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Master Project in Mathematics

A STUDY OF W_5 - CURVATURE TENSOR IN LP- SASAKIAN MANIFOLD

Research Report in Mathematics, 08, 2021

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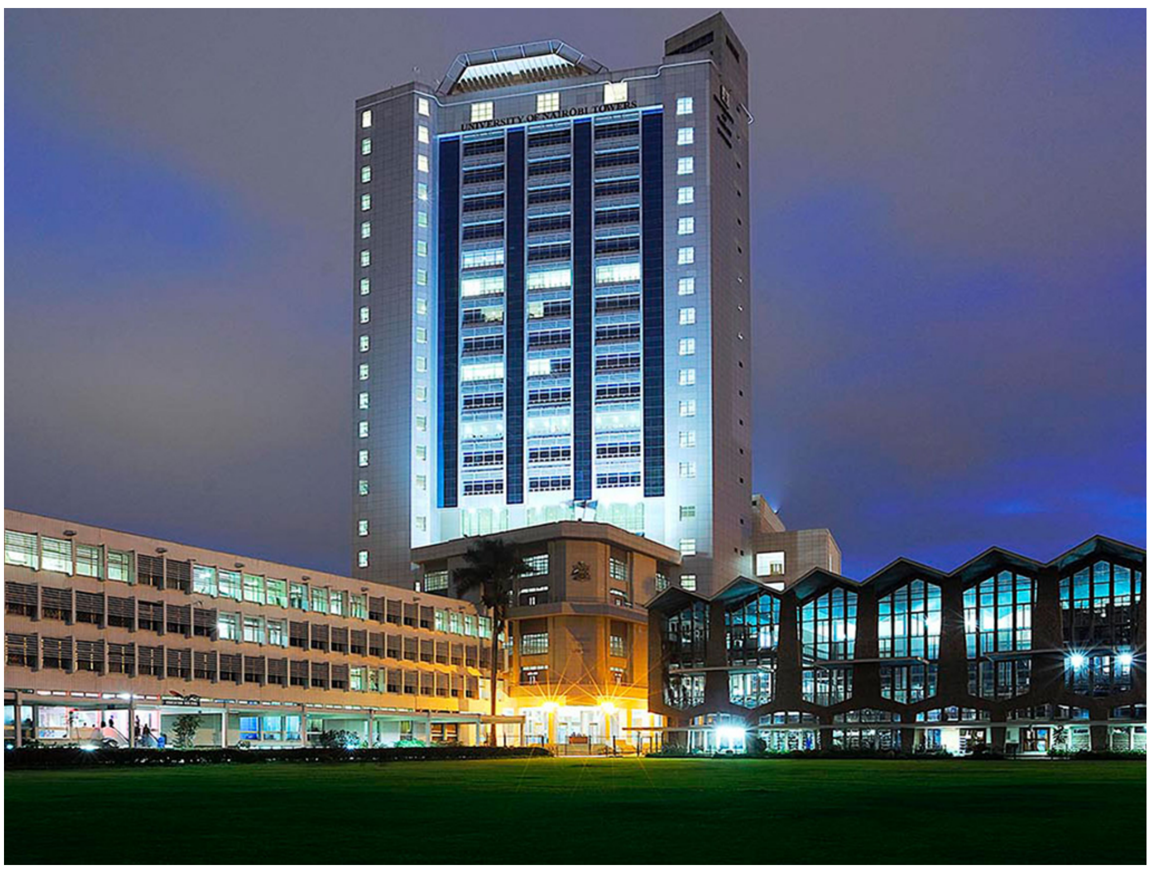

Submitted to the School of Mathematics in partial fulfilment for a degree in Master of Science in Applied Mathematics

Master Project in Mathematics **University of Nairobi**
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A STUDY OF W_5 - CURVATURE TENSOR IN LP-SASAKIAN MANIFOLD

Research Report in Mathematics, 08, 2021

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Master Thesis
Submitted to the School of Mathematics in partial fulfilment for a degree in Master of Science in Applied Mathematics

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Abstract

The goal of this project, is to study the properties of W_5-Curvature tensor in LP -Sasakian manifold and the following theorem are proved.

- A W_5 -flat LP-Manifold is a flat manifold.

- A W_5 -Semisymmetric LP-Sasakian manifold is said to be W_5-flat manifold.

- A W_5 -Symmetric and W_5 -flat LP-Sasakian manifold is a flat manifold.

-A W_5-Recurrent LP-Sasakian manifold with $R(X,Y)W_5(Z,U)V = 0$ and $A(X)g(Y,Z) - (2 - \frac{1}{n-1})g(X,Z)A(Y) = 0$ is a W_5-Symmetric space.

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Declaration and Approval

I the undersigned declare that this dissertation is my original work and to the best of my knowledge, it has not been submitted in support of an award of a degree in any other university or institution of learning.

19/08/2021

Signature Date

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In my capacity as a supervisor of the candidate's dissertation, I certify that this dissertation has my approval for submission.

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Dedication

This project is dedicated to my late father, Lucas Simmitta the most hardworking man who made me beleive in myself.To my mothers Josephine Owinga and Domtila Adhiambo for their prayers, Brother Sylvester Okoth for his wisdom,direction and support, you have been a great influence in my life.Bro Ben and to my sisters Betty and Quinter.

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Special thanks to the University of Nairobi, through the school of Mathematics for accepting me to be one of their students.
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1 Introduction

Riemannian geometry is the branch of differential geometry that studies Riemannian manifolds, smooth manifolds with a Riemannian metric.

After emergence of Riemannian Geometry, studies of results concerning the geometry of surfaces and the behaviour of geodesics on the surfaces have been developed.It has also enhanced the development of algebraic and differential topology, more so, the idea of smooth manifold admitting Riemannian metric has helped solve the problem of differential topology.

Though the concept of metric tensor was known to some mathematicians such as Carl Gauss from the 19^{th} century,Gregio Ricci - Curbastro and Tullio Levi - Civita in the early 20^{th} century understood the properties of the metric tensor.

In 1960, a Japanese, Shigeo Sasaki, started the study of almost contact structures in terms of certain tensor fields.Later in 1962 what is now called Sasakian manifold first appeared under the name of normal contact metric structures.In 1965 the term Sasakian structure and Sasakian manifold were frequently used replacing the original terms.

The study of Sasakian manifold brings together many field in mathematics from differential and algebraic topology through complex algebraic geometry to Riemannian manifold with special holonomy.

1.0.1 Notations and Definitions

Definition 1.0.1 *Consider an n-dimensional manifold M.If we let ρ be a point on the manifold, then $V\rho$ is the set of all vector field defined at p Therefore, $V\rho$ is also an n-dimensional space.*

Definition 1.0.2 *A 1- form vector $\vec{r}$ defined at ρ is a linear scalar operator acting on a vector space $V\rho$ to real number $\Re$.*
This then means
(1) $\vec{r} : V\rho \longrightarrow \Re$
(2) *For any* $\vec{u}$, $\vec{v}$ $\in Vp$ *and if a,b* $\in \Re$
$\Rightarrow \vec{r}$ (a$\vec{u}$ + b$\vec{v}$)=a $\vec{r}(\vec{u})$+b$\vec{r}(\vec{v})$
The set of all 1-forms defined at ρ is called a **co-vector** or a dual space of $V\rho$, and it is denoted by V_ρ^*.This is also an **n-dimensional vector space**.

Definition 1.0.3 *Any vector $\vec{u} \in V_\rho$ can be associated with a linear scalar operator acting on 1-form $\vec{u} \in V_\rho$ to $\Re$.*
i.e $\vec{u}\,\vec{r} \neq \vec{r}\,\vec{u} : V_\rho^* \to \Re$

Definition 1.0.4 *An (k,l)- type tensor defined at point ρ is a linear scalar operator with l slots for 1- form from V_ρ^* and k slots from V_ρ.Such tensor can also be defined as **l-times contravariant** and **k-times covariant**.*
*The total number of slots, r=l+k, is called **the rank of the tensor**.*
Thus

(1). Any vector is a (1,0) -type tensor.

(2). Any 1-form is a (0,1) -type tensor

Remark

Tensors therefore are a generalization of vectors and 1-form covectors.

A tensor of (k,l) at ρ is a multi-linear map which takes k vectors and l covectors (1-forms) and gives a real number.

A tensors (or tensor field) T, of type (k,l) is denoted with k superscripts and l subscripts (T_l^k) and is said to be of rank k+l.

Definition 1.0.5 *Let M be smooth manifold a tangent vector at a point $\rho \in M$ is a map X_ρ : C^∞ (M)$\to \Re$ which satisfies .*

1.X_ρ (f+g)= $X_\rho(f)$ + $X_\rho(g)$

2. X_ρ = 0 (for constant map)

3. X_ρ (fg) = f(ρ)Xg + g(ρ) X_ρ

f $\forall$ f,g $\in C^\infty$ (M) on the common domain.

*The set of all tangent vectors to an n-dimensional manifold M at a point P$\in$ M form an n-dimensional vector space which is called the **tangent space**, denoted by T_ρ M.*

Definition 1.0.6 *Let M be a smooth manifold , then by a Riemannian metric tensor g on M, we have a smooth assignments of an inner product to each tangent space of M.This then means, for each $\rho \in M$, $g_\rho : T\rho\ M \times T\rho\ M \to \Re$ is symmetric, positive definite and bi-linear map.That is, for any smooth vector field X and Y on M.*

$P \mapsto g_p$ (X_ρ,X_ρ) is a smooth function.

It is (2,0)- tensor, g$\in T_0^2$ (M).

