THE UNIVERSITY OF NAIROBI

COLLEGE OF BIOLOGICAL AND PHYSICAL SCIENCES

SCHOOL OF PHYSICAL SCIENCES

DEPARTMENT OF PHYSICS

REPRESENTATION OF QUANTUM MECHANICS IN THE FORMULATIONS OF SPECIAL RELATIVITY

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This project is submitted for the partial fulfillment of the requirement for the admission of Master of Science in Physics degree of the University of Nairobi.

DATE: 6TH, MAY 2016

Declaration

I declare that this project is my original work and has never been submitted elsewhere for examination, award of a degree or publication. Where other people's work or my own work has been used, this has properly been acknowledged and referenced in accordance with University of Nairobi requirements.

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This project has been submitted with my approval as the supervisors.

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Signature…………………………..

Date………………………………….

Dedication

I would like to dedicate this work my beloved wife Elizabeth and to my sons Phillip and Michael. I know your personality is enough like mine that you have had your share of struggles in seeing this project succeed. I see you growing continually, and I know that you are experiencing the victories that come from the joy of having a family in you.

I love you and I am proud of you. Keep pressing on!

Acknowledgement

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Abstract

Quantum theory and Relativity are not basically able to get along well with each other even though the laws of Physics are universal. Trying to synchronize the two theories is our interest and target since their harmonization has become a big challenge. In this research, we are trying to bridge the gap between these two theories using a simple triangular model. The modification can only be achieved if we intensively examine the representation of Quantum Mechanics in formulation of special relativity. Our main aim therefore is to think of the successful correlation of postulates of Quantum Mechanics and those of Special Relativity which can lead us to new possibilities of getting new postulates to solve the problem.

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CHAPTER ONE

1.0 Introduction

Quantum theory and theory of Special Relativity have existed from the need to understand the physical experimental results that could not be explained theoretically. However, even though the theories have been proved correct, our basic problem still remains that they do not sail together up to date. If both are right representations, how can they be so different, contradicting each other, and yet the laws of Physics are universal? [10] Through a new interpretation of Special Theory of Relativity and with a model given for the physical space time, we can find a way to understand the basic principles of Quantum Mechanics consistently from Classical Theories.

The most successful approach to this problem starts when we consider a classical relativity field theory rather than just a single relativity matter alone. Classically, when quantization is done, the result obtained realizes a consistent combination of quantum mechanics and relativity. Achieving the goal of developing Quantum Mechanics, the renunciation of the casual space-time mode of description has to be applied. This also assists in characterizing the classical physical theories experiencing such intensive clarification through the theory of relativity [9].

In this paper, we attempt to synchronize Special Relativity and Quantum Mechanics using a simple triangular model.

1.1 Statement of the problem

In recent studies Special Relativity has not been successfully synchronized with Quantum Mechanics. Currently, there is no clear relationship between Quantum theories and theory of Special Relativity. So far research has been done towards the unification of the two physical theories. However, harmonization of the two has become a big challenge. In this research, we are trying to bridge the gap between these two theories using a simple triangular model.

1.2 Justification

Physics is universal, laws of Physics are applied everywhere (invariant), yet some of the approaches do not describe each other, for instance, Quantum Mechanics is said to be successful, but quantization of gravity has not been achieved. Should we have an approach that can seek a unification of Quantum theory and the theory of Special Relativity since there is a correspondence between the two fields? There is a need for the incorporation of the postulates of Special Relativity with the postulates of Quantum Mechanics, thus, the need to reformulate Quantum theories that are inclusive of Special Relativity.

In order to design a model in science, it requires a proper understanding of mathematical skills, geometry, algebra and statistical methods and the applications. In this project we explore a triangular representation used in the Lorentz transformations in Special Relativity postulates to derive the postulates of Quantum Mechanics. This shows that there is a close correspondence between these two fields of Physics.

This project can successfully be used to correlate postulates of Quantum Mechanics and those of Special Relativity. The knowledge contained in this project can be useful in technological advancement of quantum computers and in the current research on the mathematical model for the Brain activity.

1.3 Objectives

- To have a representation or physical theory(s) that explains the diverse Relativity, formulations of quantum mechanics and other theories.
- To modify a model in science, there is need for understanding of mathematical skills and the applications in physics.
- To logically understand and come up with the applications of Quantum Mechanics and Relativity in real life experience.
- To formulate equations in both special relativity and Quantum Mechanics and compare how they correlate with each other.
- To reformulate Quantum Mechanics that can incorporate Special Relativity.

