School of Mathematics

ISSN: 2410-1397

Masters Project in Biometry

MODELLING THE CURE RATES OF FEMALE SEX WORKERS WITH STIs USING A MOVER-STAYER MARKOV CHAIN MODEL

Research Report in Biometry, Number 44, 2020

Onchwati, Felisters Kerubo

November 2020

Submitted to the School of Mathematics in partial fulfilment for a degree in Master of Science in Biometry

School of Mathematics

Masters Project in Biometry **School of Mathematics**
November 2020

MODELLING THE CURE RATES OF FEMALE SEX WORKERS WITH STIs USING A MOVER-STAYER MARKOV CHAIN MODEL

Research Report in Biometry, Number 44, 2020

Onchwati, Felisters Kerubo

School of Mathematics
College of Biological and Physical sciences
Chiromo, off Riverside Drive
30197-00100 Nairobi, Kenya

Master Thesis

Submitted to the School of Mathematics in partial fulfilment for a degree in Master of Science in Biometry

Submitted to: The Graduate School, University of Nairobi, Kenya

Master Thesis in Mathematics at the University of Nairobi, Kenya.
ISSN 2410-1397: Research Report in Mathematics

DISTRIBUTOR: School of Mathematics, University of Nairobi, Kenya

DECLARATION

Declaration by the Candidate

I, the undersigned, declare that this project is my original work and has not been presented for a degree in any other University.

Onchwati, Felisters Kerubo
Reg No. I56/11165/2018 Signature ______________ Date ______________

Declaration by the Supervisor

This project has been submitted for examination with my approval as the Supervisor.

Dr Lydiah Musiga
School of Mathematics
University of Nairobi
Box 30197, 00100
Nairobi
Kenya Signature ______________ Date ______________

Acknowledgments

I am grateful to the Almighty God for my academic journey.My deepest appreciation goes to my supervisor Dr. Musiga L.A. for her thorough guidance, patience, availability and humility. I also acknowledge the University of Nairobi for giving me the opportunity to pursue my Master of Science Degree Program. I thank my fellow classmates who offered the much-needed assistance throughout my project. Many thank for the contribution of Rev. Fr. Mogaka P.J. I also thank my foster parents Peters F. and Hesselink R. for their support. Last but not least, I am grateful to all who made this work a success.

Dedication

This project is dedicated to my mother Rosemary Bochaberi
whose curtains closed before the play could begin and to my siblings and step siblings.

Abstract

Generally human beings undergo different types of diseases. One such are sexually transmitted infections(STIs)which are either chronic or curable. In this study, a Markov model, specifically a Mover-Stayer Markov Chain model is used to determine the cure rates of female sex workers who have chronic and curable STIS. The maximum likelihood estimation method is used to obtain the proportion of stayers and movers in the Mover-Stayer Markov Chain model. Octave computational software is used to analysis and data was obtained from the Kenyan Ministry of Health. The study found that 100% of female sex workers who contract chronic STIs do not get healed while 78.78% of female sex workers who contract STIs eventually get healed. It is recommended that further research be conducted on interventions that can prevent female sex workers from contracting chronic STIs as well as interventions that drastically reduce incidences of contracting curable STIs. Furthermore, health facility workers can be sensitized on humanely and confidentially handling the female sex workers so that more female sex workers can visit health facilities more frequently.

Contents

List of Tables

CHAPTER 1
GENERAL INTRODUCTION

1.1 Introduction

Generally human beings undergo different types of diseases. One such are sexually transmitted infections (STIs) which are either curable or non-curable. Curable infections are those for which after medication the individual becomes free of malaise and symptoms. Non-curable infections are those that become lifelong once contracted. Sex workers more so, female sex workers are at a risk of contracting STIs, especially those from Third world countries. In most instances, the female sex workers go through violence and rape which go unreported further elevating the risk of infection. Furthermore, because of strict moral judgement based on religion and culture, the female sex workers are unlikely to visit health facilities freely for medical checkups for fear of being judged harshly. At times even when sick, the female sex workers prefer over the counter medication rather than visit health facilities for proper checkups.

1.2 Problem Statement

Every individual has a right to sexual as well as reproductive health. However, not everyone gets to enjoy the right to sexual health and one such case is the female sex worker. Female sex workers face harsh judgement and discrimination from those around them such that seeking health care is anathema. In this study, the problem is the determination of cure rates for female sex workers who contract chronic and curable STIs.

1.3 Objectives

1.3.1 Main Objective

The main objective of this study is to model the cure rates of female sex workers with STIs using a Mover-Stayer Markov Chain model.

