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Kenyan Mortality and Survivorship Tables

Research Report in Mathematics, Number 14, 2019

Mbugua Francis Ndung'u

June 2019

Submitted to the School of Mathematics in partial fulfilment for a degree in Master of Science in Actuarial Science

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Master Thesis

Submitted to the School of Mathematics in partial fulfilment for a degree in Master of Science in Actuarial Science

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Abstract

Mortality tables are one of the oldest statistical tools widely used by demographers , medics and actuaries.This is because,they enables the representation of mortality in terms of probability.

These tables are constructed from vital registration data e.g census data basing on reference years and can be broadly classified into two,i.e complete life table and abridged life tables.

Complete life table considers all the single years of age from birth to the last applicable year.This table is also known as unabridged life table. The latter is constructed using age intervals.In most cases,these time intervals are usually of ten or five years.

Life tables provides an effective way of presenting and evaluating survival and mortality data.They provide a method of analysing summary tables ,mortality curves ,survival plots and life expectancies.

It is of great benefit to understand the basics of life table construction not only to scholars but also to industrial players like pension schemes,hospitals ,life offices and even in social security as through examination of mortality rates and survival rates,these sectors are able to make the right cause of action.

In this research work,we reviewed the basic methodology of mortality table analysis with a specific focus on applying the Kenyan insured lives experience data.Thereon we constructed complete Kenyan life tables by age and gender and compared them with the standard English Life Tables.

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Declaration and Approval

I the undersigned declare that this dissertation is my original work and to the best of my knowledge, it has not been submitted in support of an award of a degree in any other university or institution of learning.

Signature Date

MBUGUA FRANCIS NDUNG'U
Reg No. I56/7874/2017

In my capacity as a supervisor of the candidate's dissertation, I certify that this dissertation has my approval for submission.

Signature Date

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Dedication

I dedicate this work to Mrs Jostine Kariuki, my twin brother Patrick Mwangi, and to my dear parents,Mr J.M Njoroge and Mrs Lucy Mbugua.

Acknowledgments

First, I thank the Almighty Lord for the chance and ability to come up with this project.It's by his grace,that we have achieved this.
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May I also express my gratitude to all the lecturers and staff in the school of mathematics for imparting the knowledge and skills requisite to undertake this project.
To my fellow classmates,thank you for the laughs we shared and the mutual support and encouragement.Wish you all the best in your future endeavours.
Lastly but not least, I acknowledge the patience, love and hearty encouragement from my family specially mum and dad.
Thank you very much and may The Lord bless you all.

Mbugua Francis Ndung'u

Nairobi, 2017.

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Abbreviations and Acronyms

HP Model	Heligman Pollard Model
LF	Life Table
TASK	The Actuarial Society of Kenya
UK	United Kingdom
ELT 12	English Life Table Number 12
ELT 15	English Life Table number 15
CLT	Cohort Life Table
TS Model	Theile and Steffeson Model
UDD	Uniform Distribution of Death
CFOM	Constant Force of Mortality
CPHM	Cox Proportional Hazard Model
SF	Survival Function

1 INTRODUCTION

1.1 Background of the Study

The study of mortality in a population is of essential importance in any society as it enables the understanding of death rates in the society in a given year.This enables the determination and comparison of different mortality determinants within the society as well as the society's life expectancy at any age.Specifically,insurance firms applies this knowledge to make accurate actuarial calculations such as premiums to be charged on policies as well as in doing actuarial valuations.

This understanding helps authorities,regulatory and supervisory agencies to make proper planning for their resources.It is this knowledge that enables pension firms to make right decisions in the development of policies.It also helps governments,financial institution and even school systems to make mortality and survival projections.

In general ,a life table is an important tool that aids the establishment of public policies like health, education, economic planning, workforce allocation, social security, insurance and investment planning thus an important area of study.

1.2 Problem Statement

In our modern societies,there exist a big need for an accurate and reliable mortality tables. This is because insurance firms ,Annuity and Pension funds heavily rely on these tables while carrying out Actuarial valuations for policies as well as in premium calculations. Failure to accurately provide for mortality can expose these firms including the governing authorities in un expected losses in their provision for liabilities. Thus, the understanding of mortality and survivorship concepts is of invaluable importance. These mortality and survivorship concepts varies among societies,gender,between racial groups and even between policy holders as well as between those without insurance covers.Therefore,mortality and survivorship tables matching mortality assumptions and mortality experience of a population are a crucial pillar for insurance and pension firms.

1.3 Objectives

1.3.1 General Objective

Model and construct mortality and survivorship tables based on insured experience data in Kenya

1.3.2 Specific Objectives

- Calculate life expectancy at birth in Kenya
- Estimate the mortality and survival indices in Kenya
- Construct a complete life table for Kenyan population.
- Compare the constructed tables with the Kenyan and England tables.

1.4 Significance of the Study

For each age life tables show the likelihood of of dying and living.These manifests the death rates and survival rates respectively.
These tables also exhibit the number of persons living as well as the number of persons dying at each age in the cohort.They also provide the future expectation of life in the population.
Generally, life tables describe the mortality patterns of the population under consideration.Thus,a life table is an important tool that help the insurers,pension funds and financial institutions in general to run.
Similarly, demographers and the government heavily depends on the this tool to establish public policies such as health, investment planning,education economic planning,social security and insurance.

Therefore,gaining this knowledge on mortality rates,life expectancies and survival rates,is of great importance.

1.5 Scope of the Study

Mortality studies are vast areas of study,in this research work ,we focus on the period 1920 to year 2000. In chapter two,we review the past literature on mortality and survivorship life table construction.In the third chapter ,we look into mortality and survivor ship assumptions as well as the common life table functions.In the fourth chapter, we analyse the data and present the important findings and discussions.Lastly but not least we look at the conclusions and recommendations.

2 Literature Review

2.1 Introduction

Here,we seek to look into the major mortality constituents specifically mortality graduation techniques,mortality distributions and models and as well as give some insight on some of the past and recent literature on mortality and mortality tables.

2.2 Review of Mortality tables

A mortality table is a concise way of showing the likelihoods of an individual in a given cohort surviving to or dying at a specific age.According to Shryock (1975),a life table is a form of summing up mortality rates of a given population at different ages into a unit statistical model.Principally ,life table gives the mortality experience of a given population in a specific time bound.By this,it enables the best expression of human mortality pattern as the population gets depleted by death at each age.
This life tables concepts was started hundreds of years ago by John Graunt in his publication Natural and Political Observations upon bills of mortality'. He assumed that population is stationary and proved that age pattern can be represented graphically.Other publications followed Grant's ideas until the first life table was developed by Edmond Halley (1656-1742). In 1783 Dr. Price Northampton came up with a more developed life table that could be used to determine premiums for life assurance covers. Other scholars came up with life tables like Mine (1815) who developed the{ Carlisle table.}
Dr. Farr revolutionised the life tables idea by constructing English Life Tables 1,2 and 3 using Census data.Later on The England government constructed the ELT tables we use today.

In Kenya,we got the first published mortality tables in year 2009 where experience data from assured lives in Kenya was used. This study recommended a regular adjustment of the actuarial tables we use in order to ensure that the mortality assumptions are in line with the Kenyan Mortality experience.
According to The Actuarial Society of Kenya,the biggest challenge in construction of the tables is incompleteness and inconsistency of the data .This compromises the quality of the tables.
Mikhala (1985) used life table technique to study adult mortality differentials in Kenya.He confirmed the existence of mortality differentials in the country and concluded that the quality of death registration data in most districts was better for males than for women.

He also constructed mortality tables and generally he concluded that males had a higher life expectancy than females. In her paper, An Experience Constructing Complete and Abridged Life Table Using a Mathematical Formular For a Small Population Helena (2002)used a lineal one (1 4) to construct complete life tables for data from nine provinces in Castilla y leon.She compared her results to those of Spain.In general she found out that mortality rates were much lower in Castilla y leon provinces than in Spain.Again Castilla leon provinces had a higher life expectancies both at birth and at age 65.
In his studies ,'Modelling and forecasting Mortality and Longevity risk' Bett(2017)used Lee cater model to model and construct abridged life table using Kenyan population data.He created a Kenyan life table upto a maximum age interval of 80+.From the research,he found out that infant mortality rate was much higher than all other age groups.Adult males showed a higher death rate compared to females however old age mortality was uncertain due to extreme limitation of data.
Machau (2014)modelled Kenyan mortality experience using graduation techniques.She found out that higher order polynomials graduate crude rates in the best way.By comparing the Kenyan mortality with English life tables she found out that the standard tables depicted a higher mortality with females mortality being higher than for males.

In their journal 'A review of Mortality table Construction' ,Ilker Etikan et al (2017),categorises life tables into the following;

- **Cohort life table**
  A cohort refers to a group of lives born in the same time interval,precisely born in the same calender year. This table is constructed in such a way that its cohort is developed sequentially say from $Q_1, Q_2, Q_3, Q_4, ...Q_n$.
  This table reflects the mortality-experience of a real population from birth upto the death of last member in the population.
  This is the most common type of life table .It is also referred to as a longitudinal life table or age specific mortality table.

- **Current life table**

  This life table is also called specified life table or period life table .
  It uses data of a unit cross section of time to represent an entire population.
  A sequence of $P_0, P_1, P_2, P_3, P_4, P_, ...P_n$ showing the current mortality trend is used to represent the death sequence $Q_0, Q_1, Q_2, Q_3, ...Q_n$ in the cohort.

- Complete Life table.
  This is a life table that has all the single years of age from birth to the last applicable year.This table is also known as unabridged life table.It is constructed basing on a reference year.

- Abridged life table
  This table is also constructed basing on a reference year where age intervals of ten or five years in most cases are considered except for the initial years.

- Single decrement life table
  This is a life table that considers only one cause of death at a time.It considers only one characteristic at a time. Generally ,it gives a general experience of a cohort by age.

- Multi decrement life table
  This is a life table that considers more than one cause of death at a time.It may consider many features at ago.

2.3 Mortality Graduation

Mortality graduation refers to the process of smoothing the mortality rates in order to obtain a monotonically increasing mortality series after some age.On average, this age is usually 30 years. These data needs to be graduated in order to remove the irregularity on crude rates.Actuaries achieve this by revising the initial estimates and thus they are able to describe the actual but unknown mortality pattern (London 1995).Basically,graduation has two antagonistic characteristics,goodness of fit and smoothness,both can't be achieved at a go,to achieve one ,one has to forego the other. Goodness of fit refers to how well the data fits into the model under consideration.It is mathematically defined as ;

$$F = \sum_{x=1}^{n} w_x (u_x - v_x)^2 \tag{2.1}$$

where$u_x$are the initial values and$v_x$are the graduated values and
$w_x$is the reciprocal of initial estimates usually referred to as the weights.

Smoothness is defined as the process of of removing volatility or any noise by the use of an algorithm.
It is defined as;

$$S = \sum_{x=1}^{n} (\Delta^3 V_x) \tag{2.2}$$

Graduation methods are many but are broadly classified either as parametric or non parametric.
Parametric methods are those that allow one to describe mortality using a set of equations with given parameters.This is achieved by assigning some mortality characteristics to the model parameters.They enables one to capture the mortality behaviour for a population using models like mortality models,generalised linear models,splines and junction interpolation.

Considering graduation by mortality models,Takis (2004)highlights these models as

1. Gompertz model
2. Heringman and pollard model
3. Makeham model
4. Opperman model
5. Theile and Steffeson
6. Beard model
7. Barnett model

These models require their parameters to be estimated by standard methods like maximum likelihood estimation. Suc
Non parametric methods transform crude rates into smooth curves either by running averages or medians.The most common non parametric methods are'

1. Graphical method
   Here, one uses a free hand to fit a smooth curve that passes near the initial estimates as close as possible.
2. Weighted moving averages
   Under this method,each graduated value is produced as a weighted average of 2m+1 of the initial crude rates.
3. Graduation with reference to a standard population
   In this method,the graduated values are assumed to follow a similar trend to that of the standard mortality rate.
   Other non parametric methods include
   (a) Whitaker and Henderson method ,where the graduated values are minimised by the following equation.

$$N = G + hS \tag{2.3}$$

$$= \sum_{x=1}^{n} w_x (u_x - v_x)^2 + h \sum_{x=1}^{n} (\Delta^3 V_x) \tag{2.4}$$

   (b) Kernel method where Kernel estimators are used to graduate the crude mortality rates.
   (c) Graduation using theoretical ideas
   (d) Summation and adjusted average methods
   (e) The oscillatory method

2.3.1 Factors guiding the graduation methods

The choice on which graduation method to use ,lies on the discretion of the actuary.Some of the important factors to be considered are;

(a) The desired level of smoothness
Non parametric methods give a smoother graduated values than parametric methods. Again the level of smoothness varies among the different non parametric methods.

(b) Range and form of the Actuarial data
If the data to be graduated is grouped graduated values for each age ,then parametric methods are considered over non parametric one. Again if the initial estimates are few then the best graduation method in that case should be graphical.

(c) The model Parameters
For simplicity in calculating the model parameters ,the actuary may consider the model with simpler parameters.

Nevertheless ,parametric and non parametric can be fused together by starting with a non parametric methods which in turn feeds a parametric process.

2.4 Mortality Distributions

Poison distribution and binomial distribution are the most commonly used distributions to model mortality and life survivor-ship.
For instance,the death of a single individual can be described using a Bernoulli(0,1) distribution.For more than one death,the death distribution can be expressed binomially since the summation of independent Bernoulli random variables results to a Binomial distribution B(Nx, qx), where Nx refers to the number of lives surviving at age x, qx is the probability that a life aged x will collapse and die at exact age x.
For a continuous time , we consider a continuous random variable and therefore a poison distribution becomes a good model for this.
These two models are almost the same when the individuals exposed to risk of death are many and when the probability to die is close to zero.The death probabilities can be estimated using a maximum likelihood method,we now consider the likelihood functions of these distributions.
For a Binomial distribution;

The poison distribution has a likelihood funtion given by

$$L(p) = \Pi_{x=0}^{t} \frac{(p_x)_x^Q \exp^- p_x}{O_x!} \tag{2.5}$$

$$L(p) = \Pi_{x=0}^{t} \frac{(E_x q_x)_x^O \, exp^-(E_X, q_x)}{O_x!} \tag{2.6}$$

$$\tag{2.7}$$

By taking logarithm and simplifying the equation we have the Maximum Likelihood Equation as ;

$$L(p) = \Pi_{x=0}^{t} exp\left[O_x lnE_x + lnq_x - E_x q_x - lnO_x\right]$$

2.5 The Heligman - Pollard Model Mortality Law

This parametric model was developed by Heligman Larry and John H Pollard in 1980 It consists of three terms each representing a given mortality characteristic.This model is of different types depending on the number of parameters.The original model contained eight parameters but it has undergone modifications to a nine parameter model with different types.a good model for describing all human mortality experiences.The initial eight parameter model is is given as;

$$q(x) = A^{(B+x)^C} + D\exp - E(lnx + lnF)^2 + GH^x \tag{2.8}$$

This model was modified as below and named Heligman Pollard Model of type II

$$q(x) = A^{(B+x)^C} + D\exp - E(lnx + lnF)^2 + \frac{GH^x}{1+GH^x} \tag{2.9}$$

The eight parameter Heligman Pollard model of type II was developed into nine parameter model of type III by adding another parameter K as seen below

$$q(x) = A^{(B+x)^C} + D\exp - E(lnx + lnF)^2 + \frac{GH^x}{1+KGH^x} \tag{2.10}$$

This nine parameter model of type three has been modified further to take the form,

$$q(x) = A^{(B+x)^C} + D\exp - E(lnx + lnF)^2 + \frac{GH^{x^K}}{1+GH^{x^K}} \tag{2.11}$$

For simplicity,the eight parameter H-P mortality model of type II can be reduced to a five parameter form.This removes the mortality due to accident hump in young adults.This is given as;

$$q(x) = A^{(B+x)^C} + \frac{GH^x}{1+GH^x} \tag{2.12}$$

The Heligman Pollard of type II is said to give better mortality estimates and therefore in this work we have considered it.

$$q(x) = A^{(B+x)^C} + D\exp - E(lnx + lnF)^2 + \frac{GH^x}{1+GH^x} \tag{2.13}$$

Specifically,we dissect it as below; The first term

$$q(x) = A^{(B+x)^C}$$

represents the declining child mortality during the early childhood as children adapt to the environment and gaining immunity from diseases.

The second term

$$D\exp -E(lnx+lnF)^2$$

represents the accident hump or the mortality rate in young adults.

The third component

$$\frac{GH^x}{1+GH^x}$$

represents the middle and old age mortalities.
Parameter A,B and C describe the child mortality These three parameters takes a value between 0 and 1 and they have a decreasing significance as age increases. For adults above the age of 70 years ,the parameters are totally insignificant.A and C are the scale parameters and B is the location parameter.

Parameters D,E and F measures the 'hump' resulting from young adult mortality where D measures it's level or severity and it takes values between 0 and 1. E measures it's amplitude i.e it's spread and it always takes positive values.F gives the location i.e the position of the hump.