CHAPTER TWO

2.0 LITERATURE REVIEW

In 1899, Max Plank initially presented the quantum speculation. Up until Plank, it was believed that matter could be partitioned into littler and littler particles unbounded yet at the same time be holding the fundamental attributes of the substance. Board presented the idea that electromagnetic fields must be comprised of little unified units. Quantum mechanics was developed by several people with different theories such as; Bohr’s atomic spectra, Black body radiation by Max Plank, The light quanta (photoelectric effect) by Albert Einstein, De-Broglie and Schrodinger matter waves (wave-particle duality) [3].

Coming full circle a thorough hypothesis discharged by Weiner Heisenberg in 1925 and Erwin Schrodinger in 1926. It was however understood that quantum mechanics was a deficient hypothesis since it could not account precisely for electromagnetic forces furthermore violated the laws of special relativity [1].

In 1948, another hypothesis created by Richard Feymann, Sun Itiro Tomonaga and Julian Schrodinger introduced quantum field hypothesis which today remains the best for magnetic forces and is the present theory for Nuclear and radiation constrains all the more regularly called the strong and weak forces[6,11].

In 1905, Albert Einstein stunned the world by issuing three papers in particular; Brownian Motion-matter is comprised of molecules thus the nuclear conjecture as crucial piece of quantum mechanics, The photoelectric effect ("photon"), Special Relativity-The velocity of light was a flat out steady. Everybody who measures the velocity of light will get the same number, paying little respect to how quick they are moving and how quick the light source is moving and nothing can go speedier than light. Special Relativity likewise attests that space and time exist as a union, which is called Space-time. [2]

CHAPTER THREE

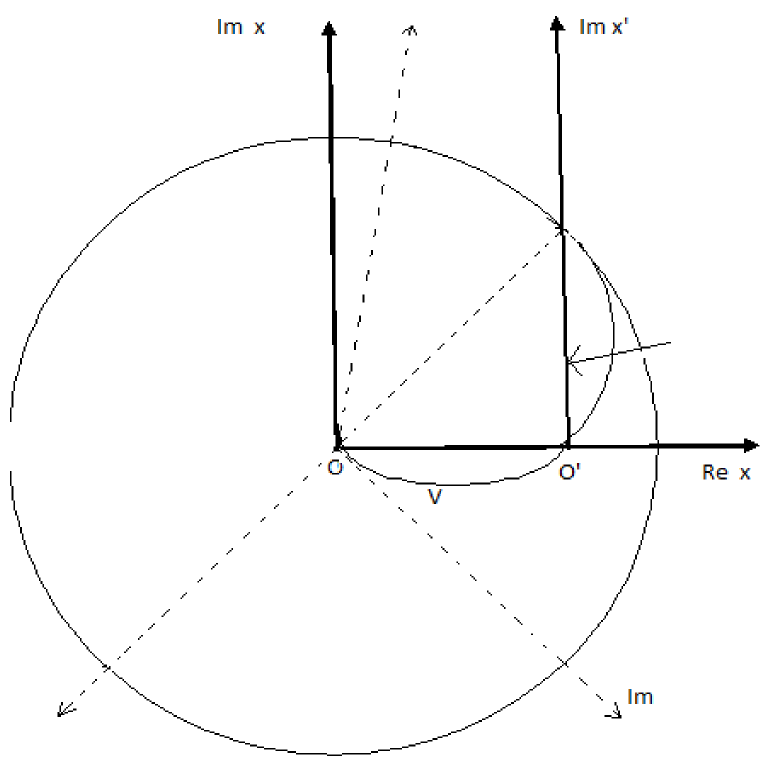
3.0 FORMULATIONS

3.1 SPECIAL RELATIVITY

We know the formulations of Special Relativity based on Lorentz transformations of space and time, which include $t' = (L_0/v)\sqrt{1-\beta^2}$, $L' = L_0\sqrt{1-\beta^2}$. The postulates of Special Relativity such as light principle, inertial frames and universal application of physics laws which were generated by Galileo, improved by Newton and clarified by Einstein can be inferred from figure 1 below. The Special Relativity in Lorentz transformations can be derived in many ways.