In a cordinate system, we may write,

$$g = g_{ij}\, dx^i \otimes dx^j.$$

Then the pair (M,g) will be called ***Riemannian manifold.***

Definition 1.0.7 *By S and R, where S denote Ricci tensor and R, Riemannian curvature tensor of an n-dimensional Riemannian manifold (M,g), then S can be defined as*

$$S(X,Y) = \sum_{i=0}^{n} g(R(e_i,x)Y,e_i)$$

where $e_1, e_2, ,, , e_n$ are orthonormal basis vector fields in T_M and X,Y Z$\in T_M$.

Definition 1.0.8 *Let M be smooth manifold an Affine Connection (Levi- Civita)* ∇ *on M is a differential operator, sending smooth vector fields X and Y to a smooth vector field* $\nabla_X Y$ *which then satisfies the following conditions*

1.∇_{X+Y}Z = ∇_XZ + ∇_YZ

2.∇_X(Y + Z) = ∇_XY + ∇_XZ

3.∇_{fX} = f ∇_XY

4.∇_X(fY) = X(f) + f(∇_XY)

$\forall$ vector fields X,Y and Z and real valued function f on M.
The vector field ∇_X is known as the **covariant derivative** of the vector field Y along X with respect to to ∇.

Definition 1.0.9 *A curve* γ *(s) is a geodesic if its tangent vector* $\vec{\gamma}$*(s) at each point are parallel.*

Definition 1.0.10 *A homeomorphism f:X* $\rightarrow$ *Y is continous bijection whose inverse* $f^{-}1$*: Y* $\rightarrow$ *X is also continous.*

Definition 1.0.11 *Let M be an n-dimensional contact manifold with contact for* η*,that is,* $\eta(d\lambda)^n \neq 0$*, then, a contact manifold admits a vector field* ξ *called* ***characteristics vector*** *such that* $\eta(\xi)=1$ *for any field X*$\in \chi(M)$*.Further , if M admits a Riemannian metric g and a tensor field* ϕ *of type (1,1) such that,*

$$\phi^2X = X - \eta(X)\xi$$

$$g(X,\xi) = \eta(X)$$

$$g(X,\phi Y) = d\eta(X,Y)$$

The we say that (ϕ,η,ξ,g) is a **contact metric structure**.

Definition 1.0.12 *A contact metric manifold is said to be* ***sasakian*** *if* $(\nabla_X\phi)Y = g(X,Y)\xi - \eta(Y)X$.
where

$$\nabla_x\xi = -\phi X$$

$$R(X,Y)\xi = \eta(Y)X - \eta(X)Y$$

For all vectors fields $X,Y \in M$.

Definition 1.0.13 *An n-dimensional manifold M is said to admit an* ***almost para-contact*** *Riemannian structure* (ϕ,η,ξ,g) *such that*

$$\phi^2 X = X - \eta(X)\xi,$$

$$\phi\xi = 0, \eta(\xi)=1, \eta(\phi X) = 0,$$

$$g(X,\xi) = \eta(X),$$

$$g(\phi X,\phi Y) = g(X,Y) - \eta(X)\eta(Y)$$

$\forall$ *vectors fields* X,T *on M.*

If (ϕ,η,ξ,g) *satisfy the equation*

$$d\eta = 0, \nabla_X\xi = \phi X$$

$$(\nabla_X\phi) = -g(X,Y)\xi - \eta(Y)X + 2\eta(X)\eta(Y)\xi$$

Then M is called **Para-sasakian manifold**.

If M admits 1-form η,such that $(\nabla_X \eta)Y = -g(X,Y) + \eta(X)\eta(Y)$, for all X,Y $\in$ M.The **para-sasakian manifold** is said to be a **special manifold.**

Definition 1.0.14 *An n-dimensional differentiable manifold M^n is **Lorentizian Para-Sasakian manifold** if it admits a (1,1) tensor field ϕ, vector field ξ, 1-form η and Lorentizian metric g which satisfies*

$$\phi^2(X) = X + \eta(X)\xi$$

$$\phi(\xi) = 0,\ \eta(\xi) = -1,\ \eta(\phi X) = 0,$$

$$g(X,\xi) = \eta(X),$$

$$g(\phi X,\phi Y) = g(X,Y) + \eta(X)\eta(Y),$$

$$(\nabla_X)Y = g(X,Y)\xi + \eta(Y)X + 2\eta(X)\eta(Y)\xi,$$

$$\nabla_X \xi = \phi X$$

where X and Y are arbitrary vector fields, ∇_X denote covariant differentiation in the direction of X with respect to g.

Lorentizian Para Sasakian manifold satistify the following relations

$$\phi\xi = 0 \qquad \eta(\phi X) = 0$$

$$\text{rank } \phi = \text{n-1}$$

Also an LP-sasakian manifold M is said to be η -Einstein if its Ricci tensor S is of the form

$$S(X,Y) = ag(X,Y) + b_\eta(X)\eta(Y)$$

for any vector fields X,Y where a,b are functions on M.

Further on such an LP- sasakian manifold with (ϕ, η, ξ, g) structure , the following relations holds

$$g(R(X,Y)Z,\xi) = \eta(R(X,Y)Z) = \eta(R(X,Y)Z) = g(Y,Z)\eta(X) - g(X,Z)\eta(Z)$$

$$R(\xi,X)Y = g(X,Y)\xi - \eta(Y)X$$

$$R(\xi,X)\xi = X + \eta(X)\xi$$

$$R(X,Y)\xi = \eta(Y)X - \eta(X)Y$$

$$R(X,Y)\xi = (n-1)\eta(X)$$

$$S(\phi X,\phi Y) = S(X,Y) + (n-1)\eta(X)\eta(Y)$$

for any vector fields X,Y and Z where R(X,Y)Z is the Riemannian curvature tensor.

Definition 1.0.15 The projective curvature tensor P on LP Sasakian manifold M of dimensional N is defined as
$P(X,Y)Z = R(X,Y)Z - \frac{1}{n-1}[g(Y,Z)QX - g(X,Z)QY]$

for all vectors fields X,Y,Z on M where Q is the Ricci operator defined by

S(X,Y)=g(QX,Y)

The manifold is said to be projectively flat if P vanishes identically on M.

Definition 1.0.16 *The Weyl projective curvature $\overline{P}$ of type (1,3) on LP-sasakian manifold M of dimensional n is defined by*

$$\overline{P}(X,Y)Z = R(X,Y)Z - \frac{1}{n-1}[S(Y,Z)X - S(X,Z)Y]$$

for all vectors X,Y,Z on M.

2 Chapter 2

2.1 Preliminaries.

In this chapter, we will be discussing in summary some concepts that we will majorly apply in this dissertion.Specifically, we are going to define tensor, we will discuss manifolds,connections ,sasakian manifolds and complex manifolds .

2.1.1 Differentiable manifolds

An non-empty paracompact Hausdorf space M is said to be an n-dimensional topological manifold,if every point x$\in$ M has an open neighbour u in M, that is homeomorphic to an open subspace of the n-dimensional euclidean space $\Re^n$.

Definition 2.1.1 *A chart on M is an embedding ϕ: u $\rightarrow \Re^n$ of an open subspace u of M into $\Re^n$ such that ϕ(u) is an open subspace of $\Re^n$.*

Let $p_i(t_1,t_2,......,t_n) = t_i \ \forall \ t \in \Re^n$, then for every chart ϕ: u$\rightarrow \Re_n$, the composition ϕ_i = $p_i \circ \phi$: u $\rightarrow \Re$ is called the **ith cordinate** of the point x$\in$ U with respect to ϕ.

The chart $\phi : u \rightarrow \Re^n$ is called the **local co-ordinate system** in $u \forall x \in U$ and the n real numbers $(t_1, t_2, ..., t_n) = (\phi_1(x), \phi_2(x), .., \phi_n(x))$ are said to be the co-ordinates of the point x with respect to ϕ.

A function f: W $\rightarrow \Re$ defined on a non-empty space W of $\Re^n$ is said to be of class

1. C^o iff it is continous

2. C^k, k=1,2,... iff it has continous partial derivatives of all orders r $\leq$ k.

3. C^∞ or smooth if it is of class C^k for every positive integer k.

4. C^ω if it is an analytic function.

Definition 2.1.2 *An atlas of class C^k is a collection α of charts on M, such that the domains of all the charts in α cover the n-manifold M, that is $\bigcup_{\phi \in \alpha}$ Domain = M and for any two charts $\phi : U \rightarrow \Re$ and $\psi : W \rightarrow \Re^n$ with $U \cap W$ not empty, function $f_{(\phi,\psi)}$ (t) = $\psi(\phi^{-1}(t))$ is of class C^k.*

The function $f_{(\phi,\psi)}$ is the **connecting function** of the two charts ϕ and ψ and $\forall$ x $\in$ U $\cap$ W, we have $f_{(\phi,\psi)}(\phi(x)) = \psi(x)$.Hence $f_{(\phi,\psi)}$ is called the **transformation for the change of local co-ordinates system from ϕ to ψ.**

Let $C^k(M)$ be set of all atlases on M of class C^k.If k$\neq 0$, this set $C^k(M)$ may be empty.The relation $\sim$ on M, defined by $\alpha \sim \beta$ iff $\alpha \cup \beta$ is an atlas in $C^k(M)$ for any two atlases α and β in C^k (M), is an equivalence relation in C^k (M) partitioning it into disjoint equivalence classes. Each of these equivalance classes is called a **differentiable structure.**
Two atlases are said to be **compatible** if their union is an atlas.

