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PENTANOMIAL LATTICE MODELS IN OPTION PRICING

Research Report in Mathematics, Number XX, 2018

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December 2018

Submitted to the School of Mathematics in partial fulfilment for a degree in Master of Science in Acturial Mathematics

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Master of Science Project

Submitted to the School of Mathematics in partial fulfilment for a degree in Master of Science in Acturial Mathematics

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Abstract

Options are derivatives which is an agreement linking two persons or more of vested interest whose worth is based on an agreed-upon underlying financial asset. An agreement defines the buyer the right, but not the commitment, to purchase or dispose the specified asset at a discussed price during a determined period of time on a specified later date. Asian option is an option which depends on the past knowledge whose payment is based on the mean-price during a secure duration of time before it matures. How much to spend on option contract is the main problem at the task in pricing options. This becomes more complex when it comes to the case of projecting the future possible price of the option. This is attainable if the probabilities of princes swelling are known, remaining the same or lessening. Each investor's wishing to maximize profit.

This proposal looks into pentanomial lattice model used in pricing Asian Option models. A Lattice representation is a discontinuous time presentation of evolution of the underlying asset price. The model also takes into account the Kurtosis and skewness of the underlying asset. It splits a certain time interval into n equal strides. The lattice is constructed using positive branch probabilities and takes into account the matching procedures the limiting distribution of lattice model is called compound Poisson process. The lattice model is used to price options more efficiently and easily. It estimates the spread of the underlying asset cost each time step.

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Declaration and Approval

This research project is my original work and has not been submitted or presented for a degree or diploma for examination in any other University

Signature Date

MISATI ONDARI EDWIN
Reg No. I56/74470/2014

This research project has been submitted for examination with our approval as University supervisor

Signature Date

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Dedication

I dedicate this project to my Dad Joseph and Mum Ann and my siblings for love and encouragement

Contents

Acknowledgments

I thank Almighty God for giving me good health, strength and inspiration throughout my studies. I also thank my supervisor Dr.Philip Ngare for the proper guidance and support.

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Misati Ondari Edwin

Nairobi, 2018.

1 INTRODUCTION

1.1 The framework of the study

In the last 30 plus years, derivatives such as options became extremely important in the world of finance. The idea of an option contract can be drawn back to before 1973. The option contracts were seen as Over-the-counter (OTC). (Wilmott,1995). This means that option trading had an intermediary or a broker, which is the option broker. This option broker is the person who negotiates the price of the option between the seller and the buyer every time an option was to be bought or sold. These option contracts were not handled properly since the contracts were not standardized in expressions of its conditions.

The OTC could manage to handle it because few companies were involved. Later in 1973 official exchange begun when the modern financial option market came into the market and quickly replace OTC. In the same 1973,Black Scholes model was formulated to price options.

Options are derivatives which is commitment linking two or more person of vested interest whose worth is found on an discussed-upon underlying financial asset sold by an option seller to an option buyer. In return for granting the option, the buyer pays an amount of money to the seller that is called the premium or option price. Options can be classified on the underlying of exercise dates i.e. on the underlying of when the option is exercised i.e. American & European option. European options can be utilized only on the lapse date, while American options can be utilized at any time prior to the lapse date.

Also, an option may classify on the basis of if one has to purchase or dispose an underlying asset. One is a call option that offers the buyer the justified reason to purchase the asset. The other is a put option that provides with the buyer the precise reason to dispose the financial belonging.

In the option price theory, a breakthrough was made in 1973, when Black & Scholes (1973) gifted the first acceptable equilibrium option pricing model based on the risk neutral arbitrage argument. It is referred as Black-Scholes Model. Robert extended their representation for many subsequent studies (e.g., Merton, 1973). Black-Scholes representation has an assumption that the representation is the logarithmic asset cost should come after a Brownian Motion with a mean. It makes it Unfortunately, the Black-Scholes Model can be used to cost easy or simple options such as European options; it cannot be used to value more sophisticated options such as American options. Moreover, as the model provides a closed-form solution to a partial differential equation, it limits its application when a PDE cannot be derived for a particular option contract.

Therefore, different alternative techniques have been proposed for pricing the options among many option pricing techniques, one well-known method is the lattice approach. Cox (1979) have come up with a binomial lattice approach using the fundamental economic principles of option valuation which is no arbitrage arguments. Lattices for option pricing were pioneered in 1979 in the originally work of Cox (1979) and Rendleman & Bartter (1979). Particularly, Cox employed a binomial model of lattice to price Brownian motion and comprise the Poisson process. The only positive characteristic of their representation is that the binomial lattice for Brownian motion is very compatible with the Black–Scholes formula for European options.

Since that pioneer effort, lattices have become a level and major equipment in pricing of options, commonly for pricing American options. Because of coherence and versatility of representations, a number of appendages have been explored in regards to the basic model of pricing options. Lattices for pricing of options have raised or developed for more than a single underlying asset, and in regards to more than single or more intricate models of a single underlying asset Yamada & Primbs (2001), (Rubinstein (1998), Amin (1993) and Boyle, (1988).

In distinct, lattice models have been employed tremendously to remove implied volatility surfaces as in the gifted works of Rubinstein (1994), Derman & Kani (1994), Dupire (1994). In addition to that, lattice models have been deemed as the only procedure that captures Kurtosis and skewness of the underlying asset. To be more specific, Rubinstein (1994) suggested an inclusive lattice model that integrates Kurtosis and skewness by using advancement of central limit theorem that controls errors.

When models of such have been suggested, they are lesser in counting than representations that have been suggested for European pricing of options under kurtosis and skeweness. In this concern sector, researchers have initiated exponential Levy process (Carr & Madan,(1999) Chan, (1999).

The binomial lattice approach, also known as CRR model, can value a wide range of options if return of the underlying asset ascends to a normal distribution. The model has been developed by matching the two moments (mean and variance) of a discrete random variable over small time interval with those of a continuous random variable. It is the thinking of investors or individuals operating within the context of financial markets to hold these factors constant. After this flourishing attempt, many multinomial lattice methods have been proposed that can be used to value more complex options on several underlying variables. Boyle (1988) has extended the CRR binomial lattice to a trinomial lattice for a single underlying variable.