3.1.1 Time Dilation and Length Contraction

In special theory of Relativity, one assumption is about the velocity of light (c) in vacuum which is the same for all inertial frames [7].



$\sqrt{1-\beta^2}$

Figure 1: Dimensional complex space

If the propagation of light follows the imaginary axis in an inertial complex space, then the speed of light is independent of a Relativistic constant of motion on real axis because of the orthogonality between the real and imaginary axis. Since the time interval is the same on the imaginary and real axis of an inertial frame, the scales in both axes are also the same, the wave front of light in the imaginary axis is corresponded to the real axis with the exactly same coordinate as the imaginary because the change of energy in vacuum is mapped to real world.[11]

In the figure1, there are two observers – A at the origin and B with the velocity (v) in x- direction. When the moving observer B passed through the origin, they synchronized their watches to zero and a light source started radiating from the origin. One second later, the observer A at the origin of O- frame recognizes that the moving observer's time is $\sqrt{1-\beta^2}$ because the light signals in O'-frame propagated from O' to p_i' through the imaginary x'axis and the time interval equals to the imaginary time interval in the inertial complex space. However, this time dilation effect is the same to observer B since the effect is relativistic. With a similar procedure as above we can also show the length contraction effect. Let us say, in O- frame a rod is appended at the origin and extended to x $= L_0$ – proper length. At $t = L_0/v$, the origin of O'- frame is at x $= L_0$ and $t' = (L_0/v)\sqrt{1-\beta^2}$. Definitely observer A, thus, surmise that observer B estimated the length of the rod as $L' = L_0\sqrt{1-\beta^2}$, where; $\frac{1}{\sqrt{1-\beta 2}} = \gamma$ and $\beta^2 = v^2/c^2$ [9].

Therefore, we can rewrite that;

$L' = L_0\gamma$ and its reciprocal is;

$$\frac{L_0}{L^{'}} = \sqrt{1-v^2/c^2} \qquad (3.1.1.1)$$

, such that under modification we can achieve;

$$\left|\frac{L_0}{L^{'}}\right|^2 = \left(\frac{c^2}{v^2}-1\right) \qquad (3.1.1.2)$$

3.2 QUANTUM MECHANICS

The triangle (Dr. Okeng'o and Dr. Awuor) used for Special Relativity, can be modified to bring out postulates of Quantum Mechanics. [8].

Consider the diagram below,

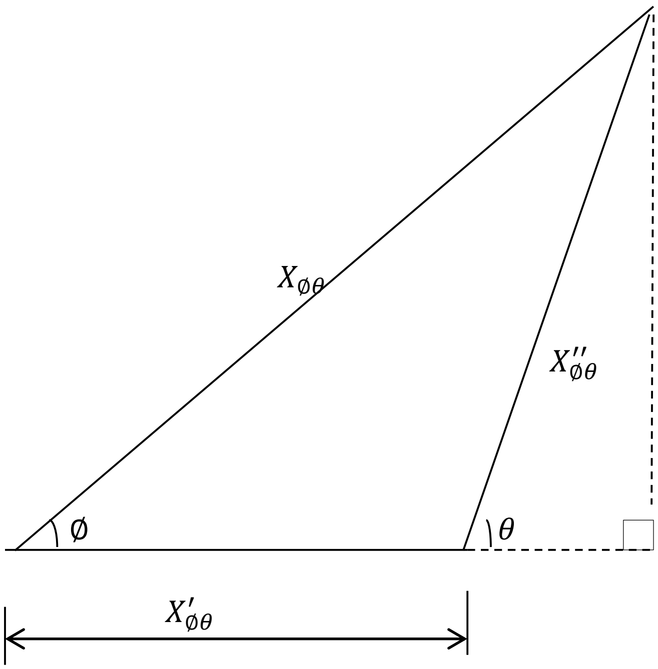


Figure2; *The general rotation in 2-D (Okeng'o and Awuor triangle)*

Given a complex number,

$$X = X' + iX'' \tag{3.2.1}$$

From the equation

$$X = X'_{\phi} + X''_{\theta} \tag{3.2.2}$$

Let

$$X''_{\phi\theta} = \xi X'_{\phi\theta} \tag{3.2.3}$$

and using the complex (from equation (3.2.2))

$$X = X^{'} + iX^{''} \quad (3.2.4)$$

Let us now substitute equations (3.2.3) and (3.2.4) into equation (3.2.2), we have

$$X_{\phi\theta} = X^{'}_{\phi\theta} + \xi X^{'}_{\phi\theta} e^{i\theta} \quad (3.2.5)$$

where $i = e^{i\theta}$, $(\cos\theta + i\sin\theta = e^{i\theta}, \textit{that is,}\ \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = e^{i\theta})$ and i is positive quarter turn.

