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## Modelling Exchange Rate Volatility of Kenya's Top Five Trade Partners

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# **Modelling Exchange Rate Volatility of Kenya's Top Five Trade Partners**

**Research Report in Mathematics, Number 15, 2018**

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## Abstract

The world has become a global village and trade among countries has also become inevitable. Since most countries use their own domestic currencies, how much of one currency should be exchanged for another has slowly become a big issue. As a result of this exchange rate and exchange rate volatility has become a topic of interest among traders, Investors and other stakeholders.

This research papers seeks to find the best GARCH family of models that fits into the top five Kenya's trading partners currencies, which are USD, EUR, GBP, JPY and UGX and also checks whether there is co-integration in the trade volume data in USD which accounts for over 60 percent of Kenya's imports and exports. The data used for both the exchange and trade volume span from January 2005 to December 2017.

The results show that  $GARCH(1,2)$  fits KES/USD data and KES/GBP while  $GARCH(2,2)$  fitted KES/EUR , KES /JPY and UGX/KES. The test for co-integration in the trade volume data showed that there is a co-integrating relationship in the trade volume data and hence conventional statistical analysis is unsuitable for this kind of data.



## Declaration and Approval

I the undersigned declare that this dissertation is my original work and to the best of my knowledge, it has not been submitted in support of an award of a degree in any other university or institution of learning.

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Signature

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In my capacity as a supervisor of the candidate's dissertation, I certify that this dissertation has my approval for submission.

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## Dedication

This project is dedicated to myself.

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## LIST OF ABBREVIATIONS AND ACRONYMS

ADF- Augmented Dickey- Fuller

AIC - Akaike Information Criterion

APARCH - Asymmetric Power Auto Regressive Conditional Heteroscedasticity

ARCH - Auto Regressive Conditional Heteroscedasticity

ARMA - Auto Regressive Integrated Moving Average

BOP - Balance of Payments

CBK - Central Bank of Kenya

EGARCH - Exponential Generalised Auto Regressive Conditional Heteroscedasticity

FOREX - Foreign Exchange

GARCH- Generalised Auto Regressive Conditional Heteroscedasticity

GDP - Gross Domestic Product

GJR GARCH - Glosten- Jagannathan Runkle Generalised auto Regressive Conditional

JPY - Japanese Yen

MAE - Mean Absolute Error

MAPE - Mean Absolute Percentage Error

RMSE - Root Mean Square Deviation

STP - Sterling Pound

UGX - Ugandan Shilling

USD - United States Dollar

WAEMU - West African Economic Monetary Unions

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# 1 INTRODUCTION

By definition, exchange rate is the value of one currency in relation to another currency for conversion purposes. Different countries use different currencies while some countries share currencies. Some of the major global currencies include the US Dollar, British pound, Euro, Japanese Yen among others. Globally almost every country has its own currency however some countries share currencies with the dollar being the most preferred currency due to its acceptance and stability. Some regions also use a common currency an example is the Euro region which uses Euro as the region's currency. In other regions without a common currency it is not unusual to find several currencies being acceptable in different countries. Commercial banks also create a market for these currencies by selling and buying different currencies, this also makes trading between parties of different origins possible.

Central banks adopt different currency regimes, these regimes are ways through which countries manage the exchange rates of their currencies in relation to other currencies. There are three common exchange rate regimes used globally. The first one is called the Fixed exchange rate regime which is a regime where the central bank intervenes to influence the exchange rate to stay close to the target exchange rate. In this regime, the central bank sets a target exchange rate against a major currency with most countries preferring the dollar, every time the effective exchange rate deviates significantly from the set target, Central banks intervene by either selling or buying the domestic currency. The main aim in this regime is to keep the current exchange rate as close as reasonably possible to the set exchange target. In the late 1980's and early 1990's, countries also used commodities to set the exchange rate value, the most used commodity was gold, this was called the gold standard. Another used means although not common is the Fiat system where a country makes it illegal to trade the domestic currency at any other rate apart from the stipulated one, this is however difficult to implement since it leads to currency black markets.

For a Floating exchange rate regime a country lets the market forces take charge of the exchange rate, these are the demand and supply forces. In the recent years, most countries have resulted to using this regime, the major currencies following this regime include the Swiss Franc, Japanese Yen, British Pound, Australina dollar among other major currencies. Although unpredictable, economists have advocated for this regime since it enables a country to automatically adjust and absorb any impact of foreign business cycles and also preempt possibility of having a crisis in balance of payments. In this regime, central banks hand through the use of Monetary policies is felt as compared to a fixed regime

where the use of monetary policies is impossible. This regime however comes with its own demerits which includes increasing currency volatility. In situations where the domestic currency either significantly strengthens or weakens, Central Banks usually intervene. Most countries in North and South America use this regime.

In a Pegged exchange rate regime which is commonly used by developing countries and similar to the fixed exchange rate regime however with the fluctuation band is allowed to be wider, countries peg their currency to another currency mostly the dollar. The main reason why countries peg their currency to the dollar is because much of their income is paid in dollars hence pegging the domestic currency to the dollar brings stability and makes the currency less volatile. Most of the countries in the Caribbean islands use this regime with the dollar as the peg. It is also possible to peg a currency to a basket of currencies, an example is the Chinese Yuan which is pegged to a basket of currencies with the dollar being one of the currencies in the basket.

In Kenya immediately after independence, Kenya adopted a fixed exchange rate regime and later a floating regime, KES/USD. Over the period that Kenya was using the fixed rate which was until the early nineties, the country struggled to boost its exports which were forcing currency devaluation often. Currently Kenya uses a floating exchange rate regime. Central Bank of Kenya which by law is mandated to keep the currency on check and facilitate stability uses both Monetary and Fiscal policies to keep the Kenya Shilling reasonably stable. With a blend of both Fixed and Floating regimes, Kenya has enjoyed a rather stable Currency in the recent past apart from the Year 2011 when the macro economic environment forced the Shilling to weaken to an all time low of KES/USD 107 against the US Dollar. The Kenyan Central Bank through its market operations is also mandated to control the currency's volatility. A very strong Kenyan Shilling makes Kenya's exports expensive and a very weak Kenyan Shilling makes imports expensive, this means that a balance between the two has to be struck. Daily, the Central bank trades with commercial banks in the Foreign exchange space either mopping up the Shilling or mopping up the Foreign currencies. The aspect of predictability of the Exchange rate is also paramount when it comes to international trade, this is because of the time it takes for imports or exports to reach their intended destination, this means that volatility of the exchange rates has to be put in check also. Nevertheless, currency traders rely on this volatility to make their returns, currency hedgers also rely on currency volatility for returns. As a result, statistical models have become very common among the various traders and users of derivatives to try and make returns from this 'inefficiency' in the currency market.

Apart from exchange rates and exchange rate returns, several studies in time series have been done in the area of exchange rate volatility. Statistically, sample standard deviation which is the square root of variance is used to measure volatility. Two measures of volatility

are majorly used in finance, these are Historical and implied volatility. Standard deviation of recorded daily exchange rates is used to measure historical volatility while forecast by market participants on what is likely to happen in the future is used in implied volatility.

With a series of daily data on exchange rates  $x_t$ , the daily return is calculated as below:

$$\Upsilon_t = \frac{(x_t - x_{t-1})}{x_t} \text{ where } \Upsilon_t \text{ is the return on day } t$$

The volatility of the returns which is calculated using the sample standard deviation is calculated as below:

$$\delta \hat{=} \sqrt{\frac{1}{N-1} \sum_{t=1}^N (\tau_t - \mu)^2} \text{ where } \mu \text{ is the average return over } N \text{ days}$$

There are various aspects to time series volatility, these include persistence, as such, a time series's volatility is said to be persistent if the return today has a large impact on the forecast variance many periods in the future. When volatility is persistent, volatility takes a while to revert to the normal levels. Since exchange rates are mostly driven by fundamentals as compared to sentiments, exchange rate volatility exhibits persistence.

The negative volatility between asset returns and volatility is explained by leverage effects. According to Black (1976), stocks returns volatility is higher following negative shocks as compared to after positive shocks. In terms of the currency returns volatility, when the currency depreciates, what follows is higher volatility as compared to when the currency appreciates. Engle, 2004 explains that high volatility is usually followed by high volatility and low volatility by low volatility. This means that when volatility is high it is likely for it to stay high over long periods of time.

