

" RATIO ANALYSIS AND THE PREDICTION  
OF BUSINESS FAILURE "

by

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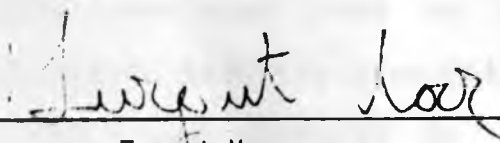
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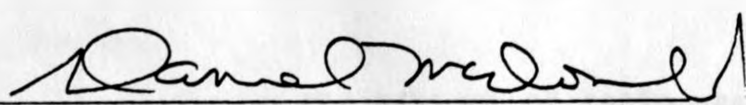
Title of Project: Ratio Analysis and the Prediction of Business Failure

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## ABSTRACT

Economic survival was once merely a minor concern to business enterprises -- usually only new companies. However, as the current recession continues, survival is quickly becoming the dominant corporate objective. As businesses fail, they leave outstanding debts, unpaid employees, reduced government revenues, and dissatisfied owners, in addition to increasing unemployment. In 1981 alone there were over 8,000 corporate bankruptcies with liabilities in excess of 1.1 billion dollars. With the number of failures increasing dramatically, models to predict survival become an important tool in managements' arsenal, and have developed from an ad hoc base to complex computerized techniques. Multiple Discriminate Analysis [MDA] is one of the latter, and attempts to quantify a company's "riskiness" into a "Z-score." These Z-scores can then be used in credit-worthiness decisions, and most importantly, predictions of economic survival.

This paper discusses the history of ratio analysis, up to the current usage of Financial Statement Information. In addition, a comparison is made between the predictive models developed by Edward Altman and Gordon Springate using American and Canadian data respectively.

Finally, the paper discusses the limitations of this research, and suggests further areas of research.

Dedicated to those who stuck with me; John and Edna, Al and Lynda, and especially my Chairman, Dr. Kenji Okuda, and his Administration.

For those who did not....

*C. J. Chambers,  
Accounting Evaluation and Economic  
Behavior, 1968, p. 346.*

---

To my good friend

Shiraz

Thank you for all your help,  
kindness, and kinship.

Best wishes always. May the  
winds of peace be at your back!

---

Terry.

REFERENCES.

The "Actual State of Accounting is not that it has no theories, but that it has an almost inexhaustible quantity of implicit, partial, and contradictory theories. Ockham's razor has not been near the stubble.

R.J. Chambers,  
Accounting Evaluation and Economic Behavior, 1966, p. 346.

## ACKNOWLEDGEMENTS

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## Introduction

The costs of Bankruptcy, and other market imperfections make financial distress an undesirable state of affairs. [Gordon, 1971, p. 354]

In the complex and competitive business world, survival can be seen as one measure of success. Unfortunately, the economic downturn of the 1980s has resulted in elevating survival to the single, most important corporate goal. Therefore, information that will reduce uncertainty and risk often becomes vital to the success of a business venture. Such information is obtained through the accounting models and reporting systems of each firm and is used by insiders (management) and outsiders (financial analysts). As such, it is necessary to be able to interpret available data, especially financial statements, to determine a particular company's financial status (healthy, growing, poor, etc.). One method commonly used, is a comparative technique known as ratio analysis.

The number of business failures:<sup>1</sup> bankruptcies, forced sales, etc., in Canada has risen dramatically in the last few years. Springate [1978, p. 8] mentioned that "the economic deficiency to the Canadian community will be well over three hundred million dollars in 1978" (see Table I). This of course does not include the "indirect" costs such as staff retraining,

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<sup>1</sup> For a discussion of "failure", economic, financial, technical, insolvency, etc., see Weston and Brigham [1977, pp. 542-44].

forced retirements, increased Unemployment Insurance payments and other opportunity costs. In other words, the costs of business failures are substantial, and there is a need to look at potential reduction of these costs. As a minimum, research should be focused towards controlling these costs.

Business failures will always be a part of any free-enterprise system, and a model which can be used to "accurately predict" any such failure may help minimize exposure to these costs. Information on predicted corporate failures will benefit corporations in the extension of credit, employees of enterprises, potential investors, governments, and the business itself. Of the current developments in this area Horrigan [1968, p. 294] indicates that:

The most striking aspect of the present state of ratio analysis is the absence of an explicit theoretical structure. Under the dominant approach of 'pragmatical empiricism', the user of ratios is required to rely upon the authority of an author's experience. As a result, the subject of ratio analysis is replete with untested assertions about which ratios should be used and what their proper levels should be.

This implies that there is a need for further research in this area, but to begin this we should look at the development of ratio analysis, from its beginnings to its present state. Basically there are two types, univariate analysis, in which one, or a series of individual ratios is developed and analysed by the "prudent business person". A more sophisticated type, multivariate analysis, calls for the simultaneous analysis of these same ratios. Thereby, trends and interrelationships can be determined, and used to develop predictive models of business

failure. By using Multiple Discriminate Analysis [MDA] the "best" ratios, those which result in the most predictive ability, can be determined and then applied to existing data for grouping companies into "failed" or "non-failed," in order to develop a riskiness index.

This paper will therefore examine the development of ratio analysis, paying particular attention to the use of ratios for failure prediction. Secondly, the implications of this development for future research will be discussed. However, before the latter can be done, we should look at the reasons for corporate failure. If in fact the reasons can be determined, they may also have some implications for further research in this area.

## Business Failures in Canada

The recent poor performance of our nation's economy has been marked by a rash of business failures in all sectors.

[Altman, 1971, p. 333]

Before looking at ratio analysis, it might be useful to look at the size of the problem. To be specific, what are the costs mentioned in the introduction? As Table I indicates, the number of bankruptcies in Canada has increased 115% in the last five years, (1977-1981) from 3745 to 8055. For an indication of the costs, a look at liabilities outstanding at point of failure, as declared by the debtor,<sup>2</sup> and therefore potential forgone assets in other enterprises, indicate that these failures in 1981 represent close to \$1.15 billion. This figure alone has risen 72% since 1977, and by all accounts in current news releases will continue to increase in the 1980s.<sup>3</sup>

Table II indicates the areas that firms were engaged in at time of failure. In a sense, it indicates the "riskiness" of different industry sectors. For instance, until 1978 it would appear that the "Finance" and "Service" sectors have been relatively stable. However "Primary" and "Manufacturing"

<sup>2</sup> This of course does not include shareholders equity.

<sup>3</sup> The Vancouver Sun (B.C. Bankruptcies Soar, by Rod Nutt, Sun Business Writer, April 13, 1982), reported that business and personal bankruptcies in B.C. "surged 44% and 62% respectively in the first three months of this year [1982] from the same period a year earlier." The businesses that declared bankruptcy during this period reported liabilities of \$65 million.

indicate their volatility as failures declined in the mid 1970s, a period of economic expansion, and then showed large increases in 1977 and 1978. Moreover, the statistics indicate continued increases in bankruptcies in these areas.

Unfortunately, the areas of "Primary" and "Construction" (the latter up 223% in 1978 over 1977) are two of the main industries in British Columbia. If an accurate failure prediction model can be established, specific industry models could be formulated, and hopefully used effectively in this province.\*

Year	Primary	Construction	Total
1976	124	108	232
1977	152	128	280
1978	188	288	476

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\* A model could be used by bank managers, and other business people who wish to make credit decisions involving a business enterprise. This model would not, of course, make the decision, but may add valuable information. This would be especially true if the decisions are being based on ad hoc methods.

**TABLE I**

**Business Bankruptcies in Canada**  
 (Under the Bankruptcy and Winding Up Act)

Year	no. of fail-ures	liabil-ities ('000s)	Size of failure with liabilities:				
			under 5000	5000 24999	25000 49999	50000 99999	over 100000
1969	2699	211105	88	1250	612	365	384
1970	3281	255884	85	1421	813	471	491
1971	3270	322654	68	1376	799	555	472
1972	3046	307192	56	1244	766	548	432
1973	2945	296710	64	1068	788	529	496
1974	2853	325560	29	1004	724	537	559
1975	2091	325297	35	641	468	428	519
1976	2631	1220895	21	711	576	600	723
1977	3745	663558	20	842	867	955	1061
1978	4882	628369	23	1441	1209	1223	1286
1979	5648	573730					
1980	6595	782966					
1981	8055	1146099					
1982 <sup>1</sup>	9962	2134316					

Source: Government of Canada, Statistics Canada Reports  
 Department of Industry, Trade and Commerce.  
 (Section 61-002) Ottawa. Survey ceased in 1978.  
 1979-1981 figures for comparative purposes only  
 (see Table II).

1. For an eleven month period (January to November). Note: for comparison, there were 28,289 consumer bankruptcies in the same period.

**TABLE II****Business Bankruptcies by Industry**

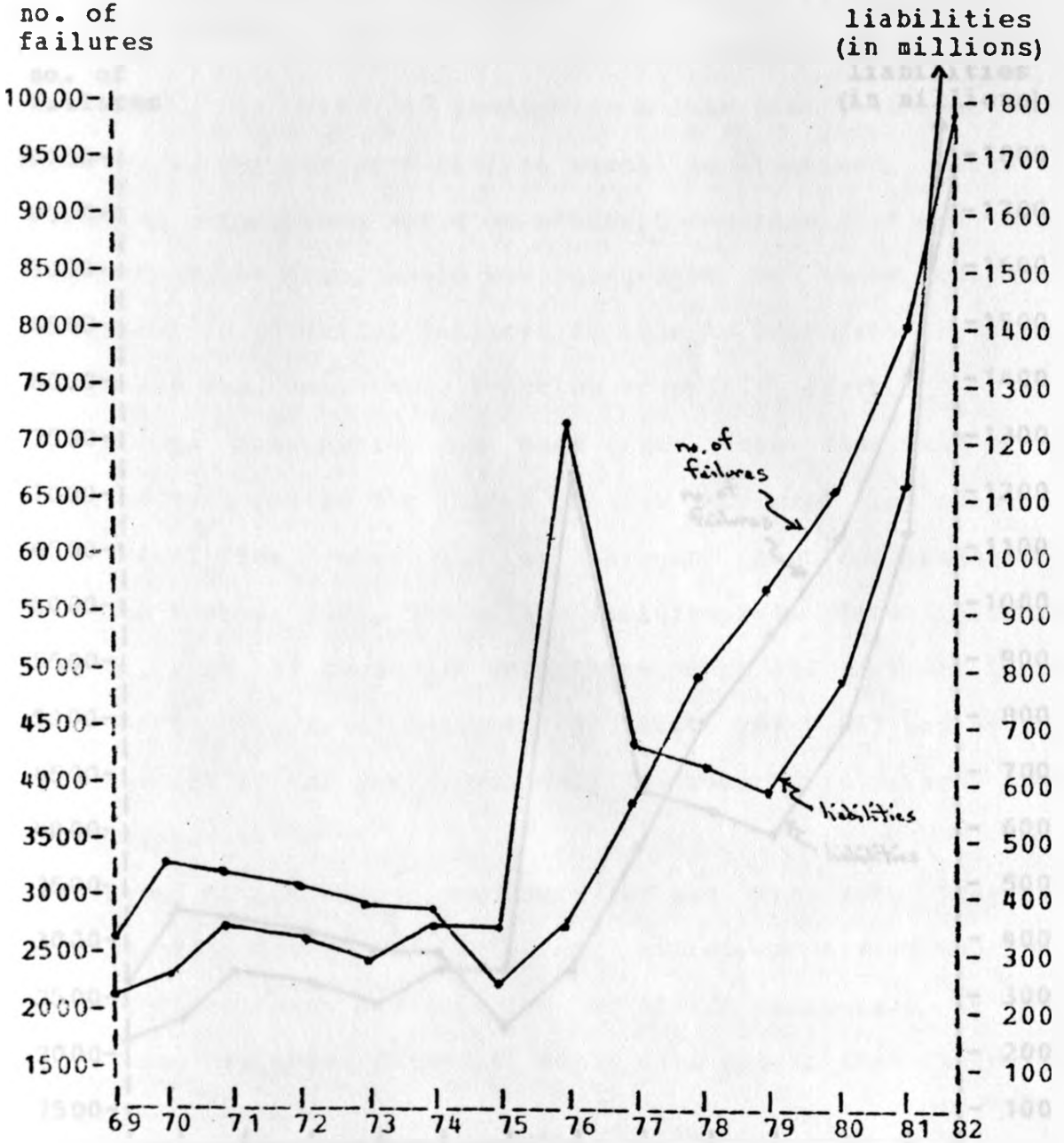
<u>Year</u>	<u>Prim.</u>	<u>Manu.</u>	<u>Const.</u>	<u>Trans.</u>	<u>Trade</u>	<u>Finance</u>	<u>Service</u>
1969 <sup>1</sup>	111	278	440	203	1150	100	417
1970	166	349	490	242	1411	118	505
1971	182	320	465	258	1463	115	467
1972	150	226	556	224	1347	76	467
1973	114	217	598	253	1294	61	408
1974	86	197	568	351	1236	60	352
1975	46	241	364	209	784	49	398
1976	63	255	489	192	1075	51	506
1977	95	349	506	191	1102	51	518
1978	109	410	1128	288	1757	149	1041
1979 <sup>2</sup>	200	534	1145	343	1882	203	1341
1980 <sup>3</sup>	312	532	1301	431	2120	254	1645
1981	427	681	1404	551	2560	262	2170
1982 <sup>4</sup>	593	946	1526	720	3033	476	2668