Trinomial model has been used in few options like American option, European option and also in exotic options like lookback, barrier and Asian option. Among these exotic options, we have the Russian option which is rarely in the stock market. Trinomial tree model is just an extension of Binomial model from having two branches of either price of a share going up or decreasing to having three branches of either price of the stock increases, remaining the same or decreases. When a trinomial tree model is recombined, we get a trinomial lattice. This means that all nodes that end up with the same prices at the same time will be taken to be one node. In this lattice every node has three possible paths to follow, going up, going down or remaining stable. This is out of the results of multiplying the stock price at the node by either of the three factors.These are ratios whereby is greater than one, is less than one and is equal to one. Apart from these ratios, we have the risk-neutral probabilities which tell us the chance that a price has to increase, decrease or remain the same.

Bhat & Kumar (2012) advanced a Markov tree model for option costing utilizing a non-iid process which is a an alteration of binomial model of option pricing that take into account Markov behavior of first order. Next Markov tree model and wind up that the mixture of two normal distribution.

The general goal of this study is to find optimal price of the option contract at the beginning of the contract using the pentanomial lattice method.

1.3.2 A Specific Goals of Research

- **i** To estimate the transition probabilities.
- **ii** To calculate the option prices at different nodes.
- **iii** To compare the pentanomial method prices with trinomial method and Black-Scholes method prices.

2 LITERATURE REVIEW

The below chapter illustrates a brief preface to the primary concepts of the option and some models and methods for the option pricing. An option is ideally a concert linking a purchaser and a vendor that permits its buyer a right, but not an obligation, to buy or sell an underlying asset at a particular cost on or before a specified date. In replacement for granting the option, the seller collects an amount of money from the buyer that is referred the option premium or option price. There are different techniques available for option pricing. The most commonly used techniques are the Black-Scholes model, Monte Carlo simulation technique, limited difference methods and the lattice approaches.

2.1 Literature Review more about options

An option is a financial tool that offers a right to the holder, but not a responsibility, to be engaged in a future transaction of the underlying asset at price specified at any time on or prior to a given occasion (Hull, 2006). The cost which is specified is also referred as the strike cost and implied date is referred the lapse date or ripening date. Based on when an option can be exercised, there are two major types of options: European & American options.

European options can be performed only on the lapse date, But American options can be exercised at any time until the ripening date. Options can also be categorized based on the justifiable reason to buy or sell an asset, such as a call option and a put option. A call option is an option to purchase asset at a consented cost on or prior to specific date, while a put option offers the buyer the justifiable reason to dispose an asset.

The costing of options employing lattice models was pioneered in 1979 in the indigenous effort of Cox.(1979) and Bartter (1979).Suggestively, Cox deployed a binomial lattice to represent Brownian motion in the option pricing incorporation the Poisson process. In binomial lattice model, they use geometric Brownian motion which is very stable with the standard Black-Scholes formula for European options. The concept of mismatch does not occur when pricing European Options compared to pricing American options where the early exercise rate is not favourable. Boyle (1977) came up with the Trinomial pricing model. This was modified by Boyle, Boyle and Lau (1994), Kimrad and Ritchen (1995) and showed their solutions. They suggest that there would be three varying prices of the underlying asset at a certain time step which are either price moving up, remaining the same or moving down. Here they prove that trinomial tree model is more realistic than binomial model. This makes trinomial model better option for pricing options since it is more accurate in solution and at the same time converge faster than binomial hence making trinomial tree model widely used in pricing different types of options. Han put on effort on a three branch tree representation(trinomial) for costing options on particular cases in numerical methods and did contrast with the binomial model and got that three branched tree representation.

Xiong (2012) developed a three branch tree model(trinomial) for costing options which was constructed on Markov chain Monte Carlo procedure and employed to compare and contrast to Black-Scholes model, binomial tree model, trinomial tree model and the warrant cost utilizing the actual facts from warrant market. Yang and Yuen (2010) came up with a tree representation to cost simple and exotic options in Markov Because of the simplicity of lattice models, a lot of expansions and additions have been advanced in relation to the original model.. Models of Lattices have been improvised to single underlying asset, In addition to that, lattice representations have been deployed widely to get the suggested volatility like the early efforts of Rubinstein (1994). Derman & Kani (1994). In addition to that, lattice representations have been suggested as a procedure of expressing skewness & kurtosis in the underlying asset. Notably. Rubinstein (1994) suggested a representation of lattice that absorbs skewness & kurtosis. While such representations have been suggested, they are lesser in number than models that have been proposed for European optioncosting under skewness and kurtosis. Researchers have proposed in that sector exponential Levy process representations- (Carr & Madan(1994), Chan (1999).

This paper/project addresses the idea of including kurtosis and skewness involving moment matching procedures in modeling the lattice, and traverse the pricing outcomes of the above mentioned model. It also prices Asian by gaining control over the first four moments and the lattice model is capable to equate kurtosis and skewness, and thus can come up with smirks and volatility smiles. When it comes to wide range of kurtosis and skewness values, branch probabilities are provided with positive conditions. Possible limits of the model in continuous time are also analyzed. The model is characterized, featured and its limitations are also looked into. This leads to probability-distributions which are employed to cost European options in a way which is in accordance with the assumptions of made in regard to lattice. Fourier transforms methods (Carr and Madan. (1999) are employed to effectively cost European options deploying the limiting disseminations. Formulas can also be derived. Notably from an easy to understand lattice model, one can develop a complete approach to price American and Asian options.

The paper continuous as follows. In the following section we come up with a basic lattice model, binomial model and its probabilities, Trinomial lattice model and its probabilities, Quadranomial lattice model and then the five branch lattice model for positivity of probabilities conditions. A limiting distribution is also computed.

3 RESEARCH METHODOLOGY

3.1 Model

Pentanomial tree method is more like trinomial tree with slight difference. From each node the price of the underlying stock (share) branch into three new prices, that is either being more than the previous price, being less than the previous price or remaining the as the previous price. This majorly depend on the ratios we use to multiply our current price and at the same time the probabilities that we consider.

3.2 Assumptions

i Time steps are all equal.

ii The interest rate used is the risk free rates.

iii Probabilities remain the same throughout

The Lattice Model
The form of exponential Levy process model takes the form below

$S_t = S_0 e^{X_t}$

Which is arrived by developing a lattice model that equates to X, The exponential model gives a lattice model for S, Before creating problem-generating for X. we begin with trying to come up with a composition of coming up with a non-continuous random variable that equates to a set of moments described.

