



UNIVERSITY OF NAIROBI

"Multi Agent Based Traffic Controls"

Henry Ndirangu Mungai

P58/70513/2008

Supervisor

Mr. Lawrence Muchemi

Submitted in partial fulfillment of the requirements of the Master of Science in Computer Science

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DECLARATION

This project as presented in this report is my original work and has not been presented for any other university award.

Student: Henry Ndirangu Mungai

Registration No: P58/70513/2008

Signature:  _____ Date: 09/03/2012

This project has been submitted in partial fulfillment of the requirements of the Master of Science in Computer Science of the University of Nairobi with my approval as the university supervisor.

Signature:  _____ Date: 9th March 2012

Supervisor: M.r Lawrence Muchemi

DEDICATION

To my fiancée Anne, son Matthew and parents thank you for your support when I needed it most.

ABSTRACT

The traffic jams are common in the modern world and are mostly attributed to increase in population and the continual expansion of the urban centers. Traffic jams cost the economy a lot of money in wasted time in traffic and fuel consumption and also result to air pollution.

Advances in the field of artificial intelligence have made it suitable to use agents in the management and control of traffic due to their ability to control and coordinate their activities.

This research project has implemented a multi agents based traffic control system that is able to manage the traffic flow based on the prevailing conditions on the roads. Multi agent systems are best suited for such environments since they are able to perceive the environment they are located in and make decisions accordingly by negotiating and cooperating to ensure smooth flow of traffic regardless of the traffic densities. The agents negotiate based on the average waiting time and queue lengths such that agents whose junction has a maximum waiting time and queue lengths are given preference.

By use of a simulator comparisons in performance have been done between a pre-timed traffic control system and a multi agent based traffic control system. Results show that the performance of both systems deteriorates with increase in traffic volumes. Regardless of this a multi agent based traffic control system is able to perform better than the pre-timed traffic control system regardless of the traffic situation on the roads by attaining 33% improvement.

ACKNOWLEDGEMENT

To almighty God for His grace, love and providence throughout this course.

To my supervisor Mr. Lawrence Muchemi for his continued support, guidance and availability throughout this project. It would not have been easy without your support.

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CHAPTER ONE

INTRODUCTION

1.0 Background

Traffic jams are common in the modern world and are mostly attributed to increase in population and the continual expansion of the urban centers. They cost the economy a lot of money in terms of wasted time in traffic, fuel consumption and also result to air pollution.

As traffic jams become a common occurrence different methods have been put in place to try and tackle the occurrence. Expansion of the current infrastructure is one of the ways put in place to cope with the large number of cars. However in the long run this may not be economically feasible due to the high cost incurred in the expansions.

As the number of vehicles continue to increase there is the need to come up with better automated methods of controlling traffic on our roads that are responsive to the changing conditions which in turn helps in improving traffic flow in the roads. Thus a need arises for the development of a traffic management system that is able to address the changing traffic conditions on our roads.

This problem can be best tackled by using the latest technology in the field of information technology to manage the existing road infrastructure.

Different methods have been used in trying to optimize the traffic flow by means of controlling the traffic lights. For instance pre timed traffic control whereby all control parameters change their status after a given clock cycle, actuated control whereby the traffic signal is adjusted from real time traffic conditions that is observed by detector.

Advances in the field of artificial intelligence have made it suitable to use agents in the management and control of traffic due to their ability to control and coordinate their activities.

This proposes a multi agent based traffic control system that is able to manage the traffic flow based on the prevailing conditions on the roads. Multi agent systems are best suited for such environments since they are able to perceive the environment they are located in and make decisions accordingly by negotiating and cooperating to ensure smooth flow of traffic regardless of the traffic densities.

1.1 Definitions of important terms

i. Agent

This is a system that is that is situated in some environment and is capable of autonomous action in this environment so as to meet its design objectives (Wooldridge, 2002).

ii. Multi agent System

This is a computer system that consists of many agents that interact with each other with the aim of solving a given problem.

iii. Reinforcement Learning

Reinforcement learning assumes that the agent lives in a Markov process and receives a reward in certain states. The goal is to find the right action to take in each state so as to maximize the agent's discounted future reward. (Vida l, 2007).

Reinforcement learning is learning what to do, how to map situations to actions so as to maximize a numerical reward signal (Sutton et al, 2005).

iv. Queuing Theory

This is the mathematical study of *queues* that enables mathematical analysis of several related processes, including arriving at the (back of the) queue, waiting in the queue and being served at the front of the queue.

1.2 Problem Statement

Commuters in most urban centers can experience long travel times due to inefficient traffic lights. Most traffic lights use time intervals that do not necessarily consider the prevailing traffic conditions. The interval may change at certain durations during peak hours but this does not completely solve the traffic jam problem. In most cases the traffic lights may turn to green even when the lane in which the vehicles are moving to congested such that the vehicles end up blocking the junction. This is as a result of poor or no interaction between the existing traffic control systems.

Pre timed traffic control systems are mainly used in our road systems and in circumstances where the traffic density is high they tend not to respond effectively in the reduction of traffic delays since the traffic lights will always be turned on and off in response to the set cycle regardless of whether a given lane has traffic or not.

This research address this shortfall of pre timed traffic signals by introducing the aspect of multi agents whereby the agents are able to detect traffic densities on lanes that are joining the junction thus they respond effectively by controlling the traffic lights so as to reduce wait times and queue lengths.

1.3 Objectives

This research project aims to achieve the following objectives

- i. Design a protocol that will be used by the agents to allow vehicles to use a given intersection.

This project makes use of multi agents that are located at different intersections which interact with each other with the main aim of controlling the traffic flow. From these interactions the agents need to have policies in place that govern the interaction from which agents are able to gain priorities to use the junction.

- ii. Develop and test a simulated platform on which the designed protocols can be tested
The designed protocol that govern how the agents interact in order to gain access to use a particular junction will need to be put to test on a simulation platform from which different traffic conditions can be put to test.
- iii. Evaluate the performance of collaborative agent controlled traffic signal against a pre-timed traffic signal.

From the simulation platform the multi agent based traffic control system was put under the same test condition as the pre timed traffic control system from which their performance was compared.

1.4 Purpose of Study

The purpose of this study was to assess the effect of using multi agent systems in the regulation of traffic flow.

1.5 Research Question

- i. What are the limitations of the pre timed traffic control systems currently being used?

- ii. What will be the effect of adding queuing theory to reinforcement learning in terms of improving traffic flow?
- iii. Does a multi agent based traffic control system outperform a pre timed based traffic control system in terms of efficiency?

1.6 Justification of the study

Cases of congestion Kenyan roads are a common occurrence due to inefficient traffic control systems. This research project addresses this shortcoming by effectively using multi agent systems to control traffic flow in the junctions whereby the agents effectively negotiate with each other thus reducing congestion. This in turn is beneficial to road users such that it helps in reducing the costs associated with traffic delays.

1.7 Scope of the study

This study is limited to three specific objectives that focus on determining the effects of using multi agent based traffic control system as opposed to using a pre timed control system. The study will be limited to studying an intersection in the City of Nairobi.

1.8 Assumption

In this project it is assume that

- i. The traffic jam is as a result of inefficient traffic lights and not caused by road closure, accidents or any other factor that may cause traffic jams.
- ii. All vehicles have the same priority to use the junction.
- iii. The vehicles do not change direction.

1.9 Chapter Summary

This chapter gives a brief introduction of the study by giving the problem statement, objectives, research questions that the research seeks to answer, scope and assumptions of the project.

Chapter two provides a detailed literature review of previous work done by other researchers on the same study area of multi agent based control traffic lights.

CHAPTER TWO

LITERATURE REVIEW

To solve congestion in highways many methods have been used to try and solve the problem. In some places the numbers of lanes have been increased or even increasing the speed limits, coming up with better ways of optimizing traffic lights at intersections.

Traffic lights optimization is a complex problem where and several intelligent algorithms such as fuzzy logic, reinforcement learning, evolutionary algorithm have been used to try and solve this problem (Weiring et al 2004).

Traffic controllers can be classified according to the method in which they allocate green time for each phase and can be roughly classified into the following types of control:

- ▶ *Fixed-time control*: A signal timing plan is selected according to a fixed schedule. The duration and order of all green phases remain fixed and are not adapted to fluctuations in traffic demand.
- ▶ *Actuated control*: In order to adapt the control scheme to fluctuations in traffic demand, traffic detectors are placed that indicate the presence or absence of vehicles. Using this information green phases are extended or terminated depending on the current traffic demands.
- ▶ *Adaptive control*: A traffic control system that continuously optimizes the signal plan according to the actual traffic load. Changes to the active signal plan parameters are automatically implemented in response to the current traffic demand as measured by a vehicle detection system.

(Katwijk, 2008)

When a group of agents engage to solve a problem cooperatively they must have joint commitment to attain the overall goal of the system, as well as their individual commitments to localized goal. The main principle of partial global planning is that cooperating agents exchange information in order to reach common conclusions about the problem solving process. Planning is partial because the system does not generate a plan for the entire problem. Partial global planning involves three iterated stages.

1. Each agent decides what its own goals are, and generates short-term plans in order to achieve them.

2. Agents exchange information to determine where plans and goals interact.
3. Agents alter local plans in order to better coordinate their own activities.

(Wooldridge, 2002 pg 202)

Wolpert et al his research on COLlective INTelligences (COINS) elaborates the relationship between the private utility function and the global utility function in a multi agent environment. They propose a relationship between agents whereby as each agent tries to maximize its private utility it does not decrease world utility. If the separate agents have high personal utilities, by luck or by design, then they have not frustrated each other, as far as the world utility is concerned.

The management and optimization of traffic provide an ideal environment to study how multi agent systems promote desired system level behavior. They represent a special class of problems where the individual actions of the agents are neither “good” nor “bad” for the system; instead it’s the interaction among agents that leads to desirable or undesirable outcomes. Therefore agents need to learn how to coordinate their actions with those of other agents so as to improve the overall system performances. (Multi agent systems for traffic and transport engineering, pg 248). Cooperative distributed problem solving highlights how agents can work together to solve problems that are beyond their individual capabilities hence without cooperation the agent cannot even archive its own individual goal.

Congestion problems are characterized by having the system performance depend on the number of agents that select a particular action, rather on the intrinsic value of those actions. In those problems, the desirability of lanes, paths or sides depends solely on the number of agents having selected them. Hence, multi-agent approaches that focus on agent coordination are ideally suited for these domains where agent coordination is critical for achieving desirable system behavior. (Bazzan and Klüglv, 2009).

According to Denise et al, to have a configured traffic light, there must be a control that determines the stage, the splits, the cycle time and, the offset time (a time delay between two successive intersections that allows vehicles to pass successive intersections without stopping). Each of these stages must have a relative green duration.

Multi-agent learning algorithms address congestion problems in traffic and transportation domains effectively.

Vicente et al (2005) proposes a multi agent system that works with traffic management strategies to support road managers to manage and control traffic in case of meteorological incidents such as accidents, road constructions by carrying out negotiations with other agents (Tomas and Garcia, 2005).

Lior et al (2008) propose a multi agent system that uses reinforcement learning approach to traffic control by cooperative learning and explicit coordination among agents. They make the assumption that an agent is affected only by those agents with a direct influence on its environment, thus the global coordination problem may be decomposed into a set of local coordination problems and can be solved with the use of coordination graphs. Max-plus which estimates the optimal joint action by sending locally optimized messages among connected agents is used. It also allows the agents to report their current best action at any time.

Visit et al (2009) proposes a multi agent based traffic control system that uses rule based reasoning to control traffic; whereby each agent observes the current traffic condition surrounding its junction and uses this information to reason with condition action rules in its knowledge base. As a result of the rule application, it may result to change in the traffic light or the need for the agent to collaborate with the neighboring agent so as to control the traffic lights more effectively.

Simulations are done using Netlogo to compare performance between collaborative agents and agents that are not collaborating. Results show that the average delay of each car at each traffic light for collaborative agents is better than the agents that are not collaborating.

