UNIVERSITY OF NAIROBI

UNDERSTANDING THE COSMOLOGICAL CRISIS ON THE SHAPE OF THE UNIVERSE USING PLANCK DATA

BY

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DECLARATION

I hereby confirm that this MSc thesis is my own unique work, and has not been presented at any other university for examination or as a research project. All sources utilized have been accredited by the use of proper references.

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DEDICATION

I devote this masterpiece to my dear mother for her continuous motivation in pursuing this uphill task and to my friend John Ndirangu who developed an interest in this research work, this motivated me to push forward in this endeavor.

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First, I wish to thank my supervisor Dr. Geoffrey O. Okeng'o, for his continued support and guidance through the research project. Without his support and regular assessments, this research would not have been a success.

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ABSTRACT

The shape of the universe has been a major topic of debate for scientists over the years. Since the beginning of time human beings have been seeking answers to intriguing questions concerning the nature of the universe, its shape, origin and its ultimate fate. Isaac Newton believed in a static and infinite universe according to his theory. The renowned physicist hypothesized that, distribution of matter was uniform and infinite and hence the universe was in balance. Einstein used the Riemannian geometry to describe the geometry of space. This has three spatial dimensions and another dimension which is temporal. Einstein achieved this by formulating the Einstein Field Equations (EFEs). In our present age cosmologists have been conducting experiments and developing mathematical models and theories to explain this cosmological phenomenon with many observations leading to antagonistic conclusions. In the past models favoring a flat universe have been widely accepted and applied by cosmologists around the globe. However recent results from analyses of 2018 data taken by the European Space Agency Planck satellite point out that our universe should be closed. These results have caused a lot of concern amongst cosmologists as such findings might be plunging us to a cosmological crisis. Detailed analysis of the cosmic microwave background (CMB) data confirmed the presence of enhanced lensing amplitude when in comparison with the ΛCDM model prediction. These discrepancies have triggered a number of studies that attempt to keenly reassess the level of discordance whereas some try to settle it by introducing new physics.

Keywords; Planck Satellite, Lambda Cold Dark Matter (ΛCDM) model

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LIST OF ABBREVIATIONS/ ACRONYMS AND SYMBOLS

$\mathbf{A_{lens}}$ **-** Lensing amplitude

a - scale factor

BAO - Baryon Acoustic Oscillation

BBN - Big Bang Nucleosynthesis

c - Speed of light

CAMB - Code for Anisotropies in the Microwave Background

CamSpec - cam specification

CfA- Center for Astrophysics

CL - Confidence/cadence level

CMB - Cosmic Microwave Background Radiation

$\mathbf{h}^2{}_b$- Density of Baryonic Matter

$\mathbf{E}^3$**-** Euclidean space

EFE - Einstein Field Equations

$\mathbf{G_N}$ **-** Newton's universal gravitational constant.

GR – General Relativity

$\mathbf{G_{\mu\nu}}$ **-** Einstein tensor

$\mathbf{g_{\mu\nu}}$ **-** Metric of manifold

H - Hubble Parameter

$\mathbf{H}^3$**-** hyperbolic space

HST - Hubble Space Telescope

I - Light intensity

k - (kappa) Space curvature

KBC - Keenan-Berger-Cowie

Kpc – Kiloparsec

LHS – Left hand side

Mpc - Megaparsec

ρ - Matter density of the Universe

PL - Planck

Plik - Planck likelihood

R - Ricci scalar

$R_{\mu\nu}$ - Ricci curvature tensor

S^3 - 3-sphere

SDSS - Sloan Digital Sky Survey

SMICA - Spacelab Mission Implementation Cost Assessment

T_B - Temperature Brightness/ Intensity

TT/ TE/ TTTEEE - Temperature Angular Power Spectrum

$T_{\mu\nu}$ - Energy-momentum tensor

Λ - Cosmological constant

ΛCDM - Lambda-Cold Dark Matter model

WMAP - Wilkinson Microwave Anisotropy Probe

z – Redshift

Ω - Density parameter

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CHAPTER 1: INTRODUCTION

1.1. Background

Cosmology which is a field in astronomy mainly entails the study of the birth of the Universe together with its large-scale structures, its evolution and eventually its fate. The shape of the universe is a very interesting and intriguing question in this branch of science. The Universe possesses a shape that is primarily traced by its spatial curvature and is expounded in depth in the general theory of relativity (GR). GR was developed by Albert Einstein's describes the nature of space- time, how it is curved/warped and affected by matter and energy.

Earlier studies aimed at investigating the universe curvature from the WMAP experiment pointed out a flat universe. In cosmology there exist various models which describe the shape of our universe. The Friedman-Lemaitre-Robertson-Walker model developed by the four scientists Friedman, Lemaitre, Robertson and Walker is a model that is currently widely used by cosmologists. In this model there are a series of mathematical formulations identified as the Friedman equations which govern the expansion of space assuming that we have a homogeneous and isotropic universe. In 1922 Alexander Friedman used Einstein's field equations in general relativity to derive these equations. In this model arguments which are consistent with data used lead to the conclusion that we reside in a universe that is flat and infinite. Modeling of inflation theory indicates that after the Big Bang the universe expanded in a flat manner where by two parallel lines would never meet. From the onset of modern cosmology, it has been conventional to view the universe as flat.

New data from the Planck satellite dictates that there is a need to reconsider our current understanding of the Universe. Recent studies on gravitational lensing have been found to be in conflict with theoretical models that describe our universe. More experiments strongly suggest that there might be a paradigm shift in our view of cosmology.

This research project is an ideal example of the treacherous ground that characterizes research. Over the years scientific knowledge especially in astronomy has been acquired from fragmenting pieces of evidence. The dialectic process where scientists present different arguments supported by valid evidence has proven to be a useful tool in the progress of science.

1.2. Statement of the problem and justification

Findings by Planck point out to a closed universe this is contrary to the previously prevailing models of a flat universe. It is also evident that different models and observations have obtained different results on the shape of the universe.

Scientists believe that there is an urgent need to elucidate whether the mentioned disagreements should be attributed to new physics, or to unaccounted systematics, or merely a statistical deviation. This research project is an effort to understand the prevailing cosmological crisis in the shape of the universe.

1.3. Objectives

1.3.1. Main Objective

To review and understand the cosmological crisis on the shape of the universe presented by the Planck 2018 results.

1.3.2. Specific Objectives

1. To investigate the exact shape of the universe.
2. To analyze Planck 2018 results on gravitational lensing due to Cosmic Microwave Background Radiation
3. To study structure formation

CHAPTER 2: LITERATUE REVIEW

2.1 Lambda Cold Dark Matter (ΛCDM) model

The FLRW model of the universe is closely linked to the Lambda Cold Dark Matter (ΛCDM) version of the universe. The ΛCDM approach presents a reasonable account of the features present in the universe at large scales and its accelerating expansion as well as the existence of the CMB radiation and its structure. These properties of the universe are key in understanding the shape of our universe.

Measurements of the CMB radiation from Planck satellite reveal occurrence of a phenomenon known as gravitational lensing. This is where light travelling from distant galaxies is bent due to encounter with massive objects in space in accordance to Einstein's theory of general relativity where mass creates a distortion/warping in the space-time fabric. According to the data there is more lensing than expected pointing out to occurrence of more dark matter in the universe. Only a closed universe is able to give a suitable account to this effect since a closed universe can sustain more dark matter than a flat one.

Three researchers Joseph Silk, Alessandro Melchiorri and Eleonora Di Valentino came up with the conclusions which disagree with conventional theory. These findings were presented in a paper (see Alessandro Melchiorri *et al.*2019,) after studying data from Planck space observatory. According to the paper there was more lensing than that forecasted in the standard ΛCDM model**.** The satellite was mapping cosmic microwave background radiation from 2009 to 2013. They argued that data displayed a conflict between the concentration of dark energy, dark matter and outward expansion. They claimed that such an imbalance would result in the universe collapsing in on itself. Some researchers have challenged this study saying that the Planck observatory data is just a statistical fluctuation. The three researchers have acknowledged challenges such as incompetence in measuring the Hubble constant precisely and problems with reconciling a flat model of the universe with surveys of dark energy that have been conducted. They pointed out that there is a need of more sophisticated equipment which can be able to study microwave background radiation with more clarity.

Will Handley (2019) expressed his views on this matter in a paper by stating that more research is necessary to demystify why the CMB alone firmly prefers a closed universe, whilst other datasets yield quantitatively contradictory constraints. When investigating the geometry of the universe cosmologists also study baryon acoustic oscillations (BAO). The primordial plasma of the early universe contained acoustic density waves which led to density fluctuations of visible baryonic matter (normal matter). The nature of dark energy can be understood by studying these oscillations. This mysterious energy contributes to the inexplicable spreading out of the universe which has been detected to be accelerating. Cosmologists achieve this by applying constraints to cosmological parameters.