- Sources: 1. 1969 to 1978: Government of Canada, Statistics Canada Reports Department of Industry, Trade and Commerce. (Section 61-002) Ottawa.
2. 1979: Canadian Business Failure Record, Dun and Bradstreet, 1979.
3. 1980 to 1981: Insolvency Bulletin, Superintendent of Bankruptcy, Consumer and Corporate Affairs, Canada.
4. 1982 figures for an eleven month period (January to November) Insolvency Bulletin, Superintendent of Bankruptcy, Consumer and Corporate Affairs, Canada.



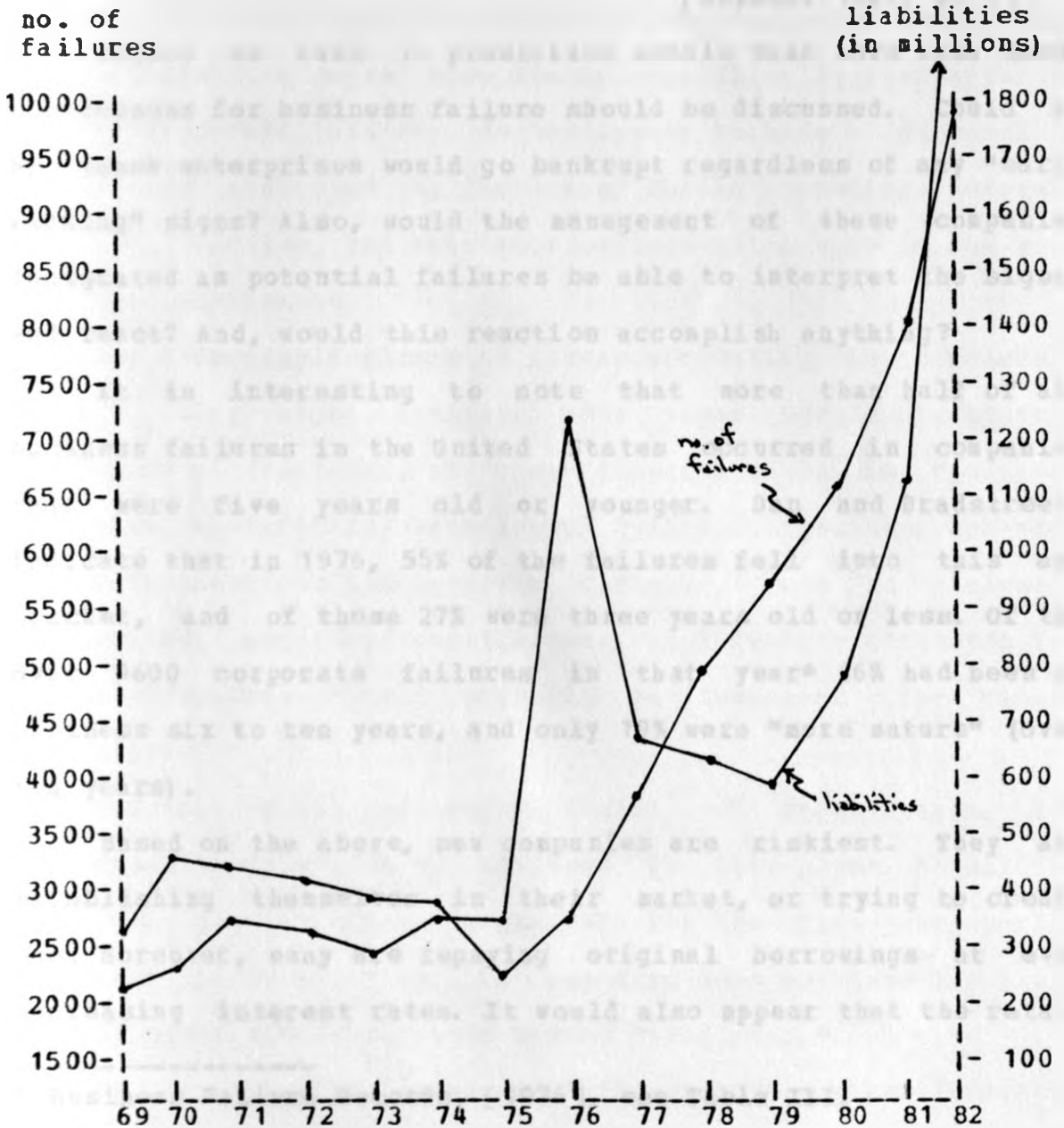
**FIGURE I**

**Bankruptcies in Canada**



**FIGURE I**

**Bankruptcies in Canada**



## Reasons for Failure

Bankruptcy, it appears, can be viewed as an intervention in market processes designed to facilitate efficient adjustments.

[Nelson, 1981, p. 3]

Before we turn to prediction models that have been used, the reasons for business failure should be discussed. Could it be these enterprises would go bankrupt regardless of any "early warning" signs? Also, would the management of these companies designated as potential failures be able to interpret the signal and react? And, would this reaction accomplish anything?

It is interesting to note that more than half of all business failures in the United States occurred in companies that were five years old or younger. Dun and Bradstreet<sup>5</sup> indicate that in 1976, 55% of the failures fell into this age bracket, and of these 27% were three years old or less. Of the over 9600 corporate failures in that year<sup>6</sup> 26% had been in business six to ten years, and only 19% were "more mature" (over ten years).

Based on the above, new companies are riskiest. They are establishing themselves in their market, or trying to create one. Moreover, many are repaying original borrowings at ever increasing interest rates. It would also appear that the retail

-----  
<sup>5</sup> Business Failure Records, [1976], see Table III.

<sup>6</sup> See Table IV.

industry is more susceptible to early catastrophe. The Dun and Bradstreet studies indicate that over 34% of the retail failures occurred in the first three years, whereas only 15% occurred in firms over ten years of age. In comparison to the retail industry, 25% of the failures in manufacturing were in the earlier period, and 25% in the latter. The indications are that those industries deemed more stable, specifically manufacturing and the wholesale industry, do not appear to have a discernible "life," as indicated by the age of failed companies. However, the retail service, and construction industries vary in the ages of failed businesses.

Are there implications as far as accounting is concerned? Firstly, they might indicate that there should be greater disclosure of financial, and non-financial data in "younger" companies specifically relating to future obligations, non-arms length transactions and so forth. Moreover, from an analyst's perspective, solvency requirements and liquidity standards for credit decisions should probably be increased for these companies.

Secondly, youth may short-circuit, or even negate, the prediction models to be reviewed in this paper. As will be shown, the studies generally use data for the five-year period preceding failure. Thus, if the firm does not have the track record, other models may be a better "predictor."

If, however, the higher risk characteristic prevails before the requisite period can be completed, other intangible factors,

will have to be closely examined (such as directors' abilities, management's past track records, etc.). Again, this may indicate that other items should be disclosed in financial statements.

In other words, should a question be posed regarding a trade-off between supplementary information and a "track record"? On the other hand, one could also speculate that the information disclosed in the initial years may still be of value to statement users as the company matures. It may, therefore, be sufficient to say that the year of start-up should be clearly disclosed as an integral part of the financial statements. Regardless, this point should be kept in mind when evaluating predictive models. Possibly different models should be used, based on the stage of the companies' development cycle. For instance, the critical levels could be changed depending on the "maturity" of the particular enterprise, or different weights should be attached to certain data points.

Age, by itself however, can not be the cause of failure. According to Dun and Bradstreet, records for both Canada and the United States indicate that the causes of bankruptcy can be categorized into four main areas; "financial control," "uninsured disasters," "neglect," and "premeditated disasters."

The main grouping has been given the title of "financial control," and includes apparent causes such as poor location, competitive weakness, receivables and inventory difficulties, and the most obvious -- inadequate sales. Of the four groups, this one deals with the actual management abilities within the

company. Thus, the statistics indicate that "better" management of the finances might have "saved" a large portion of the failed companies. On the other hand, it also indicates that the majority of these failures are due to the firm's inability to avoid certain economic conditions. Also, only a few failures were brought on by fraud, or "natural disasters". These latter causes can therefore be disregarded.

The above indicates that timely financial statements, as the record of economic events, are of extreme importance. Therefore, an effective financial reporting system must be in place before a company "opens up for business." The flow of information must be relevant, timely, and accurately present the financial position of the enterprise to management and/or owners.

Moreover, this information may be used to predict the future of the company. In fact, the Financial Accounting Standards Board in the United States indicates that information on past earnings (i.e. previous years net income) can be utilized to predict future cash flows [FASB, 1978]. Regardless, the point is that financial information can be analyzed and can form the basis of predictions for future events -- budgeting being the best example. It therefore is reasonable to assume this information can be used as an "early warning" mechanism of impending disaster.

Of note is that the majority of bankruptcies fell into the category of "inadequate sales" (44% in the U.S., and 65% in

Canada). This might negate any possible value of failure prediction models.<sup>7</sup> After all, if the company can not sell its goods and services, collapse is imminent -- regardless of past earnings and any predictions of future life. Moreover, the economist would argue that the company should not be in business anyway!