Duan et al (2010) proposes multi-objective reinforcement learning for traffic signal where data is exchanged between the vehicles and roadside equipment via a vehicle ad hoc wireless network.

The multi-objective control algorithm considers 4 types of traffic conditions

- i. **Free Traffic Condition** where they aim in minimizing the number of stops thus the cumulative number of stops is selected as the optimizing objective.
- ii. **Medium Traffic condition** where the focus is on the overall waiting time of each car before a traffic light.
- iii. **Congested traffic condition** where the focus is queue length.
- iv. Priority control for buses and emergency vehicles.

A practical road network was modeled using simulation software called Paramics where the control effects of the four types control algorithms were tested under different traffic conditions

by changing the volume of traffic entering the network and comparisons were made against fixed control, actuated control and Wierings method. Results showed that when the traffic volume was low in multi-objective reinforcement learning the number of stops was less than the other methods but with increase in volumes, multi-objective reinforcement learning changes objectives from number of stops to queue length.

In congested conditions multi-objective reinforcement learning can prevent queue spill over that avoid large traffic jams.

Aditya et al (2009) proposes a method for coordinating multirobotic/multi-agent traffic control at intersections. The robotic agents move guided by a potential field along the lanes whereby at the intersections an intersection agent controls the flow of traffic by assigning priorities to the agents that are about to enter the intersection. The priorities are computed based on the density of robotic agents in a lane and the rate of traffic flow in those lanes. The robotic agents integrate these assigned priorities into their potential field computations which help them to move through the intersection avoiding collisions. Simulation results show that time spent at the intersection is less through mixed autonomy where intersection agent and robotic agents share decision making as opposed to leaving the decision making only to the intersection agent.

Dresner et al (2005) proposes a system whereby vehicles request an agent to reserve sometime in the intersection during which they may pass. The proposed system consists of an intersection manager and driver agent where the intersection manager is responsible for directing traffic at the intersection while driver manager is responsible for controlling the vehicles they are assigned. The driver agent request to use the intersection by sending a reservation message to the intersection manager beforehand that determines whether to grant the request or not. To determine if the request will be granted the reservation manager simulates the journey the vehicle will take through the intersection and determines if another vehicle is occupying the path that the vehicle will take, if not it reserves the space for the requester. Results after comparison between the reservation based method and the traffic lights showed reduction in delays when using the reservation based method.

Samah et al (2010) propose a Q-learning based signal control system that uses variable phasing sequence. Three Q – Learning models are developed each considering different possible states. The models are tested and compared with the pre-timed control strategy as the benchmark.

Results show that the Q-learning approach out performs the pre-timed signal plan by reducing delay by 36%.

Chapter Summary

This chapter has focused on similar work done by other researchers on the area of multi agent based control traffic lights. It high lights how other researchers have implemented their research work and their findings that is similar to multi agent based traffic control.

Different methods like Q learning, coordinating multi robotic/multi-agent traffic control, multi-objective reinforcement learning have been used by different researchers showing how to effectively use these methods in the implementation of traffic control systems.

CHAPTER THREE

METHODOLOGY

3.0 Introduction

The project was based on Distributed Artificial intelligence (DAI) whereby agents coordinate and cooperate with the aim of controlling traffic at various intersections. Due to the limitation of the resource (in this case the intersection), certain protocols have to be put in place to regulate which agent is given priority to use the intersection.

Traffic control systems must have the capability of optimizing traffic flow by coordinating their activities with neighboring traffic lights. The proposed system consist of an intelligent agent located at the road intersection that controls the traffic lights based on the current condition of the lanes they are manning by negotiation and cooperation with other agents located at different intersections with the aim of maintaining an acceptable level of traffic saturation.

Sensors are located at strategic locations to gather information that is fed to the agent for the purpose of making an informed decision based on the prevailing traffic condition. These sensors collect information on the saturation levels of the lanes and the arrival rate at of the vehicles at the intersection.

This project has three objectives

- i. Design a protocol that will be used by the agents to allow vehicles to use a given intersection.
- ii. Develop and test a simulated platform on which the designed protocols can be tested.
- iii. Evaluate the performance of collaborative agent controlled traffic signal against a pre-timed traffic signal.

To achieve these objectives this section outlines how the listed objectives will be met.

3.1 Application of Queuing Theory

Since traffic flow is greatly affected by performance of neighboring intersection and the rate at which traffic arrive at the intersection; designing the protocols to govern how the agents

negotiate access to the junction will be essential for the working of the model.

By using an open queuing network (M/M/1) with Poisson arrival rate, exponential service time (average time spent by a vehicle at the junction) and a single server (in this case a single junction); from the calculations, the average waiting time of the vehicles, and mean number of vehicles in the queue, the probability of an arriving vehicle finding the queue full can be estimated. From these computations reinforcement learning can be complemented such that the agents can be able to negotiate with neighboring agents on using the intersection from the values they get from the calculations.

Considerations will be made to the lane that the vehicles intend to use after leaving the junction and determine the queue length before more vehicles are released. From these calculations rules that govern how the traffic lights will be regulated based on the results of the agents' negotiation can be formulated.

The road network figure 1 with agents stationed at the junctions can represent as figure 2 whereby consideration is made to only incoming traffic from one roundabout.

Figure 1 : Location of agents in the junction illustration

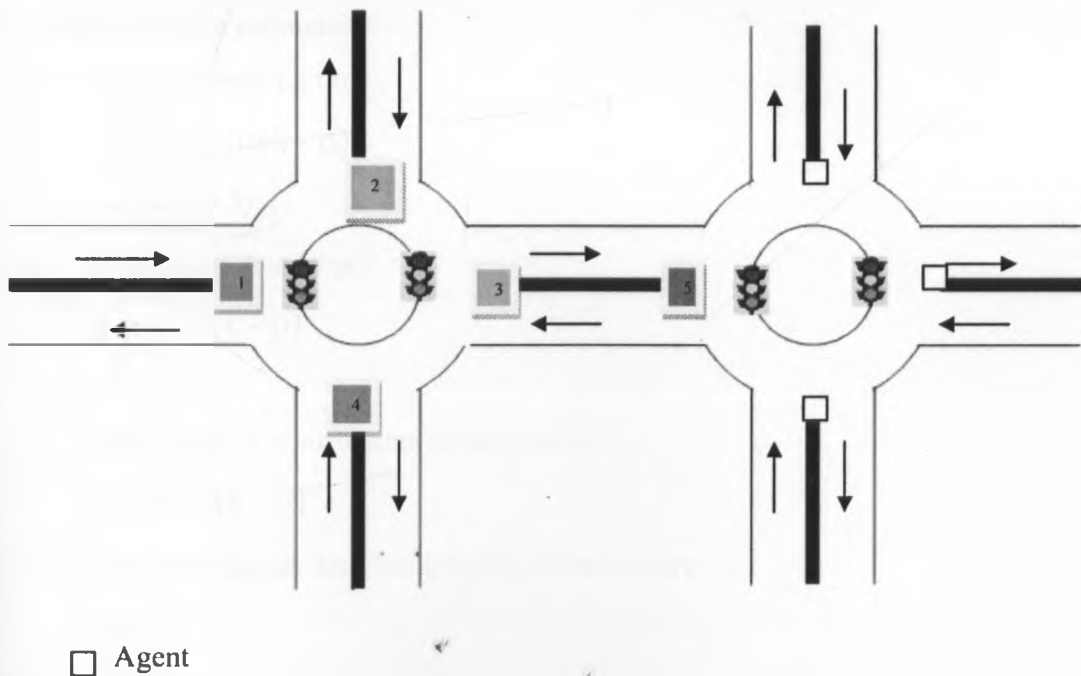
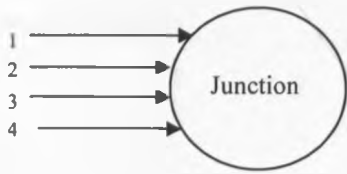


Figure 2: Illustration of the incoming traffic to the junctions as a queue



On the time based traffic control system where traffic is release to use the junction in a round robin with different arrival rates some queues will grow longer than others. In such a scenario the service time can adjusted i.e. increasing the time allocated to use the junction by lanes with high arrival rates and still at the same time try to maintain the waiting time of other junction to a favorable time.

By using an open queuing network with

- μ as the service time per unit time
- λ as the average arrival rate per unit time

Little's law is a mathematical theory that deals with queuing systems. Little law says that under steady state condition the average number of items in the queuing system equals the average rate at which items arrive multiplied by the average time that the item spends in the system (Chhajed et al 2008)

from Little's law the following values from which the agents will use as their basis for negotiation can be estimated

- i. the mean waiting time

$$WT = \rho / (\mu(1 - \rho))$$

$$\text{where } \rho = \lambda/\mu$$

- ii. the mean queue length

$$q = \rho^2 / (1 - \rho)$$

- iii. probability of n customers in the system

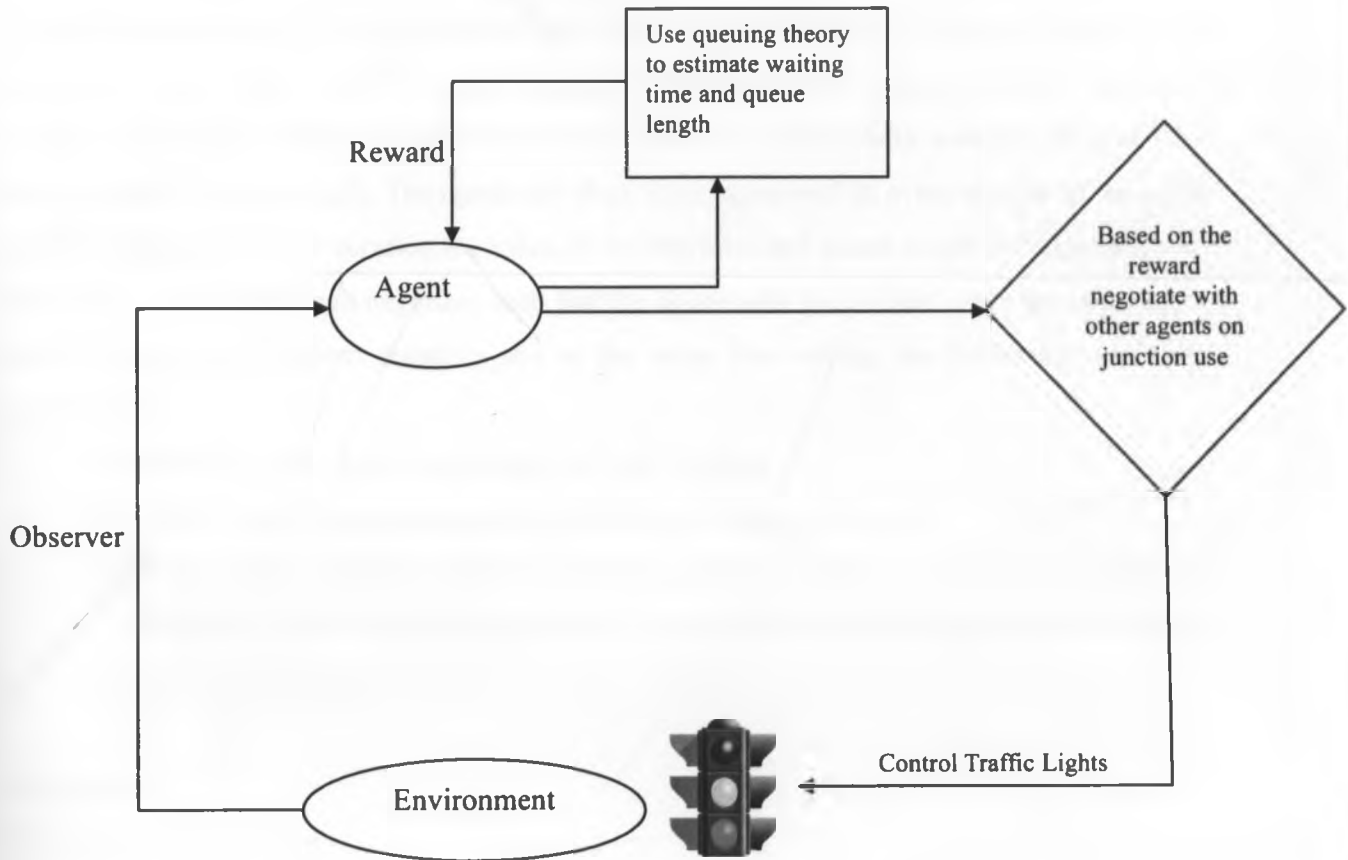
$$\rho_n = \rho^n / (1 - \rho)$$

To ensure that the queue does not grow to ∞ we require

$$\lambda * \mu < 1$$

3.2 Data Flow Diagram

Figure 3 : Conceptual diagram



Agent communication is only restricted to the agents immediate neighbor for instance in the figure 1 above agent 1 can only communicate with 2,3 and 4, while 3 can communicate with 1,2,4 and 5. This way information is propagated from one agent to the next such that when traffic is moving from intersection with agent 1 to 5, then agent 1 negotiates with agent 3 then agent 3 negotiates with agent 5.

