University of Nairobi

School of Mathematics

ISSN: 2410-1397

Master Project in Statistics

STOCHASTIC APPLICATION OF ABSORBING MARKOV CHAIN TO SECONDARY SCHOOLS: a case study of Monrovi Consolidated Schools System (MCSS), Monrovia-Liberia.

Research Report in Mathematics, Number 33, 2021

Joseph Kolee Kortu

July 2021

Submitted to the School of Mathematics in partial fulfilment for a degree in Master of Science in Social Statistics

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Master Thesis

Submitted to the School of Mathematics in partial fulfilment for a degree in Master of Science in Social Statistics

Submitted to: The Graduate School, University of Nairobi, Kenya

Abstract

Since the end of the prolonged civil war in Liberia in 2003, general enrolment in secondary schools has increased, a scenario that necessitates a significant amount of resources. Enrollment forecasting is a critical component of resource allocation budgeting and overall education sector growth planning. Education in primary and secondary schools receives a significant amount of public funding. It's critical to keep track of Secondary School students' overall development from enrollment to graduation in the three years allotted. The research was carried out in the Monrovia Consolidated School System (MCSS) to demonstrate the stochastic process of absorbing Markov Chain Techniques in studying the progress of students from class to class, to determine secondary completion and dropout rates, retention rates, and the expected duration of schooling by gender in the secondary schools of the Monrovia Consolidated School System (MCSS) from 2nd to 12th grades.

The Markov Model concept has been widely utilized in single institutions of learning across the world, mostly in elementary and secondary schools, as well as universities and colleges, although there is no documented history of such use in Liberia. From 2017 to 2020, the target demographic was secondary enrolled students at the MCSS's William V. S. Tubman, G.W. Gibson, and D-Twe high schools. Our findings indicated that male students complete more classes than female students, and female dropout rates were greater at all levels when compared to male peers. Female students had a lower duration rate of schooling than their male counterparts, and the overall conclusion revealed that retention rates were lowest in 10th grade and greatest in 12th grade.
Data source: Office of the Director for public affairs and media services; MCSS.

Master Thesis in Mathematics at the University of Nairobi, Kenya.
ISSN 2410-1397: Research Report in Mathematics

DISTRIBUTOR: School of Mathematics, University of Nairobi, Kenya

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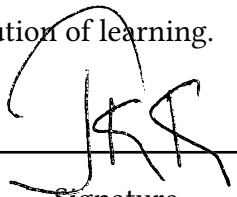
I the undersigned declare that this dissertation is my original work and to the best of my knowledge, it has not been submitted in support of an award of a degree in any other university or institution of learning.



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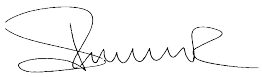

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Dedication

This project is dedicated to me.

Hon. Clarence Kortu Massaquoi, Atty Wilmot Foday Billy , CAID and My Family

Contents

Acknowledgments

First, I thank **Honorable Clarence Kortu Massaquoi**, **Atty. Wilmot F. Billy**, **Mom. Kpana Kpehe**, **Dr. Allen Johnny Borley**, **Mr. John S. Sallay**, **Min. Morris Saryon**, **Mr. Eric S. Ngafua**,**Robert G. Kamara**, **Mohammed Kamara, AKA Gandhi** , **THE MCSS** and **CAID FAMILY**

Liberia Group: **Mr. Thomas B. Kanneh**, **Mr. Jibba Alexander Kanneh**, **Mr. Abraham Billy**, **Mr. Thomas B. Jallah**, **Mr. Richard N. Korlison**, **Mr. Gweh Gaye Tarwo** , **Mss. Bendu Kawala**, **Mr. Daniel Koikoi**, **Mr. Isaac Tamba Charlison**, **Mr. Armah Blamah Kamara**,**Sam Borbor Lomax**, **Mr. Dennis M. Bolay**, **Mr. Samuel Nyenuh**, **Ma. Esther Kardar**, **Mr. Daniel Tydeh**, **Mr. Samuel Carmo**, **Mr. Joseph Kollie Yarkoi**, **Mrs. Satta S. Kamara**

Kenya Group: **Prof. Patrick G. O. Weke**, **Prof. JAM Ottieno**,**Dr. George Muhua**, **Mutua Robert**, **Denis Owiti** ,**Anne Ngala**, **Dominic Mutiso**

USA Group: **Mr. Hamidou Kabre**, **Mr. William M.S. Rogers**, **Mr. Smith O. Roberts**, **Mr. James Momoh**, **Mr. Ali Lakis**, **Mr. Namah A. Kamara**, **Mr. Olando S. Yolobah**, **Mrs. Aletha Kiawu**, & **Mr. Losene Boakai Kamara**.

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Joeph Kolee Kortu

Nairobi, 2021.

1 INTRODUCTION

Education is widely valued as central factor in economic, Social and political development of any country; therefore, Secondary school completion provides essential links for further training and to tertiary education, which helps reduces dependency ratio, improve health and human advancement. It's prudent to provide enough resources to the educational program to improve youth's lives by providing them an opportunity to advance in the 21st century. Despite the importance of the vital role of education and the need for expansion, the cost of quality education is skyrocketing, World Bank report on education 1980. Therefore, to meet the need for the current demand for quality and improve learning facilities for learners, there is an urgent need to put proper measures in attaining this goal. It's also good to acknowledge that formal education has long been a critical part of human development's program that's hugely driven by underlining economic factors.

The free compulsory Primary education in many African countries is achieving despite falling behind their target and time. Many African nations have made significant impacts. Free primary education has reviled different specific issues in other nations, (Riddell 2003-4) in his research of five countries of Sub-Sahara (Tanzania, Malawi, Kenya, and Uganda). Uganda was selecting four children only from family; Tanzania first set the age range at 7-13, but implementation was from 7-10 years; Zamia abolished the use of tuition and uniform but became selective in the process.

Liberia is a signatory to ECOWAS 1975 treaty of 93 different articles along with the rights to education and abolishment of discrimination for all Liberians. Liberia compulsory and primary education and didn't achieve in implementation, later she added the secondary education program and of recent she added all public vocational institutions and Universities. The failure to implement compulsory primary education and later added up with tertiary education became more laughable; this, it's rare from implementation. The country is locked behind in performance due to low fiscal budget allocation, lack of qualified workforce, teaching materials, learning facilities, among others.

Since the declaration of free and compulsory primary education in Liberia in 2001, there has been a great challenge of the influx of students couple with a lack of qualified teachers and infrastructures for learners. This declaration was reinforced in 2011 by Education Art Liberia (2011). It develops numbers or programs for the inclusiveness of all citizens

in Liberia. This free and compulsory education program was introduced to help reduce the illiteracy rate caused by the prolonged 14-year civil conflict. In 2010, a slight increase in literacy programs was achieved, thereby making the government of Liberia through the ministry of education pronounce free secondary education for all public institutions; by 2018, the president of Liberia declared all public Universities accessible to all citizens. This pronouncement of free basic education for primary and secondary came as an intervention for parents and caregivers to children; many saw this declaration as a worthy initiative by the government.

The saddest part of these programs is that the government has failed to implement the policy leaving many children out of school, relatively high dropout rate, poor retention ratio, and many other factors. From many indications, it is term as rhetoric or gimmick for government to join international bandwagon: Millennium Development Goals, UNESCO, etc. (Waydon et al., 2016). This situation quite different from some nations in Africa, like Kenya and Nigeria. According to the world bank report on education, Policymakers are implementing plans on education and are paying close attention, thereby improving their education systems.

1.1 BACKGROUND OF THE STUDY

The free education program is backed by the January 1986 constitution of the Republic of Liberia, which grants all Liberians from age 6 to 18 years the right to free and compulsory primary and secondary education. The law also compelled parents, guardians to send their children to the recognized institution of learning or risk of being fined. The law forbids the collection of fees from students as the national government and some Non-governmental Organisations are responsible for tuition and fees. This law enforcement aspect remains a challenging issue to dropout, retention, failure, etc. Many parents or guardians still believed in child labor during school-going hours; nothing has been done to address that issue in Monrovia, not to mention rural areas in Liberia. This impedes government plans toward its target in manpower planning and development of Liberia she wishes based on vision 2030.

It's established that Secondary completion has direct links to vocational or University education, which is very cardinal in achieving the dreams of students, parents, and upliftment of a nation or the world we wish to see. If one acquired primary and Secondary education, it is assumed that it reduces some significant problems such as joblessness, running into legal challenges, loss of self-esteem, etc. (McDill et al., 1985).

The Monrovia Consolidated School System (MCSS) was established by the government through the Act Amend by the education Law in 1964. The purpose is to provide essential primary and secondary education in the metropolitan area of Monrovia. Its vision is to provide the environment for teaching, research, entrepreneurship training in science and technology for Liberia's industrial and socio-economic development. It also offers services to the community and opened the opportunity to all Liberians intending to attract scholars. Before the establishment of MCSS, there was no organized government school system in Monrovia and Liberia.

Liberia's government initiated a free secondary education program to address the country's high illiteracy rate, which was exacerbated by the country's protracted civil war. This program isn't designed to reduce dropout rates, and it's also not tied to limitations like no repeating classes; MCSS, Monrovia's single most extensive government school system, is also subject to this restriction. In Kenya, the no-repetition technique in secondary school aids the government in achieving a low level of dropout in the education system. As a result, leaving the education system unregulated encourages students to drop out at any moment.

It also observed that difficulties in Liberia's formal education sector, such as a scarcity of trained instructors, high dropout rates, a lack of ideal learning conditions, and adolescent pregnancies, contribute to high dropout rates and low retention rates. Enrollment is expected to rise as the population grows, but sustaining low school dropout rates remains the most challenging problem. School dropout is seen as a loss by government parents and non-governmental organizations (NGOs), who fund all public educational institutions.

UNESCO concluded that with the hope of achieving universal Primary education, sub-Sahara countries Liberia inclusive required several increases in a foreign donation for Primary education, 14% of GNP goes to education to obtain 60% enrolment primary and secondary schools. To obtain 100% enrolment, government needs 78% GNP to obtained the gross enrolment. Nearly out of 5, only three benefit early childhood education, and the overall enrolment is rare. Nearly 50% of students enrolled are in primary schools are above the age of 6 and most often don't enter primary education until age ten and above.

UNESCO also stated that school enrolment statistics are the basis of many investments in decisions of education, once the total enrolment of students are forecasted, the required number of teaching staff to handle the instruction effectively can be projected using the

students' ratio method.

Students enrolment in schools is influenced by numbers of factors which include; child-age- and sex, the household factor, the value placed to the child's time by parents, school-based factor, curriculum, the capacity of the school, qualification, and experience of teachers, Student-Teacher ratio, etc. Environmental factor, rural and urban setting, the distance from school to house among other factors.