The ΛCDM model encounters one major drawback which is referred to as the cosmological constant challenge. ΛCDM model involves the use of a constant denoted by lambda (Λ). This is the energy bulk retained in space (vacuum energy), it is associated with dark energy and also referred to as the cosmological constant. This constant was introduced by Einstein in his field equations after the discovery that the equations pointed out an expanding universe. He did this to achieve a static universe and to counterbalance the effects of gravity.

$$G_{\mu\upsilon} \equiv R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R + g_{\mu\upsilon} \Lambda = 8\pi G_N T_{\mu\upsilon} \qquad (2.1)$$

On the LHS is the Einstein tensor $G_{\mu\nu}$ where an additional term that incorporates the constant **Λ** has been plugged. The extreme right-hand consists of G_N the Newton's universal gravitational constant. In the equation $g_{\mu\nu}$ is the metric of the manifold where these relations hold. It also contains the Ricci tensor and scalar $R_{\mu\nu}$ and R respectively.
$T_{\mu\nu}$ is the energy-momentum tensor and;

$$T_{\mu\upsilon} = (\rho + p)\frac{u_\mu u_\upsilon}{c^2} - g_{\mu\upsilon} p \qquad (2.2)$$

In equation (2.2) $\boldsymbol{u_\alpha}$ is the macroscopic velocity of the medium, ρ denotes energy density while p the fluid pressure. The matter content and the geometry of space-time are related by the equation

$$G_{\mu\upsilon} - g_{\mu\upsilon}\frac{\Lambda}{c^2} = \frac{8\pi G_N}{c^4} T_{\mu\upsilon} \qquad (2.3)$$

In the above equations c =1.

Initially before adding the cosmological constant the equation was given by;

$$G_{\mu\upsilon} = R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R \tag{2.4}$$

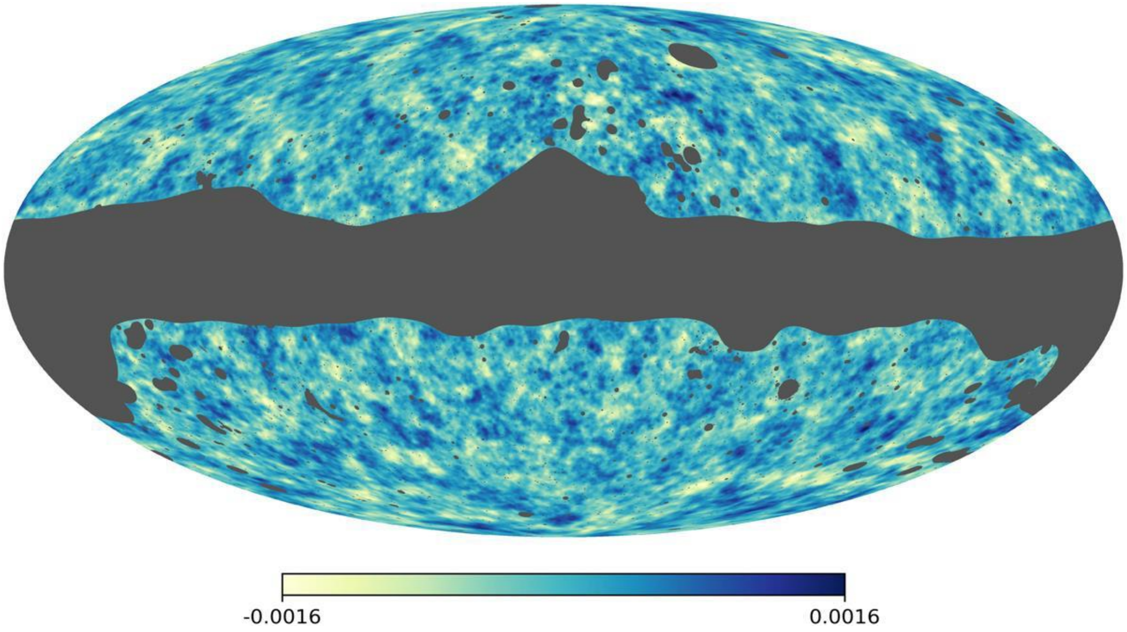
Later on, he came to realize it was a terrible mistake after Hubble observatory confirmed that indeed the universe was expanding. However, the cosmological constant is invoked to explain dark energy which dictates how the universe expands. Observations made by cosmologists show a huge discrepancy in the value of this vacuum energy when compared with that predicted in quantum field theory. The theoretical prediction is up to 120 orders of magnitude in excess to that of the observed value. In physics history this has been addressed as the worst theoretical prediction. This phenomenon is the cosmological constant problem. Many scientists point out that solving the cosmological constant problem is the answer to comprehending the true nature of the universe.

In general relativity Einstein (1916) outlined that gravity is a consequence of the distortion of spacetime fabric according to him it is incorrect to perceive gravity as a force. This phenomenon is known as warping of spacetime, every mass creates a little depression in the fabric of cosmos (like a mattress). Massive objects cause this distortion in space and hence other objects are drawn towards this depression in the spacetime fabric so that they appear as if they are attracting each other. General Relativity predicted that light would bend when passing by massive objects and the more the mass the larger it bends an occurrence known as gravitational lensing. This is the main principle used in the Planck 2018 findings on gravitational lensing.

2.2 Planck 2018 data on Gravitational lensing

When light propagates through space, photons get involved in gravitational interactions with matter and hence their trajectories become deflected. These deflections contain insights about dark energy and the growth of large-scale structure. Cosmic microwave background radiation is substantially characterized and produced at a well-known redshift hence yielding a distinct source for lensing measurements. This radiation provides a substantial amount of data since it has propagated almost the entire observable universe. The lensing signal is generated from a wide range of redshifts (z), distinctively in the range $0.1 < \mathbf{z} < 5$ (Lewis& Challinor2006). Lensing causes CMB anisotropies to appear stretched and contracted.

The following images show some of the data that was acquired by Planck satellite.



Luminosity (L)

Figure 1: CMB Anisotropies

The lensing amplitude A_{lens} corresponding to the degree of lensing was obtained from this map. Blue regions are cooler than the brighter regions which have higher temperatures the dark region is the galactic plane.

The Mollweide projection used above is a technique of map projection mostly used for global maps of the world or night sky used to achieve accuracy of proportions in area.

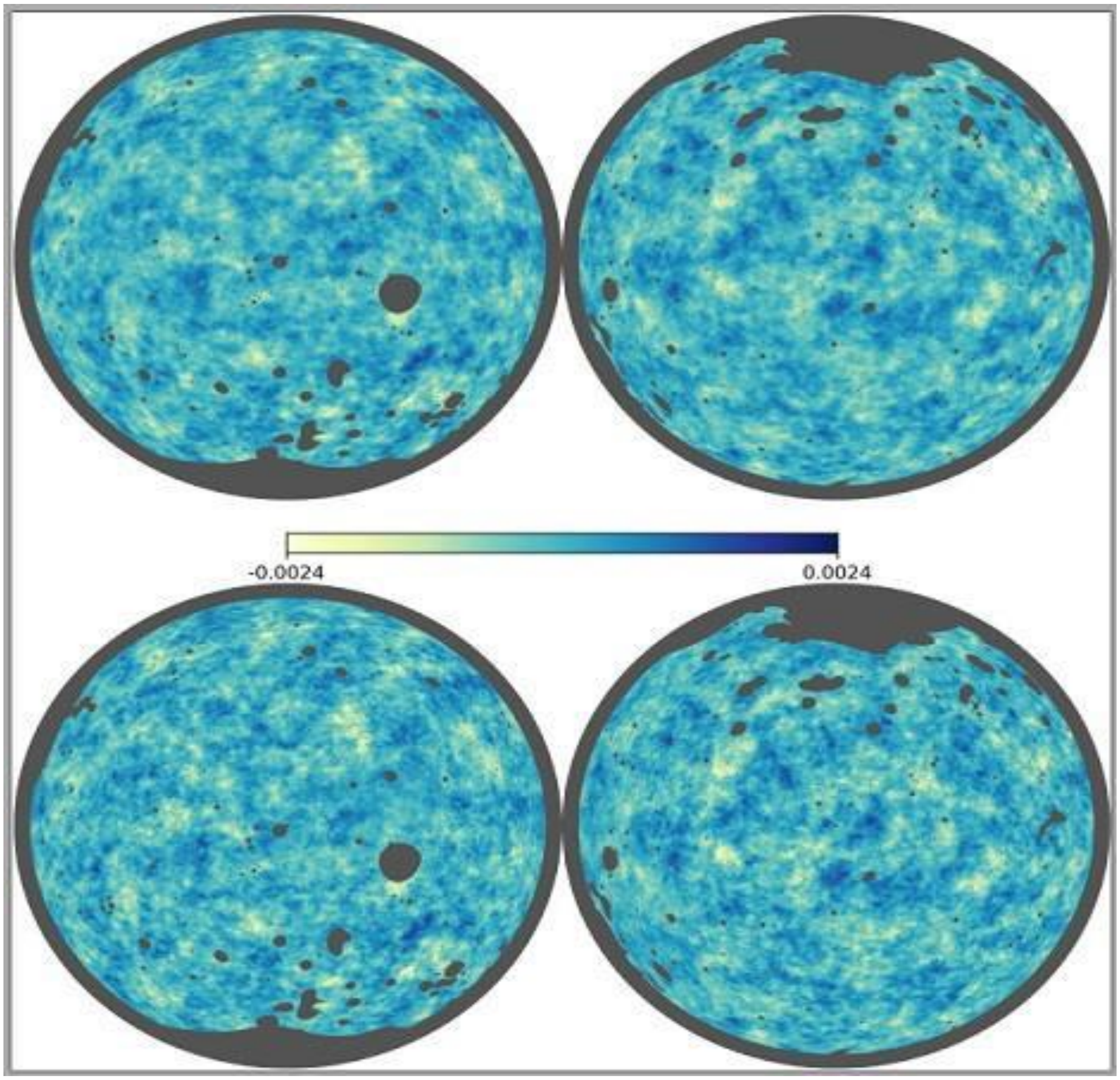


Figure 2

In this figure the left panel represents the north galactic poles while the right panel represents south Galactic poles, respectively.

