IMPROVEMENT OF VOLUME ESTIMATION OF STOCKPILE OF EARTHWORKS USING A CONCAVE HULL-FOOTPRINT

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SUMMARY

In the estimation of volume of stockpiles of earthworks, the question is no longer whether the data collected is dense and accurate (equipment and techniques capable of accurate data measurement are available), but how to manipulate the data to yield accurate volume estimation. Although surface modeling through TIN yields more accurate volumes than grid modeling, the delineation of footprint of the stockpile remains one of the main sources of errors in volume determination due to spurious surfaces created within the convex hull of the TIN model. In this paper, an approach for automatic delineation of the stockpile footprint based on a concave hull is introduced. A concave hull as a geometry (usually point data) container is realized by minimizing the enclosing planimetric area and it is usually not unique. Several algorithms for creating concave hulls are suggested, in this paper an algorithm based on Delaunay triangulation and linear referencing was used to create the concave hull. A comparison of volume estimations of stockpiles taking into consideration the footprint via convex hull, concave hull and manually delineated outline showed that volumes based on the concave hull are closer in value to volumes based on manually delineated footprint. Therefore in the absence of points manually picked to represent the outline of a footprint, the concave hull can be relied on.

Key words: Volume of Earthworks, TIN, Concave hull, Linear referencing, Footprint delineation.

1. INTRODUCTION

Estimation of volume of excavated and hauled materials is one of the most significant and common aspects of most engineering earthwork projects,

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such as route alignment, dam and tunnel construction and mining - among others. Precise and reliable planning, profit or loss depends on such estimations.

Data collection for volume estimation using conventional methods and techniques can be difficult, time consuming and largely inaccurate because stockpiles are usually uneven, on sloppy ground, and frequently not easy be fully accessed while taking measurements. With the current state-of-the art aerial and terrestrial laser scanning technology, it is increasingly becoming more probable that more accurate measurements can be obtained, and with more accurate surface modeling techniques, more accurate volume determinations are made possible. This possibility has made it necessary for almost all Geographic Information System (GIS) and Computer Aided Drawing and Design (CAD) applications to incorporate techniques for volume computation; in fact, there is no longer any difference in the volume estimated using either GIS or CAD tools, because they all use Delaunay triangulation, which produces unique results.

Although the use of laser scanners afford faster and accurate point data measurement, a new challenge of data filtering is introduced, which if not carefully handled can lead to erroneous results. Similarly, the availability of easy-to-use tools in both GIS and CAD applications for volume estimation sometimes one may assume to critically evaluate the results.

Estimation of volume of a stockpile would ordinarily involve data collection and the subsequent computation. Both these two processes present opportunities for errors. During data collection, the accuracy of the measurements (equipment and measurement techniques), and the sampling of representative ground points (sampling can be done after field measurements through filtering) influence the accuracy of the estimated volume. During the computations, the method of computation (for example, raster or vector modeling) affects accuracy of the volume determination.

The purpose of this paper is to evaluate the effect on volume of unclear delineation of the footprint of a stockpile. If no effort is made to delineate the stockpile footprint at the time of data collection or during data processing then errors in the resulting estimations should be expected. The next section gives an outline of the array of techniques of data collection and computation for volume estimation. This is followed by discussing and demonstrating how a footprint of a stockpile can be delineated using a concave hull. The effect of unclear delineation of the footprint on volume estimation is then demonstrated on some experimental datasets.

2. VOLUME ESTIMATION

2.1 Data Collection

Stockpiles and storage areas in most engineering earthwork projects are generally measured by manual techniques. This is often tedious and risk prone especially if measurements involve climbing up and down and round high mounds of materials. This is because of the limitations imposed by the measuring equipment and techniques involved. However, with the current plethora of survey measuring techniques, fast and accurate data collection is possible.

There are multiple surveying options for measuring stockpiles, mineral deposits or waste dumps. Methods that can be used to obtain three dimensional coordinates that define the formation surface of the stockpile include: tacheometry, Real Time Kinematic (RTK) GPS, aerial photogrammetry, and air-borne and terrestrial laser scanning.

Tacheometry is a conventional method of surveying, where distances and heights are determined from instrument readings alone, from which three-dimensional coordinates can be derived. It is an indirect way of measurement. Depending on the technique adopted, either a conventional theodolite or a total station may be used.