3.3 Designing Agent Negotiation Protocols

Agent's negotiation to use the junction will be based on the queue length of the lane joining the junction and the waiting time such that agents with a longer queue length and waiting time get

preference to use the junction.

As the value of waiting time and queue length increases an agent gets a + ve score. The agent aims to maximize its score by reducing its queue length and waiting time such that for every reduction it gains a - ve score with the aim of reaching an optimal score of zero that indicates that there is no traffic buildup behind the junction.

To ensure that lanes with shorter queues do not wait for a very long period to use the junction, an agent gets a much higher score for every increase in waiting time as opposed to the increase in the queue length. For instance an agent can score 10 for every increase in waiting time and 1 for every increase in queue length. The agents get their score computed on every change of the cycle whereby after a predefined duration the value of waiting time and queue length is computed.

From these scores agents can negotiate such that the agent with the highest score gets priority to use the junction for a given duration and at the same time taking the following rules into consideration.

- i. If the agent has the highest score turn the light to green.
- ii. If the lane the agent is manning has no traffic the light remains red
- iii. If the lane into which the vehicles are being released to do not have more space to accommodate more vehicles the light turns to red and the priority is assigned to the agent with the next lower score.

Pseudo code

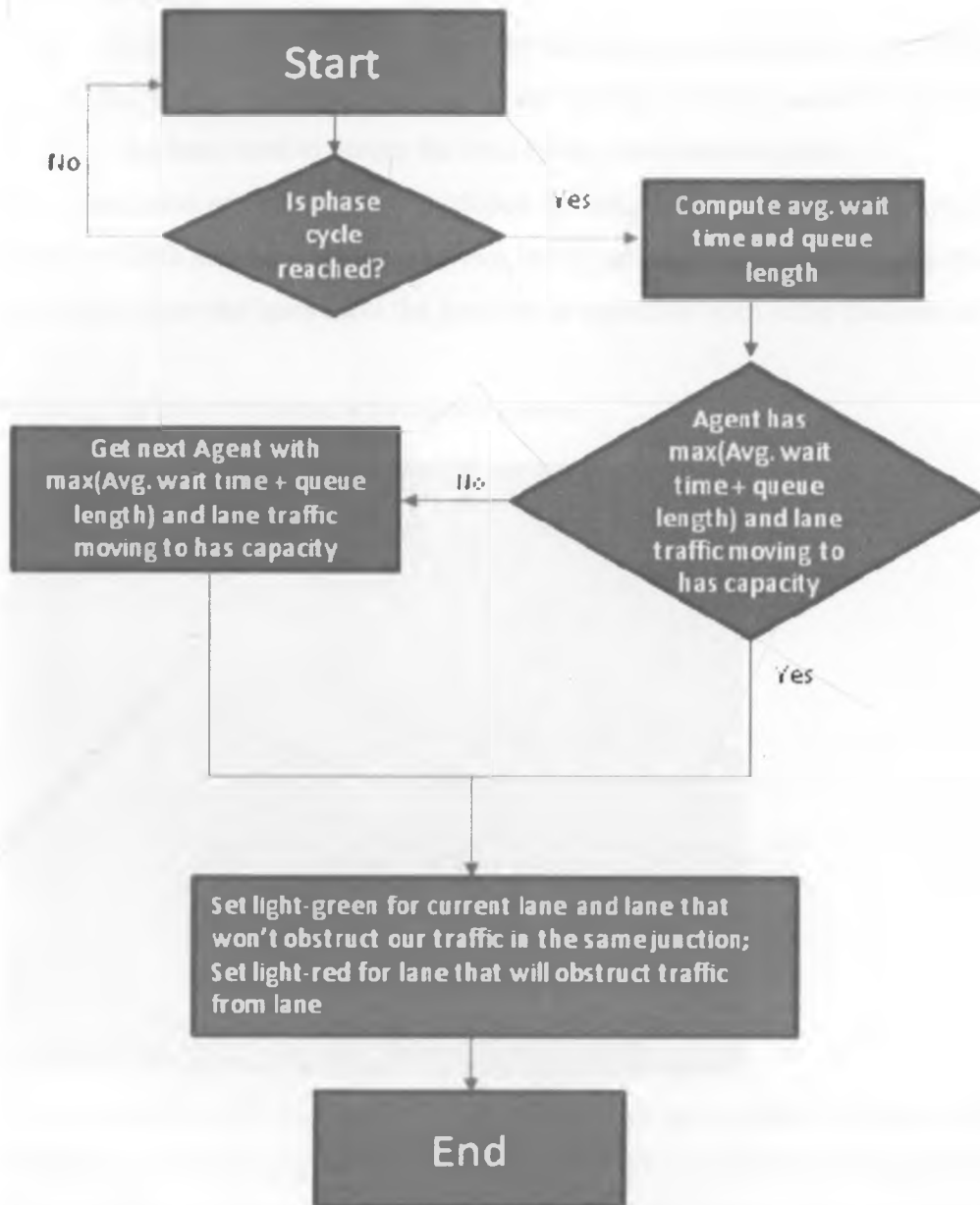
```
//Declare variable
int wait-time
int queue_length
const x = 10 //multiplying factor to make waiting time have a
high score

if phase cycle reached then
{
    wait_time = (get no. of stopped cars) * x) / no. of stopped
cars
    queue_length = get no. of stopped cars
```

```
if agent_with_max (wait_time + queue_length) and lane
traffic moving to has capacity then
{
    Set light-green for current lane and lane that won't
    obstruct our traffic in the same junction;
    Set light-red for lane that will obstruct traffic from
    lane with priority;
}
Else get next agent_with_max (wait_time + queue_length) and
lane traffic moving to has capacity then
{
    Set light-green for current lane and lane that won't
    obstruct our traffic in the same junction;
    Set light-red for lane that will obstruct traffic from
    current lane in the same junction;
}
```


Agent Negotiation Overview

Figure 4 Decision Flow



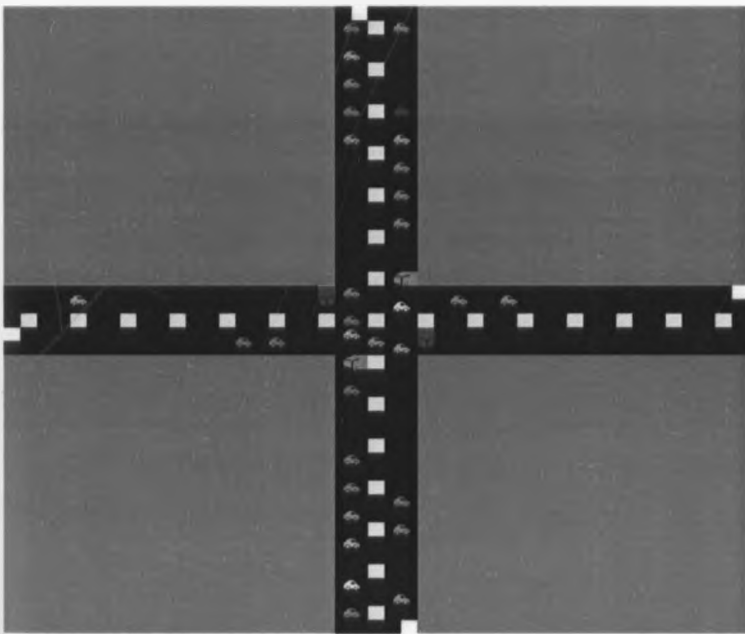
3.4 Designing the simulation platform

The simulated platform has been designed using Netlogo version 4.1.2 which is free and open source software that is best suited for modeling multi agent systems since the modeler can give instructions to hundreds of agents all operating independently. The Netlogo world is made up of agents. In the model we make use of two agents in the Netlogo world

- i. **Turtles** - These are agents that move around in the world. In the model we use the turtles as cars.
- ii. **Patches** - The world is two dimensional and is divided into a grid of patches. Each patch is a square piece on the ground over which the turtles can move on. In the model patches have been used to design the road system and the intersection

The simulation environment is developed by designing a junction with two lanes running from north to south and east to west with two lanes that have traffic moving in opposite directions. At the point where the lanes meet the junction an agent has been setup that acts as the traffic signal.

Figure 5 Illustration of the junction on the simulated platform



The vehicles in each lane can either move north for northbound vehicles, south for south bound vehicles i.e. on reaching the junction the vehicles are not allowed to change direction.

The arrival rate of the vehicles per lane can be varied by adjusting the slider linked to each lane. From this we can be able to test the model using different traffic densities in different lanes.

Each car has been designed such that they can be able to detect if the car in front of it is moving if not it stops to avoid collision, the cars are also able to detect the state of the traffic lights which determines if they can move ahead or stop.

To ensure that vehicles with a short queue length do not stop for long behind the junction we determine the average waiting time of all stationery vehicle behind the queue.

The simulation platform has an on and off switch that we use to test the proposed model and the time based scenario respectively.

3.5 Evaluation

Three scenarios were simulated and comparisons made between a pre-timed system and the proposed model will be made. The three scenarios include

- i. There are no vehicles arriving from north/south but for east bound there is heavy traffic
- ii. The arrival rate of traffic from north/south bound vehicles is greater than that for east/west bound vehicles and vice-versa
- iii. The arrival rate of traffic from north/south bound vehicles is the same for east/west bound vehicles

Simulation software was used to analyze the proposed system from which results were be collected.

Data was collected by observing a current pre-timed system in the roads at a given junction over a period of time. The data that was collected was the rate of arrival of vehicles at the junction at peak hours and also observations were made on the queue length and the time taken before a vehicle enters the junction and the average speed of the vehicles as they approach the junction.

By varying arrival rate of vehicles on the simulation comparisons could be made against the data collected from the pre timed system and the results to be obtained from the simulated system.

The results will be presented in charts and graphs. A summary conclusion will also be made based on the findings.

3.6 Chapter Summary

This chapter has focused on how each clearly explains each object that has been given in Chapter one. It outlines the methods to be used to achieve each objective by giving how the agent negotiating protocols will be designed, how the simulation will be carried out and the results expected to be collected from the simulation.

CHAPTER FOUR

RESULTS

Observation of the current pre timed system at the Nyayo stadium junction shows that the cycle time in which the lights change was two minutes whereby the green light has a duration of 40 seconds, red light has a duration of 76 seconds and orange light has a duration of 4 seconds that is split into 2 seconds after the green light and 2 seconds after the red light.