Exchange rates deviate from the normal distribution and hence their modelling is done with models which relax the normality assumption. They exhibit fat tails or excess kurtosis. Models like  $GARCH(p, q)$  which are symmetric do not capture the fact that in financial returns data like exchange rates negative effects on the returns result into higher conditional volatility compared to positive effects. To capture this, asymmetric models like  $EGARCH$ ,  $GJR - GARCH$ ,  $NGARCH$ , models are used. These models solve this by either shifting the origin or rotating the returns curve along the origin.

## 1.1 Statement of the problem

The world is slowly becoming a global village and it is inevitable for any country to interact with other countries either economically or socially. To trade with another country there must be a consensus as to which currency to use if the two countries do not share a common currency. Strengthening of one country's currency will mean that the country uses less/more money for imports/exports and weakening will mean that the country uses/gets more for imports/exports. The difference between the amount used for imports and the amount used for exports is called the Balance of Payments (BoP). This makes fluctuation in the exchange rates crucial in economists' decision making.

After the collapse of Bretton Woods agreement between 1968 and 1973, the issue of exchange rate and exchange rate volatility became a crucial part in any country's economic welfare. In the paper *Modelling Exchange Rate Volatility using GARCH Models: Empirical Evidence from Arab Countries*. (Zakaria and Abdalla, 2012). The class of GARCH models can be used to model exchange rate volatility. High volatility over a long period of time indicates ineffectiveness of central bank in ensuring price stability, (Maana et. al, 2010).

Kenya being part of the global village trades with various countries and hence KES stability is very crucial. For both the importers and exporters, the issue of exchange rate volatility is always a big headache and for the not so sophisticated traders who do not use derivatives to hedge against currency volatility understanding currency volatility is paramount. Understanding the volatility trends and how trade volumes are affected by currency movements would be a step towards breaking down this problem. Although Kenya trades with many countries, most of these trades are denominated in approximately five major currencies. On average over the last five years, among the top five trade currencies in volume, approximately 70 percent of both the imports and exports was in USD, 10 percent in EUR, 6 percent in JPY, 5 percent in GBP and 4 percent in UGX.

In trying to understand and forecast trade volumes data trends, most economists have been using conventional economic theories to model this. The study tries to check whether there is a co-integrating relationship in the trade volume data which would lead to spurious correlations if conventional economic theories like regression analysis was to be done on the trade volume data. In trying to understand the currency volatility, the project also fits the best *GARCH* family of models to each of the five partners.



## 1.2 Objectives

### General Objective:

To perform volatility analysis on Kenya's top five trading partners' currencies.

### The Specific Objectives of the study are:

1. To model the volatility of Kenya's top five trading partners' currencies using *GARCH* family of models
2. To check for co-integration relationship in the trade volume data in USD

## 2 Literature Review

Engle (1982), introduced autoregressive conditional heteroscedastic (ARCH) models to estimate exchange rate volatility. To generalize the unlikely assumption in *ARIMA* models that variance is constant ARCH models are introduced. These models assume zero mean, non-constant autoregressive conditional variances but with constant unconditional variances. In these models, past forecast errors are used to predict the non-constant forecast variance. Since financial assets portfolios are held as a function of expected means and variances of the rate of return, assuming that the mean follows a standard regression will force the variance to remain constant over time which is inappropriate, for this Engle proposes the use of these models in monetary and Finance theory. The model is used in the UK inflation data where estimated variances are found to be substantially high during the chaotic seventies.

Bollerslev(1986) appreciated the progress made from convectional time series data to *ARCH* models in modelling econometric data relaxing the assumption of constant variance hence allowing the conditional variance to change overtime. As an extension to the *ARCH* models, Bollerslev introduced the generalized form of the *ARCH* models,*GARCH* which assumes that the error variance takes *ARMA* model.*GARCH*( $p, q$ ) as denoted has  $p$  representing the number of autoregressive terms or the *ARCH* terms where  $q$  denotes the number of moving average lags or the *GARCH* terms. Some of the properties of *GARCH*( $p, q$ ) models as highlighted by Bollerslev are constant unconditional variance, conditional heteroskedasticity on the back of mean reversion of the model.

In exploring the characteristics of financial data,(Palm ,1996), acknowledged that financial data is largely non-stationary meaning they have a unit root but the returns are generally stationary. The data also exhibits conditional variance which vary with time. Palm also noted the symmetric nature of the *GARCH* models in that both negative and positive shocks has the same impact on conditional variance.

Cyprian et.al (2017) in modeling USD/KES exchange rate volatility using GARCH Models, used daily exchange rates starting 3rd January 2003 to 31st December 2015 to model volatility. Exploratory data analysis concludes that the data is not stationary. Testing for stationarity, the paper uses Augmented Dickey–Fuller test (ADF) to test for a unit root. After the test, the paper fails to reject the null hypothesis and concludes that the data has a unit root. The data is modelled using *GARCH*(1, 1),*GARCH*(1, 1),*GJR – GARCH*(1, 1) and *APARCH*(1, 1). The conclusion is that *APARCH* models, *GJR – GARCH* and *EGARCH* models are adequate to model the data’s volatility.

Nwankwo (2014), an *ARIMA* model (1,0,0) is fitted on Naira to Dollar exchange data from 1982- 2011 based on its low *AIC*, and since the model is an *AR*(1) this is a random walk.

Nwankwo (2014), further concludes that the *ARIMA* model identified if appropriately parameterized can provide a better understanding of the underlying system. Since Kenya and Nigeria fall under the same category of Emerging markets which are faced by the same head winds, fitting an *ARIMA* model in the Kenyan data (Kenya Shilling to dollar) will be suitable.

The ability of *GARCH* models to capture the volatility by its conditional variance of being non-constant through the time makes it more preferred as compared to *ARIMA* Models, Roslindar et al. (2011). The paper uses Box-Jenkins and *GARCH* Models in forecasting crude oil prices. The paper arrives at an *ARIMA*(1, 2, 1) model to forecast the oil prices. However, due to volatility in the data series this model is inadequate. A *GARCH*(1, 1) model is further fitted to the data due to its ability to accommodate volatility. Although *ARIMA* models have extensively been used in the field of finance to do forecasting, data series volatilities have with time rendered the models inadequate since *GARCH* models have lower *RMSE*, *MAE*, and *MAPE* values.

By fundamentals, exchange rates all over the world are non-stationary, noisy and deterministically chaotic, (Box et al., 1994). According to economics, this would mean that Exchange rates exhibit Weak form efficiency which means that one cannot use past exchange prices to predict future prices. This means that forecasting exchange rates using past exchange rates would need more than just technical analysis.

Razman et al. (2012), sought to explore and understand the theoretical and empirical working of *GARCH* models and choose among *ARMA*, *ARCH*, *GARCH* and *EGARCH* models which one gives the best prediction of Exchange rates. The paper uses monthly exchange rates of Pakistan for the period July 1981 to May 2010 which was obtained from State Bank of Pakistan. Since the data is found to be nonstationary, the exchange rates are transformed to exchange rate returns to make the data stationary and an *ARMA*(1, 1) model fitted. To remove the persistence in the data volatility, *GARCH*(1, 2) is found to be the best, while *EGARCH*(1, 2) best deals with the leverage effect. The three models can be said to profoundly deal with the expected aspects of the data series and be used for forecasting.

For the calculation of cash flows of a company, Interest rate risk, exchange rate risk, business cycle risk and inflation risk must be considered (John and Campbell, 1999). The paper further explains that the priorities when adjusting for risk depend on the size of a company with large multinational companies placing exchange rate risk ahead of the other risk factors. This order makes a lot of sense since Small companies mostly deal with

the local currency with multinationals being more exposed to foreign currencies. This further reinforces on the importance of exchange rate forecasting.

In the paper Modelling Exchange Rate Volatility using *GARCH* Models: Empirical Evidence from Arab Countries.(Suliman,2012) , daily observations on exchange rates for nineteen Arabic countries is used and subjected to both symmetric and asymmetric models. In particular, *GARCH* family of models are used to model the exchange rate volatility. Using *GARCH*(1,1), ten out of the sampled nineteen currencies lead to the conclusion that volatility is an explosive process since their estimated persistence coefficients sums exceed one. Seven out of the nineteen currencies, there is evidence of persistence and hence the process has a mean reverting variance. Fitting asymmetric *EGARCH*(1,1) to the data shows leverage effect for most of those currencies indicating that negative shocks at time  $t - 1$  lead to higher volatility at time  $t$  as compared to negative shocks. Suliman concludes that the class of *GARCH* models can be used to model exchange rate volatility.