A total station can be used in conjunction with or without a reflector. Here the operator is required to identify and mark suitable instrument stations around the stockpile that will afford full coverage (or view) of the surface. At each instrument set up, points (x,y,z coordinates) are picked at the foot of the pile to define the ground surface followed by points on the surface of the pile to define the formation surface. The instrument operator should pick the points that define the formation and foot of the stockpile more carefully to minimize errors, similarly as many points as possible should be picked to accurately define the formation surface. The number and distribution of the points, influenced by the complexity of the surface, determine the accuracy of the estimated volume.

The Global Positioning System (GPS) and of course other Global Navigation Satellite Systems (GNSS) are used to locate points on the Earth's surface without using terrestrial targets. Depending on the application, the GNSS receiver can be used in different measurement modes (El-Rabbany, 2006), but ultimately yielding three-dimensional coordinates of a point just like tacheometry. RTK positioning technique is the obvious choice where fast but accurate data are required. RTK GPS positioning is capable of delivering accurate real-time positions (about 2-5 cm) in the field with a possibility of improvement if a longer period of station observation (i.e., about 30 second) is adopted. RTK GPS can be operated by one person alone and is faster than a total station. The downside of this technique is that the GNSS receivers cannot work especially for materials under sheds or dense tree canopies or under high voltage power lines and it is dangerous to climb up high mounds of materials.

Using aerial photogrammetry, volume of material can be determined from stereo photographs of material heaps. This is an efficient method of data collection for medium scale projects because the climbing up and down of material is completely avoided. If conventional photogrammetric procedures are employed, then this is a relatively expensive technique and the data processing is quite elaborate; besides, it is not appropriate for materials under sheds or trees. However, if completely near-real-time processing of all data on-the-fly from an aerial photogrammetry mission is possible after landing, then products like ortho-images and elevation data (x, y and z-coordinates) are ready-for-use. Such real-time phogrammetric systems are commonly referred to as UAV-Based photogrammetric mapping systems (Wu et al, 2004). In these systems, photographs are taken with digital cameras, and simultaneously registering of the projection centre co-ordinates and the rotation angles (φ, ω, κ) using GPS and IMU (Inertia Measurement Units) techniques, in what is generally called direct georeferencing.

Laser scanning is an active measurement method that allows measurements in either daytime or at night (Vosselman and Maas, 2010). Laser scanning is now a common technique for generating high quality 3D representations of the landscape by capturing 3D point clouds. The fundamental concept of laser distance measurement and scanning applies to both air-bone and terrestrial systems, respectively referred to as Air-borne Laser Scanning (ALS) and Terrestrial Laser Scanning (TLS) respectively. Both ALS and TLS have relative advantages and disadvantages depending on the problem (Young et. al., 2010). Laser scanning is capable of measurement accuracies ranging from 5-10 mm (Karsidag and Alkan, 2012).

Terrestrial Laser Scanning (TLS), which is similar, to some extent, to the technique of using reflector-less total station, can afford fast results with a single operator, and is capable of high accurate results. The downside of laser scanning and especially for terrestrial measurements is the likelihood to miss sunken points that may not be visible from the instrument station, thus giving a wrong impression of the measurements as illustrated in Figure 1.

Airborne Laser Scanning (ALS) is similar to airborne photogrammetry in several respects, in which point data is measured from an airborne sensor. In both aerial photogrammetry and ALS, point coordinates are automatically picked, resulting in what is called a point-cloud. It has been established that the point density (number of points per unit of area) required to generate an accurate surface modeling, most commonly a Digital Terrain Model (DTM), depends on the complexity of the terrain being represented. Therefore, point datasets from such systems can withstand substantial data reduction while maintaining adequate accuracy for elevation predictions (Liu et al., 2007).

Figure 1: Laser scanned positions shown with arrows in green, positions that have been missed out ("shadowed") shown by arrows in red; the observer is at the left hand side of the heap in the figure.

Photogrammetry and airborne laser scanning techniques are compared with respect to 3D mapping for volume measurement of stockpiles in Table 1.

Table 1: Comparison of photogrammetry and laser scanning

Photogrammetry	Laser scanning
Fast but not for real time applications	Fast and for real-time applications
High accuracy when sophisticated algorithms are combined	Possible data loses when the resolution is low
Multi image configuration	Not applicable at high altitudes
Amount of information can be controlled	Huge amount of information

2.2 Methods of Volume Estimation

The three general methods for calculating earthworks include: volume from cross-sections; volume from contours; and volume from spot heights (e.g., Bannister and Raymond, 2005;, Schofield, 1993).

