DURATION DEPENDENCE IN STOCK PRICES: A DURATION ANALYSIS OF BULL AND BEAR MARKETS ON THE NAIROBI STOCK EXCHANGE.

ANDATI SAMUEL OBULEMIRE.

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DEPARTMENT OF ECONOMICS

UNIVERSITY OF NAIROBI

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DECLARATION

This research project is my original work and has not been presented for any degree award in any University.

Signed: ........................................... Date: 17-9-2004

Andati Samuel Obulemire.

This research paper has been submitted for examination with our approval as University supervisors.

Dr. Rose W. Ngugi
Signed: ........................................... Date: 17/9/2004

Mr. R.M. Kabando
Signed: ........................................... Date: 03/02/05

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You took me to High School and encouraged me to read.

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2.0 LITERATURE REVIEW

2.1 INTRODUCTION

The behaviour of asset prices has always been at the centre of academic, media and business attention. One issue that attracts the attention of financial analysts and policy makers is whether the emerging stock markets exhibit similar general characteristics, regarding the distribution behaviour of stock returns as developed stock markets. Specifically, the characteristics of interest are those predicted by Fama (1972) in the Efficient Market Hypothesis (EMH). Fama argues that there are two approaches to predicting stock prices that are commonly espoused by market professionals. These are “chartist” or “technical” theories and the theory of fundamental/intrinsic value analysis.

Chartist techniques attempt to use the knowledge of the past behaviours of a price series to predict the probable future behaviour of the series. The techniques of the chartist have always been surrounded by a certain degree of mysticism, however, and as a result market professionals have found them suspect. Thus, it is probably safe to see that the pure chartist is rare, among stock market analysts. Rather, the typical analyst adheres to fundamental analysis technique. The assumption of the fundamental analysis approach is that at any point in time an individual security has an intrinsic value (or in the terms of the economist, an equilibrium price) which depends on the earning potential of the security. The earning potential of the security depends in turn on such fundamental factors as quality of management, outlook for the industry and the economy. Through a

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*Market efficiency is defined at three levels: weak form efficiency, semi-strong efficiency and strong-form efficiency.*
careful study of these fundamental factors, the analyst should in principle be able to
determine whether the actual price of a security is above or below its intrinsic value. If
actual prices tend to move toward intrinsic values, then attempting to determine the
intrinsic value of a security is equivalent to making a prediction of its future price; and
this is the essence of the predictive procedure implicit in fundamental analysis. Thus the
EMH predicts that expected return is unpredictable from the past returns or other past
proxy variables. It is argued that the best forecast of returns is its historical mean; and the
deviations of the expected value of return are equal to zero, Fama (1991).

Contrary to EMH predictions, empirical results mainly based on developed stock markets
indicate that stock returns are predictable, non-normally distributed, while second
moments exhibit dependence, Fama (1991). In addition, stock price changes have been
shown to respond asymmetrically to shocks. However, it is debatable whether a
stochastic or chaotic process generates stock prices. The debate also considers whether
predictability is a response to irrational bubbles as rational swings in expected returns, or
a spurious effect (Fama & French, 1988; and Fama, 1991). Fama (1991) also notes that
stock prices adjust to firm-specific information including investment decision, dividend
changes, changes in capital structure or financing decisions and changes in corporate
control transactions. However, it is not clear why returns portray a high dispersion around
the event period. While most of the literature is based on developed stock markets little is
known about the behaviour of stock returns in emerging markets especially during the
evolution and growth of these markets.
2.2 THEORETICAL LITERATURE

2.2.1 THEORIES ON PREDICTABILITY OF STOCK RETURNS

Random Walk Theory

This is based on the notion that current price of a security fully reflects available information. This implies that successive price changes are independent and identically distributed with constant mean and volatility. The independence of the price changes implies that the random walk is also a fair game, but in a much stronger sense than the martingale: independence implies not only that changes are uncorrelated, but that any nonlinear functions of the changes are also uncorrelated. Most simply, the theory of random walk means that a series of stock price changes has no memory – the past history of the series cannot be used to predict the future in any meaningful way. The future path of the price level of a security is no more predictable than the path of a series of cumulated random numbers. A random walk process with a drift captures the permanent fluctuations in stock prices. It is unlikely that the random walk hypothesis provides an exact description of the behaviour of stock market prices. For practical purposes, however the model may be acceptable even though it does not fit the facts exactly. The martingale condition unlike random walk does not restrict the nonlinear dependence of second moments, so that with a martingale process, stock price changes are unpredictable while the second moment could exhibit dependence like in an ARCH process. Thus, literature on predictability of the stock returns tests for the stochastic properties of stock returns, which could be a random walk process or martingale process and not violate the EMH.
Mean Reversion Hypothesis

Fama & French (1988) propose long horizon returns analysis to capture the mean reverting component of prices i.e., regression based mean reversion tests. Tests of the predictability of returns based on the mean reversion hypothesis captures the slowly decaying temporary component of stock prices that portray the long temporary swings that prices take away from fundamental values. Cochrane (1988) on the other hand uses variance ratio of long differences which measures the size of random walk component in a series. The variance ratio at lag $k$ is defined as the $1/k$ times the variance of $k$-period return divided by the variance of the one period return. If stock prices are a random walk process, then the variance of the $k$-period returns are equal to $k$-times the variance of one period return. If the variance ratio is less than one, then negative correlation is implied, while a variance ratio greater than one indicates positive series correlation. A pure stationary process is reflected when the variance ratio approaches zero.

The regression-based test considers the autocorrelation function over increasing return horizons. Mean reversion is inferred from positive autocorrelation for short-run horizons and negative autocorrelation for long-run horizons. Empirical test of mean reversion using the variance ratio and the regression-based approach is constrained by their demand for long data set. The variance ratio, for example, requires $k$ to be large in order to capture mean reversion over the long horizon (Richards, 1996). Studies such as Fama & French (1988) revert to using overlapping data to estimate the regression based model. This, however, makes it difficult to make inferences from the $t$-statistic because the
approximating asymptotic distribution performs poorly (Gallagher, 1999). The implication is that the short data set for emerging markets limits the choice of the method to test for predictability of stock returns.

The mean reversion hypothesis is also tested using vector autoregressive (VAR) methods of decomposing the stock price into temporary and permanent components. For example Gallagher (1999) applies a technique developed by Blancard and Quah (1989) to decompose the temporary and permanent component of stock process in a multivariate time series context. The model identifies permanent and temporary shocks by imposing long-run restrictions on the VAR. Gallagher (1999) uses real consumer prices and real stock prices in the estimation. Lee (1998) identifies the fundamental variables (i.e., earnings, dividends and discount factors) and non-fundamental variables and then decomposes each variable into temporary and permanent components, and infers their contribution to temporary and permanent component of stock prices.

Volatility Clustering and Persistence Theories

Engle (1982, 2001) presents the ARCH model which specifies the conditional variances as a distributed lag over past squared innovations that measure time-varying volatility. Although the model captures volatility clustering, it has only one memory period; there is difficulty in selecting the optimal lag length and ensuring the non-negativity of the coefficients of the conditional variance. Due to the above limitations of ARCH (p) model, Bollerslev (1986) presents a generalized version, the GARCH (p, q) with a lagged
conditional variance to introduce long memory to the ARCH (p) model. The GARCH (p, q) model captures not only volatility clustering but also the persistence of volatility over time. The sum of volatility clustering and persistence parameters is expected to be equal to one and current shocks persist indefinitely in conditioning future variance. If the sum is greater than one the system is defined as explosive and volatility increases over time (Choudhry, 1996; De Santis et al., 1998; Koutmos, 1999). If the sum is less than but closer to unity, it implies that shocks to volatility are more persistent and therefore have a slower decaying rate. This model is viewed as a reduced form of more complicated dynamic structure for time varying conditional second order moments.

The GARCH model assumes martingale conditions, depicting unbiased expectations and clustering tendency of volatility. If we assume that the conditional variance influences the mean return ex ante, the GARCH-M model is used. Conditional mean is expressed as a function of the conditional variance as illustrated by Choudhry (1996), Fraser & Power (1997), Al-Loughai & Chappel (1997), Henry (1998) and Poshakwale & Murinde (2000). The GARCH model also imposes symmetric response of stock prices to shocks which may not be appropriate for modelling and forecasting stock return volatility.

**Asymmetric Response Theories**

There are arguments that stock prices respond asymmetrically to shocks. For example, Nelson (1991) argues that the sign on returns influences the future volatility being negatively correlated with the direction of actual price changes. Black (1976) and Christie (1982) point out that stock returns tend to be negatively correlated with changes in volatility, so that a reduction in the equity value of the firm raises its debt to equity
ratio, hence raising the riskiness of the firm, as manifested by an increase in future volatility. If returns are less than expected, they tend to increase future volatility, and if higher than expected, they tend to decrease future return volatility. Glosten et al. (1993) observe that investors may not require a high risk premium if the risky time periods coincide with periods when investors are better able to bear particular types of risks. Again, if the future seems risky, the investor may want to save more in the present, thus lowering demand for larger premiums. If transferring income to the future is risky and the opportunity of investment in a risk-free asset is absent, then the price of a risky asset may be raised considerably, reducing the risk premium. Thus, it is possible to have a positive and a negative relationship between current returns and current variance. The versions of GARCH that are used to capture asymmetric response of conditional variance to different shocks include, for example, the exponential GARCH (or EGARCH) by Nelson (1991), generalized quadratic ARCH (or GQARCH) by Sentena (1992), and the Glosten, Jaganathan & Runkel (GJR) model as in Glosten et al. (1993).

Bollerslev & Mikkelsen (1999) on the other hand find evidence that long-run dependence of stock market volatility is best described by a slowly mean reverting fractionally integrated GARCH process (or FIEGARCH). These models have been applied to emerging stock markets. Henry (1998) finds that GQARCH model is better than other GARCH versions in explaining volatility in Hong Kong market. Kuotmos (1999) and De Santos & Imrohorolus (1997), however, note that Gaussian GARCH does not account for the leptokurtosis characteristics of stock returns, a characteristic that emerging markets' stock returns exhibit. In addition, the assumption of a martingale process by the GARCH model implies no autocorrelation. However, given thin trading, emerging markets are
characterized by autocorrelation potentially induced by non-synchronous trading. De Santis & Imrohorolus (1997), Kim and Singal (2000), Yadav et al. (1999) and Papachristou (1999), model the conditional mean, with a lagged value to take care of autocorrelation induced by infrequent trading.

Chaotic Models

Following some results that stock returns have non-linear behaviour, various studies test the proposition that non linearity portrays a chaotic dynamic process (See for example, Serletis & Soundergard, 1996; Yadav et al., 1999 and Barkoulas & Travlos, 1998). The argument is that stock returns may not follow a stochastic process, but portray deterministic chaos. The presence of chaos indicates possibility of improved short-term but not long-term predictability, implying that a profitable non-linear trading rule would exist at least in the short-run. Tests of deterministic chaotic dynamics are based on calculating correlation integrals corresponding to different embedding dimensions, while the BDS statistic is used to infer the deterministic chaotic dynamics. ⁹

2.2.2 THEORIES ON DURATION DEPENDENCE

The theoretical literature has examined two strands of thought on duration dependence. The first and main strand of literature examines the NBER reference cycle turning point dates. Employing a battery of nonparametric tests, Diebold & Rudebusch (1990) conclude that both duration independence in expansions and duration dependence in contractions are, by and large, consistent with the data. Diebold et al., (1993), estimating

⁹ To see how chaotic dynamics are measured refer to Serletis & Soundergard (1996).
a parametric hazard model using NBER reference cycle turning points, find significant duration dependence in post-war contraction and pre-war expansions.

A second strand of literature finds its basis in regime switching time series models. In his pioneering work, Hamilton (1989) estimates a two-state Markov chain model of output growth, where the two states are interpreted as expansions and contractions. His Markov chain assumes duration independence; as the current phase of the business cycle ages, the probability of moving into the alternative phase remains constant. Compared to the traditional models, there are at least three advantages of this model. First, the model sorts data endogenously into regimes. Second, unlike the traditional models, in this latent variable model, we do not need to assume the information of the researcher, i.e., the econometrician, coincides with that of the agent in the market. Thirdly, empirically, among other studies, Ryden et al. (1998) show that the Markov switching model can explain the temporal and distributional properties of stock returns. The model has drawn a lot of attention in modelling structural changes in dependent data. In economics, it has been used to identify business fluctuations, see Hamilton (1989); to study the changes in real interest rates, Garcia (1998). Recently, the model has been used extensively in finance area especially to model the nonlinear structure in time series data. Turner et al. (1989) use the model to explain the time-varying risk premium in stock returns. Methodologically, they consider a Markov switching model which allows either the mean or the variance or both to differ between the two regimes. However, Hamilton (1989) first order Markov switching model would not capture duration dependence in states. Ignoring duration dependence, results in a failure to capture important properties of stock returns.

