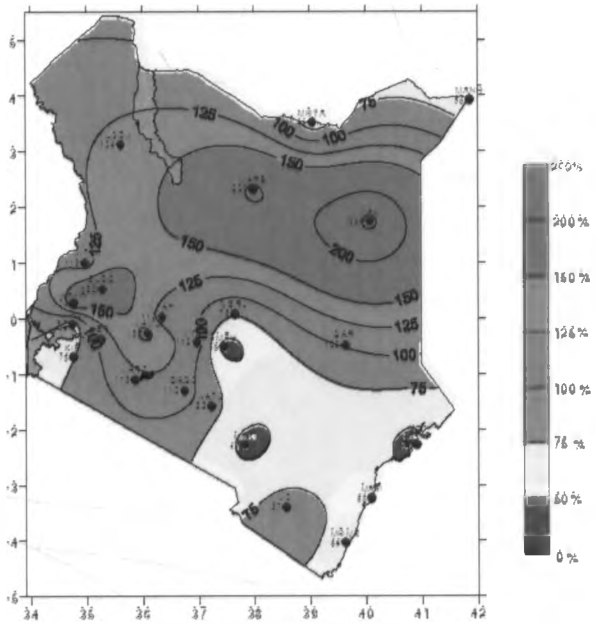


Fig. 3.9: November 2008 Rainfall Performance

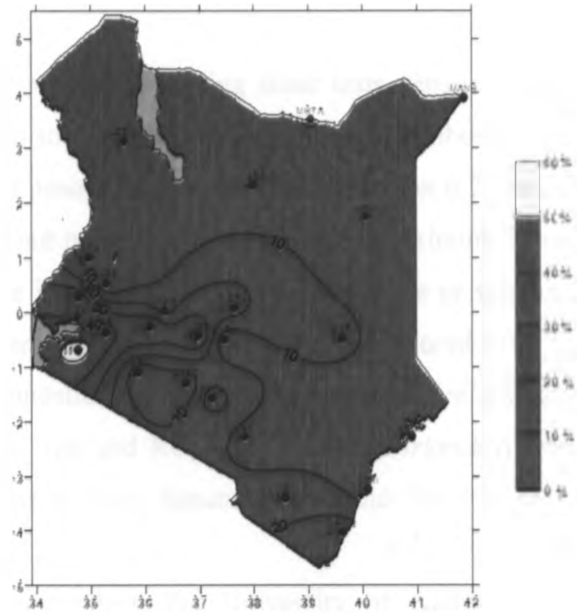


Fig. 3.10: December 2008 Rainfall Performance

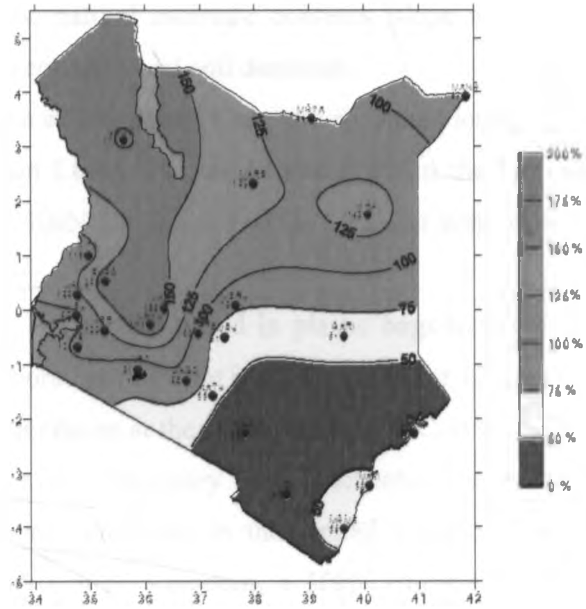


Fig. 3.11: The October-November-December (OND) 2008 Rainfall Performance

3.3 Sampling

Roots were sampled for conducting shear tests, tensile strength tests, pull-out tests and the effect of strain rate on stress-strain relationships in the laboratory. Selection was random, and roots species having root penetration into soil of more than 0.2m were excavated. Selection was random, from depths of not more than 1m, being the maximum depths of shallow landslides in which the sliding surface is located within the soil mantle or weathered bedrock (typically to a depth from few decimeters to 1 meter). Samples were favored from areas where there was great effect of the shallow landslides. These plant species were grasses: Switch grass (*Panicum virgatum*) from Sasumua Site and Red oats (*Themeda triandra*) from Mung'aria Site; shrubs: Saltbush (*Atriplex halimus*) from Sasumua Site and Tree ferns (*Asparagus species*) from Munga'ria Landslide Site.

Testing was conducted in the University of Nairobi geotechnical laboratory. The investigations that took place included the determination of root area ratio (RAR), pull-out strengths, shear strengths, natural moisture contents, slope angles, angle of shear distortion, internal angle of friction, cohesion and soil densities.

For conducting the effect of strain rate and specimen length on stress-strain relationships, roots were collected from Luton, UK, and transported to the University of Portsmouth, UK, where a computer aided 50kN Universal Tensile Machine with a variable strain rate adjuster is installed.

After excavation, roots were stored in plastic bags to preserve their moisture contents. Roots were taken to a laboratory and kept in a cold room for 12 hours to conserve their moisture content. The average temperature at the cold room was 7°C. A 2m^3 soil sample was excavated at random and transported to the laboratory to conduct laboratory pull-out and shear testing from both sites, except for tests conducted in the United Kingdom where soil samples were not required. For tests in the United Kingdom, a 50kN Universal Testing Machine was used to investigate the effect of strain rate on stress-strain relationships of roots. Details are given in Section 3.8.

3.4 Laboratory apparatus and testing procedures

3.4.1 Shear test

The standard testing method adopted for laboratory shear testing was BS 1377-7: 1990,

Roots were selected from three different plant species, i.e. Switch grass, Saltbush and Tree fern. The roots were manually inserted into the apparatus and soil material introduced and rammed in three layers. The length of the roots inside the apparatus was 150mm and outside the apparatus was 350mm. Ramming of soil specimen was in three layers, with each layer being 60mm before ramming and 50mm after ramming. Ramming was by a 4.5 Kg rammer, giving 25 equally distributed blows around the specimen. Moisture contents were determined at each layer by taking samples and keeping them in an oven, giving an average value of 14% for all the layers after 24 hours of oven-drying.

The test samples were then repeatedly submerged in water which was left until saturation was reached. Immediately saturation point was reached, water was bailed from the shear apparatus and instruments installed. A manually driven CBR jack was used to shear-test the soil blocks. All shear tests were carried out with the test block submerged and under a normal load of 150 kg equivalent to an overburden pressure at the potential shear plane of a 1-m-thick soil. Strain was applied at a constant rate to give an approximate shear displacement rate of 1 cm/min. Load cells with resolution of 0.01kN were used to measure the shear force. Shearing was done close to the maximum displacement capacity of the jack which was approximately 90mm. The procedure was repeated for all samples.

3.4.2 The testing facility

A field set-up to perform *in-situ* shear tests on soil with and without roots was presented by Jagath, Ekanayake and Chris (2003), and was used to test soil samples in New Zealand. The above experimental set-up formed the basis for apparatus devised and adopted with minimum modifications, to perform laboratory tests to meet the objectives of this research. The modifications included the introduction of 1000mm x 1000mm x 1000mm box simulating earth and using shear displacement rate of 1 cm/min rather than the 2 cm/min as used by Jagath, Ekanayake and Chris (2003), which was argued to be quite fast and could not allow the sample to shear at its weakest point. Fig. 3.12 is the sectional drawing of the set-up.

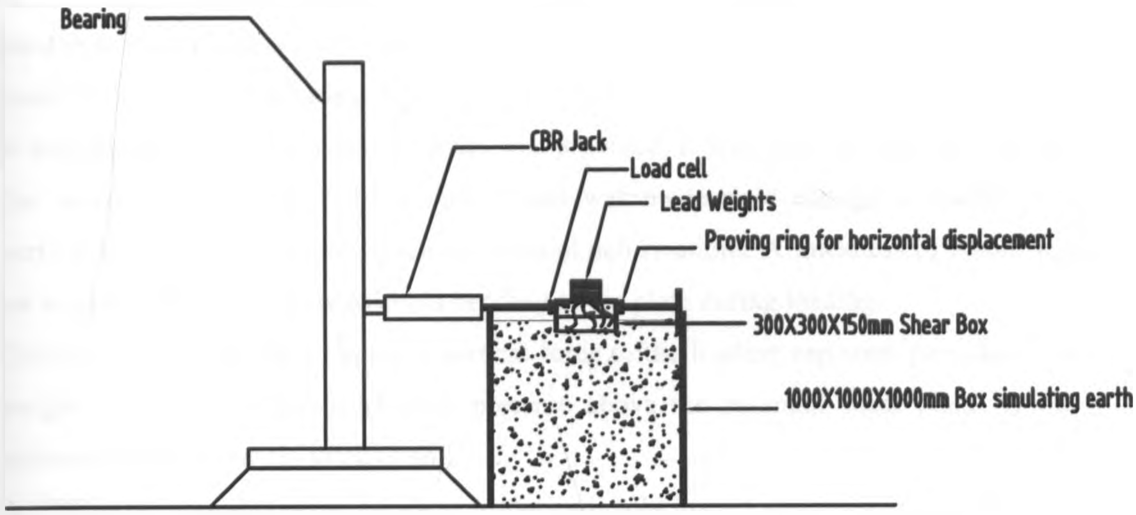


Fig. 3.12: The laboratory set-up

The essential features of the apparatus thus follow:

1. A container for the specimen, which was termed the 'shear box', which was square in plan, was 300mm square in plan and 150mm deep divided horizontally into two equal parts which enabled a square prism of soil to be laterally restrained and sheared along a mechanically induced horizontal plane while subjected to a pressure applied normal to this plane. The upper part, which was 75mm deep where an increasing horizontal force was applied, thus causing the prism to shear along the dividing plane of the box. The lower part was also 75mm deep, and was not fitted with a baseplate like in the ordinary small shear box apparatus, thus making it an open-ended box. This enabled easier placing of prepared soil blocks.
2. The materials comprising the root-reinforced soil testing apparatus were all resistant to corrosion by electro-chemical reaction with each other.

3. The upper part of the container was fitted with a porous plate of corrosion-resistant material, about 2mm smaller than the internal plan dimensions of the shearbox. Its porosity allowed free drainage of water throughout the test and prevented the intrusion of soil into the pores. The thickness of the plate was made 20mm which was sufficient enough to prevent breakage under load, and was made to be of negligible compressibility under the loads applied during the test.
4. A loading cap to cover the porous plate was provided. It was 2mm smaller in plan than the internal dimensions of the shearbox, and was made rigid enough to transmit the vertical load uniformly to the specimen without deformations. A thickness of 20mm was set to give sufficient rigidity to avoid bending of the plate during loading.
5. Calibrated Lead weights to apply a vertical force to the loading cap were provided. The weight was equal to the overburden pressure above the potential shear plane. It was estimated to be about 1m thick of soil.
6. A CBR jack to apply horizontal shear to the vertically loaded specimen at constant rates of displacement from which a rate to suit the soil being tested was provided.
7. Calibrated force measuring load rings were provided to measure the horizontal shear forces.
8. Calibrated dial gauge to measure the relative horizontal displacement of the two halves of the shearbox was provided.
9. Calibrated dial gauge to measure the vertical deformation of the specimen during the test was provided.

The shear test results are analysed in Section 4.1 of Chapter 4.

3.5 Root tensile testing

The Hounsfield Tensometer apparatus (Fig. 3.13a and 3.13b) was used to determine root tensile strength, T. The schematic diagram of the Hounsfield Tensometer apparatus is shown in Fig. 3.14.

Fig. 3.13a – Image of Hounsfield Tensometer apparatus

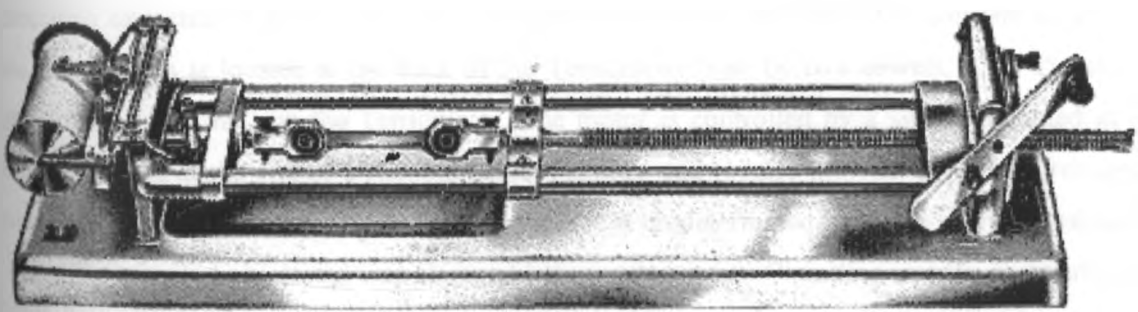


Fig. 3.13b – Artist's impression of Hounsfield Tensometer apparatus

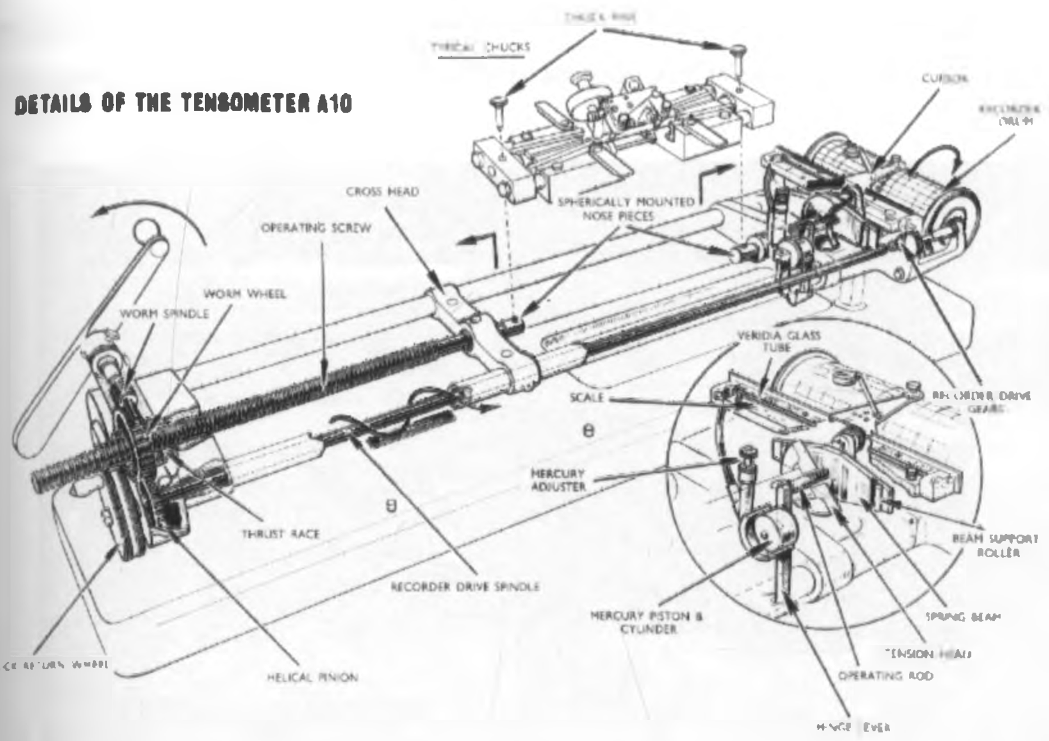


Fig. 3.14: The schematic diagram of the Hounsfield A10 Tensometer

The machine is motor driven to achieve a constant rate of extension for the large extensions expected of plant roots. The standard motor drive unit (M.D.U.) consists of a cast base plate which is located at the back of the Tensometer base by two dowels. A 9” Pulley is fitted to the drive shaft on the Tensometer. The motor is controlled by a switch mounted in a small metal box at the end of a flexible cable. Forward and reverse limit switches are provided which clip on to the rear standard and stop the motor at predetermined positions of the crosshead.

The sample to be tested was clamped between two grips. Clamping was the most critical issue when measuring root strength. In the tests, the roots were clamped using wedge grips as shown in Fig. 3.15. They were self gripping, were quick to use, and did not have to be removed from the machine for each test. To improve the clamping and avoid slippage, fine sand paper was attached to the grips. Roots were clamped into entire wedge grip length in order to achieve a superior grip which could avoid slippage during testing.

After clamping the roots into wedge grips, the motor was driven manually to apply initial tension into the roots, and the mercury scale set at zero. Root diameters at either ends were taken, and the initial length of the exposed root also recorded. The motor drive unit was then put on subjecting the sample to a movement of the clamps at a constant rate of 10mm/min., and testing commenced. Loading was recorded at every 30 seconds until failure occurred.

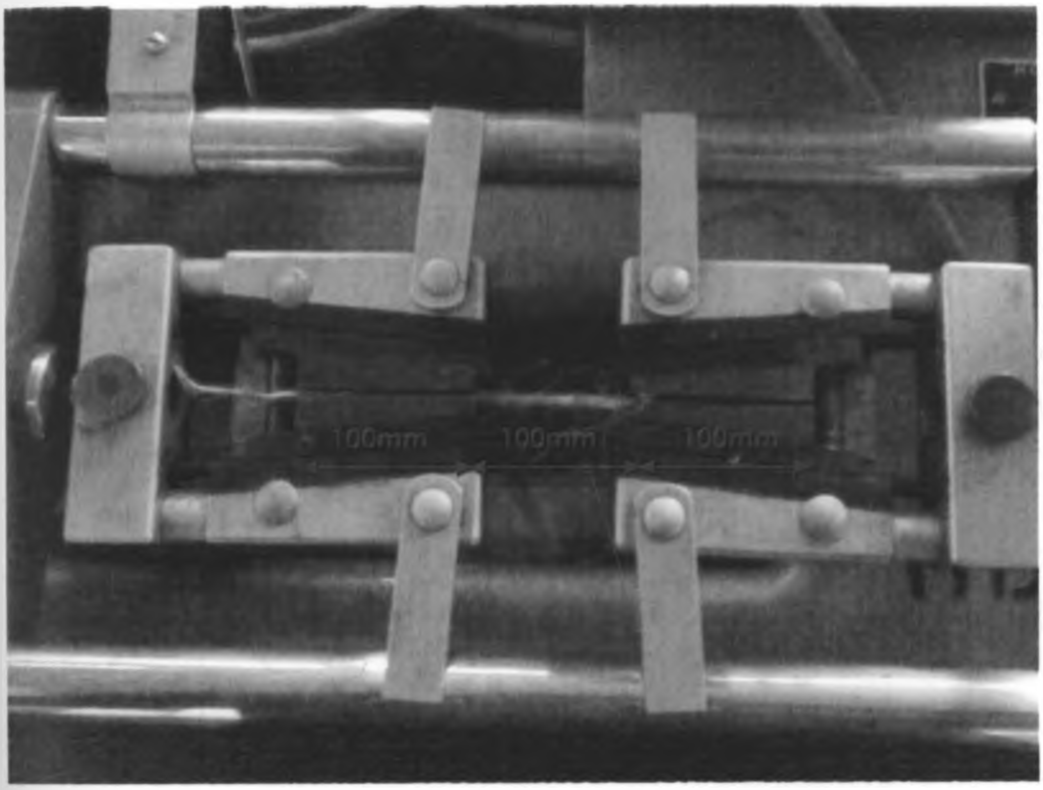


Fig. 3.15: Clamping of the roots using wedge grips

The elongation of the roots was recorded by simply taking the length between the grips at failure. Equation 3.1 was then used to calculate the root tensile strength, T (De Baets S. et al., 2008).

$$T = \frac{F_{max}}{\pi\left(\frac{D^2}{4}\right)} \qquad \textbf{Equation 3.1}$$

Where;

(i) F_{max} is the maximum force (N) needed to break the root, and

(ii) D is the mean root diameter (mm) before stretching.

Results of roots that failed at the grips were discarded as rupture occurred below their actual strength. The root tensile tests results are shown in Section 4.2.

3.6 Pull-out tests

The apparatus was specially designed and fabricated for this research (Fig. 3.16).

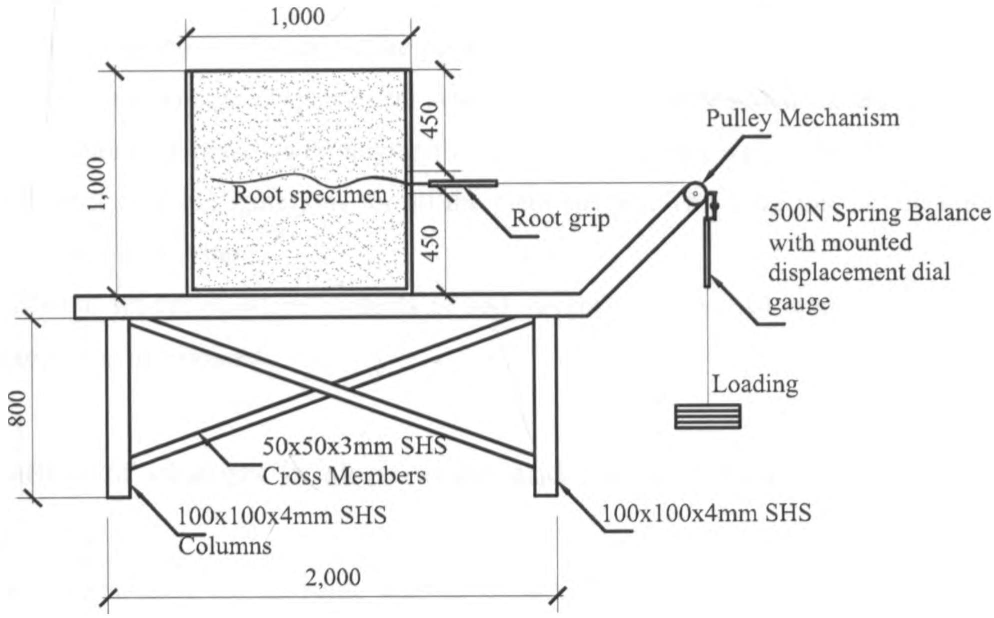


Fig. 3.16: The Pull-out test Apparatus

The main features of the apparatus are:

1. A box measuring 1m x 1m x 1m filled with soil material from the site under investigation, where roots are embedded for pull-out.
2. A pulley mechanism with loading cap to apply a horizontal pull-out force on the root. The wire-rope to be inelastic and to accommodate large pull-out forces of up to 100N, which is the estimated maximum pull-out force for small-rooted vegetation.
3. A steel table where the box is fixed high enough to allow the pulley movement to take place during the pull-out displacement
4. Loading weights measuring 1Kg each

The sample to be tested was clamped between two grips. To ensure that no slippage occurred during each pull-out test, the root crown was gripped using a specially designed wedge and barrel system similar to those used for tensile testing, but capable to grip thicker roots.

Soils were remoulded into the box after knowing the weight required to fill the box. When remoulding reached halfway the box, the root was embedded, and carefully soil was filled around it, rammed around it while avoiding hitting the root. Soil was filled to the entire box. After clamping the roots into wedge grips, the pulley mechanism was mounted, and the loading started, at 1Kg weight intervals. Displacement was recorded for every added weight until root sudden pull-out failure took place. A Photograph of the pulled out root was taken. This test was repeated for different root configurations for all the plant species. The force just before failure was recorded for each root system.

In a different set-up, moisture contents of soil samples were varied, and the maximum pull-out resistance was determined.

3.7 The effect of changes in slope angle and the determination of critical slope angle

The area surrounding the landslide was surveyed to determine why sliding occurred on one side of the slope only, while other areas remained stable. Slope angles at 5 different points, including the landslide spot, were determined by use of a Topcon GTS-235 total station (Fig. 3.17) to measure coordinates and angles by recording the values in a data collector.