Dahiru and Joseph (2013), modelled exchange rate volatility in Nigeria using monthly Naira/US dollar exchange rate returns from 1985 to 2011 and data from 2004 to 2011 on Naira/British pound. Initial results show positive skewness in the British pound returns and negative skewness in the dollar returns. Using augmented Dickey–Fuller (*ADF*) and Phillips– Perron (*PP*) unit root tests, the paper rejects the null hypothesis that there are unit roots and hence the data is concluded to be stationary and hence no differencing is needed. In the paper all the asymmetric models used find no evidence of existence of leverage effects except for *GARCH* models with volatility breaks.

Maana et. al. (2010),used daily exchange rates data from 1993 to 2016 on four currencies namely US Dollar, Euro, Sterling pound and Japanese Yen (100) is used to model exchange rate volatility. The data is split into three time periods 1993 - 2006, 2000 – 2006 and 2003 – 2006 for comparison purposes. In the initial data analysis, there is evidence of positive skewness in the data and higher excess kurtosis as compared to the normal distribution. *GARCH* (1,1) model is fitted to the data and the conclusion is that the quality of fit in the different time periods varies.

Hensen(1995),estimated by co-integration techniques and Generalised method of moments looked at goods and labour markets in the Danish private non-agricultural sector for the period 1971 - 1990. He deduced that some of the variables in the model are integrated of order one. He used Johansen method to test for co-integration in the first step. In the second step he simultaneously estimated the other parameters not covered in step one by Generalised method of moments and came up with a model to cast light on unemployment in Dutch.

Ouedraogo and Hassane(2014) examined co-integration and causality between GDP, employment and energy consumption using empirical evidence on the WAEMU countries.

They used the data from West Economic and Monetary Unions zone for the period 1979 to 2010. The goal was to examine the causal relationship between energy consumption in the region WAEMU ,Gross Domestic Product per employee and again the relationship between energy and employment. The tests used were Phillips-Perron tests which tested the level of integration between the variables of interest. The series were found to be stationary apart from GDP per employment which was found to be integrated of order 1.

## 3 METHODOLOGY

### 3.1 Time series models

These are models which analyze time series data and consequently use past time series data points to forecast future values. Before analysis, there is need to understand the properties of the data which include stationarity. The data series  $X_t$  is said to be stationary if the joint probability density function, j.p.d.f

$$X_{t_1}, X_{t_2}, \dots, X_{t_n} \quad (1)$$

is equal to the j.p.d.f

$$X_{t_1+k}, X_{t_2+k}, \dots, X_{t_n+k} \quad (2)$$

For  $t_1, t_2, \dots, t_n$  and for any lag  $k$ . For a Univariate distribution i.e.  $n = 1$  the distribution of  $X_t$  is same as that of  $X_{t+k}$ . The mean and Variance properties of the two distributions are:

$$E[X_t] = E[X_{t+k}] \quad (3)$$

$$Var(X_t) = Var(X_{t+k}) \quad (4)$$

A time series is said to be second order stationary if the mean function and the variance are constant over time and if auto covariance is not dependent on time.

In Finance and Investments, there is a negative correlation between returns at time  $t - 1$  and volatility at time  $t$ . This means that good and bad news affect returns volatility differently. Both *EGARCH* and *GJR - GARCH* analyze these effects with the *EGARCH* model looking at the asymmetric conditional heteroskedasticity arising from both the good and bad news while *GJR - GARCH* differentiates the effects by adding a seasonal term.

#### 3.1.1 ARCH(q)

An Autoregressive conditional Heteroscedasticity (*ARCH*) model deals with the variance aspect of a time series. These models test volatility of a time series, mostly when variance changes over a short period of time and hence the term heteroscedasticity. The Autoregressive term comes into play since the models are autoregressive models in squared returns and conditional since volatility in the next period is conditional on the current information. With the nature of Exchange rates where sentiments and change in economic fundamentals of a country can result into significant changes in exchange rates, it would

be prudent to use  $ARCH(q)$  models in the analysis. With  $y_t$  being the daily exchange rate return at time  $t$ , modeling the variance of the daily exchange rates and assuming that  $\varepsilon_t$  are the error terms at time  $t$ . The model is expressed as:

$$y_t = x_t \xi_t + \varepsilon_t \quad (5)$$

for  $t=1, 2, \dots, T$

$$\delta_t^2 = \alpha_0 + \sum_{j=1}^q \alpha_j \varepsilon_{t-j}^2 \quad (6)$$

Where  $\varepsilon_t = \delta_t z_t$  and  $z_t \sim N(0, 1)$

The simplest  $ARCH$  model is  $ARCH(1)$  and models return as below:

$$y_t = \delta_t \varepsilon_t \quad (7)$$

which is also commonly written as

$$y_t = \sqrt{\delta_t^2} \varepsilon_t \quad (8)$$

where the variance is  $\delta_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2$  and  $\alpha_0 > 0, \alpha_1 \geq 0$  and

$\varepsilon_t \sim iid.N(0, 1)$

Using conditional distribution and assuming normality i.e.  $y_t | y_{t-1} \sim N(0, \delta^2)$  Where the conditional variance is expressed as

$$\delta_{t|t-1}^2 = \alpha_0 + \alpha_1 y_{t-1}^2 \quad (9)$$

Some of the limitations of  $ARCH$  models are that they assume that both negative and positive shocks have the same effect on volatility and that the model is quite restrictive on the values which  $\alpha_1$  can take.

### 3.1.2 GARCH

The process of Generalized Autoregressive Conditional Heteroscedasticity is mostly used in econometrics to describe the volatility that is inherent in financial markets.. Since in  $ARCH(q)$  the value  $q$  is practically very large, the law of parsimony directs us to

$GARCH(p, q)$  model which has an infinite  $ARCH$  specification.  $GARCH$  models written as  $GARCH(p, q)$  are  $ARCH(\infty)$ . A series  $x_t$  which is stationary is said to be  $GARCH(p, q)$  and expressed as:

$$\delta_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \dots + \alpha_q \varepsilon_{t-q}^2 + \beta_1 \delta_{t-1}^2 + \dots + \beta_p \delta_{t-p}^2 = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \delta_{t-j}^2 \quad (10)$$

The model uses both the lagged residuals and conditional variances. Where,

$$p \geq 0, q \geq 0, \alpha_0 \geq 0, \alpha_1 > 0,$$

for  $i=1,2,3,\dots,p$

$$\beta_j > 0,$$

for  $j=1,2,3,\dots,q$

Using the Lag operator,  $B$ , this model can be expressed as:

$$\alpha_0 + (\alpha_1 B + \dots + \alpha_q B^q) \varepsilon_t^2 = (1 - \beta_1 B - \dots - \beta_p B^p) \delta_{t-1}^2 \quad (11)$$

Under this, when the value of  $p = 0$ , then the process is  $ARCH(q)$ , when both  $p$  and  $q$  are zero i.e.  $p = q = 0$  then the process reduces to white noise. To explore further the properties of  $GARCH(p, q)$  we consider  $GARCH(1, 1)$  which has over time been used in analysis of financial time series data. It is written as:

$$x_t^2 = \delta_t^2 z_t^2 = \alpha_0 + (\alpha_1 + \beta) x_{t-1}^2 - \beta v_{t-1} + v_t \quad (12)$$

where

$v_t = \delta_t^2 (z_t^2 - 1)$  and  $z_t^2$  can be white noise

For this equation, the mean can be expressed as

$$\Gamma_t = \mu + \varepsilon_t \quad (13)$$

Where  $\mu$  is the mean and  $\varepsilon_t$  is the error term.

The limitations of the  $GARCH$  models are that they assume both negative and positive residuals have the same effect on the conditional variance and that all coefficients are greater than zero. This makes its applicability limited.



### 3.2 Unit Roots and Co-integration

A time series  $x_t$  is said to be integrated order zero i.e  $I(0)$  if it can be represented in a moving average format.

$$\sum_{k=0}^{\infty} |b_k|^2 < \infty \quad (14)$$

Where  $b_k$  is a vector of moving average parameters. From this we can conclude that the auto covariance is decaying to zero quickly. All stationary series are  $I(0)$  but stationarity is not implied by  $I(0)$ .

