University of Nairobi

School of Mathematics

ISSN: 2410-1397

Master Project in Pure Mathematics

Hybrid GARCH(1,1) European Option Pricing Model with Ensemble Empirical Mode Decomposition

Research Report in Mathematics, Number 02, 2020

Isaac Mwaura Njoroge

June 2020

Submitted to the School of Mathematics in partial fulfilment for a degree in Master of Science in Actuarial Science

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Master Thesis
Submitted to the School of Mathematics in partial fulfilment for a degree in Master of Science in Actuarial Science

Submitted to: The Graduate School, University of Nairobi, Kenya

Abstract

Despite the option pricing importance in risk management and selection of portfolios, it is challenging to accurately price options due to unpredictable feature of asset prices. There are numerous risks in the financial markets, mainly emanating from the inaccurate computation of option prices. The inaccuracy is mostly attributed to volatility. Using GARCH or other stochastic processes directly is unsuitable for option pricing. There is the need to decompose original series with some properties to attain more financial time series aspects. E-E-M-D generally performs well in capturing volatility and option pricing of financial data with non-linearity and non-stationarity properties. We construct a hybrid GARCH(1,1) model with the ensemble empirical mode decomposition in European option pricing. Using E-E-M-D, we decompose the original daily returns into low frequency, high frequency, and trend terms, and use these terms in the hybrid GARCH(1,1) European option pricing model in options pricing. We obtain option prices for different maturities by applying Monte Carlo simulation. Our empirical results clearly illustrates that the hybrid GARCH(1,1) European option pricing model effectively predicts volatility features and performs better than BSM73 and GARCH-M(1,1). The performance of the hybrid GARCH(1,1) European option pricing model incorporating just the low-frequency term further depicts the significance of decomposing the original returns using E-E-M-D by reducing option pricing errors significantly. Therefore, the hybrid GARCH(1,1) European option pricing model is a highly innovative and effective method of option pricing.

Master Thesis in Mathematics at the University of Nairobi, Kenya.
ISSN 2410-1397: Research Report in Mathematics

DISTRIBUTOR: School of Mathematics, University of Nairobi, Kenya

Declaration and Approval

I the undersigned declare that this dissertation is my original work and to the best of my knowledge, it has not been submitted in support of an award of a degree in any other university or institution of learning.

Signature Date

ISAAC MWAURA NJOROGE
Reg No. I56/11804/2018

In my capacity as a supervisor of the candidate's dissertation, I certify that this dissertation has my approval for submission.

Signature Date

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Dedication

I dedicate this project to my Mum, Dad, brothers and sister. Despite the challenges we have been through, they have encouraged me and provided all the support I needed.

Contents

Abstract ii

Declaration and Approval iv

Dedication vi

Figures and Tables ix

Acknowledgments xi

1 Introduction 1

1.1 Background of the Study 1
1.2 Statement of the Problem 2
1.3 Research Objectives 3
1.3.1 General Objective 3
1.3.2 Specific Objectives 3

2 Literature Review 4

3 Methodology 7

3.1 Introduction 7
3.2 Black-Scholes Model(BSM73) 7
3.2.1 Put call parity 7
3.2.2 BSM73 Assumptions 8
3.3 GARCH-M(1,1) model 9
3.4 Empirical Mode Decomposition 9
3.5 Ensemble Empirical Mode Decomposition(E-E-M-D) 11
3.6 Hybrid GARCH(1,1) European Option Pricing Model with Ensemble Empirical Mode Decomposition 12
3.6.1 Unconditional variance of the hybrid GARCH(1,1) option pricing model 14
3.6.2 covariance of the GARCH-type hybrid model 15
3.6.3 The Hybrid GARCH(1,1) European Option Pricing Model with Z_t^l and $\eth_t$ 16
3.7 Model Selection 16
3.7.1 Conditional Error Distributions 16

4 Data Analysis and Results 18

4.1 Introduction 18
4.2 Conventional Facts of Daily Returns 18
4.3 Daily Returns Descriptive Statistics 22
4.4 ARCH Effects Test 23
4.5 Conditional Error Distributions Selection Criteria 23
4.6 Conditional Error Distribution Selection Using Information Criteria 28
4.6.1 Information Criteria 28

4.7 Ensemble Empirical Mode Decomposition(E-E-M-D) 28
4.8 European Option pricing 32
4.8.1 Monte Carlo Simulation 32
4.8.2 Impacts of Decomposing Returns Using E-E-M-D on Option Prices 44

5 Conclusion 52

5.1 Future Research 52

Bibliography 53

Appendices 56

Black Scholes 56
GARCH-M(1,1) 56
Ensemble Empirical Mode decomposition 58
GARCH-H(1,1) 58

Figures and Tables

Figures

Figure 1. Time Series plot of Facebook Adjusted Closing Prices 18
Figure 2. Volatility clustering 19
Figure 3. Volatility Analysis 20
Figure 4. The daily returns histogram 21
Figure 5. Normal Q-Q plot of Facebook returns 22
Figure 6. Normal Distribution QQ Plot 24
Figure 7. Student-t Distribution QQ Plot 25
Figure 8. Skewed Student-t Distribution QQ Plot 26
Figure 9. Generalized Error Distribution QQ Plot 27
Figure 10. $C_1(t)$ 28
Figure 11. $C_2(t)$ 29
Figure 12. $C_3(t)$ 29
Figure 13. $C_4(t)$ 29
Figure 14. $C_5(t)$ 30
Figure 15. $C_6(t)$ 30
Figure 16. $C_7(t)$ 30
Figure 17. $C_8(t)$ 31
Figure 18. $C_9(t)$ 31
Figure 19. $R_{10}(t)$ 31
Figure 20. Low frequency term of X_t 32
Figure 21. High frequency term of X_t 32
Figure 22. BSM73 and Market Option Prices when τ=14 days 34
Figure 23. GARCH-M and Market Option Prices when τ=14 days 34
Figure 24. GARCH-H and Market Option Prices when τ=14 days 35
Figure 25. BSM73 and Market Option Prices when τ=63 days 36
Figure 26. GARCH-M and Market Option Prices when τ=63 days 36
Figure 27. GARCH-H and Market Option Prices when τ=63 days 37
Figure 28. BSM73 and Market Option Prices when τ=126 days 38
Figure 29. GARCH-M and Market Option Prices when τ=126 days 38
Figure 30. GARCH-H and Market Option Prices when τ=126 days 39
Figure 31. TH-GARCH and Market Option Prices when τ=14 days 47
Figure 32. TH-GARCH and Market Option Prices when τ=63 days 48
Figure 33. TH-GARCH and Market Option Prices when τ=126 days 48
Figure 34. GARCH-LH and Market Option Prices when τ=14 days 49
Figure 35. GARCH-LH and Market Option Prices when τ=63 days 49
Figure 36. GARCH-LH and Market Option Prices when τ=126 days 50

Tables

Table 1. Descriptive statistics of daily returns 23
Table 2. Information Criteria 28
Table 3. GARCH-M(1,1) and GARCH-H(1,1) Parameters 33
Table 4. Call option prices and error criterion of the three models when τ=14 days 40
Table 5. Call option prices and error criterion of the three models when τ=63 days 41
Table 6. Call option prices and error criterion of the three models when τ=126 days 42
Table 7. Root Mean Squared Error (RMSE) of the call options 43
Table 8. Root Mean Squared Error (RMSE) of the put options 43
Table 9. Mean Absolute Error (MAE) of the call options 43
Table 10. Mean Absolute Error (MAE) of the put options 43
Table 11. Call option prices and error criterion of the three hybrid GARCH(1,1) models when τ=14 days 44
Table 12. Call option prices and error criterion of the three hybrid GARCH(1,1) models when τ=63 days 45
Table 13. Call option prices and error criterion of the three hybrid GARCH(1,1) models when τ=63 days 46
Table 14. Root Mean Squared Error (RMSE) of the call options 50
Table 15. Root Mean Squared Error (RMSE) of the put options 51
Table 16. Mean Absolute Error (MAE) of the call options 51
Table 17. Mean Absolute Error (MAE) of the put options 51

Acknowledgments

First, I thank God for peace, good health and sustaining me throughout this research period. Sincerely thanks to my supervisor Prof. Mwaniki for guiding, supporting and encouraging me during this research.
1 am deeply grateful to my family supporting me morally and their encouragement. May the Almighty God heal you mum and bless you all abundantly.

Isaac Mwaura Njoroge

Nairobi, 2020.

1 Introduction

1.1 Background of the Study

Options contracts have been existing for many centuries but remained obscure financial instruments before 1973, where such contracts were seen as over-the-counter (OTC). Option trading had a broker who was an intermediary between the buyer and the seller of the contract. These contracts were not accurately priced since they were not standardized based on their conditions. According to[Arora et al., 2011] there was no standard agreed-upon option pricing before the Black-Scholes model was discovered. Traders relied on their intuition to price options. With the discovery of the Black-Scholes formula, traders could use a simple equation with few inputs to price options. The formal exchange of new financial options began in the market, replacing OTC. In 1973, Black and Scholes and Merton had a breakthrough in the field of finance by formulating the first satisfactory equilibrium pricing model(BSM73) of derivative securities in options markets. Option pricing theory is crucial in investment decision making and risk management. Worldwide growth in financial markets in derivative has exploded following the prevailing theory of BSM73. The significant contribution of the Black and Scholes (1973) and Merton (1973) seminal work was the introduction of an option pricing model that does not involve an investor's risk preference and subjective views. [Cox and Ross, 1976] studies the structure of option valuation problems and introduces a new technique of jump-diffusion processes that had not been used in the previous models. The method finds an explicit formula to evaluate options and solve past problems in the securities valuation with payouts and possible bankruptcy. [Cox et al., 1979] propose a simple discrete-time binomial option pricing model. This method assumes that stock prices move either upwards or downwards only and that the magnitude of these movements is the same during the study period. Volatility is considered to be a function of the stock price and directly determined by the stock price.[Hull and White, 1987] assume a continuous-time stochastic volatility model in European option pricing and examine the impacts of stochastic volatility on option prices. stronger than no arbitrage.

Traditionally, financial economics research has focused on expected market returns. Scholars and professionals are enthusiastic about addressing the instability of the predicted market returns. Market stability is fundamental in the market, and thus concentrating on the stock volatility is of great importance. The impact of volatility on the expected returns has forced researchers to give attention to the intensity and stationarity of volatility. Researchers have shifted their interest in developing and improving econometric models capable of producing accurate projections of returns volatility. Researchers have established numerous models to forecast stock volatility. The following univariate volatility models

are among the most important. [Engle, 1982] autoregressive conditional heteroskedastic (ARCH) model, and [Bollerslev, 1986] generalized ARCH (GARCH) model,
[Hua et al., 2018] Numerous studies present various perceptions for understanding pricing of underlying assets. Non-linearity and non-stationarity are some of the special characteristics possessed by the financial time series. Occurrence of undesirable events such as financial crises or disaster shocks asset prices and may even cause a jump. In reality, numerous factors are capable of altering asset price series. Financial time series is studied under categorization according to seasonal factors, regression terms and trend terms. Accurate classification of factors affecting asset price series leads to better structures of financial time series data. Application of stochastich volatility and GARCH-type models directly is unsuitable for option pricing. This gives rise to need to decompose original series with some properties to attain more financial time series aspects. Scholars world wide are committed to developing financial time series. Wavelet based modelling technique and neural network are among trendy approaches. But since [Kumar et al., 2016] highlights that one of the most common statistical properties violated by time series data is stationarity, the above models require stationarity hence have a complexity in dealing with complicated financial data. [Huang et al., 1998] develops a new empirical mode decomposition (E-M-D) for analysing data with non-linearity and non-stationarity features. E-M-D is capable of decomposing complex data into finite intrinsic mode functions admitting Hilbert transforms. E-M-D is highly efficient because of its applicability in processes with non-linearity and non-stationarity. Despite E-M-D being highly usefulness, mode mixing phenomenon remains to be its most annoying unresolved difficulty. In order to solve the mode mixing phenomenon, ensemble empirical mode decomposition (E-E-M-D) is used. E-E-M-D is a noise-assisted analysis that involves sifting an ensemble of white noise-added data and treating the mean as the final true answer. E-E-M-D utilizes statistical properties of noise to the fullest by disturbing the original data and cancelling itself out after use. E-E-M-D skilfully improves E-M-D by eliminating the mode mixing problem while preserving physical uniqueness of decomposition by adding white noise.

1.2 Statement of the Problem

There are numerous risks in the financial markets. One of the major risks in option trading is inaccurate computation of option prices. The inaccuracy is mostly attributed to volatility. Inappropriate determination of volatility makes investors vulnerable to suffer a financial loss. This has necessitated researchers to focus on developing econometric models that accurately forecasts swings in returns volatility. The most significant objective of trading is to minimize uncertainties and maximize the expected returns. It is natural for investors to invest in a business with a possibility of maximum expected returns and minimal risks. Investor's success is deeply found on the ability to make good investment decisions and mitigate existing risks. Option pricing is dependant on the existing closed-form models. However, accuracy of option prices raises a significant question. Occurrence of undesirable events such as financial crises or disaster shocks asset prices and may even cause a jump. In reality, numerous factors are capable of altering asset price series. Accurate classification of factors affecting asset price series leads to better structures of financial time series. There is need to decompose original series with some properties to attain more financial time series aspects. E-M-D and E-E-M-D generally performs well in capturing volatility and option pricing in financial time series with non-linearity and non-stationarity features. However, there is limited application of E-M-D and E-E-M-D in option pricing and in the existing studies. Option prices are highly sensitive to volatility, it is therefore important to come up with an appropriate model that captures comprehensive fluctuation features from financial data with non-linearity and non-stationarity features. Filling this gap is our main research contribution.

1.3 Research Objectives

1.3.1 General Objective

To study the proposed hybrid GARCH(1,1) European option pricing model with ensemble empirical mode decomposition.

1.3.2 Specific Objectives

1. To analyse the underlying features of asset returns by applying E-E-M-D.
2. To construct the hybrid GARCH(1,1) European option pricing model describing the underlying asset returns.
3. To compare option prices from the proposed model with the already existing models.
4. To show the impact of decomposing returns using E-E-M-D

2 Literature Review

Option pricing is a highly celebrated theory that was in existence long before Bachelier made a publication in 1900. After [Black and Scholes, 1973] presented their path-breaking model (BSM73), option pricing theory has underwent a major revolutionary change . The BSM73 is mathematically tractable and compact making it popular in the financial markets. However, some scholars prove that some of the BSM73 assumptions fails to hold. The BSM73 assumption of normal distribution of returns and constant volatility are unrealistic. Returns indeed fails to justify normality test and also exhibits volatility clustering. Volatility is a major factor in pricing of options. Volatility is clearly non constant when we practically determine volatility on the basis of observed market prices. Worldwide research on option markets imply that the constant volatility of the BSM73 assumption fails to hold since from empirical studies on log-returns of stock indicates that volatility of time series returns is non constant. There is very much fluctuation of financial asset prices and volatility vary over time during stress times on the market. This limitation of constant volatility undermines the accuracy and applicability of BSM73 since volatility is a major factor in pricing of options. However, the BSM73 European option pricing model is highly applied world wide despite violation of some of its assumptions. Option pricing theory has improved greatly from BSM73 by relaxing some assumptions and introducing other more complex option pricing models.
A research done by [Cox and Ross, 1976] present jump and diffusion processes model to find explicit option pricing formulas. The new process had not been considered in the previous studies in pricing of options. [Cox et al., 1979] presents a simplified discrete-time option pricing model. This was the first approach to implement binomial model to evaluate options presuming a log-normal process. This method possess the ability to assess options using the no-arbitrage and risk-neutral principles. A simulation strategy is proposed where the asset prices either goes up or down. This method is inefficient since it only takes into up or down price movements and the market is nor always perfect.
Econometric modelling gained plausible success following the seminal work of Engle(1982). Autoregressive conditional heteroscedastic (ARCH) generalizes the unreliable assumption of constant variance in traditional econometric models is introduced [Engle, 1982] . ARCH allows non constant conditional volatility. Many researchers have extended the idea of ARCH model subsequently.
[Bollerslev, 1986] proposes a generalized form of the ARCH model called Generalized Autoregressive Conditional Heteroskedastic (GARCH) that allows past conditional variances to be a function of current conditional variance. Since approximating an entirely free lag distribution leads to contravention of the non-negative constraint, GARCH model is more appropriate as it allows for more flexible lag structure.