In the last component of the model,GH represents the adult mortality pattern.G gives the base level of adult mortality and H is the rate of increase in adult mortality. Specifically;
q_x is the probability of dying at age $x = 0,1,2,3\cdots$

A gives the level of mortality during early life period i.e infant mortality rate.

or the aproximation of probability q_0

B Measures the mortality rate of 1 year old children.
C Measures the decline in childhood mortality upto early adult life. D Gives the level or severity.
E Measures the amplitude of the accident hump.
F Gives the position of the accident hump.
G Gives the level of senescent mortality i.e the level of aging in mortality .
H Measures the geometric rise in death rates at older ages.
The biggest challenge is to estimate all the eight parameters because they are highly correlated. Jones(2005)refers it as a beast to fit as the estimation of these parameters results to tremendous identifiability difficulties.

Nevertheless,this model gives a continuous curve which is applicable to all age groups ie for age;$0 \leq X \leq \infty$.
It has been widely applied in mortality studies as well as in construction of mortality tables. Mario De Oliveira et al(2010) applied the eight parameter H-P model to construct the Brazilian Survival and Mortality tables. In their paper,'Modelling Hospital Mortality Data 'Pasca et al(2017) applied the H-p model on inpatient hospital mortality data in a hospital in Indonesia and compared the results to the mortality rates given by Indonesia mortality table,they found a higher mortality rate in males as compared to females. In general they concluded that the HP model gives similar mortality rates as the Indonesia Mortality table.

2.6 Brief development of frailty modelling

Onchere(2013)extensively studied frailty models and their applications to pension schemes.Frailty models are extensions of the Cox proportional hazard model.These models provides a statistical way of accounting for heterogeneity in assured population.They enable the measurement of unobservable risk characteristics by adding random effects.This makes it possible to measure the un observable covariates Frailty models are based on mixtures of distributions and survival analysis.
Mixing distributions involve combining one distribution with another .This is realised by takng n different distributions with probability densities

$$f_1(x), f_2(x), ...f_n(x)$$

with mixing weights of $w_1, w_2, ...w_n$. Each weight should be greater than zero and their summation be equal to 1. According to Johnson et.al 2005,the resulting density or mass function is a finite mixture .It is given by ;

$$f(x) = \sum_{j=1}^{n} w_j f_j[x] \quad (2.14)$$

For a random variable X and a constant parameter θ,
the probability distribution of this random variable and the parameter θ is denoted by $f(X;\theta)$.If the parameter θ is also varying,we have a joint probability distribution denoted by $f(X,\theta)$.
This joint probability distribution is defined by;

$$f(x) = \int f(x/\theta)g(\theta)d\theta \quad (2.15)$$

The function $f(x/\theta)$ is the conditional distribution defined by

$$f(x/\theta) = \int f(x/\theta)g(\theta)d\theta \quad (2.16)$$

This mixture distribution considers a scenario where the origin of the random variable is not known therefore instead of considering the parameter θ,we consider an unknown variable hence the use of mixture distributions in measurement of risk factors.

2.6.1 Functions of survival time

There exist four important functions of time to an event.They include the following;

4. The probability density function. It is given by ;

$$f(x) = Pr(X = x) \tag{2.17}$$

5. The cumulative distribution function It is defined by;

$$\begin{aligned} F(x) &= Pr(X \leq x) && (2.18)\\ &= 1 - Pr(X > x) && (2.19)\\ but Pr(X > x) &= S(x) && (2.20)\\ &= 1 - S(x) && (2.21) \end{aligned}$$

It is the complement of survival function.

6. The survival function
It is defined by.

$$\begin{aligned} S(x) &= Pr(X > x) && (2.22)\\ &= \int_0^\infty f(x)d(x) && (2.23) \end{aligned}$$

The survival function is equally to l(x) i.e the proportion of lives alive in a cohort at a certain age class x

7. The hazard function
It is denoted by h(x). It is equal to the force of mortality.

$$\begin{aligned} h(x) &= \lim_{\Delta x \to 0} Pr\left(\frac{x \leq X \leq x + \Delta x | X > x}{\Delta x S(x)}\right) && (2.24)\\ f(x) &= Pr\left(\frac{x \leq X \leq x + \Delta x | X > x}{\Delta x}\right) && (2.25)\\ h(x) &= \frac{fx}{Sx} && (2.26) \end{aligned}$$

2.6.2 The Cox Proportional Hazard Model

This model was developed by cox in 1972 .It describes the relation between the survival life time of an individual and certain explanatory variables.It is popular because it enables the handling of censoring and truncation with a lot of ease as it is interpreted as a risk that changes with time. For an individual j at time t,this model is is given by;

$$\begin{aligned} h_j(t) &= h_0(t)exp^{\vec{B}1}X1 \\ &= h_0(t)exp^{B_1X1+B_2X_2B_3X3+\cdots+B_kX_k} \end{aligned}$$

Where;

- $h_j(t)$ is the hazard function at time t
- $h_0(t)$ is the baseline hazard function at time t
- $\vec{B}$ is the column vector of coefficients
- $\vec{X}$ is the vector of covariates

This model enables the consideration of heterogeneity of the insured populations thus eliminating the error that could arise due to the assumption that populations are homogeneous. This is of importance to both the insured and the insurer as it enables correct pricing of insurance contracts.

2.6.3 The Non central Gamma frailty model

In his work ,Onchere (2013) proved that the non central gamma frailty model gives improved estimates of insurer rates.This model has a probability density function given as ;
Consider Y as the mixing distributions of $X_1, X_2, X_3, X_4 \cdots X_N$

i.e Let

$$Y = X_1, X_2, X_3, ...X_N$$

where X's $\sim Gamma(n, 1)$ and N $\sim poisson(\lambda)$

Then pdf is a convolution with weights $\frac{e^{-\lambda}(\lambda)^n}{n!}$

$$Prob(Y = j) = \sum_{j=0}^{\infty} Prob(X_1, X_2, X_3, ...X_j/N) Prob(N = j) \tag{2.27}$$

$$Prob(Y = j) = \sum_{j=0}^{\infty} \frac{X^(j-1)e^-x}{\Gamma(j)} * n * \frac{\lambda^j e^-\lambda}{j!} \tag{2.28}$$

$$= \sum_{j=0}^{\infty} \frac{X^(j+n-1)e^-x}{\Gamma(j+n)} \frac{\lambda^j e^-\lambda}{j!} \tag{2.29}$$

$$f(x, n, /\lambda) = \sum_{j=0}^{\infty} \frac{X^(j+n-1)e^-x}{\Gamma(j+n)} \frac{\lambda^j e^-\lambda}{j!} \tag{2.30}$$

Where the $\Gamma(n)$ is the Complete Gamma function with $n > 0, \lambda > 0, x \geq 0$ The hazard function becomes a special case of power variance function with r=-1.It is given by;

$$h(t) = \frac{h_0(t)}{(1 + 1/2 * \sigma^2 H_0(t)^2)} \tag{2.31}$$

2.6.4 Gompertz -non Central Gamma frailty Model

Actuary Benjamin Gompertz (1825) developed the mortality law commonly known as The Gompertz equation;He argued that the force of mortality increases with age in such a way that it's logarithm grows linearly.This function is given as;

$$\mu(x) = \alpha * exp^(\beta(x)) \tag{2.32}$$

Using this function as the baseline function we have

$$h_0(t) = \alpha * exp^(\beta(x)) \tag{2.33}$$

$$H_0(x) = \frac{\alpha}{\beta}(exp^(\beta x) - 1) \tag{2.34}$$

Thus the hazard function becomes;

$$h(t) = \frac{\alpha * exp^(\beta(x)}{(1 + \frac{1}{2} * \sigma^2 * \frac{\alpha}{\beta}(exp^(\beta x) - 1)}$$

2.7 Criteria for model choice

According to Betrao et al (2004) ,a life table model should exhibit the following properties;

(a) Should be simple and easy to use.

(b) For real population, the model should be able to describe any age specific mortality pattern.

(c) For comparison between real and predicted mortality,the model should provide the best adjustment possible.

In their book,*Brazilian Mortality and Survivorship Life Tables,Mario De Oliveira et al(2010)* stated that the methodology to be used should be in line with the following *desiderata*;

(a) Parsimony criteria ,given a choice ,use the simplest theory available to solve the problem.

(b) Intelligibility criteria which states that one should use the methedology that is easy to understand and communicate.

(c) Replicability Criteria -It states that the results given by the methodology used should concur the results of other researchers .

(d) Stability Criteria i.e the methodology used should be universally tested and accepted.

(e) Transparency Criteria,this means that the methodolgy used should be fully documentable.

(f) The methodology used should be independent of experimental softwares.

(g) The methodology used should be flexible to allow compatibility between dynamic and static life tables.

Having considered these factors and that the Heligman -Pollard Model captures mortality experience as the Gompertz -non central Gamma model, I settled on the H-P of type II model.

3 RESEARCH METHODOLOGY

Mortality tables are mainly used to describe mortality characteristics specifically summarising health status of certain populations as well as pointing out the death rates of the population in a given time period .Their construction is pegged on census data.

3.1 Mortality assumptions

Actuarial valuation requires the use of fractional age assumptions when valuing remittances that are not necessarily restricted to integer lifetimes. These fractional life time assumptions are subdivided into three;

(a) Uniform Distribution of Death(UDD)
Under this assumption,deaths are assumed to occur constantly .That is,for a given two intervals in a year people die uniformly.

(b) Constant force of Mortality(CFOM).
This assumptions means that the force of mortality between the beginning of year x and tear x+1 is constant.

(c) The hyperbolic assumption. Under this assumption,the force of mortality is assumed to take the form;

$$\mu_x = \frac{q_x}{1-(1-s)q_x} \tag{3.1}$$

This is a decreasing function and therefore this assumption is rarely used. In this project we apply Uniform distribution of death since it gives similar results to constant force of mortality.

3.2 Mortality measures

Under this section we concentrate on the different ways of estimating mortality.
In life tables,x refers to either the exact age or age interval of the cohort in concern.It starts from zero upto them maximum age limit of an individual denoted by Omega.ie $0 \leq x \leq \omega$.

(a) $l_{(0)}$
This refers to the number of lives in the cohort at the beginning .It is the original number of lives in the cohort at birth.It is also called radix and usually it is taken as 100,1000,10,000 or 100,000 lives.

(b) l_x
This refers to the proportion of lives alive in a cohort at a certain age class x.This probability is estimated by the following equation.

$$l_x = \frac{n_x}{d_x} \tag{3.2}$$

(c) $l_{(x+1)}$
This refers to the number of people alive in the cohort at age class x+1. It is found by;

$$l_{x+1} = \frac{n_{x+1}}{d_{x+1}} \tag{3.3}$$

(d) n_x
This refers to the number of individuals in a cohort of age class x.

(e) $d_{(x)}$
These refers to the number deaths occurring between age classes x and x+1.This estimate sum up to unit. It is given by ;

$$d_x = lx - lx + 1 \tag{3.4}$$

(f) q_x

This refers to the probability of a life that has survived upto the beginning of age class x dying before attaining age class x+1.It measures the rate of mortality.Mathematically it is defined as

$$q_x = \frac{d_x}{l_x} \tag{3.5}$$
$$= \frac{l_x - l_{x+1}}{l_x} \tag{3.6}$$
$$= \frac{l_x}{l_x} - \frac{l_x + 1}{l_x} \tag{3.7}$$
$$= 1 - \frac{l_{x+1}}{l_x} \tag{3.8}$$
$$= 1 - p_x \tag{3.9}$$

(g) p_x

This is the probability of a person aged x surviving upto age class x+1.
It is given by

$$p_x = \frac{l_{x+1}}{l_x} \tag{3.10}$$

(h) **mx**

This is the central death rate between age x and **x+1**.It refers to the number of deaths occurring in a year divided by the average number of persons alive in that year.
It is given by

$$m(x) = \frac{dx}{\int_0^1 l_{x+1} dt} \tag{3.11}$$

(i) L_x

These refers to the person years lived between age class x and age class x+1.It is the aggregate number of years lived by the population in the cohort between exact age x and x+1.It is also referred to as the life table population. In a stationary population the life table population is always the same for all the years while for a uniformly distributed deaths,it becomes the mid year population.By integration,it is given by;

$$L(x) = \int_0^1 l_{x+1} dt \tag{3.12}$$

This can be estimated by trapezoidal rule ;

$$L_x = 0.5l_x + 0.5l_{x+1} \tag{3.13}$$

$$= 0.5(l_{x+l_{x+1}}) \tag{3.14}$$

$$= l_x + 0.5d_x \tag{3.15}$$

(3.16)

This holds for a life aged two or more years.

(j) T_x

This refers to the person years lived after age x.It is the number of years lived by a cohort after attaining age x.In other words,it is the time left until the death of a life age x. It is given by ;

$$T_x = L_x + L_{x+1} + L_{x+2} + L_{x+3} + L_{x+4} + L_{x+5} + ...L_{w-1} \tag{3.17}$$

(3.18)

But

$$T_{x+1} = L_{x+1} + L_{x+2} + L_{x+3} + L_{x+4} + L_{x+5} + ...L_{w-1} \tag{3.19}$$

(3.20)

Therefore,

$$T_x = L_x + T_{x+1} \tag{3.21}$$

(3.22)

In terms of summation,it is expressed as

$$T_x = \sum_{n=x}^{w} L_n \tag{3.23}$$

(3.24)

(k) e_x^0

This is referred to as the complete expectation of life .It is the expected value of time until death of a life aged x.It is given by;

$$e_x^0 = E(T(x)) \tag{3.25}$$

$$= \int_0^\infty {}_tP_x dt \tag{3.26}$$

3.3 The force of Mortality

The force of mortality refers to the instantaneous rate of mortality that a certain age experiences measured on an annualised basis.It's is the frequency at which death occurs expressed as number of deaths per unit time . For a person aged X in year t and assuming the Uniform Distribution of death,the force of mortality is actuarially defined as;

$$\mu_x(t) = \lim_{\Delta x \longrightarrow 0} \frac{x < T_0(t-x) \leq x + \Delta x \mid T_0(t-x) > x}{\Delta x} \tag{3.27}$$

Where $T_0(t-x)$ is the future life time remaining for an individual alive at time t-x.

3.4 Crude death rate

This refers to the ratio of number of deaths in a year to the mid year population of the same year expressed in terms of thousands.

3.5 The Central death rate

This refers to the ratio of number of deaths between ages x and x+n to the mean population alive at that age.

3.6 Age Specific Death rate

This refers to the probability of a person aged x dying in an years' time.Specifically,it is the ratio of number of deaths in an age group to the number of persons alive in that age group.

3.7 Infant Mortality rate

This refers to the number deaths of children under the age of one year per a thousand life births.

3.8 The curtate expectation of future life time

This is the average number of complete years lived by a cohort after age class x by each individual attaining that age.It represents the expectation of further life for a person aged x now.
It is expressed as;

$$e_x = \frac{L_x + L_{x+1} + L_{x+2} + ...L_n}{l_x} \tag{3.28}$$

$$\tag{3.29}$$

Where n is the maximum attainable age.i.e the Omega. Curtate expectation of life is related to survival probability by;

$$e_x = p_x * 1 + e_x \tag{3.30}$$

3.9 The complete expectation future life time

It is the future life time of an individual after attaining age x.It is denoted by e_x^0.
It means the extra number of years a person aged x now will be expected to live in line with the current mortality trend.It can be mathematically expressed as ;

$$e_x^0 = \frac{T_x}{l_x} \tag{3.31}$$

$$= e_x + \frac{1}{2} \tag{3.32}$$

Where e_x is the curtate future life time.

3.10 The Number of persons alive at age x

This refers to number of individuals alive at a given age note,typically it starts at 100,000.It is also called radix.

3.11 The probability of surviving beyond age x

It is usually denoted as ${}_nP_x$

3.12 The Average life time lived

This is the average duration of existence of an individual .It is usually denoted as a_0.

3.13 The Number of Person years lived

It is the number of persons alive at age last birthday X in given population. It is usually denoted as ${}_nL_x$.

4 DATA ANALYSIS

4.1 Data Description

In this work we have made use of the Kenya ,England and Wales mortality experience data all spanning from the 1920 to year 2000. These data was obtained from The Kenyan Mortality tables and from the England life. tables,specifically ELT 15 males and ELT 15 females This data covered the age group 20 to 100 years for both males and females.

We used the number of deaths data in those time periods and the number of lives that were exposed to the risk of death for each gender. Irrespective of the missing data (age 0 to age 20) for Kenya,we applied it in fitting the Heligman Pollard Model. The ELT 15 data was used only for comparison purposes.

4.2 Data Analysis tools used

In this research we equally employed R version 3.6.0 and Microsoft Excel to carry out most of the data analytics.Particularly Excel functions and installed R packages specifically HPbayes.