CHAPTER 3: THEORETICAL BACKGROUND

3.1 The Shape of the Universe

The local geometry and global geometry of the universe as a whole are the crucial factors that are considered by cosmologists when determining the shapes of the universe. Spatial curvature in the observable universe describes the local geometry while its global geometry is the topology of the universe in its entire scale.

Topology is concerned with the study of spatial objects such as curves, surfaces, nature of space-time in general relativity, manifolds, fractals, knots, phase spaces and symmetry groups. Most of these objects have the same basic spatial properties as the space we call our universe. Topology is a useful tool used to outline the inherent connectivity of objects without considering their detailed form.

In cosmology there is a clear distinction between the universe that we can observe and the Universe as a whole in its entire scale. The cosmological principle aids cosmologists to extrapolate the behavior of the entire universe simply by studying the observable universe. The principle makes the assumption that when the universe is viewed on an adequately large scale it will appear homogeneous and isotropic. In other words, this principle intimates that the universe will appear the same for all observers located at different points in the universe. As a result, scientists argue that the observable universe is a reasonable sample of the whole universe and the same laws of physics apply throughout.

The shape of the universe is mainly described by its curvature (how the geometry of space varies locally from that of a flat space). There are mainly three types of curvature namely;

1. Flat universe which has zero curvature; a triangle drawn in such a space will have angles adding up to 180^o and obeying the Pythagorean theorem, Euclidean space **(E^3)** is used to model this type of space.

2. Open universe this has a negative curvature; a triangle on this space will have angles adding up to less than 180^o modeling such a space is achieved using hyperbolic space **(H^3)**.

3. Closed universe with a positive curvature; a triangle on this space will have angles adding up to an excess of 180^o a region of a 3-sphere **(S^3)** is the most suitable to describe this space.

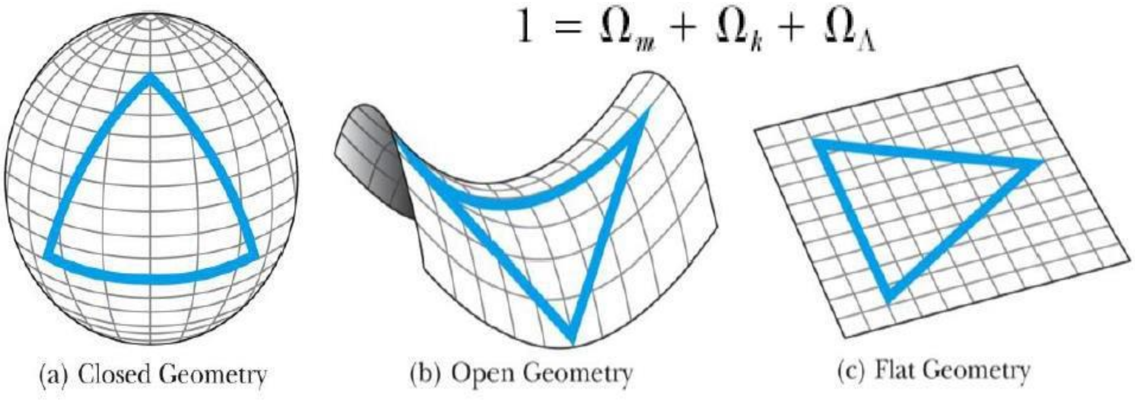


Figure 3: Universe Geometry

The density parameter denoted by **(Ω)** is used to specify the geometry of our universe locally this is expressed by.

$$\Omega_o = \frac{\rho}{\rho_c} \qquad (3.1)$$

Where **ρ** represents average density of the universe whereas **ρ_c** corresponds to the critical energy density. Critical energy density is the mass energy required to make the universe flat. A flat universe will have a density parameter Ω equal to 1, when the universe is closed the parameter is greater than 1 and less than 1 when the universe is open.

The density parameter can also be expressed as

$$\Omega_o = \Omega_\rho + \Omega_k + \Omega_\Lambda \qquad (3.2)$$

The term **Ω_ρ** stands for matter density, **Ω_Λ** is the cosmological density and **Ω_k** denotes the curvature density. Dark energy density is also another name for **Ω_k**.

3.2 The Standard Cosmological Model

Currently the Lambda Cold Dark Matter model of the universe ΛCDM has been widely recognized by cosmologists as the model that best suits our universe. This model is formulated in conformity with the Einstein's GR theory. The model has succeeded in explaining a great record of the accompanying properties of the universe. For example, the expansive hierarchy configuration in the circulation of galaxies, the presence and structure of the CMB. It also gives a reasonable account of the abundances of elements such as hydrogen, including deuterium, helium, and lithium. However, the ΛCDM is presently confronted with incompetence in explaining the discovered late-time enhanced growth of the universe. This model also requires inclusion of the cosmological term in order to be in harmony with observations. The spherically symmetric space-time metric can be defined as

$$ds^2 = c^2dt^2 - a^2\frac{dr^2}{1-kr^2} - a^2r^2(d\theta^2 + sin^2\theta d\varphi) \qquad (3.3)$$

Where, φ, θ and r are spherical coordinates.

$a = a(t)$ is better known as the scale factor which relates the separation amidst two bodies undergoing isotropic and homogeneous expansion.

The continuously extending universe is modeled with the aid of Friedmann equation which contains a parameter k known as the curvature density parameter. The parameter dictates whether the rate of expansion is increasing or decreasing. The future fate of the universe can be deducted from this parameter.

The Friedmann equation is given by;

$$H^2 = {}^{8\pi\rho}/_{3} + {}^{\Lambda}/_{3} - {}^{k}/_{a^2} \qquad (3.4)$$

Where H is the Hubble parameter

$$H = {}^{1}/_{a}\left({}^{da}/_{dt}\right) \qquad (3.5)$$

If k = 0 the spatial sections of the space-time have no curvature (flat) and the density of this type of universe is equal to a critical value whereby the universe will experience a decelerating expansion that continues forever.

When k is greater than zero the space-time fabric is positively curved (spherical/closed). This is a gravitationally bound universe known as a closed universe. It has a density that is sufficiently high meaning that the expansion will eventually be halted by gravitational attraction; this type of universe will collapse in an occurrence known as the big crunch.

When k is less than zero the curvature is negative (hyperbolic/open). This type of universe has insufficient density for gravitational attraction to stop its expansion and hence will expand forever.

CHAPTER 4: RESEARCH METHODOLOGY

4.1 Literature review

First and foremost, I was involved in conducting thorough literature review on different models of the universe and observations that have been conducted in various experiments. I extensively reviewed a wide range of scientific journals published in this area of research. This aided in achieving some of the objectives of this research.

I indulged in understanding Einstein field equations together with Friedmann equations.

4.2 Data Acquisition

The research project mainly entailed intense analysis of the 2018 Planck data release. This was followed by analysis of data from different observatories which have contributed hugely in this field of cosmology such as the Planck satellite, WMAP probe, Atacama Cosmology Telescope (ACT) and Hubble Space Telescope (HST). Most of the data was readily available online which was an added advantage.

4.3 Analyzing CMB power spectra

This was the final step where, I generated the CMB power spectra for various cosmological models by using Python CAMB software which is available online. Then I compared the results I obtained with those from Planck satellite and other satellites as well.

CHAPTER 5: RESULTS AND DISCUSSION

5.1 Results

5.1.1 The Shape of the Universe

A good number of the experiments conducted by cosmologists over the years have concluded that our universe is flat. We begin by understanding what pertains to a flat shape. Just picture being in a rectangular or square room moving about the four corners, by making four 90-degree turns you'll automatically be back to where you started. Then such a room is considered as flat. This is Euclidian geometry. A similar maneuver can be made on the Earth’s surface and this will yield different results. Starting from the equator, immediately take a ninety degree turn, march towards the North or the South Pole, then make another right angled change of direction, provided you are following a perfectly straight path you will return to the equator. On arriving at the equator making a final 90-degree change of direction will take you back to the original starting point.

In the first scenario, only four turns were required to take you back to your starting point, the second case only required three. The reason being that the topology of the surface you were navigating dictated the outcome when making a right-angled turn. Another analogy is considering two parallel lines originating at the poles, they are expected to deviate while trailing the topology of the Earth and eventually approach each other such lines will be back together and intersect at a point. On a flat surface such parallel lines would never meet.