Observation was focused on the rate of arrival of vehicles to the Nyayo stadium junction from Mombasa road, Langata Road, Lusaka road and from Uhuru highway during different peak hours. Data was collected on 14/05/2011 by counting the number of vehicles arriving for a period of 5 minutes

Table 1: Observed arrival of vehicles

	Number of Vehicles per 5 min			
	Mombasa Road	Langata Road	Lusaka Road	Uhuru Highway
9.00 am – 9.30 am	200	115	40	190
9.30 am – 10.00 am	178	90	60	222
10.30 am – 11.00 am	190	97	25	198
11.30 am -12.00pm	164	72	42	204
12.00 pm – 1.00pm	61	13	5	72
1.00 pm – 1.30pm	151	27	13	66
1.30 pm –2.00pm	193	11	10	47
2.00 pm – 2.30pm	51	5	7	42
3.00 pm – 3.30pm	70	13	3	38
3..30 pm – 4.00pm	62	25	17	48
4.00 pm – 4.30pm	66	21	14	36

In real life situation where pre timed system is used little's law can be used to estimate the queue length of traffic on a given lane. From the above observations in table 1 Little's laws will be used by taking the average arrival of vehicles for the first 10 minutes from Mombasa road(which has 4 lanes) to compute the following

- μ as the service time
- λ as the average arrival rate
- ρ mean number of vehicles in the system or fraction of time the server is busy

$$\mu = (40/120) = 0.33$$

$$\lambda = (378 / (4*600)) = 0.16$$

$$\rho = (0.16/0.33) = 0.48$$

The mean waiting time

$$WT = \rho / (\mu(1 - \rho))$$

where $\rho = \lambda/\mu$

$$WT = 0.48 / (0.33(1-0.48)) = 2.82 \text{ sec}$$

The mean queue length

$$q = \rho^2 / (1 - \rho) = 0.44$$

From these calculations the agents have information that governs how they negotiate to gain access to the junction.

Three scenarios will be simulated and comparisons between a pre-timed system and the proposed model. These three scenarios are

- There are no vehicles arriving from north/south but for east/west bound vehicles there is heavy traffic
- The arrival rate of traffic from north/south bound vehicles is less than that for east/west bound vehicles
- The arrival rate of traffic from north/south bound vehicles is the same for east/west bound vehicles

The reason as why the three scenarios were opted for is that comparisons in the performance of the proposed model against the pre timed system will be effectively evaluated by looking at the

worst case scenario when there is heavy traffic from all lanes joining into the junction, performance in moderate traffic and how the systems performances when there is no traffic from a given lane.

The first 100 seconds of the simulation was used to collect test run data thus the output is not used in the final results. Data was collected from the 900 seconds and the results are used to compare performance of the proposed model and the pre-timed system.

Scenario one: North/South bound lanes with no vehicles while East/West bound there are vehicles.

In this scenario the arrival rate of vehicles that are North/South bound is zero while for East/West bound vehicles there is moderate traffic. The reason for this is to validate the performance of the systems.

The arrival rate per tick for the junction North = 0, south = 0, east = 50 and west = 45 and green light duration = 10 ticks

From the simulation results, the average waiting for the vehicles after 900 ticks was computed after three successive runs as shown in Table 2 below

Table 2 Average wait time scenario 1

Pre-timed system

Run	Average Wait Time (Secs)			
	North bound	South bound	East bound	West bound
1	0	0	38.484	35.582
2	0	0	35.699	37.223
3	0	0	34.689	35.494

Proposed Model

Run	Average Wait Time (Secs)			
	North bound	South bound	East bound	West bound
1	0	0	3.28	2.77
2	0	0	3.009	2.846
3	0	0	3.004	2.137

The combined Average for all junctions in Table 3 shows the overall wait time for the three successive runs

Table 3 Combined average wait time scenario 1

Average Wait Time (Secs)	
Pre-timed system	Proposed Model
18.097	0.14205

The graphs figure 6 and 7 show the average wait time for the Pre timed system and proposed model respectively

Figure 6: Average wait time pre timed system, scenario 1

Pre-timed system

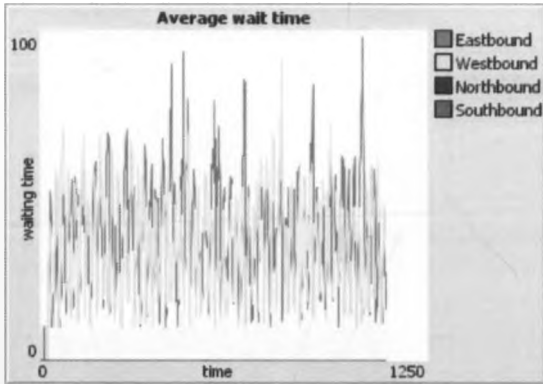
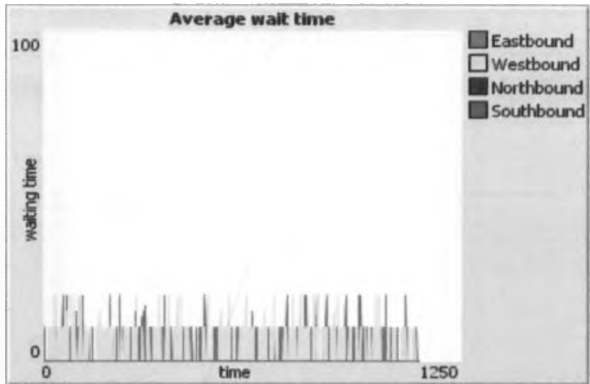


Figure 7 : Average wait time proposed system scenario 1

Proposed Model



Number of Cars

From the simulation results, the average number of the vehicles after 900 ticks was computed after three successive runs as shown in Table 4 below.

Table 4 : Average number of cars Scenario 1

Pre-timed system

Average number of cars				
Run	North bound	South bound	East bound	West bound
1	0	0	10.99	10.218
2	0	0	10.9	10.25
3	0	0	10.58	10.35

Proposed Model

Average number of cars				
Run	North bound	South bound	East bound	West bound
1	0	0	0.382	0.289
2	0	0	0.351	0.32
3	0	0	0.392	0.296

The combined Average for all junctions in Table 5 shows the overall number of waiting vehicles for the three successive runs

Table 5 : Combined Average number of cars Scenario 1

Average number of cars	
Pre-timed system	Proposed Model
5.302	0.1691

The graphs figure 8 and 9 shows the number of waiting vehicles for the Pre timed system and proposed model respectively

Figure 8 : No. of waiting cars pre timed system scenario 1

Pre-timed system

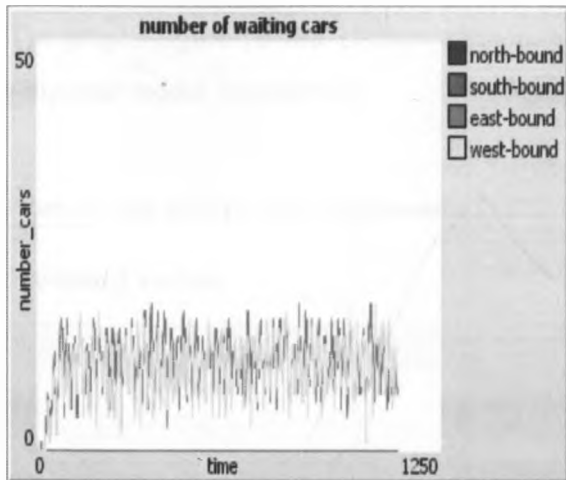
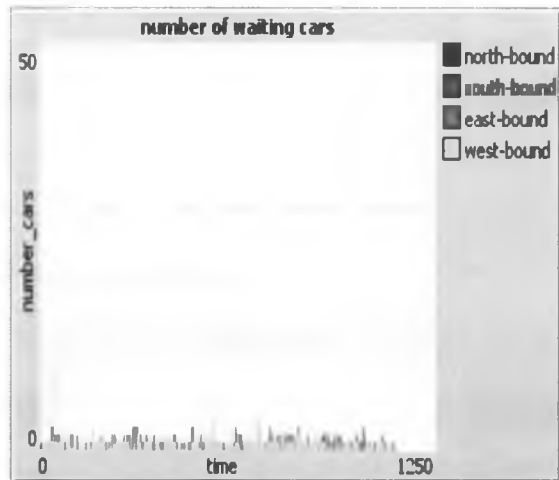


Figure 9 : No. of waiting cars proposed system scenario 1

Proposed Model



Average Speed

From the simulation results, the average speed the vehicles are able to attain after 900 ticks was computed after three successive runs as shown in Table 6 below

Table 6 Average speed Scenario 1

Pre-timed system

Average speed				
Run	North bound	South bound	East bound	West bound
1	0	0	0.329	0.355
2	0	0	0.335	0.352
3	0	0	0.345	0.35

Proposed Model

Average speed				
Run	North bound	South bound	East bound	West bound
1	0	0	0.960	0.964
2	0	0	0.962	0.964
3	0	0	0.958	0.966

The combined Average for all junctions in Table 7 shows the overall speed the vehicles are able to attain for the three successive runs

Table 7 Combined Average speed

Average speed	
Pre-timed system	Proposed Model
0.171	0.481

The graphs figure 10 and 11 shows the number of waiting vehicles for the Pre timed system and proposed model respectively

Figure 10 : Avg. speed pre timed system scenario 1

Pre-timed system

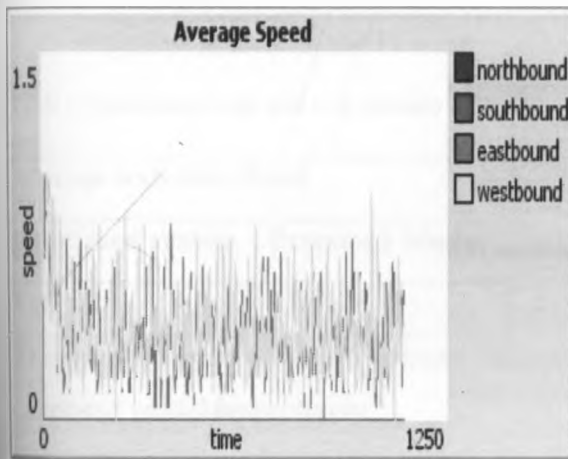
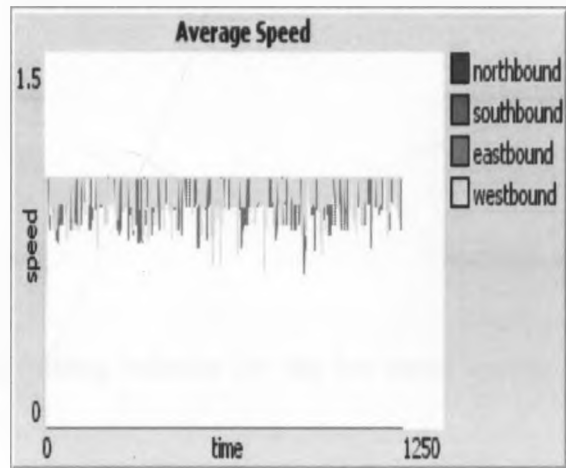


Figure 11 : Avg. speed proposed system scenario 1

Proposed Model



Scenario two: Arrival rate of North/South bound vehicles is much lower than that of East/West bound vehicles.

In this scenario the arrival rate of vehicles that are North/South bound is lower than that of East/West bound vehicles. The reason for this is to validate performance of the proposed system against the performance of the pre timed systems under such conditions.