1.3.2 Specific Objectives

The specific objectives are;

1. To determine the proportion of movers and stayers among female sex workers using maximum likelihood estimation method.
2. To compute the cure rates for female sex workers with chronic and curable STIs.

1.4 Methodology

In this work, the theory of Markov Chains is used, specifically the Mover-Stayer Markov Chain model. Octave software is used for analysis of the Mover-Stayer Markov Chain model. Data for this study was obtained from the Kenyan Ministry of Health.

1.5 Significance of the Study

This study will identify factors pertaining to STIs contracted by female sex workers in order to develop the necessary interventions which will safeguard both public health and the health of female sex workers.

CHAPTER 2
LITERATURE REVIEW

Many researchers have studied different aspects pertaining to female sex workers. In this chapter, summaries of some of the previous research works are presented.

Ramesh et al. (2008) carried out their study with an aim of investigating HIV prevalence in four South States and its determinants among the female sex workers in India. By the use of cluster sampling technique, a total of 10096 female sex workers were interviewed from 23 districts. Bivariate and multilevel logistic regression models were employed in the analysis of data to test the association between HIV prevalence (outcome) and sociodemographic as well as the characteristic of a sex worker (predictor) variables. It was found that marital status and the place of soliciting had a significance on HIV prevalence and therefore determined the HIV status of a sex worker.

Phrasisombath et al. (2012) did a survey involving six districts in Thailand and Vietnam to look at the health seeking behavior and barriers thereof among the female sex workers in Laos. Questionnaire method was used in data collection and a total of 407 who were female sex workers participated in the interview. Multiple logistic regression model was applied in data analysis. It was established that the hindrance to seeking of health include; long hours of waiting, health facilities not being located at a convenient place, ignorance of the existence of health services and negativity of health care givers. The pattern of care seeking among the interviewees was related to the duration an interviewee had worked as a sex worker in that some got the knowledge about health from fellow workers and friends and some, only when they attended a health facility. The type of healthy facility that they visit ranged from, health centres, publicly owned hospitals, privately owned clinics, chemists and traditional healers.

According to Scorgie et al. (2013) in their research on culture, health and sexuality, based on major towns of four African countries, Kenya, Uganda, Zimbabwe and South Africa, violence is ranked top as compared to sexually transmitted infections and HIV.Trained sex workers conducted 55 in depth interviews and 12 focus group discussions with 106 females, 26 males and 4 transgender sex workers. It was found that interviewees were denied treatment by health care givers who are also hostile to them because in the aforementioned countries, commercial sex work is termed as a crime and therefore punishable by law. Some felt that their right to confidentiality is not granted especially in public health facilities. Others were found to be ignorant of their status and in some fear played a great role in barring them from seeking medical help.

King and Maman (2013) carried out another study with an aim of looking at the motivators behind the seeking of health services by female sex workers and the hindrance facing them in hospitals and NGOs at St. Petersburg, Russia. Theirs was a qualitative study that employed methods like observation and interviews for data collection in which a total of 29 female sex workers participated. From this study, it emerged that lack of an unofficial system of registering patients, patients being poor and lack of privacy were major barriers to seeking health services by the female sex workers in Russia. The motivators were found to be, family members' support, being referred from the NGOs and the connection of a sex worker to a particular hospital. This meant that they only visited a particular hospital in which they had a connection without which, they could not make a visit to any health facility.

Kaur et al. (2014) did a study of analyzing the role of female sex workers on the dynamics of HIV transmission in India.They investigated a non-linear model for studying the transmission dynamics of HIV/AIDS epidemic with emphasis on the role of female sex workers. They divided the population under study into three categories (male, female and female sex workers) with an assumption that the rate of recruitment was different for every category. The basic reproduction number (R0) was computed and they concluded that an equilibrium is either greater than or less than 1, in which case its either disease free or endemic respectively.

Mitchell et al. (2016) carried out a research on the impact of Pre-exposure prophylaxis on HIV transmission among female sex workers and homosexuals in Bangalore. Using Bangalore data, a mathematical model was parameterized and fitted; female sex workers, clients, homosexuals and groups with lower risk of transmission. Further, they calculated the fraction of population attributed to transmission and assessed the impact and cost-effectiveness of Pre-exposure prophylaxis. The result indicated a decline in the fraction of population attributed to transmission. It was also found that Pre-exposure prophylaxis prevents HIV infection among female sex workers and homosexuals. Efficiency was found to have increased.