[Scott, 1987] examines the European call options pricing on stocks with variances that changes randomly. Stock and two options need to be used in developing a hedge. However, the risk-less hedge flops in forming a unique option pricing function. The equilibrium asset pricing model application is a requirement in deriving an exceptional function in pricing of option. The integral of the BSM73 formula is the resulting solution as well as the distribution function for the standard deviation of the stock price. Applying the model in actual prices setting and using Monte Carlo simulations can be accurately compute accurate prices of the options.
[Melino and Turnbull, 1990]] investigates the effects of stochastic volatility in foreign currency option pricing. They propose and examine a diffusion model with stochastic volatility. Allowing volatility to be stochastic improves the accuracy of option price prediction. Bivariate diffusion models for option valuation main drawback is that face their volatility rate is unobservable and thus call for provisions
[Nelson, 1991] proposes a new form of ARCH model (EGARCH) that meets GARCH objections. GARCH-type models employed in modelling the relationship between conditional variance with the risk premium of assets have some major drawbacks in asset pricing applications. EGARCH model may be appropriate in existence of a correlation of variance stock price.
[Robert and Victor, 1993] measures the extent of new information incorporation in volatility estimation by defining news impact curve. Option pricing is mainly dictated by the shape of the news impact curve. Innovative diagnostic tests are presented that emphasize on how volatility asymmetry reacts to news. Finding out how fast the news impact curve increases and whether it is symmetric or not is very important. However, [Hull and White, 1987] simulations proves how mispricing of option prices is dependant on volatility measures and correlation between option price and volatility.
[Duan, 1995] develops a model for valuing options valuation based on GARCH. The model introduces the locally risk neutral valuation relationship(LRNVR) by generalizing the concept of risk-neutral. Under assumptions of distribution and combination of preferences, the LRNVR is shown to be valid. The GARCH option pricing model preserves the conditional volatility of the underlying stock price. Statistical analysis implies that the GARCH option pricing model attempts to justify some well-documented limitations of the BSM73.
[Härdle and Hafner, 2000] extends the Duan's model to a volatility estimation that is more flexible. Options that are out-of-the-money are robustly dependant on volatility specifications. Duan's results of 1995 are advanced to the threshold generalized autoregressive conditional heteroskedastic (TGARCH) process notion and compared with Monte Carlo simulated Garch and the BSM73 prices. There are numerous progressive reviews to the GARCH models family to overcome drawbacks of each proposed GARCH model. [Huang et al., 2003] develops a new method applied to financial data, the Hilbert-Huang Transform (HHT). HHT analyses data with non-linearity and non-stationarity features. It is comprised of the E-M-D and classical Hilbert spectral analysis. E-M-D is observed to increase accuracy and efficiency of reflecting changes in the market volatility.
[Zhang et al., 2009] proposes to use E-M-D analysis to estimate how major events im-

pacts on the prices of crude oil. [Wu and Huang, 2009] improves the traditional E-M-D by E-E-M-D. E-E-M-D main strength is introducing white noise to disturb the original signal and cancel it away thus eliminating mode decomposition in the E-M-D.
[Byun et al., 2015] incorporates variance premium as well as the jump risk premium in the GARCH option pricing model.
[Zhu et al., 2015] apply E-M-D in decomposing the carbon prices and then reconstructs the IMFs into three components. [Tang and Diao, 2017] improves the accuracy of option pricing. They use BSM73 integrated with the hidden Markov model (HMM)to price options. Using the historical data of the underlying stock process, they train HMM in two states. They use HMM in prediction of the hidden state the hidden state of next time by the HMM. Finally, they forecast volatility on the basis of a conforming GARCH-type model. The empirical results obtained indicates better performance compared to historical volatility classical models and GARCH models.
[Li and Chu, 2017] incorporates the time-dependent correlation between underlying assets, which is a common phenomenon in financial practice but considered in few previous researches. Dynamic copula with time-dependent correlation is used to depict the dynamic nature of option price. In order to formulate the heavy-tailed characteristic of financial derivatives, they also improve the GARCH process by applying Tukey's H -distribution family. In numerical experiment, reformulated GARCH process expresses more precise curves than general GARCH process.
[Kannadhasan et al., 2018] investigates the presence and pattern of the volatility clustering by applying GARCH-type models. In addition, this study examines GARCH family of models with reference to out-of-sample forecast accuracy. the obtained results the appropriateness of GARCH (1,1) in accurate return series prediction.
[Liu and Huang, 2019] combines the GARCH model with the BSM73 to improve on the constant volatility assumed by the BSM73. After option pricing, they realize that combining the GARCH model with the BSM73, improves the accuracy of carbon option pricing results to a certain extent, and can provide reference for the research of carbon option pricing.
[Jiang and Hua, 2019] proposes threshold GARCH with generalized error distribution constructed with improved E-E-M-D.

3 Methodology

3.1 Introduction

This chapter briefly describes BSM73, GARCH-in-mean model and introduces GARCH-H(1,1). We propose to use the E-E-M-D in the decomposition of returns process. Decomposition process aims at extracting all variability information from the original financial time series data. Additionally, we construct GARCH-H(1,1) and show existence of an equivalent probability measure.

3.2 Black-Scholes Model(BSM73)

The development of [Black and Scholes, 1973] framework was significant in offering theoretical estimation of European call options prices. The BSM73 offers a closed-form solution for European option pricing making the model attractive. For non-dividend paying stock, the BSM73 European calls and put option prices are given below.

$$C_t = S_t\Phi(d_1) - Ke^{-r\tau}\Phi(d_2) \tag{3.2.1}$$

where

$$d_1 = \frac{\log\left(\frac{S_t}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}, \tau = T - t$$

$$d_2 = d_1 - \sigma\sqrt{\tau}$$

3.2.1 Put call parity

Put-call parity is a major concept in pricing of options, it shows the relationship of puts, calls prices, and the underlying asset. The put call parity states that:

$$C_t = S_t + P_t - Ke^{-r\tau} \tag{3.2.2}$$

By put call parity , we substitute equation 3.2.1 in equation 3.2.2, the put option can be obtained as follows:

$$\begin{aligned}
P_t &= C_t - S_t + Ke^{-r\tau} \\
&= S_t\Phi(d_1) - Ke^{-r\tau}\Phi(d_2) - S_t + Ke^{-r\tau} \\
&= S_t(\Phi(d_1) - 1) + Ke^{-r\tau}(1 - \Phi(d_2)) \\
&= -S_t(1 - \Phi(d_1)) + Ke^{-r\tau}(1 - \Phi(d_2)) \\
&= Ke^{-r\tau}\Phi(-d_2) - S_t\Phi(-d_1)
\end{aligned}$$

Thus,

$$P_t = Ke^{-r\tau}\Phi(-d_2) - S_t\Phi(-d_1) \tag{3.2.3}$$

The call and put option prices notation are given below.

- S_t - Current stock price
- K - Strike price of the option
- r - Risk free rate of interest
- τ - Expiration time of the option content
- σ - The underlying asset volatility
- Φ - A normal distribution

3.2.2 BSM73 Assumptions

1. Stock returns are log-normally distributed.
2. Frictionless market.
3. An option is exercised only on expiration time.
4. No dividend is payable by the stock during the option's life.
5. Stocks move in a random walk manner that is not predictable hence the markets are efficient.
6. The interest rates and volatility are known constant.
7. There is no arbitrage to avoid an opportunity to make a risk-less profit.

Some of the BSM73 assumptions used are unrealistic. BSM73 assumes that returns are Log-normally distributed, from observation returns are commonly leptokurtic exhibiting fat tails. Constant volatility assumption is questionable since volatility keeps fluctuating with changes in demand and supply. Furthermore, BSM73 assumes that the market has neither taxes nor transaction costs; that the rate of interest rate is constant which is hardly ever the case; that there are no dividends which is not the reality since options buying and selling is focused on returns. Such assumptions may lead to the option prices deviating from the real market prices where these assumptions are unrealistic.

3.3 GARCH-M(1,1) model

[Bollerslev, 1986] proposed GARCH to address the limitation of the ARCH model. GARCH gives an allowance of the past conditional variances into the function of the current variance. The model is more flexible and persistent than ARCH model since it allows more flexible lag structure.
Stock returns are dependant on their volatility (risk). [Duan, 1995] developed GARCH-M model. GARCH-M adds a heteroskedastic term into the mean equation. Supposing that the original asset price sequence is $X_t, \quad t = 0, 1, 2, \ldots, n$, and at time t the return under probability measure $\mathbb{P}$ is conditionally log-normally distributed. That is,

$$\ln\left(\frac{X_t}{X_{t-1}}\right) = r + \lambda\sigma_t - \frac{1}{2}\sigma_t^2 + \varepsilon_t, \tag{3.3.1}$$

Where, under probability measure $\mathbb{P}$

- ε_t - residue
- σ_t - conditional variance
- λ - constant risk premium
- r - risk free rate

Assuming that under probability measure $\mathbb{P}$,ε_t follows [Bollerslev, 1986] process of GARCH(p,q),

$$\varepsilon_t|\mathscr{F}_{t-1} \sim N(0, \sigma_t^2) \qquad \text{under measure}\mathbb{P}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_j \varepsilon_{t-j}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2, \tag{3.3.2}$$

where $\mathscr{F}_t$ is the information set up to and including time t; $\omega \geq 0$; $q \geq 0, p \geq 0$; $\alpha_i \geq 0, i = 1, \ldots, q$; $\beta_j \geq 0, j = 1, \ldots, p$. When p=1 and q=1, GARCH-M model becomes GARCH(1,1)-M model given as:

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{3.3.3}$$

3.4 Empirical Mode Decomposition

[Huang et al., 1998] proposes a new promising adaptive and efficient method to analyse non-linear and non-stationary data. This method can decompose time series data into finite and intrinsic mode functions(IMFs) which are usually small in number. IMFs are simple oscillatory modes with varying amplitude and instantaneous frequencies. The conditions that an IMF should satisfy are:

1. the numbers of extrema and zero-crossings are the same, or differ by not more than one all through the entire IMF.

2. the mean value of the envelope defined by the local maxima and the envelope defined by the local minima must be zero at any data location.

E-M-D analyses data by the sifting process, which decomposes the original data into IMFs level by level. Suppose that the original data is $x(t) =, t = 0, 1, 2, \ldots, n$; the following is an overall description of the sifting process for a time series $x(t)$.

1. Find the local extrema of the original signal and compute the lower and upper envelopes. The local mean $(m_i(t), i = 1, 2, \ldots, k)$ is obtained by finding the mean of both the upper and lower envelopes. Define $w_i(t), i = 1, 2, \ldots, k$ as the difference between $x(t)$ and $m_i(t), i = 1, 2, \ldots, k$. That is,

$$w_i(t) = x(t) - m_i(t), i = 1, 2, \ldots k \quad t = 0, 1, 2, \ldots, n$$

   The following is the first sifting process;

$$w_1(t) = x(t) - m_1(t), t = 0, 1, 2, \ldots, n$$

   As a result, the first sifting process is accomplished. Ideally, $w_1(t)$ should be an IMF. The sifting process works by exterminating riding waves.

2. Assuming $w_1(t)$ to be the original signal in accordance with step one, we can obtain the second component $w_2(t)$

$$w_2(t) = w_1(t) - m_2(t)$$

   This step is repeated for k times, until $w_k(t)$ is an IMF.

$$w_k(t) = w_{(k-1)}(t) - m_k(t)$$

3. If $h_k(t)$ meets the IMF conditions, designate $C_i(t) = w_k(t), i = 1, 2, \ldots N, t = 0, 1, 2, \ldots, n$ The stoppage criterion of decomposition is defined by index DS as follows:

$$DS = \sum_{t=0}^{T} \left[\frac{w_{k-1}(t) - w_{(k)}(t)}{w_{k-1}(t)} \right]^2 < \varsigma, \qquad \textbf{(3.4.1)}$$

   where ς is a predetermined constant value.

4. The residue $R_i(t)$ can be obtained by subtracting $C_i(t)$ from the original signal.

$$R_i(t) = x(t) - C_i(t), \qquad i = 1,2,\ldots,N \quad t = 0,1,2,\ldots,n \tag{3.4.2}$$

Assuming the residue $R_i(t)$ to be the original data, repeat the first two steps. We obtain the second IMF. If no more IMFs can be attained and $R_i(t)$ becomes a monotonic function, and $R_i(t)$ with $C_i(t)$ becomes smaller than the predetermined value, the sifting procedure stops. If that is not the case, the last step is repeated. The final $R_i(t) = R_N(t)$ becomes the mean trend of $x(t)$. $x(t)$ represents a summation of IMFs with the residue ($R_N(t)$).

$$x(t) = R_N(t) + \sum_{i=1}^{N} C_i(t) \tag{3.4.3}$$

The number of iterations is given by N. Index DS as shown above defines the stoppage criterion of decomposition.

3.5 Ensemble Empirical Mode Decomposition(E-E-M-D)

Empirically, mode mixing the most significant drawback of E-M-D which appears when the data has intermittency. IMF ceases to have physical meaning on its own when mode mixing occurs. Financial time series usually contains a certain level of random noise. Eliminating mode mixing problem is obligatory in order to analyse financial time series data more accurately. E-E-M-D is capable of overcoming the phenomenon of mode mixing.

The major principle in E-E-M-D is adding uniformly distributed white noise to the to the original signal and then decomposes the data using E-M-D as described above. The following is a description of E-E-M-D algorithm:

1. Adding uniformly distributed white noise to the to the original signal $x(t), \quad t = 0,1,2,\ldots,n$ with ($\sigma = 0.1$),we get $\hat{x}(t), \quad t = 0,1,2,\ldots,n$

2. Decomposing the new signal $\hat{x}(t), t = 0,1,2,\ldots,n$ into IMFs using E-M-D.

3. Repeating the first two steps over while but using various white noise series every time.

4. Obtaining the consequent IMFs and the residue.

Applying fine-to-coarse technique by [Zhang et al., 2009] to reconstruct the IMFs, apply t-test in identifying from which IMF $C_i(t), i = 1,2,\ldots,N$, the mean of the sum of $C_1(t)$

to $C_N(t)$ is significantly departing from zero. We thus find the high and low frequency sequences $Z_k^h(t)$ and $Z_k^l(t)$ respectively.

$$Z_k^h(t) = \sum_{j=1}^{k} C_i(t), \quad k < N$$

$$Z_k^l(t) = \sum_{i=k+1}^{N} C_i(t)$$

3.6 Hybrid GARCH(1,1) European Option Pricing Model with Ensemble Empirical Mode Decomposition

In this section, we construct the proposed hybrid GARCH(1,1) European option pricing model with E-E-M-D. The proposed model incorporates risk premium, Z_t^l and $R_N(t)$. Suppose that the original index sequence is $x_t, \quad t = 0, 1, 2, \ldots, n$, and at time t the return is X_t, then,

$$X_t = ln\left(\frac{x_t}{x_{t-1}}\right)$$

The following is an adopted hybrid GARCH(1,1) European option pricing model:

$$X_t = r + a_1 z_t^l + a_2 \eth_t + \lambda\sigma_t + \varepsilon_t, \varepsilon_t | F_{t-1} \sim N(\mu, \sigma^2)$$

$$\implies \sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{3.6.1}$$

where,

$$\omega > 0, \quad \alpha \geq 0, \quad \beta \geq 0,$$
$$\beta + \alpha < 1$$

- r - risk free rate of interest
- $\eth_t$ - $(R_N(t) - r)$
- λ - high frequency volatility unit price for risk premium
- a_1 - risk premium from z_t^l
- a_2 - risk premium from $\eth_t$
- $\mathscr{F}_{t-1}$ - information set up to time $t-1$

When $a_1, a_2 = 0$, the hybrid GARCH(1,1) European option pricing model model becomes GARCH-M(1,1). In such a special case, the original data series seems to fluctuate only

around the constant r(risk free rate of interest) with heteroskedasticity. Extreme events and the trend terms does not have much effect on the original data series. Classical BSM73 is equivalent to our hybrid model when $\alpha = \beta = \lambda = a_1 = a_2 = 0$. Generally, financial time series reacts to factors such as; geopolitics, major economies monetary policies, economic cycles, and financial crises. Therefore, it is fundamental to consider long-term effects of such important factors and trend term. Such consideration easily leads to significant parameter estimation and accuracy in applications. Our hybrid model facilitates to capture the fluctuating components of the historical options data by revealing various systematic risk sources.
Applying the LRNVR, proving availability of transformation measure is obligatory. By so doing, it enables us in obtaining fair options price. According to [Duan, 1995] and also in [Mwaniki et al., 2015]

Definition 3.6.1. *A pricing measure $\mathbb{Q}$ is said to satisfy the LRNVR if measure $\mathbb{Q}$ is mutually and absolutely continuous with respect to measure $\mathbb{P}$ and satisfies the following conditions:*

(I) $X_t|\mathscr{F}_{t-1}$is normally distributed under $\mathbb{Q}$;

(II) $E^{\mathbb{Q}}\left[\frac{x_t}{x_{t-1}}|\mathscr{F}_{t-1}\right] = e^r$;

(III) $Var^{\mathbb{Q}}\left[ln\left(\frac{x_t}{x_{t-1}}|\mathscr{F}_{t-1}\right)\right] = Var^{\mathbb{P}}\left[ln\left(\frac{x_t}{x_{t-1}}|\mathscr{F}_{t-1}\right)\right]$ almost surely with respect to measure $\mathbb{P}$.

Under both measures,the conditional variances must be equivalent in accordance with the LRNVR definition. This is desirable since it makes it possible to make an estimation of conditional variance under measure $\mathbb{P}$. Equivalent martingale measure existence implies non-existence of arbitrage opportunities. The LRNVR under measure $\mathbb{Q}$ implies that;

$$\begin{aligned} X_t &= \log\left(\frac{x_t}{x_{t-1}}\right) \\ &= \upsilon_t + \xi_t \end{aligned}$$

Where $\upsilon_t = r - \frac{1}{2}\sigma^2$ is the conditional mean, X_t is the returns, and ξ_t is a normal random variable under measure $\mathbb{Q}$ and by:

$$E^{\mathbb{Q}}\left[e^{\upsilon_t+\xi_t|\mathscr{F}_{t-1}}\right] = E^{\mathbb{Q}}\left[\frac{x_t}{x_{t-1}}|\mathscr{F}_{t-1}\right]$$

Under martingale measure $\mathbb{Q}$ a discounted price process $\hat{X}_t$ is martingale with respect to $\mathscr{F}_t$,i.e

$$E^{\mathbb{Q}}\left[\hat{X}_t|\mathscr{F}_{t-1}\right] = \hat{X}_{t-1}$$

$$
\begin{aligned}
\Rightarrow E^{\mathbb{Q}}\left[e^{-rt}x_t|\mathscr{F}_{t-1}\right] &= e^{-r(t-1)}x_{t-1}\\
\Rightarrow E^{\mathbb{Q}}\left[\frac{x_t}{x_{t-1}}|\mathscr{F}_{t-1}\right] &= e^{r}\\
\Rightarrow E^{\mathbb{Q}}\left[e^{X_t}|\mathscr{F}_{t-1}\right] &= e^{r}
\end{aligned}
$$

Given,

$$X_t = r + a_1 Z_t^l + a_2 \eth_t + \lambda\sigma_t - \frac{1}{2}\sigma_t^2 + \xi_t, \qquad \xi_t|\mathscr{F}_{t-1} \sim N(0,\, \sigma_t^2) \tag{3.6.2}$$

ξ_t is a normal random variable , under measure $\mathbb{Q}$, then the proposed hybrid GARCH(1,1) European option pricing model becomes;

$$\sigma_t^2 = \omega + \alpha(\xi_{t-1} - a_1 Z_{t-1}^l - a_2\eth_t - \lambda\sigma_{t-1})^2 + \beta\sigma_{t-1}^2 \tag{3.6.3}$$

3.6.1 Unconditional variance of the hybrid GARCH(1,1) option pricing model

Theorem 3.6.2. *Unconditional variance of the hybrid GARCH(1,1) option pricing model Under probability measure $\mathbb{Q}$, when Z_t^l and $\eth_t$ are stationary with mean zero and are mutually independent, can be attained .*

Under measure $\mathbb{Q}$;

$$
\begin{aligned}
X_t &= r + a_1 Z_t^l + a_2\eth_t + \lambda\sigma_t - \frac{1}{2}\sigma_t^2 + \xi_t, \qquad \xi_t|\mathscr{F}_{t-1} \sim N(0,\, \sigma_t^2)\\
\sigma_{t-1}^2 &= \omega + \alpha(\xi_{t-1} - a_1 Z_{t-1}^l - a_2\eth_t - \lambda\sigma_{t-1})^2 + \beta\sigma_{t-1}^2
\end{aligned}
$$