The order of integration is the number of differencing needed to make a non-stationary series stationary and hence a stationary series is integrated of order 0.

Considering a variable  $x_t$  expressed as:

$$x_t = x_{t-1} + \varepsilon_t \quad (15)$$

this process is integrated order 1.

Using the lag operator,  $L$ , a series is integrated of order  $d$  if  $(1 - L)^d X_t$  is a stationary series. Where

$$(1 - L)^d X_t = X_t - X_{t-1} = \Delta X_t \quad (16)$$

Considering two non-stationary series  $X_t$  and  $Y_t$ , we can model  $Y_t$  as a linear combination of series  $X_t$ .

$$Y_t = \beta_0 + \beta_1 X_{1,t} + \beta_2 X_{2,t} + \dots + \beta_k X_{k,t} + \varepsilon_t \quad (17)$$

This regression model will produce spurious results since it emanates from a non-stationary time series unless the residuals of the linear combination of both  $Y_t$  and  $X_t$  are stationary i.e.

$$Y_t + \gamma_1 X_{1,t} + \gamma_2 X_{2,t} + \dots + \gamma_k X_{k,t} \sim I(0) \quad (18)$$

Cointegration points to the hunch that in the long-run, there is some steady relationship tying individual variables together which can be expressed in a linear relationship between the variables.

With two series  $X_t$  and  $Y_t$ , cointegration suggests the presence of a series holding the two series together.

$$X_t = \gamma_0 + \gamma_1 Z_t + \varepsilon_t \sim I(0) \quad (19)$$

$$Y_t = \delta_0 + \delta_1 Z_t + \eta_t \sim I(0) \quad (20)$$

$$Z_t \sim I(0)$$

$$\eta_t, \varepsilon_t \sim I(0)$$

this means that the residuals are stationary

Although the two series  $X_t$  and  $Y_t$  are non-stationary, the residuals of their linear combination are stationary. Simply put, the two series are co-integrated if there exists  $\alpha$  where

$$\mu_t = \gamma_t - \alpha X_t \text{ is stationary}$$

If a series co-integrates, relationship between variables are interpreted as a long run equilibrium since any deviations from this eventually revert to the mean.

Testing for co-integration between two series the conditions for this are that the series are integrated of order 1 and that a linear combination of the series is integrated of order zero. Simply put, two set of variables are co-integrated if a linear combination of the said two variables has a lower integration order.

There are various tests for integration, the project will be using Engle–Granger also called the EG test which is a two step method.

Verification that both  $X_t$  and  $Y_t$  are  $I(1)$  is needed before running the regression  $y_t = \hat{\theta}x_t + \hat{\varepsilon}_t$ , after running the model estimate the parameter values of  $\hat{\theta}$  by OLS and then test  $\hat{\varepsilon}_t = y_t - \hat{\theta}x_t$  for unit roots

### 3.2.1 Dickey - Fuller test

This is the most used test for co-integration as it helps in understanding all the other tests. The test has three versions each with its own critical value which is dependent on the sample size. Assuming the model being tested is  $AR(1)$  i.e  $y_t = \rho y_{t-1} + \mu_t$ . In this case  $\rho$  is a co-efficient and  $\mu$  is the error term. The process is stationary if  $\rho = 1$  and this would mean presence of a unit root.

Using the Lag operator this can be written as:

$$\Delta y_t = (\rho - 1)y_{t-1} + \mu_t = \delta y_{t-1} + \mu_t \quad (21)$$

- Test with Intercept Only

$$\Delta y_t = A_0 + \delta y_{t-1} + \mu_t \quad (22)$$

- Trend and Intercept

$$\Delta y_t = A_0 + A_1 t + \delta y_{t-1} + \mu_t \quad (23)$$

- No trend no intercept

$$\Delta y_t = \delta y_{t-1} + \mu_t \quad (24)$$

The null hypothesis in each of the three tests is that  $\delta = 0$  (stationary) which would reduce the part  $\delta y_{t-1}$  to zero meaning the series returns to a constant (the mean). This means that huge returns are followed by small returns and small returns by huge returns (Mean reversion). This implies that current returns can be used to predict future returns and their co-efficient is negative.

### 3.2.2 Pedroni test for Panel cointegration

The hypothesis being tested is:

$H_0$ : There is a unit root

$H_1$ : No unit root

This uses four panel statistics and three group panel statistics testing the above hypothesis. Using the panel statistics, the assumption is that first order autoregressive term across all the cross sections is the same. For group panel statistics, the parameter varies in the cross sections.

In the Panel Case, rejection of null hypothesis means there is cointegration for all the sectors while in the Group panel case rejection of null hypothesis means there is cointegration among the variables.

The test is based on the premise that in the spurious regression equation, if there is cointegration then the residuals are  $I(0)$  and  $I(1)$  if there is no cointegration.

Using the equation:

$$y_{it} = \alpha_i + \delta_{it} + \beta_{1i}x_{1i,t} + \beta_{2i}x_{2i,t} + \dots + \beta_{Mi}x_{Mi,t} + \varepsilon_{i,t} \quad (25)$$

where  $t = 1, 2, \dots, T$   $i = 1, 2, \dots, N$  and  $m = 1, 2, \dots, M$

Under the null hypothesis, the residuals will be  $I(1)$ , this boils down to testing whether the residuals are  $I(1)$  using the equation:

$$\varepsilon_{it} = \rho_i \varepsilon_{it-1} + \mu_{it} \quad (26)$$

Pedroni panel cointegration statistic ( $\Psi_{N,T}$  where  $T$  is number of observations,  $N$  is number of cross-sectional units,  $M$  is number of regressors) which is asymptotically distributed.

$$\frac{\Psi_{N,T} - \mu\sqrt{N}}{\sqrt{v}} \Rightarrow N(0, 1) \quad (27)$$

is constructed using residuals from regression equation.

## 4 DATA ANALYSIS AND RESULTS

### 4.1 Data

Data on Kenya's exports and imports has been used to model trade volumes of Kenyan shilling. The monthly data spans from January 2005 to December 2017, this data was obtained from the Kenya National Bureau of Statistics leading economic indicators bulletin.

On the exports a total of eleven countries are in the bulletin as Kenya's trade partners, these are Uganda, Tanzania, United Kingdom, Pakistan, Netherlands, Egypt, Germany, Rwanda, United States, United Arab Emirates and France. Kenya largely Imports from eleven namely United Arab Emirates, United Kingdom, South Africa, Saudi Arabia, Japan, India, Unites States, Germany, Netherlands, France and China. These countries use eight different currencies namely *USD*, *EUR*, *JPY*, *GBP*, *UGX*, *ZAR*, *TSHS*, and *RWF*.

Trade volumes data for both the imports and exports have been combine to get the total consideration on trade and the top five currencies in volume (In Kenya Shillings) have been considered. These currencies are *USD*, *EUR*, *JPY*, *GBP* and *UGX* which account for more than 90% of the total trade in volume. Data on daily currency exchange rates for the five currencies has been obtained for the same period, January 2005 to December 2017. This was obtained from the Central Bank of Kenya data release page on Central Bank's website.

For the five currencies, the variable being modelled  $r_t$  is given by:

$$r_t = \log\left(\frac{e_t}{e_{t-1}}\right) \quad (28)$$

where  $e_t$  is the current exchange rate and  $e_{t-1}$  is the previous day's exchange rate.

## 4.2 Exploratory Data Analysis

The time plot of trade volumes in Kenya Shillings done in different currencies:



Figure 4.1: Volatility graph USD

In Figure 4.1 above, it is clear that the *KES/USD* series is characterized by random, rapid changes and is said to be volatile. The volatility seems to change over time, period of low volatilities then periods of high volatilities. For example, *KES/USD* exchange rate experienced a decline from 2004 to 2008 then a sudden and steady rise with a drop 2012 then increases steadily up to 2017. The rates become more volatile in 2011. The rates are high but low volatility between 2016 and 2017.