World bank record revealed that the Provision of good quality of secondary education is a critical tool to generate the opportunities and benefits of social and economic developments. Educating people means putting series of opportunities into their hands. Thus it's recognized as one of the best anti-poverty strategies. It's also the ways of ensuring a country's economic prosperity and competitiveness. The higher the quality of education, the higher the opportunities are put in students' hands, so the strategies applicable to the reduction of poverty in any country. It is forecasted that based on higher education obtained by any country's citizens, earnings also increase by 11% with each additional year of education. Also, each additional year of education about maternal education reduces childhood mortality by 8%.

According to Ministry of Education data on enrolment of students in Liberia, within the last three decades, enrolment rises from 303168 to 1.46 million from 1981 to 2015, most of which primary and junior high school students. It's forecasted that enrollment will continue to increase as the population increases.

"The ministry of education of Liberia never finalizes the quality standard of schools non – it has developed uniform tools for school inspection and monitoring of quality. The MOE has several policy documents and concept papers offering guidance, but it has never been finalized".

The lack of many policy implementations has caused student dropout to increase over the years. From this backdrop, we choose the Senior Secondary School of Monrovia Consolidated School System (MCSS) as a case study to the Application of Abaosrbing Markov Chain to investigate students' dropout, successful graduate and retention by gender. The excellent persistence performance of Students in school has long been government and school administrations concerned, but the repercussions of student attrition are critical for both students and academic staff. The likelihood of students graduating is lower than that of dropouts (Brezavšček et al., 2017). Therefore, our research study placed retention

under serious concern.

We choose the MCSS as a case study in the Application of the Absorbing Markov Chain to education because it's the most prominent government School system within Monrovia, and she also part of the free education system offered by the national government. It's also known that students' behaviours have stochastic characteristics. It's also established that students' behaviours in learning institutions can be modelled using the Andry Markov Model of stochastic. It's a unique probability attached with a random pattern. Based on research, little has been done to reduce student attrition, leading to dropout, and this subject has received little publicity. As a result, we developed this model to estimate: dropout, retention, graduate, and study period by sex, etc. Student dropouts can be equivalent to a waste of time for students and a waste of resources for both parents and government. Determing the likelihood of certain people be involve in an event (Kehinde, et al., 2002).

The double Absorbing Markov model education takes the probability of Pupils in a single cohort with specific characteristics of moving from one state to another, maybe expanded to forecast all individual students with those same characteristics within the same cohort of this research. It was shown that all students distributed in various states remained in their states at time t or the starting time and that the probability of students transitioning is $t+1$, which is referred to as the Markov chain's the following time. This practice is repeated until the cohort has achieved the maximum qualification.

(Beck 1983), in his Markov process used in medical research, defined the expected time as the total average time with the same cohort with the same characteristics. He further estimated how the Markov model could be used in evaluating the survival time in transient before being absorbed by the Markov Model, where the estimated survival of the subject of the cohort.

(Silverstein et al.,1988) emphasized that the predicted survival utilizing the Markov model analysis should be through an extrapolated survival on the premise that constant transition probabilities continue to apply shortly. The predicted period spent in short and anticipated survival may be obtained with the Markov Matrix solution, cohort simulation, and Multi Carlo simulation.

(Makridakis, 1990), In his book forecasting, planning and strategy for the 21 century, warned that we shouldn't accept any form of predictions based on a hidden formula or

ambiguous formula model that can't be explained, which could be beyond the understanding of the planner and manager.

(Nobel et al., 2007) the research of prediction of the college experience, the early identification fo students who are likely to drop out of studies are crucial to success to any retention strategies. Such information allows school administration and planners to undertake time actions once pupils are at risk of dropout terminal. Any increase in administration efforts to retain will also increase students' chances of retention to complete their studies successfully.

Our research aimed to establish retention, completion, dropout, and estimated duration of schooling of Senior Secondary education Monrovia Consolidated School System for the reason of obtaining tremendous insight in areas where there might be an improvement is necessary. That will be accepted based on data obtained from the MCSS office of public relations.
For all academic institutions of learning, an increase in retention had since been the primary achievement target. Still, the outcomes of student attritions are significant to students and academic staff. It very obvious to be necessary to students as they see colleagues progress and they are still in the same state of class after a year. Thus school dropouts are more likely to earn very little as compare with those who complete their studies.The Bureau of Labor statistics indicated that people with some of higher education earn more money and have a lower chances of being unemployed.
There is a less known research history of Liberia about getting insight into high dropout and measures to mitigate dropout. Little has been done in the fight to minimize the high dropout rates of students of the MCSS since the introduction of a free secondary education program by the government. We, therefore, planned this research to establish an Absorbing Markov model to determine, by gender, senior secondary students about completion ratio, dropout ratio, retention ratio, and total duration of schooling by grades of the MCSS.

1.2 STATEMENT OF THE PROBLEM

We assumed that the progress of students in their academic studies from time of enrolment in 10-grade, 11-grade and proceeding to their final class of graduation 12-grade after the expected duration of 3-year in senior secondary education. The government of Liberia estimated time for students to graduate from senior secondary schools is within 3-year, but it's exceeding that. Some of those reasons might be blamed on the lack of law to placed students under severe screening, such as there should be no repetition of classes since it's done on free primary and Secondary education basis. The over-stay, repetition,

dropout, low retention of students at secondary education in Liberia have also been a concern by planners, parents, and school administrators (Waydon et al., 2016). For many students, this has not been the case due to unforeseen circumstances such as death, pregnancies, early marriage, orphanages, (Uche 1980).
For quality and equality to be achieved, an understanding of Secondary Schools students enrolment trends, the rate of completion , retention , dropouts , successful graduates per class and expected duration of studies by gender need to be properly discussed and planned by stakeholders in the education sector.

The Markov chain Model has widely been used in different fields including Education(Musiga et al., 2012).There is no known history of applying the Markov chain model to education in Libeira; therefore, we used the Markov Absorbing Chain to estimate the completion, retention, duration for schooling, and dropout ratio by gender of students of Secondary Schools of MCSS.

1.3 OBJECTIVE

To study the Markov absorbing chain; to help achieve the following:

1.3.1 Specific Obejective

1.To determine Secondary School completion by Gender of the Monrovia Consolidated School System (MCSS).

2.To establish expected duration to completion of secondary School by gender of the Monrovia Consolidated School System (MCSS).

3. To establish the absorbing rates by gender secondary School of the Monrovia Consolidated School System (MCSS).

1.4 RESEARCH QUESTIONS

1.What's the Secondary School completion rate by Gender of the Monrovia Consolidated School System (MCSS)?
2. What is the expected duration of Study for Secondary School by Gender of the Monrovia Consolidated School System (MCSS)?
3. What is the expected absorption rate by Gender of Secondary School of the Monrovia

Consolidated School System (MCSS)?

1.5 SIGNIFICANCE AND IMPORTANCE OF THIS STUDY

In Liberia, it is based on school enrollment that the Nation Legislature of Liberia gives budgetary support to the various government school districts through the Ministry of Education. Since the announcement of free public primary, secondary, and University education in Liberia, there has been a great change in enrollments in various learning institutions. It is therefore good to understand the enrollment of students within MCSS for an administrator to help plan the workload of staffs, upgrade and expansion of schools and facilities for learners, to planned teacher's workload, and also get budgetary support from the national government in each fiscal year. The enrollment projection is helpful in the calculation of the workload of staff and even estimates support fund requires in the area of teaching.

Base on this research study, the enrollment forecasting can directly control budget and program planning for the Monrovia consolidated school system to help mitigate some of the challenges available; thereby, producing quality education pledged by the national government.

Enrollment data from 2017 will be used to estimates the total number of students that dropouts of the school system by gender before attaining maximum qualification, Nyandwaki et al., (2014). The dropout and completion ratios of students of the MCSS will give stakeholders an informed decision about the senior secondary education system since the introduction of the free education system. There is a need to investigate, understand, and report the current MCSS since the introduction of the free education basis of secondary schools. We selected 2017 data of enrollment because they benefited from the free education introduced by the government, and they are the most recent graduates from MCSS.

1.6 LIMITATION OF THE STUDY

The research's limitations;

1.The MCSS Extension (adult Literacy) Program was not included in the study.

2. Due to the limited data and duration of the trial, the researchers did not pinpoint the causes of high drop-out, low retention, and low success rates. The Markov model cannot be applied with some of the data since it is socioeconomic.

3. The study didn't differentiate between permanent and temporary dropout's

1.7 SOME BASIC ASSUMPTIONS OF THE MARKOV MODEL

The model is based on the following assumptions to help give a unique output;

1 The research targeted population is within (Central Monrovia, the Regular Monrovia Consolidated School System enrolment) we assumed that the School is closed, that is no immigration and out-migration of students within Monrovia and elsewhere. If such takes place, the difference is insignificance, the prevailing situation are assumed to hold into the future.
2. Enrollment takes place at once and in 10th grade only.
3.In any cycle of the analysis (X +X+1) X, dropouts are considered to be non-uniformly distributed.
4. Those who don't passed are not allowed to proceed to the next level with cohort.
5. The transition from Junior High to Senior Secondary Education of MCSS is assumed constant it does not change over time.
6. The cohort diminishes due to attrition. (Brezavščeket et al., 2017) Attrition are assumed to occur due to death, drop out, economic factors among others.

1.8 BASIC TOOLS OF MARKOV CHAIN ABSORBING

The research is designed around the theorem of the Markov Chain of stochastic Process, therefore, we introduced some basic concepts of the Markov Chain with are related to our topic under discussiion.

1.8.1 DEFINITION OF STOCHASTIC

The Stochastic Process contains random Variables $X(t) : t \in T$ defined on a specific probability space; this space is index by time variable t, and t varies over an index. according to (Feller 1971)
The stochastic Process $X(t) : t \in T$ is a discrete or finite time process provided only if T is a countable example, that's. $T = (1,2,3,\dots)$ this result into discrete-time process with corresponding values in respective to variables that's, $X(0), X(1), X(2)\dots$ with corresponding time $0,1,2\dots$ respectively
The process of stochastic $X(t) : t \in T$ becomes continuous-time process if and only T isn't countable or does have finite property i.e. $T = [0,\infty]$ or $T = [o,F]$ for some F The state space is represented by S; it's the setting in which the stochastic Process is within. The state S too is countable or discrete, or else it becomes continuous Stochastics process,(Feller 1971).
the Process of stochastic $X(t)$ becomes stationary provided all set of time is instantaneous, $T = (0,1,2,\dots,t_n)$ with time difference T, hence the joint probability distribution function does change as time change or change with a chance. The total number of

enrolled students per section or class in the senior secondary School of MCSS varies randomly; this is due to a different rate of dropout with time. We, therefore, conclude that the Senior Secondary enrollment can be represented in a stochastic process.