Then;

$$
\begin{aligned}
E^{\mathbb{Q}}[\sigma_t^2] &= \omega + \alpha E^{\mathbb{Q}}(\xi_{t-1} - a_1 Z_{t-1}^l - a_2\eth_t - \lambda\sigma_{t-1})^2 + \beta E^{\mathbb{Q}}[\sigma_{t-1}^2]\\
&= \omega + \alpha E^{\mathbb{Q}}[\xi_t^2] + \alpha E^{\mathbb{Q}}[(a_1 Z_{t-1}^l + a_2\eth_t)^2]\\
&\quad - 2\alpha E^{\mathbb{Q}}[\xi_{t-1}|E^{\mathbb{Q}}(a_1 Z_{t-1}^l + a_2\eth_t + \lambda\sigma_{t-1}]\\
&\quad + 2\alpha\lambda E^{\mathbb{Q}}[a_1 Z_{t-1}^l + a_2\eth_t]E^{\mathbb{Q}}[\sigma_{t-1}] + \beta E^{\mathbb{Q}}[\sigma_{t-1}^2]
\end{aligned}
$$

Let

$$y_t = \frac{\xi_t}{\sigma_t},\; y_t|\mathscr{F}_{t-1} \sim N(0,\, 1)$$

$$
\begin{aligned}
\Rightarrow E^{\mathbb{Q}}[\xi|\mathscr{F}_{t-1}] &= E^{\mathbb{Q}}\left[\left(\frac{\xi_t}{\sigma_t}\right)|\mathscr{F}_{t-1}\right]E^{\mathbb{Q}}[\sigma_t^2]\\
&= \left[\frac{1}{2\pi}\int_{-\infty}^{\infty} y^2 e^{\frac{-y^2}{2}}\,dy\right]E^{\mathbb{Q}[\sigma_t^2]}\\
&= E^{\mathbb{Q}}[\sigma_t^2]\\
\Rightarrow E^{\mathbb{Q}}[\xi|\mathscr{F}_{t-1}] &= 0
\end{aligned}
$$

Since Z_t^l and $\eth_t$ are mutually independent and stationary with mean zero, then the unconditional variance is independent of time t

$$E^{\mathbb{Q}}[\sigma_t^2] = \omega + [\alpha(1+\lambda^2)+\beta]E^{\mathbb{Q}}[\sigma_t^2] + \alpha E^{\mathbb{Q}}[a_1^2 {Z_{t-1}^l}^2 + a_2^2 {\eth_t}^2]$$

The unconditional variance of GARCH-H(1,1) is given by

$$Var^{\mathbb{Q}}(\xi_t) = \frac{\omega + \alpha E^{\mathbb{Q}}[a_1^2 {Z_{t-1}^l}^2 + a_2^2 {\eth_t}^2]}{1-\alpha(1+\lambda^2)-\beta}$$

Let $E^{\mathbb{Q}}[a_1^2 {Z_{t-1}^l}^2 + a_2^2 {\eth_t}^2]$ be K, $K \geq 0$

$$\Rightarrow Var^{\mathbb{Q}}(\xi_t) = \frac{\omega + \alpha K}{1-\alpha(1+\lambda^2)-\beta}$$

3.6.2 covariance of the GARCH-type hybrid model

$$\begin{aligned}
Cov_{t-1}^{\mathbb{Q}}\left(\frac{\xi}{\sigma_t}, \sigma_{t+1}^2\right) &= E_{t-1}^{\mathbb{Q}}\left(\frac{\xi_t}{\sigma_t}\sigma_{t+1}^2\right) \\
&= E_{t-1}^{\mathbb{Q}}\left[\frac{\xi_t}{\sigma_t}(\omega + \alpha(\xi_t - (a_1 Z_t^l + a_2\eth_t + \lambda\sigma_t)))^2 + \beta\sigma_t\right] \\
&= \omega E_{t-1}^{\mathbb{Q}}\left[\frac{\xi_t}{\sigma_t}\right] + \alpha E_{t-1}^{\mathbb{Q}}\left[\frac{\xi_t}{\sigma_t}(\xi_t^2 - 2\xi_t(a_1 Z_t^l + a_2\eth_t + \lambda\sigma_t) + (a_1 Z_t^l + a_2\eth_t + \lambda\sigma_t)^2)\right] \\
&+ \beta\sigma_t E_{t-1}^{\mathbb{Q}[\xi_t]} \\
&= \omega E_{t-1}^{\mathbb{Q}}\left[\frac{\xi_t}{\sigma_t}(a_1 Z_t^l + a_2\eth_t + \lambda\sigma_t)^2)\right] + \alpha E_{t-1}^{\mathbb{Q}}\left[\frac{\xi_t}{\sigma_t}(a_1 Z_t^l + a_2\eth_t + \lambda\sigma_t)^2\right] \\
&+ \beta\sigma_t E_{t-1}^{\mathbb{Q}[\xi_t]}
\end{aligned}$$

When $\xi_t|\mathscr{F}_{t-1} \sim N(0, \sigma_t^2)$, and $y_t = \frac{\xi_t}{\sigma_t}$, $y_t|\mathscr{F}_{t-1} \sim N(0, 1)$

$$\begin{aligned}
\Rightarrow Cov_{t-1}^{\mathbb{Q}}\left(\frac{\xi}{\sigma_t}, \sigma_{t+1}^2\right) &= E^{\mathbb{Q}} Cov_{t-1}^{\mathbb{Q}}\left(\frac{\xi}{\sigma_t}, \sigma_{t+1}^2\right) \\
&= E^{\mathbb{Q}}\left[\alpha E_{t-1}^{\mathbb{Q}}\left[\frac{\xi_t}{\sigma_t^3}\right] - 2\lambda\alpha E_{t-1}^{\mathbb{Q}}[\xi_t^2]\right] \\
&= -2\alpha\lambda E^{\mathbb{Q}}[\sigma_t^2]
\end{aligned}$$

3.6.3 The Hybrid GARCH(1,1) European Option Pricing Model with Z_t^l and $\eth_t$

In order to prove the significance of decomposing original historical financial data, we consider GARCH-H(1,1) with Z_t^l alone and GARCH-H(1,1) with $\eth_t$ alone.
The GARCH-LH(1,1) model is obtained by considering Z_t^l alone:

$$X_t = r + a_1 Z_t^l + \lambda \sigma_t^2 - \frac{1}{2}\sigma_t^2 + \varepsilon_t, \tag{3.6.4}$$

$$\varepsilon_t | \mathscr{F}_{t-1} \sim N(0, \sigma_t^2)$$

Thus,

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_{t-1}\sigma_{t-1}^2 \tag{3.6.5}$$

TheGARCH-TH(1,1) model is obtained by considering $\eth_t$ alone.

$$X_t = r + a_2 \eth_t + \lambda \sigma_t^2 - \frac{1}{2}\sigma_t^2 + \varepsilon_t, \tag{3.6.6}$$

$$\varepsilon_t | \mathscr{F}_{t-1} \sim N(0, \sigma_t^2)$$

Thus, under measure $\mathbb{P}$

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_{t-1}\sigma_{t-1}^2 \tag{3.6.7}$$

3.7 Model Selection

Model specification determines evaluation and inferences on real life data. Optimal model selection is fundamental in data analysis and,ultimately returns accurate forecasting results. In this section, the optimal model for forecasting GARCH-H(1,1) model is selected on the basis of the conditional error distribution with information selection criteria.

3.7.1 Conditional Error Distributions

3.7.1.1 Normal Distribution

The density function of the normal distribution is given by:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2 / 2\sigma^2}$$

3.7.1.2 Student-t Distribution

The density function of the student-t distribution is given by:

$$f(x) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\Gamma\left(\frac{\nu}{2}\right)}\left(1+\frac{x^2}{\nu}\right)^{-\frac{\nu+1}{2}}$$

3.7.1.3 Skewed Student-t Distribution

The density function of the skewed student-t distribution is given by:

$$f(x) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\sqrt{\pi(n-2)h_t}}\left[1+\frac{x^2}{(\nu-2)h_t}\right]^{-}\frac{\nu+1}{2}$$

3.7.1.4 Generalized Error Distribution(GED)

The density function of the generalized error distribution that is also known as Generalized Gaussian Distribution (GGD) is :

$$f(x) = \frac{\lambda s}{2\Gamma\left(\frac{1}{s}\right)}\exp(-\lambda^s|x-\mu|^s)$$

4 Data Analysis and Results

4.1 Introduction

In this chapter I present the obtained empirical results on Facebook options data obtained on 14th February 2020 and historical data between June 2012 and February 2020. I also give an interpretation of results from fitting the GARCH-H(1,1), GARCH-M(1,1) and the BSM73.

4.2 Conventional Facts of Daily Returns

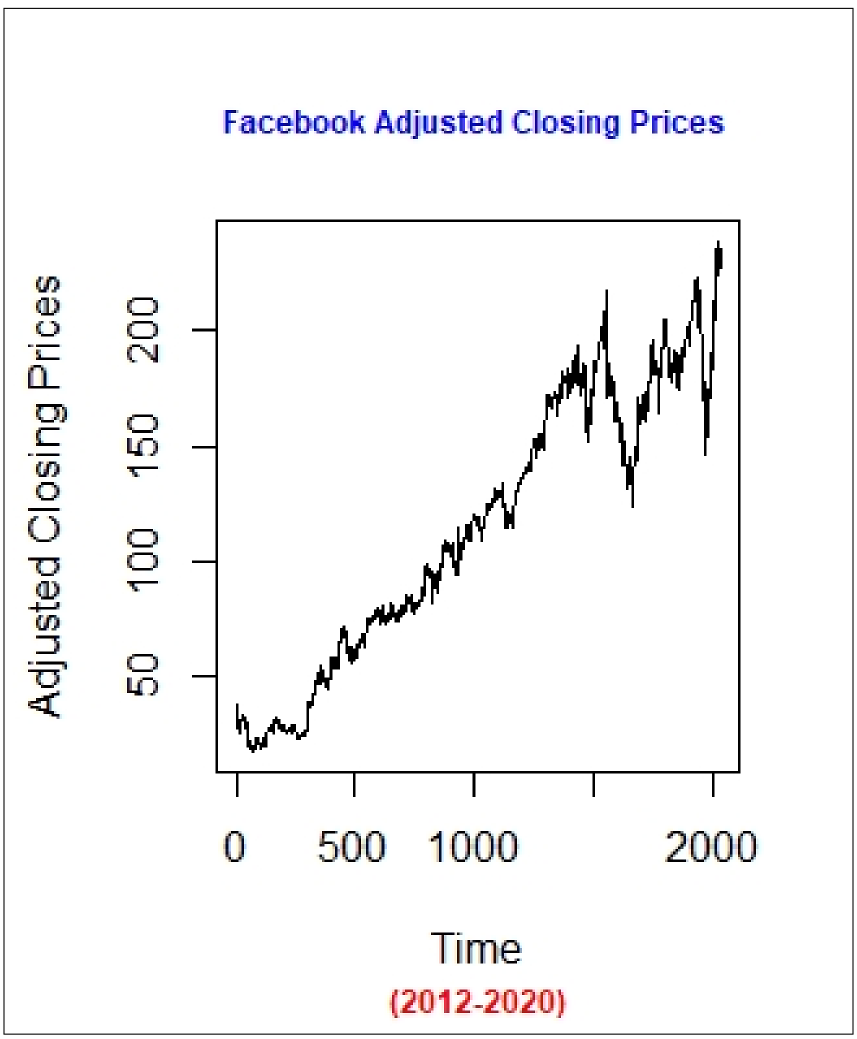

Figure 1. Time Series plot of Facebook Adjusted Closing Prices

From Figure 1, different trends are observed from different time intervals. Changing variability is obvious from the above time series plot. There is need for pricing the options with a model that allows for non constant volatility.

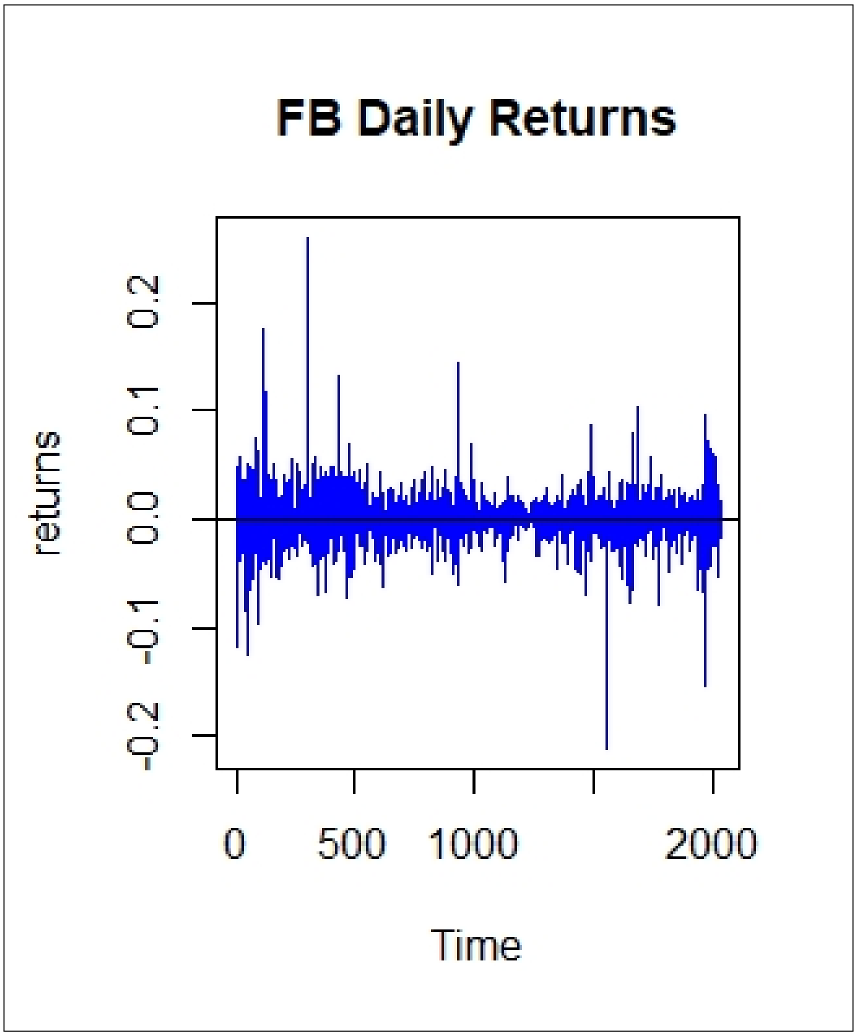

Figure 2. Volatility clustering

Figure 2 shows that the values the daily returns series change rapidly over time in unpredictable way indicating volatility. Volatility clustering is present where the tendency of either low volatility or high volatility usually persists. This behaviour is known as autoregressive conditional heteroskedasticity (ARCH). The Facebook daily returns seems to fluctuate around zero randomly, implying little or no autocorrelation.
Formally, we conduct ADF test for stationarity at significance level of 5% in order to fit a time series model.
The hypotheses are;

H_0 : The daily returns are non stationary

H_1 : The daily returns are stationary

If the p-value < 0.05, we reject the null hypothesis. The ADF test indicates that the p-value = 0.01 which is < 0.05 implying that the returns are stationary.

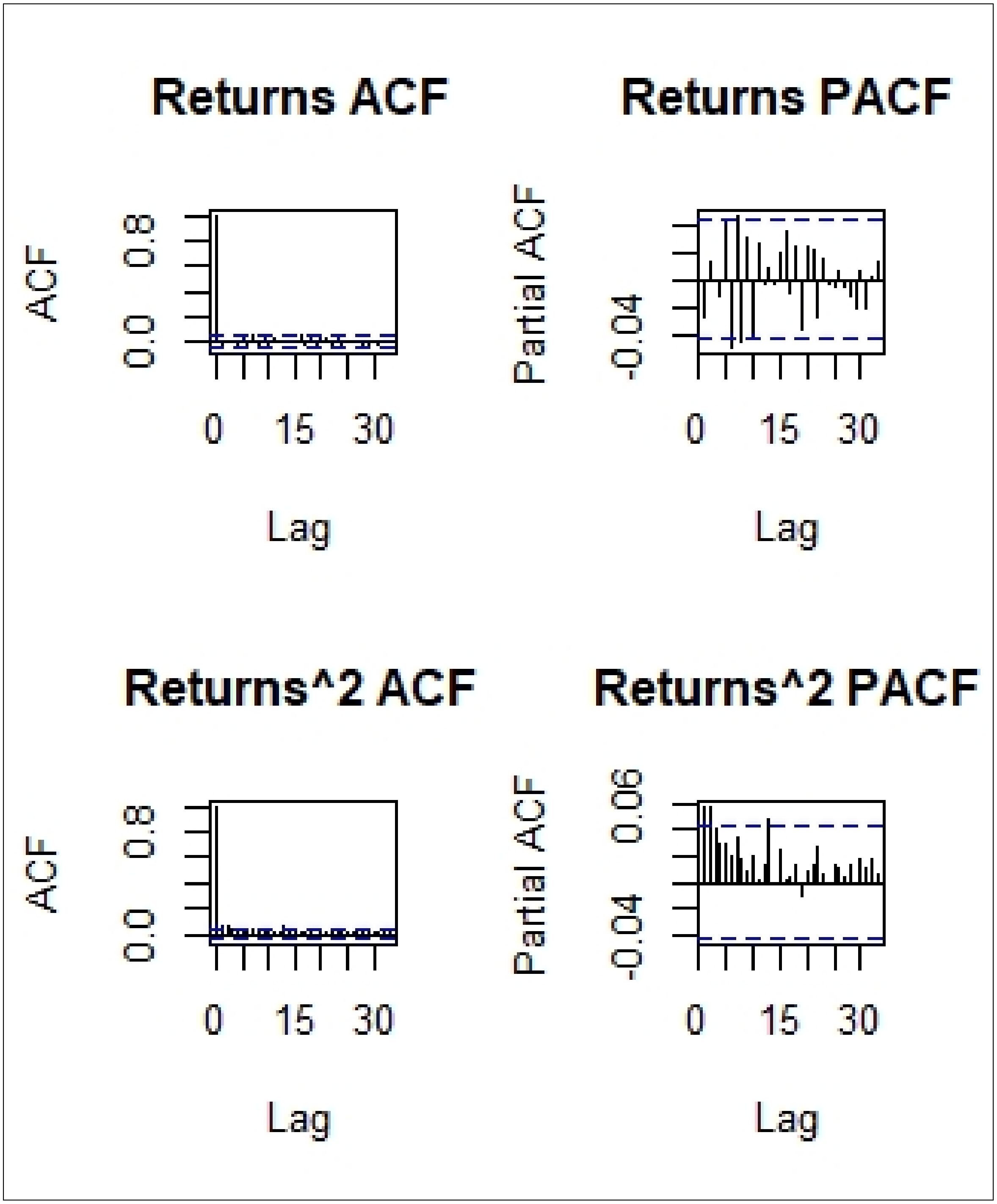

Figure 3. Volatility Analysis

From Figure 3 the returns are serially uncorrelated. However, the squared returns indicates significant autocorrelations. This implies that the returns are neither correlated nor independent.