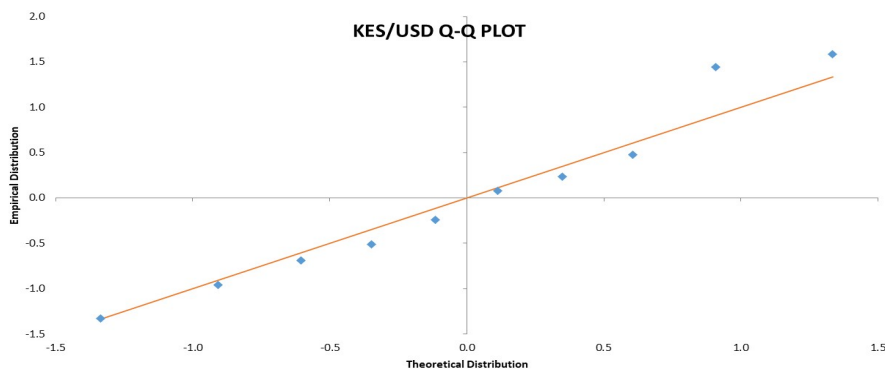


Figure 4.2: QQ plot of KES/USD data

The QQ plot of  $KES/USD$  data from January 2005 to December 2017 shows that the data is not normally distributed. The data points on exchange rates do not fall on a straight line and hence the data is not considered to be normal.

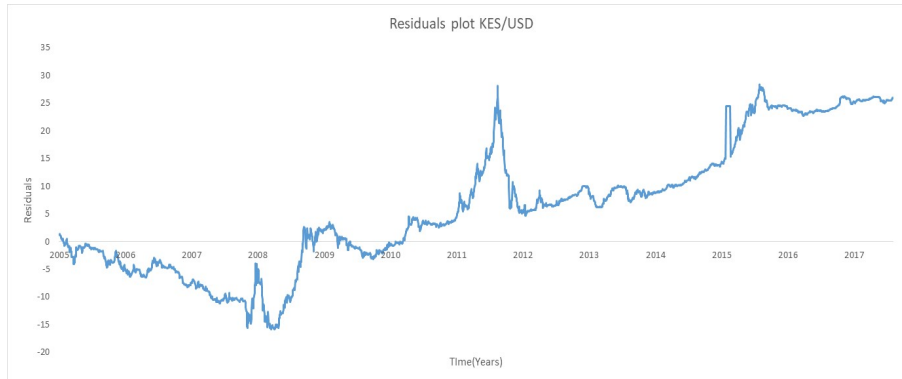


Figure 4.3: Residual plot - USD

Testing for clustering volatility in the  $KES/USD$  data, from Figure 4.3 above, clustering volatility is evident: The periods of high volatility are followed by periods of high volatility whereas periods of low volatility are followed by period of low volatility. Low volatility from 2007 followed by low volatility in 2008 – 2013, the trend then changes, high volatility in 2014 and 2015. Therefore, the residuals (error term) is conditionally heteroscedastic and can be represented by *GARCH* model.

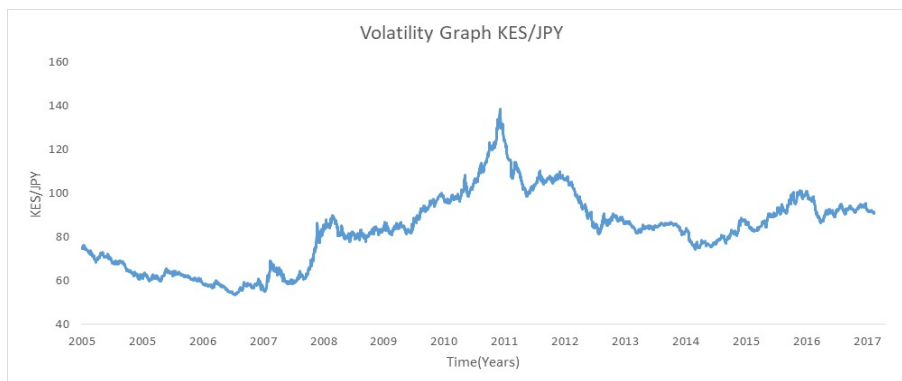


Figure 4.4: Volatility graph (JPY)

The  $KES/JPY$  rates in the period under study are characterized by random, rapid changes and are said to be volatile. The volatility seems to change over time, relatively low volatilities are witnessed between 2005 and 2008. The rates become more volatile in 2011 then decreases until 2015 before a steady rise.

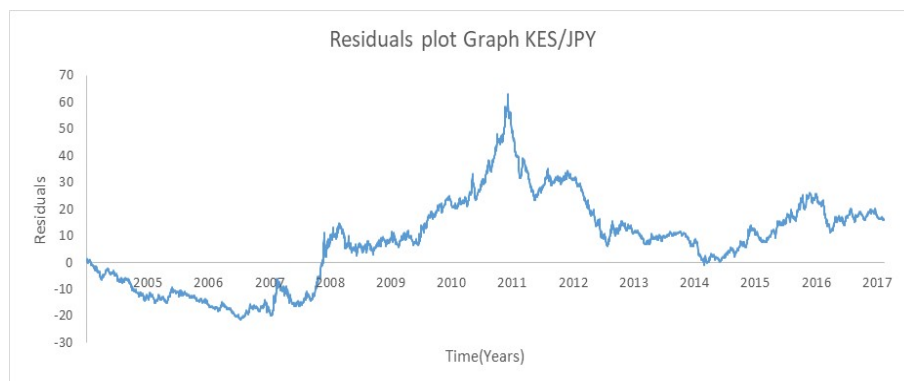


Figure 4.5: Residual plot -KES/ JPY

The exploratory test done for  $KES/JPY$  was to explore clustering volatility and from the residual plot, clustering volatility is present; low volatility is witnessed in the year 2007 followed by prolonged period of low volatility up to 2009. On the other hand, from the year 2013, high volatility is evident and this is followed by a prolonged period of high volatility up to 2016. These periods of high volatility are followed by periods of high volatility whereas periods of low volatility are followed by periods of low volatility. Therefore, the residuals (error term) is conditionally heteroscedastic and can also be represented by *GARCH* model.

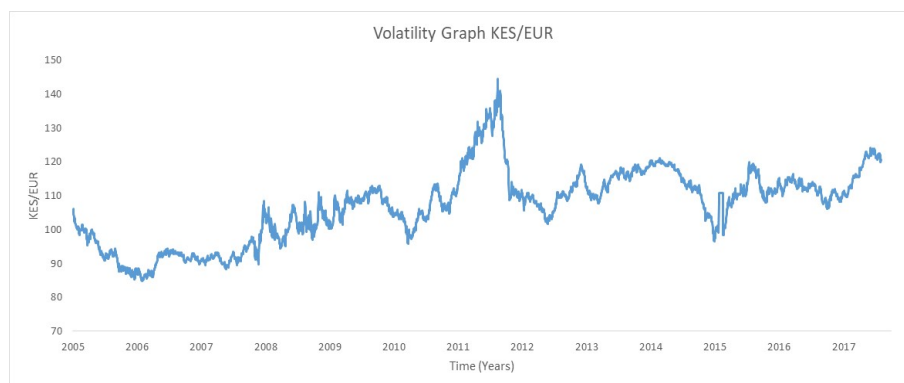


Figure 4.6: Volatility Graph-EUR

From Figure 4.6, the graph of  $KES/EUR$ , volatility is high between 2011 and 2012. Then the rates experience a relatively sedate period from 2013 to 2015; however, the rate remains relatively high within the same period. The period between 2005 and 2008 is characterized by low exchange rates.



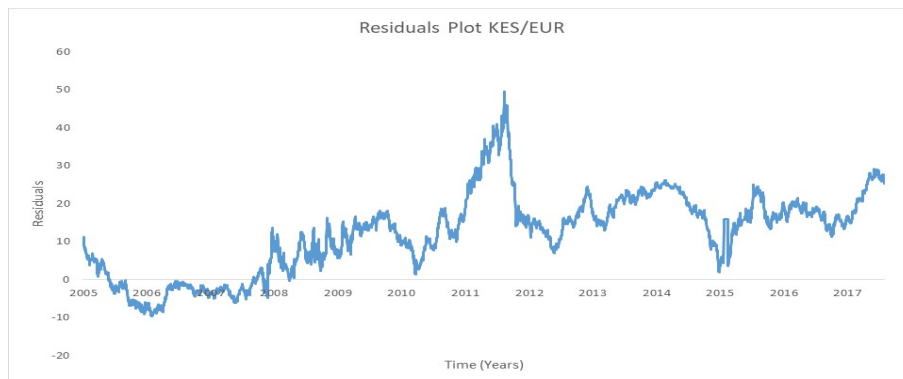


Figure 4.7: Residual Plot - EUR

From the residual plot, clustering volatility is present; there is low volatility from the year 2010 followed by prolonged period of low volatility up to the year 2012. Moreover, from the year 2013, high volatility is evident and this is followed by a prolonged period of relatively high volatility up to the year 2016. Therefore, the residuals (error term) is conditionally heteroscedastic and can also be represented by *GARCH* model.