1.8.2 THE MARKOV CHAINS

The Markov Chain is a special probability found in the stochastics process where the future state of a system doesn't depend on the previous post, but it depends on the current value of the state. The probability of being in the $(k+1)^{st}$, is a discrete-time process in stochastic i.e.$X = X(t), wheret \geq 0$, depends on state at k^{kt} step.
For every state space $S = 1,2,3,\ldots,N$, hence the Markov chain is defined by sequence of random variables i.e. $X_i \in S$, where $i = 1,2,3\ldots$ in that,

$$prob(X_{k+1} = x_{k+1}/X_1 = x_1 \ldots, X_k = x_k) = prob(X_{k+1} = x_k + 1/X_k = x_k)$$

This is gives rise to $N \times N$ matrix of the Markov chain.
were;

$$P = P_{ij}$$

defined by,

$$p_{ij} = prob(X_{k+1} = j|X_k = i)$$

The Markov Stochastic process doesn't take into account where the system is coming from; it's memory-less, the current state of the system is sufficient to predict the future state. This implies that it doesn't matter how many times students of MCSS passed, failed, or withdrawal before enrolling in the senior secondary education of MCSS. Therefore, prediction is heavily dependent on the current behavior of students; hence there is a strong Markov property, (Nelson 1997) .
The Markov Stochastic process can be defined by characteristics based on the state space being measured. In discrete finite space, it's assumed and implies that there is a finite number of states which can be reached at the ending of the Process (Azaïs et al., 2002) .
The Markov process is mainly classified by a time interval of the Observation of the Process in each state; this implies that the Process may be observed at ether restricted or finite interval or sometimes continuous (Littell et al., 1971)
***Special Note*:** The Markov Chain is used to describe a process observed at the finite interval, and also the Markov Process describes the Process observed continuously as time move on.
Therefore, the Markov chains are a special case within continuous Markov processes;

hence, when the Process is at the continuous-time, a Markov chain is observed at a discrete-time interval. The Markov Processes can be summarized, including the Markov Chains and continuous Process; (Brigo, et al., 2016).
Therefore, in Continous Markov Chain, we have $X(t) : t \geq 0$ it's continuous time discrete value with random process for $t_0 \leq t_1 \leq t_2 \leq, \ldots t_n \leq t$, thus, with t and $t_r \geq 0$ and $r = 1, 2, 3 \ldots n$.

1.8.3 THE PROPERTIES OF TRANSITION MATRIX, P

p_{ij} are conditional probabilities
$P_{ij} \geq o, \forall, i, j$
$\sum_{k\in s}^{N} P_{ik} = 1$ if, $\sum_{i\in s}^{N} P_{ik} = 1$, hence, P becomes double stochastics P
For $X_o = i$ then the distribution of X_n is expressed by,
$P_{ij}^{(}n) = P(X_n = j|X_0 = i) = P(X_{n+m} = j|X_m = i)$
The equation partition matrix P have the following pattern,

$$\begin{aligned} P_{ij} &= P(X-1 = j|X_o = i) \\ P(X_2 &= j|X_1 = i) \\ P(X_3 &= j|X_2 = i) \\ P(X_4 &= j|X_3 = i) \ldots \end{aligned}$$

This is summarized into an equation;

$$\sum_{k=1}^{n} P_{ij} = \sum_{k=1}^{n} P(X_{n+m} = J|X_m = i)$$
$$P_{ij} = P\big(X_{n+m=j}|X_m = i\big); \quad P_{ij} \geq 0 \quad for \quad all \quad i$$

1.8.4 MARKOV ABSORBING CHAIN

1. A Markov Chain becomes an absorbing chain provided there exists at least a single absorbing state or more and also if there are possibilities of every state to go to absorbing states, i.e. ($P_{ij} = 1$). A state S_i is called an absorbing state-provided, it is possible to leave from state i to state j not always in single-step.
2. In every absorbing Markov chain, the state which isn't absorbing is termed a Transient state.
The transition matrix P for absorbing the Markov chain is to the powers until it reaches an absorbing state called the matrix solution; hence it stays there. Once absorbed, it can't be returned to the system. This implies that once a student graduates or dropped can't return to that identical class. The i and j states give the transition probability of

absorption from state i to state j; if it's for the first time, it's called the initial time.

The Canonical form of an absorbing Markov Chain If we have r as an absorbing state and we have t as a transient state, hence the transition matrix P shall develop the canonical structure;

$$p = \begin{bmatrix} Q & R \\ 0 & I \end{bmatrix}$$

where;
1. Q = is the $t \times t$ matrix,q_{ij} gives the transition probability of a pupil who is in class i at $(t-1)$ will be in cass j at time t or n year, were $i, j = 1, 2, 3 \ldots t$
2. R= $(t \times r)$ it is the non-zero matrix, where r_{ik} is the probability of a students in class i at time $t-1$ will successfuly graduate from the last level of education k at time t.
3. 0= is the $(r \rtimes r)$ this is the zero matrices, that gives the conditional probability from absorbing to non-absorbing state in n trajectory or step.
4. I = is the $(r \times r)$ or the identity matrix. this gives the transition probability of absorbing state in n step or trial.

Note : The first t states are the transient stats; thus, the last r states are the absorbing states.
Hints, the ij^{th} entry, P_{ij}^{n} of the matrix P^n, is the likelihood of a Markov chain starting from state S_i will be in state S_j after n trial of step this can be expressed by: *Chapman-Kolmogorov Equation*

The canonical form of the partition Martix P^n is given;

$$P^n = \begin{bmatrix} Q^n & R^n \\ 0 & I \end{bmatrix}$$

where;
1.Q^n is a $(t \times t)$ matrix that gives the transition probability of a pupil who is state i shall be in class j , in n year, $i, j = 1, 2, 3, \ldots, t$

2.R^n = $(I + Q + Q^2 + \ldots, Q^{n-1}R(t \times r))$ matrix is also known as foundamental matrix which gives the conditional probability of Pupils in class i shall final year of completion in education k at n within years later; where, $i = 1, 2, 3 \ldots t$, and $k = 1, 2, 3 \ldots r$ this is often

called the completion rate.

0= is the $(r \times t)$ Matrix of zero, which gives the conditional probability for absorbing state to non-absorbing state in n trajectory or step.
I=$(r \times r)$ identity matrix it gives the transition probability within absorbing states in n steps; (Beck et al. 1983) in their medical analysis research.
(Silverstein et al. 1988) P matrix summarizes the transition probability of students of the cohort hence;, the transient state's analysis allows us to predict or forecast an individual behavior, giving the beginning states, recent state, and the entire circle of the absorbing process.
In absorbing Markov Chin the probaility of being absorbed is 1 of $P_{ij} = 1$

$$\lim_{n \to \infty} Q^n = 0$$

.
This implies that every entry of Q^n must approach zero as n approaches infinity too.
Fundamental Matrix
in the case of the Markov absorbing chain, Matrix N donates the fundamental Matrix in some cases, some use a different letter of the alphabet of their choice, but we decided to use N, in this case,

$$N = (I - Q)^{-1} = (I + Q + Q^2 + \dots)$$

where;
I gives the identity Matrix, whereas Q gives the partition matrix. we say that ij^{th} is the entry of N_{ij} of the Matrix N represents the estimated number of the period the process takes in transient states S_j, meaning that the process started in-state S_i.
Therefore, N gives the total average of circles before being absorbed, given a specified time when the process started.

1.8.5 ERGODIC MARKOV CHAIN

The Markov chain is often termed as ergodic ChainChain if there exist possibilities of communication from every state to every other state (it is not necessary has to occur in a single step).
The Markov chain is regular if there exist some positive elements in the transition matrix; in other words, if for some n it's possible to go from one to any other state in just exactly n-step.

Note : Every regular Markov chain is ergodic, but not all ergodic ChainChain is regular Markov Chain.

1.8.6 Classification of Markov Chain

(Yeoh et al., 2001) our definition is kneen of communication or accessibility, from one state to another. this implies that if there exists a possibility to walk from state i to state j or $(i \to j)$
Two distinct states communication, state i to j and vice versa, if there exists communications from state i to state j and also state j to state i or $(i \leftrightarrow j)$.
Persistent (recurrent) states is that which accessible from all states, if from i(i is recurrent if $i \to j$ implies that $j \to i$).
Transient state or all states are recurrent. For a finite-state Markov chain, either all states in class are recurrent or transient; hence, we can't have recurrent and transient at the same time in a single Class.
A period state i is donated by $d(i)$ which is the greater common divisor (gcd) of those values of n for which $P_{ii}^{n} \geq 0$. Therefore, if the period is less than 1, it's Aperiodic and Periodic if it is greater than 1. conclusion: Taking the difference of years in study 10th, 11th, and 12 grade of the regular senior secondary education of MCSS as 3 states and the probability of moving to a higher class of the study is donated by P_{ij} for i, j =1,2 and 3; hence, we conclude that the progress of students in senior secondary education can be modeled as Markov chains with transient states.

1.8.7 EIGEN VECTORS AND EIGENVALUES

The square matrix A, we say that the number λ is Eigenvalue of A provided there exists a non-zero vector $\prod$, satisfying the condition,(Philippe et al., 1992)

$$A\prod = \lambda \prod$$

. The above equation can be explained as,
$\prod$ is the eigenvector of A, with corresponding eigenvalue of λ. We therefore, say that the Eigenvalues of matrix A are the solutions to the question.

$$det(A - \lambda I) = 0$$

where;
$I = (r \times r)$ identity matrix of the size of A. If we say that λ is the Eigenvalue of A,bthen the eigenvector corresponding to λ is also non-zero solution to the homogeneous system.

$$(A - \lambda I) \prod = 0$$

,

The steady-state vector $\prod$ of the regular Markov chain is the same as the eigenvector of the transition matrix corresponding to eigenvalue 1 in that..

$$A \prod = \prod$$

,

where;

$\prod$=$(\pi_1, \pi_2, \pi_3 \ldots), \pi_i \geq o, \forall i$

and

$$\sum_i \pi = 1$$

We, therefore, conclude that $\prod$ is the probability distribution function of the absorbing Markov chain with the transition Matrix A.

2 LITERATURE REVIEW

2.0.1 INTRODUCTION

This chapter examines what has been accomplished in the implementation of the Absorbing Markov Chain in the field of education, highlighting some significant accomplishments as well as some shortcomings in the Absorbing Markov Chain. As we all know, Andry Markov introduced the Markov Model in 1907 in the sense of determining random probability. It has since been widely used in various fields associated with certain random characteristics pattern, but it is limited in this case to education.

2.0.2 THEORETICAL FRAMEWORK

Gani (1963) described the theoretical structure of the Markov Model as "formulae for projecting university enrolments and degrees awarded." The paper aimed to look into the planning of student enrolment and degrees to be awarded in Australian universities over five years. The research primarily focused on bachelor's degree programs. He used the words "first years," "second years," "third years," and "fourth years" (Honor), Year 5 of masters, year 6 of Ph.D., and year 7 of Ph.D. However, since the University did not have enough data to do detailed calculations, he opted to use data from 1955 to 1960 to estimate the number of degrees to be awarded. According to his findings, complete enrolment at University is a concept of entries into a single year and a fixed time during that year. He mentioned that there was often a percentage of students who drop out of classes at the end of the first, second, or third term and that others may drop out at the end of the examination; however, he had difficulty modeling this theoretically. He concluded by stating that undergraduate students have three options: pass to advance to If you fail to retake a lesson, you will be kicked out of the system. He suggested that to calculate pass, loss, and repeat rate, his formula required data. He estimated the number of degrees for students who met the age of university entry at 18. He accurately estimated the number of bachelor's degrees that will be awarded over five years.

