TOWARDS AN EFFICIENT SHEAR REINFORCEMENT FOR RECTANGULAR CONCRETE BEAMS

By

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DECLARATION

This thesis is my original work and has not been presented for a degree in any other university.

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This thesis has been submitted for examination with my approval as University Supervisor.

7/3/88

Prof. W.M. ONSONGO
(Supervisor)

To

My Wife SARASWATHY

and

Children JAI & KAVI

ABSTRACT

In order to provide shear resistance to reinforced concrete beams subjected to flexure and shear, a new type of shear reinforcement to be known as 'WAVE REINFORCEMENT' is examined experimentally and analytically. The experimental study is based on five beams incorporating wave reinforcement and/or vertical stirrups. It is aimed at resolving the acceptability and efficiency of wave reinforcement. The analytical study is aimed at verifying the adoptability of truss analogy for the design of wave reinforcement. Within the limitations of test programme, it is now confirmed that the wave reinforcement is acceptable as an alternate type of shear reinforcement and is also more efficient, functionally and economically than the vertical stirrups which are presently popular. It is also verified that the truss analogy can be safely used for the design of wave reinforcement. Again the compression field theory applied to a beam with wave reinforcement gives a good agreement between the predictions and test data. Detailing methods for the wave reinforcement and recommendations for computing the shear strength provided by them are proposed.

CONTENTS

LIST OF PLATES

NOTATION

The symbols listed below are those occurring frequently in this report. Other symbols used in one or two places only are defined where they occur and hence not included here.

A	cross-sectional area of member (=bh)
A_s	area of bottom longitudinal steel (tensile)
A'_s	area of top longitudinal steel (compressive)
A_{sv}	area of two legs of vertical stirrups
A_{sw}	cross-sectional area of wave reinforcement
A_{we}	equivalent area of wave reinforcement in the typical section
a_v	shear span from beam reaction to the first concentrated load point
b	width of section
d	distance from extreme compression fibre to centroid of (bottom) longitudinal tension reinforcement or effective depth of tension reinforcement
d'	depth from extreme compression fibre to centroid of (top) longitudinal compression reinforcement
E_c	static secant modulus of elasticity of concrete

E_s	modulus of elasticity of longitudinal tensile steel
E'_s	modulus of elasticity of longitudinal compressive steel
F	ultimate load
f'_c	peak compressive stress for standard concrete cylinder test
f_{cp}	principal compressive stress in concrete
f_ℓ	stress in bottom longitudinal steel ($f_{\ell y}$ = yield stress)
f'_ℓ	stress in top longitudinal steel ($f'_{\ell y}$ = yield stress)
f_{yv}	characteristic strength of vertical stirrups
f_{yw}	characteristic strength of wave reinforcement
h	overall depth of beam
jd	lever arm for internal couple resisting bending moment
M	bending moment due to ultimate load for combined flexure and shear
M_p	pure moment value
q	shear flow
s_v	spacing of vertical stirrups along the member
s_w	spacing of vertical components of wave reinforcement along the member

T	tension force in bottom longitudinal steel
V	shear force at section
v	shear stress
v_c	nominal shear stress in concrete
v_s	nominal shear stress resisted by vertical stirrups
v_u	ultimate shear resistance of reinforced concrete beam
y	neutral axis depth ($y = y_p$ for pure flexure and $y = y_n$ for combined flexure and shear)
α	angle between the inclined leg of the wave reinforcement and longitudinal axis of beam
ε_{co}	concrete strain corresponding to peak concrete stress
ε_{cp}	principal compressive strain in concrete
ε_{ct}	extreme top fibre compression strain
ε_{ℓ}	strain in bottom longitudinal steel ($\varepsilon_{\ell y}$ = yield strain)
ε'_{ℓ}	strain in top longitudinal steel ($\varepsilon'_{\ell y}$ = yield strain)
ε_t	principal tensile strain in concrete
ε_w	strain in wave reinforcement (ε_{wy} = yield strain
θ	principal angle of compression measured with respect to the beam longitudinal axis
Ω_c	concrete strain ratio defined as $\varepsilon_c/\varepsilon_{co}$ Similarly $\Omega_{cp}=\varepsilon_{cp}/\varepsilon_{co}$ and $\Omega_{ct}=\varepsilon_{ct}/\varepsilon_{co}$

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TOWARDS AN EFFICIENT SHEAR REINFORCEMENT

FOR

RECTANGULAR CONCRETE BEAMS

CHAPTER 1 INTRODUCTION

The aim of shear design of reinforced concrete beams is to produce economically, beams whose ultimate strengths are governed by flexure rather than by shear and thus to forestall the possibility of a shear failure. Shear failures are non-ductile and the object of good design is to suppress such a failure. The lack of ductility produces a brittle or sudden failure and to provide an adequate safety against such a failure shear reinforcement (also known as web or transverse reinforcement) is used.

Various types of shear reinforcement have been successfully tested in the past (refer Sec. 2.5). However, despite tremendous research efforts on this subject a rational answer to when, where, what kind and how much shear reinforcement is required to satisfy its functions, is still elusive. But vertical stirrups are at present most popular.

In the search for an efficient type of shear reinforcement, a new type to be known as 'WAVE REINFORCEMENT' is proposed for the present study.

The wave reinforcement is experimentally and analytically studied, bearing in mind the restrictive factors such as time and resources available, for its adoptability to structural concrete beams.

By comparative experimental study on five beams using wave reinforcement and/or vertical stirrups, the thesis aims at resolving the acceptability of the wave reinforcement from the point-of-view of ultimate strength and deformations such as extent of cracking and deflection under service conditions. The study is also aimed at establishing the efficiency of wave reinforcement over vertical stirrups, both functionally and economically.

Many researchers have developed empirical equations for the design of shear strength of reinforced concrete members subjected to combined bending and shear. But the Codes [3] and [4] have accepted empirical approaches to compute the shear strengths provided by concrete and by the shear reinforcement. The latter utilize the 45 deg truss analogy. Although the above design approach is not based on a completely rational theory, the design equations have resulted in structures being designed which have performed satisfactorily during their intended life. This thesis describes the theoretical framework for computing the shear strength provided by wave reinforcement based on truss analogy and accordingly the design recommendations have been proposed. The use of proposed design recommendations is illustrated by means of design examples.

The only relatively rational theoretical approach for design for shear available is the compression field theory. The compression field theory was originally developed by Collins [5] and was further studied by means of additional equations by Onsongo and Shitote [19 and 24] for a structural concrete beam with vertical stirrups subjected to flexure and shear. The above equations are suitably modified for the wave reinforcement and a computer program is prepared to predict the response characteristics of the beam. The study is also aimed at verifying the response characteristics predicted by the compression field theory and test data.

According to the knowledge of the author there is no similar work in this specific field at this date.

CHAPTER 2 LITERATURE REVIEW

Regarding the role of web reinforcement, Kani [12] made the following comments:

> "the main question with respect to the so-called shear strength of a reinforced concrete beam which confronts the designer is: where and what kind of shear reinforcement is to be used to prevent premature diagonal failure?"

In reply Swamy and others [12] felt that, "the main important question which need to be answered is 'when and how much' shear reinforcement is required to prevent diagonal failure and produce a ductile failure". It is relevant therefore, that in any study concerning shear reinforcement an answer should be found to when, where, what kind and how much shear reinforcement is required to satisfy its functions. In spite of tremendous research efforts a rational answer to the above is still elusive.

In this chapter the basic modes of shear failure, functions of shear reinforcement, current methods of analysis of shear strength of beams and the types of shear reinforcement presently in use are reviewed. Though this review is not directly related to the present study of wave reinforcement it roughly establishes the current state-of-the-art.

2.1 MODES OF SHEAR FAILURE

The modes of shear failure in beams, other than deep beams, without web reinforcement (Fig. 2.1) as generally understood [1, 11, 13, 21 and 25] can be classified as follows:

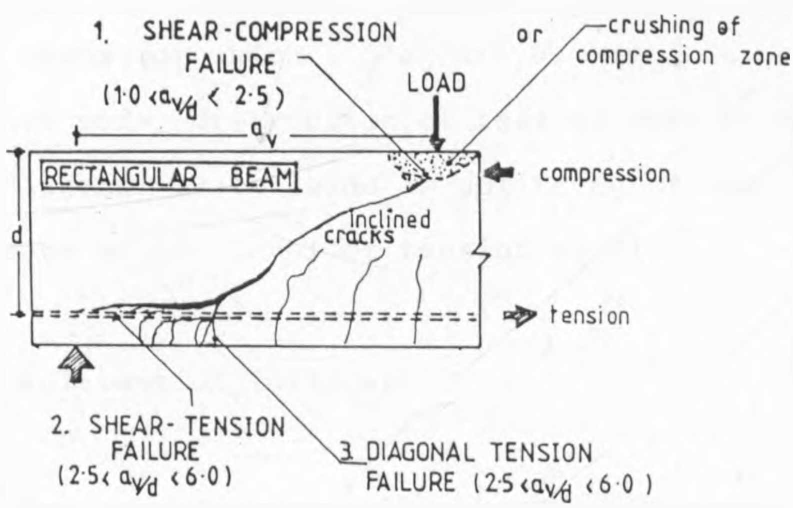


Fig. 2.1 POSSIBLE MODES OF SHEAR FAILURE IN BEAMS WITHOUT WEB REINFORCEMENT

(a) Shear-compression failures:

These are typically observed for short beams for which $1.0<a_v/d<2.5$, where a_v is the shear span and d is the effective depth of tension reinforcement. In this failure mode, destruction of compression zone above the inclined crack takes place and ultimately the compression zone crushes.

(b) Shear-tension failures:

These are typically observed for normal and long beams for which $2.5<a_v/d<6.0$. In this failure mode, destruction of tension zone below the diagonal crack leads to splitting of the concrete at the level of tension steel.

(c) Diagonal-tension failures:

These are also typically observed for beams stated in (b) above. In this failure mode, destruction of tension zone below the diagonal crack results in the collapse.

Again, the diagonal-tension cracks also known as inclined cracks may be:

(a) Primary cracks - where tension cracks open first near the mid-depth of the beam,

Either (i) independantly in the vicinity of the neutral axis, when a_v/d is small (known as web-shear cracks);

or (ii) as a development of an existing flexural crack which is extended in an inclined direction, when a_v/d is larger (known as flexure-shear cracks).

(b) Secondary cracks - where, upon further loading the inclined cracks may extend at either end leading to failure.

Taub, J., and Neville, A.M., [25] have observed that the modes of shear failure in reinforced concrete beams without web reinforcement are usually similar to that of reinforced concrete beams with web reinforcement, but in the latter case, the ultimate load is considerably higher and sudden collapse of the beam does not take place.

2.2 FUNCTIONS OF SHEAR REINFORCEMENT

As Taub and Neville [25] have put it, the generally accepted function of shear reinforcement is to resist the opening or widening of the diagonal tension cracks and thus to prevent the failure of a reinforced concrete beam due to shear. It has also been noted [1] that the shear reinforcement in its secondary role to enhance the shear resistance of a beam:

(a) carries part of the shear
(b) transfers the shear across a potential inclined crack
(c) holds the longitudinal bars and increases their dowel capacity.

Again, it is interesting to note the following statements regarding the functions of shear reinforcement:

(a) Evans and Kong [8] have observed that "an important function of stirrups is to prevent the pressing down of the longitudinal reinforcement due to dowel-action force, and the consequent splitting of the concrete at the level of such reinforcement".

(b) Kani [12] has noted that "the purpose of web reinforcement is to provide reactions for the internal concrete arches which support the compressive zone of the beam".

(c) Park and Paulay [20] have noted that "suitably detailed web reinforcement will preserve the integrity and therefore the strength of the beam mechanism, allowing additional shear forces to be resisted by the truss mechanism".

(d) Collins and Mitchell [6] have stated that "the primary function of the stirrups is to hold the beam together in the lateral direction".

Whatever be the observations regarding the function, the shear reinforcement substantially augments the shear resistance of beams by carrying a part of the shear and in turn increases the ductility of the beam and eliminates the danger of a premature non-ductile failure. This warrants the provision of a suitable type of shear reinforcement.

2.3 SHEAR TRANSFER IN BEAMS WITH TRANSVERSE REINFORCEMENT

Though the mechanism of shear transfer in beams is a subject of controversy, the mechanism proposed in the ASCE-ACI Committee 426 [1] explains basically the internal forces at a typical inclined crack (Fig. 2.2).

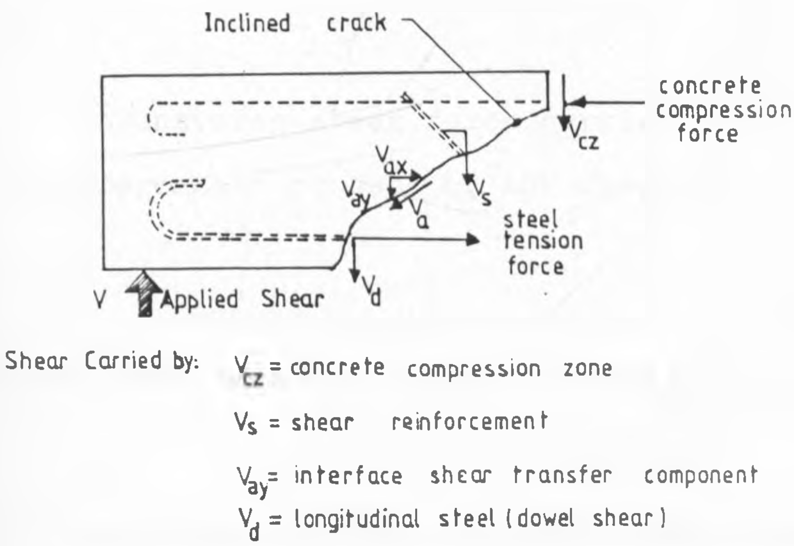


Fig. 2.2 INTERNAL FORCES AT INCLINED CRACK DUE TO APPLIED SHEAR (Based on [1])

These mechanisms are such, that the applied shear force V, is resisted by:

(a) the shear force carried across the uncracked concrete compression zone, V_{cz};

(b) the transverse component of the force due to interlocking of aggregate particles (interface shear transfer) across a crack, V_{ay};

(c) the transverse shear force induced in the main flexural reinforcement by dowel action, V_d; and

(d) the transverse shear force carried by the shear reinforcement crossed by the diagonal crack, V_s.

Hence the applied shear force, $V = (V_{cz}+V_{ay}+V_d)+V_s$

$$= V_c+V_s \quad (2.1)$$

where V_c is equal to the bracketed quantities in the above expression. This is defined as the 'concrete contribution' or the shear carried by the concrete.

Again, typical contributions [1] of the various internal forces with properly detailed stirrups are shown in Fig. 2.3.

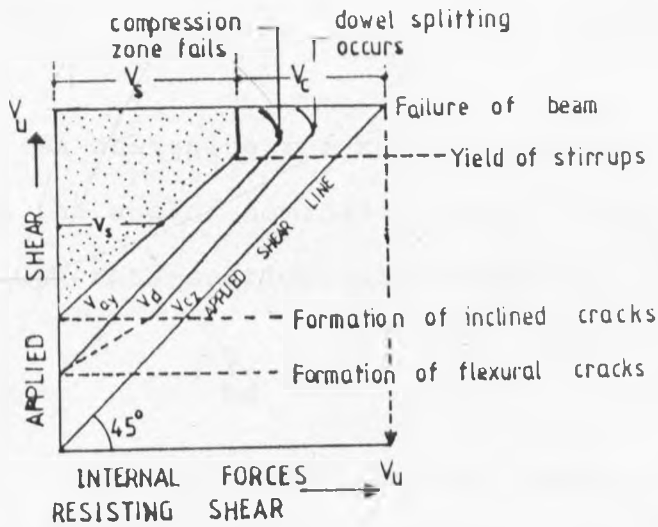


Fig. 2.3 SHEAR TRANSFER IN BEAMS WITH WELL-ANCHORED REINFORCEMENT (Based on [1])

It is however noted that there is still disagreement as to the presence and roles of these components of shear transfer in beams and the mechanisms of shear transfer is not yet well understood [1].

2.4 METHODS OF ANALYSIS OF SHEAR STRENGTH OF BEAMS

In designing for shear, the codes [3 and 4] adopted the use of nominal or design shear stress, v, at any cross-section calculated as,

$$v = \frac{V}{bd} \qquad (2.2)$$

Again $v = v_c + v_s$ where v_c, is the shear stress resisted by concrete and v_s, is the shear stress resisted by transverse reinforcement. The value of v_c is assumed to be equal to the shear strength recorded in a similar beam without transverse reinforcement when a diagonal crack was either first noted or was estimated to have traversed the neutral axis and empirical equations are proposed to compute its value. The value of v_s is customarily based on the 'truss analogy'.

2.4.1 The Truss Analogy

Ritter and Mörsch's theory commonly referred to as the 45 deg truss analogy makes use of a truss model for the design of shear reinforcement in reinforced concrete beams. According to this analogy, after the formation of the diagonal cracks the beam tends to behave like a pin-jointed plane truss: the bottom longitudinal bars and the stirrups constitute the tension members, while the top flange and the inclined concrete struts in the web are the compression members. The forces in the truss were then determined from the consideration of equilibrium.

The ASCE-ACI Committee 426 [1] which is one of the important references concerning shear transfer mechanisms noted that the truss analogy does not take into consideration the presence of several significant factors associated with shear transfer namely V_{cz}, V_{ay} and V_d (refer section 2.3). Thus in the Codes [3] and [4] this added shear capacity was taken as the 'concrete contribution'.

The ASCE-ACI Committee 426 [1] has suggested that regardless of the short-comings the truss analogy is "an excellent conceptual tool in the study of beams with shear reinforcement". Again, Rensaa in his contribution to the paper by Taub and Neville [25] has stated that,

> "the truss analogy is an excellent method of indicating the function of shear reinforcement and it gives quite accurate results provided realistic slopes of the 'compression diagonals' are being used".

It is verified by many researchers that the slope of compression diagonals vary along the length of beam. But the problem lies in its accurate determination. Incorporating the compatability of strains which was otherwise neglected in the truss analogy, Collins [5 and 6] has proposed a rational theory to compute the inclination of the diagonal compression based on the compression field theory.

It should be noted that the Codes [3]and[4] utilize the truss analogy for the design of shear reinforcement.

2.4.2 The Compression Field Theory

The compression field theory [5 and 6] assumes that after cracking, the reinforced concrete beam can resist no tension and that the shear will be carried by a field of diagonal compression which resulted in the following expression for the angle of inclination of the diagonal compression (θ). Using the compatability condition of strains, it was proved that (refer equation 3.13):

$$\tan^2\theta = \frac{\varepsilon_\ell + \varepsilon_{cp}}{\varepsilon_v + \varepsilon_{cp}} \tag{2.3}$$

where ε_ℓ = longitudinal tensile strain

ε_v = transverse tensile strain

ε_{cp} = diagonal compressive strain

Collins [6] has also applied the equilibrium equations of truss analogy, the compatability conditions of strains and the stress-strain relationships of concrete to the design of shear reinforcement rationally and accurately.

The compression field theory was used to predict the full behavioural response of prestressed and non-prestressed concrete beams in shear and torsion [6]. The rationale behind this theory has attracted many researchers [18, 19, 24, and 29]. Onsongo [18] has extensively studied the compression field theory and used it to predict the response of reinforced concrete beams subjected to combined torsion, flexure and axial load. It was further extended [19 and 24] to study the behavioural response of structural concrete subjected to flexure and shear.

In this present study, the truss analogy and the compression field theory are suitably modified to form the basis of the theoretical framework for the wave reinforcement (Chapter 3).

2.5 TYPES OF SHEAR REINFORCEMENT

Various types of shear reinforcement which have been adopted successfully in practice are:

(a) vertical stirrups

(b) inclined stirrups

(c) bent-up bars

(d) combination of vertical stirrups and bent-up bars

(e) welded wire fabric

In this section, a brief attempt is made to identify and compare their effectiveness in enhancing the shear strength of beams.

2.5.1 Vertical Stirrups

Vertical stirrups are most commonly used to attain the various functions stated in Section 2.1. These are also known as lateral reinforcement or web reinforcement or links which are bent into recommended shapes and located perpendicular to axis of member. A typical arrangement of vertical stirrups is shown in Fig. 2.4.

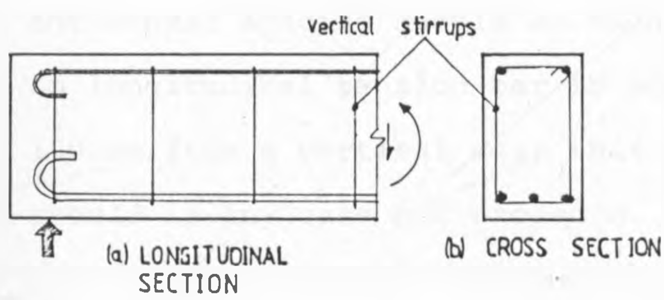


Fig. 2.4 VERTICAL STIRRUPS

Various codes specify empirical spacing limits for vertical stirrups. Table 2.1 gives the extract of spacing limits specified in ACI 318M-83 [4] and in BS 8110: Part 1: 1985 [3].

Table 2.1

SPACING LIMITS FOR VERTICAL STIRRUPS
ACI 318 M-83 [4]: Spacing of shear reinforcement placed perpendicular to axis of member shall not exceed $\frac{d}{2}$ in non-prestressed members (and $\frac{3}{4}h$ in prestressed members, nor 600 mm).
BS 8110: Part 1: 1985 [3]: Spacing of links in the direction of the span should not exceed 0.75d, the horizontal spacing should be such that no longitudinal tension bar is more than 150 mm from a vertical leg; this spacing should in any case not exceed d.

The vertical stirrups should tightly surround the longitudinal tensile steel and anchored at both ends to develop the design yield strength at a critical section and to attain the following beneficial effects [31]:

(a) extra concrete force contribution due to the prevention of diagonal crack propagation; and

(b) extra dowel action force from longitudinal tensile reinforcement due to the support.

Also, Park and Paulay [20] have noted that the vertical stirrups will prevent the opening of the bond-generated splitting of concrete along the longitudinal tensile steel in the shear span. Taub and Neville [25] observed that "vertical stirrups alone cannot always economically guarantee a full protection from shear failure of beams because they do not lie in a direction normal to the diagonal tension cracks".

2.5.2 Inclined Stirrups

Inclined stirrups (Fig. 2.5) making an angle of 45^{o} or more with longitudinal tension reinforcement is used as a shear reinforcement [4, 6]. From the test results [25] and study of free-body diagrams, it was inferred that stirrups inclined at 45^{o} to the axis of the beam was most effective in resisting the widening of diagonal tension cracks provided slippage at the contact between the inclined stirrups and the longitudinal tensile reinforcement is eliminated.

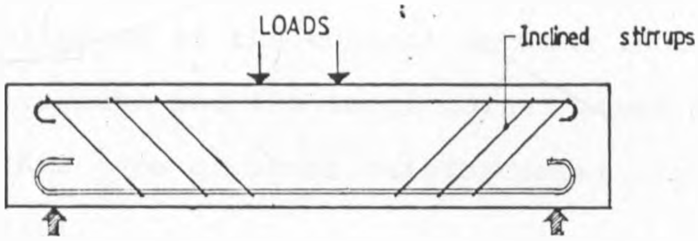


Fig. 2.5 INCLINED STIRRUPS

ACI Building Code [4] specifies that the inclined stirrups should be so spaced that every 45 deg line extending towards the reaction from the middepth of member d/2 to longitudinal tension reinforcement should be crossed by at least one line of shear reinforcement.

The following observations regarding the use of inclined stirrups are worth-noting:

(a) That maximum shear resistance is attainable with a smaller amount of inclined stirrups and it is therefore economical to provide such a reinforcement [8 and 25].

(b) That the difficulty of elimination of slippage at the contact between inclined stirrups and the longitudinal steel makes this type of shear reinforcement impracticable [16].

(c) That the inclined stirrups are effective only in one direction and hence should not be used when there is a possibility of load reversal unless it is provided in both directions [20].

2.5.3 Bent-up Bars

Main steel is utilised for shear reinforcement where it is no longer required in the tension face of the beam. Thus the longitudinal tensile reinforcement is bent upwards (Fig. 2.6) from the tensile flange into the compression flange, normally near the ends of the beam where the shear force is large and the bending moment is small. ACI Building Code [4] specifies that the longitudinal reinforcement with bent portion making an angle of 30 deg or more with the longitudinal tension reinforcement and spaced as explained in Sec. 2.5.2 may be used as a shear reinforcement.

The bent-up bars alone as shear reinforcement create stress complications in the concrete resulting in a lower shear strength and much wider cracks (than similar beams with vertical stirrups) and produce excessive compression and hence bursting of the concrete near the bends [25]. Kani [12] has also noted that the inclined bent-up bars in preventing a diagonal failure over vertical stirrups is undisputed but from practical considerations a combination of vertical stirrups and bent-up reinforcement should be used. A typical combination of vertical stirrups and bent-up bars is shown in Fig. 2.6.

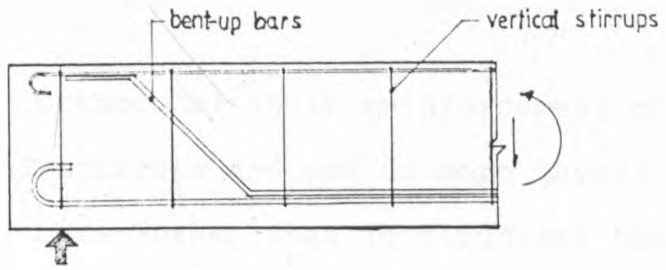


Fig. 2.6 COMBINATION OF VERTICAL STIRRUPS AND BENT-UP BARS

B.S. Code of Practice [3] specifies that when bent-up bars are used, at least 50% of shear resistance provided by the steel should be in the forms of links. In beams with a combination of bent-up bars and vertical stirrups, it was noted [25] that the vertical stirrups prevent the pressing down of the tensile bar at the lower end of the diagonal crack and the resulting splitting of the concrete. Also the vertical stirrups carry a part of the diagonal tension thus relieving some of the force in the bent-up bars and in the tension reinforcement.

2.5.4 Orthogonal Reinforcement

An orthogonal shear reinforcement consists of vertical stirrups and one or more layers of horizontal bars (other than longitudinal tensile steel) which are placed at right angles (to vertical stirrups). A typical orthogonal shear reinforcement with one layer of horizontal reinforcement is shown in Fig. 2.7.

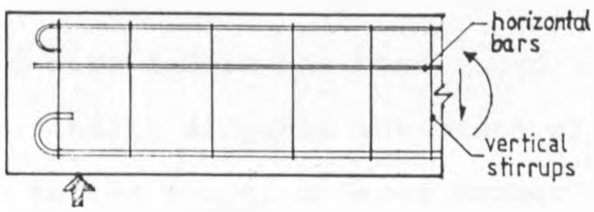


Fig. 2.7 ORTHOGONAL REINFORCEMENT

It was noted [20] that the horizontal bars strengthen the contribution of the concrete but will have no effect on the shear strength of a beam (since truss mechanism is unaffected). It was noted [25] that the orthogonal shear reinforcement is efficient especially for deep beams.

2.5.5 Welded Wire Fabric

Welded wire fabric (either smooth or deformed) as shear reinforcement consists of a series of longitudinal and transverse cold-drawn steel wires placed at right angles to each other and welded together at all points of intersection. Welded wire fabric is placed perpendicular to axis of member, the size and spacing depending on the requirements of the design.

Welded wire fabric has been found to increase the number of, and to decrease the width of, cracks. It may be due to the use of a large number of smaller bars in preference to a few large bars. It is effective for crack control and was observed that [26] the welded wire fabric offers promise in slender webs of prestressed deep girders and for resisting torsion.

The ASCE-ACI Committee 426 [1] has also recorded that the welded wire mesh stirrups with mats of 50 to 100 mm spacing of stirrup bar were the best with respect to crack widths and compressive stresses in the web.

2.6 CHOICE OF SHEAR REINFORCEMENT

For beams with identical dimensions and steel content the maximum width of shear cracks for average types of shear reinforcement was found by Leonhardt [15] to be the smallest in beams with closely spaced inclined stirrups followed by a beam with vertical stirrups. The widest cracks were observed in a beam with bent-up bars (Fig. 2.8).

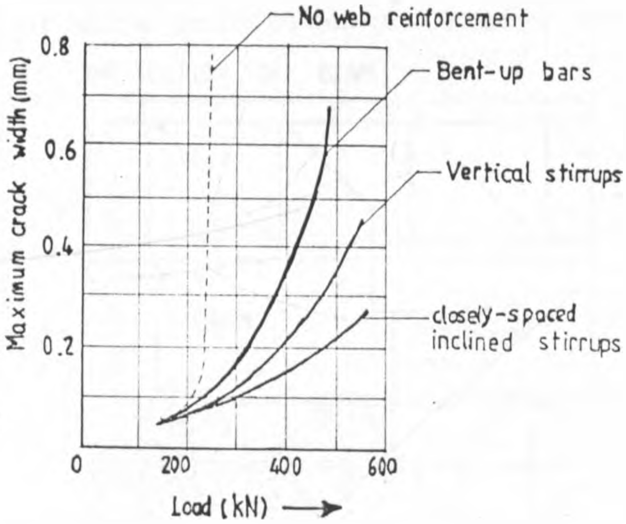


Fig. 2.8 TYPICAL LOAD Vs MAXIMUM CRACK WIDTH CURVES (Based on [15]

Taub and Neville have concluded after extensive experimental study [25] that "beams differing in web reinforcement only exhibit the first diagonal crack under the same load. It is only after the cracking has started that the behaviour of a beam depends on the type of web reinforcement used and the further development of cracks and the ultimate load are a function of this reinforcement".

A typical combination of vertical stirrups and inclined stirrups shown in Fig.2.9 is modified to derive the advantages of both. Hence the evolution of wave reinforcement (refer Sec. 3.1).

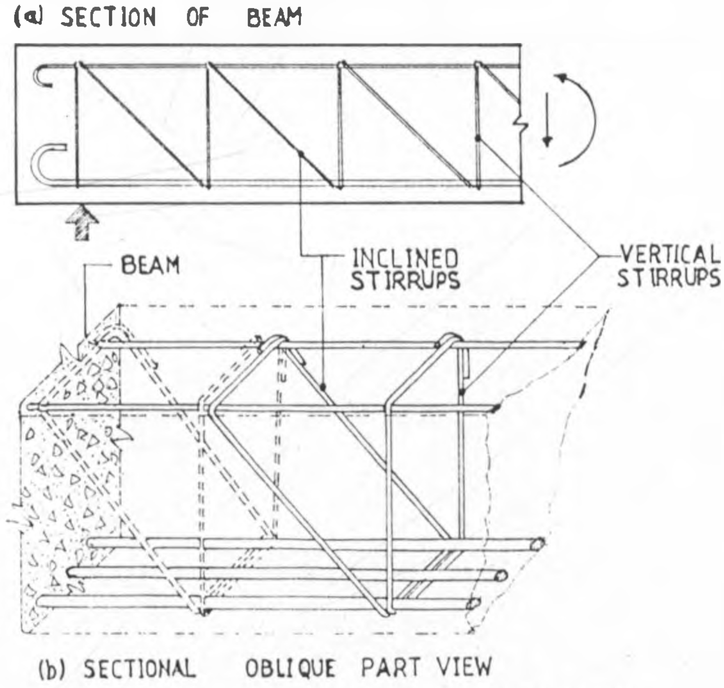


Fig. 2.9 TYPICAL COMBINATION OF INCLINED AND VERTICAL STIRRUPS

In the next chapter, the theoretical framework is established to reinforced concrete beams with wave reinforcement subjected to flexure and shear.

CHAPTER 3 THEORETICAL FRAMEWORK

This chapter aims in introducing the wave reinforcement as used in the present experimental study (refer Sec. 4.4.1), alongwith the detailing methods. In addition, the equations pertaining to the computation of nominal shear strength provided by wave reinforcement based on truss analogy are derived. Also explained is compression field theory for predicting the behaviour of beams with wave reinforcement.

3.1 WAVE REINFORCEMENT

The wave reinforcement is a steel bar consisting of vertical and inclined legs forming acute angles with two or more bends in the opposite directions (in the same plane). A typical wave reinforcement with bending dimensions is shown in Fig. 3.1.

The wave reinforcement can be identified by degrees of arc α , between the horizontal and inclined leg , namely α deg wave reinforcement. But in preparing the bar bending schedule, it can preferably be defined in terms of lengths A and B (refer also Sec. 3.2). Minimum hook allowance should also be considered during bar bending.

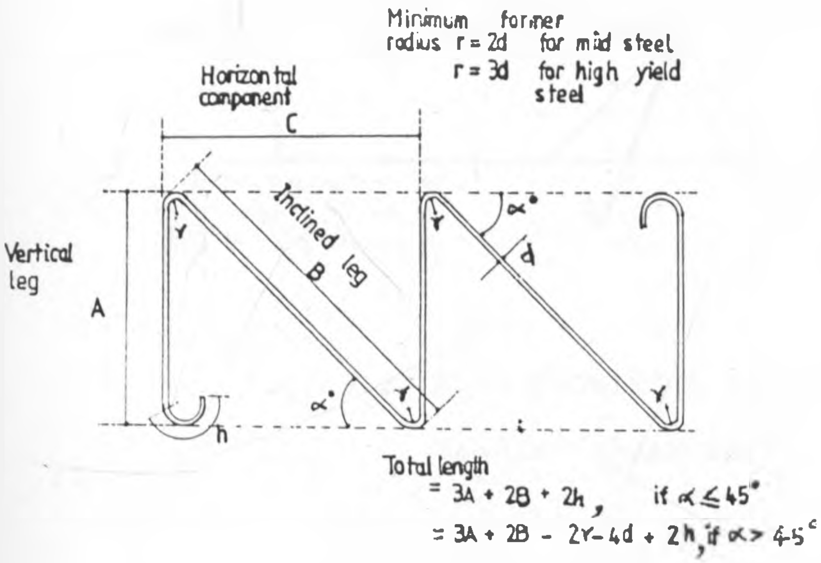


Fig. 3.1 TYPICAL BENDING DIMENSIONS FOR WAVE REINFORCEMENT

The bending of wave reinforcement can be achieved by making use of the same bar bending arrangement used for vertical stirrups (Fig. 3.2). By marking a control line (at α deg as shown) the bending of α deg wave reinforcement can be carried out. It can be seen that an angle α equals to 45 deg is easy to conceive by the bar bender at all construction sites.

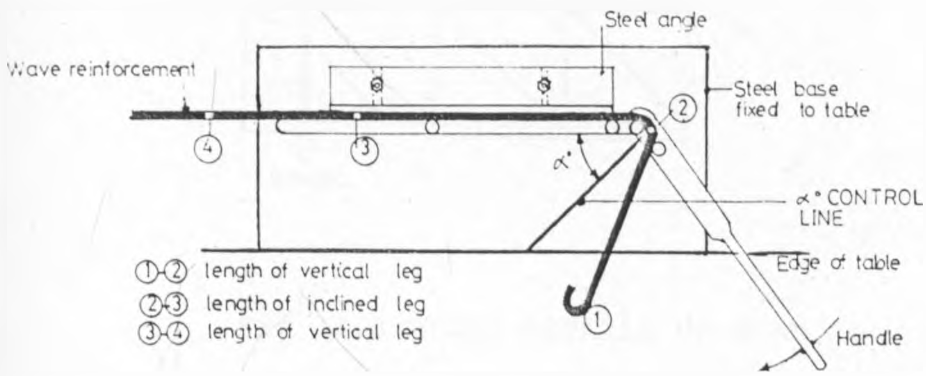


Fig. 3.2 PLAN SHOWING A TYPICAL BAR BENDING ARRANGEMENT

3.2 DETAILING OF WAVE REINFORCEMENT

Refer to the part details of beam shown in Fig. 3.3 (also refer Fig. 4.4 for more details). The wave reinforcement (bar mark 3) can be identified either as:

(a) 45^{o} deg wave reinforcement ($\alpha = 45^{o}$)

or (b) 2Y803 -370 -525, where

2Y803 denotes two units of wave reinforcement using Y8 bars of bar mark 3

370 denotes the dimension A, in mm (which is kept always equal to (d-d'))

525 denotes the dimension B, in mm

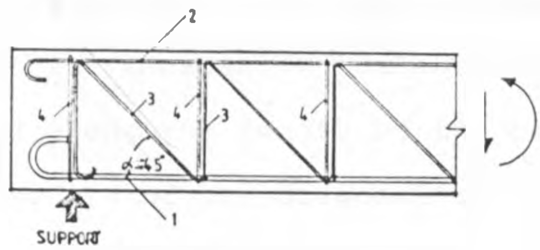


Fig. 3.3 PART DETAILS OF BEAM

(Showing bar marks)

A typical bar bending schedule for the above beam is shown in Fig. 3.4.

Member	Bar mark	Type and size	No. of mbrs	No. of bars in each	Total No.	Length of each bar (mm) +	Shape	A* (mm)	B* (mm)	C* (mm)
	1	Y25	1	3	3	3150	A	2600		
	2	Y8	1	2	2	2775	A	2600		
	3	Y8	1	4	4	3125	B A	370	525	370
	4	R6	1	8	8	1075	A B	365	115	
+ Specified in multiples of 25 mm } As per BS 4466 (1981) * Specified in multiples of 5 mm										

Fig. 3.4 TYPICAL BAR BENDING SCHEDULE

The wave reinforcement should be properly placed so that the tension in the shear reinforcement is distributed along the length of the beam balancing the outward thrusts of the diagonal concrete compressions. Again the inclined legs of the wave reinforcement should be so placed as to cross the anticipated inclined cracks effectively (Fig. 3.5a).

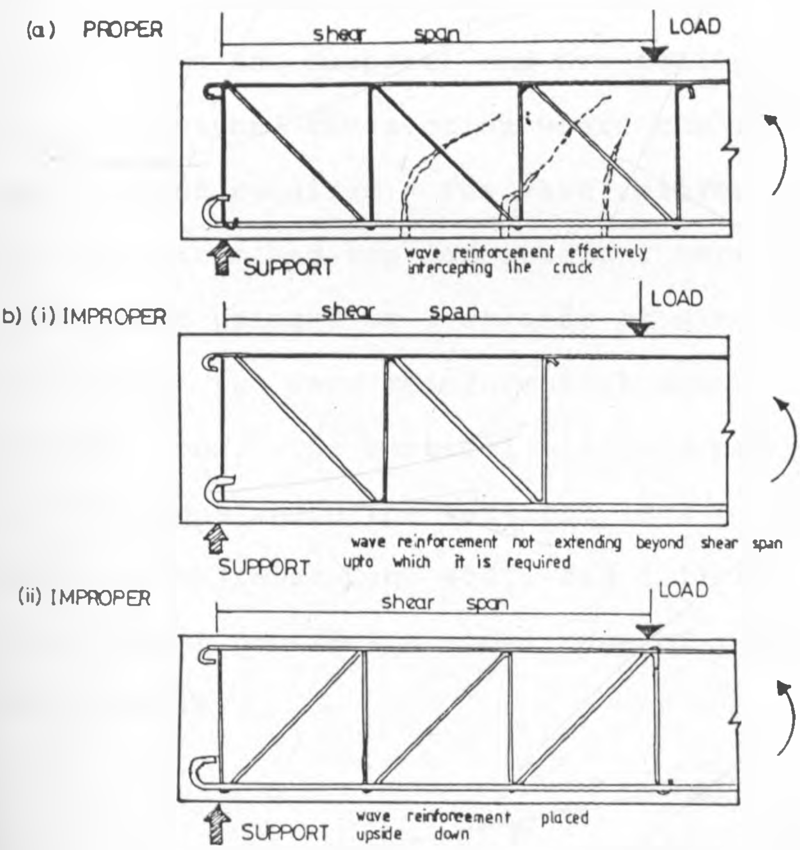


Fig. 3.5 PLACEMENT OF WAVE REINFORCEMENT

Improper placing such as wave reinforcement not extending beyond the section where the shear reinforcement is not required (Fig. 3.5b(i)) and the inclined legs placed in the direction nearly parallel to the anticipated crack direction (Fig. 3.5b(ii)) should be avoided for they defeat the very purpose of providing shear reinforcement to beams. Again, the wave reinforcement is placed to the inner side of the outer longitudinal tensile bar as shown in Fig. 3.6 such that it starts at the bottom of the beam (above the support) and eventually ends at the top, but beyond the section where the shear reinforcement is not required. The wave reinforcement is tied to the bottom and top longitudinal bars at their points of contact using 2 or 3 strands of steel wires. A minimum of two wave reinforcement should be placed at all sections. The vertical stirrups are provided at all sections where the vertical legs of wave reinforcement exist (refer Sec. 4.4.1 and 5.1) and this arrangement provides a steel cage of required shape and rigidity.

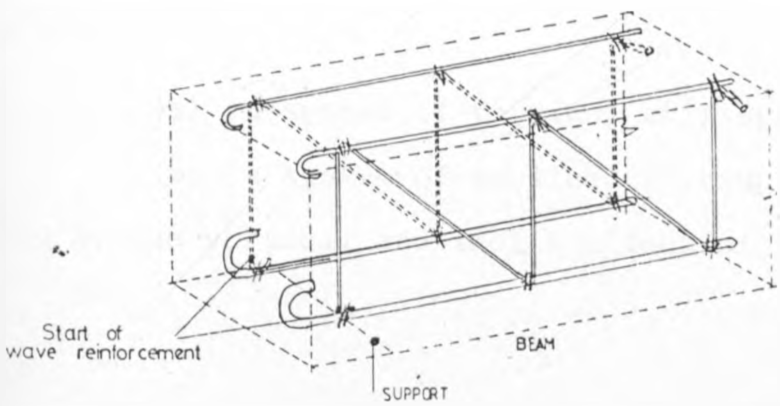


Fig. 3.6 CAGE WITH WAVE REINFORCEMENT
(Without vertical stirrups)

As Park and Paulay [20] have stated, the "detailing based on an understanding of and feeling for the structural behaviour of reinforced conrete is likely to require as much creative power as the derivation of structural actions by mathematical analysis". It is therefore suffice to say that the detailing of wave reinforcement should be properly understood in order to derive the best results.

3.3 TRUSS ANALOGY FOR WAVE REINFORCEMENT

Consider a truss model for a cracked beam (Fig. 3.7) where θ, is the arbitrary angle (in degrees) of inclination of concrete diagonals in compression measured with respect to the longitudinal axis of the beam and α, is the angle between the inclined leg of the wave reinforcement and longitudinal axis of beam.

The wave reinforcement may be assumed to constitute:

(a) a system of vertical stirrups, and

(b) a system of inclined stirrups

formed by the vertical and inclined legs of the wave reinforcement respectively. When the bars crossed by the crack reach the yield strength f_{yw}, the nominal shear strength provided by the vertical legs V_{wv}, and the inclined legs V_{wi}, can be computed using truss analogy [3, 8, and 20] but by considering one inclined leg for each wave reinforcement over a typical region, as follows:

$$V_{wv} = A_{sw} f_{yw} \cot\theta \left(\frac{d-d'}{s_w}\right) \qquad (3.1)$$

and, $$V_{wi} = A_{sw} f_{yw} (\cos\alpha + \sin\alpha \cot\theta) \left(\frac{d-d'}{s_w}\right) \qquad (3.2)$$

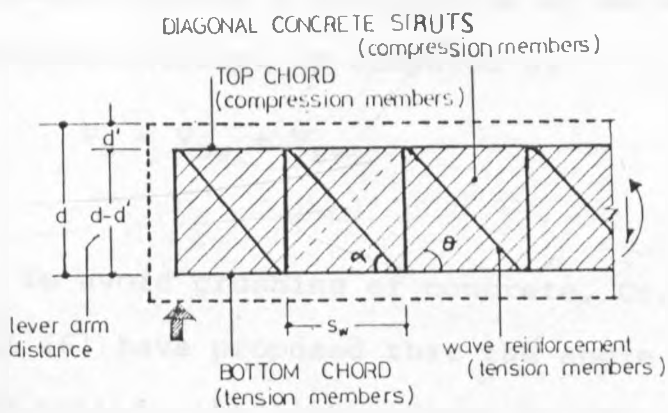


Fig. 3.7 TRUSS MODEL FOR A BEAM WITH WAVE REINFORCEMENT

Thus the nominal shear strength provided by the wave reinforcement V_{sw} can be computed as:

$$V_{sw} = V_{wv} + V_{wi} \qquad (3.3)$$

It can be noted in Sec. 4.4.1 that additional vertical stirrups are also provided at all sections where vertical legs of wave reinforcement exist. The necessity of such a provision is well-established based on test results as outlined in Sec. 5.1. Therefore, the additional shear strength V_{sv}, provided by the vertical stirrups would be,

$$V_{sv} = A_{sv} f_{yv} \cot\theta \left(\frac{d-d'}{s_w}\right) \qquad (3.4)$$

Hence the nominal shear strength V_s, provided by shear reinforcement having a combination of wave reinforcement and vertical stirrups is computed as

$$V_s = V_{sw} + V_{sv} \qquad (3.5)$$

To avoid crushing of concrete, Collins and Mitchell [6] have proposed that the angle of inclination, θ , must satisfy the limits given by the following:

$$10 + \frac{35(\tau_n/f'_c)}{0.42-50\varepsilon_\ell} < \theta < 80 - \frac{35(\tau_n/f'_c)}{0.42-65\varepsilon_t} \qquad (3.6)$$

where τ_n is the nominal shear stress given by $V_n/b(d-d')$. For highly stressed members, it was also recommended [6] that θ will be restricted to a narrow range of values close to 45 deg while for lightly loaded members the range of allowable angles will be very wide (in the limit 10 deg $\leq \theta \leq$ 80 deg); and for convenience a value of θ somewhat larger than the smallest allowable angle but constant over the length of the beam may be chosen. For the present study the 45 deg truss analogy accepted in Codes [3] and [4] is chosen in order to compute the shear strength provided by wave reinforcement.