Sharma et al. (2017) did a research to assess the correlates of utilizing health care provision under certain provided interventions among female sex workers in Andhra, India. By the use of survey and cluster sampling method, a total sample of 1973 participants was obtained from five districts in Andhra, Pradesh. Chi-Square tests as well as negative binomial regression were used to test the association between predictors and outcome. From the results they obtained, it indicated that education, residence and symptoms of an infection were major determinants of visiting a particular health facility by a sex worker. The facilities ranged from, chemists, hospitals and privately owned clinics by NGOs.

Borquez et al. (2019) did a study in Lima, Peru which involved simulation and mathematical modelling to assess the impact and cost-effectiveness of combining strategies for preventing HIV transmission among transgender women. By the use of deterministic compartmental model, a model of transmission of HIV, clients and stability with partners was used to make an estimation of cost-effectively combining interventions as compared to the standard care to reduce incidence of HIV in a period of 10 years. While taking into account the different positions of anal intercourse, use of condom by partners, a simulation was made for transmission of HIV, fitting the model to surveillance data by the use of Latin hypercube sampling and a log-likelihood of 50 best fit employed to find the maximum and minimum values for the combinations of interventions. The interventions included; increasing condom use with clients and stability with partners, increasing antiretroviral treatment coverage at CD4 count lower, equal to or greater than 500 cells per mm^3 and also increasing the Pre-exposure prophylaxis coverage by the use of generic and branded formula. The result indicated that a considerable percentage of infection would be reduced and cost-effectiveness increased.

In summary,various methods have been used to study different aspects affecting female sex workers. In this work, the Mover-Stayer Markov chain Model will be used to determine the cure rates of female sex workers who contract chronic and curable STIs.

CHAPTER 3
THE MOVER-STAYER MARKOV CHAIN MODEL

3.1 Introduction

Generally events occur randomly thus being stochastic in nature. A stochastic process is a system which evolves in time or space in accordance with probabilistic laws. A state refers to the possible position which may be occupied by the system thus the state space comprises of all the possible states of the system.Occasionally, some events move between states whereas other events cease to change states at particular points in time.As follows are a few definitions after which the Mover-Stayer Markov Chain model is presented.

3.2 Definition of Terms

(i) Stochastic Process
A Stochastic process X(t), $t\varepsilon T$, is a collection of random variables indexed by the time parameter t. Hence $X(t)$ is the state of the process at time t.

(ii) Markov Process
A Markov process is a stochastic process such that the last state depends only on the immediate former state, that is,

$$P[X(t) \leq x|X(t_n) = x_n, X(t_n - 1) = x_n - 1, ..., X(t_0) = x_0] = P[X(t) \leq x|X(t_n) = x_n] \tag{3.1}$$

(iii) Discrete Time Markov Chain
A Discrete Tie Markov Chain is a Markov process with discrete state space and discrete parameter space.

(iv) Markov Property
The Markov Property states that the present state depends on the immediate past state and not on the remote past state.

3.3 Mover-Stayer Markov Chain Model

In this model, a population is segmented into two groups; Stayers who never leave their initial state and movers who make transitions according to a Markov chain. Assume that the movers move between states according to a Markov chain with transition probability matrix $P = ((P_{ij})), i, j = 1, 2, ..., m$. Assume that s_i is the percentage of the population who are stayers in state i, S is the diagonal matrix with entries $\{s_1, s_2, ..., s_m\}$, $(I - S)$ is a diagonal matrix with entries which indicate the proportion in a state who are potentially mobile and P is the transition matrix for mobile individuals.The assumptions of the mover-stayer Markov chain model are:

(i) there is a proportion of the population in each state that never moves.

(ii) the population which is mobile is homogeneous in its pattern of movement and follows a Markov process.

(iii) the process is stationary.

3.3.1 The One-Period Transition Probability Matrix

The one-period transition probability matrix of the Mover-Stayer Markov Chain model is given by:

$$P^{(1)} = S + (I - S)\,p \tag{3.2}$$

Where the transition probability matrix P comprises transition probabilities

$$P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1m} \\ p_{21} & p_{22} & \cdots & p_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ p_{m1} & p_{m2} & \cdots & p_{mm} \end{bmatrix} \tag{3.3}$$