The arrival rate per tick North = 13, south = 22, east = 50 and west = 45 and green light duration = 10 ticks

From the simulation results, the average waiting for the vehicles after 900 ticks was computed after three successive runs as shown in Table 8 below

Table 8: Average wait time Scenario 2

Pre-timed system

Average wait time (Secs)				
Runs	North bound	South bound	East bound	West bound
1	12.038	41.252	37.314	41.252
2	12.6831	35.425	37.811	39.202
3	12.563	36.233	37.545	40.233

Proposed Model

Average wait time (Secs)				
Runs	North bound	South bound	East bound	West bound
1	15.355	33.822	38.618	40.937
2	15.762	41.0732	38.9178	42.268
3	15.031	38.660	38.9459	43.839

The combined Average for all junctions in Table 9 shows the overall wait time for the three successive runs

Table 9 Combined average wait time Scenario 2

Average wait time (Secs)	
Pre-timed system	Proposed Model
31.9625	33.6024

The graphs figure 12 and 13 shows the average waiting vehicles for the Pre timed system and proposed model respectively

Figure 12: Avg. Wait time pre timed system scenario 2

Pre-timed system

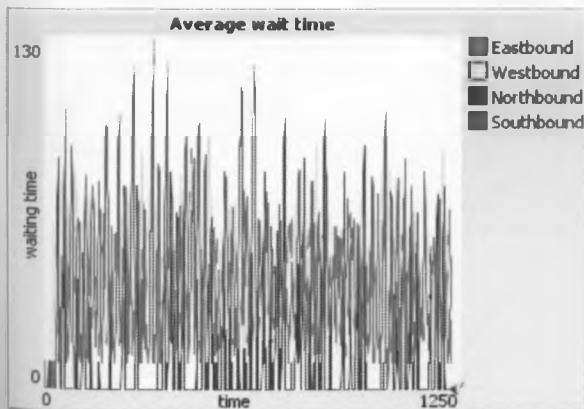
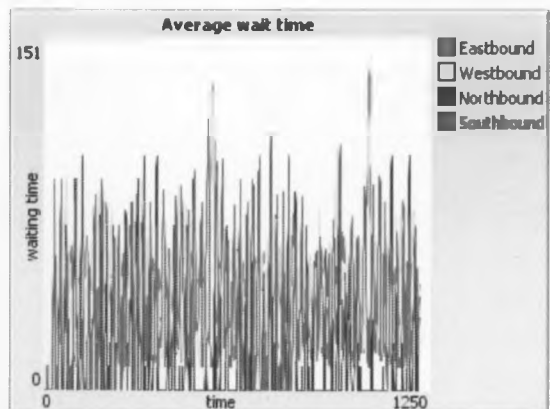


Figure 13 : Avg. wait time proposed system scenario 2

Proposed Model



Number of Cars

From the simulation results, the average number of waiting vehicles after 900 ticks was computed after three successive runs as shown in Table 10 below

Table 10: Average number of cars Scenario 2

Pre-timed system

Average number of cars				
Runs	North bound	South bound	East bound	West bound
1	0.619	9.1	11.06	12.356
2	0.809	8.054	11.048	12.153
3	0.578	6.528	11.42	11.881

Proposed Model

Average number of cars				
Runs	North bound	South bound	East bound	West bound
1	0.766	5.245	11.163	11.986
2	0.6404	6.214	11.227	11.652
3	0.721	5.671	11.345	12.357

The combined Average for all junctions in Table 9 shows the number of waiting vehicles for the three successive runs.

Table 11 Combined Average number of waiting cars

Average number of cars	
Pre-timed system	Proposed Model
7.9671	7.4156

The graphs figure 14 and 15 shows the number of waiting vehicles for the Pre timed system and proposed model respectively

Figure 14 : No. of waiting cars pre-timed system scenario 2

Pre-timed system

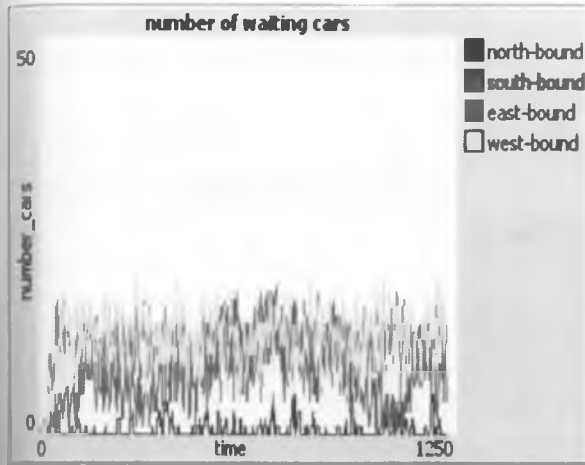
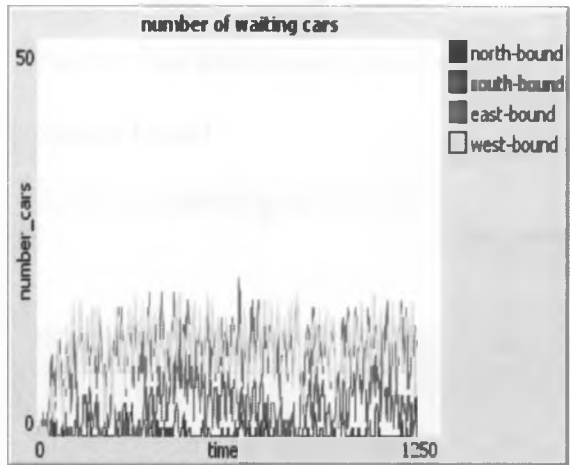


Figure 15: No of waiting cars proposed system scenario 2

Proposed Model



Average Speed

From the simulation results, the average speed attained by the vehicles after 900 ticks was computed after three successive runs as shown in Table 12 below.

Table 12 : Average speed scenario 3

Pre-timed system

Average Speed				
Runs	North bound	South bound	East bound	West bound
1	0.788	0.325	0.325	0.269
2	0.803	0.539	0.311	0.286
3	0.774	0.389	0.323	0.278

Proposed Model

Average Speed				
Runs	North bound	South bound	East bound	West bound
1	0.759	0.505	0.321	0.284
2	0.8123	0.4436	0.321	0.298
3	0.813	0.465	0.315	0.271

The combined Average for all junctions in Table 13 shows the number of waiting vehicles for the three successive runs.

Table 13 Combined Average Speed scenario 3

Average Speed	
Pre-timed system	Proposed Model
4508	0.4673

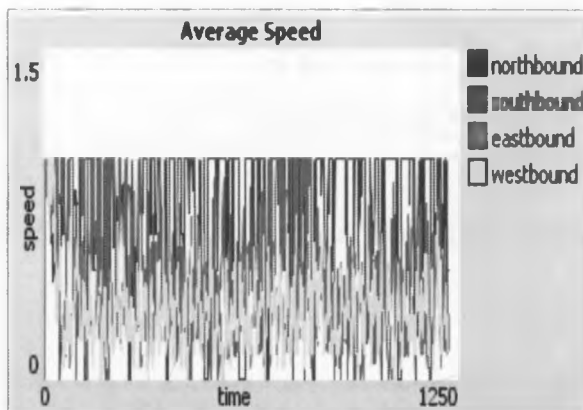
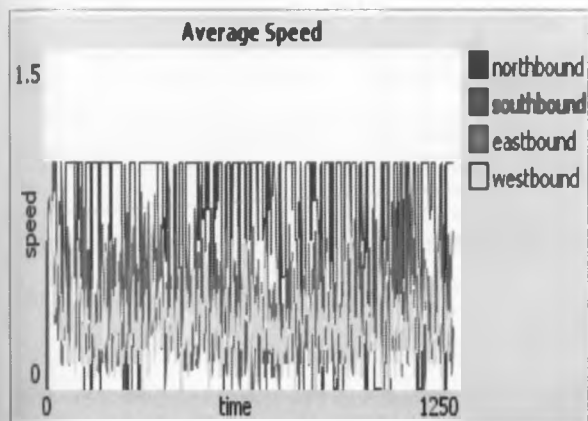
The graphs figure 16 and 17 shows the average speed attained by the vehicles for the Pre timed system and proposed model respectively

Figure 16: Avg. speed pre-timed system scenario 2

Figure 17 : Avg. speed proposed system scenario 2

Pre-timed system

Proposed Model



Scenario three: Arrival rate of North/South bound vehicles is the same as that of East/West bound vehicles.

In this scenario the arrival rate of vehicles that are North/South bound is the same as that of East/West bound vehicles. From this performance between the proposed system and that of the pre timed system can be validated when put under the same traffic densities from all lanes

The arrival rate per tick North = 50, south = 43, east = 50 and west = 46 and green light duration = 10 ticks

Average Wait Time

From the simulation results, the average waiting for the vehicles after 900 ticks was computed after three successive runs as shown in Table 14 below

Table 14 Average wait time scenario 3

Pre-timed system

Average Wait Time (Sec)				
Runs	North bound	South bound	East bound	West bound
1	41.06	44.759	38.515	42.16
2	37.664	42.181	38.21	45.21
3	38.612	42.551	37.928	44.819

Proposed Model

Average Wait Time (Sec)				
Runs	North bound	South bound	East bound	West bound
1	43.67	47.406	57.703	64.57
2	38.305	45.543	55.862	60.511
3	36.406	43.232	58.671	62.063

The combined Average for all junctions in Table 15 shows the average wait time of the vehicles for the three successive runs

Table 15 Combined Average wait time scenario 3

Average Wait Time (Sec)	
Pre-timed system	Proposed Model
41.1391	51.162

The graphs figure 18 and 19 shows the average speed attained by the vehicles for the Pre timed system and proposed model respectively

Figure 18 : Avg. wait time pre-timed system scenario 3

Pre-timed system

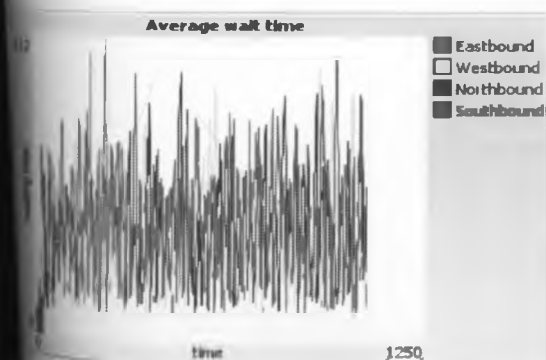
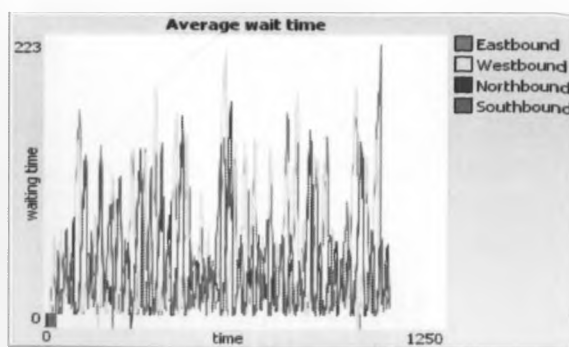


Figure 19 : Avg. wait time proposed system scenario 3

Proposed Model



Number of waiting Cars

From the simulation results, the average number of waiting vehicles after 900 ticks was computed after three successive runs as shown in Table 16 below

Table 16 Average number of cars scenario 3

Pre-timed system

Number of waiting Cars				
Runs	North bound	South bound	East bound	West bound
1	12.231	12.919	12.395	12.558
2	11.978	13.124	12.502	12.163
3	11.882	12.546	12.755	12.165

Proposed Model

Number of waiting Cars				
Runs	North bound	South bound	East bound	West bound
1	10.65	11.254	12.722	12.562
2	10.319	10.778	13.304	13.117
3	10.381	11.09	13.595	13.216

The combined average for all junctions in Table 17 shows the average number waiting vehicles for the three successive runs

Table 17 Combined average number of cars scenario 3

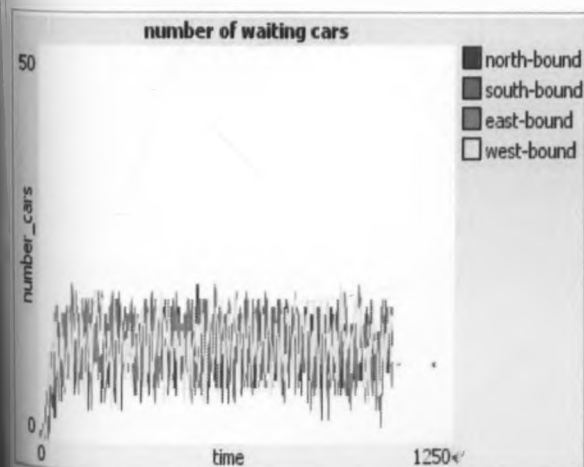
Number of waiting Cars	
Pre-timed system	Proposed Model
12.435	11.916