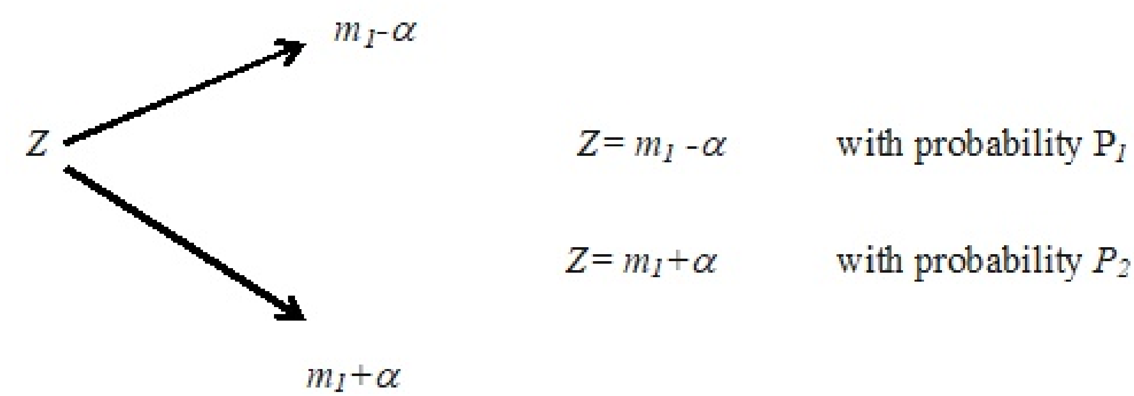
To set up, moments of random variable X are equated with a discrete r.v. Z. Let **Z** denote a non-continuous r.v.

$Z = m_1 + (2l - L - 1)\alpha$ *with probability* p_1, *i= 1,2,....L*

where
A = (distance between two outcome)-Jump size.
m_1= average of X.
L= lattice nodes in numbers.
α = positive real number.

3.3 Binomial-Lattice

When the branches are two i.e. L=2, we have a two dimensional lattice. Hence



Matching the equations of Z to the first two central moments of X the below is yielded

$$(-\alpha)P_1 + (\alpha)P_2 = \mu 1$$

$$(-\alpha)^2 P_1 + (\alpha)^2 P_2 = \mu 2$$

and

$$P_1 + P_2 = 1$$

In matrix form we have

$$\begin{bmatrix} 1 & 1 \\ -\alpha & \alpha \\ (-\alpha)^2 & (\alpha)^2 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} 1 \\ \mu_1 \\ \mu_2 \end{bmatrix} \qquad (1)$$

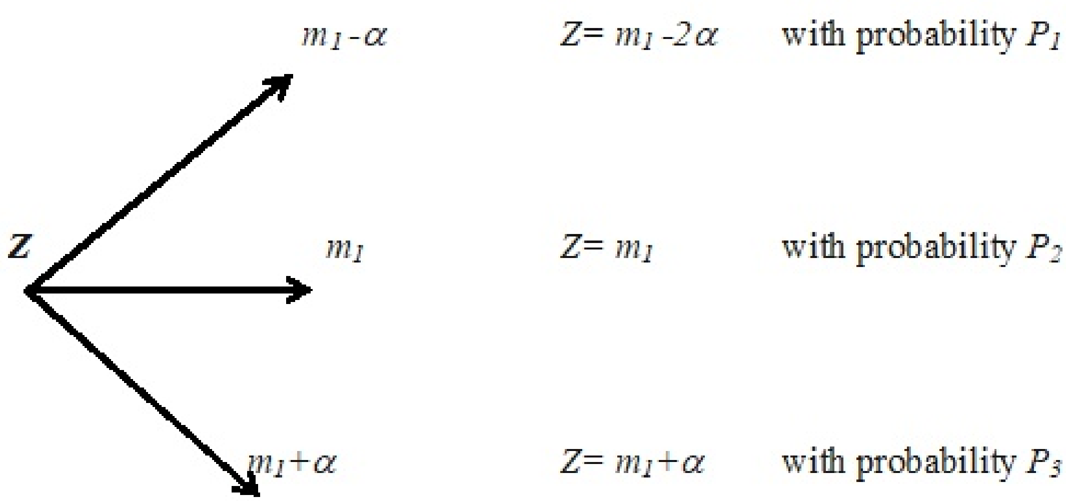
The above is simplified to give $\alpha = \sqrt{\mu_2}$ and

$P_1 = (1 - \frac{\mu_1}{\sqrt{\mu_2}})$

$P_2 = (1 + \frac{\mu_1}{\sqrt{\mu_2}})$

3.4 Trinomial- Lattice

When the branches are three i.e. L= 3 we have the trinomial lattice. Therefore



Matching the first three moments we have

$(-2\alpha)P_1 + 0P_2 + (2\alpha)P_3 = \mu_1,$

$(-2\alpha)^2 P_1 + 0^2 P_2 + (2\alpha)^2 P_3 = \mu_2,$

$(-2\alpha)^3 P_1 + 0^2 P_2 + (2\alpha)^3 P_3 = \mu_3.$

And
$P_1 + P_2 + P_3 = 1$

Contrasting the above methodology with (20.10). In matrix form we get

$$\begin{bmatrix} 1 & 1 & 1 \\ (-2\alpha) & 0 & (2\alpha) \\ (-2\alpha)^2 & 0 & (2\alpha)^2 \\ (-2\alpha)^3 & 0 & (2\alpha)^3 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} 1 \\ \mu_1 \\ \mu_2 \\ \mu_3 \end{bmatrix} \quad (2)$$

