



UNIVERSITY OF NAIROBI

SCHOOL OF MATHEMATICS

PRICING ON INDIVIDUAL HEALTH INSURANCE

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A Research Project submitted to the Graduate School in Partial Fulfilment for the Requirement of the award of Post-graduate Diploma in Actuarial Science in the school of Mathematics, University of Nairobi.

UNIVERSITY OF NAIROBI

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DECLARATION AND RECOMMENDATION

Declaration

This research project is my original work and has not been submitted or presented for examination in any other institution.

Signature: í í í í í í í í í í í

Date: í í í í í í í í í .

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I46/69037/2013

Supervisor

This research project has been submitted for examination with our approval as University of Nairobi supervisors.

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DEDICATION

I wish to dedicate this work to my late dear Dad and my loving Mum who made invaluable sacrifice to ensure I reach this far, my sisters and brothers for their support.

ACKNOWLEDGEMENT

I would like to thank God for everything He has done for me up to this point.

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I also pass my regards to all those who have made worthy contribution to this work. A lot of thanks goes to my colleagues who have been so supportive and provided guidance whenever needed.

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ABSTRACT

How much is the main problem at task in the health care insurance. This becomes more complex situation when it comes to the case of projecting on future possible financial scenarios. Due to the rising cost of health insurance, insurers argue that pooling of risk is a possible way of eventually reducing the cost of medical financing in the long run. The main goal of every individual who offers health insurance to himself or his family members is to find a precise and accurate estimate of premiums to be paid to the insurance company in the provision of essential medical service. This work thus comes up with a mathematical estimation procedure stipulating the theoretical premium amount that is contributed to medical insurers to offset the rising financial costs based on past experience on claims. Results for predictions of premiums to be paid in current times are based on claim experiences. This work then performs comparative studies on future credibility premiums based on both the Buhlmann's and the Buhlmann-Straub procedures. Result shows that the Buhlmann-Straub procedure yields higher premium amount.

CHAPTER ONE

INTRODUCTION

1.1 Background information

Insurance is a business where people exposed to the same risk are pooled together, contributing into a mutual fund that will be used to compensate those who incur the specified risk and at the same time make profit. That is, an insured, or a policyholder, pays premiums to transfer his/her risks to the insurance companies, or the insurer.

The insurer will therefore have to determine how much the policyholder will have to pay as premiums. These premiums are paid mostly in advance (that is paying before using) and in different periods like monthly, yearly and even on daily basis depending on what the insured prefer or comfortable with.

Premium calculation is rather pluralistic in nature. Several aspects must be considered in premium calculation:

- Cost of paying benefits.
- Cost of administering the program ó collecting premiums, adjudicating claims, issuing policies, filing annual statements etc.
- Cost of marketing and distributing policies ó this includes the commissions paid to agents and brokers.
- Company need to cover its cost of capital and maintain adequate financial reserves in case cost are higher than they expect.

For insurer to come up with the suitable and optimal price of premiums, he/she will have to take into consideration many factors concerning that individual or group paying the premium. Among these factors are the variables like age, health, education, income, gender, marital status and even rate of saving.

Variables are units of data that can change between different cases. The different values that a variable can take affect the type of analysis that is possible. Variable can be analyzed on their own (univariate analysis), with one another variable (bivariate analysis) or with a number of other variables (multivariate analysis).

Long before, the only method of paying healthcare costs was from the pocket of patients, under the fee-for-service business model where services were paid for separately and thus payment was dependant on the quantity instead of quality of service. Later on things changed and the traditional accident insurance evolved into modern health insurance programs.

Health insurance is a financing method that caters for the cost of healthcare entailing the spread of the risk of incurring healthcare cost over certain selected population. It's not everyone who will qualify to take up insurance and so a person wanting to take up health insurance will have to be screened for existing condition as well as inherent factors such as genetics and may be lifestyle variables. Nevertheless there are options like rating up i.e they will have to pay higher premiums than the other policyholders for those who are not viable. Based on the results of screening, premiums are then calculated.