Assuming θ = 45 deg, the shear strength (V_s) provided by α deg wave reinforcement combined with vertical stirrups as explained earlier can be computed as

$$V_s = [A_{sv}f_{yv} + A_{sw}f_{yw} + A_{sw}f_{yw}(\cos\alpha + \sin\alpha)]\left(\frac{d-d'}{s_w}\right) \quad (3.7)$$

If same diameter bars are used, $A_{ss} = A_{sv}/2 = A_{sw}$ and $f_{ys} = f_{yv} = f_{yw}$, then

$$V_s = A_{ss}f_{ys}\left(\frac{d-d'}{s_w}\right)(3+\cos\alpha+\sin\alpha) \quad (3.7a)$$

where A_{ss} = the cross-sectional area of each bar of shear reinforcement

f_{ys} = characteristic strength of shear reinforcement

For a typical case of 45 deg wave reinforcement ($\alpha = 45^{o}$) combined with vertical stirrups as stated earlier, noting that d - d' = s_w and θ = 45 deg, the above equations can be used to obtain,

$$V_s = A_{sv} f_{yv} + A_{sw} f_{yw} + 1.414 \ A_{sw} f_{yw}$$

$$= A_{sv} f_{yv} + 2.414 \ A_{sw} f_{yw} \qquad (3.8)$$

Again, if equal diameter bars are chosen

$A_{ss} = A_{sw} = \frac{1}{2} A_{sv}$ and $f_{ys} = f_{yw} = f_{yv}$, then

$$V_s = 4.414 \ A_{ss} f_{ys} \qquad (3.8a)$$

In a general form, for a 45 deg wave reinforcement in combination with vertical stirrups of same diameter,

$$V_s = A_{we} f_{ys} \qquad (3.9)$$

where A_{we} is defined as the equivalent area of shear reinforcement over a typical region of width, s_w (Fig. 3.11). Again it can be proved that,

$$A_{we} = A_{ss} \ (2N + 2.414 \ n) \qquad (3.10)$$

where N is the number of two-legged vertical stirrups and n the number of wave reinforcement, at the section considered.

For one vertical stirrup and two wave reinforcement provided in the region considered, Eq. 3.10 becomes

$$A_{we} = 6.828\ A_{ss} \qquad (3.10a)$$

It is to be noted that due to the concrete 'surround' at the bends (Fig. 3.8), the slippage of inclined legs of wave reinforcement does not exist, which was otherwise a problem observed with inclined stirrups (refer Sec. 2.5.2). Thus due to the 'anchorage' provided by the concrete, the wave reinforcement can develop the design yield strength.

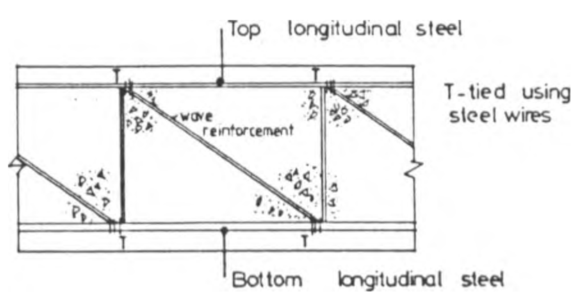


Fig. 3.8 'ANCHORAGE' PROVIDED BY THE CONCRETE

The adequacy of the equations has been verified and elaborated in Sec.5.1.2. The simplicity of truss analogy for wave reinforcement as outlined is illustrated in Appendix A using a design example. In addition, Chapter 7 provides the proposed design recommendations for shear with wave reinforcement based on truss analogy.

3.4 COMPRESSION FIELD THEORY

The compression field theory which is well-established [5, 6, 18, 19 and 24] can be used to rationally explain the behaviour of a uniformly cracked reinforced concrete beam. The compression field theory incorporates:

(a) compatability conditions in terms of average strains; and

(b) equilibrium conditions in terms of concrete stresses.

Shitote [24] has successfully applied the compression field theory for the analysis of reinforced concrete beams under combined flexure and shear, using vertical stirrups as shear reinforcement. In this section the above theory as applicable to wave reinforcement is explained.

3.4.1 Compatability Conditions in Terms of Average Strains

In formulating the compatability conditions, Collins [5] assumed that the beam is uniformly cracked over the region and the direction of average principal compressive stress in the concrete coincided with the direction of the average principal compressive strain, θ. It is also assumed that all strains at a point in a plane are compatible.

The compatability requires that the strains in the reinforcement and the concrete equal those in the member, which can be represented by Mohr's circle of strain (Fig. 3.9). The strain circle is based on:

(a) average principal compressive strain, ε_{cp} (assumed positive when compressive);

(b) angle θ , between the principal compressive strain direction and the longitudinal direction;

(c) average strain in the longitudinal direction, ε_{ℓ} (assumed positive when tensile);

(d) average strain in the transverse direction, ε_{v} (assumed positive when tensile).

The average strains are obtained by averaging the strains measured over a region involving uncracked and cracked portions.

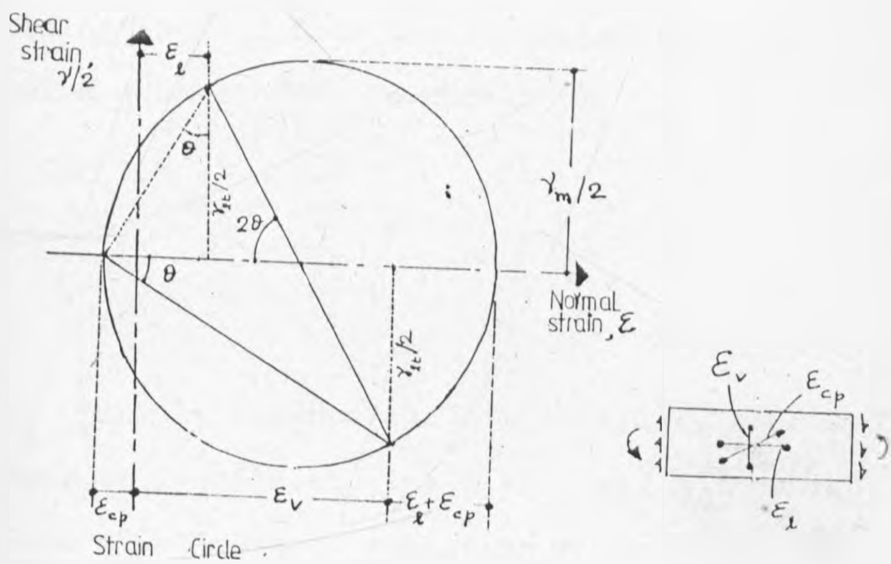


Fig. 3.9 COMPATABILITY CONDITIONS FOR AVERAGE STRAINS FOR CONCRETE (Based on COLLINS [5])

From the geometry of the strain circle, it can be seen that:

$$\gamma_{\ell t}/2 = (\varepsilon_\ell + \varepsilon_{cp})/\tan\theta \tag{3.11}$$

and,
$$\varepsilon_v = \frac{\gamma_{\ell t}/2}{\tan\theta} - \varepsilon_{cp} \tag{3.12}$$

By eliminating $\gamma_{\ell t}$ from the above two equations, the following equation can be obtained:

$$\tan^2\theta = \frac{\varepsilon_\ell + \varepsilon_{cp}}{\varepsilon_v + \varepsilon_{cp}} \tag{3.13}$$

This represents the compatability condition in terms of average strains ε_ℓ, ε_v and ε_{cp} which provides the necessary condition to determine the value of θ.

In addition, the following relationships can be obtained:

(i) the principal tensile strain, ε_t (tension positive):

$$\varepsilon_t = \varepsilon_{cp} + \varepsilon_\ell + \varepsilon_v \tag{3.14}$$

(ii) the shear strain relative to the longitudinal and transverse directions, $\gamma_{\ell t}$:

$$\gamma_{\ell t} = 2\sqrt{(\varepsilon_{\ell} + \varepsilon_{cp})(\varepsilon_{v} + \varepsilon_{cp})} \tag{3.15}$$

(iii) the maximum shear strain, γ_m :
The diameter of the circle is a measure of the maximum shear strain. Thus,

$$\gamma_m = \varepsilon_{\ell} + \varepsilon_{v} + 2\varepsilon_{cp} \tag{3.16}$$

3.4.2 Equilibrium Conditions in Terms of Concrete Stresses

It was established [5 and 19] that for a cracked beam model subjected to a shear force V and a flexural moment M, an element on the plane of shear flow will be subjected to shear stress τ, longitudinal stress $\sigma_{c\ell}$, and a transverse strain, σ_{ct} (Fig. 3.10).

Onsongo [19] has shown from the Mohr's circle of strain that:

(a) $\tau = \frac{1}{2}f_{cp}\sin 2\theta$ (3.17)

(b) $\sigma_{ct} = \tau \tan\theta$ (3.18)

(c) $\sigma_{c\ell} = \tau/\tan\theta$ (3.19)

(d) $f_{cp} = q(1+\tan^2\theta)/b\tan\theta$ (3.20)

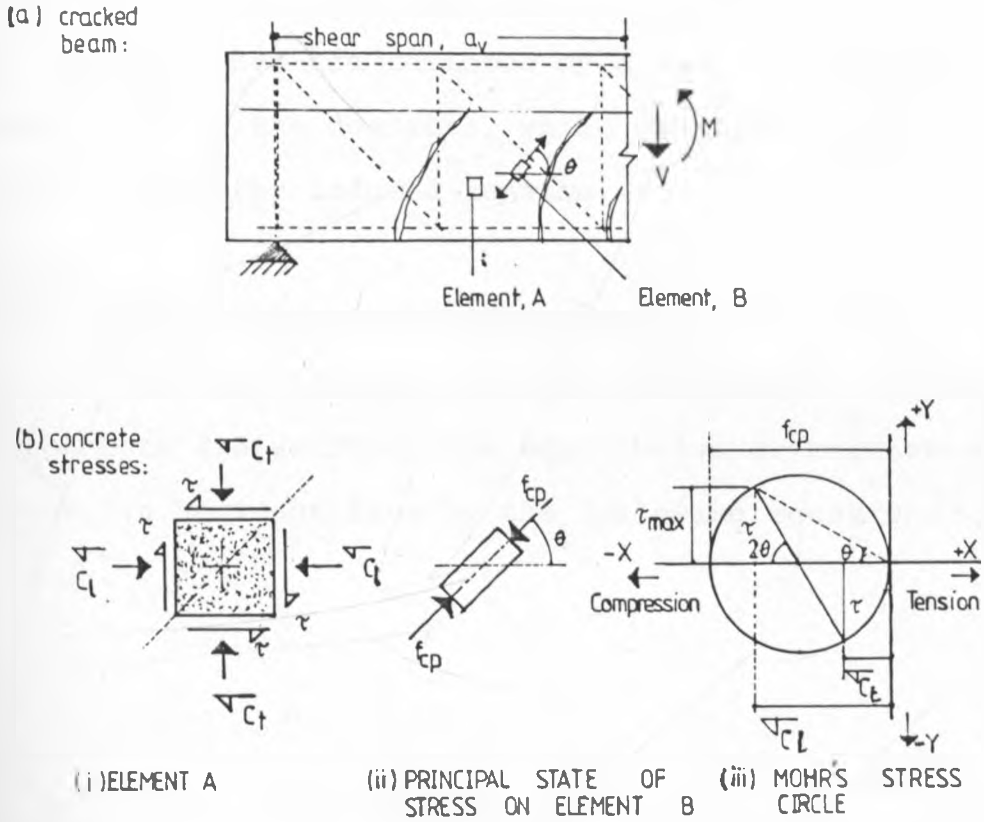


Fig. 3.10 CRACKED BEAM MODEL

(Based on Onsongo [19])

The equations obtained [19 and 24] based on the equilibrium in the longitudinal direction and the shear flow variation are given in Appendix B.

The transverse equilibrium conditions can be obtained by assuming that the section is fully cracked in the region below the neutral axis under the loading and the concrete can carry no tension. It implies that in the post-cracking loading range, the applied loads are resisted by a field of diagonal compression in the concrete, while the reinforcing steel resists the induced tension.

Assuming that within the spacing, s_w (Fig. 3.11), the applied shear and the internal compression stresses in concrete are uniform, the equilibrium of transverse forces can be identified by the following equations.

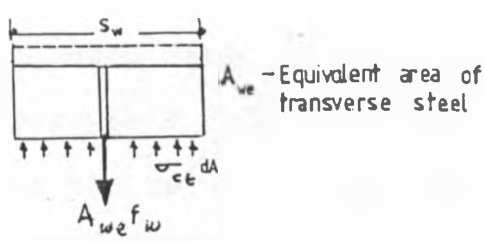


Fig. 3.11 RESULTANT TRANSVERSE FORCES

$$A_{we} f_w = \int \sigma_{ct} \, dA$$

$$= (\tau \tan\theta)(b s_w)$$

$$= \tau b \tan\theta \; s_w$$

$$= q \tan\theta \; s_w$$

Thus

$$\frac{A_{we} f_w}{s_w} = q \tan\theta \qquad (3.21)$$

Substituting $t_w = A_{we}/s_w$, equation (3.21) becomes

$$\tan\theta = \frac{t_w f_w}{q} \qquad (3.22)$$

3.4.3 Stress-Strain Relationships of Steel and Concrete

The local stress- local strain relationships determined from standard material tests may vary in a structural concrete beam under loading depending on the bond between the reinforcement and the concrete, the distribution of the reinforcing bars within the concrete and the discontinuities occurring at cracks. Due to the fact that the 'actual' stresses and strains along the member is difficult to obtain, [5] for simplicity the idealised stress-strain relationships as explained below are considered in the prediction of the behavioural response of the beam.

The idealised stress-strain curve for the reinforcement (Fig. 3.12a) is based on the simple bi-linear function defined by Young's Modulus E_s, namely:

$$\text{stress} \quad f_s = E_s \varepsilon \leq f_y \qquad (3.23)$$

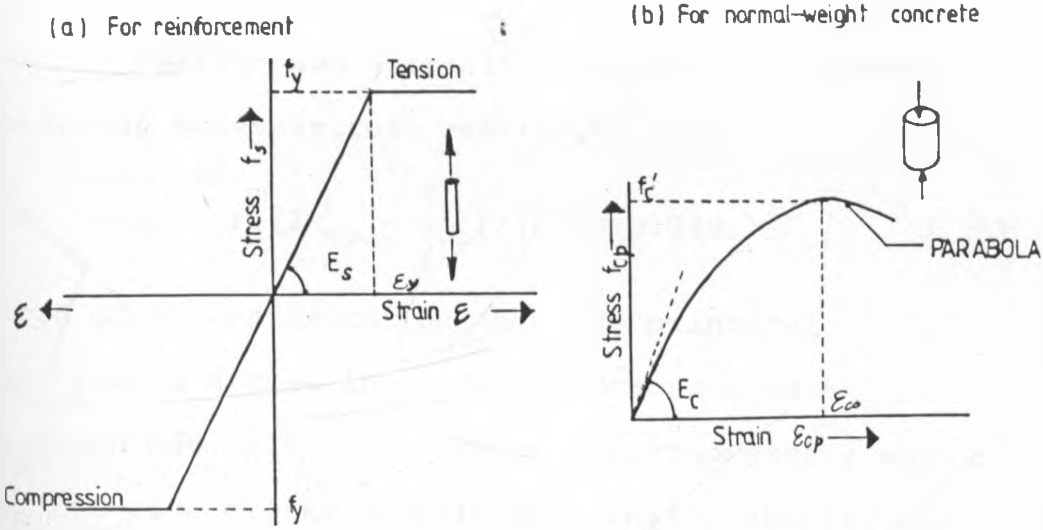


Fig. 3.12 IDEALISED STRESS-STRAIN CURVES

For the normal-weight concrete, the compressive stress-strain curve (Fig. 3.12b) is based on the parabolic expression, namely:

$$\text{stress} \quad f_{cp} = f_c' \, (2\Omega_{cp} - \Omega_{cp}^2) \qquad (3.24)$$

where $\Omega_{cp} = \varepsilon_{cp}/\varepsilon_{co}$. This expression is much simpler and reasonably accurate than the more complex relationships (of Kabaila, Hognestad and Desayi and Krishnan). Again, it was noted that for low strains (roughly, $\varepsilon_{cp}<0.3\varepsilon_{co}$) the straight-line expression may be a reasonable approximation and for high strains (roughly, $\varepsilon_{cp}>1.2\varepsilon_{co}$) the parabolic expression under-estimates the stresses.

Collins and Mitchell [6] have suggested the following stress-strain relationship:

$$f_{cp} = f'_c(2\Omega_{cp} - \Omega^2_{cp})/(0.80+0.34\Omega_t) \qquad (3.25)$$

based on the verification that the principal compressive stress in a cracked concrete is a function not only of the principal compressive strain ε_{cp} but also of the co-existing tensile strain, ε_t. The above equation is used in the solution technique.

3.4.4 Solution Technique

A solution technique for the prediction of the behaviour of a beam using the compression field theory is given in Appendix B along with the steps and equations used. A sample calculation is also provided to illustrate the procedure. Again, a computer program based on the above solution technique is provided in Appendix C.

CHAPTER 4 EXPERIMENTAL STUDY

Bearing in mind the restrictive factors such as the type of equipment available, machine capacity, laboratory space and resources, the experimental study was planned as explained in detail in the following sections.

4.1 OBJECTIVES OF THE EXPERIMENTAL STUDY

The primary purpose of the experimental study was to verify whether the wave reinforcement can be a better alternative to the vertical stirrups as shear reinforcement, to rectangular reinforced concrete beams. Accordingly the following objectives were aimed at:

(a) To test whether the wave reinforcement can be an acceptable type of shear reinforcement and also a better alternative to the vertical stirrups from the point-of-view of ultimate strength and cracking.

(b) To verify the response characteristics predicted by the compression field theory model, in particular the strains and angles of principal compression (refer Chapter 3).

The experience of experimental study was also to be used in comparing the efficiency of wave reinforcement with vertical stirrups and in formulating design recommendations involving wave reinforcement.

4.2 PARAMETERS OF THE TEST PROGRAMME

The type of shear reinforcement, namely vertical stirrups and/or wave reinforcement, the bending details associated with wave reinforcement and the amount of steel content were the basic parameters varied in the tests. Details regarding the choice of test specimens are given in Section 4.3.

4.3 CHOICE OF TEST SPECIMENS

All the specimens were rectangular non-prestressed reinforced concrete beams. The beams were simply-supported to yield a constant shearing force (neglecting self-weight), over the outer regions (Fig. 4.1). The shear force is therefore negligible in the central region.

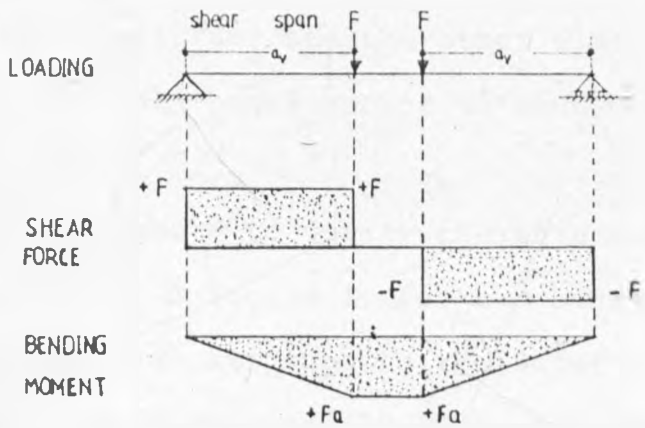


Fig. 4.1 LOADING, S.F and B.M DIAGRAMS

The cross-section of specimens were kept constant at 150 mm x 400 mm with an effective span of 2400 mm. The total length of the beams were 2700 mm. All the beams were over-reinforced to insure that shear distress would occur before flexural failure. The shear span was 1000 mm on either side.

The choice of test specimens was made to incorporate the following:

(a) Type I beams - to verify the objective (a) stated in Sec. 4.1, for two identical beams, one with vertical stirrups and the other with an approximately equal amount of wave reinforcement .

(b) Type II beams - to verify the objective (a) stated in Sec. 4.1, for two beams, each with vertical stirrups in the right shear spans and 45 deg and 30 deg wave reinforcement respectively in the left shear spans.

(c) Type III beam - to verify the objectives (a) and (b) stated in Sec. 4.1, for a beam with 45 deg wave reinforcement.

The details of manufacture of the above test specimens are explained in Section 4.4.

4.4 TEST SPECIMENS

4.4.1 Manufacture of Test Specimens

As has been noted in Section 4.3, the test specimens were grouped under three categories, and for which the details of manufacture are explained below.

(a) Type I beams (Fig. 4.2)

The two beams cast under this category were:

(i) BVS-1 (Beam with Vertical Stirrups-serial number 1) Only vertical stirrups were provided as shear reinforcement.

(ii) BWR-1 (Beam with Wave Reinforcement, serial number 1) Only 45 deg wave reinforcement was provided as shear reinforcement. But three vertical stirrups (two at the ends, one in the middle of the beam) were used to form the cage for handling purposes.

The beams BVS-1 and BWR-1 were identical, in that the reinforcing steel used were cut adjacent (from one single length) and used alternatively for the two beams. The amount of shear reinforcement was approximately equal while the amount of longitudinal reinforcement was exactly the same. The concrete produced in each batch was placed in the beams alternatively in equal quantities. The vibration and curing were carried out under similar conditions.

(b) Type II beams (FIg. 4.3)
The two beams cast under this category were:

(i) BVWR-1 (Beam with Vertical Stirrups and Wave Reinforcement - serial number 1) Vertical stirrups in the right shear span and 45 deg wave reinforcement in the left shear span were the types of shear reinforcement used.

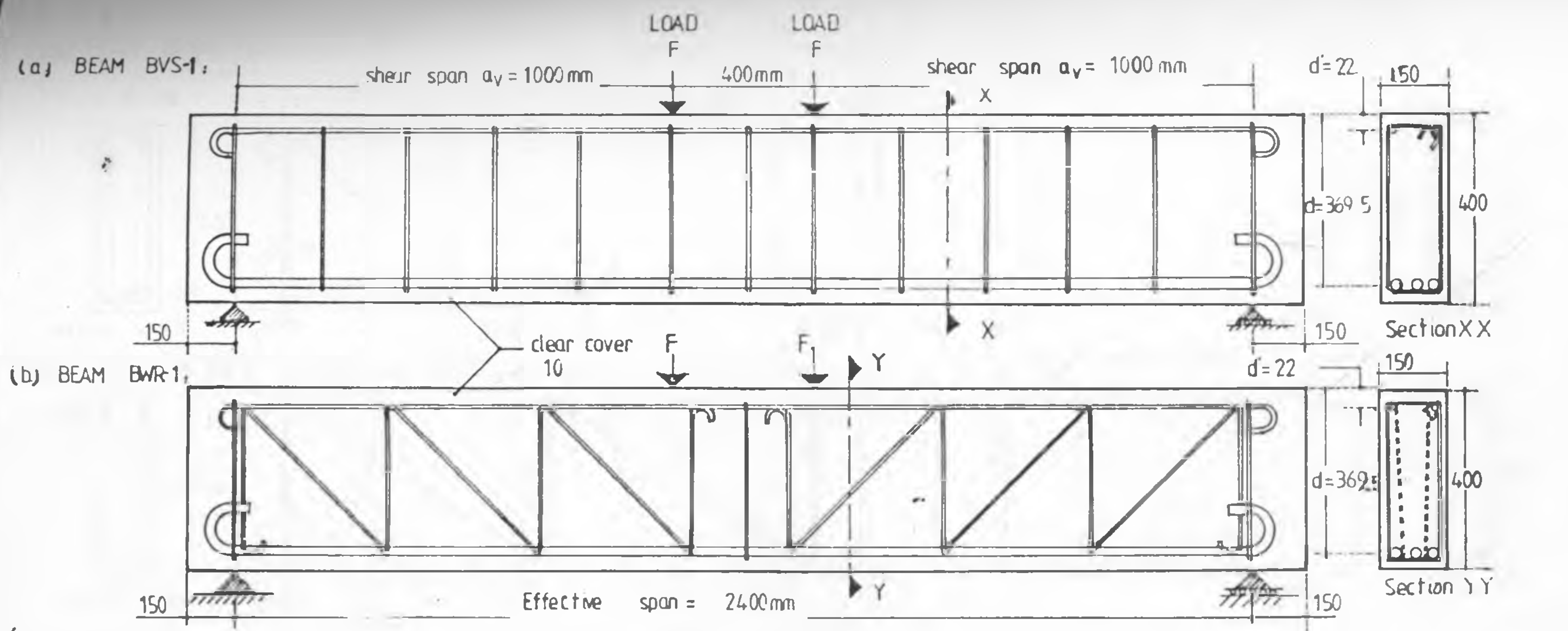
(ii) BVWR-2 (Beam with Vertical Stirrups and Wave reinforcement - serial number 2) Vertical stirrups in the right shear span and 30 deg wave reinforcement in the left shear span were the types of shear reinforcement used.

In the left shear spans of both the beams, vertical stirrups were used at all the sections where the vertical legs of the wave reinforcement existed.

(c) Type III beam (Fig. 4.4)

Under this category, a beam BWR-2 (Beam with Wave Reinforcement - serial number 2), was cast to have 45 deg wave reinforcement as shear reinforcement in addition to the vertical stirrups introduced at all sections where the vertical legs of the wave reinforcement existed.

The details of the general arrangement of steel and further beam properties are shown appropriately in Figures 4.2 to 4.4.

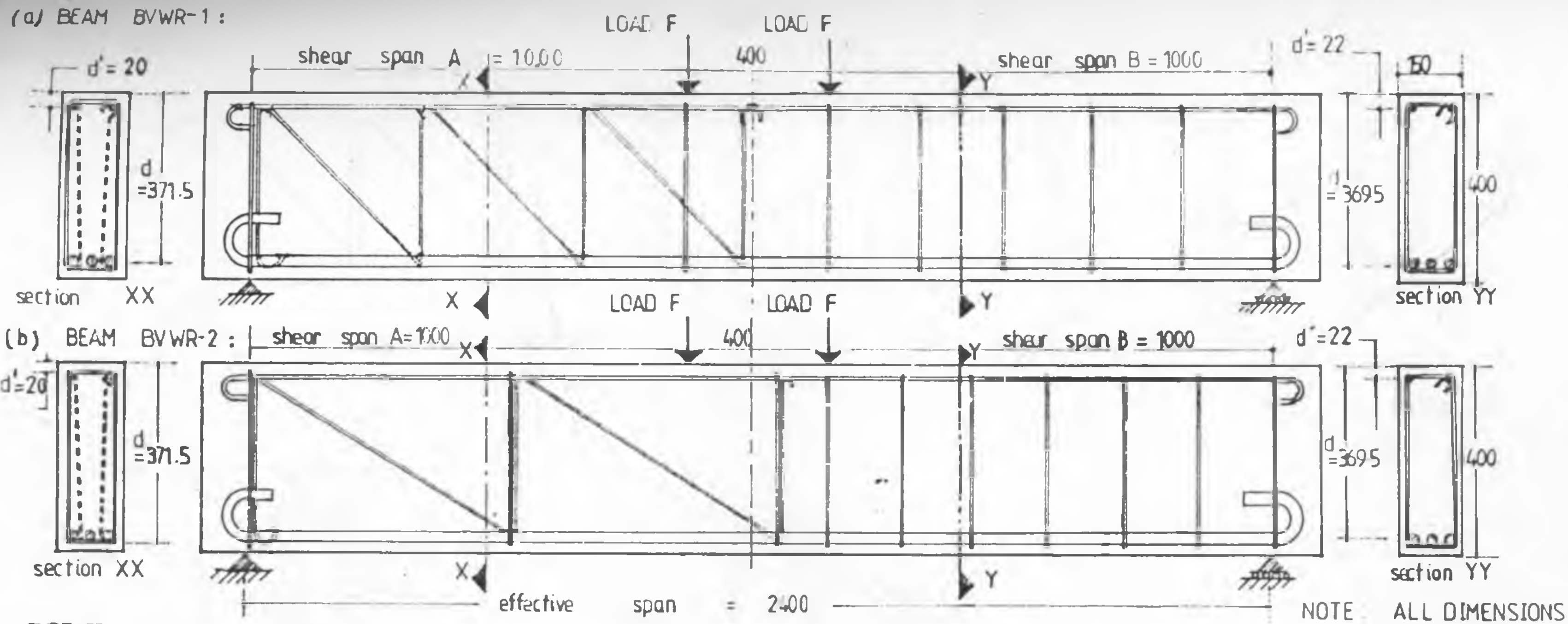


(c) FURTHER BEAM PROPERTIES :

PROPERTIES		BVS-1	BWR-1
shear span / depth ratio a_v/d		2.71	2.71
LONGITUDINAL STEEL	Top	2Y8 (A_s = 101 mm^2)	2Y8
	Bottom	3Y25 (A_s = 1470 mm^2)	3Y25
	A_s/bd	2.65 %	2.65 %
SHEAR REINFORCEMENT	Vertical stirrups ...	13Y8 - 200	3Y8
	Wave reinforcement	—	2Y8 - 370-525 (α = 45°)
	Total mass. kg	5.78	5.80

NOTE: ALL DIMENSIONS IN mm

Fig. 4.2 DETAILS OF TYPE I BEAMS

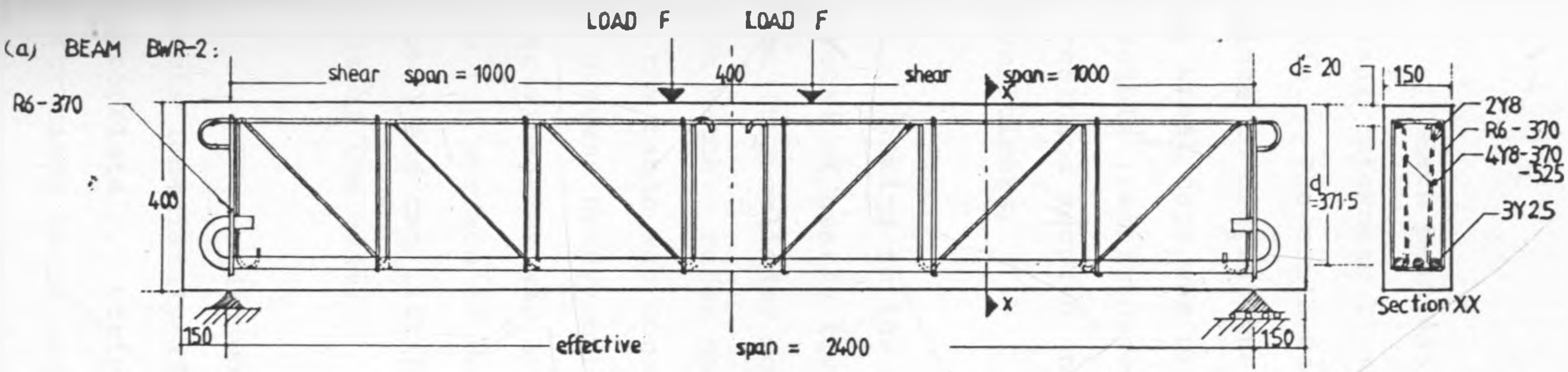


(c) FURTHER BEAM PROPERTIES :

PROPERTIES		BVWR-1		BVWR-2	
	shear span / depth ratio a_v/d	2.71		2.71	
LONGITUDINAL STEEL	Top	2Y8 (A'_s = 101 mm²)		2Y8	
	Bottom	3Y25 (A_s = 1470 mm²)		3Y25	
	A_s/bd	2.65 %		2.65 %	
SHEAR REINFORCEMENT	Shear span	A	B	A	B
	Vertical stirrups	4R6-370	7R8-200	3R6-640	6R8-165
	Wave reinforcement	2R8-370 - 525 $\alpha°$ = 45°	——	3R8-370 - 740	——
	Total mass. kg	3.28	3.11	3.78	3.56

Fig 4.3 DETAILS OF TYPE II BEAMS

(a) BEAM BWR-2:



NOTE: ALL DIMENSIONS IN mm

(b) FURTHER BEAM PROPERTIES:

PROPERTIES		BWR-2
Shear span/depth ratio a_v/d		2·69
LONGITUDINAL STEEL	Top.......	2Y8 ($A_s' = 101 mm^2$)
	Bottom.......	3Y25 ($A_s = 1470 mm^2$)
	A_s/bd.....	2·65%
SHEAR REINFORCEMENT	Vertical stirrups	8R6-370
	Wave reinforcement	2Y8-370-525 ($\alpha = 45°$)
	Total mass, kg	6·99

Fig 4.4 DETAILS OF TYPE III BEAM

Each beam was cast in an wooden mould (plank thickness 19 mm) which was set on a vibrating table. The mould was properly sealed at the joints to prevent the escape of cement slurry. The steel cage was prepared to shape as rigidly as possible (reinforcement suitably tied using steel wires) and secured into position by 10 mm thick cover blocks.

Owing to the small capacity of the mixer, a total of nearly five batches were used for each beam (two cylinder moulds were also filled from each batch - refer Sec. 4.4.2.1). Proper vibration of the table was done at each of these casting stages to prevent honey-combing of the beam (and cylinders). Over-vibration was avoided to prevent segregation. The top surface of beam was finished smooth with a trowel and two lifting hooks were placed near the top ends of the beam.

The wooden mould was wrapped with polythene sheets immediately after casting and also wetted appropriately. Stripping was done after about forty eight hours and the beam kept wet, covered with hessian for 28 days. Thereafter the beam was kept in laboratory temperature and during that time targets for strain measurements were fixed using 'araldite' quick fix. The beam was also painted white using slaked lime to facilitate the observation of cracks and for clear photography.

Beams BVS-1 and BWR-1 were cast at the same time. The beams BVWR-1, BVWR-2 and BWR-2 were cast at different times as per the planned laboratory schedule. The method of manufacture of all the specimens (placing, vibrating, curing and preparation in readiness for testing) were the same as explained earlier. Age at test varied between 30 to 45 days for all the specimens.

4.4.2 Materials

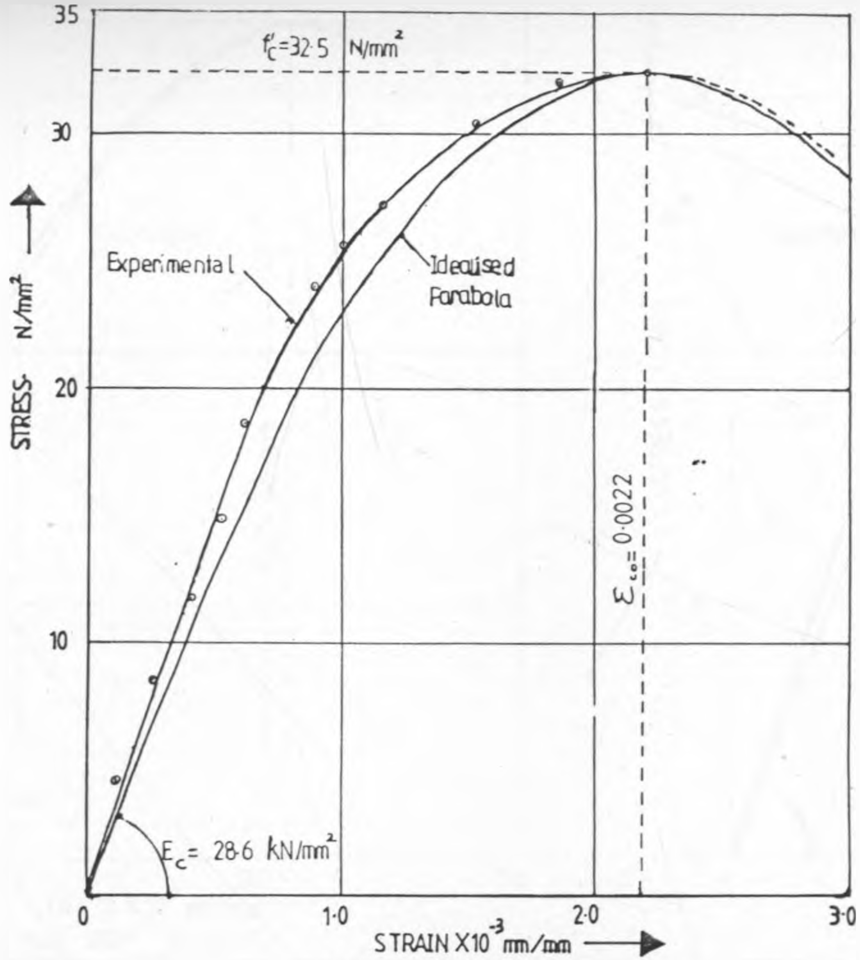
4.4.2.1 Concrete

Ordinary Portland cement, natural sand, and 9.5 mm (3/8") and 19 mm (3/4") crushed stone (normal weight aggregates), and water which was fit for drinking were used for concrete production. No admixtures were added. All concrete mix design was according to a method outlined by

Neville [17], the aggregates grading having been approximated to one of the four curves in Road Note No. 4.

Concrete compression test specimens were cast from actual batches used in casting the beams, using a standard cylinder of 150 mm (6") in diameter, 300 mm (12") long. Cylinder specimens were compacted in two layers using the bench vibrator used for the beam and the top surface finished with a trowel. For each batch of concrete, two cylinder specimens were made and hence a total of ten specimens for each beam. The specimens were standard-cured for 28 days and during that time targets were fixed at 200 mm apart on two side faces (diagonally opposite). Also the cylinders were capped with plaster-of-paris, a few hours before the test.

The cylinders were tested in the same 'as-cast' positions and 28-days stress-strain curves obtained. A standard compression testing machine was used and loads at 50 kN increments were applied without shock. The strains measured on the two pairs of targets were averaged in the plotting of the compressive stress-strain curves (Figures 4.5 to 4.7). The parabolic stress-strain relationships used to predict the behaviour of beams (to approximate the actual stress-strain relationship) are also plotted alongwith. The peak compressive cylinder strength of concrete f'_c, the corresponding strain ε_{co}, and the modulus of elasticity of concrete E_c, are also marked appropriately. The various properties of the concrete cylinder specimens are summarised in Table 4.1.



FOR BEAMS BVS-1 & BWR-1

Fig. 4·5 CONCRETE CYLINDER COMPRESSIVE STRESS - STRAIN CURVES

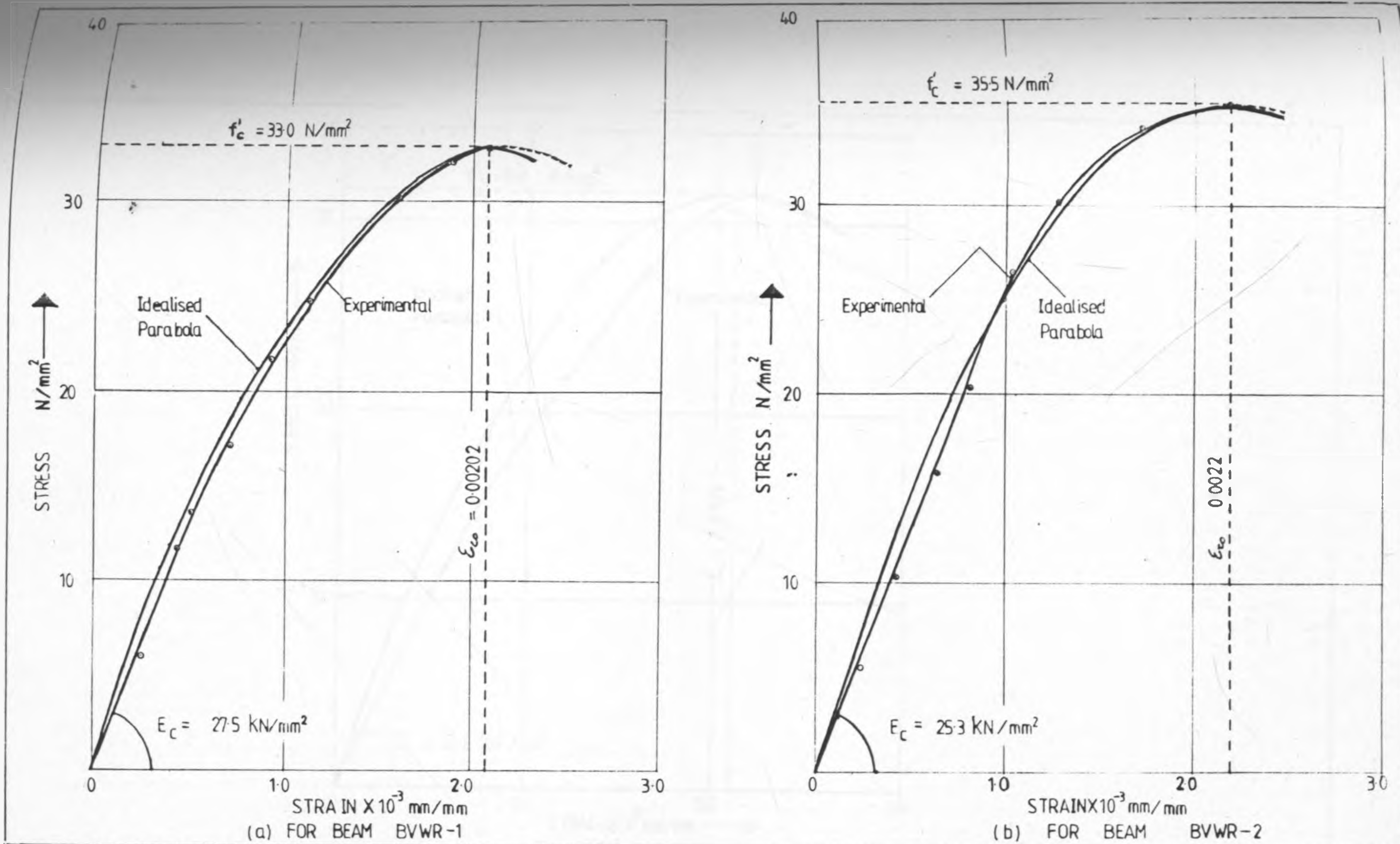
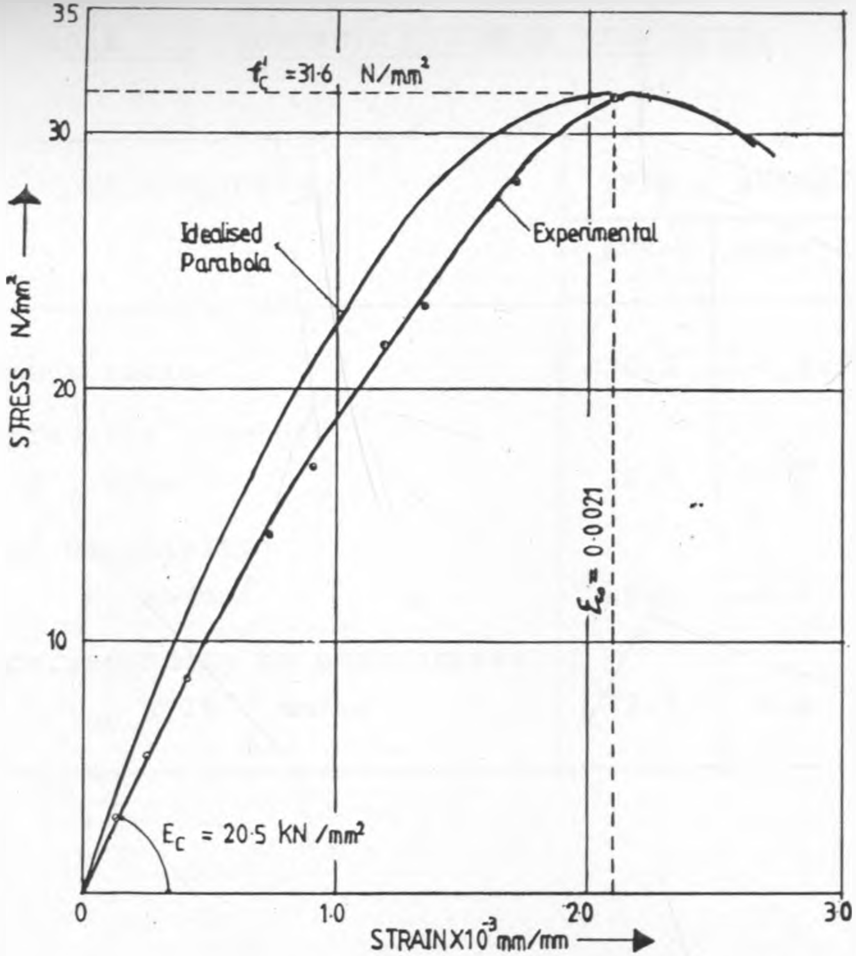


Fig. 4·6 CONCRETE CYLINDER COMPRESSIVE STRESS —STRAIN CURVES



FOR BEAM BWR-2

Fig 4.7 CONCRETE CYLINDER COMPRESSIVE STRESS-STRAIN CURVES

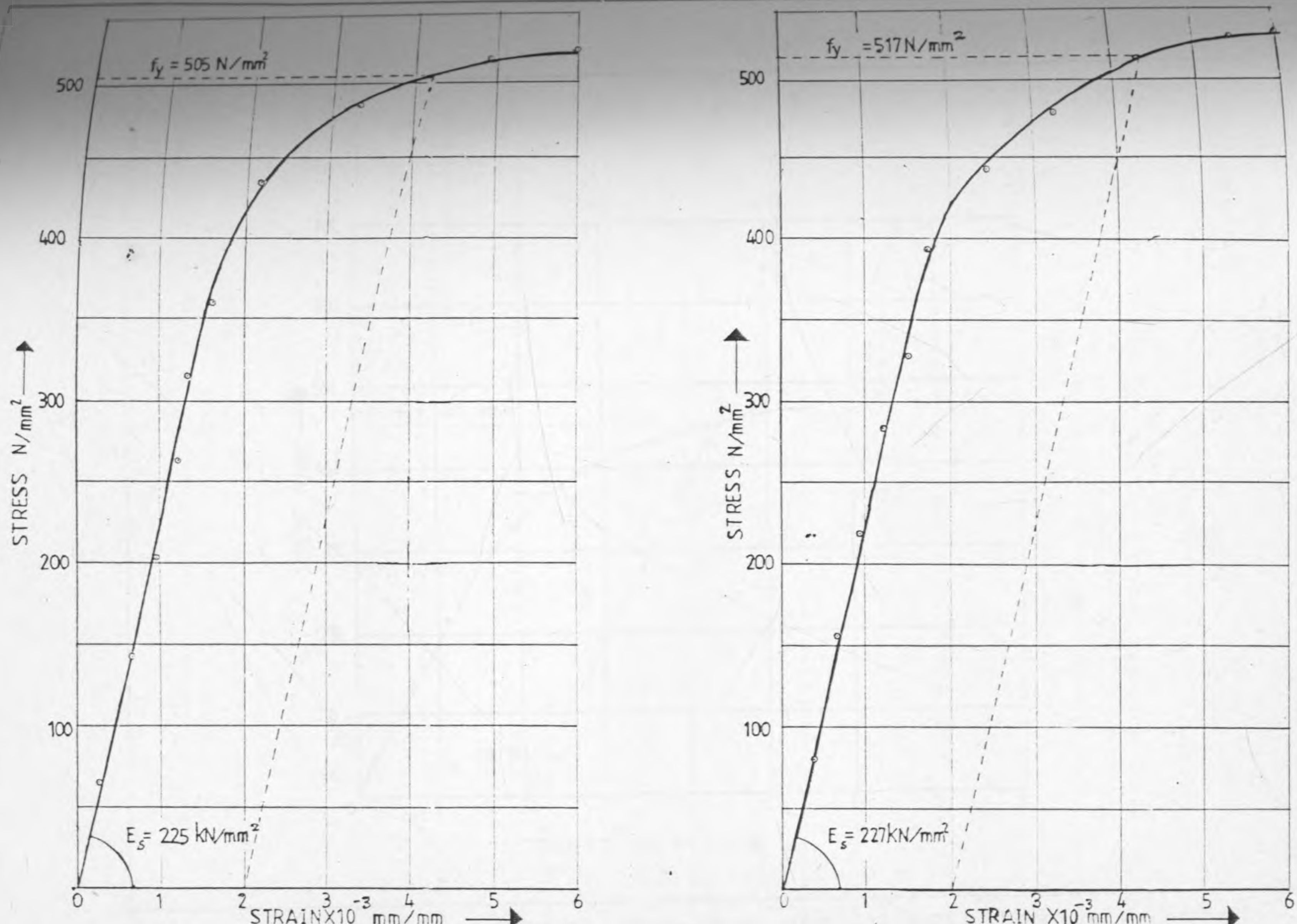
TABLE 4.1 CONCRETE CYLINDER PROPERTIES

DESCRIPTION	TYPE I BEAMS		TYPE II BEAMS		TYPE III BEAM
	BVS-1	BWR-1	BVWR-1	BVWR-2	BWR-2
Water-cement ratio	0.5	0.5	0.5	0.5	0.5
Peak compressive strength f'_c N/mm^2	32.5	32.5	33.0	35.5	31.6
Modulus of elasticity E_c kN/mm^2	28.6	28.6	27.5	25.3	20.5
Strain corresponding to peak stress $\varepsilon_{co} \times 10^{-3}$ mm/mm	2.2	2.2	2.02	2.2	2.1

4.4.2.2 Steel

The tensile testing of bars, which were representative pieces cut from the actual reinforcing bars used to make the steel cages for the specimens, consisted of straining test pieces by tensile stress to fracture. Non-proportional test pieces (or full section test pieces) with incised marking were used. At least three specimens were tested for each bar size and type used in the beam and the average was used in obtaining the tensile stress-strain curve.