Specifically, some research has been conducted on identifying market states. Borrowing the method from Turner et al. (1989), Schaller and van Norden (1997) find strong evidence for switching behaviour in stock returns in the US market. By allowing for switch in both means and variances, they discover two distinct states: in one state, the excess returns are low and the variance is low; in the other state, the excess return goes to negative and the variance is high. The results agree with Brock et al. (1992) and Maheu and McCurdy (2000a).

Pagan & Sossounov (2000) considers a definition of a bull and bear states based on cumulative price changes. Their study characterises movement in stock prices through the average duration and amplitude of bull and bear markets. The Pagan-Sossounov dating method is based on a modification of the Bry-Boschan algorithm and seeks out local peaks and troughs within a predetermined window of data points subject to a set of censoring rules that restrict both the minimum length of the bull/bear market (4 months).
as well as the minimum duration of a full cycle (16 months). The focus on local peaks and troughs will allow us to concentrate on the systematic up and down movements in stock prices and to filter out short-term noise. This is an important consideration for data as noisy as daily stock price changes. While some arguments can be made in favour of imposing an additional minimum duration constraint, this also adds an extra layer of complexity and means that the data has to be followed through a recursive pattern recognition algorithm, as explained by Pagan and Sossounov (2000). Instead this approach will model both short and long durations, but allows the hazard rate to differ across durations. The filter in our study, however, does not use restrictions on the minimal length of the bull/bear states but requires choosing the value of the initial state and setting threshold values for the size of cumulative movement in stock prices that trigger a switch between these states.

2.3 EMPIRICAL LITERATURE

Ngugi et al (2000) provides a summary of empirical results from both the developed and developing markets that show contrasting evidence on stock return predictability. For example, they show that Al-Loughani & Chappell (1997) testing for the implied assumption of random walk series where residuals are assumed to be IID variables found out that their results are consistent with the random walk hypothesis with the series stationary at first difference. However, the residuals do not suggest existence of some unexplained structure in the data. After modelling the stock price series as a GARCH (1, 1), the BDS test confirms IID residuals. They concluded that weak form efficiency is not

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10 To see the Pagan-Sossounov dating method refer to appendix B of their paper.
tenable for UK. However, Chelly-Steeley & Pentecost (1994), by controlling for firm size, find that small firms in the UK market are inefficient while large firms are efficient.

Lee et al. (1998) reject the EMH with respect to non-European countries including Canadian and US financial markets. The results are consistent with the findings by Lee (1998) who uses the mean reversion hypothesis on Standard Poor’s composite stock price (US), that stock returns are predictable. Fama & French (1988) used the regression-based method to demonstrate a negative autocorrelation for NYSE market found similar results.

Gallagher (1999) using VAR method finds support for the mean reversion hypothesis in various developed stock markets indicating predictability of stock prices. However, some studies fail to confirm mean reversion in emerging stock markets, for example in Korea (see Titman & Wei, 1999; De Santos & Imrohorolus, 1997) and Malaysia (De Santis & Imrohorolus, 1997). Stock returns show non-normal distribution for both the developed and developing stock markets (See Bekaert & Harvey, 1997; De Santis & Imrohorolus, 1997; Choudhry, 1996). This supports the general view that emerging markets may be characterized by non-normal distribution (Richards, 1996) and points to similarities in distribution of returns for both the developed and developing markets.

Volatility clustering is also evident in both the developed and developing markets. De Santis & Imrohorolus (1997), using GARCH model and assuming generalized error distribution (GED) of the conditional density function, show predictability, clustering and

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persistence in conditional volatility of returns in emerging markets. Similarly, Fraser & Power (1997), and Choudhry (1996) find evidence of volatility clustering for both developed and emerging markets. Yadav et al. (1999) find that stock returns exhibit significant non-linear dependence in UK, as in US market, and conclude that differences in institutional arrangements do not affect the time series dynamics stock returns.

Evidence from both the developed and developing markets also shows that stock returns response to shocks are asymmetric (see, for example, Kuotomos, 1999; and Fraser & Power, 1997). Kuotomos (1999), tests for asymmetric response in stock prices in five emerging markets including Korea, Singapore, Malaysia, Philippines and Taiwan. The EGARCH model is applied assuming GED distribution to take care for the leptokurtic standardized residuals obtained from ARCH-type models. Results show asymmetric response of stock returns to past information. Fraser & Power (1997) studying the Pacific Rim, UK, and US markets also document substantial asymmetries in the dynamics of price changes both within and across markets in developed markets. The findings by Shields (1997), however, demonstrate non-existence of asymmetric response in emerging markets.

Other studies show evidence of time-varying risk premium. Fraser & Power (1997) find a significantly negative coefficient for Malaysia investors, which they interpret as showing that investors in Malaysia are predominantly risk lovers. Choudhry (1996) using the GARCH-M model confirms no time varying risk premium in several emerging markets and where it is significant the sign is negative, indicating risk averse investors.
Song et al (1998) use GARCH models to analyse the relationship between returns and volatility in the Shanghai and Shenzhen Stock Exchanges in China and find that there exists volatility transmission between the two markets (the volatility spill-over effect). Similarly, Booth et al (1997) show evidence of price and volatility spill-overs, among the Danish, Norwegian, Swedish and Finnish stock markets. The impact of good news (market advances) and bad news (market retreats) is described by an EGARCH model. Volatility transmission is asymmetric, spill-overs being more pronounced for bad than good news. Significant price and volatility spill-overs exist but they are few in number. Studies that have looked at the chaotic response of stock returns show weak support for both, the developed, and developing markets, an indication that stock returns are generated by a stochastic process (see Barkuolas & Travlos, 1998 and Yadav et al. 1999).

Generally, various factors are identified to influence the distributional characteristics of returns. Fama & French (1988) observe that the slowly decaying price component could be explained by models of irrational market in which stock prices take long temporary swings away from fundamental values, and time varying equilibrium expected returns generated by rational pricing in an efficient market. But as noted by Fama (1991), factors behind the predictability of returns are not conclusive as to whether predictability indicates irrational bubbles in prices or large rational swings in expected returns.

Ferson & Harvey (1991) attribute predictability to economic variables. They use a multi-beta asset pricing model with macroeconomic variables, including an unexpected
inflation, consumer expenditures, and interest rates that proxy for risk factor in the stock market. Their results indicate that most of the predictable variation in asset returns can be explained by shifts in the assets' risk exposures (beta) and by shifts in the market's compensation for holding these exposures (risk premiums). Both betas and risk premiums change predictably over time. The stock market risk premium is, however, found to be the most important for capturing predictable variation of the stock portfolios. The evidence suggests that investors rationally update their assessments of expected return. Thus predictability is associated with sensitivity to economic variables. Reichenstein & Rich (1993) show a more consistent relationship between risk premium and S & P stock returns than either dividend yield or earnings - price ratio. They conclude that risk premium predicts long-horizon stock returns more than other variables as it mirrors movement in the unobservable market risk premium.

Gallagher (1999) observes that deviation of the market value of stock from their fundamental values, with a reversion to their mean, could be explained by such theories as noise trading, limited arbitrage, fads and speculative bubbles. Fisher (1996), Scholes & Williams (1977), Kuotomos (1999) attribute the presence of serial correlation in returns to non-synchronous trading in portfolios of small stocks and thin markets. Papachristou (1998) and De Santis & Imrohorolus (1999) introduce a lagged return variable in the conditional return model to capture serial correlation induced by thin trading. However, Cochrane & De Fina (1995) and Lee (1998) attribute predictability of stock returns to the activities of noise traders and inefficiencies in pricing of securities. Lee (1998) concludes that predictability of excess stock returns is a fad rather than a bubble factor. However,
Shefrin & Statman (1994) deduce existence of price efficiency in the presence of noise traders, using a behavioural theory of capital asset prices and the volume of trade.

Durland & McCurdy (1994) introduce duration dependence into the Hamilton model by allowing the transition probabilities to depend upon the age of the current phase of the business cycle. They developed a parsimonious implementation of a high-order Markov chain that allowed state transition probabilities to be duration dependent. They called the model duration dependent Markov switching (DDMS) model. In that model duration influenced the conditional mean through the hazard functions. That is, duration determined the persistence of state specific conditional mean by influencing when the states are switched. Analysing GNP growth in the US, they inferred duration dependence in post-war GNP growth rates using the estimated relationship between the transition probabilities and the age of the current phase. They find that as a contraction ages the probability of moving into an expansion increases and that this increase is significant.

McQueen & Thorley (1994) study speculative bubbles that take the form of sequences of small positive abnormal returns interrupted by rare but large negative abnormal returns in a crash state. Their bubble implies that the probability that a run of positive abnormal returns comes to an end declines with the length of the sequence. In empirical tests on monthly stock returns over the period 1927-1991, they find evidence of negative duration
dependence (declining hazard\textsuperscript{11}) for positive runs (high returns) while there appears to be no duration dependence in negative runs (low returns). Thus, they showed that a rational stochastic bubble will display negative duration dependence. Data limitations mean that they consider runs of at most six months’ duration. This study would use high frequency daily data from 2/1/1992 to 30/6/2004 that allows us to consider hazard rates at both much shorter and longer durations. This study is also different from the other in that it sorts stock prices probabilistically into the alternative states (bull and bear market states).

Maheu & McCurdy (2000a) use DDMS model to capture nonlinear structure in both the conditional mean and the conditional variance of stock returns. The model sorts returns into a high return stable state and a low return volatile state. The model is an extension of Durland & McCurdy (1994) model. In addition to duration dependent hazards, duration also enters directly as a conditioning variable in both the mean and variance.\textsuperscript{12} Now, given persistence in a particular state, the conditional mean and variance can change with duration. This allowed them to investigate dynamic behaviour for the mean return and variance within each state. In addition to revealing some interesting state specific path dependence, this model also captures ARCH effects. They treat the state as unobserved variable and classify monthly stock returns into two latent states based on a Markov switching model extended to account for duration dependent state transitions. Their Markov switching approach endogenously identifies a high mean, low variance bull state and a low mean, high variance bear state. When the regime switching model is restricted

\textsuperscript{11} Declining hazard means that the probability of observing the end of a run of high returns will decline with duration.

\textsuperscript{12} Note that allowing volatility to vary with duration captures volatility clustering.
to only occur in the mean, in a "decoupled" model a state with a large negative mean return and a state with a small positive mean return are identified. Their empirical results find declining hazard functions (negative duration dependence) in both the bull and bear markets using monthly data from 1834-1995. This means that the probability of switching out of the state declines with the duration in that state. Despite the declining hazards, the best market gains come at the start of a bull market. That is, returns in the bull market state are a decreasing function of duration. Volatility in the bear market state, however, is an increasing function of duration. They identified four main explanations for negative duration dependence in stock returns. First, is the fundamentals themselves. If dividends are positively related to the business cycle, they are likely to display positive duration dependence. Secondly, a possible explanation for the declining hazards could be irrational investors, such as noise traders or fads. Both models allow stock prices to deviate from fundamental prices. Thirdly, the declining hazards found in all models could be interpreted as a momentum effect in the market. For example, as a bull market persists, investors could become more optimistic about the future and hence wish to invest in the stock market. This results in a decreasing probability of switching out of the bull market. Similarly, the length of the bear market could be related to the amount of pessimism about future returns by investors. This would lead to a substitution from equity into other expected high return instruments, such as the treasury bills. Finally, the evidence also shows that a rational stochastic bubble will display negative duration dependence.