Deducing the equations yields to

$$\alpha = \frac{1}{2}\sqrt{\frac{\mu_3}{\mu_1}} \quad (3)$$

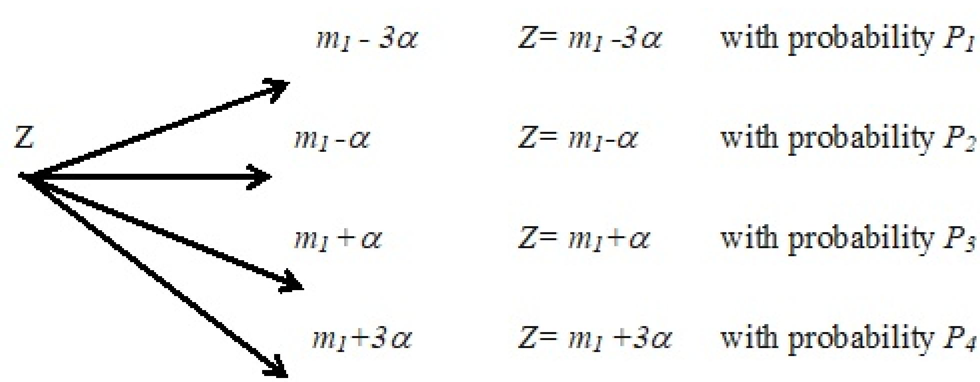
$$P_1 = \frac{1}{2}\left(\frac{\mu_1\mu_2}{\mu_3} - \sqrt{\frac{\mu_1^3}{\mu_3}}\right) \quad (4)$$

$$P_2 = 1 - \frac{\mu_1\mu_2}{\mu_3} \quad (5)$$

$$P_3 = \frac{1}{2}\left(\frac{\mu_1\mu_2}{\mu_3} + \sqrt{\frac{\mu_1^3}{\mu_3}}\right) \quad (6)$$

3.5 The Quadrinomial-Lattice

When the branches are four i.e. L = 4 we have a quadrinomial lattice. Therefore



Matching the first L moments we have

$(-3\alpha)P_1 + (-\alpha)P_2 + (\alpha)P_3 + (3\alpha)P_4 = \mu_1$

$(-3\alpha)^2 P_1 + (-\alpha)^2 P_2 + (\alpha)^2 P_3 + (3\alpha)^2 P_4 = \mu_2$

$(-3\alpha)^3 P_1 + (-\alpha)^3 P_2 + (\alpha)^3 P_3 + (3\alpha)^3 P_4 = \mu_3$

$(-3\alpha)^4 P_1 + (-\alpha)^4 P_2 + (\alpha)^4 P_3 + (3\alpha)^4 P_4 = \mu_4$

and
$P_1 + P_2 + P_3 P_4 = 1$

In matrix form we have.

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ (-3\alpha) & (-\alpha) & (\alpha) & (3\alpha) \\ (-3\alpha)^2 & (-\alpha)^2 & (\alpha)^2 & (3\alpha)^2 \\ (-3\alpha)^3 & (-\alpha)^3 & (\alpha)^3 & (3\alpha)^3 \\ (-3\alpha)^4 & (-\alpha)^4 & (\alpha)^4 & (3\alpha)^4 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix} = \begin{bmatrix} 1 \\ \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{bmatrix} \tag{7}$$

Ignoring the last row of the matrix , it is easier to find expressions for the probabilities as functions of α
$\mathrm{P}_1 = \frac{1}{16}(-1\frac{\mu_1}{3\alpha} + \frac{\mu_2}{\alpha^2} - \frac{\mu_3}{3\alpha^3}),$

$P_2 = \frac{1}{16}(3 - \frac{3\mu_1}{3\alpha} + \frac{\mu_2}{3\alpha^2} + \frac{\mu_3}{3\alpha^3}),$

$P_3 = \frac{1}{16}(3 + \frac{3\mu_1}{\alpha} - \frac{\mu_2}{\alpha^2} - \frac{\mu_3}{3\alpha^3}).$

$P_4 = \frac{1}{16}(-1\frac{\mu_1}{3\alpha} + \frac{\mu_2}{\alpha^2} + \frac{\mu_3}{3\alpha^3}),$

To achieve equality from the last row so that it can be true, the following condition must be satisfied by α

$$\mu_4 = 81\alpha^4 P_1 + \alpha^4 P_2 + \alpha^4 P_3 + 81\alpha^4 P_4$$
$$= -9\alpha^4 + 10\mu_2\alpha^2 \tag{8}$$

Determining the above equation of this expressions for α yields to four- roots

$\alpha_1 = \frac{1}{3}(5\mu_2 + \sqrt{25\mu_2^2} - 9\mu_4)$

$\alpha_2 = \frac{1}{3}(5\mu_2 + \sqrt{25\mu_2^2} - 9\mu_4)$

$\alpha_3 = \frac{1}{3}\sqrt{(5\mu_2 - \sqrt{\sqrt{25\mu_2^2} - 9\mu_4)}}$

$\alpha_3 = -\frac{1}{3}\sqrt{(5\mu_2 - \sqrt{\sqrt{25\mu_2^2} - 9\mu_4)}}$

We can make another access through finding $lpha$ by ignoring first row. This yields to a set of different probability equations:

$P_1 = \frac{1}{16}\big(\frac{3\mu_1}{3\alpha} - \frac{\mu_2}{9\alpha^2} - \frac{\mu_3}{3\alpha^3} + \frac{u_4}{9\alpha^4}\big)$

$P_2 = \frac{1}{16}\big(-\frac{9\mu_1}{\alpha} + \frac{9\mu_2}{\alpha^2} + \frac{\mu_3}{\alpha^3} - \frac{\mu_4}{\alpha^4}\big)$

$P_3 = \frac{1}{16}\big(\frac{9\mu_1}{\alpha} + \frac{9\mu_2}{\alpha^2} - \frac{\mu_3}{\alpha^3} - \frac{\mu_4}{\alpha^4}\big)$

$P_4 = \frac{1}{16}\big(\frac{\mu_1}{3\alpha} + \frac{\mu_2}{9\alpha^2} + \frac{\mu_3}{3\alpha^3} - \frac{\mu_4}{9\alpha^4}\big)$

The imposed condition is given by in the first row as

$$I = p_1 + p_2 + p_3 + p_4$$
$$= \frac{10}{9}\frac{\mu_2}{\alpha_2} - \frac{\mu_4}{9\alpha^4} \tag{9}$$

This can be taken as:

$\alpha^4 - \frac{10}{9}\mu_2\alpha^2 + \frac{1}{9}\mu_4 = 0,$

Which gives the same solution for αa**3**(**6**.8)*.Pentanomiallatticemodel*

3.6.1 A Non-continuous Moment-generating r.v.

Firstly, we come up with matching moments set up for r.v. X that has a discontinuous random variable Z. Lets take into account a r.v X. Let M_J indicate its kth raw-moment, its kth central moment, and $c3$ its kth row. A non-continous r.v Z is constructed for random variable X that matches its moments.