The tests for Y25 bars were performed using Avery-Denison testing machine and for Y8, R8 and R6 bars using 'Instron' testing machine. Both plotted load-elongation curves from which the nominal stress-strain curves were prepared (Figures 4.8 to 4.10). The yield stress, the corresponding yield strain and the modulus of elasticity are also marked appropriately. For bars which did not have a distinct yield point, the yield stress was taken as 0.2% proof stress. The modulus of elasticity was obtained as the slope of the linear elastic portion of the curve. The various properties of reinforcing steel specimens are summarised in Table 4.2.



(a) FOR TYPE I AND III BEAMS

(b) FOR TYPE II BEAMS

Fig 4.8 TENSILE STRESS – STRAIN CURVES FOR Y25 BARS

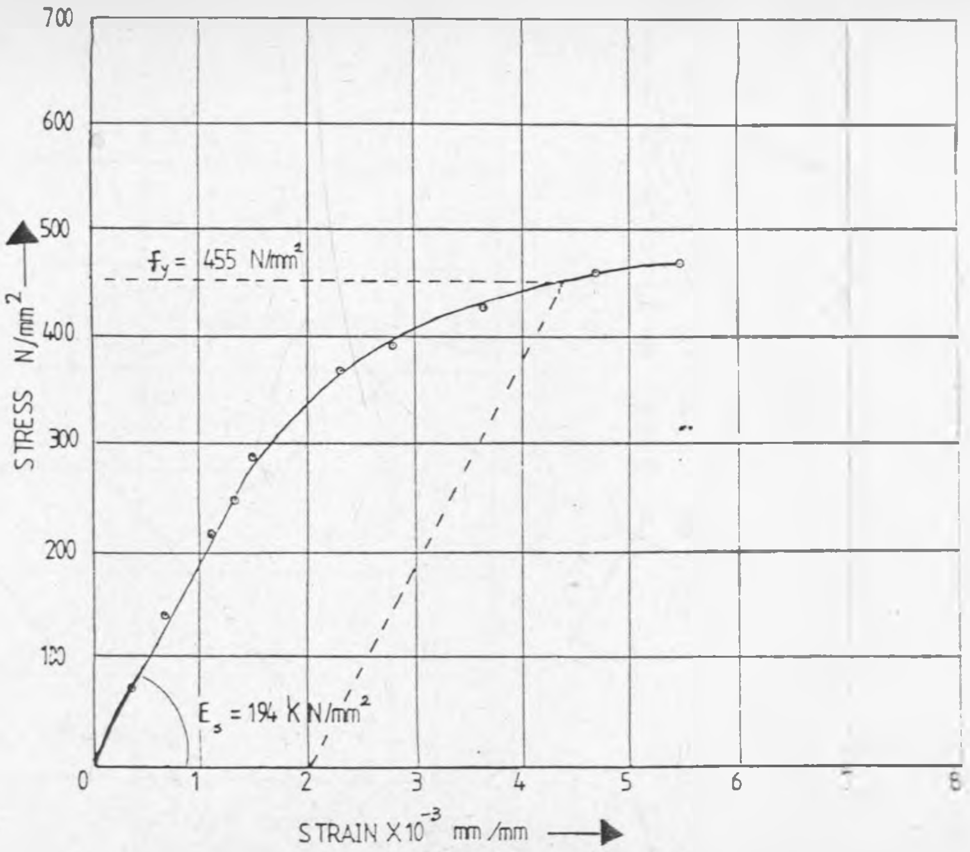
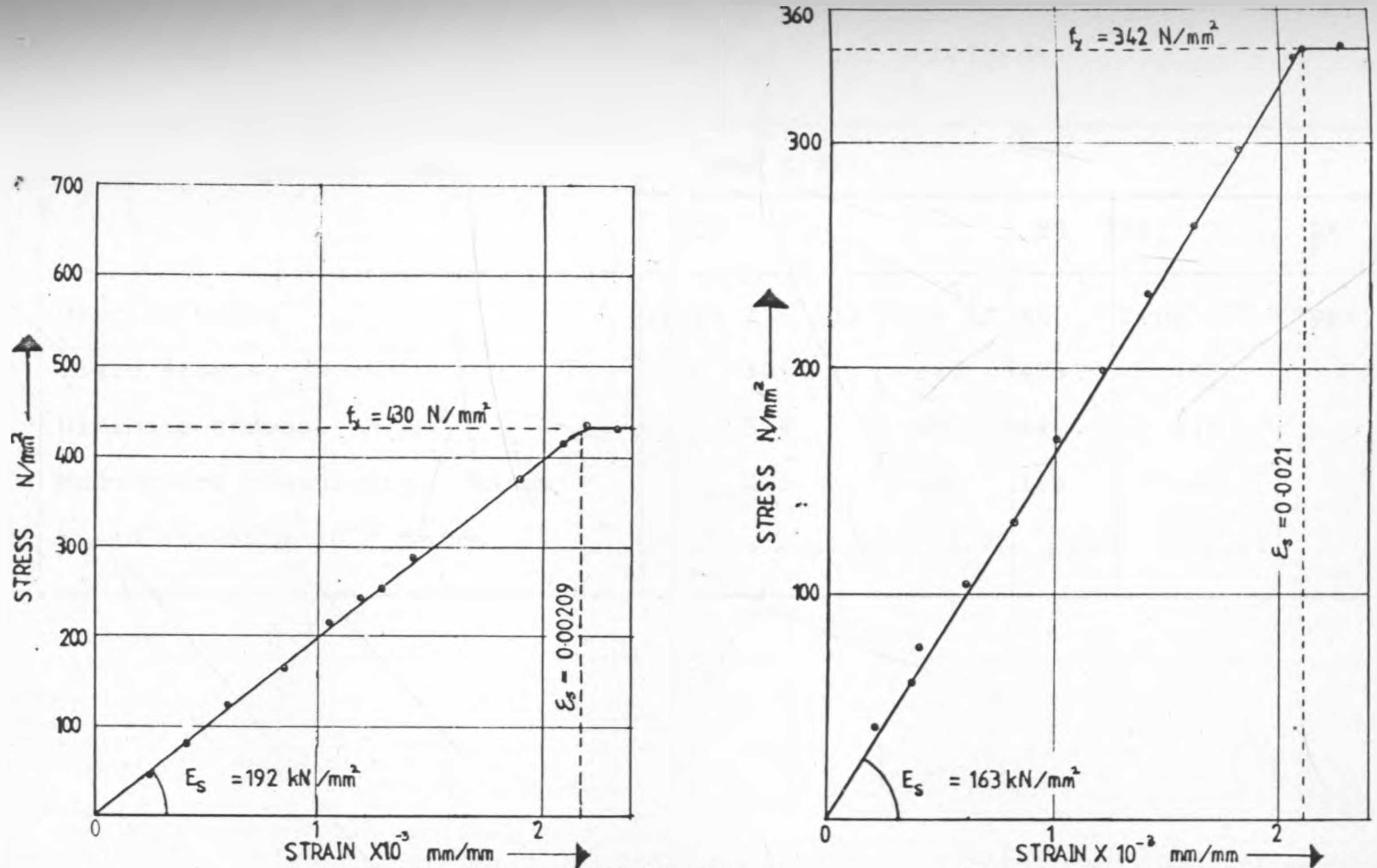


Fig. 4.9 TENSILE STRESS—STRAIN CURVE - Y8 BARS



(a) FOR TYPE II & III BEAMS – R8 BARS

(b) FOR TYPE II & III BEAMS – R6 BARS

Fig · 4·10 TENSILE STRESS—STRAIN CURVES FOR ROUND BARS

TABLE 4.2 PROPERTIES OF REINFORCING STEEL SPECIMENS

DESCRIPTION	BAR SIZES				
	Y25		Y8	R8	R6
Used in beams	Types I & III	Type II	ALL	Type II	Types II & III
Yield stress, N/mm^2	505	517	455	430	342
Ultimate stress, N/mm^2	574	602	642	430	342
Modulus of elasticity, kN/mm^2	225	227	194	192	163
Yield strain x 10^{-3} mm/mm	2.24	2.28	2.35	2.09	2.09

4.5 TEST RIG

A testing machine transmitting loads to two hydraulic loading jacks was used to test the beam. The loading jacks were positioned 400 mm apart (centre to centre) and held in stable vertical positions by suitable tie rods. The verticality was checked using plumb-bobs attached to the sides of the loading jacks. The supports to the test beam were provided by two horizontal I-beams which were bolted to rectangular framework.

The loading from the jacks was transferred to the beams through rollers kept over steel plate. It was noted that the rollers permitted the loading jack to follow the curvature of the beam easily. The readings were taken in 'pounds per square inch' and then converted into kilonewtons (kN). The typical test arrangement is shown in Fig. 4.11 and in Plate 4.1. A load sensor introduced between the beam and the loading jacks (Plate 4.2) indicated that the jacks transmitted the loads equally to the beam and that the machine reading against the transmitted load compared well.

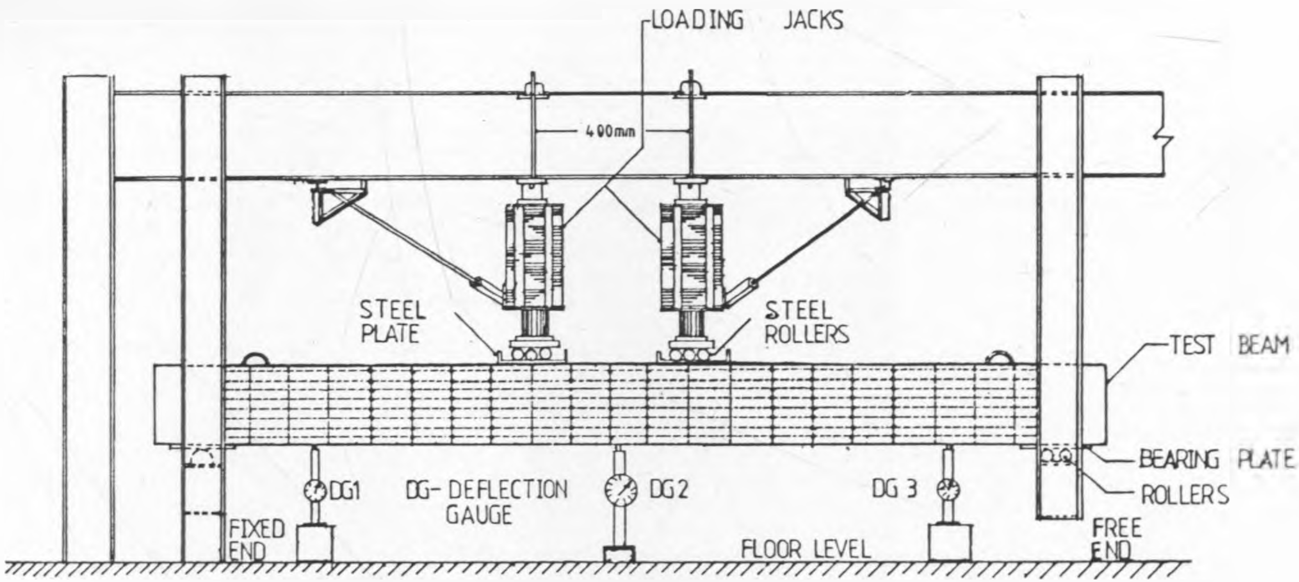


Fig. 4.11 TYPICAL TEST ARRANGEMENT

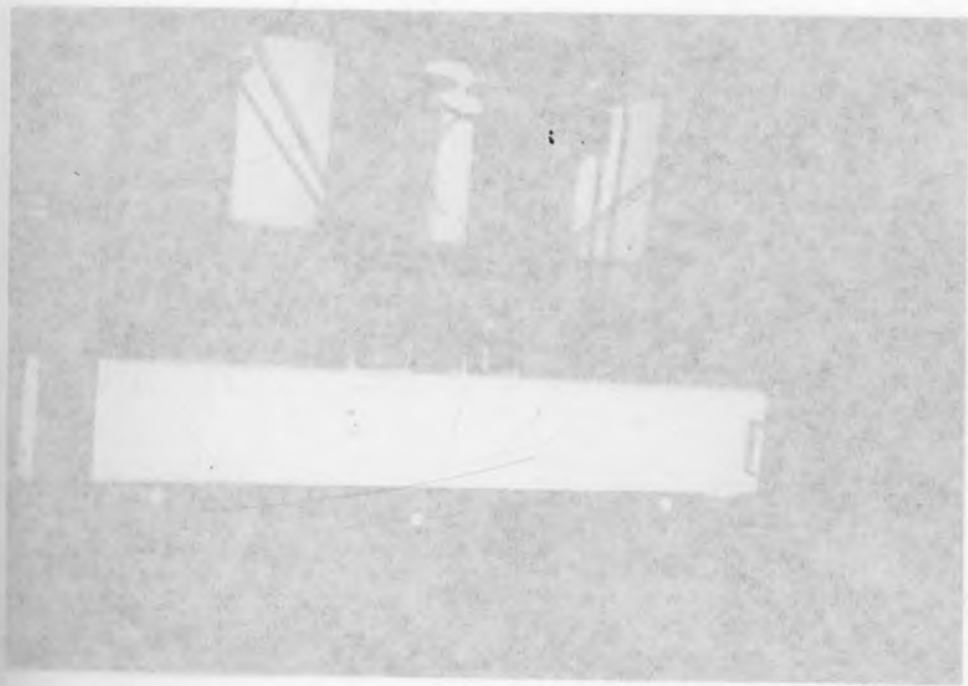


PLATE 4-1 TYPICAL TEST ARRANGEMENT

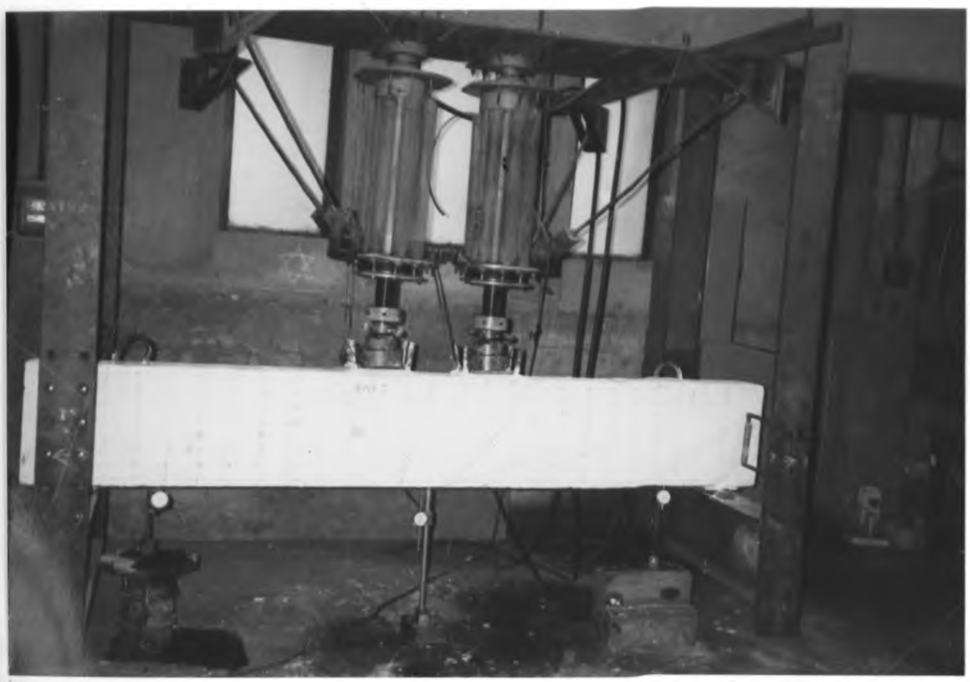
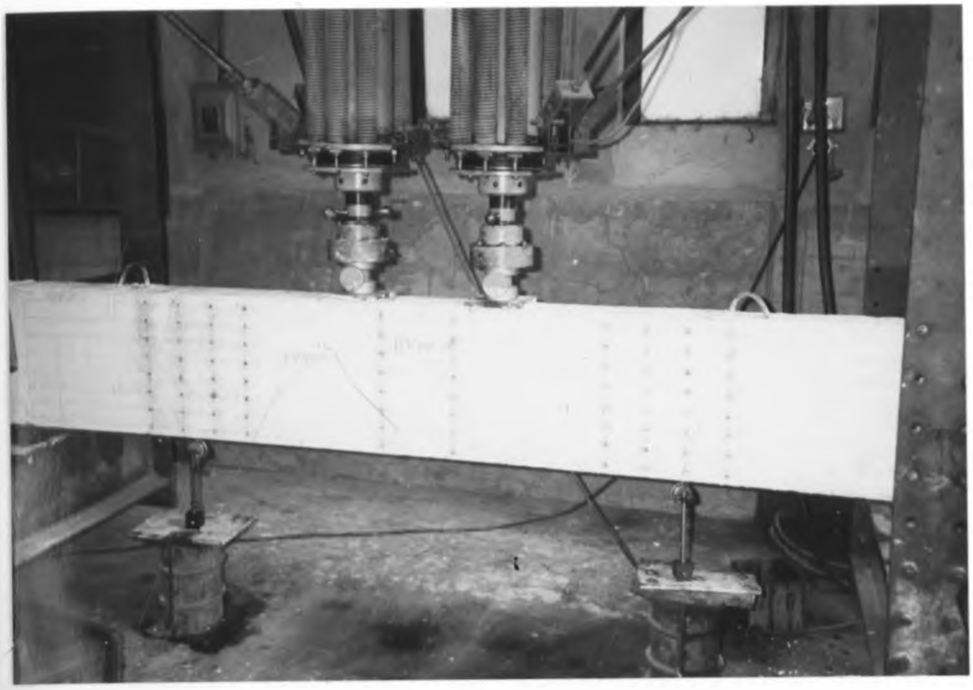


PLATE 4.1 TYPICAL TEST ARRANGEMENT

(a) with load sensors



(b) without load sensors

PLATE 4·2 TYPICAL LOADING

4.6 INSTRUMENTATION

The following instruments were used during the testing of beams:

(a) Hand-held strain gauges

Two types of hand-held strain gauges were used to measure strains between demec or target points. They are:

(i) strain gauge with 1 division = strain of 0.81 x 10^{-5}; and

(ii) strain gauge with 1 division ≡ 0.00254 mm (converted from 0.0001 inches)

The demec points were stuck using quick-fix adhesive pastes choosing a pattern of targets as shown in Fig. 4.12a, and left dry for a day before the test. The target pattern was maintained constant for all the beams. The regions of the beams approximately within a radius equal to the effective depth are normally affected by the effects of flexural moment and local disturbances of point loads [5] and hence such regions were neglected from strain measurements.

(b) Deflection gauges

Baty Push-off (magnetic base) deflection gauge with a least count of nearly 0.025 mm (converted from 0.001 inches) was used to measure deflection of the beam bottom, at selected locations. (refer Sec. 4.7).

(c) Hand-held crack microscope

A hand-held microscope with built-in reticule to read widths to the nearest 0.05 mm was used to measure crack widths. A hand-held magnifying glass was also used to locate the existence of the cracks which were not visible to the naked eye.

4.7 TESTING PROCEDURE

In order to check the general performance of the test arrangement, the beam was loaded a day earlier to the actual test to about 15% of the calculated flexural failure load and then unloaded. During the actual test, load was applied in increments of about 1/10th of the calculated flexural failure load though the increment was reduced around the flexural cracking, shear cracking and failure load.

The following observations were made at zero load and thereafter at each increment of load:

(a) Strain observations

The longitudinal, transverse and diagonal strains were measured between the targets (refer Fig. 4.12b).

(b) Deflection observations

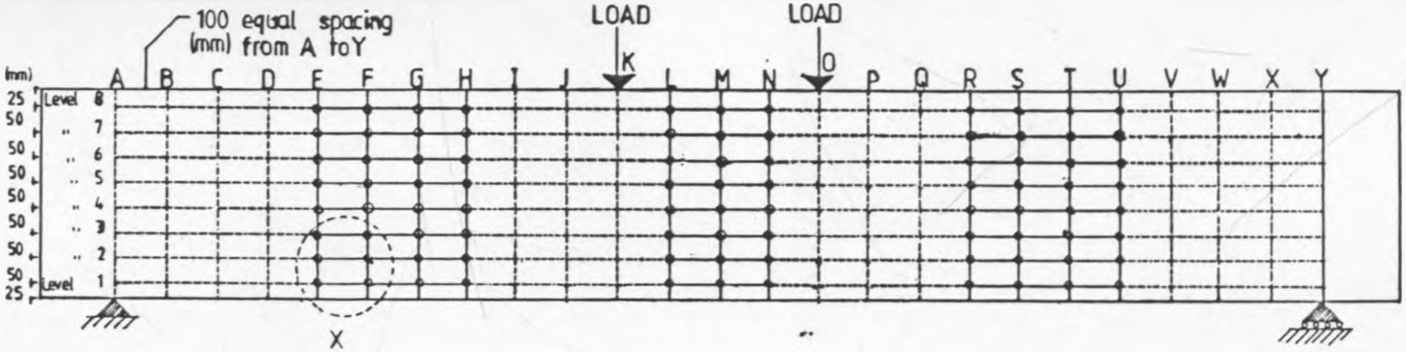
For each beam the deflection readings were taken at the bottom face, one at the centre and two more on either side in the shear spans but equidistant from the supports. Just prior to the ultimate failure the deflection gauges were removed to avoid any damage.

(c) Crack observations

The crack width at various levels was measured at 90 deg to the propagation direction and recorded. The propagation pattern of cracks was marked on both sides of the beam using a black felt pen and recorded fairly accurately. The corresponding applied moment was also marked at the locations where the cracks formed first. Photographs were also taken at different stages of crack formation and on ultimate failure.

A period of about 45 minutes was spent at every load stage for the observations and recordings. Enough care was taken to maintain the load constant during this period. The average time for the complete test on a beam from zero load to ultimate failure load was about seven hours and with an active participation of about five technicians (Typical 'observation' activity is shown in Plate 4.3). Test on each beam was performed without any interruption. The ultimate failure load was recorded for all the beams.

(a) TARGET POSITIONS ON SIDE FACE



(b) DETAILS OF STRAIN MEASUREMENTS AT X FOR A PARTICULAR LOAD

LONGITUDINAL :	E1F1, E2F2 ; E3F3, & C
TRANSVERSE :	E1E3, F1F3, E2E4; F2F4, & C
DIAGONAL :	E1F3, E2F4, F1E3 ; F2E4 ; & C

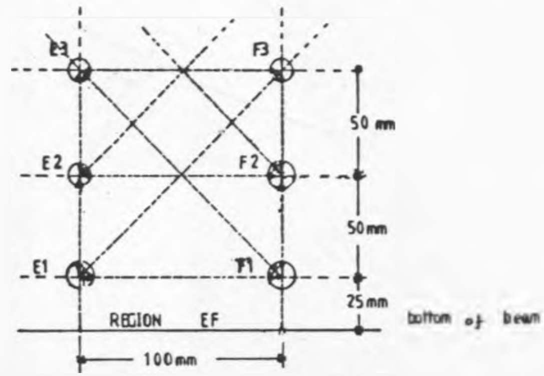


Fig. 4·12 TARGET ARRANGEMENT FOR BEAMS

PLATE 4·3 TYPICAL OBSERVATION ACTIVITIES

CHAPTER 5 ANALYSIS OF TEST RESULTS AND DISCUSSION

The chief items of behaviour of a beam which are of practical interest are the ultimate strength and deformations such as the extent of cracking and deflection under service conditions. Keeping this in mind, the data collected are appropriately analysed and discussed in this chapter. The experimental strain patterns and other related aspects are also compared.

Plates 5.1 to 5.6 provided at the end of this Chapter shows the arrangement of steel,the pattern of cracks and other details for the beams tested.

5.1 ULTIMATE STRENGTHS OF BEAMS

5.1.1 Flexural Strength

The ultimate flexural strengths of all the beams tested and their type of failures are indicated in Table 5.1. While the experimental ultimate flexural moment (M_{ue}) for each beam is obtained from the load recorded in the testing machine, the computed ultimate flexural strength (M_{uf}) is based on the flexural formula using the actual material properties.

TABLE 5.1 COMPARISON OF EXPERIMENTAL AND COMPUTED FLEXURAL STRENGTHS AND TYPE OF FAILURES

BEAMS	ULTIMATE STRENGTH (kN m)		M_{ue}/M_{uf}	Type of failure
	Experimental (M_{ue})	Computed (M_{uf})		
<u>TYPE I BEAMS</u>				
BVS-1	268.9	225.5	1.19	Shear-compression
BWR-1	242.0	225.5	1.07	Shear-tension
<u>TYPE II BEAMS</u>				
BVWR-1	220.3	217.9	1.01	Shear-compression
BVWR-2	239.2	222.2	1.08	Shear-compression
<u>TYPE III BEAM</u>				
BWR-2	240.9	212.0	1.14	Shear-compression

Based on the results of Type I beams (Table 5.1) the following observations can be made:

(a) beam BWR-1 with 45 deg wave reinforcement recorded lower experimental ultimate flexural strength than the identical beam BVS-1 with vertical stirrups.

(b) beam BWR-1 failed by shear-tension by splitting of the concrete at the level of longitudinal tensile steel by destruction of bond (Fig. 5.1b), while BVS-1 failed by shear-compression (Fig. 5.1a).

(c) the ratios M_{ue}/M_{uf} are greater than unity in both cases.

From the observations (a) and (b) mentioned above, it can be noted that the lower ultimate strength of beam BWR-1 may be attributed to the absence of vertical stirrups and hence the reduced capacity of longitudinal bars in developing bond forces necessary to share in the shear mechanism. It can also be suggested that the high concentration of inclined compression stresses would have attributed to the concrete failure near the joints of the transverse reinforcement and the longitudinal tensile steel leading to the type of failure mentioned above.

The tests on beam BVS-1 and BWR-1 have confirmed that beam BWR-1 with wave reinforcement has satisfied the ultimate flexural strength criteria and hence wave reinforcement is functionally a viable alternative type of shear reinforcement. The remedy for the splitting failure and the associated lower ultimate strength of beam BWR-1 was thought of by placing vertical stirrups at all sections where the vertical legs of the wave reinforcement exist. This combination of vertical stirrups and wave reinforcement was adopted in Type II and Type III beams. It should be noted that the vertical stirrups facilitate fabrication of a rigid cage and this eliminates the necessity of providing additional vertical stirrups exclusively for forming a cage.

The following observations may be made from the results of beam BVWR-1 and BVWR-2 (for details of beams refer to Sec. 4.4.1):

(a) both beams have satisfied the ultimate strength criteria.

(b) both beams failed by shear-compression and not by splitting at the level of longitudinal tensile steel, thus supporting the necessity of vertical stirrups alongwith wave reinforcement.

Results of beam BWR-2 confirmed that the 45 deg wave reinforcement combined with vertical stirrups has satisfied the ultimate flexural strength criteria. Also, the absence of splitting of the longitudinal tensile steel is a noteworthy feature.

5.1.2 Shear Strength

For the beams with wave reinforcement the nominal shear strength V_n is computed from $V_n = V_c + V_s$ (refer equation 2.1), where the values of V_c and V_s are worked out from the equations 7.1 and 7.5 respectively. It is also assumed that θ is equal to 45 deg. The ultimate shear strengths V_{ue} attained in all the tests for the beams with wave reinforcement are shown in Table 5.2.

TABLE 5.2 COMPARISON OF CALCULATED NOMINAL SHEAR STRENGTH V_n AND EXPERIMENTAL ULTIMATE SHEAR V_{ex}

BEAMS	CALCULATED VALUES (kN)			EXPERIMENTAL	RATIO
	V_c	V_s	V_n	V_{ex} (kN)	V_{ex}/V_n
BWR-1	52.66	110.50	163.16	242.0	1.48
BVWR-1	53.07	129.86	182.93	220.3	1.20
BVWR-2	55.04	129.86	184.90	239.2	1.29
BWR-2	52.21	136.25	188.46	240.9	1.28

It can be noted from Table 5.2 that the ratio V_{ex}/V_n is greater than unity for the beams BVWR-1 and BWR-2 (beams with 45 deg wave reinforcement combined with vertical stirrups). Thus the shear strength calculated using Eq. 3.5 compared well with the actual shear strength of the specimens listed. This confirms the validity of equations derived for the design of wave reinforcement.

5.2 CRACKING

The vital stages of formation of cracks for all the beams tested are as follows:

(a) Initially, flexural cracks developed and they were almost vertical.

(b) Subsequently, with the addition of loads, inclined shear cracks developed. Further widening of these cracks and flexural cracks led to secondary cracks.

(c) Finally, with the exception of beam BWR-1, the failure of beams occurred with the extension of critical shear crack into the compression zone. The concrete above the upper end of the crack crushed, resulting in shear-compression failure. But the beam BWR-1 failed by the splitting of the concrete at the level of longitudinal tensile steel and destruction of bond, resulting in shear-tension failure.

The cracks in the pure moment section were vertical whereas the cracks in the shear spans curved towards the applied load points. It is also worth noting that the inclined cracks formed at a distance larger than the depth, d of the beams from the support and this may be attributed to the effect of vertical compression due to support reactions in the immediate vicinity of the supports.

The crack pattern for Type I to III beams alongwith the critical crack and the load in kN, at which a particular crack first formed, are illustrated in Fig. 5.1 to 5.3.

Referring to Fig. 5.2a and b, it can be seen that the critical crack in beam BVWR-1 formed in the shear span with vertical stirrups whereas in beam BVWR-2, it formed in the shear span with 30 deg wave reinforcement. Hence it may be suggested that 45 deg wave reinforcement is superior to either 30 deg wave reinforcement or vertical stirrups. The number of cracks are nearly the same in both the shear spans of beams BVWR-1 and BVWR-2.

Referring to Fig. 5.3 it can be seen that the beam BWR-2 developed fewer cracks than Type I and II beams.

From the typical maximum crack width (flexural and shear) against the ratio of applied load to ultimate load plotting (Fig. 5.4), it is evident that maximum crack width was the least in beam BWR-2.

Based on the extent of cracks and the formation of critical cracks in all the test beams, it can be suggested that the 45 deg wave reinforcement with the combination of vertical stirrups (beams BVWR-1 and BWR-2) is an acceptable type of shear reinforcement and is also a better type than vertical stirrups or 30 deg wave reinforcement.

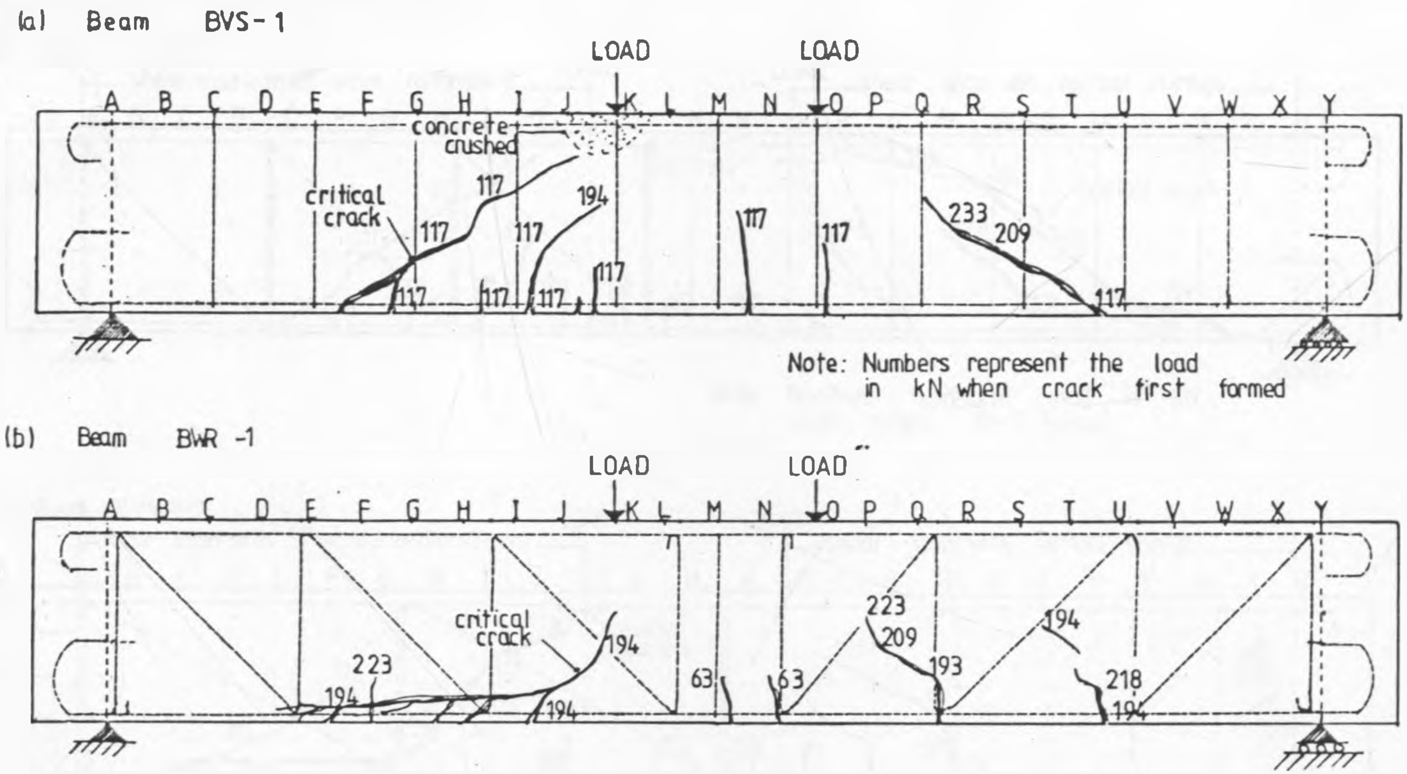


Fig. 5.1 CRACK PATTERN FOR TYPE I BEAMS

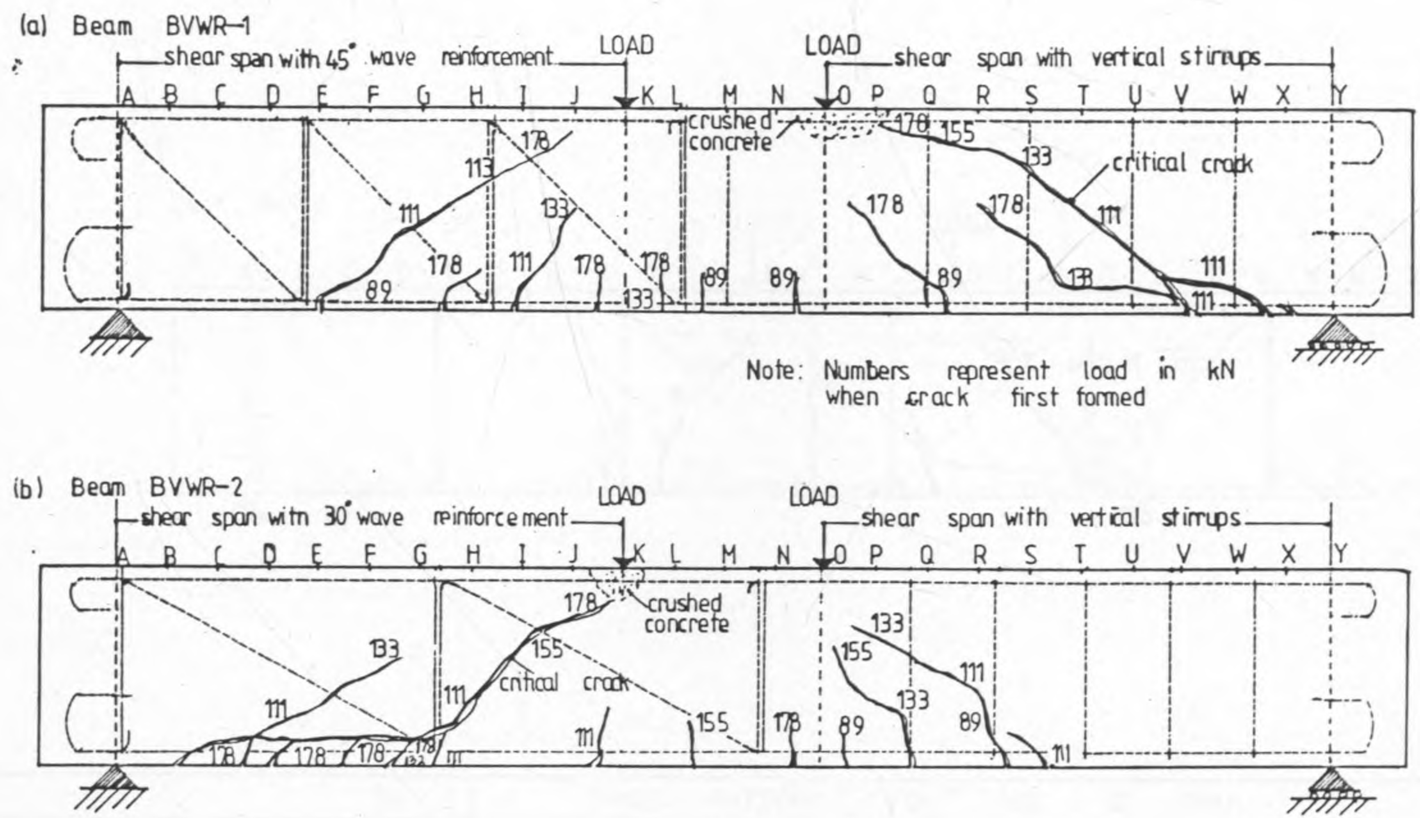


Fig. 5.2 CRACK PATTERN FOR TYPE II BEAMS

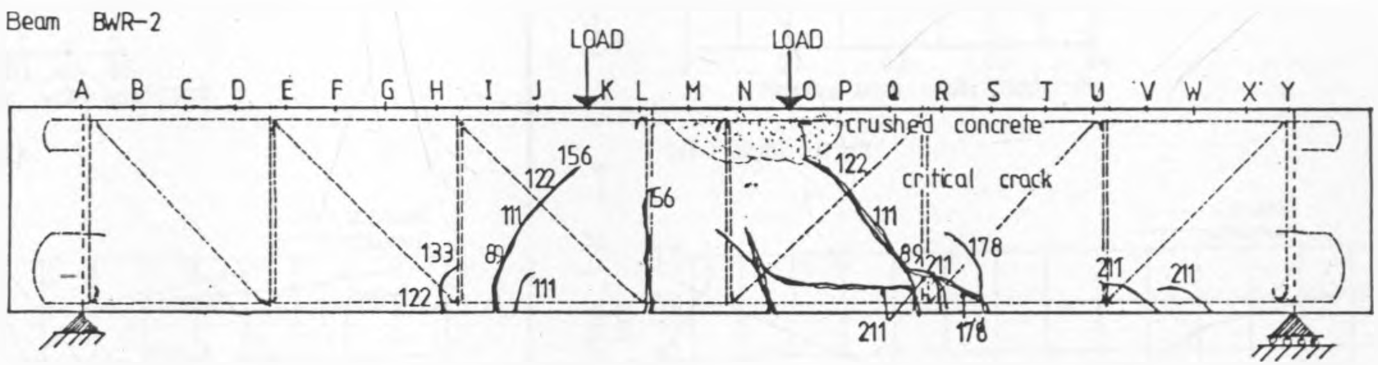


Fig. 5.3 CRACK PATTERN FOR TYPE III BEAM

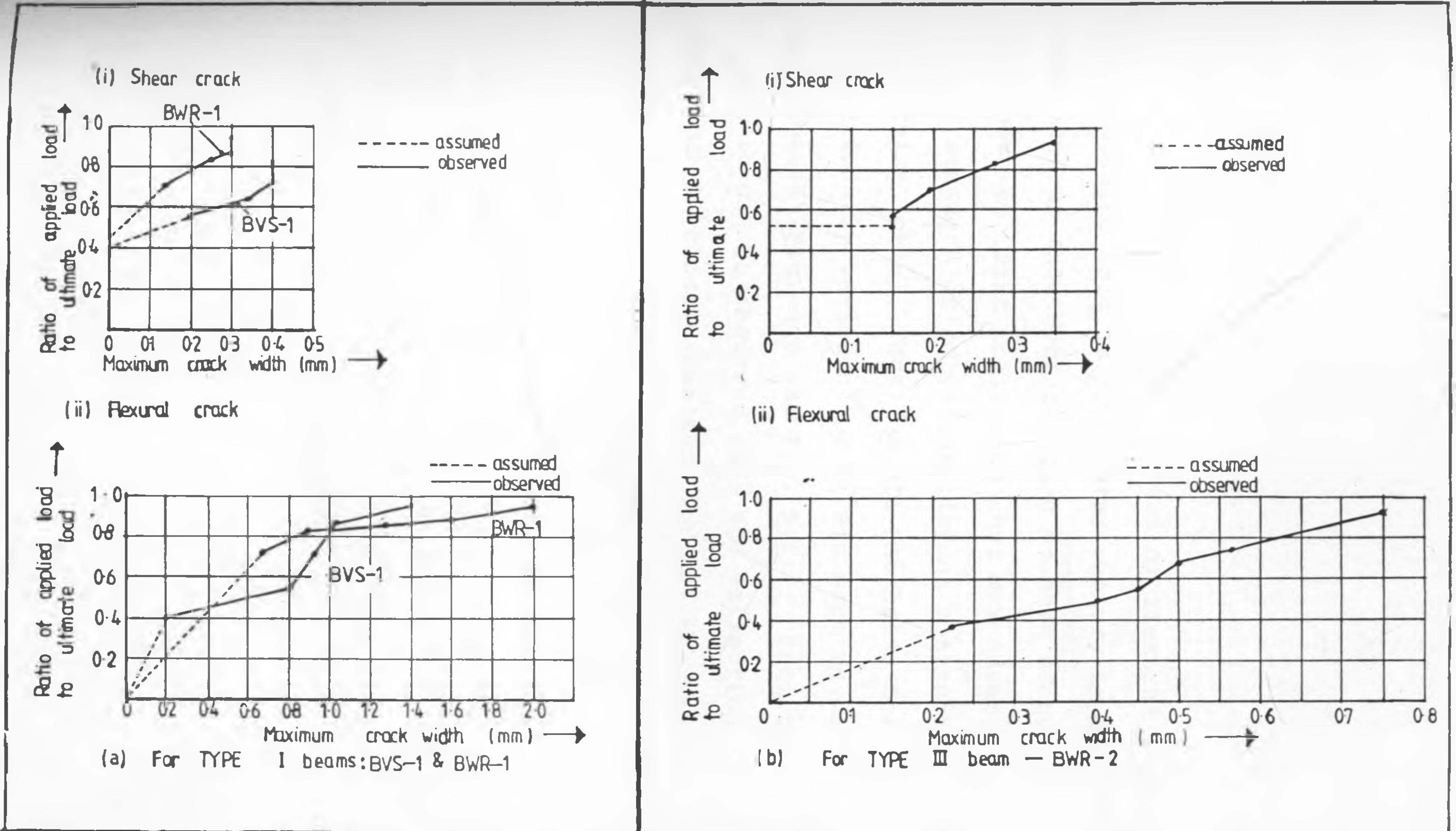


Fig. 5.4 MAXIMUM CRACK WIDTH AGAINST RATIO OF APPLIED LOAD TO ULTIMATE LOAD

5.3 DEFLECTION

The study of deflections of flexural members under service load is of greater sensory importance. Such members should be designed to have adequate stiffness to limit deflection that may adversely affect strength or serviceability of a structure at service loads. Thus for all the beams tested, deflections at various load levels were measured and plotted (Fig. 5.5 and Fig. 5.6). From the graphs it can be observed that the deflection behaviour is closely the same for all the beams tested.