Chiu (1974) used the estimation probability of students' flow in the Orange State University (OSU). The research aimed to test whether the transition probabilities have stationary or non-stationary using the Markov chain models. Before his research, several findings of the application of Markov from Goodman and Halperin, both from 1957 to 1966, respectively, said the transition Matrices were discrete. He decided to use the sta-

tionary transition probabilities with aggregate statistics with the least square techniques and derived the matched estimation data very close to regular transition probability. He tried the Non- Stationary, which by divided into three parts: the causative matrix, linear, and composite of the three mentioned; linear regression was selected best for estimation in the forecasting of transition matrices of Orange State University. The research reveals that the Markov model isn't only for students' flow in institutions and in a diversified field such as economic activities, stock market, marketing, etc. He didn't have enough data for the studies to conclude the model. The hypothesis set for the model requires sufficient samples to validate his claim, and some new departments were formed just two years of the studies, so they had no data for that research. **Uche (1980)** proposed a mathematical model of single-channel academic survival for Nigerian primary schools in his Mathematical model of academic survival for Nigerian primary schools. He used the Markov chain model, estimating that graduation will take six years. He had two consuming or absorbing states: graduates and dropouts. He said that dropouts were likely to run contrary to educational objectives, so they were a significant problem for administrators and planners. This study focused on dropouts, leading to financial difficulties, poor health, illness, or death, among other things. The study found that due to dropouts, the cost of completion rates was poor, and the cost of completing primary education decreased for both sexes. In his single-step study, females had the lowest transition probability rate, and direct students had three options: Passed failed and drop. He applied his findings to the entire Nigerian educational system, from primary schools to universities, but no data from universities was available.

Mashat and Khedra (2012) wrote a paper titled decision support system-dependent Markov model for performance assessment of student's flow in the institution because there were many factors to assess students' performance due to various studies and the average time it took for students to graduate. The study was focused on an effective decision-making method on an absorbing. The research hypothesis is based on a Markov Model that incorporates seven rules for students enrolled at ate University. Freshman, sophomore, junior, senior, graduate, dropped, and not graduated were the seven student classes. Since there were seven states in the transition matrix, two were consuming states: Graduate and Dropped. Graduates were not permitted to return to any of the states. Similarly, students who dropped out were unable to be replaced by the system in any way. The cohort ran from 2006-2007 to 2011-2012, and students were required to complete 140 credit hours before graduating in four years. A total of 221 students were screened for this study. According to the findings, various levels of and curriculum necessitate different average life spent up to the semester to graduate. At that program, girls from the IT college performed better than boys. In that research, the Markov chain model assisted in the resolution of several decision problems. This study could not be generalized because the data was collected from only one university department with 221 students.

Mohammed, Auwalu, and Saliu (2013). The emphasis of this study was on the theatrical model of a schooling application of finite Markov chains in Nigeria. The study focused on a single school, with data obtained from that school and a sample of 474 students. They had a particular assumption in their theoretical works, like every other Markov model, the likelihood of transition, the decision to proceed to the next point, repeat, or drop is based on the most recent history rather than the previous past. Evidence that the transition phase was of awareness of one outcome is essential in predicting the next backed up the conclusion. Even though the chosen school had a higher than average norm, its discrete Markov chain produced satisfactory results. When the available data is compared to the outcome, From the available information, when compared with the product, there were suitable for prediction of the progress of sample of 474. The development of the finding couldn't be generalized for all universities because the selected school is based above average level and the sample data is too small.

Musiga, Owino, and Weke (2013), In their research paper titled Hierarchical structure with and implementation of Markov Chain containing double absorbing states in single-channel, assumes that absorbing becomes permanent to returning within the department's system. Data was collected from the University of Nairobi's Department of Actuarial Science, School of Mathematics, and Actuarial Science, Kenya. Data of School of Mathematics, the University of Nairobi from 2004 to 2008, the study was conducted in four classes: I, I, III, and IV. The results of the transfer matrices revealed that Year I students had a 15 percent chance of finishing, Year II students had a 25 percent chance of spending, and Year III students had an 87.31 percent chance of finishing in 2008, indicating that those who dropped out did not receive a diploma. Year two had an 88 percent dropout rate, year three had an 87.81 percent dropout rate, and year four had an 87.88 percent dropout rate. Since both absorbing states (dropouts and graduates) were grouped together in the same category, the model provided valuable information but was constrained. There was no clarification as to whether or not most students transited after the study.

Hlavaty and Domeova (2014) focused on the Markov chain of students' progress in the examination of the Czech University of Life Sciences in the capital of Prague. Various students were required to go through the regular guide as required by departments to show students' performance after the semester examinations, the final grades were assumed to be absorbed, it's one state, and the probability of transitioning from one state to another was depending on the behavior of a single student It was convened to identify the critical phases of the entire course work, as well as to allow time for preparation to assess students. A total of 990 students were assessed, and to receive the requisite credits, students needed to score at least 660 points out of a possible 110. A condition was established that students who scored 900 points would receive one bonus. A single

student's misbehavior was called to describe the crucial phases of the entire course work and provide time for planning so that students could be assessed. A total of 990 students were evaluated. Students needed to score at least 660 points out of a possible 110 to earn the necessary credits. It was decided that students who received 900 points would earn a prize. The probability of failing on the second attempt is 0.84. This indicates that students who fail the first time have a higher risk of passing the second time. There is a low chance of failing twice; however, if they do, it indicates that they lack the necessary skills for postgraduate studies. One significant finding was that only an attempt was made at the test that would decide whether or not students pass or fail, and to advance, you must have a 2.5 GPA. Students who fielded twice were given one opportunity, in which they had to drop out due to poor results. They were unable to address dropout issues among students of low and high performance and their suggestions for improving the level of dropout and final graduates in their research studies.

Elsayed, Dawood, and Karthikeyan (2017). Survival time of students in Zagazig University's engineering faculty. This study aimed to figure out how to create a model and estimate the survival and Separation of engineering students from the time they enroll to the time they graduate. Survival and Separation are the two absorption states in their model. The cohort's mathematical model was built on the assumption that students who started in 2008 would graduate with a bachelor's degree in 2012. The expectation was that students studying in engineering departments had completed 5 years and 65 days of schooling in the previous year, with a likelihood of 0.95 for the first year. Students who enrolled in the second year will have to wait for another 4 years and 65 days in a year with a likelihood of 0.95, while those who enrolled in the third year will have to wait for another 3 years and 102 days in a year with a probability of 0.95. with a 0.98 chance Those who advance to the fourth year will have to wait an additional year in their research, or 102 days, with a probability of 0.99, and those who progress to the fifth year will have to wait an additional two years, or 102 days, with a probability of 0.98. One year and 107 days, one year and 102 days, one year and 102 days, and one year were included in the calculations. The Markov model provided valuable knowledge that could be applied to Zagging University's engineering faculty. The findings of the study yielded an estimate of the estimated number of engineers and those who would be enrolled the following year, and the study was extended to other Saudi Arabian universities with similar profiles. There was no information available about what caused Separation in their research.

Brazavsceck, Bach, and Baggia (2017) In their study of students' performance and academic progress in Slovenian higher education used stochastic processes, especially Markov chain model analysis, to investigate students' flow and performance. Their study spanned eight years, from 2008/2009 to 2016/2017, and included student data from all eight years. In the study, the Markov model had seven states, with two of them being

absorbing states: withdrawal and graduates. They also had some assumptions in certain states, such as the fact that if you quit, you won't be able to finish the program, and that if you graduate, you won't be able to reapply under the same system. Based on the Markov model, the researchers were able to estimate the length of time spent by each student at each point of their studies, as well as the flow of transfer probabilities, or the chances of students progressing or withdrawing from Slovenia's higher educational institutions. They were also able to estimate three years' worth of enrollments in high schools using the Markov model. The studies were conducted to provide valuable information that would have an effect on higher education institutions and to plan for better educational programs, the probability of survival calculation.

Boumi, Vela and Chini (2020) Improved graduating rate estimation using frequent updating multi-level absorbing Markov Chains is the title of their paper. This mathematical module study was carried out at American universities using a six-year graduating rate and student data to determine the graduation and dropout rates. The average time to complete a degree at a university in the United States is six years, according to all American universities. Since some students enrolled full-time and others part-time, it's commonly referred to as norm six in their scheme. The frequency counting of many graduate form programs within six years was derived using the Markov Chain, and the tradition matrix was derived by dividing the number of graduates by the total number of students enrolled at the same time. This is the standard approach for estimating the percentages of graduates and dropouts. They said that this approach was biased and underestimated the number of graduates. They implemented multi-level updating regularly to prevent underestimating graduates. Freshman, sophomore, junior, senior, graduate, and dropouts were the six states in their model. Dropouts and graduates were the two absorbing states. With a transfer likelihood matrix based on the University of Florida, the cohort had 10,000 students. From 2008 to 2016, the Department of Physics had a cohort available. The Markov chain model's flaw was that it couldn't account for students who took different semesters for another program or degree before returning to finish the program in a later year. They used the binomial distribution with a parameter theta to change the Markov model, but their model was constrained because it is more effective for standard six and not all university programs, so the findings couldn't be generalized. They calculated the projected number of students enrolled in the faculty. Nonetheless; they extrapolate the results to other Saudi Arabian universities and determine those who will enroll in the faculty for the first time, based on profiles and other factors.

2.0.3 SUMMARY OF THE LITERATURE REVIEW

The Markov model implementations, which are based on a theoretical framework, have produced positive results in various fields, including education. He estimated the num-

ber of degrees to be awarded in Australian universities in the next five years, similar to **(Gani's 1963)** Markov Model; 'formulae for prediction of enrolments and degrees awarded in universities,' and the outcome of his calculation reasonably matched the total degrees awarded in the next five years. **(Thousand 1967)** used a similar theoretical Markov model to forecast Norwegian university attendance, and the results were accurate. **(Uche 1980)** proposed a mathematical model of single-channel academic survival for Nigerian primary schools in his Mathematical model of academic survival for Nigerian primary schools. He used a Markov chain model to predict graduation time and then applied the results to all Nigerian educational institutions. The numbers of certificates awarded were very close to what he had expected. Implementations of the Markov model, built on a theoretical framework, have yielded promising results in a wide range of fields, including education. Similar to **(Gani 1963)** Markov Model; 'formulae for estimation of enrolments and degrees awarded in universities,' he projected the number of degrees to be awarded in Australian universities over the next five years, and the result of his calculation reasonably matched the total degrees awarded over that period.