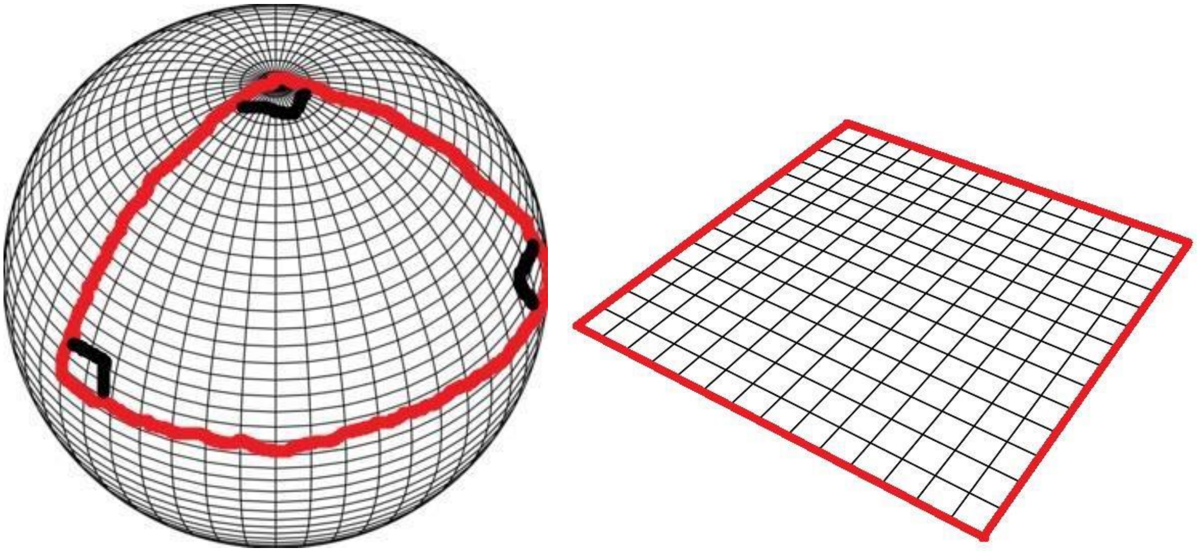


Figure 4: Topology manuever

The same analogy can be applied to the case of the universe itself. Consider venturing into space in a spacecraft able to travel for millions or billions of light-years, making the same 90 degree turns and arriving back at the origin. It would be the assumed that these can't be achieved in three, or five turns, one would require four. This prediction is quite intuitive which would mean that our universe is flat.

Testing for the curvature of our Universe would require an observer to travel a significantly enormous distance way. The cosmic microwave background radiation is a perfect example of the largest possible observation which astronomers put into consideration. This is the photosphere of the early Universe during the Big Bang, which is detected in every direction in the universe as a red-shifted, diminishing instant in time. It was until three hundred and eighty thousand years after its birth that the universe ceased to be opaque the CMB was then released after it became transparent.

The universe had a temperature of about 3000K when this radiation was being emitted. This was favorable enough for photons to freely traverse the universe. These photons have been stretched out by the expansion of the universe as they continue to traverse the universe 13.8 billion year later. Consequently, this has shifted them to around 2.7K hence moving down from the visible spectrum into the microwave spectrum.

Studying the CMB radiation enables astronomers to observe tiny variations in the temperature. They are detected using some of the most sophisticated equipment such as space-based telescopes. These miniscule temperature differences correspond to the largest structures that can be observed in the cosmos. A section which was warmer by just a small degree grew to an immense galaxy cluster which is millions of light-years across.

Any curvature in our universe would result into the distortion of these temperature variations compared to the actual form of the structures that has been recorded until date.

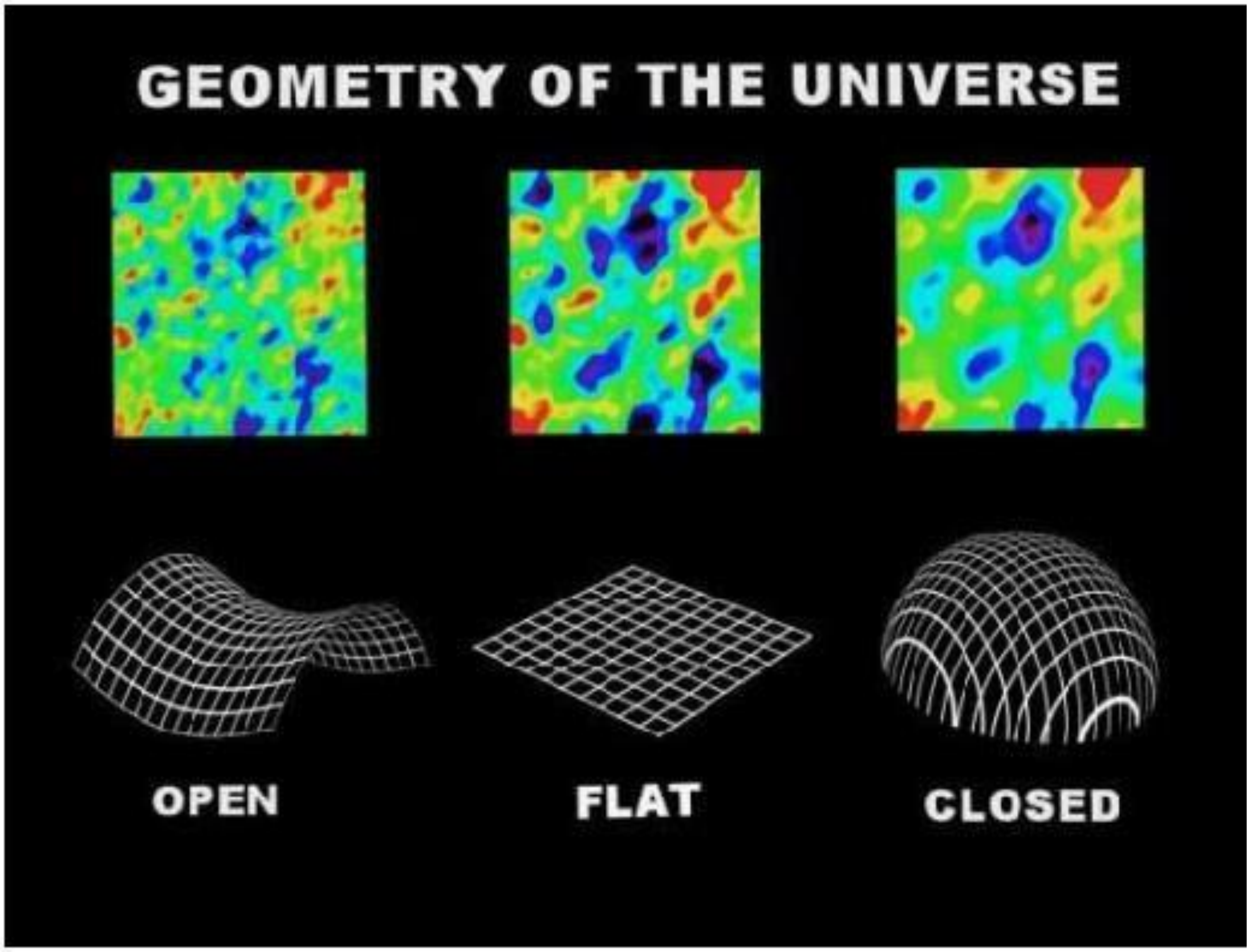


Figure 5: Geometry of the Universe

As illustrated in the images above in the case where there is curvature there is less detail in the CMB due to distortion as compared to the case where there is no curvature.

5.1.2 CMB Maps

The Planck Satellite launched by the European Agency’s was mainly devoted to probe the early universe. This satellite analyzed the microwave together with the submillimeter sky subsequently between 12 August 2009 and 23 October 2013. It produced maps in nine frequency bands (from 30 to 857 Giga Hertz) which were deep and of high-resolution.

