UNIVERSITY OF NAIROBI

COLLEGE OF PHYSICAL AND BIOLOGICAL SCIENCES

SCHOOL OF MATHEMATICS

ANALYSIS OF CAPITAL CALCULATION MODELS FOR A LIFE INSURER

BY

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the Degree of Master of Science in Actuarial Science of

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DECLARATION

This is my original work and has never been submitted for a degree to any University

Sign.. Date..

GLADYS MUBIRI

Reg No.: I56/68371/2013

This project has been submitted for examination with our approval as The University of Nairobi

Sign... Date.......................................

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DEDICATION

I dedicate this project to my family.They have given me moral, emotional and financial support through out this journey.Thank you.

I thank my friends for being there for me in moments of distress and being understanding.

Above all I thank God for the good health, sound mind and for making this possible for me.I do not take it for granted. THANK YOU GOD.

ACKNOWLEDGMENT

I wish to thank my supervisor Prof. Patrick G.O Weke for the expertise, valuable academic guidance and encouragement while I was doing this project.

My classmate Erastus Kimani for the great help and guidance through out this project.

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ABSTRACT

The amount of capital required to start any business entity is very vital. The methods used to determine it are also very important as they will determine whether the company stays afloat or not.

The subject of this study is to analyze risks surrounding capital calculation for an insurer our main focus being Life Insurance. The study looks at what other countries use like Australia and United Kingdom. Analyze the various capital calculation risks that insurers face in Kenya and the world in general.In the research we shall calculate the reserve that is to be set aside to ensure a life insurance company stays solvent as claims is one of the major risks that insurers face.

Chapter 1

INTRODUCTION

Insurance initially started through insurance agents who acted for foreign insurance companies. British and Indian companies were the insurers at the time. Lloyds of London was also represented by such agents, as Smith Mackenzie and Co. whose operations started as early as 1901, Sydne and C. Fichart who were established as land and estate agents in 1905, but in addition acted as sole agents in British East Africa and Uganda for the Norwich Union Fire Assurance Society. Later on East African Underwriters started business as chief agents in 1954 for a number of Indian Insurance companies such as United India Fire and General Insurance Co. Ltd and Oriental Fire and General Insurance Co. My focus in this project will be life insurance or assurance as most commonwealth countries would refer to it.

1.1 Background of the Problem

Life insurance has become an increasingly important part of the financial sector over the past 40 years, providing a range of financial services for consumers and becoming a major source of investment in the capital market. It has several key benefits discussed as follows;

- Asset protection

  The core benefit of life insurance is that the financial interests of ones family remain protected from circumstances such as loss of income due to critical illness or death of the policyholder. Insurance products also have a strong inbuilt wealth creation proposition. The customer therefore benefits on two counts and life insurance occupies a unique space in the landscape of investment options available to a customer.

- Provides income while your family is adjusting

  When an income provider dies, there is a significant impact on the finances of the surviving family. However, the death benefits of a life insurance policy can prevent this from happening or at least keep the impact to a minimum by replacing income lost with the demise of the breadwinner. That is why it is good the insured is protected to ensure that the surviving family will have financial support during the period they need to reconcile with their grief, get back on their feet, find other sources of family income, and adjust to their new income level.

- Funding specific financial goals

  In addition to providing survivors with income, proceeds from a life insurance policy can also provide funds to achieve specific goals that the insured may have planned for his family. These could include accumulating funds for college education of the children, the purchase of a home and many other goals.

- It covers medical and funeral expenses

  It is very likely that the insured will incur huge medical expenses prior to death. A protracted illness can easily run up to several hundred of thousands or more than a million. Adequate life insurance proceeds ensure that these final expenses are well taken care of.

In Africa insurance penetration levels remain very low and this is largely due to a lack of trust from the public who find insurance companies unable to pay claims when they arise and this brings us to ask the question of what is considered in calculating the minimum capital required by a life insurerso as to stay solvent.

In Kenya, the current system used to measure solvency is based on a solvency margin requirement that ignores specific risks faced by a company. Studies have pointed this out as a major weakness of the system. For example, it ignores the transfer of risk through reinsurance agreements. It also ignores the company size, variability of claim size and the riskiness of an investment portfolio held by a life insurance company.

Capital provides a cushion that allows a life insurance company to remain solvent through certain adverse selection fluctuations in results. The more capital a scheme holds, the less likely it is to go insolvent. An insurer's probability of ruin (or probability of insolvency) depends on the risks that it faces, as well as the amount of capital it holds. Thus two companies with different risk profiles but same capital levels would have different probabilities of ruin. By changing the amount of risk and/ or capital held, insurers can influence their probability of ruin, making this a three way relationship.

Risk-Based Capital (RBC) is a method of measuring the minimum amount of capital appropriate for a reporting entity to support its overall business operations in consideration of its size and risk profile hence limiting the amount of risk a company can take. Capital provides a cushion to a company against insolvency.

Capital calculation of a Life insurance is driven by three processes:

(i) Risk classification and measurement

A life insurance company is solvent if its assets exceed its liabilities. It must consider all material risks that may have an impact on the firm's ability to meet its liabilities to policyholders. The Risk Based Capital for a firm would depend on the risks that the company is faced with, it is risk appetite and regulatory requirements. Risk appetite here can be looked at as the probability of survival within a specified time period. For countries in Europe and also South Africa their probability of survival is 99.5

The various risks faced by a Life insurer based on solvency are:

1. Market risk
   Under this type of risk we look at interest rate, equity, property, spread, currency, concentration and illiquidity.

2. Operational Risk

3. Insurance Risk
   Under this risk we look at mortality, morbidity, longevity, lapse and

expenses.
There is widespread customer dissatisfaction in the insurance industry, stemming from insurers failure to satisfy customers needs. In addition, poor capitalization of insurance companies and a lack of proper risk management have led to the collapse many life insurance companies.

(ii) Stress test calibration

Stress testing is calibrated in terms of risk driver moves. Setting the severity of the extreme events at the required confidence level involves analysis of historic moves and judgment to formulate a view about what is a 1-in-200 year event. Because of the complexity of stress test calibration this is often determined by the regulator.
For the purpose of deriving the level of stress tests for each risk, companies must determine how they will measure risks in terms of the variability of outcomes. Commonly used risk metrics are Value at Risk and Tail Value at Risk.
Value at Risk here is defined as the potential loss in a portfolio in a year at a specified confidence level. Tail Value at Risk defined simply as the value at risk of a portfolio plus the expected loss above the Value at Risk. We can therefore say at a specified confidence interval, Tail Value at Risk is greater than Value at Risk.

(iii) Capital Aggregation Technique

Capital aggregation refers to the task of incorporating the financial impact of multiple types or sources of risks into a single capital figure. There are a number of capital aggregation techniques that have been implemented in different regulatory regimes. The most common being:

a. Correlation Method
b. Monte Carlo Simulation
c. Risk Geographies
d. Copulas

Due to the reasons above there has been an increased interest by insurance regulators across Africa to introduce risk based capital (RBC) in Africa.