Modifying triangle (Figure 2) to give $sin\theta$ and $cos\theta$ by making it right angle triangle (complementarity) then if the angle $\theta = 90^0$, then the probability in Quantum Mechanics can successfully be derived. If $\theta = \frac{\pi}{2}$, then we have $X_{\phi\frac{\pi}{2}}$ as complex..

Therefore, substituting for *i*, we get

$$X_{\phi\frac{\pi}{2}} = X^{'}_{\phi\frac{\pi}{2}} \left[1 + \xi e^{i\theta}\right] \quad (3.2.6)$$

Consider the energy conservation of system above, the sum of kinetic energy (k.e) and potential energy (p.e) is a constant.

Taking the complex form of the sum above, we have

$$E = E_k + iE_p \quad (3.2.7)$$

From similarity addition;

$$E_k = \xi_k e^{i\theta_k} \quad (3.2.8)$$

$$E_p = \xi_p e^{i\theta_p} = 1\ , \quad (3.2.9)$$

Iff

$$\xi_p = e^{-i\theta_p} , \tag{3.2.10}$$

Such that,

$$\xi_p \xi_p^* = 1 . \tag{3.2.11}$$

$\xi_k = n_k e^{-i\theta_k}$, where n_k is a constant of proportionality.

Thus equation (3.2.6) takes the form;

$$X = X^{'} \left(\xi_p^* \xi_p + n_k \xi_k^* \xi_k \right) \tag{3.2.12}$$

where $\left(\xi_p^* \xi_p + \xi_k^* \xi_k \right)$ are finite numbers then, $X^{'} n_k \xi_k^* \xi_k$ is a fraction of X.

$$X = X^{'} \xi_p^* \xi_p \left[1 + n_k \frac{\xi_k^* \xi_k}{\xi_p^* \xi_p} \right] \tag{3.2.13}$$

$$X = X^{'} \left(1 + n_k \xi_k^* \xi_k \xi_p \xi_p^* \right)$$

$$X = X^{'} \left(1 + n_k \psi^* \psi \right)$$

$$\frac{X}{X^{'}} = 1 + n_k \psi^* \psi$$

$$\psi^* \psi = \frac{1}{n_k} \left(\frac{X}{X^{'}} - 1 \right)$$

$$\left| \psi \right|^2 = \frac{1}{n_k} \left[\frac{X}{X^{'}} - 1 \right] \tag{3.2.14}$$

3.3 OTHER POSSIBILITIES

Suppose that we have two observers with the knowledge of Quantum mechanics each with a possibility space, then, can one understand Quantum? To what level? In which form? (Is it possible to understand everything?)

The diagram below corresponds to the Galilean and Newtonian Relativity for the case of a single observer in which the laws of motion hold in all frames of reference provided the velocity is constant (no acceleration). It can also be equivalent to the Schrodinger's wave function in which the function describes all the characteristics of the physical observables.[3]