4.3 General Data analysis

For each age x ,the death probabilities were obtained by dividing the number of deaths by the population size in the respective age.Survival probabilities were found by applying the formula $p_x + q_x = 1$. The total person years lived after age x was found by the formula

$$L_x = l_(x+1) + 0.5d_x$$

The Future life time was obtained by summing up all the total person years lived by a life aged X.ie

$$T_x = \sum_{n=20}^{100} L_{20} \tag{4.1}$$

For every age X,the complete expectation of life was realised by applying the formula ;

$$e_x^0 = \frac{T_x}{l_x} \tag{4.2}$$

The central death rate was obtained by dividing the number of deaths by the total person years lived for each age X.Finally,for each age X,the force of mortality was estimated by;

$$\mu_x(t) = -0.5(log(p_(x+1) + logp_(x)) \tag{4.3}$$

This applied to both males and females

4.4 Graphics

From the graph below,it is evident that over the time period,male mortality in Kenya has been increasing with age ,reaching the highest points between ages 70 to 80 years.

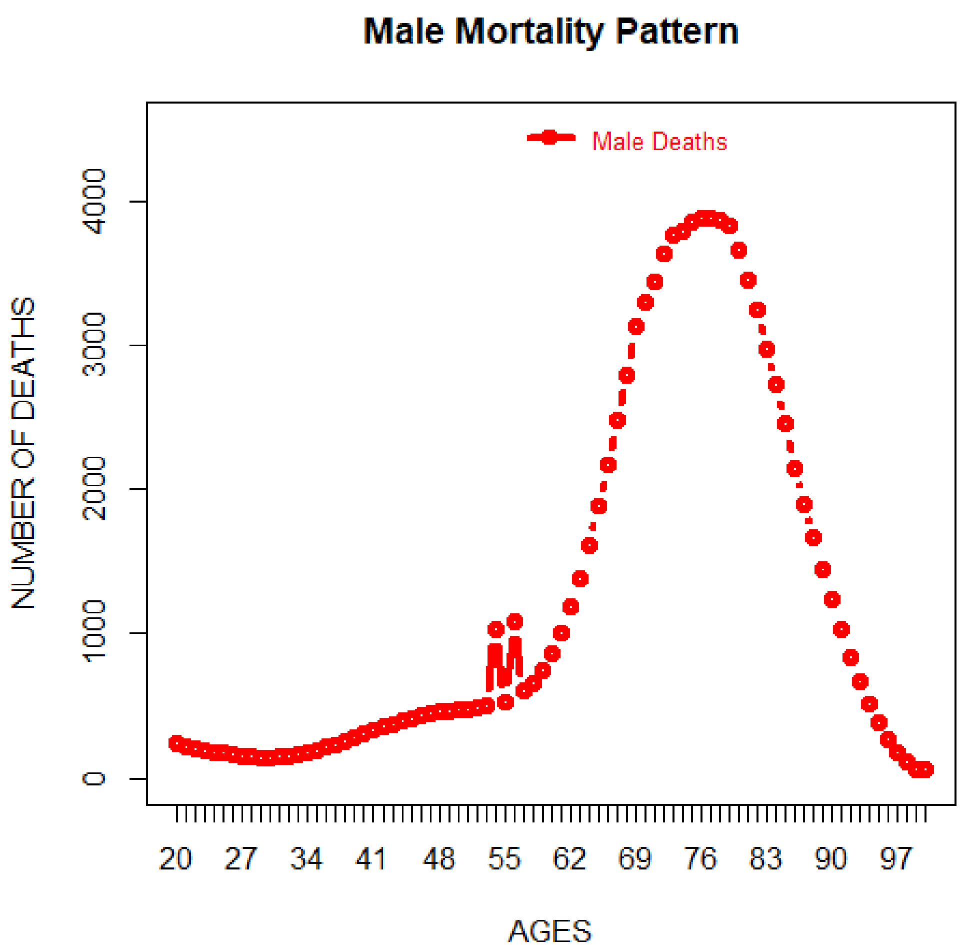

Figure 1. Line Graph for Male Mortality in Kenya

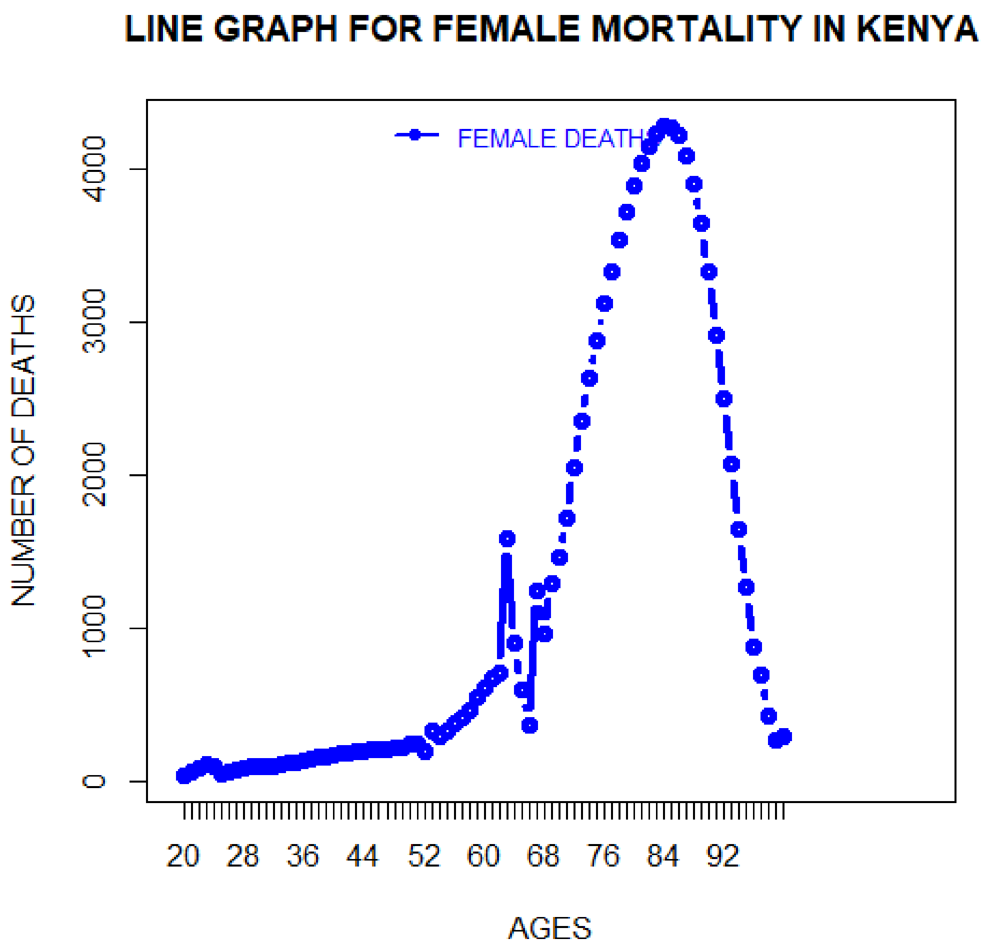

Figure 2. Line Graph for Female Mortality in Kenya

Similar to male mortality,female mortality increases with age ,getting to the peak at the age of 80 to 85 years. This shows that male mortality is higher than female mortality.

From the plot below,it is observable that for all the ages,males in Kenya have a higher death rate than females.This means that males die earlier than females.

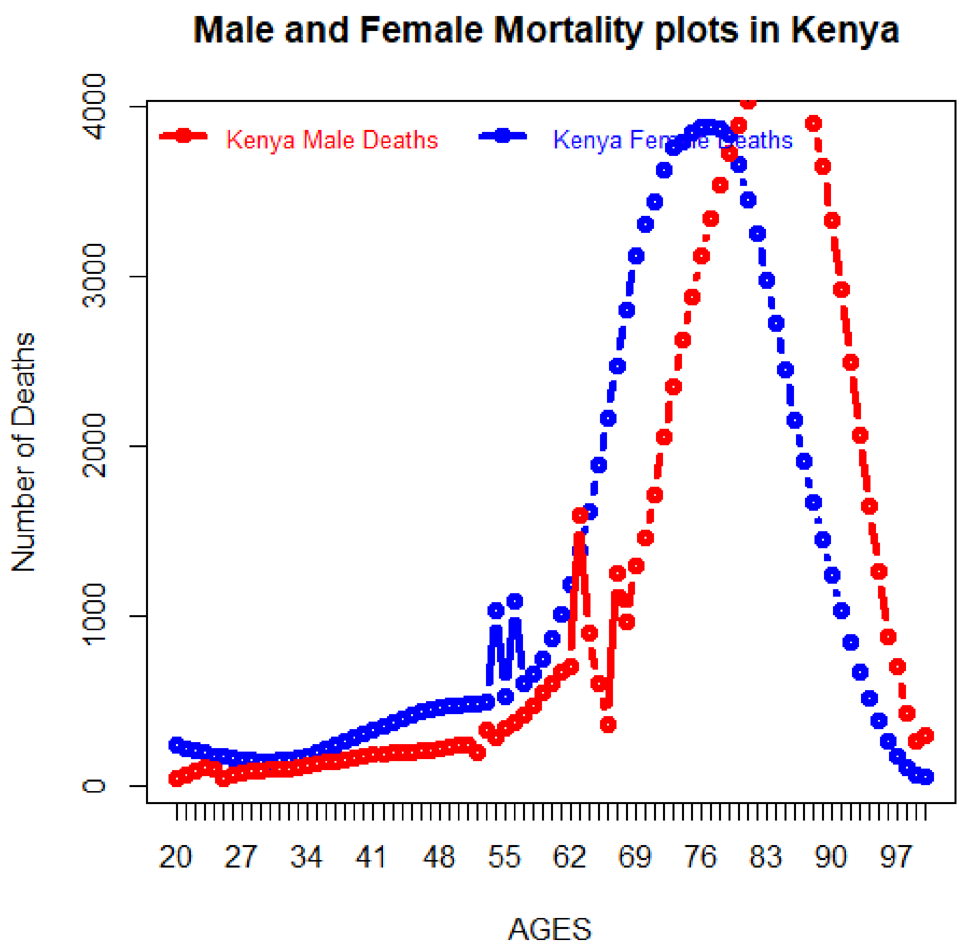

Figure 3. Male and Female Mortality Graphs in Kenya

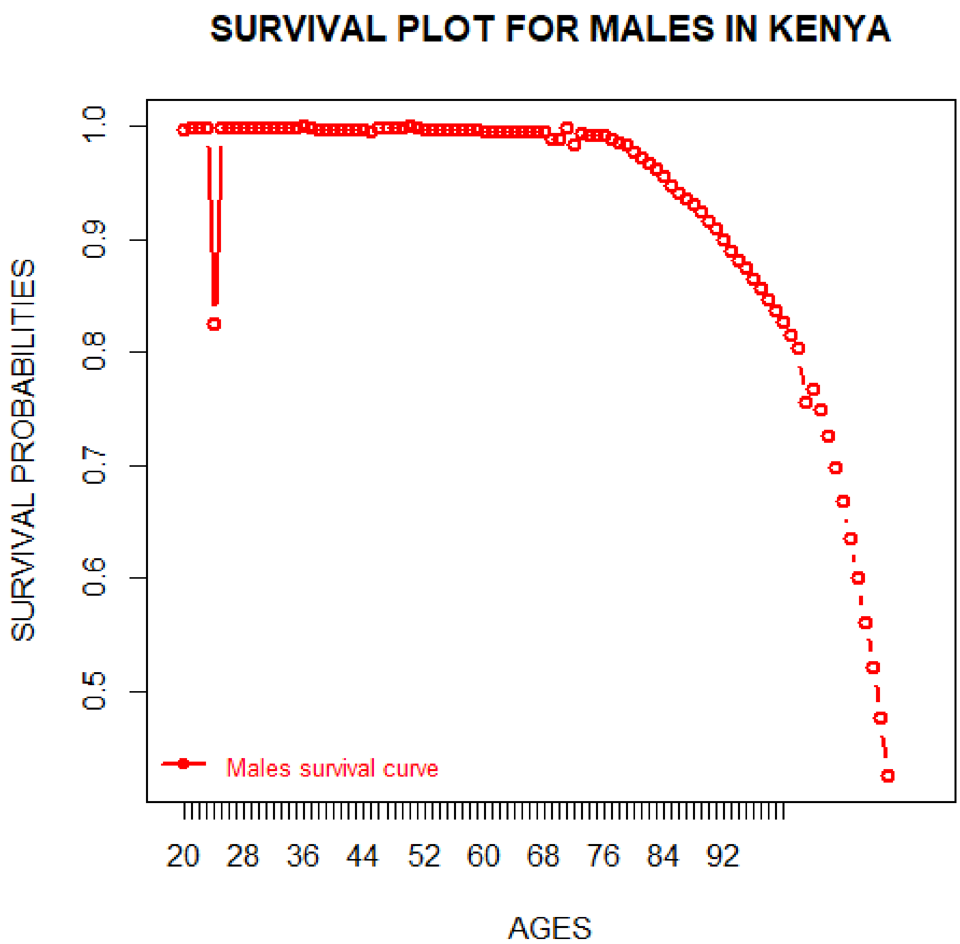

Figure 4. Survival Plot for males in Kenya

From the plot,we find out that as the age increases,the males survival rates decreases.

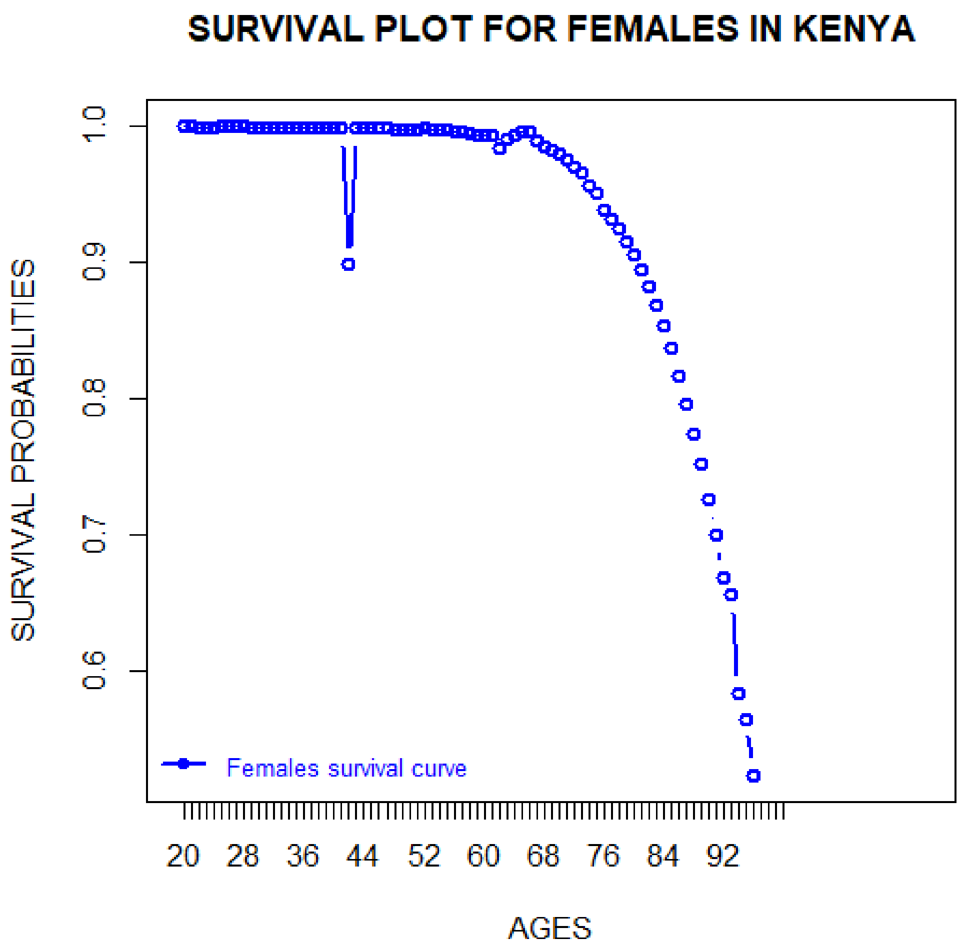

Figure 5. Survival Plot for females in Kenya

Similar to the males survival plot,as the age increases,the females survival rates decreases.

