Trust and Distrust: A Reputation Ratings Approach

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Abstract: Agents’ reputation ratings in a social network form a real valued matrix which is discounted with singular value decomposition (SVD) to estimate the trust and distrust levels of agents. SVD eliminates noise as future expected trust and distrust are based on current reputation ratings. A discounting of 20 percent is optimal, further discounting does not improve error reduction. Reputation and trust are closely related. Distrust is different from trust and reputation. Distrust is similar to trust negation; and trust is similar to distrust negation.

Keywords: Trust, distrust, reputation ratings, singular value decomposition.

I. INTRODUCTION

There is growing interest in the role of trust and distrust in agent networks as the agents enhance their interaction outcomes in these networks. Much of the research work is concerned with trust and/or distrust in online interactions or situations where agents do not know each other [1, 3, 6, 12].

We consider a social network of agents familiar with each other in day to day interactions. A social network is a set of people, each of whom is acquainted with some subset of the others [4]. They are popular platforms for communication, interaction and collaboration between friends [16]. Social networks continue to permeate our lives as no agent lives in a vacuum; it must interact with other agents to achieve its goals [12, 13]. Networks play a central role in the transmission of information as they; allow for repeated interactions of the agents; and captures the dynamics of societal cooperation gradually [16].

For the agents to interact, they require information on whom to trust. Knowing whom to distrust is equally important but is trickier to compute in a satisfying way [1]. Trust is transitive while distrust is certainly not transitive. Trust continues to be a fundamental building block for today’s most successful recommendation and e-commerce systems. Thus, a network of people connected to each other thrives on the basis of trust ratings [6, 12]. We treat trust and distrust in a social network where the agents are interdependent, cooperate and dynamically change the perception of each other on the basis of the interaction outcome. This is achieved through use of peer to peer reputation rating system in the network and gives rise to a real valued matrix. A good reputation system collects, distributes and aggregates the feedback about the agents past behaviour [8, 12].

Trust and trustworthiness are positively correlated across societies [10]. Trust is a particular level of subjective probability with which an agent asesses that another agent or group of agents will perform a particular action, both before he can monitor such action [2]. Thus, trust is a subjective probability or expectation an agent has about another’s future behavior. Reputation is a perception that an agent has of another’s intentions and norms.

It is a social quantity calculated based on actions by a given agent and observations made by others in an ‘embedded social network’ [5].

On the basis of the two definitions, we refer to reputation as the current value and trust as the expected future value assigned to an agent. We therefore use the network discounted reputation ratings with SVD to estimate the agents trust and distrust levels. SVD, a matrix approximation method is used as it is widely researched and a common tool used heavily in recommendation systems, bioinformatics, computer vision and text processing [6, 11, 14, 20]. Our contribution is the estimation of trust and distrust levels of the agents in a social network using discounted peer-peer reputation ratings.

II. RELATED WORK

Social networks have been found crucial in sharing of information and thus key in the formation of opinions and beliefs, which shape behavior [7]. The social interaction structures that emerge tend to separate the agents into small interaction groups [18]. The dynamics of the societal cooperation gradually incorporate more and more information about social network structures [19].

A method for computing trust based on path probability in a random graph is developed by [1]. For each pair of users (x, y), they placed an edge between them with some probability that depends on the direct trust value between them, denoted by $I_{x,y}$. A set of n users each optionally expressing some level of trust and distrust for any other user is the work of [6]. The expressions become entries for a real valued matrix that is used to predict an unknown trust and distrust value between any two users. Compared to trust, distrust is much trickier to compute in a satisfying way [1].

A framework for describing semantics of general trust and belief networks is the work of [21]. Trust and distrust was incorporated by use of direct information over inferred information. They explored realistic models of trust and distrust based on partially ordered discrete values and further proposes a framework which is sensitive to local,
relative ordering of values rather than their magnitudes. The work of [17] hypothesized that it is possible to use bootstrapping to measure trust from human social behavior. They developed a model suitable for mobile social networking applications to manage trust. Initial trust values are generated and used to evaluate unknown mobile users to enable impromptu social networking.