The method of volumes from sections is capable of general application only when the formations have a constant width and side slopes, as illustrated, for example, in Figure 2 a), with the red outline. Once the length, width and height of the stockpiles have been measured, the volume is then computed by simply multiplying the length by the width by the height or applying the different formulae as found in most surveying textbooks. Cross sections are, as a rule, selected at intervals of 5, 10, 20, 50 and possible 100 m, depending on the segmentation of the shape. This method is only an approximation, and the formations are rarely of uniform shapes (see Figure 2 b)), and is mostly used on narrow works, such as roads, railways, canals, embankments.


Figure 2 a) stockpile with regular surface Figure 2 b) stockpile with irregular surfaces

The method of volume determination by contours assumes that the contours have already been created. The method depends on the area between any two successive contours and the difference in height between the two contour lines. The stockpile is divided into layers using horizontal planes crossing the pile at the contour line. Similar to the vertical cross section, Simpson's rule (Equation 1) can be used. The rule can be interpreted as follows: one third the distances between the ordinates, multiplied by the sum of the first and the last ordinates, plus four times the sum of even ordinates, plus twice the sum of the odd ordinates. This equation requires an odd number of ordinates; however with some slight modification an even number of ordinates can also be used. This method is however rarely used owing to the fact that contours are derivatives of basic measurements, and would therefore not be as accurate.

$$V_i = \frac{w}{3}\left[(h_1 + h_7) + 4(h_2 + h_4 + h_6) + 2(h_3 + h_5)\right] \qquad (1)$$

Volume determination from spot heights is the most common method used particularly for large open excavations or heaps, and takes point data as input. Traditionally, the method entails dividing the area into squares or rectangles and then taking the levels at each of the corner points. Only a level and leveling staff are required to measure the levels. This way, the third coordinate (height) is associated with a temporarily horizontal surface (x, y coordinates). It therefore means that every time a volume has to be determined, a temporary horizontal surface has to be assumed, which means that stockpiles that are far apart cannot be easily referred to a common horizontal reference, and it is nearly impossible to make incase measurements have to be repeated. If however, the spot heights are irregularly spaced, then volume computation is determined from a vector- or raster-based surface model.

3. VOLUME ESTIMATION THROUGH SURFACE MODELING

Surface modeling has become an important element in the processing and visualization of three-dimensional geographic information. Models are created from a finite sample of data points over the area of interest. The techniques used for surface modeling can be broadly divided into raster-based interpolation methods and vector-based triangulation methods. In a raster, a DTM is structured as a regular grid consisting of a rectangular array of uniformly-spaced equally-sized cells with sampled or interpolated z-values. In vector, a more advanced, more complex, and more common form of DTM is the Triangular Irregular Network (TIN), which is constructed as a set of irregularly located nodes with z-values, connected by edges to form a network of contiguous, non-overlapping triangular facets. Both raster and vector surfaces are created using two main methods: interpolation and triangulation, respectively.

According to Meenar and Sorrentino, (2009), in TIN modeling, there is a possibility of higher resolution in areas where the surface is more complex, and therefore the TIN creation process makes it more reliable than the grid approach. TIN modeling, which preserves the original data upon modeling, is mostly used in smaller areas, for more detailed, large-scale applications. On the other hand, in grid modeling there is loss of initial data, due to interpolation, and is commonly used in more regional, small-scale applications.

Grid and TIN surface structures have dimensional properties between 2D and 3D with no underlying or overlying information; they are sometimes described as 2.5D data. Therefore, their usefulness is limited to basic queries, such as slope and aspect calculations, contouring, hill-shading and view-shed analysis.

These surfaces are not considered as true 3D structure. This is because they do not contain multiple z-values at the same (x,y) location, therefore they cannot be used to model overhangs and tunnels, and support accurate volumetric calculations. To be useful for volume determination, two surfaces (raster or TIN) are required, with one surface functioning as the formation surface (upper surface) and the other as a reference/datum surface (ground surface). Mass points and break lines are collected that describe the upper surface. Volume of a surface is usually determined relative to a given base height, or reference plane, can be another surface.