The graphs figure 20 and 21 shows the number of vehicles for the Pre timed system and proposed model respectively

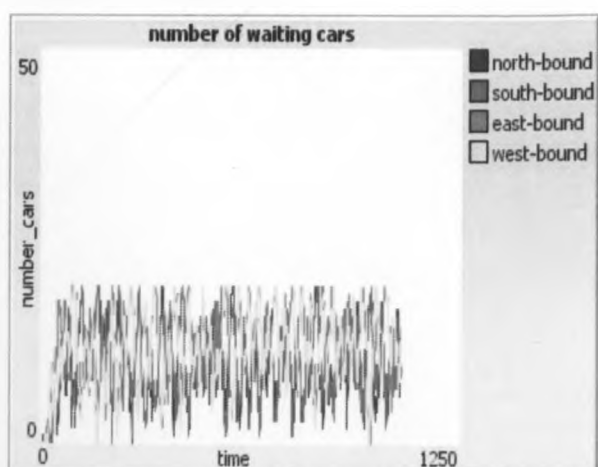
Figure 20 : No. of cars waiting cars pre-timed system scenario 3

Figure 21 : No. of waiting cars proposed system scenario 3

Pre-timed system



Proposed Model



Average Speed

From the simulation results, the average speed attained by the vehicles after 900 ticks was computed after three successive runs as shown in Table 18 below

Table 18 Average speed scenario 3

Pre-timed system

Average Speed				
Runs	North bound	South bound	East bound	West bound
1	0.284	0.237	0.260	0.272
2	0.277	0.228	0.269	0.276
3	0.28	0.251	0.26	0.277

Proposed Model

Average Speed				
Runs	North bound	South bound	East bound	West bound
1	0.344	0.312	0.269	0.272
2	0.353	0.325	0.243	0.247
3	0.353	0.317	0.230	0.240

The combined average for all junctions in Table 19 shows the average speed attained by the vehicles for the three successive runs.

Table 19 Combined Average speed scenario 3

Average Speed	
Pre-timed system	Proposed Model
0.26425	0.2921

The graphs figure 22 and 23 shows the average speed attained by the vehicles for the Pre timed system and proposed model respectively

Figure 22 : Avg. speed pre-timed system scenario 3

Pre-timed system

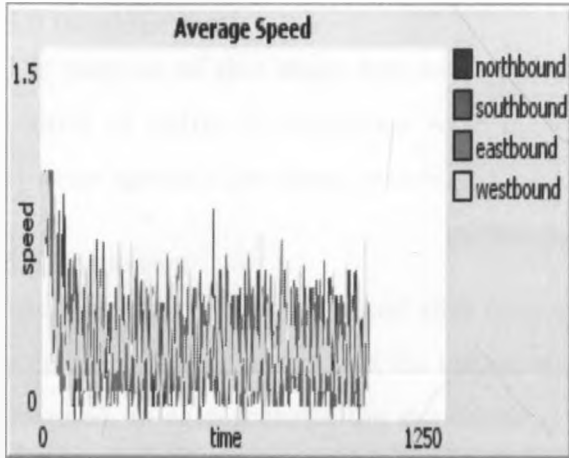
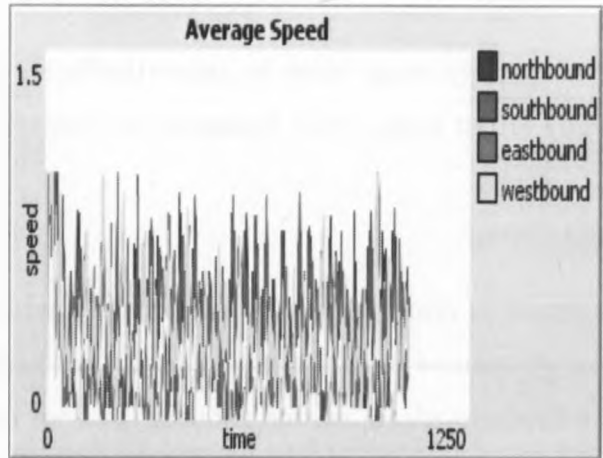


Figure 23 : Avg. speed proposed system scenario 3

Proposed Model



Chapter Summary

This chapter has focused on the results obtained from the simulation by comparing the performance of the proposed traffic control system against a pre timed traffic control system.

The results obtained include the average waiting time of vehicles, number of waiting vehicles and the average speed that the vehicles are able to attain. These results have been clearly presented in tables and graphs for easier interpretation.

CHAPTER FIVE

DISCUSSION, CONCLUSION AND RECOMMENDATIONS

5.0 Introduction

The purpose of this study was to investigate the effectiveness of multi agent systems in the control of traffic. Comparisons were made between the proposed multi agent traffic control systems against a pre timed system.

5.1 Discussion

The results have been collected after running three separate runs in each scenario to ensure the accuracy of the data such that the results obtained are averaged so that a more accurate figure is obtained. In each scenario the simulation is run for 1000 seconds but the results obtained from the first 100 seconds is considered as test run data thus it's not included in the final computation of the simulation results.

From the results obtained in the three scenarios, as the number of vehicles increases there is a drastic increase in the average wait time and the number of cars waiting behind the junction.

Table 20 Summary of Performance

			Pre-timed System	Proposed Model	% Improvement
Average Waiting Time	Scenario 1		18.097	0.14205	99%
	Scenario 2		31.9625	33.6024	-5%
	Scenario 3		41.1391	51.162	-24%
Number of waiting vehicles	Scenario 1		5.302	0.1691	97%
	Scenario 2		7.9671	7.4156	7%
	Scenario 3		12.435	11.916	4%
Average Speed	Scenario 1		0.171	0.481	64%
	Scenario 2		0.4508	0.4673	4%
	Scenario 3		0.26425	0.2921	10%

Scenario 1 where North/South bound lanes with no vehicles while East/West bound there are vehicles shows that the proposed model outperforms the pre-timed system in terms to higher average speed attained, lower number of waiting vehicles and lower waiting time. This is

attributed to the fact that the pre-timed system does not consider the lanes that do not have traffic thus it turns the traffic lights to green on the North/south bound lanes thus the peaks in the graph as traffic moving from east/west has to wait. This translates to 87% improvement in terms of waiting time, average speed and number of waiting vehicles that the proposed model is able to achieve against the pre-times system

Scenario 2 shows that the proposed model attains slightly better performance than the pre-timed system since the vehicles tend to attain slightly higher average speeds, fewer number of waiting cars and slightly lower waiting time. This translates to 8% improvement in terms of waiting time, average speed and number of waiting vehicles that our proposed model is able to achieve against the pre-times system.

Scenario 3 shows that the proposed model shows better performance than the pre-timed system in terms of higher average speed attained by the vehicles and fewer waiting vehicles. The average waiting time in the proposed models is slightly higher than the pre-timed system thus it can be concluded that the vehicles have to wait slightly longer but when given access to use the junction they use the junction for a longer period such the lights are able to release almost all stationary vehicles behind the junction such that arriving are able to attain higher speeds. This translates to 6% improvement in terms of waiting time, average speed and number of waiting vehicles that the proposed model is able to achieve against the pre-times system.

5.2 Conclusions

This project aimed to address the inefficiency of pre timed traffic signals by introducing the aspect of multi agents whereby the agents are able to detect traffic densities and respond effectively by controlling the traffic lights so as to reduce wait times and queue lengths.

The research was guided by the following research questions

- i. What are the limitations of the pre timed traffic control systems currently being used?
- ii. What will be the effect of adding queuing theory to reinforcement learning in terms of improving traffic flow?
- iii. Does a multi agent based traffic control system outperform a pre timed based traffic control system in terms of efficiency?

From the results obtained, the multi agent traffic control system is able to effectively control traffic regardless of the traffic densities as opposed to the pre timed traffic control system whose performance deteriorates with increase in traffic.

5.2.1 Limitation of pre timed traffic control system

In a pre timed traffic control system there is poor or no interaction between the existing traffic control systems such that the traffic lights may turn to green even when the lane in which the vehicles are moving to congested hence the vehicles end up blocking the junction.

On the other hand the pre timed traffic control systems used in the road systems tend not to respond effectively in circumstances where the traffic density is high since the traffic lights will always be turned on and off in response to the set cycle regardless of whether a given lane has traffic or not thus this results to traffic build up at the intersection.

This has been clearly highlighted in scenario one where the agent based system is able to attain an improvement of 87% as opposed to the pre timed system due to this limitation.

5.2.2 Effects of adding queuing theory to reinforcement learning.

In all the three scenarios the agent based traffic control that makes use of reinforcement learning, and queuing theory to estimate the queue length from which the agent are able to make informed decision shows improved performances. By using the arrival rate the agents can estimate the queue length from which they will base their negotiations to gain preference to use the junction.

5.2.3 Performance difference between the systems

The results in table 20 show that the performance of both systems deteriorates with increase in traffic volumes. Regardless of this a multi agent based traffic control system can perform better than the pre-timed traffic control system regardless of the traffic situation on the roads. From the results this is about 33% improvement of the whole system in the three scenarios that the proposed system is able to attain as opposed to the pre-timed system.

These findings are in agreement with El-Tantawy et al (2010) whose acyclic Q-learning approach outperforms the pre-timed signal plan by reducing total delay by 36%.

Comparisons in performance between the research and that done using fuzzy logic (Adunya 2011) show that fuzzy logic is able to attain 22% improvement in overall waiting time as

opposed to the pre-timed system while the research shows an improvement of 23% in the waiting time.

This is attributed to the fact that multi agent systems are able to observe the prevailing traffic conditions, negotiate amongst themselves and adjust accordingly to ensure that the traffic is moving effectively.

It can be concluded that multi-agent based systems traffic controls can be effectively implemented and this will result to drastic improvement in traffic flow.

5.3 Limitations

In this research vehicles are not allowed to change direction thus the results we obtain may be limiting in the sense that the time a vehicle may take to change direction.

Presently the research focuses on a single junction; this could be extended by increasing the number of junctions in the system and observing how the agents will interact with each other and in return the effect this will have on the overall performance of the road network.

5.4 Recommendation

Multi agent based traffic controls can be effectively implemented in the control of traffic. From the results it shows tremendous benefits can be achieved from using the agent based traffic control system as opposed to the pre timed system in terms of reduced waiting time and queue lengths.

Future research could be conducted on how multi agent systems can be used in offering alternative routes to drivers since the agents located at the junctions have enough information about the traffic density in that section thus by sharing information with neighboring agents they can give a road user an idea of what to expect before they reach a given junction and also offer alternative routes to the users destination.

5.5 Chapter Summary

This chapter has focused on the results obtained from the simulation by comparing the performance of the proposed traffic control system against a pre timed traffic control system in a more summarized form, it also highlights the how the research questions have been answered from the results obtained and recommendations for future work.