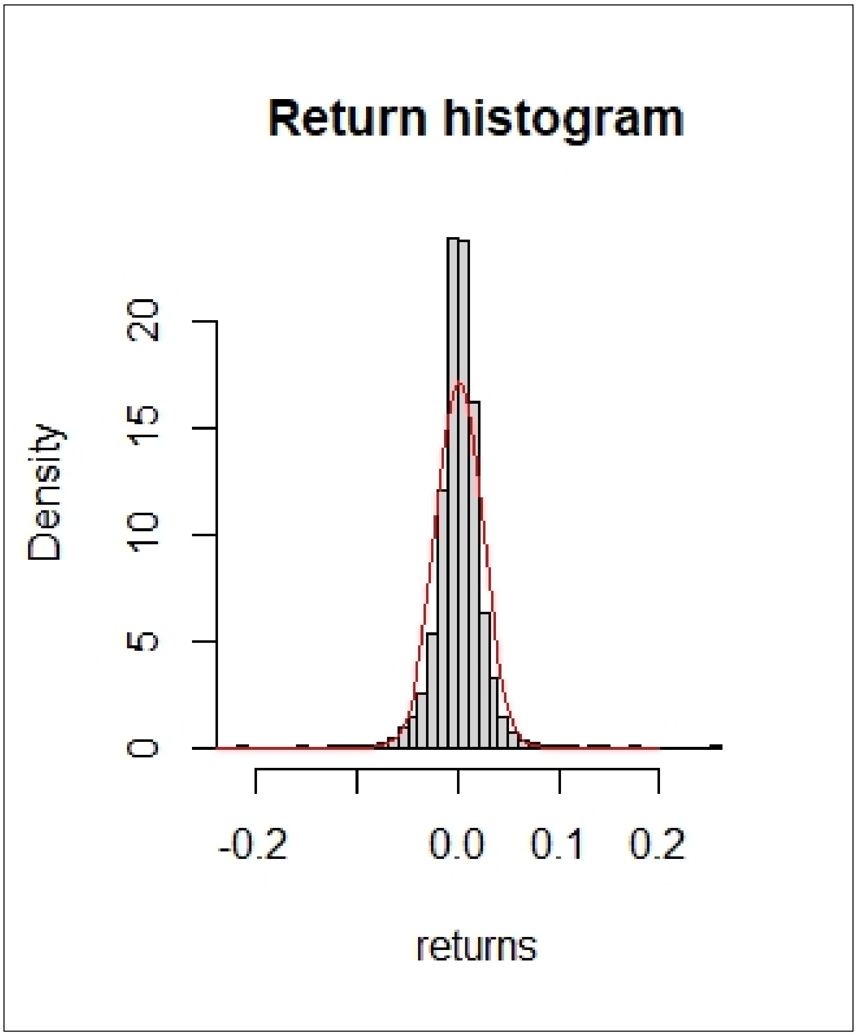

Figure 4. The daily returns histogram

Looking at Figure 4 and comparing it with the normal distribution with same mean and variance shows presence of excess kurtosis and fat tails.

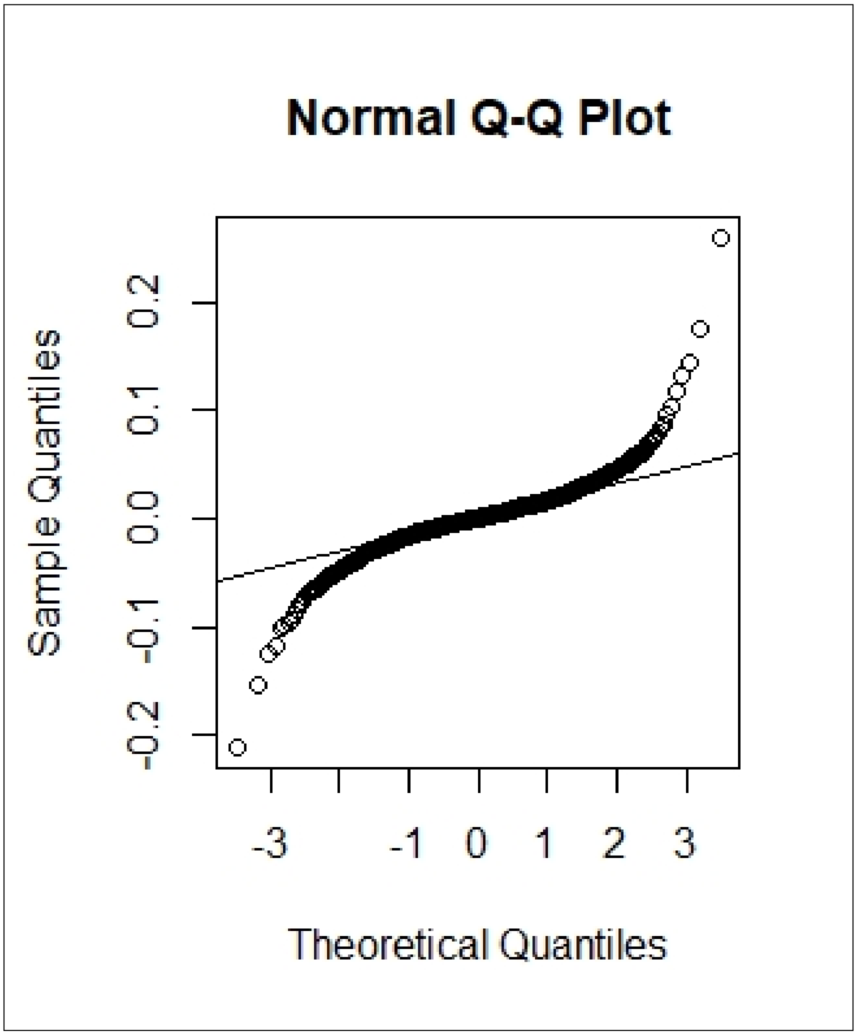

Figure 5. Normal Q-Q plot of Facebook returns

QQ-plot in Figure 5 illustrates that there are fat tails since the empirical quantiles against those from of a normal distribution should have formed a roughly straight line. Nonconformity with the straight line implies non normality of the daily returns. We can further confirm the normality using the Shapiro test for normality. In Shapiro-Wilk's test of normality, the p-value= 0.00000000000000022 which is < 0.05 confirming non normality of the returns. Non normality is a major characteristic of daily returns.

4.3 Daily Returns Descriptive Statistics

Table 1. Descriptive statistics of daily returns

Minimum value	-0.21
Maximum value	0.26
Mean value	0
Median	0
Lower quartile	0.01
Upper quartile	-0.01
Variance	0
Standard deviation	0.02
Skewness	0.35
Kurtosis	16.12978

From Table 1 kurtosis is 16.12978 exhibits that returns are leptokurtic .Since the value is larger than 3, daily returns exhibit heavy tails. The value of skewness is 0.35 which is greater than zero inferring that the daily returns are skewed to the right.

4.4 ARCH Effects Test

We use the ARCH test to check for ARCH effects. We use $\alpha = 0,05$ significance level for the null hypothesis test. The hypotheses for ARCH test are ;

H_0 : No ARCH effect in the returns
H_1 : ARCH effect in the returns

The p-value=0.002307 which is < 0.05, we therefore reject the null hypothesis. Presence of ARCH effects from the daily returns is evident.
ARCH test enables us to detect time varying conditional volatility phenomenon and thus we are able to suggest on the appropriateness of using ARCH/GARCH models. Since the ARCH effects are present in the daily returns, we can exploit the ARCH/GARCH models to capture those dynamics.

4.5 Conditional Error Distributions Selection Criteria

We use QQ plots and information criteria to realize from which distribution the daily returns conforms to.

4.5.0.1 QQ Plots of Conditional Error Distributions in the GARCH(1,1) Model

We use QQ plots to us if the daily returns plausibly come from a certain theoretical conditional error distribution. QQ plot results are as follows:

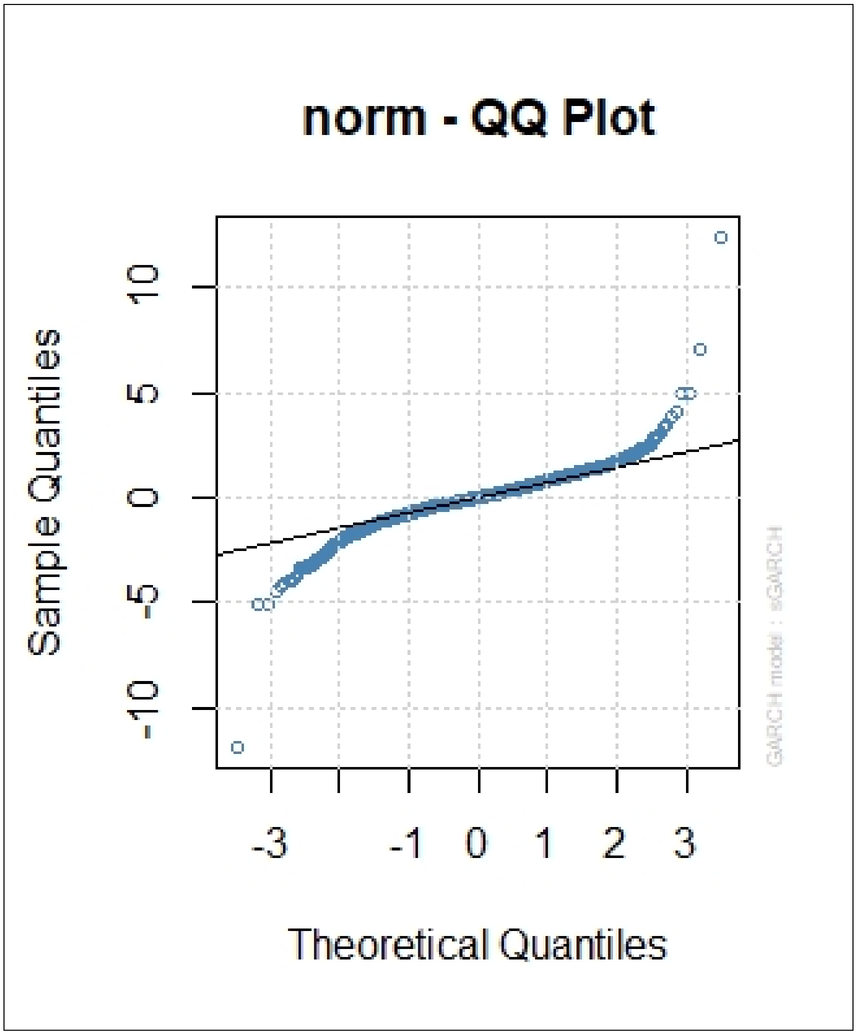

Figure 6. Normal Distribution QQ Plot

Figure 6 clearly indicates poor fit of the normal distribution since the daily returns data fails to be linear especially at the tails.

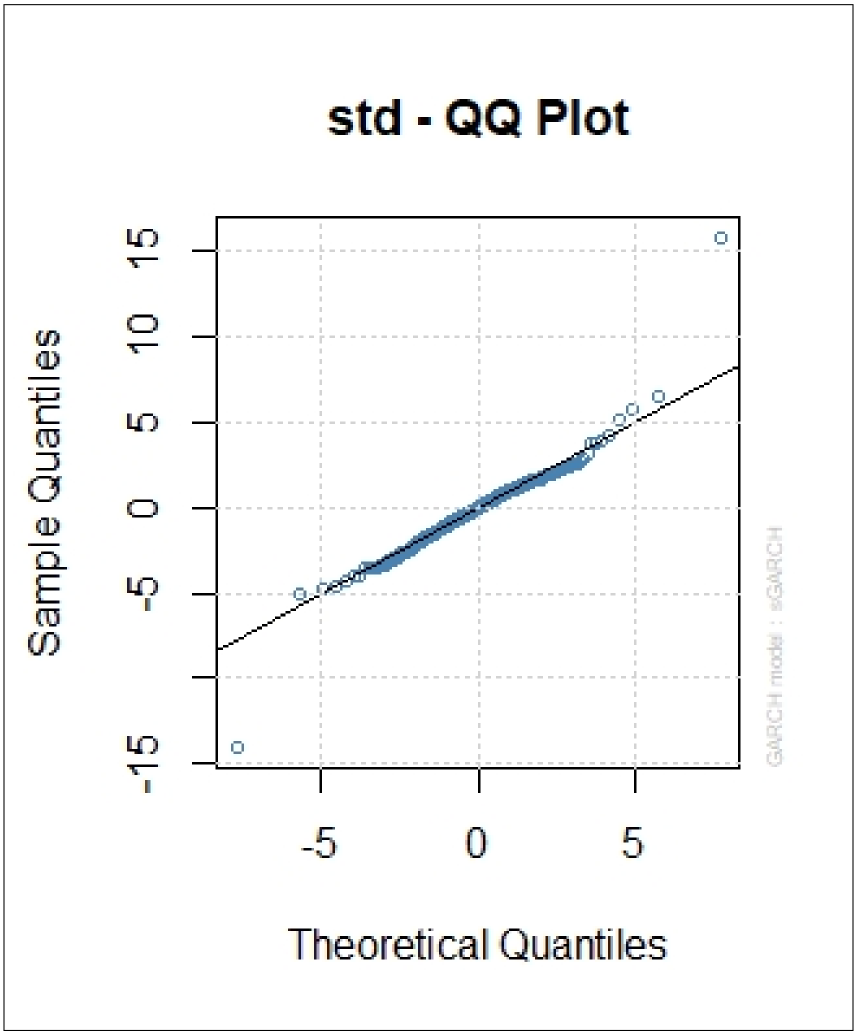

Figure 7. Student-t Distribution QQ Plot

Figure 7 shows that the std-QQ plot fairly fits with std.

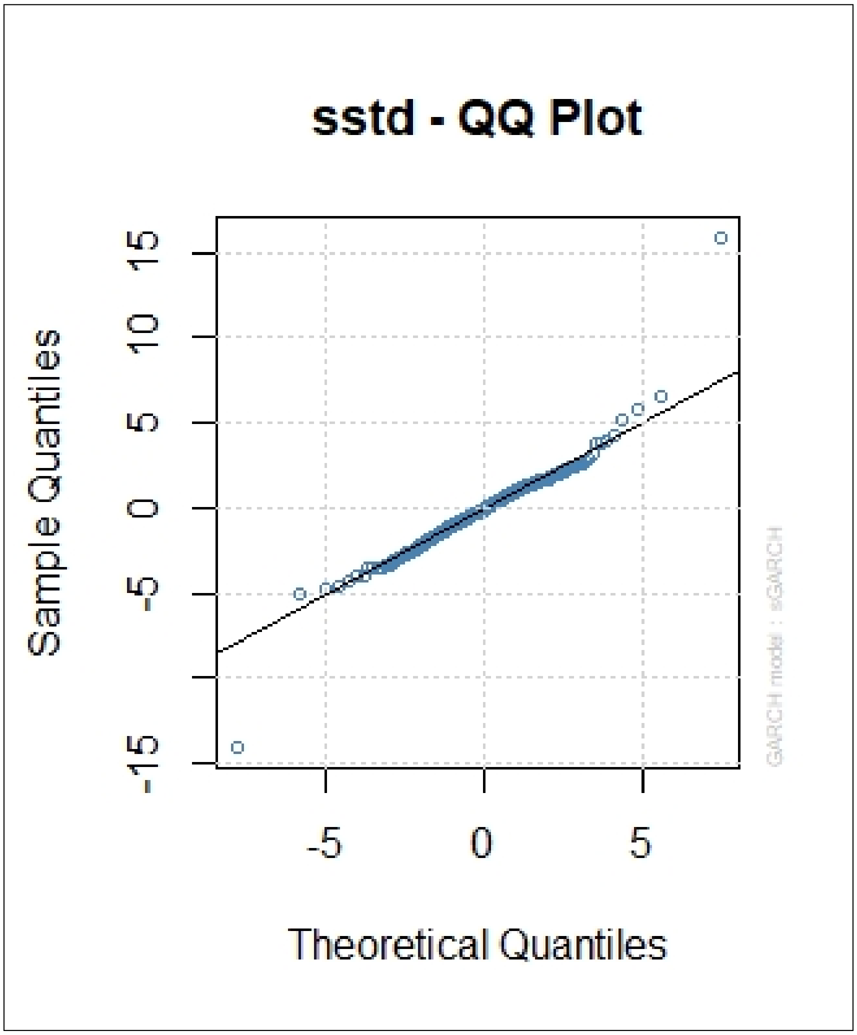

Figure 8. Skewed Student-t Distribution QQ Plot

Figure 8 shows that the sstd-QQ plot fairly fits with sstd.