From the equation(3.3) above, we see that transitions are possible from a state, to the next state or to the previous states. Thomas et al.(2000) states that in order to fit the model to data, we need to estimate $s_i, i = 1, 2, ..., m$ and p_{ij} , $i, j = 1, 2, ..., m$. The estimators of the s_i and $p_{i,j}$ are not straight forward because there is a probability $(p_{i,i})^T$ that a mover who starts in state i in period 1 will stay there for the rest of the time and so has the same behavior as a stayer. If $(\hat{p_{ii}})^T$ is the estimator for that behavior, then the maximum likelihood estimator $\hat{s_i}$ for s_i satisfies:

$$\hat{s}_i = \begin{cases} \frac{ns_i - n_i(0)p_{ii}^T}{n_i(0)[1-(p_{ii})]^T}, & ns_i - n_i(0)\,(p_{ii})^T > 0 \\ & 0, \quad \text{otherwise} \end{cases} \tag{3.4}$$

and $\hat{p_{i,i}}$ satisfies

$$\hat{p}_{ii}\,(n_i - Tns_i) = (n_{ii} - Tns_i) + (\hat{p}_{ii})^T\,(n_i - Tn_i\,(0)\,\hat{p}_{ii} - n_{ii} + Tn_i\,(0)) \tag{3.5}$$

but

$$\lim_{T\to\infty} (\hat{p}_{i,i})^T \to 0$$

implying that

$$\hat{p}_{i,i} = \frac{n_{ii} - Tns_i}{n_i - Tns_i} \tag{3.6}$$

Hence $\hat{p}_{ik}$ is obtained iteratively by

$$\hat{p}_{ik} = \frac{n_{ik}\left(1 - \hat{p}_{ii} - \sum\limits_{r=1,r\neq i}^{k-1} \hat{p}_{ir}\right)}{\sum\limits_{r=k,r\neq i}^{m} n_{ir}} \tag{3.7}$$

If one has data for T periods, $t = 1, 2, ..., T$ then in order to estimate $\hat{p}_{i,i}$, $\hat{s}_i$ and $\hat{p}_{i,k}$ let the following symbols have the following meanings

- T represents the periods, $t = 1, ...T$
- n represents the number of individuals in the sample
- s_i represents the proportion of individuals who are in a state i for all T periods.
- $(I - S_i)$ represents the proportion of individuals who are movers.
- n_{ii} represents the total number of transitions from state i to state i
- ns_i represents the number of individuals permanently observed in state i
- $n_i(t)$ represents individuals in state i at time t
- $n_i(0)$ represents individuals in state i at the initial time.
- $n_i = \sum\limits_{t=1}^{T} n_i(t)$ represents the number of visits by individuals to state i in periods 1 to T

- $n_{i,j}(t)$ represents the number of i, j transitions made at time t
- $n_{i,k}$= $\sum_{t=1}^{T} n_{i,j}(t)$ represents total number of i, j transitions from state i to state j

3.3.2 The n-Period Transition Probability Matrix

Equation 3.2 gives us the One-Period Transition Probability Matrix. In order for predictions over a multiple number of periods the n-Period Transition Probability Matrix is given by

$$P^{(n)} = S + (1 - S)\, p^n \tag{3.8}$$

Since the entries in matrix P are transition probabilities, then the asymptotic behaviour of matrix P satisfies the equation,

$$\lim_{n \to \infty} p^n = \begin{bmatrix} 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 0 \end{bmatrix} \tag{3.9}$$

In the following chapter we apply the Mover-Stayer Markov Chain Model to female sex workers' cohort data pertaining their health facility visits.

CHAPTER 4
APPLICATION OF THE MOVE-STAYER MARKOV CHAIN MODEL

4.1 Introduction

This chapter deals with the application of the Mover-Stayer Model to analyze data, the results obtained and the discussion.

4.2 Data for the Study

The data for the study is 2011 female sex workers' cohort data which was obtained from the Kenyan Ministry of Health. The sample data captured 1076 female sex worker's health facility visit records, specifically the number of subsequent visits. It was observed that all the female sex workers made a subsequent second visit and a maximum of up to twelve subsequent health facility visits were considered, as shown in Table 4.1. Seven states can be occupied by the female sex workers, that is, 0, 1, 2, 3, 4, 5 and 6. State 0 is the state of visiting the health facility hence the female sex workers who are in state 0-0 continue visiting the health facility and thus are the stayers. State 0-1 represents the female sex workers who move from the state of visiting the health facility to the state of missing 1 subsequent visit. State 2-3 represents the female sex workers who move from the state of missing 2 subsequent visits to the state of missing 3 subsequent visits, while state 6-6 represents the female sex workers who have missed 6 subsequent visits. Three assumptions are made pertaining the data:

i A female sex worker who misses 6 subsequent visits to a health facility is healed.

ii Female sex workers who continue visiting the health facility have chronic STIs.

It is also worth noting that a female sex worker can only miss a visit at a time and in order, for instance, a female sex worker cannot skip a visit from zero to say, two or three, or skip a visit from four to zero or even four to two.