A decoupled model is a model in which transition probabilities associated with the conditional mean are
Maheu & McCurdy (2000b) further study a model where regimes are present in the volatility but not in the mean. They use DDMS model, which is a discrete state stochastic volatility model which incorporates a parsimonious high-order Markov chain to allow for duration dependence. Their model is useful for capturing shifts and turning points in the volatility due to for example, policy changes or news arrivals that are difficult to accommodate with the ARMA structure implicit in the GARCH. Their model is also suited to exploiting the persistence associated with volatility clustering. This is achieved due to the important features of the specification of DDMS that include: First, the duration variable provides a parsimonious parameterization of potential high-order dependence. Secondly, unlike the GARCH case, persistence is permitted to be time varying by allowing the duration of a state to affect the transition probabilities. Thirdly, including duration as a conditioning variable in the conditional variance specification allows the model to capture a broad range of volatility levels. They also examined alternative benchmark models (MS-ARCH\textsuperscript{14} and GARCH models) to capture and forecast time varying volatility in weekly returns from exchange rates of Germany and UK currencies with the US dollar. They used a sample size of 1304, covering the time period from 2/1/1974 to 23/12/1998. Their findings indicate that DDMS parameterization provides a good description of both the unconditional and conditional distributions of foreign exchange returns, unlike the benchmark models.

\textsuperscript{14} MS-ARCH stands for Markov switching ARCH model.
2.4 LITERATURE OVERVIEW

The analysis of the predictability of stock prices has advanced over the period; the random walk, VAR, GARCH, EGARCH, GQARCH, FIEGARCH and duration dependence. The advancement is intended to capture the non-normally distributed returns, time varying volatility, asymmetries in volatility and small sample size problems. There is concern that a random walk model can be incorrectly rejected and therefore its rejection does not imply that there is no weak form efficiency. This suggests that other methodologies such as duration analysis technique in country specific case studies may provide adequate answers to the issues of the non-normally distributed returns, small sample size problems and time-varying risk premium.

The literature also points out that the studies of duration dependence in stock prices should be in a dynamic setting to capture time-varying state variables. These state variables capture risk factors and economic environment. It is also pointed out in the literature that studies on predictability of stock prices should capture market fundamentals such as dividends, price-earnings ratio, term structure variables in the presence of non-synchronous trading, noise traders both in developed and emerging markets. There are various studies carried out from north to south and empirical results from both the developed and emerging markets show contrasting evidence on stock return predictability and duration dependence as shown in table 1.
<table>
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<th>Market</th>
<th>Non-normal distribution</th>
<th>Volatility Clustering</th>
<th>Time varying risk premium</th>
<th>Predictability of returns</th>
<th>Asymmetric response</th>
<th>Chaos</th>
<th>Duration dependence</th>
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**Table 1: A SUMMARY OF PREDICTABILITY TEST RESULTS AND DURATION DEPENDENCE**

**Source:** This table is largely borrowed from Ngugi et al (2000).

**GEND**

presence of the characteristic indicated, * = absence of the characteristic indicated

negative duration dependence in both the bull and bear markets, -0 = negative duration dependence for high returns and no duration dependence for low returns. BH = Bekaert & Harvey (1997);

*Dermis* & Travlos (1998); C = Choudhry (1996); DI = De Santis & Imrohorulus (1997);

*Fraser & Power* (1997); G = Gallagher (1999); K = Koutmos (1999); L = Lee (1998);

*McKee & McCurdy* (2000a); MT = McQueen & Thorley (1994); M = Muriu (2003);

*Nguki* (2002); PM = Poshakwale & Murinde (2000); R = Richards (1996); SS = Serletis & Sondergard (1996);

*Titman & Wei* (1999); Y = Yadav et al. (1999)
3.0 METHODOLOGY

3.1 THEORETICAL FRAMEWORK
Duration dependence in stock prices involves modelling time series dependence in stock prices that allow bull and bear hazard rates i.e., the probability that a bull or bear market spell terminates next period to depend on the age of the market. To define the spells, the underlying trend in stock prices is classified into bull and bear markets spells because durations of these spells are key components of the risk and return characteristics of the stock returns. So, it is important to understand durations of these market spells and their effects on hazard rate. Having information that stock prices have been in a particular state for a certain length of time affects the conditional distribution of the stock returns and would therefore help in determining optimal investment durations and performance.

The theoretical literature on duration dependence in stock prices suggests that there is duration dependence in stock prices in both the bull and bear markets. For instance, during the long bull market of the nineties in the US concern was often expressed that this bull market was at greater risk of terminating because it has lasted too long by historical standards. This indicates a belief that the bull market hazard rate depend positively on its duration. The opposite view is that the bull market gains momentum: the longer a bull market has lasted, the more robust it is and hence the lower its hazard rate, indicating negative duration dependence. On the other hand, as a bear market persists, investors could become more pessimistic about the future and lead to a substitution from equity into other expected high return instruments, such as treasury bills. This results in an increasing probability of switching out of the bear market.
The theoretical literature also suggests that interest rate levels or changes closely track state of the business cycle and appears to be a key determinant of the stock returns. Thus the relationship between interest rate levels or changes and hazard rate may be direct or indirect through the business cycle. Evidence also suggests that non-normally distributed returns and time-varying risk premium also affect duration distribution of returns. Hence, the study considers the age of the market spell, non-normally distributed returns, constant and time-varying interest rate levels/changes; constant or time-varying risk premium on the bull and bear market hazard rates.

The estimation of the discrete hazard rate has been carried out before using duration analysis technique with a complementary log log and logit model specifications, Sueyoshi (1995). This study adopted a logit link hazard model specification in capturing duration dependence in stock prices. This is because a logit link model ensures that the probabilities are in (0, 1) range, the model is convenient and easier to interpret as it gives estimates that are robust.

This study would follow closely studies by Maheu & McCurdy (2000a) and Pagan & Sossounov (2000). The study by Maheu & McCurdy (2000a) treats a bull or a bear market state as unobserved variable and classifies monthly stock returns into two latent states based on a Markov switching model extended to account for duration dependent state transitions. While in this study regimes are defined according to cumulative movements in stock prices and thus track local peaks and troughs in stock prices, their

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15 An extensive comparison of such link functions is found in Sueyoshi (1995).
Markov switching approach endogenously identifies a high mean, low variance bull state and a low mean, high variance bear state. Our definitions of bull and bear states focuses on the direction or drift of the stock prices and provides an observable bull market indicator. This allows us to sort stock prices probabilistically into bull and bear market spells and consider hazard rates at both much shorter and longer durations. The study by Pagan & Sossounov (2000) on the other hand, considers a definition of a bull and bear states based on cumulative price changes. Their study characterises movement in stock prices through the average duration and amplitude of bull and bear markets and imposes both the minimum length of the bull/bear market to 4 months as well as the minimum duration of a full cycle to 16 months.\textsuperscript{16} Our approach modelled both short and long durations and allowed hazard rate to differ across durations. Hence, adoption of both studies in developing the methodology would therefore help in analysing clearly the duration dependence in stock prices.

3.1.1 DEFINITION OF BULL AND BEAR MARKETS

Financial analysis and stock market commentators frequently classify the underlying trend in stock prices into bull and bear markets although there is no generally accepted formal definition of bull and bear markets in the finance literature. One of the few sources that attempt a definition of bull and bear markets is Sperandeo (1990) who defines bull and bear markets as follows: \textbf{“Bull market” - A long-term upward price movement characterized by a series of higher intermediate highs interrupted by a series}

\textsuperscript{16} To see the Pagan-Sossounov dating method refer to appendix B of their paper.
of higher intermediate lows. **Bear market** - A long-term downtrend characterized by lower intermediate lows interrupted by lower intermediate highs.”

In a more recent contribution, Chauvet and Potter (2000) offer a similar definition. To formalize the idea of a series of increasing highs interrupted by a series of higher intermediate lows, let \( I \) be a bull market indicator variable taking the value of one if the stock market is in a bull state at time \( t \), and zero otherwise. We assume that time is measured on a discrete scale and that the stock price at the end of period \( t \) is \( P_t \). Suppose that at \( t_0 \) the stock market is at a local maximum and define the stochastic process \( P_{t_0}^{\text{max}} = P_{t_0} \), where \( P_{t_0} \) is the stock price at time \( t_0 \). Let \( \lambda \) be a scalar defining the threshold of the movements in stock prices that trigger a switch between bull and bear markets. Also let \( \tau_{\text{max}} \) and \( \tau_{\text{min}} \) be stopping time variables defined by the following conditions:

\[
\tau_{\text{max}} \left( P_{t_0}^{\text{max}}, t_0, \lambda \right) = \inf\left\{ t_0 + \tau : P_{t_0 + \tau} \geq \left(1 - \lambda\right)P_{t_0}^{\text{max}} \right\},
\]

\[
\tau_{\text{min}} \left( P_{t_0}^{\text{max}}, t_0, \lambda \right) = \inf\left\{ t_0 + \tau : P_{t_0 + \tau} \leq \left(1 - \lambda\right)P_{t_0}^{\text{max}} \right\},
\]

where \( \tau \geq 1 \). Then \( \min(\tau_{\text{max}}, \tau_{\text{min}}) \) is the first time the price process crosses one of the two barriers \( \left(1 - \lambda\right)P_{t_0}^{\text{max}} \). If \( \tau_{\text{max}} < \tau_{\text{min}} \), we update the local maximum in the current bull market state:

\[
P_{t_0 + \tau_{\text{max}}} = P_{t_0 + \tau_{\text{max}}},
\]

and the bull market continued between \( t_0 + 1 \) and \( t_0 + \tau_{\text{max}} : I_{t_0 + 1} = \ldots = I_{t_0 + \tau_{\text{max}}} = 1 \).

Conversely, if \( \tau_{\text{min}} < \tau_{\text{max}} \) so that the stock price at \( t_0 + \tau_{\text{min}} \) has declined by a fraction \( \lambda \) since its local peak

\[
P_{t_0 + \tau_{\text{min}}} \lambda P_{t_0}^{\text{max}},
\]
then the bull market has switched to a bear market that prevailed from \( t_0 + 1 \) to
\[ t_0 + \tau_{\min} : I_{t_0 + 1} = \ldots = I_{t_0 + \tau_{\min}} = 0. \] In the latter case, we set \( P_{t_0 + \tau_{\min}}^{\min} = P_{t_0 + \tau_{\min}} \).

If the starting point at \( t_0 \) is a bear market state, the stopping times get defined as follows:

\[
\tau_{\min}(P_{t_0}^{\min}, t_0, \lambda) = \inf\{ t_0 + \tau : P_{t_0 + \tau} \leq P_{t_0}^{\min} \}, \quad (4a)
\]

\[
\tau_{\max}(P_{t_0}^{\min}, t_0, \lambda) = \inf\{ t_0 + \tau : P_{t_0 + \tau}(1 + \lambda)P_{t_0}^{\min} \}, \quad (4b)
\]

This definition of bull and bear states partitions the data on stock prices into mutual exclusive and exhaustive bull and bear market subsets based on the sequences of first passage times. It focuses on the direction or drift of the stock prices and provides an observable bull market indicator. The resulting indicator function, \( I_t \), gives rise to a random variable, \( T \), which measures the duration of bull and bear markets. This is simply given as the time between successive switches in \( I_t \).

Other studies considered a range of values for \( \lambda \). The smaller the value of this parameter is set at, the more bull and bear market spells we expect to see. This is likely to improve the power of our statistical tests as the sample size used in the duration analysis increases.

However, there are also limits to how \( \lambda \) can be set since too small values will lead the analysis to capture short-term dynamics in stock price movements. A value of \( \lambda = 0.20 \) is conventionally used in the financial press so we normally entertain this along with smaller values. However, in this study we will consider a filter size \( (\lambda = 10) \), expressed in percentage terms, since we are dealing with an emerging market.
Naturally such a filter is related to a long literature on technical trading rules that models local trends in stock prices, Brock et al. (1992), Brown et al. (1998) and Sullivan et al. (1999). However, the similarities between technical trading rules and duration measures are only superficial. Technical trading rules search for patterns in prices conditional on a time horizon that is typically quite short. For example, the value of a 100-day moving average of prices may be compared to the value of a 25-day moving average. In contrast, we do not condition on the time of a particular movement but instead explicitly treat this as a random variable whose distribution we are interested in modelling.