RBC is intended to be a minimum regulatory capital standard and not necessarily the full amount of capital that an insurer would want to hold to meet its safety and competitive objectives. RBC will also help improve risk assessment of life insurers which builds confidence on how these companies are managed. This measure will align local insurers with internal regulatory approaches.

There is a relationship between Capital, Risk and Probability of Ruin. A life insurers probability of ruin/probability of insolvency depends on the risks that it faces as well as the amount of capital it holds.

Under the Risk Based Capital system the regulator would choose the maximum probability of ruin and then, by examining a schemes risk profile, determine the minimum amount of capital required by that scheme to achieve that probability of ruin. This then makes the relationship between the three simpler.

The Risk Based Capital Formula was developed as an additional tool to assist regulators in the financial analysis of insurance companies. The purpose of the formula is to establish a minimum capital requirement based on the types of risks to which a company is exposed. Separate RBC models have been developed for each of the primary insurance types: Life, Property/Casualty, Health and Fraternal. This reflects the differences in the economic environments facing these companies.

The risk factors for the NAICs RBC formulas focus on three major areas:

1. Asset Risk

2. Underwriting Risk

3. Other

The emphasis on these risks differs from one formula to the next. As a generic formula, every single risk exposure of a company is not necessarily captured in the formula. The formula focuses on the material risks that are common for the particular insurance type. For example, interest rate risk is included in the Life RBC formula because the risk of losses due to changes in interest rate levels is a material risk for many life insurance products. Under the RBC system, regulators have the authority and statutory mandate to take preventive and corrective measures that vary depending on the capital deficiency indicated by the RBC result. These preventive and corrective measures are designed to provide for early regulatory intervention to correct problems before insolvencies become inevitable, thereby minimizing the number and adverse impact of insolvencies.

The NAIC RBC formula generates the regulatory minimum amount of capital that a company is required to maintain to avoid regulatory action. There are four levels of action that a company can trigger under the formula: company action, regulatory action, authorized control and mandatory control levels. Each RBC level requires some particular action on the part of the regulator, the company, or both. For example, an insurer that breaches the Company Action Level must produce a plan to restore its RBC levels. This could include adding capital, purchasing reinsurance, reducing the amount of insurance it writes, or pursuing a merger or acquisition.

The main aim of a risk based system is to give financially weak companies an incentive to reduce the danger of insolvency. It should encourage those schemes for which market incentives are insufficient to reduce risk and/or hold more capital. There is a large variation in life insurance consumption across countries. The reason for this still remains unclear. Therefore, further research to improve the industrys understanding of service quality is imperative. Capital aggregation refers to the task of incorporating the financial impact of multiple types or sources of risks into a single capital figure. There are a number of capital aggregation techniques that have been implemented in different regulatory regimes. The most common being:

1.2 Statement of the Problem

Insurance penetration in Kenya remains as low as 3%.This is due to lack of trust from the public who find insurance companies unable to pay claims when they arise and also insurers low capitalization level and inadequate systems to manage the risks to which they are exposed. In view of this, this project will examine risks that affect capital calculation of a life insurer in Kenya.

1.3 General Objective

In this project:

- We aim to calculate capital using various models.We shall look at models used in developing countries like United Kingdom.

1.3.1 Specific Objectives

i.Explain the UK Risk Based Capital system capital calculation method.

ii.Find the minimum reserve required to cover risk using Value at Risk Method

iii.Find the minimum reserve required to cover risk using Tail Value at Risk Method

iii.Find the minimum reserve required to cover risk using Conditional Value at Risk Method

1.4 Implications of the Study

The amount of Capital and reserve is vital to keep Life Insurance Companies solvent and any other financial institution for that matter. Many life insurers have found it difficult to stay solvent due to certain risks that surround the business.This research paper will look at Risk Based Capital and how it can assist in capital calculation for a life insurer in Kenya and Value at Risk looking at the various risks faced by life insurers.

Chapter 2

LITERATURE REVIEW

2.1 Introduction

Cocozza and Di Lorenzo Solvency of Life Insurance Company: Methodological Issues Volume 13, 2006) think that solvency of a company should be evaluated using three main steps:

- Recognizing relevant risks. Life Insurers face several risks but the two main ones are:
  - (i) Demographic Risks which arise due to difference in assumed frequencies and actual frequencies.
  - (ii) Financial Risks. This are due to divergence between the actual return on assets purchased
- Measuring this risk. This step helps to show hazards faced by a life insurance company. The system of measurement should be able to state the potential danger and hence limit these dangers through capital requirements.
- Defining capital requirements to absorb these losses. Regulators use two main approaches to determine capital requirement for Insurance companies. They use:
  - (i) **Fixed ratio system**.

    It is a formula method that calculates solvency margin requirements through a fixed percentage of a risk exposure proxy, usually a financial item. It is mostly used in EU countries. Despite the advantage that this system is simple,in-expensive and non-discretional, it has several disadvantages.

(ii) **Risk- Based system**.

This is based on evaluation of risk components that are then used to calculate capital requirements that reflect the insurance companys size and overall risk exposures. This system was introduced in the US by National Association of Insurance Commissioners (NAIC) in the 1990s. Its objective was to calculate the main risks faced by insurers. For life insurers the risks are asset risk, insurance risk, interest rate risk and business risk. Coccoza and Di Lorenzo established that the RBC system is more comprehensive and consistent with the specific company risk profile. For all these risks, different factors are applied to the corresponding items on the financial statement to express the risk potential as likely loss.

2.2 Various Capital Calculation Models

Bob Brown (1998) says that the NAIC RBC formula was first introduced in 1993. It's a very basic type of RBC formula that has four risk components:

- Asset risk C_1
- Insurance risk C_2
- Interest rate risk C_3
- Business risk C_4

The total of all those risks adds up to less than the sum of its parts because there's a covariance adjustment. The NAIC RBC formula produces a minimum capital standard meant to separate companies that are adequately capitalized from companies that aren't. It was based on the best available data at the time. Where data were not available to determine a particular factor for a risk component, factors were developed by piggybacking on the modeling that was done for other factors for which data were available.

Kendal J (2004) The Australian RBC has to fulfill two requirements. The solvency standard which requires insurers to hold an amount of capital (i.e. the solvency reserve) that will enable the fund to meet its current obligations and the Capital Adequacy standard. This requires the fund to hold sufficient capital (i.e. the capital adequacy reserve) to continue to meet its obligations as a going concern. The capital adequacy reserve is expected to exceed the solvency reserve.

The method used to calculate the solvency reserve; the capital adequacy reserve is calculated similarly. The difference between the solvency requirement and the reported liabilities of the fund is the solvency reserve. Solvency reserve is the minimum amount of capital an insurer is required to hold under the solvency standard.