There are three major ways of obtaining health insurance:

- Health insurance purchased individually. This is normally taken up by individuals who are not covered by their employers or are self-employed individuals. Also can be taken by individuals who feel that their current insurance cover is inadequate to them.
- Health insurance provided by the employer. It is also known as group medical insurance. This is health insurance cover provided by the employer to its employees. In Kenya this is an incentive to workers since they pay only twenty percent out-of-pocket in contribution towards offsetting medical cost on drugs whenever a member falls sick.
- Health insurance provided by government. The government provide fund to some organizations like the National Health Insurance Fund (NHIF) in Kenya in which will provide cover to its members. Government can also have some supplement initiative for certain special groups like the elderly and the disable.

If an insured falls sick, he/she seeks medical redress in a medical facility and depending on his/her medical cover the insurer will meet the medical costs. This costs incurred by the insurer instead of insured is called the claims borne by the insurer.

Many at a time there will be delay in payment of this claims, this is due to reasons like;

- Requirement of some documents before processing a claim
- Legal issues concerning the interpretation of medical cover
- Doing post-claim underwriting if need be by the insurer in order to confirm a pre-existing condition not disclosed by the insured during application of the policy.

The delay in payment of these claims by the insurer causes need of the insurer to have some reserves and thus forecasting outstanding claims and setting up suitable reserves to meet those claims is a vital part of insurance business.

1.2 Statement of the problem

For an individual health insurance, the focus is on the aggregate cost of an individual (i.e individual as a person or as the group like a family that an individual is responsible for their healthcare). Insurance company focus on the historical claim levels for the individuals and also health of the specific person when applying for the policy. Insurer will make payments on behalf of an individual who in turn is responsible for payments of premiums. We then ask ourselves "how much of a premium is to be paid projected from the past expenditure history on health insurance" This is a factor considered on selection of insurance provider by concept of optimal pricing. An individual will seek to find health insurance provider with a minimal premium cost but with effective and satisfactory service provision to the policyholder or those under its umbrella. This will largely lead to saving or investing of large amount of money that would have otherwise been used to cover medical costs, hence the need for charging "the right price" to the customer in the health insurance business is paramount.

1.3 Objectives

1.3.1 General Objective

The general objective of this study is to find the optimal price of premiums paid to an insurer to cover for individual healthcare expenses based on historical expenditures.

1.3.2 Specific Objectives

- ✓ To determine the ultimate reserve to set aside to meet the increasing future claims
- ✓ To generate a credible risk premiums values of individual claim experience to be used as an average premium regulator.

- ✓ To determine the loss ratios to be used in the expected loss ratio reserving method

1.4 Justification

The cost of healthcare in Kenya is increasing each coming year due to many factors that affect the economy. They include the cost of inflation and technological advancement. People hire the service of insurance companies in providing medical cover to them or those under their care. A good number of insurance companies would want to take advantage of the lack of expertise that determines the correct premiums. This has led to unscrupulous insurance companies taking advantage of clients by charging high cost of insurance and hence allowing them to make super normal profits. The aim of this study is to provide an optimal premium paying function obtained by experience rating, premiums are based on individual's own experience. Past year's claims are projected forward and used as basis for this year's premiums. The obtained limits of premium payments would help the individual in determining the 'fair' price that is charged by the health insurance company.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

The genesis of insurance have been said to originate from members of community assisting each other on times when one of them incurred unexpected loss such as death or illness in the family or loss of property. This was good in intention but sometimes could fail to meet the threshold, like when the loss is too big for the community to raise the amount or when the loss is too small such that the sufferer is just taking advantage of the generosity of community members to get rich. Due to this reason and others, there came the study of insurance in the early 20th century during the famous London's fire. After this London's fire residents made contribution to a group account in form of savings. Some insurance companies directly paid for some of London's city fire brigades. Persons who paid insurance companies to insure their homes were given a "fire plate" showing the insurance company's logo. The fire plate was fixed at the front door of their house.