3.1 Volume Calculation from TIN Surface Model

TIN model is the most appropriate model for computation of earthworks. The Delaunay triangulation is most commonly used approach to construct a TIN rather than other, less restrictive triangulations. In a Delaunay triangulation, the circumscribing circle of any triangle contains no other vertices (Shewchuck, 1996). Delaunay triangulation of a set of vertices is unique; this is an important quality, which allows one to repeat the calculations and to verify the results independently.
To calculate the volume enclosed by two TIN surfaces, let the planimetric surface of a triangle I in the upper TIN surface be Ai (Figure 3), href be the height of the horizontal reference plane (lower surface), and h_i be the elevations of the three vertices of triangle *i*. The volume generated by one triangular prism is determined by the prismodial equation:

$$V_i = \frac{A}{3}\sum_{i=1}^{3}(h_i - h_{ref}) \qquad (2)$$

The sum of the individual triangular prisms represents the volume enclosed by the two surfaces. If the input surface is a raster, its cell centres are connected into triangles. These are then processed in the same fashion as the TIN triangles.

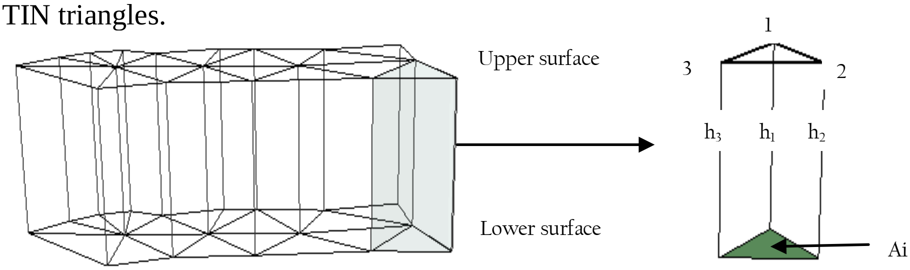


Figure 3: Illustration of a two surface TINs and a triangular prism

3.2 Error in Volume Computation from a TIN Model

The TIN model approximates the terrain surface using a network of triangles connected together. The difference between the TIN and the modeled surface are considered to be model errors. These errors are partly due to errors in the data and errors in approximation (Hao and Pan, 2011).The error in data is stochastic, and is introduced in the original data, as a result of the survey equipment and the method of measurement. The data errors are both in height and planimetric measurements (Wulf, et. al., 2012). The model (or representation) error is determined by the quantity and distribution of sampling points, which in turn are an important indicator of the accuracy of the TIN model.
Another source of error in volume estimation when using TIN modeling is the unclear definition of the stockpile footprint/outline. This is commonly the problem if the data points are picked automatically or hand digitized, it is not explicitly indicated which points represent the footprint.

4. STOCKPILE FOOTPRINT DELINEATION

During TIN modeling, the footprint of stockpiles, if not explicitly indicated, it is represented by the planimetric convex hull of the points. The convex hull is perhaps the most basic and common geometry container that is used in computational geometry. The convex hull has been applied in many fields, for example, business, engineering, science, daily life and so on.
The convex hull is used in particular when the only objective is to minimize the outline length. If used to represent the outline of stockpile, then extra area and volume are included. To define the footprint more precisely, a hull with minimum area should be used. Such a hull is called non-convex (or better- a concave hull).

4.1 Concave Hull

A concave hull is a concave polygon that encloses all geometries within a set, but has less area compared to the convex hull. Because of minimizing the area, the concave hull’s line length is longer than the corresponding convex hull. A concave hull could be suitable for some real-world problems, for example, finding the boundary of a city based on the amalgamation of the land parcel boundaries.
Computing the concave hull is considered one of the complicated problems in geometry, and as such, there are many variations of it (Sunday, 2006),

which can be used depending on the intended application. However, there are currently no algorithmic fundamentals that exist for the creation of a concave hull. This is because the algorithms for concave hulls are much more complicated than convex hulls- because several variations dependent on constraints are possible. Moreover, for any given set of points, there may be lots of different concave hulls. In this regard, we present an approach for the creation of a concave polygonal hull based on the concept of linear referencing. The approach was motivated by the need to delineate the outline of a set of points where this has not been done by manual means. After a review of the few algorithmic efforts to construct a concave hull, some theory on linear referencing is presented followed by a discussion on the algorithmic implementation of the concave hull.

The concave hull approach is a more advanced approach used to capture the exact shape of the surface of features contained in a dataset. However, producing the concave hull is difficult; this is because of several possible and often conflicting objectives. Little work has focused on concave hull algorithms.

Galton and Duckham (2006) suggested 'Swing Arm' algorithm based on gift-wrapping algorithm. In the 'Swing Arm' algorithm, the polygon hull is generated by a sequence of swings of a line segment of some constant length, r (the swing arm). The initial line segment is anchored at an external point, and at each subsequent step, the line segment is anchored to the last point added to the hull, and rotated clockwise until it hits another point in the hull. If r is not less than the longest side of the hull perimeter, then the procedure will generate the convex hull; but if r is shorter, the resulting polygon is concave. The 'Swing Arm' Algorithm may produce separated concave hulls instead of single one, a situation that may not be desirable.