In essence, this indicates that firms should include probabilistic budgeting,<sup>8</sup> complementary to survival forecasting models in their arsenal. This includes breakeven analysis and cost-volume-profit models.

A sound knowledge of cost-volume-profit behavior and cost interrelationships is essential to many business decisions. Information which is easily understood but may not represent reality can lead to costly errors in judgement in business decisions.

[Raun, 1964, p. 927]

The utilization of such budgeting models, allows for a degree of flexibility in the control of revenues and costs. By using, for example, regression techniques the firm may be able to forecast sales, based on available data. In other words, a theory of sales and earnings could be developed.

This would allow management to be prepared to respond to differences in sales volume, and to choose among available alternatives. [Jaedicke and Robichek, 1964, p. 917] Moreover, probabilistic models can utilize measures for relative risk of

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<sup>7</sup> Unless, of course, a model can be used to effectively predict future sales.

<sup>8</sup> As opposed to "traditional" single scenario budgeting.

available alternatives.\*

Thus, in times of recession as the present, the number of failures, and therefore risk, increase dramatically. This in itself does not mean that "good" predictive models are even more necessary. On the contrary, it could make the exercise irrelevant. What happened last period, may of course have no relevance to the current period. If the company suddenly loses its' market or the market itself disappears, any predictive model is of no help to the managers of the business.

1964	27.5	27.5	27.5
1965	27.6	27.6	27.6
1966	27.7	27.7	27.7
1967	27.8	27.8	27.8
1968	27.9	27.9	27.9
1969	28.0	28.0	28.0
1970	28.1	28.1	28.1
1971	28.2	28.2	28.2
1972	28.3	28.3	28.3
1973	28.4	28.4	28.4
1974	28.5	28.5	28.5
1975	28.6	28.6	28.6
1976	28.7	28.7	28.7
1977	28.8	28.8	28.8
1978	28.9	28.9	28.9
1979	29.0	29.0	29.0
1980	29.1	29.1	29.1
1981	29.2	29.2	29.2
1982	29.3	29.3	29.3
1983	29.4	29.4	29.4
1984	29.5	29.5	29.5
1985	29.6	29.6	29.6
1986	29.7	29.7	29.7
1987	29.8	29.8	29.8
1988	29.9	29.9	29.9
1989	30.0	30.0	30.0
1990	30.1	30.1	30.1
1991	30.2	30.2	30.2
1992	30.3	30.3	30.3
1993	30.4	30.4	30.4
1994	30.5	30.5	30.5
1995	30.6	30.6	30.6
1996	30.7	30.7	30.7
1997	30.8	30.8	30.8
1998	30.9	30.9	30.9
1999	31.0	31.0	31.0
2000	31.1	31.1	31.1
2001	31.2	31.2	31.2

BY THE BOARD OF DIRECTORS OF THE COMPANY

\* See also Parker and Segora [1971], and Ferrara and Hayya [1970].



Table III

Failure Age in United States  
1950 - 1976

<u>Year</u>	<u>% in Business</u> <u>5 Years or less</u>	<u>% in Business</u> <u>6 to 10 Years</u>	<u>% in Business</u> <u>Over 10 years</u>
1950	68.2%	19.0%	12.8%
1951	63.2	23.5	13.3
1952	59.9	25.8	14.3
1953	58.5	26.7	14.8
1954	57.2	27.3	15.5
1955	56.6	26.0	17.4
1956	58.6	23.1	18.3
1957	58.9	21.8	19.3
1958	57.2	21.4	21.4
1959	57.1	22.3	20.6
1960	58.6	20.8	20.6
1961	56.2	22.4	21.4
1962	55.4	22.2	22.4
1963	55.4	21.7	22.9
1964	56.0	21.5	22.5
1965	56.9	21.4	21.7
1966	57.4	21.5	21.1
1967	55.3	22.5	22.7
1968	53.9	23.3	22.8
1969	53.2	24.4	22.4
1970	54.9	22.7	22.4
1971	54.2	22.2	23.6
1972	55.7	22.4	21.9
1973	57.0	22.4	20.6
1974	59.9	20.8	19.3
1975	57.4	22.7	19.9
1976	54.8	26.0	19.2

Source: The Business Failure Record compiled by the Business Economic Division of Dun and Bradstreet [1976].

Table IV

Failure Age by Industry  
1976

<u>Industry</u>	<u>3 Years or Less</u>	<u>4-5 Years</u>	<u>6-10 Years</u>	<u>Over 10 Years</u>
Manufacturing	24.6%	23.6%	26.4%	25.4%
Wholesale	23.4	25.9	24.0	26.7
Retail	34.5	28.4	22.6	14.5
Construction	16.6	29.7	30.9	22.8
Service	<u>23.9</u>	<u>27.8</u>	<u>31.8</u>	<u>16.5</u>
Total	27.2%	27.6%	26.0%	19.2%
Cumulative		54.8%	80.8%	100.0%

Source: The Business Failure Record compiled by the Business  
Economic Division of Dun and Bradstreet [1976].

Table V

Apparent Causes of Failure

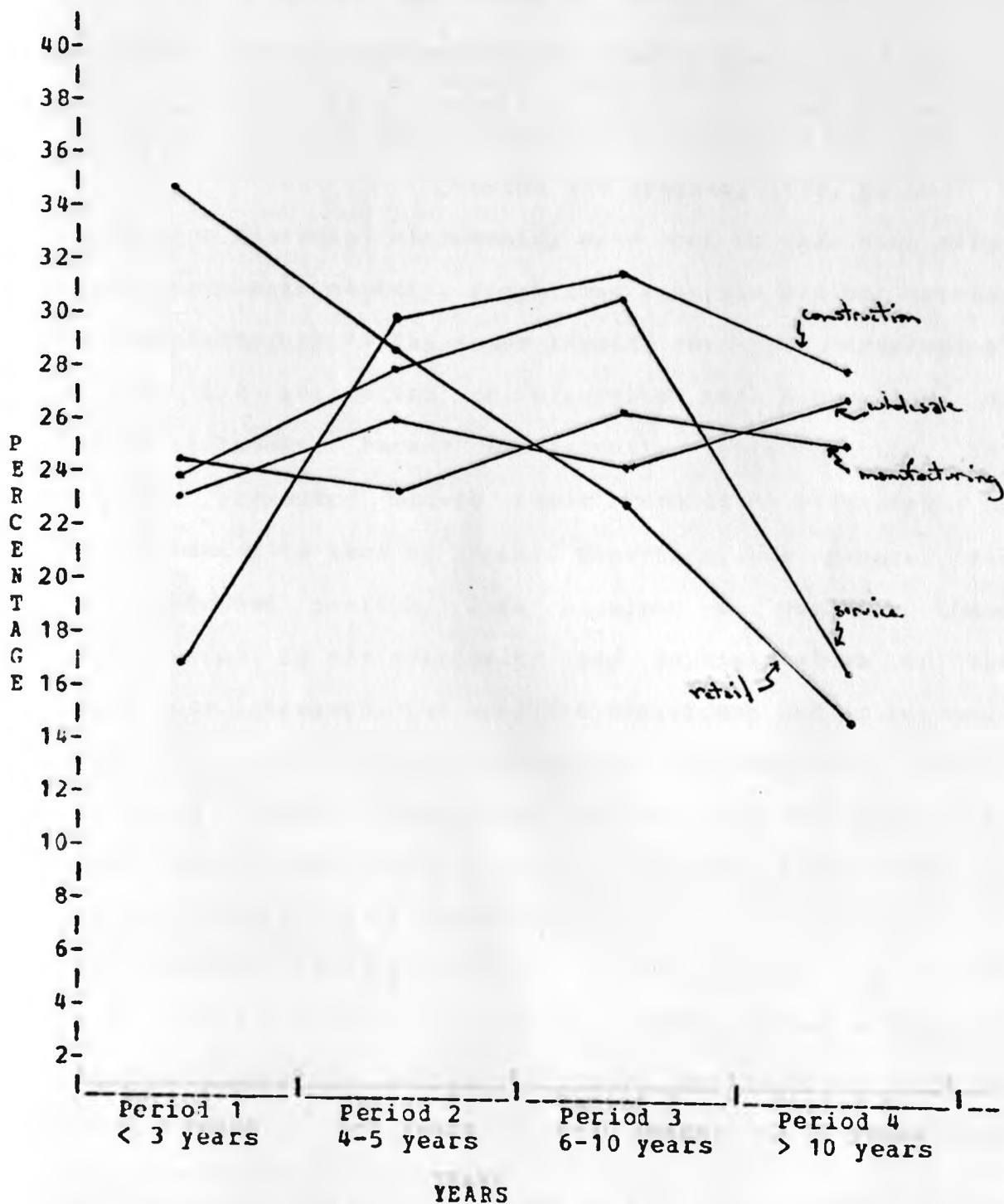
<u>Financial</u>	<u>U. S. <sup>1</sup></u>	<u>Canadian<sup>2</sup></u>
Inadequate Sales	49.9%	65.5%
Competitive Weakness	25.3	3.3
Heavy Operating Expenses	13.0	23.2
Receivables Difficulties	8.3	2.4
Inventory Difficulties	7.7	2.1
Excessive Fixed Assets	3.2	0.8
Poor Location	2.7	1.1
Other	<u>1.1</u>	<u>0.3</u>
	111.2%	98.7%
Uninsured Disasters	0.8%	0.8%
Neglect	0.8	1.5
Premeditated Disasters	<u>0.5</u>	<u>0.3</u>
	113.3%	101.3%

"Because some failures are attributed to a combination of apparent causes, the totals exceed 100%."