If a house caught fire, the fire brigade will use the fire plate on the front of the door to check if it's their company and thus fight fire otherwise will leave the house burning down. Those people who needed the fire brigades to help them when their home caught fire had to put in a little money to help pay for the fire brigades to protect their houses. The problem with this is that, the population was heterogeneous in nature, that is residents were of different social classes in terms of wealth and lifestyle each lived varies largely. The main issue here was "how much one is expected to pay monthly or yearly that is proportional to the value of the house". Since this era, actuaries have developed several theories to determine the premium

amount per period of insurance. Among these theories is the credibility theory ó measure of predictive value attached to a particular class of data based on experience rating and premium rating ó process of determining premium estimates of expected values of future costs per unit time of exposure for group of risks. Other theories are like the Chain-ladder and the Bornhuetter-Ferguson methods.

2.2 Credibility Theory

Credibility theory is a technique that can be used to determine premiums or claim frequencies (number of claims) in general insurance. This technique uses;

- ✓ Historical data related to the actual risk
- ✓ Data from other related but relevant sources commonly referred to as collateral data.

The credibility premium formula as derived by Waters (1987) is of the form;

$$m = zX + (1 - z)\mu \quad (2.1)$$

Where;

m is the premium, z is the weight or credibility factor and is usually between zero and one. The credibility factor here is an increasing function for large value of n . The mean parameter x is the observed mean claim amounts per unit risk exposed for individual contract/risk itself. μ is the parametric estimate of the proposed data in the case than an assumption of the underlying distribution is made. For a series of risks, μ is the corresponding portfolio (set of risks) mean.

Here are some features of credibility formula;

- ✓ It is a linear combination of estimates to a pure premium policy based on observed data from the risk itself and the other based on projected risks.
- ✓ The credibility factor z , shows the degree of reliability of the observed risk data in the sense that high values of z implies high reliability.
- ✓ The credibility z is a dependent function of the number of claims. This implies, the higher the claim number the larger the credibility factor.
- ✓ The value of credibility factor lies between zero and one.

2.2.1 Credibility theory development

Credibility theory was originally developed for a long time by actuaries from North America in the early 20th century. Mowbray (1914) put it into practical solution to premium calculation and it came to be called the American credibility theory. It is sometimes referred to as "limited credibility theory" or "the Fixed effect credibility". In this work it was assumed that the annual claims X_1, X_2, \dots, X_n are independently and identically distributed random variables from a probabilistic model with means $m(\cdot)$ and variance $s^2(\cdot)$. The assumption is that the data follows a normal distribution.

Whitney (1918) and other researchers criticized a lot this theory. Whitney proposed that claims are random in nature and hence assumption of fixed effects model was invalid. In addition, the theory also faced the problem of partial credibility since it was difficult to determine the value of the credibility factor. After the World War II revolution, Whitney's random effect model came into place.

Later on, Nelder and Verall derived credibility functions by the generalized linear model approach and consequently included the random effects model. This has provided a wide range of actuarial application among them is premium rating and reserving. Though a lot of research was done that yield several findings, it was found that the fixed effect credibility was not able to solve the problem of credibility. It is said that part of it was due to undeveloped or poor statistical background.

In 1967 and 1970, the real thing came when Bulhmann derived the credibility premium formula in a distribution free-way such that there was no assumption of prior distribution of claims. Bulhmann later clarified in this work the several assumptions of using the credibility premium formula (see Bulhmann 1971). This major breakthrough has seen much of the research tilting to the development of Bayesian estimation techniques by Jewell (1974, 1975), Hachmeister (1975), Devylder (1976, 1986) and Gooverts and Hoogstad (1987). Jewell (1974) showed that for exponential family distribution, the best linear approximation to Bayesian estimate is obtained using quadratic loss functions. Hachmeister (1975) extended the Bulhmann Straub model by use of matrix methods.

CHAPTER THREE

METHODOLOGY

3.1 Credibility Theory

The credibility premium is given by the linear function of the form;

$$m = (1 - z)\mu + z\bar{X} \quad (3.1)$$

Here, z is the amount of credibility placed to a certain data set created out of the past experience data. The main question is how much observation are required in order to attain 100% credibility. From this question, we end up determining conditions necessary to attain full credibility and partial credibility. In practical situation, full credibility rarely occur/happen. Mowbray (1914) using the fixed effects model came up with a criterion for determining the sample size required for partial credibility.