It can also be noted that the maximum deflection at 0.5 times the ultimate load was about L/450 for beams with wave reinforcement and about L/350 for beams with vertical stirrups (where L is the effective span of the member).

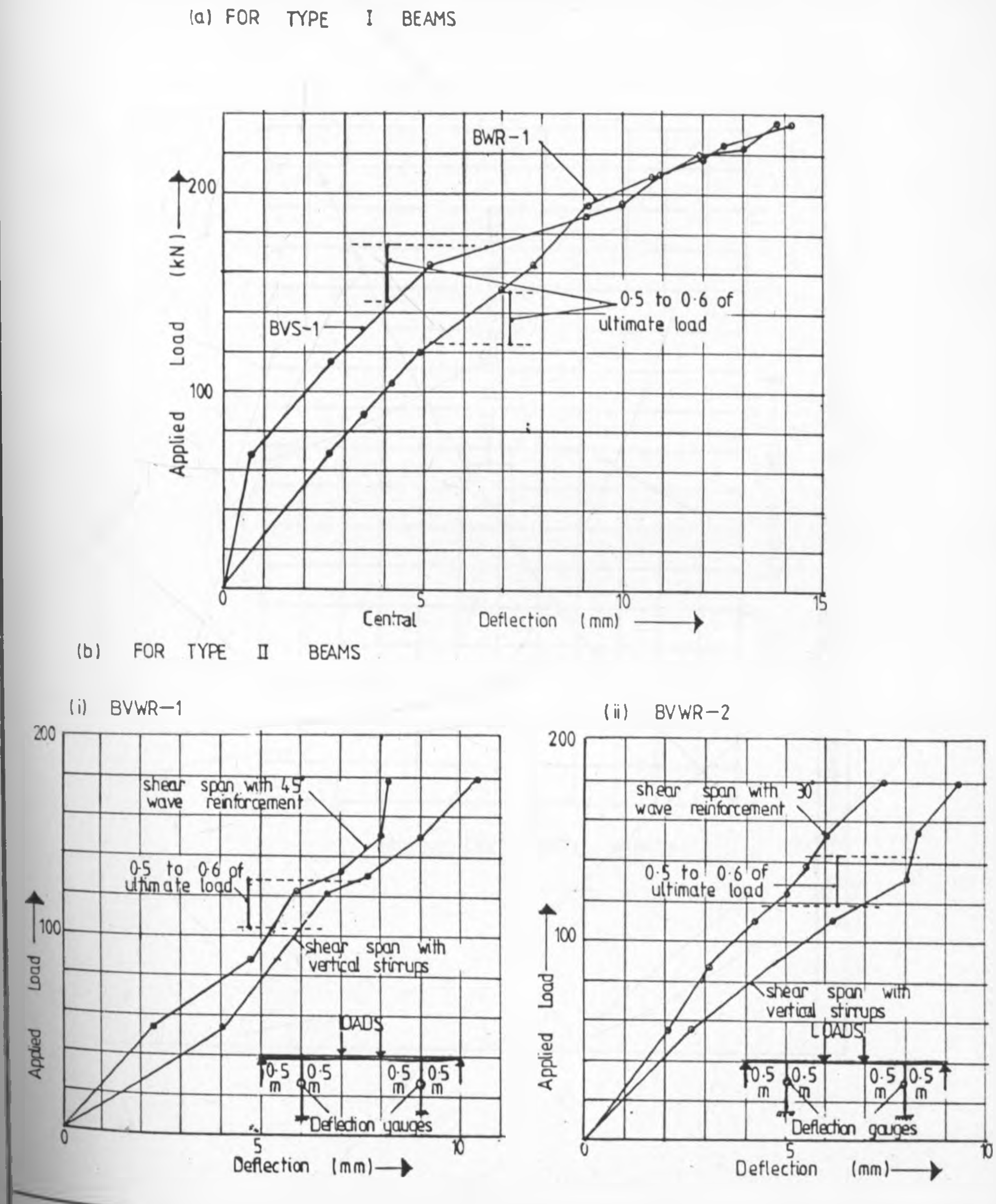


Fig. 5.5 MEASURED LOAD —VERTICAL DEFLECTION BEHAVIOUR

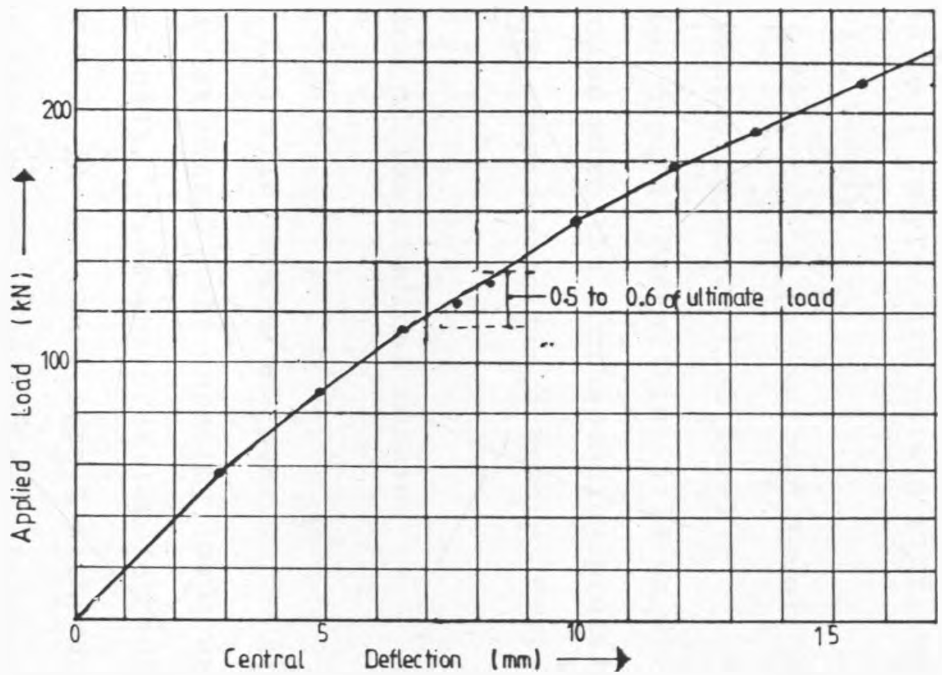


Fig. 5.6 MEASURED LOAD—VERTICAL DEFLECTION BEHAVIOUR - BWR- 2

5.4 STRAIN PATTERNS

The average longitudinal and transverse strain and diagonal compressive strain profiles at two load levels, namely

(a) at nearly half capacity load level, and

(b) at nearly capacity load level

are plotted for typical cases (Fig. 5.7a to d). They are compared with the corresponding profiles predicted using compression field theory (CFT).

(a) Variation of longitudinal strains

Fig. 5.7a illustrates the average longitudinal strain profile at nearly half capacity loads and full capacity loads. From the profiles for beams BVWR-1 and BVWR-2, in the shear span A with wave reinforcement the average longitudinal strain curves are linear over the depth of the beam at both the load levels.

For the beam BWR-2 (Fig. 5.7b) the experimental and predicted profiles are fitted for half capacity and full capacity load levels. Certain disparities exist between the two profiles, which may be due to the non-uniform cracking behaviour or due to the influence of loading. But from Fig. 5.10 it can be noted that longitudinal strain profile at pure moment section is linear with correlation coefficient close to unity and provides a clear picture of strain variation over the depth of beam.

(b) Variation of transverse strains:

A typical transverse strain profile for the beam BWR-2 is illustrated in Fig. 5.7c. Again the influence of loading system and possible inaccuracy in demec gauges would have caused the disparity in shapes but at approximately half capacity load level the curves are identical.

(c) Variation of diagonal compressive strains:

The diagonal compression strain profile (Fig. 5.7d) at the above mentioned two load levels has a similarity in shape. Again the predicted curve agreed closely with the experimental results.

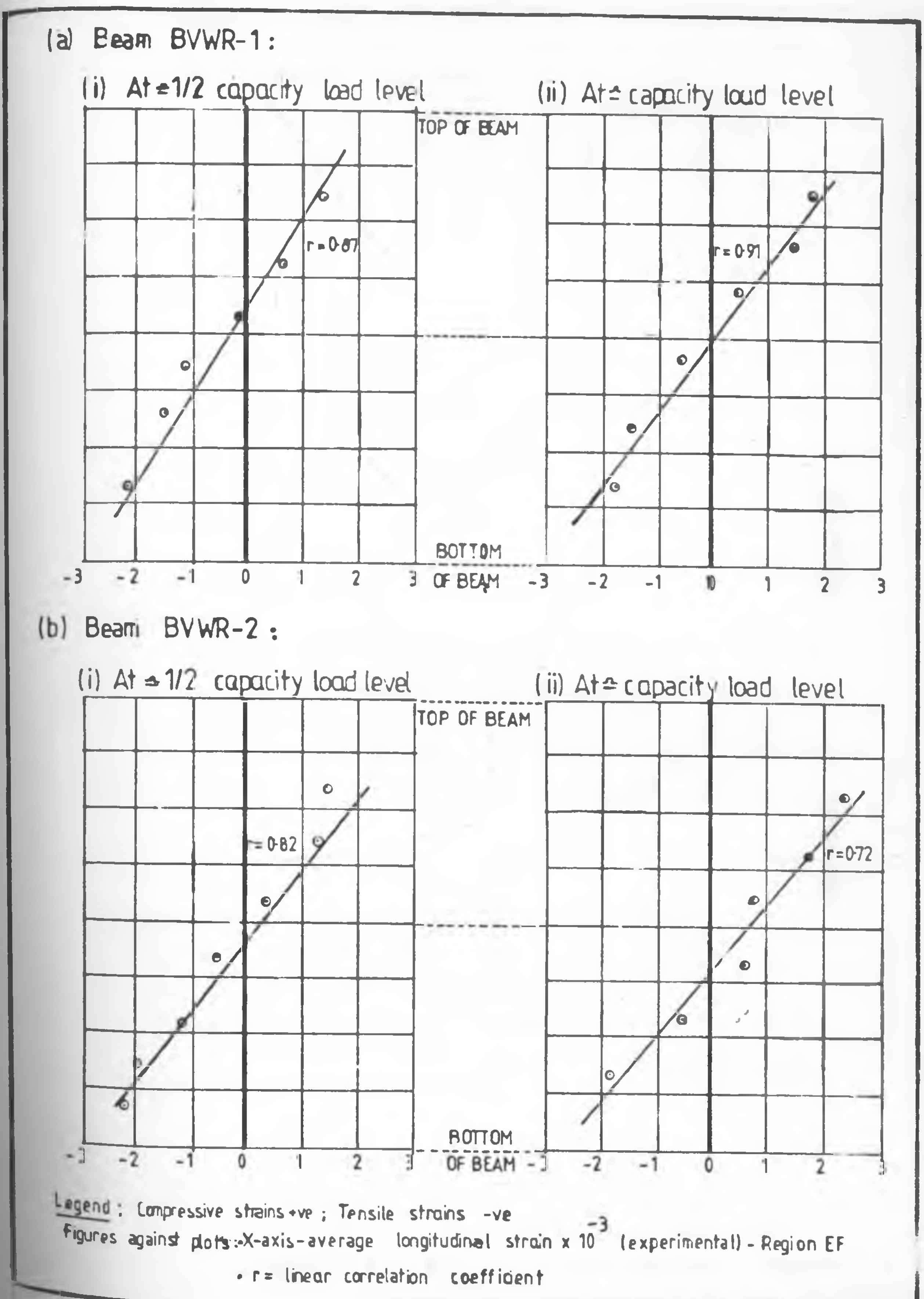
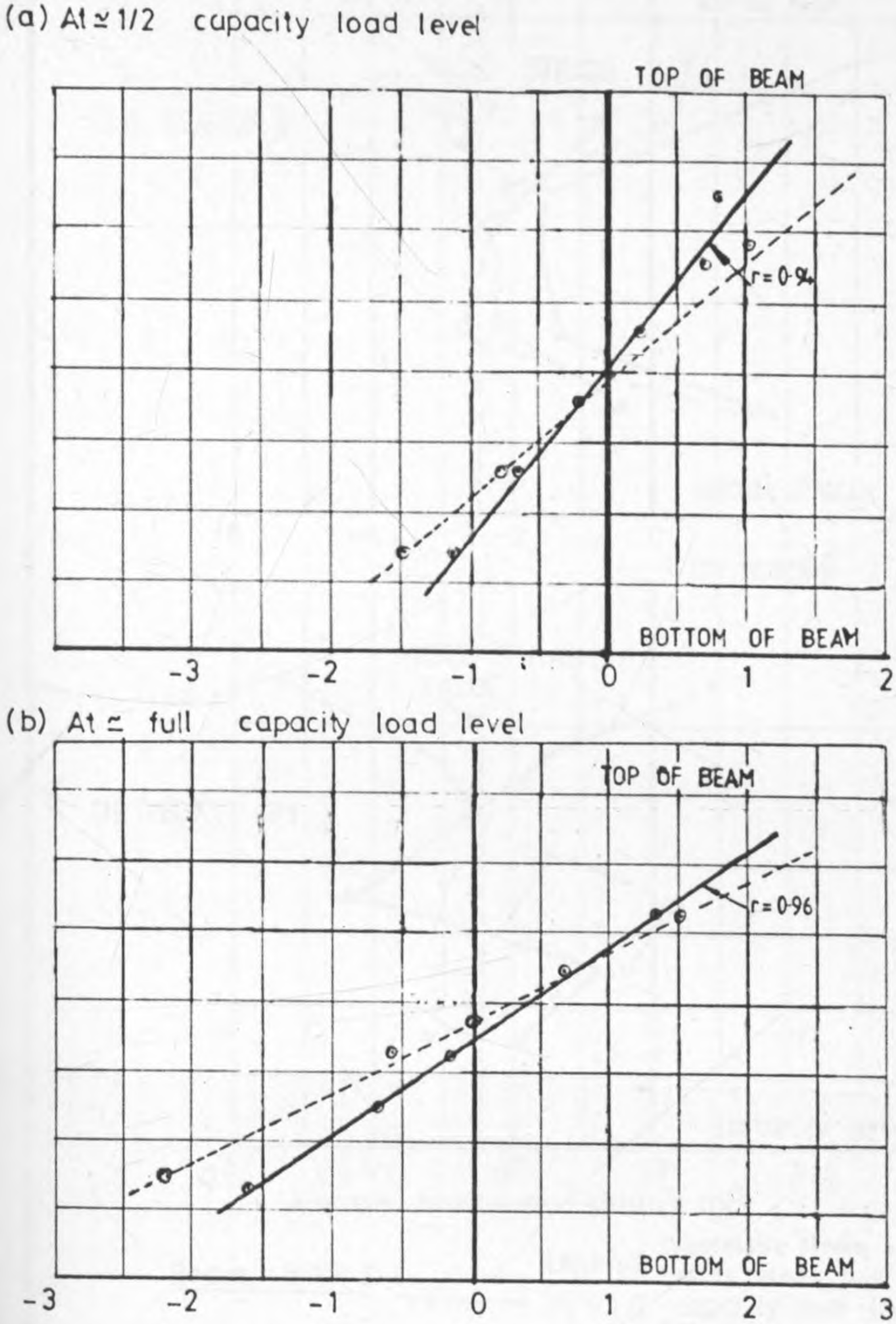


Fig. 5·7 (a) AVERAGE LONGITUDINAL STRAIN PROFILES



Beam BWR–2

- Compressive strains +ve
- Tensile strains -ve
- EXPERIMENTAL ——o——
- PREDICTED By ----⊗---- CFT

Legend:

Figures against plots:

- X-axis - average longitudinal strain X 10^{-3}
- r - linear correlation coefficient

Fig. 5·7 (b) AVERAGE LONGITUDINAL STRAIN PROFILES

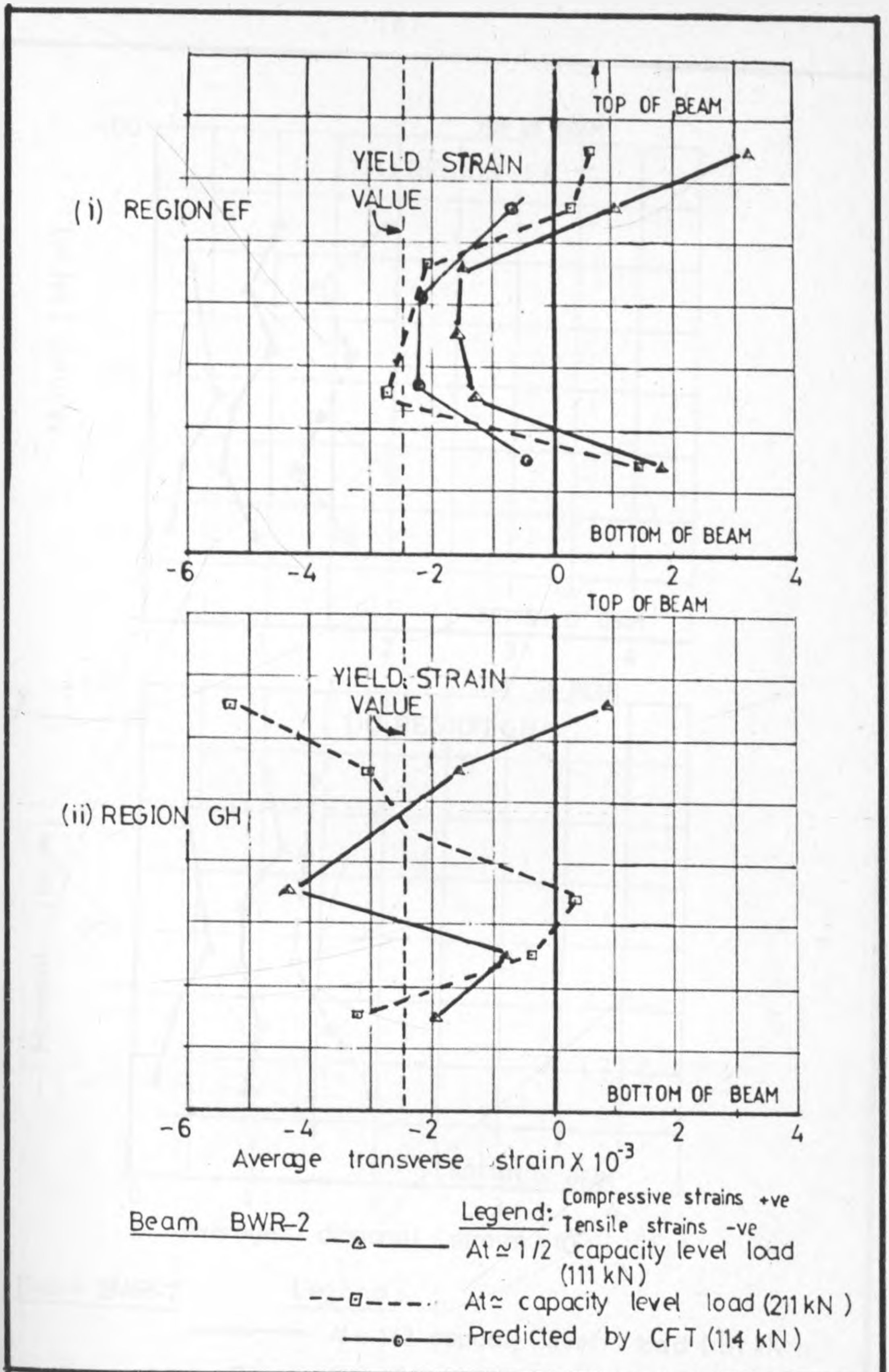


Fig. 5.7 (c) TRANSVERSE STRAIN PROFILE

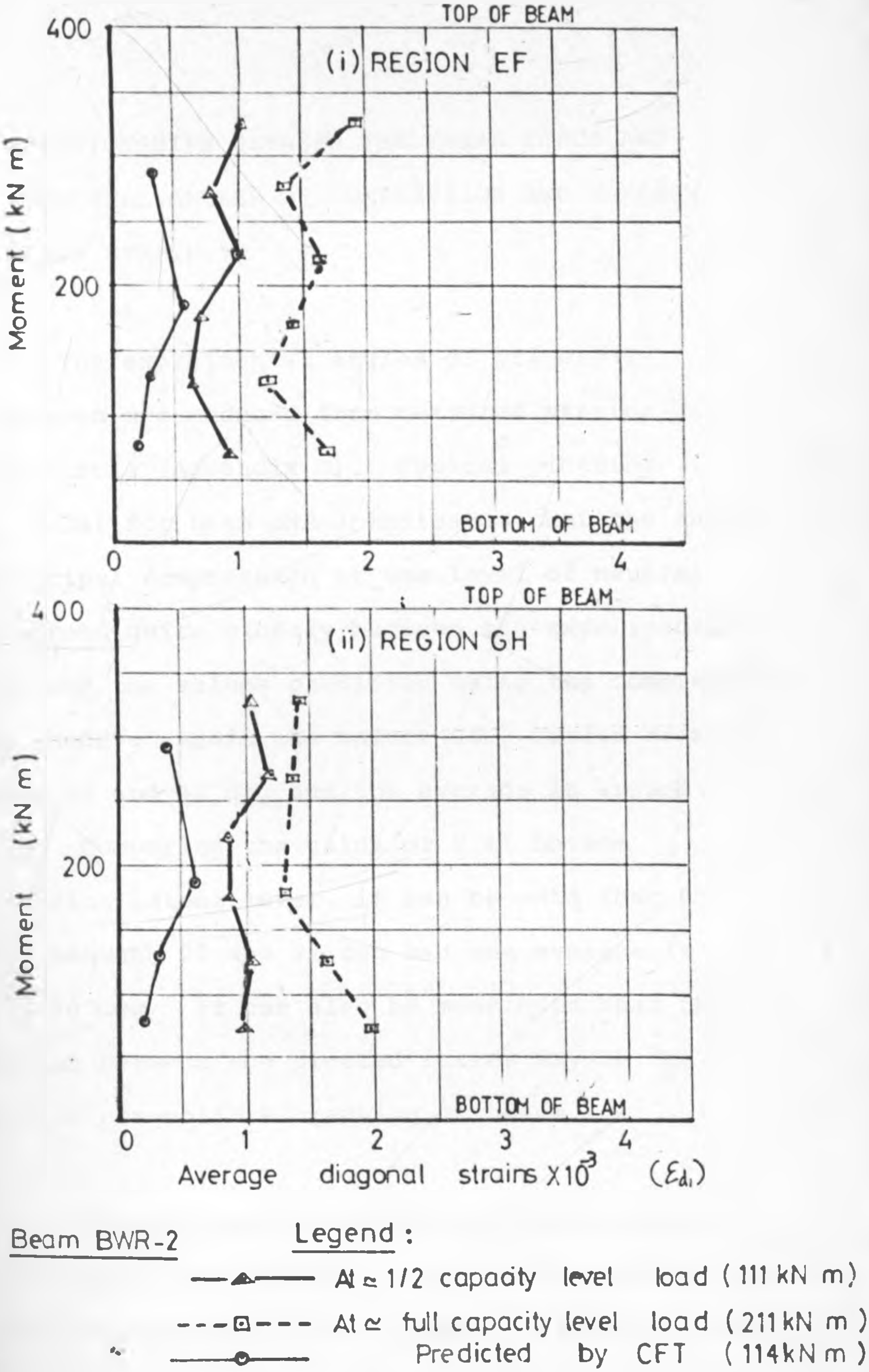


Fig. 5.7 (d) DIAGONAL COMPRESSIVE STRAIN PROFILES

5.5 RELATIONSHIPS BETWEEN THE SHEAR FORCE AND PRINCIPAL ANGLES OF COMPRESSION AND MAXIMUM SHEAR STRAIN

The experimental angles of principal compression are deduced from measured strains using Mohr's circle (Appendix D). Typical plotting (Fig. 5.8a) for beam BWR-2 indicates that the angles of principal compression at the level of neutral axis agreed quite closely between the experimental values and the values predicted using the compression field theory. Again the values of θ varied nearly between 20 and 40 deg and the average is around 31 deg. Comparing the value of θ at bottom longitudinal steel level, it can be said that θ varied between 22 and 62 deg and the average is around 40 deg. It can also be mentioned that the variation between the plotted curves may be due to effect of non-uniform cracking of beams.

Typical shear force-maximum shear strain relationships are plotted in Fig. 5.8b. Within the range of experimental data a close similarity exists between the experimental and predicted curves, though the predicted shear strains are generally greater than those obtained from experiment. Hence the theory predicts maximum shear strains conservatively.

5.6 RELATIONSHIP BETWEEN MOMENT-CURVATURE ($M - \phi$ curves)

Based on the test results, the experimental M-ϕ curves are obtained for the test beams BVS-1, BWR-1 and BWR-2 (Fig. 5.9a and b). Using the strain profiles (some of which are shown in Fig. 5.10) the curvature ϕ for a given section of the beam corresponding to the moment M is calculated as,

$$\phi = (\varepsilon_{ct} + \varepsilon_{\ell})/d$$

where ε_{ct} is the concrete strain at the extreme compression fibre and ε_{ℓ} is the strain at longitudinal tensile steel.

The theoretical M-ϕ curves for flexure are fitted by calculating the moment and curvature corresponding to a range of ε_{ct} values. For an assumed ε_{ct}, the neutral axis depth is adjusted until the stresses in the concrete and steel result in internal forces to balance. The material properties are used in computing the stresses. Strains are calculated on the assumption that the plane sections before bending remain plane after bending. Another M-ϕ curve based on the compression field theory (Appendix B and C) for the beam BWR-2 is also fitted. It can be noted that the experimental and the theoretical curves compared quite closely and the ultimate loads predicted correspond with the experimental results.