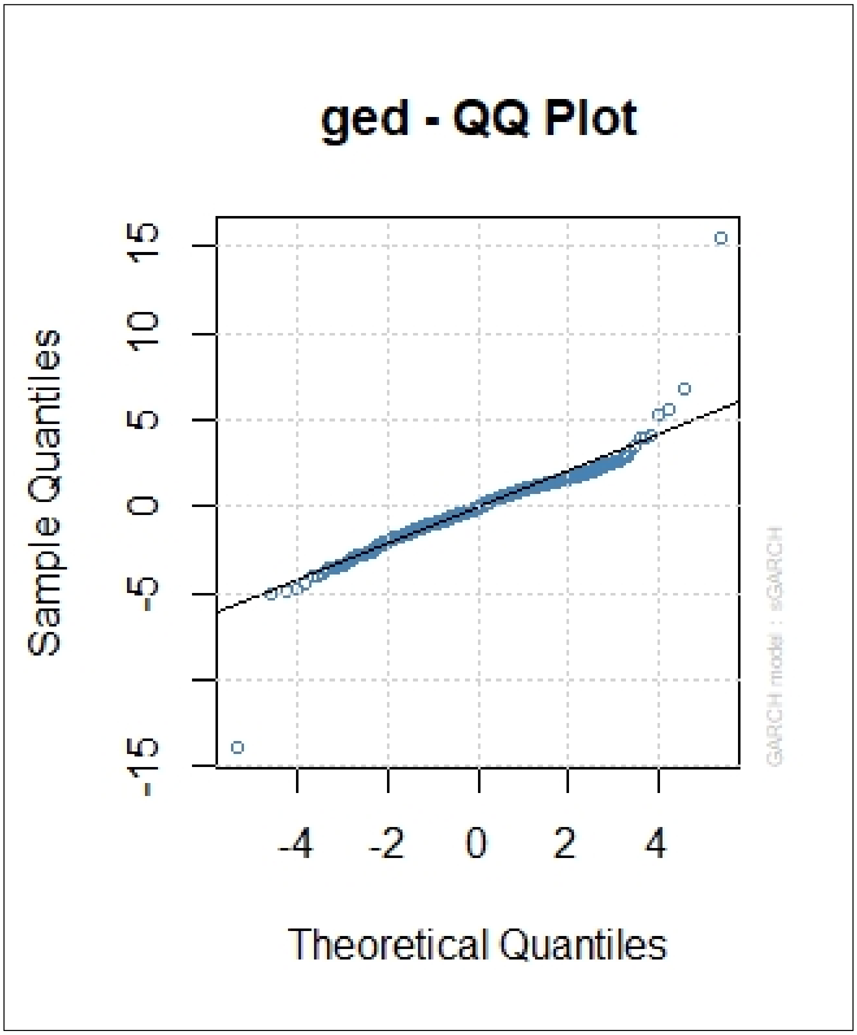

Figure 9. Generalized Error Distribution QQ Plot

Figure 9 shows that the ged-QQ plot fairly fits with generalized error distribution. Evidently std, sstd and ged seems to fit the data into linear model. Although std distribution seems to fit the best among the three distributions that fits the data relatively fair, it is not very clear which distribution fits the data hence need to use information criteria.

4.6 Conditional Error Distribution Selection Using Information Criteria

We will show information criteria after fitting the GARCH-H(1,1) with the norm, std, sstd and ged and select the distribution that describes the returns optimally.

4.6.1 Information Criteria

We will select the optimal model on the basis of AIC,BIC,SIC and HQIC and then select the best distribution to use in the model. Information criteria results shown from the four error distribution is summarised in the following table:

Distribution	AIC	BIC	SIC	HQIC
NORM	-4.8322	-4.8184	-4.8322	-4.8271
STD	-5.1250	-5.1085	-5.1251	-5.1190
SSTD	-5.1244	-5.1051	-5.1245	-5.1173
GED	-5.0926	-5.0760	-5.0927	-5.0866

Table 2. Information Criteria

The model that has the least information criteria value is the most optimal. From Table 2 Student-t distribution has the least value of the four information criteria. We therefore choose Student-t distribution to be used in the GARCH(1,1) model.

4.7 Ensemble Empirical Mode Decomposition(E-E-M-D)

The following are results obtained after decomposing the daily returns using the E-E-M-D.

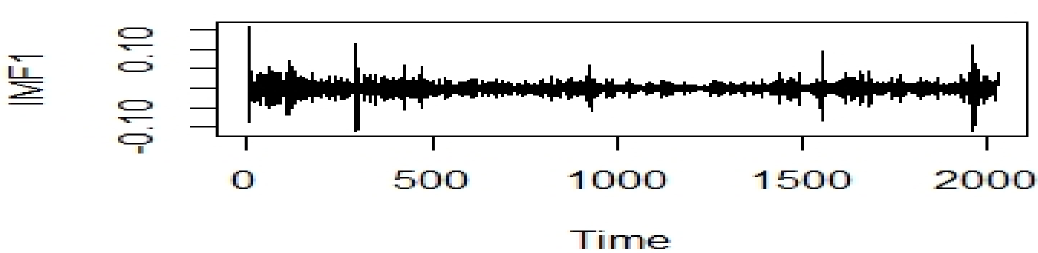

Figure 10. $C_1(t)$

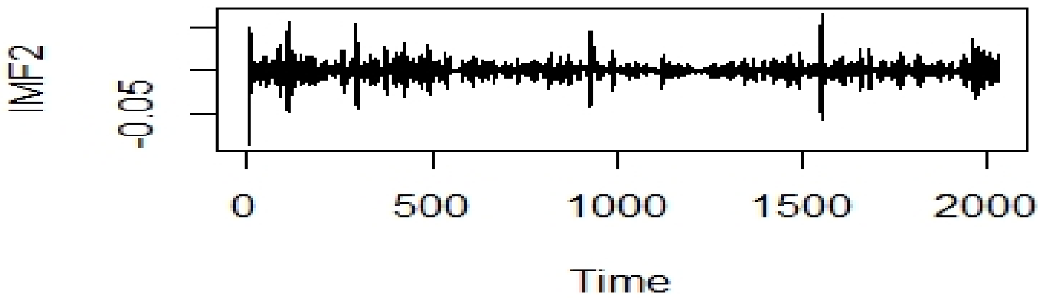

Figure 11. $C_2(t)$

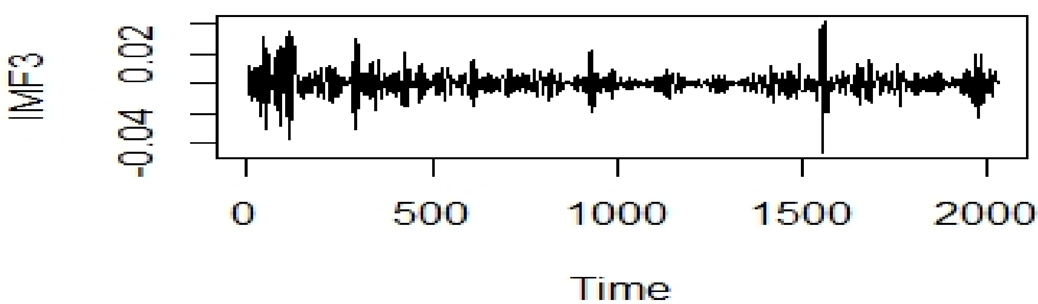

Figure 12. $C_3(t)$

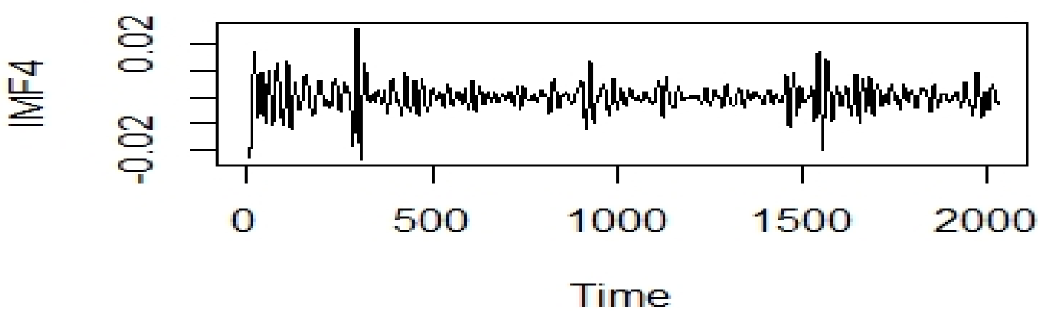

Figure 13. $C_4(t)$

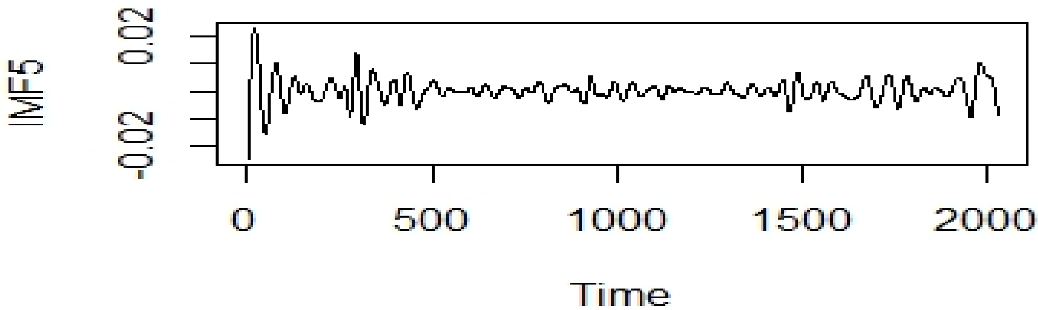

Figure 14. $C_5(t)$

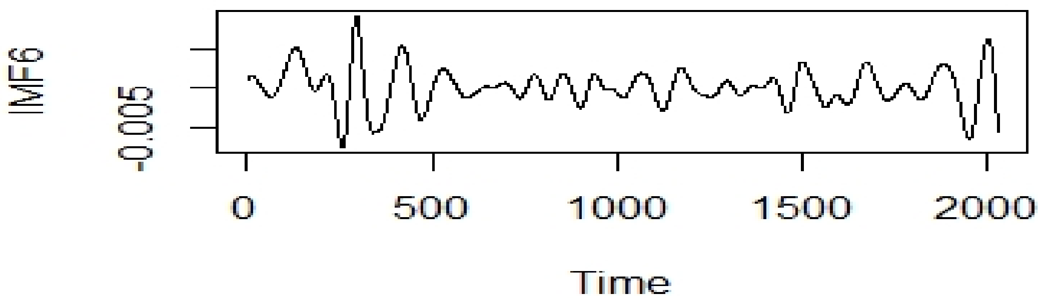

Figure 15. $C_6(t)$

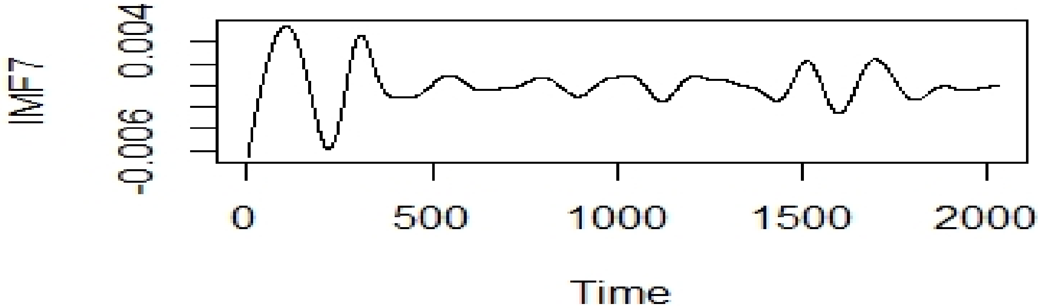

Figure 16. $C_7(t)$

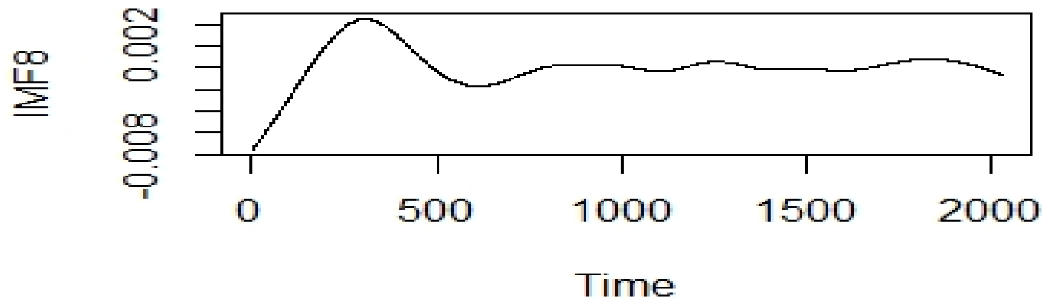

Figure 17. $C_8(t)$

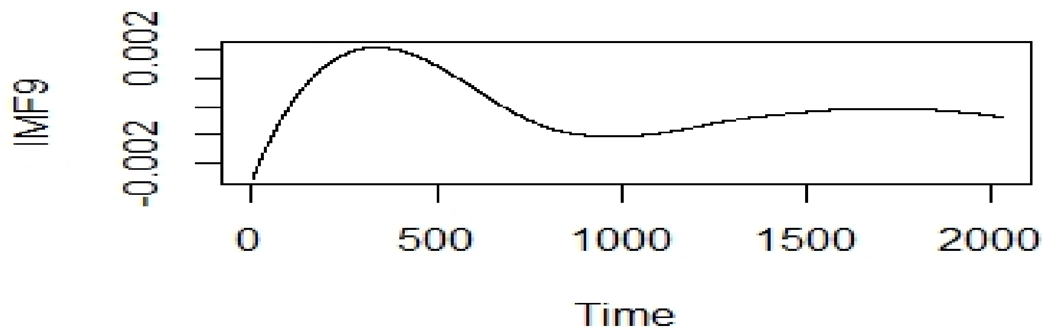

Figure 18. $C_9(t)$

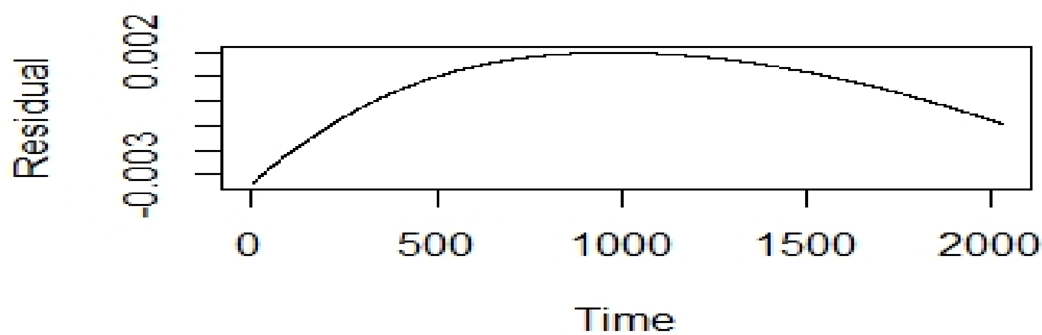

Figure 19. $R_{10}(t)$

By the [Zhang et al., 2009] fine-to-coarse reconstruction, we Exploit t-tests in obtaining the low frequency term $Z_k^l = \sum_{j=1}^{7} C_j(t)$, and the high frequency term $Z_k^h = \sum_{j=8}^{9} C_j(t)$ as follows.

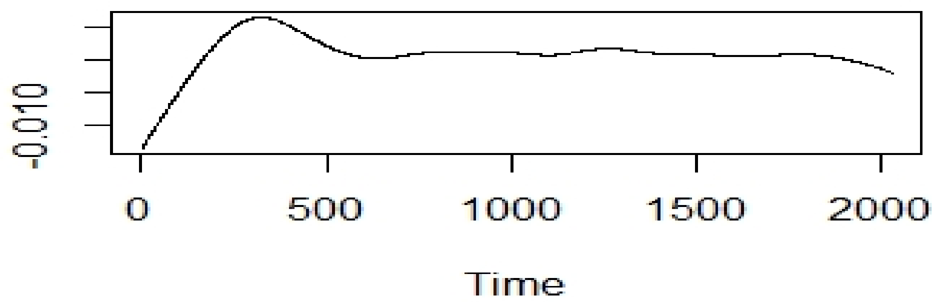

Figure 20. Low frequency term of X_t

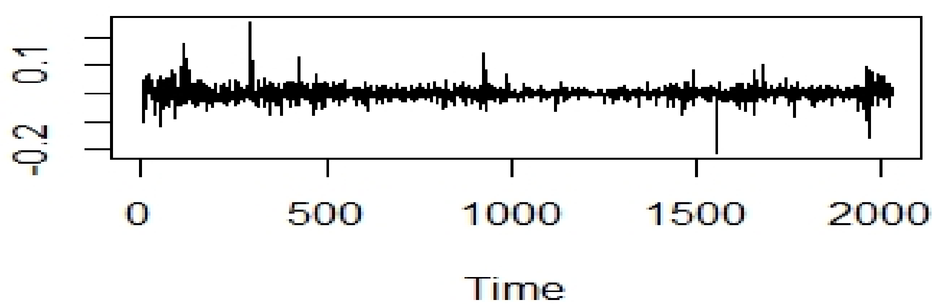

Figure 21. High frequency term of X_t

Figure 21 shows that the Z_t^h posses volatility clustering. $Z_t^h = \sum_{j=1}^{7} C_j(t)$ represents the noise term. Figure 20 shows the low frequency term $Z_t^l = \sum_{j=8}^{9} C_j(t)$, describing large returns' fluctuation. Apart from low and high frequencies term, trend term is present. Figure 19 shows the trend term illustrating equilibrium return of the period from May 2012 to Jan 2020.

4.8 European Option pricing

4.8.1 Monte Carlo Simulation

Monte Carlo simulation is an effective way of pricing complex options with no analytical solution. On the basis of GARCH-M(1,1) and GARCH-H(1,1), call option price under measure $\mathbb{Q}$ at maturity is given by:

$$C_t = e^{-\tau r}\mathbb{E}^{\mathbb{Q}}(max\{S_T - K, 0\}|\mathscr{F}_t) \tag{4.8.1}$$

Where,

- T - maturity time
- K - strike price
- r - risk free rate
- τ - time to maturity

By 3.2.2 put options can be obtained. We simulate the payoff distribution function $max\{S_T - K, 0\}$ at maturity by generating n stock price processes as given by:

$$S_{T,j} = S_t \exp(\sum_{i=t+1}^{T} x_{i,j}), j = 1, 2, 3, \ldots, n. \tag{4.8.2}$$

$x_{i,j}$ indicates return at time i in j_{th} replication. The call option is thus given as:

$$C_t = \exp^{-\tau r} \frac{1}{n} \sum_{i=1}^{n} (max\{S_T - K, 0\}) \tag{4.8.3}$$

The following table shows the parameter estimations of model 3.3.3 and 3.6.3.

Table 3. GARCH-M(1,1) and GARCH-H(1,1) Parameters

	ω	α	β	a_1	a_2	λ
GARCH-M	0.0000037153	0.055352019810	0.940549740254			0.002721
GARCH-H	0.0000036658	0.05456816	0.941255	-1.37353	-2.690293	0.0027667

Parameters in table 3 are all significantly not zero at the 5% level. The coefficients condition of $\alpha + \beta$<1 is obeyed as shown by the alpha and beta values, $0.05535202 + 0.9405497 = 0.9959018 < 1$ and $0.05456816 + 0.941255 = 0.9958231$<1

Having estimated the model parameters, we simulate option prices using the GARCH-M and GARCH(1,1) hybrid type model. We simulate option prices for 14 days, 63 days and 126 days. We now show the option prices using BSM73, GARCH-M(1,1), and GARCH-H(1,1) and then give their comparisons.