Solvency Requirement comprises of the following:

1. Liability Risk

   Solvency Liability It is the value of insurers liabilities on a conservative basis. It is equal to the reported liabilities of the insurer plus a loading for those components of the insurers liabilities that are unknown and had to be estimated.

   Solvency net claims It is based on two quantities:

   (a) Outstanding claims

   (b) The maximum of

      a. Contributions received in advance (Loss ratio)

      b. Contributions received in advance.

2. Asset Risk

   Inadmissible Assets Reserve This reserve means that if an insurer has holdings in an entity that is required to hold a minimum quantity of capital (i.e. a prudentially regulated entity), that insurer will be required to hold a reserve to the extent that the balance sheet value of that holding includes part of the entity's capital requirement. This ensures that the assets supporting the capital requirement of the subsidiary entity are not used to meet the capital requirement of the insurer that owns it.

   Expense Reserve In a runoff situation, contributions will cease but the insurer will continue to incur certain expenses.

3. Additional Obligations

Resilience Reserve The resilience reserve establishes a reserve against adverse movements in asset values relative to liability values. In simple language, resilience reserve is that quantity of capital that ensures an insurers assets still exceed its liabilities after an economic shock that decreases the value of the insurers assets relative to its liabilities.

Management Capital Amount Management Capital amount imposes a fixed dollar minimum on both the solvency and capital adequacy reserves. The minimum solvency reserve is $10 Million while the minimum capital adequacy reserve is $1.5 Million.

This ensures that the insurer meets its obligations in a run-off situation under a range of adverse circumstances affecting assets as well as liabilities.

Capital Adequacy Requirement comprises of the following:

- Capital Adequacy Reinsurance accrued liabilities
- Capital adequacy net claims liability

  Capital adequacy net claims liability is essentially the solvency net claims liability with the 10% being replaced by margin.

  Margin is the capital adequacy margin and it is determined by the characteristics of the insurer in question.

- Capital adequacy other liabilities

  Capital Adequacy other liabilities is the value of all other liabilities at their balance sheet values. Capital adequacy net claims liability is the sum of the following two quantities:

  1. 1+Margin Outstanding
  2. Maximum of

     (i)1+Margin Contribution received in advance loss ratio

     (ii) Contributions received in advance

Renewal Option Reserve

Renewal Option Reserve is the Net Present Value of cash outflows less cash inflows over the twelve months following the valuation date, subject to minimum value of zero.

Current contribution levels may be insufficient to cover claims and expenses over the coming year. This may take the insurer sometime to realize and adjust premiums. The renewal option reserve requires insurers to hold a quantity of capital to absorb such losses.

Business Funding Reserve

This is basically the extra quantity of capital required to ensure that the fund will meet the solvency requirement over the three years following the valuation date, less any capital that the insurer has entered into binding arrangements to raise externally in the future.

Mutuli M (2014) says Insurance Regulators have an increased interest in introducing Risk Based Capital System in Kenya. This mainly was as a result of the economic turmoil in 2008 and also the need by regulators to see African Insurers review their underlying risks and manage them independently. Regulators feel RBC would enable insurers improve their risk assessment which results to the public having confidence in these companies. In addition he mentions that African Insurance Regulators have realized the importance of RBC as it will play an important role for the insurer in early detection of events that may lead to insolvency and help identify necessary action to take. This system would put African Insurers in a strategic position to compete with other countries that are also looking at ways of dealing with the ever rising risks in insurance sector. Mutuli looks at UK RBC in detail.

The UK Risk Based Capital system

The UK Risk Based System is broad because it considers all the material risks faced by a company.

Capital Calculation of a life Insurer is driven by three processes:

i. Risk Classification

ii. Stress test calibration

iii. Net asset value function and capital aggregation methodology

1. **Risk Classification and Measurement**
   The risks include:

   - Market risk (volatility in equity, bond, property, interest rates)
   - Insurance risks (Risk of under-estimation of insurance liabilities),
   - Credit Risk (failure of counter parties to honor end of the bargain),
   - Surrender Value (addresses lapse risk in excess of level of reserves)
   - Operational risk (failure of internal systems and human error/fraud)

2. **Stress test calibration**
   Mathematical techniques are used in the quantification of the Risk Based Capital, where 99.5% non-ruin probability over one-year is the bottom line. That is, capital is held such that there is at most a 0.5% chance of company being insolvent over a one year period.
   Stress test calibration methods used are:

   i. Value at Risk (VaR) generalizes the likelihood of under performing by providing a statistical measure of downside risk. It can be determined as:

   $$VaR(x) = -\mu - \sigma \times z_{\alpha} \tag{2.1}$$

   VaR assesses the potential losses on a portfolio over a given future time period with a given degree of confidence.

   ii. Tail-VaR expected shortfall below a certain level. For a continuous random variable, the expected shortfall is given by:

   $$Expectedshortfall = E[max(L - X, 0)] = \int (L - X)f(x)dx \tag{2.1}$$

3. **Capital Aggregation technique**

   This method is applied to arrive at the capital requirement for individual risks. There are a number of capital aggregation techniques that have been implemented in different regimes. These include:

- Correlation Method
- Monte Carlo Simulation
- Risk Geographies
- Copula

2.3 Summary of Literature Review

In this study we shall also look at Stress test calibration where we shall look at the following models:

- Value at Risk Model to determine a level of loss that one is reasonably sure will not be exceeded
- Conditional Value at Risk to determine the mean of the losses that exceed VaR
- Tail Value at Risk

Chapter 3

METHODOLOGY

3.1 Correlation Method

It is the most widely used method to calculate diversified capital requirement. This approach has been implemented in South Africa, UK and has also been adopted for Solvency II and SAM standard formula.

The assumptions of Correlation Method include:

1. Risk drivers have a multivariate elliptically contoured distribution
2. The firms net assets is a linear function of risk drivers

A multivariate normal distribution is an example of an elliptically contoured distribution and is most widely used because its easily understood.

The solvency method calculates the Solvency Capital Requirement (SCR) based on the formula:

$$SCR = \sqrt{\sum CORR_{i,j} \times SCR_i \times SCR_j} \tag{3.1}$$

SCR_i is the impact on net assets after individual stressing of risk i
$CORR_{\mathrm{i,j}}$ matrix of correlation between risk i and j

3.1.1 Asset and Liability Valuations

Some valuation rules proposed are as follows:

Valuation of Assets

- Assets are to be valued at their market price
- Assets that are unlisted and without a market value , a discounted cash flow model that is marked to market is used

Valuation of Liabilities

This requires insurers to determine the best estimates of these liabilities. This is determined by projecting future cash flows using realistic assumptions and then discounting these cash flow streams at appropriate interest rates. Assumptions shall include expenses, mortality, morbidity and lapses.