This approach never last due to a lot of criticism due to its fixed effects. The Mowbray's results was seen as a criterion for full credibility of ∞ implying setting $z = 1$. This bring in a wide area of research where experience rating problems was like a matter of estimating the random variables θ from observed mean of information, \bar{x} of individual data sets. The major objective was to minimize the mean square error.

$$\rho(\bar{m}) = E \left[m(\theta) - \bar{m}x \right] \quad (3.2)$$

This mean square error gave restriction on distribution function and so it was modified to avoid much restriction on distribution function. This eventually gives rise to a linear credibility function of the form;

$$\hat{m} = a - b\bar{m}(X) \quad (3.3)$$

$$\bar{m} = E[m(\theta)] + \frac{\text{cov}[m, \hat{m}]}{\text{var}[\hat{m}]} (\hat{m} - E(\hat{m})) \quad (3.4)$$

And thus the linear bayes risk is given by

$$\bar{\rho} = \text{var}(m) - \frac{\text{cov}^2[m, \hat{m}]}{\text{var}(\hat{m})} \quad (3.5)$$

The linear Bayes measures the accuracy of the Linear Bayes estimator. Linear estimator make sense only when the Linear Bayes risk approach 0 as data X increases.

Since

$$\bar{\rho} \leq \text{var}(\hat{m}) - \text{var}(\hat{m})$$

Sufficient condition for $\bar{\rho}$ is

- i. $\text{var}(\hat{m}) - \text{var}(\hat{m}) \rightarrow 0$
- ii. $\text{cov}^2[m, \hat{m}] \rightarrow 0$
- iii. $\text{cov}^2[m, \hat{m}] = \text{var}(\hat{m})$

Due to this conditions being in place, then

$$\text{var}(\hat{m}) = \text{var}(\hat{m}) \quad (3.6)$$

$$\text{cov}^2[m, \hat{m}] = \text{var}(\hat{m}) \quad (3.7)$$

$$\text{cov}^2[m, \hat{m}] = \text{var}(\hat{m}) + \text{cov}^2[m, \hat{m}] \quad (3.8)$$

Thus is given by

$$w = \frac{VHM}{VHM + EPV} \quad (3.9)$$

Alternatively,

Take weight to be proportional to its reciprocal of the variance. That is

$$w \propto \frac{1}{VHM}$$

For it to be with 0 and 1, use denominator as

$$\frac{1}{VHM} + \frac{1}{EPV}$$

Hence

$$w = \frac{n}{EPV} \div \left(\frac{n}{EPV} + \frac{1}{VHM} \right)$$

$$= \frac{n}{n + \frac{EPV}{VHM}} \quad \text{B.10}$$

And so

$$w = \frac{n}{n + \frac{EPV}{VHM}} = 1 - \frac{\frac{EPV}{VHM}}{n + \frac{EPV}{VHM}}$$

Thus the familiar Buhlmann credibility formula with credibility

$$\hat{\theta} = \frac{\sum_{i=1}^n x_i}{\sum_{i=1}^n x_i + \sum_{i=1}^n \theta}$$

There are many more models suggested for calculation for the credibility premiums in the literature of empirical Bayes credibility. The assumptions of this model are, the aggregate claims are independent and identically distributed in nature. This is not true in most life situations because to analyze for risk we need different variables that are not necessarily dependent on each other.

3.2 Empirical Bayes Credibility Theory

Considering a single risk, X_1, X_2, \dots, X_n are successive values of random variables representing the quantity in which we are interested (aggregate claims). Distribution of X_i depends on parameter(s) θ whose value is fixed but unknown. X_1, X_2, \dots, X_n are unconditionally identically distributed while $X_1|\theta, X_2|\theta, \dots, X_n|\theta$ are independent and identically distributed.

Notion

\underline{x} denotes the data X_1, X_2, \dots, X_n

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

$$E[X_i|\theta] = \theta$$

$$E[X_i^2|\theta] = \theta^2 + \sigma^2$$

Assumptions are

μ_j and

$$\frac{\sum_{j=1}^n \mu_j^2}{n}$$

don't depend on j

The problem here is to determine the value of μ_j . Without any data from the risk, the best estimate is μ_j mean of the prior. With data X_j , best estimate is $\frac{\sum_{i=1}^n X_{ij}}{n}$ mean of the posterior. Given the data X_j produce a credibility estimator which uses data from the risk itself and some other (collateral) information.