3.2 EMPIRICAL MODEL

Duration data needs to be characterized in terms of the conditional probability that the bull or bear state ends in a short time interval following some period \( t \), given that the state lasted up to \( t \). For the \( i^{th} \) duration, \( T_i \), this is measured by the discrete hazard function

\[
\lambda_i(t|X_i) = \Pr(T_i = t|T_i > t, X_i), \quad t = 1, \ldots, \Delta,
\]

This is the conditional probability of termination in the interval \([a_{i-1}, a_i)\) given that this interval was reached in the first place. \( X_i = \{x_{i1}, \ldots, x_{i\Delta}\} \) is a vector of additional conditioning information, which will depend on duration. Hypotheses on the probability that a bull or bear market is terminated as a function of its age are naturally expressed in terms of the shape of this hazard function. For example, the natural null hypothesis is that the duration of the current state does not affect the hazard rate.
The probability that a bull or bear market lasts for a certain period of time can still be derived from these hazard rates. This is given as the discrete survivor function, which measures the probability that a bull or bear market survives on the interval \([a_{i-1}, a_i)\):

\[
S_i(t|X_{it}) = \Pr(T_i > t|X_{it}) = \prod_{s=1}^{t} (1 - \lambda_i(s|X_{it})), \quad t = 1, ..., A.
\]  

(6)

Common choices of hazard models are the Probit, Logit and Double Exponential link. In the paper we use a Logit link, because it is empirically relevant and has nice properties i.e.

\[
\lambda_i(t|X_{it}) = F(x_i'\beta) = \frac{\exp(x_i'\beta)}{1 + \exp(x_i'\beta)}.
\]  

(7)

We consider two separate models for these hazard rates. The first is a static model that assumes that the underlying parameters linking the covariates or state variables to the hazard rate do not vary over time and that the covariates are fixed from the point of entry into a state. This makes our results directly comparable to the large literature on univariate dynamics in stock prices surveyed in Chapter 2 in Campbell et al (1997).

Under this assumption, the data takes the form of \(\{t_i, x_i; i = 1, ..., n\}\), where \(t_i\) is the survival time and \(x_i\) is a covariate (or state variable) observed at the beginning of the interval \([a_{i-1}, a_i)\). However, switches between bull and bear market states are likely to be caused by changes in the underlying economic environment. For example, the drift in stock prices may turn from positive to negative as a result of increased interest rates or worsening economic prospects. The effect of such covariates may well depend on the age of the current bull or bear market. To account for this possibility, our second model extends the setup and allows \(x_{it}\) being a vector that incorporates time-varying covariates.

Now the data for the \(i^{th}\) duration spell takes the form
Since the data is discretely measured, the covariates follow a step function with jumps at the follow-up times, \( a_t \). Within the interval \([a_{t-1}, a_t)\) the history of covariates

\[
X_{it} = (x_{i1}, x_{i2}, ..., x_{it})
\]

is allowed to influence the hazard rate \( \lambda_t(\cdot|X_{it}) \).

To allow for the possibility that the effect on the hazard rate of these covariates could depend on the age of the current state, we consider an episode splitting approach that allows the parameters of the time-varying covariates to be constant:

\[
\lambda_t(\cdot|X_{it}) = F(x_t; \alpha_t)
\]  

(8)

The vector \( \alpha_t = (\gamma_{0t}, \gamma') \) comprises both the baseline and the covariance parameters. We use the first-order random walk as our choice of transition equation determining the evolution in \( \alpha_t \):

\[
\alpha_t = \Phi \alpha_{t-1} + \xi_t, \quad \Rightarrow \left( \begin{array}{c} \gamma_{0t} \\ \gamma' \end{array} \right) = \left( \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right) \left( \begin{array}{c} \gamma_{0t-1} \\ \gamma'_{t-1} \end{array} \right) + \left( \begin{array}{c} \xi_{0t} \\ \xi'_{t} \end{array} \right)
\]

where \( \left( \begin{array}{c} \xi_{0t} \\ \xi'_{t} \end{array} \right) \sim N_{p+1}(0, \text{diag}(\sigma_0^2, \sigma_1^2, ..., \sigma_p^2)), \quad p = \text{dim}(\gamma'), \quad \alpha_0 \sim N_p(a_0, Q_0). \)

\( N_{p+1}(\cdot) \) is the \((p+1)\)-dimensional standard normal distribution, \( a_0, Q_0 \) are mean and variance matrices for the parameters, respectively. This random walk specification has the advantage of not imposing mean reversion on the parameters which are not allowed to differ across durations.

### 3.3 ESTIMATION PROCEDURE

The estimation of the static model would be carried out in two steps while in estimation of the dynamic model we add a third step. These steps are outlined as follows:
In the first step we would identify the bull and bear markets, their durations, and their respective hazard rates and hence survivor functions along the definitions as given in sections 3.1.1 and 3.2. To better illustrate the individual episodes we would plot the natural logarithm of the nominal stock price index against duration.

In the second step, to estimate both the static and dynamic models of duration dependence we would set up the log-likelihood function using notation from the literature on discrete choice models.\(^\text{17}\) Consider the following discrete indicator variable:

\[
y_{it} = \begin{cases} 
1, & \text{ith bull or bear market terminates in } [a_{s-1}, a_s) \\
0, & \text{ith bull or bear market survives through } [a_{s-1}, a_s)
\end{cases}
\]

\(s = 1, \ldots, t_i\), and \(i = 1, \ldots, n\). Each bull or bear spell, \(i\), thus generates a string

\[y_i = (y_{i1}, \ldots, y_{it}) = (0, \ldots, 0, 1), i = 1, \ldots, n.\]

Using this notation, the contribution to the likelihood function from the \(i^{th}\) observation is

\[
L_i \propto \prod_{s=1}^{t_i} \lambda_i(s|x_i) \left(1 - \lambda_i(s|x_i)\right)^{1 - y_{is}}.
\]

For every spell the bull or bear market lives through it therefore contributes to the likelihood with the survivor probability \(1 - \lambda_i(s|x_i)\). Summing across duration spells, the total log-likelihood function for the model \(\lambda_i(t|x_i) = F(x_i'\beta)\) is given by

\[
\ln L \propto \sum_{i=1}^{n} \sum_{s=1}^{t_i} y_{is} \ln(\lambda_i(s|x_i)) + (1 - y_{is}) \ln(1 - \lambda_i(s|x_i))
\]

An approach of treating the covariate parameters as fixed effects is only appropriate if the number of intervals is very small. In applications such as ours without enough intervals to apply continuous time techniques, maximum likelihood estimates of a large number of parameters in the hazard functions of an unrestricted hazard model can be expected to
have very poor sampling properties. To get around this problem, we follow Jenkins (1995) and adopt episode splitting to split survival time for each subject into sub-periods within which there is one period for each week at risk of failure. Thus we create multiple records for each subject with one record per sub-period. The episode splitting has two advantages. First, it ensures that duration data is in expanded form for ease estimation of the discrete hazard models since a correct likelihood function for each subject would be created. Second, the reorganised data format allows us to incorporate time-varying covariates into the analysis. To test for the goodness of fit of the model we would use the Hosmer-Lemeshow test statistic that tests the null hypothesis of goodness of fit against the alternative of no goodness of fit.

Finally, to estimate the dynamic model and shed light on how the hazard rates depend on the underlying state of the economy we will include interest rates and volatility as time-varying covariates. Interest rates have been widely documented to closely track the state of the business cycle and appear to be a key determinant of the stock returns at the monthly horizons. Interest rate levels \( i_t \), may be affected by a low frequency component and therefore might not contain the same information over a sample like ours, while interest rate changes, \( di_t \), are more likely to track business cycle variation across the full sample. For this reason we include both levels and changes in nominal interest rates and time-varying volatility, \( q_t \). So the set of covariates is \( X_u = (a, r, i_t, di_t, q_t) \).

\(^{17}\) For literature on discrete choice models see Greene (2000).

The hazard specification which allows time-varying interest rate and volatility effects to vary with the age of the current state is

\[ \lambda_i(t|x_i(t)) = F(\gamma_{0i} + \gamma_{1i}i_t + \gamma_{2i}d_i + \gamma_{3i}q_i) \] (13)

We will also consider the interest rate and volatility effects on the state (bull or bear) and on its age. To do this we consider the effects of interest rate level, interest rate changes and volatility on the hazards. In both cases the hazard rates will be computed as

\[ \lambda_i(t|x_{it}) = F(\gamma_{0i} + \gamma_{1i}i_t + \gamma_{2i}d_i + \gamma_{3i}q_i) = \frac{\exp(\gamma_{0i} + \gamma_{1i}i_t + \gamma_{2i}d_i + \gamma_{3i}q_i)}{1 + \exp(\gamma_{0i} + \gamma_{1i}i_t - \gamma_{2i}d_i + \gamma_{3i}q_i)} \] (14)

While the impact of changes in interest rate level, interest rate change and volatility on stock prices depends on

\[ \frac{\partial \lambda_i(t|x_{it})}{\partial i_t} = \gamma_{1i} \lambda_i(t|x_{it})(1 - \lambda_i(t|x_{it})) \] (15)

\[ \frac{\partial \lambda_i(t|x_{it})}{\partial d_i} = \gamma_{2i} \lambda_i(t|x_{it})(1 - \lambda_i(t|x_{it})) \] (16)

\[ \frac{\partial \lambda_i(t|x_{it})}{\partial q_i} = \gamma_{3i} \lambda_i(t|x_{it})(1 - \lambda_i(t|x_{it})) \] (17)

The duration dependence of these effects is indicated through their \( t \) subscripts.

Before estimating both static and dynamic models in bull and bear markets, each model is tested for frailty (unobserved heterogeneity) using likelihood ratio test. This is important because frailty model leads to misleading inferences about duration dependence and potentially misleading inferences about the effects of included explanatory variables. If there is frailty we correct for it before carrying out the estimation. Each model is also tested whether it fits the data correctly using Hosmer-Lemeshow test statistic. This ensures that the model fits the data correctly.
Having ascertained that the models are correctly specified and frailty is not a serious problem, we first tested for the predictability of stock returns and random walk model using autocorrelation test and unit root test. This is significant since it lays a foundation for interpretation of results. We then carry out estimation of static and dynamic models of both bull and bear markets. After estimating the model the following null hypotheses are tested using Z-statistic:

1. Duration of the current market spell does not affect the hazard rate in the bull or bear market.
2. Interest rate levels/changes do not affect the hazard rate in the bull and bear market.
3. Time-varying volatility does not affect the hazard rate in the bull and bear market.

3.4 MEASUREMENTS OF VARIABLES

As motivated by the literature review the following variables are used:

a) A binary dependent variable for discrete hazard model, dead.

b) A censoring indicator, terminated that is equal to one if the bull/bear market terminated and zero otherwise.

c) A time-varying spell week (natural logarithm of spell week identifier (t)), ln(t), that captures pure duration dependence.

d) Age (survival time in weeks) of the bull/bear market, a.

e) Non-normally distributed return, r, which captures mean reverting component in stock prices.
f) Interest rate level in percentage, $i$, that tracks behaviour of the business cycle and is assumed to be fixed.

g) Interest rate changes in percentage, $d_i$, which closely tracks the behaviour of the business cycle and is assumed to be constant.

h) Volatility in stock returns, $q$, which is assumed to be constant.

i) Time-varying interest rate change in percentage ($d_i$), tvdi2, which closely tracks the behaviour of the business cycle.

j) Time-varying volatility in percentage ($q$), natural logarithm of $q$, $\ln tvq$ that captures time-varying volatility or risk premium.

The above variables are captured in each of the bull and bear markets that are identified as per section 3.3 in the first step. Dead is used as a binary dependent variable while all other variables are explanatory variables.

3.5 DURATION DATA

To investigate the properties of bull and bear markets identified along the definition in section 3.1.1, we have constructed a data set of nominal daily stock index on the NSE from 1/2/1992 to 6/30/2004. We used the 90-day Treasury bill rate provided by the CBK. This rate is provided on a monthly basis and we converted it into a high frequency daily series by simply applying the rate reported for a given month to each day of that month.

Much of standard survival analysis in economics and finance assumes continuously measured data. However, since we use daily data and do not follow price movements continuously, our data is interval censored and the termination of our durations is only
known to lie between consecutive follow ups. Effectively the measurement of $T$, the
duration of bull and bear markets is divided into $A$ intervals.