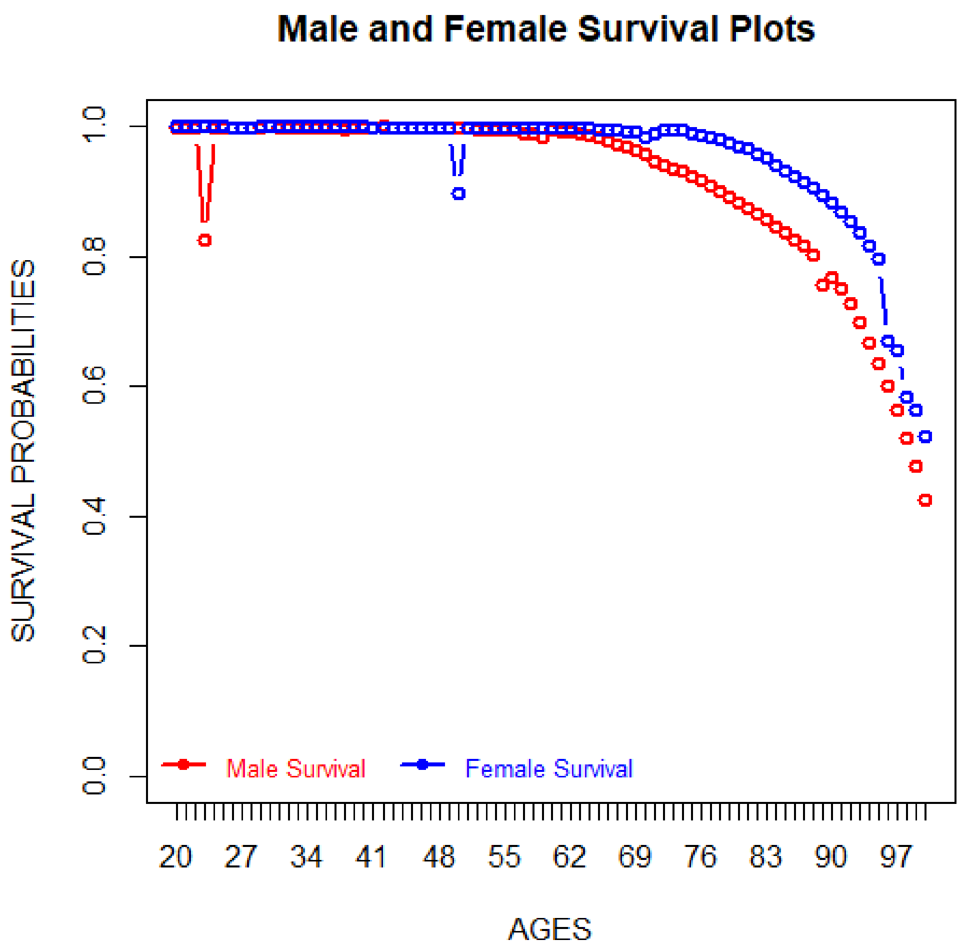

Figure 6. Survival Plot for Both Male and Females in Kenya

Interesting to observe that,from this superimposed survival plots,females in Kenya have a longer life span as compared to the males.

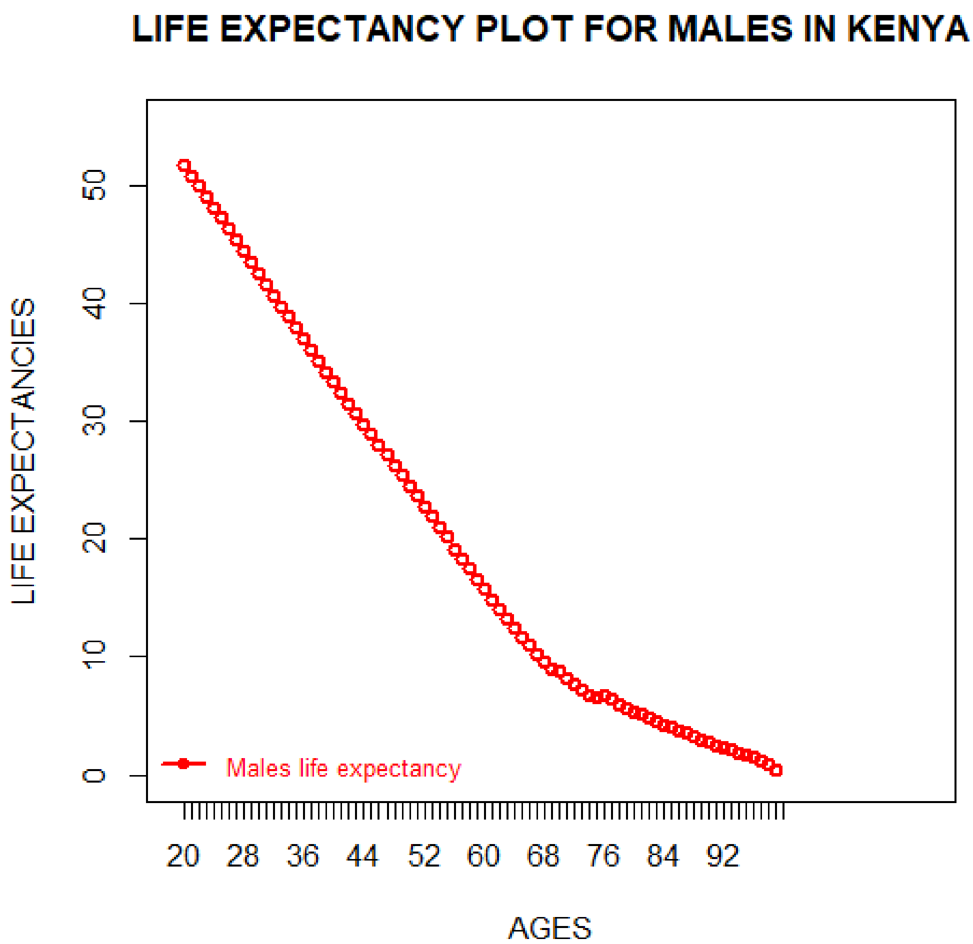

Figure 7. Male expectancy in Kenya

As per this data,male expectancy at age zero is 51.68 years.As an individual ages ,life expectancy decreases showing that this plot is a decreasing function.

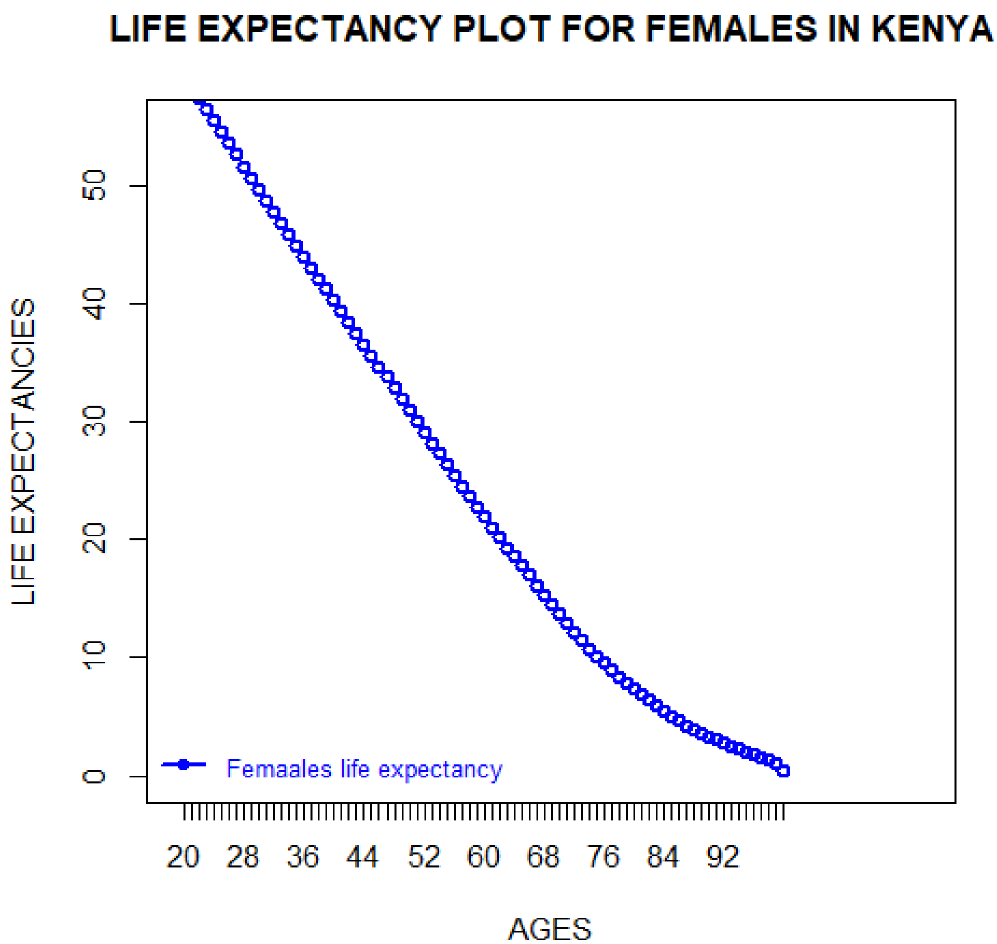

Figure 8. Female expectancy in Kenya

From this plot,it is evident that females in Kenya have a longer life span than males.Specifically,females have a life expectancy of 59.30 years ,8 years higher than that for males. Similarly, female life expectancy is a decreasing function of age.

Figure 9. Male and Female expectancy in Kenya

Interesting,male expectancy plot is much lower than the female expectancy plot,physically proving that females live longer than males.

4.5 Parameters estimation

Estimation of parameters was done by minimising the sum of squares of the proportional difference between fitted and observed mortality rates.
This was achieved in R-3.5.1 by installing the package 'Bayesian Melding with Incremental Mixture Importance Sampling' in the R application.
R,applied Bayesian procedures enabling the estimation of all the 8 parameters and converted them into age -specific death likelihoods and later into corresponding life tables. That is minimising the function;

$$S^2 = \sum_{x=20}^{100} \left(\frac{q_x - q_x^0}{q_x^0}\right)^2 \tag{4.4}$$

where;
q_x is the fitted mortality rate at age x
q_x^0 is the observed mortality rate at age x

Table 1. The estimated HP parameters for Males'data

Number	Parameter	Estimate
1	A	0.0009
2	B	0.0016
3	C	0.000
4	D	0.00785
5	E	11.0139
6	F	42.9934
7	G	0.0000
8	H	1.21530

Number	Parameter	Estimate
1	A	0.0006
2	B	0.0041
3	C	0.000
4	D	0.0008
5	E	8.0010
6	F	2.135
7	G	0.0000
8	H	1.11437

Table 2. The estimated HP parameters for Females' data

4.6 Explanation of the HP parameters

Parameter A measures the infant mortality rate,this rate was higher for males at 0.0009 compared to females at 0.0006 .Parameter B measures the mortality rate for one year old children.
It was higher for females at 0.0041 compared to males at 0.0016.Meaning that male children aged one have a higher survival rate compared to female children aged one. Parameter C estimates the decline in childhood mortality upto early adult life.For both cases,it was insignificant.

Comparing the sum of these three parameters we realise that females have a higher value of 0.0047 compared to that for males at 0.0025 implying that females in Kenya experience higher childhood mortality rates compared to males children.

The parameters D,E and F measures the accident 'hump' resulting from the young adult mortality .Parameter D is higher for males at 0.00785 compared to females at 0.0008.This means that the severity of deaths resulting from accidents is higher for males than for females. E measures the amplitude or the spread of the hump ,For this data,it was more pronounced for males at 11.0139 compared to females at 8.0010.Parameter F gives the position of the accident hump,for females it was less pronounced at 2.135 than for males at 42.99. In general,the accident hump was more pronounced in males compared to females.

The third component and last one measures the middle and old age mortalities depicted by parameters G and H.Parameter G measures the level of ageing in mortality ,in this case for both male and females,it was insignificant but parameter H is more pronounced for males than for females at 1.2153 and for the later at 1.11437.This implies that the rate of increase in adult mortality is higher for males than for females.

4.7 Model Fitting

4.7.1 Helingman Pollard Model Plot on Male Assured Lives

For each gender,deaths data was regressed against the exposure to risk data in R.For males,the following model was fitted.

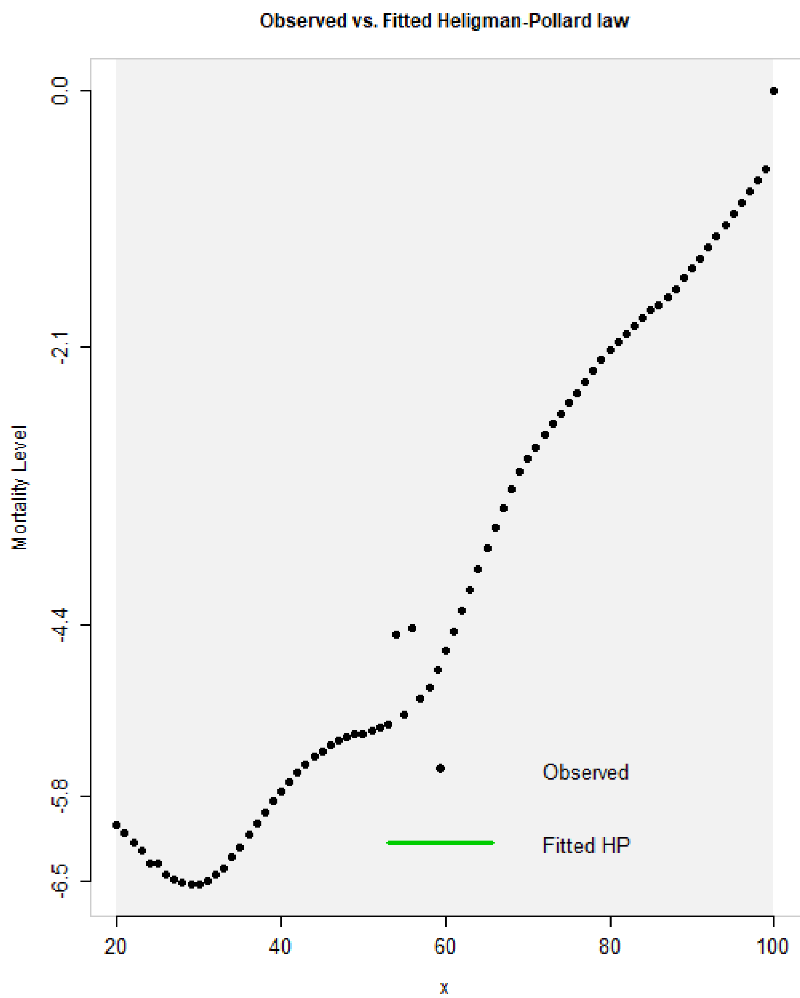

Figure 10. Helingman Pollard Model Graph on Assured Male Lives in Kenya

The male fitted equation becomes

$$q_x = 0.0009^{(0.016+x)^0.0} + 0.00785\exp -11.01(lnx + ln(42.9934)^2 + \frac{0.00000e * 1.2153e^x}{1 + 0.00000e * 1.2153e^x} \quad \textbf{(4.5)}$$

$$= 0.0009 + 0.00785052\exp -11.01(lnx + ln(42.9934)^2) + \frac{0}{1+0} \quad \textbf{(4.6)}$$

$$= 0.0009 + 0.00785052\exp -11.01(lnx + ln(42.9934)^2) \quad \textbf{(4.7)}$$

The resultant equation becomes a quadratic function.

4.7.2 Helingman Pollard Model Plot on Female Assured Lives

On the data range,20 to 100 years,the plot took the following shape.