2.0.4 GAPS OF THE LITERATURE REVIEW

Generally, most of the studies are carry out in primary schools level, uinversities and single institutions of learning, nothing has been done in modeling the progress of students of secondary school in Liberia with the used of Markov model.

3 RESEARCH METHODOLOGY

3.0.1 INTRODUCTION

This chapter considers all the techniques required to achieve the objective of the absorbing Markov chain of the senior secondary education of the Monrovia Consolidated School System, such as the design of the study, targeted population, location of the research study, sampling techniques, and model development. The model is absorbing Markov Chain, with two absorbing states, graduate and dropout. It's aided by a steady-state transition matrix which we used to model the progress of senior secondary education of MCSS.

3.0.2 DESIGN OF THE STUDY

This research design is based on cross-sectional studies coming up with the study sample and its methodology to making an inference.

3.0.3 LOCATION OF STUDY

This study takes place in central Monrovia and Bushrod Island. The three MCSS Regular Senior Secondary schools, William V.S. Tubman, Gariston Wilmot Gibson, and D-The High Schools, are located. Liberia's capital and largest city, Monrovia, is located in West Africa. Monrovia is situated on Bushrod Island and Cape Mesurado. It is bordered on the south by the Atlantic Ocean, home to the Freeport, a massive seaport. The city of Monrovia is split into two sections: Central and Greater Monrovia. Monrovia receives 4624mm (182 in) of annual rainfall and an average annual temperature of 25 to 31 degrees Celsius. Monrovia is surrounded mostly by mangrove swamps with black soil and heavy rains. Administration, industry, and education, among other things, are the city's main activities. It has a population of 1.02 million people and has one public university, the University of Liberia, as well as several private universities, including African Episcopal Methodist University, United Methodist University, and others.

3.0.4 TARGETED POPULATION OF THE STUDY

The study's target population was the Monrovia Consolidated School System (MCSS); three regular Senior Secondary Schools, D-Two, Garrestone Wilmot Gibson, and William V.S. Tubman High Schools. Data source: The Office of the Director of Public Relations and Media Services at MCSS.

3.0.5 SAMPLE OF THE RESEARCH

The research sample was the enrollment of students in Senior Secondary Schools of Monrovia Consolidated School System beginning with class 10. The cohort started in 2017 which ended in 2020 with three years of studies. The reason why we choose this cohort is that it the most recent class and it's benefited from free education offered by the government of Liberia.

3.0.6 DEVELOPMENT OF THE MODEL

The Markov Chain Model is based on the underlying Process in which the system in one state S_i shall move subsequently in-state S_j in n-step later time. The state is often referred to as recurrent stat and the succeeding state. The Process of moving from one state to another is referred to as a transition or Step.
The stochastics process with Markov Characteristics is often called the Markov chain or Markov Process. The chances or probability of leaving from any state of study in an academic year to another state of the study in the next year doesn't depend on past in succession. Students progress doesn't depend on all of the past behaviors but the current state of the study, this defined the Markov Process.
In our research, the Markov Chain with t non-absorbing state $1,2,3\ldots t$ is arranged in line with the class of education system, while r an absorbing state corresponds with various final qualification was considered.
This means that $N = t + r$ where N represents the total number of possible states within the education system.
The transition probabilities from absorbing state to another absorbing state are represented by 1; we use this with the identity matrix. It's clearly stated that transition from an absorbing state to a non-absorbing state is impossible. This is represented by a matrix of Zero. This implies that you can't graduate from secondary education and be re-enrolled. However, there is a possibility of transition from a non-absorbing state to an absorbing state. Elsewhere, there is a possibility of transition from a non-absorbing state to another non-absorbing state within the same system. This is called backward and forward movement in the Markov chain process but ofter explained in graphs.

THE TRANSITION AND INITIAL STATE MATRICES

The Markov Stochastics process depends on two matrices: The transition Matrix and the initial matrix.

THE TRANSITION PROBABILITY MATRICES (Musiga et al., 2011) and Nyandwakai et al., (2016) The transition matrix is $n \times n$ which contains the transition probabilities,P_{ij} this represents the transition probability of movements from the current state in progress to the Next state. Assuming homogeneity in time, the transition probability P of the absorbing Markov chain is represented in the canonical form;

$$p = \begin{bmatrix} Q & R \\ 0 & I \end{bmatrix}$$

where,
Q, R, I and 0 are all submatrices whose elements are probabilities distribution.

In this reseach, double absorbing state is taking into consideration of Markov chain and the state areof education are donated by integer $1, 2, 3 \ldots, n$ at time $t = 0, 1, 2 \ldots,$ while. P_{ij} gives the trasition probaibity that a students in state or class i at time $t-1$ shall be in class j at time t thean the transition Matrix. $P_{ij} = ((P_{ij})), i, j = (1, 2, 3, \ldots, N)$

The non-absorbing state or (transient state) were three and donated by the value of 1, and 2, thus is explain that the Q Part of the transition matrix P is a 3×3 matrix; therefore, the number of absorbing states are 2, graduate and dropout represented by 4 and 5.
The state of absorbing 4 represents graduation after attaining maximum qualification, and the state 5 donates drop out of the education system before attaining maximum qualification. We, therefore, use the R and component of matrix P were 3×2 matrix.

(Armacost et al. 2002) The main function of the transition matrix is to represent the probability of movement between states within a single time period. We assumed that students will progress to a certain state at the ending of each year of the study.

Initial Transition Matrix :

By letting ij^{th} to represent the number of pupils in class i at time $t-1$ shall be in class j at time t and $n_i(t-1)$ the number of pubpil in class i at time $t-1$, and by assuming the multinumial distribution , the initial trasition probabilities were estimated by,
$P_{ij} = n_i j(t)/n_i(t) \quad i, j = 1,2,3\ldots t$
Were;

p_{ij} is represent ratio of pupils who were in class i at time $t-1$ end ended being in class j at time t.

The n-Step Trasition (Beck et al., 1983) used the $n-staep$ tranition probability that gives the conical structure on matrix

$$p^n = \begin{bmatrix} Q^n & R^n \\ 0 & I \end{bmatrix}$$

The solution of this n-step transition matrix donates the states of pupils in n-step (a year later). The n-step transition probability matrix components represent the probabilities that a pupil in a given state or class shall be in the following condition after n-step (a year later).

3.0.7 Rate of Completion

(Nyandwaki et al., 2016) and Nicholls (2009) both of them defined dropout rate of students from class i in n year later by,

$$r_{ik}^n = \sum_{j=1}^{n} q_{ij}(n-1) r_{ik}$$

&

$$i, j = 1,2,3,\ldots,s$$

were;
$q_{ij}(n-1)$ this donates the transitio probbility that a pupil in class i shall be in class j in $n-1$ year later.
and;
r_{ik} this gives the transition probability that pupil in i at time $t-1$ shall graduate with final education k at time t, thus this the $(i,k)^{th}$ characteristics or elements of the product

$$Q(n-1)R$$

.
Hence, the total number of pupils who have dropped out ratio within y year from class i will be donated by;

$$r_{ik}^{y} = \sum_{n=1}^{y} r_{ik}$$

$$i = 1,2,3,\ldots,\&k = 1,2,\ldots,r,$$

Therefore $r_i k^y$ is the $(ik)th$ element of $(I+Q+Q^2+,\ldots,+Q^(y-1))R$

Absorbing Markov Chain : Dorrach et al., (1965) and Bowen et al., (2013) While holding all factors constant, we assumed that students shall continue to existin the system forever, then the absorbing ratio is donated by;

$$\begin{aligned} r_{i1}^{\infty} &= \sum_{n=1}^{\infty} r_{i1}^{n} \\ &= (I+Q+Q^2+,\ldots,)R \\ &= (I-Q)^{-}1 \end{aligned}$$

(Dworkin's 2005) Retention ratio in secondary education is assumed to be in the best public interest; however, since the cost of education is borne by the government and parents, this aspect is not intended for students in learning institutions. This is more social and economical, and it has an effect on the community, so it's important to consider the retention rate of students in educational institutions; (Aitken 1982) gives important information about students retention in learning institutions in his specification and estimation of structural model"

Significant Notations .
Some important notations used we met to distinguish the general group from the gender category.
P_g= donates the transition probability of general students.
P_f= donates the transition probability of female students.
P_m= donates the transition probability of male students.
Q_g= donates the transient state of the genera group of students.
Q_f= donates the transient state of female students

Q_m= donates the transition probability of male students.

4 ANALYSIS OF DATA

4.1 INTRODUCTION

This chapter presents the Absorbing Markov chain model we used in our study based on data we acquired from the Monrovia Consolidated School System through the office of the public relation of MCSS on enrolment in Regular Senior Secondary Education. We drew up a table and divided the enrolment data into gender categories. Using my Excel matrix multiplication function, we created the Initial transition matrix and solved the absorbing Markov chain, Mmult (array 1, array 2). This function can be used to find the ordinary product between two arrays. We also got the matrix inverse function, Minverse (array) where, the array we used is were square matrix. This method aided us in obtaining our numerous products through multiplication, addition, and subtraction of matrices.

4.2 INITIAL TRANSITION PROBABILITIES OF THE ABSORBING MARKOV CHAIN

By letting ij^{th} to represent the number of pupils in class i at time $t-1$ shall be in class j at time t and $n_i(t-1)$ the number of pubpil in class i at time $t-1$, and by assuming the multinumial distribution , the initial trasition probabilities were estimated by,
$P_{ij} = n_i j(t)/n_i(t) \quad i, j = 1,2,3\ldots t$
Were;
p_{ij} is represent ratio of pupils who were in class i at time $t-1$ end ended being in class j at time t.

4.2.1 INITIAL TRANSITION PROBABILITIES MATRIX OF THE ABSORBING MARKOV CHAIN FOR THE MONROVIA CONSOLIDATED SCHOOL SYSTEM

This section took the general MCSS data of student enrolment obtained from the office of the public relation of MCSS.Student enrolment beginning from 10, 11, and 12 grades are starting from the academic year 2017 to 2020.

$326/1156 = 0.282007$ dropout ratio of 10-grade candidates in general before the completion of academic year 2017/2018
$112/830 = 0.134940$ dropout ratio of 11-grade candidates in general before the comple-

tion of academic year 2018/2019.
$26/718 = 0.036112$dropout ratio of the 12-grade candidate in general before attaining maximum qualification 2019/2020.

To obtain the initial transition ratio, we subtract the dropout ratio from 1 to obtain the value. this direct probability which values lie between 0 and 1.

$1 - 0.282007 = 0.717993$ the transition probability of 10-grade candidate, in general, to progress to 11 grade after attaining maximum qualification of the academic year 2017/2018.
$1 - 0.134940 = 0.865060$ the transition probability of 11-grade candidate, in general, to progress to 12 grade after attaining maximum qualification of the academic year 2018/2019.
$1 - 0.036112 = 0.963888$ the transition probability of 12-grade candidate in general to progress to graduate successfully after attaining maximum qualification of the academic year 2019/2020.