3.2.1 Derivation of credibility premium

Taking that the estimator must be linear in the data, then

$$\mu_j + \sum_{i=1}^n a_i X_{ij} = \text{estimator} \quad (3.11)$$

To minimize the square error loss function

$$\sum_{i=1}^n \sum_{j=1}^n (\mu_j + \sum_{i=1}^n a_i X_{ij} - \mu_j)^2 \quad (3.12)$$

$$\text{If } \mu_1 = \mu_2 = \dots = \mu_n$$

Then (3.12) is

$$\sum_{i=1}^n \sum_{j=1}^n (\sum_{i=1}^n a_i X_{ij} - \mu_j)^2 = \sum_{i=1}^n \sum_{j=1}^n (X_{ij} - \bar{X}_j)^2 \quad (3.13)$$

To show that (3.13) holds, we let

A denote $\frac{1}{2}(a+b) - \frac{1}{2}(a-b)$ and B denote $\frac{1}{2}(a+b) + \frac{1}{2}(a-b)$

So

$$A+B = \frac{1}{2}(a+b) - \frac{1}{2}(a-b) + \frac{1}{2}(a+b) + \frac{1}{2}(a-b)$$

Then

$$\frac{1}{2}(a+b) + \frac{1}{2}(a-b) = \frac{1}{2}(a+b) + \frac{1}{2}(a-b) + \frac{1}{2}(a+b) + \frac{1}{2}(a-b)$$

$$= \frac{1}{2}(a+b) + \frac{1}{2}(a-b) \text{ since } \frac{1}{2}(a+b) + \frac{1}{2}(a-b) = 0$$

$$= \frac{1}{2}(a+b) + \frac{1}{2}(a-b) - \frac{1}{2}(a+b) - \frac{1}{2}(a-b) + \frac{1}{2}(a+b) + \frac{1}{2}(a-b) - \frac{1}{2}(a+b) - \frac{1}{2}(a-b)$$

A is not a function of a or b implying that values of a and b which minimize

$$\frac{1}{2}(a+b) - \frac{1}{2}(a-b)$$

Minimizes also

$$\frac{1}{2}(a+b) + \frac{1}{2}(a-b) - \frac{1}{2}(a+b) - \frac{1}{2}(a-b)$$

So

$$\frac{1}{2}(a+b) + \frac{1}{2}(a-b) - \frac{1}{2}(a+b) - \frac{1}{2}(a-b) = 0$$

$$\rightarrow \frac{1}{2}(a+b) - \frac{1}{2}(a-b) = 0$$

but $\frac{1}{2}(a+b) - \frac{1}{2}(a-b) = \frac{1}{2}(a-b)$

$$\rightarrow \hat{\theta} = \frac{\sum_{i=1}^n x_i}{n} - \frac{\sum_{i=1}^n x_i^2}{\sum_{i=1}^n x_i} \quad (3.14)$$

$$\frac{\partial}{\partial \theta} \left(\sum_{i=1}^n x_i^2 - n\theta - \frac{\sum_{i=1}^n x_i^2}{\theta} \right) = 0$$

$$\rightarrow \sum_{i=1}^n 2x_i^2 - n - \frac{\sum_{i=1}^n x_i^2}{\theta^2} = 0$$

But

$$\sum_{i=1}^n x_i^2 = \sum_{i=1}^n x_i^2 + \frac{(\sum_{i=1}^n x_i)^2}{n}$$

And

$$\frac{\partial}{\partial \theta} \left(\frac{\sum_{i=1}^n x_i^2}{\theta} \right) = -\frac{\sum_{i=1}^n x_i^2}{\theta^2}$$

$$\rightarrow \hat{\theta} = \frac{\sum_{i=1}^n x_i^2}{\sum_{i=1}^n x_i^2 + \frac{(\sum_{i=1}^n x_i)^2}{n}}$$

$$= \frac{\sum_{i=1}^n x_i^2}{\sum_{i=1}^n x_i^2 + \frac{(\sum_{i=1}^n x_i)^2}{n}} \equiv \hat{\theta} \quad (3.15)$$