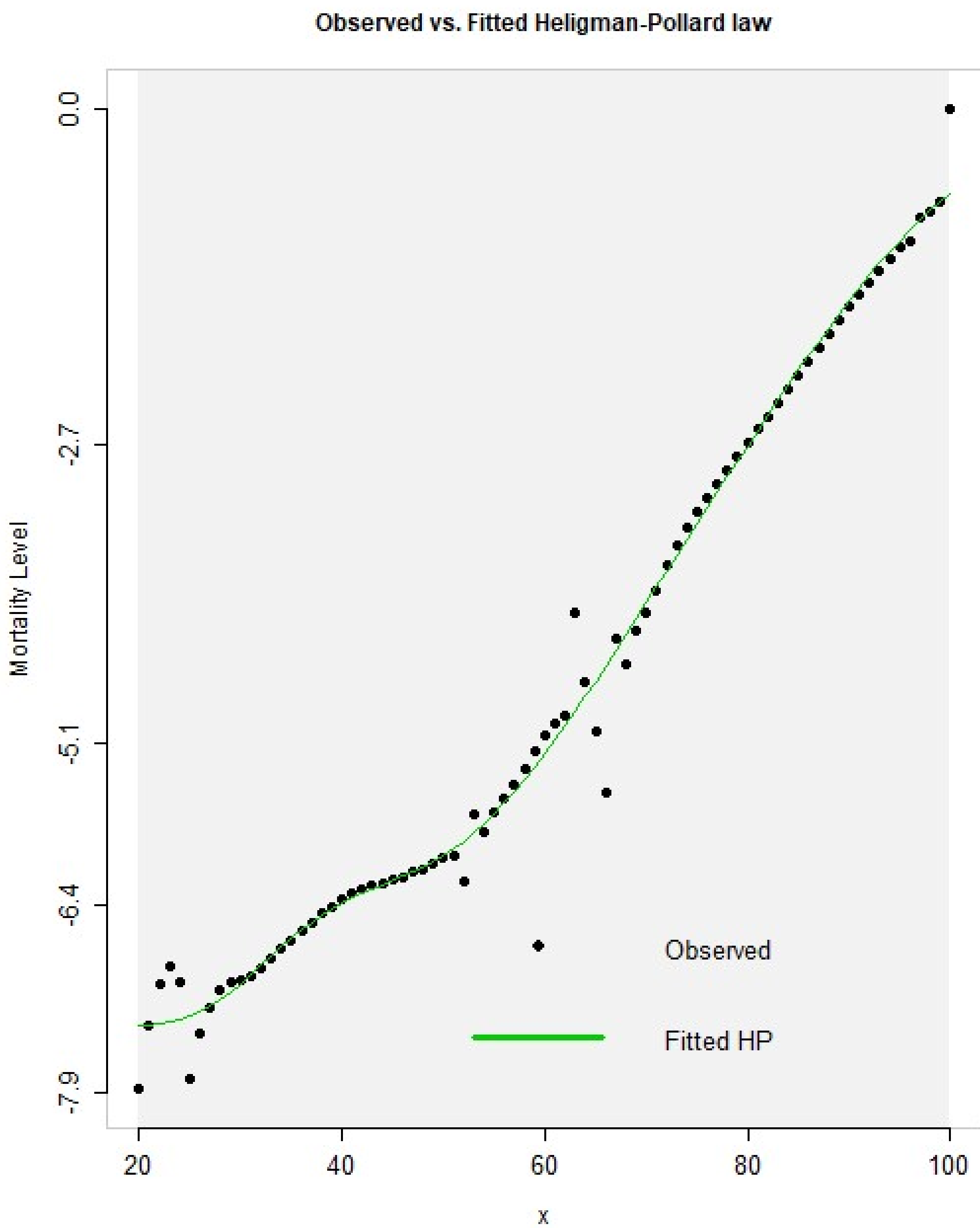

Figure 11. Helingman Pollard Model Graph on Assured Female Lives in Kenya

For female,we fitted the following model

$$q(x) = 0.0006^{(0.0041+x)^{0}.0} + 0.0008\exp -8.001(lnx + ln(2.135)^{2} + \frac{0.0000 * 1.1437^{x}}{1 + 0.0000 * 1.11437^{x}} \quad \textbf{(4.8)}$$

$$= 0.0006 + 0.0008\exp -8.001(lnx + ln(2.135)^{2} + \frac{0}{1+0} \quad \textbf{(4.9)}$$

$$= 0.0006 + 0.0008\exp -8.001(lnx + ln(2.135)^{2}) \quad \textbf{(4.10)}$$

Similar to the males' fitted equation ,females' fitted equation also turns out be quadratic.

4.8 Comparison of Mortality Indices in Kenya and England

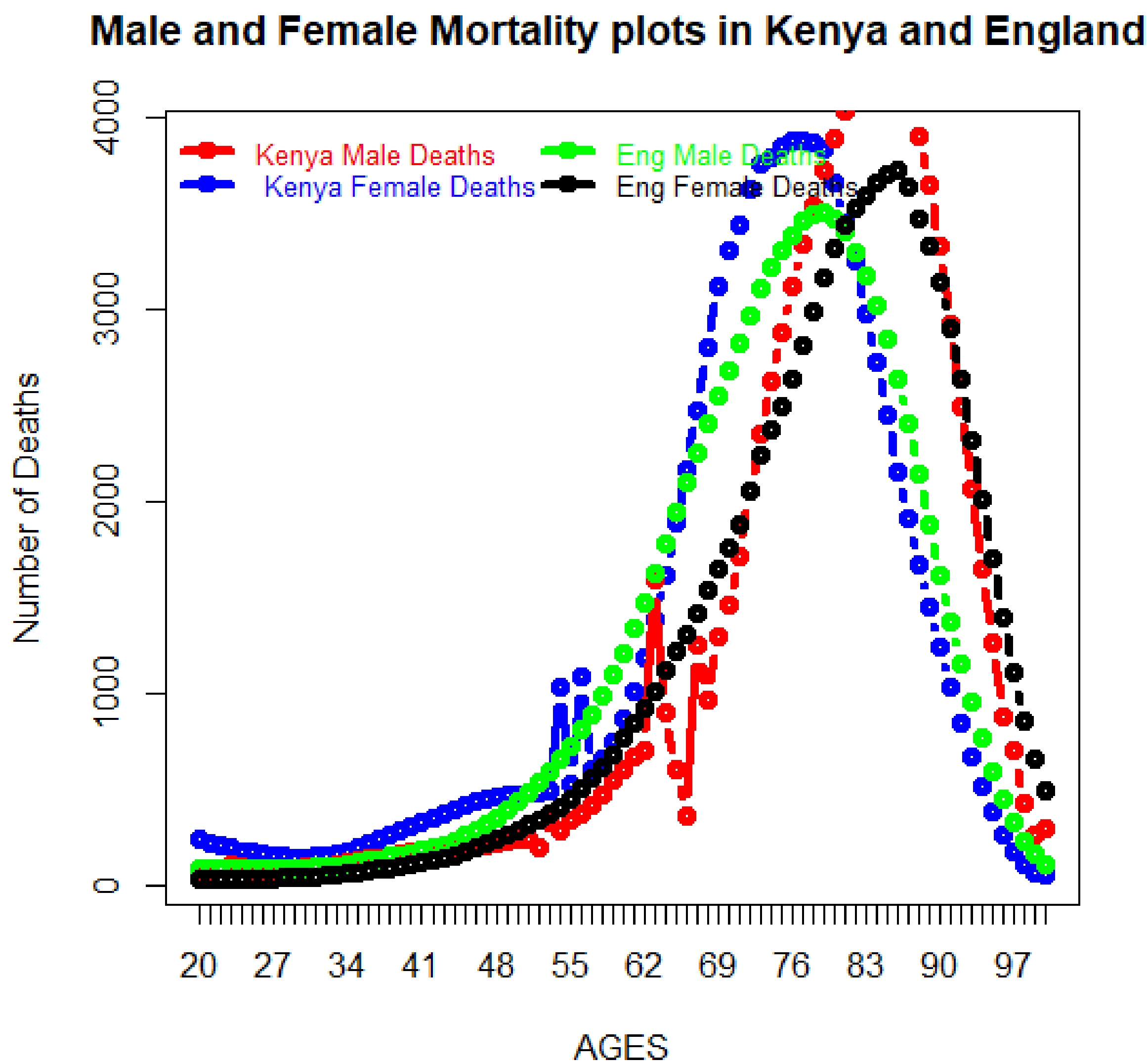

Figure 12. Male and Female Mortality Graphs in Kenya and in England

Kenyan males record the highest number of deaths followed by women in Kenya.English women record the lowest number of deaths.

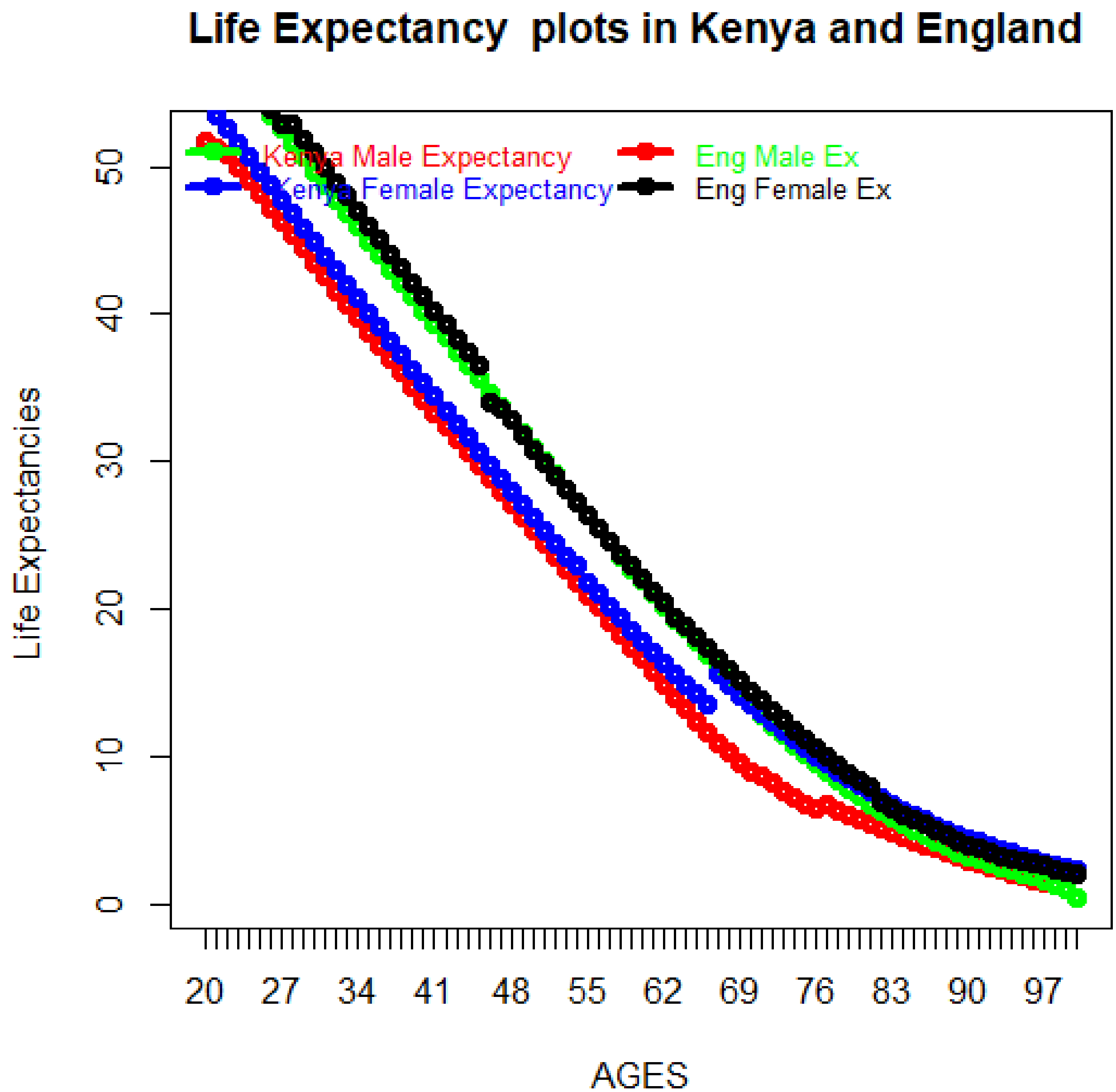

Figure 13. Male and Female Life Expectancies in Kenya and in England

It is interesting to note that ,from this visual plot,women in England have the highest life expectancy,followed by English males. A wide difference is observable between life expectancy in Kenya and that for England.

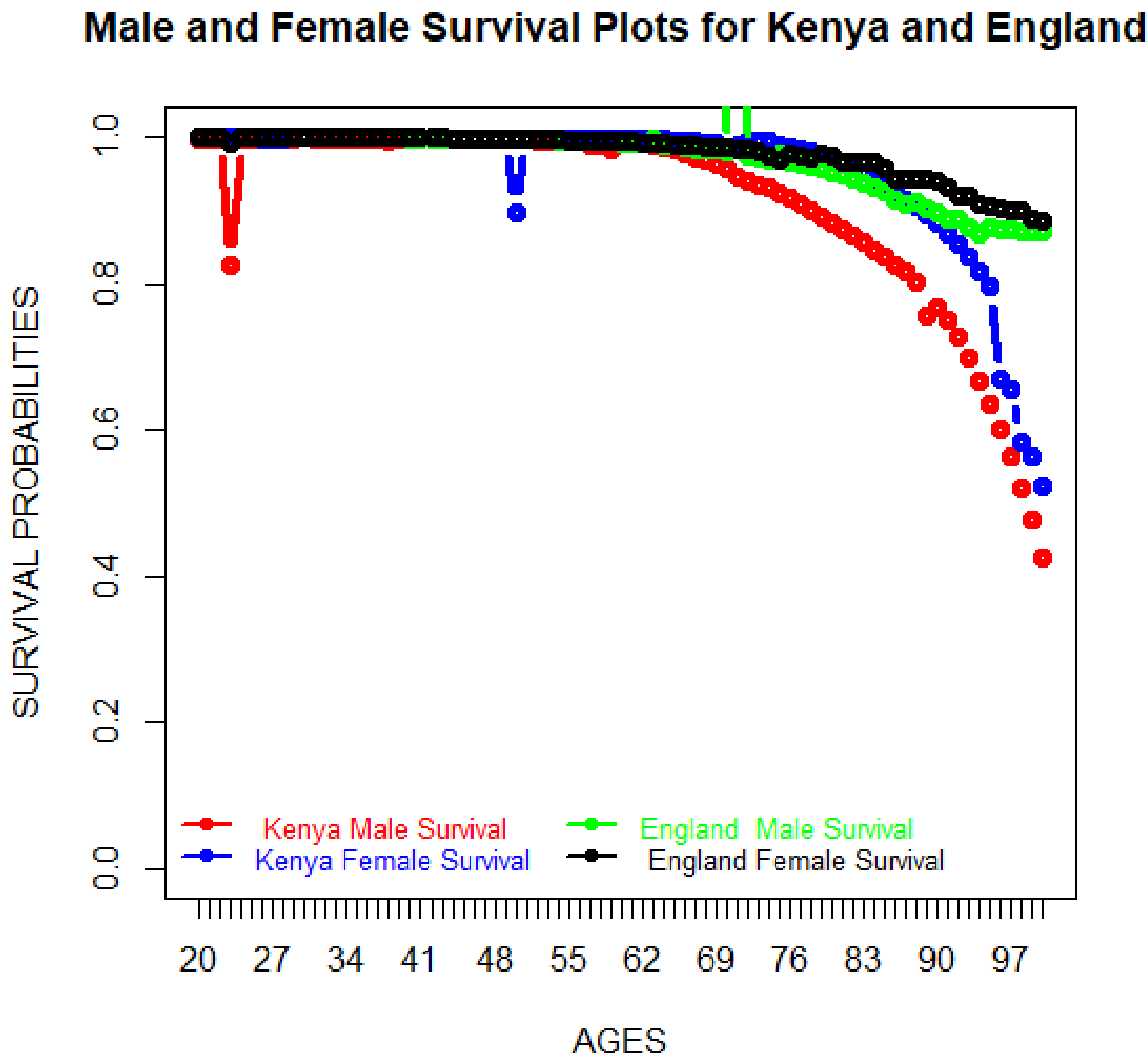

Figure 14. Male and Female Survival plots in Kenya and in England

It is interesting to note that women in both countries have a higher survival rates compared to the males,With English women having the highest survival rates.Simillary,English males have a longer survival rates than Kenyan males.

5 CONCLUSION AND RECOMMENDATION

5.1 Conclusion

Insured lives data from Kenya was used in this work,where life expectancies for both male and female were calculated at age zero, Females had a higher life expectancy as compared to males with 59.54 years and the later with 52.82 years.This is an average of 7 years difference.Meaning, for a male and female born at the same year ,the man will depart 7 years earlier than the woman.This explain why we have many widows than widowers in Kenya as well as many females in Kenya than males.As per 2009 census Kenya had 19,192,458 male and 19,417,639 women population.As of year 2018,male/female sex ratio at age 55 and above was 0.805.

Using R version 3.6.0,we calculated the mortality rates for both male and female lives for the ages 20 to 100 years.For both male and female,mortality rates increased with age getting to the peak at age interval 75-80 years for males and between ages 80 -85 years and decelerating to zero.Comparing the two mortality rates,females had a lower mortality rate than males concurring with the finding that females have a higher life expectancy.

Using R ,we run and fitted survival rates and survival plots for both males and females.Both plots concurred with our expectation that as age increases,survival rates reduces .Comparing the two survival curves,males had a lower survival function compared to females confirming the result that females survive for a longer periods compared to males.Using the same age bracket ,we fitted survival functions for England population for both male and female.Females had a higher survival function compared to English males.Comparing the two countries,females in England have the highest survival rates followed by English males, Kenyan females,then males in Kenya had the lowest survival rates.Generally,females have higher survival rates in both countries than males.

Using the same data,we fitted the Helignman Pollard model of type II. On testing how well the model fits the data,we accepted the hypothesis that the HP mortality model is adequate for describing the age pattern of mortality.

Again using R .6.0 and applying the same model and data,we constructed complete life tables for both males and females. This was consistent with the existing Kenyan Life tables. The analysis ,specifically, the construction of mortality tables was also done using Microsoft Excel and a high correlation was observed between the two tables proving the power in Microsoft spreadsheets.

In general,The Heligman Pollard model describes the age pattern of mortality well by considering a wide range of experiences.Specifically, it captures the three aspects of mortality;that for young children adapting to new environment ,the accident mortality in young adults and that for ageing population.This model also allows the comparisons

of mortality by age and sex across the country over time as it is applicable to the entire age range.All its parameters have a demographic interpretation and together they fully describe the age mortality pattern in our country.
Lastly,R produces exact life tables like those produced by Microsoft Excel,only that it prints the first five rows and the last three rows.This confirms the accuracy of the excel generated life tables.These two tools prove to be powerful in data analytics.