Definition 2.1.3 *An n-manifold M together with a given differentiable structure σ of class C^k on M, is called a **differentiable n-manifold**.*

Let X and Y be differentiable m and n manifolds respectively of class C^k with differentiable structures ξ and η where $k = 0, 1, ..., \infty$.An arbitrary function $f : X \to Y$ is said to be **differentiable of class C^h**, $h \leq k$ if for every chart $\phi : U \to \Re^m$ in the maximal atlas of ξ and every chart $\psi : W \to \Re^n$ with $A = U \cap f^{-1}(W) \neq \emptyset$, the function $f_{(\phi,\psi)}: \phi(A) \to \Re^n$ defined by $f_{(\phi,\psi)}(t) = \psi(f\phi^{-1}(t))$ $\forall$ t $\in \phi(A)$ where $\phi(A)$ is an open subspace of $\Re^m$ is of class C^h.

A differentiable curve of class C^k in M is differentiable mapping of class C^k of a closed interval $[a,b]$ of $\Re$ into M, which is essentially the restriction of a differentiable function of class C^k of an open interval containing $[a,b]$ into M.

2.1.2 Manifold

A (real) n-dimensional manifold is a topological space M for which every point $x \in M$ has a neighbourhood homeomorphism to Euclidean space $\Re^n$.

Definition 2.2.1 *Let M be a topological space and $U \subseteq M$ an open set. Let $V \subseteq \Re^n$ be open. A homeomorphism $\phi : U \to V$, $\phi(u) = (x_1(u),...,x_n(u))$ is called a cordinate system on U, and the function $x_1,.....,x_n$ the cordinate function.*

The pair (U,ϕ) is called a chart on M. Also the inverse map ϕ^{-1} is a parametization of U.

Definition 2.2.2 *An atlas on M is a collection of charts (U_α,ϕ_α) such that U_α covers M.*
The homeomorphism $\phi_\beta \phi_\alpha^{-1} : \phi_\alpha(U_\alpha \cap U_\beta) \to \phi_\beta(U_\alpha \cap U_\beta)$ are the transformation maps or cordinate transformation.

Definition 2.2.3 *Let X be a non-empty set. A collection τ of subsets of X is called a **topology** on X. We call the pair (X,τ) a topological space. Often, we denote the topological space of X instead of (X,τ).*

Definition 2.2.4 *A mapping $f : X \to Y$ between two topological spaces is called continous if for every $U \subseteq Y$ open in Y the inverse image $f^{-1}(u)$ is open in X. We also say that f is a map.*

Definition 2.2.5 *A topological space X is said to be Hausdorff if for any two distinct points x,y $\in$ X (x $\neq$ y) there exists two disjoint open subset u,v (u $\cap$ v = $\emptyset$) such that x $\in$ U and y $\in$ V. This is an example of a separation axiom since one thinks of the U , V as "separating" the points x and y.*

Definition 2.2.6 *Let M be a topological Haudorff space with a countable basis M is called a topological manifold if there exists an n $\in$ N (natural number) and for every point p$\in$ M an open neighbourhood U_p of p which is homeomorphic to some open subset $V_p \in \Re^n$.The integer n is called the dimension of M and we write M^n to denote that M has dimension n.*

Definition 2.2.7: *Let M be a topological manifold. A open cover M is a collection of open (subsets) $U \subset M$ whose union is M.*
i.e M = $\bigcup_{\phi \in I} U_\alpha$
A chart of M is a pair (U,ϕ) such that $U \subset M$ is an open set in M and ϕ is a homeomorphism from U onto an open set in $\Re^n$ i.e $\phi : U \to \Re^n$
An atlas for M mean a collection of charts $\{U_\alpha, U_\alpha | \ \alpha \in I\}$ such that $\{U_\alpha | \alpha \in$ I$\}$ is an open cover of M.

Definition 2.2.8 *A manifold M is called a differential manifold of class C^k if there is an atlas of M $\{U_\alpha, \phi_\alpha \mid \alpha \in I\}$ such that, for any $\alpha, \beta \in$ I, the composites*

$$\phi_\beta \circ \phi_\alpha^{-1} : \phi_\beta(u_\alpha \cap u_\beta) \to \Re^n$$

is differentiable of class C^k.
The atlas $\{(u_\alpha, \phi_\beta) \mid \alpha \in I\}$ is called a differential atlas of class C^k on M. If instead, the atlas is of class C^∞, then M is said to have a differentiable (smooth) structure and is called **smooth (differential) manifold.**

Definition 2.2.9 *Let M and N be two smooth manifolds. A smooth map $f : M \to N$ is called a* ***diffeomorphism*** *if f is one - to - and onto and if smooth inverse $f^{-1} : N \to M$ exists.*

2.1.3 Sub - manifolds.

Definition 2.3.1 *A sub - manifold M is a subset S which itself has the structure of a manifold, and for which the inclusion map $S \to M$ satisfies certain properties such that connection properties.*

2.1.4 Charts and Atlases

Let X be a topological space.A smooth n-dimensional atlas on X is a collection $\{(U_\alpha, \phi_\alpha)\}$, where U_α are an open cover of X and

$$\phi_\alpha : U_\alpha \to V_\alpha$$

where $V_\alpha \subset \Re^n$ are open, such that $\phi_\alpha \circ \phi_\beta^{-1}$ is C^∞ where defined (i.e on $\phi_\beta(u_\beta \cap u_\alpha)$).Each (u_α, ϕ_α) is known as a **chart**.

Let X be a topological space and $\{(u_\alpha, \phi_\alpha)\}$ a smooth atlas.Let (u, ϕ) be such that $u \subset X$ is open, $\phi : U \to V \subset \Re^n$ a homeomorphism such that $\phi \circ \phi_\alpha^{-1}, \phi_\alpha \circ \phi^{-1}$ are C^∞ where defined then $\{(u_\alpha, \phi_\alpha)\} \cup \{(u, \phi)\}$ is again an atlas.

Definition 2.4.1 *Let $u \subset \Re^n$ be an open set.Then , a Riemannian metric on u is a C^∞ function $g : u \to M_{n\times n}$ (where the matrix represents an inner product on that space) such that*

· *g(x) is a symmetric non-degenerate matrix,*

· *g(x) is positive definite.*

Example
1.Set $g(x) := I_{n\times n}$ for all x.This is the standard Riemannian metric on $\Re^n$

Definition 2.4.2 *A C^∞ map $f : u \to \Re^m$ is an immersion if df_x is injective for all $x \in U$.The induced Riemannian metric is denoted f^*h and is given by*
$f^*h_x(u, v) = h(df_x(u), df_x(v))$

Definition 2.4.3 *A topological space X is locally Euclidean if or all $x \in X$, there exists $d \geq 0$, $d \in Z$, an open set $u \subset \Re^d$ and homeomorphism $f : u \to x$.*

Definition 2.4.5 *A topological space X is second countable if X admits a countable basis of open sets.*

Definition 2.4.6 *A basis for a topological space X is a collection of sbsets V_α so that*
(i) $x = u_\alpha v_\alpha$

(ii) For every α, β, one can cover $v_\alpha \bigcap v_\beta$ = $u_\gamma v_\gamma$

REMARK

If X a topological manifold, every connected component is of X will be a locally Euclidean, Hausdorff,second countable space.So one can define a topological manifold to be something satisfying these three properties.

Definition 2.4.7 *A space X is paracompact if every open covers admit a locally finite refinement.*

2.1.5 Compatible charts

Suppose $(u, \phi : u \to \Re^n)$ and $(v, \psi : v \to \Re^n$ are two charts of a topological manifold.Since $u \bigcap v$ is open in U and $\phi : u \to \Re^n$ is a homeomorphism onto an open subset of $\Re^n$, the image $\phi(u \bigcap v)$ will also be an open subset of $\Re^n$. Similarly ,$\psi(u \bigcap v)$ is an open subset of $\Re^n$.

Definition 2.5.1 *Two charts $(u, \phi : u \to \Re^n)$, $(v, \psi : v \to \Re^n)$ of a topological manifold are C^∞ compatible if the two maps*

$$\phi : \psi^{-1} : \psi(u \bigcap v) \to \phi(u \bigcap v),\ \psi \circ \phi^{-1}\colon \phi(u \bigcap v) \to \psi(u \bigcap v) \text{ are } C^\infty.$$

These two maps are called the *transition function* between the charts. If $u \bigcap v)$ is empty, then the two charts are automatically C^∞

Definition 2.5.2 *Two charts (u,x) and (v,y) of a topological manifolds are called flower compatible if either*
(i) $u \bigcap v = \phi$

(ii) $u \bigcap v \neq \phi$

2.1.6 Connection.

Let M be a C^∞ manifold.A connection, infinitesimal connection or covariant differentiation on M is an operator ∇ that assigns to each pair of C^∞ vector fields X, Y with domain A a C^∞ field ∇_XY with domain A. If Z is a C^∞ field on A while f is a C^∞ real valued function on A, then ∇ satisfies the following properties
1. ∇_X(Y + Z) = ∇_X Y + ∇_X Z.

2. ∇_{X+Y}[Z] = ∇_XZ + ∇_YZ.

3. ∇_{fX}[Y] = f∇_XY.