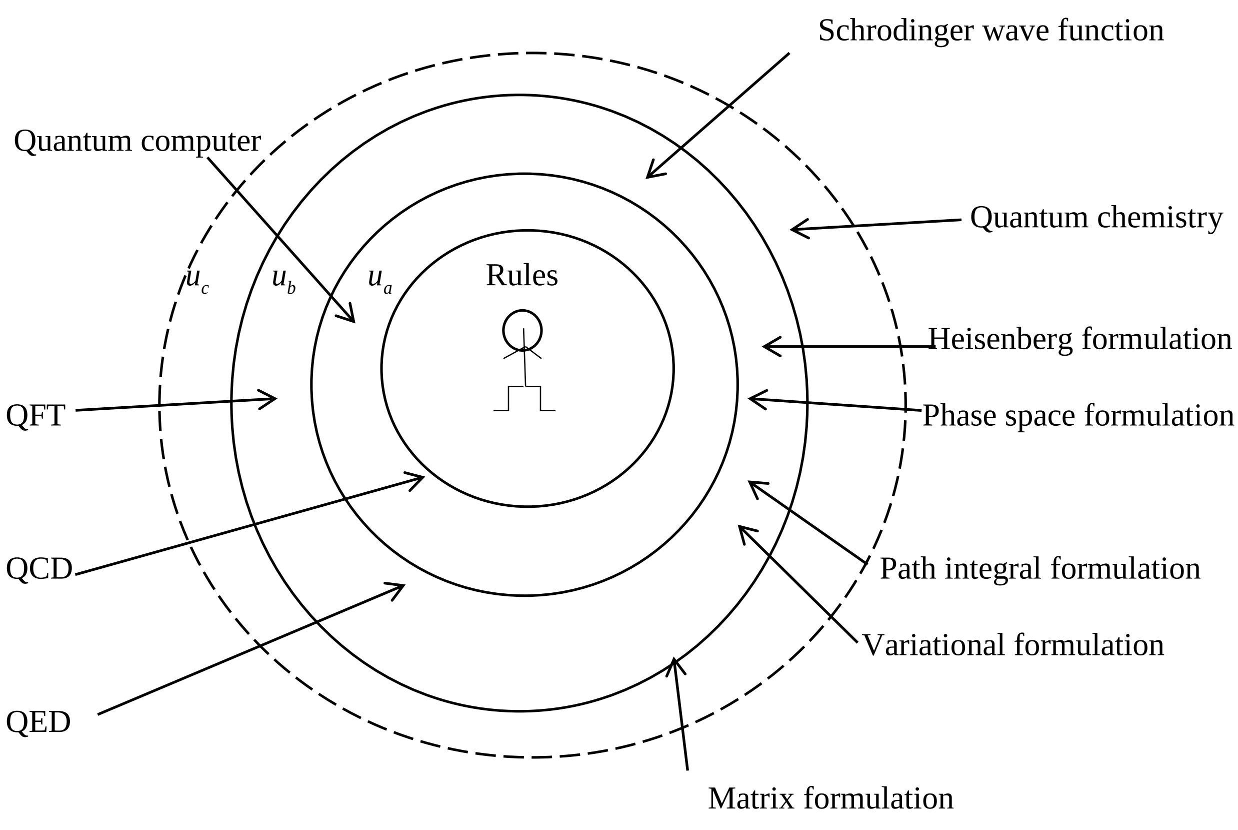


Figure 3 *(Single observer)*

QFT- Quantum Field Theory

QCD- Quantum Chromodynamics

QED- Quantum Electrodynamics

$u_a < u_c$

$u_b < u_a$

u_a -Observable universe

u_b -Observers universe (state of the mind)

u_c -The unseen universe (if it exists)

u_a -The observable universe comprises of physical observable properties which can be indemnified such as length/size, color, shape, momentum, Energy.

Suppose, we have two observers who understand quantum mechanics well, can they absolutely agree or disagree and to what level? Two observers with relative motion are condemned to eternal disagreements about times and lengths or are they? They will disagree over the arbitrary times measured in seconds and length measured in meters. They can also disagree on clock ticks, atomic vibrations and light wavelengths.[10]

Let us now consider the diagram 4 below which gives an insight of Einstein's Special Relativity for two observers A and B under uniform motion. Their measurement of time and length are related by Lorentz transformation. However, the observers can only agree on the measurement of the speed of light c.[7].

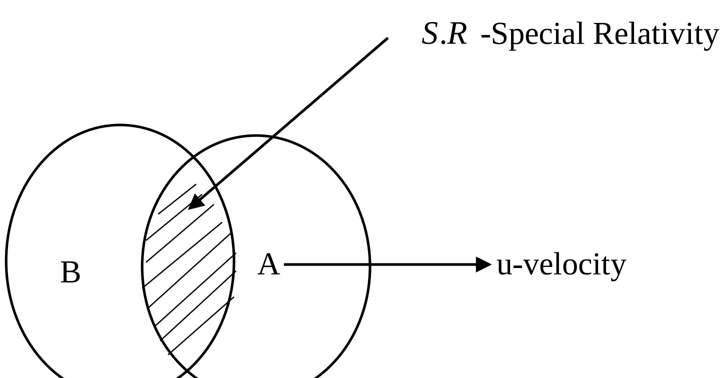


Figure 4*: (two observers)*

The intersection of A and B represents the region of agreement about their knowledge of Quantum Mechanics, that is, the properties of the observables such as length, time, and mass .in the universe.