Let Z be a variable that is not continuous as indicated below

$Z = m_1 + (2l - L - l)x$, $l = 1,.,L$ that has probability density functions pi
Where x is a framework and m_1 is the mean of X.

Therefore, Z is a random variable which is non-continuous that may take on L range.

Theorem
Moment of Equations of Z

The moments of Z must match with the moments of X. X moments are considered and the below identifications should be kept constant:

$$\sum_{l=1}^{L}((2l - L - 1)X)jpl = \mu_j \tag{10}$$

Definitions: μ_j refers to the $_jth$ central moments of X and μ_1=0.

Matching the four moments of the Model.

To continue further, four moments that match are taken into examination or into account, and let L=5. The lattice has five branches. Four moments are taken into consideration to faced problems in finance. Because of this, it is ideal to take into account kurtosis and skewness of the yield-distributions of the asset in consideration which incorporates the ideas of the first four moments. In regards to four moments, quadranomial lattice , is applied or used in solving the pricing problem, but the condition of recombining and also taking into account the condition requirement of positive probabilities brings about problems in regards to the range of kurtosis and skewness which is taken into account or captured. A pentanomial lattice accepts a more accommodative character and information with less problem or complexity which is our main goal.

Solving (1) for pl, with $l = 1,.,5$ yields.

$$Z_5 = \begin{cases} m_1 - 4xp_1 = \frac{(\mu_4 - 4x^2\mu_2 - 4x\mu_3)}{384x^4} \\ m_1 - 2xp_2 = \frac{(-\mu_4 + 16x^2\mu_2 - 2x\mu_3)}{96x^4} \\ m_1 p_3 = 1 + \frac{(-20x^2\mu_2 + \mu_4)}{64x^4}(2) \\ m_1 + 2xp_4 = \frac{-2x\mu_3 - \mu_4 + 16x^2\mu_2}{96x_4} \\ m_1 + 4xp_5 = \frac{(\mu_4 - 4x^2\mu_2 + 4\mu_3 x)}{384x^4} \end{cases}$$

Therefore, an assumption is made that $x > 0$. A question arises. For what are the values of x which have positive probabilities?(probability density functions).

The below theorem addresses the inquisition.

Theorem 1: Given that ${}_2\mu_4 \geq 3\mu_3^2$ and $25\mu_2^2 \geq 16\mu_4$ (or equal to $k \geq 3s^2 - 3$ and $k \geq 3s^2$ - $\frac{23}{16}$ where s =$\frac{\mu_3}{\mu_2^{3/2}}$ refers to skewness and $K = \frac{\mu_4}{\mu_2^2} - 3$ is kurtosis),there prevails an area of values of x derived by

$$\frac{1}{16\mu_2}(3 + (\mu_3^2 + 16\mu_2\mu_4)\frac{1}{2}) \leq x \leq \frac{1}{4\mu_2}(-2\mu_{3+2}(\mu_3^2 + \mu_2\mu_4)^{\frac{1}{2}}) \quad \textbf{(11)}$$

Which exclusively includes

$$x = \sqrt{\frac{\mu_4}{12\mu_2}} = \sigma \sqrt{\frac{3+k}{12}}$$

for which all the probabilities $P_t.1 = 1, , ..., 5$ are not negative.

Theorem 1 gives us with a strong case for positivity of the probabilities. Furthermore, it will be derived that α will yield in non-negative probabilities for figures of μ_2, μ_3, and μ_4, clarifying that $2\mu_4 \geq 3\mu_3^2$ and $3\mu_2^2 \geq 2\mu_4$ (or equal to $k \geq 3s^2 - 3$ and $k \geq -\frac{3}{2}$). Which are more

accommodative than the other conditions However, (3.11) does not keep up with these more conditions that are all inclusive. For values which are more of kurtosis less than - $\frac{23}{16}$,the positivity conditions are more composite. These conditions are not included as p excess positive kurtosis $(K > 0)$ is a great area of concern in finance or unexplored area in finance.

The tails are heavier and higher picks. The proposition describes/determines a variation of kurtosis and skewness figures that are adaptable with a pentanomial. It determines a large area of Kurtosis and skewness and that much compatible with a lattice-model. The restrictions on the spacing between results of **Z** which is the r.v inclined by the described parameter. Ultimately, a simple formula for that spacing is described or introduced. Consequently, this simple proposition gives the basis of modeling of a lattice model, and also governs it characteristics and disadvantages.

One can up with resembling research as in theorem 1 other than for more than one-branch lattice which employ more than five branches. Although, unexpectedly five-branches, the logic used to come up with the conditions in theorem one is very long and tiresome, and with eve more extra branches, it would lead to a lot of work. When only kurtosis and skewness are seized, the pentanomial lattice is an exact model to grasp most of the ranges of interest of parameters, and allows for easier understandable characteristic. Pentanomial captures both complexity and practicability.

How to create a lattice model: We deduce how to equate the moments of a r.v. X with a non-continous r.v. **Z**. To reduce this to a lattice- model, It's believed that A'1 is a Levy process. For any time-given **t** the obtained answers from the existing part shows how to equate the moments of X with a non-continuous r.v **Z(t)**. Since A'1 is a Levy process. Its cumulants scale is in line with time and accordingly we define its cumulants at any time t by defining its yearly- cumulants. That is, let C_j be the $_j$th cumulants of X_1, then the $_j$th cumulant of X_1, is C_jt.

Let τ be an increment in t that provide us with the step size of the lattice. To come up with lattice model, we come up with each increment X_π with the discrete r.v $Z(\tau)$ that equals its moments. This leads to the below model which explains the lattice.