4. ∇_X(fY) = (Xf) Y + f ∇_XY

2.1.7 Linear connections

A linear connection on M is a connection on TM i.e a map
$\nabla : \tau(M) \times \tau(M) \to \tau(M)$ satisfying the properties of connection

Lemma 2.6.1 Let ∇ be a linear connection, and let $X, Y \in \tau(M)$ be expressed in terms of a local frame by $X = X^i E_i, Y = Y^j E_j$. Then
$\nabla_X Y = (XY^k + X^i Y^j \Gamma_{ij}^k) E_k$

Proof
Using the defining rules for connection we have

$\nabla_X Y = \nabla_X (Y^j E_j)$

$\nabla_X Y = (XY^j) E_j + Y^j \nabla_{X^i E_i} E_j$

$\nabla_X Y = (XY^j) E_j + X^i Y^j \nabla_{E_i} E_j$

$\nabla_X Y = XY^j E_j + X^i Y^j \Gamma_{ij}^k E_k$
Renaming the dummy index in the first term yields
$\nabla_X Y = (XY^k + X^i Y^j \Gamma_{ij}^k) E_k$

2.1.8 Riemannian manifold

Let T_p be the tangent space at the point p of a differentiable manifold M.If we single a real valued bilinear, symmetric and positive definite function g on the ordered pairs of tangent vectors at each point p in M, then M is called **Riemannian manifold** and g is called the **metric tensor** of M.Thus, for two vectors X,Y in T_p, we have

1. g(X,Y) $\in \Re$

2. g(X,Y) = g(Y,X)

3. g(aX + bY, Z) = ag (X,Z) + bg (Y,Z)

4. g(X,X) > 0

5. If X and Y are C^∞ fields with domain A, then g(X,Y) is a C^∞ function on A.

2.1.9 A Contact Metric Manifold.

Definition 2.9.1 *Let (M,ϕ, ξ, η,,g) be an n = (2m +1) - dimensional almost contact metric manifold consisting of a (1,1) tensor ϕ, a vector field ξ, a 1-form η and a Riemannian metric g.*

Let χ(M) be the lie algebra of vector in M. Then consider X,Y,Z,V,W $\in$ (M). If M_n is a k-contact Riemannian manifold, then

$\nabla_X \xi = -\alpha\phi X + \beta(X - \eta(X)\xi)$, $\nabla_X \xi = - \phi X$

$(\nabla_X \eta)Y = - g(\phi,X,Y)$

$S(X,\xi) = (n-1)\eta X$

$\eta(R(X,Y)Z) = g(Y,Z)\eta(X) - g(X,Z)\eta(Y)$

2.1.10 Tensors

Let M be an n-dimensional smooth manifold, a tensor of type (r,s) at p is an $(r+s)$ linear valued function on $(T_p)^r \otimes (T_p)^s$ and the vector space of this product is denoted by $T^r_{\rho s}$

Let V be a fixed vector space over a field F, then T^r = V $\otimes\otimes$.....$\otimes$V (r times tensor product) is called the **contravariant tensor space of degree r.** Similarly T_s = $V^* \otimes V^* \otimes ... \otimes V^*$ (s times tensor product) is called the **covariant tensor space of degree s)**. By convection T^1 = V, $T_1 = V^*$ and $T^0 = T_0 = \Gamma$.

A mixed tensor space of type (r,s) or a tensor space of contravariant degree r and covariant degree s is the tensor product $T^r \otimes T^s = V \otimes V \otimes ... \otimes V^* \otimes V \otimes ... \otimes V^*$.

An element of T^r_s is called the a **tensor of type** (r,s) or tensor of contravariant degree r and covariant degree s.

Let T_p (M) be the tangent space to a manifold M at ρ and T^r_s is called a **tensor of type** (r,s) on a subset N and M, is an assignment of tensor $K_x \in T^s_r$(X) to each point x of N.

Definition 2.10.1 *The number of indices of tensor components reveal all the general information about tensor as operators.For example, if a tensor T has components T^{ik}_{jl}*

This then tells us that
1. T is a 4^{th}.

2. T is -(2 2) type tensor.

3. Its 1^{st} *and* 3^{rd} *slots are for 1-form whereas the* 2^{rd} *and* 4^{th} *slots are for vectors.*

2.1.11 Spaces of N dimensions

In three dimensional space a point is a set o three numbers, called *cordinates*, determined by specifying a particular cordinate system or frame of reference.For example (x,y,z),(ρ,ϕ,z),(r,θ,ϕ) are cordinates of a point in rectangular, cylindrical and spherical cordinate system respectively.A point N dimensional space is by analogy a set of N number denoted by $(x^1,x^2,....,x^N)$ where 1,2,....,N are taken not as exponents but as superscripts.

2.1.12 Coordinate transformations

Let $(x^1,x^2,......,x^N)$ and $(x^1,x^2,......,x^N)$ be cordinates of a point in two different frames o references.Suppose there exists N independent relations between the cordinates of the two systems having the form
$\bar{x}^k = \bar{x}^k(x^1,x^2,......,x^N)$
where it is supposed that the functions involved are single valued,continous and have contionous derivatives.Then conversely to each set of $(x^1,x^2,......,x^N)$ there will correspond a unique set $(x^1,x^2,......,x^N)$ given by
$x^k = x^k(\bar{x}^1,\bar{x}^2,......,\bar{x}^N)$,k=1,2,....., N

The relations above define a transformation of co-ordinates from one frame to another.

2.1.13 Summation Convection

In writing an expression such as $a_1x^1 + a_2x^2 + ... + a_nx^n$ we can use the short notation $\sum_{j=0}^{n} a_jx^j$.An even shorter notation is simply to write it as a_jx^j, where we adopt the convection that whenever an index is repeated in a given term we are to sum over that index from 1 to n unless otherwise specified.This is called **summation convection.**

Clearly, instead of using yje index j we could have used another letter say p, and the sum could be written a_px^p.Any index which is repeated in a given term, so that the summation convection applies is called **dummy index or umbral index**

2.1.14 Properties of tensors

(i) Outer product
The outer product of two tensor of two tensor is equal to tensor whose rank is the sum of rank of given tensor and it also involve multiplication of components of the tensor.

(ii) Contraction
If we set one covariant index of tensor equal to one contravariant index then the resulting tensor will be of rank two less than original tensor.This process is called contraction.

(iii) Inner multiplication
The outer multiplication of two tensors followed by contraction will result to a tensor known as inner product of given tensor.

(iv) *Addition and substraction of tensor of same rank and type result in tensor of same rank and type.*

N/B - Two operation are defined only for tensor of same rank and type.
For us to verify whether functions would form components of tensor, we can use transformation laws of which they can be cumbersome of instead we can use the quotient law which is more convenient.

2.1.15 Quotient Law

If an inner product of any quantity x with arbitrary tensor is also a tensor then X is also a tensor.
A tensor Q of type (r,0) is said to be symmetric in h^{th} and k^{th} place if $S_{h,k}(Q)$= Q and skew symmetric if $S_{h,k}(Q)$= -Q where $1 \leq h < k \leq r$ and $S_{h,k}$ is a linear mapping which interchanges vector at h^{th} and k^{th} places.
Note it is also applies to a tensor of type (0,1).

2.1.16 Curvature tensor.

Definition 2.16.1 *Consider a connexion* ∇ *then the operator* R_{XY} *defined by* $R_{XY} = [\nabla_X, \nabla_Y]$ - $\nabla_{X,Y}$ *is called the* ***curvature operator.***
Then curvature R of the connexion ∇ is defined by

R(X,Y,Z) = R_{XY}Z
which can be written as
R(X,Y,Z) = $[\nabla_X, \nabla_Y]$Z - $\nabla_{X,Y}$Z
= $\nabla_X \nabla_Y$Z - $\nabla_Y \nabla_X$Z - $\nabla_{X,Y}$Z
The curvature tensor R satisfy two identities

(i) R(X,Y,Z) + R(Y,X,Z) + R(Z.X,Y) = 0 and

(ii) $(\nabla_X R)$(Y,Z,W) + $(\nabla_Y R)$(Z,X,W) + $(\nabla_Z R)$(X,Y,W) = 0
which are the Bianchi first and second identities respectively.

Properties of Riemannian curvature tensor

Definition 2.10.2 *Let (M,g) be Riemannian manifold with Levi-Civita connection* ∇. *The Riemannian curvature R is a correspondence that associates to* $X, Y \in \Gamma$*(TM) a map R(X,Y).*Γ*(TM)* $\rightarrow\Gamma$*(TM)defined by*
R(X,Y)Z = $\nabla_Y \nabla_X Z$ - $\nabla_X \nabla_Y Z$ + $\nabla_{[X,Y]} Z$.

The Riemannian curvature tensor is a linear over the ring of smooth function are coefficient on the right hand side and satisfy the following properties

(i) R(X,Y)Z = -R(Y,X)Z and if f is smooth function then

(ii)R(fX,Y)Z = -fR(Y,X)Z where ∇ is Riemannian connexion.

Let us define $'R(X,Y,Z,W)$ = g(R(X,Y,Z)W) then $'R$ skew symmetric in the two slots and the last two slots.The Riemannian curvature tensor R satisfy Binanchi's first identity and Bianchi's second identity.

2.1.17 Conformal curvature tensor

The tensor V defined by

V(X,Y,Z)= K(X,Y,Z) + $\frac{1}{n-2}$[Ric(Y,Z)X - Ric(X,Z)-g(X,Z)RY + g(Y,Z)RX] + $\frac{r}{(n-1)(n-2)}$[g(Y,Z)X - g(X,Z)Y]

is same for manifolds in conformal correspondence.This tensor is called **conformal curvature tensor**.

A manifold whose conformal tensor vanishes at every point is said to be ***conformally - flat***.
Aconformal curvature V satisfies Bianchi's first identity.
V(X,Y,Z)+ V(Y,Z,X) + V(Z,X,Y) =0

2.1.18 Concircular curvature tensor

The concircular curvature tensor is defined by
C(X,Y,Z) = K(X,Y,Z) - $\frac{r}{n(n-1)}$[g(Y,Z)X-g(X,Z)Y]

2.1.19 Conharmonic curvature tensor

The conharmonic curvature tensor is defined by
L(X,Y,Z) = K(X,Y,Z)-$\frac{1}{n-2}$[Ric(Y,Z)X - Ric(X,Z)Y + g(Y,Z)RX

2.1.20 The Weyl Projective curvature tensor

This is defined by
W(X,Y,Z) = R(X,Y,Z) + $\frac{1}{n+1}$[L(X,Y) - L(Y,X)]Z + $\frac{n}{n^2-1}$[L(X,Y)Y - L(Y,Z)Y] + $\frac{1}{n^2-1}$[L(Z,X)Y - L(Z,Y)X]

It can be shown that symmetric connexion which are projectively related have the same curvature tensor.
The Weyl's projective curvature tensor W satisfies the following properties
(i) W(X,Y,Z) = -W(X,Y,Z)

(ii) (tr W)(X,Y) = $(C_3^{'}W)$(X,Y) = 0

(iii)W(X,Y,Z) + W(Y,Z,X) + W(Z,X,Y) = 0

In Riemannian manifold the Weyl projective tensor reduces to

W(X,Y,Z) = R(X,Y,Z) - $\frac{1}{n-1}$[Ric(X,Z)Y - Ric(Y,Z)X]

2.1.21 Lie brackets

Vector field can be thought of a derivations on functions.For two vectors X and Y it may not always be true that X(Y(f)) = Y (X(f)) for all f.This leads to the definition of the Lie brackets or commutators of two vector fields.