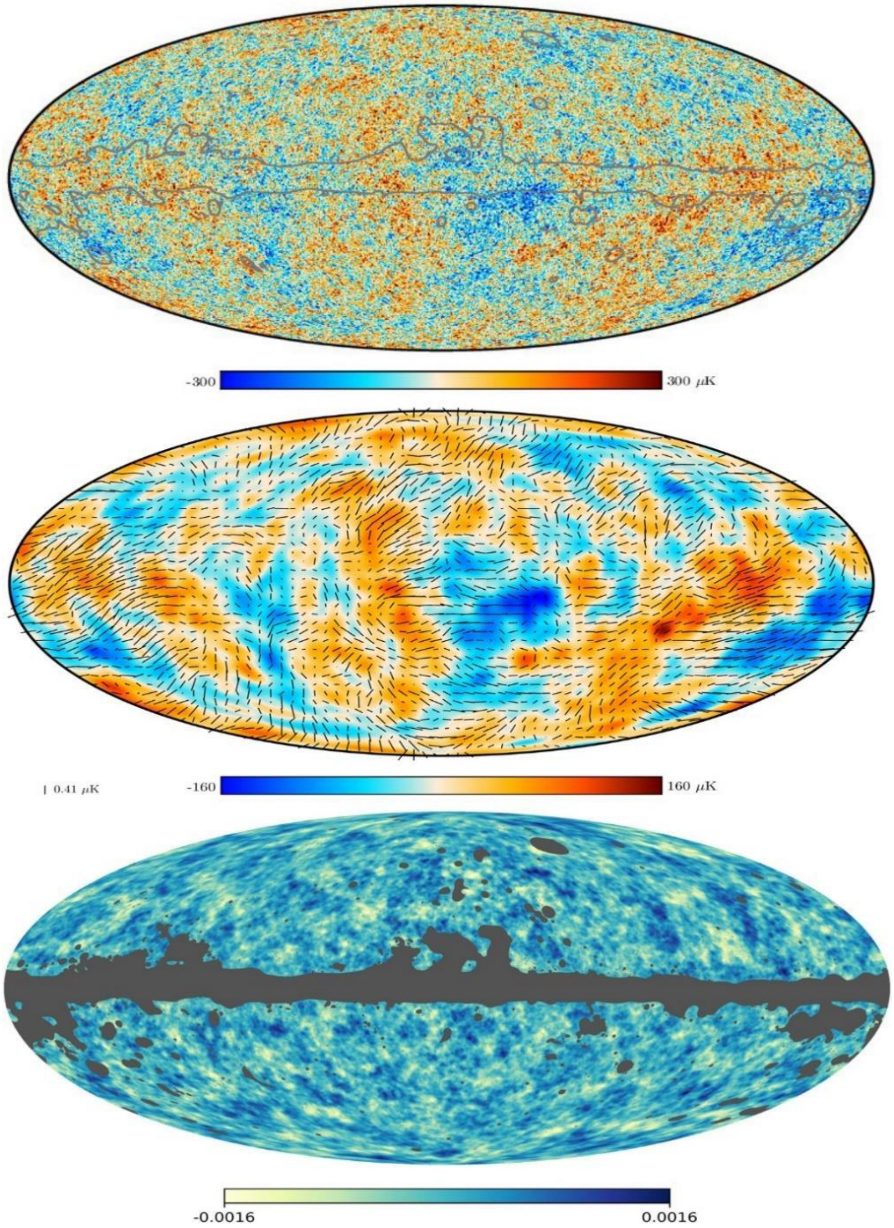


Figure 6: CMB Maps

The figure above illustrates the CMB sky as recorded by Planck. Top panel: 2018 SMICA temperature map. SMICA (Spacelab Mission Implementation Cost Assessment) is a method of enhancing raw CMB maps by linearly combining Planck input channels with multiple dependent weights. The second map in the middle illustrates rods of different lengths which represent the polarization field which is superimposed on the map representing the temperature distribution. The third panel at the bottom is the Planck lensing map recreated using SMICA 2018 foreground-cleaned maps. This was done by combining maps of high frequency. This involves the use of minimum variance; Wiener filter applied on both temperature and polarization maps. A mask which represents the Galactic plane is outlined by a grey line and a dark shade in the top and the bottom map. The area which is unmasked covers 80.7% of the sky.

Majority of the signal in the first and second map is contributed by processes occurring at a redshift $z \simeq 103$. Large scale structures possess gravitational potentials which cause the deflection of CMB photons. This in turn modifies the signals detected by Planck satellite. Determining the impact of lensing in the CMB is the method used to quantify the lensing amplitude $\mathbf{A_{lens}}$. This is illustrated in the map at the bottom which provides sensitivity to the lower-redshift Universe. This sheds some light on the gravitational instability in the early Universe.

The second map highlights the polarization signal. In order to increase legibility this is implemented using a comparatively low angular resolution of 5°. Bars which have different orientation and length describe the polarization field. The temperature signal is greater in amplitude than the signal due to polarization. In the second figure, it can be seen that the polarization signal is overlaid on the temperature anisotropies. The temperature was found to be correlated with the polarization signal.

The statistical properties associated with CMB maps makes them useful in the study of our Universe. It is evident that the initial CMB anisotropies have a distribution that is remarkably similar to Gaussian distribution (see e.g., Planck Collaboration VII and IX 2020). Nevertheless, a couple of fluctuations from Gaussian distribution are present. This is in line with what most inflationary models predict. The information from the CMB is not obtained from the precise locations of individual features, but is rather encoded in its statistical properties which are also isotropic. The correlation function and the mean can fully describe a Gaussian field. This correlation function or the power spectra contains relevant information on the CMB anisotropies.

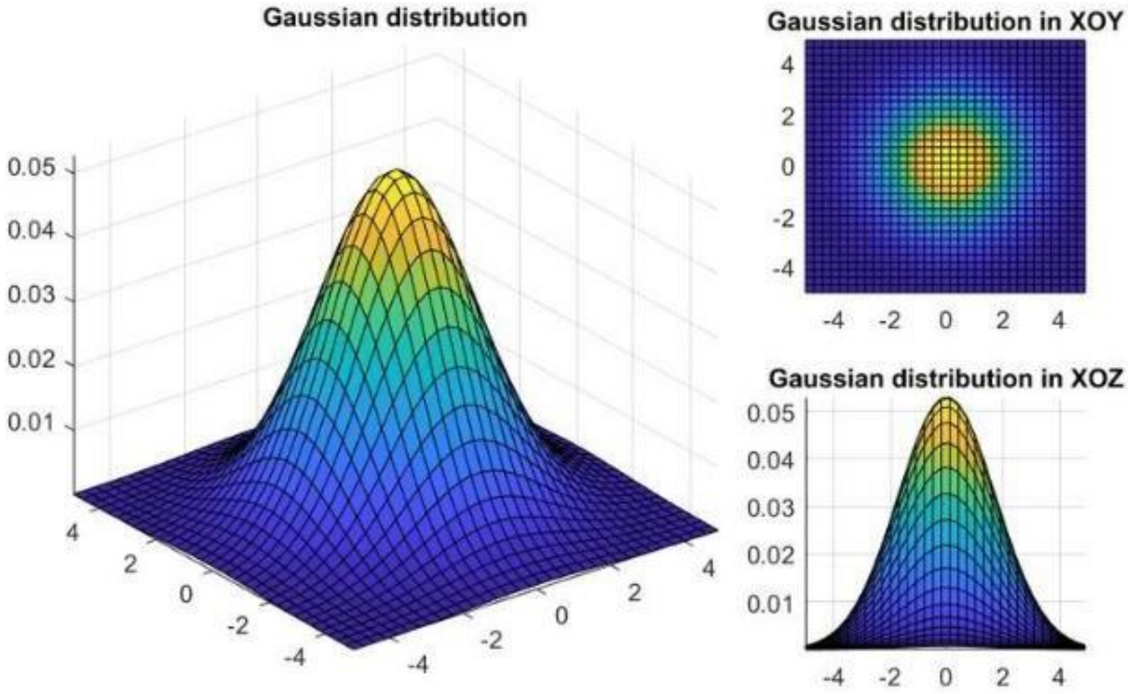


Figure 7: Gaussian distribution

The ΛCDM model can be completely described using only 6 independent parameters. These parameters include the Hubble constant H_0, optical depth to reionization τ, matter density Ω_m, the age of the Universe t_0, the scalar power law index n_s, and the fluctuation amplitude σ_8. More additional parameters can also describe this model e.g, the cold dark matter density Ω_ch2, redshift at recombination, age of the Universe, baryon density Ω_bh2 amongst other parameters.

Data shown in the table below illustrates the values of some of the parameters in harmony with the Planck 2018 CMB data. This table also compares the two scenarios with and without addition of baryon acoustic oscillation data. Measurement of baryon acoustic features is achieved by studying celestial objects such as galaxies and the Ly α forest as tracers in the redshift range $0 < z < 2.5$.

Table 1: Cosmological parameters from CMB maps

Parameter	Planck alone	Planck + BAO
Ω_bh2	0.02237 ± 0.00015	0.02242 ± 0.00014
Ω_ch2	0.1200 ± 0.0012	0.11933 ± 0.00091
τ	0.0544 ± 0.0073	0.0561 ± 0.0071
n_s	0.9649 ± 0.0042	0.9665 ± 0.0038
H_0	67.36 ± 0.54	67.66 ± 0.42
Ω_Λ	0.6847 ± 0.0073	0.6889 ± 0.0056
Ω_m	0.3153 ± 0.0073	0.3111 ± 0.0056
σ_8	0.8111 ± 0.0060	0.8102 ± 0.0060
z_{re}	7.67 ± 0.73	7.82 ± 0.71
Age [Gyr]	13.797 ± 0.023	13.787 ± 0.020
Ω_K	−0.0096 ± 0.0061	0.0007 ± 0.0019

The Planck scientific collaboration is a team of scientists who have contributed to the mission by Planck satellite and they were also the first to consume and exploit the Planck data. They interpreted this data by adopting the ΛCDM model this required the scientists to make several assumptions. The ΛCDM model applies the cosmological principle which is the underpinning theory of cosmology. This has the implication that the universe looks identical at each location (i.e., homogeneous) as well as in all directions (i.e., isotropic). In simpler terms the laws of physics should be the same regardless of the location of an observer in the Universe. The model embraces general relativity (GR) as the most adequate description of gravity.

In this model the spatial curvature is very small. Scientists argue that since the Universe is expanding then there exist a time in the distant past when everything in it was contained in a very small point. This state is known as singularity which was then followed by a rigorous explosion referred to as the Big Bang. There were variations in density which are, adiabatic, and nearly scale invariant. These fluctuations were present everywhere with a distribution that is Gaussian, as predicted by inflation.

This model is composed of five basic cosmological constituents:

- The Big Bang photosphere or simply the photons detected as the CMB.
- Dark matter which is stable, it can only be detected through its gravitational interactions with normal matter. It is not involved in electromagnetic interactions and hence it was given the name dark since it cannot be detected through visible light. This form of matter is hypothesized to be without pressure (in line with the role of structure formation).
- Dark energy which causes enhanced outward growth of the Universe. Normally matter is expected to be drawn towards each other due to gravity, contrary to this it has been noted that matter in the Universe is flying away from each other. This deduction has been attributed to the dark energy.
- Regular atomic matter which behaves in a similar manner as terrestrial matter.
- Neutrinos that hypothesized to be of very low masses about 0.06eV.