5.2 Recomedations

In this work, we have fitted the Helingman Pollard Model of type II.It can be interesting to know how other mortality models can fit the Kenyan data and the the resemblance of the resulting mortality tables.
In the analysis,we found that that males in both countries ,Kenya and England exit earlier and at a higher rate than females ,this can be an area for further study to establish the social,economic and mortality factors behind this phenomena and its impacts in the society.
In the year 2009,The Actuarial Society of Kenya (TASK),applied the Brass Logit Model on Kenyan experience data on assured lives to construct Kenyan life tables.These matched the mortality assumptions and the mortality experience of Kenyan population.The government and schools specially, tertiary institutions should make provisions for the use of the tables instead of the foreign specially, the United Kingdom table we use today.The industry should as well embrace the tables in their Actuarial valuations and calculations.

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6 Appendices

Table 3. Kenyan Male Survival Rates

Age	Population	survival Rate	Age	Population	survival Rate
20	100000	0.99761	41	95927.2	0.996576049
21	99761	0.997775985	42	95598.75	0.996317943
22	99539.13	0.997931969	43	95246.75	0.996067057
23	99333.28	0.998075066	44	94872.15	0.995827964
24	99142.07	0.998253416	45	94476.34	0.99560599
25	98968.91	0.998262586	46	94061.21	0.995405013
26	98796.96	0.998403999	47	93629	0.995232033
27	98639.28	0.998473935	48	93182.58	0.995089962
28	98488.75	0.998520034	49	92725.05	0.994983988
29	98342.99	0.998540008	50	92259.94	0.994920005
30	98199.41	0.998531967	51	91791.26	0.994790027
31	98055.25	0.998492992	52	91313.03	0.994690024
32	97907.48	0.998423001	53	90828.16	0.994549928
33	97753.08	0.998318007	54	90333.14	0.988572854
34	97588.66	0.998177042	55	89300.89	1.005894566
35	97410.76	0.998004943	56	89827.28	0.987926608
36	97216.42	0.997806029	57	88742.76	0.993240012
37	97003.13	0.997583995	58	88142.86	0.99258
38	96768.77	0.997346045	59	87488.84	0.99149
39	96511.95	0.997094971	60	86744.31	0.990055947
40	96231.58	0.996837005	61	85881.72	0.988263975

Table 4. Kenyan Male Survival Rates

Age	Population	survival Rate	Age	Population	survival Rate
62	84873.81	0.986072029	83	20656.05	0.855999574
63	83691.69	0.983438977	84	17681.57	0.846000101
64	82305.67	0.980363443	85	14958.61	0.836000136
65	80689.47	0.976656681	86	12505.4	0.827999904
66	78805.91	0.972496987	87	10354.47	0.81600024
67	76638.51	0.967699007	88	8449.25	0.803000266
68	74163.01	0.962272028	89	6784.75	0.785999484
69	71364.99	0.956239047	90	5332.81	0.76800036
70	68241.99	0.951600034	91	4095.6	0.748998926
71	64939.08	0.947000019	92	3067.6	0.725687834
72	61497.31	0.940999858	93	2226.12	0.697985733
73	57868.96	0.935000041	94	1553.8	0.667853006
74	54107.48	0.930000067	95	1037.71	0.635167821
75	50319.96	0.923499939	96	659.12	0.599814905
76	46470.48	0.916599958	97	395.35	0.561628936
77	42594.84	0.90900001	98	222.04	0.52053684
78	38718.71	0.900000026	99	115.58	0.476379997
79	34846.84	0.890000069	100	55.06	0.42904
80	31013.69	0.881999852			
81	27354.07	0.874000103			
82	23907.46	0.864000191			

Table 5. Kenyan Female Survival Rates

Age	Population	Survival Rates	Age	Population	Survival Rates
20	100000	0.99962	41	97877	0.998171174
21	99962	0.999369761	42	97698	0.998096174
22	99899	0.99911911	43	97512	0.998051522
23	99811	0.998978069	44	97322	0.997996342
24	99709	0.999097373	45	97127	0.997951136
25	99619	0.999588432	46	96928	0.997905662
26	99578	0.9994075	47	96725	0.997828896
27	99519	0.999266472	48	96515	0.997762006
28	99446	0.99915532	49	96299	0.997684296
29	99362	0.999104285	50	96076	0.997543611
30	99273	0.999083336	51	95840	0.997527129
31	99182	0.999052247	52	95603	0.997970775
32	99088	0.999000888	53	95409	0.996551688
33	98989	0.998919072	54	95080	0.996992007
34	98882	0.998826885	55	94794	0.996497669
35	98766	0.998744507	56	94462	0.996083081
36	98642	0.998631415	57	94092	0.995589423
37	98507	0.998548327	58	93677	0.99502546
38	98364	0.998434387	59	93211	0.994195964
39	98210	0.998350473	60	92670	0.99349304
40	98048	0.998255956			

Table 6. Kenyan Female Survival Rates

AGE	POPULATION	SURVIVAL RATES	AGE	POPULATION	SURVIVAL RATES
61	92067	0.992787861	81	53078	0.924074004
62	91403	0.992297846	82	49048	0.915450171
63	90699	0.982502563	83	44901	0.90572593
64	89112	0.989922794	84	40668	0.894806728
65	88214	0.993198359	85	36390	0.882577631
66	87614	0.995891068	86	32117	0.868792228
67	87254	0.985708392	87	27903	0.853420779
68	86007	0.988849745	88	23813	0.836181917
69	85048	0.984808579	89	19912	0.816994777
70	83756	0.982568413	90	16268	0.795611015
71	82296	0.979136289	91	12943	0.774472688
72	80579	0.974571538	92	10024	0.751296887
73	78530	0.970049663	93	7531	0.725534458
74	76178	0.965488724	94	5464	0.699121523
75	73549	0.960815239	95	3820	0.668848168
76	70667	0.955891717	96	2555	0.655968689
77	67550	0.950643967	97	1676	0.58353222
78	64216	0.944966986	98	978	0.564417178
79	60682	0.938713292	99	552	0.523550725
80	56963	0.931797834	100	289	0.4765

Table 7. Kenyan Male Life table in Microsoft Excel

x	lx	qx	px	dx	Lx	Tx	ex	mx	Ux
20	100000	0.0024	0.9976	239	99880.5	5278596.9	52.79	0.0024	0.0010
21	99761	0.0022	0.9978	221.87	99650.065	5178716.4	51.91	0.0022	0.0009
22	99539.13	0.0021	0.9979	205.85	99436.205	5079066.335	51.03	0.0021	0.0009
23	99333.28	0.0019	0.9981	191.21	99237.675	4979630.13	50.13	0.0019	0.0008
24	99142.07	0.0017	0.9983	173.16	99055.49	4880392.455	49.23	0.0017	0.0008
25	98968.91	0.0017	0.9983	171.95	98882.935	4781336.965	48.31	0.0017	0.0007
26	98796.96	0.0016	0.9984	157.68	98718.12	4682454.03	47.39	0.0016	0.0007
27	98639.28	0.0015	0.9985	150.53	98564.015	4583735.91	46.47	0.0015	0.0007
28	98488.75	0.0015	0.9985	145.76	98415.87	4485171.895	45.54	0.0015	0.0006
29	98342.99	0.0015	0.9985	143.58	98271.2	4386756.025	44.61	0.0015	0.0006
30	98199.41	0.0015	0.9985	144.16	98127.33	4288484.825	43.67	0.0015	0.0006
31	98055.25	0.0015	0.9985	147.77	97981.365	4190357.495	42.73	0.0015	0.0007
32	97907.48	0.0016	0.9984	154.4	97830.28	4092376.13	41.80	0.0016	0.0007
33	97753.08	0.0017	0.9983	164.42	97670.87	3994545.85	40.86	0.0017	0.0008
34	97588.66	0.0018	0.9982	177.9	97499.71	3896874.98	39.93	0.0018	0.0008
35	97410.76	0.0020	0.9980	194.34	97313.59	3799375.27	39.00	0.0020	0.0009
36	97216.42	0.0022	0.9978	213.29	97109.775	3702061.68	38.08	0.0022	0.0010
37	97003.13	0.0024	0.9976	234.36	96885.95	3604951.905	37.16	0.0024	0.0011
38	96768.77	0.0027	0.9973	256.82	96640.36	3508065.955	36.25	0.0027	0.0012
39	96511.95	0.0029	0.9971	280.37	96371.765	3411425.595	35.35	0.0029	0.0013
40	96231.58	0.0032	0.9968	304.38	96079.39	3315053.83	34.45	0.0032	0.0014
41	95927.2	0.0034	0.9966	328.45	95762.975	3218974.44	33.56	0.0034	0.0015
42	95598.75	0.0037	0.9963	352	95422.75	3123211.465	32.67	0.0037	0.0017
43	95246.75	0.0039	0.9961	374.6	95059.45	3027788.715	31.79	0.0039	0.0018
44	94872.15	0.0042	0.9958	395.81	94674.245	2932729.265	30.91	0.0042	0.0019
45	94476.34	0.0044	0.9956	415.13	94268.775	2838055.02	30.04	0.0044	0.0020
46	94061.21	0.0046	0.9954	432.21	93845.105	2743786.245	29.17	0.0046	0.0020
47	93629	0.0048	0.9952	446.42	93405.79	2649941.14	28.30	0.0048	0.0021
48	93182.58	0.0049	0.9951	457.53	92953.815	2556535.35	27.44	0.0049	0.0022
49	92725.05	0.0050	0.9950	465.11	92492.495	2463581.535	26.57	0.0050	0.0022
50	92259.94	0.0051	0.9949	468.68	92025.6	2371089.04	25.70	0.0051	0.0022

Table 8. Kenyan Male Life table in Microsoft Excel

x	lx	qx	px	dx	Lx	Tx	ex	mx	Ux
51	91791.26	0.005209973	0.994790027	478.23	91552.145	2279063.44	24.83	0.0052	0.0023
52	91313.03	0.005309976	0.994690024	484.87	91070.595	2187511.295	23.96	0.0053	0.0023
53	90828.16	0.005450072	0.994549928	495.02	90580.65	2096440.7	23.08	0.0055	0.0037
54	90333.14	0.011427146	0.988572854	1032.25	89817.015	2005860.05	22.21	0.0115	0.0012
55	89300.89	0.005894566	0.9875	526.39	89564.085	1916043.035	21.46	0.0059	0.0014
56	89827.28	0.012073392	0.987926608	1084.52	89285.02	1826478.95	20.33	0.0121	0.0041
57	88742.76	0.006759988	0.993240012	599.9	88442.81	1737193.93	19.58	0.0068	0.0031
58	88142.86	0.00742	0.99258	654.02	87815.85	1648751.12	18.71	0.0074	0.0035
59	87488.84	0.00851	0.99149	744.53	87116.575	1560935.27	17.84	0.0085	0.004
60	86744.31	0.009944053	0.990055947	862.59	86313.015	1473818.695	16.99	0.01	0.0047
61	85881.72	0.011736025	0.988263975	1007.91	85377.765	1387505.68	16.16	0.0118	0.0056
62	84873.81	0.013927971	0.986072029	1182.12	84282.75	1302127.915	15.34	0.014	0.0067
63	83691.69	0.016561023	0.983438977	1386.02	82998.68	1217845.165	14.55	0.0167	0.0079
64	82305.67	0.019636557	0.980363443	1616.2	81497.57	1134846.485	13.79	0.0198	0.0094
65	80689.47	0.023343319	0.976656681	1883.56	79747.69	1053348.915	13.05	0.0236	0.0112
66	78805.91	0.027503013	0.972496987	2167.4	77722.21	973601.225	12.35	0.0279	0.0132
67	76638.51	0.032300993	0.967699007	2475.5	75400.76	895879.015	11.69	0.0328	0.0155
68	74163.01	0.037727972	0.962272028	2798.02	72764	820478.255	11.06	0.0385	0.0181
69	71364.99	0.043760953	0.956239047	3123	69803.49	747714.255	10.48	0.0447	0.0205
70	68241.99	0.048399966	0.951600034	3302.91	66590.535	677910.765	9.93	0.0496	0.0226
71	64939.08	0.052999981	0.947000019	3441.77	63218.195	611320.23	9.41	0.0544	0.025
72	61497.31	0.059000142	0.940999858	3628.35	59683.135	548102.035	8.91	0.0608	0.0278
73	57868.96	0.064999959	0.935000041	3761.48	55988.22	488418.9	8.44	0.0672	0.0304
74	54107.48	0.069999933	0.930000067	3787.52	52213.72	432430.68	7.99	0.0725	0.033
75	50319.96	0.076500061	0.923499939	3849.48	48395.22	380216.96	7.56	0.0795	0.0362
76	46470.48	0.083400042	0.916599958	3875.64	44532.66	331821.74	7.14	0.087	0.0396
77	42594.84	0.09099999	0.90900001	3876.13	40656.775	287289.08	6.74	0.0953	0.0436
78	38718.71	0.099999974	0.900000026	3871.87	36782.775	246632.305	6.37	0.1053	0.0482
79	34846.84	0.109999931	0.890000069	3833.15	32930.265	209849.53	6.02	0.1164	0.0526
80	31013.69	0.118000148	0.881999852	3659.62	29183.88	176919.265	5.7	0.1254	0.0565

Table 9. Kenyan Male Life table in Microsoft Excel

x	lx	qx	px	dx	Lx	Tx	ex	mx	Ux
80	31013.69	0.118000148	0.882	3659.62	29183.88	176919.265	5.7	0.1254	0.0565
81	27354.07	0.125999897	0.874	3446.61	25630.765	147735.385	5.4	0.1345	0.061
82	23907.46	0.135999809	0.864	3251.41	22281.755	122104.62	5.11	0.1459	0.0655
83	20656.05	0.144000426	0.856	2974.48	19168.81	99822.865	4.83	0.1552	0.0701
84	17681.57	0.153999899	0.846	2722.96	16320.09	80654.055	4.56	0.1668	0.0752
85	14958.61	0.163999864	0.836	2453.21	13732.005	64333.965	4.3	0.1786	0.0799
86	12505.4	0.172000096	0.828	2150.93	11429.935	50601.96	4.05	0.1882	0.0851
87	10354.47	0.18399976	0.816	1905.22	9401.86	39172.025	3.78	0.2026	0.0918
88	8449.25	0.196999734	0.803	1664.5	7617	29770.165	3.52	0.2185	0.0999
89	6784.75	0.214000516	0.786	1451.94	6058.78	22153.165	3.27	0.2396	0.1096
90	5332.81	0.23199964	0.768	1237.21	4714.205	16094.385	3.02	0.2624	0.1201
91	4095.6	0.251001074	0.749	1028	3581.6	11380.18	2.78	0.287	0.1324
92	3067.6	0.274312166	0.7257	841.48	2646.86	7798.58	2.54	0.3179	0.1477
93	2226.12	0.302014267	0.698	672.32	1889.96	5151.72	2.31	0.3557	0.1657
94	1553.8	0.332146994	0.6679	516.09	1295.755	3261.76	2.1	0.3983	0.1862
95	1037.71	0.364832179	0.6352	378.59	848.415	1966.005	1.89	0.4462	0.2095
96	659.12	0.400185095	0.5998	263.77	527.235	1117.59	1.7	0.5003	0.2363
97	395.35	0.438371064	0.5616	173.31	308.695	590.355	1.49	0.5614	0.267
98	222.04	0.47946316	0.5205	106.46	168.81	281.66	1.27	0.6306	0.3028
99	115.58	0.523620003	0.4764	60.52	85.32	112.85	0.98	0.7093	0.3015
100	55.06	0.47638	0.5236	55.06	27.53	27.53	0.5		