Fig. 3.17: The Topcon GTS-235 total station

Equipment parts;

1. Yellow case with total station, serial cable, plumb-bob, raincover, battery charger for total station, power cord, tool kit.
2. Prism, data collector, USB cable and prism holder.
3. 5m Range pole
4. Tripod
5. Staff

The total station was mounted with 2 pole prisms. The total station combined an electronic transit with a laser distance measurer. The pole-mounted prism was then sighted. Horizontal (relative to a previously determined baseline) and vertical angles from the station to the prism were measured. By combining the angles with the measured slope distance, horizontal distances and elevation distances were calculated

The areas surveyed and angles taken are given below;

Area 1 – slope angle 23°

Area 2 – slope angle 25°

Area 3 – slope angle 33°

Area 4 – slope angle 15°

Landslide area – slope angle 46°

Fig. 3.18 is a site layout of the area under investigation at Murang'a.

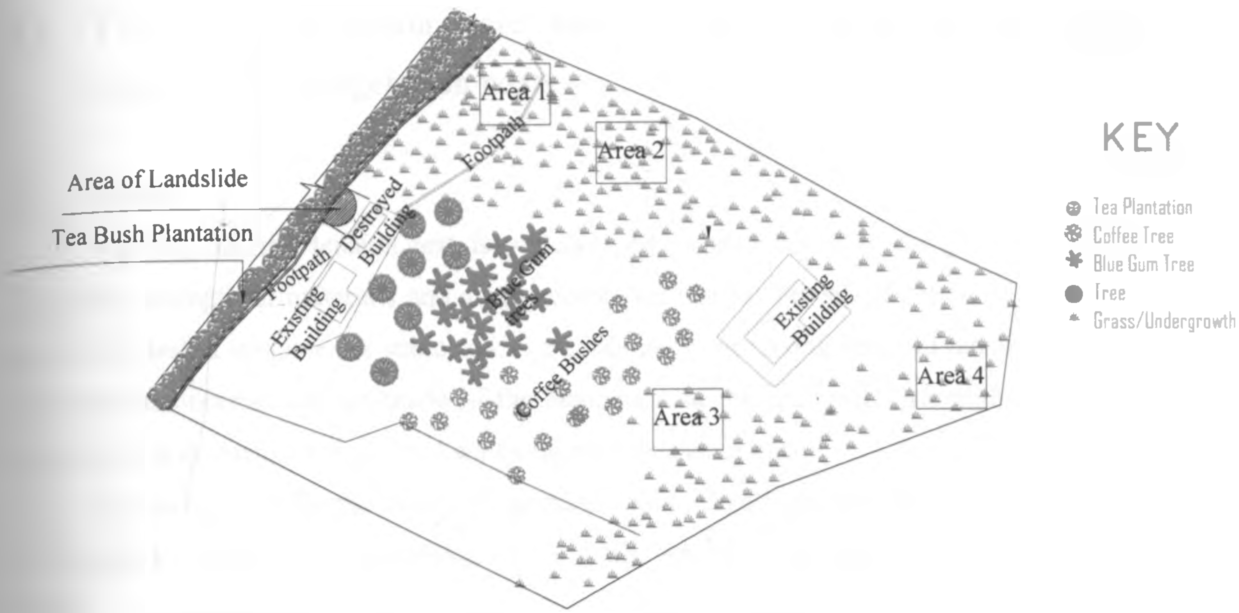


Fig. 3.18: Murang'a site layout

The number and type of vegetation found at each area was noted, together with the number and diameter of roots crossing at depths of 1m, where majority of shallow landslides occurs.

3.8 The effect of strain rate and specimen length on stress-strain relationship of vegetation roots

(a) Theory

The engineering tension test is widely used to provide basic design information on images/the strength of materials and as an acceptance test for the specification of materials. In the tension test, a specimen is subjected to a continually increasing uniaxial tensile force while simultaneous observations are made of the elongation of the specimen. An engineering stress-strain curve is constructed from the load elongation measurements.

The strain used for the engineering stress-strain curve is the average linear strain, which is obtained by dividing the elongation of the gauge length of the specimen, d, by its original length.

The parameters, which are used to describe the stress-strain curve of a metal, are the tensile strength, yield strength or yield point, percent elongation, and reduction of area. The first two are strength parameters while the last two indicate ductility. Tests on non-metals are conducted using specialized jaws to grip the specimens, as there is tendency of specimens slipping from the jaws during testing.

(b) Equipment

A 50kN Universal Testing Machine was used to investigate the effect of strain rate on stress-strain relationships of roots. The equipment assembly is shown in Fig. 3.19.

This equipment is suitable for tensile, compression, bending, shearing, peeling and tearing test of metal and nonmetal materials, such as rubber, plastic, electrical wire and cable, composite, profiled bar, bar metal, board, spring, components and so on. It confirms to ISO 75001, ASTM A370, ASTM E4 ASTM E8 and JIS, DIN, BSEN testing standards.

The machines comes with high capacity self-tightening wedge grips (Fig. 3.20) designed to ensure quick and easy clamping of a wide variety of components and materials. Their capstan action speedily loads the sample securely in the jaws prior to the test. The wedge grips are supplied fitted with integral jaw faces to maximize the clamping force and reduce slippage.

Fig. 3.19: The 50kN Universal Testing Equipment assembly

Fig. 3.20: Wedge grips

(c) **Equipment specifications**

Parameters	*Ranges*
Accuracy	≤±1% (0.5% as special order)
Measuring range of force (KN)	0.4%--100%FS
Resolution of force	1/300000 of maximum force
Accuracy of displacement measuring	±0.5%
Resolution of displacement	0.01μm
Accuracy of deformation measuring	±0.5%
Speed adjusting range of displacement	0.01-500mm/min
Speed adjusting range of force	0.005-5%F.S / S
Speed adjusting range of deformation	0.005-5%FS/S
Speed adjusting range of constant displacement, force and deformation	0.5%-100%FS/S

Speed controlling accuracy	$\leqq \pm 1.0\%$ of the set value
Valid tension testing space	800
Compression testing space	950
Valid width of testing space	370
Power supply	220, 1.5 kW, 50/60Hz
Working environment	Room temperature 10°C ~ 30°C, relative humidity≤80%
Over Dimension and Weight	740×500×1780mm, 580kg

(d) **Equipment components**

- A Load frame consisting of two strong supports for the machine.
- Load cells - A force transducer or other means of measuring the load.
- Cross head - A movable cross head (crosshead) controlled to move up or down at a constant speed, called a *constant rate of extension* (CRE) machine capable of programming the crosshead speed at a constant force and a constant deformation. Electromechanical, servo-hydraulic, linear drive and resonance drive to be used.
- Extensometers to measure extension.
- Computer interface for analysis and printing.
- A controlled room or a special environmental chamber during testing.

- Test fixtures, specimen holding jaws and related sample making equipment.

(e) **Testing procedure**

Vegetation roots of English broom shrubs (*Cytisus scoparius*) comprising in total of 4 samples were collected from Luton, England, and transported to the University of Portsmouth laboratory where the 50kN Universal Testing machine is installed. The samples were stored in a cold room to maintain their moisture content for a maximum period of 12 hours but in any case testing was conducted the same day of obtaining the specimens. The average temperature at the cold room was 10°C.

Individual roots from each sample were cut and their skin peeled off. This was essential because the wedge grips could not tightly grip the roots when their skins were on. Straight and uniform diameter specimens were with gauge lengths of 70mm, 100mm and 120 and 150mm were prepared.

The sample to be tested was clamped between two grips. Clamping was the most critical issue when measuring root strength. In our tests, the roots were clamped using wedge grips as shown in Fig. 3.20. They are self gripping, are quick to use, and do not have to be removed from the machine for each test. To improve the clamping and avoid slippage, fine sand paper was attached to the grips. The 70mm samples were carefully put in between the jaws before tightly fastening the wedge grips for conducting the variable strain rate test. The upper wedge grip was connected to a cross-head which was moved upwards at a pre-programmed rate to apply tension to the sample. The resistive force between the apparatus and the tensioned sample was measured. During the tensile test the cross-head movement and resulting force were logged. The data was then used to calculate material properties. Test speeds were varied, i.e., 2mm/min, 4mm/min, 8mm/min, and 16mm/min for each 70mm fresh sample and the resulting graphs recorded for analysis. Tests were repeated for samples which failed prematurely when as a result of slippage at the grips.

Lastly, gauge lengths were varied i.e., 100mm, 120 and 150mm, using a 2mm/min. test speed. The results are presented in Section 4.7.

Chapter four

Results and discussion

4.1 Shear test results

Stress-strain curves are shown in Figs. 4.1 for test samples of each plant species and for unvegetated soil.

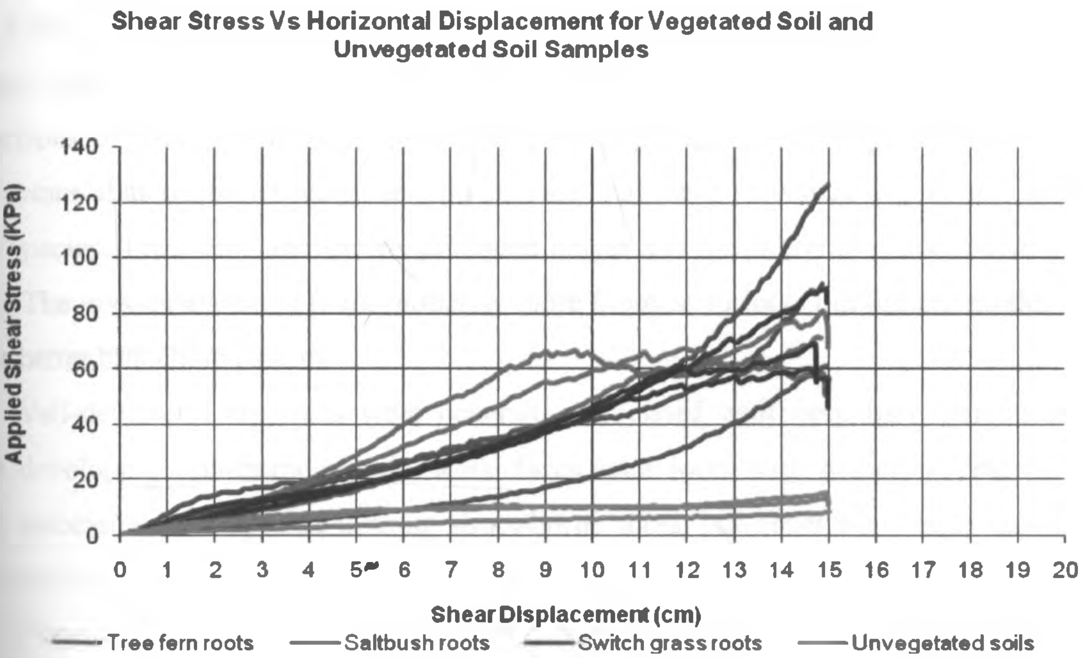


Fig. 4.1: Shear Stress vs. horizontal displacement for representative samples of vegetated and unvegetated soils

The shear stress in different root systems varied as testing was being done. This is due to different root densities and orientations of the root matrix.

The results suggest that unvegetated soil achieved a maximum shear stress of 16 kPa. For the vegetated soils, the maximum shear stress for switch grass roots was 90 kPa, saltbush roots was 80 kPa and tree fern roots was 120 kPa. Averagely the highest strength obtained was 96.7

kPa. The rooted soils were stronger in shear by a factor of 6. It was noted the varying shear strengths for different soil samples for each species. This varying degree is attributed to root biomass density.

Shear stresses were still increasing at the end of the test for the rooted samples. Thus root tensile failure was not happening. Observations of the roots after test indicates roots elongated. This elongation can be related to the root biomass density to explain why varying strengths were obtained for different samples of a same species.

4.2 Root tensile testing

4.2.1 Root morphology

Differences among plant species in morphology and patterns of growth are assumed to influence their ability to acquire resources and, consequently, their competitive ability. Comparisons of root morphology, growth rate and topology of seedlings of nine herbaceous plant species that occurs in early to mid-successional fields revealed significant differences among species. Tree ferns and shrubs produced longer and more branched roots than did grass species. The grasses allocated proportionately more biomass to roots but did not produce deeper roots or better branching pattern.

Well-developed tap roots were generally associated with deep root systems and with species developing root-borne shoots, while fasciculate roots with numerous fine roots were usually associated with species forming shoot-borne roots, but never root-borne shoots. In the studied species, shoot-rooting was related to laterally spreading root systems reaching moderate depths. These features are clearly adaptive for those species, which tend to spread horizontally. Fig. 4.2 – 4.10 shows the growth patterns on some of the studied roots.

Fig. 4.2: Tap root system with laterals from Sasumua (Panicum virgatum)

Fig. 4.3: Tap root system from Sasumua (Atriplex halimus)

Fig. 4.4: Fibrous root system from Sasumua (Agrostis gigantean)

Fig. 4.5: Fibrous root system from Sasumua (Asparagus species)

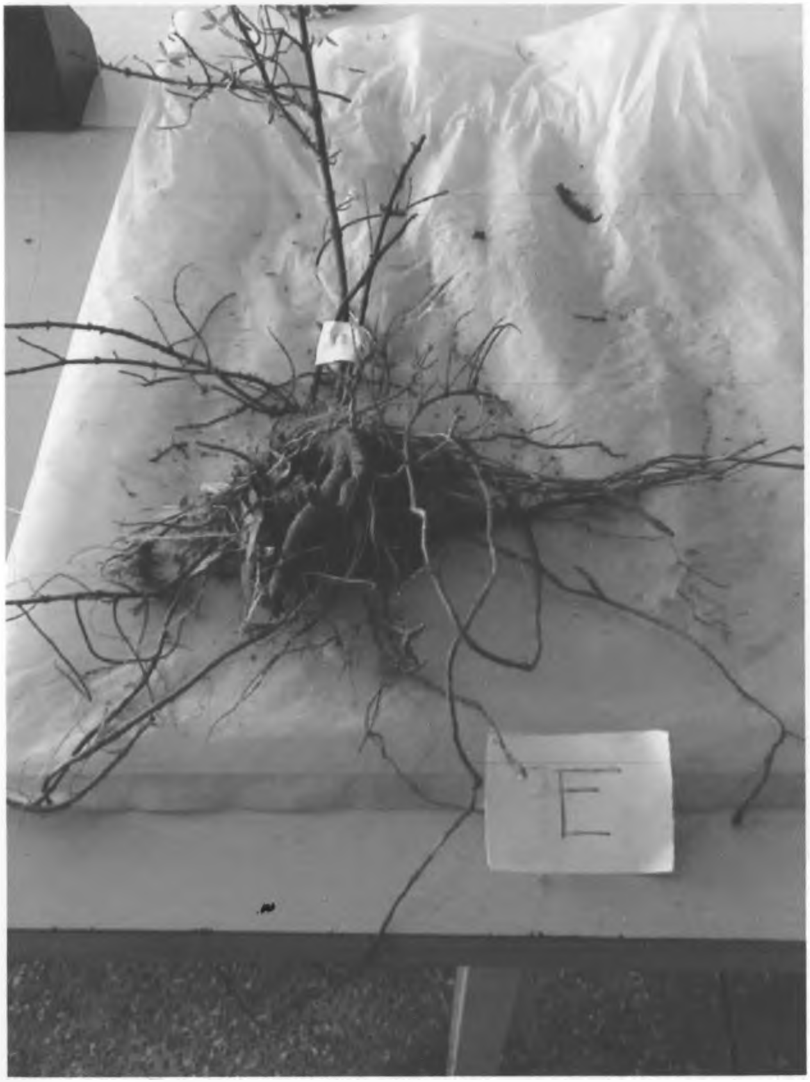


Fig. 4.6: Fibrous root system from Sasumua (Agrostis canina)

Fig. 4.7: Tap root system laterals from Sasumua (Agropyron repens)

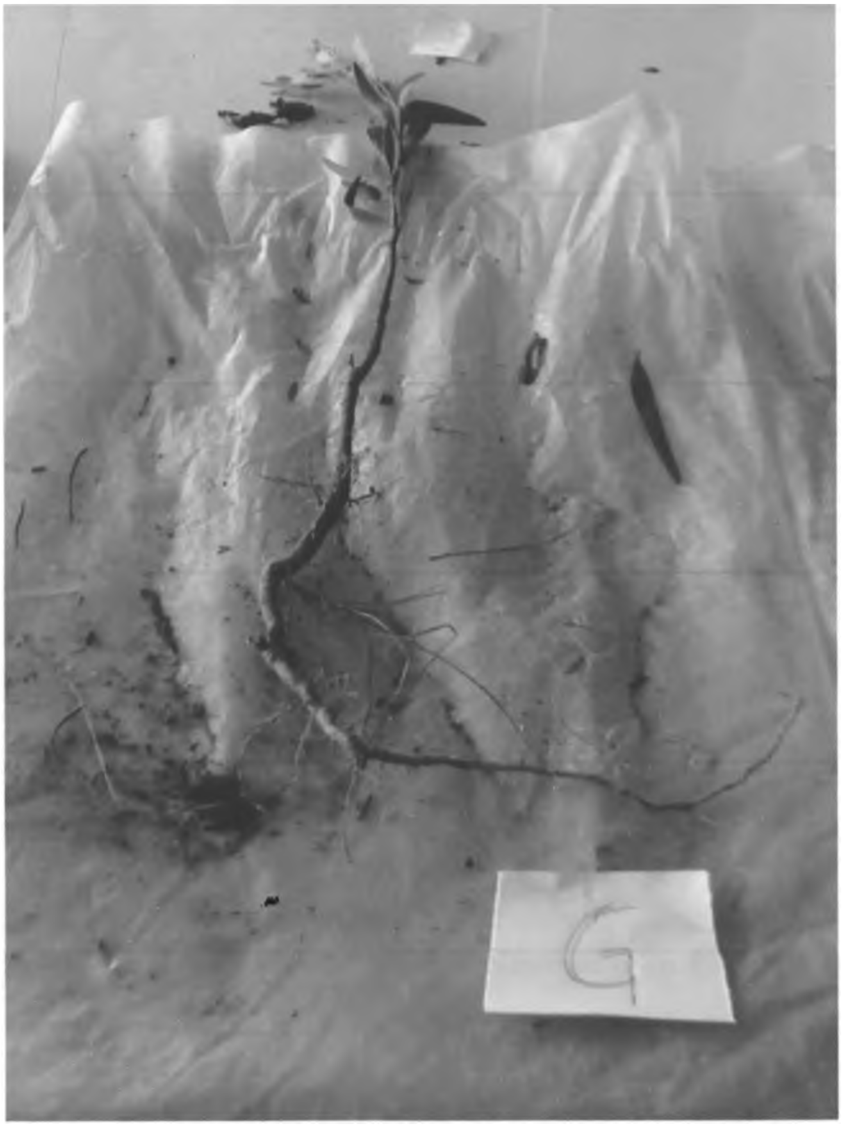


Fig. 4.8: Tap root system from Sasumua (Ailanthus altissima)

Fig. 4.9: Fibrous root system from Sasumua (Festuca glauca)

Fig. 4.10: Fibrous root system from Sasumua (Pinus sylvestris)

4.2.2 Tensile force - diameter relationship

The relationship between root tensile force and diameter is linear as shown in Fig 4.11 – 4.13.

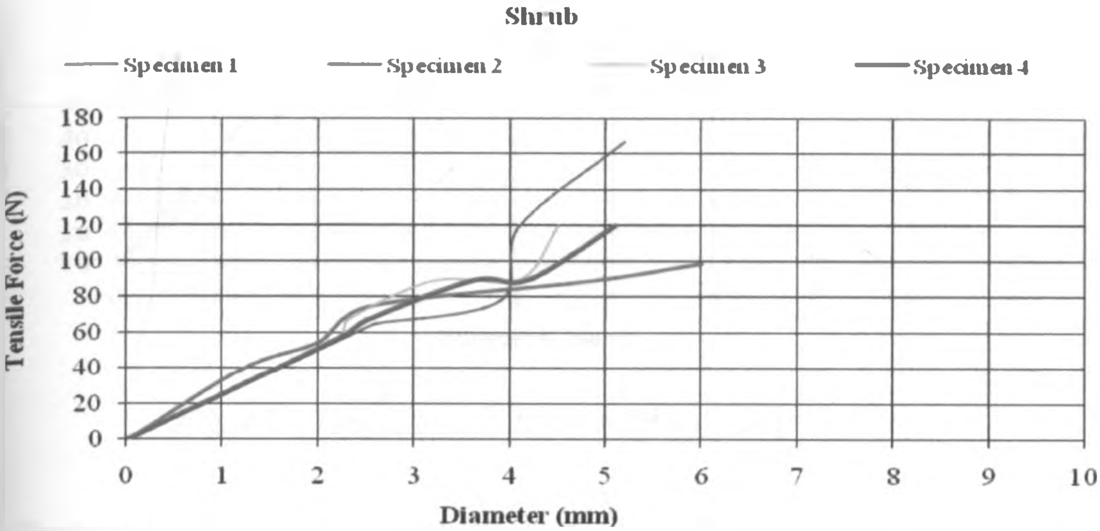


Fig. 4.11: Tensile force – Diameter relationship for shrubs

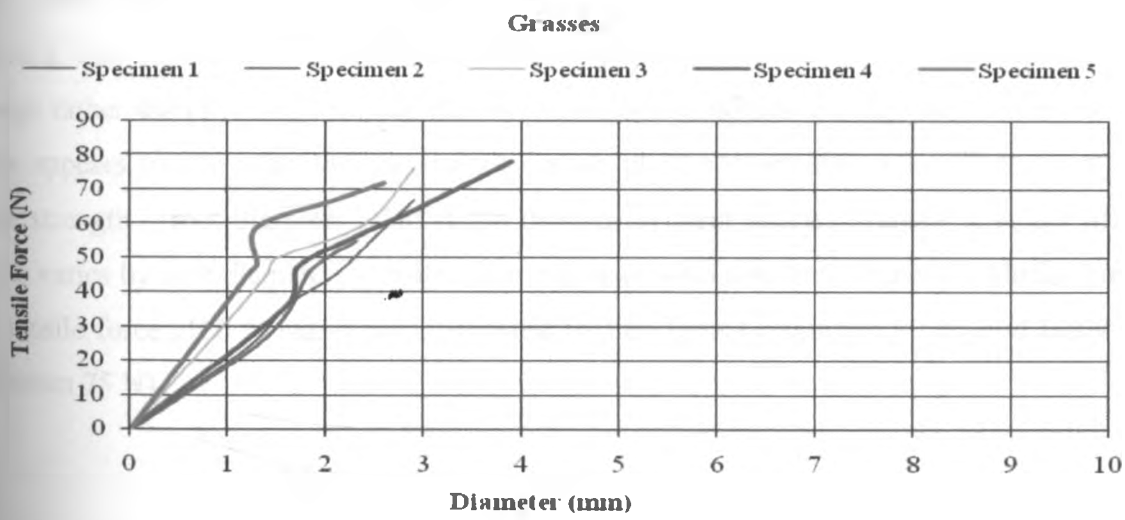


Fig. 4.12: Tensile force – Diameter relationship for grasses

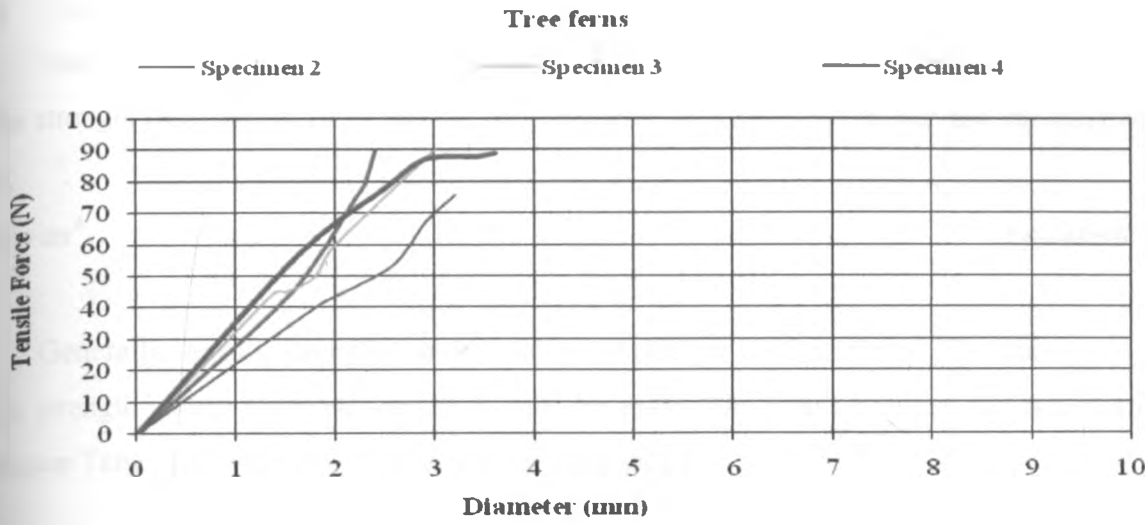


Fig. 4.13: Tensile force – Diameter relationship for ferns

Thicker roots fail at higher tensile forces than narrow roots within a similar species. The graphs show that root failure is abrupt after undergoing plastic behaviour., explaining why during shear test of root reinforced soils, shear stresses still increase at the end of the test, indicating that root tensile failure do not occur during the shear tests. Root elongation or slippage rather than breakage is thus the most common condition during failure. This mode of failure appears to allow for the survival of certain plant species after a landslide event. Root tensile strength – root diameter relationship depend on plant species. Tensile strength within a species varies by root diameter. Tensile force increases with diameter. Generally shrubs break at high tensile force (160 N maximum), followed by tree ferns (maximum 90 N) and lastly grass (maximum 75 N).