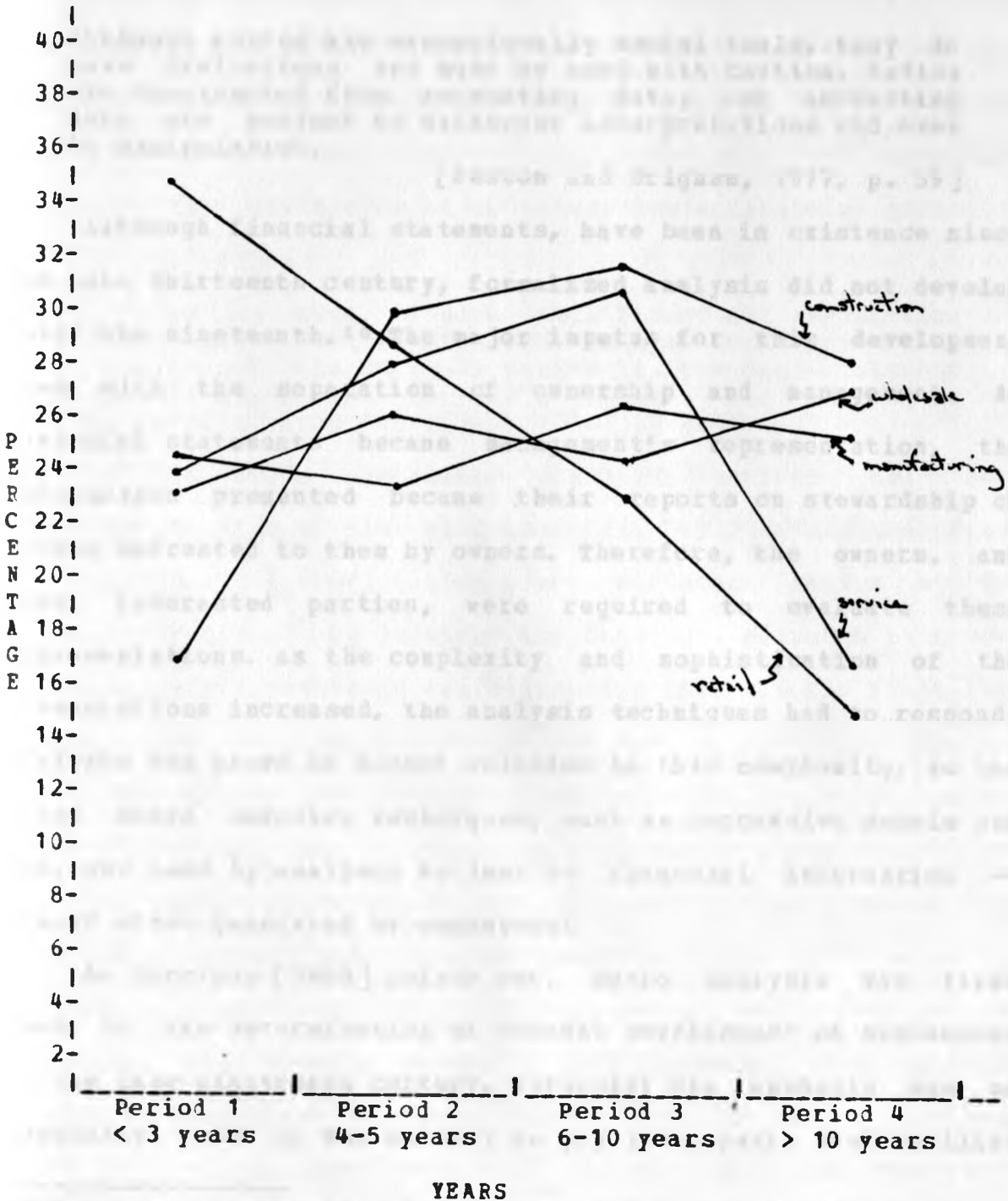
<sup>1</sup> Dun and Bradstreet [1976].

<sup>2</sup> Springate, [1978, p. 4].

**FIGURE II**  
**Age of Failed Businesses by Industry -- 1976**



**FIGURE II**  
**Age of Failed Businesses by Industry -- 1976**



## Ratio Analysis, Univariate Models

Although ratios are exceptionally useful tools, they do have limitations and must be used with caution. Ratios are constructed from accounting data, and accounting data are subject to different interpretations and even to manipulation.

[Weston and Brigham, 1977, p. 59]

Although financial statements, have been in existence since the late thirteenth century, formalized analysis did not develop until the nineteenth.<sup>10</sup> The major impetus for this development came with the separation of ownership and management. As financial statements became management's representation, the information presented became their reports on stewardship of assets entrusted to them by owners. Therefore, the owners, and other interested parties, were required to evaluate these representations. As the complexity and sophistication of the presentations increased, the analysis techniques had to respond. Analysis has grown in direct relation to this complexity, to the point where computer techniques, such as regression models and MDA, are used by analysts to look at financial information -- itself often generated by computers!

As Horrigan [1968] points out, ratio analysis was first used in the determination of "credit worthiness" of businesses in the late nineteenth century. Naturally the emphasis was on liquidity, and on the ability to pay (or repay). Profitability

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<sup>10</sup> See Chatfield [1977], Lev [1974], and Horrigan [1968].

was also of prime interest to the analysts as an indicator of a basic "credit risk". As the sophistication of business continued a new concept of current and non-current assets and liabilities was established. Foulke [1961, p. 181] indicates that working capital, first appeared in financial data in 1891. With this change in presentation, working capital became a very important part of the evaluation of an enterprise's financial position. The current ratio,

... was to have a more significant and longlasting impact upon financial statement analysis than any other ratio. Truly, the usage of ratios in financial statement analysis can be said to have begun with the advent of the current ratio [Horrigan, 1968, p. 285].

By the 1920s systematic analysis routines had been established. At that time Alexander Wall [1919] wrote his "Study of Credit Barometrics", taking seven different ratios of 981 firms, stratified by industry and location. Although this work has been criticized on academic grounds, this was the first real breakthrough in ratio analysis. It would appear that this was the first general recognition of the benefits of a "multi-ratio" type of analysis. In the same year, the DuPont Company developed a model using multi-ratios for managerial use, called the "triangle" system [see Kline and Hessler, 1955]. This system had "return on investment" (profits/total assets) on the top of the triangle, and "profit margin" (profits/sales) and "capital turnover" (sales/total assets) ratios along the sides. Although this system was not generally accepted then, the concept of Return on Investment [ROI] with its two components has become

the basis of "responsibility accounting".

By 1930, the emphasis of analysis shifted, from "credit-worthiness", to one of failure prediction. In addition, the formation of the SEC in the United States, and the publishing of Dun and Bradstreet services on a regular basis improved the "data base" and increased the demand for more, and better information.

At this stage, writers began to compare ratios of failed versus non-failed firms in an attempt to set out a predictive model, concentrating on the change in ratios in the year preceding failure. Another impetus for the research was to clarify ratios of importance to determine which should be highlighted in any evaluation of financial data. Ramser and Foster [1931] studied the information of 173 companies in Illinois, and examined eleven types of ratios. At first, their conclusions appear trivial as they stated that

... firms which turned out to be less successful, and those which failed, tended to have ratios which were lower than the more successful firms [Horrigan, 1968, p. 289].

Nevertheless, they determined that sales to net worth, and sales to total assets were opposite to their general conclusion.

Winaker and Smith [1930] also looked at ratios as indicators of "financial difficulties". Their work consisted of taking ten-year trends for companies failing in 1923 to 1931. Their findings, based on the evaluation of 21 ratios, was that the best indicator of difficulty was the ratio of net working capital to total assets. They also saw a steady decline in this



ratio beginning ten years before an individual firms' collapse. Moreover, the rate of decline increased as failure approached, and was the basis of their conclusions. However as pointed out by many writers there were two main flaws with their review. First, they did not have a "control group", of non-failed firms with which to compare results. Second, they did not reconcile their analysis with the general economic events of that time (being the beginning of the "great depression"). Thus, although their conclusions are not theoretically valid, they succeeded in confirming the importance of working capital. Interestingly, in more sophisticated multivariate techniques, the working capital to total asset ratio remains the most important, and "best" predictive statistic in the analysis of business failure.<sup>11</sup> More specifically, the ratio of current assets to total assets is determined, using Wilks' Lambda, [Wilks, 1967] to be the "best" predictor of failure.

A third study [Fitzpatrick, 1931], improved on these earlier works. It was the first of its type to "match" failed and non-failed firms, and was done over the nine-year period of 1920-1929. This study analysed nineteen pairs of firms (using asset size and sales, for the matching) and studied the three to five year trends of the groups prior to failure. The study found that the twenty ratios selected<sup>12</sup>, accurately predicted failure.

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<sup>11</sup> See the studies in the following section on multivariate models, and MDA for discussion of Wilks' Lambda.

<sup>12</sup> See Appendix I.

Moreover, the "best" indicators were profits to net worth, net worth to debt, and net worth to fixed assets. The study concluded [Fitzpatrick, 1931, p. 731] that ratios deteriorated as failure approached. Therefore the ratios were valid indicators of disaster.

Merwin's [1941] study brought together ideas of previous works, as he looked at matched firms for the six year period preceeding failure. He compared failed firms with what he called "estimated normal" ratios. In essence these latter ratios were based on the projected financial position of the failed companies had they maintained the same average ratios as the surviving firms. Using 900 firms, with assets less than \$250,000, his results indicated that three ratios in particular could be used as predictors of failure from five years before "discontinuance" of operations. These ratios were: current ratio, net worth to total assets, and net working capital to total assets. As Horrigan [1968, p. 89] indicates, "Merwin's study was the first really sophisticated analysis of ratio predictive power, and the findings of the study still appear to be credible".

One other study of note was done in 1959 by James Walker. He proposed that the funds statement ratio could be used instead of working capital for predictive purposes. He theorized that net cash flows could be used to determine "technical solvency", which he defined as the ability to meet current liabilities ("obligations maturing within twelve months"). Of this work,

Comerford wrote:

Walter demonstrated an alternative to working capital position as an indicator of technical solvency.... This new type of ratio, the funds statement ratio, brought net cash flows and related considerations into prominence. [Comerford, 1976, p. 62]

According to Horrigan [1968, p. 292] Walter was the first to specifically incorporate the funds flow statement into ratio analysis.<sup>13</sup>

William Beaver [1967] wrote what is considered as the benchmark study on bankruptcies using univariate analysis.<sup>14</sup> He collected data on failed<sup>15</sup> firms from 1954 to 1964 and matched them with non-failed corporations; based on industry, as well as asset size, over a five year span. He took thirty ratios for each year and concluded that six could be useful in the development of a "profile analysis". This analysis was not used to strictly determine failure potential, but to outline the "general relationships" existing between the matched firms.

Beaver used mean values in his analysis, such that for each matched pair of firms, the ratio of the non-failed firms is deducted from the ratio of the matched firm to "mitigate the potentially disruptive effects of industry and asset size"

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<sup>13</sup> This is interesting in light of the FASB's assertion that net income is a good indicator of future cash flows. See also Spector [1981].

<sup>14</sup> Cf. this work, Horrigan [1968, p. 291] wrote "this study will undoubtedly become a landmark for future research in ratio analysis".

<sup>15</sup> Failed meaning bankruptcy, or default on either bond payments or preferred stock dividends.

[Beaver, 1967, p. 75]. Thus, instead of using the absolute ratios, Beaver looked at the difference of the ratios to examine for trends and predictors. The mean of the ratios was computed for the failed and non-failed firms in each of the years before failure. The comparison of these mean values Beaver called the "profile analysis". This analysis "... facilitates a comparison with previous studies, since their analysis was based entirely upon the mean values" [Beaver, 1967, p. 79]. In other words, the profile allowed testing for more than one specific attribute, and its overall effect on the firm, its "profile analysis" effect, whereas previous studies looked only at changes in a particular mean value.<sup>16</sup>

Beaver [1968, p. 80] put forth four concepts with respect to the "liquid-asset-flow", or cash-flow, of the enterprise. He hypothesized that the probability of failure decreased as:

- the "reservoir" of current assets increased
- cash-flow increased,
- outstanding debt decreased, and
- "fund expenditures" from operations decreased.

Using the above, he developed a set of six ratios to be tested, and determined that the ability to predict failure was strongest in the cash-flow to total-debt ratio. He indicated that the propositions held, and the ratios were as predicted for five years before failure. Moreover, their change in direction correctly indicated imminent failure. He concluded by saying

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<sup>16</sup> Beaver asserted that this work could "... convey useful information in determining solvency for at least five years before failure." [Beaver, 1967, p. 98].

that:

Even with the use of ratios, investors will not be able to completely eliminate the possibility of investing in a firm that will fail. This is a rather unfortunate fact of life, since the costs in that event are high [Beaver, 1968, p. 91].

With this high cost in mind, and the indications that more than one ratio is required, it would appear, and Beaver agrees, that univariate analysis is limited. A change in a particular ratio may predict an occurrence with a degree of probability, but it does not take account of the interrelationships of the financial statement model. In other words, the changes in combined ratios would probably be of better predictive quality in looking at the possibility of failure.