$[a_0, a_1), [a_1, a_2), ..., [a_{q-1}, a_q), [a_q, \infty)$ where $q = A - 1$,

and only the discrete time duration $T \in \{1, ..., A\}$ is observed, where $T = t$ denotes
termination within the interval $[a_{r-1}, a_r)$. Although we draw on approaches from the
literature on economic duration data (see, e.g., Kiefer (1988), Kalbfleisch & Prentice
(1980) and Lancaster (1990)) this also means that we have to be careful in modifying the
standard tools from continuous time analysis.
CHAPTER FOUR

4.0 EMPIRICAL RESULTS

4.1.0 EFFECT OF POLICY CHANGE ON MARKET SPELLS AND STOCK RETURNS

Table 2a: Mean Number of Bull and Bear Markets and their Mean Durations under Different Policy Changes

<table>
<thead>
<tr>
<th>Part 1: Entry of Foreign Investors</th>
<th>Part 2: Entry of Foreign Investors and Change in Trading System from T+7 to T+5</th>
<th>Part 3: Change in Trading System from T+7 To T+5</th>
<th>Part 4: Entire Sample Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before Entry</td>
<td>After Entry</td>
<td>After Entry and Change</td>
<td>Before Change</td>
</tr>
<tr>
<td>Mean</td>
<td>1/2/92 to 12/31/94</td>
<td>8/1/95 to 6/30/04</td>
<td>1/2/92 to 7/31/00</td>
</tr>
<tr>
<td>No. of Bulls</td>
<td>4</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>No. of Bears</td>
<td>3</td>
<td>7</td>
<td>13</td>
</tr>
<tr>
<td>Duration of Bulls (weeks)</td>
<td>19.5</td>
<td>6.5</td>
<td>7.6</td>
</tr>
<tr>
<td>Duration of Bears (weeks)</td>
<td>10</td>
<td>17</td>
<td>17.8</td>
</tr>
</tbody>
</table>

From Table 2a, mean number of bulls increase and mean durations of these bulls fall after entry of foreign investors from 4 to 6 and 19.5 weeks to 6.5 weeks respectively. The number of bulls and their durations, however, increase to 12 and 7.6 weeks respectively, in the period characterized jointly by the entry of foreign investors and change in the trading system from t+7 to t+5. Thus, both entry of foreign investors and change in trading system increase the number of bull markets and the survival time of these market spells. The period characterized by a change in the trading system shows different trends with mean number of bulls and their durations declining from 10 to 6 and 11.7 weeks to 8.7 weeks, respectively. This means that a change in the trading system decreases bull market spells and their durations. The entire sample period though small as compared to
other emerging markets has a mean number of bulls and their mean durations of 16 and 10.6 weeks, respectively.

Also, table 2a shows that both the mean number of bears and mean durations of these bears increase after entry of foreign investors from 3 to 7 and 10 weeks to 17 weeks respectively. Thus entry of foreign investors increases both the number and survival time of bear markets. The number of bears and their durations also increase to 13 and 17.8 weeks respectively, in the period characterized jointly by the entry of foreign investors and change in the trading system from t+7 to t+5. Thus, both entry of foreign investors and change in trading system increase the number of bear markets and the survival time of these market spells. The period characterized by a change in the trading system only shows different trends with mean number of bears and their durations declining from 10 to 6 and 17.6 weeks to 15.6 weeks, respectively. This implies that a shift from t+7 to t+5 trading system only, decreases bear market spells and their durations as in the bull market. The entire sample period has a mean number of bears and their mean durations of 16 and 17.5 weeks, respectively. Thus from table we conclude that bear markets tend to be longer than bull markets, except before the entry of foreign investors.

The results of predictability and random walk tests for stock returns are shown in table 2b below. The results are corroborated by autocorrelation test, test for random walk model and unit root test. The test results are important as they help us in interpretation of results from hazard function estimates.
Table 2b: Autocorrelation, Random Walk and Unit Root Tests for Daily Stock Returns under Different Policy Changes

<table>
<thead>
<tr>
<th>Panel 1: Autocorrelation Test</th>
<th>PART 1: Entry of Foreign Investors</th>
<th>PART 2: Entry of Foreign Investors and Change in Trading System from T+7 to T+5</th>
<th>PART 3: Change in Trading System from T+7 To T+5</th>
<th>PART 4: Entire Sample Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before Entry</td>
<td>After Entry</td>
<td>After Entry and Change</td>
<td>Before Change</td>
<td>After Change</td>
</tr>
<tr>
<td>1/2/92 To 12/31/94</td>
<td>1/2/95 To 7/31/00</td>
<td>8/1/95 To 6/30/04</td>
<td>1/2/92 To 7/31/00</td>
<td>8/1/00 To 6/30/04</td>
</tr>
<tr>
<td>ρ(1)</td>
<td>0.149</td>
<td>0.172</td>
<td>0.299</td>
<td>0.289</td>
</tr>
<tr>
<td>Q-statistic</td>
<td>16.798</td>
<td>41.445</td>
<td>200.28</td>
<td>180.31</td>
</tr>
<tr>
<td>Probability</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel 2: Random Walk Test (r_t = \beta_1 + \beta_2 r_{t-1} + \epsilon_t); r is stock return, (\beta) is coefficient, (\epsilon) is error term</th>
<th>R-squared</th>
<th>0.400996</th>
<th>0.250694</th>
<th>0.206267</th>
<th>0.376809</th>
<th>0.154035</th>
<th>0.344618</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope coef. ((\beta_2))</td>
<td>-0.633243</td>
<td>-0.500799</td>
<td>-0.454170</td>
<td>-0.613867</td>
<td>-0.392410</td>
<td>-0.587045</td>
<td></td>
</tr>
<tr>
<td>Probability</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td></td>
</tr>
<tr>
<td>F-statistic</td>
<td>501.4097</td>
<td>466.0541</td>
<td>578.9895</td>
<td>1297.569</td>
<td>177.8941</td>
<td>1644.267</td>
<td></td>
</tr>
<tr>
<td>Prob. (F-stat.)</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>751</td>
<td>1395</td>
<td>2230</td>
<td>2148</td>
<td>979</td>
<td>3129</td>
<td></td>
</tr>
</tbody>
</table>

|----------------------------------------------------------------------------------|------------------------------|---------------------------------------------|--------------------------------|---------------------------------|-----------------------------|-----------------------------|-----------------------------|

Note: \(\rho(1)\) is the autocorrelation of first order and MacKinnon cv stands for the MacKinnon 1% critical value for rejection of hypothesis of a unit root.

Table 2b panel 1, reports autocorrelation, Box-Pierce Q statistic, and probability value for daily stock returns for entire and sub-periods. From the table returns for the period before entry of foreign investors has a first order negative serial correlation, which is significant as indicated by the Q-statistic and probability value. This indicates long-run predictability. The other periods characterised by the entry of foreign investors and/or change in the trading system have highly significant positive serial correlation confirming
short-run predictability of stock returns, thus rejecting weak form efficiency. The entire sample period has positive insignificant serial correlation. The sub-periods serial correlations demonstrate that the sign and significance of serial correlation is an artefact of the sub-period of the data influenced by the policy changes. Thus entry of foreign investors and the shift from $t+7$ to $t+5$ trading system increases predictability tremendously since the rejections of non predictability are slightly stronger. To the extent that such predictability has been a source of abnormal profits its increase is consistent with the fact that NSE being an emerging market is still uncompetitive.

To develop a sense of economic significance of serial correlations from table 2b panel 2, observe that the R-squared of regression of returns on a constant and its first lag is the square of the slope coefficient, which is simply the first order autocorrelation. Therefore an autocorrelation of 39.6% implies that 15.6816% of variation in daily returns is predictable using the preceding days return. The random walk model is rejected at 5% significance level for the entire period and all sub-periods as shown by the F-statistic and is associated probability value.

Unit root test results for temporary nature of shocks to returns from table 2b panel 3 indicated that we strongly reject unit root hypothesis at all significance levels as shown by the Phillips-Perron test statistic and MacKinnon 1% critical value for the rejection of hypothesis of a unit root. This implies weak form efficiency. Hence, we cannot conclude conclusively for weak form efficiency. Thus, despite the non-normally distributed returns the random walk model, which is a benchmark model for stock returns is rejected and we would therefore compare these results with those from the duration models.
4.1.1 IDENTIFICATION OF BULL AND BEAR MARKETS AND THEIR DURATIONS

Insight into how our definition partitions nominal stock prices into bull and bear spells is gained from figures 4a and 4b that uses the nominal stock price index to show the sequence of consecutive bull and bear market durations over the full sample period 1/2/1992-6/30/2004. These figures use a barrier, $\lambda$, of 10% that splits the sample into 16 bull and 16 bear markets. To better illustrate individual episodes, we plot in four separate windows the natural logarithm of the nominal stock price index. Many of the bull markets are very short; the shortest lasted for one week, lasting from 4/8/2004 to 4/21/2004. The longest bull market lasted for 38 weeks, lasting from 1/26/93 to 2/11/94. The bear markets are long; the longest lasted for 121 weeks, lasting from 1/13/1999 to 5/27/2002.

The longest bull market is followed by short bear markets while the longest bear market is followed by short bull markets. On average, bear markets last longer than bull markets. This is because there is relatively higher volatility in the bull markets than bear markets. Also, the Nairobi Stock Exchange is an emerging market as characterised by low market size, low turnover ratio and few listed companies. This infact implies that the market is relatively inefficient since the economy has been led by bank driven growth and not market driven growth. High volatility in the bull markets indicates that they are at a greater risk of termination and thus investors taking into the trade-off between risk and return prefer to invest in risk-free securities that are also high yielding. This further explains why bear markets survive longer than bull markets.
FIGURE 4A: BULL AND BEAR MARKETS BASED ON NOMINAL NSE STOCK PRICE INDEX


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FIGURE 48: BULL AND BEAR MARKETS BASED ON NOMINAL NSE STOCK PRICE INDEX
4.1.2 DESCRIPTIVE STATISTICS FOR BULL AND BEAR MARKETS

Tables 3a and 3b present descriptive statistics for the distribution of bull and bear market durations and other economic variables. Properties of the bull and bear market states are reported in weeks although it should be recalled that our analysis was carried out using daily data. In this study we focus on the 10% filter. With a filter of 10% the mean bull market duration is 10.5625 weeks against 17.5 weeks for bear market durations. The corresponding median values are 6.5 and 8.5 weeks for bull and bear markets, respectively. While the shortest bull and bear markets each lasted for only a week, the longest bear market, at 121 weeks, lasted more than three times longer than the longest bull market (38 weeks). Partly as a result of this, the dispersion of bear market durations is about three times greater than that of bull markets. Overall, the stock market spends roughly two thirds of the time in the bear state and one third in the bull state.

To see how much returns vary across bull and bear states, Tables 3a and 3b also reports return statistics for these states. Mean returns are 0.004847 and -0.0037409 per week in bull and bear markets, respectively. A larger asymmetry shows up in the median return which is 0.0052798 and -0.0023671 per week for bull and bear markets, respectively. Although bull states last much shorter than bear markets, the upward drift in bull markets is thus stronger than the downward drift during bear markets. On a volatility basis the standard deviation of returns is lower in bear states (0.0061669) and higher in bull states (0.0088495). This in overall identifies a high mean return, high volatile bull market state and a low mean return, low volatile bear market state.

19 These figures are computed as the mean return per bull or bear market converted into a weekly number.
### Table 3a: Summary Statistics for Bull Market

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Mean</th>
<th>Median</th>
<th>Standard Deviation</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bull Market Durations(weeks)</td>
<td>10.5625</td>
<td>6.5</td>
<td>10.15197</td>
<td>1</td>
<td>38</td>
</tr>
<tr>
<td>Interest Rate (%)</td>
<td>17.39338</td>
<td>17.81202</td>
<td>13.83891</td>
<td>1.15827</td>
<td>55.51869</td>
</tr>
<tr>
<td>Interest Rate Change (%)</td>
<td>-0.2557883</td>
<td>-0.1181175</td>
<td>1.260539</td>
<td>-4.110706</td>
<td>1.995139</td>
</tr>
<tr>
<td>Stock Market Return</td>
<td>0.004847</td>
<td>0.0052798</td>
<td>0.0031134</td>
<td>0.0012955</td>
<td>0.0130378</td>
</tr>
<tr>
<td>Volatility</td>
<td>0.0088495</td>
<td>0.007502</td>
<td>0.005912</td>
<td>0.0035695</td>
<td>0.02713</td>
</tr>
</tbody>
</table>

### Table 3b: Summary Statistics for Bear Market

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Mean</th>
<th>Median</th>
<th>Standard Deviation</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bear Market Durations(weeks)</td>
<td>17.5</td>
<td>8.5</td>
<td>28.73094</td>
<td>1</td>
<td>121</td>
</tr>
<tr>
<td>Interest Rate (%)</td>
<td>14.8855</td>
<td>17.75148</td>
<td>9.336178</td>
<td>1.4582</td>
<td>28.64064</td>
</tr>
<tr>
<td>Interest Rate Change (%)</td>
<td>-0.1903293</td>
<td>0.0833894</td>
<td>1.083755</td>
<td>-3.262706</td>
<td>1.54849</td>
</tr>
<tr>
<td>Stock Market Return</td>
<td>-0.0037409</td>
<td>-0.0023671</td>
<td>0.0033098</td>
<td>-0.0100106</td>
<td>-0.0005934</td>
</tr>
<tr>
<td>Volatility</td>
<td>0.0061669</td>
<td>0.00562</td>
<td>0.00232</td>
<td>0.0034873</td>
<td>0.0113365</td>
</tr>
</tbody>
</table>

The correlation matrix in table 4a for the bull market below shows that interest rate level is negatively correlated with stock market returns while interest rate change is positively correlated with the stock market returns. This means that in the bullish market, when interest rate levels are moving upward the stock returns tend to fall since investors substitute treasury bills for equity. The interest rate changes, however, move in the same direction with stock returns.