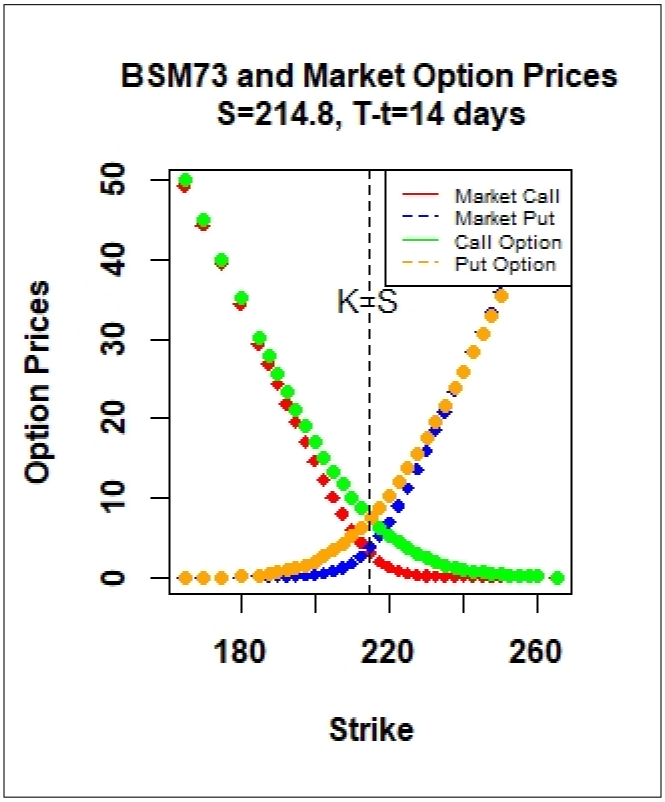

Figure 22. BSM73 and Market Option Prices when τ=14 days

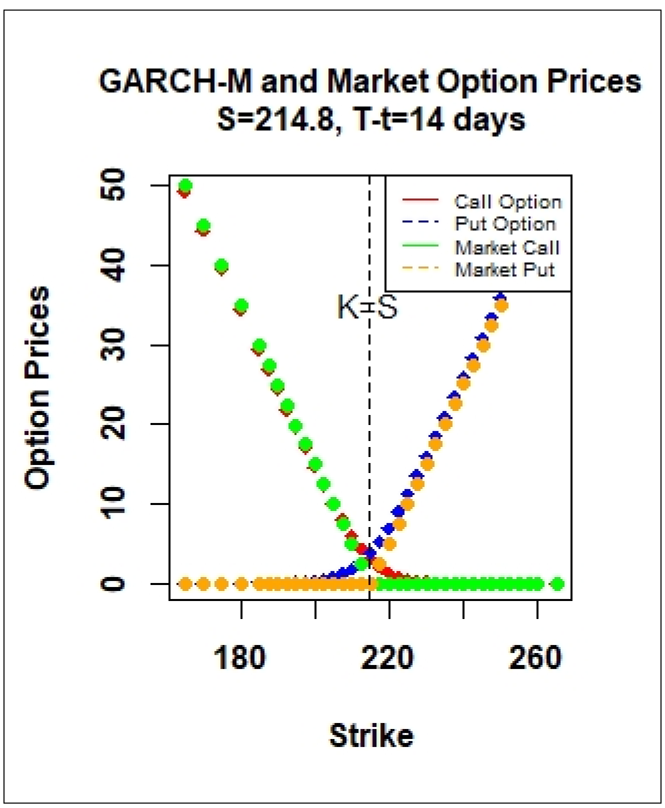

Figure 23. GARCH-M and Market Option Prices when τ=14 days

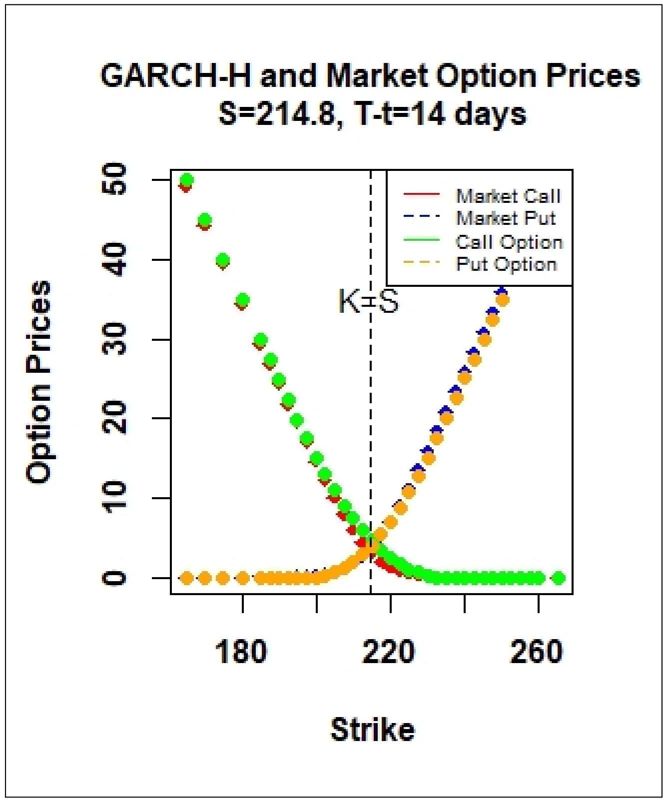

Figure 24. GARCH-H and Market Option Prices when τ=14 days

Figure 24 shows the best fit compared to Figure 23 and Figure 22 for the market option prices. GARCH-H(1,1) predicted option prices seems to be mapping on the actual market prices.

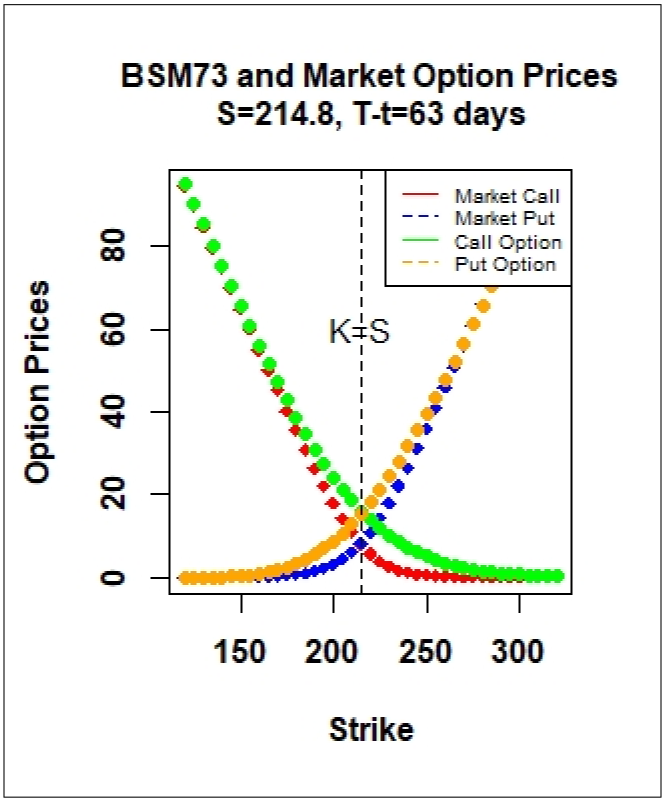

Figure 25. BSM73 and Market Option Prices when τ=63 days

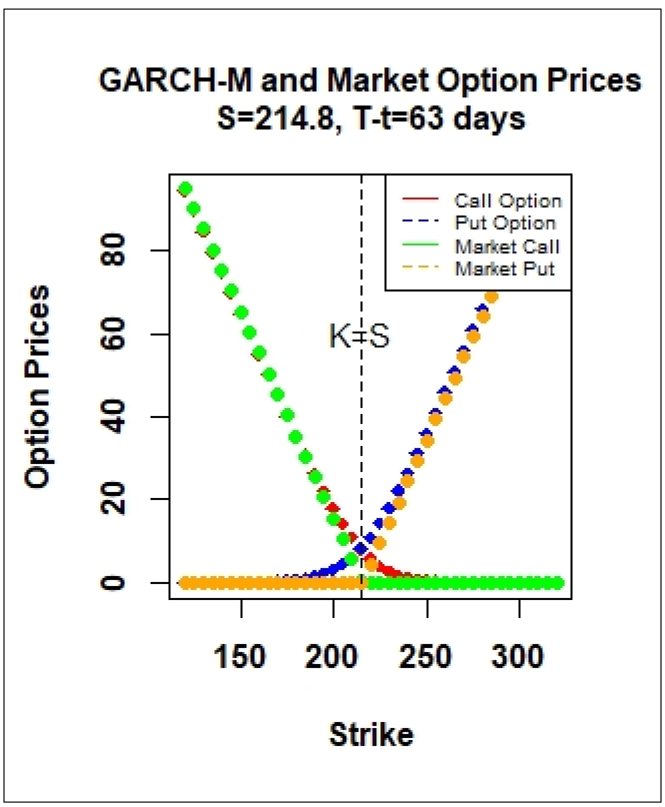

Figure 26. GARCH-M and Market Option Prices when τ=63 days

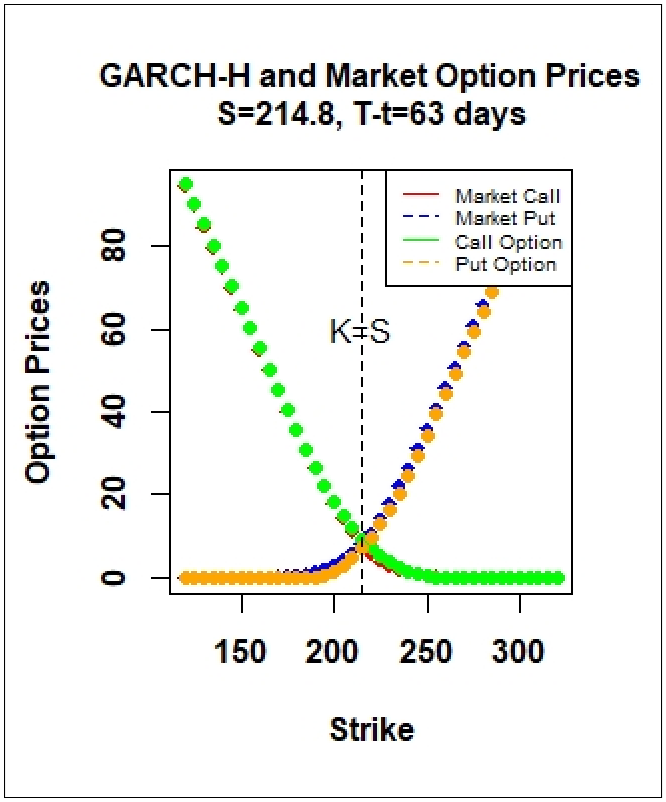

Figure 27. GARCH-H and Market Option Prices when τ=63 days

Figure 27 shows the best fit compared to Figure 26 and Figure 25 for the market option prices. GARCH-H(1,1) predicted option prices seems to be mapping on the actual market prices. The precision of the fits for the three models seems to be reducing with increasing time to maturity τ.

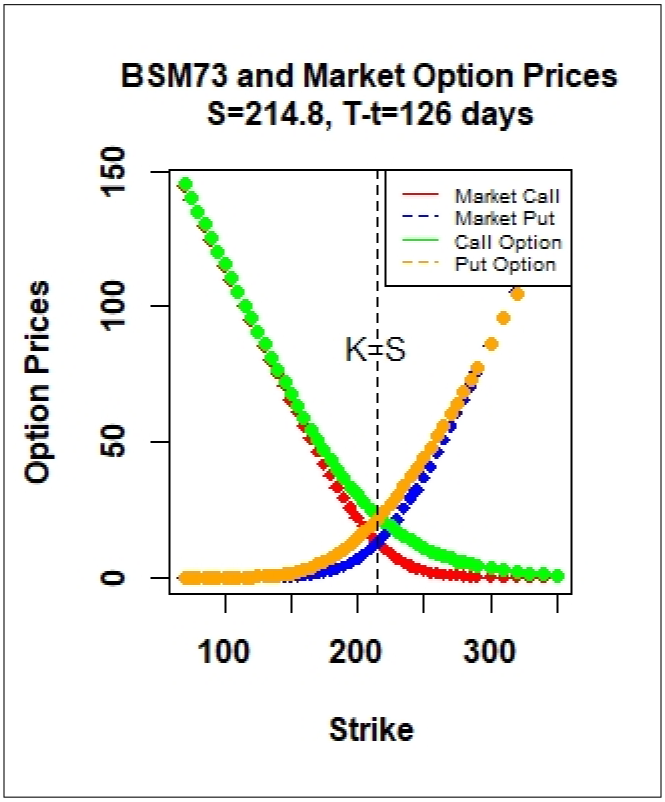

Figure 28. BSM73 and Market Option Prices when τ=126 days

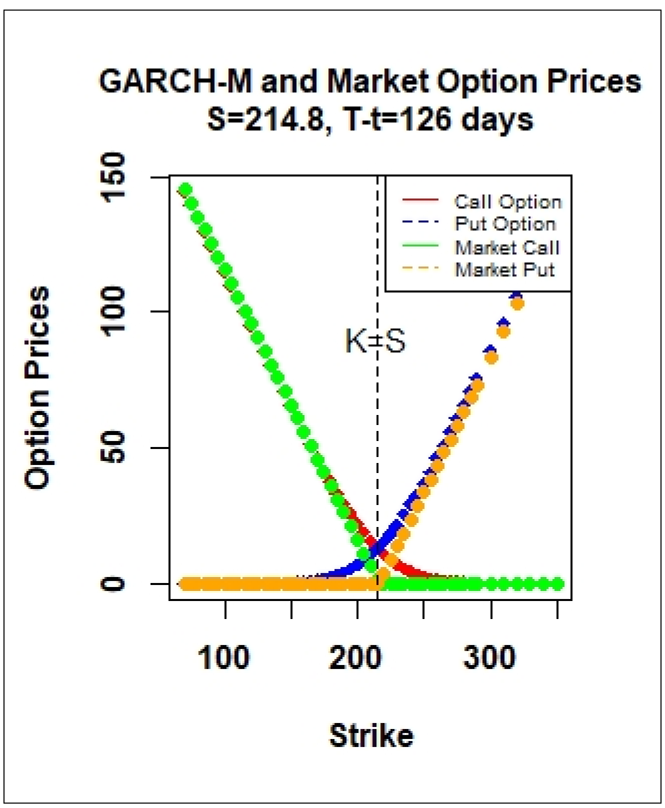

Figure 29. GARCH-M and Market Option Prices when τ=126 days

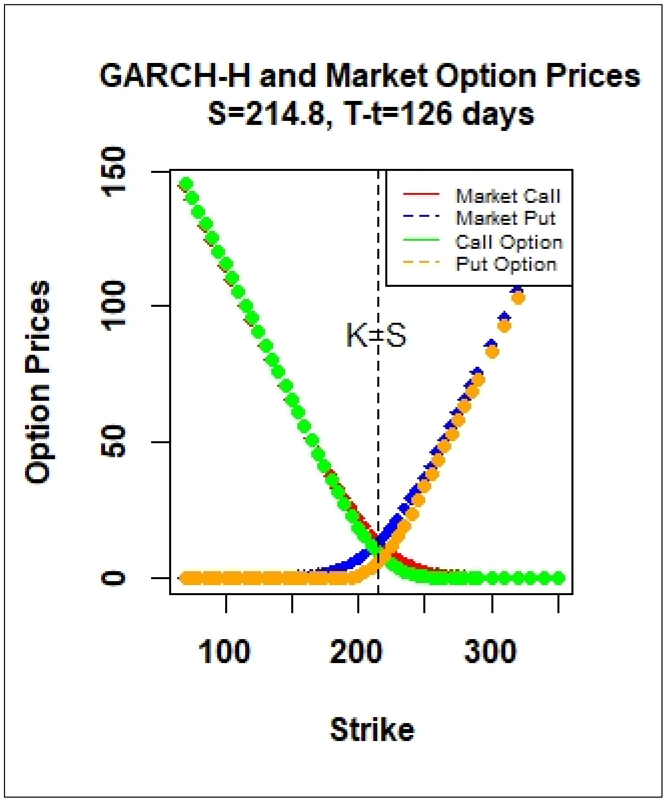

Figure 30. GARCH-H and Market Option Prices when τ=126 days

Figure 30 shows the best fit compared to Figure 29 and Figure 28 for the market option prices. GARCH-H(1,1) predicted option prices seems to be mapping on the actual market prices.. The precision of the fits for the three models seems to be reducing with increasing time to maturity τ.

From the above figures it is clear that GARCH-H(1,1) performs significantly better than the others. It is important to use statistically show how the models perform. We use error criterion to numerically find the performance of the three models. The error criterion we use to obtain goodness-of-fit criterion is given by the following relative residuals:

$$\mu_t = \frac{C_t - C_{market,t}}{C_{market,t}}$$

Where C_t is call option price of either BSM73, GARCH-M(1,1) or HG-GARCH(1,1). Considering a error criterion squared, i.e

$$E = \sum_{t=1}^{k} \mu_t^2$$

Results obtained from the three models is as follows.