## Ratio Analysis, Multivariate Models

Under the dominant approach of 'pragmatical empiricism,' the user of ratios is required to rely upon the authority of an author's experience. As a result, the subject of ratio analysis is replete with untested assertions about which ratios should be used and what their proper levels should be; and similarly, the expected relationships of the various ratios with a quantification of some desired, or undesired end have generally not been formulated.

[Horrigan, 1968, p. 294]

As previously mentioned this analytical approach looks at all selected ratios in an attempt to build a more relevant predictive model. "The major feature of the multivariate approach to failure prediction is the simultaneous consideration of several indicators in the prediction process" [Lev, 1974, p. 145].

Meir Tamari [1966] was one of the first to look at ratio analysis in this way. His idea was to weight the ratios such that those deemed important would be given more emphasis. The weightings Tamari gave to six ratios of importance were calculated to yield an index of the individual firm's risk. Thus, "...each [ratio] is given a weight according to its importance in the eyes of financial analysts, economists, and credit men..." [Tamari, 1966, p. 19].

By applying these weightings, he developed a critical threshold such that firms under a "low level" (set at 30 points) had a good chance of failing, whereas firms over a "high level" (60 points) had a low probability of failure. It is interesting

to note that Tamari's objective was not to develop failure prediction model, but to see if ratios could be used in financial analysis. In other words, he did not intend his study to be deemed as a "scientific" indication of financial difficulties. He concluded that previous studies, like

do not prove that companies with weak ratios on the Altman index necessarily go bankrupt or fail to do so eventually. It is therefore necessary to study the probability of firms with different indices going bankrupt or improving their financial position [Tamari, 1966, p. 20].

This, then, is the basis of the discriminate analysis used in the development of failure prediction. If ratios of financial ratios could be determined, and weights attached, the result would be an indication of the firms relative "risk".

On the matter of these "weights", Tamari assigned his weights, in that "no mathematical technique was employed" [Botheras, 1979, p. 18]. However, he recognized that a more advanced technique of assigning weights should be used for a true predictive model. MDA succeeds in that it mathematically determines weightings for those ratios that are most relevant. The firm's riskiness is then calculated. The idea of a critical level, can be determined (see comments in the following section).

Edward Altman indicated in his Ph.D. dissertation that the technique of MDA was first used in the 1930s by Fischer, for the grouping of plant types, and by Altman in identifying variations of measurements of Egyptian stocks [Altman, p. 27]. In more recent studies Paul Hoel [1962] developed

method in his analysis of races of insects, and the first study in the business field was completed in 1964 by Neuwirth and Shegda.<sup>17</sup> This latter study examined credit decision-making, and its objective was to group loans as "good" or "bad".

Financial ratios used for failure prediction were examined by Edmister in 1972. In using different analysis techniques for predictive power, he concluded that MDA could settle the dual problem of developing appropriate ratios and assigning appropriate weights. He also found that as a result of MDA a simple function could be obtained in building the critical levels [Botheras, 1979, p. 20].

Marc Blum, developed a 'Failing Company Model', in 1974, to predict failure, as soon as possible before bankruptcy.<sup>18</sup> Using 115 bankrupt firms from 1954 to 1958 with assets in excess of \$1 million, matched with 115 non-failed companies, he developed a model for the five years preceding failure. He then applied discriminate analysis to see if it could distinguish between failing and non-failing firms. His accuracy was in the range of 94 percent one year before failure, 80 percent two years before, and 70 percent for three to five years before failure [Blum, 1974, p. 2]. Beyond five years, discrimination was not found to be statistically significant, and:

<sup>17</sup> Botheras [1979] notes that David Durand pioneered MDA in evaluation of credit in installment financing.

<sup>18</sup> His objective in development of a model was to analyse the legalistic "Failing Company Doctrine", which allows failing companies, in the U.S., to merge with non-failed ones, thereby avoiding anti-combines laws.



...in comparison with other studies of business failure, the Failing Company Model was demonstrated to be more reliable than a reported multivariate model. However, its accuracy was only approximately that of the leading univariate study published to date [Beaver's]. This conclusion simply reinforces the need for further research [Blum, 1974, p. 14].

In 1972, Edward Altman published his study of the predictors of failure, using financial ratios. In this study, Altman selected 33 pairs of manufacturing firms using industry, and asset size ranging from \$0.7 to \$25.9 million [Altman, 1972, p. 61] for the matching criteria. Twenty-two indicators, (financial ratios and non-financial variables), were computed for each firm in the sample. Of these indicators, five were chosen to have the best predictive power,<sup>19</sup> using "...a classification test somewhat similar to Beaver's" [Lev, 1979, p. 146]. Altman's model, using the five variables, correctly predicted 95 percent of the firms in the first year, and 72 percent two years preceeding failure. His results indicated that accuracy diminishes beyond the two years, and that the model becomes unreliable for predictive ability [Altman, 1972, p. 73].

In his general remarks on the model, Altman [1972, p. 82] stated that due to the small sample size, and the long period reviewed, one model for all manufacturing firms was inappropriate. Moreover, he indicated that particular industry groupings, or groups of related industries, would be more representative (and presumably "better") of the type of firm and

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<sup>19</sup> Or as Lev [1974, p. 146] states "...the combination [that] did the best overall job in discriminating the bankruptcy status of the sampled firms."

its environment.

Of further research, Altman suggested that Meyer and Pifer [1970] is of "primary relevance to the subject". They developed a linear regression model, similar to MDA, to predict commercial bank failures. This study involved 39 pairs of banks; with size, location and age as the criteria used in the matching. For the regression model 32 variables were computed, some quite specific; for instance, growth of loans to total assets and coefficient of variation of total loans. Similar to Altman, they concluded that a reliable model could only be developed for a two year time period before failure. Approximately 80 percent of the sample was correctly grouped, but the model was not accurate for two years preceeding failure or earlier [Lev, 1974, p. 148].

In 1973, Balmeister and Jones duplicated Altman's study of railroad failures, using MDA with companies of a larger size. It was indicated that although the predictive ability of their model was not as "good" as Altman's, MDA could be used to point out those companies "which possess a profile which is significantly similar to firms which have failed in the past." [Balmeister and Jones, 1973, p. 15]

Robert Libby [1974] utilized the MDA technique in two separate studies. The first study looked at decision-making by loan officers and the second at the use of MDA as a predictor. His initial study concerned itself with the behavioral aspects of decision-makers. A summary of his findings indicate that a decline towards failure will continue, even if there are

substantial changes within the firm, for example management shake-up or merger. Thus:

...the author concluded that MDA models accurately predicted failure even when the decision-maker's processes intervened in the usual routine of firms [Springate, 1978, p. 27].

In the second study, Libby used a step-wise method of determining the "best" predictors with ratios used by Beaver. The resulting ratios<sup>20</sup> are similar to those finally selected by Altman. Libby summarized that the predictive capacity of a model with as few as five ratios, is only slightly less than one with the fourteen initially selected,<sup>21</sup> and that five appeared to be the optimal number.

In a newspaper article in 1977, Richard Taffler indicated that, using MDA, he had developed a "risk profile index" for English companies. He generated this index by using "off-setting" ratios such that a critical index value could be generated. The critical value was set equal to zero, and thus a firm with a negative value (similar therefore to a Z-score) would be classed as a failed firm, and non-failed enterprises would have a positive score.

Current research in bankruptcy Prediction models is including current value accounting informatin in its analysis. Interestingly research has indicated that including data adjusted for inflation does not improve the predictive ability.

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<sup>20</sup> See Appendix II.

<sup>21</sup> Predictive power declined from 90% to 85% [Springate, 1978, p. 28].

Norton and Smith [1979] indicated that including general price level [GPL] financial information does not add to the predictive quality over conventional accounting models.<sup>22</sup>

Mensah [1983] included specific price-level adjusted [SPL] data for prediction and compared SPL to historic cost [HC] accounting models (in addition to an HC/SL model) using MDA, in a one-tailed t-statistic, and logistic regression.

The results of the discriminant analysis show that there is no difference in predictive accuracy between the SPL and combined HC/SPL models on one hand, and the HC model on the other at any of the visual significance levels. When costs are accounted for, the SPL model dominates the HC model, but both dominate the HC/SPL model.  
[Mensah, 1983, p. 241]

With Canada moving towards current cost accounting, with the issuance of section 4510 of the CICA Handbook in 1982,<sup>23</sup> research is now being focused on the influence of "inflation-accounting" data predictive models. It will be interesting to see replications of studies using MDA with the adjusted data. However it will take some time, before preliminary results are made available. Hopefully the conclusions of Norton and Smith [1979] can be discounted, and the expense of adjusting accounting data for the effects of changing prices is of some benefit.

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<sup>22</sup> See also Solomon and Beck [1980] and Ketz [1978] who indicated that the Norton and Smith study did not examine the use of GPL information in the supplementary statement form.

<sup>23</sup> Currently requiring current cost information, for certain corporations, on a supplementary basis.

## Multiple Discriminate Analysis

The operation performed by MDA is essentially to identify the variables, and their relationships to each other, which best distinguish between groups but which are most similar within groups.

[Comerford, 1967, p. 71]

In a sophisticated method of multivariate analysis, Multiple Discriminate Analysis, is used to discriminate (or group) a sample into a priori groupings. It is effective when there is more than one independent attribute, or variable, and can be used to divide the sample into a maximum of  $X-1$  groups, (where  $X$  is the number of variables). As the discrimination is based on all the variables simultaneously, it is a more advanced technique than the rather simplistic methods used in univariate models.

As this analysis looks at combinations of the variables, it continues to improve the predictive ability, by adding variables when they increase the difference between the means of the groups. In other words, MDA will seek combinations that best differentiate the groupings, without regards to the individual variables' predictive ability. Added to these specific variables will be the weightings, generated mathematically to maximize the discrimination.

From the resultant linear equation developed, a Z value for each firm is calculated. A critical index is then established "at a point where there is a minimum number of

misclassifications. The accuracy of the function is determined from the number of misclassifications of the samples." [Springate, 1978, p. 35] For a geometric interpretation of MDA, see Comerford [1967, p. 224] and for a mathematical interpretation, see Tatsuoka [1970, p. 25-38].

It is important at this point to look at the assumptions involved in the utilization of MDA. First, it is assumed that each group is identifiable and discrete. For failure prediction purposes, two a priori groups are used, "failed" and "non-failed" firms. Obviously, this discrete assumption does not compromise the "reality" of the problem. The second important assumption is that all observations (firms) can be described by a set of accurate measurements. Again, this particular assumption does not jeopardise the analysis, as the model uses data from audited financial statements only.

The third basic assumption is that the critical value indices, or "Z" scores, for each group has a normal distribution. A normal distribution is required in order to determine probabilities of incorrect classification of observations from either group. No difficulties are seen with this assumption, and to assume otherwise would severely limit the use of MDA.