### Table 4a: Correlation Matrix for Bull Market

<table>
<thead>
<tr>
<th></th>
<th>Bull Market Durations</th>
<th>Interest rate level in %</th>
<th>Interest rate change in %</th>
<th>Stock Market Return</th>
<th>Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bull Market Durations</td>
<td>1.000000</td>
<td>0.505096</td>
<td>0.186542</td>
<td>-0.261188</td>
<td>0.498358</td>
</tr>
<tr>
<td>Interest rate level in %</td>
<td>0.505096</td>
<td>1.000000</td>
<td>0.198743</td>
<td>-0.010878</td>
<td>0.527217</td>
</tr>
<tr>
<td>Interest rate change in %</td>
<td>0.186542</td>
<td>0.198743</td>
<td>1.000000</td>
<td>0.143915</td>
<td>0.321022</td>
</tr>
<tr>
<td>Stock Market Return</td>
<td>-0.261188</td>
<td>-0.010878</td>
<td>0.143915</td>
<td>1.000000</td>
<td>0.530512</td>
</tr>
<tr>
<td>Volatility</td>
<td>0.498358</td>
<td>0.527217</td>
<td>0.321022</td>
<td>0.530512</td>
<td>1.000000</td>
</tr>
</tbody>
</table>
The correlation matrix in table 4b for the bear market below shows that interest rate level is positively correlated with stock market returns while interest rate change is negatively correlated with the stock market returns. This means that unlike the bull market, in bearish market when interest rate changes are moving upward the stock returns tend to fall since investors substitute treasury bills for equity. This is because the return on treasury bills is higher than stock market return. The interest rate levels, however, move in the same direction with stock returns, indicating that investors are optimistic that bear spell would terminate. Also bull durations are positively correlated with volatility while bear durations are negatively correlated. This implies that bull market spells tend to be more volatile than bear market spells and therefore are at a greater risk of termination than bear market spells as illustrated by the Kernel density functions in figures 5a and 5b.

Table 4b: Correlation Matrix for Bear Market

<table>
<thead>
<tr>
<th></th>
<th>Bear Market Durations</th>
<th>Interest rate level in %</th>
<th>Interest rate change in %</th>
<th>Stock Market Return</th>
<th>Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bear Market Durations</td>
<td>1.000000</td>
<td>0.104556</td>
<td>0.015813</td>
<td>0.355538</td>
<td>-0.058810</td>
</tr>
<tr>
<td>Interest rate level in %</td>
<td>0.104556</td>
<td>1.000000</td>
<td>0.106152</td>
<td>0.502789</td>
<td>0.055884</td>
</tr>
<tr>
<td>Interest rate change in %</td>
<td>0.015813</td>
<td>0.106152</td>
<td>1.000000</td>
<td>-0.110623</td>
<td>-0.251108</td>
</tr>
<tr>
<td>Stock Market Return</td>
<td>0.355538</td>
<td>0.502789</td>
<td>-0.110623</td>
<td>1.000000</td>
<td>-0.105173</td>
</tr>
<tr>
<td>Volatility</td>
<td>-0.058810</td>
<td>0.055884</td>
<td>-0.251108</td>
<td>-0.105173</td>
<td>1.000000</td>
</tr>
</tbody>
</table>

A more systematic picture of bull and bear durations in weeks is provided in figure 5a (bull market) and 5b (bear market) which plots the estimated densities of bull and bear market durations using a Gaussian kernel smoother and a 10% filter. From the figure there are significant differences in the duration profile of bull and bear markets. In the bull markets the density starts off much lower (1%) and peaks at a higher point (9%) than the density in the bear market (1.25% and 4.75% respectively). Bull markets thus appear to be at greater risk of termination than the bear markets.
Figure 5a: Smoothed Density of Bull Market Durations based on a Gaussian Kernel

Figure 5b: Smoothed Density of Bear Market Durations based on a Gaussian Kernel
4.2 DIAGNOSTIC TESTS

Before we carried out the estimation of hazard models we tested for frailty using likelihood ratio test and also tested for goodness of fit of the models using Hosmer-Lemeshow test. The test results are shown below.

4.2.1 FRAILTY TEST

Since frailty (unobserved heterogeneity) resulting from the misspecification of the model underestimates (overestimates) the degree of positive duration dependence (negative duration dependence), attenuates the proportional response of the hazard to variation in each covariate at any survival time and frailty models are relatively fragile (hard to fit) we have tested for it. To carry out the test we assumed that the random error term that has zero mean and finite variance assumes a frailty Gaussian distribution. The results of the test indicated that frailty is not statistically significant. The results are corroborated by inspection of the rho value (estimate of the frailty distribution variance) which is near zero and the p values for the likelihood ratio test are very high (close to one). Note that sigma_u reported is the standard deviation of the heterogeneity variance and rho is the ratio of heterogeneity variance to one plus heterogeneity variance. So since the hypothesis that rho is zero cannot be rejected then frailty is unimportant. The results of the test are shown in table 5a for both static and dynamic models of bull and bear markets in panel 1 and 2, respectively.
Table 5a: Frailty Test Results for Static and Dynamic Models of Bull and Bear Markets

<table>
<thead>
<tr>
<th>Panel 1: Bull Market</th>
<th>STATIC MODEL</th>
<th>DYNAMIC MODEL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dead</td>
<td>Coefficient</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Std. Error</td>
</tr>
<tr>
<td>sigma_u</td>
<td>.00009119</td>
<td>.4152941</td>
</tr>
<tr>
<td>Rho</td>
<td>2.53e-07</td>
<td>1.00007</td>
</tr>
</tbody>
</table>

Static Model's Likelihood ratio test of rho=0: chibar2(01) = 0.00 Prob >= chibar2 = 1.000

Dynamic Model's Likelihood ratio test of rho=0: chibar2(01)= 0.74 Prob >= chibar2 = 0.195

Panel 2: Bear Market

<table>
<thead>
<tr>
<th>Panel 2: Bear Market</th>
<th>STATIC MODEL</th>
<th>DYNAMIC MODEL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dead</td>
<td>Coefficient</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Std. Error</td>
</tr>
<tr>
<td>sigma_u</td>
<td>.00009119</td>
<td>.5044962</td>
</tr>
<tr>
<td>Rho</td>
<td>2.53e-07</td>
<td>.000085</td>
</tr>
</tbody>
</table>

Static Model’s Likelihood ratio test of rho=0: chibar2(01) = 0.00 Prob >= chibar2 = 1.000

Dynamic Model’s Likelihood ratio test of rho=0: chibar2(01)= 0.00 Prob >= chibar2 = 1.000

4.1.4 GOODNESS OF FIT TEST

Next, before estimating the model we have also tested whether the model fits the data correctly using Hosmer-Lemeshow (HL) test statistic. The test groups observations on the basis of predicted probability. The idea underlying the test is to compare fitted expected values to the actual values by group. If those differences are large we reject the model as providing an insufficient fit to the data. The HL Chi-square test results of the models against the null that the models fit the data cannot be rejected decisively and therefore the models fit the data correctly. The results of the test are shown in table 5b for the bull and bear markets.

Table 5b: Goodness of Fit Test Results for Static and Dynamic Models of Bull and Bear Markets

<table>
<thead>
<tr>
<th>Panel 1: Bull Market</th>
<th></th>
<th>Static Model</th>
<th>Dynamic Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hosmer-Lemeshow chi2(8)</td>
<td>0.29</td>
<td>1.09</td>
</tr>
<tr>
<td></td>
<td>Prob &gt; chi2</td>
<td>1.0000</td>
<td>0.9976</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel 2: Bear Market</th>
<th></th>
<th>Static Model</th>
<th>Dynamic Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hosmer-Lemeshow chi2(8)</td>
<td>6.12</td>
<td>1.44</td>
</tr>
<tr>
<td></td>
<td>Prob &gt; chi2</td>
<td>0.6335</td>
<td>0.9936</td>
</tr>
</tbody>
</table>
4.3 DURATION DEPENDENCE WITH FIXED COVARIATES

Using the estimation techniques and hazard models from section three, we first estimated the hazard function for the bull and bear markets in a model without time-varying covariates. The logit estimates for the static model of the bull and bear markets are shown in table 6 below.

Table 6: Logit Estimates of the Static Model for the Bull and Bear Markets

<table>
<thead>
<tr>
<th>Logit Estimates</th>
<th>Bull Market</th>
<th>Bear Market</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Obs.</td>
<td>359</td>
<td>449</td>
</tr>
<tr>
<td>LR Chi2(6)</td>
<td>74.31</td>
<td>68.24</td>
</tr>
<tr>
<td>Prob&gt;Chi2</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Pseudo R2</td>
<td>0.5680</td>
<td>0.4940</td>
</tr>
<tr>
<td><strong>Dead</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lnt</td>
<td>8.43736</td>
<td>5.00343</td>
</tr>
<tr>
<td>A</td>
<td>-0.5598566</td>
<td>-0.3579665</td>
</tr>
<tr>
<td>R</td>
<td>-1289.613</td>
<td>675.1287</td>
</tr>
<tr>
<td>Q</td>
<td>-457.1885</td>
<td>-181.176</td>
</tr>
<tr>
<td>I</td>
<td>-1926681</td>
<td>0.0565828</td>
</tr>
<tr>
<td>Di</td>
<td>0.1148712</td>
<td>0.0518365</td>
</tr>
<tr>
<td>_cons</td>
<td>-10.02948</td>
<td>-9.800757</td>
</tr>
<tr>
<td><strong>Std.Err</strong></td>
<td>2.21145</td>
<td>1.367127</td>
</tr>
<tr>
<td></td>
<td>0.14263</td>
<td>0.1048278</td>
</tr>
<tr>
<td></td>
<td>-3.58</td>
<td>3.32</td>
</tr>
<tr>
<td></td>
<td>-2.23</td>
<td>-1.40</td>
</tr>
<tr>
<td></td>
<td>-2.78</td>
<td>1.67</td>
</tr>
<tr>
<td></td>
<td>0.40</td>
<td>0.19</td>
</tr>
<tr>
<td></td>
<td>2.78679</td>
<td>2.467789</td>
</tr>
<tr>
<td>**P&gt;</td>
<td>z</td>
<td>**</td>
</tr>
<tr>
<td></td>
<td>0.000</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>0.026</td>
<td>0.010</td>
</tr>
<tr>
<td></td>
<td>0.005</td>
<td>0.095</td>
</tr>
<tr>
<td></td>
<td>0.689</td>
<td>0.851</td>
</tr>
<tr>
<td></td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Note: For the bull market, 94 failures and 0 successes completely determined.
Note: For the bear market, 126 failures and 0 successes completely determined.