Table 4. Call option prices and error criterion of the three models when τ=14 days

S/K	Price MG	Price BSM73	Price HG	μ_t MG	μ_t BSM73	μ_t HG
τ=14						
1.301818	49.932010	49.926452	49.926452	0.013334	0.013221	0.013221
1.263529	44.948428	44.930283	44.930283	0.014065	0.013656	0.013656
1.227429	39.986062	39.934115	39.934115	0.015519	0.014200	0.014200
1.193333	35.069960	34.937947	34.937947	0.021703	0.017857	0.017857
1.161081	30.242930	29.941779	29.941779	0.031302	0.021033	0.021033
1.145600	27.881925	27.443695	27.443695	0.036503	0.020212	0.020212
1.130526	25.568711	24.945611	24.945611	0.050050	0.024460	0.024460
1.115844	23.314217	22.447527	22.447527	0.065793	0.026173	0.026173
1.101538	21.130074	19.949443	19.949443	0.080822	0.020432	0.020432
1.087595	19.028200	17.451359	17.487074	0.116023	0.023540	0.025635
1.074000	17.020306	14.953275	15.155055	0.159816	0.018962	0.032712
1.060741	15.117381	12.455191	12.976803	0.229055	0.012617	0.055025
1.047805	13.329179	9.957106	10.964868	0.322995	-0.011702	0.088324
1.035181	11.663765	7.459022	9.091519	0.457971	-0.067622	0.136440
1.022857	10.127140	4.960938	7.433926	0.667019	-0.183385	0.223692
1.010824	8.722995	2.462854	5.987917	0.982499	-0.440260	0.360890
0.999070	7.452583	0.000000	4.684785	1.443470	-1.000000	0.535995
0.987586	6.314746	0.000000	3.536940	2.133869	-1.000000	0.755305
E=$\sum_{t=1}^{k}\mu_t^2$				8.469518	2.236675	1.072662

Table 5. Call option prices and error criterion of the three models when τ=63 days

S/K	Price MG	Price BSM73	Price HG	μ_t MG	μ_t BSM73	μ_t HG
τ=63						
1.790000	95.219483	95.213287	95.213287	0.006815	0.006749	0.006749
1.718400	90.244449	90.230507	90.230507	0.007193	0.007037	0.007037
1.652308	85.276915	85.247727	85.247727	0.007406	0.007061	0.007061
1.591111	80.322227	80.264948	80.264948	0.008440	0.007721	0.007721
1.534286	75.388215	75.282168	75.282168	0.009551	0.008131	0.008131
1.481379	70.485678	70.299388	70.299388	0.010910	0.008238	0.008238
1.432000	65.628676	65.316608	65.316608	0.012788	0.007972	0.007972
1.385806	60.834579	60.333829	60.333829	0.016026	0.007663	0.007663
1.342500	56.123789	55.351049	55.351049	0.022291	0.008216	0.008216
1.301818	51.519160	50.368269	50.368269	0.030383	0.007365	0.007365
1.263529	47.045117	45.385489	45.385489	0.040821	0.004104	0.004104
1.227429	42.726581	40.402710	40.402710	0.061530	0.003794	0.003794
1.193333	38.587790	35.419930	35.421595	0.084688	-0.004359	-0.004312
1.161081	34.651130	30.437150	30.533763	0.124124	-0.012582	-0.009448
1.130526	30.936087	25.454371	25.907062	0.175160	-0.033072	-0.015876
1.101538	27.458394	20.471591	21.655929	0.248109	-0.069473	-0.015640
1.074000	24.229451	15.488811	17.796766	0.349830	-0.137114	-0.008537
1.047805	21.256018	10.506031	14.366766	0.494272	-0.261439	0.009966
1.022857	18.540184	5.523252	11.303723	0.700934	-0.493280	0.037039
0.999070	16.079595	0.540472	8.618925	0.997465	-0.932861	0.070674
0.976364	13.867871	0.000000	6.372287	1.411804	-1.000000	0.108224
0.954667	11.895173	0.000000	4.475540	1.992496	-1.000000	0.125922
E=$\sum_{t=1}^{k} \mu_t^2$				7.938708	3.207416	0.035326

Table 6. Call option prices and error criterion of the three models when τ=126 days

S/K	Price MG	Price BSM73	Price HG	μ_t MG	μ_t BSM73	μ_t HG
τ=126						
2.261053	120.461623	120.453244	120.453244	0.005313	0.005243	0.005243
2.148000	115.504953	115.487625	115.487625	0.005484	0.005333	0.005333
2.045714	110.555524	110.522006	110.522006	0.005965	0.005660	0.005660
1.952727	105.617519	105.556387	105.556387	0.006360	0.005778	0.005778
1.867826	100.696603	100.590769	100.590769	0.006714	0.005656	0.005656
1.790000	95.800081	95.625150	95.625150	0.007626	0.005786	0.005786
1.718400	90.936956	90.659531	90.659531	0.009009	0.005931	0.005931
1.652308	86.117874	85.693912	85.693912	0.010477	0.005502	0.005502
1.591111	81.354961	80.728294	80.728294	0.012192	0.004396	0.004396
1.534286	76.661549	75.762675	75.762675	0.014713	0.002815	0.002815
1.481379	72.051820	70.797056	70.797056	0.014814	-0.002858	-0.002858
1.432000	67.540391	65.831437	65.831437	0.028403	0.002382	0.002382
1.385806	63.141870	60.865819	60.865819	0.027115	-0.009909	-0.009909
1.342500	58.870408	55.900200	55.900200	0.051257	-0.001782	-0.001782
1.301818	54.739291	50.934581	50.934581	0.063415	-0.010499	-0.010499
1.263529	50.760569	45.968962	45.968962	0.088115	-0.014599	-0.014599
1.227429	46.944759	41.003344	41.003344	0.123080	-0.019059	-0.019059
1.193333	43.300619	36.037725	36.061595	0.143251	-0.048509	-0.047879
1.161081	39.835005	31.072106	31.205995	0.191773	-0.070393	-0.066388
1.130526	36.552798	26.106487	26.637385	0.229699	-0.121733	-0.103873
1.101538	33.456908	21.140869	22.266040	0.300560	-0.178198	-0.134459
1.074000	30.548336	16.175250	18.212720	0.372959	-0.273022	-0.181451
1.047805	27.826286	11.209631	14.586599	0.464541	-0.410019	-0.232284
1.022857	25.288316	6.244012	11.409820	0.590460	-0.607295	-0.282401
0.999070	22.930523	1.278394	8.664141	0.733877	-0.903335	-0.344866
0.976364	20.747736	0.000000	6.362924	0.912234	-1.000000	-0.413555
0.954667	18.733724	0.000000	4.495019	1.122802	-1.000000	-0.490649
0.933913	16.881398	0.000000	3.028948	1.428978	-1.000000	-0.564180
E=$\sum_{t=1}^{k} \mu_t^2$				5.609460	4.482441	1.052278

Using the error criterion from the above tables, it is easy to observe that GARCH-H(1,1) model outperforms GARCH-M(1,1) and BSM73 at different $\frac{S}{K}$ and different maturities. BSM73 is commonly known to under-price out of the money options. The above results shows that the GARCH-H(1,1) model significantly outperforms the BSM73 in-the-money and out-of-the money. Further to confirm on the models' performance, we use root mean square error and mean absolute error as shown in the following tables.

Table 7. Root Mean Squared Error (RMSE) of the call options

τ	GARCH-M	BSM73	GARCH-H
14	0.8277008	2.259593	0.8216258
63	2.055431	4.045267	1.901062
126	3.717032	5.437031	2.966974

Table 8. Root Mean Squared Error (RMSE) of the put options

τ	GARCH-M	BSM73	GARCH-H
14	1.221131	1.783126	1.188731
63	2.599729	3.481463	2.298423
126	4.36859	4.6987	4.361075

Table 9. Mean Absolute Error (MAE) of the call options

τ	GARCH-M	BSM73	GARCH-H
14	0.5232432	1.757394	0.5215843
63	1.122546	3.068789	1.07557
126	2.161218	4.244515	1.871367

Table 10. Mean Absolute Error (MAE) of the put options

τ	GARCH-M	BSM73	GARCH-H
14	0.9215652	1.30572	0.9105411
63	1.854336	2.453804	1.739143
126	2.940007	3.387053	2.937426

The two measures clearly shows that GARCH-H(1,1) model performs significantly well in pricing call options and put options in comparison with BSM73 and GARCH-M(1,1) model.

4.8.2 Impacts of Decomposing Returns Using E-E-M-D on Option Prices

In addition to the analysis shown above, we will further show the significance of decomposing returns. After decomposition, the original signal is divided into low frequency term, high frequency term and the trend term. The three parts represents important information of the original signal.
We consider GARCH-LH(1,1) by considering 3.6.4 in 3.6.5, and GARCH-TH(1,1) by considering 3.6.6 in 3.6.7. We now use the two models to price options and compare them with the GARCH-H(1,1) to show how they perform.

Table 11. Call option prices and error criterion of the three hybrid GARCH(1,1) models when τ=14 days

S/K	Price HG	Price LHG	Price THG	μ_tHG	μ_t LHG	μ_t THG
τ=14						
1.301818	49.926452	49.926452	49.926452	0.013221	0.013221	0.013221
1.263529	44.930283	44.930283	44.930283	0.013656	0.013656	0.013656
1.227429	39.934115	39.934115	39.934115	0.014200	0.014200	0.014200
1.193333	34.937947	34.937947	34.937947	0.017857	0.017857	0.017857
1.161081	29.941779	29.941779	29.941779	0.021033	0.021033	0.021033
1.145600	27.443695	27.443695	27.443695	0.020212	0.020212	0.020212
1.130526	24.945611	24.945611	24.945611	0.024460	0.024460	0.024460
1.115844	22.447527	22.447527	22.447527	0.026173	0.026173	0.026173
1.101538	19.949443	19.949443	19.949443	0.020432	0.020432	0.020432
1.087595	17.487074	17.451359	17.451359	0.025635	0.023540	0.023540
1.074000	15.155055	14.953275	14.953275	0.032712	0.018962	0.018962
1.060741	12.976803	12.504386	12.455191	0.055025	0.016617	0.012617
1.047805	10.964868	10.253738	9.957106	0.088324	0.017741	-0.011702
1.035181	9.091519	8.201819	7.459022	0.136440	0.025227	-0.067622
1.022857	7.433926	6.362231	4.960938	0.223692	0.047281	-0.183385
1.010824	5.987917	4.815913	2.711443	0.360890	0.094526	-0.383763
0.999070	4.684785	3.491927	1.146608	0.535995	0.144894	-0.624063
0.987586	3.536940	2.347942	0.256966	0.755305	0.165232	-0.872473
E=$\sum_{t=1}^{k} \mu_t^2$				1.072662	0.065041	1.340785

Table 12. Call option prices and error criterion of the three hybrid GARCH(1,1) models when τ=63 days

S/K	Price HG	Price LHG	Price THG	μ_tHG	μ_t LHG	μ_t THG
τ=63						
1.790000	95.213287	95.213287	95.213287	0.006749	0.006749	0.006749
1.718400	90.230507	90.230507	90.230507	0.007037	0.007037	0.007037
1.652308	85.247727	85.247727	85.247727	0.007061	0.007061	0.007061
1.591111	80.264948	80.264948	80.264948	0.007721	0.007721	0.007721
1.534286	75.282168	75.282168	75.282168	0.008131	0.008131	0.008131
1.481379	70.299388	70.299388	70.299388	0.008238	0.008238	0.008238
1.432000	65.316608	65.316608	65.316608	0.007972	0.007972	0.007972
1.385806	60.333829	60.333829	60.333829	0.007663	0.007663	0.007663
1.342500	55.351049	55.351049	55.351049	0.008216	0.008216	0.008216
1.301818	50.368269	50.368269	50.368269	0.007365	0.007365	0.007365
1.263529	45.385489	45.385489	45.385489	0.004104	0.004104	0.004104
1.227429	40.402710	40.402710	40.402710	0.003794	0.003794	0.003794
1.193333	35.421595	35.419930	35.419930	-0.004312	-0.004359	-0.004359
1.161081	30.533763	30.437150	30.437150	-0.009448	-0.012582	-0.012582
1.130526	25.907062	25.454371	25.454371	-0.015876	-0.033072	-0.033072
1.101538	21.655929	20.636238	20.471591	-0.015640	-0.061989	-0.069473
1.074000	17.796766	16.304500	15.488811	-0.008537	-0.091671	-0.137114
1.047805	14.366766	12.534704	10.506031	0.009966	-0.118826	-0.261439
1.022857	11.303723	9.278803	5.758802	0.037039	-0.148734	-0.471670
0.999070	8.618925	6.518243	2.354974	0.070674	-0.190280	-0.707457
0.976364	6.372287	4.282097	0.447780	0.108224	-0.255288	-0.922125
0.954667	4.475540	2.506094	0.000000	0.125922	-0.369536	-1.000000
E=$\sum_{t=1}^{k} \mu_t^2$				0.035326	0.288308	2.667144

Table 13. Call option prices and error criterion of the three hybrid GARCH(1,1) models when τ=63 days

S/K	Price HG	Price LHG	Price THG	μ_tHG	μ_t LHG	μ_t THG
τ=126						
2.261053	120.453244	120.453244	120.453244	0.005243	0.005243	0.005243
2.148000	115.487625	115.487625	115.487625	0.005333	0.005333	0.005333
2.045714	110.522006	110.522006	110.522006	0.005660	0.005660	0.005660
1.952727	105.556387	105.556387	105.556387	0.005778	0.005778	0.005778
1.867826	100.590769	100.590769	100.590769	0.005656	0.005656	0.005656
1.790000	95.625150	95.625150	95.625150	0.005786	0.005786	0.005786
1.718400	90.659531	90.659531	90.659531	0.005931	0.005931	0.005931
1.652308	85.693912	85.693912	85.693912	0.005502	0.005502	0.005502
1.591111	80.728294	80.728294	80.728294	0.004396	0.004396	0.004396
1.534286	75.762675	75.762675	75.762675	0.002815	0.002815	0.002815
1.481379	70.797056	70.797056	70.797056	-0.002858	-0.002858	-0.002858
1.432000	65.831437	65.831437	65.831437	0.002382	0.002382	0.002382
1.385806	60.865819	60.865819	60.865819	-0.009909	-0.009909	-0.009909
1.342500	55.900200	55.900200	55.900200	-0.001782	-0.001782	-0.001782
1.301818	50.934581	50.934581	50.934581	-0.010499	-0.010499	-0.010499
1.263529	45.968962	45.968962	45.968962	-0.014599	-0.014599	-0.014599
1.227429	41.003344	41.003344	41.003344	-0.019059	-0.019059	-0.019059
1.193333	36.061595	36.037725	36.037725	-0.047879	-0.048509	-0.048509
1.161081	31.205995	31.072106	31.072106	-0.066388	-0.070393	-0.070393
1.130526	26.637385	26.106487	26.106487	-0.103873	-0.121733	-0.121733
1.101538	22.266040	21.140869	21.140869	-0.134459	-0.178198	-0.178198
1.074000	18.212720	16.425408	16.175250	-0.181451	-0.261779	-0.273022
1.047805	14.586599	12.336075	11.209631	-0.232284	-0.350733	-0.410019
1.022857	11.409820	8.856494	6.260879	-0.282401	-0.442988	-0.606234
0.999070	8.664141	5.971378	2.456167	-0.344866	-0.548478	-0.814279
0.976364	6.362924	3.668657	0.419761	-0.413555	-0.661875	-0.961312
0.954667	4.495019	1.934352	0.000000	-0.490649	-0.780810	-1.000000
0.933913	3.028948	0.755969	0.000000 -0.564180	-0.891227	-1.000000	
$E=\sum_{t=1}^{k}\mu_t^2$				1.052278	2.585600	4.252311

Figures 11, 12 and 13 shows that GARCH-H(1,1) outperforms both GARCH-LH(1,1)GARCH-TH(1,1). It is an indication of the importance of the long run factors on option pricing. However GARCH-LH(1,1) performs better than GARCH-TH(1,1) since it characterizes the actual trend of the returns.