A discriminate function is calculated such that variables and weightings maximize the difference between the means of the groupings relative to the variance of the groups. Thus "... the quality to be maximized is a familiar term in the analysis of

variance, vis. the ratio of the variance between groups to the variance within groups" [Springate, 1978, p. 34]. The linear discriminate functions developed in MDA, one less than the number of groups, yield the Z score that is used to group the particular observation and to calculate the "critical index". Therefore, as there are two groups, the result is one linear function in the form:<sup>24</sup>

$$Z_i = a_1 X_{1i} + a_2 X_{2i} + \dots + a_n X_{ni}$$

where:  $Z_i$  = the discriminant score of  $i$ th firm

$a_j$  = the discriminant coefficient for the  $j$ th variable

$X_{ji}$  = the  $i$ th firm's value of the  $j$ th independent variable

$n$  = number of a priori groups.

This function thereby transforms the values of the individual variables into a Z score, which will be used in relation to the scores of all firms in the sample. Thus the individual firm's Z score will indicate its' group in the analysis, depending on which side of a "critical value" the score falls. The critical value is thereby set at a point such that "misclassification" is minimized. The results of the grouping is then compared to the original sample, to determine the accuracy of the predictive model. Accuracy is measured in the format of determining Type I and Type II errors and is presented in a table form:

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<sup>24</sup> See Morrison [1969, p. 156].

Actual Outcome

		failed	non-failed
Classified outcome	failed	correctly classified	Type I
	non-failed	Type II	correctly classified

From a creditor's point of view, it is felt that a Type I error is more serious than a Type II error -- and should therefore be minimized. Logically this is intuitive, as, for example, any firm classed as "failed" will not receive credit desired. However, the costs of loaning funds to a firm classed as "non-failed" which does indeed fail are very real. Thus, the firm would be investing in an unsuccessful business, which of course is what entire credit departments in large firms attempt to minimize.

The total number of misclassifications is called the Overall Error, which consists of the average of the two error types. Simultaneously an "F" value is calculated to indicate that the observations do not come from one homogenous sample. The F value is determined by looking at the means of the (two) groups, called "centroids". In other words, the value determines the significance of the difference between the means of each



ratio.<sup>25</sup>

Calculation of the common F ratio found in analysis of variance determines the overall discriminating power of the model. The F value is the ratio of the sum of squares between groups to the within-groups sum of squares [Springate, 1978, p. 40].

Thus, if the F value is found to be statistically insignificant, the hypothesis that the sample is from (in our case) two groups is accepted, as the firms in the two groups "possess characteristics which show that they come from different groups, on a multivariate basis" [Springate, 1978, p. 41].

The basic equation allows calculation of the Z score of each firm--enabling the model to discriminate groups and thus determine the misclassifications (errors). However, this equation uses all the available data, and therefore in Altman's case contains 22 ratios, or in Springate's model 19. To simplify this equation, and to reduce the number of variables, some ratios that do not enhance the predictive ability of the model must be eliminated.

As previously mentioned, many writers do this arbitrarily, [Altman, 1968, p. 47]. The disadvantages are that it requires more computation space, and time.<sup>26</sup> Therefore, the selection of variables is done on an empirical basis, with the specific

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<sup>25</sup> As Altman suggests a good indication of how successful the MDA will be is to investigate the average values in each of the groups for particular ratio measures. If they are significantly different from each other, there is a good possibility they will be helpful contributors to the overall discriminating power of a complete profile of variables. [Altman, 1971, p. 335]

<sup>26</sup> As variables are selected and tested against every other possible combination of variables--as Altman chose to do.

ratios selected rationalized to be the "best" predictors.

A mathematical alternative is Wilks' Step-Wise Method [Wilks, 1967]. This technique uses a step-wise approach in the evaluation of specific variables when added to the model, and stops when the statistical output yields a discrimination with an acceptable degree of misclassifications. Therefore, the model initiates its iterative procedure by selecting the variable which best discriminates, and calculates the Wilks' Lambda. In the second iteration, the model will evaluate all the variables and select a second variable that, along with the first chosen, improves the discriminating ability.

In the step-wise method, the variable with the largest F-value is added at each iteration. [Springate, 1978, p. 35]. In addition, Wilks' method develops a measurement, called Lambda, of "unused discriminating capability [that] remains in the unused variables. The larger the Lambda, the less likely that all the discriminating capability of the variables has been utilized." [Springate, 1978, p. 36].

According to Morrison [1969, p. 157] another statistic that differentiates, called a Mahalanobis  $D^2$  (which is a transformed F-statistic) can be calculated.

After a transformation this  $D^2$  statistic becomes an F statistic, which is then used to see if the two groups are statistically different from each other. In fact this test is simply the multidimensional analog of the familiar t-test for the statistical significance of the

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<sup>27</sup> See also Comerford [1976, pp. 139-144], and Mensah [1983, 234]. The latter for an explanation of the one-tailed t-statistic, and Lachenbruch's u-method.

difference between one sample mean  $x_1$  and another sample mean  $x_2$ . The  $D^2$  statistic tests the difference between the  $n$ -dimensional mean vector  $X_1$  for group 1 and the corresponding  $n$ -dimensional mean vector  $x_2$  for group 2. However, the statistical significance per se of the  $D^2$  statistic means very little. [Morrison, 1969, p. 157]

Thus, this step-wise method looks at the variables available and decides upon entering the variable which minimizes Lambda. Moreover, the variable to be transferred is determined by its F-value.<sup>28</sup> For the two group case, the relationship between the F-value and Lambda is:

$$F = \frac{1-\lambda}{\lambda} \times \frac{N-P-1}{P}$$

where:  $N$  = total sample size

$P$  = number of variables

and for which

$n_1$  (degrees of freedom of numerator) =  $P$ .

$n_2$  (Degrees of freedom of denominator) =  $N-P-1$ .

[Cooley and Lohnes, 1971, p. 227].

Thus ratios selected for inclusion into the "basic equation" are those with the largest F-value.<sup>29</sup>

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<sup>28</sup> Wilks' Lambda can be expressed as:

$$\lambda = \frac{P_w(O_n)}{P_n(O_n)}$$

where  $\lambda$  = Wilks' Lambda

$P$  = probability density function

$P_w(O_n)$  = least upper bound of  $P$  in set

$P_n(O_n)$  = least upper bound of  $P$  in set

[Springate, 1978, p. 36].

<sup>29</sup> F-value can be stated as:

$$F = \frac{SS_b / (k-1)}{SS_w / (N-k)} = \frac{SS_b}{SS_w} \times \frac{N-k}{k-1}$$

<sup>29</sup> This, of course, is the same as selecting the variable that minimizes Wilks' Lambda. In other words, the routine uses both measurement (and a third if D<sup>2</sup> is included) to calculate the additive variable at each step. In addition, the step-wise method calculates the marginal value of each variable in the "new" function. However, the disadvantage with this method is that:

If in the event of entering a variable, or a series of variables, the F ratio of a used variable is greatly decreased, the analysis will delete that variable in the following iteration to minimize Lambda. This may cause cycling of the analysis. When this method is used, it is prudent to specify the number of steps which the analysis should perform. [Springate, 1978, p. 37]

Using Wilks' method with all nineteen variables Springate calculated a function of:

$$Z = 1.52 \text{ VAR2} - 3.30 \text{ VAR5} + 4.73 \text{ VAR8} + 1.01 \text{ VAR9} + 0.39 \text{ VAR18}$$

where: VAR 2 = working capital to total assets  
VAR 5 = income to sales  
VAR 8 = EBIT to net assets  
VAR 9 = income to current liabilities  
VAR 18 = sales to total assets.<sup>30</sup>

It should be noted that two variables, VAR 1 (current

<sup>29</sup>(cont'd)

where:

k = number of groups

n = total number of variables

SSb = sum-of-squares between groups

SSw = sum-of-squares within groups.

<sup>30</sup> See Appendix IV. For comparison to Altman's model see Appendix III, where his discriminate function is:

$$Z = .012X1 + .014X2 + .033X3 + .066X4 + .999X5.$$

asset/current liabilities) and VAR 3 (current asset/total liabilities), were deleted from this function as VAR 2 was "more highly correlated with all unused variables where there was a significant correlation, except in the case of VAR 12". As the function indicates, the non-failed group has more positive ratios than the failed group. In other words, the Z value is more positive for non-failed firms used in the sample. The exception to this is in VAR 5, negative in the overall function, indicating that it is more negative (or lower) for non-failed firms. Springate analysed the function to see if VAR 5 (Income to Sales) should be deleted from the function, and concluded that VAR 5 could be left out with only a small decrease in discriminating power, as the components of the ratio are actually part of VAR 18 and VAR 8 [Springate, 1978, p. 46]. He deemed the resultant equation, with four all-positive variables, to be easy for a layman to use, and therefore would be acceptable from a practical point of view. Using step-wise MDA the function yielded was:

$$Z = 1.03VAR2 + 3.07VAR8 + .66VAR9 + .40VAR18$$

The overall error of both models was the same, and distributions were similar. The critical index decreased from 1.070 to .862, and the distance from this index to the centroids was "almost identical", indicating a very slight decline in discriminating ability in the former variable function. Thus, of the five categories of ratios, three are represented: liquidity (VAR 2),

profitability (VAR 8 and 9) and activity (VAR 18). The deletion of another variable would mean that one of these three categories would be left out, and Springate, considered this unacceptable.

With this equation, he proceeded to evaluate the model, using his data from a sample of twenty failed and twenty non-failed firms. He concluded that the Z scores could be used in the development of "risk profiles" in addition to assessing potential failure [Springate, 1978, p. 63].

**TABLE VI**  
Discriminate Function Using Wilks' Method  
as Calculated by Springate

<u>STEP</u>	<u>ENTERING VARIABLE</u>	<u>CENTROID</u>	<u>WILKS' LAMBDA</u>	<u>TYPE I ERROR</u>	<u>TYPE II ERROR</u>	<u>OVERALL ERROR</u>
1	VAR 8	0.7029	0.4933	10.0 %	10.0 %	10.0 %
2	VAR 18	.7402	.4381	10.0	10.0	10.0
3	VAR 9	.7737	.3861	5.0	5.0	5.0
4	VAR 2	.7949	.3519	10.0	5.0	7.5
5	VAR 5	.8194	.3113	10.0	5.0	7.5
6	VAR 1	.8375	.2805	5.0	5.0	5.0
7	VAR 3	.8684	.2266	5.0	5.0	5.0
Overall		.9289	.1149	0	5.0	2.5

[Source: Springate p. 43]

## Discussion

The dichotomy bankruptcy versus no bankruptcy is at the most, a very crude approximation of the payoff space of some hypothetical decision problem.

[Ohlson, 1980, p. 111]

One of the major limitations of failure prediction analysis is the lack of a consistent definition of "failure." This problem was discussed by Altman [1971, pp. 2-4], Lev [1974, p. 133], Weston and Brigham [1977, pp. 542-544] and Foster [1978].

Many view failure as insolvency, or forced liquidation. For instance, to owners of a business, insolvency may force them out of business. However, only legal (statutory) bankruptcy statistics are compiled by government agencies, as seen in Table I. Thus, mergers, re-organizations, and closing-sales would not be included as "failures." In other words, the idea of a business failure may have to be limited strictly to voluntary and involuntary bankruptcies only. Data would not be captured on temporary insolvent companies, or for other kinds of ceased operations.

As such, this lack of data about voluntary liquidations may bias the sample. Also, the predictive model would not account for firms altering their strategies to forestall impending failure.