Will their Quantum measurements be Lorentz transformed? Will their measurement of speed of light be constant?

How can we measure the level of agreement or disagreement? We use correspondence-Measurement of length and time. Hence, Einstein’s theory of relativity and the Lorentz transformations are applicable. This answers the question, “does quantum mechanics contain relativity?”

CHAPTER FOUR

4.0 DISCUSSIONS AND RESULTS

From equations (3.1.1.2) and (3.2.14), it is clear that there is a close analogy between the ratios of lengths in Special Relativity and probability in Quantum Mechanics.

Imagine that we have observers.

In Special Relativity Einstein referred to classical measurements made by two observers. These measurements are reconciled. Each observer's measurements are related by Lorentz transformations in relative motion in relation to inertial reference frame.

Einstein measurements are classical in which *speed of light* is constant so that no absolute reference frame, no absolute direction, that is, there is isotropy and homogeneity in terms of reference frames. Suppose that these two observers are making quantum measurements rather than the classical one, would then space and time values be related by Lorentz transformations?

Relations between Quantum and Classical measurements are accounted for by correspondence principle with reference to quantum numbers (microscopic) and large quantum numbers (macroscopic) refer to the classical measurements. However, Bohr statement on correspondence is a single observer's context. A single correspondence will agree with both Bohr and Bohm. For two observers the law of correspondence tells us that large measurements will give measurements that are Lorentz transformed. We therefore need to identify each observer, the extent to which they identify quantum observables since each has a personal universe of possibilities. How would these two possibility spaces be related? Relation between quantum values whether there are new postulates to be formulated, but at the moment, postulates are covered by rules of symmetry (rules of invariant), complementarity, and conservation laws. Further rules may be found.

We know that Special Relativity relates classical measurements made by distinct observers, in quantum phase we wish to have quantum measurements made by two distinct observers.

The nature of measurements in Quantum Mechanics differs from those classical ones in ways yet to be discovered. We incorporate concepts such as the chameleon effect where observer interferes with the outcome.

The other difference is uncertainty principle which limits the extent to which we cannot simultaneously get the pairs of values such as kinetic energy and potential energy. Classically, the observer does not interfere with the outcome of a well-defined experiment. Classical measurements result into deterministic values while Quantum measurements give the probability of the expectation value.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATIONS

From this analysis, Relativity endorses space time continuum, while quantum hypothesis recommends a discrete world. It is shown further that Relativity is deterministic while quantum is probabilistic in nature. There are potential outcomes of getting new issues with new postulates, and Lorentz transformations can be done at the next level.

REFERENCES

1. Ballentine, L.E. *Quantum Mechanics: A Modern Development,* Singapore, World Scientific, 1998.
2. D'Ariano, G.M.*Where the mathematical structure of Quantum Mechanics comes from* Work presented at the conference *Beyond the Quantum* held at the Lorentz Centre in Leiden (2006) (quant-ph/0612162).
3. Einstein, A.,Podolsky,B. and Rosen, N., phys. Rev. **47,** 777-780 (1935)
4. Feyerabend, P.K. phil. of Science **35**, 309-331 (1968)
5. Gertsbakh, I., Gertsbakh, E. – *Measurement Theory for Enginee*rs – Springer (2003)
6. Harlow, R.H., Thompson, R.L., Dotson, C.- *Fundamentals of Dimensional Metrology*- Thomsom Delmar Learning (2003).
7. Khots, B., Khots, D., Observer's Mathematics- Mathematics of Relativity, *Applied Mathematics and Computations,* v. 187, issue 1, pp 228-238, 2007.
8. Okeng'o G.O and Awuor J.B on the Complexification of Minkowskian Spacetime *African Journal of Physical Sciences* 2(2015) 73-76 ISSN 2313-3317.
9. Reichenbach, H. *Philosophical Foundations of Quantum Mechanics,* University of California Press, 1944.
10. Rocchi, P., Gianfagna, L. – *An Introduction to the Problem of the Existence of Classical and Quantum Information* – Proc. Quantum Theory: Reconsideration of foundations (QTRF-3), 248-258 (2006)
11. Von Neumann, J.: *Mathematical Foundations of Quantum Mechanics*, Princeton University Press, Princeton, NJ, 195.