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Appendix 1 Sample of collected simulation Data

Scenario one

Average Wait Time

Table 21 Sample Output, Average Wait Time Scenario 1

Pre-timed system

Time	East bound	West Bound	North Bound	South Bound
100	55	14.44444	0	0
101	50	18.88889	0	0
102	51.66667	20	0	0
103	37.5	22	0	0
104	16	27.77778	0	0
105	17.5	28.18182	0	0
106	22	32.22222	0	0
107	30	30	0	0
108	35.55556	34.16667	0	0
109	42	45.55556	0	0
110	48.18182	44.16667	0	0
111	50.76923	53.33333	0	0
112	55.83333	47.5	0	0
113	50	46.92308	0	0
114	46.42857	45	0	0
115	43.63636	50	0	0
116	36.66667	48.33333	0	0
117	40.90909	43	0	0
118	50	46.66667	0	0
119	41.11111	39.09091	0	0
120	36.25	42.85714	0	0
121	27.5	20	0	0
122	15	11.42857	0	0

Proposed Model

Time	East bound	West Bound	North Bound	South Bound
100	1	0	0	0
101	2	0	0	0
102	0	0	0	0
103	0	2	0	0
104	2	1	0	0
105	1	0	0	0
106	0	0	0	0
107	1	2	0	0
108	0	1	0	0
109	0	0	0	0
110	0	0	0	0
111	1	0	0	0
112	1	0	0	0
113	1	0	0	0
114	2	0	0	0
115	1	0	0	0
116	0	0	0	0
117	1	0	0	0
118	0	1	0	0
119	0	0	0	0
120	0	0	0	0
121	1	0	0	0
122	0	0	0	0

123	12.5	12.85714	0	0
124	12.5	14.44444	0	0
125	18.57143	18.88889	0	0
126	19	20.76923	0	0
127	23.33333	27.5	0	0
128	31.81818	33.63636	0	0
129	33.57143	34	0	0
130	40.76923	42	0	0

123	0	0	0	0
124	0	0	0	0
125	0	0	0	0
126	0	1	0	0
127	0	0	0	0
128	0	0	0	0
129	0	1	0	0
130	0	0	0	0

Number of Cars

Table 22 Sample Output, Average number of cars Scenario 1

Pre-timed system

Time	North Bound	South Bound	East bound	West bound
100	0	0	8	9
101	0	0	7	9
102	0	0	6	9
103	0	0	8	10
104	0	0	5	9
105	0	0	8	11
106	0	0	10	9
107	0	0	9	12
108	0	0	9	12
109	0	0	10	9
110	0	0	11	12
111	0	0	13	12
112	0	0	12	12
113	0	0	12	13
114	0	0	14	12
115	0	0	11	11

Proposed Model

Time	North Bound	South Bound	East bound	West bound
100	0	0	1	0
101	0	0	0	1
102	0	0	0	0
103	0	0	0	0
104	0	0	0	0
105	0	0	1	0
106	0	0	0	0
107	0	0	0	0
108	0	0	0	0
109	0	0	0	1
110	0	0	1	0
111	0	0	0	0
112	0	0	0	1
113	0	0	3	0
114	0	0	2	0
115	0	0	0	2

116	0	0	12	12
117	0	0	11	10
118	0	0	7	9
119	0	0	9	11
120	0	0	8	7
121	0	0	4	7
122	0	0	6	7
123	0	0	4	7
124	0	0	8	9
125	0	0	7	9
126	0	0	10	13
127	0	0	12	12
128	0	0	11	11
129	0	0	14	15
130	0	0	13	15

116	0	0	1	1
117	0	0	0	0
118	0	0	0	0
119	0	0	0	0
120	0	0	1	0
121	0	0	1	2
122	0	0	0	1
123	0	0	0	0
124	0	0	0	0
125	0	0	0	1
126	0	0	0	0
127	0	0	0	2
128	0	0	0	1
129	0	0	0	2
130	0	0	0	1

Table 23 Sample Output, Average speed Scenario 1

Average Speed

Pre-timed system

Time	North Bound	South Bound	East Bound	West Bound
100	0.0000	0.0000	0.4660	0.3973
101	0.0000	0.0000	0.5313	0.4356
102	0.0000	0.0000	0.5700	0.3980
103	0.0000	0.0000	0.4271	0.3313
104	0.0000	0.0000	0.6386	0.3980
105	0.0000	0.0000	0.4653	0.2660
106	0.0000	0.0000	0.3320	0.3980
107	0.0000	0.0000	0.3980	0.2494
108	0.0000	0.0000	0.3987	0.2481

Proposed Model

Time	North Bound	South Bound	East Bound	West Bound
100	0.0000	0.0000	0.9091	1.0000
101	0.0000	0.0000	0.8173	1.0000
102	0.0000	0.0000	0.9982	1.0000
103	0.0000	0.0000	1.0000	0.8000
104	0.0000	0.0000	0.8000	0.8878
105	0.0000	0.0000	0.8980	0.9989
106	0.0000	0.0000	0.9990	1.0000
107	0.0000	0.0000	0.9000	0.8000
108	0.0000	0.0000	0.9990	0.8878

109	0.0000	0.0000	0.3333	0.4350
110	0.0000	0.0000	0.3125	0.2500
111	0.0000	0.0000	0.1875	0.2494
112	0.0000	0.0000	0.2929	0.2929
113	0.0000	0.0000	0.2929	0.1869
114	0.0000	0.0000	0.2217	0.2481
115	0.0000	0.0000	0.3500	0.3113
116	0.0000	0.0000	0.2929	0.2494
117	0.0000	0.0000	0.3106	0.3307
118	0.0000	0.0000	0.5600	0.4363
119	0.0000	0.0000	0.4369	0.2653
120	0.0000	0.0000	0.4640	0.5300
121	0.0000	0.0000	0.7293	0.4971
122	0.0000	0.0000	0.5693	0.5293
123	0.0000	0.0000	0.7107	0.5300
124	0.0000	0.0000	0.4271	0.3973
125	0.0000	0.0000	0.5313	0.4350
126	0.0000	0.0000	0.3738	0.2341
127	0.0000	0.0000	0.2929	0.2918
128	0.0000	0.0000	0.3518	0.3506
129	0.0000	0.0000	0.1759	0.1661
130	0.0000	0.0000	0.2335	0.1656

109	0.0000	0.0000	1.0000	0.9989
110	0.0000	0.0000	1.0000	1.0000
111	0.0000	0.0000	0.9091	1.0000
112	0.0000	0.0000	0.9082	1.0000
113	0.0000	0.0000	0.9082	1.0000
114	0.0000	0.0000	0.8173	1.0000
115	0.0000	0.0000	0.9082	1.0000
116	0.0000	0.0000	0.9990	1.0000
117	0.0000	0.0000	0.9091	1.0000
118	0.0000	0.0000	0.9990	0.8750
119	0.0000	0.0000	1.0000	0.9986
120	0.0000	0.0000	1.0000	1.0000
121	0.0000	0.0000	0.9091	1.0000
122	0.0000	0.0000	0.9990	1.0000
123	0.0000	0.0000	1.0000	1.0000
124	0.0000	0.0000	1.0000	1.0000
125	0.0000	0.0000	1.0000	1.0000
126	0.0000	0.0000	1.0000	0.8750
127	0.0000	0.0000	1.0000	0.9986
128	0.0000	0.0000	1.0000	1.0000
129	0.0000	0.0000	1.0000	0.8571
130	0.0000	0.0000	1.0000	0.9986

Scenario two

Average Wait Time

Table 24 Sample Output, Average wait time Scenario 2

Pre-timed system

	East bound	West bound	North bound	South bound
100	47.7778	73.3333	30.0000	61.6667

Proposed Model

	East bound	West bound	North bound	South bound
100	17.1429	11.6667	46.6667	80.0000

101	44.0000	48.3333	40.0000	62.8571
102	18.5714	28.8889	0.0000	58.5714
103	15.7143	24.4444	0.0000	61.6667
104	17.5000	22.5000	0.0000	55.0000
105	23.7500	17.5000	0.0000	40.0000
106	27.7778	21.0000	0.0000	32.5000
107	32.2222	25.8333	0.0000	12.5000
108	38.0000	33.3333	0.0000	10.0000
109	35.7143	37.1429	0.0000	10.0000
110	45.3846	47.6923	0.0000	16.0000
111	49.2857	52.8571	0.0000	20.0000
112	46.1538	55.6250	0.0000	10.0000
113	47.5000	60.7692	10.0000	10.0000
114	40.6667	59.2857	20.0000	17.5000
115	46.9231	48.5714	30.0000	24.0000
116	42.5000	42.9412	40.0000	34.0000
117	36.0000	50.0000	50.0000	44.0000
118	28.8889	47.1429	35.0000	41.4286
119	26.2500	51.6667	45.0000	50.0000
120	33.3333	44.1667	55.0000	53.7500
121	23.3333	51.1111	65.0000	49.0909
122	11.6667	30.0000	50.0000	63.0000
123	13.3333	25.8333	0.0000	73.7500
124	15.0000	27.0000	10.0000	78.5714
125	17.7778	20.8333	0.0000	71.4286
126	20.0000	25.0000	0.0000	64.2857
127	21.0000	30.9091	0.0000	51.2500
128	27.0000	28.7500	0.0000	37.5000
129	34.0000	36.2500	0.0000	21.2500
130	33.5714	44.6667	0.0000	14.0000

101	20.0000	12.0000	56.6667	90.0000
102	10.0000	12.5000	26.6667	40.0000
103	11.6667	12.8571	30.0000	10.0000
104	17.5000	16.6667	20.0000	15.0000
105	18.5714	15.5556	20.0000	20.0000
106	22.5000	22.5000	10.0000	10.0000
107	31.4286	25.5556	0.0000	10.0000
108	31.0000	30.0000	0.0000	0.0000
109	39.0000	35.4545	0.0000	0.0000
110	42.7273	39.2308	0.0000	0.0000
111	48.3333	45.0000	0.0000	0.0000
112	44.5455	40.7143	10.0000	0.0000
113	38.1818	42.3077	15.0000	0.0000
114	43.0000	35.7143	25.0000	0.0000
115	28.3333	36.4286	35.0000	0.0000
116	20.0000	37.2727	45.0000	0.0000
117	16.6667	24.0000	55.0000	0.0000
118	14.2857	22.1429	65.0000	10.0000
119	18.0000	21.0000	75.0000	10.0000
120	20.0000	19.0909	85.0000	20.0000
121	22.2222	17.7778	95.0000	30.0000
122	12.8571	19.0000	100.0000	25.0000
123	14.1667	21.6667	0.0000	0.0000
124	20.0000	25.8333	10.0000	10.0000
125	24.6667	25.5556	0.0000	0.0000
126	32.5000	20.0000	10.0000	0.0000
127	36.1538	24.1667	0.0000	0.0000
128	40.0000	29.2857	0.0000	0.0000
129	46.2500	36.1538	0.0000	0.0000
130	50.5556	40.0000	0.0000	0.0000

Number of Cars

Table 25 Sample Output, number of cars Scenario 2

Pre-timed system

Time	North bound	South bound	East bound	West bound
100	1	6	9	9
101	1	7	5	6
102	0	7	7	9
103	0	6	7	9
104	0	6	8	8
105	0	4	8	8
106	0	4	9	10
107	0	4	9	12
108	0	1	10	12
109	0	4	14	14
110	0	5	13	13
111	0	3	14	14
112	0	1	13	16
113	1	3	12	13
114	1	4	15	14
115	1	5	13	14
116	1	5	12	17
117	1	5	10	15
118	2	7	9	14
119	2	7	8	12
120	2	8	3	12
121	2	11	3	9
122	1	10	6	10
123	0	8	6	12
124	1	7	8	10

Proposed Model

Time	North bound	South bound	East bound	West bound
100	3	4	7	6
101	3	4	6	5
102	3	1	3	4
103	2	2	6	7
104	2	2	4	3
105	1	1	7	9
106	1	1	8	8
107	0	1	7	9
108	0	0	10	10
109	0	0	10	11
110	0	0	11	13
111	0	0	12	14
112	1	0	11	14
113	2	0	11	13
114	2	0	10	14
115	2	0	6	14
116	2	0	11	11
117	2	0	6	10
118	2	1	14	14
119	2	1	10	10
120	2	1	9	11
121	2	1	9	9
122	1	2	7	10
123	0	0	12	12
124	2	1	13	12

125	0	7	9	12
126	0	7	8	12
127	0	8	10	11
128	0	8	10	16
129	0	8	10	16
130	0	5	14	15

125	0	0	15	9
126	1	0	12	10
127	0	0	13	12
128	0	0	15	14
129	0	0	16	13
130	0	0	18	14