Another approach is based on alpha shapes, first described by Edelsbrunner (1981). Alpha shapes are considered as a generalization of the convex hull and a sub-graph of the Delaunay triangulation. They can be used in place of simple convex hulls to create a polygonal boundary containing the geometric objects within it. Mathematically, alpha shapes are defined as a family of shapes that can be derived from the Delaunay triangulation of a given point set with some real parameter, "alpha" controlling the desired level of detail. For sufficiently large alpha, the alpha shape is identical to the convex hull, while for sufficiently small alpha, the alpha shape is empty. As such, the resulting shape is neither necessarily convex nor necessarily connected. Alpha shapes maybe good, but sometimes they are not flexible enough because the alpha parameter is fixed. The "alpha-shape" algorithm based on Delaunay triangle suggested by Duckham et al. (2008) is similar to the concept of alpha shapes, and has the same weaknesses.

Adriano and Yasmina (2007) suggested a concave hull algorithm based on the k-nearest neighbours approach. The algorithm, although fundamentally designed for a set of points, can be used for other geometry primitives. The undesirable feature of the algorithm is that holes are produced in the resulting concave hull even when they are not expected.

4.2 Concave Hull through Linear Referencing

In the approach presented in this paper, the concept of using linear referencing (Curtin et al., 2007) to create a concave hull of a set of polygon features is described in Siriba (2012).Linear referencing as a process consists of a number of steps. The typical steps for a linear referencing are as follows:

a) Identifying the underlying linear feature (or the route structure) to which events can be referenced;
b) Defining and identifying measurements along the identified route (linear feature);
c) Output of linearly referenced events.

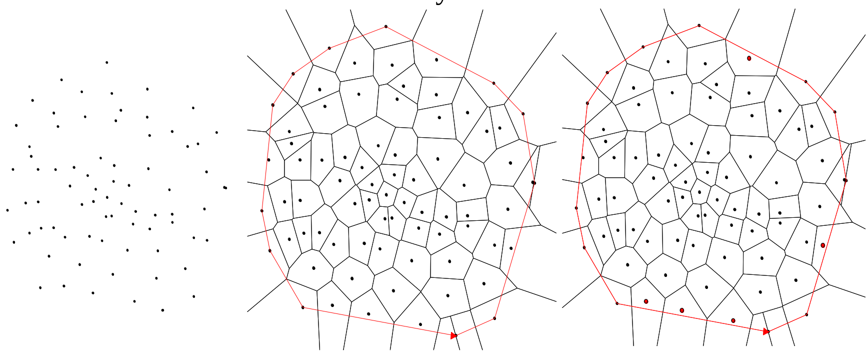
a) Identifying the underlying linear feature

The first step in linear referencing is to define the reference linear feature or network. However, a dataset consisting only of a set of points (Figure 3a)) does not consist of a linear network or linear features. The representative linear features from such dataset would include extracting the outlines of all the polygons. In this technique, such an outline, as the initial reference linear feature, is approximated by the convex hull of the polygon (Figure 3 b)).

b) Defining and making measurements along the linear feature

The event data (points) to be referenced and the direction of measurement are identified. The points to be referenced should include all “outer points”. This is achieved by identifying the points whose Thiessen polygon intersects with the reference linear feature, initially approximated by the convex hull. The Thiessen polygons are used as a means to identify the points that will eventually form the concave hull because each Thiessen polygon technically represents one individual point, and for all candidate points represented in red (Figure 3 c) of interest, their Thiessen polygons should definitely overlap (intersect) with required concave hull.

After the first iteration, points whose Thiessen polygons intersect with the convex hull (red outline in Figure 3 c) are identified. These points are then referenced to the reference linear feature, the outline of the convex hull represented by the red outline in the figure. The starting and end point (and therefore its direction) of the reference linear feature are depicted by the red arrow. Linear referencing is done as follows: for each identified point, its distance from the start of the reference linear feature and its offset from the reference linear feature are calculated and this constitutes the reference information. Then, a new reference linear feature is created, from all the identified points (red dots). This new reference feature is used to create the outline of a new concave hull, which effectively replaces the earlier one. This procedure is done iteratively until no more points can be identified and the concave hull cannot be modified any further.