Definition 2.21.1: *Let X and Y be vector fields on space M.We define the Lie bracket (at times known as The Jacobi-Lie bracket, or commutator) [X,Y] to be operator.*
$[X,Y] = XY - YX$

As it turns out, the bracket of two vector fields is again a vector field is again a vector field, meaning it is a first order differential operator. In components, letting

$$X = X^i \frac{\partial}{\partial X^i} \text{ and } Y = Y^j \frac{\partial}{\partial X^j}$$

$$[X,Y] = X^i \frac{\partial Y^j}{\partial X^i} - Y^i \frac{\partial X^j}{\partial X^i} \frac{\partial}{\partial X^j}$$

$$= X(Y^j)\frac{\partial}{\partial X^j} - Y(X^j) \frac{\partial}{\partial X^j}$$

$$= XY - YX$$

Thus $[X,Y]$ is the vector field.

2.1.22 Lie brackets and covariant derivatives

Let X,Y,Z be C^∞ vector field on M_n.Then Lie brackets is mapping
[]: $M_n \times M_n \rightarrow M_n$ such that
$[X,Y]$ = X(Yf) - Y(Xf) f being C^∞ function.
This satisfies the following properties

(i) [X,Y] $(f_1 + f_2)$ = [X,Y]f$_1$ + [X,Y]f$_2$

(ii)[X,Y](f$_1 f_2$)=f_1[X,Y]f_2 + f_2[X,Y]f_1

(iii)[X,Y] + [Y,X] = 0

(iv) [X+Y ,Z] = [X,Z] + [Y,Z]

(iv) [f_1X,f_2Y] =$f_1 f_2$[X,Y + $f_1(X, f_2$Y - $f_2(Y f_1)$X and further it satisfy identity i.e
$[X,[Y,Z]] + [Y,[Z,X]] + [Z,[X,Y]]$ = 0

The covariant derivative $\triangle$ is a mapping $\triangle$: $T^r_s \rightarrow T^r_{s+1}$ such that $\triangle_p(a_1, \ldots\ldots\ldots, a_r, x_1, \ldots\ldots\ldots, x_{s+1})$
= $\triangle_{s+1} p(a_1, \ldots., a_r, x_1, \ldots., x_s)$ where
$p \in T^r_s : a_1, \ldots\ldots a_r \in T_{(P)}$ and $x_1, \ldots\ldots, x_s \in T^*_{(P)}$

2.1.23 Lie brackets and Exterior Derivatives

Let X be C^∞ vector field on an open set.A Lie derivative via X is a type preserving mapping
$L_x : T^r_s \rightarrow T^r_s$ such that

(i)L_xf $\rightarrow$ xf, where f is C^∞

(ii) L_xa = 0, $a \in \Re$

L_xY = [X,Y], $Y \in T_{(P)}$
$(L_x A)$(Y) = X(A(Y)) - A([X,Y])
where A$\in T^*_p$ and $(L_{xp})(A_1, \ldots.., A_r, x_1, \ldots\ldots, x_s)$ = X(p($A_1, \ldots\ldots., X_s$)....p($A_1, \ldots\ldots., [X, X_s]$)
where $p \in T^r_s$. Let V_ρ be C^∞ ρ form an open set A.Then the mapping

$d : V_\rho \rightarrow V_{\rho+1}$ by
$(df)(X)$ = Xf where $X \in T_\rho$ and f is C^∞ function on A thus from above its clear now we

can define the following as
$(dA)\,(x_1,\ldots\ldots,x_{p+1}) = x_1(A(x_2,\ldots\ldots,x_{p+1})) + x_2(A(x_1,x_3,\ldots\ldots,x_{p+1}))$ +,....,+$x_{p+1}(A(x_1,x_2,\ldots\ldots,x_p))$
- $A([x_1,x_2]x_3,\ldots,x_{p+1})$ - $A([x_1,x_3]x_2x_4,\ldots,x_{p+1})$ - $A([x_2,x_3]x_1,x_4,\ldots\ldots,x_{p+1})$.....for all arbitrary C^∞ field $X \in V$ and $A \in V_p$ is called **exterior derivatives.**

2.1.24 Lie Algebra

Let M be the set of all infinity vector field on A the btackets [] is defined by mapping
[]: $M \times M \rightarrow$M such that for x,y in M,
and
$[x,y]$ f = xyf - yxf
where f is smooth function for x,y,z in M
we have
(i) $X,Y]$ = -$[Y,X]$ skew commutative (symmetric)

(ii) $[X+Y,Z]$ = $[X,Z]$ + $[Y,Z]$

(iii) $[fX,gY]$ = fg$[X,Y]$ + f(XgY) - g(Yf)X

(iv) $[[X,Y],Z]+[[Y,Z],X]+[[Z,X],Y]$ = 0
The last equation is known as **Jacobs Identity.**

2.1.25 Riemannian Connection

Definition 2.25.1 *A connection ∇ is said to be Riemannian if*

1. ∇ is symmetric or torsion free that is $\nabla_X Y$ - $\nabla_Y X$ = $[X,Y]$.

2. g is covariant constant with respect to ∇ or $\nabla_{X g}$=0.

Definition 2.25.2 *The torsion tensor of a connexion ∇ is a vector valued bilinear function T which assigns to each pair of C∞ fields X,Y with domain A, a C∞ vector field T(X,Y) with domain A defined by*

$$T(X,Y) = \nabla_X Y - \nabla_Y X - [X,Y]$$

If T(X,Y) = 0, then the connexion ∇ is said to be torsion free or symmetric.

2.1.26 An Affine Connection

Definition 2.26.1 *Let M be smooth manifolds.An affine connection (Levi-Civita) connection ∇ on M is a differential operator,sending smooth vector field $\nabla_X Y$, which satisfies the following conditions*

1.$\nabla_{X+Y} Z = \nabla_X Z + \nabla_Y Z$, $\nabla_X (X + Y) = \nabla_X Y + \nabla_X Z$

2.$\nabla_{fX} Y = f \nabla_X Y$, $\nabla_X (fY) = X(f)Y + f(\nabla_X Y)$
for all smooth vector fields X,Y and Z and real valued function f on M.

A vector field ∇_XY is known as the covariant derivative of the vector field Y along X (with respect to the affine connection ∇).

2.1.27 Complex manifolds

Smooth manifold is a space in which some neighbourhood of every point is homeomorphic to an open subset of $\Re^n$ such that the transitions between those open sets are given by smooth functions.

Complex manifolds is a space in which some neighbourhood of every point is homeomorphic to an open subset of C^n such that the transitions between those open sets are given by holomorphic fuctions.

The study of complex manifolds has two different subfields

(1). Function theory concerned with properties of holomorphic functions on open subsets $D \subseteq C^n$.

(2). Geometry: concerned with global properties (for instance compact) complex manifolds.

2.1.28 Holomorphic functions

Definition 2.28.1 *Let D be an open subset of* C^n*, and let* $f : D \to C$ *be a complex valued function on D. Then f is holomorphic in D if each point* $a \in D$ *has an open neighbourhood U, such that th function f can be expanded into a power series*

$$f(z) = \sum_{k_1,...k_n=0}^{\infty} ck_1, ..., ck_n(z_1 - a_1)^{k_1} .....(z_n - a_n)^{k_n}$$
which converges for all $z \in U$.

We denote the set of all holomorphic functions on D by the symbol $\vartheta(D)$
More generally, we say that a mapping $f : D \to E$ between open sets $D \subseteq C^n$ and $E \subseteq C^n$ is holomorphic if its m coordinate functions $f_1, ..., f_m : D \to C$ are holomorphic functions on D.

Definition 2.28.2 *A geometric structure ϑ on the topological space X is a collection of sub-rings $\vartheta(U) \subseteq C(U)$, where U runs over the open sets in X, subject to the following conditions*

1. *The constant functions are in $\vartheta(U)$.*

2. *If $f \in \vartheta(U)$ and $V \subseteq U$ then $f|_v \in \vartheta(v)$*

3. *If $f_i \in \vartheta(U_i)$ is a collection of functions satisfying $f_i|U_i \cap U_j = f_j|U_i \cap U_j$ for all $i, j \in I$ then there is a unique $f \in \vartheta(U)$ such that $f_i = f|U_i$, where $U = U_{i \in I} U_i$*

The pair (X, ϑ) is called a geometric space functions in $\vartheta(U)$ is sometimes called *distinguished.*

Definition 2.28.3 *A morphism $f : (X, \vartheta_X) \to (Y, \vartheta_Y)$ of geometric spaces is a continous map $f : X \to Y$ with the following additional property , whenever $C \subseteq Y$ is open , and $g \in \vartheta_Y(U)$, the composition $g \circ f$ belongd to $\vartheta_X(f^{-1}(U))$.*

Definition 2.28.4 *A complex manifold is ageometric space (X, ϑ_X) in the every point has an open neighbourhood $U \subseteq X$, such that $(U, \vartheta_X|_U) \simeq (D, \vartheta)$ for some open subset $D \subseteq C^n$ and some $n \in N$.*

2.1.29 Complex submanifolds

Let (X, ϑ_X) be a geometric space and $z \subseteq X$ any subset.There is a natural way to make z into a geometric space. First, we give z the induced topology. We call a continous function $f : V \to C$ on an open subset $V \in z$ *distunguished* if every point $a \in Z$ admits an open neighbourhood U_a in X, such that there exists $f_a \in \vartheta_x(U_a)$ with the property that $f(z) = f_a(z)$ for every $z \in V \cap U_a$.