Average Speed

Table 26 Sample output Average speed scenario 2

Pre-timed system

Time	North Bound	South bound	East Bound	West bound
100	0.7500	0.3333	0.3987	0.4363
101	0.7500	0.2222	0.6633	0.5967
102	1.0000	0.1250	0.5307	0.3980
103	1.0000	0.2488	0.4986	0.3973
104	1.0000	0.3322	0.4647	0.4633
105	1.0000	0.5533	0.4653	0.4633
106	1.0000	0.5980	0.3987	0.3313
107	1.0000	0.5970	0.3980	0.2488
108	1.0000	0.8856	0.3744	0.2488
109	1.0000	0.5990	0.1759	0.1759
110	1.0000	0.4990	0.2335	0.2341
111	1.0000	0.6644	0.1753	0.1759
112	1.0000	0.8856	0.2761	0.1106
113	0.5000	0.6656	0.2918	0.2756
114	0.5000	0.5556	0.1656	0.1759
115	0.5000	0.4444	0.2329	0.2206
116	0.5000	0.4444	0.2918	0.1042
117	0.5000	0.5000	0.3725	0.1650

Proposed Model

Time	North Bound	South bound	East Bound	West bound
100	0.5714	0.2000	0.4979	0.5686
101	0.5714	0.2000	0.5362	0.6393
102	0.5000	0.7450	0.7646	0.6892
103	0.6650	0.4975	0.5369	0.4600
104	0.5980	0.3333	0.6892	0.7821
105	0.7980	0.6633	0.4993	0.4000
106	0.8317	0.6633	0.4264	0.4640
107	1.0000	0.5000	0.4979	0.3980
108	1.0000	0.9950	0.2850	0.3313
109	1.0000	1.0000	0.2843	0.3113
110	1.0000	1.0000	0.2129	0.1869
111	1.0000	1.0000	0.1993	0.1759
112	0.6667	1.0000	0.3106	0.2206
113	0.3333	1.0000	0.2647	0.2329
114	0.3333	1.0000	0.3320	0.1747
115	0.3333	1.0000	0.5960	0.2200
116	0.3333	1.0000	0.2660	0.3494
117	0.3333	1.0000	0.6206	0.4417

118	0.0000	0.3636	0.4350	0.2206
119	0.0000	0.3627	0.4640	0.2918
120	0.3333	0.3333	0.7960	0.2924
121	0.3333	0.1538	0.7843	0.4338
122	0.5000	0.2300	0.5369	0.3719
123	0.9950	0.3325	0.5686	0.2481
124	0.5000	0.4158	0.4647	0.3719
125	1.0000	0.4158	0.3980	0.2481
126	1.0000	0.4158	0.4640	0.2481
127	1.0000	0.3325	0.3731	0.3506
128	1.0000	0.3317	0.3725	0.1100
129	1.0000	0.3317	0.4100	0.1100
130	1.0000	0.6108	0.1759	0.1650

118	0.3333	0.7500	0.1753	0.1747
119	0.3333	0.7475	0.3719	0.4076
120	0.3333	0.8000	0.4344	0.3094
121	0.3333	0.8000	0.4350	0.4344
122	0.5000	0.6667	0.5581	0.3731
123	0.9950	0.9980	0.2924	0.2488
124	0.0000	0.8000	0.2329	0.2488
125	0.9900	0.9980	0.1165	0.4344
126	0.0000	1.0000	0.2912	0.3725
127	0.0000	1.0000	0.2335	0.2929
128	0.0000	1.0000	0.1656	0.1759
129	0.0000	1.0000	0.1568	0.2335
130	0.0000	1.0000	0.0521	0.1753

Scenario three

Average Wait Time

Table 27 Sample output average wait time scenario 3

Pre-timed system

Time	East bound	West bound	North Bound	South Bound
100	50.7692	26	42.1429	53.8889
101	34.4444	27.5	46.1538	63.5294
102	26.3636	14.2857	50	64
103	21.8182	14	45.7143	68.3333
104	24.1667	18.75	41.4286	68.8235
105	26.4286	19.2857	48.3333	68.3333
106	32.1429	27.6923	46.1538	74.1176
107	38.3333	33.5714	48.1818	80
108	34.1667	42.3077	46.6667	80.7143
109	34.2857	46.1538	47.5	78.3333

Proposed Model

Time	East bound	West bound	North Bound	South Bound
100	134.0000	128.0000	12.5000	12.8571
101	144.0000	138.0000	12.5000	12.0000
102	151.0526	141.6667	12.5000	12.2222
103	150.0000	140.5556	12.0000	15.0000
104	161.8750	143.1250	16.0000	16.2500
105	165.7143	148.6667	20.0000	21.1111
106	150.6667	150.7692	22.5000	24.5455
107	166.1538	149.0000	27.7778	30.8333
108	144.0000	117.5000	30.9091	36.1538
109	117.5000	117.2727	39.0909	41.5385

110	40	52.3077	39	76
111	46.9231	53.0769	36.25	60
112	36.875	58.5714	31.6667	44
113	36.6667	55.3846	22.5	34.5455
114	37.1429	52.8571	15.5556	18.8889
115	32.6316	49.375	18.1818	20
116	37.6471	56.4286	25.4545	26.9231
117	40.5882	54.6667	29.0909	31.4286
118	44.1667	55.4545	30.7143	37.8571
119	37.2727	48.4615	38.4615	48.1818
120	35.3846	52.5	44.6154	44.2857
121	38.3333	51.6667	46.6667	50.6667
122	46.6667	47	48	56.4286
123	25	44	56.6667	48.4615
124	17.2727	42	46.25	43.75
125	21.5385	21	55.3846	46.25
126	27.6923	22.2222	45.7143	43.8462
127	31.4286	24.1667	48.5714	40.7692
128	34.6154	30.8333	50.7143	40.7143
129	37.8571	36.1538	52.5	44
130	42.6667	38.125	49.0909	38.1818

110	88.8889	86.6667	48.1818	45.0000
111	73.3333	65.0000	50.7692	55.3846
112	30.9091	65.7143	56.4286	57.3333
113	12.0000	44.2857	65.7143	63.1250
114	11.0000	14.0000	67.5000	69.4118
115	14.4444	10.0000	77.5000	75.5556
116	15.0000	12.0000	82.9412	85.5556
117	11.6667	12.0000	87.7778	95.5556
118	12.0000	13.3333	93.1579	105.5556
119	12.5000	15.0000	98.5000	105.0000
120	10.0000	13.3333	108.5000	115.0000
121	12.8571	12.5000	118.5000	125.0000
122	12.8571	13.3333	124.2105	126.6667
123	15.0000	13.3333	125.8824	128.1250
124	19.0000	15.0000	118.3333	130.0000
125	23.3333	20.0000	114.0000	118.6667
126	27.0000	30.0000	110.0000	130.7692
127	36.6667	30.0000	101.6667	120.0000
128	34.6154	37.7778	94.1667	117.0000
129	41.4286	43.0000	95.5556	100.0000
130	52.3077	42.3077	92.5000	64.0000

Number of waiting Cars

Table 28 Sample output Number of cars scenario 3

Pre-timed system

Time	North bound	South bound	East bound	West bound
100	14	18	13	10
101	13	17	9	8

Proposed Model

Time	East bound	West bound	North Bound	South Bound
100	4	7	20	20
101	4	5	20	20

102	12	20	11	7
103	14	18	11	10
104	14	17	12	8
105	12	18	14	14
106	13	17	14	13
107	11	16	12	14
108	12	14	12	13
109	8	12	14	13
110	10	10	13	13
111	8	10	13	13
112	6	10	16	14
113	8	11	15	13
114	9	9	14	14
115	11	13	19	16
116	11	13	17	14
117	11	14	17	15
118	14	14	12	11
119	13	11	11	13
120	13	14	13	12
121	15	15	12	12
122	15	14	6	10
123	12	13	12	10
124	16	16	11	10
125	13	16	13	10
126	14	13	13	9
127	14	13	14	12
128	14	14	13	12
129	12	10	14	13
130	11	11	15	16

102	4	9	19	18
103	5	8	19	18
104	5	8	16	16
105	6	9	14	15
106	8	11	15	13
107	9	12	13	10
108	11	13	10	12
109	11	13	8	11
110	11	14	9	9
111	13	13	6	10
112	14	15	11	7
113	14	16	5	7
114	16	17	10	5
115	16	18	9	5
116	17	18	4	5
117	18	18	6	5
118	19	18	5	6
119	20	20	4	4
120	20	20	5	3
121	20	20	7	4
122	19	18	7	3
123	17	16	8	3
124	18	14	10	6
125	15	15	9	8
126	15	13	10	6
127	12	12	9	9
128	12	10	13	9
129	9	9	14	10
130	8	10	13	13

Average Speed

Table 29 Sample Output Average speed scenario 3

Pre-timed system

Time	North bound	South bound	East bound	West bound
100	0.0521	0.1863	0.596	0.4676
101	0.0521	0.1759	0.6627	0.3113
102	0	0.1667	0.4981	0.31
103	0.1042	0.1656	0.5307	0.3725
104	0.0521	0.2912	0.4633	0.4969
105	0.2206	0.4088	0.3725	0.2488
106	0.1106	0.22	0.4356	0.1863
107	0.1747	0.275	0.3119	0.2481
108	0.1747	0.2912	0.3113	0.1863
109	0.3106	0.2912	0.2494	0.2341
110	0.5967	0.2912	0.2494	0.1661
111	0.466	0.1856	0.1875	0.1656
112	0.3987	0.4331	0.2929	0.2616
113	0.332	0.4956	0.1661	0.165
114	0.3313	0.2924	0.2206	0.3306
115	0.3973	0.1747	0.1106	0.165
116	0.398	0.165	0.1568	0.385
117	0.3313	0.275	0.055	0.2918
118	0.2653	0.11	0.275	0.2335
119	0.2494	0.055	0.1747	0.31
120	0.2929	0.11	0.275	0.4344
121	0.2935	0.0526	0.3494	0.6613
122	0.1875	0	0.3494	0.435
123	0.31	0	0.3494	0.31

Proposed Model

Time	North bound	South bound	East bound	West bound
100	0.6658	0.4979	0.0000	0.0000
101	0.6900	0.6386	0.0000	0.0000
102	0.6900	0.3980	0.0495	0.0990
103	0.6131	0.4633	0.0495	0.0521
104	0.6407	0.4633	0.1568	0.1563
105	0.5700	0.3980	0.2211	0.1656
106	0.4653	0.2653	0.1661	0.2761
107	0.3987	0.2494	0.2335	0.4094
108	0.2653	0.1869	0.4088	0.2929
109	0.2653	0.2341	0.4975	0.3100
110	0.3119	0.1753	0.4350	0.4344
111	0.1875	0.2335	0.6219	0.3313
112	0.1244	0.1171	0.3113	0.5300
113	0.1244	0.1106	0.6607	0.4979
114	0.0588	0.0556	0.3725	0.6393
115	0.1111	0.0000	0.3960	0.6393
116	0.1053	0.0000	0.7287	0.6400
117	0.0521	0.0000	0.5980	0.6123
118	0.0000	0.0526	0.6633	0.5362
119	0.0000	0.0000	0.7114	0.6885
120	0.0000	0.0000	0.6400	0.7669
121	0.0000	0.0000	0.4592	0.6650
122	0.0000	0.0521	0.4964	0.7475
123	0.1042	0.1100	0.4257	0.7677

124	0.1171	0	0.2912	0.2475
125	0.1244	0	0.1747	0.2329
126	0.2475	0.1042	0.165	0.1747
127	0.198	0.1106	0.165	0.2918
128	0.4344	0.1106	0.055	0.2918
129	0.3307	0.2341	0.0521	0.3322
130	0.31	0.3113	0.1042	0.1042

124	0.0521	0.2211	0.3320	0.5377
125	0.1650	0.1171	0.3973	0.4279
126	0.1650	0.2335	0.3313	0.5693
127	0.2918	0.2481	0.3980	0.4000
128	0.2481	0.3725	0.1869	0.3987
129	0.4350	0.4350	0.1244	0.3327
130	0.4647	0.3313	0.1863	0.1869