The Q component of the matrix p, whose states are transient, represents the proportion of pupils who transited to grades 10, 11, and 12, respectively.

$$p_g = \begin{bmatrix} 0.000000 & 0.717993 & 0.000000 & 0.000000 & 0.282007 \\ 0.000000 & 0.000000 & 0.865060 & 0.000000 & 0.134940 \\ 0.000000 & 0.000000 & 0.000000 & 0.963888 & 0.036112 \\ 0.000000 & 0.000000 & 0.000000 & 1.000000 & 0.000000 \\ 0.000000 & 0.000000 & 0.000000 & 0.000000 & 1.000000 \end{bmatrix}$$

The initial transition component represented by;
Q_g

$$Q_g = \begin{bmatrix} 0.000000 & 0.717993 & 0.000000 \\ 0.000000 & 0.000000 & 0.865060 \\ 0.000000 & 0.000000 & 0.000000 \end{bmatrix}$$

The initial Absorbing Component is given by; R_g

$$R_g = \begin{bmatrix} 0.000000 & 0.282007 \\ 0.000000 & 0.134940 \\ 0.963888 & 0.036112 \end{bmatrix}$$

4.2.2 THE INITIAL TRANSITION PROBABILITIES MATRIX BY SEX

To obtain the initial transition ratio, we subtract the dropout ratio from 1 to the obtain value. This method is a direct probability which values lie between 0 and 1

$167/563 = 0.296625$ dropout ratio of female candidate in 10 grade before completing the academic year 2017/2018
$62/396 = 0.156566$ dropout ratio of female candidate in 11 grade before completing the academic year 2018/2019
$15/334 = 0.044940$ dropout ratio of female candidate in 12 grade before attaining maximum qualification of academic year 2019/2020

We subtract the dropout ratio from 1 to obtain the value to the initial transition ratio. this direct probability which values lie between 0 and 1.

$1 - 0.296635 = 0.703375$ the transition probability of 10-grade female candidate to progress to 11 grade after attaining maximum qualification of the academic year 2017/2018.
$1 - 0.156566 = 0.843434$the transition probability of 11-grade female candidate to progress to 12 grade after attaining maximum qualification of the academic year 2018/2019.
$1 - 0.044910 = 0.955090$ the transition probability of 12-grade female candidate to successfully graduate after attaining maximum qualification of the academic year 2019/2020
.
The $Q - f$ component of the matrixp_f, whose states are transient, represents the proportion of pupils who proceeded on to grades 10, 11, and 12, respectively.

$$P_f = \begin{bmatrix} 0.000000 & 0.703375 & 0.000000 & 0.000000 & 0.296625 \\ 0.000000 & 0.000000 & 0.843434 & 0.000000 & 0.156566 \\ 0.000000 & 0.000000 & 0.000000 & 0.955090 & 0.044910 \\ 0.000000 & 0.000000 & 0.000000 & 1.000000 & 0.000000 \\ 0.000000 & 0.000000 & 0.000000 & 0.000000 & 1.000000 \end{bmatrix}$$

Q component of female students is given by;

$$Q_f = \begin{bmatrix} 0.000000 & 0.703375 & 0.000000 \\ 0.000000 & 0.000000 & 0.843434 \\ 0.000000 & 0.000000 & 0.000000 \end{bmatrix}$$

R_f component of female studets is given by;

$$R_f = \begin{bmatrix} 0.000000 & 0.296625 \\ 0.000000 & 0.156566 \\ 0.955090 & 0.044910 \end{bmatrix}$$

Likewise, the male, initial transition probability;
$159/593 = 0.268128$ dropout ratio of the male candidate in 10 grade before completing the academic year $2017/2018$.
$50/434 = 0.115207$ dropout ratio of the male candidate in 11 grade before completing the academic year $2018/2019$.
$11/354 = 0.028645$ dropout ratio of female candidates in 12 grade before attaining maximum academic year qualification $2019/2020$.

We subtract the dropout ratio from 1 to obtain the value to obtain the initial transition ratio. This method is a direct probability which values lie between 0 and 1.
$1 - 0.268128 = 731872$ the transition probability of 10 grade male candidate to progress to 11 grade after attaining maximum qualification of the academic year $2017/2018$.
$1 - 0.115207 = 0.884793$ the transition probability of 11-grade female candidate to progress to 12 grade after attaining maximum qualification of the academic year $2018/2019$.
$1 - 0.028646 = 0.971354$ the transition probability of 12-grade female candidate to successfully graduate after attaining maximum qualification of the academic year $2019/2020$.
The transition probability matrix P for a male candidate in the double absorbing Markov Chain, considering it to be uniform, is donated by;

$$P_m = \begin{bmatrix} 0.000000 & 0.731872 & 0.000000 & 0.000000 & 0.268125 \\ 0.000000 & 0.000000 & 0.884793 & 0.000000 & 0.115207 \\ 0.000000 & 0.000000 & 0.000000 & 0.971354 & 0.028646 \\ 0.000000 & 0.000000 & 0.000000 & 1.000000 & 0.000000 \\ 0.000000 & 0.000000 & 0.000000 & 0.000000 & 1.000000 \end{bmatrix}$$

Q_m Represents the male commponent and it's given by;

$$Q_m = \begin{bmatrix} 0.000000 & 0.731872 & 0.000000 \\ 0.000000 & 0.000000 & 0.884793 \\ 0.000000 & 0.000000 & 0.000000 \end{bmatrix}$$

$$R_m = \begin{bmatrix} 0.000000 & 0.268125 \\ 0.000000 & 0.115207 \\ 0.971354 & 0.028646 \end{bmatrix}$$

the below table represents the matrix of partition of Markov Chain

4.3 COMPLETION RATE

The completion rate of the regular 12th grade is combined with those who dropped from the beginning of the academic studies to the final stage and those who successfully graduated. That's the rate of completion and dropout before attaining maximum qualification are in one category. Thus; the completion rate is given by $(i,k)^(th)$ element of;
$I+Q+Q^2+\cdots+Q(y-1)$
within the first year of enrollment in grade 10 in 2017, our completion rate was donated by;

The identiy matrix I and trasient matrix Q are of the form;

$$(I+Q_g+)R_g = \begin{bmatrix} 1.000000 & 0.717993 & 0.000000 \\ 0.000000 & 1.000000 & 0.865060 \\ 0.000000 & 0.000000 & 1.000000 \end{bmatrix} \begin{bmatrix} 0.000000 & 0.282007 \\ 0.000000 & 0.134940 \\ 0.963888 & 0.036112 \end{bmatrix}$$

We multiply the transient and the absorbing states to obtain the first one year completion agfter enrollement in 10-Grade.
by;

$$(I+Q_g)R = \begin{bmatrix} 1.000000 & 0.717993 & 0.000000 \\ 0.000000 & 1.000000 & 0.865060 \\ 0.000000 & 0.000000 & 1.000000 \end{bmatrix} \begin{bmatrix} 0.000000 & 0.282007 \\ 0.000000 & 0.134940 \\ 0.963888 & 0.036112 \end{bmatrix}$$

From the result of one year study, the trasient and the absorbing were multiplyed and added and its result is given by;

$$(I+Q_g)R_g = \begin{bmatrix} 0.000000 & 0.378893 \\ 0.833821 & 0.166179 \\ 0.963888 & 0.036112 \end{bmatrix}$$

The funishing component is given by;

The multiplication of Q^2 with is the maximun times for the state to be absorbed.

$$(Q^2) = \begin{bmatrix} 0.000000 & 0.717993 & 0.000000 \\ 0.000000 & 0.000000 & 0.865060 \\ 0.000000 & 0.000000 & 0.000000 \end{bmatrix} \begin{bmatrix} 0.000000 & 0.717993 & 0.000000 \\ 0.000000 & 0.000000 & 0.865060 \\ 0.000000 & 0.000000 & 0.000000 \end{bmatrix}$$

The result of trasient matrices square product is given below by;

$$(Q^2) = \begin{bmatrix} 0.000000 & 0.000000 & 0.621107 \\ 0.000000 & 0.000000 & 0.000000 \\ 0.000000 & 0.000000 & 0.000000 \end{bmatrix}$$

We therefore added the identity matrix I to the trasient matrix Q along with the square transient matrix Q^2 to obtained the below reseult;

$$(I+Q+Q^2) = \begin{bmatrix} 1.000000 & 0.703375 & 0.621107 \\ 0.000000 & 1.000000 & 0.843434 \\ 0.000000 & 0.000000 & 1.000000 \end{bmatrix}$$

We carried on the completion by solving the matrix in the following operations or steps given by the formula below;
$(I+Q_g+Q^2)R_g$

$$(I+Q_g+Q^2)R_g = \begin{bmatrix} 1.000000 & 0.717993 & 0.621107 \\ 0.000000 & 1.000000 & 0.865060 \\ 0.000000 & 0.000000 & 1.000000 \end{bmatrix} \begin{bmatrix} 0.000000 & 0.282007 \\ 0.000000 & 0.134940 \\ 0.963888 & 0.036112 \end{bmatrix}$$

The final completion by the general group is given below after series of matrices operations were performed!

$$(I+Q_g+Q^2)R_g = \begin{bmatrix} 0.598678 & 0.401322 \\ 0.833821 & 0.166179 \\ 0.963888 & 0.036112 \end{bmatrix}$$

Table 1. Regular Monrovia Consolidated School System Completion Tabale

Year (y)	10-Grade		11-Grade		12-Grade	
	completion	dropout	completion	dropout	completion	dropout
1	0.000000	0.282007	0.000000	0.134940	0.963888	0.036112
2	0.000000	0.378893	0.833821	0.166179	0.963888	0.036112
3	0.598678	0.401322	0.833821	0.166179	0.963888	0.036112
4	0.598678	0.401322	0.833821	0.166179	0.963888	0.036112

40.1332% of the students who enrolled in 10-Grade have dropped from the MCSS in 2020 before attaining maximum qualification. In 2020, about 59.8678% who enrolled in 10-grade in 2017/2018 were expected to graduate successfully from the system in 2019/2020 after attaining maximum qualification.
From the same data, the same cohort that entered in 11-grade in 2018/2019 about 16.6179% dropped and 83.33821% successfully entered after attaining in 12-grade in after attaining maximum qualification. In their final year in 2019/2020, about 3.6112 dropped from the school system before attaining maximum qualification; likewise, 96.3880% were expected to graduate successfully after attaining maximum qualification.