Table 4.1. Female Sex Workers Health Facility Visits

	1st visit	2nd visit	3rd visit	4th visit	5th visit	6th visit	7th visit	8th visit	9th visit	10th visit	11th visit	12th visit	13th visit
0-0	1076	1076	756	582	472	410	353	320	288	260	235	219	200
0-1	0	0	320	174	110	62	57	33	32	28	25	16	19
0-2	0	0	0	0	0	0	0	0	0	0	0	0	0
0-3	0	0	0	0	0	0	0	0	0	0	0	0	0
0-4	0	0	0	0	0	0	0	0	0	0	0	0	0
0-5	0	0	0	0	0	0	0	0	0	0	0	0	0
0-6	0	0	0	0	0	0	0	0	0	0	0	0	0
1-0	0	0	0	0	0	0	0	0	0	0	0	0	0
1-1	0	0	0	0	0	0	0	0	0	0	0	0	0
1-2	0	0	0	320	174	110	62	57	33	32	28	25	16
1-3	0	0	0	0	0	0	0	0	0	0	0	0	0
1-4	0	0	0	0	0	0	0	0	0	0	0	0	0
1-5	0	0	0	0	0	0	0	0	0	0	0	0	0
1-6	0	0	0	0	0	0	0	0	0	0	0	0	0
2-0	0	0	0	0	0	0	0	0	0	0	0	0	0
2-1	0	0	0	0	0	0	0	0	0	0	0	0	0
2-2	0	0	0	0	0	0	0	0	0	0	0	0	0
2-3	0	0	0	0	320	174	110	62	57	33	32	28	25
2-4	0	0	0	0	0	0	0	0	0	0	0	0	0
2-5	0	0	0	0	0	0	0	0	0	0	0	0	0
2-6	0	0	0	0	0	0	0	0	0	0	0	0	0
3-0	0	0	0	0	0	0	0	0	0	0	0	0	0
3-1	0	0	0	0	0	0	0	0	0	0	0	0	0
3-2	0	0	0	0	0	0	0	0	0	0	0	0	0
3-3	0	0	0	0	0	0	0	0	0	0	0	0	0
3-4	0	0	0	0	0	320	174	110	62	57	33	32	28
3-5	0	0	0	0	0	0	0	0	0	0	0	0	0

3-6	0	0	0	0	0	0	0	0	0	0	0	0	0
4-0	0	0	0	0	0	0	0	0	0	0	0	0	0
4-1	0	0	0	0	0	0	0	0	0	0	0	0	0
4-2	0	0	0	0	0	0	0	0	0	0	0	0	0
4-3	0	0	0	0	0	0	0	0	0	0	0	0	0
4-4	0	0	0	0	0	0	0	0	0	0	0	0	0
4-5	0	0	0	0	0	0	320	174	110	62	57	33	32
4-6	0	0	0	0	0	0	0	0	0	0	0	0	0
5-0	0	0	0	0	0	0	0	0	0	0	0	0	0
5-1	0	0	0	0	0	0	0	0	0	0	0	0	0
5-2	0	0	0	0	0	0	0	0	0	0	0	0	0
5-3	0	0	0	0	0	0	0	0	0	0	0	0	0
5-4	0	0	0	0	0	0	0	0	0	0	0	0	0
5-5	0	0	0	0	0	0	0	0	0	0	0	0	0
5-6	0	0	0	0	0	0	0	320	174	110	62	57	33
6-0	0	0	0	0	0	0	0	0	0	0	0	0	0
6-1	0	0	0	0	0	0	0	0	0	0	0	0	0
6-2	0	0	0	0	0	0	0	0	0	0	0	0	0
6-3	0	0	0	0	0	0	0	0	0	0	0	0	0
6-4	0	0	0	0	0	0	0	0	0	0	0	0	0
6-5	0	0	0	0	0	0	0	0	0	0	0	0	0
6-6	0	0	0	0	0	0	0	0	320	494	604	666	723

4.3 The Stayer Matrix

In this section we compute the transition probabilities of the stayer matrix. The values for$\hat{p}_{ii}$; i = 0 and 6, that is, transition probability from state i to state i are computed using equation (3.2). In the equation, n_{ii} represents the total number of transitions from state i to state i, T represents the total number of the subsequent visits, ns_i represents the number of female sex workers permanently observed in state i and n_i represents the total number of visits to state i in periods 1 to T. Considering state 0 - 0, that is for i = 0, n_{00} = 6247, T = 12, ns_0 = 200 and n_0 = 6247, thus

$$\hat{p}_{00} = \frac{6247 - 12(\frac{200}{1076}) * 1076}{6247 - 12(\frac{200}{1076}) * 1076} = \frac{3847}{3847} = 1 \tag{4.1}$$