Definition 2.29.1 *A subset Z of a complex manifold (X, ϑ_X) is called smooth if, for every point $a \in Z$, there exists a chart $\phi : U \to D \subseteq C^n$ such that $\phi(U \cap Z)$ is the intersection of D with linear subspace of C^n*

2.1.30 A sasakian manifold

Definition 2.30.1 *Let (M,ϕ, ξ, η, g) be an n = (2m + 1)- dimensional almost contact metric manifold consisting of a (1,1) tensor ϕ, a vector field ξ, a 1-form η and a Riemannian metric g.*

Let $\kappa(M)$ be the lie algebra of vector in M. Now considering X,Y,Z,V,W $\in$ (M), we have

ϕ^2 = -1 + η(X) ξ, $\eta(\xi)$ = 1, $\eta \circ \phi$ = 0,$\phi \in$ = 0

$g(X,\xi) = g(X,Y) - \eta(X)\eta(Y)$

$g(X,\xi) = \eta(X)$

$(X\phi)Y = g(X,Y)\xi - \eta(Y)X$

$(X_\xi) = -\phi X$

Thus, M is sasakian manifold
Also
$R(X,Y)Z = g(Y,Z)X - g(X,Z)Y$
$R(X,Y)\xi = \eta(Y)X - \eta(X)Y$

$R(\xi,X)Y = g(X,Y)\xi - \eta(Y)X$

$R(.)\xi = \eta(X)\xi - X$

$S(X,\xi) = (\eta - 1)\eta(X)$

$\phi\xi = (\eta - 1)\xi$

2.2 Statement of the problem

The aim of this study is to investigate W_5 Curvature tensor on LP-Sasakian manifold.The motivation is to generate new ideas with emphasis on producing new geometric results having physical meaning.

2.3 Objectives

The project aims to give a detailed study on the properties of W_5 curvature tensor on the LP-Sasakian manifold.
The study focuses on generating new ideas and emphasizing new geometric results.We also investigate the basic propeties of various LP-Sasakian spaces and investigate the results obtained and use them to put forward some new ideas.

3 Literature Review

Differential geometry builds on the following disciplines as its prerequisites: the anaytic geometry of Descartes and Calculus(Leibniz 1646 - 1716), Newton 1645 - 1727).

Mishra (1969)studied some properties of the Riemannian curvature tensor as well as the Weyl projective curvature tensor and conharmonic curvature tensor in Sasakian manifold. He showed that a concircular symmetric Sasakian manifold is a manifold of constant curvature and that the concirclar and Riemannian curvature tensor do not vanish in a Sasakian manifold.

Pokhariyal and Mishra and also Pokhariyal (1979), defined Weyl tensor to define the relativistic significance of curvature tensor. The Weyl's projective curvature tensor was then defined on the basis of geodesic correspondence due to a particular type of distribution of vector field found in it.This tensor was then given by the equation

$$W(X,Y,X,T) = R(X,Y,Z,T) + \frac{1}{n-1}[g(X,Y)Ric(Y,T) - g(X,T)Ric(Y,Z)] \qquad \textbf{(3.0.1)}$$

The relativistic signicance of Weyl's projective curvature tensor was also studied by Singh et al,(1965).

De U.C (1976) studied projective curvature tensor on k-contact and proved that a projectively semisymmetric flat k-contact is a sasakian manifold.

De U.C and De A (2011) proved that a projectively pseudosymmetric k-contact manifold and pseudoprojectively flat k-manifold are sasakian manifold respectively.

The notion if a Lorentzian Para Sasakian manifold was introduced by Matsumoto K(1989).Muhai I and Risca R. (1992) defined the same notion independently and they obtained several results on this manifold.Also LP-Sasakian manifold equipped with projective curvature tensor were studied by Teleshian A. and Asghari N.(2011).

De U.C,Jae B and Abul K. (2008) have studied quasi-coformally semi-symmetric Sasakian manifolds and proved that a Sasakian manifold is quasi -cinformally flat if and only if it is locally isometric with the unit sphere $S^n(1)$

Pokhariyal and Mishra (1970) have introduced new tensor field, called W_2 -Curvature

tensor as
$W_2(X,Y)Z = R(X,Y)Z + \frac{1}{n-1}[g(X,Z)QY - g(Y,Z)QX]$
in a Riemannian manifold and studied their properties .Further , Pokhariyal (1982) studied some properties of these tensor field in Sasakian manifold.

Matsumoto ,Ianus and Mihai (1986) have studied P-Sasakian manifold admitting W_2 and E-tensor field.

Pokhariyal (2001) studied W_2 -Curvature tensor , its associated symmetric and skew-symmetric tensor in an Einstein Sasakian manifold.

Pokhariyal G.P(1982) gave a new curvature tensor known as W_5 -Curvature tensor as
$W(X,Y)Z = R(X,Y)Z + \frac{1}{n-1}g(X,Z)QY - S(X,Z)Y$

A k-contact manifold is always a contact metric manifold,but the converse is not true in general.Pradip M and De U.C (2013)studied on concircular curvature tensor on k-contact manifold established that a $(2n-1)$ - dimensional ϕ - concircularly flat k-contact manifold ($n \geq$1) is Einstein manifold of scalar curvature equal to $2n(2n+1)$. In the same study,they proved that, a concircularly semisymmetric k-contact manifold of dimension $(2n+1)$, $n \geq 1$ is a Sasakian manifold.

Khan(2006) studied Einstein Projective Sasakian manifold.He showed that a projectively flat a projectively flat Sasakian manifold is an Einstein manifold and is of constant curvature.He also showed that if an Einstein Sasakian manifold is projectively flat, then it is locally isometric with the unit sphere $S^n(1)$.

Prakasha D.G Vasant C, and Kakasals M(2016) established taht a $\phi - W_5$-flat generalized Sasakian space-form is conformally flat and that it is $\phi - W_5$-semi-symmetric if and only if it is W_5 flat

Dwivedi M. and Kim J.(2011) studied on coharmonic curvature tensor in k-contact and Sasakian manifolds.They showed that a quasi - coharmonically flat k-contact manifold of dimension $(2n+1)$ has a vanishing scalar curvature.
They established that $(2n+1)$ -dimensional quasi-projectively flat k-contact is an Eistein manifold while a quasi-coharmonically flat Sasakian manifold was shown to be η- Einstein though.

In 1970 Pokhariyal and Mishra defined some of the tensors which included

$$W_1(X,Y,Z,T) = R(X,Y,Z,T) + \frac{1}{n-1}[g(X,T)Ric(Y,Z) - g(Y,T)Ric(X,Z)] \quad \textbf{(3.0.2)}$$

$$W_2(X,Y,Z,T) = R(X,Y,Z,T) + \frac{1}{n-1}[g(X,Z)Ric(Y,T) - g(Y,Z)Ric(X,T)] \quad \textbf{(3.0.3)}$$

$$W_3(X,Y,Z,T) = R(X,Y,Z,T) + \frac{1}{n-1}[g(Y,Z)Ric(X,T) - g(Y,T)Ric(X,Z)] \quad \textbf{(3.0.4)}$$

$$W_4(X,Y,Z,T) = R(X,Y,Z,T) + \frac{1}{n-1}[g(X,Z)Ric(Y,T) - g(X,Y)Ric(Z,T)] \quad \textbf{(3.0.5)}$$

$$W_5(X,Y,Z,T) = R(X,Y,Z,T) + \frac{1}{n-1}[g(X,Z)Ric(Y,T) - g(Y,T)Ric(X,Z)] \quad \textbf{(3.0.6)}$$

$$W_6(X,Y)Z = R(X,Y)Z + \frac{1}{n-1}[g(X,Z)Y - S(X,Y)X] \quad \textbf{(3.0.7)}$$

Later, Pokhariyal G.P defined new tensor field W^* on a Riemannian manifold as

$$W^*(X,Y,Z,U) =' R(X,Y,Z,U) - \frac{1}{2(n-1)}[S(Y,Z)g(X,U) - S(X,Z)g(Y,U)] \quad \textbf{(3.0.8)}$$

where
$W^*(X,Y,Z,U)$ = g$(W^*(X,Y)Z,U)$

Pokhariyal (1982) gave the definition of W_5, W_7, W_8, W_9 curvature tensor, where he gave its definition as

$$W_5(X,Y,)Z = R(X,Y)Z + \frac{1}{n-1}[g(X,Z)\phi Y - S(X,Z)Y] \quad \textbf{(3.0.9)}$$

$$W_7(X,Y,Z,T) = R(X,Y,Z,T) + \frac{1}{n-1}[g(Y,Z)Ric(X,T) - g(X,T)Ric(Y,Z)] \quad \textbf{(3.0.10)}$$

$$W_8(X,Y)Z = R(X,Y)Z + \frac{1}{n-1}[S(X,Y)Z - S(Y,Z)X] \quad \textbf{(3.0.11)}$$

where

$$S(X,Y) = g(QX,Y) = (n-1)g(X,Y)$$
$$= R(X,Y)$$

and Q is the Ricci Operator, i.e the linear endomorphism of tangent space at each of its points or equivalently

$$W^{'}(X,Y,Z,U) =^{'} R(X,Y,Z,U) - \frac{1}{n-1}[R(X,Y)g(Z,U) - R(Y,Z)g(X,U)] \quad \textbf{(3.0.12)}$$

$$W_9(X,Y,Z,T) = R(X,Y,Z,T) + \frac{1}{n-1}[g(Z,Y)Ric(X,Y) - g(Y,Z)Ric(X,T)] \quad \textbf{(3.0.13)}$$

4 STUDY OF W_5-CURVATURE TENSOR IN LP-SASAKIAN MANIFOLD

4.0.1 Introduction

In this section, we study the W_5-curvature tensor on LP-Sasakian mnifold.The following geometrical properties of W_5 -Curvature tensor are being investigated; flatness,semi-symmetric and symmetric on LP-Sasakian manifold.