Table 10. Kenyan Female Life table in Microsoft Excel

x	lx	qx	px	dx	Lx	Tx	ex	mx	Ux
20	100000	0.00038	0.9996	38	99981	5930087	59.3	0.000380072	0.0002
21	99962	0.000630239	0.9994	63	99930.5	5830106	58.32	0.000630438	0.0003
22	99899	0.00088089	0.9991	88	99855	5730175.5	57.36	0.000881278	0.0003
23	99811	0.001021931	0.999	102	99760	5630320.5	56.41	0.001022454	0.0003
24	99709	0.000902627	0.9991	90	99664	5530560.5	55.47	0.000903034	0.0002
25	99619	0.000411568	0.9996	41	99598.5	5430896.5	54.52	0.000411653	0.0002
26	99578	0.0005925	0.9994	59	99548.5	5331298	53.54	0.000592676	0.0002
27	99519	0.000733528	0.9993	73	99482.5	5231749.5	52.57	0.000733797	0.0003
28	99446	0.00084468	0.9992	84	99404	5132267	51.61	0.000845036	0.0003
29	99362	0.000895715	0.9991	89	99317.5	5032863	50.65	0.000896116	0.0003
30	99273	0.000916664	0.9991	91	99227.5	4933545.5	49.7	0.000917084	0.0003
31	99182	0.000947753	0.9991	94	99135	4834318	48.74	0.000948202	0.0003
32	99088	0.000999112	0.999	99	99038.5	4735183	47.79	0.000999611	0.0003
33	98989	0.001080928	0.9989	107	98935.5	4636144.5	46.83	0.001081513	0.0004
34	98882	0.001173115	0.9988	116	98824	4537209	45.89	0.001173804	0.0004
35	98766	0.001255493	0.9987	124	98704	4438385	44.94	0.001256281	0.0004
36	98642	0.001368585	0.9986	135	98574.5	4339681	43.99	0.001369523	0.0004
37	98507	0.001451673	0.9985	143	98435.5	4241106.5	43.05	0.001452728	0.0005
38	98364	0.001565613	0.9984	154	98287	4142671	42.12	0.00156684	0.0005
39	98210	0.001649527	0.9984	162	98129	4044384	41.18	0.001650888	0.0005
40	98048	0.001744044	0.9983	171	97962.5	3946255	40.25	0.001745566	0.0006
41	97877	0.001828826	0.9982	179	97787.5	3848292.5	39.32	0.0018305	0.0006
42	97698	0.001903826	0.9981	186	97605	3750505	38.39	0.00190564	0.0006
43	97512	0.001948478	0.9981	190	97417	3652900	37.46	0.001950378	0.0006
44	97322	0.002003658	0.998	195	97224.5	3555483	36.53	0.002005667	0.0006
45	97127	0.002048864	0.998	199	97027.5	3458258.5	35.61	0.002050965	0.0006
46	96928	0.002094338	0.9979	203	96826.5	3361231	34.68	0.002096533	0.0007
47	96725	0.002171104	0.9978	210	96620	3264404.5	33.75	0.002173463	0.0007
48	96515	0.002237994	0.9978	216	96407	3167784.5	32.82	0.002240501	0.0007
49	96299	0.002315704	0.9977	223	96187.5	3071377.5	31.89	0.002318389	0.0008

Table 11. Kenyan Female Life table in Microsoft Excel

x	lx	qx	px	dx	Lx	Tx	ex	mx	Ux
50	96076	0.002456389	0.9975	236	95958	2975190	30.967	0.002459409	0.000770279
51	95840	0.002472871	0.9975	237	95721.5	2879232	30.042	0.002475933	0.000675184
52	95603	0.002029225	0.998	194	95506	2783510.5	29.115	0.002031286	0.000942396
53	95409	0.003448312	0.9966	329	95244.5	2688004.5	28.173	0.003454268	0.000981149
54	95080	0.003007993	0.997	286	94937	2592760	27.269	0.003012524	0.001047141
55	94794	0.003502331	0.9965	332	94628	2497823	26.35	0.003508475	0.001184644
56	94462	0.003916919	0.9961	370	94277	2403195	25.441	0.003924605	0.001331935
57	94092	0.004410577	0.9956	415	93884.5	2308918	24.539	0.004420325	0.001502255
58	93677	0.00497454	0.995	466	93444	2215033.5	23.645	0.004986944	0.001737563
59	93211	0.005804036	0.9942	541	92940.5	2121589.5	22.761	0.005820928	0.001970834
60	92670	0.00650696	0.9935	603	92368.5	2028649	21.891	0.0065282	0.00219278
61	92067	0.007212139	0.9928	664	91735	1936280.5	21.031	0.007238241	0.002367977
62	91403	0.007702154	0.9923	704	91051	1844545.5	20.18	0.00773193	0.004576581
63	90699	0.017497437	0.9825	1587	89905.5	1753494.5	19.333	0.017651868	0.003887547
64	89112	0.010077206	0.9899	898	88663	1663589	18.669	0.010128238	0.002445841
65	88214	0.006801641	0.9932	600	87914	1574926	17.853	0.006824852	0.001541327
66	87614	0.004108932	0.9959	360	87434	1487012	16.972	0.004117391	0.003520055
67	87254	0.014291608	0.9857	1247	86630.5	1399578	16.04	0.014394468	0.003812018
68	86007	0.011150255	0.9888	959	85527.5	1312947.5	15.266	0.011212768	0.004400504
69	85048	0.015191421	0.9848	1292	84402	1227420	14.432	0.015307694	0.005292838
70	83756	0.017431587	0.9826	1460	83026	1143018	13.647	0.017584853	0.006278802
71	82296	0.020863711	0.9791	1717	81437.5	1059992	12.88	0.021083653	0.00764305
72	80579	0.025428462	0.9746	2049	79554.5	978554.5	12.144	0.025755928	0.009121632
73	78530	0.029950337	0.97	2352	77354	899000	11.448	0.030405667	0.010617054
74	76178	0.034511276	0.9655	2629	74863.5	821646	10.786	0.035117247	0.012154893
75	73549	0.039184761	0.9608	2882	72108	746782.5	10.154	0.039967826	0.01377555
76	70667	0.044108283	0.9559	3117	69108.5	674674.5	9.547	0.04510299	0.015512876
77	67550	0.049356033	0.9506	3334	65883	605566	8.965	0.05060486	0.017402706
78	64216	0.055033014	0.945	3534	62449	539683	8.404	0.056590178	0.019496036
79	60682	0.061286708	0.9387	3719	58822.5	477234	7.865	0.063224106	0.021836338
80	56963	0.068202166	0.9318	3885	55020.5	418411.5	7.345	0.070610045	0.024478071

Table 12. Kenyan Female Life table in Microsoft Excel

x	lx	qx	px	dx	Lx	Tx	ex	mx	Ux
80	56963	0.0682	0.9318	3885	55020.5	418411.5	7.35	0.0706	0.0245
81	53078	0.0759	0.9241	4030	51063	363391	6.85	0.0789	0.0275
82	49048	0.0845	0.9155	4147	46974.5	312328	6.37	0.0883	0.0309
83	44901	0.0943	0.9057	4233	42784.5	265353.5	5.91	0.0989	0.0348
84	40668	0.1052	0.8948	4278	38529	222569	5.47	0.111	0.0393
85	36390	0.1174	0.8826	4273	34253.5	184040	5.06	0.1247	0.0445
86	32117	0.1312	0.8688	4214	30010	149786.5	4.66	0.1404	0.0505
87	27903	0.1466	0.8534	4090	25858	119776.5	4.29	0.1582	0.0575
88	23813	0.1638	0.8362	3901	21862.5	93918.5	3.94	0.1784	0.0656
89	19912	0.183	0.817	3644	18090	72056	3.62	0.2014	0.075
90	16268	0.2044	0.7956	3325	14605.5	53966	3.32	0.2277	0.0853
91	12943	0.2255	0.7745	2919	11483.5	39360.5	3.04	0.2542	0.0968
92	10024	0.2487	0.7513	2493	8777.5	27877	2.78	0.284	0.1104
93	7531	0.2745	0.7255	2067	6497.5	19099.5	2.54	0.3181	0.126
94	5464	0.3009	0.6991	1644	4642	12602	2.31	0.3542	0.1448
95	3820	0.3312	0.6688	1265	3187.5	7960	2.08	0.3969	0.1588
96	2555	0.344	0.656	879	2115.5	4772.5	1.87	0.4155	0.1987
97	1676	0.4165	0.5835	698	1327	2657	1.59	0.526	0.2404
98	978	0.4356	0.5644	426	765	1330	1.36	0.5569	0.2802
99	552	0.4764	0.5236	263	420.5	565	1.02	0.6254	0.0716
100	289		0.4773		144.5	144.5	0.5		

Table 13. Kenyan Male Life table In R

x	mx	qx	ax	lx	dx	Lx	Tx	ex
20	0.0024	0.0024	0.5	100000	239	99881	5282393	52.82
21	0.0022	0.0022	0.5	99761	222	99650	5182512	51.95
22	0.0021	0.0021	0.5	99540	20	99437	5082862	51.06
23	0.0019	0.0019	0.5	99334	191	99238	4983425	50.17
24	0.0017	0.0017	0.5	99143	173	99056	4884187	49.26
25	0.0017	0.0017	0.5	98970	172	98884	4785130	48.35
98	0.4795	0.3809	0.46	475	181	377	685	1.44
99	0.5236	0.4076	0.46	294	120	229	308	1.05
100	1	0.6321	0.46	174	174	79	79	0.46

Table 14. Kenyan Female Life table In R

x	mx	qx	ax	lx	dx	Lx	Tx	ex
20	0.0004	0.0004	0.5	100000	38	99981	5953640	59.54
21	0.0006	0.0006	0.5	99962	63	99931	5853659.56	58.56
22	0.0009	0.0009	0.5	99899	88	99855	5753729	57.6
23	0.001	0.001	0.5	99811	102	99760	5653874	56.65
24	0.0009	0.0009	0.5	99709	90	99664	5554113	55.7
25	0.0004	0.0004	0.5	99619	41	99599	5454449	54.54
98	0.435	0.3531	0.46	1708	603	1384	2579	0.51
99	0.4764	0.379	0.46	1105	419	879	1195	1.08
100	1	0.6321	0.46	686	686	316	316	0.46

6.1 R CODES USED IN THE ANALYSIS

```
##Male Survival
ages=c(20:100)
ages
survm=c(0.99761,0.99778,0.99808,0.825,0.99826,0.99840,0.99808,
0.99852,0.99849,0.99842,0.99998,0.99781,0.99753,0.99735,0.99684,
0.99658,0.99632,0.99607,0.99583,0.99842,0.99832,0.99818,0.99998,
0.99781,0.99758,0.99735,0.99709,0.99684,0.99684,0.99658,0.99632,
0.99607,0.99583,0.99541,0.99498,0.99492,0.99455,0.98857,0.98891,
0.98324,0.99324,0.99258,0.99250,0.9883,0.9861,0.9834,0.9767,0.9725,
0.9677,0.9623,0.9562,0.9470,0.9410,0.9350,0.93,0.9235,0.9166,0.909,
0.9,0.89,0.882,0.874,0.8640,0.856,0.8460,0.8360,0.826,0.816,0.803,
0.756,0.768,0.749,0.7257,0.698,0.6679,0.6352,0.5998,0.5616,0.5205,
0.4764,0.426)

Ukmalesurv=c(0.99916,0.99915,0.99914,0.9914,0.99912,0.99914,0.99915,0.99915,0.99913,0.9991,0.99914,

0.99915,0.99915,0.99913,0.9991,0.99909,0.99906,0.99903,0.99901,0.99898,0.99873,0.99862,0.99851,

0.9984,0.99828,0.99814,0.99799,0.99781,0.99781,0.9976,0.99703,0.99668,0.99668,0.99629,0.99585,
0.99536,0.99481,0.99423,0.99423,0.99358,0.99286,0.99203,0.9911,0.99605,0.98888,0.98757,0.98608,
0.9844,0.9844,0.98251,0.98035,097801,0.97553,0.97289,0.97003,0.97708,0.96708,0.96398,0.9607,
0.95689,0.95255,0.94783,0.94303,0.93803,0.93223,0.92582,0.915,0.909,0.91162,0.90384,0.89589,
0 .88721,0.88721,0.8775,0.8673,0.87765,0.873,0.87250,0.87200,0.87198,0.87190)
Ukmalesurv
Ukfemalesurv=c(0.99920,0.99920,0.99919,0.9918,0.99917,0.99916,0.99916,0.99915,0.99914,0.99915,
0.99913,0.99915,0.99914,0.999135,0.99909,0.99909,0.99908,0.99907,0.99907,0.99908,0.99906,
0.99905,0.99904,0.99901,0.9981,0.9980,0.9984,0.9974,0.99861,0.99800,0.99791,0.99701,
0.99668,0.99650,0.99642,0.99589,0.99541,0.99480,0.99500,0.99488,0.99342,0.99338,
0.99300,0.99289,0.99100,0.98900,0.98960,0.98870,0.98670,0.98500,0.98490,0.98380,
0.98200,0.98061,0.98075,0.987,0.987990.97500,0.97300,0.97600,0.97556,0.96600,
0.96451,0.96452,0.96589,0.95600,0.94289,0.94233,0.94230,0.94177,0.94101,0.9300,0.9200,
0.9187,0.9085,0.9055,0.9015,0.89900,0.89888,0.88700,0.88600)
Ukfemalesurv
data10=data.frame(ages,survm,survf,Ukmalesurv,Ukfemalesurv)
data10
attach(data10)
```

#Female Survival

```
survf=c(0.99965,0.99963,0.99962,0.9993,0.9991,0.99897,0.99852,0.99849,
0.99842,0.99998,0.999,0.999588,0.99941,0.99945,0.99942,0.99926,
0.99916,0.9991,0.99908,0.99905,0.999,0.99891,0.99882,0.99874,
0.99854,0.99843,0.99835,0.99825,0.99817,0.9981,0.898,0.99799,
0.99795,0.9979,0.99782,0.99776,0.99763,0.99768,0.99754,0.99752,
0.99797,0.99655,0.99699,0.99649,0.99608,0.99558,0.99419,0.99349,
0.99278,0.99229,0.9828,0.9899,0.99319,0.99589,0.9957,0.98884,0.9848,
0.9825,0.9791,0.9745,0.9700,0.9654,0.9558,0.9506,0.9387,0.9317,0.92407,
0.9154,0.9057,0.8948,0.8825,0.86787,0.8534,0.8361,0.8169,0.7956,0.6688,
0.6559,0.5835,0.5644,0.5235)
survf
data2=data.frame(ages,survm,survf)
data2
attach(data2)
```

```
plot(survm,main='Male and Female Survival Plots ',type='b',lwd=2,
ylab='SURVIVAL PROBABILITIES',xlab='AGES',xaxt='n',ylim=c(0,1),col='red')
axis(1,at=1:length(ages),labels=ages)
lines(survf,col='blue',type='b',lwd=2)
legend('bottomleft',legend=c('Male Survival','Female Survival'),
lty=1,lwd=2,pch=21,col=c('red','blue'),ncol=2,bty='n',cex=0.8,
text.col=c('red','blue'),inset=0.01)
```

#Expectancy Comparison

malelifeexp=c(51.6835124,51.23,50.80613406,49.91826506,49.02067535,48.11425457,
47.1975624,46.27883662,45.35201646,44.42056819,43.48566548,42.54851597,41.61033535,
40.67238244,39.73583431,38.8019399,37.87189023,36.94659832,36.02673692,35.11277755,
34.20488276,33.30308169,32.40716689,31.51679081,30.63141845,29.7503915,28.87293644,
27.99815764,27.12509458,26.25265033,25.37972074,24.50514687,23.62771554,22.74884138,
21.86761286,20.98470606,20.22149359,19.10592517,18.33330708,17.4546805,16.58142467,
15.7194522,14.87231532,14.04299224,13.23428294,12.44883251,11.68852008,10.95551381,
10.25120432,9.576691467,8.932562942,8.766989462,8.229661346,7.714306599,7.215835592,
6.721328077,6.52759883,6.74469208,6.369848195,6.022053363,5.70455386,5.400855704,
5.107385728,4.832621193,4.561475876,4.300798336,4.046408751,3.783102853,3.523409178,
3.265140941,3.017993328,2.778635609,2.542241492,2.314214867,2.099214828,1.894561101,
1.695578954,1.49324649,1.268510178,0.976379997,0.5)

malelifeexp

femaleexpectancy=c(59.30087,58.32322282,57.35968829,56.40981956,55.46701401,54.51667353,
53.53891422,52.57035842,51.60858154,50.65178841,49.69675038,48.74188865,47.7876534,
46.83494631,45.88508525,44.93838973,43.99425194,43.05385912,42.11572323,41.18097953,
40.24819476,39.31763846,38.38875924,37.46103044,36.5331888,35.60553193,34.67760606,
33.74933575,32.82168057,31.89417855,30.96704692,30.04207012,29.11530496,28.17348992,
27.26924695,26.3500116,25.44086511,24.53894061,23.64543591,22.76114944,21.89110823,
21.03121097,20.1803606,19.33311834,18.66851827,17.85346997,16.97231036,16.04027323,
15.26558885,14.43208541,13.64699842,12.88023719,12.14403877,11.44785432,10.78586994,
10.1535371,9.547235626,8.964707624,8.404182758,7.864506773,7.345320647,6.84635819,
6.367802969,5.909745885,5.47282876,5.057433361,4.663776193,4.292602946,3.944001176,
3.618722378,3.317310057,3.041064668,2.781025539,2.536117381,2.30636896,2.083769634,
1.867906067,1.585322196,1.3599182,1.023550725,0.5)

femaleexpectancy

Ukmaleexp=c(54.452,53.497,52.54,51.58,50.63,49.67,48.72,47.76,46.8,45.84,44.88,43.92,
42.96,42,41.04,40,39.19,38.185,37.237,36.292,35.349,34.409,33.473,32.53,31.609,30.684,
29.765,28.852,27.97,27.049,26.159,25.279,24.408,23.54,22.96,21.85,21.02,20.21,19.49,
18.6,17.85,17.09,16.35,15.64,14.9,14.26,13.612,15.64,14.93,14.2,13.612,12.97,12.36,
11.76,11.18,10.62,10.08,9.55,9.05,8.57,8.1,7.6,7.2,6.8,6.4,6.07,5.7,5.3,5.0,4.76,
4.478,4.21,3.96,3.73,3.508,3.285,3.07,2.872,2.69,2.53,2.381)