4.2.3 Tensile strength - diameter relationship

The *T-D* relationships for these species are shown in Fig. 4.14 – 4.16. It is noted that root tensile strength decreases with increasing root diameter, and follows a power law equation of the form;

$f(x) = ax^k$ **Equation 4.1**

Generally, tensile strength can be well predicted by root diameter. The maximum root tensile strength values recorded was 39 N/mm^2 for grass. The a and k values and the R^2 values are shown Table 4.1, where n is the number of roots tested.

Table 4.1: *a and k values and R^2 values for the power relationships for the root tensile strengths*

Vegetation type	Diameter range (mm)	a	k	n	R^2
Shrub					
	1.2-6.0	49.21	-1.45	3	0.99
	2.4-5.2	38.87	-0.76	5	0.70
	2.2-4.5	38.49	-0.63	4	0.99
	2.2-5.1	46.05	-1.17	5	0.98
Grass					
	1.3-2.3	22.45	-1.55	5	0.79
	1.2-2.9	25.53	-0.87	5	0.98
	1.1-2.9	44.08	-1.28	5	0.98
	1.6-3.9	25.53	-1.23	5	0.97
	1.3-2.9	36.72	-0.87	5	0.97
Tree fern					
	1.4-2.4	30.5	-1.53	3	0.94
	1.8-3.2	39.29	-0.88	4	0.95
	1.4-3.0	55.86	-1.44	5	0.99
	1.5-4.3	42.66	-1.22	6	0.98

It is noted that strong roots have high *a*-values and low *k*-values and vice versa. Shrubs generally have high *a*-values, but a great variation is noted within individual plant species.

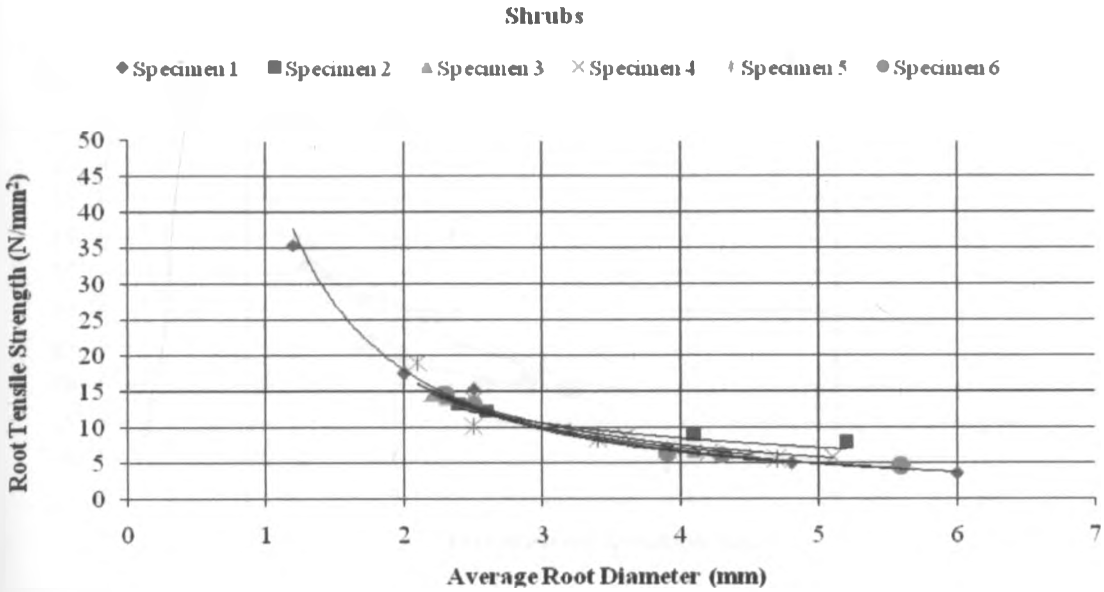


Fig. 4.14: T-D relationship for shrubs. The curves show increasing tensile strength with decreasing root diameter following power law

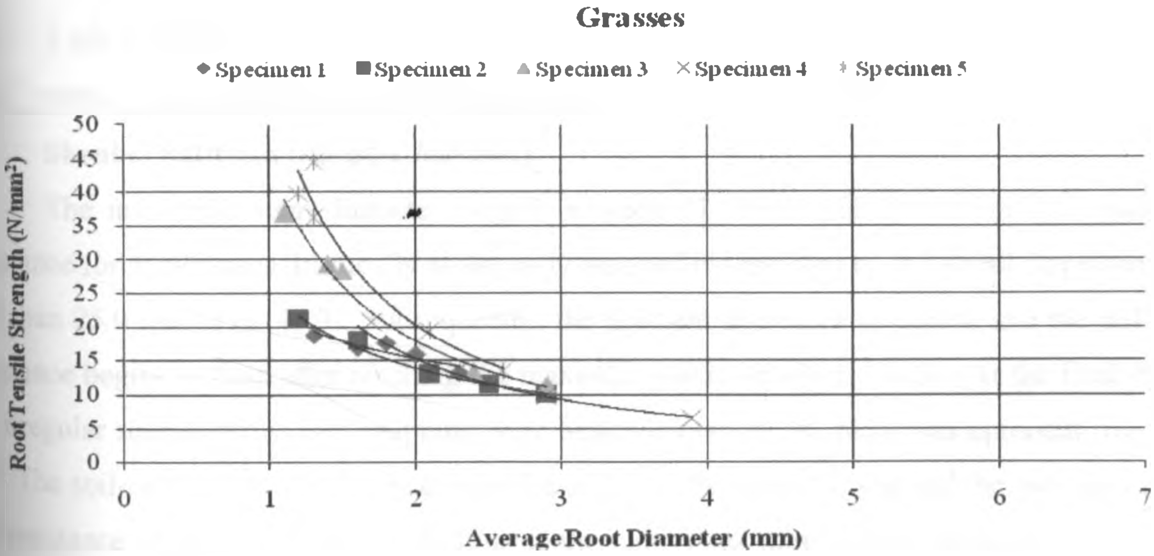


Fig. 4.15: T-D relationship for grasses. The curves show increasing tensile strength with decreasing root diameter following power law

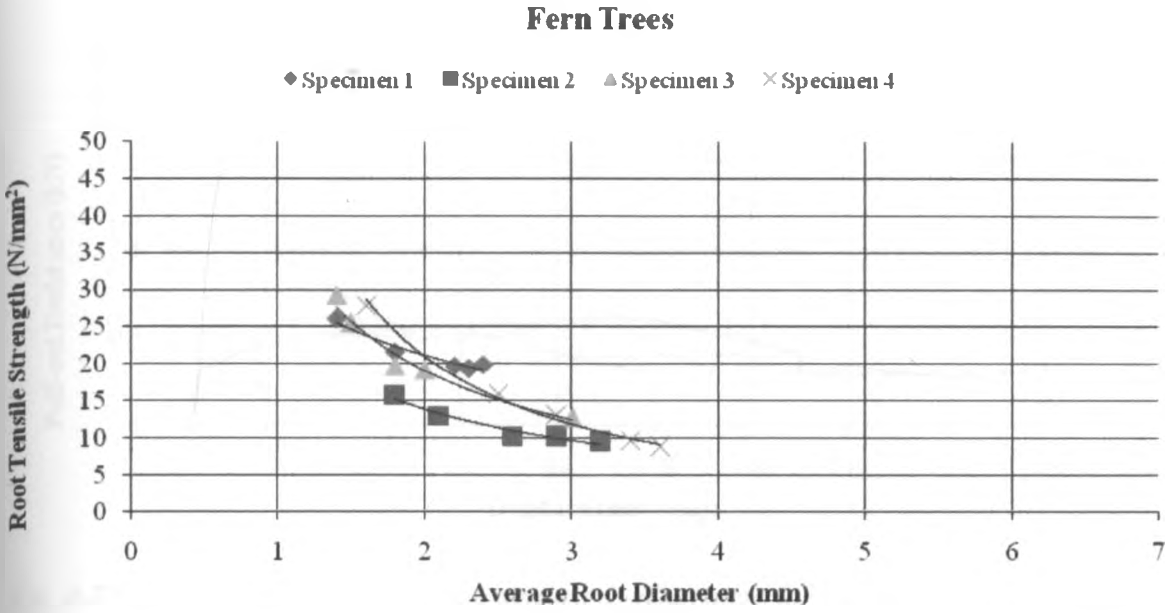


Fig. 4.16: T-D relationship for tree ferns. The curves show increasing tensile strength with decreasing root diameter following power law

4.3 Pull-out tests

4.3.1 Pull-out resistance against displacement

(a) Shrubs: Saltbush (*Atriplex halimus*):

The maximum stem diameter ranged between 17.55mm and 31.55 mm. The pull-out resistance force increases drastically at the early stage of the test that is, at a small displacement, less than 25.0 mm (Fig. 4.23). Subsequently, the gradient decreases gradually and the pull-out resistance begins to drop after reaching the maximum value, at about 0.9 kN. At the final stage, the irregular sounds of the root snapping were heard just before the plant was uprooted from the soil. The soil-root interaction resulted from the adhesion between the soil and the root, and from the resistance of the soil to lateral motion of the root. Thus roots behave as perfectly flexible fibers resisting forces in pure tension. Therefore, the roots could only fail in tension and not in shear. This mechanism is consistent with the one made by McOmber (1981).

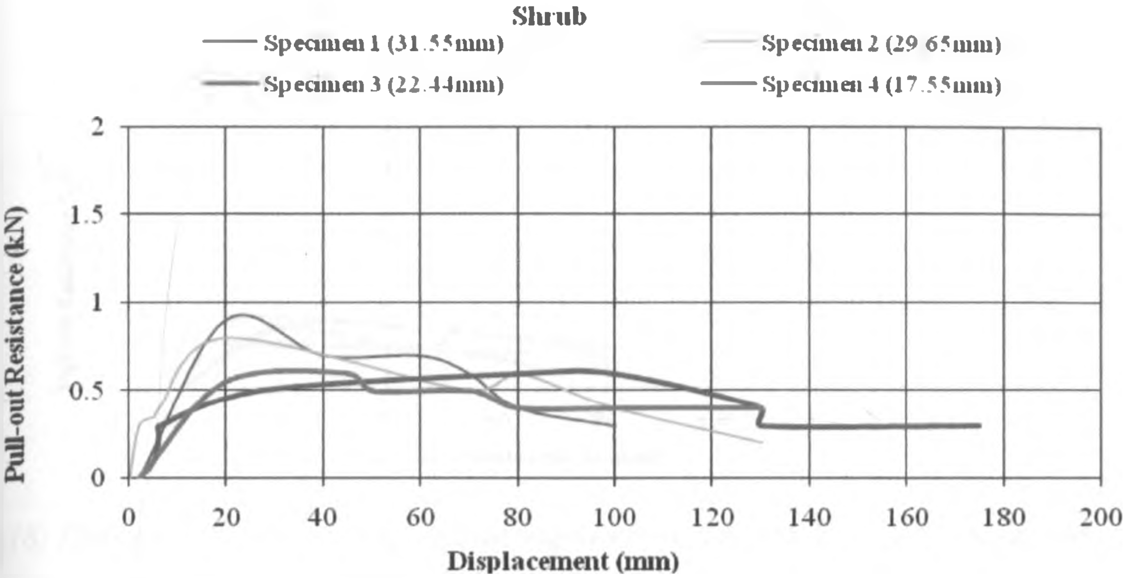


Fig. 4.17: Plots of pull-out resistance against displacement for Saltbush (Atriplex halimus)

(b) Switch grass (*Panicum virgatum*)

The stem diameters of the tested samples ranged from 9.50 mm to 15.70 mm. The lateral roots spread not more than 15 mm from the primary root and the depth was up to 38.5 cm. The gradient decreases gradually and the pull-out resistance begins to drop after reaching the maximum value, at about 0.5 kN. At the final stage, the irregular sounds of the root snapping were heard just before the plant was uprooted from the soil. This observation indicates that the tap root plays a major role in providing the maximum pull-out resistance to this type of plant. This behavior is shown in Fig. 4.24.

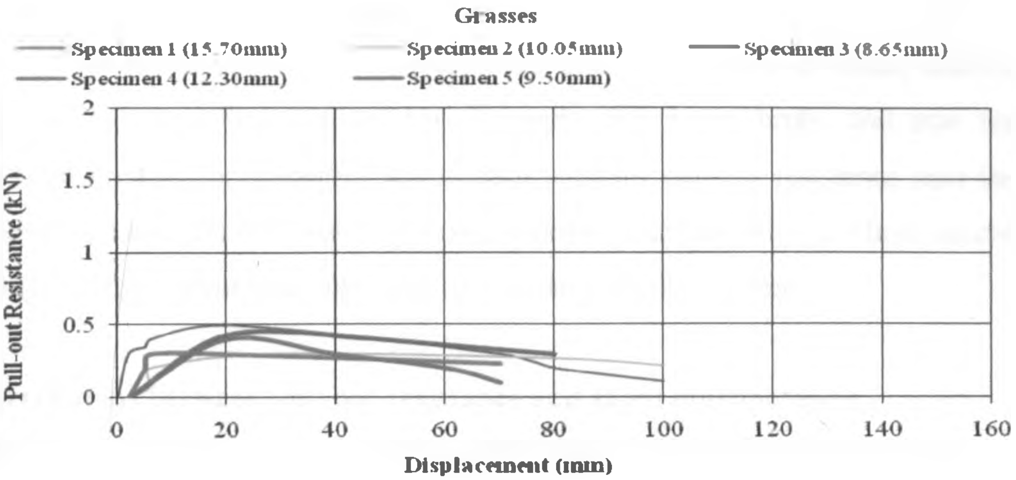


Fig. 4.18: Plots of pull-out resistance against displacement for Switch grass (Panicum virgatum)

(c) Tree ferns (*Asparagus species*)

The stem diameter ranged between 10.50 mm and 30.50 mm. The roots spread not more than 32mm from the primary root. Similar to the previous species, *Asparagus species* exhibits gradual increase in the pull-out resistance against displacement. The lateral and fibrous roots of the plant contribute most of the pull-out resistance force to the plant. The value of the maximum pull-out resistance for *Asparagus species* ranged from 0.6 to 0.9 kN (Fig. 4.25). The displacement increases as the resistance increases and drops drastically at a certain point due to breaking or dislodging of most of the lateral roots that is, when the plant was completely uprooted.

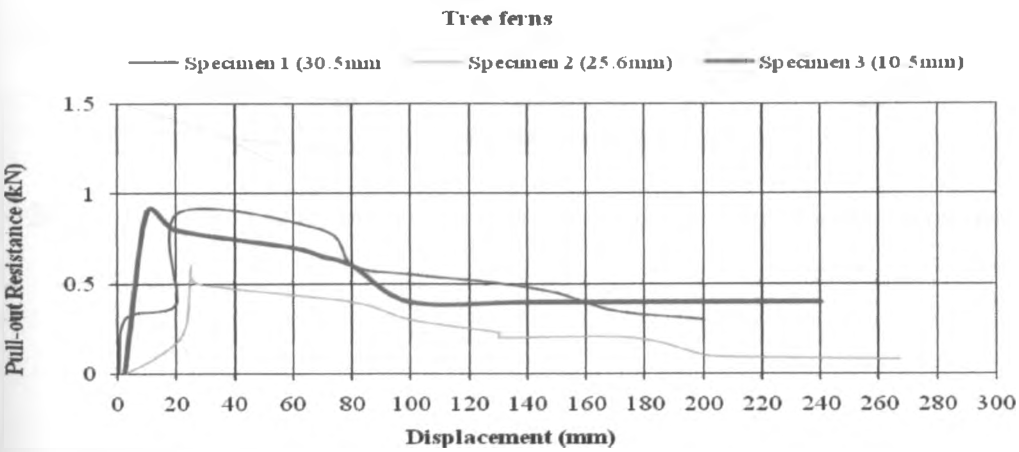


Fig. 4.19: Plots of pull-out resistance against displacement for Tree ferns (Asparagus species)

Understanding of the pull-out resistance of a plant is useful in our assessment of the ability of a plant to sustain environmental stress and forces such as wind, landslide, mass movement and soil creeping. Overall results imply that both shrubs and tree ferns show promising great reinforcing properties due to their superior pull-out resistance than the grasses. They both show similar trend in which only single peak value can be seen. These species acquire the maximum strength of pull-out resistance from mainly the lateral roots.

4.3.2 Correlations between pull-out resistance and shoot morphologies

There are many factors that influence the pull-out resistance-displacement relationship which include the development and conditions of the root system. In order to have a better understanding of these influences, some properties of the root system are correlated to the pull-out resistance. In this study, pull-out tests have been conducted on plants of different stem sizes. The correlation between the pull-out capacity against the maximum stem size is illustrated (Figure 4.26). A strong linear relationship is observed between pull-out capacity and maximum diameter that is, the pull-out resistance increases as the diameter increases.

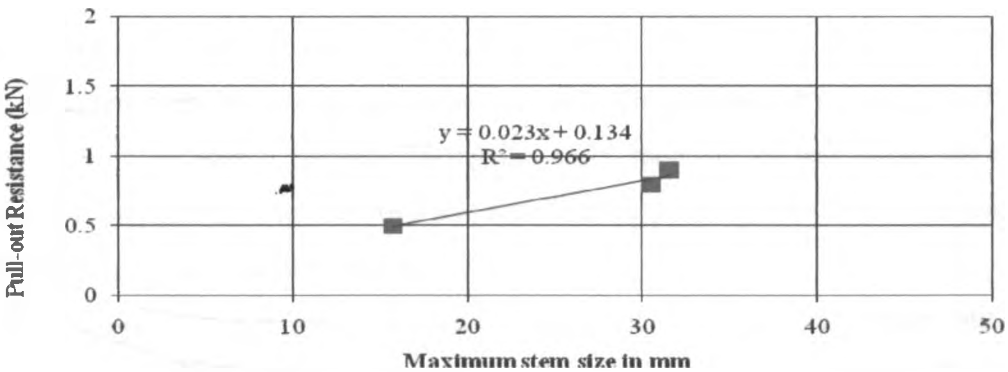


Fig. 4.20: Plots of pull-out resistance against maximum stem size for the three species

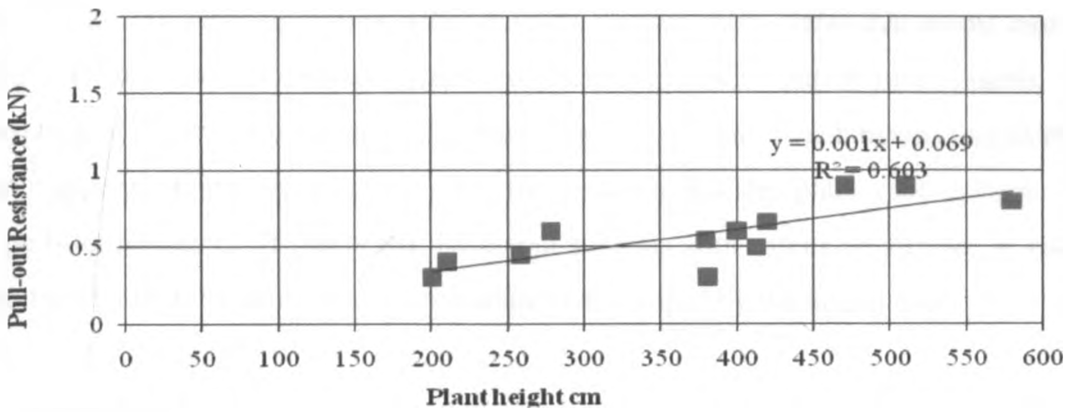


Fig. 4.21: Plots of pull-out resistance against plant height for all species

The result is anticipated as the length and the root density seem to be correlated to the plant age which can be estimated from the plant height. The results also, arguably, imply that the higher the plant age, the higher the plant diameter and height, thus the higher pull-out resistance (Fig. 4.27).

4.3.3 Pull-out resistance force versus root morphologies

It was observed that during the test, the lateral roots play a very important role in providing the pull-out resistance especially during the early stage of the test. After the end of each test, number of lateral roots was recorded and results clearly show that the pull-out capacity varies linearly with number of lateral root (Fig. 4.28).

After each pull-out test, the plant (together with the root) was completely taken out of the soil and the total length of all roots was then measured. The root length density (RLD) was calculated as total root length / soil volume. This parameter reflects the intensity of the root system and hypothesized to have a positive effect on the pull-out capacity. It is evidently shown that the relationship is linear where the pull-out resistance directly depends on the root length density (Fig. 4.29).

Overall correlation and regression analysis show that the pull-out resistances of all species have a positive, either weak or strong, linear relationships with all the morphological properties of the plants. Bigger plants can resist pull-out force better than the smaller plants. The increase in plant size will normally generate high pull-out resistance.

Taller plants will resist uprooting better than the shorter ones. Plants that invest more in their above ground parts would also invest more in the proliferation of their root systems. The pull-out resistance of plant is dependent on the plant shoot dry weight, which means that as more development happens on the stem section, the more developed the plant root system. The increase in pull-out resistance of plants that have root systems with extensive number of lateral root is due to the fact that the stronger soil-anchorage is developed by the lateral roots.