In equation (3.4) which is used to estimate $\hat{S}_i$, ns_i represents the number of individuals permanently observed in state i, $n_i(0)$ represents the number of individuals in state i initially and $(\hat{p}_{00})^T$ is the probability that a mover who starts in state i in period 1 will stay there for the rest of the time and so has the same behavior as a stayer. Considering state $0-0$,

$$s_0 = \frac{1076 * (\frac{200}{1076}) - (1076)(1)^{12}}{1076(1-(1^{12}))} = \frac{200-(1076)}{1076(1-1)^{12}} = \frac{-876}{0} = 0 \tag{4.2}$$

Because of the condition in equation 3.4, the numerator is less than 0 and therefore returns value (0) for S_0 above.

considering state 6-6, $i =$,n_{66}=2807,T=12, ns_6=0, n_6=3563, then

$$\hat{p}_{66} = \frac{2807 - 12(1076 * (\frac{0}{1076})}{3563 - 12(1076 * \frac{0}{1076})} = \frac{2807}{3563} = 0.7878 \tag{4.3}$$

$$S_6 = \frac{1076 * (\frac{0}{1076}) - 320(0.7878)^{12}}{320(1-(0.7878)^{12})} = \frac{-18.2870}{301.713} = 0 \tag{4.4}$$

Because the numerator is > 0 we return a value (0) for S_6. Thus the Stayer Matrix is given by,

$$S = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{4.5}$$

4.4 The Mover Matrix

Now we compute the transition probabilities of the mover matrix which are represented by$\hat{p}_{ik}$ as shown in equation (3.3), where n_{ik} represents the total number of i; j transitions from state i to state j,$\hat{p}_{ii}$ is the estimate of the transition probability from a state to the same state,$\hat{p}_{ir}$ is the estimate of the transition probability from a state to another different state and n_{ir} represents the total number of transitions from a state to another different state. Thus the transition probability from state 0 to 1 is given by;

$P_{01} = \frac{n_{01}(1-P_{00}) - \sum_{r=1,r\neq 0}^{1-1} .\hat{P}_{0r}}{\sum_{r=1}^{6} .n_{0r}}$

Thus

$$P_{01} = \frac{876(1-1) - \sum_{r=1}^{0} .\hat{P}_{0r}}{876+0} = \frac{876*0}{876+0} = 0 \tag{4.6}$$

$$P_{12} = \frac{857(1-0) - \sum_{r=2,r\neq 1}^{2-1} .\hat{P}_{1r}}{(0+857+0+0+0)} = \frac{857}{857} = 1 \tag{4.7}$$

$$P_{23} = \frac{841(1-0) - \sum_{r=1,r\neq 2}^{3-1} .\hat{P}_{2r}}{(0+0+841+0+0+0)} = \frac{841}{841} = 1 \tag{4.8}$$

$$P_{34} = \frac{816(1-0) - \sum_{r=1,r\neq 3}^{4-1} .\hat{P}_{3r}}{(0+0+0+816+0+0)} = \frac{816}{816} = 1 \tag{4.9}$$

$$P_{45} = \frac{788(1-0) - \sum_{r=1,r\neq 4}^{5-1} .\hat{P}_{4r}}{(0+0+0+0+788+0)} = \frac{788}{788} = 1 \tag{4.10}$$

$$P_{56} = \frac{3563(1-0.7878) - \sum_{r=1,r\neq 5}^{6-1} .\hat{P}_{5r}}{(0+0+0+0+0+756)} = \frac{756.0686}{756} = 1.000 \tag{4.11}$$

Thus, assuming time homogeneity, the Mover Matrix is;

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.78780 \end{bmatrix} \tag{4.12}$$

4.5 One-Period Transition Probability Matrix

The one-period transition probability matrix of the Mover-Stayer Markov Chain model was given by equation (3.2) as
$P^{(1)} = S + (I - S)\,p$

Hence the one-period transition probability matrix is obtained as,

$P^1 =$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.78780 \end{bmatrix} \right)^1$$

$=$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.78780 \end{bmatrix} \tag{4.13}$$

4.6 n-Period Transition Probability Matrix

The n-period transition probability matrix was given in equation (3.8) by
$P^{(n)} = S + (1-S)\,p^n$
As an example, the 3-period transition probability matrix is obtained as
$P^3 = S + (I-S)P^3$ Thus;

$$P^3 =$$

$$\begin{bmatrix} 0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0 \end{bmatrix} + \begin{bmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&0&0&0\\ 0&0&1&0&0&0&0\\ 0&0&0&1&0&0&0\\ 0&0&0&0&1&0&0\\ 0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1 \end{bmatrix} \left(\begin{bmatrix} 1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0\\ 0&0&0&1&0&0&0\\ 0&0&0&0&1&0&0\\ 0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0.78780 \end{bmatrix} \right)^3$$