Trust and reputation are closely linked. Reputation is the rating a member of a group receives from others; trust is a more complex social relationship than reputation [1]. Trust and distrust are referred to as positive and negative trust and [20] analysis indicates that distrust is not the negation of trust for distrust has added value over trust. Distrust is at least as important as trust [6]. A method for computing distrust is developed by [1], but acknowledges that computing distrust is complex. Distrust is incorporated by resolving conflicting trust and distrust information through a nonlinear optimization. Distrust is a new dimension of trust, but some social scientists still consider distrust as the darker side of trust [20]. Further, distrust information is publicly unavailable and social media services rarely implement distrust mechanism in their networks.

The work of [15] represents trust in the interval (0,1) where 0 represents complete distrust and value 1 blind trust. The model reflects members of social network and differentiates them according to their disposition to trusting somebody. A value of 1 indicates that the agent is highly trusted and hence blind trust. Both trust and distrust help a decision maker reduce the uncertainty and vulnerability or risk associated with decision consequences [20]. Generally, trust and distrust are complex measures representing people’s multidimensional utility functions [6,8].

Trust and reputation rating systems have wide applications with many different types of mechanisms. The basic criteria for judging the quality and soundness of reputation computation engines are summarized by [8]. No single solution is known to exist that is suitable in all contexts and applications. They further discuss the number of reputation computational engines that exist.

We model the trust and distrust levels of agents in the social network from the real valued matrix of the discounted reputation ratings in the network using SVD. The initial reputation values are discounted because they are current perceptions about an agent while trust (distrust) are the expectations that an agent will perform (not perform) a particular task in future as expected.

III. SOCIAL NETWORKS

We assume that reputation ratings about current interactions are captured and distributed and agents are willing to provide the ratings. Consider a set of $N = \{1,2,\ldots,n\}$ agents whose state and interactions in a social network evolve in discrete time $t$. We assume that the agents are connected to each other at any given time $t \in \{0,T\}$ and thus we have a peer to peer review system for the agents’ reputation ratings in the network. Let $R_i = \{r_{i1},r_{i2},\ldots,r_{in}\}$ be the reputation ratings agent $i$ receives from the other $N-1$ agents in the social network. This peer to peer reputation rating is based on a five star scale: 1-lowest, 2-low, 3-medium, 4-good and 5-high, that is, $R \in \{1,2,3,4,5\}$ where:

$$ R = \begin{cases} r_{ij} = 5 & \text{If } i = j \\ 1 \leq r_{ij} \leq 5 & \text{If } i \neq j \end{cases} $$

Each agent is expected to rate the other $N-1$ agents. In ideal real life situations, we would rate ourselves with a score of 5 which forms the diagonal of matrix $R$. The ratings form the entries of a real valued matrix, $\hat{R}$ and are bidirectional. We take $\hat{R}$ to be the reputation ratings; $r_{ij}$ is the reputation rating that agent $i$ holds for agent $j$. Let $\tilde{R}$ be a real valued matrix with $r_{ij} = 5$ for $i, j = 1,2,\ldots,N$, the maximum score possible. Let $T$ be the matrix of ‘raw’ trust values of the agents. As trust is the future expectations based on the current reputation ratings, we express trust as, $E(T) = \alpha R = R_k$. We estimate the ‘raw’ trust values of the matrix $T$ by discounting the singular values obtained from the SVD with a factor $\alpha$. This eliminates the noise which represents the future expectations based on current observations in trust estimation. Noise is eliminated by adopting an accuracy threshold from 10% to 90% and thus chooses a reduced rank approximation with error defined as [9]

$$ \alpha = 1 - \frac{\sum_{i=1}^{k} \sigma_i^2}{\sum_{i=1}^{N} \sigma_i^2} $$

This is the relative error for a sum of the first $k$ terms of the SVD outer product expansion [9]. For distrust estimation, let $D$ be the matrix of ‘raw’ agents distrust levels. Let $\hat{R} = \hat{R} - R$ be the difference between maximum possible rating and the reputation ratings. Similar to discounting in the ‘raw’ trust values, we express distrust as $E(D) = \beta \hat{R} = \hat{R}_k$ where $\beta$ (or $\alpha$) represent the noise that is eliminated using SVD.