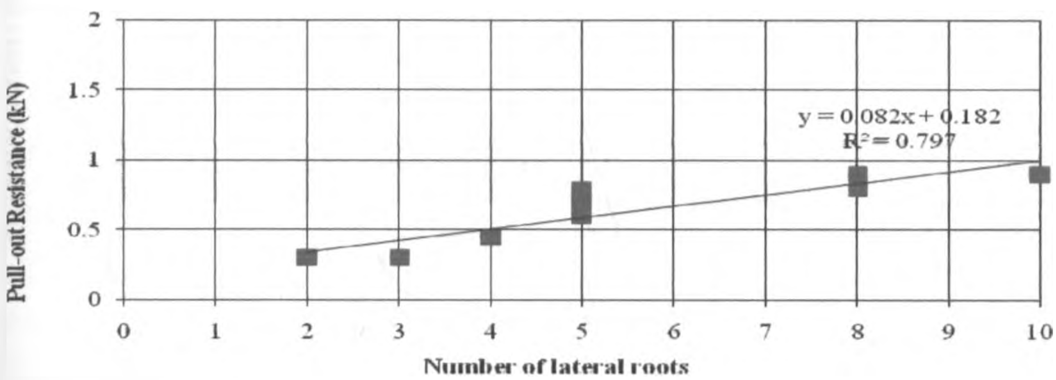


Fig. 4.22: Plots of maximum pull-out resistance against number of lateral roots for all specimen species

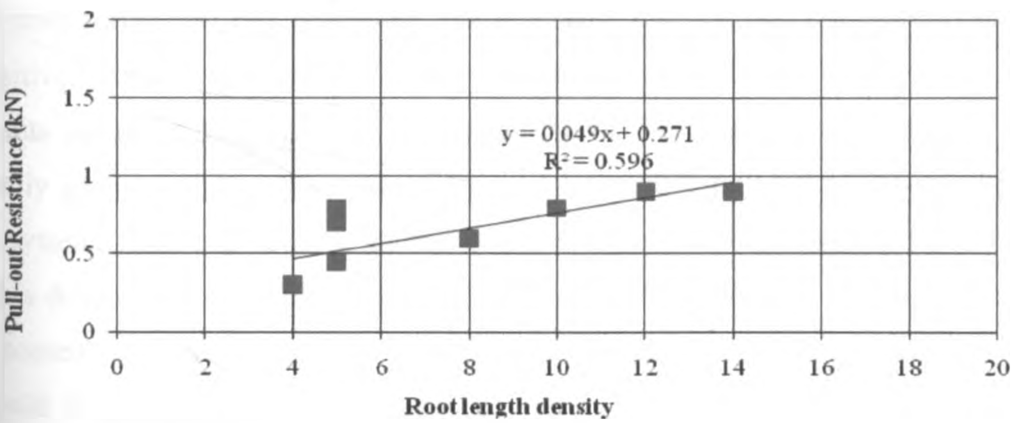


Fig. 4.23: Plots of maximum pull-out resistance against root length density for all the species

4.3.4 Pull-out resistance against soil moisture content

In a different set-up, moisture contents of soil samples were varied, and the maximum pull-out resistance was determined for shrubs, grasses and trees. Fig. 4.30 demonstrates the relationship between pull-out resistance and moisture content of soil.

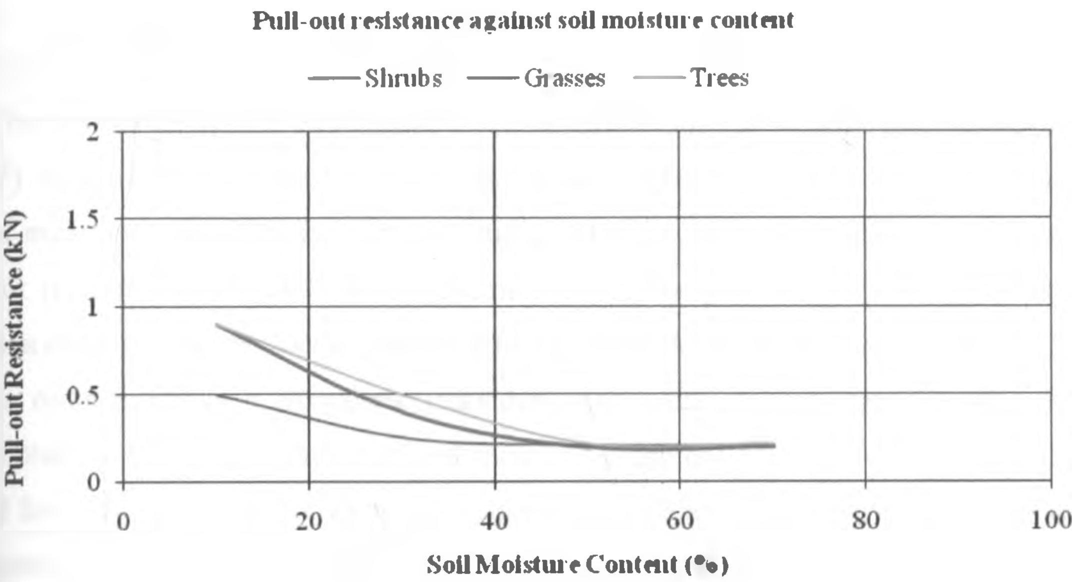


Fig. 4.24: Plots of pull-out resistance against soil moisture content

All species tested give similar trend in the pull-out resistance and displacement relationship, where only one peak value is observed.

Overall correlation and regression analysis show that the pull-out resistances of plants have a positive, either weak or strong, linear relationships with all the morphological properties. Bigger plants can resist pull-out force better than the smaller plants. The increase in plant size will normally generate high pull-out resistance. The increase in pull-out resistance of plants that have root systems with extensive number of lateral root is due to the fact that the stronger soil-anchorage is developed by the lateral roots. The pull-out resistance force decreased drastically as moisture content was increased. It was observed that at moisture content averagely 55%, most slopes could be at the brink of collapse as the root reinforcing mechanism is severely compromised.

4.4 Root area ratio and root distribution

Biomechanical characteristics of the root system were assessed by measuring Root area ratio (RAR) values and tensile strength of root specimens. RAR (the ratio of the sum of the root areas to the area of soil profile they intersect) values of the roots were obtained using profile trenching method as a function of soil depth in order to estimate root contribution to soil strength.

Mean cross-sectional area occupied by roots were obtained for soil depth intervals of 0.05m.The number of roots at intervals of 0.05m is then multiplied with the mean cross-sectional area (m^2) of a root of a certain species. This value is then divided by the horizontal cross-sectional area determined by the vertical orthogonal projection of the above-ground biomass, being our reference area. RAR or root biomass concentration as a function of soil depth is required in order to estimate root contribution to soil strength (De Baets *et al.*, 2008).

In order to obtain RAR values, individual roots were counted manually after the plant was uprooted from the soil. The number of roots and depth at which they occur were taken and recorded for each species. RAR values were obtained at depth increments of 0.05m of all roots with diameter more than 1mm. The diameters of roots intersecting the soil profile were measured by a Vernier caliper.

The distribution with depth for shrubs, grasses and trees is shown in Fig. 4.17 – 4.19. From the plot, it is observed that values of RAR show great variability with depth. Maximum RAR values were located within 0.1 m for all the species, with maximum rooting depth of 0.7 m for fern tree. Shrubs species showed high RAR values between 0.1 – 0.3 m depth. In general, vegetations growing in the Sasumua backslope have shallow roots (maximum root depth 0.65 m) therefore unable to reinforce the soils to stop the landslide occurring at 1 m depths. It therefore follows that the vegetation in this area was inappropriate for slope stabilization and hence the slippage.

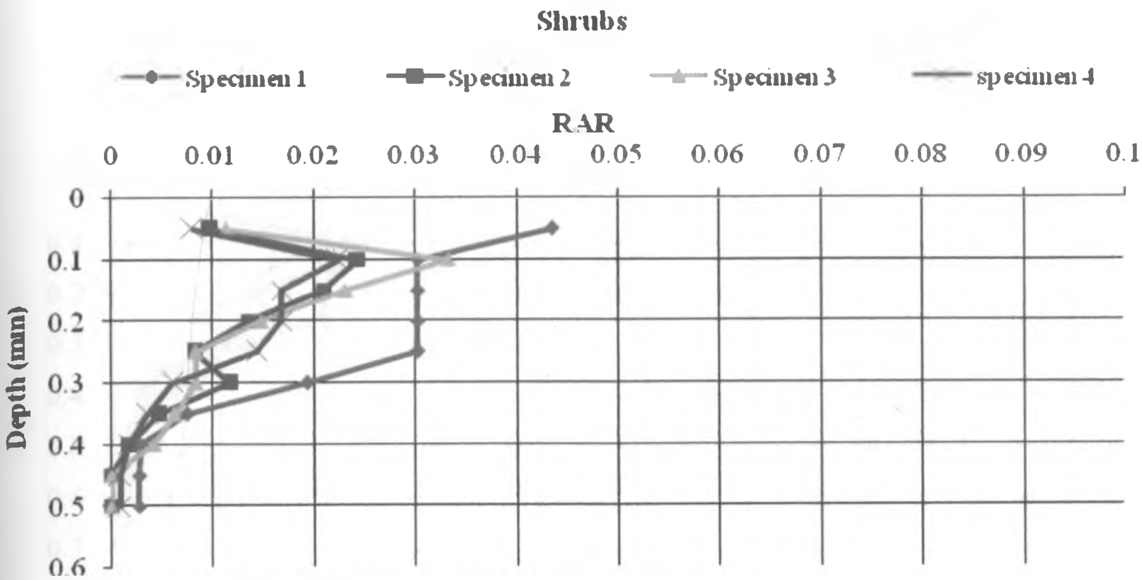


Fig. 4.25: Root area ratio (RAR) distribution with depth for shrubs

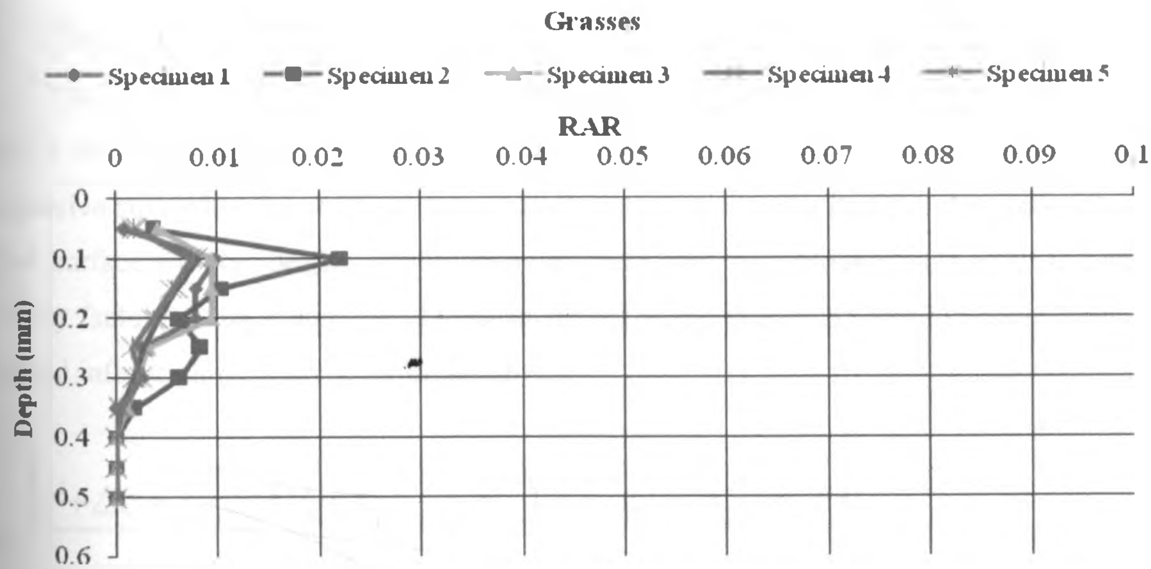


Fig. 4.26: Root area ratio (RAR) distribution with depth for grasses

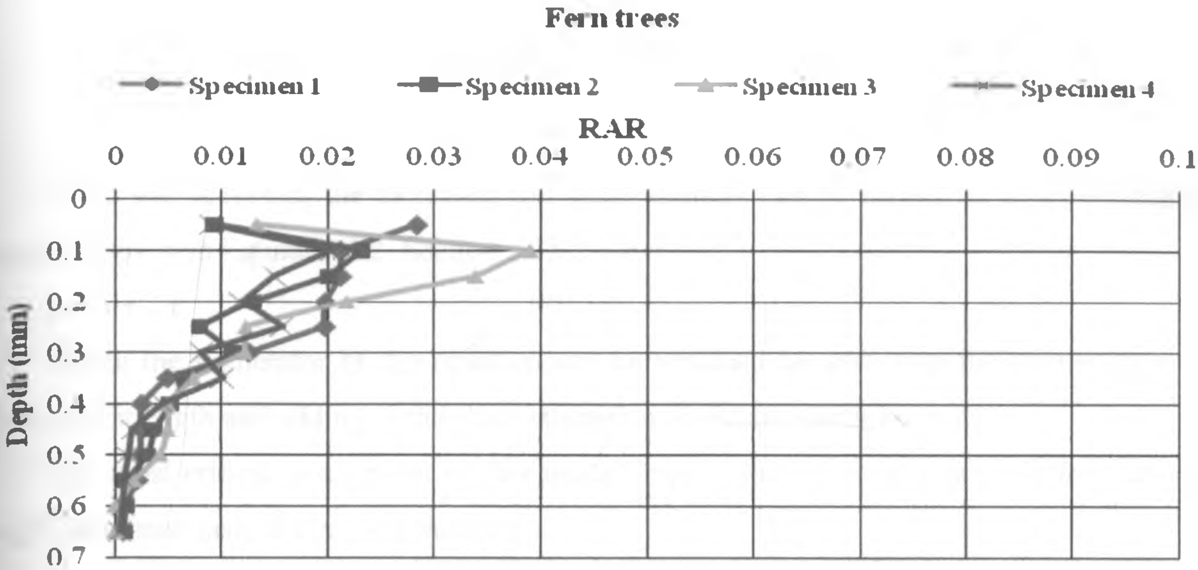


Fig. 4.27: Root area ratio (RAR) distribution with depth for tree ferns

4.5 Root cohesion

The Wu et al. (1979) model was used to assess the shear strength due to roots. According to Wu et al. (1979), the effect of vegetation roots on soil shear strength can be taken as part of the cohesive strength component of the soil-root system. Assuming that the phreatic surface is at the soil surface and the location of the potential shear plane is *z* distance below the soil surface, the safety factor (the minimum possible shear strength / the maximum possible shear stress) for a vegetated infinite slope is given by Equation 4.2;

$$SF = \frac{\left[c' + \Delta c + \left(z\cos^2(\alpha)(\gamma_{sat} - \gamma_w) + w_t \cos\alpha\right)\tan\phi'\right]}{\left[z\gamma_{sat}\cos\alpha\sin\alpha + w_t\sin\alpha\right]}$$

Equation 4.2

Where;

(i) c' and ϕ' are the effective soil strength parameters,

(ii) Δc is the increased cohesion due to tree roots,

(iii) α is the slope angle,

(iv) w_t is the vegetation surcharge (weight / unit area),

(v) γ_{sat} is the saturated unit weight of soil, and

(vi) γ_w is the unit weight of water.

If the soil is rooted, the increased soil shear strength can be expressed as an additional cohesion as given in Equation 2.2, Section 2.12.2, i.e;

$s_r = s + Cr$

Hence the numerator of the equation can be obtained by assessing the contribution of roots to soil strength and adding to the shear strength mobilization without roots.

The most critical assumption of this model implies that all roots attain ultimate tensile strength simultaneously during soil shearing.

From Wu *et al.* (1979) model, the predicted shear strength increase from a full mobilization of root tensile strength is given by Equation 2.3, section 2.12.2, i.e.;

$Cr = t_R(\sin\theta + \cos\theta\tan\phi)$

Laboratory tests conducted on soil specimen yielded a soil friction angle of 22° and angle of shear distortion was taken as 45° for Murang'a landslide soils. Substituting to Equation 2.3 yields Equation 4.3;

$Cr = 0.993t_R$ **Equation 4.3**

t_R can be calculated from a modified model of Wu et al. (1979) to give Equation 4.4.

$$t_R = \frac{\sum Ta}{A}$$ **Equation 4.4**

Where;

(i) T is root tensile stress (MPa),

(ii) a is the root cross-sectional area (m^2), and

(iii) A is the reference area of soil occupied by roots (m^2, determined by the vertical projection of the above-ground biomass of the plant).

T was tested for 5 roots in each species and plotted against Diameter, D. From graph, T was obtained for all the roots.

It has been established that root tensile strength decreases with increasing root diameter, and follows a power law equation of the form of Equation 4.5;

$f(x) = ax^k$ **Equation 4.5**

Generally, tensile strength, T, can be well predicted by root diameter.

From Figs. 4.14, 4.15 and 4.16 as well as Table 4.1, tensile strength can be predicted using equations for different species as shown in Equations 4.6, 4.7 and 4.8, by taking the lowest a values from the population tested.

$T = 38d^{-0.76}$ for Grass **Equation 4.6**

$T = 22d^{-0.155}$ for Shrub **Equation 4.7**

$T = 30d^{-1.53}$ for Tree ferns **Equation 4.8**

From average T values, t_R can be predicted at 1m depth, hence Cr can be calculated for each species.

The potential of soil reinforcement by roots was estimated for each species as shown in Fig. 4.20 – 4.22.

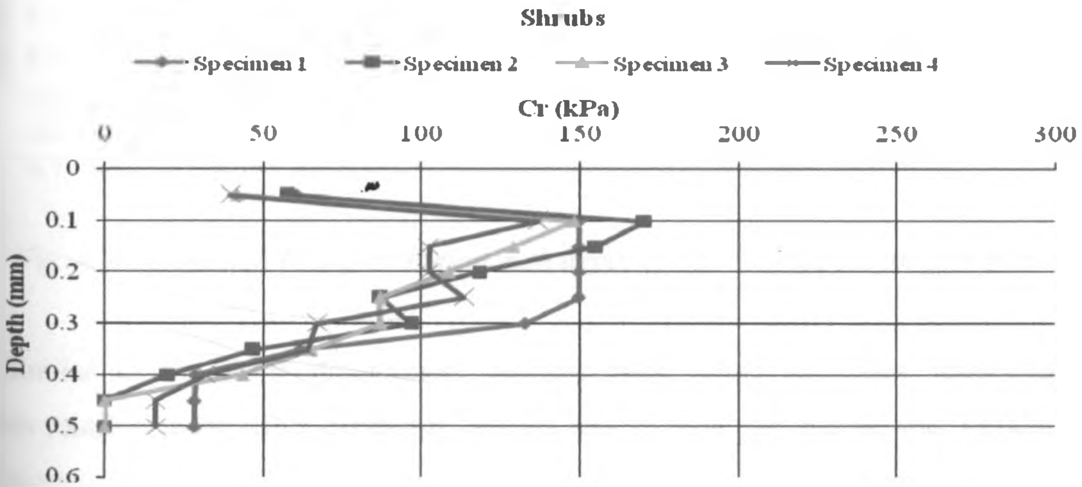


Fig. 4.28: Cohesion (C_r) distribution with depth by the root system for shrubs

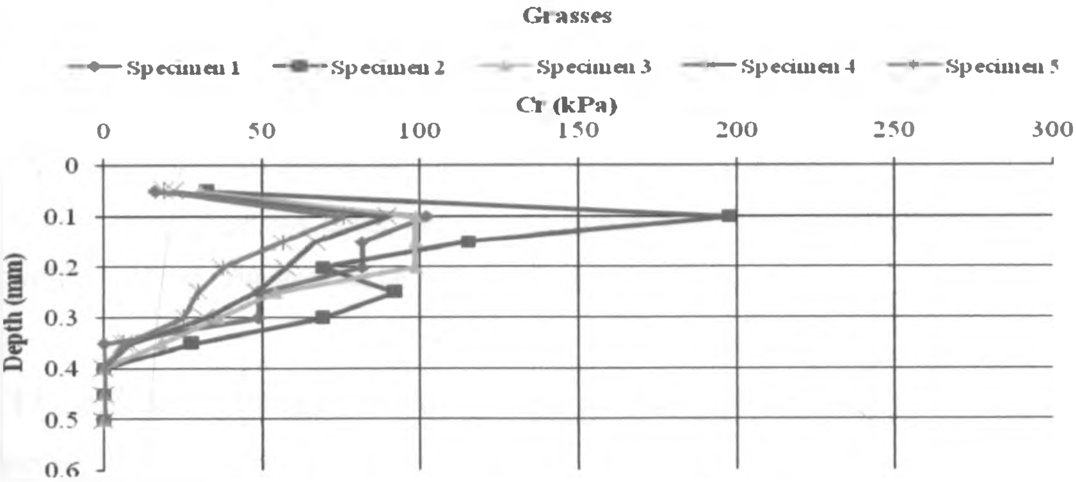


Fig. 4.29: Cohesion (C_r) distribution with depth by the root system for grasses

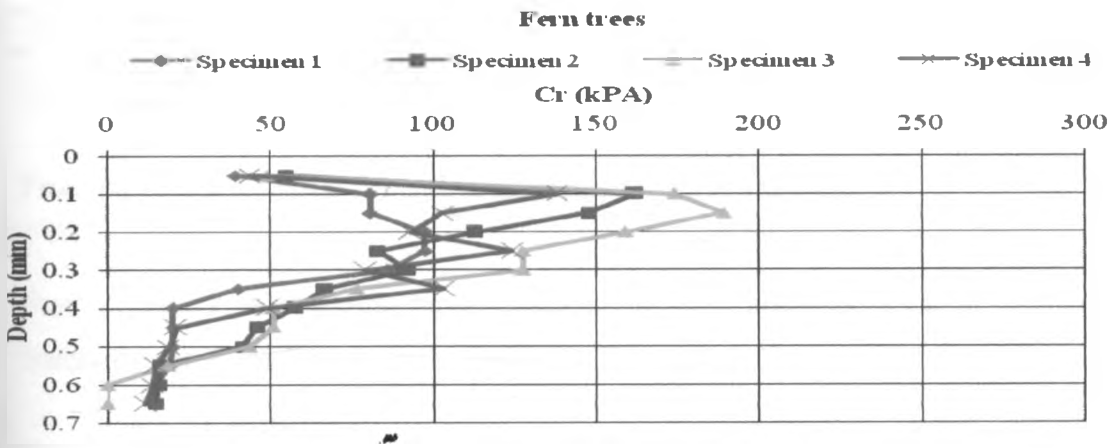


Fig. 4.30: Cohesion (C_r) distribution with depth by the root system for fern trees

Similar to the RAR distribution, reinforcement effect decreases with depth. The maximum reinforcement effect exerted by shrubs was 155kPa, for grasses was 197kPa and for tree ferns was 188kPa. High values of cohesion is noted at low depths, explaining why there are shallow landslides occurring at 1 m depth, where there are no roots to offer sufficient cohesion.

The simple perpendicular model of Wu was used to calculate soil reinforcement by action of roots. It was observed that, generally, smaller diameter roots have higher tensile strengths, but there was a wide variation in the decline in tensile strengths among individual species. Maximum

RAR values were located within 0.1 m for all the species, with maximum rooting depth of 0.7 m for fern tree. Shrubs species showed high RAR values between 0.1 – 0.3 m depth. In general, vegetations growing in the backslope have shallow roots (maximum root depth 0.7 m) therefore unable to reinforce the soils to stop the landslides occurring at 1 m depths.

Shrubs have a typically dense and fibrous root system. Tree ferns have a typical tap root system with branching. The general behaviour of root density is to decrease with depth.

4.6 The effect of changes in slope angle and its determination

From Section 4.2.3, it has been established that

$$Cr = 0.993 t_R \quad \textbf{Equation 4.9}$$

And t_R is calculated from a modified model of Wu et al. (1979) as

$$t_R = \frac{\sum Ta}{A} \quad \textbf{Equation 4.10}$$

From past results, tensile strength is predicted for shrubs as follows;

$$T = 38d^{-0.76} \quad \textbf{Equation 4.11}$$

From average T values, t_R can be predicted at 1m depth, and thus Cr can be calculated.