4.3.1 COMPLETION RATE BY GENDER

From the general Analysis in the previous section, the results are classified into two sections (Male and Female)students. It's arranged based on students who dropped before attaining maximum qualification along with those who successfully graduated from the system in 2020. Thus; the completion rate is given by $(i,k)^(th)$ element of;
$I+Q+Q^2+\cdots+Q^(y-1)$

Female within a one year the completion rate of female is given by;

$$(I+Q_f)R_f = \begin{bmatrix} 1.000000 & 0.703375 & 0.000000 \\ 0.000000 & 1.000000 & 0.843434 \\ 0.000000 & 0.000000 & 1.000000 \end{bmatrix} \begin{bmatrix} 0.000000 & 0.296625 \\ 0.000000 & 0.156566 \\ 0.955090 & 0.044910 \end{bmatrix}$$

The From the result of one year study, the trasient and the absorbing were multiplied and added and the result is given by;

$$(I+Q_f)R_f = \begin{bmatrix} 0.000000 & 0.406750 \\ 0.805555 & 0.194445 \\ 0.955090 & 0.044910 \end{bmatrix}$$

To obtained the final completion of Schooling year of MCSS by female students, we must mutply the trasient states as many as possible until maximum qualifucation is attained by the pocess.
Q^2 represents transient states to be twice multiply.

$$(Q^2) = \begin{bmatrix} 0.000000 & 0.703375 & 0.000000 \\ 0.000000 & 0.000000 & 0.8434434 \\ 0.000000 & 0.000000 & 0.0000000 \end{bmatrix} \begin{bmatrix} 0.000000 & 0.703375 & 0.000000 \\ 0.000000 & 0.000000 & 0.8434434 \\ 0.000000 & 0.000000 & 0.0000000 \end{bmatrix}$$

The result of femle trasient matrices square product is given below by;

$$(Q^2) = \begin{bmatrix} 0.000000 & 0.000000 & 0.592350 \\ 0.000000 & 0.000000 & 0.000000 \\ 0.000000 & 0.000000 & 0.000000 \end{bmatrix}$$

We therefore added the identity matrix I to the trasient matrix Q along with the square transient matrix Q^2 to obtained the below reseult;

$$(I+Q+Q^2) = \begin{bmatrix} 1.000000 & 0.703375 & 0.592350 \\ 0.000000 & 1.000000 & 0.843434 \\ 0.000000 & 0.000000 & 1.000000 \end{bmatrix}$$

We carried on the completion of female students by solving the matrix in the following operations by the formula below;
$(I+Q_g+Q^2)R_g$

$$(I+Q_f+Q^2)R_f = \begin{bmatrix} 1.000000 & 0.703375 & 0.592350 \\ 0.000000 & 1.000000 & 0.843434 \\ 0.000000 & 0.000000 & 1.000000 \end{bmatrix} \begin{bmatrix} 0.000000 & 0.296625 \\ 0.000000 & 0.156566 \\ 0.955090 & 0.044910 \end{bmatrix}$$

The final completion for female category was done by multipliying and adding to obtained the result given by;

$$(I+Q_f+Q^2)R_f = \begin{bmatrix} 0.566607 & 0.433390 \\ 0.805555 & 0.194445 \\ 0.955090 & 0.044910 \end{bmatrix}$$

From table two, where the female category is arranged, it's seen that 43.3393 % of the

Table 2. Regular Monrovia Consolidated School System female Completion Tabale

Year (Y)	10-Grade		11-Grade		12-Grade	
	completion	dropout	completion	dropout	Completion	dropout
1	0.000000	0.296625	0.000000	0.156565	0.955090	0.044910
2	0.000000	0.406750	0.805555	0.194445	0.955090	0.044910
3	0.566607	0.433393	0.805555	0.194445	0.955090	0.044910
4	0.566607	0.433393	0.805555	0.194445	0.955090	0.044910

female students who enrolled in 10-Grade have dropped from the MCSS in the year 2020 before attaining maximum qualification. In 2020, about 56.6607% who enrolled in 10-grade in 2017/2018 were expected to graduate successfully from the system in 2019/2020 after attaining maximum qualification.
From the same data, the same cohort the proportion of females that entered in 11-grade in 2018/2019 about 19.4445% dropped from the school system and 80.5555% successfully entered after attaining in 12-grade in after attaining maximum qualification. In their final year 1in 2019/2020, about 4.4910% dropped from the school system before attaining maximum qualification likewise 95.5091% were expected to graduate from the system successfully after attaining maximum qualification.

Elsewhere, the completion of male candidates is in the below matrices.
completion rate after one year of study in 2017 and 2018 academic year
within one year, the completion rate of male is given by;
We first obtained our identity matrix I and the transient matrix Q my adding both to obtain the below matrix;

$(I+Q)$

$$(I+Q_m) = \begin{bmatrix} 1.000000 & 0.731872 & 0.000000 \\ 0.000000 & 1.000000 & 0.884793 \\ 0.000000 & 0.000000 & 1.000000 \end{bmatrix}$$

within a one year the completion rate of male is given by;

$$(I+Q_m) = \begin{bmatrix} 1.000000 & 0.731872 & 0.000000 \\ 0.000000 & 1.000000 & 0.884793 \\ 0.000000 & 0.000000 & 1.000000 \end{bmatrix} \begin{bmatrix} 0.000000 & 0.268128 \\ 0.000000 & 0.115207 \\ 0.971354 & 0.028646 \end{bmatrix}$$

within one year, the completion rate of the female is given by;
the result is added and summed up to;

$$(I+Q_m)R_m = \begin{bmatrix} 0.000000 & 0.352445 \\ 0.859447 & 0.140553 \\ 0.971354 & 0.028646 \end{bmatrix}$$

To obtained the final completion of Schooling year of MCSS by malem students, we must mutply the trasient states as many as possible until maximum qualifucation is attained by the pocess.
Q^2 represents transient states to be twice multiply.

$$(Q^2) = \begin{bmatrix} 0.000000 & 0.731872 & 0.000000 \\ 0.000000 & 0.000000 & 0.884793 \\ 0.000000 & 0.000000 & 0.000000 \end{bmatrix} \begin{bmatrix} 0.000000 & 0.731872 & 0.000000 \\ 0.000000 & 0.000000 & 0.884793 \\ 0.000000 & 0.000000 & 0.000000 \end{bmatrix}$$

The result of male students trasient matrices square product is given below by;

$$(Q^2) = \begin{bmatrix} 0.000000 & 0.000000 & 0.647555 \\ 0.000000 & 0.000000 & 0.000000 \\ 0.000000 & 0.000000 & 0.000000 \end{bmatrix}$$

We therefore added the identity matrix I to the trasient matrix Q along with the square transient matrix Q^2 to obtained the below reseult;

$$(I+Q+Q^2) = \begin{bmatrix} 1.000000 & 0.731872 & 0.592350 \\ 0.000000 & 1.000000 & 0.884793 \\ 0.000000 & 0.000000 & 1.000000 \end{bmatrix}$$

We carried on the completion by solving the matrix in the following operations or steps given by the formula below;
$(I+Q_m+Q^2)R_g$

$$(I+Q_m+Q^2)R_m = \begin{bmatrix} 1.000000 & 0.731872 & 0.647555 \\ 0.000000 & 1.000000 & 0.884793 \\ 0.000000 & 0.000000 & 1.000000 \end{bmatrix} \begin{bmatrix} 0.000000 & 0.268128 \\ 0.000000 & 0.115207 \\ 0.971354 & 0.028646 \end{bmatrix}$$

The completion ratio after enrolment in 10-grade is summarizes in into one matrix with dropout ratio and successful completion of MCSS students.

$$(I+Q_m+Q^2)R_m = \begin{bmatrix} 0.629005 & 0.370995 \\ 0.859447 & 0.140553 \\ 0.971354 & 0.028646 \end{bmatrix}$$

Table 3. Regular Monrovia Consolidated School System male Completion Table

Year (Y)	10-Grade		11-Grade		12-Grade	
	Completion	Dropout	Completion	Dropout	Completion	Dropout
1	0.000000	0.268128	0.000000	0.115207	0.971354	0.028646
2	0.000000	0.352445	0.859447	0.140553	0.971354	0.028646
3	0.629005	0.370995	0.859447	0.140553	0.971354	0.028646
4	0.629005	0.370995	0.859447	0.140553	0.971354	0.028646

From table three, where the male category is arranged, it's seen that 37.0995% of the male students who enrolled in 10-Grade have dropped from the MCSS in the year 2020 before attaining maximum qualification. In 2020, about 62.9005% who enrolled in 10-grade in 2017/2018 were expected to graduate successfully from the system in 2019/2020 after attaining maximum qualification.
From the same data, the same cohort proportion of males students that entered in 11-grade in 2018/2019 about 14.0553% dropped from the school system and 85.9447% successfully entered after attaining in 12-grade in after attaining maximum qualification. In their final year 1in 2019/2020, about 2.8646% dropped from the school system before attaining maximum qualification; likewise, 97.1354% were expected to graduate successfully after attaining maximum qualification.

4.4 THE ESTIMATED DURATION OF STUDY

In finding the expected duration of studies, we used the fundamental matrix N to get the desirable results after series of mathematical operations. This matrix gives the number of times that pupils spent in their learning cycle before being absorbed by the system. This circle became from 10-grade and ended to 12-grade. The starting time for schooling is fixed.
The fundamental matrix $N$is given by;

$N+I+Q+Q^2+\cdots=(I-Q)^{-}1$ The general group calculated result of N is given by;

$$(I+Q_g)=\begin{bmatrix}1.000000 & -0.717990 & 0.647555\\ 0.000000 & 1.000000 & -0.865060\\ 0.000000 & 0.000000 & 1.000000\end{bmatrix}$$

We used the matrix inverse to get our result is given below;

$$(I+Q_g)^{-}1=\begin{bmatrix}1.000000 & 0.731872 & 0.621107\\ 0.000000 & 1.000000 & 0.884793\\ 0.000000 & 0.000000 & 1.000000\end{bmatrix}$$

We added the matrix by rows to obtain our exected results as (Beck et al.,1983).

$$(I+Q_g)^{-}1=\begin{bmatrix}2.339100\\ 1.865060\\ 1.000000\end{bmatrix}$$

The expected duration of the study for both males and females in the Senior Secondary Education of MCSS is donated from the above matrix. The study duration for students from the start until completion of high school, from 10, 11, and 12-grade is 2.339100, 1.865060, and 1.00000 years.

4.4.1 THE ESTIMATED DURATION OF THE STUDY BY GENDER

From the general solution obtained with the same fundamental matrix N; subdivided into gender category (male and female). Therefore, this explained the number of cycle pupil

resides in transient states before being absorbed by the system; hence, the completion rates donated by particular starting state. It's also double absorbing states.
Thus the fundamental matrix N.
$N+I+Q+Q^2+\cdots=(I-Q_f)^-1$
The female completion rate is donated by the matrix N;

$$(I+Q_f)^-1=\begin{bmatrix}1.000000 & -0.703375 & 0.000000\\ 0.000000 & 1.000000 & -0.843434\\ 0.000000 & 0.000000 & 1.000000\end{bmatrix}$$

We used the matrix inverse to get the our result below;

$$I+Q_f)^-1=\begin{bmatrix}1.000000 & 0.703375 & 0.593250\\ 0.000000 & 1.000000 & 0.843434\\ 0.000000 & 0.000000 & 1.000000\end{bmatrix}$$

We added the matrix by rows to obtain our expected results (Beck et al.,1983).

$$I+Q_f)^-1 \quad (Famale)=\begin{bmatrix}2.296625\\ 1.843434\\ 1.000000\end{bmatrix}$$

The expected duration of the study for the female category in the Senior Secondary Education MCSS is donated from the above matrix. The duration of study for students until completion of high school, from 10, 11, and 12-grade is 2.296625, 1.843434, and 1.00000 years, respectively.