3.1.2 Capital Requirements

The framework discussed is meant to show the strength or weakness of a company and enable regulators intervene if need be.

Capital Cover Ratio measures the adequacy of the capital available in the insurance fund to support the capital required.

$$CapitalCoverRatio = \frac{CapitalAvailable}{CapitalRequired} \times 100\% \tag{3.2}$$

3.1.3 Capital Available

It is composed mainly of the core capital available to an insurer. Regulators need to ensure that the capital is available to meet any losses arising from the risks that insurers are exposed to.

3.1.4 Capital Required

This consists of five components and is calculated as follows:

$$CR = Max\{C_1 \sum (C_2 + C_3 + C_4 + C_5)\} \tag{3.3}$$

The components are defined as follows:

C_1: Surrender value capital requirement

C_1: Credit risk capital requirement

C_1: Market Risk capital requirement

C_1: Operational Market risk capital requirement

Determination of capital requirement components

1. **Surrender Value Capital Requirement C_1**

   This addresses lapse risk in excess of the levels assumed in calculation of reserves for a life insurer. This means, if there are policies that have been surrendered what is the chance that the payment will exceed reserves set aside.

   This is defined as:

   $$C_1 = Max\{0, SurrenderValueofinforcebusiness - PolicyReserves\}$$

   Example: A policyholder is paid Kshs. 500,000 as surrender value if policy is terminated. The amount of reserve set aside is Kshs. 485,000 therefore C1 of the policy is Kshs. 15,000. The company is required to calculate the aggregate C1 for all the policies in force taking into account the probability of lapse and the expected surrender value.

2. **Credit Risk Capital Requirement C_2**

   This aims to mitigate risk of losses from:

   - Asset defaults
   - Inability/Unwillingness of a counterparty to meet its contractual obligations

   $$C_2 = \sum (Exposure_i \times Credit\ risk\ charge_i) \tag{3.4}$$

   (i)refers to the different exposures to counterparties in the fund

3. **Market Risk Capital Requirement C_3**

   This mitigates risks of financial losses arising from the level or volatility of market prices of financial assets.

   Market movements will affect both assets and liabilities and consists of the following risks:

   - Equity Risk

- Property Risk
- Interest Rate Risk

$$C_3 \;=\; \sqrt{\sum (CorrMrt_{i,j} \times C_{3,i} \times C_{3,j})}$$

4.**Insurance Risk Capital Requirement** C_4

This aims to mitigate the risk of under-estimation of risk liability. The life insurance risk requirement consists of the following risk factors:

- Mortality
- Longetivity
- Morbidity
- Lapse
- Expenses

The formula for Insurance Risk Capital Requirement is as follows:

$$C_4 \;=\; \sqrt{\sum (CorrLife_{i,j} \times C_{4,i} \times C_{4,j})}$$

5. **Operational Risk Capital Requirement** C_5

This aims to mitigate risk of losses as a result of the following:

- Inadequate or failed internal processes
- Personal and systems
- External events

3.2 Value at Risk

VaR is a most often used risk measure in practice. It refers to a loss we are fairly sure will not be exceeded over the given time horizon. It can be also expressed in an amount of money.

$$\begin{aligned}\{VaR_\alpha\} &= \; inf\{l \in R : P(L \; < \; l) \leq 1 - \alpha\} \\ &= inf\{l \in R : F_L(l) \geq \alpha\}\end{aligned}$$

Given $\alpha \epsilon$ (0,1], the VaR of a life insurer at confidence level α is given by the smallest number l such that the probability that the loss L exceeds l is no larger than 1-α

VaR is thus simply a quantile of the loss distribution. Typical values of α are 0.95 or 0.99. In market risk management the time horizon is usually 1 or 10 days. In credit risk management and operational management the time horizon is usually 1 year.

Value at Risk for Normal distributions

$$VaR = \mu + \sigma$$

VaR has two basic parameters:

- Significance level (or confidence level 1-)
- Risk horizon

3.3 Tail Value at Risk

Let X be an uncertain variable and $\alpha \epsilon$ (0,1] being the confidence level. The VaR is the function TVaR :(0,1) such that

$$TVaR(\alpha) = \frac{1}{1-\alpha} \int_{\alpha}^{1} VaR(\beta) d\beta$$

3.4 Conditional Value at Risk (CVaR)

Conditional Value at Risk is the mean of the losses that exceed Value at Risk. It is also known as Expected Tail Value at Risk. We shall define the following under CVaR:

$CVaR^{+}$ = expected losses strictly exceeding VaR i.e Mean excess loss and expected shortfallCV

Conditional Value at Risk is a weighted average of VaR and CVaR^{+}

$$CVaR \ = \ \lambda VaR \ + \ (1-\lambda)CVaR^{+} \quad ; 0 \leq \lambda \leq 1$$

A risk measure attempts to assign a numerical value to a random functional loss. Risk measurement is used as an input in many decisions such as the amount of holding capital for an insurance company or prices of different type of insurance services.

It is very important for risk managers to detect in which areas of their insurance business there is a deviation from what could be considered as a normal activity. In this study VaR will be used to detect when the claims are too many in insurance to alert the risk manager.

Steps to compute Value at Risk

1. We arrange data yearly and monthly
2. Test for normality for both yearly and monthly data
3. Compute Value at Risk at different confidence levels i.e alpha=1%, 5%, 10% for both Yearly and Monthly data
4. Compute Conditional Value at Risk at different confidence levels for both Yearly and Monthly data
5. Compute Tail Value at Risk different confidence levels for both Yearly and Monthly data

We use claims data from 2004 to 2014. Here is a sample of the yearly and monthly claims data:

Table 3.1: Yearly Claims Data

Year	**Claim Amount**
2004	238,130
2005	627,119
2006	5,872,874
2007	12,923,917
2008	90,710,673
2009	910,541,340
2010	1,778,536,741
2011	1,899,481,425
2012	2,973,481,873
2013	2,997,134,112
2014	327,189,504

Table 3.2: Monthly Claims Data

Monthly date	**Claim Amount**
2/1/2005	335,630
7/1/2005	130,000
8/1/2005	80,000
12/1/2005	100,000
1/1/2006	223,500
2/1/2006	1,452,361
3/1/2006	122,380
4/1/2006	453,278
5/1/2006	591,278
6/1/2006	482,100
7/1/2006	458,224

We consider claims data for a Kenyan Insurer for claims that occurred in the period 2004-2014

Table 3.3: Descriptive Statistics for Yearly Data

Mean	999,703,428
Standard Error	363,652,212.40
Median	327,189,504
Standard Deviation	1,206,097,943
Sample Variance	1.45E+18
Kurtosis	-0.967641053
Skewness	0.826587334
Range	2,996,895,982
Minimum	238,130
Maximum	2,997,134,112
Sum	10,996,737,708
Count	11