Safety Factor is obtained from the following Equation 4.12;

$$SF = \frac{[c' + c_r + \delta \tan\phi']}{[z\gamma_{sat}\cos\alpha\sin\alpha + w_t\sin\alpha]} \quad \textbf{Equation 4.12}$$

At the landslide site, the above parameters were obtained and are summarised in the Table 4.2;

Table 4.2a: *Landslide strength parameters*

c' (kN/m^2)	z (m)	γ_{sat} (kN/m^2)	w_t (kN/m^2)	α (deg)	δ (kN/m^3)	ϕ'
2	1	16	17	46	17	13

Table 4.2b: *Landslide strength parameters*

T (kN/m^2)	a (m^2)	A (m^2)	t_r (kN/m^2)	n (No.)	$\sum t_r$ (kN/m^2)	c_r (kN/m^2)
2,200	3.14×10^{-4}	0.9	0.69	19	13.12	13

By substituting the values into Equation 4.12, the safety factor is obtained as 1.08. This is close to unity, explaining why the landslide occurred at this point.

Safety Factors for the other regions adjacent to the landslide are calculated and tabulated in Table 4.3;

Table 4.3: *Safety factors with Slope angles*

	Area 1	Area 2	Area 3	Area 4	Landslide spot
Slope angle	23	25	33	15	46
SF	1.8	1.6	1.3	2.6	1.1

The relationship between slope angle and safety factor for this type of soil and vegetation is shown in Fig. 4.31. The safety factor decreases with increasing slope angle until the slope fails at a critical angle, and follows a power law equation of the form;

$$f(x) = ax^k$$

Although the actual slope angle at the landslide was 46°, it was very close to the critical angle and failure could have been triggered by a slight sudden movement of the ground when there was a heavy downpour.

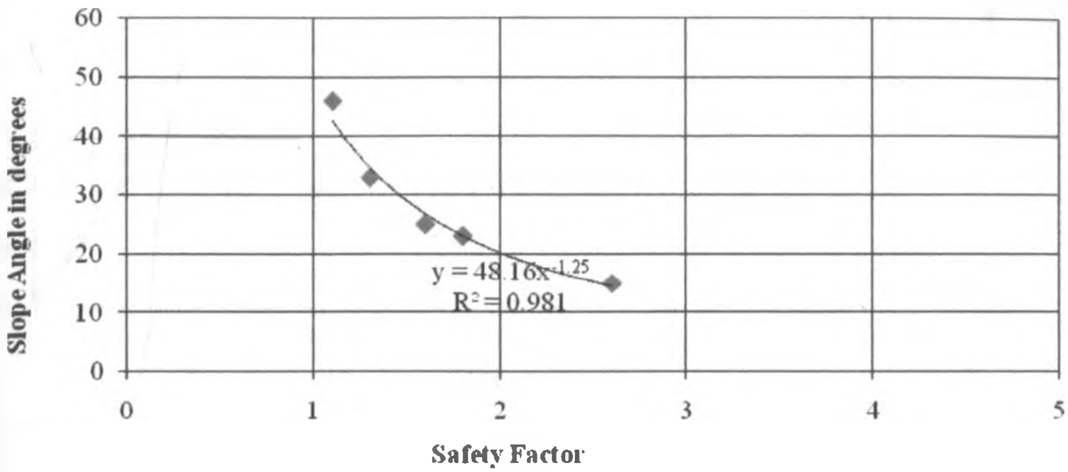


Fig. 4.31: Plots of slope angles versus Safety Factors

4.7 The effect of strain rate on stress-strain relationship of vegetation roots

(a) Strain rate variations

Fig. 4.32 is the stress-strain plots of identical specimens but varying test speeds. The stress-strain curves indicate that Young's modulus for these specimens is consistent at 2.2 – 2.8 kPa and that each specimen behaves in the same manner in the elastic region (Fig. 4.34). Failure is seen to occur at different levels of stress. As test speed was increased from 2mm/min to 16mm/min, the samples' ultimate tensile stresses increased from 53kPa to 62kPa while the yield stresses improved significantly from 23 – 60kPa. The increase in strain rate did not significantly affect the Young's modulus of elasticity.

Given that all specimens behave identically in the elastic region, it is clear that the strain rates have an effect on mainly the plastic deformation behavior. It is interesting to note that vegetation roots thus can increase their yield stress and the ultimate tensile stress during sudden ground movements, a characteristic where strain rates are increased, induced by earthquakes or precipitation, and thus mitigating against landslide occurrence on slopes susceptible to failure.

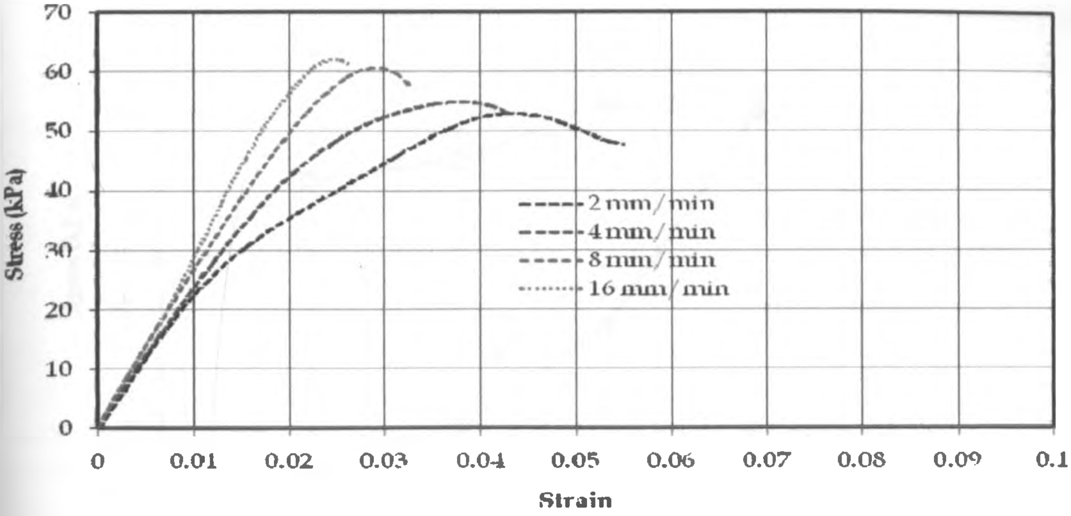


Fig. 4.32: Stress-strain curves for several strain rates

Table.4.4 and Fig 4.33 shows the ultimate tensile strength (UTS) and yield strength (YS) for the strain rate domain. With increasing strain rate, the stresses tend in general to increase.

As speed was increased from 2 – 16mm/min, elongation was reduced by almost half. The UTS increased by 17%.

Table 4.4: Corresponding Speed, UTS and Elongation parameters

Speed (mm/min)	**UTS (kPa)**	**Elongation (mm)**
2	53	3.85
4	55	3.0
8	61	2.3
16	62	1.85

As strain rates approached 16mm/min, it is observed that the difference between UTS and YS diminishes. This implies that specimens ruptured without undergoing plastic and significant deformation.

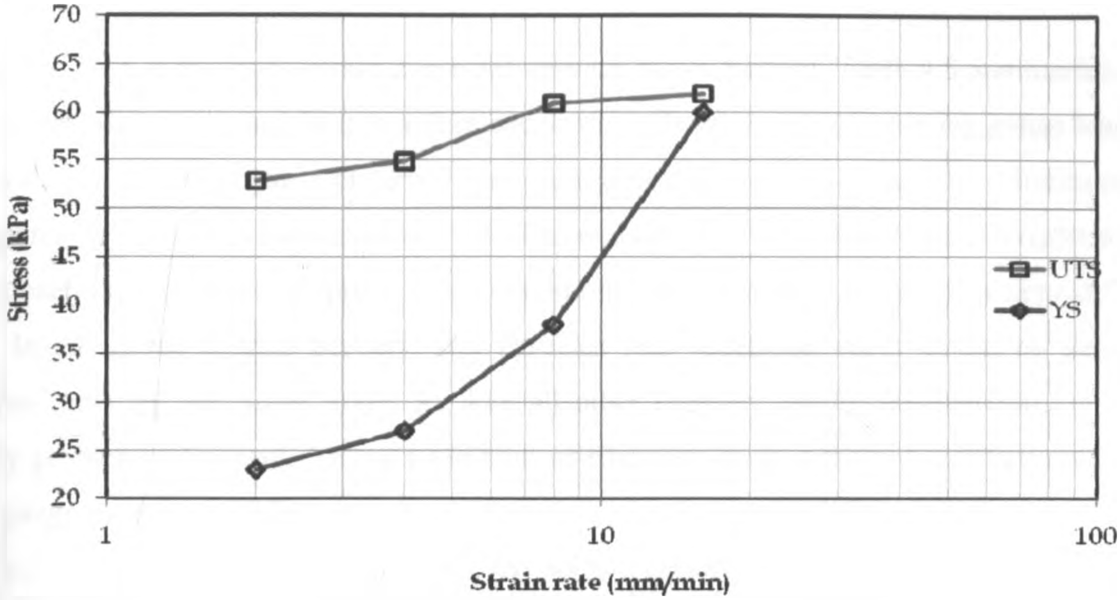


Fig. 4.33: Behaviour of the ultimate tensile strength (UTS) and yield strength (YS) against strain rates for four specimens

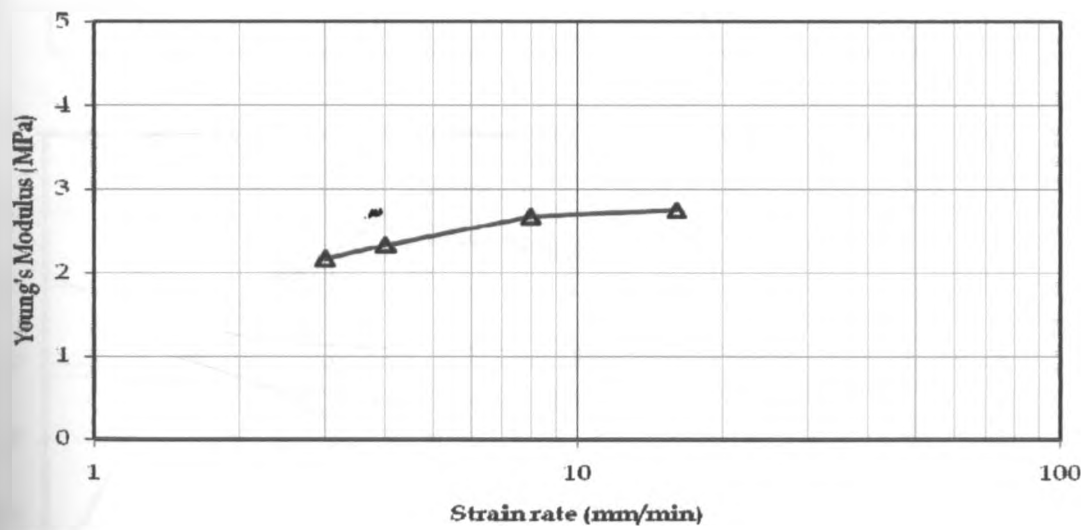


Fig. 4.34: Young's modulus of elasticity (E) against strain rates for four specimens

(b) Gauge length variations

Fig. 4.35 is the stress-strain curve for varying gauge lengths. Table 4.5 summarizes the observations and shows that there is a decrease in strength values with increasing gauge lengths. A 13% decrease in strength is observed for increase in gauge length from 70 – 150mm and a corresponding increase in elongation of 32%. These result show that shorter gauge lengths which more closely approximate the plane strain conditions of confinement in soil that approach the 'zero' length gauge lengths between soil particles and vegetation roots would be stronger, therefore implying that roots would act like all other ordinary reinforcing materials in slope stability problems. However, Young's Modulus of Elasticity fairly remained constant regardless of the gauge used (Fig. 4.35).

Table 4.5: Corresponding Gauge Length, UTS and Elongation parameters

Gauge Length (mm)	UTS (kPa)	Elongation (mm)
70	53	3.6
100	48	4.0
120	46	4.5
150	46	4.75

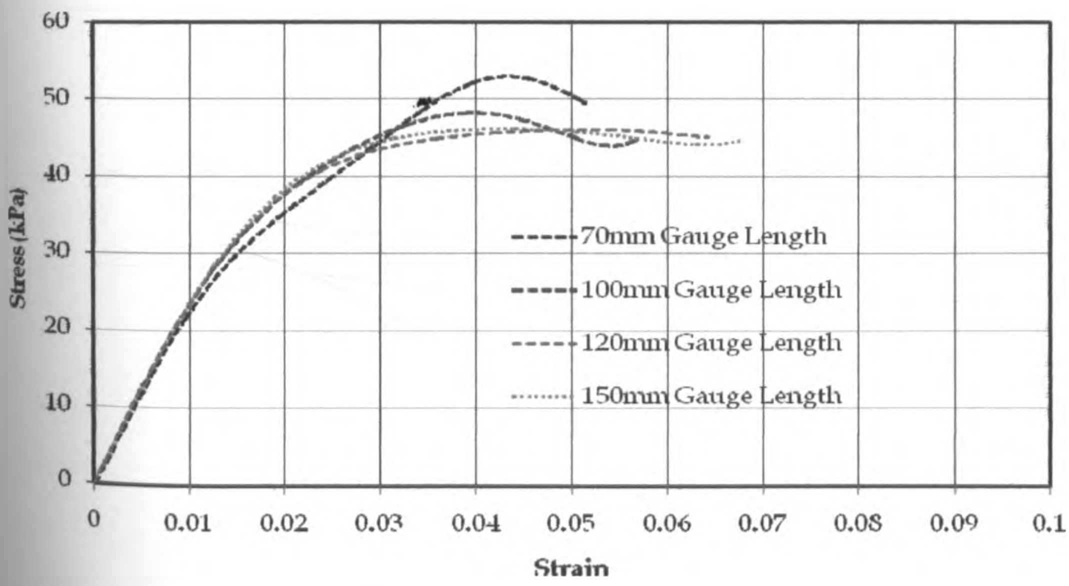


Fig. 4.35: Stress-Strain curves for constant test speed and varying gauge lengths

The difference between UTS and YS generally diminish as gauge lengths increases (Fig. 4.36).

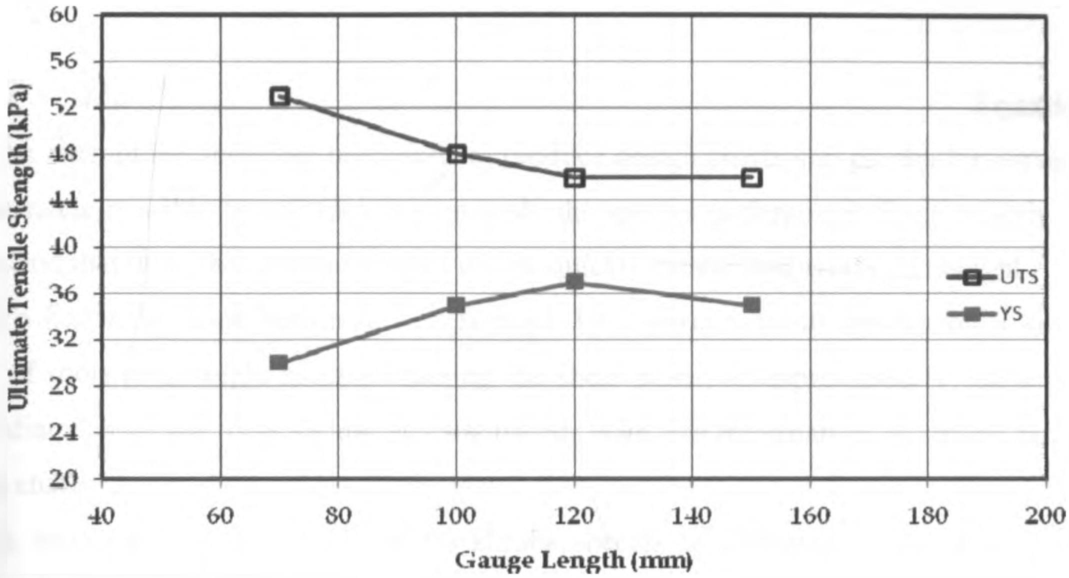


Fig. 4.36: UTS and YS against gauge lengths

Chapter five

5.1 Design tables and chart for shrubs

It has been established that shrubs species reinforce soils better than all other species tested and generally, their tensile strengths follow a power law curve (equation 5.1);

$$T = 22d^{-0.155}$$ **Equation 5.1**

An attempt has therefore been done to produce design charts and graphs for Shrubs. This has been made possible by the fact that most shrubs species undergo relatively orderly growth patterns, and that their shoot morphologies can be quickly established with plant height.

To know the contribution to soil strength by a certain shrub species on a slope, the number of roots of a single species crossing the shear plane, at approximately 1m depth, and average diameter of the roots should be established. With this information, t_R values can picked from the charts.

t_R values are obtained quickly for shrubs species by referring to the design table in Appendix 3. Fig. 5.1 illustrates the behaviour of the diameter against t_R values in a logarithmic chart.

Equation 5.2 is then used to obtain C_r (kPa), which is the increase in shear strength due to the presence of roots.

$$C_r = Kt_R$$ **Equation 5.2**

Where;

(i) K is $(\sin\theta + \cos\theta \tan\phi)$

(ii) φ is the soil friction angle (°),

(iii) θ is the angle of shear distortion in the shear zone, and

(iv) t_R is the total mobilized tensile stress of roots fibres per unit area of soil.

The design table in Appendix 4 gives K values for the range of 1° – 50° θ values and the same range for ϕ values. K values should be between 0.5 and 0.999 for a better result for a root reinforced soil design. High K values are associated with high ϕ values, which are also associated with hard soil or rock, and thus reinforcement is not necessary.

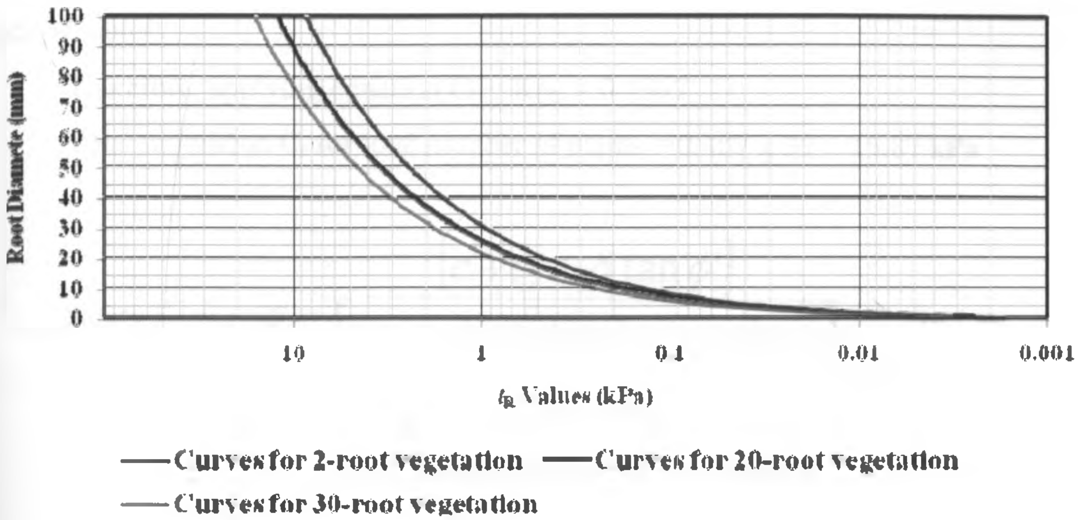


Fig. 5.1: Power law curves for root diameter against t_R Values

5.2 Application of the design tables to assess safety factors for the Murang'a landslide

Consider the Murang'a landslide, whose parameters have been given in Section 3.7 and 4.4;

Site exploration yielded the following physical parameters;

a) Landslide site Safety Factor computation

• Number of plants	39 (manually counted)
• Diameter, average	30mm
• Number of roots	19
• Measured angle of distortion, θ	5°
• Soil friction angle, ϕ	13°
• Cohesion, c'	2 kN/m^2
• Normal stress, δ	17 kN/m^3
• Slope angle, α	46°
• Vegetation surcharge (weight / unit area), w_t	17 kN/m^2
• γ_{sat} is the saturated unit weight of soil, γ_{sat}	16 kN/m^2

Using the given diameter (30mm) and number of roots for each plant (19), $t_R = 1.3113$ obtained from Appendix 3 tables.

Using the angle of distortion and soil friction angle, K = 0.3102 obtained from Appendix 4 tables.

Therefore, applying equation $C_r = Kt_R$, = 0.4067

Total mobilized C_r for total number of plants = 0.407 x 39 = 15.87 kPa

$$\text{Safety Factor, SF} = \frac{[c' + c_r + \delta \tan \phi']}{[z\gamma_{sat} \cos\alpha \sin\alpha + w_t \sin\alpha]}$$

$$= \frac{[2 + 15.87 + 17\tan 13]}{[1 \times 16 \cos 46 \sin 46 + 17 \sin 46]}$$

= 1.0778, agreeing with the values computed in Section 4.4.

b) Area 2 Safety Factor computation

• Number of plants	40 (manually counted)
• Diameter, average	32mm
• Number of roots	20
• Measured angle of distortion, θ	2°
• Soil friction angle, ϕ	13°
• Cohesion, c'	2 kN/m^2
• Normal stress, δ	17 kN/m^3
• Slope angle, α	25°
• Vegetation surcharge (weight / unit area), w_t	17 kN/m^2
• γ_{sat} is the saturated unit weight of soil, γ_{sat}	16 kN/m^2

Using the given diameter (32mm) and number of roots for each plant (20), $t_R = 1.4771$ obtained from Appendix 3 tables.

Using the angle of distortion and soil friction angle, K = 0.259 obtained from Appendix 4 tables.

Therefore, applying equation $C_r = K t_R$, = 0.383

Total mobilized C_r for total number of plants = 0.383 x 40 = 15.2 kPa

$$\text{Safety Factor, SF} = \frac{[2 + 15.2 + 17\tan 13]}{[1 \times 16\cos 25 \sin 25 + 17 \sin 25]}$$

= 1.58, agreeing with the values computed in Section 4.4

5.3 Conclusion

An attempt has been made to produce design charts and graphs for Shrubs.

Appendix 3 gives the t_R values while Appendix 4 gives K values for the range of 1° – 50° θ values and the same range for ϕ values. These parameters are then used to design root reinforced slopes.

Chapter six

Conclusions, recommendations and further research

6.1 Conclusions

6.1.1 General

The main objective of this investigation was to provide information about the contribution of plant roots to soil shear strength. This information will help designers and land managers to make;

(i) Qualitative assessment, and

(ii) To perform quantitative slope stability analyses in vegetated landfills, riverbanks, shorelines, embankments, cut slopes and retaining walls, and other reclamation applications where the mechanical contribution of root reinforcement is important to predicting soil behaviour.

To obtain the objectives, the following investigations were conducted;

(iii) Investigations into the behaviour of an individual root located across a shearing zone. Testing was performed to determine the behaviour of the soil and root below the shearing zone and what effect this behaviour had on the soil being sheared.

(iv) Investigations about individual root tensile strength of typical plant species and their contribution to soil shear strength. Root area ratio (RAR) values were obtained of the individual species to determine their distribution in the soil in a typical site where shallow landslide problems were rampant.

(v) Investigations into root pull-out strengths were conducted.

(vi) Investigations into the effect of strain rate and specimen length on stress-strain relationships of typical vegetation roots used in slope stability problems were conducted. By varying the strain rate, we established the behaviour of these roots during sudden movements, as in landslides, when the strain rates were increased. This investigation yielded information that lead to establishment of a standardized mechanism of testing of roots and the establishment of a generalized design criteria using design charts.