Ukmaleexp

Ukfemale=c(59.748,58.76,57.7,56.8,55.8,54.84,53.86,52.86,52.87,51.89,50.91,49.93,

```
plot(survm,main='Male and Female Survival Plots for Kenya and England ',type='b',lwd=4,
ylab='SURVIVAL PROBABILITIES',xlab='AGES',xaxt='n',ylim=c(0,1),col='red')
axis(1,at=1:length(ages),labels=ages)
lines(survf,col='blue',type='b',lwd=4)
lines(Ukmalesurv,col='green',type='b',lwd=4)
lines(Ukfemalesurv,col='black',type='b',lwd=4)
legend('bottomleft',legend=c(' Kenya Male Survival','Kenya Female Survival', 'England
Male Survival',' England Female Survival'),
lty=1,lwd=2,pch=21,col=c('red','blue','green','black'),ncol=2,bty='n',cex=0.8,
text.col=c('red','blue','green','black'),inset=0.01)
```

Code for fitting male survival curve.

```
survm=c(0.99761,0.99778,0.99793,0.99808,0.825,0.99826,0.99840,0.99847,0.99808,0.99852,0.99854,0.99853,0.99
survm plot(survm,type=' b',lwd=2,ylab='SURVIVAL PROBABILITIES',xaxt='n',xlim=c(0,100),col='red',xlab=
PLOT FOR MALES IN KENYA') axis(1,at=1:length(ages),labels=ages) legend('bottomleft',legend=c('Males
survival curve'), lty=1,lwd=2,pch=21,col=c('red'),ncol=2,bty='n',cex=0.8,text.col=c('red'),inset=0.01)
```

Codes for fitting male death plot plot(maledx,type='b',lwd=2,ylab='NUMBER OF DEATHS',xaxt='n',xli
FOR MALES IN KENYA') axis(1,at=1:length(ages),labels=ages) legend('bottomleft',legend=c('Male
Deaths'), lty=1,lwd=2,pch=21,col=c('red'),ncol=2,bty='n',cex=0.8,text.col=c('red'),inset=0.01)

Codes for fitting female deaths plot

```
f=plot(femaledx,type='b',lwd=4,ylab='NUMBER OF DEATHS',xaxt='n',xlim=c(0,100),col='blue',xlab='AGES
GRAPH FOR FEMALE MORTALITY IN KENYA') f axis(1,at=1:length(ages),labels=ages) leg-
end('topright',legend=c('FEMALE DEATHS'), lty=1,lwd=2,pch=21,col=c('blue'),ncol=2,bty='n',cex=0.8,text.
```

Codes for fitting male and female deaths plot

```
data=data.frame(ages,femaledx,maledx) data attach(data) plot(femaledx,main='Male and
Female Mortality Charts',type='b',plot(maledx,type='b',lwd=2,ylab='NUMBER OF DEATHS',xlab='AGES',l
GRAPH FOR MALE MORTALITY IN KENYA',col='red') axis(1,at=1:length(ages),labels=ages)
legend('topright',legend=c('Male Deaths'), lty=1,lwd=2,pch=21,col=c('blue'),ncol=2,bty='n',cex=0.8,text.col
OF DEATHS') axis(1,at=1:length(ages),labels=ages) lines(maledx,col='blue',type='b',lwd=2)
legend('topright',legend=c('FEMALE DEATHS','MALE DEATHS'), lty=1,lwd=2,pch=21,col=c('red','blue'),nco
```

Code for fitting male survival curve.

```
survm=c(0.99761,0.99778,0.99793,0.99808,0.825,0.99826,0.99840,0.99847,0.99808,0.99852,0.99854,0.99853,0.99
survm plot(survm,type=' b',lwd=2,ylab='SURVIVAL PROBABILITIES',xaxt='n',xlim=c(0,100),col='red',xlab=
PLOT FOR MALES IN KENYA') axis(1,at=1:length(ages),labels=ages) legend('bottomleft',legend=c('Males
survival curve'), lty=1,lwd=2,pch=21,col=c('red'),ncol=2,bty='n',cex=0.8,text.col=c('red'),inset=0.01)
```

Code for fitting female survival curve.

```
survf=c(0.99962,0.9993,0.9991,0.99897,0.999,0.999588,0.99941,0.99926,0.99916,0.9991,0.99908,0.99905,0.999,0.99891,0
survf plot(survf,type=' b',lwd=2,ylab='SURVIVAL PROBABILITIES',xaxt='n',xlim=c(0,100),col='blue',xlab='AGES',n
PLOT FOR FEMALES IN KENYA') axis(1,at=1:length(ages),labels=ages) legend('bottomleft',legend=c('Females
survival curve'),lty=1,lwd=2,pch=21,col=c('blue'),ncol=2,bty='n',cex=0.8,text.col=c('blue'),inset=0.01)
```

Code for fitting male survival curve

```
survm=c(0.99761,0.99778,0.99793,0.99808,0.825,0.99826,0.99840,0.99847,0.99808,0.99852,0.99854,0.99853,0.99849,0.9
survm plot(survm,type='b',lwd=2,ylab='SURVIVAL PROBABILITIES',xaxt='n',xlim=c(0,100),col='red',xlab='AGES',
PLOT FOR MALES IN KENYA') axis(1,at=1:length(ages),labels=ages) legend('bottomleft',legend=c('Males
survival curve'), lty=1,lwd=2,pch=21,col=c('red'),ncol=2,bty='n',cex=0.8,text.col=c('red'),inset=0.01)
```

Code for fitting superimposed male and female survival curve

```
ages=c(20:100) ages survm=c(0.99761,0.99778,0.99808,0.825,0.99826,0.99840,0.99808,0.99852,0.99849,0.99842,0.9999
,0.99753,0.99735,0.99684,0.99658,0.99632,0.99607,0.99583,0.99842,0.99832,0.99818,0.99998,0.99781,0.99758.99735,0.9
survm survf=c(0.99965,0.99963,0.99962,0.9993,0.9991,0.99897,0.99852,0.99849,0.99842,0.99998,0.999,0.999588,0.9994
survf data2=data.frame(ages,survm,survf) data2 attach(data2)
```

Codes for fitting male expectancy plot

```
malelifeexp=c(51.6835124,50.80613406,49.91826506,49.02067535,48.11425457,47.1975624,46.27883662,45.35201646,4
malelifeexp plot(malelifeexp,type=' b',ylim=c(0,55),lwd=2,ylab='LIFE EXPECTANCIES',xaxt='n',xlim=c(0,100),col='
EXPECTANCY PLOT FOR MALES IN KENYA') axis(1,at=1:length(ages),labels=ages) leg-
end('bottomleft',legend=c('Males life expectancy'), lty=1,lwd=2,pch=21,col=c('red'),ncol=2,bty='n',cex=0.8,text.col
```

Codes for ploting female expectancy in Kenya

```
malelifeexp=c(51.6835124,50.80613406,49.91826506,49.02067535,48.11425457,47.1975624,46.27883662,45.35201646,4
malelifeexp plot(malelifeexp,type=' b',ylim=c(0,55),lwd=2,ylab='LIFE EXPECTANCIES',xaxt='n',xlim=c(0,100),col='
EXPECTANCY PLOT FOR MALES IN KENYA') axis(1,at=1:length(ages),labels=ages) leg-
end('bottomleft',legend=c('Males life expectancy'), lty=1,lwd=2,pch=21,col=c('red'),ncol=2,bty='n',cex=0.8,text.col
```

Codes for plotting male and female expectancies

```
ages=c(0:100) ages malelifeexp=c(51.6835124,50.80613406,49.91826506,49.02067535,48.11425457,47.1975624,
46.27883662,45.35201646,44.42056819,43.48566548,42.54851597,41.61033535,40.67238244,
39.73583431,38.8019399,37.87189023,36.94659832,36.02673692,35.11277755,34.20488276,
33.30308169,32.40716689,31.51679081,30.63141845,29.7503915,28.87293644,27.99815764,
27.12509458,26.25265033,25.37972074,24.50514687,23.62771554,22.74884138,21.86761286,
20.98470606,20.22149359,19.10592517,18.33330708,17.4546805,16.58142467,15.7194522,
14.87231532,14.04299224,13.23428294,12.44883251,11.68852008,10.95551381,10.25120432,
9.576691467,8.932562942,8.766989462,8.229661346,7.714306599,7.215835592,6.721328077,
```

6.52759883,6.74469208,6.369848195,6.022053363,5.70455386,5.400855704,5.107385728,
4.832621193,4.561475876,4.300798336,4.046408751,3.783102853,3.523409178,3.265140941,
3.017993328,2.778635609,2.542241492,2.314214867,2.099214828,1.894561101,1.695578954,
1.49324649,1.268510178,0.976379997,0.5) malelifeexp femaleexpectancy=c(59.30087,58.32322282,57.3596882
femaleexpectancy data3=data.frame(ages,survm,survf) data3 attach(data3) plot(malelifeexp,main='Male and Female Expectancy Plots ',type='b',lwd=2,ylab='LIFE EXPECTANCY',xlab='AGES',xaxt='n',ylim=c(0,60
axis(1,at=1:length(ages),labels=ages) lines(femaleexpectancy,col='blue',type='b',lwd=2) legend('bottomleft',legend=c('Male Expectancy','Female Expectancy'), lty=1,lwd=2,pch=21,col=c('red','blue'),r

Codes for fitting male death plot

plot(maledx,type='b',lwd=2,ylab='NUMBER OF DEATHS',xaxt='n',xlim=c(0,100),col='red',xlab='AGES',ma
FOR MALES IN KENYA') axis(1,at=1:length(ages),labels=ages) legend('bottomleft',legend=c('Male Deaths'), lty=1,lwd=2,pch=21,col=c('red'),ncol=2,bty='n',cex=0.8,text.col=c('red'),inset=0.01)

Codes for fitting female deaths plot

f=plot(femaledx,type='b',lwd=4,ylab='NUMBER OF DEATHS',xaxt='n',xlim=c(0,100),col='blue',xlab='AGES
GRAPH FOR FEMALE MORTALITY IN KENYA') f axis(1,at=1:length(ages),labels=ages) legend('topright',legend=c('FEMALE DEATHS'), lty=1,lwd=2,pch=21,col=c('blue'),ncol=2,bty='n',cex=0.8,text.

Codes for fitting male and female deaths plot

data=data.frame(ages,femaledx,maledx) data attach(data) plot(femaledx,main='Male and Female Mortality Charts',type='b',plot(maledx,type='b',lwd=2,ylab='NUMBER OF DEATHS',xlab='AGES',l
GRAPH FOR MALE MORTALITY IN KENYA',col='red') axis(1,at=1:length(ages),labels=ages)
legend('topright',legend=c('Male Deaths'), lty=1,lwd=2,pch=21,col=c('blue'),ncol=2,bty='n',cex=0.8,text.co
OF DEATHS') axis(1,at=1:length(ages),labels=ages) lines(maledx,col='blue',type='b',lwd=2)
legend('topright',legend=c('FEMALE DEATHS','MALE DEATHS'), lty=1,lwd=2,pch=21,col=c('red','blue'),nc

Codes for fitting male life table

year=1920 year ages=20:100 ages maledx=c(239,221.87,205.85,191.21,173.16,171.95,157.68,150.53,145.76,143.
maledx Dx=maledx malesEx=c(100000,99761,99539.13,99333.28,99142.07,98968.91,98796.96,98639.28,98488.
malesEx Ex=malesEx Ex fit=MortalityLaw(x=age,Dx=maledx,Ex=malesEx,law='HP',opt.method='LF2')
ls(fit) summary(fit) plot(fit) LT1=LifeTable(ages,Dx=Dx,Ex=Ex) LT1

Codes for fitting female life table

year=1920 year ages=20:100 ages femaledx=c(38,63, 88, 102, 90, 41, 59, 73, 84, 89, 91, 94, 99, 107, 116, 124, 135, 143, 154, 162, 171, 179, 186, 190, 195, 199, 203, 210, 216, 223, 236, 237, 194, 329, 286, 332, 370, 415, 466, 541, 603, 664, 704, 1587, 898, 600, 360, 1247, 959, 1292, 1460, 1717, 2049, 2352, 2629, 2882, 3117, 3334, 3534, 3719, 3885, 4030, 4147, 4233, 4278, 4273, 4214, 4090, 3901, 3644, 3325, 2919, 2493, 2067, 1644, 1265, 879, 698, 426, 263, 289) femaledx Dx=femaledx Dx femaleEx=c(100000,99962,99899,99811,99709,99619,99578,99519,99446,99362,
99273,99182,99088,98989,98882,98766,98642,98507,98364,98210,98048,97877,
97698,97512,97322,97127,96928,96725,96515,96299,96076,95840,95603,95409,95080,
94794,94462,94092,93677,93211,92670,92067,91403,90699,89112,88214,87214,87245,

82600,85048,83756, 82296, 80579, 78530, 76178, 73549, 70667, 67550, 64216,
60682, 56963, 53078, 49048, 44901, 40668, 36390, 32117, 27903, 23813,
19912, 16268,12943,10024,7531, 5464, 3820, 2555, 1676, 978, 552,
289) femaleEx Ex=femaleEx Ex fit=MortalityLaw(x=age,Dx=femaledx,Ex=femaleEx,law='HP',opt.method='LF2')
fit ls(fit) summary(fit) plot(fit) LT2=LifeTable(ages,Dx=Dx,Ex=Ex) LT2

Codes for fitting deaths plot in Kenya and England

femaledx=c(38,63, 88, 102, 90, 41, 59, 73, 84, 89, 91, 94, 99, 107, 116, 124, 135, 143, 154, 162, 171, 179, 186, 190, 195, 199, 203, 210, 216, 223, 236, 237, 194, 329, 286, 332, 370, 415, 466, 541, 603, 664, 704, 1587, 898, 600, 360, 1247, 959, 1292, 1460, 1717, 2049, 2352, 2629, 2882, 3117, 3334, 3534, 3719, 3885, 4030, 4147, 4233, 4278, 4273, 4214, 4090, 3901, 3644, 3325, 2919, 2493, 2067, 1644, 1265, 879, 698, 426, 263, 289) femaledx maledx=c(239,221.87,205.85,191.21,173.16,171.95,157.68, 150.53,145.76,143.58,144.16,147.77,154.4,164.42,177.9,194. 34,213.29,234.36,256.82,280.37,304.38,328.45,352,374.6,395.81 ,415.13,432.21,446.42,457.53,465.11,468.68,478.23,484.87,495.02, 1032.25,526.39,1084.52,599.9,654.02,744.53,862.59,1007.91,1182.12, 1386.02,1616.2,1883.56,2167.4,2475.5,2798.02,3123,3302.91, 3441.77,3628.35,3761.48,3787.52,3849.48,3875.64,3876.13,3871.87, 3833.15,3659.62,3446.61,3251.41,2974.48,2722.96,2453.21,2150.93, 1905.22,1664.5,1451.94,1237.21,1028,841.48,672.32,516.09,378.59, 263.77,173.31,106.46,60.52,55.06) maledx Ukmaledx=c(83,85,87,87,87,84,83,83,85,87,89,91,95,97,103,113,124, 133,145,155,166,179,194,210,230,255,283,315,352,391,436,485,537,594, 656,727,806,892,987,1091,1207,1334,1472,1625,1783,1940,2097,2255,2403, 2543,2674,2819,2969,3109,3218,3301,3386,3455,3494,3502,3474,3400,3300, 3175,3023,2839,2637,2407,2144,1873,1608,1369,1154,953,762,590,442,322,229, 157,105) Ukmaledx Ukfemaledx=c(31,32,32,33,32,34,34,35,38,39,43,46,51,57,61,68,74,81, 88,96,105,114,126,138,154,173,192,212,234,257,283,312,342,372,406,450, 499,554,614,683,761,839,915,1007,1117,1218,1308,1417,1533,1647,1751,1876, 2056,2239,2366,2487,2634,2812,2984,3158,3314,3435,3526,3596,3655,3706, 3724,3634,3475,3330,3143,2903,2631,2321,2008,1702,1395,1102,853,653,488) Ukfemaledx data4=data.frame(a
data4 attach(data)

plot(maledx,main='Male and Female Mortality plots in Kenya and England ',type='b',lwd=4,ylab='Number of Deaths',xlab='AGES',xaxt='n',col='blue') axis(1,at=1:length(ages),labels=ages) lines(femaledx,col='red',ty
lines(Ukmaledx,col='Green',type='b',lwd=4) lines(Ukfemaledx,col='black',type='b',lwd=4)
legend('topleft',legend=c('Kenya Male Deaths',' Kenya Female Deaths','Eng Male Deaths','Eng Female Deaths'), lty=1,lwd=4,pch=21,col=c('red','blue','Green','black'),ncol=2,bty='n',cex=0.8,text.col=c('rec