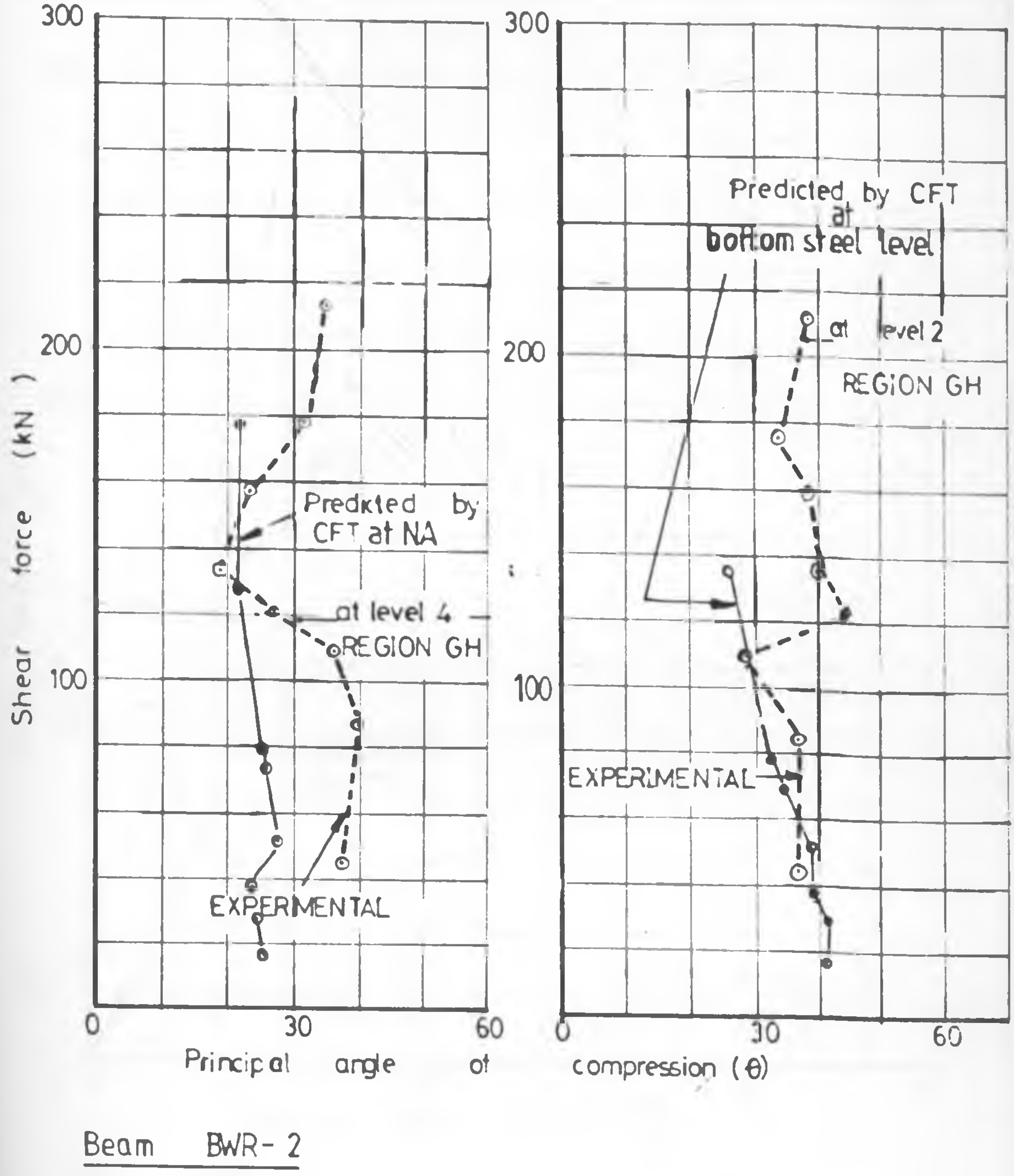


Fig. 5.8(a) - RELATIONSHIP BETWEEN SHEAR FORCE AND PRINCIPAL ANGLE OF COMPRESSION

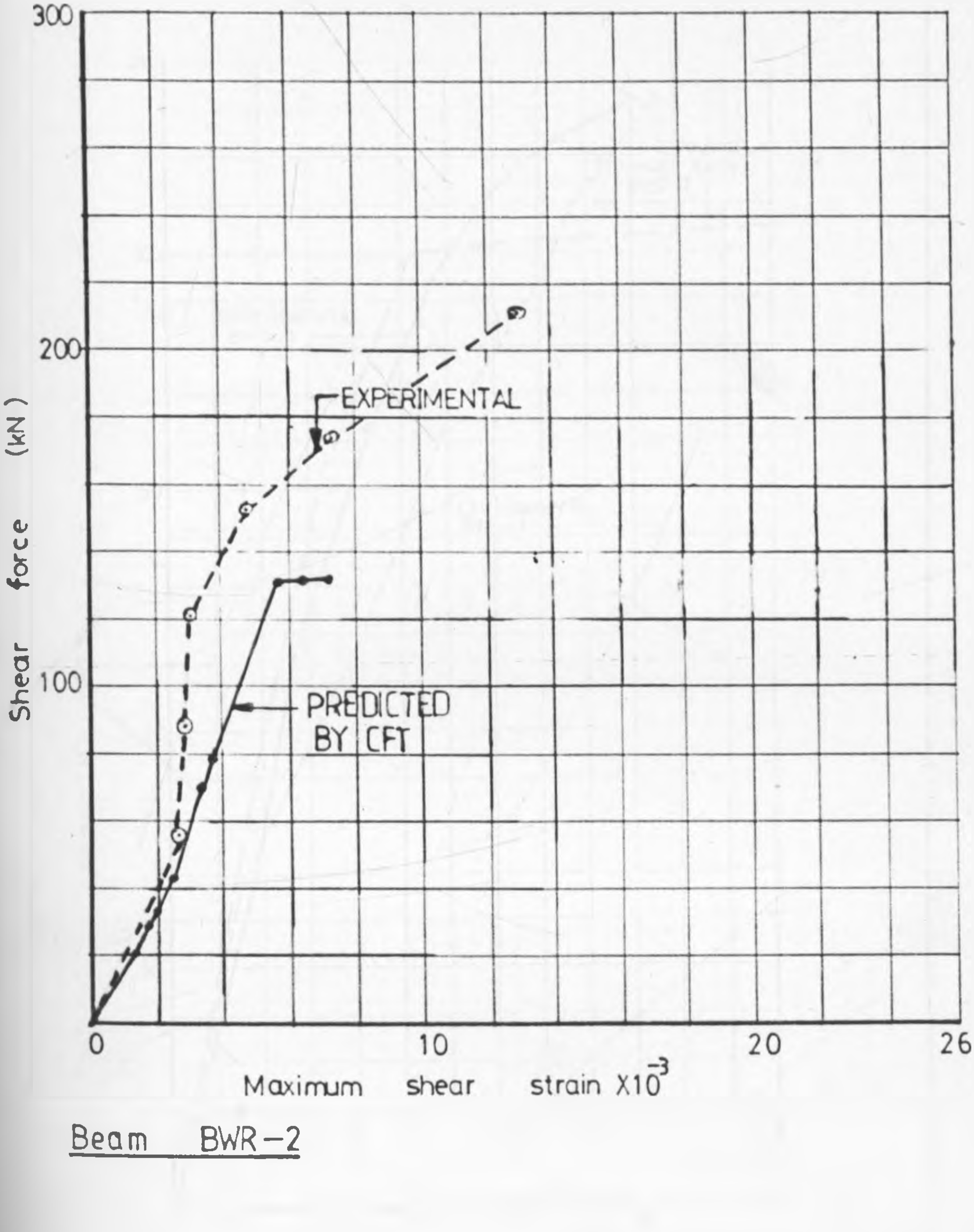


Fig. 5.8 (b) RELATIONSHIP BETWEEN SHEAR FORCE AND MAXIMUM SHEAR STRAIN AT NEUTRAL AXIS

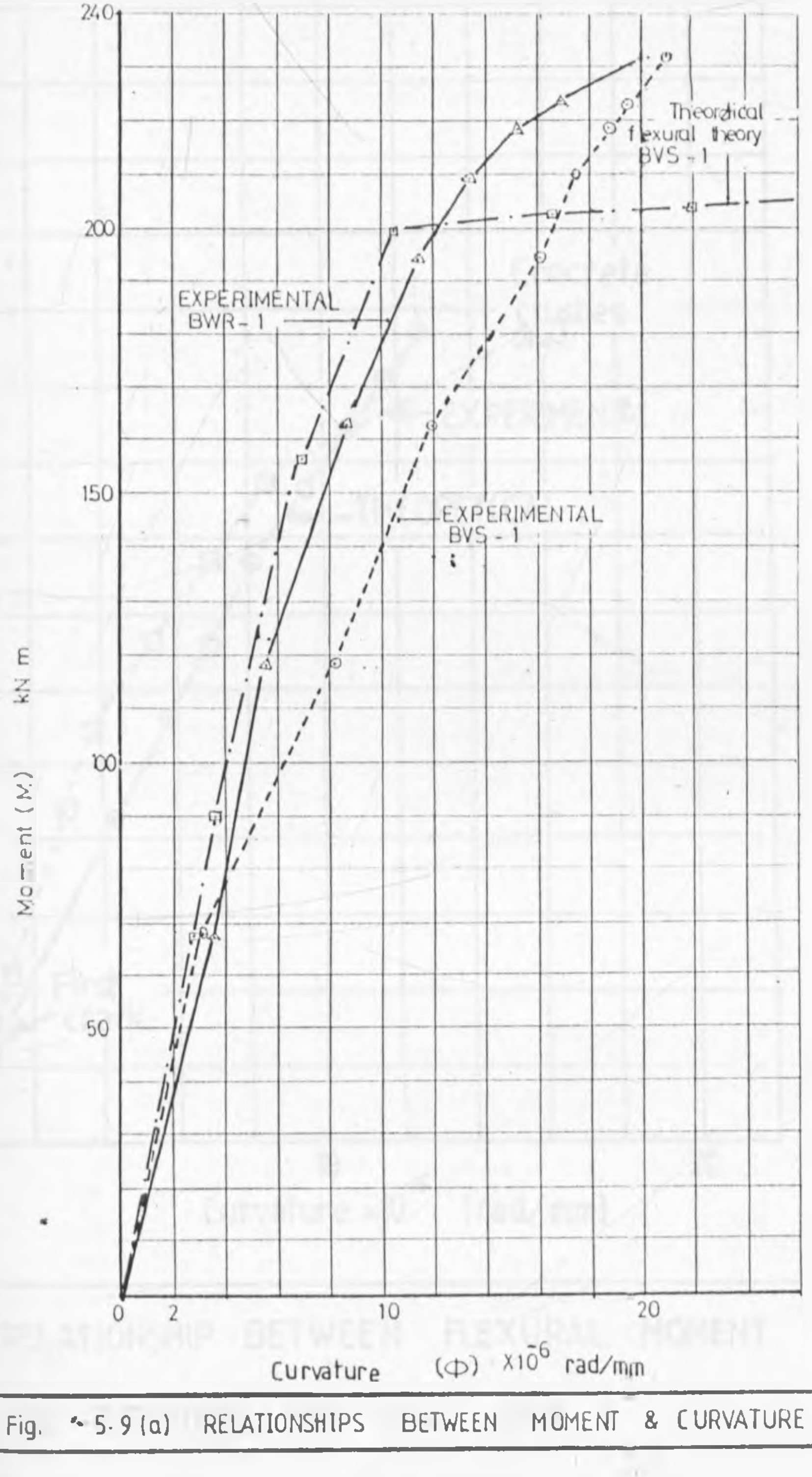


Fig. 5.9 (a) RELATIONSHIPS BETWEEN MOMENT & CURVATURE

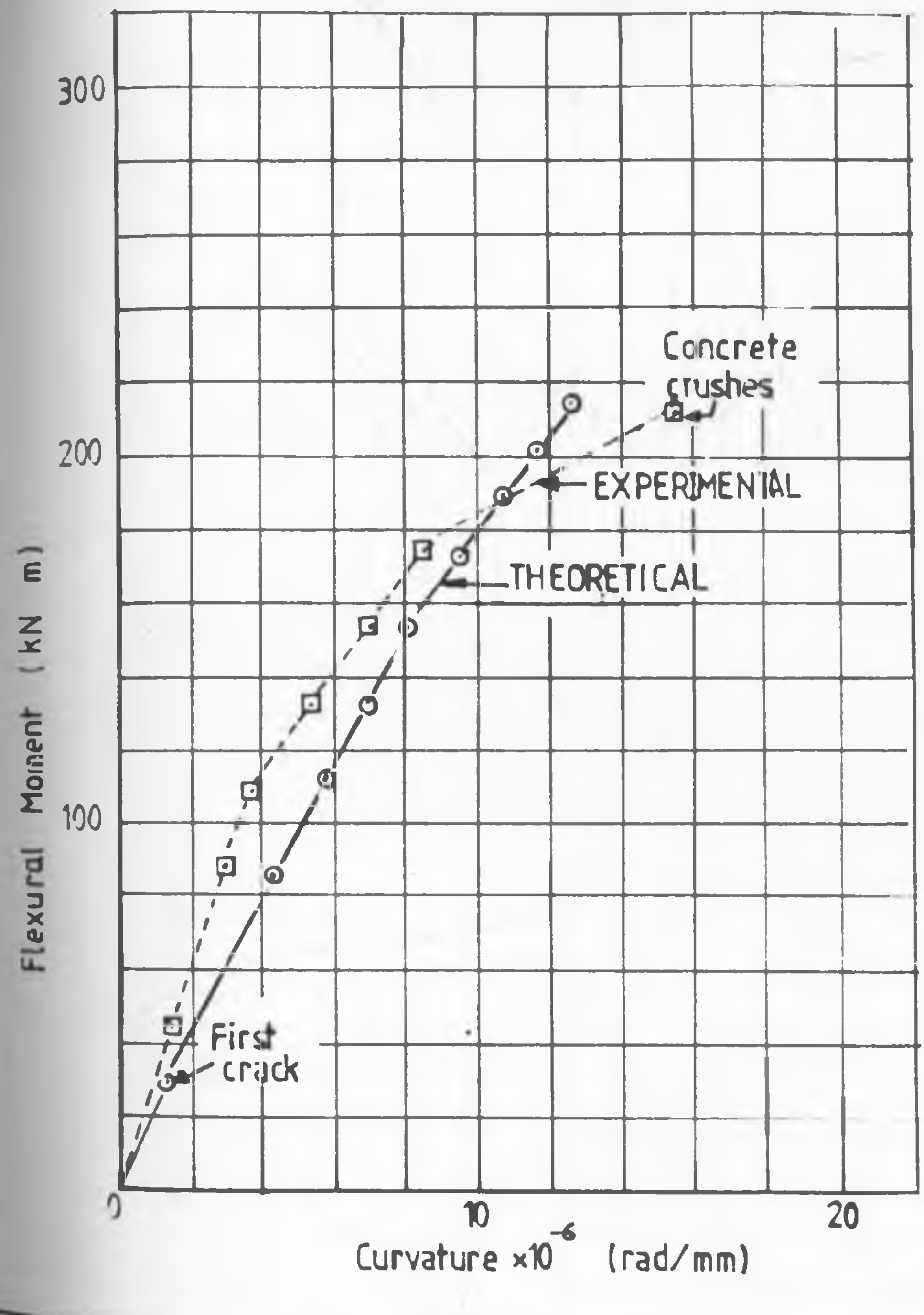


Fig. 5-9(b) RELATIONSHIP BETWEEN FLEXURAL MOMENT AND CURVATURE FOR BEAM BWR-2

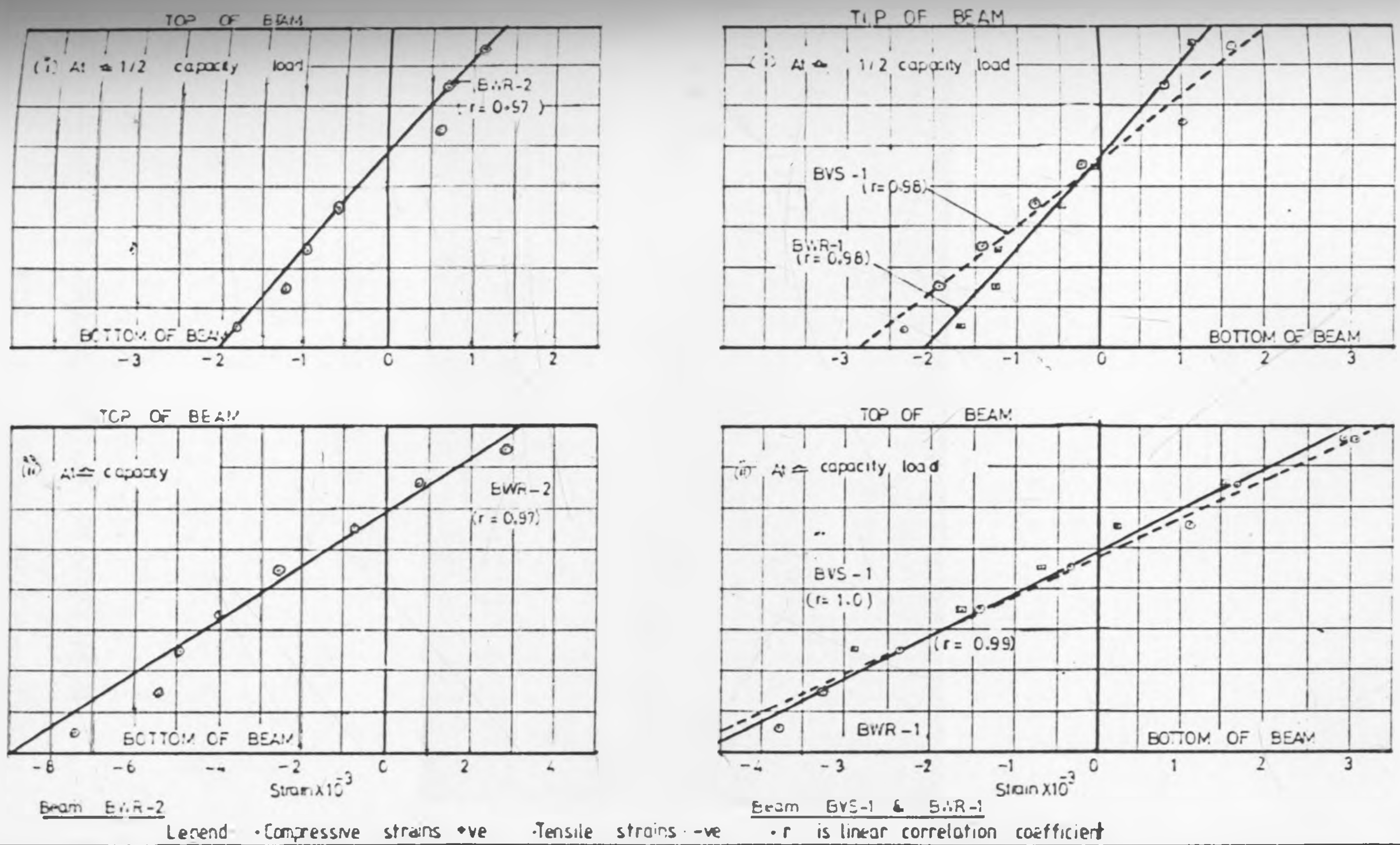


Fig 5.10 LONGITUDINAL STRAIN PROFILE IN PURE MOMENT SECTION

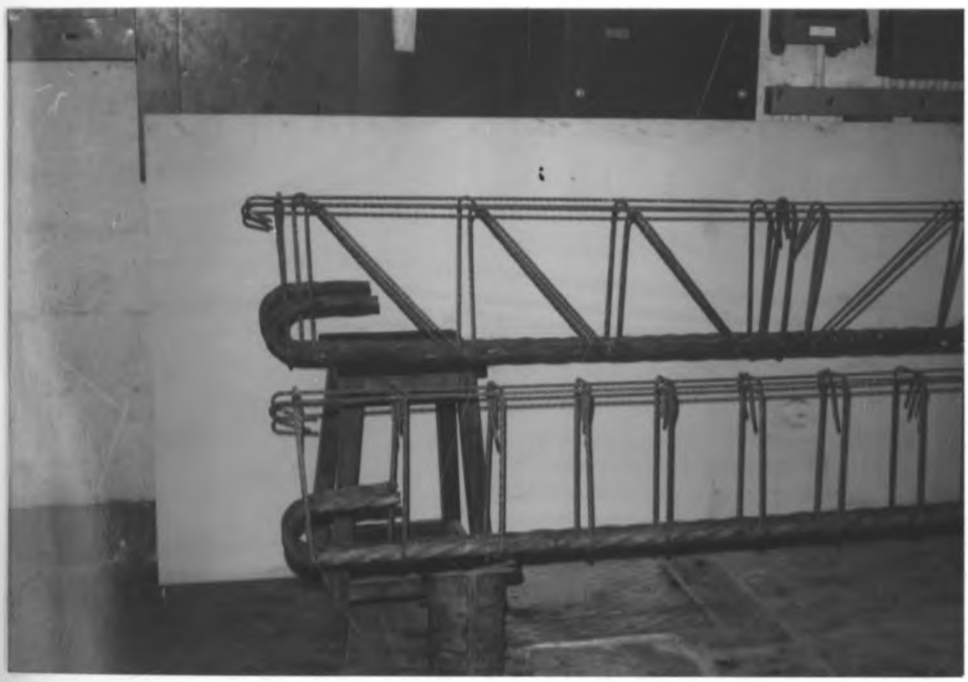
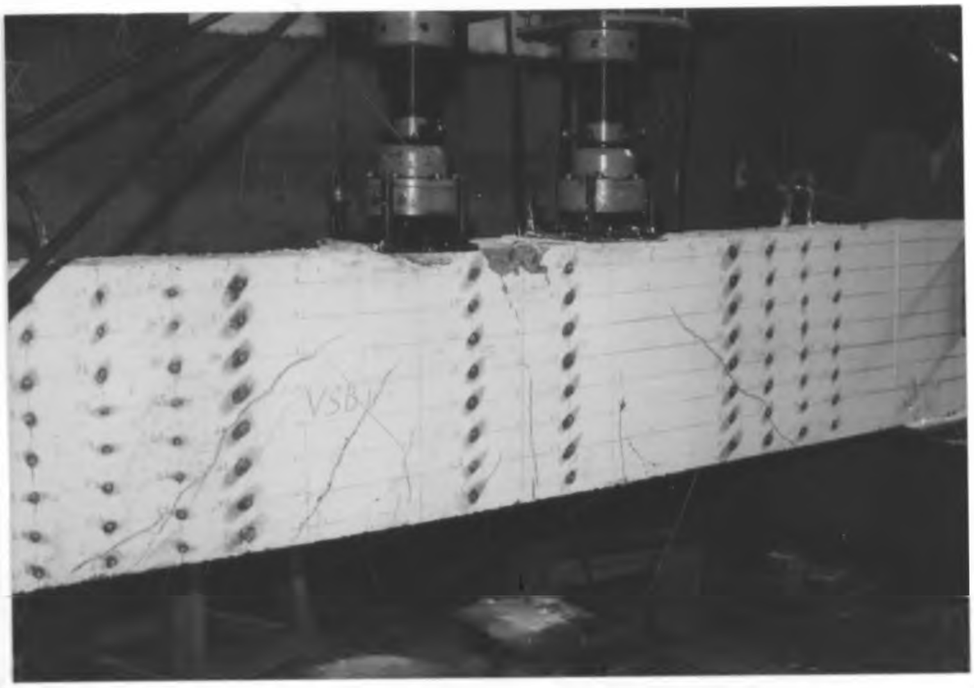


PLATE 5·1 ARRANGEMENT OF REINFORCEMENT

FOR TYPE I BEAMS (BVS-1 and BWR-1)

(a) For beam BVS-1 (old name VSB-1)



(b) For beam BWR-1 (old name DSB-1)

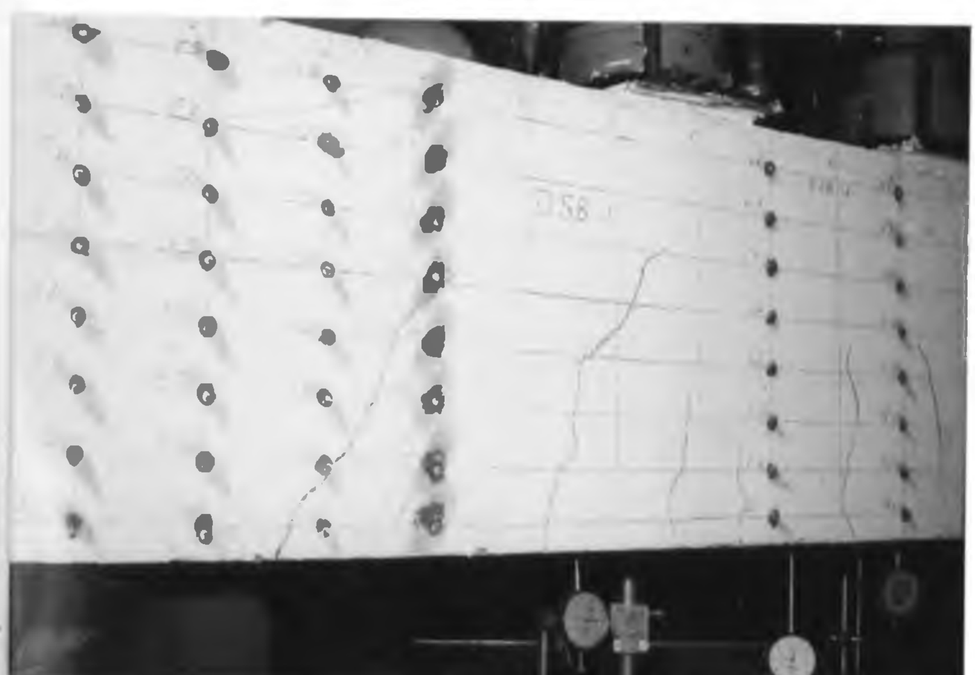
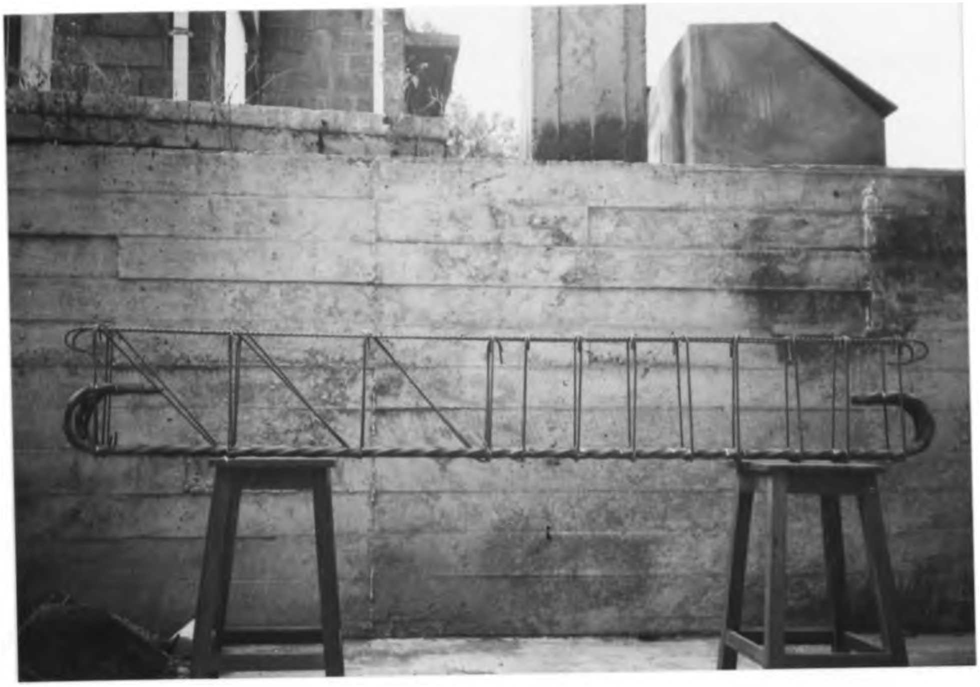


PLATE 5·2 CRACK PATTERNS
FOR TYPE I BEAMS

(a) Arrangement of reinforcement



(b) Crack pattern

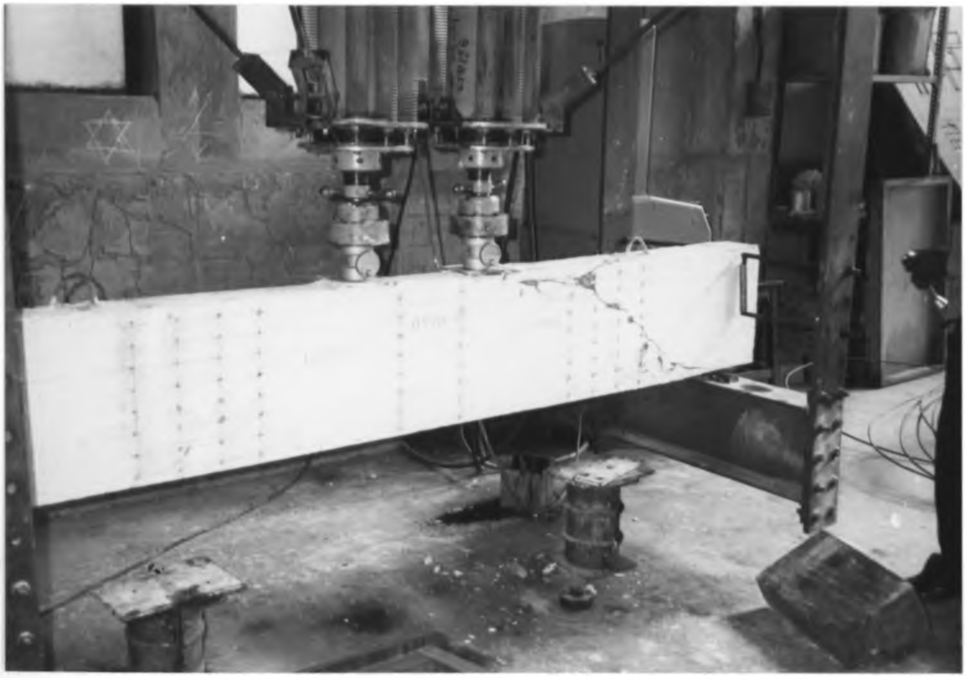
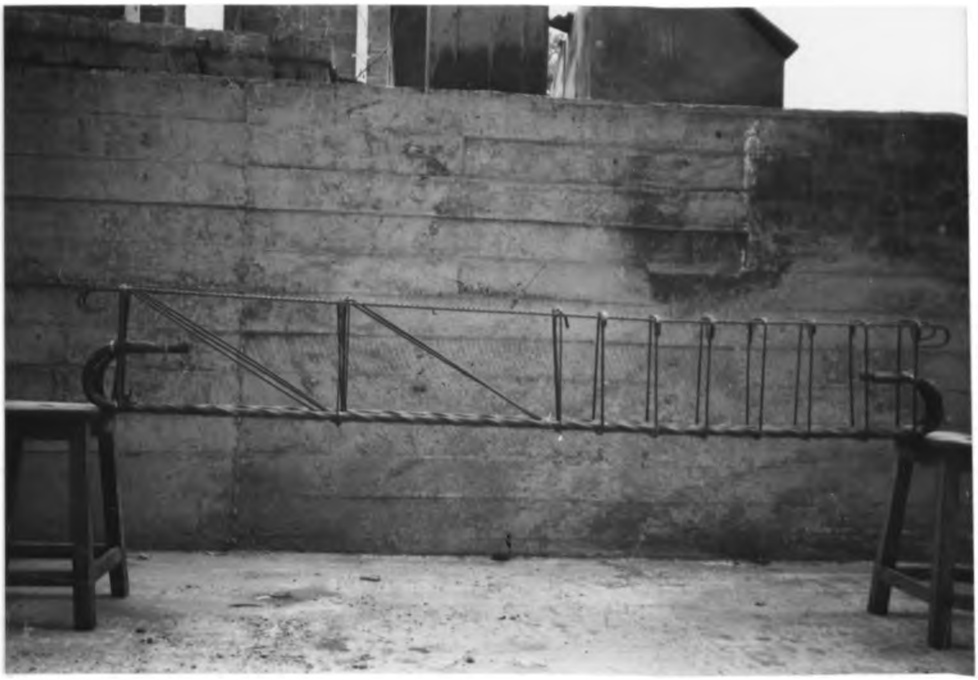


PLATE 5·3 BEAM BVWR-1

(a) Arrangement of reinforcement



(b) Crack pattern

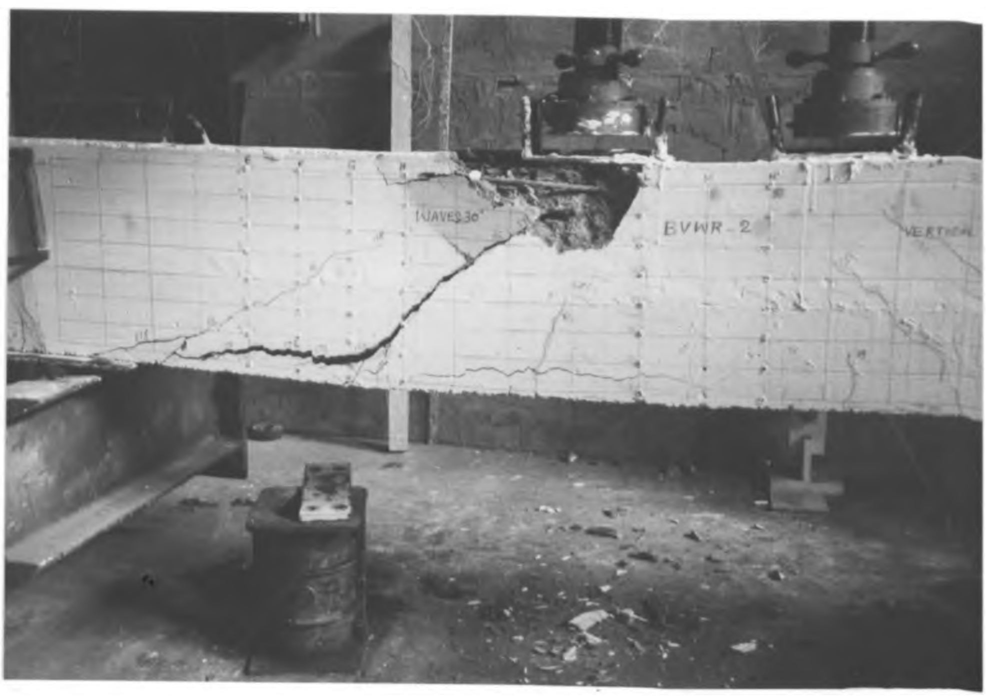


PLATE 5·4 BEAM BVWR - 2

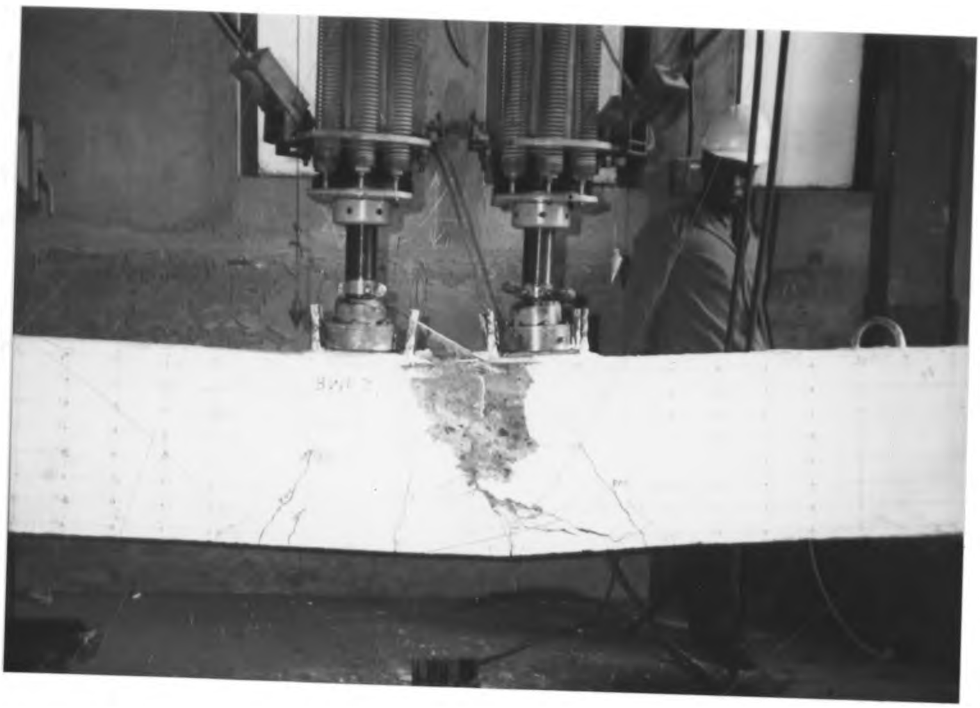


PLATE 5.5 CRACK PATTERN IN BEAM BWR-2

PLATE 5·6 BEAM BWR – 2:

REINFORCEMENT PARTLY EXPOSED

CHAPTER 6 EFFICIENCY OF WAVE REINFORCEMENT

To evaluate the 'best' choice between vertical stirrups and wave reinforcement the efficiency concept with respect to function and economy is applied. The details of which are explained in the following sections.

6.1 FUNCTIONAL EFFICIENCY

Function refers to the use or uses performed, including the esteem features provided. Any shear reinforcement should play an effective role in resisting the opening or widening of inclined cracks (Sec. 2.2) and accordingly the vertical stirrups and wave reinforcement are studied as under:

(a) percentage chance in not intercepting inclined cracks, and
(b) crack interception efficiency.

6.1.1 Percentage Chance in Not Intercepting Inclined Cracks

The Code [4] requires that the stirrups must be spaced such that every 45 deg line representing a potential inclined crack must be crossed in the tension side of the beam by at least one stirrup which limits the maximum spacing to d/2.

Fig. 6.1a to c show the arrangement of steel for a beam with vertical stirrups, 45 deg wave reinforcement and 30 deg wave reinforcement respectively.

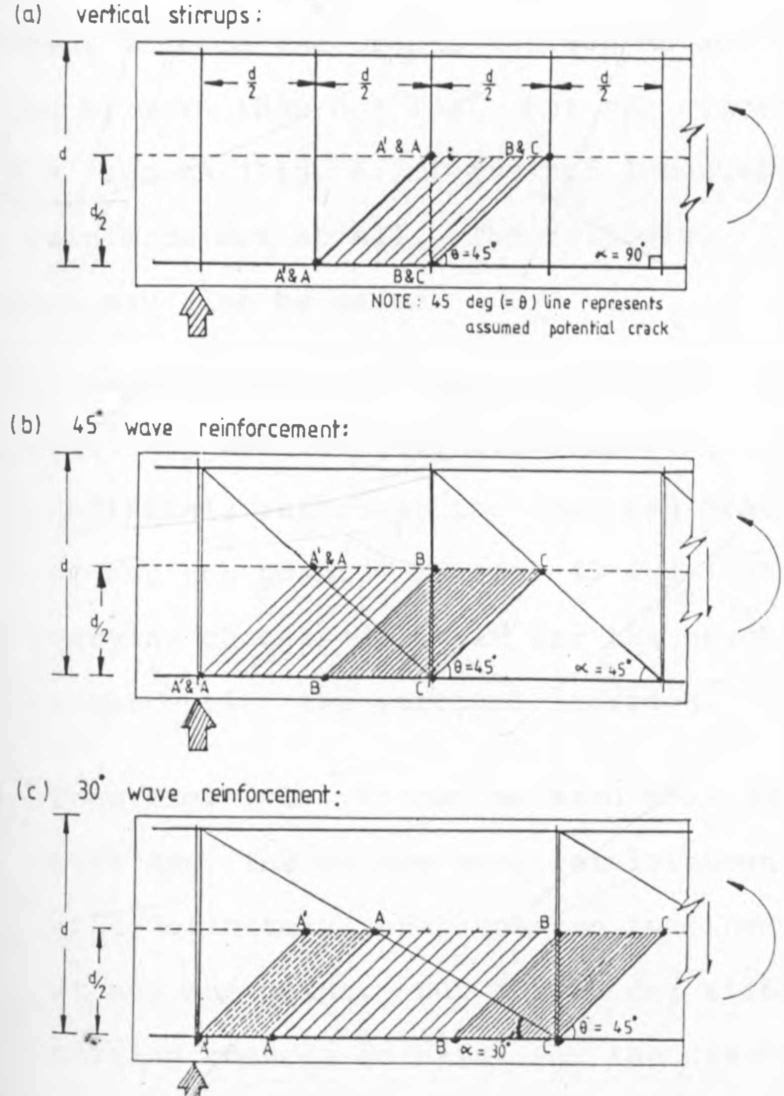


Fig. 6.1 ARRANGEMENT OF SHEAR REINFORCEMENT

A typical case of 45 deg line (assumed inclined crack angle) extending towards the reaction from middepth of member d/2 to longitudinal tension reinforcement is also indicated.

It may be noted that the cracks between AA and BB are intercepted by at least one leg of shear reinforcement whereas the cracks between BB and CC are intercepted by more than one leg. But the cracks between A'A' and AA (Fig. 6.1c) are not intercepted by shear reinforcement at all. The following observations may also be made:

(a) From Fig. 6.1a, it can be seen that for $\theta \leq 45$ deg the vertical stirrups will definitely intercept the inclined crack at at any one point. But for 45 deg $< \theta \leq 60$ deg varying chances do exist for the cracks not intercepting the vertical stirrups.

(b) From Fig. 6.1b, it can be seen that for $\theta \leq 45$ deg, the 45 deg wave reinforcement will definitely intercept the inclined crack at any one point. But for 45 deg $< \theta \leq 60$ deg varying chances do exist for the cracks not intercepting the 45 deg wave reinforcement.

(c) From Fig. 6.1c, it can be seen that for $\theta \leq 30$ deg the 30 deg wave reinforcement will definitely intercept the inclined crack at any one point but for 30 deg $< \theta \leq 60$ deg varying chances do exist for the cracks not intercepting the 30 deg wave reinforcement.

Table 6.1 shows the percentage chance that the transverse reinforcement not intercepting the inclined cracks developing at an assumed constant angle θ deg.

TABLE 6.1 PERCENTAGE CHANCE OF TRANSVERSE REINFORCEMENT NOT INTERCEPTING THE INCLINED CRACKS

TYPES OF SHEAR REINFORCEMENT	VALUE OF θ (in deg)						
	30	35	40	45	50	55	60
Vertical stirrups	NIL	NIL	NIL	NIL	20	45	80
45 deg wave reinforcement	NIL	NIL	NIL	NIL	6	14	21
30 deg wave reinforcement	NIL	7	15	20	24	29	33

NOTE: The above values of percentage chance are calculated from (A'A/A'C) x 100

It is evident from Table 6.1 that 45 deg wave reinforcement is more effective in intercepting the inclined cracks (for the ranges of θ indicated) than the vertical stirrups and also 30 deg wave reinforcement.

6.1.2 Crack Interception Factor

Defining crack interception factor, k_{CI} as the ratio of the areas of the portion where the cracks cut more than one leg (BBCC) to the area of the portion where the cracks cut at least one leg (AABB), different values of k_{CI} are obtained for various values of θ. The results are shown in Table 6.2.

TABLE 6.2 CRACK INTERCEPTION FACTOR, k_{CI}

TYPES OF SHEAR REINFORCEMENT	VALUE OF θ (in deg)						
	30	35	40	45	50	55	60
Vertical stirrups	0	0	0	0	0	0	0
45 deg wave reinforcement	6.0	2.5	1.6	1.0	0.9	0.7	0.4
30 deg wave reinforcement	1.0	0.8	0.7	0.6	0.5	0.4	0.3

It is evident from Table 6.2, for $\theta \leq 45$ deg, the values of k_{CI} are greater than 1.0 for 45 deg wave reinforcement whereas for vertical stirrups, $k_{CI} = 0$ and for 30 deg wave reinforcement k_{CI} 1.0. Even for 45 deg $< \theta \leq 60$ deg, k_{CI} value is higher for 45 deg wave reinforcement than for 30 deg wave reinforcement. Fig. 6.2 (based on Table 6.2) shows the variation of k_{CI} against θ.

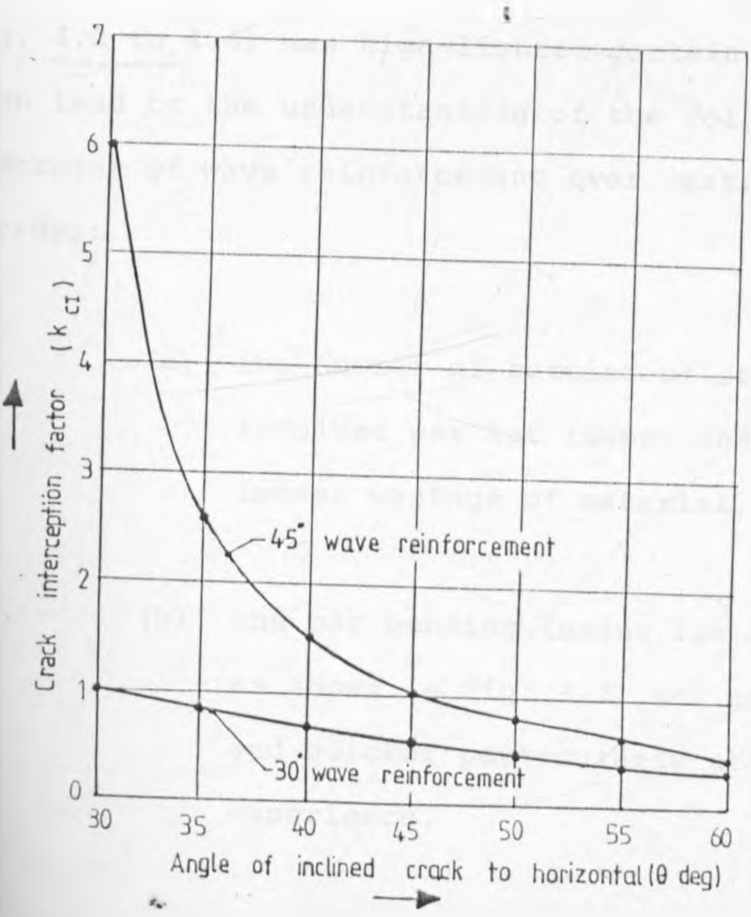


Fig. 6.2 CRACK INTERCEPTION FACTOR AGAINST ANGLE OF INCLINED CRACK

This again confirms that 45 deg wave reinforcement is more efficient in intercepting the inclined cracks than either vertical stirrups or 30 deg wave reinforcement.

6.1.3 Other Pertinent Facts

The experience of placing and fixing transverse reinforcement to the various test beams (Fig. 4.2 to 4.4) has high-lighted certain facts which lead to the understanding of the following advantages of wave reinforcement over vertical stirrups:

(a) the number of cutting of steel involved was far fewer; and hence lesser wastage of material, if any.

(b) the bar bending (using the arrangement as shown in Fig. 3.2) was much easier and quicker particularly after some experience.

(c) the arrangement of reinforcement provided lesser congestion and hence better access for the pouring of concrete.

With poker vibrators, this advantage may lead to a better compaction.

(d) the time taken for bending, placing and fixing was much shorter.

It can also be observed that larger diameter bars can be bent and used for wave reinforcement which was otherwise a limitation for vertical stirrups (due to number of hooks and/or bends involved).

Summarising, it can be said that 45 deg wave reinforcement is functionally more efficient than the vertical stirrups.

6.2 ECONOMIC EFFICIENCY

Though the consideration of function will be a dominant feature in the whole exercise of evaluating the "best" choice between vertical stirrups and wave reinforcement, the cost advantages technique may be used to identify the economic efficiency. Economic efficiency is studied for:

(a) Case I: beam with vertical stirrups as shear reinforcement

(b) Case II: identical beam with 45 deg wave reinforcement combined with vertical stirrups (as explained in Sec. 5.1.1) as shear reinforcement

The details of which are explained below.

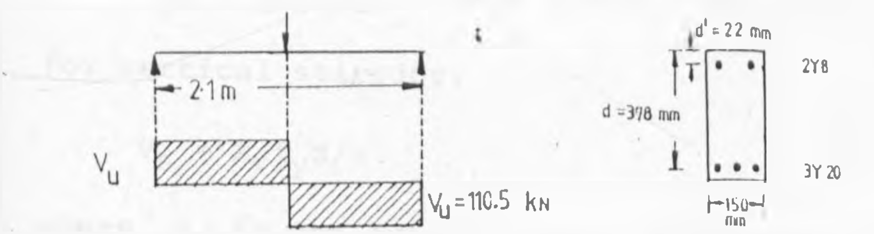
6.2.1 Economy of Material Used for Shear Reinforcement

For a typical case of 45 deg wave reinforcement and $\theta = 45^{o}$, it can be seen that (Eq. 3.7),

nominal shear strength provided by an inclined leg of wave reinforcement
= 1.414 times the nominal shear strength provided by vertical stirrups

This proves that the 45 deg wave reinforcement can provide more shear strength than the vertical stirrups having same area and hence economical.

Consider a case loading for a simply-supported doubly-reinforced rectangular concrete beam with a central point load as shown in Fig. 6.3.



(a) Loading and S.F. Diagrams (b) Beam cross-section

Fig. 6.3 BEAM DESIGN DATA

The beam is to be designed for shear using

(a) ACI 318M code [4] – for vertical stirrups
(b) BS8110 code [3] – for vertical stirrups
(c) Proposed design recommendations (Chap. 7) - for 45 deg wave reinforcement

Assume

$f'_c = 25\ N/mm^2$

$f_{ys} = 250\ N/mm^2$ (for transverse steel)

(a) ACI 318M Code:

factored shear force V_u = 110.5 kN

nominal shear strength V_n = 110.5/0.85 = 130 kN

permissible $V_c = (\sqrt{f'_c}/6)bd$

$= (\sqrt{25}/6) \times 150 \times 378$ N

= 47.25 kN

Since $V_u > \phi V_c$, stirrups must be designed for,

$$V_s = V_n - V_c = 130 - 47.25 = 82.75 \text{ kN}$$

For vertical stirrups,

$$V_s = A_v f_y d/s$$

where A_v is the area of shear reinforcement within a distance s.

Try R8 two-legged stirrups

$$s = 101 \times 250 \times 378/(82.75 \times 10^3) \text{ mm}$$

$$= 115.3 \text{ mm}$$

Choose 115 mm spacings which is less than d/2 = 378/2 = 189 mm, permissible maximum.

Provide 20 R8 - 115 two-legged stirrups

Total length of transverse steel per stirrup

= 2(A+B) +20d

= 2(370+114) + 20 x 8 mm

= 1128 mm

Assuming a standard mass of 0.395 kg/m, total mass of vertical stirrups

= 20 x (1128/1000) x 0.395 kg

= 8.91 kg

(b) BS 8110 Code:

Design concrete shear stress (v_c) is calculated as follows:

$100A_s/bd$ = 100 x 943/(150x378)

= 1.66

The relevant table from the code is partially extracted and given below (Table 6.3).

TABLE 6.3 DESIGN CONCRETE SHEAR STRESS (v_c)

$100A_s/bd$	Effective depth (mm)	
	300	>400
	N/mm^2	N/mm^2
1.50	0.78	0.72
2.00	0.86	0.80

By suitable interpolation, v_c = 0.76 N/mm^2

The design shear stress v = $110.5 \times 10^3/(150 \times 378)$ N/mm^2

= 1.949 N/mm^2

Since $(v_c+0.4) < v < 5\ N/mm^2$, shear reinforcement should be provided.

$$\frac{A_{sv}}{s_v} \geq \frac{b(v-v_c)}{0.87 f_{yv}}$$

$$\geq \frac{150(1.949-0.76)}{0.87 \times 250}\ \text{mm}$$

$$\geq 0.820\ \text{mm}$$

Try R8 two-legged stirrups.

$$s_v = 123.2\ \text{mm}$$

Choose 120mm spacings which is less than $0.75d = 0.75 \times 378 = 283.5$ mm, permissible maximum.

Provide 19 R8 -120 two-legged stirrups. Using the procedure given in (a) above, the total mass of vertical stirrups can be calculated as 8.47 kg.

(c) Proposed design recommendations for 45 deg wave reinforcement:

Shear reinforcement in the form of 45 deg wave reinforcement should be placed perpendicular to the axis of member in combination with vertical stirrups placed at sections where the vertical legs of the wave reinforcement exist (refer Sec. 7.2.1).

permissible $V_c = (\sqrt{f'_c}/6)bd$

$= (\sqrt{25}/6) \times 150 \times 378$ N

$= 47.25$ kN

$V_s = V_n - V_c = (110.5/0.85) - 47.25$ kN

$= 82.75$ kN

Since $\alpha = 45$ deg, $\theta = 45$ deg and $d-d' = s_w$, and same diameter for wave reinforcement and vertical stirrups, from Eq. 3.8,

$V_s = 4.414\ A_{ss} f_{ys}$

Therefore,

$A_{ss} = 82.75 \times 10^3/(4.414 \times 250)\ mm^2$

$= 74.99\ mm^2$

which is for one 45 deg wave reinforcement and one vertical stirrup (two-legged).

Provide R10 (area 78.5 mm^2)

as 1R10 - 355 - 500

and

8R10 - 355

Alternatively,

provide two 45 deg wave reinforcement and one vertical stirrup (two-legged). From Eq. 3.10

$A_{we} = A_{ss}(2N + 2.414\,n)$

where N=1 and n=2,

$A_{we} = 6.828\ A_{ss}$

From Eq. 3.9, $V_s = A_{we} f_{ys}$ leading to

$$= 6.828\ A_{ss} f_{ys}$$

Therefore $A_{ss} = 82.75 \times 10^3 / (6.828 \times 250)\ mm^2$

$$= 48.48\ mm^2$$

Provide 4R8 - 355 - 500
and (Area provided
8R8 - 355 $= 50.3\ mm^2$)

The latter is preferable. Refer Sec. 7.2.2.2.

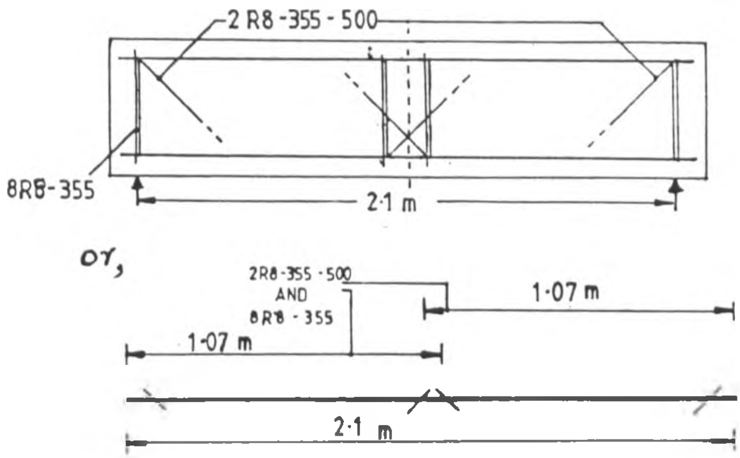


Fig. 6.4 DETAILS OF TRANSVERSE STEEL

Assume a standard mass of 0.395 kg/m.

Mass of wave reinforcement provided

$$= 4(4 \times 355 + 3 \times 500 + 2 \times 100) \times 10^{-3} \times 0.395\ kg$$

$$= 4.93\ kg$$

Mass of vertical stirrups

$$\doteq 8[2(370+114)+20 \times 6] \times 10^{-3} \times 0.395\ kg$$

$$= 3.44\ kg$$

Hence the total mass provided

= <u>8.37 kg</u>

Table 6.4 shows the various design features for shear reinforcement.

TABLE 6.4 COMPARISON OF SHEAR REINFORCEMENT REQUIRED

S.No	DESIGN METHOD	SHEAR REINFORCEMENT	MASS REQUIRED,kg
1	ACI 318M Code	Vertical stirrups 21R8-100	8.91
2	BS 8110 Code	vertical stirrups 25R8-90	8.47
3	Proposed design recommendations	45 deg wave reinforcement 4R8-355-500 and vertical stirrups 8R8-355	8.37

Based on the above calculations it can be worked out that the proposed 45 deg wave reinforcement combined with vertical stirrups requires:

(a) 93.9% of the amount of shear reinforcement required by the ACI method.

(b) 98.8% of the amount of shear reinforcement required by the BS method.

Thus the proposed type of shear reinforcement is relatively more economical, from the point-of-view of material usage, than the vertical stirrups. Additional example given in Appendix A also confirms the above statement.

6.2.2 Cost Advantages

The costs associated (direct and indirect) can be calculated for case I and case II as shown in Table 6.5.

TABLE 6.5 ASSOCIATED COSTS

ITEMS	COSTS FOR	
	Case I	Case II
(a) material for shear reinforcement	C_1	K_1C_1
(b) labour for bending of shear reinforcement, their placing and fixing	C_2	K_2C_2
(c) cost associated with vibration of concrete	C_3	K_3C_3
(d) others	C_4	C_4

Note: 1. value of K_1 is less than 1.0 (refer Sec.6.2.1)

2. values of K_2 and K_3 are less than 1.0, though they are too general to be identified specifically (refer Sec. 6.1.3)

Since the beams in case I and II serve exactly the same purpose, the benefits (B) may be assumed to be same for both cases.

The benefit-cost ratio (B/C) can thus be calculated as follows:

(1) Case I: $(B/C)_I = B/(C_1+C_2+C_3+C_4)$

(2) Case II: $(B/C)_{II} = B/(K_1C_1+K_2C_2+K_3C_3+C_4)$

It can be seen that $(B/C)_I < (B/C)_{II}$. Also, the net difference between benefits and costs for Case II is more than that of Case I. Hence it may be stated that 45 deg wave reinforcement combined with vertical stirrups is economically justifiable.

Based on Sec. 6.1 and 6.2, it may be concluded that the 45 deg wave reinforcement combined with vertical stirrups is more efficient than vertical stirrups alone from the point-of-view of functional and economic efficiency and hence it is the 'best' choice.

CHAPTER 7 PROPOSED DESIGN RECOMMENDATIONS FOR SHEAR USING WAVE REINFORCEMENT

In this chapter specific design recommendations are proposed for the shear resistance of non-prestressed concrete beams with wave reinforcement subjected to flexure and shear. Existing code specifications [3] and [4] adopted for the design of wave reinforcement are also used accordingly.

7.1 SHEAR STRENGTH PROVIDED BY CONCRETE (V_c)

7.1.1 - The shear strength V_c should be computed by the provisions made in the approved design codes.

In the absence of a more detailed calculation, the following equation may be considered adequate:

$$V_c = (\sqrt{f'_c}/6)\,bd \text{ ----- (SI units)} \qquad (7.1)$$

or

$$V_c = [(\sqrt{f'_c} + 120\,\rho_w \frac{V_u d}{M_u}) \div 7]\,bd \qquad (7.2)$$

but not greater than $0.3\sqrt{f'_c}\,bd$ and $\frac{V_u d}{M_u}$ should not be greater than 1.0 where M_u is factored moment occurring simultaneously with factored shear force V_u at section considered and $\rho_w = A_s/bd$.

7.2 SHEAR STRENGTH PROVIDED BY WAVE REINFORCEMENT (V_s)

7.2.1 Wave Reinforcement

7.2.1.1 Shear reinforcement in the form of wave reinforcement should be placed perpendicular to axis of member in combination with vertical stirrups, placed at sections where the vertical legs of the wave reinforcement exist preferably of same diameter bars.

7.2.1.2 The inclined leg of the wave reinforcement should make an angle of 45 deg with longitudinal tension reinforcement. Any angle more than 45 deg may be considered, if verified suitable.

7.2.1.3 The wave reinforcement should end with a vertical leg and a standard hook and this should be continued at least a distance d, past the point where they theoretically are no longer required to restrain inclined cracks starting at that point.

7.2.1.4 End anchorages in the form of hooks or bends should comply to specific code requirements.

7.2.1.5 Design yield stress f_{yw}, of shear reinforcement should not exceed 400 N/mm^2 or yield stress of wave reinforcement or yield stress of vertical stirrups whichever is the lesser.

7.2.2 Design of Wave Reinforcement

7.2.2.1 For $v < v_c/2$, a minimum of two wave reinforcement combined with vertical stirrups in accordance with Section 7.2.1.1, should be provided in all beams of structural importance.