4.0.2 Preliminaries

An n-dimensional real differentiable manifold M_n is said to be Lorentzian Para(LP) LP-Sasakian manifold if it admits a (1,1)tensor field F,a C^∞ 1-form A and a Lorentzian metric g which satisfy [Mishra 1]

$$A(T) = -1 \tag{4.0.1}$$

$$\overline{\overline{X}} = X + A(X)T \tag{4.0.2}$$

$$g(\overline{X}, \overline{Y}) = g(X,Y) + A(X)A(Y) \tag{4.0.3}$$

$$g(X,Y) = A(X), \qquad D_X T = \overline{X}, A(Y) \tag{4.0.4}$$

$$(D_X F)(Y) = g(X,Y) + A(X)A(Y)T + X + A(X)A(Y) \tag{4.0.5}$$

where $\overline{X} = F(X)$ and D_X denotes the covariant differentiation with respect to g,and X and Y are any arbitrary vector fields on M.

In LP -Sasakian manifold M_n with structure (F,T,A,g) it can be seen that Pokhariyal[2]

$$T = 0 \qquad A(\overline{X} = 0 \tag{4.0.6}$$

$$rank(F) = n-1 \tag{4.0.7}$$

If we put

$$F^{'}(X,Y) = g(\overline{X}, Y) \tag{4.0.8}$$

then the tensor field $F^{'}(X,Y)$ is symmetric in X and Y, thus, we have $F^{'}(X,Y) = F^{'}(Y,X)$

In an n-dimensional LP-Sasakian manifold with the structure (F,T,A,g), we have

$$R^{'}(X,Y,Z,T) = g(X,T)g(Y,Z) - g(Y,T)g(X,Z) \tag{4.0.9}$$

where $g(X,Z)$ is the metric tensor representing potential and
$Ric(X,Y) = g(QX,Y) = (n-1)g(X,Y)$ is the Ricci tensor representing the matter tensor.
$S(X,Y) = Ric(X,Y), \qquad S(T,T) = R(T,T) = -(n-1)$

where R is the Riemannian (0,4) Curvature tensor,S=Ric(...) is the Ricci tensor.

4.0.3 W_5-Curvature tensor in LP-Sasakian manifold.

Mishra and Pokhariyal[3] gave the definition of W_5-Curvature tensor as

$$W_5(X,Y)Z = R(X,Y)X + \frac{1}{n-1}[g(X,Z)QY - S(X,Z)Y]$$

or

$$W_5'(X,Y,Z,T) = R'(X,Y,Z,T) + [g(X,Z)S(Y,T) - g(Y,T)S(X,Z)]$$

Definition 4.3.1 A LP-Sasakian manifold M_n is said to be flat if the Riemannian Curvature tensor vanishes identically i.e
$R(X,Y)Z = 0$

Definition 4.3.2 A LP-Sasakian manifold M_n is said to be W_5-flat if W_5 -Curvature tensor vanishes identically i.e
$W_5(X,Y)Z = 0)$

Theorem 4.3.3 A W_5-flat LP-Sasakian manifold is a flat manifold.

Proof

If LP-Space is W_5-flat then W_5 =0 in

$$W_5'(X,Y,Z,T) = R'(X,Y,Z,T) + \frac{1}{n-1}[g(X,Z)S(Y,T) - g(Y,T)S(X,Z)]$$

If LP-Space is W_5-flat then we have

$$0 = R'(X,Y,Z,T) + \frac{1}{n-1}[g(X,Z)S(Y,T) - g(Y,T)S(X,Z)]$$

where S(X,Y)=g(QX,Y)=(n-1)g(X,Y)

we have

$$R'(X,Y,Z,T) = \frac{1}{n-1}[g(Y,T)S(X,Z) - g(X,Z)S(Y,T)]$$

$= \frac{1}{n-1}[g(Y,T)(n-1)g(X,Z) - g(X,Z)(n-1)g(Y,T)]$

$R^{'}(X,Y,Z,T) = [g(Y,T)g(X,Z) - g(X,Z)g(Y,T)]$

$R^{'}(X,Y,Z,T) = 0$
Hence the theorem proved.

4.0.4 W_5 -Semisymmetric LP-Sasakian Manifold.

U.C De and N.Guha [4] gave the definition of semisymmetric as R(X,Y)R(Z,U)V=0

Definition 4.4.1 A LP-Sasakian manifold is said to be W_5 -Semisymmetric if $R(X,Y)W_5(Z,U)V = 0$

Theorem 4.4.2 A W_5 -Semisymmetric LP-Sasakian manifold is said to be W_5-flat manifold

Proof
If LP-Space is a W_5-Semisymmetric then

$R(X,Y)W_5(Z,U)V = 0$

$\Rightarrow g(R(X,Y)W_5(Z,U)V,T) = R^{'}(X,Y,W_5(Z,U)V,T)$

$= g(X,T)g(Y,W_5(Z,U)V) - g(Y,T)g(X,W_5(Z,U)V)$

$= A(X)g(Y,W_5(Z,U)V) - A(Y)g(X,W_5(Z,U)V)$

$= A(X)W_5^{'}(Y,Z,U)V - A(Y)W_5^{'}(X,Z,U)V = 0$
But since $A(X) \neq 0$ and $A(Y) \neq 0$ then it follows that $W_5^{'}(Y,Z,U)V = 0$ and $W_5^{'}(X,Z,U)V = 0$ hence the theorem proved.

4.0.5 W_5 Symmetric LP-Sasakian Manifold

Chaki and Gupta (1963) gave the definition of a conformally symmetric manifold as

$\nabla_u C = 0$ where C is conformal curvature tensor.

Definition 4.5.1 A LP-Sasakian manifold is said to be W_5 -Symmetric if

$\nabla_u W_5(X,Y)Z = W_5^{'}(U,X,Y)Z = 0$

Theorem 4.5.2 A W_5-Symmetric and W_5-Semisymmetric LP-Sasakian manifold is a flat manifold.

Proof
From the previous theorem a W_5-Semisymmetric LP-Sasakian manifold is a W_5-flat manifold and if LP-Sasakian space is W_5-Symmetric this implies

$\nabla_u W_5(X,Y)Z = R(X,Y,W_5(Z,U,V)) - W_5(R(X,Y,Z),U,V) - W_5(Z,R(X,Y,U),V) - W_5(Z,U,R(X,Y,V)) = 0$

Computing each of the above four terms and subject them to same conditions we have:

$R(X,Y,W_5(Z,U,V)) = R'(X,Y,W_5(Z,U,V),T)$

$R'(X,Y,W_5(Z,U,V),T) = g(X,T)g(Y,W_5(Z,U,V)) - g(Y,T)g(X,W_5(Z,U,V))$

$$A(X)W_5'(Y,Z,U,V) - A(Y)W_5'(X,Z,U,V) \tag{4.0.10}$$

Again
$W_5(R(X,Y,Z),U,V) = W_5'(R(X,Y,Z),U,V,T)$

$W_5'(R(X,Y,Z),U,V,T) = R'(R(X,Y,Z),U,V,T) + \frac{1}{n-1}[g(R(X,Y,Z),V)S(U,T) - g(U,T)S(R(X,Y,Z),V)]$

then using S(X,Y)=(n-1)g(X,Y) we get

$W_5'(R(X,Y,Z),U,V,T) = R'(R(X,Y,Z),U,V,T) + \frac{n-1}{n-1}[g(R(X,Y,Z),V)g(U,T) - g(U,T)g(R(X,Y,Z),V)]$

$W_5'(R(X,Y,Z),U,V,T) = R'(R(X,Y,Z),U,V,T)$

$R'(R(X,Y,Z),U,V,T) = g(U,V)g(R(X,Y,Z),T) - g(R(X,Y,Z),V)g(U,T)$

$$= g(U,V)R'(X,Y,Z,T) - A(U)R'(X,Y,Z,V) \tag{4.0.11}$$

Also
$W_5(Z,R(X,Y,U),V) = W_5'(Z,R(X,Y,U),V,T)$

$W_5'(Z,R(X,Y,U),V,T) = R'(Z,R(X,Y,U),V,T) + \frac{1}{n-1}[g(Z,V)S(R(X,Y,U),T) - g(R(X,Y,U),T)S(Z,V)]$

then using S(X,Y)=(n-1)g(X,Y)

$$W_5'(Z,R(X,Y,U),V,T) = R'(Z,R(X,Y,U),V,T) + \frac{n-1}{n-1}[g(Z,V)g(R(X,Y,U),T) - g(R(X,Y,U),T)g(Z,V)]$$

$$W_5'(Z,R(X,Y,U),V,T) = R'(Z,R(X,Y,U),V,T)$$

$$= g(R(Z,Y,U),V)g(Z,T) - g(Z,V)g(R(X,Y,U),T)$$

$$= R'(X,Y,U,V)A(Z) - g(Z,V)R'(X,Y,U,T) \quad \textbf{(4.0.12)}$$

Also
$W_5(Z,U,R(X,Y,V)) = W_5'(Z,U,R(X,Y,V),T)$

$$W_5'(Z,U,R(X,Y,V),T) = R'(Z,U,R(X,Y,V),T) + \frac{1}{n-1}[g(Z,R(X,Y,V))S(U,T) - g(U,T)g(Z,R(X,Y,V)]$$

then using S(X,Y)=(n-1)g(X,Y)

$$W_5'(Z,U,R(X,Y,V),T) = R'(Z,U,R(X,Y,V),T) + \frac{n-1}{n-1}[g(Z,R(X,Y,V))g(U,T) - g(U,T)g(Z,R(X,Y,V))]$$

$$W_5'(Z,U,R(X,Y,V),T) = R'(Z,U,R(X,Y,V),T)$$

$$R'(Z,U,R(X,Y,V),T) = g(U,R(X,Y,U))g(Z,T) - g(Z,R(X,Y,V))g(U,T)$$

$$R'(X,Y,V,U)A(Z) - R'(X,Y,V,Z)A(U) \quad \textbf{(4.0.13)}$$

Next we put together equations (4.0.10),(4.0.11),(4.0.12) and (4.0.13)