It was assumed that the CMB radiation was that of a blackbody made up of photons and neutrinos with a temperature T = 2.7260K.

The CMB radiation fit perfectly to that of a blackbody that it cannot be made by stars. The absorption lines and band edges in stars qualifies them as poor blackbodies. A star should possess a temperature gradient in its outer regions for it to radiate.

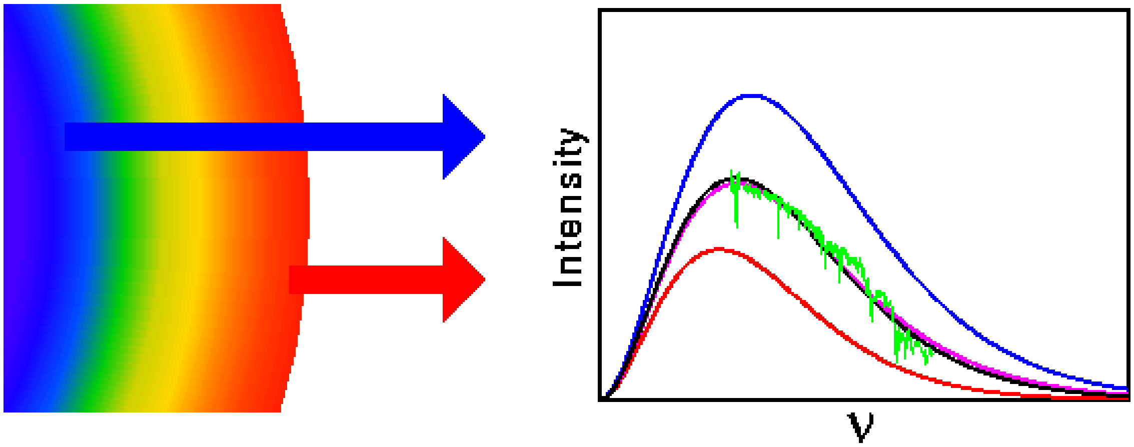


Figure 8: Blackbody Spectrum

The gradient in temperature makes light from stars to be seen as a mixture of radiation from the hotter lower levels (blue) and the cooler outer levels (red). The purple curve represents blackbodies when the temperatures are mixed. This is precisely not equivalent to the blackbody curve (black); however, it is very close. A typical star such as our sun (class G2V) has a spectrum that is similar to the one shown by the green lines. In cosmology, the question of how an almost ideal blackbody CMB radiation was produced by the Big Bang is quite intriguing. In the Big Bang model where the Universe is homogeneous no temperature gradients exist. The early survey of CMB revealed that the primordial universe is nearly uniform and fluctuations from this uniformity were not yet detected. However, recently scientists have discovered anisotropies proving that the universe was not fully uniform after all. Tiny perturbations were present in the cosmic plasma.

The blackbody spectrum is a fundamental subject in physics. German physicist Max Planck studied blackbody radiation in depth. This led him to introduce the concept of *quanta*, and he described the quantum of action $\boldsymbol{h}$ known as the Planck's constant. This fundamental constant explains the nature of quantum elements on the atomic scale. The study incorporated the wave-particle aspect of light. Light and other electromagnetic radiation is generated in discrete quantities, scientifically known as quanta or energy packets. These are integral multiples of the constant h. Every photon possesses energy E which is directly proportional to its vibrational frequency;

$$E = h\upsilon \qquad (5.1)$$

Where the Greek letter ν, represents the frequency of radiation. The universality of the blackbody spectrum makes it easy to determine its *brightness temperature* at its characteristic wavelength. The brightness temperature denoted by T_B describes the amount of radiation. A blackbody spectrum has a unique character such that at all wavelengths the brightness temperature is the same. Planck's law for a black body is given by;

$$I_\upsilon = \frac{2h\upsilon^3}{c^2}\frac{1}{e^{\frac{h\upsilon}{kT}} - 1} \qquad (5.2)$$

Where the brightness which is the same as intensity is denoted by I_v , this quantifies the energy emitted per unit surface area per unit time per unit solid angle. In the equation k represents the Boltzmann's constant and T the temperature of the black body. Considering a source with known spectral radiance and using the inverse of the Planck function yields the brightness temperature T_B of the radiation.

$$T_B = \frac{hv}{k} ln^{-1} \left(1 + \frac{2hv^3}{I_v c^2}\right) \qquad (5.3)$$

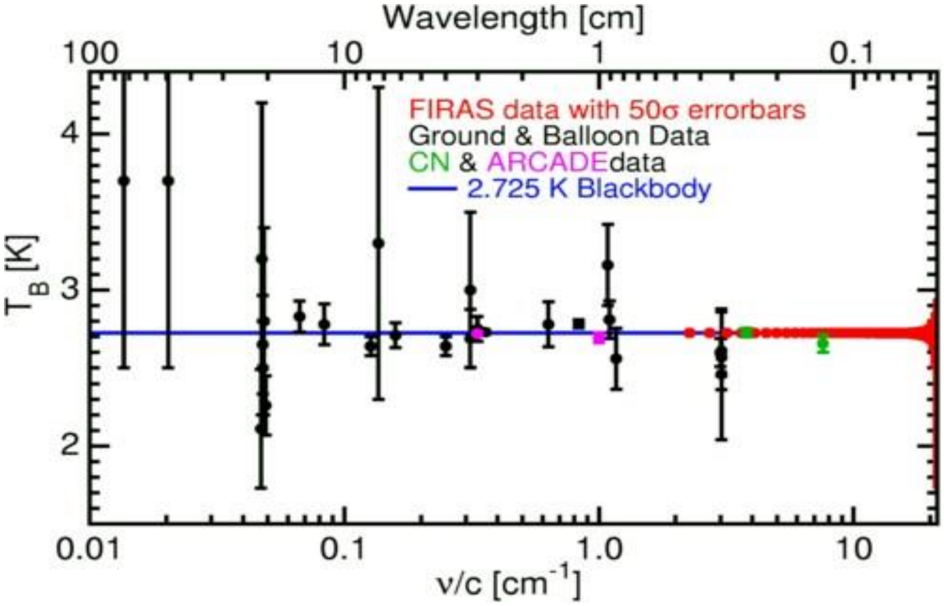


Figure 9: Brightness Temperature

The above diagram illustrates the brightness temperature T_B of the CMB as detected from various wavelengths. It can be seen clearly $T_B = 2.725\ K$.

An object must be opaque, non-reflective and isothermal for the spectrum of a blackbody to be produced. The hot deeper layers and cooler outer layers of a star can be seen and hence a star being opaque does not produce a blackbody spectrum. The Universe has a temperature that keeps on changing while the Universe evolves, with

$$T_{CMB} = T_o\ (1 + z) \qquad (5.4)$$

The distant Universe which is warmer seems to have a similar temperature as the Universe today because of the redshift. Consequently, this makes the Universe to appear isothermal.

$\mathbf{T_B}$ may either be independent or dependent on, the wavelength of the radiation. Any subsequent absorption and the nature of the source of radiation will also dictate the temperature brightness. The power spectrum of the CMB anisotropies is characterized by features such as acoustic peaks. These features have enabled scientists to extract substantial knowledge concerning the estimates of fundamental cosmological parameters. This has contributed significantly to our understanding of the cosmos (de Bernardis et al. 2000; Hanany et al. 2000).

Analysts of CMB data acknowledge that there is an exorbitant computational price of producing CMB power spectra and matter transfer operations of the entire grid points in multidimensional parameter space. Scientists then develop codes and algorithms that sample the space selectively. Markov Chain Monte Carlo (MCMC) algorithm is one of the approaches employed to search hunt for the parameter space of a cosmological model. Cosmologists then employ theoretical models which are backed by probabilistic statistical methods such as Bayesian statistics. In these analytical processes, different parameters are permitted to float as the other parameters are kept at given set values. Scientists then determine the most probable values for the cosmological parameters being tested.

The cosmological parameters are however not ascertained to absolute precision by using CMB data only. There exist degeneracies between particular combinations of parameters which pose as a challenge since this leads to identical temperature fluctuation spectra (Efstathiou & Bond 1999). This CMB data can therefore be combined with other data sets such as baryon acoustic oscillations (BAO) and the power spectrum of galaxy clustering (as shown in Table 1 pg. 18). These sets of data are then compared with those predicted in theoretical models which help in breaking the degeneracies. Applying this knowledge together with the assumptions enables scientists to make predictions using these parameters and to derive other parameters for example since it has been discovered that $|\Omega_K| << 1$ then Ω_Λ can be computed from;

$$\Omega_\Lambda = 1 - \Omega_m \qquad (5.5)$$

The matter density Ω_m quantifies the mass density of visible material in the universe. Baryons constitute most of this visible matter. A baryon is a subatomic particle that is studied in particle physics which participates in the strong interaction, neutrons and protons are an example of this type of matter. The word “barýs” is a Greek word for heavy in which the term baryon was derived from. Baryons were heavier than other elementary particles which were known at the time of their naming. Almost all matter we encounter daily is baryonic matter, and this type of matter has the property of mass. They contribute about 4% of the critical density. The Big Bang is assumed to have produced a Universe with similar amounts of baryons and antibaryons. Baryons exceeded their antimatter particles in an event known as baryogenesis. Theoretically the non-conservation of baryon number in the primordial universe made baryons to exceed the antibaryons. Unfortunately, this has not been clearly grasped.