Also, the output from this exercise is the baseline hazard rates plotted in figures 6a (bull market) and 6b (bear market) from the hazard estimates shown in table 7a below for bull and bear markets, respectively. These measure the pure duration dependence of the bull and bear market termination probabilities. The vertical lines at each point show the confidence band at 5% significance level.
Table 7a: Hazard Rate Estimates of the Static Model for the Bull and Bear Markets

<table>
<thead>
<tr>
<th>Interval</th>
<th>Beg. Total</th>
<th>Hazard</th>
<th>Std. Error</th>
<th>Beg. Total</th>
<th>Hazard</th>
<th>Std. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bull Market</td>
<td></td>
<td></td>
<td></td>
<td>Bear Market</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Beg. Total</td>
<td>Hazard</td>
<td>Std. Error</td>
<td>Beg. Total</td>
<td>Hazard</td>
<td>Std. Error</td>
</tr>
<tr>
<td>1 2</td>
<td>359</td>
<td>0.0056</td>
<td>0.0040</td>
<td>449</td>
<td>0.0022</td>
<td>0.0022</td>
</tr>
<tr>
<td>3 4</td>
<td>356</td>
<td>0.0065</td>
<td>0.0049</td>
<td>446</td>
<td>0.0068</td>
<td>0.0039</td>
</tr>
<tr>
<td>5 6</td>
<td>350</td>
<td>0.0146</td>
<td>0.0049</td>
<td>440</td>
<td>0.0231</td>
<td>0.0073</td>
</tr>
<tr>
<td>5 6</td>
<td>335</td>
<td>0.0188</td>
<td>0.0077</td>
<td>425</td>
<td>0.0585</td>
<td>0.0139</td>
</tr>
<tr>
<td>7 8</td>
<td>305</td>
<td>0.0475</td>
<td>0.0127</td>
<td>395</td>
<td>0.0182</td>
<td>0.0069</td>
</tr>
<tr>
<td>8 9</td>
<td>284</td>
<td>0.0299</td>
<td>0.0106</td>
<td>374</td>
<td>0.0670</td>
<td>0.0137</td>
</tr>
<tr>
<td>9 10</td>
<td>252</td>
<td>0.0364</td>
<td>0.0121</td>
<td>342</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>14 15</td>
<td>243</td>
<td>0.1892</td>
<td>0.0291</td>
<td>333</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>15 16</td>
<td>201</td>
<td>0.0806</td>
<td>0.0208</td>
<td>291</td>
<td>0.0543</td>
<td>0.0140</td>
</tr>
<tr>
<td>16 17</td>
<td>171</td>
<td>0.0000</td>
<td>0.0000</td>
<td>261</td>
<td>0.0632</td>
<td>0.0156</td>
</tr>
<tr>
<td>27 28</td>
<td>155</td>
<td>0.1908</td>
<td>0.0366</td>
<td>245</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>28 29</td>
<td>128</td>
<td>0.2456</td>
<td>0.0461</td>
<td>218</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>31 32</td>
<td>100</td>
<td>0.4493</td>
<td>0.0786</td>
<td>190</td>
<td>0.1777</td>
<td>0.1154</td>
</tr>
<tr>
<td>38 39</td>
<td>38</td>
<td>0.4493</td>
<td>0.0436</td>
<td>121</td>
<td>0.2714</td>
<td>0.0436</td>
</tr>
</tbody>
</table>

Figure 6a: Hazard Rate in Bull Market with Fixed Covariates

The baseline hazard in the bull markets is initially 0.56% per week but it slowly rises to 4.75% for markets that have lasted 8 weeks and drops after 9 weeks only to pick again after 14 weeks and drops again after 15 weeks. Thereafter it rises again to peak at the maximum of 44.93% after 32 weeks, as indicated from the hazard rate estimates for the bull market with fixed covariates in table 7a above. Observe that in the intervals 16-17 and 38-39 in which a hazard rate cannot be calculated (when there are no failures), table
7a and figure 6a shows a hazard equal to zero and no standard error or confidence band. The high hazard for long-lived bull markets entirely reflects volatility clustering. The hazard rate in the bull market tends to be increasing most of the time and hence there is positive duration dependence in the stock prices. The figure below shows the bear hazard rate drawn from table 7a above.

**Figure 6b: Hazard Rate in Bear Market with Fixed Covariates**

![Hazard Function, NSE Data (Itable)](image)

The baseline hazard in the bear markets is initially 0.22% per week but it slowly rises to 5.85% for markets that have lasted 7, drops after 8 weeks only to narrowly pick again slowly after 9 weeks, decelerates slowly but not significantly and increases to peak at the maximum of 27.14% after 39 weeks (see table 7a for bear market). The high hazard rate for long-lived bear markets here reflects volatility clustering and persistence. Thus the bear market hazard rate is essentially increasing most of the time also implying positive duration dependence in stock prices.
The long-run stock returns thus depend crucially on the difference between bull and bear markets hazard rates. A bull market has always a higher probability of instantaneous termination than a bear market of the same age as vindicated from figures 6a and 6b. In fact, in relative terms the bull market hazard rate appears to be more increasing as a function of duration and is about seven times higher for long durations (44.93%) compared with short ones (5% on average). Similarly, bear market hazard rate appears to be increasing as a function of duration most of the times but relatively less than bull market hazard rate. When compared with the insights from tables 3a and 3b, this suggests that it is the presence of very long bear markets and short bull markets that account for historically low mean returns on NSE stocks as opposed to the differences between bull and bear markets at the short end of the duration distribution.

As a means of providing a single summary measure of the attrition rates in bull and bear markets, figures 7a (bull market) and 7b (bear market) below plot the survivor functions estimated from the hazard rates shown in figures 6a and 6b and estimates shown in table 7b below.

Table 7b: Survivor Function Estimates of the Static Model for the Bull and Bear Markets

<table>
<thead>
<tr>
<th>Interval</th>
<th>Bull Market</th>
<th>Bear Market</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Beg. Total</td>
<td>Deaths</td>
</tr>
<tr>
<td>1 2</td>
<td>359  2  1</td>
<td>0.9944</td>
</tr>
<tr>
<td>3 4</td>
<td>356  3  3</td>
<td>0.9860</td>
</tr>
<tr>
<td>5 6</td>
<td>350  5  10</td>
<td>0.9517</td>
</tr>
<tr>
<td>7 8</td>
<td>335  6  24</td>
<td>0.9537</td>
</tr>
<tr>
<td>9 10</td>
<td>305  14  7</td>
<td>0.9094</td>
</tr>
<tr>
<td>11 12</td>
<td>284  8  24</td>
<td>0.8826</td>
</tr>
<tr>
<td>13 14</td>
<td>252  9  0</td>
<td>0.8511</td>
</tr>
<tr>
<td>15 16</td>
<td>243  42  0</td>
<td>0.7040</td>
</tr>
<tr>
<td>17 18</td>
<td>201  15  15</td>
<td>0.6494</td>
</tr>
<tr>
<td>19 20</td>
<td>171  0  16</td>
<td>0.6494</td>
</tr>
<tr>
<td>21 22</td>
<td>155  27  0</td>
<td>0.5363</td>
</tr>
<tr>
<td>23 24</td>
<td>128  28  0</td>
<td>0.4190</td>
</tr>
<tr>
<td>25 26</td>
<td>100  31  31</td>
<td>0.2653</td>
</tr>
<tr>
<td>27 28</td>
<td>38   38  0</td>
<td>0.2653</td>
</tr>
<tr>
<td>29 30</td>
<td>121  0  121</td>
<td>0.4748</td>
</tr>
</tbody>
</table>

66
For a given duration the bear state has the highest survival probability most of the times. Also the survivor probability declines with duration faster in bull market than in the bear market. This implies that the difference between the survival probabilities in the two market states increases as a function of duration. In the interval 14-15 weeks, for example, 83.87% of the bear markets survive as compared to only 70.4% of the bull markets that survive this long.

Figure 7a: Survivor Function in Bull Market with Fixed Covariates

![Figure 7a](image)

Figure 7b: Survivor Function in Bear Market with Fixed Covariates

![Figure 7b](image)
4.4 DURATION DEPENDENCE WITH TIME-VARYING COVARIATES

To shed light on how the hazard rates depend on the underlying state of the economy among other variables such as age of the market states and return, we next incorporated interest rate levels, interest rate changes and stock market volatility as time-varying covariates. But since interest rate changes are more likely to track business cycle variations than the interest rate levels we include both levels and changes in the interest rates and volatility as time-varying covariates. The duration dependence of these effects is indicated through their $t$ subscript attached to respective variables. Therefore, the set of covariates are $X_{it} = (a, r, i_t, d_{it}, q_t)$. The hazard specification that allows covariates to vary with duration of the current state is given in equation (14).

The logit estimates for the dynamic model of the bull and bear markets are shown in table 8 below.

Table 8: Logit Estimates of the Dynamic Model for the Bull and Bear Markets

<table>
<thead>
<tr>
<th>Logit Estimates</th>
<th>Bull Market</th>
<th>Bear Market</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log Likelihood</td>
<td>-21.732471</td>
<td>-32.87553</td>
</tr>
<tr>
<td>No. of Obs.</td>
<td>359</td>
<td>449</td>
</tr>
<tr>
<td>LR Chi2(6)</td>
<td>87.35</td>
<td>72.37</td>
</tr>
<tr>
<td>Prob&gt;Chi2</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Pseudo R2</td>
<td>0.6677</td>
<td>0.5240</td>
</tr>
</tbody>
</table>

|         | Coef.     | Std.Err | z    | P>|z| | Coef.     | Std.Err | z    | P>|z| |
|---------|-----------|---------|------|-------|-----------|---------|------|-------|
| h       | -1.655375 | .451218 | -3.67| 0.000 | -0.575894 | .155068 | -3.71| 0.000 |
| R       | -873.3364 | 239.533 | -3.65| 0.000 | 463.986   | 116.0932| 4.00 | 0.000 |
| intvg (q_t) | 8.023246 | 2.40472 | 3.37 | 0.000 | 5.428879  | 1.454383| 3.73 | 0.000 |
| tvg (d_{it}) | .0301006 | .008317 | 3.62 | 0.000 | .0034242  | .001002 | 3.42 | 0.001 |
| cons    | -8.142764 | 2.42171 | 3.62 | 0.001 | -8.51996  | 2.027633| -4.20| 0.000 |

Note: For the bull market, 150 failures and 0 successes completely determined.
Note: For the bear market, 117 failures and 0 successes completely determined.

---

Note that interest rate level as a variable is dropped due to a serious collinearity problem.
Figures 8a (bull market) and 8b (bear market) show the sequence of baseline hazards drawn from bull and bear market hazard rate estimates for the dynamic model from table 9a below.

Table 9a: Hazard Rate Estimates of the Dynamic Model for the Bull and Bear Markets

<table>
<thead>
<tr>
<th>Interval</th>
<th>Bull Market</th>
<th>Bear Market</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Beg. Total</td>
<td>Hazard</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>0.0655</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>0.0357</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>0.0392</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>0.1143</td>
</tr>
<tr>
<td>8</td>
<td>9</td>
<td>0.0714</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td>0.0870</td>
</tr>
<tr>
<td>14</td>
<td>15</td>
<td>0.3158</td>
</tr>
<tr>
<td>15</td>
<td>16</td>
<td>8</td>
</tr>
<tr>
<td>16</td>
<td>17</td>
<td>6</td>
</tr>
<tr>
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<td>121</td>
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Figure 8a: Hazard Rate in Bull Market with Time-varying Covariates
Figures 8a (bull market) and 8b (bear market) show the sequence of baseline hazards. Compared to figure 6a, it is clear that controlling for time-varying covariates has a significant effect on the shape of the bull market baseline hazard. Controlling for these effects, the baseline hazard initially drops sharply from 6.56% to 3.57% per week as the bull market duration extends beyond 4 weeks and rises until 8 weeks. Thereafter if falls and rises to peak at 31.58% after 15 weeks, drops steadily and rises again to peak at a maximum of 50% after 32 weeks. Young bull markets thus appear substantially less at risk of termination than bull market that has lasted for 15 weeks or longer. This is reflected in the survivor function in figure 9a below drawn from the survivor function estimates shown in table 9b below.
Table 9b: Survivor Function Estimates of the Dynamic Model for the Bull and Bear Markets

<table>
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<tr>
<th>Interval</th>
<th>Bid. Total</th>
<th>Deaths</th>
<th>Lost</th>
<th>Survival</th>
<th>Std Error</th>
<th>Beg. Total</th>
<th>Deaths</th>
<th>Lost</th>
<th>Survival</th>
<th>Std Error</th>
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</tbody>
</table>

Figure 9a: Survivor Function in Bull Market with Time-varying Covariates

The survivor probability in the time-varying covariates case declines with duration faster than in the fixed covariates case. Thus time-varying covariates are associated with large increases in the bull hazard rate and hence faster declining survival probability.