The ability to predict corporate failure is important from both the private and social points of view, since failure is obviously an indication of resource misallocation. An early warning signal of probable failure will enable both management and investors to take preventative measures; operating policy changes, reorganization of financial structure, and even



voluntary liquidation will usually shorten the length of time losses are incurred and thereby improve both private and social resource allocation. [Lev, 1974, p. 134]

In Canada therefore, "failed" firms are represented by companies that have declared bankruptcy (or filed proposals). It should be kept in mind that this is only one portion of "failed" companies. Not included will be firms that have sold off inventory at "fire-sale" prices and allowed leases to lapse, enterprises that sold out (or merged) with competitors, or even companies that their owners shut and left.

The second major limitation regarding failure prediction models is the lack of a theoretical base. Ohlson posed the basic question "why forecast bankruptcy?" [Ohlson, 1980, p. 111]. He concluded that "... there is no consensus varying significantly and arbitrarily across studies."

Horrigan [1968, p. 292] is much more forward in this analysis of the emergence of predictive models.

This development, which is still in a relatively embryonic stage, has been characterized by careful and well-constructed a priori analysis in contrast to the senseless proliferation of ratios which characterized the early development of ratio analysis.

Basically, there is no theory of financial failure, or the role of financial ratios as predictors of survival. Due to this absence many researchers take a heuristic approach utilizing varying numbers of ratios, and models. Because of this, it is difficult to generalize the empirical results. [Lev, 1974, p. 149]

Attempts to construct a theory of corporate failure, that is, to identify and generalize the major causes of

failure, have been meager and generally unsatisfactory because of the complexity and diversity of business operations, the lack of a well-defined economic theory of the firm under uncertainty, and a surprising reluctance by many researchers to incorporate the failure phenomenon in their models. [Lev, 1974, p. 134]

Moreover, Gordon [1971, p. 347] wrote "The academicians with first hand knowledge of the subject have left the scene of action."

Altman [1971, p. 341] suggested using aggregate data on a times series basis. He postulated a form for business failure as:

$$\Delta F.R. = f(\Delta GNP, \Delta SP, \Delta MS)$$

where:  $\Delta F.R.$  = quarterly change in Dun and Bradstreet's failure rate

$\Delta GNP$  = quarterly change in real GNP

$\Delta SP$  = quarterly change in the standard poor 500 Index of Common Policies

$\Delta MS$  = quarterly change in money supply

In the analysis of the above model, he indicated that the failure rate varied negatively with overall economic activity, stock market performance, and money supply conditions. In the context of railroad failures in the United States, Altman indicated that this model was of little information value although there were "... significant statistical values for the independent variables." [Altman, 1971, p. 342]

The need for predictive models however is still present. As indicative of the increasing number of failures. "The ratio fills that need as a simple, quick method of comparison.... Thus, the ratio is certainly a very admirable device because it is simple and it has predictive value" [Horrigan, 1968, p. 294].

There are also methodological problems with using multi-discriminate analysis for forecasting purposes. Basically, the sample is grouped, and matched a priori. Because the studies are "essentially retrospective" [Lev, 1974, p. 149] there is a bias in the sample selection.

This problem may be overcome by using non-matched, unbalanced samples. Moreover, studies of existing "live" firms should be undertaken (i.e. before knowledge of disaster is known). In addition, previous studies have restricted their samples to large corporations -- both failed and non-failed. For example, Altman included matched firms with assets of at least \$700,000. Obviously, smaller firms, and unincorporated businesses would not be included. Therefore the predictive models developed would be bias towards the larger companies.

It would be interesting to include a greater range of firm size. And also to develop prediction models specifically for the smaller enterprise. This could also have a significant impact on the current standard-setting argument in Canada. At present there is a "Big G.A.A.P. versus Little G.A.A.P."<sup>31</sup> discussion.

The development of models for smaller firms could indicate there is a need for different accounting principles for companies depending on their size. On the other hand similar models (i.e. using the same variables) may indicate that there is no requirement to change from the status quo.

Another statistical problem of forecasting is that they use failures over a period of time as opposed to one year only. For example Beavers' study used ratios between 1954 and 1964. Moreover studies like Springate's do not detail the exact periods covered for failed companies.

This problem is accentuated in Canada, due to the problems in accumulating data. Whereas in the United States companies file annually with the government, this is not necessarily so in Canada. In addition, specific data is not collected and accumulated for "failed" companies. The Department of Consumers and Corporate Affairs keeps track only of the legal bankruptcies, under the Bankruptcy and Winding Up Act. Springate [1978, p. 65] suggests that better quality data be accumulated by accounting bodies, say the Canadian Institute of Chartered

<sup>31</sup> Generally Accepted Accounting Principles -- G.A.A.P.

Accountants, to enable further research. Comparisons could then be made with the United States, and other countries. This would entail replications of studies, and generation of specific Canadian models. This latter research would also have as a by-product a test to see if indeed accounting regulations are materially different in various countries.

The readily available data would also allow for different sample configurations. As already mentioned, the matched firms approach leads to an upwards bias of failed companies. If the average failure rate of enterprises in Canada is, say 2%, then large samples with only 2% unsuccessful firms should be used.

Moreover, failed firms included in the samples should have the same date of failure. This "cross-sectional" approach would have more meaning, as they would be included with non-failed firms from the same period. This would also enable research into establishing forecasts from each year.

Finally, a bank of data would enable researchers to establish specific models for industries, and geographic areas. Due to the regional disparities in this country, this would also be an important improvement in this field.<sup>32</sup> It would be interesting to try to establish risk profiles by industry and by area with an eye to developing specific market profiles. These could be compared to similar markets in other countries, specifically the United States.

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<sup>32</sup> Regional differences, on a provincial basis, are examined by Mason and Strain [1982] using data from Consumer and Corporate Affairs, Ottawa.

The different number of ratios has already been mentioned. Beaver [1968] selected thirty, Altman [1972] twenty-two (manufacturing) and fourteen (railroads), Edmister [1972] nineteen, and Springate [1978] selected nineteen. Clearly there is no consensus to the effective mix of data. A few points have to be kept in mind however when selecting ratios.

First, multicollinearity has to be avoided (or at best minimized). This is usually done by specifically excluding variables which are essentially the same, for example "quick" assets/current liabilities and cash plus receivables/current liabilities should not be utilized simultaneously.

Second, the variables have to be "operational." If a resulting model is to be widely expected, the ratios should be straightforward and easy to calculate. For instance, a model using different measurement scales, or market values of shares may not be understood. Moreover, if not calculated by the particular enterprise, could prove costly.<sup>33</sup> On the other hand, however, the excessive limitation of the variables employed in the research entails a loss of descriptive information on the firms." [Finardi, 1982, p. 6] "Besides, data are actually difficult to collect." [Finardi, 1982, p. 8]

One approach as to which variables to use could be determined by professional analysts, stock investors, and credit granting institutions. Behavioral studies should be assimilated into failure prediction models to aid in the potential

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<sup>33</sup> See also Morrison [1969, p. 162].

operationalizing of a model. A presentation of the various models determined could be presented to these groups of people for their reactions. Moreover studies of choices using MDA, in addition to their decision models could be tested. Also, an interesting study would use MDA to "check" previous decisions!

It however should be stressed that the critical indices developed are not substitutes for other decision models. At best they can be used to complement any developed models. And, if no sophisticated model is used at all, could help to formulate a rationally based decision model for credit-granting decisions.

## Summary

We have looked at the development of ratio analysis, from its humble beginnings to the advent of computerized techniques, focusing on Multiple Discriminate Analysis. It is clear that development of the MDA is only now beginning in earnest. One impediment to MDA's progress has been an educational gap existing between theoreticians and practitioners. Now that the computing sciences, in all its manifestations, have become more widely accepted and understood, we can use computers for more "useful" tasks. By useful, we mean that in addition to regular bookkeeping activities, (recording, posting, footing and so forth) computers can (and now should) be used for budgeting, predictive analysis, and other forecasting models.

We have seen two predictive models developed in an attempt to establish a model that will accurately predict business failures. The models developed by Altman [1972] and Springate [1978] are similar in nature, and in part both agree on the ratios of importance. For maximum predictive "power" they have concluded that these ratios together can best differentiate groups of failed versus non-failed enterprises.



Table VII

Ratios in Discriminate Functions

<u>Altman</u>	<u>Springate</u>
Working Capital/Total Assets	Working Capital/Total Assets
E.B.I.T./Total Assets	E.B.I.T./Total Assets
Sales/Total Assets	Sales/Total Assets
Retained Earnings/Total Assets	
Equity Value/Debt Book Value	E.B.T./Net Worth

The next step is therefore to test the models on Canadian data, with the goal of development of a truly distinctive model for specific industries within the Canadian economy. If this could be achieved, credit granting agencies could use the model to help them in their own credit granting decisions. With the resource based economy of British Columbia, it is obvious that there is a need for models in forestry, mining, tourism, and the fishing industry.

To develop these models therefore, data will be required for both successful as well as unsuccessful enterprises in each industry. This will be the difficult part of any such research due to the present lack of Canadian information. Moreover, the research will have to be restricted to public companies--in order to obtain the necessary data on the failed enterprises.

## Appendix I

### Ratios Used in Univariate Studies

#### Smith and Winston: (1930)

##### Group 1

1. Working Capital to Total Assets
2. Surplus and Reserve to Total Assets
3. Net Worth to Fixed Assets
4. Fixed Assets to Total Assets

##### Group 2

1. Current Ratio
2. Net Worth to Total Assets
3. Sales to Total Assets
4. Cash to Total Assets

#### Fitzpatrick: (1930)

1. Net Profit to Net Worth
2. Net Worth to Debt
3. Net Worth to Fixed Assets

#### Merwin: (1940)

1. Current Ratio
2. Net Worth to Total Debt
3. Net Working Capital to Total Debt

#### Beaver: (1967)

1. Cash-flow to Total Debt
2. Net Income to Total Assets
3. Total Debt to Total Assets
4. Working Capital to Total Assets
5. Current Ratio
6. Non-credit Interval

## Appendix II

### Ratios Used in Multivariate Models

<u>Tamari (1966)</u>	
1. Equity Capital/Total Liabilities	<u>weights</u> 25
2. Profit trend	25
3. Current Ratio	20
4. Value of Production/Inventory	10
5. Sales/Receivables	10
6. Values of Production/Working Capital	10
<u>Altman (1967)</u>	
1. Working Capital/Total Assets	<u>coefficient</u> 1.2
2. Retained Earnings/Total Assets	1.4
3. E.B.I.T./Total Assets	3.3
4. Equity Value/Debt Book Value	.6
5. Sales/Total Assets	.999
<u>Taffler (1967)</u>	
1. Profits Before Taxes/Current Liabilities	
2. Current Assets/Total Liabilities	
3. Current Liabilities/Total Assets	
4. No-credit Interval	
<u>Libby (1974)</u>	
1. Net Income/Total Assets	
2. Current Assets/Sales	
3. Current Assets/Current Liabilities	
4. Current Assets/Total Assets	
5. Cash/Total Assets	
<u>Springate (1978)</u>	
1. Working Capital/Total Assets	<u>coefficient</u> 1.03
2. E.B.I.T./Net Worth	3.07
3. Profit Before Taxes/Current Liabilities	.66
4. Sales/Total Assets	.40

## Appendix III

### Ratios Used in Altman's (1968) Study

1. Current Ratio
2. Cash plus Marketable Securities/Current Liabilities
3. X1 Working Capital/Total Assets
4. Cross Profit/Sales
5. Profit Before Taxes/Sales
6. Profit After Taxes/Sales
7. Profit After Taxes Before Interest/Total Assets
8. X2 Profit Before Interest and Taxes/Total Assets
9. Number of Years Negative Profits in Last Three Years
10. Short Term Debt/Total Assets
11. Long Term Debt/Total Assets
12. Total Debt/Total Assets
13. X3 Retained Earnings/Total Assets
14. X4 Market Value Equity/Par Value Debt
15. Net Worth/Total Debt
16. Sales/Cash Plus Marketable Securities
17. Sales/Inventory
18. Cost of Sales/Inventory
19. Sales/Net Fixed Assets
20. Sales/Current Liabilities
21. X5 Sales/Total Assets
22. Working Capital/Sales

Note: these attributes were chosen on the basis of  
(i) popularity in the literature  
(ii) potential relevancy to the study, and a few  
"new" ratios developed for the study [Altman,  
1968, p. 594].