=

$$= \begin{bmatrix} 1&0&0&0&0&0&0\\ 0&0&0&0&1&0&0\\ 0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0.78780\\ 0&0&0&0&0&0&0.62063\\ 0&0&0&0&0&0&0.48893 \end{bmatrix} \quad \textbf{(4.14)}$$

The n-period transition probabilities are displayed in Tables 4.2 as shown.

Table 4.2. n-Step Transition Probabilities

Pn	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
0-0	1	1	1	1	1	1	1	1	1	1
0-1	0	0	0	0	0	0	0	0	0	0
0-2	0	0	0	0	0	0	0	0	0	0
0-3	0	0	0	0	0	0	0	0	0	0
0-4	0	0	0	0	0	0	0	0	0	0
0-5	0	0	0	0	0	0	0	0	0	0
0-6	0	0	0	0	0	0	0	0	0	0
1-0	0	0	0	0	0	0	0	0	0	0

1-1	0	0	0	0	0	0	0	0	0	0
1-2	1	0	0	0	0	0	0	0	0	0
1-3	0	0	0	0	0	0	0	0	0	0
1-4	0	0	1	0	0	0	0	0	0	0
1-5	0	0	0	0	0	0	0	0	0	0
1-6	0	0	0	0	1	0.7878	0.6206	0.4849	0.3852	0.3034
2-0	0	0	0	0	0	0	0	0	0	0
2-1	0	0	0	0	0	0	0	0	0	0
02-2	0	0	0	0	0	0	0	0	0	0
2-3	1	0	0	0	0	0	0	0	0	0
02-4	0	1	0	0	0	0	0	0	0	0
2-5	0	0	0	0	0	0	0	0	0	0
2-6	0	0	0	1	0.7878	0.6206	0.4889	0.3852	0.3034	0.2391
3-0	0	0	0	0	0	0	0	0	0	0
3-1	0	0	0	0	0	0	0	0	0	0
3-2	0	0	0	0	0	0	0	0	0	0
3-3	0	0	0	0	0	0	0	0	0	0
3-4	1	0	0	0	0	0	0	0	0	0
3-5	0	1	0	0	0	0	0	00	0	0
3-6	0	0	1	0.7878	0.6206	0.4889	0.3852	0.3034	0.2391	0.1883
4-0	0	0	0	0	0	0	0	0	0	0
4-1	0	0	0	0	0	0	0	0	0	0
4-2	0	0	0	0	0	0	0	0	0	0
4-3	0	0	0	0	0	0	0	0	0	0
4-4	0	0	0	0	0	0	0	0	0	0
4-5	1	0	0	0	0	0	0	0	0	0
4-6	0	1	0.7878	0.6206	0.4889	0.3852	0.3034	0.2391	0.1883	0.1484
5-0	0	0	0	0	0	0	0	0	0	0
5-1	0	0	0	0	0	0	0	0	0	0
5-2	0	0	0	0	0	0	0	0	0	0
5-3	0	0	0	0	0	0	0	0	0	0

5-4	0	0	0	0	0	0	0	0	0	0
5-5	0	0	0	0	0	0	0	0	0	0
5-6	1	0.7878	0.6206	0.4889	0.3852	0.3034	0.2391	0.1883	0.1484	0.1169
6-0	0	0	0	0	0	0	0	0	0	0
6-1	0	0	0	0	0	0	0	0	0	0
6-2	0	0	0	0	0	0	0	0	0	0
6-3	0	0	0	0	0	0	0	0	0	0
6-4	0	0	0	0	0	0	0	0	0	0
6-5	0	0	0	0	0	0	0	0	0	0
6-6	0.7878	0.6206	0.4889	0.3852	0.3034	0.2391	0.1883	0.1484	0.1169	0.0921

4.7 Discussion

Referring to Tables 4.1 and 4.2, we note that in 1-step transition, 100% f female sex workers who contract chronic STIs do not stand a chance of being healed, thus will continue visiting health facilities without end. On the other hand, 78.78% of female sex workers who contract STIs eventually get healed.