(vii) Critical slope angle was also determined

Ideally, this investigation and perhaps follow-on studies can help to establish both the merit and a practical approach to incorporating vegetation for its physical functions in land stabilization, including for situations where it is currently routinely disregarded.

The questions that were answered in this research report were:

- Do the roots contribute to the stability of slopes?
- Which kind of tree species should be introduced in slopes?
- What is the critical slope angle for any root-reinforced soil?
- How do we design a root-reinforced soil slope

Conclusions from results of this study are presented in Sections 6.1.2 – 6.1.8 for separate tests as they were conducted.

6.1.2 Shear strength

Unvegetated soils are weaker than vegetated slopes in shear. Tests conducted at Sasumua backslope indicated that shear values reached a 16 kPa for unvegetated soils. On the other hand, vegetated soil had maximum values of 90 kPa, 80 kPa and 120 kPa. Rooted soils were thus stronger in shear as compared to fallow soils. Within the same species, varying shear strengths values emerged indicating that root biomass density played a significant role in mobilizing shear strengths. Tree ferns reinforced soils better than the Switch grass, which was weakest, and Saltbush roots.

Shear stresses increase at the end of testing for the rooted samples. Thus root tensile failure was not happening. Observations of the roots after test indicates roots elongated. This elongation can be related to the root biomass density to explain why varying strengths were obtained for different samples of a same species.

6.1.3 Tensile strengths and root distribution

The perpendicular model of Wu *et al.* (1979) was used to calculate soil reinforcement by action of roots.

The *T-D* relationships indicate that root tensile strength decreases with increasing root diameter, and follows a power law equation of the form;

$$f(x) = ax^k$$

Strong roots have high a-values and low k-values and vice versa. Shrubs generally have high a-values, but a great variation is noted within individual plant species.

It was observed that, generally, smaller diameter roots have higher tensile strengths, but there was a wide variation in the decline in tensile strengths among individual species. Maximum RAR values were located within 0.1 m for all the species, with maximum rooting depth of 0.7 m for fern tree. Shrubs species showed high RAR values between 0.1 – 0.3 m depth. In general, vegetations growing in the Sasumua backslope have shallow roots (maximum root depth 0.7 m) therefore unable to reinforce the soils to stop the landslides occurring at 1 m depths. Root tensile strength – root diameter relationship depend on plant species. Tensile strength within a species varies by root diameter. Tensile force increases with diameter. Generally shrubs break at high tensile force (160 N maximum), followed by tree ferns (maximum 90 N) and lastly grass (maximum 75 N). Shrubs have a typically dense and fibrous root system. Tree ferns have a typical tap root system with branching. The general behaviour of root density, in any case, is to decrease with depth.

Tensile strength of root decreases with increasing root diameter. Some correlations are observed between tensile strength properties; maximum tensile resistance, tensile strength and root diameter of all species studied. The results strongly imply that shrubs species have prominent root mechanical properties and it is anticipated that these particular plants have the necessary features to be outstanding slope plants.

From Table 4.1, tensile strengths can be predicted using equations for different species as shown in Equations 4.6, 4.7 and 4.8 reproduced below, by taking the lowest a values from the population tested;

$T = 38d^{-0.76}$ for Grass

$T = 22d^{-0.155}$ for Shrub

$T = 30d^{-1.53}$ for Tree ferns

6.1.4 Pull-out strengths

The fabricated equipment has served its intended function and pull-out test had been performed successfully. All species tested give similar trend in the pull-out resistance and displacement relationship, where only one peak value is observed.

Overall correlation and regression analysis show that the pull-out resistances of plants have a positive, either weak or strong, linear relationships with all the morphological properties. Bigger plants can resist pull-out force better than the smaller plants. The increase in plant size will normally generate high pull-out resistance. Taller plants will resist uprooting better than the shorter ones. Plants that invest more in their above ground parts would also invest more in the proliferation of their root systems. The pull-out resistance of plant is dependent on the plant shoot dry weight, which means that as more development happens on the stem section, the more developed the plant root system. The increase in pull-out resistance of plants that have root systems with extensive number of lateral root is due to the fact that the stronger soil-anchorage is developed by the lateral roots

The pull-out resistance force decreased drastically as moisture content was increased. It was observed that at moisture content averagely 55%, most slopes could be at the brink of collapse as the root reinforcing mechanism is severely compromised.

6.1.5 The effect of slope angle

The safety factor decreases with increasing slope angle until the slope fails at a critical angle, and follows a power law equation of the form;

$$f(x) = ax^k$$

The critical slope angle of 48° for Murang'a site is deduced. Although the actual slope angle at the landslide was 46°, it was very close to the critical angle and failure could have been triggered by a slight sudden movement of the ground when there was a heavy downpour.

Generally, a slope angle of 30° yields a safety factor of 1.5, and is considered as a stable slope. Slopes more than 30° are in most cases unstable, and communities living in such areas should be relocated.

6.1.6 Soil reinforcement by roots

Two ways are considered in which vegetation can affect slope stability: changes in the soil moisture regime and contribution to soil strength by the roots.

Roots physically reinforce soils and resist erosion. Deep, woody roots lock the soil layers together, and lateral roots connect many plants into an interlocking grid. Fine feeder roots form a network through the upper soil layer, preventing surface erosion. Grasses have relatively shallow roots and low biomass, so they prevent surface erosion only, and do not stabilize deep soil. Trees possess deeper roots than shrubs and are essential for slope plantings. Rainfall saturates the upper soils and then seeps laterally causing slides. Deep tree roots penetrate into the compacted layer and help tie the layers together, preventing slides.

Moisture content of 50% or more is detrimental, as this can initiate slope failure. The root reinforcing mechanism is severely compromised at this moisture content, regardless of the type of plant roots present.

6.1.7 Design tables and chart

In this investigation, easy-to-use design tables and a graph have been developed to quickly determine the contribution of shrub roots to soil strength. Using design tables in Appendix 3 and 4 respectively, it is possible and easy to roughly estimate the contribution of roots to shear strength of a soil.

By estimating the number of roots crossing the potential shear plane, the average root diameter, the shear distortion angle and internal angle of friction, the contribution of roots to shear strength can be computed.

The information obtained in this study has certain limitations regarding its application to most naturally occurring root permeated soils as the tables do not give values for root diameters greater than 100mm. The growth pattern of shrubs has been assumed to be uniform so that with the same age or height, it is assumed that the number of roots crossing the shear plane is the same for all the plants. Additional testing using varying soil conditions and a variety of root configurations is necessary to gain a more complete understanding of root reinforced soil behaviour.

6.1.8 The effect of strain rate on stress-strain relationship of vegetation roots

With increasing strain rate, the stresses tend in general to increase. As speed was increased from 2 – 16mm/min, elongation was reduced by almost half. The UTS increased by 17%. The increase in strain rate did not significantly affect the Young's modulus of elasticity. Vegetation roots thus increase their yield stress and the ultimate tensile stress during sudden ground movements, a characteristic where strain rates are increased, like during earthquakes or rainfall event, and thus could offer more resistance to forces causing instability in slopes.

A 13% decrease in strength is observed for increase in gauge length from 70 – 150mm and a corresponding increase in elongation of 32%. Shorter gauge lengths which more closely approximate the plane strain conditions of confinement in soil that approach the 'zero' length gauge lengths between soil particles and vegetation roots would be stronger, therefore implying that roots would act like all other ordinary reinforcing materials in slope stability problems. Young's modulus of elasticity fairly remained constant regardless of the gauge used.

In general, strain rate and specimen length variation significantly alter the stress-strain behavior of the specimens causing substantial changes in the yield stress and specimen failure mechanism.

This investigation yielded information that lead to establishment of a standardized mechanism of testing of roots and the establishment of a generalized design criteria using design charts.

6.2 General recommendations

Based on the findings from the two sites, the following recommendations are drawn;

1. Vegetation capable of developing deeper roots (> 1 m) should be introduced generally in most slopes in order to reduce the frequency of shallow landslides occurrence. These vegetations should have high values of RAR and high tensile strengths. Shrubs species provide superior soil reinforcing properties with high values of RAR and are recommended.
2. Vegetation with potential to develop high surcharge (weight / unit area) are detrimental and should be avoided. Trees generally fall in this category and should be avoided. However, if trees are to be used for soil reinforcement, shorter families which grow to less than 5m in height and a girth of less than 30cm are recommended.

3. The vegetation density is a great issue in slope stability and vegetation should be spaced 1m apart in order to influence full mobilization of shear strength as the roots integrate with the soil mass. Conclusions from this research indicated that if root diameters of a certain slope are 100 mm and if root population is 10 per plant, then strength in the range of 7 kPa could be mobilized. For 20 of those plants, a mobilization of 140 kPa is achieved.
4. Population living in slopes greater than 30° should be resettled in gentle areas as safety factor beyond 30° is less than 1.5, and failure can be triggered during a rainy spell.
5. A proper drainage system should be designed and constructed over slopes to dissipate surface runoff immediately it occurs in order to avoid premature failure of slopes. When moisture content approaches 50%, the shear strength mobilization decreases. When water is quickly drained, the build-up of moisture content is slowed down, and slopes can remain intact.
6. When selecting plants for erosion control or slope stabilization it is important to use fast growing species that have root systems that will "hold" the soil in place. Heavy plants are not recommended for slope control because their dense foliage can actually contribute to slope erosion.
7. There should be coordination with plant suppliers before preparing areas to be planted. Depending on the species and season, not all species may be available and delays may result in substantial erosion.
8. The newly planted site will be especially vulnerable to erosion until the plants have become established. Temporary stabilization barriers should be used as needed to prevent erosion during plant establishment. These barriers may include hay bales, erosion control blankets, temporary seeding, nurse crops, and erosion control netting.
9. When the establishment site is located adjacent to water bodies (i.e. drainage ditches, canals, streams, etc.), measures should be taken to minimize movement of soil into them.
10. The easy to use design charts developed under this research should be applied for root reinforced soils design process.

6.3 Further research

In order to conclusively determine the contribution of vegetation roots to soil stability and how they can be applied, a number of further researches are necessary. The following are the areas which need immediate attention in addition to this research;

1. Comparison of tensile strengths of fibers from different plant species used in slope stabilization
2. A fundamental biological and geotechnical study of how different plant species increase slope stability. This research will provide a fundamental understanding of how roots from different plants species interact with soil to stabilise slopes, quantify the impact of root reinforcement on slope failure experimentally, and therefore determine which root system structures are most beneficial for slope stabilisation. This will enable environmentally friendly technologies for slope stabilisation by vegetation through cross-disciplinary research that links plant root biomechanical properties with modelled slope behaviour. The creation of reduced-scale root analogues will allow different root architectures to be investigated sequentially to assess their importance.
3. Using *in situ* three-dimensional plant root architecture in models of shallow-slope stability. This research shall incorporate three-dimensional digitizing to obtain accurate root system architecture data in forest stands along slopes and to obtain the effect of mass of vegetation to shear strength. These results can then be incorporated directly to equations already developed, or from charts derived under this research.
4. Centrifuge modelling of soil slopes reinforced with vegetation. This research shall be conducted for a series of geotechnical centrifuge model tests to investigate the mechanical reinforcement of slopes by vegetation. The model slopes shall contain young trees, which shall be grown in controlled conditions to provide different root distributions and mechanical properties. Slopes can be brought to failure in the centrifuge by increasing water pressures. The failure mechanisms can be investigated photographically and using post-test excavation. By measuring the soil properties and pore pressures in each test when failure occurs, slope stability calculations could be performed for each slope failure. This modelling technique is suitable for further investigation of root mechanical interactions with slopes.

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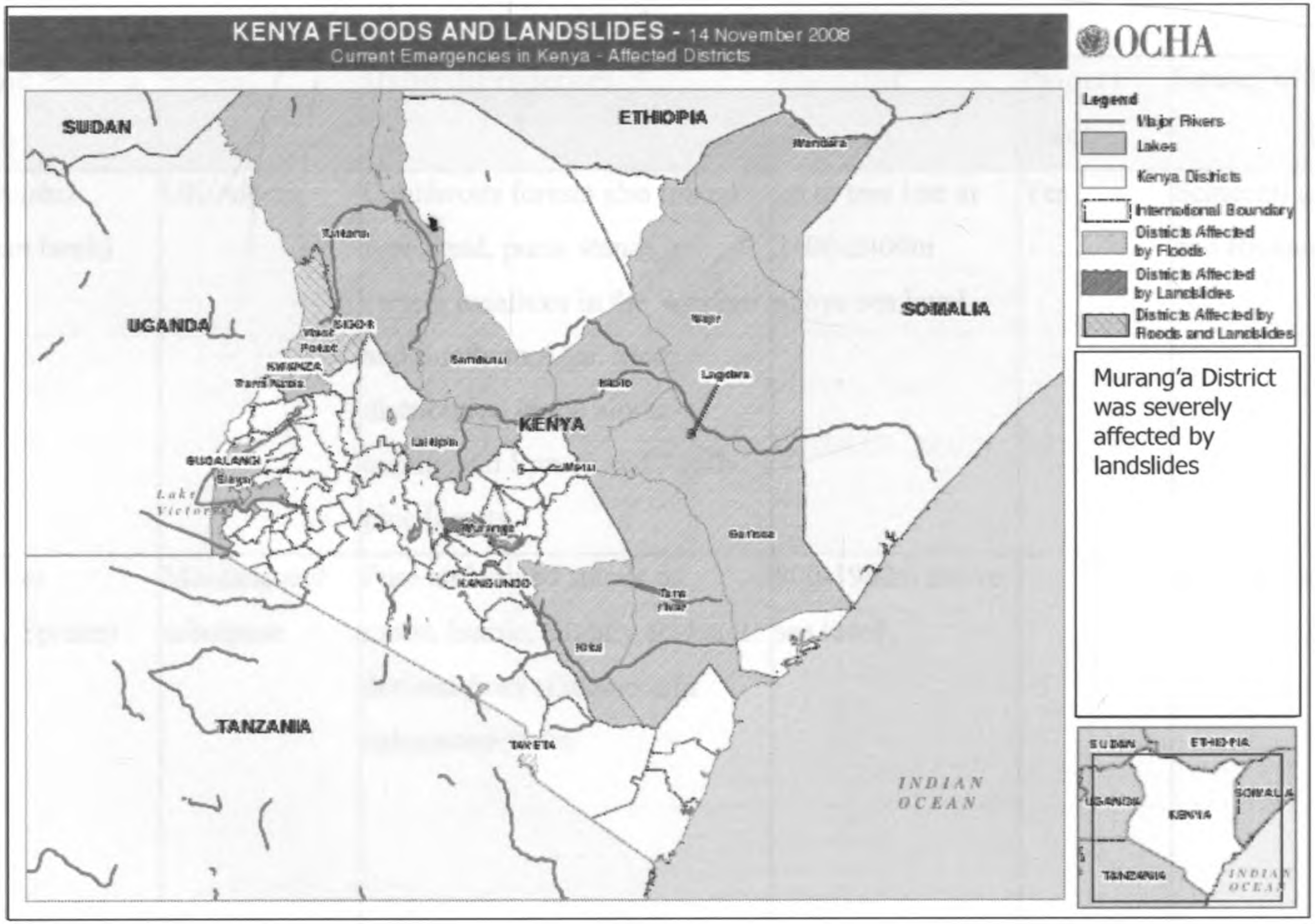
APPENDIX 1

Kenya floods and landslides situation



APPENDIX 2

List of plants suitable for slope stability

1. CONIFERS

Latin and English name	**Region**	**Habitat/Properties**	**Elevation**	**Pioneer plant**	**Source of Reference**
Larix decidua (European larch)	UK/Alpine	Coniferous forests also mixed woodland, purse stands in certain localities in the Western and Southern Alps. Main distribution in the alpine-continental Spruce and Arolla pine forests.	up to tree line at 2100-2400m above sea level	Yes	Schiechtl and Stern (1996); Coppin and Richards (1990)
Picea abies (Norway Spruce)	Montane and subalpine	Pure and mixed stands on moist, humic, slightly acid soil derived from siliceous and calcareous rocks.	800-1900m above sea level		Schiechtl and Stern (1996)

1. CONIFERS – Cont'd

Latin and English name	Region	Habitat/Properties	Elevation	Pioneer plant	Source of Reference
Pinus sylvestris (Scots pine)	UK/Alpine - subalpine	Mixed and multi-species pine forests on base-rich calcareous soil and in acid pine forest on poor acid soil. Drought resistant, frost hardy.	Up to 1900m above sea level	Yes	Schiechtl and Stern (1996)
Pinus uncinata (Mountain pine)	Submontane to subalpine	Montane and subalpine forest on very shallow, stoney calcareous soil. Optimum in Western Alps, Pyrenees. Locally in small patches in montane and subalpine boggy areas.	up to tree line at 2000-2400m above sea level	Yes	Schiechtl and Stern (1996)

2. EXOTIC

Latin and English name	**Region**	**Habitat/Properties**	**Elevation**	**Pioneer plant**	**Source of Reference**
Ailanthus altissima (Tree of heaven)	Lowlands	Foothills	Foothills to 500m		Schiechtl and Stern (1996)
Buddleia alternifolia (Buddleia)	UK	Foothills to montane	up to 800m above sea level		Schiechtl and Stern (1996)
Caragana arborescens (Caragana)	UK	Foothills to montane	up to 1000m above sea level		Schiechtl and Stern (1996)
Elaegnus angustifolia (Elaeagnus)	UK	Foothills	up to 600m above sea level		Schiechtl and Stern (1996)
Forsythia intermedia (Forsythia)	UK	Foothills to montane	up to 1500m above sea level		Schiechtl and Stern (1996)

2. *EXOTIC – Cont'd*

Latin and English name	Region	Habitat/Properties	Elevation	Pioneer plant	Source of Reference
Lycium barbarum (Duke of Argyll's Tea-tree)	UK	Foothills to montane	up to 1200m above sea level		Schiechtl and Stern (1996)
Rhus typhina, *Rhus laciniata* (Sumach)	UK	Foothills to montane	up to 1000m above sea level		Schiechtl and Stern (1996)
Robinia pseudacacia (Robinia)	UK	Foothills to montane. Good pioneer species on poor, sandy soils. N-fixer.	up to 900m above sea level	Yes	Schiechtl and Stern (1996); Coppin and Richards (1990)
Symphoricarpus racemosus (Snowberry)	UK	Foothills to montane	up to 1200m above sea level		Schiechtl and Stern (1996)

3. GRASSES

Latin and English name	Region	Habitat/Properties	Elevation	Pioneer plant	Source of Reference
Agropyron repens (Couch grass)	Alpine	Arable weed, not to be used near arable land, needs light. Dry to moist, fertile soil.	up to 900m above sea level		Schiechtl and Stern (1996)
Agrostis canina (Bent grass)	Alpine	Fens and bogs, peat areas, wet woodland, pioneer plant of open wet soil. Subsp. canina on moist, subsp. montana on dry habitat of poor fertility.	up to 1100m above sea level	Yes	Schiechtl and Stern (1996)

3. *GRASSES – Cont'd*

Latin and English name	Region	Habitat/Properties	Elevation	Pioneer plant	Source of Reference
Agrostis gigantean (Black bent, red top)	Alpine	Riverbanks and lakeshore, wet woodland, unsuitable for lawns	up to 1400m above sea level		Schiechtl and Stern (1996)
Agrostis stolinifera (Creeping bent)	UK/Alpine	Riverbanks, ditches, wet areas, moisture indicator, dense sward, can stand heavy grazing	up to 1800m above sea level	Yes	Coppin and Richards (1990); Scheichtl and Stern (1996)
Agrostis tenuis (Common bent, brown top)	Alpine	Grassy patches in woodland, moist grassland in mountainous regions, moors, recently cut forest areas indicates acid and very poor soil conditions; valuable grass for mountainous terrain.	up to 2200m above sea level		Schiechtl and Stern (1996)

3. GRASSES – Cont'd

Latin and English name	Region	Habitat/Properties	Elevation	Pioneer plant	Source of Reference
Alopecurus pratensis (Foxtail)	Alpine	Riverbanks, alluvial area, resistant to late frosts, overwatering, tolerant to long lasting snow cover, needs fertilizer and irrigation	up to 1800m above sea level		Schiechtl and Stern (1996)
Anthoxantum odoratum (Sweet vernal grass)	Alpine	Meadows and pastures, open wood land, poor mountain meadows	up to 2500m above sea level		Schiechtl and Stern (1996)
Arrhenatherum elatius (Tall oat grass)	UK/Alpine	Moist and fertile meadows, at higher elevations replaced by Trisetum sp., main species of fertilized meadows, avoids wet heavy and compacted soils.	up to 1500m above sea level		Coppin and Richards (1990); Scheichtl and Stern (1996)
Avena sativa (Oats)	Alpine	Best cover crop in humid habitats, frost tender, high water demand, desiccates the soil.	up to 1600m above sea level		Schiechtl and Stern (1996)

3. *GRASSES – Cont'd*

Latin and English name	Region	Habitat/Properties	Elevation	Pioneer plant	Source of Reference
Avenella flexuosa (Oat grass, flexible)	Alpine	Acid humus pioneer on poor soils, shade tolerant.	up to 2200m above sea level		Schiechtl and Stern (1996)
Brachypodium pinnatum (Chalk false brome)	Alpine	Indicates base rich soil and deteriorating woodland conditions, enhanced by burning	up to 1600m above sea level		Schiechtl and Stern (1996)
Bromus erectus (Upright brome)	Alpine	Semi-arid grassland on limestone, in the south on gneiss and serpentine, intolerant of fertiliser and irrigation, avoids shafe and wetness, but very resistane to dry heat, tufted.	up to 1400m above sea level	Yes	Schiechtl and Stern (1996)
Bromus inermis (Hungarian brome)	Alpine	Very drought and cold resistant	up to 1200m above sea level	Yes	Schiechtl and Stern (1996)

3. *GRASSES – Cont'd*

Latin and English name	Region	Habitat/Properties	Elevation	Pioneer plant	Source of Reference
Bromus mollis (Soft brome)	Alpine	Grassland weed, indicates poor soil, suitable cover crop on dry sites.	up to 1000m above sea level		Schiechtl and Stern (1996)
Cynodon dactylon (Couch grass)	Alpine	Pioneer for the quick stabilisation of sandy soil in low altitudes, starts late in the season, frost hardy, pasture grass, turns brown in winter.	up to 1000m above sea level	Yes	Schiechtl and Stern (1996)
Cynosurus cristatus (Crested dog's tail)	Alpine	Fertilized permanent pasture, indicates heavy soil, frost tender, shade tolerant for pastures and meadows.	up to 1500m above sea level		Schiechtl and Stern (1996)

3. GRASSES – Cont'd

Latin and English name	Region	Habitat/Properties	Elevation	Pioneer plant	Source of Reference
Dactylis glomerata (Cocksfoot)	Alpine	All round pioneer, vigorous when fertilized.	up to 1900m above sea level	Yes	Schiechtl and Stern (1996)
Deschampsia caespitosa (Tufted hair grass)	Alpine	Intermittent moist locations in woodland and meadows, wet places and spring horizons, marshes	up to 2800m above sea level		Schiechtl and Stern (1996)
Festuca arundinacea (Tall fescue)	Alpine	Indicates soil compaction and wetness, tufted, tolerant of trampling, suitable for pathways and terraces in vineyards and orchards.			Schiechtl and Stern (1996)