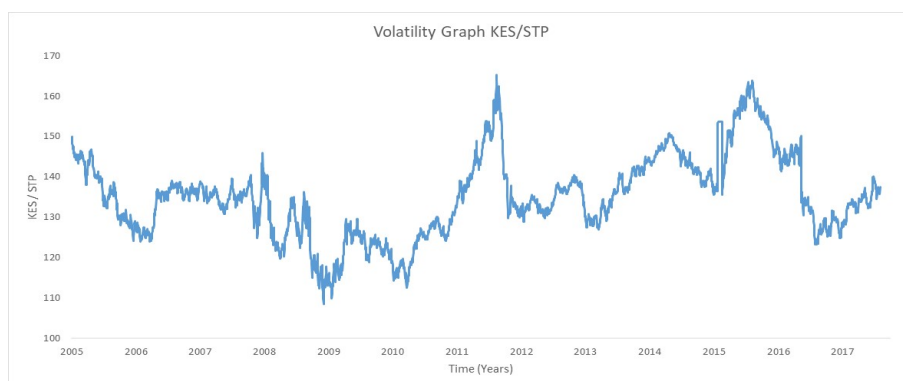


Figure 4.8: Volatility Graph - GBP

The *KES/GBP* series is evidently volatile. It is also clear that the volatility seems to change over time, period of low volatilities then periods of high volatilities. The lowest volatility is observed in 2009 and the exchange rate goes below 120, the data is also characterized by random, rapid changes.

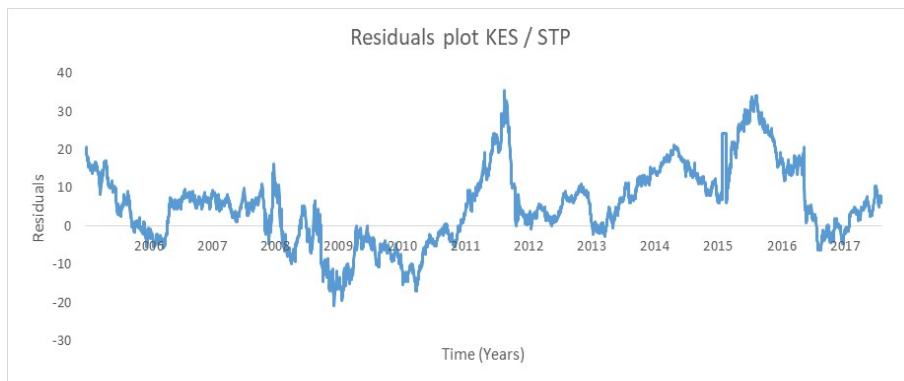


Figure 4.9: Residual Plot - GBP

From the plot of  $KES/GBP$ , there is clustering volatility since low volatility is witnessed from the year 2007 followed by prolonged period of low volatility up to the year 2008. We also note that from the year 2009, high volatility is evident and this is followed by a prolonged period of relatively high volatility up to the year 2013. Hence, the residuals (error term) is conditionally heteroscedastic and can now be represented by *GARCH* model.

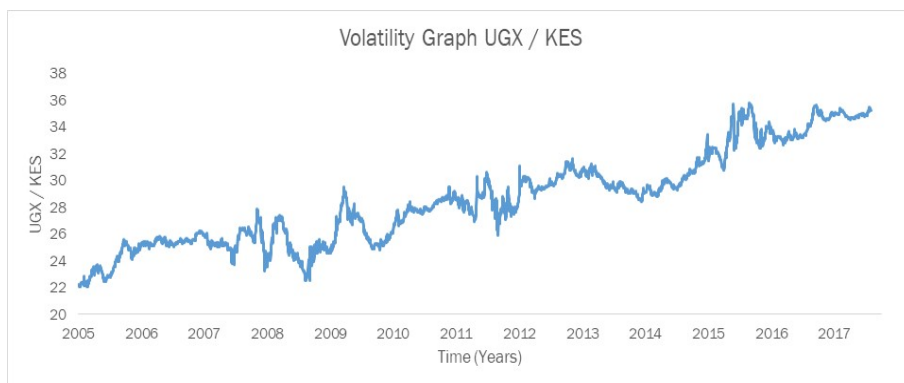


Figure 4.10: Volatility Graph UGX

The series on  $KES/UGX$  is characterized by upward and downward trends from 2004 to 2017. It is also important to notice that highest rates are observed around late 2015 and in 2017 while the lowest rates are experienced in 2004. In general volatility seems to change over time, period of low volatilities then periods of high volatilities.

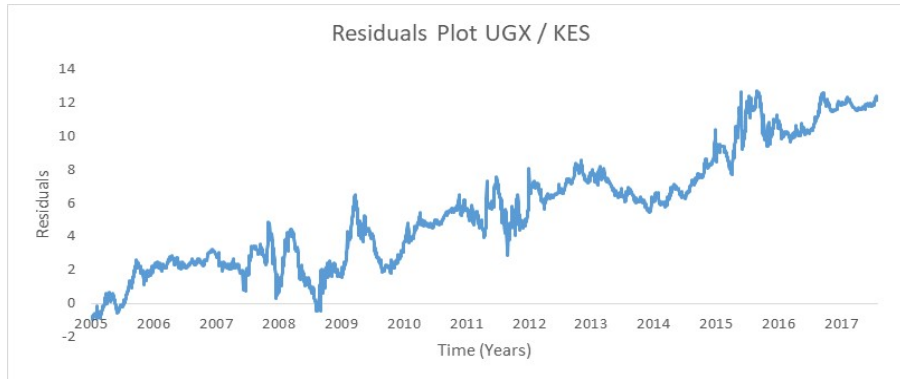


Figure 4.11: Residual Plot - UGX

The residual plot shows clustering volatility since low volatility is witnessed from the year 2007 followed by a prolonged period of low volatility up to the year 2009. High volatility is also witnessed from the year 2010, which is followed by a prolonged period of relatively high volatility up to the year 2013. Hence, the residuals (error term) is conditionally heteroscedastic and can now be represented by *GARCH* model.

Clearly, from the exploratory data analysis, the data being analysed is non-stationary and hence the variable used in *ARCH – GARCH* modelling is:

$$r_t = \log\left(\frac{e_t}{e_{t-1}}\right) \quad (29)$$

From the QQ plot in Figure 4.2 on *KES/USD*, the data on exchange rates is not normally distributed. From the plots again done on the five currencies Figure 4.1, Figure 4.4, Figure 4.6, Figure 4.8 and Figure 4.10, the data was found to be non-stationary and hence a natural log function of the first order difference is used to transform the variable of interest. The model that was used is a skewed student-t distribution this is because financial data is known to exhibit non-normal characteristics which would not be captured well by a *GARCH* model with a gaussian distribution assumption. Skewed Student-t distribution allows heavy tails and skewed data.

### 4.3 ARCH - GARCH EFFECTS TESTING

#### KES/USD

The models run are  $GARCH(1,1)$ ,  $GARCH(1,2)$  and  $GARCH(2,2)$  and the parameters in every model given and then each checked for goodness of fit where  $AIC$  for each of the models is checked.