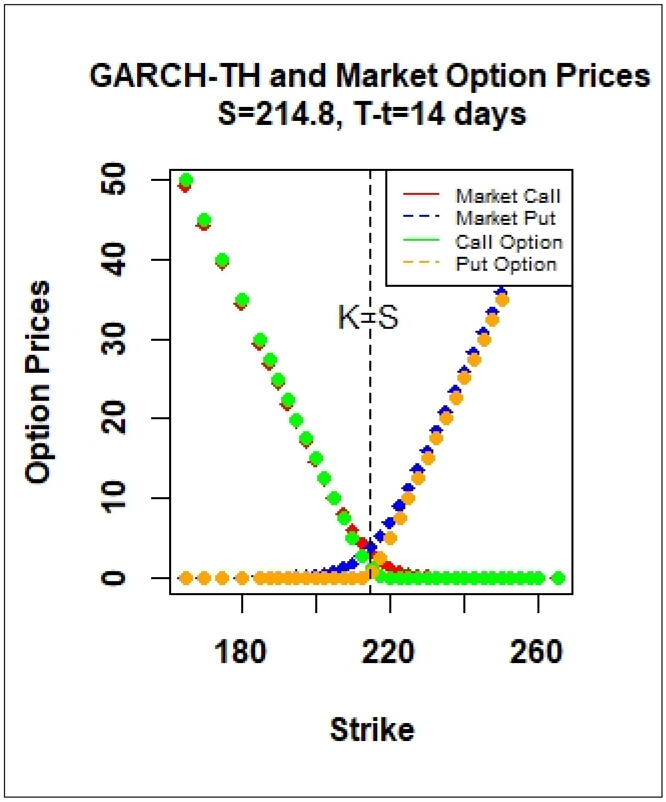

Figure 31. TH-GARCH and Market Option Prices when τ=14 days

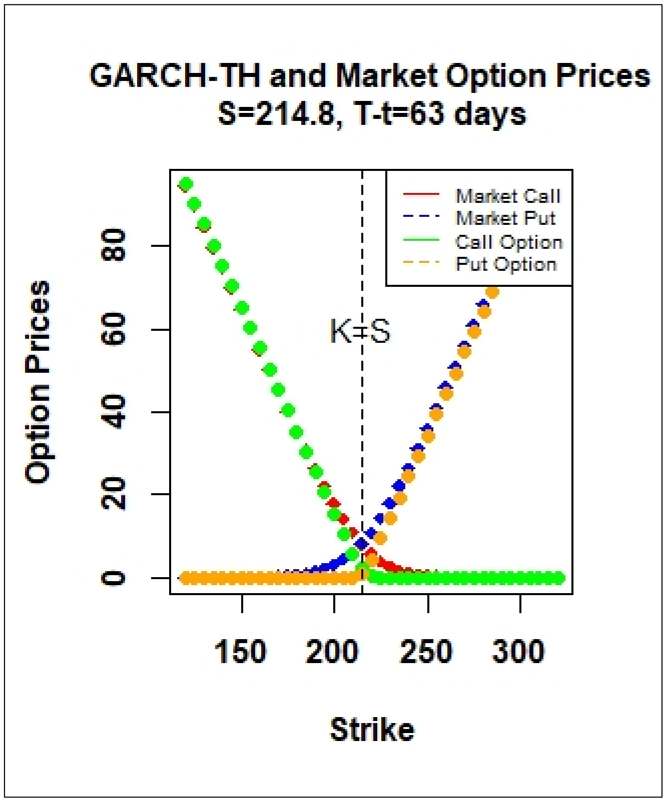

Figure 32. TH-GARCH and Market Option Prices when τ=63 days

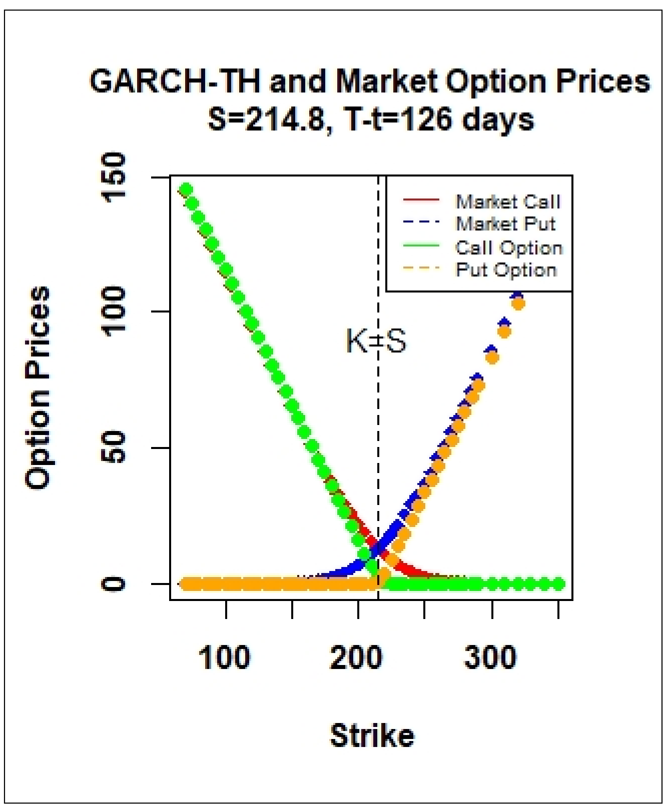

Figure 33. TH-GARCH and Market Option Prices when τ=126 days

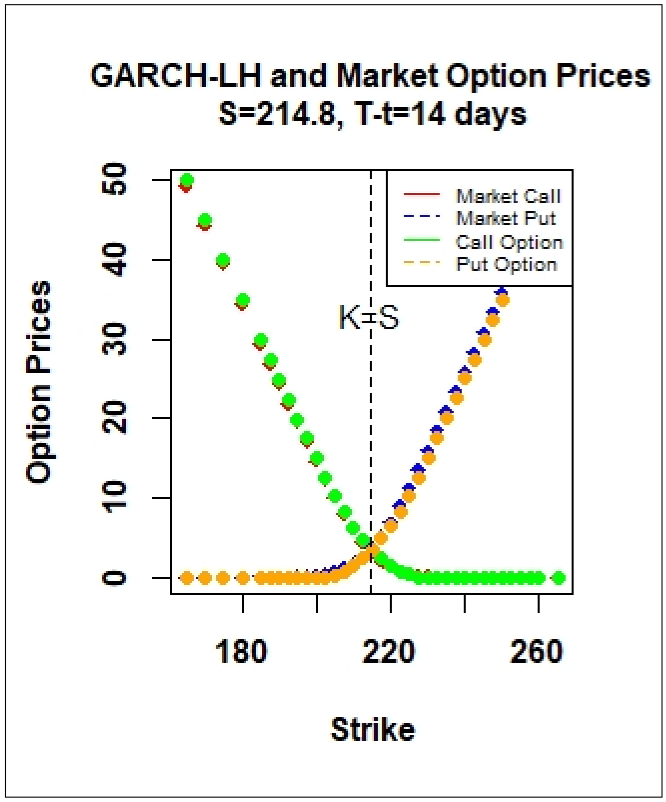

Figure 34. GARCH-LH and Market Option Prices when τ=14 days

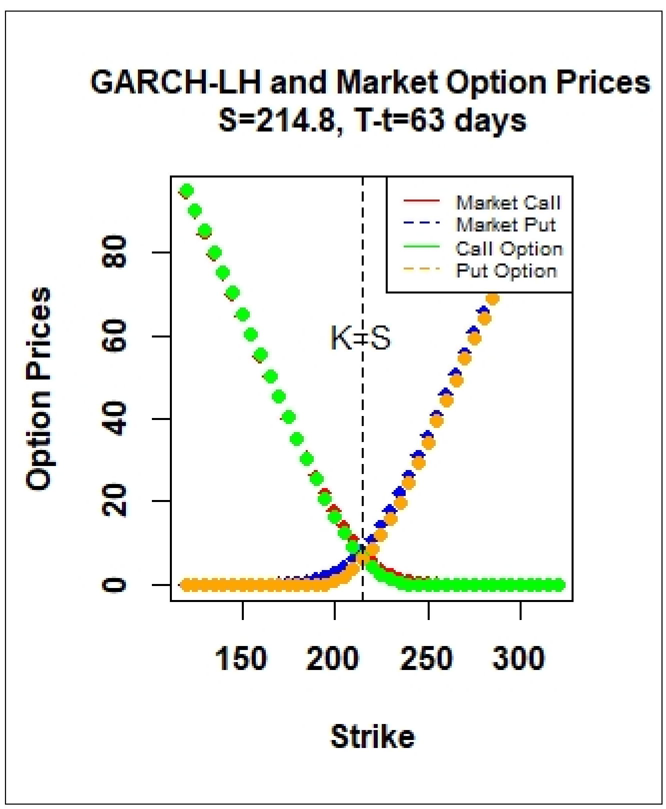

Figure 35. GARCH-LH and Market Option Prices when τ=63 days

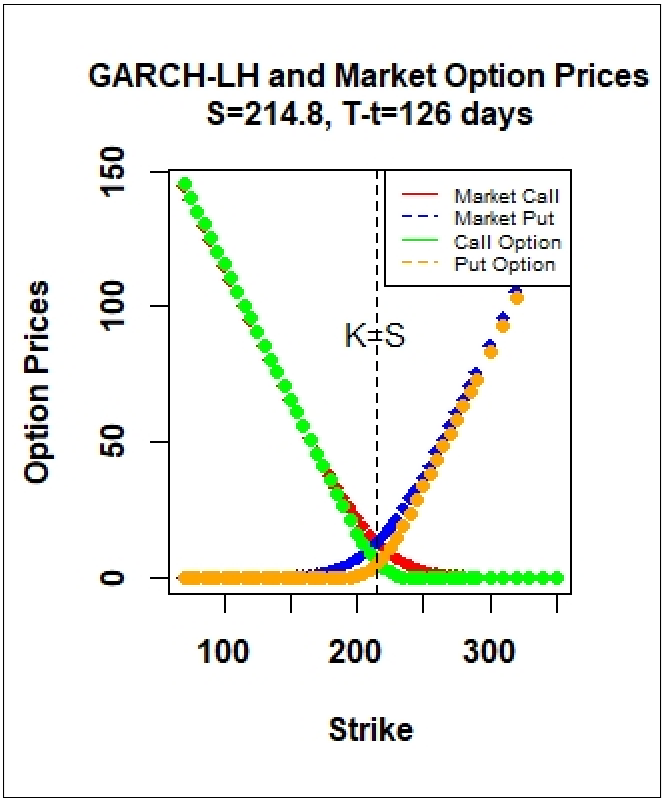

Figure 36. GARCH-LH and Market Option Prices when τ=126 days

The high frequency term characterizes impacts from short term factors such as supply and demand, inflation, policy changes, risk changes, and some emergencies. The effects of such short term factors does not last for long. Low frequency term characterizes long run factors such as economic cycles and government policies. Impacts of long run factors have extensive effects on the option price volatility. From the above figures, we can easily spot great improvement in option pricing by the GARCH-H(1,1) model. GARCH-H(1,1) significantly performs better than the other models. To further confirm the significance of decomposing returns in option pricing, we compare the MAE and RMSE of all the models to show which model performs best.

τ	BSM73	GARCH-M	H-GARCH	TH-GARCH	GARCH-LH
14	2.259593	0.8277008	0.8216258	0.6894592	0.3397197
63	4.045267	2.055431	1.901062	1.859914	0.8136603
126	5.437031	3.717032	2.966974	3.620803	2.949902

Table 14. Root Mean Squared Error (RMSE) of the call options

τ	BSM73	GARCH-M	H-GARCH	TH-GARCH	GARCH-LH
14	1.783126	1.221131	1.188731	1.17703	0.6455975
63	3.481463	2.599729	2.298423	2.543351	1.560794
126	4.6987	4.36859	4.361075	4.349598	3.66952

Table 15. Root Mean Squared Error (RMSE) of the put options

τ	BSM73	GARCH-M	H-GARCH	TH-GARCH	GARCH-LH
14	1.757394	0.5232432	0.5215843	0.4773498	0.2678409
63	3.068789	1.122546	1.07557	1.061623	0.6055375
126	4.244515	2.161218	1.871367	2.129563	1.863873

Table 16. Mean Absolute Error (MAE) of the call options

τ	BSM73	GARCH-M	H-GARCH	TH-GARCH	GARCH-LH
14	1.30572	0.9215652	0.9105411	0.9063041	0.5515168
63	2.453804	1.854336	1.739143	1.835921	1.337328
126	3.387053	2.940007	2.937426	2.93344	2.642662

Table 17. Mean Absolute Error (MAE) of the put options

From the above tables, GARCH-H(1,1) performs best, this manifest the significance of the low frequency and trend terms. GARCH-H(1,1) model with low frequency and trend terms gives the best approximations of the real option prices. From all the above analyses, decomposition using ensemble empirical mode decomposition is, indeed effective in obtaining accurate option prices. Time to maturity impacts on precision of option pricing. The error when τ=14 days is relatively lower that when τ=63 days and τ=126 days.

5 Conclusion

Previous studies have focused on the stochastic processes such as Levy processes,geometric Brownian motion and jump-diffusion processes. However, the features of underlying assets tend to vary due to various factors such as, time to maturity, economic cycles, some emergencies, monetary policies and other factors. Therefore the characteristics of the underlying assets series cannot entirely be described by the above processes from previous studies. Consequently, we decompose daily returns using E-E-M-D to obtain high frequency term, low frequency term and the trend term. Based on these terms, we construct our hybrid GARCH(1,1) option pricing model with the low frequency term and the trend term excess returns as the major components. In correspondence with our analyses, we realize that the proposed GARCH-H(1,1) model significantly performs better than BSM73 and GARCH-M(1,1). In addition, we find the impact of decomposition using E-E-M-D since GARCH-H(1,1) model with low frequency term outperforms the GARCH-H(1,1) with the excess returns of the trend term. Moreover, time to maturity of options greatly affects the accuracy of estimated option prices. From our empirical analysis, it's evident that the proposed GARCH-H(1,1) gives more accurate approximation of option prices.

5.1 Future Research

We will attempt to consider other volatility aspects and apply additional GARCH-type models in European option pricing. i

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Appendices

Black Scholes

```
BSM<-function(Sigma,r,St,K,T){
  R=r*0.01/252
  d1<-(1/(Sigma*sqrt(T)))*(log(St/K)+(R+0.5*Sigma^2)*T)
  d2<-d1-Sigma*sqrt(T)
  ct<-St*pnorm(d1)-exp(-R*T)*K*pnorm(d2)
  pt=(-St*pnorm(-d1)+exp(-R*T)*K*pnorm(-d2))
  price<-cbind(ct,pt)
  return(price)
}
####################### 14 DAYS ################
BSM14<-BSM(SIGMA,1.38,214.8,K14,14)
BSM14
ct14<-BSM14[,1]
pt14<-BSM14[,2]

##################### 63 DAYS #################
BSM63<-BSM(SIGMA,1.38,214.8,K63,63)
BSM63
ct63<-BSM63[,1]
pt63<-BSM63[,2]

##################### 126 DAYS #################
BSM126<-BSM(SIGMA,1.38,214.8,K126,126)
BSM126
ct126<-BSM126[,1]
pt126<-BSM126[,2]
##################################################
```

GARCH-M(1,1)

```
duan95<-function(FB,r,S_0,strikes,days){
  R<-0.01*r/252
  retns<-diff(log(FB[,2]))
  retns
  garchm = ugarchspec(mean.model=list(armaOrder=c(0,0),
```

```
                                        archm=T,archpow=1),
                     variance.model=list(garchOrder=c(1,1)),distribut
model1 = ugarchfit(garchm, data=retns)
model1
garchFit
coef(model1)
model1@fit$coef
resd1=model1@fit$residuals
resd1
sigma.1=model1@fit$sigma
sigma.1
gretns<-resd1/sigma.1
gretns
ArchTest(resd1)
om=model1@fit$coef[3]
om
elph=model1@fit$coef[4]
elph
bita=model1@fit$coef[5]
bita
lambda<-.01*sqrt((1-elph-bita)/elph)
lambda
elph+bita
NN<-50000
KK<-(1+days)
rz<-rnorm(KK)
RZ<-matrix(rz,nrow=(1+days),ncol=NN)
SGM<-matrix(0,nrow=(1+days),ncol=NN)
ST<-seq(0,0,length=NN)
CT<-seq(0,0,length=NN)
CT_t1<-matrix(0,nrow=(length(strikes)),ncol=NN)
MC_cec<-seq(0,0,length(strikes))
####################################################################
varQ<-om/(1-elph-bita)
####################################################################
SGM[1,]<-varQ
for(j in 2:(days+1)){
  SGM[j,]<-om+elph*(SGM[(j-1),]*RZ[j-1,])^2+bita*SGM[(j-1),]}
 r_D<-(r*0.01)/252
YT<-r_D+(sqrt(SGM)*RZ)-0.5*SGM+lambda*sqrt(SGM)
for(k in 1:NN){ST[k]<-S_0*exp(sum(YT[,k]))}
ST_tilta<-exp(days*r_D)*S_0*ST/(mean(ST))
```

```
    for(jj in 1:length(strikes)){
    for(f in 1:NN){CT_t1[jj,f]<-exp(-days*r_D)*(max(0,(ST_tilta[f]-strikes[
  }
  ########################################################################

  for(jk in 1:length(strikes)){ MC_cec[jk]<-mean(CT_t1[jk,]) }
  #list(MC_ec=MC_ec,MC_cec=MC_cec)
  return(MC_cec)
}
Gcall14<-duan95(FB,r=1.38,S_0=214.8,strikes=K14,days=14)
Gcall14
Gcall63<-duan95(FB,r=1.38,S_0=214.8,strikes=K63,days=63)
Gcall63
Gcall126<-duan95(FB,r=1.38,S_0=214.8,strikes=K126,days=126)
Gcall126
```

Ensemble Empirical Mode decomposition

```
IMF <- eemd(returns, num_siftings = 10, ensemble_size = 50, threads = 1)
plot(IMF,main="Decomposition of Returns\nUsing EEMD",cex.main=1)
plot(IMF[,1],ylab="IMF1")
plot(IMF[,2],ylab="IMF2")
plot(IMF[,3],ylab="IMF3")
plot(IMF[,4],ylab="IMF4")
plot(IMF[,5],ylab="IMF5")
plot(IMF[,6],ylab="IMF6")
plot(IMF[,7],ylab="IMF7")
plot(IMF[,8],ylab="IMF8")
plot(IMF[,9],ylab="IMF9")
plot(IMF[,10],ylab="Residual")
high=rowSums(IMF[, 1:7])
low=rowSums(IMF[, 8:9])
ct=IMF[,10]-0.0138
ct
# High frequencies
ts.plot(rowSums(IMF[, 1:7]),ylab="")
# Low frequencies
ts.plot(rowSums(IMF[, 8:ncol(IMF)]),ylab="")
```

GARCH-H(1,1)

```
du95<-function(FB,r,S_0,strikes,days){
  library(Rlibeemd)
```

```
R<-0.01*r/252
retns<-diff(log(FB[,2]))
  gspec.ru <- ugarchspec(mean.model=list(
  armaOrder=c(0,0)), distribution="std")
gfit <- ugarchfit(gspec.ru, retns)
gfit
gfit@fit$coef
om=gfit@fit$coef[2]
om
elph=gfit@fit$coef[3]
elph
bita=gfit@fit$coef[4]
bita
lambda<-.01*sqrt((1-elph-bita)/elph)
lambda
########################################################################
IMF <- eemd(retns, num_siftings = 10, ensemble_size = 50, threads = 
high=rowSums(IMF[, 1:7])
low=rowSums(IMF[, 8:9])
low
ct=IMF[,10]-0.0138
ct
lw=low[100:length(1+days)]
ctl=ct[100:length(1+days)]
mean(retns)
a1=mean(low-0.0138)*100
a2=mean(ct-0.0138)*100
##############################################################
NN<-50000
KK<-(1+days)
rz<-rnorm(KK)
RZ<-matrix(rz,nrow=(1+days),ncol=NN)
SGM<-matrix(0,nrow=(1+days),ncol=NN)
ST<-seq(0,0,length=NN)
CT<-seq(0,0,length=NN)
CT_t1<-matrix(0,nrow=(length(strikes)),ncol=NN)
MC_cec<-seq(0,0,length(strikes))
varQ<-om/(1-elph-bita)
################################################################
SGM[1,]<-varQ
for(j in 2:(days+1)){
  SGM[j,]<-om+elph*(SGM[(j-1),]*RZ[j-1,])^2+bita*SGM[(j-1),]}
```

```
  r_D<-(r*0.01)/252
  YT<-r_D+(sqrt(SGM)*RZ)-0.5*SGM+lambda*sqrt(SGM)+a1*lw+a2*ctl
  for(k in 1:NN){ST[k]<-S_0*exp(sum(YT[,k]))}
  ST_tilta<-exp(days*r_D)*S_0*ST/(mean(ST))
  for(jj in 1:length(strikes)){
    for(f in 1:NN){CT_t1[jj,f]<-exp(-days*r_D)*(max(0,(ST_tilta[f]-strikes[j
  }

  for(jk in 1:length(strikes)){ MC_cec[jk]<-mean(CT_t1[jk,]) }
  #list(MC_ec=MC_ec,MC_cec=MC_cec)
  return(MC_cec)
}
GGcall14<-du95(FB,r=1.38,S_0=214.8,strikes=K14,days=14)
GGcall14
GGcall63<-du95(FB,r=1.38,S_0=214.8,strikes=K63,days=63)
GGcall63
GGcall126<-du95(FB,r=1.38,S_0=214.8,strikes=K126,days=126)
GGcall126
```