**Appendix IV**  
**Ratios Used by Springate**

**Liquidity**

Ratio 1	VAR 1	Current Assets/Current Liabilities
Ratio 2		Cash and Marketable Securities/ Liabilities
Ratio 3	VAR 2	Current Assets-Current Liabilities/ Total Assets
Ratio 4	VAR 3	Current Assets/Total Liabilities
Ratio 5		Current Liabilities/Total Assets

**Profitability**

Ratio 6	VAR 4	Gross Profit/Sales
Ratio 7	VAR 5	Net Profit Before Taxes/Sales
Ratio 8	VAR 6	Net Profit After Taxes/Sales
Ratio 9	VAR 7	Net Profit After Taxes Before Interest/ Total Assets
Ratio 10	VAR 8	Net Profit Before Interest and Taxes/ Net Worth
Ratio 11		Net Profit After Taxes/Net Worth
Ratio 12		Number of Years Negative Profit in Last Three Years
Ratio 13	VAR 9	Net Profit Before Taxes/Current Liabilities

**Leverage**

Ratio 14		Short Term Debt/Total Assets
Ratio 15		Long Term Debt/Total Assets
Ratio 16	VAR 10	Total Debt/Total Assets
Ratio 17	VAR 11	Retained Earnings/Total Assets
Ratio 18		Net Profit Before Interest and Taxes/ Interest Charges

**Solvency**

Ratio 19	VAR 12	Net Worth/Total Debt
Ratio 20		Market Value of Equity/Par Value of Debt
Ratio 21	VAR 13	(Current Assets-Current Liabilities)/ Operating Costs
Ratio 22		Sales/Cash and Market Securities
Ratio 23	VAR 14	Sales/Inventory
Ratio 24	VAR 15	Cost of Goods Sold/Inventory
Ratio 25	VAR 16	Sales/Net Fixed Assets
Ratio 26	VAR 17	Sales/Current Liabilities
Ratio 27	VAR 18	Sales/Total Assets
Ratio 28	VAR 19	Current Assets-Current Liabilities/Sales

## References

- Altman, Edward. "Predictions of Railroad Bankruptcy." Journal of Economic and Management Science (Spring 1972).
- "Railroad Bankruptcy Propensity." Journal of Finance (May, 1971).
- "Financial Ratios, Discriminant Analysis, and the Prediction of Corporate Bankruptcy." Journal of Finance (September 1968).
- "The Prediction of Corporate Bankruptcy: A Discriminate Analysis." Ph.D dissertation, University of California, Los Angeles, 1967.
- Balmeister, Philip and Jones, Richard. "Railroads, Failures and M.D.A. Made Mergers." Mergers and Acquisitions Vol. 8, No. 2, (Summer 1973).
- Beaver, William. "Alternate Accounting Measures as Predictors of Failure." Accounting Review (January 1968).
- "Informational Content of Annual Earnings..." Accounting Research (Supplement, 1968).
- "Financial Ratios as Predictors of Failure." Supplement to the Journal of Accounting Research (January 1967).
- Blum, Marc. "Failing Company Discriminant Analysis." Journal of Accounting Research (Spring 1974).
- Botheras, Donald. "Use of a Business Failure Prediction Model for Evaluating Potential and Existing Credit Risk." Unpublished M.B.A. research project, Simon Fraser University, 1979.
- Canadian Institute of Chartered Accountants, "Reporting the Effects of Changing Prices," Section 4510 CICA Handbook, Toronto, Ontario, December 1982.
- Cooley, William and Lohnes, Paul. Multivariate Data Analysis, John Wiley and Sons, Inc., New York, New York, 1971.
- Chafield, Michael. A History of Accounting Thought. Robert G. Krieger Publishing Company, Huntington, New York, 1977.
- Comerford, Robert. "Bankruptcy as Business Strategy: A Multivariate Analysis of the Financial Characteristics of Firms Which Have Succeeded in Chapter XI Compared to Those

- which have Failed." Ph.D. dissertation, University of Massachusetts, 1976.
- Dun & Bradstreet. The Failure Record Through 1976. Dun & Bradstreet Inc., New York, New York, 1976.
- Edmister, Robert. "An Empirical Test of Financial Ratio Analysis for Small Business Failure Prediction." Journal of Financial and Qualitative Analysis (March 1972).
- Ferrara, William and Hayya, Jack. "Towards Probabilistic Profit Budgets." Management Accounting (October, 1970).
- Financial Accounting Standards Board [FASB], "Statement of Financial Accounting, Concepts No. 1, Objectives of financial Reporting by Business Enterprises." Financial Accounting Standards Board, Stanford Connecticut, 1978.
- Finardi, Alessandra d'Amico, "Forecasting Insolvency: A Preliminary Approach to the Problem." Unpublished research paper, Istituto Unwersitario Di Bergamo, Italy, 1982.
- Fitzpatrick, Paul. "Symptoms of Industrial Failures." Catholic University Press 1931.
- Poster, George. Financial Statement Analysis, Prentice Hall, Englewood Cliffs, New Jersey, 1978.
- Foulke, Roy. Practical Financial Statement Analysis McGraw-Hill Book Company, 5th Edition, New York, New York, 1961.
- Gordon, Myron J. "Towards A Theory of Financial Distress," Journal of Finance, (May 1971).
- Heath, Lloyd and Rosenfield, Paul. "Solvency: The Forgotten Half of Financial Reporting," Journal of Accountancy. (January, 1979).
- Hoel, Paul G. Introduction to Mathematical Statistics. New York: John Wiley and Sons, Inc., 1962.
- Horrigan, James. "A Short History of Financial Ratio Analysis." Accounting Review (April 1968).
- Jaedicke, Robert K. and Robichek, Alexander A. "Cost-Volume-Profit Analysis Under Conditions of Uncertainty." Accounting Review (October, 1964).
- Ketz, J. Edward. "The Effect of General Price-Level Adjustments on the Predictive Ability of financial Ratios." Journal of Accounting Research, Vol. 16 (Supplement, 1978).
- Kline, C.A., and Hessler, H.L. "The DuPont Chart System for

- Appraising Operating Performance." Readings in Cost Accounting, ed: William E. Thomas Jr. South-western Publishing Company, Cincinnati, Ohio, 1955.
- Lev, Baruch. Financial Statement Analysis: A New Approach. Prentice-Hall Inc., Englewood Cliffs, New Jersey, 1974.
- Libby, Robert. "Prediction Achievement and the Use of Simulated Decision Makers In Information Evaluation." Ph.D. dissertation, University of Illinois, 1974.
- Mason, Greg and Strain, Frank. "Bankruptcies in Canada -- An Analysis." Western Economic Review (July 1982).
- Mensah, Yaw M. "The Differential Bankruptcy Predictive Ability of Specific Price Level Adjustments: Some Empirical Evidence." Accounting Review (April, 1983).
- Merwin, Charles. Financial Characteristics of American Manufacturing Corporations. U.S. Government Printing Office, 1941.
- Meyer, P.A., and Pifer, H.W. "Prediction of Bank Failures." Journal Of Finance (September 1970).
- Morrison, Donald. "On the Interpretation of Discriminant Analysis." Journal of Marketing Research (May, 1969).
- Nelson, Philip. "Corporations in Crisis, Behavioral Observations for Bankruptcy Policy." Praeger Publishers, New York, New York, 1981.
- Neuwirth, Sidney and Shegda, Michael. "Discriminant Analysis." Management Science (March-April 1964).
- Norton, Curtis L. and Smith, Ralph E. "A Comparison of General Price Level and Historical Cost Financial Statements in The Prediction of Bankruptcy." Accounting Review (January, 1979).
- Ohlson, James A. "Financial Ratios and the Probabilistic Prediction of Bankruptcy." Journal of Accounting Research (Spring, 1980).
- Parker, George and Segura, Edilberto. "How to Get a Better Forecast" Harvard Business Review (March-April, 1971).
- Ramser and Foster. "A Demonstration of Ratio Analysis," Bulletin No. 40 University of Illinois Press, Bureau of Business Research, Urbana, Ill., 1931.
- Raun, "The Limitation of Profit Graphs, Breakeven Analysis, and Budgets." Accounting Review (October, 1964).



- Smith, Raymond. "Test Analysis of Unsuccessful Industrial Companies." Bulletin 31 University of Illinois Press, Bureau of Business Research, Urbana, Ill., 1930.
- Smith, R., and Winakor, A.H. "Changes in financial Structure of Unsuccessful Industrial Companies." Bulletin 51 University of Illinois Press, Bureau of Business Research, Urbana, Ill., 1935.
- Solomon, Ira and Beck, Paul J. "A Comparison of General Price Level and Historical Cost Financial Statements in the Prediction of Bankruptcy: A Comment." Accounting Review, (July, 1980).
- Spector, Steven. "An Examination of the FASB Assertion of Predictability of Cash Flows." Unpublished M.A. thesis, Simon Fraser University, 1981.
- Springate, Gordon. "Predicting the Possibility of Failure in a Canadian Firm: A Discriminant Analysis." Unpublished M.B.A. research project, Simon Fraser University, 1978.
- Taffler, Richard. "How to Tell Who's Going Bust." Sunday Times (February 27, 1977).
- Tamari, Meir. "Financial Ratios as a Means of Forecasting Bankruptcy," International Management Review, 1966.
- Tatsuoka, Maurice. "Discriminant Analysis: The Study of Group Differences." Selected Topics in Advanced Statistics: An Elementary Approach, Number 6. Institute for Personality and Ability Testing, Champaign, Illinois, 1970.
- Wall, Alexander. "Study of Credit Barometrics." Federal Reserve Bulletin (March 1919).
- Warkentin, Edward M. "Canadian Analysis of Bankruptcy Studies Using a Canadian and an American Discriminant Analysis." Unpublished research report, Simon Fraser University, 1978.
- Walker, James. "A Discriminant Function for Earnings Price Ratios of Large Industrial Corporations." Review of Economics and Statistics 41 (February 1959).
- Weston, J. Fred and Brigham, Eugene. Essentials of Managerial Finance, 4th Edition, The Dryden Press, Hinsdale Illinois, 1977.
- Wilks, S.S. Collected Papers Contributions to Mathematical Statistics, edited by T.W. Anderson, John Wiley and Sons, Inc., New York, New York, 1967.