3.6.2 Lattice Model

Let s_0 be the cost of the mentioned underlying asset. Then a lattice model estimating $S = S_{oe}X_t$ is given by $Sn,(\pi)$=$s_0 exp(\sum_{k=1}^{n} z^{k(t)})$

Given n refers to the count of time step size **t** and the $Z_K(t)$ are iid r.v distributed as

$$Z(\tau) = \begin{cases} C_1\tau - 4xp_1(\tau) = \frac{((c_4\tau+3c_2^2(\tau)^2)-4x^2c_2\tau-4xc_3\tau)}{384x^4} \\ C_1\tau - 2xp_2(\tau) = \frac{(-(c_4\tau+3c_2^2(\tau)^2))+16x^2c_2\tau+2xc_3\tau}{96x^4} \\ C_1\tau p_3(\tau) = 1+\frac{(-20x^2c_2\tau+(c_4\tau+3c_2^2(\tau)^2))}{64x^4}(5) \\ C_1\tau+2xp_4(\tau) = \frac{-2xc_3\tau(c_4\tau+3c_2^2(\tau)^2)+16x^2c_2\tau}{96x^4} \\ c_1\tau+4xp_5(\tau) = \frac{((c_4\tau+3c_2^2(\tau)^2)-4x^2c_2\tau+4xc_3\tau)}{384x^4} \end{cases}$$

The diagram above describes one-step of distance τ of the lattice model at an ith time. Because the lattice model mentioned is given in relation to cumulants of X_t, positivity-condition of theorem 1 in relations to cumulants is also rephrased.

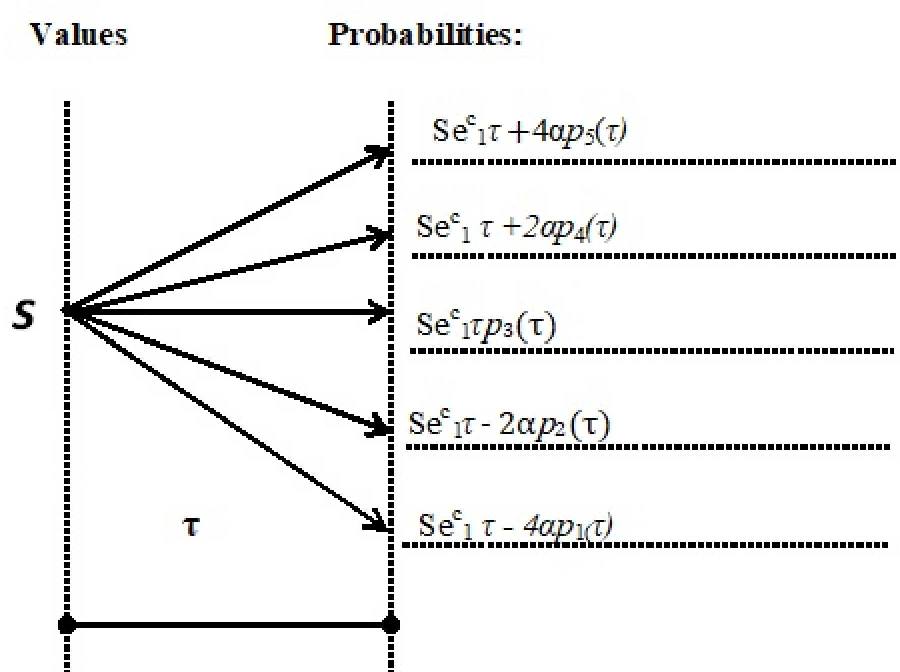


Figure 1. Pentanomial lattice model with one step

*Proposition 2.*Provided

$$c_4c_2 \geq 3c_3^2 - 3c_3^2\tau and c_4 \geq -\frac{23}{16}c_2^2\tau \tag{12}$$

there occurs a wide magnitude of α values of deduced to;

$$\frac{1}{16c_2\tau}(c_3\tau + (c_3^2\tau_2 + 16c_2\tau(c_4\tau + 3c_2^2\tau^2))^{1/2}) \leq x \leq \frac{1}{4c_2\tau}(-2c_3\tau + 2(c_3^2\tau^2 + c_2\tau(c_4\tau + 3c_2^2\tau^2))^{1/2})\ldots(7)$$

This also includes

$$\propto = \frac{1}{2}\sqrt{c_2\tau} + \frac{c_4}{3c_2} \tag{13}$$

for which the probabilities i $p_l, 1 = 1.....5$. are positive.

Comparing the results in provision with the cumulants of the process X. this explains why the circumstances scale in relation to the time-step τ. Particularly when the limit as $\tau \rightarrow 0$ which is our area of concern.

3.7 Dissadvantages of the Lattice-Model.

In the below segment, consideration is taken into account when the restrictions of the lattice model are in continuous time as $\tau \to 0$. For this valid reason, we make an assumption that the third and fourth cumulants $C_3$and C_4 are real positive numbers. If the cumulants are zero then it is deduced that the lattice will merge to a geometric Brownian motion because of the positivity condition.

In regards to discreet model to have continuous time limit, the condition of positivity should be practicable or seen as $\tau \to 0$. In the limit, (6) deduces to

$$c_4 c_2 \geq 3c_3^2 and c_4 \geq 0 \tag{14}$$

Hence, more assumptions are made that (3.14)holds. Note that the
requirement of $c_4 \geq O$ is equivalent to the expectation of positive excess of kurtosis.
We wish to have this lattice to contain an explained maximum as the size-step reaches to zero. Consequently, we deduce an assumption that has a limit as $\tau \to 0$. Let us assume that the limit is given by
$\alpha = lim_{t \to 0} \alpha$ where it is within a specified spread described.
For continuation purposes, we introduce the new probabilities (q_1, q_2, q_4, q_5) to refer to the branch probabilities as below:

$$\begin{cases} lim_{\tau} \to 0(1/\tau p_1(\tau)) = \frac{-4x_0^2 c_2 - 4x_0 c_3 + c_4}{384 x_0^4} = \lambda q_1 \\ lim_{\tau} \to 0(1/\tau p_2(\tau)) = \frac{16x_0^2 c_2 - 2x_0 c_3 - c_4}{96 x_0^4} = \lambda q_2 \\ lim_{\tau} \to 0(1/\tau p_3(\tau)) - 1 = \frac{-20x_0^2 c_2 + c_4}{64 x_0^4} = -\lambda \\ lim_{\tau} \to 0(1/\tau p_4(\tau)) = \frac{16x_0^2 c_2 - 2x_0 c_3 - c_4}{96 x_0^4} = \lambda q_4 \\ lim_{\tau} \to 0(1/\tau p_5(\tau)) = \frac{-4x_0^2 c_2 + 4x_0 c_3 + c_4}{384 x_0^4} = \lambda q_5 \end{cases}$$