Table 3.4: Computation of VaR, CVaR and TVaR Yearly Data

Confidence Interval()	**VaR**	**CVaR**	**TVaR**
1%	238,130	238,130	999,792,560
5%	238,130	223,500	999,771,636
10%	627,119	273,493	999,760,949

Figure 3.1: QQ Plot for Yearly data

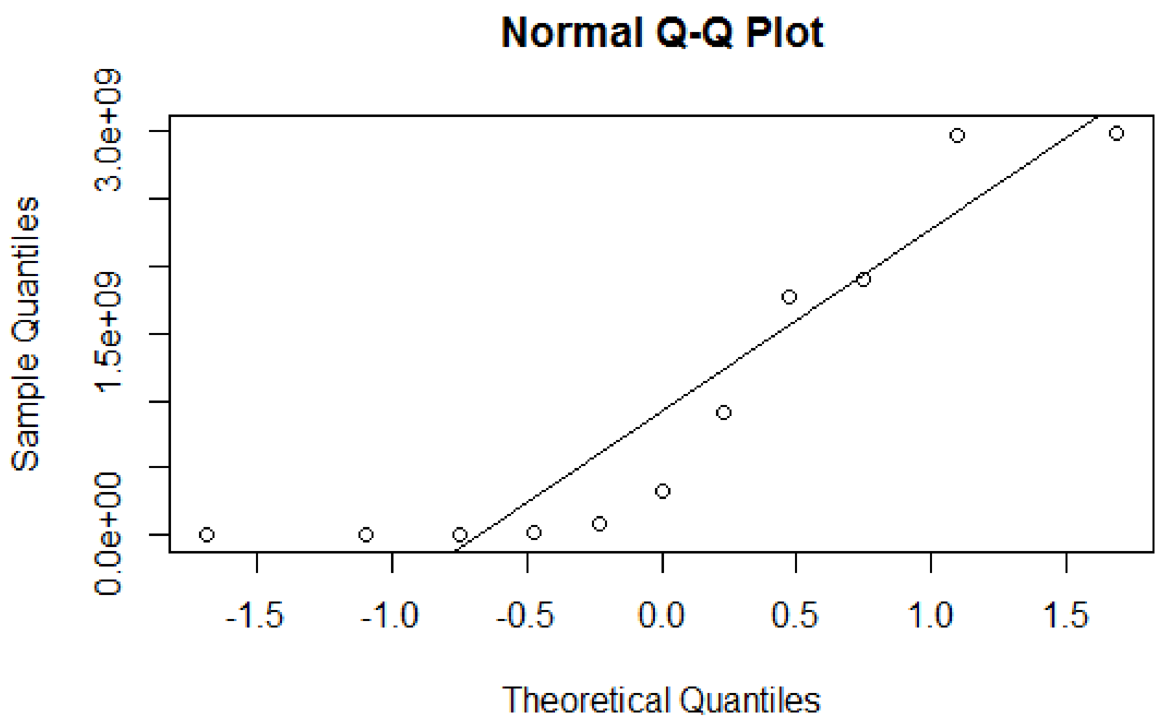


Figure 3.2: Graph for VaR yearly data

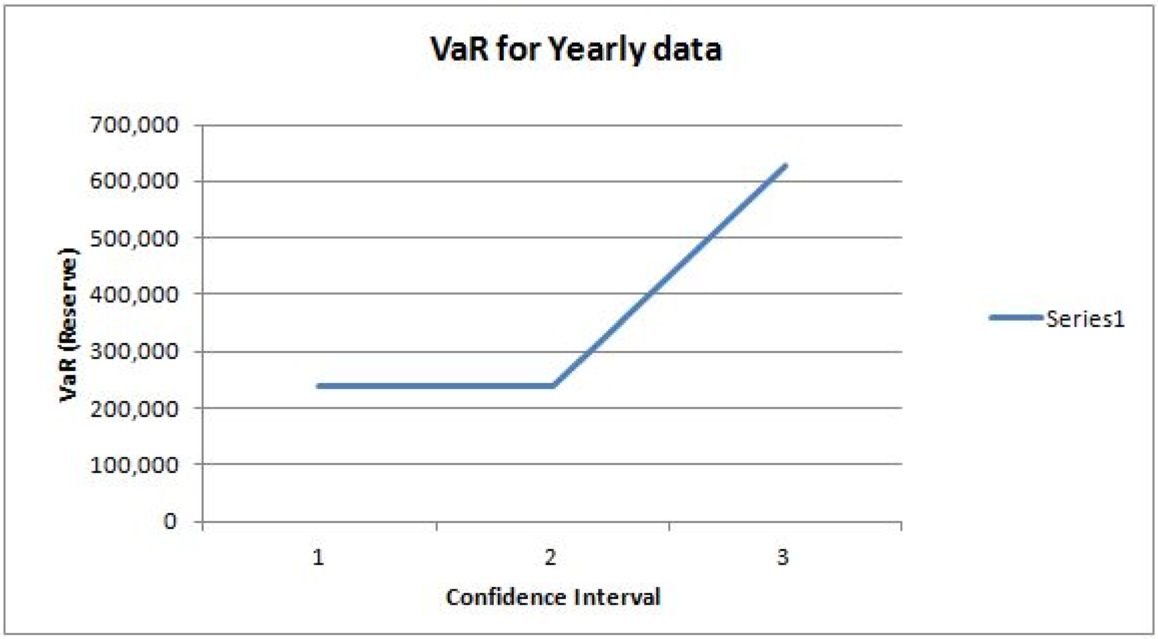


Figure 3.3: Graph for CVaR yearly data

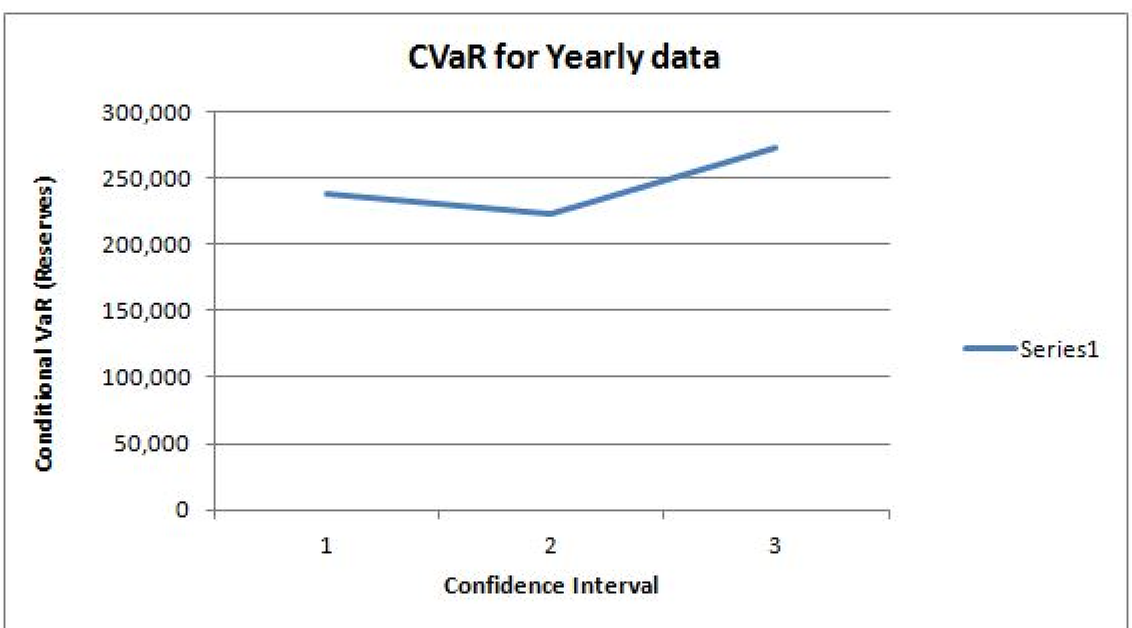


Figure 3.4: Graph for TVaR yearly data

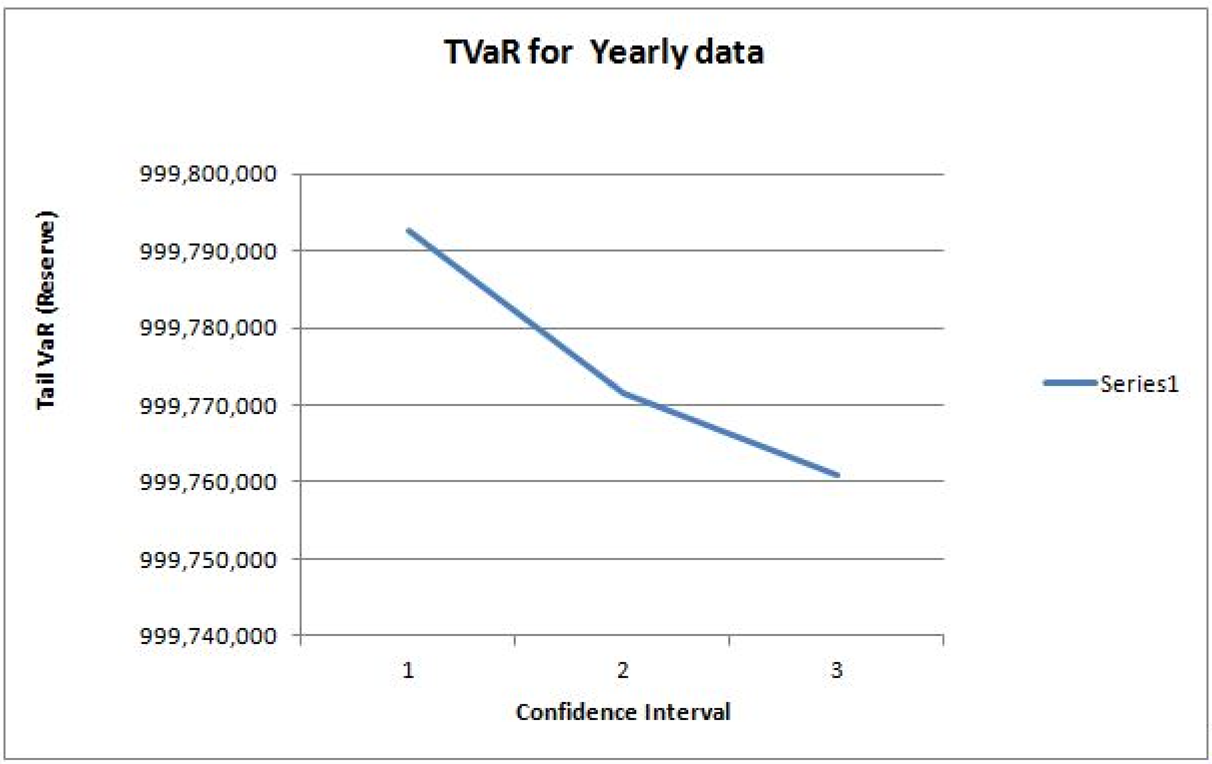


Table 3.5: Descriptive Statistic for Monthly Data

Mean	342622634.4
Standard Error	143260996.8
Median	72836898
Standard Deviation	1453940322
Sample Variance	2.11394E+18
Kurtosis	39.58005851
Skewness	6.225762033
Range	10876943789
Minimum	80000
Maximum	10877023789
Sum	35290131339
Count	103

Table 3.6: Computation of VaR, CVaR and TVaR Monthly Data

Confidence Interval()	**VaR**	**CVaR**	**TVaR**
1%	1,000	133,821	346,567,236
5%	223,500	90,050	346,579,025
10%	453,278	218,375	346,602,108