Physical baryon density of the universe $\Omega_b h^2$ obtained from the CMB data is subjected to comparison with the value determined by investigating the abundance of deuterium in distant gas clouds. Big Bang Nucleosynthesis (BBN) can be used to predict baryon content since it models the manufacture of elements that are not heavy in the primordial universe e.g., deuterium and the two values from the different approaches are in reasonable agreement. Results show that about 4-5% of the universe today is made up of baryons.

The physical CDM density $\Omega_c h^2$ is the density of non-baryonic cold dark matter which is visible inside CMB. The type of matter that is deficient in baryons is referred to as non-baryonic. Examples of this type of matter include dark matter, free electrons, neutrinos, black holes, axions and supersymmetric particles. Values of the cold dark matter density parameter Ω_c can be compared with those obtained from the relation;

$$\Omega_c \approx \Omega_m - \Omega_b \qquad (5.6)$$

Where Ω_b is the density of baryonic matter and they have been found to be consistent. The density of dark matter ω_{dm} is given by;

$$\omega_{dm} = \Omega_{dm} h^2 \qquad (5.7)$$

and

$$\Omega_{dm} = \Omega_{cdm} + \Omega_\nu \qquad (5.8)$$

Where Ω_{cdm} is the cold and Ω_{ν} the hot dark matter component, ν represents neutrinos. The quotient of dark matter in the form of massive neutrinos is given by;

$$f_{\upsilon} = {}^{\Omega_{\upsilon}}\!/_{\Omega_{dm}} \qquad (5.9)$$

The optical depth to reionization, τ, is a parameter that dictates the time when emitting sources were formed and began reionizing. The optical depth can be related to an approximate redshift, or range of redshifts, during which reionization occurred: larger values imply an earlier onset of reionization. This gives a glimpse on the era when the first stars and galaxies formed. The photons produced from these objects ionized the neutral gas which was present after recombination.

The parameter z_{re} represents the redshift of the recombination epoch. Recombination is when charged electrons became bound to protons (hydrogen nuclei) hence forming electrically neutral hydrogen atoms. The electrons then transit to a lower energy state by emitting photons which travel freely through the universe without interacting with matter. This is what is observed as the CMB today a blackbody radiation redshifted from the visible spectrum by a factor of 1100 to the microwave spectrum. Scientists believe it was emitted when the universe had a temperature of about 3000K.

The fluctuation amplitude parameter which is denoted by σ_8 is the root mean square linear density fluctuation in spheres of radius 8 h^{-1} Mpc at z = 0. If the amplitude of fluctuations is high structures form at a high rate and vice versa. Recent observations suggest amplitude that ranges from a value of σ_8 ~0.7 to a “high” value of around σ_8 ~0.9–1. This parameter strongly depends on the cosmological density. Density fluctuations of only 0.032 are expected in the ΛCDM model. These findings suggest that there is a conflict since these high values imply that structures form rapidly than expected in the standard model. According to White S.D.M, George Efstathiou et. al σ_8 ~0.52 – 0.62 for a critically dense universe and σ_8 ~1.25 – 1.58 for a spatially flat universe.

The scale factor **a** describes the expansion of the Universe. The factor is given a value of one in the present day at earlier times it was smaller since the Universe was more compact. The Hubble rate is a measure of how rapidly the scale factor changes as time progresses;

$$H(t) \equiv \frac{da/_{dt}}{a} \qquad (5.10)$$

H_o is used to denote the Hubble rate today. The value obtained by studying the recession of nearby galaxies has revealed that H_o = lOOh km s-1 Mpc -1. The Hubble parameter h which has no dimensions is used to parameterize the Hubble rate. The most recent studies suggest h of around 0.72 ± 0.08 (Freedman et, 2001). The value has been found to be discrepant with the one obtained from the Planck and CMB data which yield $H_o \approx 67.4 \pm 0.5$ Km s^{-1} Mpc^{-1}. It was this phenomenon that led to the deduction that the rate of the expansion is accelerating since the local approach of studying nearby galaxies quantifies the expansion rate today whereas the prediction from the CMB is based on the physics of the early universe. This parameter can be derived from;

$$h = \sqrt{(\omega_{dm} + \Omega_b h^2)/_{\Omega_m}} \qquad (5.11)$$

Since the universe is expanding an observer on Earth should therefore see galaxies receding. The waves produced from an object that is receding have a longer wavelength ($\lambda_{observed}$) than the original wavelength $\lambda_{emitted}$. This phenomenon is known as Doppler shift and the stretching factor is defined as the redshift z;

$$1 + z \equiv \frac{\lambda_{observed}}{\lambda_{emitted}} = \frac{1}{a} \qquad (5.12)$$

For low redshifts;

$$z = \frac{v}{c} \qquad (5.13)$$

where, v is the velocity of the incident wave and c the speed of light. Measuring the redshifts in determines the rate of recession.

The scalar power spectral index n_s explain the manner in which density deviations of the early universe vary with scale. The inflationary theory hypothesizes that the exponential increase of the scale factor **a** in the time of inflation triggered quantum variations of the inflation field to be lengthened to macroscopic scales. After departing from the horizon they froze in at subsequent stages of radiation and matter dominance and hence re-entering the horizon this set the preliminary configurations for formation of structure.

Cosmologists suggest that in models where the Universe is closed $n_s > 1$ today Planck data yields a value of $n_s = 0.96$.

In the Friedmann model of the universe, the following equation yields the density parameter **Ω**;

$$\Omega \equiv \frac{\rho}{\rho_c} = \frac{8\pi G\rho}{3H^2} \qquad (5.14)$$

Where **ρ** is the actual (or observed) density and $\boldsymbol{\rho_c}$ the critical density. This relation dictates the entire geometry of our universe. When the two are equal, the universe has a geometry which is Euclidean i.e., flat. The quantity $\boldsymbol{\rho_c}$ was hypothesized as the topographical boundary which determines whether the Universe will expand or contract. The density parameter can also be expressed as;

$$\Omega_o = \Omega_\rho + \Omega_k + \Omega_\Lambda \qquad (5.15)$$

The term $\boldsymbol{\Omega_\rho}$ stands for matter density; $\boldsymbol{\Omega_\Lambda}$ is the cosmological density and $\boldsymbol{\Omega_k}$ denotes the curvature density. Recalling the first Friedmann equation;

$$H^2 = {}^{8\pi\rho}/_{3} + {}^{\Lambda}/_{3} - {}^{k}/_{a^2} \qquad (5.16)$$

The Friedmann model assumes Λ to be zero. Considering minimal or absence of spatial curvature, i.e., $k = 0$ an expression for the critical density ρ_c can be obtained:

$$\rho_c = \frac{3H^2}{8\pi G} \qquad (5.17)$$

Today $\rho_c = 1.8788 \times 10^{-26}$ h2 kg/m3 in this case $h = H_o / (100$ km/s/Mpc). The critical density has a value of 8.5×10^{-27} kg/m3 when having a H_o of 67.4 km/s/Mpc. Results indicate that $\boldsymbol{\rho_c}$ is roughly five atoms of monatomic hydrogen per cubic meter. The mean density of ordinary matter in the Universe is estimated to be 0.2– 0.25 atoms per cubic meter. In the Universe today dark energy (Λ) dominates the total energy density contributing about 73%. Dark matter makes up about 23% of the Universe while ordinary matter (baryonic matter) detected as gas, atoms, chemical elements, and plasma is believed to make up about 4%. Recently released data from Planck cosmology probe has refined the values to 69.1% dark energy, 25.9% dark matter and 4.9% ordinary matter.

When investigating the history of the universe cosmologists have discovered that any epoch in its evolution can be associated with either temperature or time. Any period in time since the birth of our Universe can be characterized by the quantity of the scale factor **a** at that time; or by the temperature T. Today where a = 1; t ≃ 14 billion years; and T = 2.725K= 2.35 x 10-4 eV/k_B. The diagram below clearly illustrates this. Cosmologists are quite certain about milestones such as nucleosynthesis and the CMB whereas more speculative about dark matter production, inflation, and dark energy today.