3. GRASSES – Cont'd

Latin and English name	Region	Habitat/Properties	Elevation	Pioneer plant	Source of Reference
Festuca ovina (Sheep's fescue)	Alpine	Dry poor grassland on soils derived from acid rocks, indicates degradation in forests.	up to 2300m above sea level		Schiechtl and Stern (1996)
Festuca rubra rubra (Creeping red fescue)	UK/Alpine	Montane meadows and pastures, coniferous wood and deciduous forests, drought and wetness sensitive.	up to 2000m above sea level		Coppin and Richards (1990); Scheichtl and Stern (1996)
Festuca rubra subsp. Commutata (Red fescue)	Alpine	On acid soils, replaced by Nardus stricta if over used.	up to 2000m above sea level		Schiechtl and Stern (1996)
Festuca tenuifolia syn. Capillata (Fine-leaved sheep's fescue)	Alpine	On acid soils, indicates deterioration in forests.	up to 1000m above sea level		Schiechtl and Stern (1996)

3. GRASSES – Cont'd

Latin and English name	Region	Habitat/Properties	Elevation	Pioneer plant	Source of Reference
Festuca trachyphyylla syn. Longfolia (Rough leaved fescue)	UK/Alpine	Origin in the low lying areas of northern Germany on sandy soils, widespread in Central Europe and England.	up to 1000m above sea level		Schiechtl and Stern (1996)
Holcus lanatus (Yorkshire fog)	Alpine	Indicates acid soils low in nitrogen, frost tender, green during winter, in years of good rainfall very prolific on poor soils.	up to 900m above sea level		Schiechtl and Stern (1996)
Holcus mollis (Creeping soft grass)	Alpine	Arable lands and ploughed out pasture, wet soil areas, never on calcareous soil, sandy soils, troublesome weed in gardens and arable land.	up to 1500m above sea level		Schiechtl and Stern (1996)

3. GRASSES – Cont'd

Latin and English name	Region	Habitat/Properties	Elevation	Pioneer plant	Source of Reference
Lolium multiflorum subsp. italicum (Italian ryegrass)	Alpine	Only in Atlantic areas under moist mild climatic conditions, frost damage below -5oC, needs potash-rich soils, vigourous after cuts, unsuitable for permanent meadows.	up to 1700m above sea level		Schiechtl and Stern (1996)
Lolium perenne (Perennial ryegrass)	UK/Alpine	Tolerant to repeated cutting and trampling, pioneer of well-aerated and moist soil, needs fertiliser, fast growing.	up to 1000m above sea level	Yes	Coppin and Richards (1990); Scheichtl and Stern (1996)
Phleum pratense (Timothy)	Alpine	Pastures, tolerates cold climate, wetness and prolonged snow cover, wind; grazing increases vigour and yield.	up to 2600m above sea level		Schiechtl and Stern (1996)

3. GRASSES – Cont'd

Latin and English name	Region	Habitat/Properties	Elevation	Pioneer plant	Source of Reference
Poa annua (Annual meadow grass)	UK/Alpine	Tolerates trampling and heavy fertiliser applications.	up to 3000m above sea level		Coppin and Richards (1990); Scheichtl and Stern (1996)
Poa compressa (Flat-stalked meadow grass)	UK/Alpine	Not on soils derived from acid rocks, green over winter.	up to 1800m above sea level		Coppin and Richards (1990); Scheichtl and Stern (1996)
Poa nemoralis (Wood meadow grass)	Alpine	On heavy soils, grows early after melting smow, very shade tolerant, not to be planted in pure stands, does not form close sward.	up to 2300m above sea level		Schiechtl and Stern (1996)
Poa palustris (Swamp meadow grass)	Alpine	On riverbanks, early, resistant to late frost.	up to 1500m above sea level		Schiechtl and Stern (1996)

3. GRASSES – Cont'd

Latin and English name	Region	Habitat/Properties	Elevation	Pioneer plant	Source of Reference
Poa pratensis (Smooth meadow grass)	UK/Alpine	Important constituent of meadows and pastures, wide ecological amplitude, hardy, long living, grows early in spring, very suitable for first seedings.	up to 2300m above sea level		Coppin and Richards (1990); Scheichtl and Stern (1996)
Puccinella distans (Sea meadow grass)	Alpine	Saline soils, manure heaps, cattle pens, solonetzic soil.			Schiechtl and Stern (1996)
Trisetum flavescens (Yellow oat)	Alpine	Fertile meadows in montane and subalpine regions, frost sensitive, resistant to repeated cutting.	up to 2300m above sea level		Schiechtl and Stern (1996)

4. HERBS AND LEGUMES

Latin and English name	Region	habitat and properties	Elevation	Pioneer plant	Source of Reference
Achillea millefolium (Yarrow, milfoil)	UK/ Alpine	Meadows and pastures, drought resistant, indicator of fertile soil. Salt tolerant.	up to 1900m above sea level		Schiecthl and Stern (1996); Coppin and Richards (1990)
Chrysanthemum leucanthemeum (Ox-eye, dog daisy)	UK/Alpine	Pioneer on loose, well-aerated, immature soils, indicates poor fertility in meadows. Tall growth.	up to 2200m above sea level	Yes	Schiecthl and Stern (1996); Coppin and Richards (1990)
Pimpinella saxifraga (Burnet saxifrage)	UK/Alpine	Indicates poor, dry soil.	up to 2300m above sea level		Schiecthl and Stern (1996)

4. HERBS AND LEGUMES – Cont'd

Latin and English name	Region	habitat and properties	Elevation	Pioneer plant	Source of Reference
Plantago lanceolata (Ribwort plantain)	Worldwide	Many soil types throughout the world. Short growth.	up to 1800m above sea level		Schiecthl and Stern (1996); Coppin and Richards (1990)
Sanguisorba minor (Salad burnet)	UK/Alpine	Tall growth; prefers lime soils but tolerates infertile soils. Grasslands.	up to 1200m above sea level	Yes	Schiecthl and Stern (1996); Coppin and Richards (1990)
Anthyllis vulneraria (Kidney vetch)	Alpine	Frost and drought resistant.	up to 2000m above sea level	Yes	Schiecthl and Stern (1996)
Coronilla varia (Crown vetch)	UK/Alpine	Dry, sunny slopes. Wide tolerance, dense growth, slows to establish particularly in the north.	up to 900m above sea level		Schiecthl and Stern (1996); Coppin and Richards (1990)

4. HERBS AND LEGUMES – Cont'd

Latin and English name	Region	habitat and properties	Elevation	Pioneer plant	Source of Reference
Lotus corniculatus (Birds-foot tre foil)	UK	Wide soil tolerance. Salt tolerant.			Coppin and Richards (1990)
Lotus uliginosus (Large bird's trefoil)	Alpine	Semi-dry turf, fertile meadows and pastures, prefers calcareous soils, high temperature resistance	up to 2300m above sea level		Schiecthl and Stern (1996)
Lupinus albus (White lupin)	Alpine		up to 600m above sea level		Schiecthl and Stern (1996)
Lupinus luteus (Sweet lupin)	Alpine		up to 1400m above sea level		Schiecthl and Stern (1996)
Lupinus polyphyllus (Garden lupin)	UK/ Alpine	On wood land fringes and clearings. Wide soil tolerance will die out with several hard winters.	up to 1400m above sea level		Schiecthl and Stern (1996); Coppin and Richards (1990)

4. HERBS AND LEGUMES – Cont'd

Latin and English name	Region	habitat and properties	Elevation	Pioneer plant	Source of Reference
Medicago falcata (Sickle medick)	Alpine	Not cultivated as it becomes woody.	up to 1100m above sea level		Schiecthl and Stern (1996)
Medicago lupulina (Black medick)	Alpine	Dry meadows of good fertility, indicator of dry habitat, undemanding pioneer, prefers calcareous soil, frost resistant, can be heavily grazed.	up to 1500m above sea level	Yes	Schiecthl and Stern (1996)
Medicago sativa (Lucerne)	UK/Alpine	Drought tolerant, neutral/alkaline soils. Sensitive to late frosts.	up to 1000m above sea level		Schiecthl and Stern (1996); Coppin and Richards (1990)
Melilotus albus (White melilot)	Alpine	Drought resistant becomes woody, needs mowing.	up to 1800m above sea level		Schiecthl and Stern (1996)

4. HERBS AND LEGUMES – Cont'd

Latin and English name	Region	habitat and properties	Elevation	Pioneer plant	Source of Reference
Melilotus officinalis (Common melilot)	Alpine	Medicinal.	up to 1000m above sea level		Schiecthl and Stern (1996)
Onobrychis viciifolia (Common sanfoin)	UK/Alpine	Dry soil indicator in *Arrhenatheretum*, important feed plant on dry, clayey calcareous soil, sensitive to grazing. Neutral/alkaline soils, drought tolerant.	up to 2000m above sea level		Schiecthl and Stern (1996); Coppin and Richards (1990)
Phacelia tanacetifolia	Alpine		up to 1000m above sea level		Schiecthl and Stern (1996)
Pisum sativum (Garden pea)	UK/Alpine		up to 1000m above sea level		Schiecthl and Stern (1996)

4. HERBS AND LEGUMES – Cont'd

Latin and English name	Region	habitat and properties	Elevation	Pioneer plant	Source of Reference
Trifolium dubium (Lesser clover)	UK/Alpine	Fertile meadows and pastures, needs heavy nitrogen dressings.	up to 1000m above sea level		Schiecthl and Stern (1996)
Trifolium hybridum (Alsike clover)	UK/Alpine	Pioneer on moraines, tolerates moist and cool conditions, frost resistant, and tolerates prolonged snow cover, sensitive to shade and dry soil conditions. Tolerates waterlogging.	up to 2000m above sea level	Yes	Schiecthl and Stern (1996); Coppin and Richards (1990)
Trifolium pratense (Red clover)	UK/Alpine	Fertile meadows and pastures, but also moist and poor meadows, sensitive in spring to grazing pressure, important feed plant.	up to 2200m above sea level		Schiecthl and Stern (1996)

4. HERBS AND LEGUMES – Cont'd

Latin and English name	Region	habitat and properties	Elevation	Pioneer plant	Source of Reference
Trifolium repens (White clover)	UK/Alpine	Heavily used turf, meadows, parks, aerodromes in humid areas, very prolific. Requires moderate fertility.	up to 2300m above sea level		Schiecthl and Stern (1996); Coppin and Richards (1990)
Vicia sativa (Common vetch)	UK/Alpine	Valuable cover crop.	up to 1600m above sea level		Schiecthl and Stern (1996)
Vicia villosa (Fodder vetch)	UK/Alpine	Cover crop. Frost hardy if sown early.	Up to 1700m above sea level		Schiecthl and Stern (1996)

APPENDIX 3

Design tables for shrubs t_R values in kPa

Diameter	Number of roots crossing the shear plane											
mm	1	2	3	4	5	6	7	8	9	10	20	25	30
1	1.7E-03	1.7E-03	1.7E-03	1.7E-03	1.7E-03	1.5E-03	1.5E-03	1.5E-03	1.6E-03	1.4E-03	2.5E-03	2.9E-03	3.2E-03
2	0.0062	0.0062	0.0062	0.0062	0.0062	0.0053	0.0054	0.0055	0.0056	0.0052	0.0089	0.0103	0.0116
3	0.0131	0.0131	0.0131	0.0131	0.0131	0.0112	0.0115	0.0117	0.0118	0.0109	0.0187	0.0219	0.0246
4	0.0223	0.0223	0.0223	0.0223	0.0223	0.0191	0.0195	0.0198	0.0201	0.0186	0.0319	0.0372	0.0418
5	0.0337	0.0337	0.0337	0.0337	0.0337	0.0289	0.0295	0.0299	0.0303	0.0280	0.0481	0.0561	0.0631
6	0.0471	0.0471	0.0471	0.0471	0.0471	0.0404	0.0412	0.0419	0.0424	0.0393	0.0673	0.0785	0.0883
7	0.0626	0.0626	0.0626	0.0626	0.0626	0.0537	0.0548	0.0557	0.0564	0.0522	0.0895	0.1044	0.1174
8	0.0801	0.0801	0.0801	0.0801	0.0801	0.0687	0.0701	0.0712	0.0721	0.0668	0.1144	0.1335	0.1502
9	0.0996	0.0996	0.0996	0.0996	0.0996	0.0853	0.0871	0.0885	0.0896	0.0830	0.1422	0.1659	0.1867
10	0.1209	0.1209	0.1209	0.1209	0.1209	0.1036	0.1058	0.1075	0.1088	0.1008	0.1727	0.2015	0.2267
11	0.1442	0.1442	0.1442	0.1442	0.1442	0.1236	0.1262	0.1282	0.1298	0.1201	0.2060	0.2403	0.2703
12	0.1693	0.1693	0.1693	0.1693	0.1693	0.1451	0.1481	0.1505	0.1524	0.1411	0.2418	0.2821	0.3174
13	0.1962	0.1962	0.1962	0.1962	0.1962	0.1682	0.1717	0.1744	0.1766	0.1635	0.2803	0.3270	0.3679
14	0.2250	0.2250	0.2250	0.2250	0.2250	0.1928	0.1968	0.2000	0.2025	0.1875	0.3214	0.3749	0.4218
15	0.2555	0.2555	0.2555	0.2555	0.2555	0.2190	0.2236	0.2271	0.2300	0.2129	0.3650	0.4258	0.4791
16	0.2878	0.2878	0.2878	0.2878	0.2878	0.2467	0.2518	0.2558	0.2590	0.2398	0.4112	0.4797	0.5397
17	0.3219	0.3219	0.3219	0.3219	0.3219	0.2759	0.2816	0.2861	0.2897	0.2682	0.4598	0.5365	0.6035
18	0.3577	0.3577	0.3577	0.3577	0.3577	0.3066	0.3130	0.3179	0.3219	0.2981	0.5110	0.5961	0.6706
19	0.3952	0.3952	0.3952	0.3952	0.3952	0.3387	0.3458	0.3513	0.3557	0.3293	0.5646	0.6587	0.7410
20	0.4344	0.4344	0.4344	0.4344	0.4344	0.3724	0.3801	0.3862	0.3910	0.3620	0.6206	0.7240	0.8145
21	0.4753	0.4753	0.4753	0.4753	0.4753	0.4074	0.4159	0.4225	0.4278	0.3961	0.6791	0.7922	0.8913

	1	2	3	4	5	6	7	8	9	10	20	25	30
22	0.5179	0.5179	0.5179	0.5179	0.5179	0.4440	0.4532	0.4604	0.4661	0.4316	0.7399	0.8632	0.9711
23	0.5622	0.5622	0.5622	0.5622	0.5622	0.4819	0.4919	0.4997	0.5060	0.4685	0.8032	0.9370	1.0541
24	0.6081	0.6081	0.6081	0.6081	0.6081	0.5213	0.5321	0.5406	0.5473	0.5068	0.8688	1.0136	1.1403
25	0.6557	0.6557	0.6557	0.6557	0.6557	0.5620	0.5737	0.5829	0.5901	0.5464	0.9367	1.0929	1.2295
26	0.7049	0.7049	0.7049	0.7049	0.7049	0.6042	0.6168	0.6266	0.6344	0.5874	1.0070	1.1749	1.3217
27	0.7558	0.7558	0.7558	0.7558	0.7558	0.6478	0.6613	0.6718	0.6802	0.6298	1.0796	1.2596	1.4170
28	0.8082	0.8082	0.8082	0.8082	0.8082	0.6927	0.7072	0.7184	0.7274	0.6735	1.1546	1.3470	1.5154
29	0.8623	0.8623	0.8623	0.8623	0.8623	0.7391	0.7545	0.7665	0.7760	0.7185	1.2318	1.4371	1.6167
30	0.9179	0.9179	0.9179	0.9179	0.9179	0.7868	0.8032	0.8159	0.8261	0.7649	1.3113	1.5299	1.7211
31	0.9752	0.9752	0.9752	0.9752	0.9752	0.8358	0.8533	0.8668	0.8776	0.8126	1.3931	1.6253	1.8284
32	1.0340	1.0340	1.0340	1.0340	1.0340	0.8863	0.9047	0.9191	0.9306	0.8617	1.4771	1.7233	1.9387
33	1.0944	1.0944	1.0944	1.0944	1.0944	0.9380	0.9576	0.9728	0.9849	0.9120	1.5634	1.8240	2.0520
34	1.1564	1.1564	1.1564	1.1564	1.1564	0.9912	1.0118	1.0279	1.0407	0.9636	1.6519	1.9273	2.1682
35	1.2199	1.2199	1.2199	1.2199	1.2199	1.0456	1.0674	1.0843	1.0979	1.0166	1.7427	2.0331	2.2873
36	1.2850	1.2850	1.2850	1.2850	1.2850	1.1014	1.1243	1.1422	1.1565	1.0708	1.8357	2.1416	2.4093
37	1.3516	1.3516	1.3516	1.3516	1.3516	1.1585	1.1826	1.2014	1.2164	1.1263	1.9308	2.2526	2.5342
38	1.4198	1.4198	1.4198	1.4198	1.4198	1.2169	1.2423	1.2620	1.2778	1.1831	2.0282	2.3663	2.6620
39	1.4895	1.4895	1.4895	1.4895	1.4895	1.2767	1.3033	1.3240	1.3405	1.2412	2.1278	2.4824	2.7927
40	1.5607	1.5607	1.5607	1.5607	1.5607	1.3377	1.3656	1.3873	1.4046	1.3006	2.2295	2.6011	2.9263
41	1.6334	1.6334	1.6334	1.6334	1.6334	1.4001	1.4292	1.4519	1.4701	1.3612	2.3335	2.7224	3.0627
42	1.7077	1.7077	1.7077	1.7077	1.7077	1.4637	1.4942	1.5179	1.5369	1.4231	2.4395	2.8461	3.2019
43	1.7835	1.7835	1.7835	1.7835	1.7835	1.5287	1.5605	1.5853	1.6051	1.4862	2.5478	2.9724	3.3440
44	1.8607	1.8607	1.8607	1.8607	1.8607	1.5949	1.6281	1.6540	1.6747	1.5506	2.6582	3.1012	3.4889
45	1.9395	1.9395	1.9395	1.9395	1.9395	1.6624	1.6971	1.7240	1.7455	1.6162	2.7707	3.2325	3.6366
46	2.0198	2.0198	2.0198	2.0198	2.0198	1.7312	1.7673	1.7953	1.8178	1.6831	2.8854	3.3663	3.7871

	1	2	3	4	5	6	7	8	9	10	20	25	30
47	2.1015	2.1015	2.1015	2.1015	2.1015	1.8013	1.8388	1.8680	1.8914	1.7513	3.0022	3.5025	3.9403
48	2.1848	2.1848	2.1848	2.1848	2.1848	1.8726	1.9117	1.9420	1.9663	1.8206	3.1211	3.6413	4.0964
49	2.2695	2.2695	2.2695	2.2695	2.2695	1.9453	1.9858	2.0173	2.0425	1.8912	3.2421	3.7824	4.2553
50	2.3557	2.3557	2.3557	2.3557	2.3557	2.0191	2.0612	2.0939	2.1201	1.9630	3.3652	3.9261	4.4169
51	2.4433	2.4433	2.4433	2.4433	2.4433	2.0943	2.1379	2.1718	2.1990	2.0361	3.4905	4.0722	4.5812
52	2.5324	2.5324	2.5324	2.5324	2.5324	2.1707	2.2159	2.2511	2.2792	2.1104	3.6178	4.2207	4.7483
53	2.6230	2.6230	2.6230	2.6230	2.6230	2.2483	2.2951	2.3316	2.3607	2.1858	3.7472	4.3717	4.9182
54	2.7151	2.7151	2.7151	2.7151	2.7151	2.3272	2.3757	2.4134	2.4436	2.2625	3.8787	4.5251	5.0907
55	2.8085	2.8085	2.8085	2.8085	2.8085	2.4073	2.4575	2.4965	2.5277	2.3405	4.0122	4.6809	5.2660
56	2.9035	2.9035	2.9035	2.9035	2.9035	2.4887	2.5405	2.5809	2.6131	2.4196	4.1478	4.8391	5.4440
57	2.9999	2.9999	2.9999	2.9999	2.9999	2.5713	2.6249	2.6665	2.6999	2.4999	4.2855	4.9998	5.6247
58	3.0977	3.0977	3.0977	3.0977	3.0977	2.6552	2.7105	2.7535	2.7879	2.5814	4.4253	5.1628	5.8082
59	3.1969	3.1969	3.1969	3.1969	3.1969	2.7402	2.7973	2.8417	2.8772	2.6641	4.5671	5.3282	5.9943
60	3.2976	3.2976	3.2976	3.2976	3.2976	2.8265	2.8854	2.9312	2.9679	2.7480	4.7109	5.4960	6.1831
61	3.3997	3.3997	3.3997	3.3997	3.3997	2.9141	2.9748	3.0220	3.0598	2.8331	4.8568	5.6662	6.3745
62	3.5033	3.5033	3.5033	3.5033	3.5033	3.0028	3.0654	3.1140	3.1530	2.9194	5.0047	5.8388	6.5687
63	3.6082	3.6082	3.6082	3.6082	3.6082	3.0928	3.1572	3.2073	3.2474	3.0069	5.1546	6.0137	6.7655
64	3.7146	3.7146	3.7146	3.7146	3.7146	3.1840	3.2503	3.3019	3.3432	3.0955	5.3066	6.1910	6.9649
65	3.8224	3.8224	3.8224	3.8224	3.8224	3.2764	3.3446	3.3977	3.4402	3.1853	5.4606	6.3707	7.1670
66	3.9316	3.9316	3.9316	3.9316	3.9316	3.3700	3.4402	3.4948	3.5385	3.2763	5.6166	6.5527	7.3718
67	4.0422	4.0422	4.0422	4.0422	4.0422	3.4648	3.5369	3.5931	3.6380	3.3685	5.7746	6.7370	7.5792
68	4.1542	4.1542	4.1542	4.1542	4.1542	3.5608	3.6350	3.6927	3.7388	3.4619	5.9346	6.9237	7.7892
69	4.2677	4.2677	4.2677	4.2677	4.2677	3.6580	3.7342	3.7935	3.8409	3.5564	6.0966	7.1128	8.0019
70	4.3825	4.3825	4.3825	4.3825	4.3825	3.7564	3.8347	3.8955	3.9442	3.6521	6.2607	7.3041	8.2171
71	4.4987	4.4987	4.4987	4.4987	4.4987	3.8560	3.9363	3.9988	4.0488	3.7489	6.4267	7.4978	8.4350