Figure 3.5: QQ Plot for Monthly data

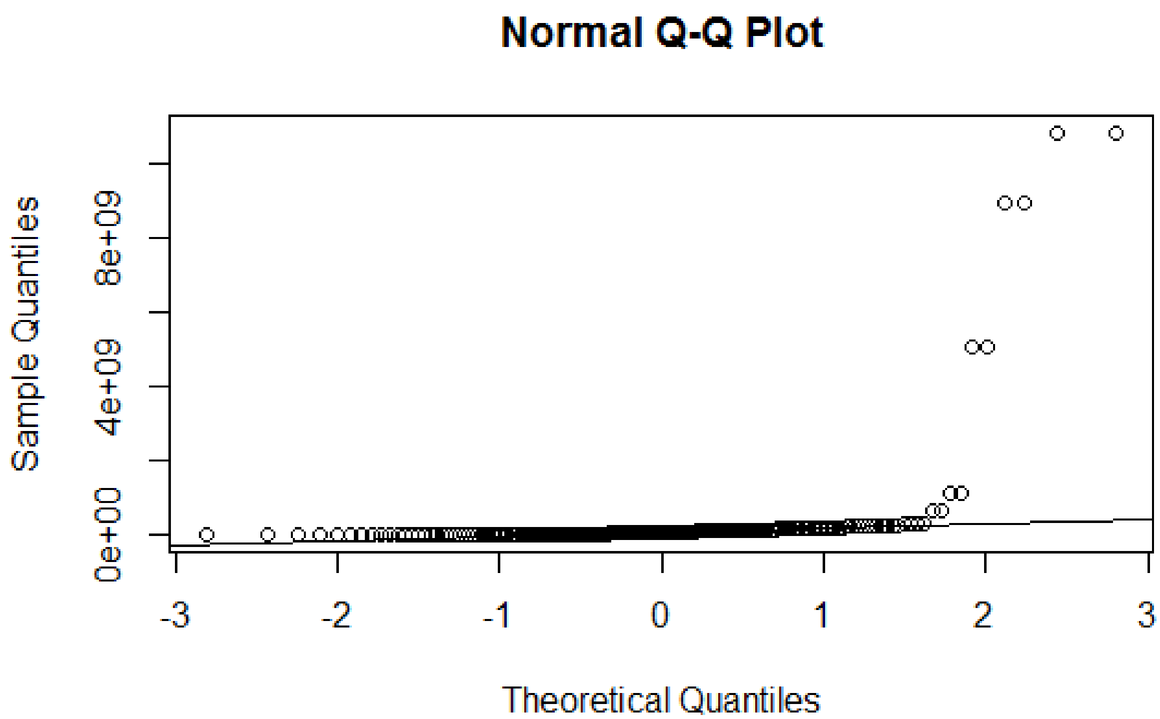


Figure 3.6: Graph for VaR monthly data

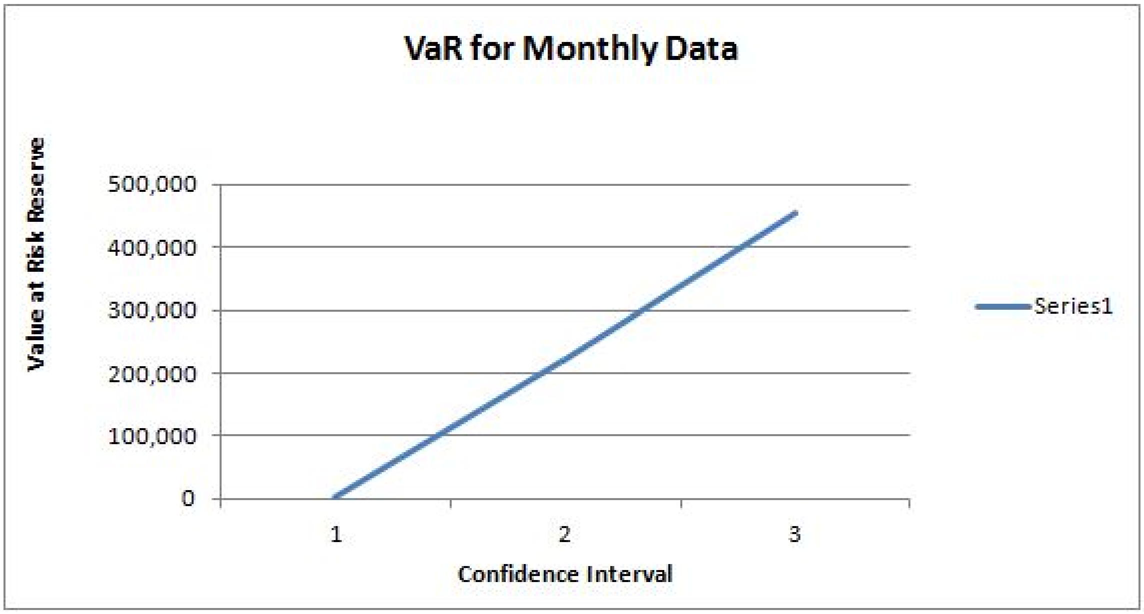


Figure 3.7: Graph for CVaR monthly data

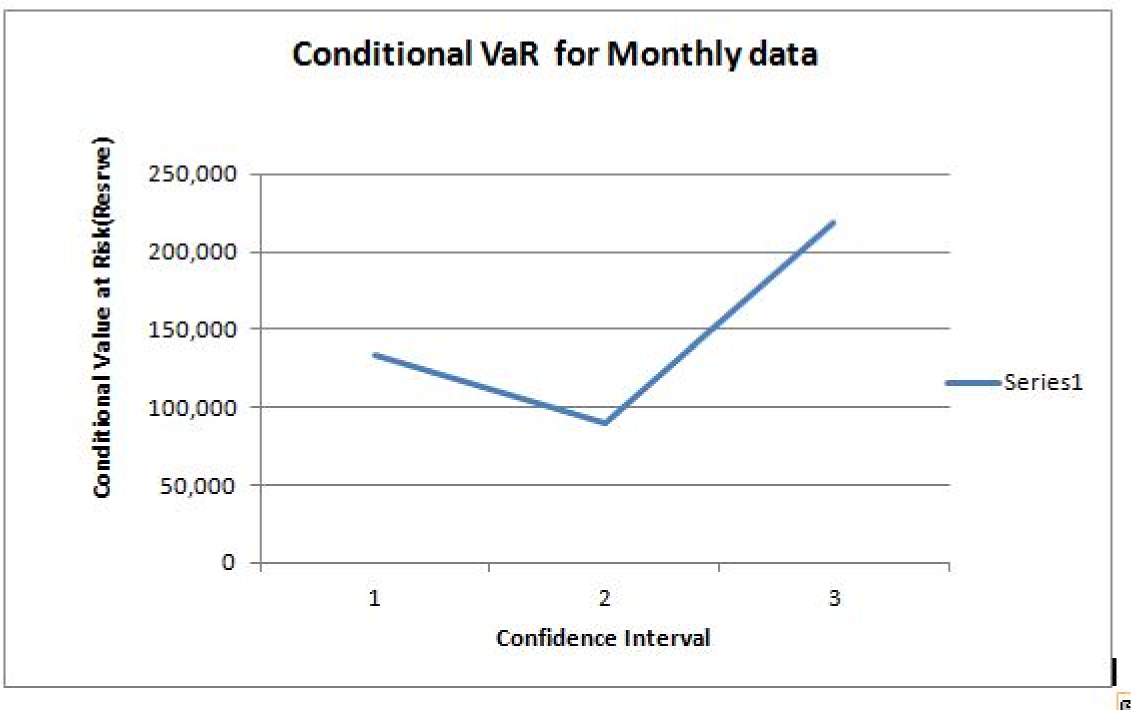
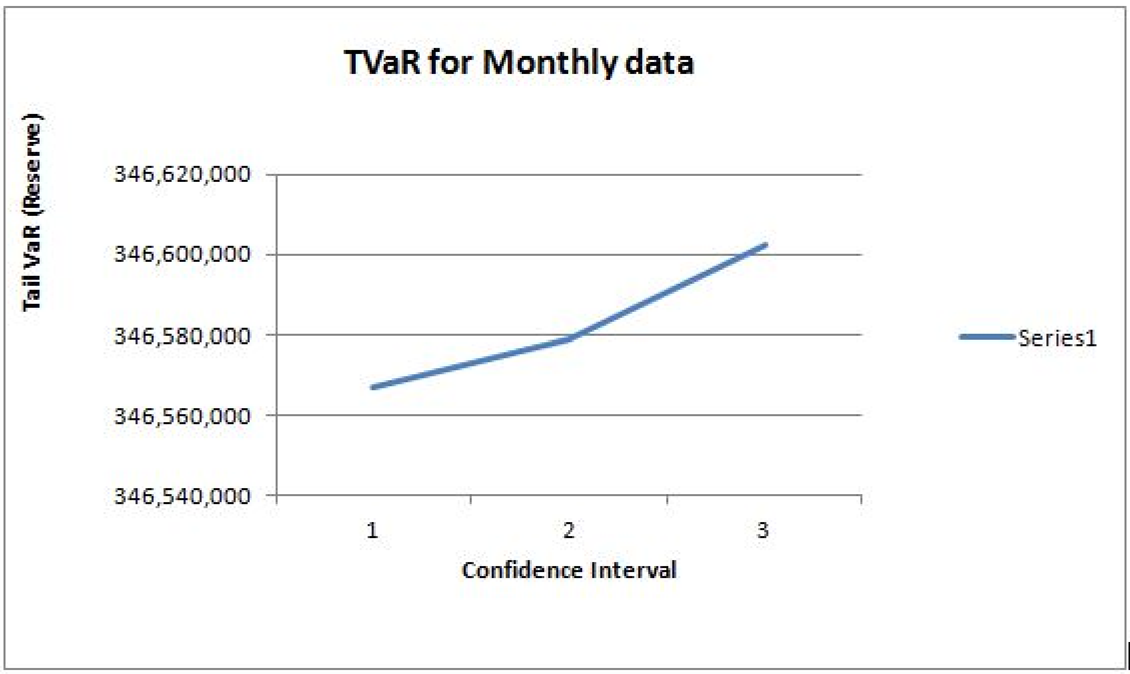