Therefore,

$$q_1 + q_2 + q_4 + q_5 = 1$$

If we use α to simplify to

$$\begin{cases} \lambda = \frac{3x_2^2}{2C_4} \\ q_1 = \frac{1}{6}(1 - c_3\sqrt{\frac{3}{c_2 c_4}}) \\ q_2 = \frac{1}{3}(1 + c_3\sqrt{\frac{3}{c_2 c_4}}) \\ q_4 = \frac{1}{3}(1 - c_3\sqrt{\frac{3}{c_2 c_4}}) \\ q_5 = \frac{1}{6}(1 + c_3\sqrt{\frac{3}{c_2 c_4}}) \end{cases} \tag{15}$$

In the limit ,$Z(\tau)$ approaches the increase of a compound Poisson process described as:

$C_1 t + \sum_{k=0}^{N_t} W_k$

N_t=Poisson process with force λ and the W_k which are iid r.v with the below given dispersal

$$W_k = \begin{cases} -4x_0 \text{ with probability } q_1 \\ -4x_0 \text{ with probability } q_2 \\ 2x_0 \text{ with probability } q_4 \\ 4x_0 \text{ with probability } q_5 \end{cases} \tag{16}$$

*Theorem 3.*Reflect a permanent range given by; $[0,T]$ of time where the count of steps n is expanding. Then the size of the steps,$\tau = \frac{T}{n}$. Our area of interest is in the r.v at time T described below as:

$X_n = \sum_{k=1}^{n} Z_k(\tau)$

Implying as $n - \infty$, X_n,yields to the below distribution

$$C_1 T + \sum_{k=0}^{N_T} W_k \tag{17}$$

Given that N_T is Poisson distribution with an average of λT and the W_k, which are iid r.v described in (3.16).

The above hypothesis gives us with the dispersion involved in pricing European options that is compatible with the lattice model. Secondly, we use or put into practice Fourier transform methods as in Carr & Madan (1999) to accurately come up with the costing of European options. Note that the characteristic function of the distribution must be known. For the r.v described by (12), the attribute result of Fourier transform (Breiman.

1992) is deduced to:

$$\phi_T(\mu) = e^{i\mu c_i T} exp\Big(\lambda T \sum_{i\varepsilon(1,2,3,4,5)} q_1(e^{i\mu(2l-6)x} - 1)\Big) \quad \textbf{(18)}$$

The above will be applied when it comes to evaluating European in the below sections.

Applying the Fourier Transform to price European Options(Calls & Puts)

In this sample. We employ or refer to the pioneer assignment of Carr & Madan (1999) on costing of options employing the Fourier transform. When risk neutral probabilities are known for Fourier Transform, the approach becomes useful. Our case scenario reverses slightly from Can and Madan (1999) since we utilize non continuous spread and therefore use the non-continuous Fourier transform.

Let $q_T(n)$ be the discrete risk neutral probability distribution of the r.v described in (12) above. Employing the limiting distribution in (12) as the risk neutral probabilities, call option amount is simply deduced as a allowable expectation of the payoff which is stated below:

$$CT(k,K) = e^{-rt} \sum_{n=k}^{\infty} (e^{c_1\tau + 2x_0^n} - K) q_\tau(n) \quad \textbf{(19)}$$

where $k > (\frac{ln(\frac{K}{s}) - C_{1T}}{2x_0})$

As said earlier, the approach made is uniform to Carr & Madan(1994), then European Call option pricing is given below.

Fourier Trans form pricing Equation & **Formula**
A cost of a European call option that does not pay money from its
dividends on the repressed holding represented as $S_{0e}X_t$ with the original cost S_0, the dissemination of X_t at lapse of time T of (12), and selling cost **K** is given or derived as

$C_\tau(k,K) = \frac{e^{-\beta\kappa}}{2\pi} \int_{-\Pi}^{\Pi} \Psi(\mu,K) e^{-i\mu k} d\mu \ldots (3.15)$
where $\beta > 0$ is a specification to describe Fourier Transform.

$\Psi(\mu,K) e^{-rT} (\frac{1}{1-e^{-(\beta+i\mu)}})[s_0 e^{c_1\tau} \phi(-i(\frac{\beta+2\alpha_0+i\mu}{2\alpha_0}) - K\phi(-i(\frac{\beta+i\mu}{2\alpha_0})] \ldots (3.16)$

And

$\phi(\mu) = E[e^{i\mu X}] = \sum_{n=-\infty}^{\infty} e^{i\mu 2\alpha_0 n} qT(n) =$

$exp(\lambda T \sum_{l\varepsilon 1,2,3,4,5} q_1(e^{(2l-6)\propto_0}) - 1) \ldots (3.17)$

is the mgf of the limiting distribution in (3.17)).
In consideration or factoring the formula described in relation to Fourier transform, the Fast-Fourier transform algorithm employed to comprehensively to compute the exact price. The use of fast Fourier transform was established by Madan (1999). In a given procedure aloft, incorporation of the rate of return (c1) given by Levy process, which is very applicable in a risk-neutral applications. This is to help in determining the value of underlying asset which doesn't pay dividend. **NOTE** The *(c1)drift* should be in considerate with the below risk neutral condition.

$e^{r^{\tau}} = [e^{X_{\tau}}] = \phi(-i) = exp(c_1 T)exp(\lambda T \sum_{l=1,2,3,4,5} q_1(e^{(2l-6)\alpha_0} - 1))$

Solving for C_i yields

$C_i = r - \lambda \sum_{l\varepsilon 1,2,3,4,5} q_1(e^{(2l-6)\propto_0} - 1)$

4 DATA ANALYSIS AND RESULTS

4.1 Introduction

In the chapter below, we introduce the outcomes reached at in the calculation of the European put option by using a Trinomial Tree Method. We also segment the time period of an American put option into equal intervals and carry out European put pricing at the end of every interval independently with assumption that every interval is the expiring date of a European put option. We then draw a line graph from these prices of different points. The curve gives us the best prices for an American put option.
The results are then compared to Black – Scholes, Trinomial tree method and Monte-Carlo simulation. The numerical solutions were obtained by implementing an R program written for the methods.