Thus credibility estimate is

$$\hat{\theta} = \frac{\sum_{i=1}^n x_i^2}{\sum_{i=1}^n x_i^2 + \frac{(\sum_{i=1}^n x_i)^2}{n}}$$

$$= \frac{\sum_{i=1}^n x_i^2}{\sum_{i=1}^n x_i^2 + \frac{(\sum_{i=1}^n x_i)^2}{n}}$$

3.3 Buhlmann - Straub credibility model

This is more of generalization of the credibility premium of Buhlmann (1969). The risk here is just one risk in a collective of N similar risks. Each has unobserved random risk parameter θ_j .

Let $X_{1j}, X_{2j}, \dots, X_{nj}$ be successive values of random variables representing the quantity in which we are interested in (aggregate claims). Each year, j, there is some non-random quantity m_j which measures risk volume.

We define $\bar{X}_{j\theta_j} = \frac{X_{j\theta_j}}{m_j}$ which imply that $X_{j\theta_j}$ is the aggregate claims in year j per unit of risk.

Distribution of $\bar{X}_{j\theta_j}$ depend on parameter θ_j whose value is fixed but unknown.

$\bar{X}_{1\theta_1}, \bar{X}_{2\theta_2}, \dots, \bar{X}_{n\theta_n}$ are independent and identically distributed.

Assumptions are;

θ_j do not depend on j and denote θ

m_j do not depend on j and denote m

Without data from the risk, the best estimate is θ and with data $X_{1\theta}, X_{2\theta}, \dots, X_{n\theta}$ denoted \bar{X} , the best estimate is $\hat{\theta}$ where $X_{1\theta}, X_{2\theta}, \dots, X_{n\theta}$ are assumed to be known. Task here is to produce an estimator that uses data from the risk itself and collateral information.

3.3.1 Derivation of Buhlmann-Straub

Taking that the estimator must be linear in the data, then

$$\hat{\theta} + \sum_{j=1}^n w_j X_{j\theta} = \text{estimator}$$

To minimize the squared error loss

$$\sum_{k=0}^n \binom{n}{k} x^k - \sum_{k=0}^n \binom{n}{k} x^k = 0$$

calculate the coefficients a_0, a_1, \dots, a_n where $a_0 \neq a_1 \neq \dots \neq a_n$

And

$$\sum_{k=0}^n \binom{n}{k} x^k - \sum_{k=0}^n \binom{n}{k} x^k = 0$$

So

$$\sum_{k=0}^n \binom{n}{k} x^k - \sum_{k=0}^n \binom{n}{k} x^k = 0$$

$$\rightarrow \sum_{k=0}^n \binom{n}{k} x^k - \sum_{k=0}^n \binom{n}{k} x^k = 0$$

but $\sum_{k=0}^n \binom{n}{k} x^k = \sum_{k=0}^n \binom{n}{k} x^k$

$$\sum_{k=0}^n \binom{n}{k} x^k = \sum_{k=0}^n \binom{n}{k} x^k \tag{3.16}$$

For $k = 1, 2, \dots, n$

$$\sum_{k=0}^n \binom{n}{k} x^k - \sum_{k=0}^n \binom{n}{k} x^k = 0$$

$$\rightarrow \sum_{i=1}^n \frac{1}{n} \left(\frac{1}{n} \sum_{j=1}^n x_{ij} - \frac{1}{n} \sum_{j=1}^n \bar{x}_{i.} - \frac{1}{n} \sum_{i=1}^n \bar{x}_{.j} + \bar{x} \right) = 0$$

$$\rightarrow \sum_{i=1}^n \frac{1}{n} \sum_{j=1}^n x_{ij} + \sum_{i=1}^n \frac{1}{n} \sum_{j=1}^n x_{ij}^2 - \sum_{i=1}^n \bar{x}_{i.} \sum_{j=1}^n \bar{x}_{i.}$$

$$- \sum_{i=1}^n \bar{x}_{i.} \sum_{j=1}^n \bar{x}_{i.} + \sum_{i=1}^n \bar{x}_{i.} \sum_{j=1}^n \bar{x}_{i.} - \frac{1}{n} \sum_{i=1}^n \bar{x}_{i.}^2 = 0$$

$$\sum_{i=1}^n \bar{x}_{i.}^2 = \sum_{i=1}^n \bar{x}_{i.} \sum_{j=1}^n \bar{x}_{i.} + \sum_{i=1}^n \bar{x}_{i.} \sum_{j=1}^n \bar{x}_{i.} - \sum_{i=1}^n \bar{x}_{i.}^2$$