The male completion rate is also donated by the trasient matrix N;

$$(I+Q_m)^-1=\begin{bmatrix}1.000000 & -0.7731872 & 0.000000\\ 0.000000 & 1.000000 & -0.884792\\ 0.000000 & 0.000000 & 1.000000\end{bmatrix}$$

The matrix inverse operation also resulted in;

$$(I+Q_m)^{-}1 = \begin{bmatrix} 1.000000 & 0.7731872 & 0.647555 \\ 0.000000 & 1.000000 & 0.884792 \\ 0.000000 & 0.000000 & 1.000000 \end{bmatrix}$$

We added the matrix by rows to obtain our exected results given below;

$$(I+Q_m)^{-}1 \quad (Male) = \begin{bmatrix} 2.379427 \\ 1.884792 \\ 1.000000 \end{bmatrix}$$

The expected duration of the study for the female category in the Senior Secondary Education of MCSS is donated by the above matrix. The duration of study for students until completion of high school, from 10, 11, and 12-grade is 2.379427, 1.884794, and 1.00000 years, respectively.

4.5 Absorbing Ratios By Group

In this category, the Absorbing ratios were divided into two columns: successful completion and unsuccessful dropout.
The general projected Double Absorbing states were donated in the long run by;
$(I-Q_g)^{-}1$

$$(I+Q_g)R_g = \begin{bmatrix} 0.598678 & 0.401322 \\ 0.833821 & 0.166179 \\ 0.963888 & 0.036112 \end{bmatrix}$$

The dropout rates of male and female students of the MCSS Senior Secondary education students from 10, 11, and 12-grade were calculated in percentages and given by: 40.1322%, 16.6179%, and 3.6112% respectively, dropouts of the school system before attaining maximum qualification. On the other hand, successful progress and graduates from 10, 11, and 12-grade were arranged in percentages: 59.8678%, 83.3821%, and 96.3888%′ successful graduate from the system.

4.5.1 AN ABSORBING RATES BY GENDER OF THE MCSS

The Absorbing ratios were also divided into two columns in the female category: successful completion and unsuccessful dropout.

The general projected double Absorbing states were donated in the long run by;
$(I-Q_g)^{-}1$.

$$(I+Q_f+Q^2)R_f = \begin{bmatrix} 0.566607 & 0.433393 \\ 0.805555 & 0.194445 \\ 0.955090 & 0.044910 \end{bmatrix}$$

The dropout rates of female students of the MCSS Senior Secondary education students from 10, 11, and 12-grade were calculated in percentages and given by: 43.33390%, 19.4445%, and 4.4910%. Respectively dropouts of the school system before attaining maximum qualification. On the other hand, successful graduates from 10, 11, and 12-grade were arranged in percentages; 56.6607%, 80.5555%, and 95.5090%′ successful graduate from the system.

The Absorbing ratios were also divided into two columns in the male category: successful completion and unsuccessful dropout.
The general projected double Absorbing states were donated in the long run by;
$(I-Q_g)^{-}1$.

$$(I+Q_m+Q^2)R_m = \begin{bmatrix} 0.629005 & 0.370995 \\ 0.859447 & 0.140553 \\ 0.971354 & 0.028646 \end{bmatrix}$$

The dropout rates of male students of the MCSS Senior Secondary education students from 10, 11, and 12-grade were calculated in percentages and given by: 37.0995%, 14.0553%, and 2.86460%. Respectively dropouts of the school system before attaining maximum qualification. On the other hand, successful graduates from 10, 11, and 12-grade were arranged in percentages; 62.9005%, 85.9447%, 97.1354%′ successful graduate from the system.

4.6 RETENTION RATES OF THE MCSS

The general (male and female) retention ratios were determined from the transition probabilities matrix. From our research, the rates obtained from the transition probability beginning from 10, 11, and 12-grade were established to be: 0.717993, 0.865060, and 0.963888, respectively
. It's established that the highest retention rate was 12-grade and the lowest retention rate was in 10-grade .
By gender, we also have the retention rations in the following categories;

Female students retention rates of the MCSS are: 0.703375, 0.843434, and 0.955090 from 10, 11 ,and 12-grade respectively.
The highest retention rate for the female category was in 12-grade 0.955090, and the lowest was in 10-grade 0.703375, respectively.
Male students retention ratios of MCSS are: 0.731872, 0.884793, and 0.971354. the highest retention from the male group was also 0.971354, and the lowest was 0.731872. the highest retention ratio was in 12-grade, and the lowest retention ratio was in 10-grade also.

5 SUMMARY, AND ANALYSIS OF RESEARCH FINDINGS, AND THE RECOMMENTATIONS

5.1 INTRODUCTION

This chapter gives detail of the summary of the research findings, conclusions, and basic recommendations derived from our major-specific objectives. Finding are therefore given in the below sections based on categories.

5.2 SUMMARY AND ANALYSIS OF THE RESEARCH FINDINGS

Based on our findings, it was established that on assumption that there is no repetition of classes in any closed education ranking or Hierarchical population a Markov Chain Model can be used to study its progress.

From our research, it was established that in a short period A Markov Chain can be used to forecast the future population with the assumption of time homogeneity. From our finding it was also established that completion rates of students from 10-Grade were very less as compared with the same cohort that migrated into in 11-Grade, and subsequently into 12-Grade for completion. This also suggests that students who enrolled in 10-Grade after this cohort are subject to the same attrition in 11, and 12-grade respectively. Hence, the completion ratio of Regular Senior Secondary of MCSS students from 10, 11 and 12-grade are: 0.598678, 0.833821 and 0.963888 respectively. This gave a clear indication that students in 12-Grade have high chances of completion or graduation, as compare to those in lower classes.

The completion rates by gender category from 10, 11, and 12-Grade in the following order: 0.566607, 0.805555, and 0.955090 for females respectively, and for male students completion ratios were: 0.629005, 0.859447, and 0.971354 for male students in the same classes and the same order. From this order, it can be analyzed that Male student completion ratios were higher as compared to female students ratios in the regular Senior secondary school of the Monrovia Consolidated School system (MCSS).

The secondary school dropout rates were also determined by the Monrovia Consolidated School system (MCSS) from 10, 11, and 12-Grade to be: 0.401322, 0.166179, and 0.036112 respectively. From the dropout ratios, it's indicated that students from 10-Grade are more likely to drop out as compare to other higher classes.

Comparing the same ratios in the gender category, we found that female students' dropout ratios from 10, 11, and 12-Grade were: 0.433393, 0.194445 and 0.044910 respectively. While male students' dropout ratios were: 0.370995, 0.140553, and 0.028646 in the same class and order. From this category, it's indicated that dropout ratios for female students are higher than male dropout ratios in all Grade levels.

In the Markov model in which there's no repetition of classes as the cohort progress in each semester or academic year, it's established that the completion and dropout rates, in the long run, are the same as the absorbing rate. This doesn't hold in the model that allows the repetition of classes (Musiga et al., 2011).

From research analysis, it was established that the Markov Mosel can be used to forecast the future enrollment of students in MCSS in a very short-term period. The model can't project in the long-run term which, is one major weakness of the Markov model.

Addressing the expected duration of studies of the senior secondary school of Monrovia consolidated school system (MCSS), it was established for the entire MCSS the expected duration of the study was: 3.339100, 1.865060, and 1.000000 years, from 10, 11, and 12-Grade respectively. The same estimated duration of study of students by gender category was: 2.296625, 1.843434, and 1.000000 years for female students from 10, 11, and 12-Grade respectively, and the estimated duration for male counterpart was: 2.379427, 1.884792, and 1.000000 years, from 10, 11, and 12-Grade respectively. From the comparison in the gender category, it was established that female students had a lower expected duration of study as compared to their male counterparts.

5.3 CONCLUSION FOR THE FINDINGS

From our research findings, the following conclusion was established;

- The expected duration of study for male students was higher as compared with female students. From the comparison amongst classes in Senior Secondary School of

MCSS, 10-Grade had the lowest retention rates and 12-Grade had the highest retention rate.

- From the research, it was established that the dropout rate of female students is higher than that of their male counterparts.

- In the short run, completion and dropout rates were found to be the same as the absorbing rates when repetition of classes is prohibited.

- Female students were found to have a lower expectation of schooling as compared with their male counterparts.

- The Markov model can be used to forecast student enrollment of the MCSS in a short interval period.

- The transition rates from 11 and 12-grade are higher for males as compared to female students. From all the transitions rates, male students of MCSS have the highest transition rates as compared with females.

5.4 RECOMMENDATION FOR THE FINDINGS

The above research lead to the following findings, conclusions, and the following recommendations are made;

1. Since all government schools in Liberia are tuition-free including the MCSS, there should be some regulation such as no repetition of classes by students; thus, this is one of the benchmarks of Kenya Secondary Certificate Education (KCSE). This method will aid the increase of retention strategies and avoid unnecessary dropouts which is a waste of government funding on fees and West African Senior Secondary Certificate Examination (WASSCE) fees for 12-Grade students.

2. There should be a serious retention strategy for students in 10-Grade, our research investigated that students in 10-Grade are subject to attrition than any classes in Senior Secondary School of MCSS.

3. Research should be done to investigate the root cause of the high dropout rate of female students of MCSS and recommend the way forward to reduce gender disparity of the MCSS. In a short closed hierarchical system, a Markov Chain model can be used to study how the school system progress with time.

4. The Absorbing Markov chain can only be used for a short period, that is one of the weaknesses of the Markov model.

5. The retention rates, there should be organized research to get into the main insight of the reason as to why there are very low retention rates in 10-Grade of the MCSS, and also female has the lower retention rate by gender category.

6. The research wasn't able to established the difference between permanent and temporary dropouts as it was a cohort study.

7. The Absorbing Markov chain Can't differentiate between permanent and temporary dropout from a system.

5.5 FURTHER RECOMMENDED RESEARCH

1. There should be research that will investigate various reasons why students drop out of the school system at each level of study; Logistic regression, etc.

2. An extension of similar research is necessary for the entire country or Republic of Liberia.

Table 4. The MCSS enrolment table from 2017-2020

Grade	2017-2018			2018-2019			2019-2020		
	Male	Female	Total	Male	Female	Total	Male	Female	Total
10^{th}	593	563	1156	676	653	1329	676	576	1252
11^{th}	422	357	797	434	396	830	512	403	915
12^{th}	234	215	449	374	318	692	384	334	718

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