On further analysis and predicting the future rates, we see that in 4-step transitions, 100% of female sex workers who will have contracted chronic STIs will still not get healed but 78.78% of female sex workers who will have missed 3 visits will get healed, 62.06% of female sex workers who will have missed 4 visits will get healed, 48.89% of female sex workers who will have missed 5 visits will get healed and 38.52% of female sex workers who will have missed 6 visits will get healed. In ten-step transitions, still 100% of female sex workers who will have contracted chronic STIs will not get healed while 30.34% of female sex workers who will have missed 1 visit will get healed, 23.91% of female sex workers who will have missed 2 visits will get healed, 18.83% of female sex workers who will have missed 3 visits will get healed, 14.84% of female sex workers who will have missed 4 visits will get healed, 11.69% of female sex workers who will have missed 5 visits will get healed and 9.21% of female sex workers who will have missed 6 visits will get healed.

In the next chapter, conclusions and recommendations are presented.

CHAPTER 5

5.1 Introduction

This chapter summarizes the work done, makes conclusions from the findings of the study and gives recommendations for further research.

5.2 Summary

The proportion of female sex workers who had chronic STIs (stayers) and thus will always have to visit health facilities as well as the proportion of female sex workers who had curable STIs were obtained using maximum likelihood estimation method. Octave software was used for analysis and subsequently cure rates were predicted.

5.3 Conclusion

In conclusion, the results indicate that 100% of female sex workers who contract chronic STIs do not get healed while 78.78% of female sex workers who contract STIs eventually get healed.

5.4 Recommendations

From this study, further research can be conducted on interventions that can prevent female sex workers from contracting chronic STIs as well as on interventions that can drastically reduce incidences of contracting curable STIs. Health facility workers can be sensitized on humanely and confidentially handling the female sex workers so that the female sex workers can visit health facilities more frequently.

References

[1] Bórquez A., Guanira J. V., Revill P., Caballero P., Silva-Santisteban A., Kelly S., Salazar X., Bracamonte P., Minaya P., Hallett T. B., and Cáceres C. F., The Impact and Cost-Effectiveness of Combined HIV Prevention Scenarios Among Transgender Women Sex-Workers in Lima, Peru: A Mathematical Modelling Study, Lancet Public Health,4(3),127-136,2019

[2] Kaur N., Ghosh M. and Bhatia S. S., Mathematical Analysis of the Transmission Dynamics of HIV/AIDS: *Role of Female Sex Workers,* Appl. Math. Inf. Sci. 8 (5), 2491-2501,2014.

[3] King E. J., and Maman S., Structural Barriers to Receiving Health Care Services for Female Sex Workers in Russia, Qual Health Res., 23(8), 1079–1088,2013.

[4] Mitchell K. M., Prudden H. J., Washington R., Isac S., Rajaram S. P., Foss A. M., Terris-Prestholt F., Boily M. C., and Vickerman P., Potential Impact of Pre-Exposure Prophylaxis for Female Sex Workers and Men who have Sex with Men in Bangalore, India: A Mathematical Modelling Study,*Journal of the International AIDS Society,* 19(1), 1-10, 2016.

[5] Olofsson P. and Andersson M., Probability, Statistics and Stochastic Processes, $2^{n}d$ Edn.,John Wiley & Sons,USA,2012.

[6] Phrasisombath K., Thomsen S., Sychareun V. and Faxelid E., Care Seeking Behaviour and Barriers to accessing Services for Sexually Transmitted Infections among Female Sex Workers in Laos: *A Cross-Sectional Study, BmC Health Services Research*,12(37), 1-9,2012.

[7] Ramesh B. M., Moses S., Washington R., Isac S., Mohapatra B., Mahagaonkar S. B., Adhikary R., Brahmam G. N., Paranjape R. S., Subramanian T. and Blanchard J. F., Determinants of HIV Prevalence among Female Sex Workers in Four South Indian States: *Analysis of Cross-Sectional Surveys in Twenty -Three Districts*,AIDS,22(5), 35-44, 2008.

[8] Scorgie F., Nakato D., Harper E., Richter M., Maseko S., Nare P., Smit J.and Chersich M., "We are despised in the hospitals": Sex Workers' Experiences of Accessing Health Care in Four African Countries , Culture, Health & Sexuality: *An International Journal for RESEARCH, Intervention and Care,* 15(4),450-465,2013.

[9] Sharma V., Suryawanshi D., Saggurti N. and Bharat S., Correlates of Health Care Utilization Under Targeted Interventions: *The Case of Female Sex Workers in Andhra Predesh, India, Health Care for Women International*, 38(11), 1188-1201, 2017.

[10] Thomas L.C., Edelman D.B and Crook J.N., Credit Scoring and its Applications, USA,2000.

[11] Wasserman L., All of Statistics: A Concise Course in Statistical Inference, Springer, USA,2005.