But $\sum_{i=1}^n \bar{x}_{i.}^2 = \sum_{i=1}^n \bar{x}_{i.} \sum_{j=1}^n \bar{x}_{i.}$

And thus

$$\sum_{i=1}^n \bar{x}_{i.}^2 = \sum_{i=1}^n \bar{x}_{i.} \sum_{j=1}^n \bar{x}_{i.}$$

Hence

$$\sum_{i=1}^n \bar{x}_{i.}^2 = \frac{\sum_{i=1}^n \bar{x}_{i.}^2 / \sum_{i=1}^n \bar{x}_{i.}^2}{\sum_{i=1}^n \bar{x}_{i.}^2 + \sum_{i=1}^n \bar{x}_{i.}^2 / \sum_{i=1}^n \bar{x}_{i.}^2} \sum_{i=1}^n \bar{x}_{i.}^2$$

$$\sum_{i=1}^n \bar{x}_{i.}^2 = \frac{\sum_{i=1}^n \bar{x}_{i.}^2}{\sum_{i=1}^n \bar{x}_{i.}^2 + \sum_{i=1}^n \bar{x}_{i.}^2 / \sum_{i=1}^n \bar{x}_{i.}^2} \sum_{i=1}^n \bar{x}_{i.}^2 \tag{3.17}$$

Credibility estimate

$$\sum_{i=1}^n \bar{x}_{i.} + \sum_{i=1}^n \bar{x}_{i.}^2 = \sum_{i=1}^n \bar{x}_{i.} - \sum_{i=1}^n \bar{x}_{i.}^2 + \sum_{i=1}^n \bar{x}_{i.}^2$$

Where

$$\sum_{i=1}^n \bar{x}_{i.} = \sum_{i=1}^n \bar{x}_{i.} + \sum_{i=1}^n \bar{x}_{i.}^2 / \sum_{i=1}^n \bar{x}_{i.}^2 \tag{3.18}$$

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1 Presentation and discussion of results

The following data was obtained from the database of Jubilee Insurance Company Ltd Kenya.

We use the data set below for our analysis.

TABLE 4.2 Aggregate claim amounts

Contract	Aug	Sept	Oct	Nov	Dec	Total
1	829245.5	983204.4	814627.6	1166578.8	567121.6	4360777.9
2	621933.9	737403.3	610970.7	874934.1	425341.2	3270583.2
3	414622.8	491602.2	407313.8	583289.4	283560.8	2180389
4	207311.3	245801.1	203656.9	291644.7	141780.4	1090194.4

TABLE 4.3 Values of $m(\theta_i)$, $s^2(\theta_i)$, $E[m(\theta)]$ and $E[s^2(\theta)]$

Contract	Aug	Sept	Oct	Nov	Dec	Total	$m(\theta_i)$	$s^2(\theta_i)$
1	829245.5	983204.4	814627.6	1166578.8	567121.6	4360777.9	872155.58	49303336325
2	621933.9	737403.3	610970.7	874934.1	425341.2	3270583.2	654116.64	27733130304
3	414622.8	491602.2	407313.8	583289.4	283560.8	2180389	436077.8	12325833545
4	207311.3	245801.1	203656.9	291644.7	141780.4	1090194.4	218038.88	3081458923
Average							545097.225	23110939774

Thus $z = \frac{n}{n + \frac{E[s^2(\theta)]}{\text{var}[m(\theta)]}}$ is given by

$$z = 0.941664748$$

We then find credibility premiums using the linear function

$$cp_i = z\bar{X}_i + (1-z)\mu$$

Table 4.4 Buhlmann's Results

Collective premium: 545097.23

Within contract variance: 23110939774

Between contract variance: 74612719170

Contract	\bar{X}_i	z_i	Cred premium
1	872155.58	0.941664752	853076.5498
2	654116.64	0.941664752	647756.9654
3	436077.8	0.941664752	442437.4752
4	218038.88	0.941664752	237117.9097