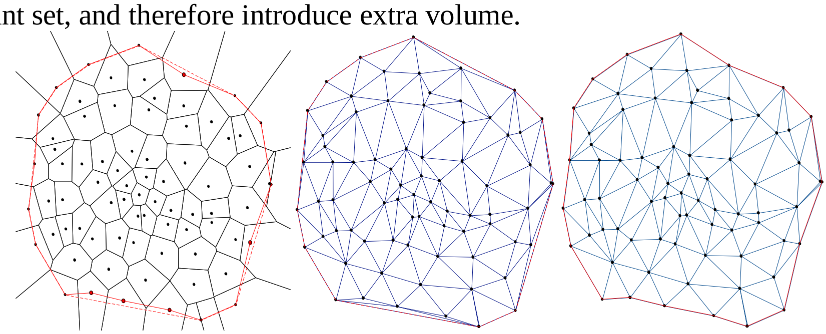


a) b) c)

Figure 4 a) point on plan b) initial approximation of the concave hull by the convex hull; b) additional candidate points for the concave hull

c) Output of linearly referenced events.

The linearly referenced points are consecutively constituted into a linear ring from which the concave hull is build. Figure 5 shows the final concave hull (red outline) created from the points identified after two iterations. The initial concave hull approximated by the convex hull is depicted by the red broken outline. The new outline delineates the footprint of the point set and is used to limit the triangulation of the points during volume estimation. The resulting triangulation based on the convex hull and the concave hull are respectively illustrated in Figure 5 b) and 5 c). It is evident from Figure 5 a) that there is extra surface at the border that is considered to be part of the point set, and therefore introduce extra volume.



a) b) c)

Figure 5 The convex hull – broken red outline and the final concave hull in red

5. EXPERIMENTATION

The spurious border polygons created as a result of unclear delineation of the stockpile introduces extra volume. With an experiment of 7 stockpiles, the spot heights were measured with RTK technique. For each pile, the footprint of each pile had been manually picked, although the points were not equally spaced. During computation of the volume, it was assumed that the footprints had not been identified, in which case the footprints were taken as the convex hull of the data points. Because of the spurious volumes at the periphery, the footprints were refined based on the concave hull of the points. The concave hull was created as described in the previous chapter. Finally, the volumes were computed using the manually delineated footprint. Table 2 shows for the seven samples, the volumes calculated based on the three sets of footprints.

Table 2: Comparison of volume based on the convex hull, concave hull and manual delineation of the stockpile footprints (units in cubic meters)

Stockpile	Convex Volume (A)	Concave Volume (B)	Manual Volume (C)	Absolute Difference || C - A ||	Absolute Difference || C – B ||
1	71.44	70.13	70.13	1.31	0.00
2	224.36	220.73	221.92	2.44	1.20
3	724.74	739.34	740.39	15.64	1.05
4	1048.74	1019.61	1023.29	25.45	3.67
5	1840.27	1857.27	1886.96	46.70	29.70
6	1887.25	1571.78	1643.94	243.31	72.15
7	4518.20	4430.77	4204.85	313.35	225.93

The difference between the manually delineated footprint and the footprint based on convex and concave hull are presented in the last two columns of Table 2. From the differences, it is clear that the volumes based on the concave hull footprint are closer to those based on the manual footprint. Although it is expected that the volume based on manual and concave hull footprints should be less than the volume based on the convex hull footprint, because spurious volumes are reduced, there is however a contradiction in sample 3 and 5. This is because in the data points representing the footprint are not uniformly spaced, thereby proving an opportunity for spurious volumes during triangulation. In particular, the convex hull approach is subjective, when setting the parameter. A unique result can only be realized if the data points representing the footprints are uniformly spaced.

6. CONCLUSIONS

A clearly delineated stockpile footprint is one way of ensuring that accurate volumes are determined. In case no deliberate effort has been made to identify it during point data collection, the resulting data can best be approximated by at least by a convex hull, but at best by a concave hull. A more representative outline, and therefore footprint can be achieved if the outline data points are more uniformly spaced – something that can be done during data collection. Prior to any manipulation of the point data to compute the volume the footprint, the boundary should be known in advance and picked during field data collection, and at uniform spacing.
The algorithm of the concave hull presented here is not based on any constraint and should therefore yield unique results and therefore more robust than those based the algorithms cited in this paper, which however depend on some constraints.

Acknowledgement
We acknowledge the East African Portland Cement Company ltd (Kenya), who provided the stockpiles of earthwork used for experimentation.

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