Turning next to the bear hazard estimates and comparing figures 9b and 6b; the shape of the baseline hazard changes as a result of controlling for time-varying covariates effects.
The baseline hazard now increases initially from 3.28% to 18.6% per week as the duration extends from one to seven weeks only to drop after 8 weeks. Thereafter it increases relatively quickly to a level of 21.43%, falls slowly and increases to the maximum of 66.67% after 39 weeks. Thus the bear market hazard rate with time-varying covariates is relatively higher than those in the fixed covariates case with the same duration. This also implies that the survival probability with time-varying covariates declines with duration faster than in the fixed covariates case. This is indicated in the survivor function in figure 9b below drawn from the survivor function estimates shown in table 9b above.

Figure 9b: Survivor Function in Bear Market with Time-varying Covariates

![Survivor Function, NSE Data & Covariates (itable)](image-url)
4.4.1 EFFECT OF A TIME-VARYING INTEREST RATE CHANGE

To demonstrate the effect on stock prices of a change in interest rates we consider how the bull and bear hazard rates change. This means capturing equations (15) and (16). While there is no effect of interest rate levels on the bear and bull hazard rates there is a systematic positive effect of time-varying interest rate changes. The effect, however, is relatively greater on the bull market hazard (3%) than on the bear market hazard (0.3%). Thus on balance a bear market tends to survive longer in a dynamic environment with increasing interest rate changes than a bull market where positive interest rate changes are associated with larger increases in the bull hazard rate. Therefore, with increasing interest rate changes overtime the survival probability in the bear market are relatively higher than those in the bull market.

Thus a higher interest rate change in a bull state leads to a relatively higher bull hazard than a bear hazard. This increases the survival probability of a low mean return state and decreases the survival probability of a high mean return state in the interest rate change scenario relative to the no interest rate change scenario. Consequently, the mean return in future periods is consistently lower in the high interest rate change scenario initiating from the bull state. There is much less of an effect on future mean returns if the initial state is a bear market, although the lower mean returns come through towards the end.

4.4.2 EFFECT OF TIME-VARYING VOLATILITY

While non-time-varying volatility is statistically significant and has a negative effect on the bull market hazard it is insignificant in the bear market. However, there is a
systematic positive effect of time-varying volatility on the bull and bear hazards as captured by equation (17). The effect of time-varying volatility on the hazard rate is, however, relatively greater in the bull state (8.823246) than in the bear state (5.428879). Thus in balance a bear market tends to survive longer with increasing time-varying volatility than a bull market of the same age. This increases the survival probability of a low volatile bear market state and decreases that of a high volatile bull market state. Hence, despite the small sample size of NSE data and without making the assumption that returns are normally distributed we have identified a high mean return, high volatile bull state and a low mean return, low volatile bear state. The termination probabilities of the two market states significantly depend on the age of market state, behaviour of the business cycle and time varying volatility.
5.0 CONCLUSIONS AND POLICY IMPLICATIONS

5.1 CONCLUSIONS

This paper used duration analysis approach to document dependence in the direction of stock prices based on the probability of exiting from bull or bears states and examines the effect of state variables and time-volatility on the hazard rates. Since the length of time spent in these states is a key determinant of the mean and risk of stock returns, it is important to study the determinants of the bull and bear durations. We find positive duration dependence in both the bull and bear markets, strong evidence contradicting standard models of stock prices even after controlling for time-varying volatility and state variables. The bull and bear hazard rates rise sharply in the end of the duration. However, while the hazard rate in the bull market is relatively higher at longer horizons, it is relatively low in the bear markets, making long bear spell more likely. The long bear spells observed may have been partly due to policy reforms such as entry of foreign investors and the shift in the trading system. Also economic decline increased risk as indicated by the high interest rate changes experienced during the period. Time-varying volatility and interest rate changes are associated with large increase in bull and bear hazard rates and hence faster declining survival rates. The age of the market state has a negative significant effect on the bull and bear hazards but the effect is relatively stronger in the bull market than bear market. This possibly explains why bull markets appear to be at greater risk of termination than bear markets.
Evidence of deviations from random walk model does not imply a rejection of the EMH. On the other hand, long-run dependencies in stock prices have important implications for both long-run risk management and for interpretation of sources of movements in stock prices. It is beyond the scope of this paper to propose an economic model that can explain duration dependence in stock prices. Instead we briefly consider explanations of duration dependence based on speculative bubbles, market fundamentals and time-varying risk premium, irrational investors, a momentum effect in the market, habit formation effects, belief distortions, "Joseph effect", bandwagon effect and contrarian strategic investors.

Maheu and McCurdy (2000a) study use duration dependent Markov switching model to capture nonlinear structure in both the conditional mean and the conditional variance of stock returns. The model sorts returns into a high return stable state and a low return volatile state and their empirical results find declining hazard functions (negative duration dependence) in both the bull and bear markets using monthly data from 1834 – 1995. This means that the probability of switching of the state declines with duration in that state. Despite the declining hazards the best market gains come at the start of the bull market. That is, returns in the bull market state are a decreasing function of duration. Volatility in the bear market state, however, is an increasing function of duration.

They identified four main explanations for duration dependence in stock returns: First, market fundamentals such as dividend payoffs and time-varying risk premiums. If dividends are positively related to the business cycle they are likely to display positive duration dependence. The response of stock prices to variation in risk premium can lead one to incorrectly infer the presence of mean reversion and excess volatility. For
example, when the risk premium and required return on the market rises over a duration stock prices will fall increasing the hazard rate in the bull market and hence positive duration dependence in stock prices. Second, a possible explanation of the declining hazard could be irrational investors such as noise traders or fads (stock prices overreact to relevant news). Asymmetry of information between noise traders and rational investors leads uninformed traders to rationally behave like price chasers. This introduces serial correlation in stock returns. If such effects are linked to the underlying state of the economy it is possible that they could affect the duration distribution of stock returns. Third, the declining hazards found in all models could be interpreted as a momentum effect in the market. For example, as a bull market persists, investors could become more optimistic about the future and hence wish to invest in the stock market. This results in a decreasing probability of switching out of the bull market. But if the bull market is highly volatile as in this study, then we expect the probability of switching out of the bull market to increase with duration. Similarly, the length of the bear market could be related to the amount of pessimism about future returns by investors. This would lead to a substitution from equity into other expected high return instruments such as the treasury bills and bonds and therefore long bear spells. Finally, the evidence also shows that a rational stochastic bubble will display negative duration dependence and we expect that the irrational one will therefore display positive duration dependence. This study has used daily data over the period 2/1/1992 – 6/30/2004 which allowed us to consider hazard rates at both much shorter and relatively longer durations. While our finding of both increasing bull and bear market hazard rates is inconsistent with Maheu and McCurdy (2000a)'s results, it is possible to appeal to the above explanations.
Pagan and Sossounov (2000) study on other hand considers a definition of bull and bear states based on cumulative price changes. They found that asset pricing model in which consumption growth follows a lognormal process with habit formation effects has promise for matching the average duration of bull and bear states, though matching the hazard function may be a more difficult test to pass. Cacchetti et al., (2000) introduce belief distortions that vary over expansions and contractions and leads to systematic predictability in stock returns. These models all seem to have some promise for explaining bull and bear durations, which needs to be explored in future research work.

Other possible explanations of the positive duration dependence are: First, Joseph effect, where in a typical behaviour of an economic variable observations in the remote past are nontrivially correlated with the observations in the distant future. The positive serial correlation of stock returns for the entire sample confirms this hypothesis. Second, bandwagon effect where individual investors see stock price rising and are drawn in a kind of bandwagon effect. For example, Shiller (2002) describes the rise in the US stock market during the late 1990s as the result of psychological contagion leading to irrational exuberance. Finally, there are investors who rely on investment techniques that rest on a contrarian strategy that is, buying the stocks or groups of stocks that have been out of favour for long periods of time and avoiding those stocks that have had large run-ups over the last several weeks. If we have such kind of investors it is possible that they could affect duration distribution of stock market returns.
5.2 POLICY IMPLICATIONS

This study analysed the duration dependence in stock prices and the effects of time-varying covariates on hazard rates in the bull and bear markets. Testing the predictions based on predictability of asset returns the study makes the following findings and derive relevant policies based on the findings in order to improve investment performance:

First, the age of the market state and non-normally distributed return significantly affect the duration distribution of stock returns. Controlling for time-varying volatility and interest rate changes the termination probability of bull and bear markets declines as a function of its age. However, the decline is higher in the bull market that has high volatility than in the bear market that has less volatility. It is therefore important to pursue policies that reduce volatility and increase efficiency of the stock market. This can be achieved by reducing information asymmetry in the market and also the need to educate the public on activities of the stock market. There is also need to tighten disclosure rules, protect investors and tightening market surveillance that would discourage irrational and contrarian investors, reduce excessive speculation and thin trading. This ensures that bull markets survive longer than bear markets and therefore positive stock returns that stimulate investments.

Second, time-varying volatility is found to have a positive significant effect on the duration distribution of stock returns. Thus increasing volatility is associated with the increased termination probability in both the bull and bear markets but the effect is higher in the bull market than bear market. The bull market is at the greater risk of termination because the rise in stock prices may reflect investors' demand for higher risk premium
and therefore the bull market tends to be very volatile than bear market. The high
volatility also indicates that NSE is still in transition and therefore adjusting to reforms.
The reforms need proper sequencing and a good policy environment in order to decrease
volatility, increase efficiency and liquidity.

Third, the results show that there is no systematic effect of market interest rate levels on
the bull and bear hazards, while there is a systematic significant positive effect of interest
rate changes. The effect is however, stronger on the bull hazard than the bear hazard.
Thus increasing interest rate changes are consistently associated with the increase in the
bull and bear hazard rates. When Treasury bill rate changes increase Treasury bill yields
are high and are risky-free. If investors are pessimistic about future stock market returns
they would substitute Treasury bills for equity leading to longer bear spells and bull
markets terminates very quickly. It is therefore important to encourage listing of
corporate bonds and reduce government bonds and Treasury bills activity in the market
overtime. This is because high concentration of these instruments may crowd out private
sector in the stock market.

Fourth, since interest rate changes also closely track the behaviour of business cycle and
their rapid increase fuels instability in the economy, it important to create a
macroeconomic stable environment. This is because economic performance has a direct
influence on investors' participation especially because it affects their earnings potential
and ability to participate in the market. Therefore an enabling environment that
encourage sustainable growth of the economy and economically empower the private
sector will pave way in improving stock market performance.
Finally, the policy implications arising from this research relate to the question of whether investment performance in the stock market can be conceived from duration dependence in stock prices or not. Since we have observed that the duration of stock returns are influenced by the age of the market state, time-varying volatility, interest rate changes that track performance of the economy and the policy environment, it is therefore possible to conceive investment performance in the stock market. Since there is a high mean return, high volatile bull market state and a low mean return, low volatile bear market state reflecting a relatively high increasing hazard rate in the bull market (low survival rate) and a relatively low increasing hazard rate in the bear market (high survival rate) it appears that it is profitable to invest at the end of a bearish state or at the beginning of bullish state, when interest rate changes are low. This is because low interest rate changes would ensure that survival rate of the bull market is relatively higher than that of the bear market and therefore positive stock returns would be realised but only in a good policy environment.

5.3 LIMITATIONS OF THE STUDY

The main limitation was the unavailability of consistent high frequency daily data on interest rates. This is because there is no continuous daily data on interest rates in Kenya. However, we used the 90-day Treasury bill rate provided by the Central Bank on a monthly basis by converting it into a daily series by simply applying the rate reported for a given month to each day of that month. But this will not significantly affect the accuracy of the results. There were also some data constraints especially on stock prices since the data on the NSE is only available commencing 1992. This means that the study
was not able to capture long bull and bear markets in order to compare the duration distribution under duration models and the benchmark models (random walk and GARCH models). This could not be avoided; however, the available data enabled us to conclusively achieve our objectives.

5.4 SUGGESTIONS FOR AREAS OF FURTHER RESEARCH

There is need for further research to capture long bull and bear markets in order to make a comparison between duration distributions under duration models and the benchmark models (random walk models and models that capture volatility clustering and persistence; using both nominal and real stock market returns. Furthermore, in future, research should be directed to examine duration dependence in stock prices in different policy regimes. This will give a clear picture of whether policy changes such as entry of foreign investors and change in the trading system have an effect on survival probabilities of bull and bear markets.
REFERENCES


Faust, J. (1992), 'When are variance ratios tests for serial dependence optimal?' Econometrica 60, 1215-1226.


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