Figure 3.8: Graph for TVaR monthly data



Chapter 4

SUMMARY AND CONCLUSION

4.1 SUMMARY

The QQ Plot for yearly data shows that the data follows a normal distribution with mean=342,622,634.4 and Standard deviation=1,453,940,322.

Computation of VaR, CVaR and TVaR is done.

VaR graph obtained from yearly data indicates that as the confidence level increases the amount of reserve remains constant then starts increasing.

CVaR graph obtained from yearly data indicates that as the confidence level increases the amount of reserve reduces at 5% confidence interval then starts increasing.

TVaR graph obtained from yearly data indicates that as the confidence level increases the amount of reserve reduces.

VaR graph obtained from monthly data indicates that as the confidence level increases the amount of reserve also increases exponentially.

The QQ Plot for monthly data shows that the data follows a normal distribution with mean=342,622,634.4 and Standard deviation=1,453,940,322.

Computation of VaR, CVaR and TVaR is done.

VaR graph obtained from monthly data indicates that as the confidence level increases the amount of reserve also increases exponentially.

CVaR graph obtained from yearly data indicates that as the confidence level increases the amount of reserve reduces at 5% confidence interval then starts increasing.

TVaR graph obtained from yearly data indicates that as the confidence level increases the amount of reserve increases.

4.2 CONCLUSION

Measurement of risk in uncertain environments like insurance sector is a problem and coping with risk management issues in decision making is very important. Value at Risk allows firms to monitor, report, and control their risks in a manner that efficiently relates risk control to desired and actual economic exposures. In addition, reliance on VAR can result in serious problems when improperly used.

VAR is useful only to certain firms and only in particular circumstances. Specifically, VAR is a tool for firms engaged in total value risk management, where the consolidated values of exposures across a variety of activities are at issue.

Dangerous misinterpretations of the risk facing a firm can result when VAR is wrongly applied in situations where total value risk management is not the objective, such as at firms concerned with cash flow risk rather than value risk.

VAR should be applied very carefully to firms selectively managing their risks. When an organization deliberately takes certain risks as a part of its primary business, VAR can serve at best as a diagnostic monitoring tool for those risks. When VAR is analyzed and reported in such situations with no estimates of corresponding expected profits, the information conveyed by the VAR estimate can be extremely misleading.

Risk management as a process encompasses much more than just risk measurement. Although judicious risk measurement can prove very useful to certain firms, it is quite pointless without a well-developed organizational infrastructure and IT system capable of supporting the complex and dynamic process of risk taking and risk control.

VAR need not be calculated by assuming variance is a complete measure of risk but in practice this often is how VAR is calculated. This assumption can be problematic when measuring exposures in markets characterized by non-normal example non-Gaussian) distributions example return distributions that are skewed or have fat tails. In such a case, generate the VAR distribution in a manner that does not presuppose variance is an adequate measure of risk. Alternatively, other summary risk measures can be calculated

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APPENDIX

R codes used in computation of VaR, CVaR and TVaR

Install fExtremes, actuar and nortest packages

install.packages(”fExtremes”)

install.packages(”actuar”)

install.packages(”nortest”)

check for normality in the yearly data

Code: claimsdata<-read.csv(”C://Users//Gladys//Documents//R//Processed Data.csv”,header=T,sep=”,”)

claimsdata

summary(claimsdata)

attach(claimsdata)

qqplot(claimsdata$Year, claimsdata$Claim.Amount,plot.it=TRUE,xlab=”Year”,ylab=”Claim

amount”,main=”QQ plot”)

read the data in vector form and check for normality

yearly data in vector form

ydata<-c(238130,627119,5872874,12923917,90710673,910541340,1778536741,1899481425,297348187

ydata

library(”nortest”)

monthly data in vector form

mdata<-c(335630,130000,80000,100000,223500,1452361,122380,453278,591278,482100,458224,1557

mdata

summary(mdata)

qqnorm(mdata)

qqline(mdata)

shapiro.test(mdata)

ad.test(mdata)

Load the fExtremes package and carry out VaR and CVaR computations

library("fExtremes")

VaR(ydata, alpha = 0.05, type = "sample", tail = c("lower"))

VaR(ydata, alpha = 0.01, type = "sample", tail = c("lower"))

VaR(ydata, alpha = 0.10, type = "sample", tail = c("lower"))

CVaR(ydata, alpha = 0.05, type = "sample", tail = c("lower"))

CVaR(ydata, alpha = 0.01, type = "sample", tail = c("lower"))

CVaR(ydata, alpha = 0.10, type = "sample", tail = c("lower"))

VaR(mdata, alpha = 0.05, type = "sample", tail = c("lower"))

VaR(mdata, alpha = 0.01, type = "sample", tail = c("lower"))

VaR(mdata, alpha = 0.10, type = "sample", tail = c("lower"))

CVaR(mdata, alpha = 0.05, type = "sample", tail = c("lower"))

CVaR(mdata, alpha = 0.01, type = "sample", tail = c("lower"))

CVaR(mdata, alpha = 0.10, type = "sample", tail = c("lower"))

Load the actuar package

library("actuar")

First convert the data from vector to class aggregatedist

Yedata¡-aggregateDist(ydata,method="normal",moments=c(999700000,1206097943))

Mydata¡-aggregateDist(mdata,method=”normal”,moments=c(346500000,1467753458))

Carry out the tail VaR computation

TVaR(Mydata)

TVaR(Yedata)