#### *GARCH Analysis on KES/USD*

<b>GARCH(1,1)</b>				
	<b>Param</b>	<b>Value</b>	<b>AIC</b>	<b>ARCH?</b>
	$\mu$	-2.977926744	11,650.0642	TRUE
	$\alpha_0$	6.176254562		
	$\alpha_1$	0.171667079		
	$\beta_1$	0.171667079		
	$\nu$	5		
<b>GARCH(1,2)</b>				
	<b>Param</b>	<b>Value</b>	<b>AIC</b>	<b>ARCH?</b>
	$\mu$	-2.977926744	11,133.3550	TRUE
	$\alpha_0$	4.759075636		
	$\alpha_1$	0.132539294		
	$\beta_1$	0.228938094		
	$\beta_2$	0.132539294		
	$\nu$	5		
<b>GARCH(2,2)</b>				
	<b>Param</b>	<b>Value</b>	<b>AIC</b>	<b>ARCH?</b>
	$\mu$	-2.977926744	11,218.7880	TRUE
	$\alpha_0$	4.759075636		
	$\alpha_1$	0.132539294		
	$\alpha_2$	0.114469047		
	$\beta_1$	0.132539294		
	$\beta_2$	0.114469047		
	$\nu$	5		

Table 4.1: GARCH analysis on KES-USD

Using the AIC criterion, the model that fits the data is  $GARCH(1,2)$  which has the lowest AIC value. The model has a long run mean of -2.97792 and the parameters of the ARCH component are  $\alpha_0 = 4.7590, \alpha_1 = 0.1325$ . The GARCH components are  $\beta_1 = 0.2289$  and  $\beta_2 = 0.1325$ . The degrees of freedom of the shock distribution is 5. The model also has ARCH (Meaning the squared error terms exhibit autocorrelation).

## KES/EUR

## GARCH Analysis on KES/EUR

<b>GARCH(1,1)</b>				
	<b>Param</b>	<b>Value</b>	<b>AIC</b>	<b>ARCH?</b>
	$\mu$	-2.47465427	10,902.6106	TRUE
	$\alpha_0$	5.879105299		
	$\alpha_1$	0.045709088		
	$\beta_1$	0.045709088		
	$\nu$	5		
<b>GARCH(1,2)</b>				
	<b>Param</b>	<b>Value</b>	<b>AIC</b>	<b>ARCH?</b>
	$\mu$	-2.47465427	1.7977E+308	TRUE
	$\alpha_0$	5.339564058		
	$\alpha_1$	0.041461212		
	$\beta_1$	0.091987153		
	$\beta_2$	0.041461212		
	$\nu$	5		
<b>GARCH(2,2)</b>				
	<b>Param</b>	<b>Value</b>	<b>AIC</b>	<b>ARCH?</b>
	$\mu$	-2.47465427	10763.41894	TRUE
	$\alpha_0$	5.339564058		
	$\alpha_1$	0.041461212		
	$\alpha_2$	0.045993576		
	$\beta_1$	0.041461212		
	$\beta_2$	0.045993576		
	$\nu$	5		

Table 4.2: GARCH analysis on KES-EUR

Using the AIC criterion, the model that fits the data is  $GARCH(2,2)$  since it has the lowest AIC value. The model has a long run mean of -2.4746 and the parameters of the ARCH component are  $\alpha_0 = 5.3395, \alpha_1 = 0.0414, \alpha_2 = 0.0459$ . The GARCH components are  $\beta_1 = 0.04146$  and  $\beta_2 = 0.04599$ . The degrees of freedom of the shock distribution is 5. The model also has ARCH effects (Meaning the squared error terms exhibit autocorrelation).

## KES/GBP

*GARCH Analysis on KES/GBP*

<b>GARCH(1,1)</b>				
	<b>Param</b>	<b>Value</b>	<b>AIC</b>	<b>ARCH?</b>
	$\mu$	-2.481847762	10,895.74	TRUE
	$\alpha_0$	5.854152243		
	$\alpha_1$	0.049488338		
	$\beta_1$	0.049488338		
	$\nu$	5		
<b>GARCH(1,2)</b>				
	<b>Param</b>	<b>Value</b>	<b>AIC</b>	<b>ARCH?</b>
	$\mu$	-2.481847762	10,756.02	TRUE
	$\alpha_0$	5.32282258		
	$\alpha_1$	0.045073055		
	$\beta_1$	0.090667856		
	$\beta_2$	0.045073055		
	$\nu$	5		
<b>GARCH(2,2)</b>				
	<b>Param</b>	<b>Value</b>	<b>AIC</b>	<b>ARCH?</b>
	$\mu$	-2.481847762	10,757.20	TRUE
	$\alpha_0$	5.32282258		
	$\alpha_1$	0.045073055		
	$\alpha_2$	0.045333928		
	$\beta_1$	0.045073055		
	$\beta_2$	0.045333928		
	$\nu$	5		

Table 4.3: GARCH analysis on KES-GBP

Using the AIC criterion, the model that fits the data is  $GARCH(1,2)$  since it has the lowest AIC value of 10,756. The model has a long run mean of -2.4817 and the parameters of the ARCH component are  $\alpha_0 = 5.3228, \alpha_1 = 0.04507, \alpha_2 = 0.04533$ . The GARCH components are  $\beta_1 = 0.04507$  and  $\beta_2 = 0.04533$ . The degrees of freedom of the shock distribution is 5. The model also has ARCH effects (Meaning the squared error terms exhibit autocorrelation).

## KES/JPY

## GARCH Analysis on KES/JPY

<b>GARCH(1,1) &amp; t-dist(v)</b>				
	<b>Param</b>	<b>Value</b>	<b>AIC</b>	<b>ARCH?</b>
	$\mu$	-2.411514308	11,075.63	FALSE
	$\alpha_0$	5.36473905		
	$\alpha_1$	0.05994206		
	$\beta_1$	0.05994206		
	$\nu$	5		
<b>GARCH(1,2) &amp; t-dist(v)</b>				
	<b>Param</b>	<b>Value</b>	<b>AIC</b>	<b>ARCH?</b>
	$\mu$	-2.411514308	1.8E+308	FALSE
	$\alpha_0$	5.082477454		
	$\alpha_1$	0.056742571		
	$\beta_1$	0.052437939		
	$\beta_2$	0.056742571		
	$\nu$	5		
<b>GARCH(2,2) &amp; t-dist(v)</b>				
	<b>Param</b>	<b>Value</b>	<b>AIC</b>	<b>ARCH?</b>
	$\mu$	-2.411514308	10,993.65	FALSE
	$\alpha_0$	5.082477454		
	$\alpha_1$	0.056742571		
	$\alpha_2$	0.026218969		
	$\beta_1$	0.056742571		
	$\beta_2$	0.026218969		
	$\nu$	5		

Table 4.4: GARCH analysis on KES-JPY

By using the AIC criterion, the model that fits the data best is  $GARCH(2,2)$  since it has the lowest AIC value of 10,993.65. The model has a long run mean of -2.4115 and the parameters of the ARCH component are  $\alpha_0 = 5.0825$ ,  $\alpha_1 = 0.0567$ ,  $\alpha_2 = 0.0262$ . The GARCH components are  $\beta_1 = 0.0567$  and  $\beta_2 = 0.0262$ . The degrees of freedom of the shock distribution is 5. The model also has ARCH effects (Meaning the squared error terms exhibit autocorrelation).

## KES/UGX

## GARCH Analysis on KES/UGX

<b>GARCH(1,1) &amp; t-dist(v)</b>				
	<b>Param</b>	<b>Value</b>	<b>AIC</b>	<b>ARCH?</b>
	$\mu$	-2.644876021	11,187.90	TRUE
	$\alpha_0$	5.952024311		
	$\alpha_1$	0.098984704		
	$\beta_1$	0.098984704		
	$\nu$	5		
<b>GARCH(1,2) &amp; t-dist(v)</b>				
	<b>Param</b>	<b>Value</b>	<b>AIC</b>	<b>ARCH?</b>
	$\mu$	-2.644876021	1.8E+308	TRUE
	$\alpha_0$	5.131267487		
	$\alpha_1$	0.085288271		
	$\beta_1$	0.137925458		
	$\beta_2$	0.085288271		
	$\nu$	5		
<b>GARCH(2,2) &amp; t-dist(v)</b>				
	<b>Param</b>	<b>Value</b>	<b>AIC</b>	<b>ARCH?</b>
	$\mu$	-2.644876021	10,961.78	TRUE
	$\alpha_0$	5.131267487		
	$\alpha_1$	0.085288271		
	$\alpha_2$	0.068962729		
	$\beta_1$	0.085288271		
	$\beta_2$	0.068962729		
	$\nu$	5		

Table 4.5: GARCH analysis on KES-UGX

By using the AIC criterion, the model that fits the data best is  $GARCH(2,2)$  since it has the lowest AIC value of 10.961.78. The model has a long run mean of -2.6449 and the parameters of the ARCH component are  $\alpha_0 = 5.3113$ ,  $\alpha_1 = 0.0853$ ,  $\alpha_2 = 0.0690$ . The GARCH components are  $\beta_1 = 0.0853$  and  $\beta_2 = 0.0690$ . The degrees of freedom of the shock distribution is 5. The model also has ARCH effects (Meaning the squared error terms exhibit autocorrelation).