	1	2	3	4	5	6	7	8	9	10	20	25	30
72	4.6163	4.6163	4.6163	4.6163	4.6163	3.9568	4.0392	4.1033	4.1546	3.8469	6.5947	7.6938	8.6555
73	4.7353	4.7353	4.7353	4.7353	4.7353	4.0588	4.1433	4.2091	4.2617	3.9460	6.7646	7.8921	8.8786
74	4.8556	4.8556	4.8556	4.8556	4.8556	4.1620	4.2487	4.3161	4.3701	4.0464	6.9366	8.0927	9.1043
75	4.9774	4.9774	4.9774	4.9774	4.9774	4.2663	4.3552	4.4243	4.4796	4.1478	7.1105	8.2956	9.3326
76	5.1005	5.1005	5.1005	5.1005	5.1005	4.3719	4.4629	4.5338	4.5905	4.2504	7.2864	8.5009	9.5635
77	5.2250	5.2250	5.2250	5.2250	5.2250	4.4786	4.5719	4.6445	4.7025	4.3542	7.4643	8.7084	9.7969
78	5.3509	5.3509	5.3509	5.3509	5.3509	4.5865	4.6820	4.7564	4.8158	4.4591	7.6441	8.9182	10.0329
79	5.4782	5.4782	5.4782	5.4782	5.4782	4.6956	4.7934	4.8695	4.9303	4.5651	7.8259	9.1303	10.2715
80	5.6068	5.6068	5.6068	5.6068	5.6068	4.8058	4.9059	4.9838	5.0461	4.6723	8.0097	9.3446	10.5127
81	5.7368	5.7368	5.7368	5.7368	5.7368	4.9172	5.0197	5.0994	5.1631	4.7806	8.1954	9.5613	10.7564
82	5.8681	5.8681	5.8681	5.8681	5.8681	5.0298	5.1346	5.2161	5.2813	4.8901	8.3830	9.7802	11.0027
83	6.0008	6.0008	6.0008	6.0008	6.0008	5.1436	5.2507	5.3341	5.4008	5.0007	8.5726	10.0014	11.2516
84	6.1349	6.1349	6.1349	6.1349	6.1349	5.2585	5.3680	5.4532	5.5214	5.1124	8.7642	10.2248	11.5029
85	6.2703	6.2703	6.2703	6.2703	6.2703	5.3746	5.4865	5.5736	5.6433	5.2253	8.9576	10.4506	11.7569
86	6.4071	6.4071	6.4071	6.4071	6.4071	5.4918	5.6062	5.6952	5.7664	5.3393	9.1530	10.6785	12.0133
87	6.5452	6.5452	6.5452	6.5452	6.5452	5.6102	5.7271	5.8180	5.8907	5.4544	9.3503	10.9087	12.2723
88	6.6847	6.6847	6.6847	6.6847	6.6847	5.7298	5.8491	5.9420	6.0162	5.5706	9.5496	11.1412	12.5338
89	6.8255	6.8255	6.8255	6.8255	6.8255	5.8505	5.9723	6.0671	6.1430	5.6880	9.7508	11.3759	12.7979
90	6.9677	6.9677	6.9677	6.9677	6.9677	5.9723	6.0967	6.1935	6.2709	5.8064	9.9539	11.6128	13.0645
91	7.1112	7.1112	7.1112	7.1112	7.1112	6.0953	6.2223	6.3211	6.4001	5.9260	10.1589	11.8520	13.3335
92	7.2561	7.2561	7.2561	7.2561	7.2561	6.2195	6.3491	6.4498	6.5305	6.0467	10.3658	12.0934	13.6051
93	7.4022	7.4022	7.4022	7.4022	7.4022	6.3448	6.4770	6.5798	6.6620	6.1685	10.5746	12.3371	13.8792
94	7.5498	7.5498	7.5498	7.5498	7.5498	6.4712	6.6060	6.7109	6.7948	6.2915	10.7854	12.5829	14.1558
95	7.6986	7.6986	7.6986	7.6986	7.6986	6.5988	6.7363	6.8432	6.9288	6.4155	10.9980	12.8310	14.4349
96	7.8488	7.8488	7.8488	7.8488	7.8488	6.7275	6.8677	6.9767	7.0639	6.5407	11.2126	13.0813	14.7165

	1	2	3	4	5	6	7	8	9	10	20	25	30
97	8.0003	8.0003	8.0003	8.0003	8.0003	6.8574	7.0003	7.1114	7.2003	6.6669	11.4290	13.3338	15.0006
98	8.1531	8.1531	8.1531	8.1531	8.1531	6.9884	7.1340	7.2472	7.3378	6.7943	11.6473	13.5886	15.2871
99	8.3073	8.3073	8.3073	8.3073	8.3073	7.1205	7.2689	7.3843	7.4766	6.9227	11.8676	13.8455	15.5762
100	8.4628	8.4628	8.4628	8.4628	8.4628	7.2538	7.4049	7.5225	7.6165	7.0523	12.0897	14.1046	15.8677

APPENDIX 4

Design tables for determining the K values

θ (degrees)	ϕ (degrees) 1	2	3	4	5	6	7	8	9	10
1	0.0349	0.0523	0.0698	0.0872	0.1045	0.1219	0.1392	0.1565	0.1737	0.1908
2	0.0524	0.0698	0.0872	0.1046	0.1219	0.1393	0.1565	0.1738	0.1909	0.2080
3	0.0699	0.0873	0.1047	0.1220	0.1394	0.1566	0.1739	0.1911	0.2082	0.2253
4	0.0874	0.1048	0.1222	0.1395	0.1568	0.1741	0.1913	0.2084	0.2255	0.2425
5	0.1049	0.1223	0.1397	0.1570	0.1743	0.1915	0.2087	0.2258	0.2428	0.2598
6	0.1225	0.1399	0.1573	0.1746	0.1919	0.2091	0.2262	0.2433	0.2602	0.2772
7	0.1402	0.1576	0.1750	0.1922	0.2095	0.2266	0.2437	0.2608	0.2777	0.2946
8	0.1580	0.1754	0.1927	0.2100	0.2272	0.2443	0.2614	0.2783	0.2952	0.3121
9	0.1758	0.1932	0.2105	0.2278	0.2449	0.2620	0.2791	0.2960	0.3129	0.3296
10	0.1938	0.2111	0.2284	0.2457	0.2628	0.2799	0.2969	0.3138	0.3306	0.3473
11	0.2118	0.2292	0.2464	0.2637	0.2808	0.2978	0.3148	0.3317	0.3484	0.3651
12	0.2300	0.2473	0.2646	0.2818	0.2989	0.3159	0.3328	0.3497	0.3664	0.3830
13	0.2483	0.2656	0.2829	0.3001	0.3171	0.3341	0.3510	0.3678	0.3845	0.4010
14	0.2667	0.2841	0.3013	0.3185	0.3355	0.3525	0.3693	0.3861	0.4027	0.4192
15	0.2854	0.3027	0.3199	0.3371	0.3541	0.3710	0.3878	0.4045	0.4211	0.4375
16	0.3042	0.3215	0.3387	0.3558	0.3728	0.3897	0.4065	0.4231	0.4396	0.4560
17	0.3231	0.3404	0.3576	0.3747	0.3917	0.4086	0.4253	0.4419	0.4584	0.4747
18	0.3423	0.3596	0.3768	0.3939	0.4108	0.4277	0.4444	0.4609	0.4774	0.4936
19	0.3617	0.3790	0.3962	0.4132	0.4302	0.4470	0.4636	0.4801	0.4965	0.5127
20	0.3814	0.3986	0.4158	0.4328	0.4497	0.4665	0.4831	0.4996	0.5159	0.5321
21	0.4013	0.4185	0.4357	0.4527	0.4696	0.4863	0.5029	0.5193	0.5356	0.5517
22	0.4214	0.4387	0.4558	0.4728	0.4896	0.5063	0.5229	0.5393	0.5555	0.5715

	1	2	3	4	5	6	7	8	9	10
23	0.4419	0.4591	0.4762	0.4932	0.5100	0.5267	0.5432	0.5595	0.5757	0.5917
24	0.4626	0.4799	0.4970	0.5139	0.5307	0.5473	0.5638	0.5801	0.5962	0.6121
25	0.4837	0.5009	0.5180	0.5349	0.5517	0.5683	0.5847	0.6009	0.6170	0.6329
26	0.5051	0.5223	0.5394	0.5563	0.5730	0.5896	0.6060	0.6222	0.6382	0.6540
27	0.5269	0.5441	0.5612	0.5780	0.5947	0.6113	0.6276	0.6437	0.6597	0.6754
28	0.5491	0.5663	0.5833	0.6002	0.6168	0.6333	0.6496	0.6657	0.6816	0.6973
29	0.5717	0.5889	0.6059	0.6227	0.6394	0.6558	0.6720	0.6881	0.7039	0.7195
30	0.5947	0.6119	0.6289	0.6457	0.6623	0.6787	0.6949	0.7109	0.7267	0.7422
31	0.6182	0.6354	0.6524	0.6692	0.6857	0.7021	0.7183	0.7342	0.7499	0.7654
32	0.6422	0.6594	0.6763	0.6931	0.7096	0.7260	0.7421	0.7580	0.7736	0.7890
33	0.6668	0.6839	0.7009	0.7176	0.7341	0.7504	0.7664	0.7823	0.7978	0.8132
34	0.6919	0.7090	0.7259	0.7426	0.7591	0.7753	0.7914	0.8071	0.8226	0.8379
35	0.7176	0.7347	0.7516	0.7683	0.7847	0.8009	0.8169	0.8326	0.8480	0.8632
36	0.7439	0.7610	0.7779	0.7945	0.8109	0.8271	0.8430	0.8586	0.8740	0.8892
37	0.7709	0.7880	0.8049	0.8215	0.8378	0.8540	0.8698	0.8854	0.9007	0.9158
38	0.7986	0.8157	0.8326	0.8491	0.8655	0.8815	0.8973	0.9129	0.9281	0.9431
39	0.8271	0.8442	0.8610	0.8776	0.8939	0.9099	0.9256	0.9411	0.9562	0.9711
40	0.8564	0.8735	0.8903	0.9068	0.9231	0.9390	0.9547	0.9701	0.9852	
41	0.8866	0.9037	0.9204	0.9369	0.9531	0.9691	0.9847			
42	0.9177	0.9348	0.9515	0.9680	0.9841					
43	0.9498	0.9668	0.9836							
44	0.9830									

	11	12	13	14	15	16	17	18	19	20
1	0.2079	0.2250	0.2420	0.2589	0.2757	0.2924	0.3091	0.3256	0.3421	0.3584
2	0.2251	0.2421	0.2590	0.2758	0.2925	0.3092	0.3258	0.3422	0.3586	0.3748
3	0.2423	0.2592	0.2760	0.2928	0.3094	0.3260	0.3425	0.3589	0.3751	0.3913
4	0.2595	0.2763	0.2931	0.3098	0.3264	0.3429	0.3592	0.3755	0.3917	0.4077
5	0.2767	0.2935	0.3102	0.3268	0.3433	0.3597	0.3760	0.3922	0.4083	0.4242
6	0.2940	0.3107	0.3274	0.3439	0.3603	0.3767	0.3929	0.4090	0.4249	0.4408
7	0.3113	0.3280	0.3446	0.3611	0.3774	0.3937	0.4098	0.4258	0.4417	0.4574
8	0.3288	0.3454	0.3619	0.3783	0.3946	0.4107	0.4268	0.4427	0.4585	0.4741
9	0.3463	0.3628	0.3793	0.3956	0.4118	0.4279	0.4438	0.4596	0.4753	0.4909
10	0.3639	0.3804	0.3968	0.4130	0.4291	0.4451	0.4610	0.4767	0.4923	0.5077
11	0.3816	0.3980	0.4143	0.4305	0.4466	0.4625	0.4783	0.4939	0.5094	0.5247
12	0.3995	0.4158	0.4321	0.4482	0.4641	0.4800	0.4956	0.5112	0.5265	0.5418
13	0.4174	0.4337	0.4499	0.4659	0.4818	0.4976	0.5132	0.5286	0.5439	0.5590
14	0.4356	0.4518	0.4679	0.4838	0.4997	0.5153	0.5308	0.5461	0.5613	0.5763
15	0.4538	0.4700	0.4860	0.5019	0.5176	0.5332	0.5486	0.5639	0.5789	0.5938
16	0.4723	0.4884	0.5043	0.5201	0.5358	0.5513	0.5666	0.5817	0.5967	0.6115
17	0.4909	0.5070	0.5228	0.5386	0.5541	0.5695	0.5847	0.5998	0.6146	0.6293
18	0.5098	0.5257	0.5415	0.5572	0.5727	0.5880	0.6031	0.6180	0.6328	0.6473
19	0.5288	0.5447	0.5605	0.5760	0.5914	0.6066	0.6217	0.6365	0.6511	0.6656
20	0.5481	0.5639	0.5796	0.5951	0.6104	0.6255	0.6404	0.6552	0.6697	0.6840
21	0.5676	0.5834	0.5990	0.6144	0.6296	0.6446	0.6595	0.6741	0.6885	0.7027
22	0.5874	0.6031	0.6186	0.6339	0.6491	0.6640	0.6787	0.6933	0.7076	0.7217
23	0.6075	0.6231	0.6385	0.6538	0.6688	0.6837	0.6983	0.7127	0.7269	0.7409

	11	12	13	14	15	16	17	18	19	20
24	0.6279	0.6434	0.6588	0.6739	0.6889	0.7036	0.7181	0.7325	0.7465	0.7604
25	0.6485	0.6640	0.6793	0.6944	0.7092	0.7239	0.7383	0.7525	0.7665	0.7802
26	0.6696	0.6850	0.7002	0.7152	0.7299	0.7445	0.7588	0.7729	0.7867	0.8003
27	0.6910	0.7063	0.7214	0.7363	0.7510	0.7654	0.7796	0.7936	0.8073	0.8208
28	0.7127	0.7280	0.7430	0.7578	0.7724	0.7867	0.8008	0.8147	0.8283	0.8417
29	0.7349	0.7501	0.7651	0.7798	0.7942	0.8085	0.8225	0.8362	0.8497	0.8629
30	0.7576	0.7726	0.7875	0.8021	0.8165	0.8306	0.8445	0.8581	0.8715	0.8846
31	0.7806	0.7956	0.8104	0.8249	0.8392	0.8532	0.8670	0.8805	0.8937	0.9066
32	0.8042	0.8191	0.8338	0.8482	0.8624	0.8763	0.8899	0.9033	0.9164	0.9292
33	0.8283	0.8431	0.8577	0.8720	0.8861	0.8999	0.9134	0.9266	0.9396	0.9523
34	0.8529	0.8677	0.8822	0.8964	0.9103	0.9240	0.9374	0.9505	0.9633	0.9759
35	0.8782	0.8928	0.9072	0.9213	0.9352	0.9487	0.9620	0.9750	0.9876	
36	0.9040	0.9186	0.9329	0.9469	0.9606	0.9740	0.9872			
37	0.9305	0.9450	0.9592	0.9731	0.9867					
38	0.9577	0.9721	0.9862							
39	0.9857									
40										

	21	22	23	24	25	26	27	28	29	30
1	0.3747	0.3908	0.4068	0.4227	0.4384	0.4541	0.4695	0.4849	0.5001	0.5151
2	0.3910	0.4070	0.4229	0.4386	0.4543	0.4698	0.4851	0.5003	0.5154	0.5302
3	0.4073	0.4232	0.4390	0.4546	0.4701	0.4855	0.5007	0.5157	0.5306	0.5454
4	0.4237	0.4394	0.4551	0.4706	0.4860	0.5012	0.5163	0.5312	0.5460	0.5606
5	0.4400	0.4557	0.4713	0.4867	0.5019	0.5170	0.5319	0.5467	0.5613	0.5758
6	0.4565	0.4721	0.4875	0.5028	0.5179	0.5328	0.5476	0.5623	0.5767	0.5910
7	0.4730	0.4885	0.5038	0.5189	0.5339	0.5487	0.5634	0.5779	0.5922	0.6063
8	0.4896	0.5049	0.5201	0.5351	0.5500	0.5647	0.5792	0.5936	0.6077	0.6217
9	0.5062	0.5215	0.5365	0.5514	0.5662	0.5807	0.5951	0.6093	0.6233	0.6372
10	0.5230	0.5381	0.5530	0.5678	0.5824	0.5969	0.6111	0.6252	0.6390	0.6527
11	0.5398	0.5548	0.5697	0.5843	0.5988	0.6131	0.6272	0.6411	0.6548	0.6683
12	0.5568	0.5717	0.5864	0.6009	0.6153	0.6294	0.6434	0.6571	0.6707	0.6841
13	0.5739	0.5887	0.6032	0.6176	0.6319	0.6459	0.6597	0.6733	0.6867	0.6999
14	0.5911	0.6058	0.6202	0.6345	0.6486	0.6625	0.6761	0.6896	0.7029	0.7159
15	0.6085	0.6230	0.6374	0.6515	0.6655	0.6792	0.6927	0.7061	0.7192	0.7321
16	0.6261	0.6405	0.6547	0.6687	0.6825	0.6961	0.7095	0.7227	0.7356	0.7483
17	0.6438	0.6581	0.6722	0.6860	0.6997	0.7132	0.7264	0.7394	0.7522	0.7648
18	0.6617	0.6759	0.6898	0.7036	0.7171	0.7304	0.7435	0.7564	0.7690	0.7814
19	0.6798	0.6939	0.7077	0.7213	0.7347	0.7479	0.7608	0.7735	0.7860	0.7982
20	0.6982	0.7121	0.7258	0.7392	0.7525	0.7655	0.7783	0.7908	0.8031	0.8152
21	0.7167	0.7305	0.7441	0.7574	0.7705	0.7834	0.7960	0.8084	0.8205	0.8324
22	0.7356	0.7492	0.7626	0.7758	0.7888	0.8015	0.8140	0.8262	0.8382	0.8499
23	0.7546	0.7682	0.7815	0.7945	0.8073	0.8199	0.8322	0.8443	0.8561	0.8676
24	0.7740	0.7874	0.8006	0.8135	0.8261	0.8385	0.8507	0.8626	0.8742	0.8856
25	0.7937	0.8070	0.8200	0.8327	0.8452	0.8575	0.8695	0.8812	0.8927	0.9038

	21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
26	0.8137	0.8268	0.8397	0.8523	0.8647	0.8767	0.8886	0.9001	0.9114	0.9224
27	0.8341	0.8470	0.8598	0.8722	0.8844	0.8963	0.9080	0.9194	0.9305	0.9413
28	0.8548	0.8676	0.8802	0.8925	0.9045	0.9163	0.9277	0.9389	0.9499	0.9605
29	0.8759	0.8886	0.9010	0.9131	0.9250	0.9366	0.9479	0.9589	0.9696	0.9800
30	0.8974	0.9099	0.9222	0.9342	0.9459	0.9573	0.9684	0.9792	0.9898	
31	0.9193	0.9317	0.9438	0.9557	0.9672	0.9784	0.9894			
32	0.9417	0.9540	0.9659	0.9776	0.9889					
33	0.9646	0.9767	0.9885							
34	0.9881									

	31	32	33	34	35	36	37	38	39	40
1	0.5300	0.5447	0.5593	0.5737	0.5879	0.6019	0.6158	0.6294	0.6429	0.6562
2	0.5450	0.5595	0.5739	0.5881	0.6022	0.6160	0.6297	0.6432	0.6565	0.6695
3	0.5600	0.5744	0.5886	0.6026	0.6165	0.6302	0.6437	0.6570	0.6700	0.6829
4	0.5750	0.5892	0.6033	0.6172	0.6309	0.6444	0.6577	0.6708	0.6837	0.6964
5	0.5900	0.6041	0.6180	0.6317	0.6452	0.6586	0.6717	0.6846	0.6973	0.7098
6	0.6051	0.6191	0.6328	0.6463	0.6597	0.6728	0.6858	0.6985	0.7110	0.7233
7	0.6203	0.6340	0.6476	0.6610	0.6742	0.6871	0.6999	0.7124	0.7247	0.7368
8	0.6355	0.6491	0.6625	0.6757	0.6887	0.7015	0.7141	0.7264	0.7385	0.7504
9	0.6508	0.6642	0.6775	0.6905	0.7033	0.7159	0.7283	0.7405	0.7524	0.7641
10	0.6662	0.6795	0.6925	0.7054	0.7180	0.7304	0.7426	0.7546	0.7664	0.7779
11	0.6817	0.6948	0.7077	0.7203	0.7328	0.7450	0.7571	0.7688	0.7804	0.7917
12	0.6972	0.7102	0.7229	0.7354	0.7477	0.7597	0.7716	0.7832	0.7945	0.8056
13	0.7129	0.7257	0.7383	0.7506	0.7627	0.7746	0.7862	0.7976	0.8087	0.8196
14	0.7288	0.7414	0.7537	0.7659	0.7778	0.7895	0.8009	0.8121	0.8231	0.8338
15	0.7447	0.7572	0.7694	0.7813	0.7931	0.8046	0.8158	0.8268	0.8376	0.8480
16	0.7608	0.7731	0.7851	0.7969	0.8085	0.8198	0.8308	0.8416	0.8522	0.8624
17	0.7771	0.7892	0.8010	0.8127	0.8240	0.8351	0.8460	0.8566	0.8669	0.8770
18	0.7935	0.8055	0.8171	0.8286	0.8397	0.8507	0.8613	0.8717	0.8818	0.8917
19	0.8102	0.8219	0.8334	0.8447	0.8556	0.8664	0.8768	0.8870	0.8969	0.9066
20	0.8270	0.8386	0.8499	0.8609	0.8717	0.8822	0.8925	0.9025	0.9122	0.9216
21	0.8441	0.8555	0.8666	0.8774	0.8880	0.8983	0.9084	0.9182	0.9276	0.9368
22	0.8614	0.8726	0.8835	0.8941	0.9045	0.9146	0.9245	0.9340	0.9433	0.9523
23	0.8789	0.8899	0.9006	0.9111	0.9213	0.9312	0.9408	0.9502	0.9592	0.9680
24	0.8967	0.9075	0.9180	0.9283	0.9383	0.9480	0.9574	0.9665	0.9753	0.9839
25	0.9147	0.9254	0.9357	0.9458	0.9556	0.9650	0.9742	0.9831	0.9917	

	31	32	33	34	35	36	37	38	39	40
26	0.9331	0.9435	0.9537	0.9635	0.9731	0.9824	0.9913			
27	0.9518	0.9620	0.9720	0.9816	0.9910					
28	0.9708	0.9808	0.9906							
29	0.9902									

	41	42	43	44	45	46	47	48	49	50
1	0.6692	0.6821	0.6948	0.7072	0.7194	0.7315	0.7433	0.7548	0.7662	0.7773
2	0.6824	0.6951	0.7075	0.7198	0.7318	0.7436	0.7552	0.7665	0.7776	0.7885
3	0.6956	0.7081	0.7203	0.7324	0.7442	0.7557	0.7671	0.7782	0.7891	0.7997
4	0.7088	0.7211	0.7331	0.7450	0.7566	0.7679	0.7790	0.7899	0.8006	0.8110
5	0.7221	0.7341	0.7460	0.7576	0.7690	0.7801	0.7910	0.8017	0.8121	0.8223
6	0.7354	0.7472	0.7589	0.7703	0.7814	0.7924	0.8030	0.8135	0.8237	0.8336
7	0.7487	0.7604	0.7718	0.7830	0.7939	0.8046	0.8151	0.8253	0.8353	0.8450
8	0.7621	0.7736	0.7848	0.7958	0.8065	0.8170	0.8272	0.8372	0.8469	0.8564
9	0.7756	0.7868	0.7978	0.8086	0.8191	0.8294	0.8394	0.8491	0.8586	0.8679
10	0.7891	0.8002	0.8110	0.8215	0.8318	0.8418	0.8516	0.8611	0.8704	0.8794
11	0.8028	0.8136	0.8242	0.8345	0.8446	0.8544	0.8639	0.8732	0.8822	0.8910
12	0.8165	0.8271	0.8375	0.8476	0.8574	0.8670	0.8763	0.8854	0.8942	0.9027
13	0.8303	0.8407	0.8508	0.8607	0.8704	0.8797	0.8888	0.8976	0.9062	0.9144
14	0.8442	0.8544	0.8643	0.8740	0.8834	0.8925	0.9014	0.9100	0.9183	0.9263
15	0.8583	0.8683	0.8780	0.8874	0.8966	0.9055	0.9141	0.9224	0.9305	0.9383
16	0.8725	0.8822	0.8917	0.9009	0.9099	0.9185	0.9269	0.9350	0.9428	0.9504
17	0.8868	0.8963	0.9056	0.9146	0.9233	0.9317	0.9399	0.9477	0.9553	0.9626
18	0.9013	0.9106	0.9196	0.9284	0.9369	0.9450	0.9529	0.9606	0.9679	0.9749
19	0.9159	0.9250	0.9338	0.9423	0.9506	0.9585	0.9662	0.9735	0.9806	0.9874
20	0.9308	0.9396	0.9482	0.9565	0.9645	0.9722	0.9796	0.9867	0.9935	
21	0.9458	0.9544	0.9627	0.9708	0.9785	0.9860	0.9931			
22	0.9610	0.9694	0.9775	0.9853	0.9928					
23	0.9764	0.9846	0.9924							
24	0.9921									