7.2.2.2 For $v_c/2 < v < (v_c+0.4)$ N/mm^2, a minimum of two wave reinforcement combined with vertical stirrups in accordance with Section 7.2.1.1, should be provided for the whole length of beam to serve as a minimum reinforcement. In such cases,

$$v_{sw} = 0.4 \ N/mm^2$$

$$A_{we} \geq 0.4 \ bs_w/f_{yw} \qquad (7.3)$$

The value of A_{we}, the equivalent area of minimum shear reinforcement (refer Eq. 3.10a) assuming same diameter for the 45 deg wave reinforcement and vertical stirrups, is given by,

$$A_{we} = 6.828\ A_{ss} \tag{7.4}$$

where A_{ss} is the cross-sectional area of each bar of shear reinforcement required at the section.

7.2.2.3(a) For $(v_c + 0.4) < v < 5\ N/mm^2$, a maximum number of wave reinforcement equal to the number of longitudinal tensile steel should be used.

(b) The shear strength V_s is calculated from (Eq. 3.7),

$$V_s = [A_{sv} f_{yv} \cot\theta + A_{sw} f_{yw} \cot\theta + A_{sw} f_{yw} (\cos\alpha + \sin\alpha \cot\theta)] \left(\frac{d-d'}{s_w}\right) \tag{7.5}$$

(c) (i) In lieu of more exact analysis, assuming θ = 45 deg, for a 45 deg wave reinforcement combined with two-legged vertical stirrups, from Eq. 3.8,

$$V_s = A_{sv} f_{yv} + 2.414\ A_{sw} f_{yw} \tag{7.6}$$

(ii) Again, if same diameter shear steel is used, $A_{ss} = A_{sv}/2 = A_{sw}$ and $f_{ys} = f_{yv} = f_{yw}$, which leads to (Eq. 3.8a),

$$V_s = 4.414\ A_{ss} f_{ys} \qquad (7.7)$$

where A_{ss} is the cross-sectional area of each bar of shear reinforcement and f_{ys} is the characteristic strength of shear reinforcement used.

(iii) In a general form, Eq. 7.7 can be stated as

$$V_s = A_{we} f_{ys} \qquad (7.8)$$

The equivalent area of shear reinforcement A_{we} is then given by (Eq. 3.10),

$$A_{we} = A_{ss}\,(2N + 2.414\ n) \qquad (7.9)$$

where N is the number of two-legged vertical stirrups and n the number of wave reinforcement, at the section considered.

7.2.2.4 Additional shear requirement, if necessary after satisfying the requirements of Section 7.2.2.3(a), may be provided in the form of additional vertical stirrups with a spacing not exceeding $s_w/2$; or the wave reinforcement may be staggered (Fig. 7.1) midway between the vertical stirrups.

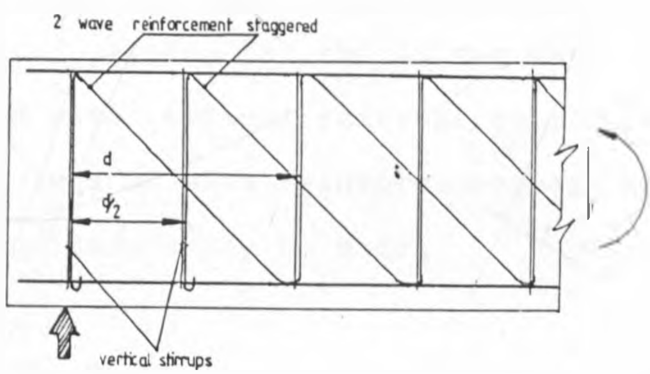


Fig. 7.1 WAVE REINFORCEMENT STAGGERED MIDWAY BETWEEN VERTICAL LEGS

CHAPTER 8 CONCLUSIONS AND RECOMMENDATIONS

8.1 CONCLUSIONS

Based on the results of experimental and analytical study it may be concluded that the proposed wave reinforcement is an acceptable type of shear reinforcement. Again, for the 45 deg wave reinforcement combined with vertical stirrups at sections where the vertical legs of wave reinforcement exist, the following conclusions may be made:

(a) It is comparable to the provision of vertical stirrups alone as shear reinforcement in ensuring that the ultimate strengths are governed by flexure rather than by shear.

(b) It is a better alternate type of shear reinforcement than only the vertical stirrups from the point-of-view of controlling the number and widths of shear cracks.

(c) It is more efficient functionally and economically, than the vertical stirrups provided alone as shear reinforcement.

(d) The 45 deg truss analogy can be used to compute the shear strength provided by wave reinforcement.

(e) The compression field theory predicts the response characteristics of a beam reasonably well.

Finally, it is the hope of the author that the contents of this report will help the designer in assessing the acceptability of 45 deg wave reinforcement combined with vertical stirrups, as explained earlier, to non-prestressed rectangular concrete beams.

8.2 RECOMMENDATIONS

Though the present study has resolved the acceptability and efficiency of wave reinforcement, the test results would not be considered as conclusive since no duplication tests were made. Also, the idea of efficiency to the totality of a situation requires a knowledge of all the facts and involving not only 45 deg wave reinforcement but with varying angles as well.

Again further research work should be conducted to verify the effects of wave reinforcement to:

(a) prestressed concrete members

(b) resist torsion

(c) fluctuating loads

(d) reversal of loadings

(e) include all reinforced concrete structures such as arches and shells.

It is therefore recommended that a sufficient number of specimens incorporating all possible situations should be tested to permit statistical interpretation of the test results and to formulate comprehensive design requirements using wave reinforcement.

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APPENDIX A DESIGN EXAMPLE

The use of proposed design recommendations (Chapter 7) is illustrated by means of a design example and material used for the proposed shear reinforcement is also compared with the B.S. code design [3] involving only vertical stirrups.

It is required to design shear reinforcement for the left-end span of a three-span continuous beam. The beam has a uniform rectangular section of breadth b = 300 mm and an effective depth of 740 mm. The material properties are:

$$f'_c = 25\ \text{N/mm}^2$$

$$f_{yv} = f_{yw} = f_{ys} = 280\ \text{N/mm}^2$$

The shear force envelope is as shown in Fig. Al. Suitable assumptions are also made.

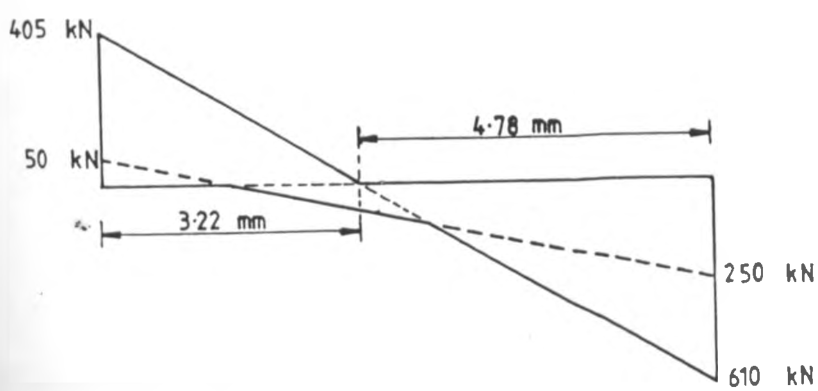


Fig. Al SHEAR FORCE ENVELOPE

Using Section 7.1.1, the following are worked out:

The shear strength provided by concrete (V_c),

$$V_c = (\sqrt{f'_c}/6)bd$$

$$= (\sqrt{25}/6)\ (300 \times 740)\ \ N$$

$$= 185 \times 10^3\ \ N$$

The design concrete shear stress (v_c),

$$v_c = 0.833\ N/mm^2$$

The shear stress diagram is shown in Fig. A2.

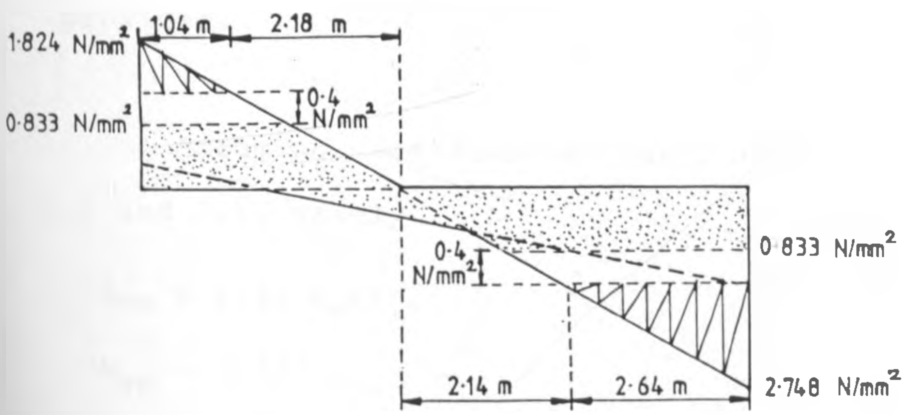


Fig. A2 SHEAR STRESS DIAGRAM

Wave reinforcement to be provided should be in the form of:

45 deg wave reinforcement
combined with
vertical stirrups
(refer Section 7.2.1.1 and 7.2.1.2).

Referring to Fig. A2, the following may be stated:

(a) For the inner portions 2.18 m and 2.14 m combining Sections 7.2.2.1 and 7.2.2.2, a minimum of two 45 deg wave reinforcement combined with vertical stirrups should be provided.

Assuming same diameter bars, using Eq. 7.3 and 7.4, namely

$$A_{we} \geq 0.4b\ s_w/f_{yw}$$

$$A_{we} = 6.828\ A_{ss}$$

the minimum shear reinforcement can be designed.

$$6.828\ A_{ss} = 0.4 \times 300 \times 740/280$$

$$A_{ss} = 46.45\ \text{mm}^2$$

Choose 8 mm bars
(area provided 50.3 mm^2)

Assuming a clear cover of 30 mm and top longitudinal steel of Y16,

$$d - d' = 740 - (30+8+8) \text{ mm}$$
$$= 694 \text{ mm}$$
$$\approx 695 \text{ mm (multiples of 5 mm)}$$

Use two 45 deg wave reinforcement

2R8 - 695 - 980

and

vertical stirrups

R8-695

(b) For the regions 1.04 m from the left support and 2.64 m from the right support (shown hatched), $(v_c+0.4)<v<5$ N/mm^2 and hence design shear reinforcement using Section 7.2.2.3.

(i) Region marked 1.04 m (near left support):

$$v_s = v_n - v_c$$
$$= 1.824 - (0.833+0.4) \text{ N/mm}^2$$
$$= 0.591 \text{ N/mm}^2$$

Hence the shear strength provided by wave reinforcement combined with vertical stirrups (V_s),

$$V_s = 0.591 \times 300 \times 740 = 131.20 \times 10^3 \text{ N}$$

Assume same diameter bars for 45 deg wave reinforcement and vertical stirrups. From Eq. 7.7,

$$V_s = A_{we} f_{ys}$$

$$A_{we} = 131.20 \times 10^3/280 \text{ mm}^2$$

$$= 468.57 \text{ mm}^2$$

From Eq. 7.9, $A_{we} = A_{ss}(2N+2.414n)$

Try three 45 deg wave reinforcement (n=3)

two 2-legged vertical stirrups (N=2).

$$468.57 = A_{ss}(2x2+2.414x3)$$

$$A_{ss} = 41.68 \text{ mm}^2$$

Choose 8 mm bars

(area provided 50.3 mm^2)

Provide three 45 deg wave reinforcement

3R8 - 695 - 980

and

2 x 2-legged vertical stirrups

2 x 3R8-695

(ii) Region marked 2.64 m (near right support):

$$v_s = v_n - v_c$$

$$= 2.748 - (0.833+0.4) \text{ N/mm}^2$$

$$= 1.515 \text{ N/mm}^2$$

Hence the shear strength provided by wave reinforcement combined with vertical stirrups (V_s),

$$V_s = 1.515 \times 300 \times 740 \quad N$$
$$= 336.33 \times 10^3 \quad N$$

$$A_{we} = V_s/f_{ys}$$
$$= 336.33 \times 10^3/280 \ mm^2$$
$$= 1201.18 \ mm^2$$

Try three 45 deg wave reinforcement and two 2-legged vertical stirrups.

$$1201.18 = A_{ss} (2 \times 2 + 2.414 \times 3)$$
$$A_{ss} = 106.85 \ mm^2$$

Choose 12 mm bars

(area provided 113 mm^2)

Provide three 45 deg wave reinforcement

3R12-695-980

and

2x2-legged vertical stirrups

2x5R12-695

The arrangement of shear reinforcement is shown in Fig. A3.

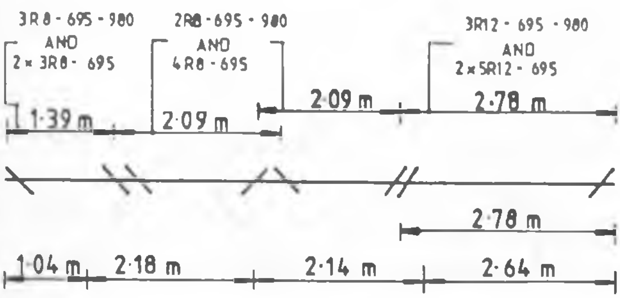


Fig. A3 ARRANGEMENT OF PROPOSED SHEAR REINFORCEMENT

Using similar procedure shown in the example in Chapter 6, the following can be obtained:

mass of wave reinforcement ≅ 35.5 kg

mass of vertical stirrups ≅ 27.5 kg

Hence, the total mass of transverse steel required using the proposed shear reinforcement = 35.5+27.5 kg

= <u>63.0 kg</u>

Using B.S Code [3], the following arrangement of transverse steel as shown in Fig. A4 can be obtained.

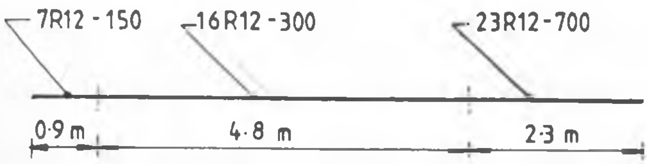


Fig. A4 ARRANGEMENT OF VERTICAL STIRRUPS

Hence, the total mass of vertical stirrups required is approximately equal to 80.0 kg.

Again, it can be seen that the proposed shear reinforcement requires only 79% of that required by the vertical stirrups alone.

APPENDIX B RESPONSE PREDICTION USING COMPRESSION FIELD THEORY

Based on the compression field theory, Shitote [24] developed the necessary equations in order to predict the post-cracking response of a structural concrete beam for a combined loading case of flexure and shear. It was originally derived for the case of beams with vertical stirrups as transverse reinforcement. In the present study, a few of the equations are suitably modified for the case of wave reinforcement and shown in Sec. 3.4. But for easy reference all the equations used are provided below in a concise manner alongwith the procedure.

B.1 SOLUTION TECHNIQUE

Assuming that the beam cross-section, the stress-strain characteristics of concrete and steel are given, then for a specified extreme compression fibre strain, ε_{ct}, and a moment to shear ratio of loading, a_v, the following solution technique is proposed.

A: <u>Pure Flexure case</u>

STEP 1: To compute rectangular stress block factors, using parabolic stress-strain relationship for the doubly-reinforced section and y_p, M_p, ϕ_p for pure flexure case.

Choose top face strain, ε_{ct} and obtain Ω_{ct}

$$\Omega_{ct} = \frac{\varepsilon_{ct}}{\varepsilon_{co}}$$

↓

Compute stress block factors α, β and $\alpha\beta$

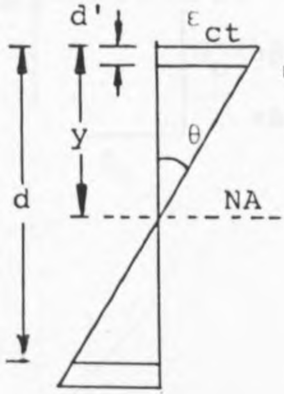
$$\alpha = 6(1-\Omega_{ct}/3)^2/(4/\Omega_{ct}-1)$$

$$\beta = (4-\Omega_{ct})/(6-2\Omega_{ct})$$

$$\alpha\beta = \Omega_{ct}-\Omega_{ct}^2/3$$

↓

Establish strain profile based on depth of NA, y
Note: for pure flexure case $y=y_p$ and $\theta=\theta_p$



STRAIN PROFILE

$$\varepsilon_{\ell}' = (y-d')\varepsilon_{ct}/y$$

$$\theta = \varepsilon_{ct}/y$$

$$\varepsilon_{\ell} = (d-y)\varepsilon_{ct}/y$$

Compute depth of neutral axis, y_p, for pure flexure case using equilibrium of forces conditions

Note: $f'_\ell = E'_s \epsilon'_\ell$

$f_\ell = E_s \epsilon_\ell$

Either (a) all steel in elastic range

$$\alpha\beta f'_c b y_p + A'_s f'_\ell = A_s f_\ell$$

or (b) only tension steel yielding

$$\alpha\beta f'_c b y_p + A'_s f'_\ell = A_s f_{\ell y}$$

or (c) only compression steel yielding

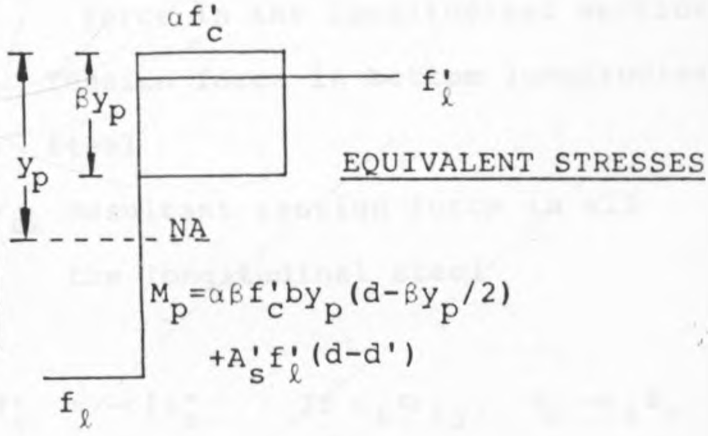
$$\alpha\beta f'_c b y_p + A'_s f'_{\ell y} = A_s f_\ell$$

or (d) all steel yielding

$$\alpha\beta f'_c b y_p + A'_s f'_{\ell y} = A_s f_{\ell y}$$

↓

Compute moment of resistance, M_p curvature, ϕ_p for the above pure flexure case



$$M_p = \alpha\beta f'_c b y_p (d - \beta y_p/2) + A'_s f'_\ell (d - d')$$

to STEP 2

B: Combined Flexure and Shear Case

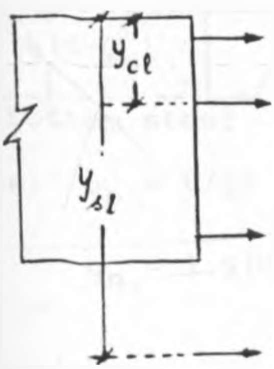
STEP 2:

(a) Assume $y_n = y_p$ and establish strain profile using,

$$\varepsilon'_\ell = (y_n - d')\varepsilon_{ct}/y_n \quad ; \quad \text{and}$$

$$\varepsilon_\ell = (d - y_n)\varepsilon_{ct}/y_n$$

(b) Compute the magnitude and the position of resultant longitudinal forces using the pure flexure moment M_p.



C_s compression force in top steel

$F_{c\ell}$ Resultant concrete compression force in the longitudinal section

T Tension force in bottom longitudinal steel

$F_{s\ell}$ Resultant tension force in all the longitudinal steel

If $\varepsilon'_\ell < 0$, $f'_\ell = -\varepsilon'_\ell E'_s$ If $\varepsilon_\ell < \varepsilon_{\ell y}$, $f_\ell = \varepsilon_\ell E_s$

If $\varepsilon'_\ell < \varepsilon_{\ell y}$, $f'_\ell = \varepsilon'_\ell E'_s$ If $\varepsilon_\ell \geq \varepsilon_{\ell y}$, $f_\ell = f_{\ell y}$

If $\varepsilon'_\ell \geq \varepsilon_{\ell y}$, $f'_\ell = f_{\ell y}$

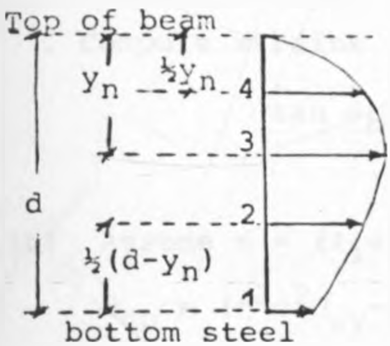
- $F_{s\ell} = T - C_s = A_s f_\ell - A'_s f'_\ell$
- $y_{s\ell} = (A_s f_\ell d - A'_s f'_\ell d')/F_{s\ell}$

(F_{cl} and $y_{c\ell}$ are calculated in Step 4)

- The internal moment lever arm,

$$jd = M_p/T$$

(c) Obtain shear flow distribution



Levels	Shear flow	longitudinal strain
Top of beam	$q_t = 0$	ε_{ct}
4	$q_4 = 0.75q_n$	$\frac{1}{2}\varepsilon_{ct}$
3	$q_3 = q_n$	0
2	$q_2 = \frac{1}{4}(q_b+3q_n)$	$\frac{1}{2}\varepsilon_{\ell}$
1 (bottom steel)	$q_1 = q_b$	ε_{ℓ}

Note: $q_b = V/jd$

$$q_n = 1.5(V/d)(1-(d-y_n)/3jd)$$

<u>STEP 3</u>: Compute compatible strains at bottom steel level (longitudinal strain = ε_ℓ)

Let $c = (A_{we}\ f_{yw})/(s_w q_b)$

$\Omega_{wy} = \varepsilon_{wy}/\varepsilon_{co}$ and $\Omega_\ell = \varepsilon_\ell/\varepsilon_{co}$

$$\ell_1 = \left(\frac{\Omega_\ell}{\Omega_{wy}} \cdot \frac{1}{c^2}\right)^{1/3} \quad \text{and} \quad \ell_2 = 1/c$$

- If $\ell_1 < 1.0$ and $\ell_2 < 1.0$ GO TO Step 3(b)

(a) Assume $f_c^* = f_c'$

- $f_{cp} = q_b(1+c^2)/bc$

- $\Omega_{cp} = \varepsilon_{cp}/\varepsilon_{co} = 1 \pm \sqrt{1-f_{cp}/f_c^*}$

  (+ve when $\varepsilon_{ct} \geq \varepsilon_{co}$)

- $\varepsilon_w = (\varepsilon_{cp}(1-c^2)+\varepsilon_\ell)/c^2 \geq \varepsilon_{wy}$

- If $\varepsilon_w < \varepsilon_{wy}$ GO TO step 3(b); otherwise

  $\Omega_{ct} = \varepsilon_t/\varepsilon_{co} = (\varepsilon_\ell+\varepsilon_{cp}+\varepsilon_w)/\varepsilon_{co}$

- Re-evaluate $f_c^* = \dfrac{f_c'}{0.80+0.34\Omega_t} \leq f_c'$

- Repeat step 3(a) until convergence in f_c^* is obtained

- Compute strains ε_w, ε_ℓ, ε_{cp} and angle θ_b from

  $$\tan\theta_b = c \quad ;$$

(b) Assume $\eta = (\ell_1+\ell_2)/2$

- $\Omega_{cp} = (\eta^3c^2\Omega_{wy}-\Omega_\ell)/(1-\eta^2c^2)$

- $f(\eta) = \Omega_{cp}z - \frac{1}{3}\Omega_{cp}^2 z-k$

  where $z = \eta c/(1+\eta^2c^2)$

  $k = q_b/f_c^*b$---- (assume $f_c^* = f_c'$)

- $$f'(\eta) = \frac{d\Omega_{cp}}{d\eta}z+\Omega_{cp}\frac{dz}{d\eta} - \frac{1}{3}(2\Omega_{cp}\frac{d\Omega_{cp}}{d\eta}z+\Omega_{cp}^2\frac{dz}{d\eta}$$

  where $\dfrac{d\Omega_{cp}}{d\eta} = \eta^2c^2(3\Omega_{wy}+2\Omega_{cp}/\eta)/(1-\eta^2c^2)$

  and $\dfrac{dz}{d\eta} = c(1-\eta^2c^2)/(1+\eta^2c^2)^2$

- If $f(\eta)$ is not close enough to zero, use the Newton-Raphson iteration method,

  $$\eta_{n+1} = \eta_n-f(\eta_n)/f'(\eta_n)$$

where η_{n+1} is a better approximation to the root of $f(\eta) = 0$

. Re-evaluate f_c^* (refer Step 3a)

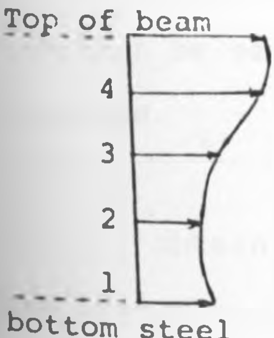
until convergence in f_c^* is achieved.

. Compute strains ε_w, ε_ℓ, ε_{cp} and θ_b from $\tan\theta_b = \eta c$

(c) Repeat Step 3 to compute compatible strains and also angle of principal compression at all other levels by altering the longitudinal strains as shown in Step 2(c).

STEP 4: Compute the longitudinal compression stresses in concrete and obtain $F_{c\ell}$ and $y_{c\ell}$

Levels — Longitudinal compression stresses in concrete



$Q_{ct} = bf_c'(2\Omega_{ct} - \Omega_{ct}^2)$

$Q_{c4} = q_4 \cot\theta_4$

$Q_{c3} = q_3 \cot\theta_3$

$Q_{c2} = q_2 \cot\theta_2$

$Q_{c1} = q_1 \cot\theta_1$

· $F_{c\ell} = (Q_{ct} + 4Q_{c4} + Q_{c3})y_n/6 + (Q_{c3} + 4Q_{c2} + Q_1)(d - y_n)/6$

· $y_{c\ell} = [(2Q_{ct}y_n + Q_{c3}y_n)(y_n/6)$

$+ (Q_{c3}y_n + 2Q_{c2}(y_n + d) + Q_{c1}d)(d - y_n)/6]$

. Equilibrium in the longitudinal direction requires that the magnitudes of $F_{c\ell}$ and $F_{s\ell}$ (Step 2b) be equal and that these internal resultant forces constitute a couple which equilibrates the applied moment, M, so that:

$$F_{s\ell} = F_{c\ell}$$

$$M = F_{s\ell}(y_{s\ell}-y_{c\ell})$$

If $F_{c\ell} \neq F_{s\ell}$, revise y_n and repeat Steps 2, 3 and 4 till convergence is obtained. Use the latest evaluated value of jd(=M/T) for each new iteration.

STEP 5:

A new extreme compression fibre strain ε_{ct} can then be selected and the steps 2, 3 and 4 are repeated.

Obtain the results M, y_n, ϕ for the section and also θ, q, ε_ℓ, ε_ω, ε_t, ε_{cp}, $\gamma_{\ell t}$ and γ_m for various levels considered .

B.2 SAMPLE CALCULATION

Some of the equations shown in B.1 are utilized hereunder for a beam having the following cross-section and material properties:

$$b = 150 \text{ mm}$$
$$h = 400 \text{ mm}$$
$$d = 371.5 \text{ mm}$$
$$d' = 20 \text{ mm}$$

$A_s = 1470 \text{ mm}^2$	$f_{\ell y} = 505 \text{ N/mm}^2$	$\varepsilon_{\ell y} = 2.24\text{x}10^{-3}$
$A'_s = 101 \text{ mm}^2$	$f'_{\ell y} = 455 \text{ N/mm}^2$	$\varepsilon'_{\ell y} = 2.35\text{x}10^{-3}$
$A_{we} = 344 \text{ mm}^2$	$f_{yw} = 455 \text{ N/mm}^2$	$\varepsilon_{wy} = 2.35\text{x}10^{-3}$
$s_w = 371.5 \text{ mm}$		

$$f'_c = 31.5 \text{ N/mm}^2$$
$$\varepsilon_{co} = 2.1\text{x}10^{-3}$$
$$E_s = 225\text{x}10^3 \text{ N/mm}^2$$
$$E'_s = 194\text{x}10^3 \text{ N/mm}^2$$
$$E_w = 194\text{x}10^3 \text{ N/mm}^2$$
$$E_c = 20.5\text{x}10^3 \text{ N/mm}^2$$

Consider a case loading with top face strain $\varepsilon_{ct} = 0.0005$.

Step 1 — Pure flexure case:

$$\Omega_{ct} = \varepsilon_{ct}/\varepsilon_{co} = 0.2381$$
$$\alpha\beta = 0.2192$$
$$\beta = 0.6810$$

Assume all steel in elastic range.

$$Y_p = 173.618 \text{ mm}$$

check:

$$\varepsilon_\ell = 0.5699 \times 10^{-3} < \varepsilon_{\ell y}$$

$$\varepsilon'_\ell = 0.4424 \times 10^{-3} < \varepsilon'_{\ell y}$$

check correct.

Hence the moment of resistance for a pure flexure case, M_p is calculated as,

$$M_p = 59.219 \times 10^6 \text{ N mm}$$

Step 2 Combined flexure and shear case:

$$\text{Guess } y_n = Y_p$$

$$= 173.618 \text{ mm}$$

$$\varepsilon_\ell = 0.5699 \times 10^{-3}$$

$$\varepsilon'_\ell = 0.4424 \times 10^{-3}$$

The magnitude and the position of resultant longitudinal forces are calculated as follows:

$$T = 188.49 \text{ kN}$$

$$C_s = 8.668 \text{ kN}$$

$$F_{s\ell} = T - C_s$$

$$= 179.82 \text{ kN}$$

$$y_{s\ell} = 196.62 \text{ mm}$$

Assuming $a_v = 1000$ mm,

$V = 59.219 \times 10^3$ N

$jd = 314.176$ mm

The shear flows can be obtained as follows:

$q_b = 188.49$ N/mm

$q_n = 188.91$ N/mm

Step 3: To compute compatible strains and principal angles of compression, etc. at bottom steel level:

$c = 2.235$

$\Omega_{wy} = 1.1168$

$\Omega_s = 0.2714$

$\ell_1 = 0.3651 < 1.0$

$\ell_2 = 0.4474 < 1.0$

Therefore the following can be calculated:

$\eta = 0.4063$

$\Omega_{cp} = 0.5860$

Assuming $f_c^* = f_c'$, $f(\eta) = 0.1947$

$f'(\eta) = 8.9451$

Using the iteration formula,

$\eta_{n+1} = 0.3845$

and hence $f(\eta) = 0.0415$

The step 3 is now repeated until convergence in $f(\eta) = 0$ is obtained and then, $\eta = 0.3775$ (approximately). Therefore, $\tan \theta_b = \eta c$ leading to $\theta_b = 40.11^o$

Also the following are calculated:

$$\varepsilon_w = 0.8871 \times 10^{-3}$$

$$\varepsilon_{cp} = 0.2092 \times 10^{-3}$$

$$\varepsilon_\ell = 0.5699 \times 10^{-3}$$

$$\Omega_t = 0.7934 \times 10^{-3}$$

Re-evaluate f_c^* as $f_c^* = 29.4457\ N/mm^2$

and step 3 is again repeated until convergence in f_c^* is obtained.

<u>Step 4</u>: Evaluation of shear flow for the levels considered and $F_{c\ell}$ and $y_{c\ell}$ and also the convergence in $F_{c\ell} = F_{s\ell}$ using the new value of jd for each new iteration, is carried out and results obtained. This warrants for convenience-a computed-aided design.

The solution technique outlined above has been programmed to enable a computer-aided design in predicting the behavioural response of rectangular reinforced concrete beams loaded in flexure and shear. The steps of the program, the full listing and output of results are given in Appendix C.

APPENDIX C COMPUTER PROGRAM

A computer program for the analysis of solid rectangular reinforced concrete beams under combined flexure and shear based on the compression field theory is outlined below. The equations used are already provided in Appendix B. For an assumed top face strain, the sequence of operations followed in the program (for the known geometric and material properties of a beam) are:

(1) input beam data
(2) input the analysis control parameters
(3) function to compute pure moment
(4) work out stress block factors
(5) compute depth of neutral axis, moment and curvature for pure moment case
(6) assume depth to neutral axis for combined flexure and shear case and compute the forces and shear flow
(7) compute strains, principal angle of compression at chosen points
(8) repeat steps (6) and (7) to obtain convergence
(9) output results

C.1 BEAM INPUT DATA

It includes the section and material properties of the beam (the symbols used in the program listing are shown on the left and the corresponding symbols as explained in 'NOTATION' are shown on the right).

(a) Section geometry:

B	b
D	d
D1	d'

(b) Material properties:

As	A_s
A1	A'_s
AW	A_{we}
SW	s_w
F1	$f'_{\ell y}$
F2	$f_{\ell y}$
FW	f_{yw}
ES	E_s
E1	E'_s
EC	E_c
YW	ε_{wy}
EO	ε_{co}

(b) Cont.d

FC f'_c

EW ε_v

(c) Control parameters:

AV shear span a_v

ET top fibre compression strain, ε_{ct}

C.2 NOTATION USED IN PRINTING RESULTS

Levels considered:

1 - level of bottom steel

2 - midway between levels 1 and 3

3 - level of neutral axis

4 - midway between level 3 and top of beam

THETA	principal angle of compression in deg
SHEAR FLOW	shear flow
ELC	average longitudinal strain
EW	average transverse strain
ECP	principal compressive strain in concrete
FCP	principal compressive stress
GAMLT	shear strain
GAM-M	maximum shear strain

The program listing and the printout of output are given in C.3 and C.4 respectively.

C.3 PROGRAM LISTING AND INPUT DATA FOR BEAMS BWR-2 AND BWR-1

```
10 LPRINT TAB(20)"TOWARDS AN EFFICIENT SHEAR REINFORCEMENT FOR"
20 LPRINT TAB(19)"CONCRETE RECTANGULAR BEAMS-RESPONSE PREDICTION
30 LPRINT TAB(29)"BY COMPRESSION FIELD THEORY"
40 LPRINT :LPRINT
50 LPRINT TAB(17)"SOURCE : P.GANAPATHI, UNIVERSITY OF NAIROBI (1988
60 LPRINT :LPRINT
70 LPRINT TAB(21)"BEAM   :BWR-2"
200 LET B=150
210 LET D=371.5
220 LET D1=20
230 LET AS=1470
240 LET A1=101
250 LET AW=299
260 LET SW=371.5
270 LET F1=455
280 LET F2=505
290 LET FW=455
300 LET ES=225000!
310 LET E1=194000!
320 LET EC=20500
325 LET YW=.00224
330 LET EO=.0021
340 LET FC=31.6
350 LET AV=1000
```

```
10 LPRINT TAB(20)"TOWARDS AN EFFICIENT SHEAR REINFORCEMENT FOR"
20 LPRINT TAB(19)"CONCRETE RECTANGULAR BEAMS-RESPONSE PREDICTION"
30 LPRINT TAB(29)"BY COMPRESSION FIELD THEORY"        ;
40 LPRINT :LPRINT
50 LPRINT TAB(17)"SOURCE : P.GANAPATHI, UNIVERSITY OF NAIROBI (1988)"
60 LPRINT :LPRINT
70 LPRINT TAB(21)"BEAM   :BWF-1"
200 LET B=150
210 LET D=371.5
220 LET D1=20
230 LET AS=1470
240 LET A1=101
250 LET AW=243
260 LET SW=369.5
270 LET F1=455
280 LET F2=505
290 LET FW=455
300 LET ES=225000
310 LET E1=194000
320 LET EC=28600
325 LET YW=.00235
330 LET E0=.0022
340 LET FC=32.5
350 LET AV=1000
```

```
360 DIM EL(4),B(4),C(4),OS(4),L1(4),L2(4)
370 DIM TH(4),F1(4),FP(4),EF(4),EW(4),OB(4)
380 DIM DF(4),DT(4),F2(4),FA(4),Z1(4)
390 DIM Z3(4),Z4(4),FT(4),EZ(4),DE(4),Z5(4)
395 DIM FZ(4),EA(4),EM(4),FC(4)
397 DIM F3(4),F4(4),F5(4)
400 FOR ET=.0005 TO .0035 STEP .00025
405 Z1=1
410 OM=ET/EO
420 BE=(4-OM)/(6-2*OM)
430 AB= OM-1/3*OM^2
440 DE=AB*FC*B
450 REM*TOP & BOTTOM STEEL YIELDING*
460 Y2=F2/ES
470 Y1=F1/E1
480 YP=(AS*F2-A1*F1)/DE
490 GOSUB 20000
500 IF S1>=Y1 AND S2>=Y2 THEN 510 ELSE 550
510 CS=A1*F1
520 CC=AB*FC*YP*B
530 M=CS*(D-D1)+(D-BE*YP*0.5)*CC
540 PH=ET/YP :GOTO 795
550 REM*BOTTOM STEEL YIELDING*
560 X1=A1*E1*ET
570 X2=(X1-AS*F2)/DE
580 YP=(-X2+(X2^2+(4*X1*D1/DE))^.5)*.5
590 GOSUB 20000
```

```
600 IF S1<Y1 AND S2>=Y2 THEN 610 ELSE 650
610 CS=A1*E1*S1
620 CC=AE*FC*YP*B
630 M=(CS*(D-D1))+(CC*(D-BE*YP*.5))
640 PH=ET/YP :GOTO 795
650 REM* NONE OF THE STEEL YIELDING*
660 X3=(AS*ES*ET+A1*E1*ET)/DE
670 X4=(AS*ES*ET*D+A1*E1*ET*D1)/DE
680 YP=(-X3+(X3^2+4*X4)^.5)*.5
690 GOSUB 20000
700 IF(S1-Y1)>0 AND(S2-Y2)>0 THEN GOTO 750 ELSE 710
710 CS=A1*E1*S1
720 CC=AE*FC*YP*B
730 M=CS*(D-D1)+CC*(D-BE*YP*.5)
740 PH=ET/YP :GOTO 795
745 REM*TOP STEEL YIELDING*
750 X5=AS*ES*ET
755 X6=(X5+A1*F1)/DE
760 YP=(-X6+(X6^2+(4*X5*D1/DE))^.5)*.5
765 GOSUB 20000
770 IF S1>=Y1 AND S2<Y2 THEN 775 ELSE 795
775 CS=A1*F1
780 CC=AE*FC*YP*B
785 M=CS*(D-D1)+CC*(D-BE*YP*.5)
790 PH=ET/YP
795 PRINT ET,M,PH,YP
796 LPRINT TAB(25)"TOP FACE STRAIN"TAB(48)"="TAB(50)ET
797 LPRINT TAB(25)"PURE MOMENT"TAB(48)"="TAB(50)M
798 LPRINT TAB(25)"DEPTH OF NA BELOW TOP"TAB(48)"="TAB(50)YP
799 LPRINT TAB(25)"PURE CURVATURE"TAB(48)"="TAB(50)PH
800 YN=YP
810 S1=(1-D1/YN)*ET
820 S2=(D/YN-1)*ET
830 IF S1<0 GOTO 860
840 IF S1>=Y1 GOTO 870
850 IF S1<Y1 GOTO 880
860 CS=-A1*S1*E1 GOTO 890
870 CS=A1*F1:GOTO 890
880 CS=A1*S1*E1
890 IF S2>=Y2 GOTO 910
900 IF S2<Y2 GOTO 920
910 T2=AS*F2:GOTO 930
920 T2=AS*S2*ES
930 FL=T2-CS
940 YL=(T2*D-CS*D1)/FL
950 JD=M/T2
960 V=M/AV
```

```
970 QS=V/JD
980 QN=1.5*V/D*(1-(D-YN)/(3*JD))
990 TW=AW/SW
995 E2=FW/YW
1000 DY=FW/(E2*EO)
1150 EL(1)=S2
1160 Q(1)=QS
1170 EL(2)=S2/2
1180 Q(2)=(QS+3*QN)/4
1190 EL(3)=0
1200 Q(3)=QN
1210 EL(4)=ET/2
1220 Q(4)=.75*QN
1230 FOR I=1 TO 4
1240 C(I)=TW*FW/Q(I)
1250 DS(I)=EL(I)/EO
1260 L1(I)=(DS(I)/(DY*C(I)^2))^(1/3)
1270 L2(I)=1/C(I)
1280 IF L1(I)<1! AND L2(I)<1! THEN 1500
1290 TH(I)=ATN(C(I))
1297 LET F5(I)=FC
1300 FP(I)=Q(I)*(1+C(I)^2)/(B*C(I))
1305 IF ET>=EO THEN 1320 ELSE 1310
1310 DP(I)=1+(1-FP(I)/F5(I))^.5 :GOTO 1330
1320 DP(I)=1-(1-FP(I)/F5(I))^.5
1330 EP(I)=DP(I)*EO
1340 EW(I)=(EP(I)*(1-C(I)^2)+EL(I))/C(I)^2
1350 YW=FW/E2
1360 IF EW(I)<YW THEN 1500
1370 DT(I)=(EL(I)+EP(I)+EW(I))/EO
1375 IF F4(I)>0 THEN 1410
1380 F1(I)=FC/(.8+.34*DT(I))
1385 F4(I)=F5(I)-F1(I)
1390 IF F4(I)<=.5 THEN 1410 ELSE 1400
1395 IF F4(I)<0 THEN 1400
1400 F5(I)=F1(I) :GOTO 1300
1410 PRINT EW(I),EL(I),EP(I),TH(I)
1430 GOTO 1770
1500 FC(I)=FC
1510 KA(I)=Q(I)/(FC(I)*B)
1520 Z1(I)=(L1(I)+L2(I))/2
1530 Z3(I)=1-(Z1(I)*C(I))^2
1540 Z4(I)=1+(Z1(I)*C(I))^2
1550 DP(I)=(Z1(I)^3*C(I)^2*DY-DS(I))/Z3(I)
1560 EZ(I)=Z1(I)*C(I)/Z4(I)
1570 FT(I)=DP(I)*EZ(I)-(DP(I)^2*EZ(I))/3-KA(I)
1580 DE(I)=C(I)*Z3(I)/Z4(I)^2
1590 DF(I)=(3*DY+2*DP(I)/Z1(I))*Z1(I)^2*C(I)^2/Z3(I)
1600 Z5(I)=(2*DF(I)*DP(I)*EZ(I)+(DP(I)^2*DE(I)))/3
1610 F3(I)=DP(I)*EZ(I)+DF(I)*DE(I)-Z5(I)
1620 IF FT(I)>=.1 THEN 1630 ELSE 1650
1630 Z1(I)=Z1(I)-FT(I)/F3(I)
1640 GOTO 1530
```

```
1650 EW(I)=Z1(I)*YW
1660 EF(I)=ED*DF(I)
1670 GT(I)=(EL(I)+EF(I)+EW(I))/EC
1675 IF F2(I)<0 THEN 1730
1680 F1(I)=FC/(.8+.34*GT(I))
1700 F2(I)=FC(I)-F1(I)
1710 IF F2(I)<=.5 THEN 1730 ELSE 1720
1715 IF F2(I)<0 THEN 1500
1720 FC(I)=F1(I) :GOTO 1510
1730 EW(I)=Z1(I)*YW
1740 EF(I)=DF(I)*EC
1750 TT(I)=Z1(I)*C(I)
1760 TH(I)=ATN(TT(I))
1770 NEXT I
1780 QT=B*FC*(2*DM-DM^2)/(.8+.34*GT(1))
1790 Q4=.75*DN*COS(TH(4))/SIN(TH(4))
1800 Q2=(QE+3*DN)/4*COS(TH(2))/SIN(TH(2))
1810 Q3=DN*COS(TH(3))/SIN(TH(3))
1820 Q1=QB*COS(TH(1))/SIN(TH(1))
1830`FF=(QT+4*Q4+Q3)*YN/6+(Q3+4*Q2+Q1)*(D-YN)/6
1840 YC=(2*QT*YN+Q3*YN)*YN/6
1850 YC=YC+(Q3*YN+2*Q2*(YN+D)+Q1*D)*(D-YN)/6
1860 YC=YC/FF
1870 FQ=FL-FF
1871 MC=FF*(YL-YC)
1873 ND=MC/T2
1875 JD=ND
1877 Z1=Z1+1
1878 IF Z1>12 THEN 1950
1880 IF FQ>5000 THEN 1900
1890 IF FQ<5000 AND FQ>-5000 THEN 1950 ELSE 1910
1900 YN=YN+.01*YP : GOTO 810
1910 YN=YN-.01*YP : GOTO  810
1950 PC=ET/ND
1955 FOR I=1 TO 4
1960 GA(I)=(2*(EL(I)+EF(I))*(EP(I)+EW(I)))^.5
1970 GM(I)=EL(I)+EW(I)+2*EF(I)
1975 NEXT I
2000 PRINT ET,MC,PC,YN :LPRINT :LPRINT
2001 LPRINT TAB(30)"COMBINED FLEXURE AND SHEAR":LPRINT
2002 LPRINT TAB(25)"MOMENT"TAB(48)"="TAB(50)MC
2003 LPRINT TAB(25)"DEPTH OF NA BELOW TOP"TAB(48)"="TAB(50)YN
2004 LPRINT TAB(25)"CURVATURE"TAB(48)"="TAB(50)PC
2005 SC=MC/1000
2006 LPRINT TAB(25)"SHEAR"TAB(48)"="TAB(50)SC
2020 PRINT I,
2022 LPRINT TAB(5)"LEVEL"TAB(20)"1"TAB(35)"2"TAB(50)"3"TAB(65)"4"
2040 FOR I=1 TO 4
2048 TH(I)=TH(I)*180/3.14159
2049 NEXT I
```

```
2052 LPRINT TAB(5)"THETA"TAB(20)TH(1)TAB(35)TH(2)TAB(50)TH(3)TAB(65)TH(4)
2082 LPRINT TAB(5)"SHEAR FLOW"TAB(20)Q(1)TAB(35)Q(2)TAB(50)Q(3)TAB(65)Q(4)
2112 LPRINT TAB(5)"ELC"TAB(20)EL(1)TAB(35)EL(2)TAB(50)EL(3)TAB(65)EL(4)
2142 LPRINT TAB(5)"EW"TAB(20)EW(1)TAB(35)EW(2)TAB(50)EW(3)TAB(65)EW(4)
2172 LPRINT TAB(5)"ECF"TAB(20)EF(1)TAB(35)EF(2)TAB(50)EF(3)TAB(65)EF(4)
2180 FOR I=1 TO 4
2182 FF(I)=FC*(2*DF(I)-DF(I)^2)/(.8+.34*DT(I))
2184 NEXT I
2202 LPRINT TAB(5)"FCP"TAB(20)FF(1)TAB(35)FF(2)TAB(50)FF(3)TAB(65)FF(4)
2232 LPRINT TAB(5)"GAMLT"TAB(20)GA(1)TAB(35)GA(2)TAB(50)GA(3)TAB(65)GA(4)
2262 LPRINT TAB(5)"GAM-M"TAB(20)GM(1)TAB(35)GM(2)TAB(50)GM(3)TAB(65)GM(4)
2264 LPRINT :LPRINT
2280 NEXT ET
2290 END
20000 S1=(1-D1/YP)*ET
20010 S2=(D/YP-1)*ET
20020 RETURN
```