$A(X)W_5'(Y,Z,U,V) - A(Y)W_5'(X,Z,U,V) - g(U,V)R'(X,Y,Z,T) + A(U)R'(X,Y,Z,V) - A(Z)R'(X,Y,U,V$
$g(Z,V)R'(X,Y,U,T) - R'(X,Y,V,U)A(Z) + R'(X,Y,V,Z)A(U)$

Terms which are coeffeicients of A(Z) and A(U) cancel out since R' is skew-symmetric with respect to the last terms
$g(Z,V)R'(X,Y,U,T) - g(U,V)R'(X,Y,Z,T) = 0$

But since $g(Z,V) \neq 0$ and $g(U,V) \neq 0$ hence

$R'(X,Y,U,T) = 0$ and $R'(X,Y,Z,T) = 0$
Hence the theorem proved.

4.0.6 W_5 Recurrent LP-Sasakian Manifold

In this part, we study some the geometrical properties of W_5 -Curvature tensor which is recurrent on LP-Sasakian manifold M.

Definition 4.6.1 If we consider an LP-Sasakian manifold M which is W_5 -Recurrent ,then we have (Pokhariyal 1996)

$$\nabla_U W_5(X,Y)Z = B(U)W_5(X,Y)Z \tag{4.0.14}$$

where B is a non-zero 1 form and W_5-Curvature tensor is given by

$W_5(X,Y)Z = R(X,Y)Z + \frac{1}{n-1}[g(X,Z)Y - S(X,Z)Y]$

$W_5(X,Y)Z = g(Y,Z)X - g(X,Z)Y + \frac{1}{n-1}[g(X,Z)Y - (n-1)g(X,Z)Y]$

$= g(Y,Z)X - 2g(X,Z)Y + \frac{1}{n-1}g(X,Z)Y$

$$= g(Y,Z)X - (2 - \frac{1}{n-1})g(X,Z)Y \tag{4.0.15}$$

$= g(Y,Z)X - (2 - \frac{1}{n-1})g(X,Z)Y$

$g(W_5(X,Y)Z,T) = g(g(Y,Z)X,T) - g(2 - \frac{1}{n-1})g(X,Z)Y,T)$

$W_5'(X,Y,Z,T) = g(Y,Z)g(X,T) - g(X,Z)g(Y,T)(2 - \frac{1}{n-1})$

$W_5'(X,Y,Z,T) = g(Y,Z)A(X) - g(X,Z)A(Y)(2 - \frac{1}{n-1})$

Theorem 4.6.2 If a LP-Sasakian manifold is W_5 -Recurrent and Ricci -recurrent ,then for the same recurrence parameter its recurrent.

Proof

Given that

$W_5'(X,Y,Z,T) = R'(X,Y,Z,T) + \frac{1}{n-1}[g(X,Z)S(Y,T) - g(Y,T)S(X,Z)]$

$\nabla_U W_5'(X,Y,Z,T) = B(U)W_5'(X,Y,Z,T)$

$\nabla_U W_5'(X,Y,Z,T) = \nabla_U R'(X,Y,Z,T) + \frac{1}{n-1}[g(X,Z)(\nabla_U S)(Y,T) - g(Y,T)(\nabla_U S)(X,Z)]$

But $(\nabla_U S)(Y,T) = B(U)S(Y,T)$ and $(\nabla_U S)(X,Z) = B(U)S(X,Z)$

$\nabla_U W_5'(X,Y,Z,T) = \nabla_U R'(X,Y,Z,T) + \frac{1}{n-1}[g(X,Z)(B(U)S(Y,T) - g(Y,T)B(U)S(X,Z)] = B(U)W_5'(X,Y,Z,T)$

$\nabla_U R'(X,Y,Z,T) = B(U)[W_5'(X,Y,Z,T) - \frac{1}{n-1}[g(X,Z)S(Y,T) - g(Y,T)S(X,Z)]$

$\nabla_U R'(X,Y,Z,T) = B(U)R'(X,Y,Z,T)$
Hence the theorem proved.

Theorem 4.6.3 A W_5-Recurrent LP-Sasakian manifold with $R(X,Y)W_5(Z,U)V = 0$ and $A(X)g(Y,Z) - (2 - \frac{1}{n-1})g(X,Z)A(Y) = 0$ is a W_5-Symmetric space.

Proof
From the definition 4.6.1, we have

$\nabla_U W_5(X,Y)Z = B(U)W_5(X,Y)Z$

$$\nabla_X W_5(Z,U)V = R(X,Y)W_5(Z,U)V - W_5(R(X,Y)Z,U)V - W_5(Z,R(X,Y)U)V - W_5(Z,U)R(X,Y)V \tag{4.0.16}$$

But we are given $R(X,Y)W_5(Z,U)V = 0$ (semisymmetric space).
This implies that we are left to show that the relations is symmetric under stated condition.
Therefore (4.0.16) becomes

$\nabla_U W_5(X,Y)Z = B(U)W_5(X,Y)Z$

$$= -W_5(R(X,Y)Z,U)V - W_5(Z,R(X,Y)U)V - W_5(Z,U)R(X,Y)V \tag{4.0.17}$$

Hence expanding each term of (4.0.17) we get
$W_5(R(X,Y)Z,U)V = g(U,V)R(X,Y)Z - (2 - \frac{1}{n-1})g(R(X,Y)Z,V)U$

$$= g(U,V)R(X,Y)Z - (2 - \frac{1}{n-1})R'(X,Y,Z,V)U \tag{4.0.18}$$

Also

$W_5(Z,R(X,Y)U)V = g(R(X,Y)U,V)Z - (2 - \frac{1}{n-1})g(Z,V)R(X,Y)U$

$$W_5(Z,R(X,Y)U)V = R'(X,Y,U,V)Z - (2 - \frac{1}{n-1})g(Z,V)R(X,Y)U \tag{4.0.19}$$

Again
$W_5(Z,U)R(X,Y)V = g(U,R(X,Y)Z) - (2 - \frac{1}{n-1})g(Z,R(X,Y)V)U$

$$W_5(Z,U)R(X,Y)V = R'(X,Y,V,U)Z - (2 - \frac{1}{n-1})g(Z,R(X,Y)V)U \tag{4.0.20}$$

Combining equation (4.0.18),(4.0.19) and (4.0.20) we have
$\nabla_U W_5(X,Y)Z = B(U)W_5(X,Y)Z$

$$= -g(U,V)R(X,Y)Z + (2-\frac{1}{n-1})R^{'}(X,Y,Z,V)U - R^{'}(X,Y,U,V)Z + (2-\frac{1}{n-1})g(Z,V)R(X,Y)U - R^{'}(X,Y,V,U \quad \textbf{(4.0.21)}$$

Terms which are coefficients of U and Z cancel out since $R^{'}$ is skew -symmetric with respect to the last two variables hence (4.0.21) becomes

$$(2-\frac{1}{n-1})g(Z,V)R(X,Y)U - g(U,V)R(X,Y)Z \quad \textbf{(4.0.22)}$$

Expanding (4.0.22) gives

$g(\nabla_X W_5(Z,U)V,T) = g(B(X)W_5(Z,U)V,T)$

$$= (2-\frac{1}{n-1}g(Z,V)[g(Y,U)X - g(X,U)Y] - g(U,V)[g(Y,Z)X - g(X,Z)Y] \quad \textbf{(4.0.23)}$$

Taking inner product of (4.0.23) with respect to T both sides yields

$g(\nabla_X W_5(Z,U)V,T) = g(B(X)W_5(Z,U)V,T)$

$$= (2-\frac{1}{n-1})g(Z,V)g(Y,U)g(X,T) - g(X,U)g(Y,T) - g(U,V)g(Y,Z)g(X,T) - g(X,Z)g(Y,T) \quad \textbf{(4.0.24)}$$

Relation (4.0.24) reduces

$g(\nabla_X W_5(Z,U)V,T) = g(B(X)W_5(Z,U)V,T)$

$$= (2-\frac{1}{n-1})g(Z,V)g(Y,U)A(X) - g(X,U)A(Y) - g(U,V)g(Y,Z)A(X) - g(X,Z)A(Y) \quad \textbf{(4.0.25)}$$

The coefficients for g(Z,V)and g(U,V) from the initial given conditions given are both equal to zero.Hence

$(\nabla_X W_5(Z,U)V) = g(B(X)W_5(Z,U)V = 0$ Hence the theorem proved.

4.1 Future Research

The aim of this project was to study W_5 -Curvature tensor on LP-Sasakian manifold.
In future,we may wish to extend the work to Sasakian maniolds,In Para -Contact manifolds,In Para-Kemontsu manifolds and in almost -Kenmotsu manifold.

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