## 4.4 CO-INTEGRATION ANALYSIS

The condition to conduct Johansen test for cointegration is that the variables to be used in cointegration analysis must be non-stationary but their first difference should be stationary. The test is done on KES/USD and trade volumes denominated in USD. The reason for choosing this currency is because more than 60% of Kenya's trade is denominated in USD.

Using the Unit root test we perform the test on the trade volume data using three criterias:

- Test with intercept only
- Trend and intercept
- No trend no intercept

### Test with Intercept only

The hypothesis to be tested in this case is

$H_0$ : Variables are non - stationary/has unit root

$H_1$ : Variables are stationary/ has no unit root

Dickey-Fuller test for unit root				Number of obs	=	3298
Test Statistic	Interpolated Dickey-Fuller					
	1% Critical Value	5% Critical Value	10% Critical Value			
$Z(t)$	-3.875	-3.430	-2.860	-2.570		
MacKinnon approximate p-value for $Z(t) = 0.0022$						
D.	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
tradevolUSD						
tradevolUSD						
l1.	-0.0090545	.0023365	-3.88	0.000	-0.0136356	-0.0044734
_cons	481.0951	126.7611	3.80	0.000	232.5567	729.6335

Table 4.6: Test for unit root - Intercept only

The absolute value of the test statistic is (3.875) which is larger than the t-value at critical value 5% in absolute terms which is 2.860, so we reject the null hypothesis and conclude that trade volume data is stationary.

## Trend and intercept

Dickey-Fuller test for unit root		Number of obs = 3298			
Test Statistic	Interpolated Dickey-Fuller				
	1% Critical Value	5% Critical Value	10% Critical Value		
$Z(t)$	-8.376	-3.960	-3.410	-3.120	
MacKinnon approximate p-value for $Z(t) = 0.0000$					
D. tradevol~D	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
tradevolUSD					
_L1.	-.0416981	.0049786	-8.38	0.000	-.0514595    -.0319367
_trend	.6584785	.0888814	7.41	0.000	.4842101    .832747
_cons	1076.798	149.2493	7.21	0.000	784.1677    1369.429

Table 4.7: Test for Unit root - Trend and Intercept

The absolute value of the test statistic is 8.376 which is larger than the critical absolute value at 5% level of confidence which is 3.410, so we reject the null hypothesis and conclude that trade volume data is stationary.

## No trend no intercept

Dickey-Fuller test for unit root		Number of obs = 3298		
Test Statistic	Interpolated Dickey-Fuller			
	1% Critical Value	5% Critical Value	10% Critical Value	
Z(t)	-0.863	-2.580	-1.950	-1.620

D. tradevolUSD	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
tradevolUSD L1.	-.0006333	.0007335	-0.86	0.388	-.0020714	.0008048

Table 4.8: Test for Unit root - No-trend and No-Intercept

The absolute value of the test statistics is 0.863 which is less than the absolute critical value at 5% significance level which is 1.950, so we fail to reject the null hypothesis and conclude that trade volume data is non-stationary.

General conclusion from the three tests is that since the first two tests on trade volume are stationary at their first difference, we conclude that trade volume data is non-stationary hence we can proceed with cointegration analysis.

## Test for Co-integration

### Test for Co-integration using Johansen Test

Test	Stat	C.V.	Passed?
<b>Trace Test (r=0)</b>	<b>0</b>		<b>r&gt;0</b>
<i>No Const</i>	<b>5.4</b>	<b>12.3</b>	<b>FALSE</b>
<i>Const-Only</i>	<b>11.2</b>	<b>15.5</b>	<b>FALSE</b>
<i>Const + Trend</i>	<b>21.8</b>	<b>18.4</b>	<b>TRUE</b>

Table 4.9: Test for Co-integration - Johansen Test

The test statistic under the 'No Constant assumption' and 'Constant only' assumption are clearly less than the boundary limit of all the possible values of the test statistic.

Considering the null hypothesis which is that there is no co-integration the values  $5.4 < 12.3$  and  $11.2 < 15.5$ . However, under the assumption 'Constant and Trend', the test statistic is more than the boundary i.e.  $21.8 > 18.4$  and this leads to the rejection of the null hypothesis that there is no co-integration. Also the test on whether each of the assumptions 'Passes' the co-integration test results into one assumption as 'True' i.e. the 'Constant and Trend' and the others as false.

Hence, using the trace test and under the three different assumptions for the input variable, the question of whether there is at least one linear combination of the inputs that yields a stationary process is answered and clearly under one of the three assumptions we get a stationary process. We can hence conclude that the variables are co-integrated.

### Test for Co-integration using Maximum Eigenvalues test

<b>Maximum Eigenvalue Test(<math>r=1</math>)</b>	<b>1</b>		<b><math>r=2</math></b>
<b>No Const</b>	<b>1.5</b>	<b>4.1</b>	<b>FALSE</b>
<b>Const-Only</b>	<b>0.7</b>	<b>3.8</b>	<b>FALSE</b>
<b>Const + Trend</b>	<b>10.1</b>	<b>3.8</b>	<b>TRUE</b>

Table 4.10: Test for Co-integration - Maximum Eigenvalues test

In the maximum eigenvalues test, the check is to make sure that the number of linear combinations are not equal to the number of input variables since if they are equal, then this would mean that the variables are stationary to begin with and the test for co-integration would be irrelevant. Under both 'No Constant' and 'Constant Only' assumption, the test that the linear combinations are not equal to two are 'FALSE'. It is only in one scenario that this passes. In this case,  $r$  which is the number of co-integrating relationships, is not two. Under the assumption 'Constant and Trend' the test is 'True'.

In conclusion, the exchange rates  $KES/USD$  and the trade volumes in  $USD$  are co-integrated. This means that in the long-run the two variables i.e. Trade volumes and  $KES/USD$  exchange rate are related.

## 5 Conclusion

Inter-country trade is inevitable in the current times, most countries can't sustain their demand for goods and services and hence the issue of foreign exchange and exchange rates becomes very important. Both the importing and the exporting countries have an interest in the volatility of exchange rates. Models that can forecast the exchange rates and their volatility would be very beneficial to the two groups.

Due to the principle of parsimony the models tested are those with low specifications of *GARCH* models as there is evidence in previous studies that these models can sufficiently capture the traits of exchange rates data.

Three models were fitted for the KES/USD exchange rate, *GARCH*(1,1), *GARCH*(1,2) and *GARCH*(2,2) using the conditional distribution of skewed student-t distribution. Using the AIC criterion the model that best fits the data for KES/USD is *GARCH*(1,2) with a skewed student-t distribution. The same three models are tested for KES/EUR exchange rate under the same assumptions, using the AIC criterion, *GARCH*(2,2) best fits the data. Similarly, for the KES/GBP data the same three models were tested and the same AIC criterion used to arrive at the best fitting model which in this case is *GARCH*(1,2). Data for KES/USD and UGX/KES was tested for the three models and for the two data sets, *GARCH*(2,2) fits both data sets.

There are suggestions among economists that the volume of trade in a specific currency is related to the exchange rates in that currency. Relationship between the two would mean that the volume of trade in a currency is affected by either the weakening or strengthening of the currency. Understanding whether there is co-integration between the *KES/USD* exchange rate and the volume of trade done in *USD*.

The data on *KES/USD* exchange rates is non-stationary as shown by the volatility plots. Using Dickey Fuller test for unit roots, data on trade volumes is tested for stationarity at first difference under the three assumptions, Intercept only, Trend and intercept and No trend no Intercept. Under the three assumptions, the data at first difference is found to be stationary and hence the test on co-integration is given a green light.

Using Johansen test for co-integration, the trace test and under the three different assumptions for the input variable, the question of whether there is at least one linear

combination of the inputs that yields a stationary process is answered and clearly under the assumption of Intercept and trend we get a stationary process. We can hence conclude that the variables are co-integrated. This means that in the long run the trade volumes as denominated in USD have a co-integrating relationship and hence non-standard statistical properties which would require non-conventional statistical analysis like stochastic trend analysis.

## 5.1 Recommendation

1. Testing for co-integration between the exchange rates and Foreign Direct Investments(FDI's)
2. Modelling trade volume data using stochastic analysis

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