TOWARDS AN EFFICIENT SHEAR REINFORCEMENT FOR
CONCRETE RECTANGULAR BEAMS-RESPONSE PREDICTION
BY COMPRESSION FIELD THEORY

SOURCE : P.GANAPATHI, UNIVERSITY OF NAIROBI (1988)

BEAM :BWR-2

TOP FACE STRAIN	=	.0005
PURE MOMENT	=	5.934791E+07
DEPTH OF NA BELOW TOP	=	173.4329
PURE CURVATURE	=	2.88296E-06
TOP FACE STRAIN	=	.00075
PURE MOMENT	=	8.617271E+07
DEPTH OF NA BELOW TOP	=	176.0106
PURE CURVATURE	=	4.261107E-06
TOP FACE STRAIN	=	.001
PURE MOMENT	=	1.110202E+08
DEPTH OF NA BELOW TOP	=	178.7204
PURE CURVATURE	=	5.595332E-06
TOP FACE STRAIN	=	.00125
PURE MOMENT	=	1.33826E+08
DEPTH OF NA BELOW TOP	=	181.5751
PURE CURVATURE	=	6.884203E-06

C.4 PRINTOUT OF OUTPUT OF RESULTS:

(a) Beam BWR-2:

TOP FACE STRAIN	=	.0015
PURE MOMENT	=	1.545191E+08
DEPTH OF NA BELOW TOP	=	184.5884
PURE CURVATURE	=	8.126187E-06
TOP FACE STRAIN	=	.00175
PURE MOMENT	=	1.73022E+08
DEPTH OF NA BELOW TOP	=	187.7762
PURE CURVATURE	=	9.319604E-06
TOP FACE STRAIN	=	.002
PURE MOMENT	=	1.892491E+08
DEPTH OF NA BELOW TOP	=	191.157
PURE CURVATURE	=	1.04626E-05
TOP FACE STRAIN	=	.00225
PURE MOMENT	=	2.031057E+08
DEPTH OF NA BELOW TOP	=	194.7526
PURE CURVATURE	=	1.155312E-05
TOP FACE STRAIN	=	.0025
PURE MOMENT	=	2.144857E+08
DEPTH OF NA BELOW TOP	=	198.5875
PURE CURVATURE	=	1.258891E-05

TOWARDS AN EFFICIENT SHEAR REINFORCEMENT FOR
CONCRETE RECTANGULAR BEAMS-RESPONSE PREDICTION
BY COMPRESSION FIELD THEORY

SOURCE : P.GANAPATHI, UNIVERSITY OF NAIROBI (1988)

BEAM :BWR-2

TOP FACE STRAIN	=	.0005
PURE MOMENT	=	5.934791E+07
DEPTH OF NA BELOW TOP	=	173.4329
PURE CURVATURE	=	2.88296E-06

COMBINED FLEXURE AND SHEAR

MOMENT	=	2.029701E+07
DEPTH OF NA BELOW TOP	=	187.3075
CURVATURE	=	4.006126E-06
SHEAR	=	20297.01

LEVEL	1	2	3	4
THETA	39.36647	34.04386	26.56507	35.47272
SHEAR FLOW	162.6247	190.1408	199.3129	149.4847
ELC	4.916847E-04	2.458424E-04	0	.00025
EW	8.161157E-04	7.857845E-04	6.095785E-04	6.515548E-04
ECP	1.763409E-04	2.076055E-04	2.031927E-04	1.642184E-04
FCP	4.887296	5.935319	6.24661	4.882973
GAMLT	1.151509E-03	9.491586E-04	5.747161E-04	8.220808E-04
GAM-M	1.660482E-03	1.446838E-03	1.015964E-03	1.229992E-03

TOP FACE STRAIN	=	.00075
PURE MOMENT	=	8.617271E+07
DEPTH OF NA BELOW TOP	=	176.0106
PURE CURVATURE	=	4.261107E-06

COMBINED FLEXURE AND SHEAR

MOMENT	=	2.89182E+07
DEPTH OF NA BELOW TOP	=	193.6116
CURVATURE	=	5.911086E-06
SHEAR	=	28918.2

LEVEL	1	2	3	4
THETA	39.38481	33.95918	26.56507	35.513
SHEAR FLOW	227.9173	277.0072	293.3706	220.0279
ELC	6.890922E-04	3.445461E-04	0	.000375
EW	1.144526E-03	1.141129E-03	8.972446E-04	9.60457E-04
ECP	2.524347E-04	3.166503E-04	2.990815E-04	2.32586E-04
FCP	6.276003	8.070301	8.413052	6.274095
GAMLT	1.621898E-03	1.388437E-03	8.459298E-04	1.204057E-03
GAM-M	2.338488E-03	2.118976E-03	1.495407E-03	1.800629E-03

TOP FACE STRAIN	=	.001
PURE MOMENT	=	1.110202E+08
DEPTH OF NA BELOW TOP	=	178.7204
PURE CURVATURE	=	5.595332E-06

COMBINED FLEXURE AND SHEAR

MOMENT	=	3.676807E+07
DEPTH OF NA BELOW TOP	=	198.3796
CURVATURE	=	7.850191E-06
SHEAR	=	36768.07

LEVEL	1	2	3
THETA	39.49219	34.00285	26.56507
SHEAR FLOW	288.6364	357.918	381.0119
ELC	8.726722E-04	4.363361E-04	0
EW	1.454985E-03	1.476867E-03	1.165287E-03
ECP	3.599287E-04	4.325687E-04	3.884289E-04
FCP	8.018452	9.897913	10.08864
GAMLT	2.115213E-03	1.821602E-03	1.098643E-03
GAM-M	3.047515E-03	2.778341E-03	1.942145E-03

35.6056
285.759
.0005
1.251652E-03
2.910398E-04
7.209521
1.562262E-03
2.333732E-03

TOP FACE STRAIN	=	.00125
PURE MOMENT	=	1.33826E+08
DEPTH OF NA BELOW TOP	=	181.5751
PURE CURVATURE	=	6.884203E-06

COMBINED FLEXURE AND SHEAR

MOMENT	=	4.389097E+07
DEPTH OF NA BELOW TOP	=	201.5484
CURVATURE	=	9.928664E-06
SHEAR	=	43890.97

LEVEL	1	2	3
THETA	38.64639	33.97493	26.56507
SHEAR FLOW	348.6229	432.6089	460.6042
ELC	1.054038E-03	5.270188E-04	0
EW	1.705144E-03	1.783186E-03	1.408712E-03
ECP	1.003966E-04	5.179195E-04	4.695705E-04
FCP	2.335141	10.86332	11.36838
GAMLT	2.041753E-03	2.19295E-03	1.328146E-03
GAM-M	2.959975E-03	3.346044E-03	2.347853E-03

4
35.76663
345.4531
6.250001E-04
1.522121E-03
3.425502E-04
7.870006
1.89956E-03
2.832221E-03

TOP FACE STRAIN = .0015
PURE MOMENT = 1.545191E+08
DEPTH OF NA BELOW TOP = 184.5884
PURE CURVATURE = 8.126187E-06

COMBINED FLEXURE AND SHEAR

MOMENT = 4.730981E+07
DEPTH OF NA BELOW TOP = 204.8932
CURVATURE = 1.279076E-05
SHEAR = 47309.81

LEVEL	1	2	3
THETA	42.23171	36.16847	34.46954
SHEAR FLOW	403.419	500.9338	533.4388
ELC	1.21971E-03	6.098549E-04	0
EW	.0022913	4.256491E-03	3.990701E-03
ECP	3.79778E-03	3.576054E-03	3.557143E-03
FCP	5.518828	7.378495	8.103573
GAMLT	7.816891E-03	8.097695E-03	7.32786E-03
GAM-M	1.110657E-02	1.201845E-02	1.110499E-02

4
35.29216
400.0791
7.500001E-04
1.732212E-03
2.36259E-04
5.410953
1.970493E-03
2.95473E-03

TOWARDS AN EFFICIENT SHEAR REINFORCEMENT FOR
CONCRETE RECTANGULAR BEAMS-RESPONSE PREDICTION
BY COMPRESSION FIELD THEORY

SOURCE : P.GANAPATHI, UNIVERSITY OF NAIROBI (1989)

BEAM :BWR-1

TOP FACE STRAIN	=	.0005
PURE MOMENT	=	5.876878E+07
DEPTH OF NA BELOW TOP	=	174.2814
PURE CURVATURE	=	2.868924E-06
TOP FACE STRAIN	=	.00075
PURE MOMENT	=	8.546926E+07
DEPTH OF NA BELOW TOP	=	176.7334
PURE CURVATURE	=	4.243681E-06
TOP FACE STRAIN	=	.001
PURE MOMENT	=	1.103111E+08
DEPTH OF NA BELOW TOP	=	179.3043
PURE CURVATURE	=	5.577113E-06
TOP FACE STRAIN	=	.00125
PURE MOMENT	=	1.332372E+08
DEPTH OF NA BELOW TOP	=	182.0046
PURE CURVATURE	=	6.867958E-06
TOP FACE STRAIN	=	.0015
PURE MOMENT	=	1.54185E+08
DEPTH OF NA BELOW TOP	=	184.8461
PURE CURVATURE	=	8.11486E-06

(b) Beam BWR-1:

TOP FACE STRAIN	=	.00175
PURE MOMENT	=	1.730866E+08
DEPTH OF NA BELOW TOP	=	187.8422
PURE CURVATURE	=	9.31633E-06
TOP FACE STRAIN	=	.002
PURE MOMENT	=	1.898674E+08
DEPTH OF NA BELOW TOP	=	191.0082
PURE CURVATURE	=	1.047076E-05
TOP FACE STRAIN	=	.00225
PURE MOMENT	=	2.044457E+08
DEPTH OF NA BELOW TOP	=	194.3618
PURE CURVATURE	=	1.157635E-05
TOP FACE STRAIN	=	.0025
PURE MOMENT	=	2.1673E+08
DEPTH OF NA BELOW TOP	=	197.923
PURE CURVATURE	=	1.263118E-05

TOWARDS AN EFFICIENT SHEAR REINFORCEMENT FOR
CONCRETE RECTANGULAR BEAMS-RESPONSE PREDICTION
BY COMPRESSION FIELD THEORY

SOURCE : P.GANAPATHI, UNIVERSITY OF NAIROBI (1988)

BEAM :BWR-1

TOP FACE STRAIN	=	.0005
PURE MOMENT	=	5.876878E+07
DEPTH OF NA BELOW TOP	=	174.2814
PURE CURVATURE	=	2.868924E-06

COMBINED FLEXURE AND SHEAR

MOMENT	=	2.0022E+07
DEPTH OF NA BELOW TOP	=	188.2239
CURVATURE	=	4.021274E-06
SHEAR	=	20022

LEVEL	1	2	3	4
THETA	38.38929	34.08457	26.56507	35.39988
SHEAR FLOW	161.0279	188.4338	197.569	149.1768
ELC	4.868569E-04	2.434284E-04	0	.00025
EW	1.001953E-03	1.001365E-03	7.758066E-04	8.270011E-04
ECP	3.816555E-04	3.966946E-04	2.586022E-04	3.387435E-04
FCP	9.455839	10.12051	7.492173	9.067379
GAMLT	1.550278E-03	1.337856E-03	7.314375E-04	1.171601E-03
GAM-M	2.25212E-03	2.038182E-03	1.293011E-03	1.754488E-03

TOP FACE STRAIN	=	.00075
PURE MOMENT	=	8.546926E+07
DEPTH OF NA BELOW TOP	=	176.7334
PURE CURVATURE	=	4.243681E-06

COMBINED FLEXURE AND SHEAR

MOMENT	=	2.89063E+07
DEPTH OF NA BELOW TOP	=	194.4067
CURVATURE	=	5.863012E-06
SHEAR	=	28906.3

LEVEL	1	2	3	4
THETA	37.95986	33.51278	26.56507	35.16522
SHEAR FLOW	225.9706	274.9212	291.2381	218.4286
ELC	6.832067E-04	3.416034E-04	0	.000375
EW	1.384518E-03	1.429769E-03	1.143623E-03	1.208546E-03
ECP	4.074904E-04	5.082501E-04	3.812076E-04	4.464269E-04
FCP	9.238212	11.52638	9.932984	10.64145
GAMLT	1.977139E-03	1.814957E-03	1.078218E-03	1.648902E-03
GAM-M	2.882706E-03	2.787873E-03	1.906038E-03	.0024764

TOP FACE STRAIN = .001
PURE MOMENT = 1.103111E+08
DEPTH OF NA BELOW TOP = 179.3043
PURE CURVATURE = 5.577113E-06

COMBINED FLEXURE AND SHEAR

MOMENT = 3.247259E+07
DEPTH OF NA BELOW TOP = 199.0278
CURVATURE = 8.826499E-06
SHEAR = 32472.59

LEVEL	1	2	3	4
THETA	37.1188	40.06344	38.30154	34.77345
SHEAR FLOW	286.6193	355.8064	378.8688	284.1516
ELC	8.665738E-04	4.332869E-04	0	.0005
EW	1.703556E-03	2.363658E-03	2.543192E-03	1.549462E-03
ECP	2.554885E-04	4.230486E-03	4.216596E-03	4.768968E-04
FCP	5.749408	2.553161	2.81504	10.55323
GAMLT	2.096745E-03	7.842651E-03	7.550271E-03	1.989745E-03
GAM-M	3.081107E-03	1.125792E-02	1.097638E-02	3.003256E-03

APPENDIX D EVALUATION OF AVERAGE STRAINS FROM EXPERIMENTAL DATA OF STRAINS

The average longitudinal, transverse and diagonal strains were calculated as explained below for the test beams, using the experimental data of strains tabulated in Appendix E.

D.1 AVERAGE LONGITUDINAL STRAINS (ε_{ℓ})

For beams BVWR-1 and BVWR-2, the regions EF and TU, and GH and RS had similar load conditions. But the pattern of shear reinforcement was different. Using the method of 3-level moving average, the average longitudinal strains were computed for all the regions and levels considered (Tables D1 and D2). For example, the average longitudinal strain ($\varepsilon_{\ell 2}$) for the region EF at level 2 was calculated as,

$$\varepsilon_{\ell 2} = \frac{1}{3} (\varepsilon_{\ell 1} + \varepsilon_{\ell 2} + \varepsilon_{\ell 3})$$

where $\varepsilon_{\ell 1}$, $\varepsilon_{\ell 2}$ and $\varepsilon_{\ell 3}$ are the longitudinal strains E1F1, E2F2 and E3F3 respectively.

For beam BWR-2, the load conditions and the reinforcement pattern were similar for regions

EF and TU, and GH and RS. Applying again the 3-level moving average method, for example, the average longitudinal strain ($\varepsilon_{\ell 2}$) for the region EF or TU atlevel 2, was calculated as,

$$\varepsilon_{\ell 2} = [\frac{1}{3}(\varepsilon_{\ell 1}+\varepsilon_{\ell 2}+\varepsilon_{\ell 3})_{EF} + \frac{1}{3}(\varepsilon_{\ell 1}+\varepsilon_{\ell 2}+\varepsilon_{\ell 3})_{TU}] \div 2$$

This procedure was repeated for all other levels for the region EF or TU and also for GH or RS (Table D3).

D.2 AVERAGE TRANSVERSE STRAINS (ε_v)

For beams BVWR-1 and BVWR-2, the average transverse strains were calculated for the regions EF and GH (regions with vertical stirrups) by a simple arithmetic mean (Tables D4 and D5). For example, the average transverse strain (ε_{v2}) for the region EF at level 2 was calculated as,

$$\varepsilon_{v2} = \tfrac{1}{2}(\text{strain E1E3} + \text{strain F1F3})$$

Considering that the beam BWR-2 had similar load conditions and pattern of shear reinforcement, the average transverse strain was assumed to represent the regions EF or TU and GH or RS. For example, the average transverse strain (ε_{v2}) for the region EF (or TU) at level 2 was computed as,

$$\varepsilon_{v2} = [\tfrac{1}{2}(\text{strain E1E3} + \text{strain F1F3}) + \tfrac{1}{2}(\text{strain T1T3} + \text{strain U1U3})] \div 2$$

This procedure was repeated for all other levels for the regions EF or TU and GH or RS (Table D6).

D.3 AVERAGE DIAGONAL STRAINS (ε_d)

For beams BVWR-1 and BVWR-2 the average diagonal strains ε_{d1} and ε_{d2}, which are mutually perpendicular, were taken as the strain values calculated from the experimental data (Tables D7 to D10). For example, in the region EF at level 2,

$$\varepsilon_{d1} = \text{strain E1F3}$$

$$\varepsilon_{d2} = \text{strain F1E3}$$

Since the shear reinforcement pattern and loading conditions were similar for the regions EF and TU, and GH and RS for the beam BWR-2, the average diagonal strains were computed by averaging the corresponding values (Table D11). For example, in the region EF (or TU) at level 2,

$$\varepsilon_{d1} = \tfrac{1}{2}(\text{strain E1F3} + \text{strain U1T3})$$

$$\varepsilon_{d2} = \tfrac{1}{2}(\text{strain F1E3} + \text{strain T1U3})$$

TABLE D1 BEAM BVWR-1

AVERAGE LONGITUDINAL STRAINS x 10^{-3} (ε_ℓ)

Moment kN m	Regions EF and TU						Regions GH and RS					
	Levels						Levels					
	2	3	4	5	6	7	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0	0	0	0	0
	4.883	2.467	2.392	0.650	-1.025	-1.125	1.392	1.975	2.658	-3.317	-3.050	-2.492
55.6	0.508	0.658	0.600	-1.375	-0.492	-0.658	0.517	0.217	0.083	-0.133	-0.216	-0.058
	2.792	2.992	2.683	-3.000	-2.925	-2.400	1.892	2.233	2.050	-2.500	-3.325	-3.258
89.0	0.550	0.833	0.742	-1.383	-0.533	-0.792	0.250	0.008	0.158	-0.008	-0.100	-0.233
	2.058	1.975	2.242	1.283	-1.667	-1.825	2.300	2.025	3.383	-3.150	-4.200	-3.617
111.3	0.300	1.008	0.700	-0.950	-0.208	-0.417	0.450	0.008	0.058	-0.083	-0.008	-0.525
	2.358	0.558	2.250	-2.292	-1.667	-2.600	1.083	1.125	1.650	-1.758	-2.858	-2.917
133.5	1.025	1.583	1.158	-0.475	-0.458	-0.325	0.517	0.042	0.075	-0.283	-0.008	-0.550
	3.000	3.242	3.000	-2.767	-2.092	-1.808	1.633	2.192	1.575	-3.863	-2.725	-3.417
155.8	0.942	1.958	1.100	-0.292	-0.592	-0.192	0.408	0.008	0.050	-1.550	-0.225	-0.267
	1.208	1.375	0.850	-0.125	-2.408	-1.917	1.817	1.900	2.125	-2.142	-2.967	-3.000
178.0	1.775	2.567	1.517	-0.833	-0.183	-0.358	0.242	0.258	0.233	-0.383	-0.117	-0.617

TABLE D2 BEAM: BVWR-2

AVERAGE LONGITUDINAL STRAINS x 10^{-3} (ε_ℓ)

Moment kN m	Regions EF and TU, Levels 2	3	4	5	6	7	Regions GH and RS, Levels 2	3	4	5	6	7
0	0	0	0	0	0	0	0	0	0	0	0	0
	0.850	0.558	0.575	-0.100	0.000	-1.800	0.392	0.733	-0.308	-0.400	-1.350	-1.100
55.6	0.700	1.392	2.033	-1.083	-1.425	-1.050	0.467	0.658	-0.358	-0.317	-0.808	-0.050
	0.175	1.008	0.800	-1.000	-0.783	-1.108	1.042	0.400	-0.383	-0.358	-1.783	-2.692
89.0	0.525	0.975	1.658	-0.817	-1.550	-1.150	1.508	0.667	-0.125	-0.492	-1.000	0.075
	0.225	0.117	0.308	-0.808	-0.108	-1.633	0.117	0.450	0.142	-1.017	-2.050	-1.483
111.3	1.117	2.475	1.992	-0.842	-1.025	-2.375	2.158	0.925	0.200	-1.317	0.700	-0.392
	0.067	0.442	0.792	-0.575	-1.375	-3.075	0.233	-0.117	-0.233	-1.525	-2.433	-2.050
122.4	0.917	2.225	2.867	-1.833	-1.583	-1.967	2.958	2.792	0.533	-1.342	-1.983	-0.458
	0.092	0.192	0.142	-1.350	-1.000	-2.917	0.800	1.067	1.217	-0.150	-0.658	-0.342
133.5	0.825	2.333	3.058	-1.908	-1.908	-1.425	3.217	3.175	0.892	-0.242	-0.917	0.525
	0.258	0.418	0.043	-1.368	0.733	-1.075	2.950	2.083	0.933	-0.542	-0.717	-0.183
155.8	0.983	2.225	1.792	-0.733	-0.853	-2.075	3.733	3.708	2.058	-0.317	-0.350	1.183
	1.125	1.267	1.617	-1.992	-1.617	-2.583	6.075	4.483	1.792	-1.142	-1.358	-0.833
178.0	0.700	2.083	2.658	-1.750	-1.350	-1.925	5.900	5.667	1.908	0.250	-1.100	-0.358
	0.942	1.150	0.783	-1.442	-1.008	-2.767	11.02	7.443	3.267	-1.483	-2.317	-1.825
211.4	0.242	1.658	1.858	-0.908	-1.025	-1.725	4.800	4.650	1.567	-0.433	0.350	0.525

TABLE D3 BEAM: BWR-2

AVERAGE LONGITUDINAL STRAINS x 10^{-3} (ε_λ)

Moment kN m	Region EF or TU Levels 2	3	4	5	6	7	Region GH or RS Levels 2	3	4	5	6	7
0	0	0	0	0	0	0	0	0	0	0	0	0
55.0	0.093	0.083	0.054	-0.066	-0.192	-0.175	0.055	0.303	0.297	-0.330	-0.946	-1.327
89.0	0.213	0.452	0.120	-0.100	-0.196	-0.383	0.575	0.854	0.391	-0.325	-1.021	-1.971
111.3	0.513	0.321	0.122	-0.135	-0.329	-0.412	0.804	0.283	0.013	-0.371	-0.742	-1.346
122.4	1.058	0.412	0.170	-0.161	-0.376	-0.825	1.040	0.648	0.104	-0.571	-1.325	-2.342
133.5	1.425	0.510	0.290	-0.168	-0.472	-0.929	1.671	1.092	0.212	-0.604	-1.029	-1.967
155.8	1.296	0.522	0.320	-0.194	-0.420	-0.975	1.271	0.600	0.215	-0.032	-0.800	-1.142
178.0	1.224	0.633	0.310	-0.211	-0.636	-1.383	1.288	0.710	0.329	-0.066	-1.788	-2.029
211.4	1.600	0.658	0.298	-0.243	-0.720	-1.521	1.243	0.775	0.350	-0.146	-1.533	-2.529

TABLE D4 BEAM: BVWR-1

AVERAGE TRANSVERSE STRAINS $\times 10^{-3}$ (ε_v)

Moment kN m	Regions EF and TU						Regions GH and RS					
	Levels						Levels					
	2	3	4	5	6	7	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0	0	0	0	0
55.6	-1.588	0.013	-1.275	-2.338	-2.938	-3.113	-0.575	-0.938	-0.038	-1.563	-2.588	0.475
	0.488	0.275	0.850	0.800	2.125	0.588	0.488	2.863	0.163	0.138	0.700	2.975
89.0	-2.100	-0.175	-1.075	-1.963	-2.850	-1.900	-0.850	-0.138	0.350	-1.188	-1.638	0.650
	0.988	0.238	0.113	0.088	0.088	0.325	-4.225	1.825	0.438	1.788	0.538	1.913
111.3	-0.300	-0.838	-1.100	-1.513	-1.950	-1.638	0.113	-0.975	1.475	-1.325	-1.450	0.925
	1.325	1.775	4.325	3.063	0.125	0.225	-1.813	0.438	0.825	2.438	-0.463	2.688
133.5	0.550	-0.013	-1.075	-2.275	-3.188	-1.613	0.288	0.350	2.563	-1.213	-0.625	0.925
	1.425	1.663	2.325	8.738	3.113	1.325	0.788	0.575	0.863	2.288	2.563	5.000
155.8	0.438	-1.288	-1.650	-1.775	-2.925	-2.813	-1.238	2.000	1.650	-0.188	-0.425	1.600
	0.475	0.450	2.388	16.45	6.375	1.975	0.613	-1.325	0.975	1.000	5.375	9.750
178.0	3.275	-1.388	-1.075	-1.663	-1.800	-1.638	-0.963	1.150	2.763	-0.188	0.125	0.100
	2.238	1.350	7.425	21.37	7.878	1.100	1.300	-0.900	1.350	3.313	9.688	16.22

TABLE D5 BEAM: BVWR-2

AVERAGE TRANSVERSE STRAINS x 10^{-3} (ϵ_v)

Moment kN m	Regions EF and TU Levels 2	3	4	5	6	7	Regions GH and RS Levels 2	3	4	5	6	7
0	0	0	0	0	0	0	0	0	0	0	0	0
55.6	-1.413	-0.088	-0.038	-2.663	-1.313	-2.513	1.788	-1.475	0.100	-0.325	0.050	0.950
	-2.05	1.463	-0.125	0.075	-1.138	0.388	-1.063	0.800	-1.425	-0.113	-2.038	-1.225
89.0	-1.675	0.713	-0.375	-2.800	-1.138	0.088	-0.488	0.425	-0.325	-1.338	-2.713	0.963
	-2.425	0.038	1.688	1.000	0.350	1.525	0.0	2.388	0.075	0.100	-0.300	-1.438
111.3	0.225	0.050	-0.125	-2.225	-1.825	-2.713	0.363	1.438	2.550	0.450	-1.025	-0.213
	-1.225	0.400	0.238	0.238	-1.038	2.100	0.763	1.213	1.088	0.025	-0.613	-3.038
122.4	-2.375	2.075	1.613	-3.075	-0.463	-2.875	-0.988	1.263	-0.613	-0.363	-1.500	0.713
	-0.738	1.375	0.100	0.188	-1.363	0.563	-1.238	2.525	1.675	0.238	-0.075	-2.800
133.5	2.225	4.175	-0.200	-3.125	-1.775	-2.388	1.425	4.300	-1.525	-0.388	-1.938	-0.313
	-1.638	0.100	1.838	0.263	-0.05	2.563	2.113	2.863	0.913	0.200	0.0	-2.038
155.8	1.363	4.788	0.550	-2.650	-1.113	-2.625	2.625	3.125	1.925	1.700	-0.688	0.900
	-1.325	1.313	2.013	0.725	-0.800	0.500	2.650	3.163	1.400	0.063	-0.250	-1.200
178.0	7.915	5.550	1.225	-2.813	-1.388	-2.513	5.423	6.388	4.425	-0.438	-0.113	1.188
	-1.050	0.600	2.150	0.813	-0.975	1.258	3.975	3.338	3.388	0.488	-0.125	-3.250
211.4	13.95	6.550	1.425	-2.688	-1.438	-2.675	13.18	10.54	9.165	0.038	-0.763	1.013
	-1.550	0.088	2.875	3.600	-1.263	1.825	5.813	4.913	3.600	2.113	1.988	-3.175

TABLE D6 BEAM: BWR-2

AVERAGE TRANSVERSE STRAINS x 10^{-3} (ε_v)

Moment kN m	Region EF or TU Levels						Region GH or RS Levels					
	2	3	4	5	6	7	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0	0	0	0	0
55.6	0.613	-1.331	-0.875	-0.581	1.263	0.494	-2.250	-0.544	-1.313	-0.300	-1.256	-2.100
89.0	0.875	-1.000	-1.338	-0.469	1.119	0.756	-1.294	-0.069	-0.181	-0.625	-0.713	-1.475
111.3	0.800	-0.694	-0.813	-0.769	0.481	1.569	-1.000	-0.325	-2.144	-0.006	-0.825	0.350
122.4	-0.006	-1.988	-0.900	-0.100	0.219	0.644	-0.094	-0.419	-1.825	0.913	-2.150	-0.750
133.5	1.569	-0.644	0.019	0.281	1.725	1.138	-0.669	0.406	-1.469	0.144	-0.475	-0.131
155.8	0.869	-0.944	0.319	1.238	1.269	1.631	-0.669	1.306	0.256	-0.063	-0.931	0.806
178.0	0.481	-0.169	-0.225	0.881	1.156	0.688	0.144	-0.113	-1.063	-0.544	-0.669	-0.588
211.4	0.613	-1.388	-1.238	-1.100	0.106	0.264	-1.588	-0.231	0.200	-1.175	-1.500	-2.638

TABLE D7 BEAM: BVWR-1

AVERAGE DIAGONAL STRAINS x 10^{-3} (ε_d)

Moment kN m		Region EF Levels 2	3	4	5	6	7	Region TU Levels 2	3	4	5	6	7
0		0	0	0	0	0	0	0	0	0	0	0	0
55.6	ε_{d1}	1.785	0.583	0.283	0.035	-0.088	-0.071	0.424	0.636	0.194	0.548	0.159	0.194
	ε_{d2}	0.336	0.106	0.053	-0.018	0.301	0.018	0.566	0.689	0.371	-8.432	2.316	0.247
89.0	ε_{d1}	1.768	-0.088	0.301	-0.018	-0.212	0.088	1.998	0.477	0.106	0.389	0.071	-0.371
	ε_{d2}	1.609	0.283	-0.053	-0.141	0.407	0.177	1.644	0.689	0.654	-8.308	2.388	0.601
111.3	ε_{d1}	3.253	0.053	0.159	0.053	-0.194	-0.035	0.407	0.689	0.247	0.133	0.106	0.159
	ε_{d2}	2.934	0.265	0.212	0.141	1.273	0.336	1.998	0.990	2.970	-6.170	2.952	0.371
133.5	ε_{d1}	1.732	0.354	0.194	0.177	0.035	-0.018	0.389	0.636	0.159	0.937	0.177	-0.053
	ε_{d2}	4.985	0.301	0.212	-0.141	0.407	0.318	2.192	0.972	5.162	-3.942	4.950	0.318
155.8	ε_{d1}	2.033	0.177	0.389	0.088	0.177	0.088	0.442	0.619	0.124	6.540	0.000	-0.336
	ε_{d2}	6.523	0.495	0.407	0.283	0.619	0.285	2.687	1.326	9.228	0.071	8.697	0.088
178.0	ε_{d1}	2.245	0.124	0.354	0.053	0.088	0.265	0.619	0.778	0.194	8.273	1.768	-0.106
	ε_{d2}	8.379	0.177	0.407	0.318	0.636	0.194	3.253	0.301	11.21	2.068	10.60	0.301

TABLE D8 BEAM: BVWR -1

AVERAGE DIAGONAL STRAINS x 10^{-3} (ε_d)

Moment kN m		Region GH Levels 2	3	4	5	6	7	Region RS Levels 2	3	4	5	6	7
0		0	0	0	0	0	0	0	0	0	0	0	0
55.6	ε_{d1}	0.442	-0.371	-2.051	-1.061	-0.018	-0.018	-0.237	0.566	-1.220	-1.892	-1.450	0.318
	ε_{d2}	-1.290	1.591	0.017	0.017	-0.017	-0.336	-0.725	0.371	-1.414	0.460	-1.679	-0.778
89.0	ε_{d1}	0.725	-0.124	-1.768	-0.694	-0.106	0.283	-1.344	0.513	-1.273	-1.838	-1.538	0.177
	ε_{d2}	-1.167	0.937	1.061	0.725	0.301	0.283	-2.600	0.477	-2.600	-0.760	-1.111	1.202
111.3	ε_{d1}	0.831	-0.124	-1.626	-0.742	-0.194	-0.018	1.167	0.760	-1.114	-1.220	-1.573	2.139
	ε_{d2}	-1.49	2.280	1.520	1.184	0.760	-0.106	0.265	0.707	-0.371	-0.672	-0.088	-0.707
133.5	ε_{d1}	0.548	1.591	-1.503	-0.035	-0.203	0.442	-1.237	0.601	0.088	-1.520	0.124	0.884
	ε_{d2}	-1.255	3.500	2.828	2.386	1.662	0.141	0.689	1.856	-0.371	-0.583	2.157	3.748
155.8	ε_{d1}	0.530	0.230	-1.839	-0.141	0.000	0.088	-1.131	0.795	-1.008	-1.255	-0.283	4.738
	ε_{d2}	-1.114	4.225	3.748	3.200	2.404	-0.124	0.672	0.566	-0.106	0.460	6.930	6.187
178.0	ε_{d1}	0.778	0.177	-1.432	-0.566	-0.366	0.230	-0.955	0.636	1.290	-1.556	-1.785	6.629
	ε_{d2}	-0.495	5.445	4.914	3.200	3.483	0.424	1.626	0.760	-1.273	0.159	9.351	8.892

TABLE D9 BEAM: BVWR-2
AVERAGE DIAGONAL STRAINS x 10^{-3} (ε_d)

Moment kN m		Region EF Levels 2	3	4	5	6	7	Region TU Levels 2	3	4	5	6	7
0		0	0	0	0	0	0	0	0	0	0	0	0
55.6	ε_{d1}	0.088	0.106	-0.866	-0.283	-1.909	-1.785	0.000	0.301	0.301	-0.177	-0.212	1.679
	ε_{d2}	0.053	-0.389	-0.018	0.354	-0.141	-0.301	0.354	0.159	0.247	0.177	1.008	0.265
89.0	ε_{d1}	0.088	0.088	1.025	-1.450	-0.124	0.371	-0.018	0.124	0.742	-0.283	-0.301	3.288
	ε_{d2}	0.141	-0.371	-0.053	0.194	-0.124	-0.990	0.407	0.088	-0.071	0.283	1.184	0.318
111.3	ε_{d1}	-0.035	0.513	-0.866	-0.265	-0.053	-0.177	-0.018	0.000	0.636	0.035	-0.460	1.538
	ε_{d2}	-2.722	-7.142	1.750	0.141	-6.283	-0.902	0.389	0.141	-0.350	0.053	-1.008	0.230
122.4	ε_{d1}	-0.124	0.972	-0.795	-0.636	-0.106	0.018	-0.124	0.053	0.477	-0.212	-0.311	1.573
	ε_{d2}	-5.620	-6.258	2.333	0.265	-0.035	-0.990	0.654	0.106	0.124	0.087	-0.177	0.247
133.5	ε_{d1}	-0.849	0.460	-1.803	-1.255	-0.849	-0.742	-0.124	0.177	0.495	-0.283	-0.601	0.053
	ε_{d2}	-5.480	-6.346	2.404	-0.354	-0.513	-1.556	0.707	0.088	0.159	0.319	1.061	0.141
155.8	ε_{d1}	-0.601	1.503	-1.750	-1.043	-0.849	-2.616	-0.371	-0.548	0.212	-0.760	-0.795	1.167
	ε_{d2}	-4.773	-5.480	3.076	-0.212	-0.477	-1.344	0.636	-0.265	-0.071	-0.035	-0.742	1.202
178.0	ε_{d1}	1.008	1.768	-1.644	-1.344	-0.902	-0.636	-0.247	-0.371	0.018	-0.530	-0.813	1.025
	ε_{d2}	-0.742	-5.657	3.253	-0.124	-0.495	-1.202	0.937	-0.283	0.247	-0.124	-0.018	1.397
211.4	ε_{d1}	1.467	2.245	-1.945	-1.467	-0.742	-0.636	-0.212	-0.619	-2.086	-0.283	-0.919	1.114
	ε_{d2}	0.141	-5.392	3.695	-0.124	-0.389	-0.760	2.475	-0.141	2.015	1.750	3.483	1.609

TABLE D10 BEAM: BVWR-2

AVERAGE DIAGONAL STRAINS x 10^{-3} (ε_d)

Moment kN m		Region GH						Region RS					
		Levels 2	3	4	5	6	7	Levels 2	3	4	5	6	7
0		0	0	0	0	0	0	0	0	0	0	0	0
55.6	ε_{d1}	0.071	-0.513	0.212	0.035	-0.742	-0.088	-1.591	-0.548	0.035	-0.301	0.071	1.078
	ε_{d2}	1.909	-0.742	3.871	1.750	-2.121	0.849	0.318	0.318	0.159	-1.697	-0.035	-0.018
89.0	ε_{d1}	-0.071	-0.424	0.141	0.212	-0.601	-0.088	-1.697	-0.619	-0.194	-0.519	0.035	-1.998
	ε_{d2}	2.316	-0.194	4.260	1.520	0.177	-0.212	1.290	-0.636	1.008	-1.255	-0.071	-0.144
111.3	ε_{d1}	-0.124	-0.460	-0.018	-0.407	-0.177	0.354	-1.874	0.159	-0.177	-0.424	-0.424	-0.071
	ε_{d2}	3.288	0.725	5.427	1.927	0.619	-0.177	2.988	1.043	2.404	-1.202	-0.071	-0.088
122.4	ε_{d1}	-0.035	-0.884	0.407	-0.301	-0.053	0.124	-0.053	0.106	-0.141	-0.902	-0.141	0.336
	ε_{d2}	3.942	2.157	5.869	2.280	0.407	0.442	2.333	3.854	3.412	-1.131	0.071	-1.556
133.5	ε_{d1}	-0.778	-1.025	-0.778	-0.123	0.265	-0.566	-1.892	-1.025	-0.301	-0.530	-0.053	-0.849
	ε_{d2}	4.296	2.068	5.975	1.785	0.212	-2.386	4.738	4.455	3.995	-6.417	0.035	-0.124
155.8	ε_{d1}	-0.636	-0.919	-0.389	0.071	-0.283	-2.104	-0.366	0.301	-0.672	-0.990	-0.601	-0.477
	ε_{d2}	6.647	4.384	8.450	2.386	0.601	-0.902	5.904	5.462	4.897	-1.008	-2.157	-0.495
178.0	ε_{d1}	2.333	-0.884	-0.389	-0.548	-0.247	-0.318	-0.831	-0.725	-0.725	-0.990	-0.477	-0.106
	ε_{d2}	12.23	9.882	13.24	2.528	0.813	0.283	6.894	6.382	5.816	-1.008	-2.192	-0.530
211.4	ε_{d1}	4.685	-0.919	-0.460	-0.247	-0.371	-0.795	-0.902	-0.548	-0.831	-0.548	-0.354	-0.689
	ε_{d2}	19.69	16.72	19.53	2.970	-0.601	0.725	9.511	8.910	8.078	-0.212	0.247	-0.159

TABLE D11 BEAM: BWR-2

AVERAGE DIAGONAL STRAINS x 10^{-3} (ε_d)

Moment kN m		Region EF or TU Levels 2	3	4	5	6	7	Region GH or RS Levels 2	3	4	5	6	7
0		0	0	0	0	0	0	0	0	0	0	0	0
55.6	ε_{d1}	0.557	0.539	0.601	0.814	0.652	0.548	0.712	0.769	0.688	0.716	0.548	0.716
	ε_{d2}	0.318	0.751	0.699	0.699	0.716	0.699	0.699	0.556	0.575	0.457	-1.220	0.336
89.0	ε_{d1}	0.787	0.800	0.707	1.079	0.920	0.875	0.990	0.706	0.858	1.061	0.946	0.973
	ε_{d2}	0.637	1.300	0.902	1.167	0.999	0.911	1.167	-0.018	0.089	0.831	-1.926	0.672
111.3	ε_{d1}	0.778	0.610	0.743	1.070	0.831	1.008	0.911	1.03	0.849	0.849	1.202	1.024
	ε_{d2}	0.866	1.459	0.800	1.246	0.929	0.752	1.415	0.928	0.876	1.786	-0.699	0.521
122.4	ε_{d1}	0.800	0.831	0.689	1.008	0.866	0.831	0.946	1.026	0.813	0.946	0.831	0.822
	ε_{d2}	1.061	1.229	1.114	1.353	1.238	0.831	1.441	1.096	0.133	1.114	-0.919	0.575
133.5	ε_{d1}	0.690	0.743	0.619	0.902	0.831	0.902	0.920	0.814	0.840	0.875	0.849	0.698
	ε_{d2}	0.981	1.291	0.981	1.229	1.344	0.875	1.794	1.079	0.283	1.115	-0.848	0.690
155.8	ε_{d1}	0.575	0.530	0.849	1.017	0.805	0.654	0.919	0.663	0.928	0.964	0.681	0.734
	ε_{d2}	1.149	1.256	1.131	1.370	1.202	1.450	2.307	0.928	0.654	1.114	-0.804	0.592
178.0	ε_{d1}	0.557	0.530	0.725	0.840	0.778	0.981	0.875	0.800	0.707	0.805	0.822	0.805
	ε_{d2}	1.291	1.353	1.415	1.459	1.247	0.929	2.696	1.927	0.681	1.247	-0.760	0.725
211.4	ε_{d1}	0.451	0.646	0.592	1.070	0.716	0.787	0.645	0.796	0.805	0.867	0.752	0.495
	ε_{d2}	1.671	1.291	1.379	1.653	1.326	1.892	2.652	1.989	0.999	1.273	-0.540	0.822

APPENDIX E EXPERIMENTAL DATA OF STRAINS

The demec readings using the targets shown in Fig. 4.10 of Chapter 4 during the test were used to compute strains. The computed values of longitudinal, transverse and diagonal strains for the beams are presented in Tables E1 to E25. The moments (in kN m) and the strains (as multiples of 10^{-3}) are tabulated appropriately for the regions and the levels considered.