Table 4.5Buhlmann-Straub data

Contract	Aug		Sept		Oct		Nov		Dec	
	Y8	P8	Y9	P9	Y10	P10	Y11	P11	Y12	P12
1	829245.2	34	983204.4	41	814627.6	34	1166578.6	48	567121.6	23
2	621933.9	26	737403.3	30	610970.7	25	874934.1	36	425341.2	18
3	414622.8	17	491602.2	20	407313.8	17	583289.4	24	283560.8	12
4	207311.3	9	245801.1	10	203656.9	8	291644.7	12	141780.4	6

Table 4.6Weights, $X_{ij} = Y_{ij}/P_{ij}$

X8	X9	X10	X11	X12
24389.56471	23980.59512	23959.63529	24303.7208	24657.46087
23920.53462	24580.11	24438.828	24303.725	23630.06667
24389.57647	24580.11	23959.63529	24303.725	23630.06667
23034.58889	24580.11	25457.1125	24303.725	23630.06667

Table 4.7 credibility estimates

Contract	X8	X9	X10	X11	X12	$m(\theta_i)$	$s^2(\theta)$
1	24389.56471	23980.595	23959.6353	24303.72083	24657.46087	24258.19536	10542983.3
2	23920.53462	24580.11	24438.828	24303.725	23630.06667	24174.65286	14293635.9
3	24389.57647	24580.11	23959.6353	24303.725	23630.06667	24172.62269	8837183.98
4	23034.58889	24580.11	25457.1125	24303.725	23630.06667	24201.12061	28386568.4
	Average					24201.64788	15515092.9

$$\text{var}[m(\theta)] = \frac{19}{315}(3222389.737 - 3826587.809)$$

$$= -36443.69321$$

This imply that $z = 0$

$$\text{Thus } c.p_i = \bar{X}$$

Table 4.8 Buhlmann-Straub Results

Collective premium: 677645.9329

contract	$c.p_i$	P_{ij} for Jan	actual prem
1	24201.64788	50	1210082.394
2	24201.64788	31	750251.0843
3	24201.64788	22	532436.2534
4	24201.64788	9	217814.8309

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusion

This study has focused mainly on computation of credibility premiums by Buhlmann and Buhlmann Straub credibility theory. These methods are rather linear approximation techniques as opposed to techniques that are normally parametric in nature.

In the case of health insurance claims, there are the risks levels being the outpatient and in-patient entities. Our procedure looks at both sides of scenario using the data of Jubilee Insurance Limited Kenya.

Real data with full information or details is of high importance in determining the financial scenario of a company. In the medical sector of life insurance, detailed data is difficult to obtain. In general, data from many insurance companies in Kenya was difficult to obtain. Data from Jubilee Insurance Ltd was of short period and less information. The reason behind this is the oath of secrecy to hold on to information that is deemed ethically private in the medical sector. If real data with full information or details is observed, then the findings may be varied due to the different claims experience. In addition, real data claims amount is inclusive of expenses like administration and commission costs that may have been incurred. Also the amount of claim may contain so errors because some claims that may be made in one month may not be paid till the next month. The claim amount is recorded in the exiting month while the claim number is in the correct month of entry of claim.

Health and age are very important factors in determining the cost of health insurance. An individual health may occur seasonally since it is a variable of time and so claim experience may

vary highly at different seasons. These fluctuations may lead to inflated premium amount. Age consideration in the premium computation is very vital for it helps in obtaining accurate credibility premium.

The Buhlmann and Buhlmann-Straub procedures is faced the problem of outliers which distort the mean and variance functions. This will in turn affect the accuracy of the credibility premium.

5.2 Recommendation

- This study recommend that the data given by any insurance firm for the purpose carrying out research should contain a reasonable information in order to get good if not best results.
- It further recommends that data should be smoothened of any outliers in order to increase accuracy of the credibility premiums.

5.3 Limitation

- There were limited information/details of the data.
- The data used was of a short period of time which is not really good since for more accurate credibility premium, a large data is required.

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