4.2 Pentanomial Solution

In using pentanomial tree method, the results tend to approach explicit solution as the stride time-numbers grow moderately. The main steps followed in pentanomial method are:

i Generate the path for the underlying asset prices by a random walk under a risk neutral world.

ii Calculate the risk neutral probabilities and take assumption to be all constant throughout.

iii Evaluate the payoff and get the highest at every node.

iv Discount the highest payoff of an interest rate of a at a risk-free asset to get a required present value, which is the price of an option.

Example 1; Pentanomial tree method

Consider valuating the price values of a European put option with S_0=100, x=110 ,r=0.05,σ=0.125 and T=$\frac{1}{2}$ years. The option price reached or acquired by the Black-Scholes equation is 11.50987. Therefore using Trinomial lattice, as the number of time steps expands the results becomes more accurate. The challenge with this is that it takes more computational time very large number of steps are experienced.
The work surface below shows the accuracy with increasing number of time steps of trinomial method. Every step is one week .

Table 1. Pentanomial Method

Number of steps	European Put Option
1	9.9900
2	9.9987
3	10.0823
4	10.2030
5	10.341
6	10.4869
7	10.6362
8	10.7861
9	10.935
10	11.083
11	11.2268
12	11.369
13	11.5085

Using the above results for half a year of the European put price, we come up with the American put price curve (boundary) that give us the max prices at different points.

Figure 2. American put option graph

4.3 Comparison of the Results

Example 2; Comparison

In this example, we make comparison of the European put prices obtained by Black-Scholes formula, Binomial tree method and Trinomial tree method. Consider example 1 above, we have frameworks, S_0=100, x =110 ,r =0.05,σ = 0.125 and T=$\frac{1}{2}$ years. We will vary the time steps and evaluate the corresponding put prices. The values obtained are shown in the table below.

Table 2. Model Comparison of the European put prices

Number of steps	Black scholes	Pentanomial model	Trinomial Model
1	9.8972	9.9900	9.8942
2	9.8441	9.9987	9.7886
3	9.8585	10.082	9.8060
4	9.8948	10.2031	9.8942
5	9.9584	10.3409	9.9183
6	10.0333	10.4869	10.0577
7	10.1139	10.6362	10.098
8	10.192	10.78610	10.233
9	10.28214	10.935	10.2523
10	10.3667	11.082	10.4090
11	10.4509	11.2268	10.4237
12	10.5333	11.3691	10.57831
13	10.614	11.5081	10.58971
14	10.6934	11.64539	10.7402
15	10.7710	11.77919	10.7485

5 SUMMARY CONCLUSION AND RECOMMENDATION

Conclusions

In the project, we have examined a pentanomial lattice representation that assimilates kurtosis and skewness. We have also controlled the states on kurtosis and skewness under continuous time. We came up with the restricting distribution which is ideal compound Poisson distribution. In the end, we came up with a formula involving Fourier transform techniques that systematically employed to compute European option prices. Thus, this explains a compatible representation for estimating American and European option prices under kurtosis and skewness.

The outcomes observed propose that a Poisson process is a rational possibility of representation of an underlying asset when pricing American and European options.

The pricing of derivatives has been made easier by the development of Black-Scholes model. The implementation of the pentanomial and trinomial tree methods made it easier to make a comparison of the results obtained by these numerical methods to the explicit answer reached by involving the use of the Black-Scholes method. We observed that the results obtained were approximately equal to the explicit solution. Pentanomial method converges to the answer two times faster than Binomial and Black-Scholes. Also Kurtosis and skewness are taken into account.

This project analyzed through a recombining pentanomial tree for coasting European options with unchanging volatilities. The above was attained by letting the time steps regular and risk neutral probabilities remaining the same for the entire contract time. This recombining trinomial tree is more flexible/ easy to use than just a tree with a lot of nodes which are more the same and thus suitable for predicting prices of options.

I recommend the use of Pentanomial lattice model in option pricing.

References

Amin. K. I. (1993) Jump diffusion option valuation in discrete time. *Journal of Finance.*48 (5).pp. 1833-1863.

Barndorff-Nielsen. 0. (1998) Processes of normal inverse Gaussian type. *Finance & Stochastics*, 2(1).pp. 41-68.

Boyle. P. (1988) A lattice framework for option pricing with two state variables.*Journal of Financial and Quantitive Analysis,* .23. pp. 1-12.

Breiman. L. (1992) Probability (Philadelphia: SIAM).

Carr. P. and Madan. D. (1999) Option pricing and the fast fourier transforms. *.Journal of Computational Finance.* 2(4).pp. 61-73.

Chan. K. (1999) Pricing contingent claims on stocks driven by Levy Processes. *Annals of Applied Prbability.*, 9.pp. 504-528.

Corrado.C. J. and Su. T. (1996) S & P 500 index option tests of Jarrow and Ruddís approximate option valuation formula. *Journal of Markets .* 6, pp. 611-629.

Corrado.C. J. and Su. T. (1997) Implied volatility skews and stock index skewness and kurtosis implied by S & P 500 Index option prices. *Journal of Derivatives.Summer.* pp. 8-19.

Cox. J. C. ci al. (1979) Option pricing: a simplified approach. *Journal of Financial Economics.*7. pp. 229-263.

Derman.E. and Kani. I. (1994) Implied trinomial trees of the volatility smile. *Goldman Sac/is Quantitative Strategies Research Notes.* February 1994a.

Derman.E. and Kani. I. (1994b) Riding on a smile. Risk.7(2).pp. 32-39.

Dupire.B. (1994) Pricing with a smile.Risk.7(1).pp. 18-20.

Jarrow.R. and Rudd. A. (1982) Approximate valuation for arbitrary stochastic processes. *Journal of Financial Economics.* 10(3).pp. 349-369.

Karl Lundengard, Carolyne Ogutu, Sergei Silverstrov and Prof Patrick Weke-Asiaon options, Jump Diffusion process on a Lattice Model and Vendermonde Matrix.

Rendleman.R. and Bartter. B. (1979) Two state option pricing. *Journal of Finance.*34. pp. 1093-1110.

Rubinstein. M. (1994) Implied binomial trees. *Journal of Finance.* 3. pp. 771-818.

Rubinstein. M. (1998) Edgeworth binomial trees. *Journal of Derivatives.* 5(3).pp. 20-27.

Yamada.Y. and Primbs.J. (2001) Construction of multinomial lattice
random walks for optimal hedges.In Proceedings of the 2001 International conference of Computational Science.