TABLE E.2 BEAM BVWR-1

LONGITUDINAL STRAINS

178.0	155.8	133.5	111.3	89.0	55.6	0	Moment kN m
0.775	0.475	0.650	0.775	1.525	1.625	0	U1T1
0.100	2.250	0.050	0.050	0.050	0.125	0	T1S1
1.125	0.950	1.375	1.350	0.675	1.000	0	S1R1
1.700	0.500	0.450	0.200	0.100	0.100	0	U2T2
2.725	2.725	3.050	2.875	2.925	0.025	0	T2S2
0.25	0.025	0.050	0.075	0.225	0.125	0	S2R2
2.850	2.850	2.675	0.325	0.225	0	0	U3T3
3.775	1.825	3.95	3.800	3.950	0.175	0	T3S3
0.150	0.250	0.125	0.075	0.100	0.425	0	S3R3
3.150	2.525	2.525	2.900	2.375	2.075	0	U4T4
0.475	0.150	0.250	0.225	-0.900	-0.075	0	T4S4
0.375	0.300	0.300	0.025	-0.100	-0.100	0	S4R4
1.450	2.075	1.725	1.125	-0.375	-0.275	0	U5T5
-0.725	0.050	-0.300	-0.150	-0.150	-0.025	0	T5S5
-0.175	-0.100	-0.050	-0.075	-0.275	-0.275	0	S5R5
-0.800	-0.425	-0.625	-1.075	-2.150	-2.325	0	U6T6
-0.225	-3.500	-3.250	-3.075	-0.500	-0.250	0	T6S6
-0.600	-4.25	-0.500	-0.350	-0.350	-0.225	0	S6R6
-0.100	-0.125	-0.275	-0.450	-0.175	-0.575	0	U7T7
-0.575	-1.375	-0.500	2.325	-0.425	0	0	T7S7
-0.425	3.675	0.575	0.250	-0.225	-0.150	0	S7R7
-0.175	-0.275	-0.625	-0.625	-0.400	-0.225	0	U8T8
-3.475	-3.800	-2.050	-2.375	-0.750	-3.775	0	T8S8
-1.675	-0.225	-1.725	1.675	-0.125	0.200	0	S8R8

TABLE E.1 BEAM BVWR-1

LONGITUDINAL STRAINS

Moment kN m	E1F1	F1G1	G1H1	E2F2	F2G2	G2H2	E3F3	F3G3	G3H3	E4F4	F4G4	G4H4
0	0	0	0	0	0	0	0	0	0	0	0	0
55.6	2.600	0.100	0.400	1.350	1.125	0.700	2.800	0.875	3.075	3.25	-0.675	-2.150
89.0	2.975	0.825	0.450	3.725	1.325	3.125	1.675	0.525	3.000	3.575	1.075	0.575
111.3	3.025	.0.525	0.875	0.425	0.375	0.700	2.725	0.575	5.325	2.775	2.950	0.050
133.5	-2.575	2.425	0.325	1.800	1.700	0.725	2.700	0.450	2.850	2.825	2.475	0.200
155.8	-2.250	0.700	0.450	3.275	0.575	·2.75	·3.475	2.500	2.600	2.975	2.575	·1.225
176.0	0.300	0	0.675	3.275	2.300	1.900	0.650	1.800	2.875	3.200	3.275	0.925

Moment kN m	E5F5	F5G5	G5H5	E6F6	F6G6	G6H6	E7F7	F7G7	G7H7	E8F8	F8G8	G8H8
0	0	0	0	0	0	0	0	0	0	0	0	0
55.6	-1.125	-2.525	-2.750	-3.575	-1.350	-5.05	-1.625	-1.475	-1.350	-1.425	-1.600	-1.057
89.0	-2.800	-2.600	-2.575	-2.625	-0.575	-4.350	-3.350	-2.725	-3.05	-1.225	-2.600	-2.375
111.3	1.225	-2.550	-4.775	-2.85	-1.500	-4.625	-0.925	+0.850	-3.200	-1.700	-2.925	-3.025
133.5	-1.225	-2.625	-2.300	-2.825	-1.575	-3.175	-0.950	-1.650	-3.100	-4.025	-2.200	-2.475
155.8	2.550	2.575	-0.900	-2.775	-1.625	-5.600	-0.950	-1.750	-1.675	-1.700	-3.400	-2.975
176.0	2.900	3.900	-2.575	-3.275	-1.725	-2.925	-1.050	-1.800	-3.400	-1.425	-2.425	-2.675

TABLE E.3 : BEAM BVWR-1
TRANSVERSE STRAINS

Moment kN m	0	55.6	89.0	111.3	133.5	155.8	178.0
U1U3	0	0.350	0.300	-1.900	-1.775	-1.725	-0.900
U2U4	0	-0.200	-0.125	-0.250	-0.225	-1.050	-0.175
U3U5	0	1.575	0.150	5.950	2.225	14.70	15.00
U4U6	0	1.675	0.225	3.475	9.725	17.00	24.28
U5U7	0	0.025	0.025	0	-0.125	-0.150	-0.125
U6U8	0	0.200	0.100	0.150	0.075	0.1000	0.175
T1T3	0	0.625	1.675	4.550	4.625	2.675	5.375
T2T4	0	0.750	0.600	3.800	3.550	1.950	2.875
T3T5	0	0.125	0.075	2.700	2.425	0.075	-0.150
T4T6	0	-0.075	-0.050	2.650	7.750	15.90	18.45
T5T7	0	4.225	0.150	0.250	6.35	12.90	15.88
T6T8	0	0.975	0.550	0.300	2.575	3.850	2.025
S1S3	0	2.725	0.050	-2.250	1.800	0.550	-1.300
S2S4	0	0.575	3.725	0.975	1.050	0.400	1.000
S3S5	0	0.150	1.050	1.375	1.450	1.675	2.825
S4S6	0	0.225	3.300	4.000	3.725	0.950	6.750
S5S7	0	0.075	-0.050	-1.525	4.825	10.60	17.45
S6S8	0	3.050	1.025	2.300	7.800	13.15	20.45
R1R3	0	-1.750	-8.500	-1.375	-0.225	0.675	3.900
R2R4	0	-0.025	-0.075	-0.100	0.100	-3.050	-2.800
R3R5	0	0.175	-0.175	0.275	0.275	0.275	-0.125
R4R6	0	0.050	0.275	0.875	0.850	1.050	-0.125
R5R7	0	1.325	1.125	0.600	0.300	0.150	1.925
R6R8	0	2.900	2.800	3.075	2.200	6.350	11.98

TABLE E.4 BEAM BVWR-1
TRANSVERSE STRAINS

Moment kN m	0	55.6	89.0	111.3	133.5	155.8	178.0
E1E3	0	-3.050	-2.875	-2.450	-2.575	-2.825	-0.200
E2E4	0	0.825	0.550	-0.825	0.775	-0.500	0.525
E3E5	0	-0.500	-1.275	-1.350	-1.325	-1.250	-1.325
E4E6	0	-2.400	-3.050	-2.500	-2.200	-2.725	-2.600
E5E7	0	-2.45	-2.475	-2.775	-2.45	-2.700	-2.45
E6E8	0	-4.825	-2.450	-2.100	-1.975	-2.100	-1.800
F1F3	0	-0.125	-1.325	1.850	3.675	3.700	6.750
F2F4	0	-0.800	-0.900	-0.850	-0.800	-2.075	-3.300
F3F5	0	-2.050	-0.875	-0.850	-0.825	-2.050	-0.825
F4F6	0	-2.275	-0.875	-0.525	-2.35	-0.825	-0.725
F5F7	0	-3.425	-3.225	-1.125	-3.925	-3.150	-1.150
F6F8	0	-1.400	-1.350	-1.175	-1.250	-3.525	-1.475
G1G3	0	0	-0.600	1.400	1.400	1.350	-0.600
G2G4	0	-0.475	-0.075	0.150	2.925	5.375	4.725
G3G5	0	1.750	1.675	4.575	6.175	6.675	9.300
G4G6	0	-1.100	-1.050	-1.400	-1.200	-3.600	-3.825
G5G7	0	-2.500	-2.275	-1.650	-2.450	-2.175	-2.200
G6G8	0	0.100	0.325	1.300	1.300	3.000	0.225
H1H3	0	-1.150	-1.100	-1.175	-0.825	-3.825	-1.325
H2H4	0	-1.400	-0.200	-2.100	-2.225	-1.375	-2.425
H3H5	0	-1.825	-0.975	-1.625	-1.050	-3.375	-3.775
H4H6	0	-2.025	-1.325	-1.250	-1.225	3.225	3.450
H5H7	0	-2.675	-2.000	-1.250	1.200	1.325	2.450
H6H8	0	0.850	0.975	0.550	0.550	0.200	-0.025

TABLE E.5 BEAM BVWR-1
DIAGONAL STRAINS

Moment kN m	E1F3	E2F4	E3F5	E4F6	E5F7	E6F8							G1H3	G2H4	G3H5	G4H6	G5H7	G6H8
0	0	0	0	0	0	0							0	0	0	0	0	0
0	1.785	0.583	0.283	0.035	-0.088	0.071							0.442	-0.371	-2.051	-1.061	-0.018	-0.018
89.0	1.768	-0.088	0.301	-0.018	-0.212	0.088							0.725	-0.124	-1.768	-0.654	-0.106	0.283
111.3	3.253	0.053	0.159	0.053	-0.194	-0.035							0.831	-0.124	-1.626	-0.742	-0.194	-0.018
133.5	1.732	0.354	0.194	0.177	0.035	-0.018							0.548	1.591	-1.503	-0.035	-0.230	0.442
155.8	2.033	0.177	0.389	0.088	0.177	0.088							0.530	0.230	-1.839	-0.141	0	0.088
178.0	2.245	0.125	0.354	0.053	0.088	0.265							0.778	0.177	-1.432	-0.566	-0.336	0.230

TABLE E.6 BEAM BVWR-1
DIAGONAL STRAINS

Moment kN m	U1T3	U2T4	U3T5	U4T6	U5T7	U6T8							S1R3	S2R4	S3R5	S4R6	S5R7	S6R8
0	0	0	0	0	0	0							0	0	0	0	0	0
55.6	0.424	0.636	0.194	0.548	0.159	0.194							-1.237	0.566	-1.220	-1.892	-1.450	0.318
89.0	1.998	0.477	0.106	0.389	0.071	-0.371							-1.344	0.513	-1.273	-1.838	-1.538	0.177
111.3	0.407	0.689	0.247	0.133	0.106	0.159							-1.167	0.760	-1.114	-1.220	-1.573	2.139
133.5	0.389	0.636	0.159	0.937	0.177	-0.053							-1.237	0.601	0.088	-1.520	0.124	0.884
155.8	0.442	0.619	0.124	6.540	0	-0.336							-1.131	0.795	-1.008	-1.255	-0.283	4.738
178.0	0.619	0.778	0.184	8.273	1.768	-0.106							-0.955	0.636	-1.290	-1.556	-1.785	6.629

TABLE E.7 BEAM BVWR-1

DIAGONAL STRAINS

Moment kN m	F1E3	F2E4	F3E5	F4E6	F5E7	F6E8							H1G3	H2G4	H3G5	H4G6	H5G7	H6G8
0	0	0	0	0	0	0							0	0	0	0	0	0
55.6	0.336	0.106	0.053	-0.018	0.301	0.018							-1.290	1.591	0.017	0.017	-0.017	-0.336
89.0	1.609	0.283	0.053	-0.141	0.407	0.177							-1.167	0.937	1.061	0.725	0.301	0.283
111.3	2.934	0.265	0.212	0.141	1.273	0.336							-1.149	2.280	1.520	1.184	0.760	-0.106
133.5	4.985	0.301	0.212	-0.141	0.407	0.318							-1.255	3.500	2.828	2.386	1.662	0.141
155.8	6.523	0.495	0.407	0.283	0.619	0.265							-1.114	4.225	3.748	3.200	2.404	-0.124
178.0	8.379	0.177	0.407	0.318	0.636	0.194							-0.495	5.445	4.914	3.200	3.483	0.424

TABLE E.8 BEAM BVWR-1

DIAGONAL STRAINS

Moment kN m	T1U3	T2U4	T3U5	T4U6	T5U7	T6U8							R1S3	R2S4	R3S5	R4S6	R5S7	R6S8
0	0	0	0	0	0	0							0	0	0	0	0	0
55.6	0.566	-0.689	0.371	-8.432	2.316	0.247							-0.725	0.371	-1.414	0.460	-1.679	-0.778
89.0	1.644	0.689	0.654	-8.308	2.388	0.601							-2.600	0.477	-2.600	0.760	-1.111	1.202
111.3	1.998	0.990	2.970	-6.170	2.952	0.371							0.265	0.707	-0.371	-0.672	-0.088	-0.707
133.5	2.192	0.972	5.162	-3.942	4.950	0.318							0.689	1.856	-0.371	-0.583	2.157	3.748
155.8	2.687	1.326	9.228	0.071	8.697	0.088							0.672	0.566	-0.106	0.460	6.930	6.187
178.0	3.253	0.301	11.21	2.068	10.60	0.301							1.626	0.760	-1.273	0.159	9.351	8.892

TABLE E.9 BEAM BVWR-2
LONGITUDINAL STRAINS

Moment kN m	0	55.6	89.0	111.3	122.4	133.5	155.8	178.0	211.4
U1T1	0	0.175	0.300	0.250	0.125	0.025	0.325	0.325	3.400
T1S1	0	3.250	0.900	0.600	2.750	5.750	4.550	5.475	8.825
S1R1	0	0.625	3.900	4.250	2.025	2.025	2.150	4.350	2.150
U2T2	0	0.575	0.500	0.375	0.325	0.450	0.175	0.250	0.225
T2S2	0	2.300	2.500	2.675	0.675	0.225	5.150	0.875	2.450
S2R2	0	0.900	0.400	0.075	4.150	4.475	3.225	8.75	6.650
U3T3	0	2.500	2.375	3.475	2.950	2.950	2.800	2.675	2.900
T3S3	0	0.875	0.200	0.025	0	0.025	3.575	3.075	0.225
S3R3	0	0.125	1.025	2.300	2.700	3.150	5.625	4.600	5.600
U4T4	0	2.250	1.050	4.325	4.050	4.500	4.050	3.825	4.325
T4S4	0	0.375	0.025	-2.075	0.375	2.300	0.175	2.325	0.050
S4R4	0	1.200	1.375	0.550	1.525	1.900	2.075	3.650	1.700
U5T5	0	-1.350	-1.550	1.825	-1.600	-1.725	-1.475	-1.475	-1.650
T5S5	0	-2.950	-3.100	-2.550	-2.525	-0.700	-1.125	-0.850	-0.825
S5R5	0	-2.150	-2.775	-2.250	-2.625	-2.375	-2.725	-2.525	-2.600
U6T6	0	-0.350	-0.150	-0.025	0.150	0.500	-0.375	-0.050	-0.050
T6S6	0	-0.075	-0.125	-0.300	-0.400	-0.200	-0.575	-0.400	-0.525
S6R6	0	0	0.075	-2.250	-2.925	-0.250	-0.300	-0.375	-0.400
U7T7	0	-3.275	-3.250	-4.875	-2.675	-4.500	-4.425	-2.625	-4.675
T7S7	0	-3.125	-3.050	-0.100	-3.150	-3.550	-3.475	-0.425	-3.425
S7R7	0	-0.275	-0.150	-2.550	-0.400	-0.125	-1.975	-0.400	-1.950
U8T8	0	-0.225	-0.350	-2.225	-3.375	-0.275	-2.375	-3.200	-0.450
T8S8	0	-0.800	-0.300	-0.050	-2.300	-0.150	-1.950	-0.200	-0.125
S8R8	0	0.125	0.450	-1.475	-1.950	-1.950	-1.875	-0.300	-0.025

TABLE E.10 BEAM BVWR-2
LONGITUDINAL STRAINS

Moment kN m	0	55.6	89.0	111.3	122.4	133.5	155.8	178.0	211.4
E1F1	0	1.000	0.925	1.200	1.100	1.150	1.750	1.625	1.700
F1G1	0	0.550	1.750	0.600	0.200	0.350	0.500	2.325	0.250
G1H1	0	1.075	0.675	0.750	0.600	0.325	2.600	4.800	10.13
E2F2	0	0.075	0.375	0.450	0.050	0.125	0.100	0.525	0.500
F2G2	0	1.350	1.350	3.575	0.075	0.600	0.400	1.450	2.025
G2H2	0	2.325	2.650	0.175	0.925	1.600	2.200	6.675	11.28
E3F3	0	1.475	1.075	0.975	1.125	1.300	1.075	1.225	3.625
F3G3	0	2.125	2.350	0.275	2.525	2.350	1.850	1.650	2.150
G3H3	0	0.075	0.200	1.275	0.825	3.675	4.050	6.750	11.65
E4F4	0	0.125	1.575	0.175	0.150	2.000	2.23	2.050	2.325
F4G4	0	0.125	0.725	0.375	0.850	0.350	4.575	0.325	0.150
G4H4	0	0.050	1.250	0.250	0.250	1.125	0	0.025	0.600
E5F5	0	-0.125	-0.250	-0.125	-1.100	-1.125	-1.025	-1.575	-0.600
F5G5	0	-2.200	-1.700	-0.975	-1.125	-0.850	-1.175	-1.250	-1.650
G5H5	0	-1.050	-0.300	-1.100	-1.275	-1.150	-1.250	-1.400	-1.250
E6F6	0	-0.300	-1.675	-2.125	-2.975	-3.175	-2.900	-2.350	-2.600
F6G6	0	-1.175	-3.175	-0.400	-1.725	-1.325	-1.500	-1.275	-1.150
G6H6	0	-0.200	-2.025	-2.200	-3.050	-0.425	-0.375	-2.050	-2.600
E7F7	0	0.175	-0.925	1.675	-2.250	-0.950	4.075	-0.925	-1.025
F7G7	0	1.100	-0.600	-1.350	0.775	-0.550	-0.775	-0.800	-0.650
G7H7	0	-2.800	-3.025	-2.850	-2.975	-0.400	-0.525	-0.625	-3.100
E8F8	0	-6.850	-0.725	-4.450	-4.000	-4.625	-4.400	-4.475	-4.675
F8G8	0	-6.225	-3.375	-4.350	-2.525	-4.175	-4.175	-4.325	-4.325
G8H8	0	-0.300	-3.025	0.600	-0.125	-0.200	0.350	0.175	0.225

TABLE E.11 BEAM BVWR-2
TRANSVERSE STRAINS

Moment kN m	U1U3	U2U4	U3U5	U4U6	U5U7	U6U8	T1T3	T2T4	T3T5	T4T6	T5T7	T6T8	S1S3	S2S4	S3S5	S4S6	S5S7	S6S8	R1R3	R2R4	R3R5	R4R6	R5R7	R6R8
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
55.6	-2.450	2.750	0.275	-0.050	1.000	0.650	-1.650	0.175	-0.525	0.200	-3.275	0.125	-2.125	1.675	-2.775	-0.275	-3.950	-2.250	0	-0.075	-0.075	0.050	-0.125	-0.200
89.0	-2.950	0.075	0.450	0.100	0.500	0.425	-1.900	0	2.925	1.900	0.200	2.625	-2.375	1.675	-3.050	-0.075	-0.375	-2.425	2.375	3.100	3.200	0.275	-0.225	-0.450
111.3	-2.250	0.575	0.375	0.125	-1.850	0.175	-0.200	0.225	0.100	0.350	-0.225	4.025	-0.400	0.475	-0.025	-0.075	-0.675	-4.525	1.925	1.950	2.200	0.125	-0.550	-1.550
122.4	0.450	2.525	0.250	-0.150	-2.425	-1.325	-1.925	0.225	-0.050	0.525	-0.300	2.450	-2.175	1.975	-0.050	0.025	-0.050	-4.450	-0.300	3.075	3.400	0.450	-0.100	-1.150
133.5	-2.000	-0.025	0.600	0.150	-0.050	0.675	-1.275	0.225	3.075	0.375	-0.050	4.450	1.800	1.825	-2.300	0.350	0.300	-2.05	2.425	3.900	4.125	0.050	-0.300	-2.025
155.8	0.700	2.400	0.725	0.975	-2.250	-1.500	-3.350	0.225	3.300	0.475	0.650	2.500	3.125	1.175	-2.625	-0.125	-0.225	-2.225	2.175	5.150	5.425	0.250	-0.275	-0.175
178.0	-1.550	0.725	1.000	0.650	-2.100	-1.600	-0.550	0.475	3.300	0.975	0.150	2.675	5.600	0.250	0.100	0.175	0.100	-4.325	2.350	6.425	6.675	0.800	-0.350	-2.175
211.4	-1.200	-0.050	2.650	2.325	-1.925	0.800	-1.900	0.225	3.100	4.875	-0.600	2.850	11.80	0.025	-2.650	3.925	3.675	-4.325	-0.175	9.800	9.850	0.300	0.300	-2.025

TABLE E.12 BEAM BVWR-2
TRANSVERSE STRAINS

Moment kN m	E1E3	E2E4	E3E5	E4E6	E5E7	E6E8	F1F3	F2F4	F3F5	F4F6	F5F7	F6F8	G1G3	G2G4	G3G5	G4G6	G5G7	G6G8	H1H3	H2H4	H3H5	H4H6	H5H7	H6H8
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
55.6	-0.900	0.050	0.175	-1.700	-0.975	-4.075	-1.925	-0.225	-0.250	-3.625	-1.650	-0.950	1.450	-0.425	-1.200	0.175	-0.150	-0.125	2.125	-2.525	1.400	-0.825	0.250	2.025
89.0	-0.850	1.700	-1.850	-1.450	-2.375	-3.750	-2.500	-0.275	-0.225	-4.150	0.100	3.925	0.350	0.100	0.150	0.125	-2.775	0.100	-1.325	0.750	-0.800	-2.800	-2.650	1.823
111.3	2.700	2.400	-0.375	-1.850	-3.750	-3.750	-2.250	-2.300	0.125	-2.600	0.100	-1.675	1.350	3.100	2.300	0.400	0.475	-0.200	-0.625	-0.225	2.800	0.500	-2.525	-0.225
122.4	-3.375	4.275	0.850	-1.925	-0.975	-4.175	-1.375	-0.125	2.375	-4.225	0.050	-1.575	-1.400	-2.225	-2.450	0.900	-0.475	0.100	-0.575	4.750	1.225	-1.625	-2.525	1.325
133.4	6.925	6.700	-2.300	-1.825	-3.600	-3.625	-2.475	1.650	1.900	-4.425	0.050	-1.150	0.675	3.175	-3.275	1.725	-0.175	-0.050	2.175	5.425	0.225	-2.500	-2.125	0.575
155.8	4.400	6.675	-1.825	-1.575	-2.200	-4.100	-1.675	2.900	2.925	-3.725	-0.025	-1.150	3.675	-0.025	-1.000	2.700	0.450	0.075	1.575	6.275	4.850	0.700	-1.825	1.725
178.0	11.63	6.625	-2.000	-1.650	-1.200	-3.725	4.200	4.475	4.45	-3.975	-1.575	-1.300	11.38	0.725	0.125	2.375	0.550	-0.150	-0.525	12.05	8.725	-3.250	-0.775	2.525
211.4	15.55	8.500	-1.725	-1.775	-1.25	-4.000	12.35	4.600	4.575	-3.600	-1.625	-1.350	26.25	0.100	-0.350	3.575	0.675	-0.450	0.125	20.98	18.68	-3.500	-2.200	2.475

TABLE E.13 BEAM BVWR-2
DIAGONAL STRAINS

Moment kN m	E1F3	E2F4	E3F5	E4F6	E5F7	E6F8	G1H3	G2H4	G3H5	G4H6	G5H7	G6H8
0	0	0	0	0	0	0	0	0	0	0	0	0
55.6	0.088	0.106	-0.866	-0.283	-1.909	-1.785	0.071	-0.513	0.212	0.035	-0.742	-0.088
89.0	0.088	0.088	1.025	-1.450	-0.124	0.371	-0.071	-0.424	0.141	0.212	-0.601	-0.088
111.3	-0.035	0.513	-0.866	-0.265	-0.053	-0.177	-0.124	-0.460	-0.018	-0.407	-0.177	0.354
122.4	-0.124	0.972	-0.795	-0.636	-0.106	0.018	-0.035	-0.884	0.407	-0.301	0.053	0.124
133.5	-0.849	0.460	-1.803	-1.255	-0.849	-0.742	-0.778	-1.025	-0.778	-1.273	0.265	-0.566
155.8	-0.601	1.503	-1.750	-1.043	-0.849	-2.616	-0.636	-0.919	-0.389	0.071	-0.283	-2.104
178.0	1.008	1.768	-1.644	-1.344	-0.902	-0.636	2.333	-0.884	-0.389	-0.548	-0.247	-0.318
211.4	1.467	2.245	-1.945	-1.467	-0.742	-0.636	4.685	-0.919	-0.460	-0.247	-0.371	-0.795

TABLE E.14 BEAM BVWR-2
DIAGONAL STRAINS

Moment kN m	F1E3	F2E4	F3E5	F4E6	F5E7	F6E8	H1G3	H2G4	H3G5	H4G6	H5G7	H6G8
0	0	0	0	0	0	0	0	0	0	0	0	0
55.6	0.053	-0.389	-0.018	0.354	-0.141	-0.301	1.909	-0.742	3.871	1.750	-2.121	0.849
89.0	0.141	-0.371	-0.053	0.194	-0.124	-0.990	2.316	-0.194	4.260	1.520	0.177	-0.212
111.3	-2.722	-7.142	1.750	0.141	-0.283	-0.902	3.288	0.725	5.427	1.927	0.619	-0.177
122.4	-5.620	-6.258	2.333	0.265	-0.035	-0.990	3.942	2.157	5.869	2.280	0.407	0.442
133.5	-5.480	-6.346	2.404	-0.354	-0.513	-1.556	4.296	2.068	5.975	1.785	0.212	-2.386
155.8	-4.773	-5.480	3.076	-0.212	-0.477	-1.344	6.647	4.384	8.450	2.386	0.601	-0.902
178.0	-0.742	-5.657	3.253	-0.124	-0.495	-1.202	12.23	9.882	13.24	2.528	0.813	0.283
211.4	0.141	-5.392	3.695	-0.124	-0.389	-0.760	19.69	16.72	19.53	2.970	-0.601	0.725

TABLE E.15 BEAM BVWR-2
DIAGONAL STRAINS

Moment kN m	U1T3	U2T4	U3T5	U4T6	U5T7	U6T8	S1R3	S2R4	S3R5	S4R6	S5R7	S6R8
0	0	0	0	0	0	0	0	0	0	0	0	0
55.6	0	0.301	0.301	-0.177	-0.212	1.679	-1.591	-0.548	0.035	-0.301	0.071	1.078
89.0	-0.018	0.124	0.742	-0.283	-0.301	3.288	-1.697	-0.619	-0.194	-0.513	0.035	-1.998
111.3	-0.018	0	0.636	0.035	-0.460	1.538	-1.874	0.159	-0.177	-0.424	-0.424	-0.071
122.4	-0.124	0.053	0.477	-0.212	-0.371	1.573	-0.053	0.106	-0.141	-0.902	-0.141	0.336
133.5	-0.124	0.177	0.495	-0.283	-0.601	0.053	-1.892	-1.025	-0.301	-0.530	-0.053	-0.849
155.8	-0.371	-0.548	0.212	-0.760	-0.795	1.167	-0.636	0.301	-0.672	-0.990	-0.601	-0.477
178.0	-0.247	-0.371	0.018	-0.530	-0.813	1.025	-0.831	-0.725	-0.725	-0.990	-0.477	-0.106
211.4	-0.212	-0.619	-2.086	-0.283	-0.919	1.114	-0.902	-0.548	-0.831	0.548	-0.354	-0.689

TABLE E.16 BEAM BVWR-2
DIAGONAL STRAINS

Moment kN m	T1U3	T2U4	T3U5	T4U6	T5U7	T6U8	R1S3	R2S4	R3S5	R4S6	R5S7	R6S8
0	0	0	0	0	0	0	0	0	0	0	0	0
55.6	0.354	0.159	0.247	0.177	1.008	0.265	0.318	0.318	0.159	-1.697	-0.035	-0.018
89.0	0.407	0.088	-0.071	0.283	1.184	0.318	1.290	-0.636	1.008	-1.255	-0.071	-0.141
111.3	0.389	0.141	-0.035	0.053	-1.008	0.230	2.988	1.043	2.404	-1.202	-0.071	-0.088
122.4	0.654	0.106	0.124	0.087	-0.177	0.247	2.333	3.854	3.412	-1.131	0.071	-1.556
133.5	0.707	0.088	0.159	0.318	1.061	0.141	4.738	4.455	3.995	-6.417	0.035	-0.124
155.8	0.636	-0.265	-0.071	-0.035	-0.742	1.202	5.904	5.462	4.897	-1.008	-2.157	-0.495
178.0	0.937	-0.283	0.247	-0.124	-0.018	1.397	6.894	6.382	5.816	-1.008	-2.192	-0.530
211.4	2.475	-0.141	2.015	1.750	3.483	1.609	9.511	8.910	8.078	-0.212	0.247	-0.159

TABLE E.17 BEAM BWR-2
LONGITUDINAL STRAINS

Moment kN m	E1F1	F1G1	G1H1	E2F2	F2G2	G2H2	E3F3	F3G3	G3H3	E4F4	F4G4	G4H4	E5F5	F5G5	G5H5	E6F6	F6G6	G6H6	E7F7	F7G7	G7H7	E8F8	F8G8	G8H8
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
55.6	2.600	1.250	1.425	0.200	0.300	0.225	1.75	0.100	1.500	2.100	1.125	0.800	-2.075	-1.175	-1.075	0	-0.100	-2.375	-1.150	-0.075	-0.650	-0.400	-2.600	-3.125
89.0	2.600	1.675	2.225	0.150	0.500	3.70	3.775	1.400	1.200	2.075	1.400	1.775	-4.200	-3.25	-2.675	-6.075	-3.850	-3.950	-1.950	0.925	-4.750	-2.300	-3.000	-1.725
111.3	1.150	1.450	1.95	0.125	1.025	1.800	3.900	0.425	1.400	2.800	1.425	1.825	-3.975	-3.25	-0.200	-4.075	-4.200	-3.950	-1.675	-1.650	-4.775	-2.300	-2.800	-3.525
122.4	1.250	1.450	2.000	1.225	1.300	0.410	4.15	0.975	0.700	4.500	3.775	0.275	-6.650	-2.700	-2.750	-6.075	-6.375	-4.000	-1.350	0.875	-4.875	-2.625	-2.750	-2.975
133.5	0.500	1.475	1.875	1.100	0.925	5.250	4.400	3.925	4.025	4.500	4.750	0.225	-4.050	-3.075	-2.85	-4.075	-3.875	-3.925	-1.325	0.875	-2.875	-0.300	-2.775	-2.750
155.8	4.125	1.550	1.725	0.975	1.500	6.325	4.500	1.150	1.700	3.850	1.075	0.275	-2.725	-0.875	-0.275	-4.425	-6.25	-1.350	-0.975	0.925	-3.125	-0.275	-2.775	-2.750
178.0	1.525	1.100	1.375	2.425	0.775	1.050	4.25	1.350	1.325	3.925	2.775	1.750	-3.975	-1.450	-0.950	-4.05	-5.525	-4.375	-1.100	-1.375	-5.375	-2.075	-2.475	-2.500
211.4	1.500	0.725	1.375	1.875	0.900	4.400	4.25	3.800	3.775	4.350	3.575	1.750	-4.225	-0.300	-2.975	-6.175	-6.500	-4.425	-1.800	-2.075	-4.875	-2.875	-2.850	-2.900

TABLE E.18 BEAM BWR-2
LONGITUDINAL STRAINS

Moment kN m	U1T1	T1S1	S1R1	U2T2	T2S2	S2R2	U3T3	T3S3	S3R3	U4T4	T4S4	S4R4	U5T5	T5S5	S5R5	U6T6	T6S6	S6R6	U7T7	T7S7	S7R7	U8T8	T8S8	S8R8
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
55.6	1.600	0.250	2.225	0.300	0.900	1.075	2.625	3.075	0.175	0.225	-0.025	0.045	-0.100	-2.900	0.125	-0.100	-0.025	0.500	-0.225	-0.100	-2.200	-0.375	-3.200	-0.050
89.0	1.475	0.150	2.375	0.100	0.575	1.125	2.525	0.100	0.175	0.125	2.175	0.250	-2.350	-2.575	-0.225	-2.775	-0.425	-0.425	-0.025	-0.525	-2.250	-0.775	-1.250	0.425
111.3	1.250	0.325	0.025	0.100	0.500	0.250	3.050	0.825	0.050	0.075	2.15	3.025	-0.525	-3.375	-0.125	-2.800	-0.025	-0.450	-0.325	-0.400	-2.250	-1.750	-3.175	-0.025
122.4	1.275	0.350	1.775	0.150	0.450	1.050	3.275	1.450	0.525	0.025	0.025	3.15	-2.100	-2.650	-0.125	-2.500	0.225	-0.225	-0.050	-0.200	-2.225	-0.450	-0.530	0.250
133.5	1.350	0.175	0.450	0.075	0.500	0.075	0.050	0.50	0.550	0.125	2.35	3.475	-1.95	-2.550	-0.300	-0.100	0.125	-0.200	-0.175	-0.325	-2.425	-2.600	-1.350	-0.025
155.8	1.375	0.400	1.300	0.025	0.150	2.200	1.275	2.525	1.375	0.050	2.175	3.325	-2.225	-2.975	-0.100	-2.600	-0.075	-0.025	-0.075	-0.300	0.025	-3.250	-3.200	0.325
178.0	1.475	1.75	1.025	0.075	0.175	2.000	2.525	3.100	1.000	0.050	2.075	3.175	-2.375	-2.700	-0.125	-2.575	-0.500	-0.050	0	-1.275	-0.050	-2.350	-1.775	-0.025
211.4	1.375	1.90	0.070	0.375	0.425	3.275	2.775	1.175	1.25	0.075	0.075	3.25	-2.275	-2.775	-0.275	-2.325	-0.250	-2.600	-0.175	-0.300	-0.050	-0.425	-1.150	-0.325

TABLE E.19 BEAM BWR-2
TRANSVERSE STRAINS

Moment kN m	U1U3	U2U4	U3U5	U4U6	U5U7	U6U8	T1T3	T2T4	T3T5	T4T6	T5T7	T6T8	S1S3	S2S4	S3S5	S4S6	S5S7	S6S8	R1R3	R2R4	R3R5	R4R6	R5R7	R6R8
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
55.6	0.825	-2.050	-2.400	+0.900	-0.450	0.025	-0.700	0.025	-1.025	-2.825	0	-0.375	-2.250	-0.550	-0.200	-0.150	2.000	1.600	-2.500	-0.075	-2.525	-0.300	0.100	-2.550
89.0	1.050	-2.150	-3.050	-1.100	-0.225	-3.025	1.150	-2.225	-1.225	-2.600	2.200	0.125	-2.425	-2.250	-0.050	0.025	2.275	-3.900	-2.325	2.075	0.125	-0.100	0.200	-0.400
111.3	-0.975	-1.850	-2.050	-1.025	-0.400	-0.075	1.425	-1.250	-0.975	-2.525	0	0.525	-4.950	0.200	-2.775	0.075	-0.050	1.225	-1.025	-0.225	-2.050	0.075	-0.150	02.475
122.4	-0.875	-1.825	-2.400	-0.900	-0.175	-0.075	1.075	-2.100	-1.400	0.250	0.350	-0.125	-2.225	2.725	-2.325	0.125	-0.025	-2.725	0.025	-2.625	-1.700	2.950	-3.750	-2.350
133.5	3.050	-1.800	-1.700	-0.600	0	-0.250	1.275	-2.275	-0.300	-2.225	0.300	-0.375	-2.15	2.950	-0.150	0.125	2.300	3.575	-0.075	0.200	-1.575	-0.075	0.175	-2.450
155.8	-0.750	-1.900	-1.725	-0.625	0.025	0.525	1.500	-2.200	1.725	0.325	0.475	0.675	1.075	2.850	2.600	0.125	0.200	3.900	-2.475	3.500	-0.800	0.400	2.600	-2.500
178.0	-5.750	0.050	-1.675	-0.875	0.075	-1.300	3.250	-2.275	1.950	0.350	0.125	-1.250	1.200	-0.500	0.075	0.050	2.225	-2.000	-0.350	0.775	-0.300	0.425	0.250	-0.300
211.4	0.075	-2.15	-2.000	-0.750	-0.425	-1.175	3.500	-0.175	-1.450	0.275	0.675	1.579	-1.000	2.525	7.75	-0.250	0.400	-1.650	-5.600	0.925	-0.225	0.825	0.150	-2.325

TABLE E.20 BEAM BWR-2
TRANSVERSE STRAINS

Moment kN m	E1E3	E2E4	E3E5	E4E6	E5E7	E6E8	F1F3	F2F4	F3F5	F4F6	F5F7	F6F8	G1G3	G2G4	G3G5	G4G6	G5G7	G6G8	H1H3	H2H4	H3H5	H4H6	H5H7	H6H8
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
55.6	-1.475	-1.150	1.250	-0.100	2.650	2.075	3.800	-2.150	-1.325	-0.300	2.850	0.250	-0.225	-2.325	-0.100	1.05	-2.025	-5.75	-4.025	0.775	-2.425	0.600	-5.100	-1.700
89.0	-0.675	0.200	0.025	0.325	0.050	1.975	1.975	0.175	-1.100	1.500	2.450	3.950	1.950	-0.875	-1.000	-1.925	-2.8	-0.250	-2.375	0.775	0.200	-0.500	-2.525	-1.350
111.3	-0.775	0.175	0.025	0.325	-0.025	2.075	3.525	0.150	-0.250	0.150	2.350	3.750	2.15	-0.400	-0.950	-0.025	-2.975	-0.050	-0.175	-0.875	-2.800	-0.150	-0.125	2.700
122.4	-0.625	-2.325	0.050	0.325	-0.075	2.175	0.400	-1.700	0.150	-0.075	0.775	0.600	4.400	-1.025	-0.500	0.725	-2.15	-0.575	-2.575	-0.750	-2.775	-0.150	-2.675	2.650
133.4	-0.425	-0.150	0.350	0.550	2.500	3.400	2.375	1.650	1.725	3.425	4.100	1.775	1.600	-1.450	-1.300	0.700	-3.100	-0.100	-2.050	-0.075	-2.850	-0.175	-1.275	-0.500
155.8	0.750	0.525	-0.400	1.800	2.875	3.225	1.975	-0.200	1.675	3.450	1.700	2.100	1.175	-1.475	1.900	0.250	-3.075	-0.575	-2.450	0.350	-2.675	-1.025	-3.450	2.400
178.0	0.975	-0.250	0.025	0.650	2.725	3.350	3.450	1.800	-1.200	3.400	1.700	1.950	1.475	-0.725	-1.250	0	-2.225	-0.175	-1.750	0	-2.775	-2.65	-2.925	0.125
211.4	-2.050	-1.575	-1.525	-3.275	-0.325	0.650	0.925	-1.650	0.025	-0.650	0.500	0	1.950	-2.475	-2.975	-1.175	-4.175	-2.175	-1.700	-1.900	-3.750	-4.100	-2.375	-4.40

TABLE E.21 BEAM BWR-2
DIAGONAL STRAINS

Moment kN m	E1F3	E2F4	E3F5	E4F6	E5F7	E6F8							G1H3	G2H4	G3H5	G4H6	G5H7	G6H8
0	0	0	0	0	0	0							0	0	0	0	0	0
55.6	0.972	0.583	0.725	0.955	0.778	0.672							0.945	1.043	0.743	0.778	0.689	0.672
89.0	1.397	1.061	0.937	1.203	1.167	1.184							1.361	1.343	1.061	1.326	1.184	1.273
111.3	1.114	0.672	0.813	1.326	1.025	1.396							1.226	1.52	1.008	1.096	1.626	1.078
122.4	0.937	0.902	0.689	1.078	1.202	1.237							1.397	1.326	0.990	1.149	1.008	0.795
133.4	1.061	0.902	0.725	1.078	1.202	1.255							1.397	1.291	0.990	1.096	1.078	0.760
155.8	0.636	0.583	1.078	1.167	1.114	0.766							1.290	1.220	1.043	1.008	0.902	0.813
178.0	0.919	0.689	1.008	1.131	1.025	1.467							1.326	1.326	0.919	1.149	1.255	0.831
211.4	0.831	0.955	0.636	1.326	0.955	1.078							1.255	1.291	1.202	1.273	1.273	0.636

TABLE E.22 BEAM BWR-2
DIAGONAL STRAINS

Moment kN m	U1T3	U2T4	U3T5	U4T6	U5T7	U6T8							S1R3	S2R4	S3R5	S4R6	S5R7	S6R8
0	0	0	0	0	0	0							0	0	0	0	0	0
55.6	0.141	0.495	0.477	0.672	0.526	0.424							0.477	0.495	0.513	0.654	0.407	0.760
89.0	0.177	0.530	0.477	0955	0.672	0.566							0.619	0.071	0.654	0.795	0.707	0.672
111.3	0.449	0.548	0.672	0.813	0.636	0.619							0.601	0.548	0.689	0.601	0.778	0.972
122.4	0.654	0.760	0.689	0.937	0.530	0.424							0.495	0.725	0.636	0.742	0.654	0.849
133.5	0.318	0.583	0.513	0.725	0.460	0.548							0.442	0.336	0.689	0.654	0.619	0.636
155.8	0.513	0.477	0.619	0.866	0.495	0.548							0.548	0.106	0.813	0.919	0.460	0.654
178.0	0.194	0.371	0.442	0.548	0.530	0.495							0.424	0.265	0.495	0.460	0.389	0.778
211.4	0.071	0.336	0.548	0.813	0.477	0.495							0.035	0.301	0.407	0.460	0.230	0.354

TABLE E.23 BEAM BWR-2

DIAGONAL STRAINS

Moment kN m	0	55.6	89.0	111.3	122.4	133.5	155.8	178.0	211.4
F1E3	0	0.088	0.495	0.495	0.849	0.831	0.9	1.096	1.326
F2E4	0	0.813	1.380	1.414	1.255	1.414	1.397	1.503	1.414
F3E5	0	0.934	1.202	1.131	1.326	1.184	1.131	1.167	1.556
F4E6	0	0.902	1.167	1.131	1.273	1.114	1.290	1.556	1.679
F5E7	0	1.061	1.114	0.955	1.308	1.343	1.255	1.220	1.308
F6E8	0	0.937	1.114	0.990	1.131	1.237	0.972	1.397	1.450
H1G3	0	0.778	1.467	1.662	1.626	1.838	1.856	2.528	3.765
H2G4	0	0.831	-0.654	1.061	1.061	0.849	-0.513	1.467	1.450
H3G5	0	0.760	-0.778	0.902	-0.583	-0.601	-0.813	-0.831	-0.354
H4G6	0	0.650	1.061	1.078	1.308	0.990	0.972	1.873	1.255
H5G7	0	-2.775	-2.722	-0.495	-0.654	-0.566	-0.636	-0.477	-0.301
H6G8	0	0.725	1.131	1.290	0.813	1.096	0.795	1.167	1.202

TABLE E.24 BEAM BWR-2

DIAGONAL STRAINS

Moment kN m	0	55.6	89.0	111.3	122.4	133.5	155.8	178.0	211.4
T1U3	0	0.548	0.778	1.237	1.273	1.131	1.397	1.485	2.015
T2U4	0	0.689	1.220	1.503	1.202	1.167	1.114	1.202	1.167
T3U5	0	0.460	0.601	0.460	0.902	0.778	1.131	1.662	1.202
T4U6	0	0.495	1.167	1.361	1.432	1.343	1.450	1.361	1.626
T5U7	0	0.371	0.884	0.902	1.167	1.025	1.149	1.273	1.343
T6U8	0	0.460	0.707	0.513	0.530	0.513	1.927	0.460	2.333
R1S3	0	0.619	0.866	1.167	1.255	1.750	2.768	2.864	1.538
R2S4	0	0.301	0.619	0.795	1.131	1.308	2.369	2.386	2.528
R3S5	0	0.389	0.601	0.849	0.849	1.167	2.121	2.192	2.351
R4S6	0	0.301	0.601	2.493	0.919	1.240	1.255	1.220	1.290
R5S7	0	0.336	-1.13	-0.902	-1.184	-1.13	-0.972	-1.043	-0.778
R6S8	0	-0.053	0.212	0.248	0.336	0.283	0.389	0.283	0.442

TABLE E.25 BEAM BWR-2
LONGITUDINAL STRAIN

FLEXURAL SPAN: LONGITUDINAL

Moment kN m	L1 N1	L2 N2	L3 N3	L4 N4	L5 N5	L6 N6	L7 N7	L8 N8
0	0	0	0	0	0	0	0	0
55.6	-1.077	-0.494	-0.281	-0.308	-0.112	-0.133	-0.322	1.372
89.0	-1.304	-0.861	-0.493	-0.721	-0.340	-0.172	-0.123	1.282
111.3	-1.377	-1.242	-0.962	-1.069	-0.531	-0.111	0.592	1.173
122.4	-1.4013	-1.512	-1.223	-1.296	-0.641	-0.194	0.271	0.791
133.5	-1.458	-1.793	-1.050	-1.547	-0.812	-0.301	0.311	0.913
155.8	-1.472	-2.450	-1.841	-2.017	-1.183	-0.462	0.332	1.972
178.0	-1.563	-3.140	-2.542	-2.714	-2.322	-0.553	0.432	2.230
211.4	-1.450	-5.374	-4.931	-4.803	-2.713	-1.072	0.721	3.072

TABLE E.25 BEAM BWR-2
LONGITUDINAL STRAIN

FLEXURAL SPAN: LONGITUDINAL

Moment kN m	L1 N1	L2 N2	L3 N3	L4 N4	L5 N5	L6 N6	L7 N7	L8 N8
0	0	0	0	0	0	0	0	0
55.6	-1.077	-0.494	-0.281	-0.308	-0.112	-0.133	-0.322	1.372
89.0	-1.304	-0.861	-0.493	-0.721	-0.340	-0.172	-0.123	1.282
111.3	-1.377	-1.242	-0.962	-1.069	-0.531	-0.111	0.592	1.173
122.4	-1.4013	-1.512	-1.223	-1.296	-0.641	-0.194	0.271	0.791
133.5	-1.458	-1.793	-1.050	-1.547	-0.812	-0.301	0.311	0.913
155.8	-1.472	-2.450	-1.841	-2.017	-1.183	-0.462	0.332	1.972
178.0	-1.563	-3.140	-2.542	-2.714	-2.322	-0.553	0.432	2.230
211.4	-1.450	-5.374	-4.931	-4.803	-2.713	-1.072	0.721	3.072