SVD is used for optimal low rank approximation and a partial SVD can be used to construct a rank $k$ approximation. Given a matrix $\hat{R}$, we can represent it as the product of two orthonormal matrices $U$ and $V$ and a diagonal matrix $S$ as $R = USV^T$. The low rank approximation is to make $\| \tilde{T} - VV^T\hat{T}_k \|$ and $\| \tilde{D} - VV^T\hat{D}_k \|$ as small as possible for the estimation of the trust and distrust levels respectively [14]. In both cases, matrices $\tilde{T} = \hat{T}_k$ and $\tilde{D} = \hat{D}_k$ can be decomposed with SVD and thus these are the discounted
matrices. We use simulation with 
\[ R \sim \bigcup (0,1) \text{ for } 1 \leq R \leq 5 \] 
based on Matlab version 7.0.1.

**TABLE 1:** Glossary of matrix names used in the study

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T )</td>
<td>'Raw' trust levels matrix</td>
</tr>
<tr>
<td>( R )</td>
<td>Reputation ratings matrix</td>
</tr>
<tr>
<td>( \tilde{T} = R_k )</td>
<td>Estimated trust levels matrix</td>
</tr>
<tr>
<td>( \hat{R} )</td>
<td>Maximum reputation ratings, ( r_g = 5 )</td>
</tr>
<tr>
<td>( \hat{R} )</td>
<td>Difference of the matrices, ( \hat{R} = R - \tilde{R} )</td>
</tr>
<tr>
<td>( D )</td>
<td>'Raw' distrust levels matrix</td>
</tr>
<tr>
<td>( \hat{D} = \hat{R}_k )</td>
<td>Estimated distrust levels matrix</td>
</tr>
</tbody>
</table>

We compare the trust, distrust and reputation values based on the simulated peer to peer reputation ratings which are a real-valued matrix. SVD extracts the trust and distrust matrices through low rank matrix approximation.

### IV. RESULTS

Table 2 Friedman test highlights that trust and reputation are similar from error estimation of 20%. Any other reduction of reputation ratings does not show any changes in trust values, that is, further reduction of the singular values has not benefit in noise reduction. In table 2, we have the first four values only as the rest do not change with increase in the error term.

**TABLE 2:** Friedman tests comparison between Trust, Reputation and Distrust

<table>
<thead>
<tr>
<th>Error term (%)</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reputation &amp; trust</td>
<td>0.018</td>
<td>0.157</td>
<td>0.157</td>
<td>0.157</td>
</tr>
<tr>
<td>Reputation &amp; Distrust</td>
<td>0.025</td>
<td>0.025</td>
<td>0.025</td>
<td>0.025</td>
</tr>
<tr>
<td>Trust &amp; Distrust</td>
<td>0.025</td>
<td>0.025</td>
<td>0.025</td>
<td>0.025</td>
</tr>
</tbody>
</table>

Figure 1 show the relationship between reputation ratings and trust in a social network. Reputation and trust are closely linked with similarities observable for all the network agents. Even after trust values are extracted from reputation values using low rank matrix approximation, the two values are highly similar which indicates that reputation is a good measure of possible trust values of network agents. This compares well with table 2 from the 20 percent noise elimination using SVD.

**Fig. 1:** Relationship between trust and reputation

Figure 2 indicate the varying difference between trust and distrust. The dynamics show that high trust is linked to low distrust and low trust with high distrust. Dissimilarity between trust and distrust is observable.

**Fig. 2:** Relationship between trust and distrust

In figure 3, reputation, trust and distrust values show the close link between trust and reputation. Distrust is different from trust and reputation values. Similar observations are noted in figure 1 and figure 2.

**Fig. 3:** Comparison of trust, reputation and distrust

Figure 4 compares trust and the distrust negation and a similarity is observable.

**Fig. 4:** Comparison of trust and distrust negation

Figure 5 highlights that distrust and trust negation are similar.

**Fig. 5:** Comparison of distrust and trust negation
These results are similar to those in figure 4 where trust and distrust negation are closely related. Thus, given reputation ratings of agents in the network, we can estimate both the trust and distrust levels.

V. CONCLUSION

Trust and distrust values in an agent network are estimated based on a real valued reputation matrix using SVD. Trust and reputation values are highly similar but dissimilarity is observed against distrust. Trust is similar to negation of distrust and distrust is the negation of trust. Both trust and distrust are the subjective probability of future expectations based on current reputation ratings.

REFERENCES