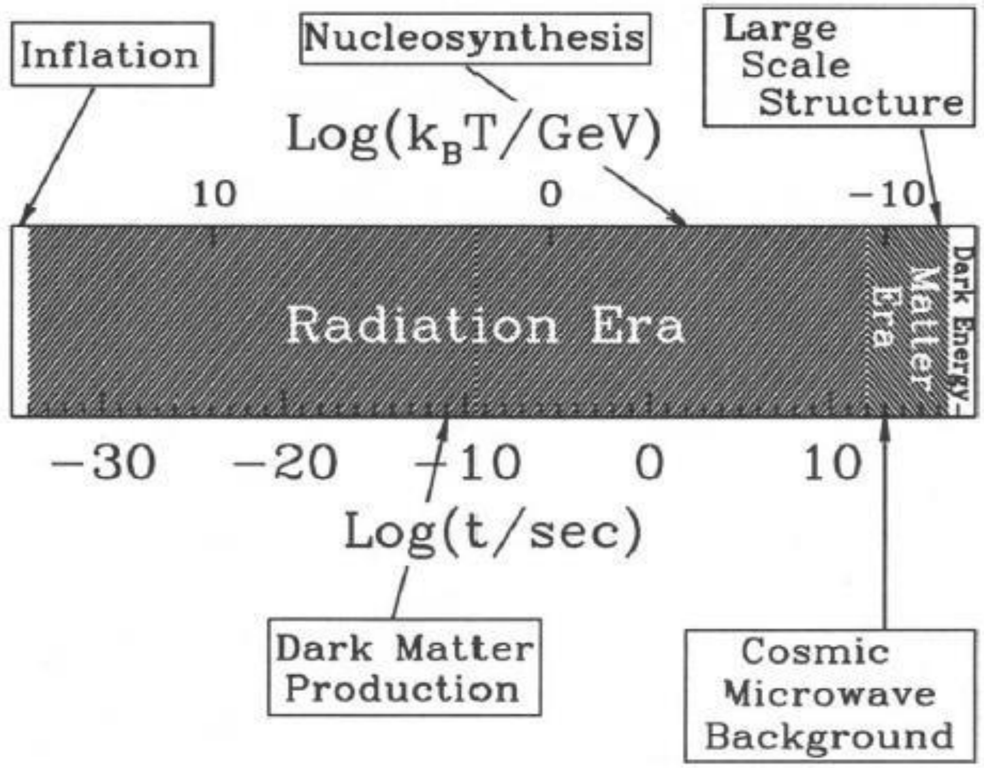


Figure 10: Evolution of the Universe with Time and Temperature

The first Friedmann equation can be expressed using the estimates of the constituent parameters at the present time;

$$\frac{\mathrm{H}^2}{\mathrm{H}_\mathrm{o}^2} = \Omega_{o,R}a^{-4} + \Omega_{o,M}a^{-3} + \Omega_{o,k}a^{-2} + \Omega_{o,\Lambda} \qquad (5.18)$$

Where the subscript "o" is used to denote the values of the given parameters today, $\boldsymbol{\Omega_{0,R}}$ denotes the radiation density , $\boldsymbol{\Omega_{0,M}}$ represents dark matter together with the baryonic matter density,

$\boldsymbol{\Omega_{0,\Lambda}}$is the vacuum energy density and $\boldsymbol{\Omega_{0,k}}$ is the spatial curvature density, where

$$\Omega_{0,k} = 1 - \Omega_0 \qquad (5.19)$$

Ω_k which specifies the curvature of the universe is given by the equation;

$$\Omega_k = -\frac{k}{a^2H^2} \qquad (5.20)$$

Where k is a constant which is equal to 1 when the universe is spherical/ closed, 0 when flat and -1 when open/hyperbolic. As a consequence of the negative sign in equation (5.20) Ω_k will be negative for a closed universe and positive for the open case, it will remain zero for a flat universe. Curvature is measured as a percentage and it is typically quantified via its partial input to the cosmic energy budget.

The best measurement made by scientists so far point that the universe nearly flat (see e.g *The evidence for a spatially flat Universe* George Efstathiou and Steven Gratton). Earlier studies by Planck spacecraft revealed that it is flat to within 0.5%. A conflicting interpretation has been presented recently in three different papers (Park & Ratra 2019; Di Valentino et al. 2019; Handley 2019). These papers claim that from statistics done on the Planck data a closed universe is more preferred. Di Valentino et al. (2019) concluded that statistics from the 2018 data favor a positive curvature at the 3.4σ level (i.e., a probability to exceed (pte) of 0.034%). From the Planck 2018 lensing data Ω_K was measured to have a value of -0.0438 implying a closed Universe with a probability ratio of about 1:41, with respect to a flat model. The scientists attributed this to either, statistical fluctuations or systematics which were undetected and if not so then there might be new physics behind these deductions.

5.1.3 Planck 2018 findings on Gravitational lensing

In general relativity Einstein portrayed gravity as a consequence of the distortion of the space-time fabric instead of a force. This phenomenon is known as warping of space-time, every mass creates a little depression in the fabric of cosmos (like a mattress). Massive objects cause this distortion in space and hence other objects are drawn towards this depression in the space-time fabric so that they appear as if they are attracting each other. All the matter and energy content that is present in the entire Universe dictates the curvature of space-time. General Relativity predicted that light would bend when passing by massive objects and the more the mass the larger it bends an occurrence known as gravitational lensing. This is the main principle which is investigated in the Planck 2018 findings on gravitational lensing.

When light propagates through space, photons get involved in gravitational interactions with matter and hence their trajectories become deflected. These deflections contain insights about dark energy and the growth of large-scale structure. Cosmic microwave background radiation is substantially characterized and produced at a well-known redshift hence yielding a distinct source for lensing measurements. This radiation provides a substantial amount of data since it has propagated almost the entire observable universe. The lensing signal is generated from a wide range of redshifts (z), distinctively in the range $0.1< \mathbf{z} < 5$ (Lewis& Challinor2006). Lensing causes CMB anisotropies to appear stretched and contracted. The image below illustrates the lensing phenomenon.

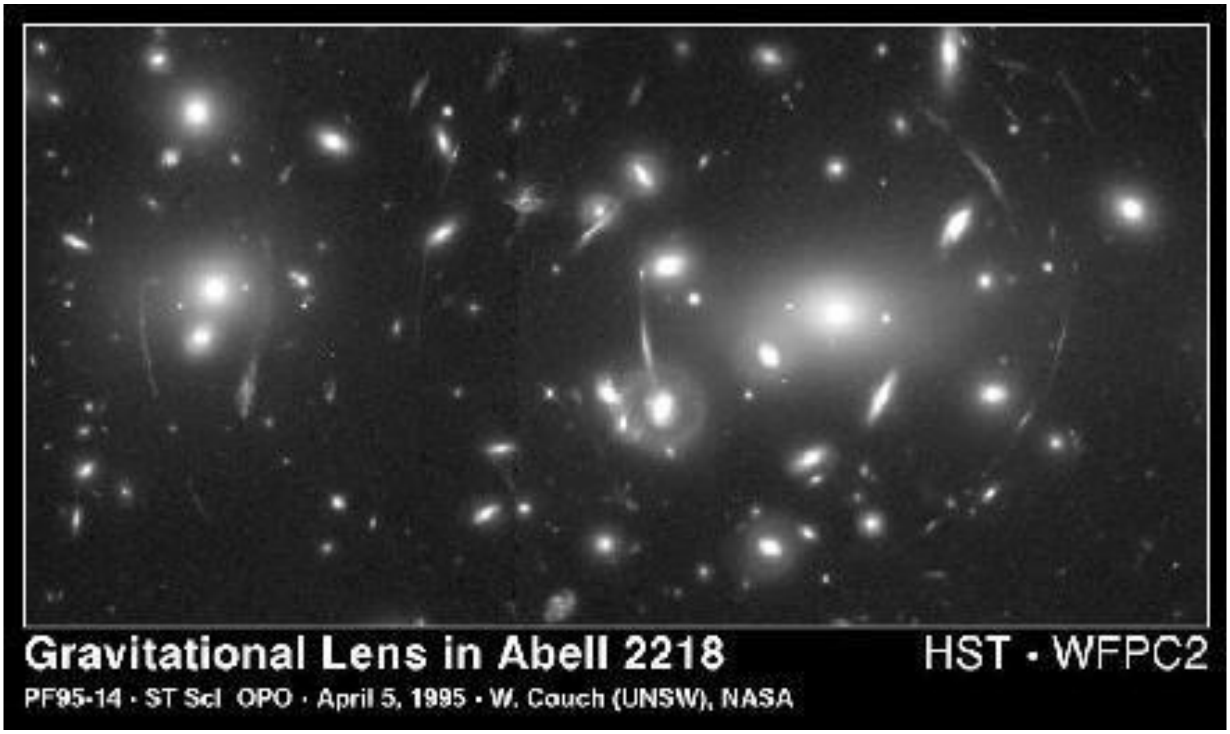


Figure 11: Gravitational Lensing

Astronomers are able to measure the mass in the cluster by quantifying how objects in the surroundings are optically deformed by the galaxy cluster in the foreground. Keen observations of gravitational lensing confirms that there is a mass discrepancy. Analysis on the extent of lensing done on this particular image reveal that mass in the cluster is more than five times greater when compared to the inferred mass in visible stars, gas and dust. Scientists have come to a conclusion that there exists an incomprehensible form of matter that exerts a gravitational pull, however this form of matter does not interact with light. This mysterious material which is not well understood was given the name dark matter.

Planck 2018 data show that studies done on gravitational lensing in the CMB are unfolding into what scientists now refer to as a “cosmological crisis”. Analysis of Planck data reveals that there is more gravitational lensing than expected. This hints that our universe could contain more matter than previously thought, this occurrence introduces curvature to the spatial geometry of the universe. Particularly these are the findings that are in contradiction with the prevailing models such as the ΛCDM model where the universe has no curvature (i.e., flat).

The pattern displayed by the CMB anisotropies is determined by the curvature of space defined by the curvature parameter *k* as intimated earlier in the Friedmann equation (3.4). The CMB can be examined by measuring the temperature variations at different angular scales. This is achieved by computing the power spectrum of the CMB map. This power spectrum describes how temperature varies at various angular scales in the sky. Scientists find it efficient to assign each angular scale a number *l* known as the multipole number. Large values of this number correspond to small angular scales while smaller quantities relate to large angular scales. The resultant power spectrum is then subjected to comparison with different power spectra, which are predicted by the various cosmological models applied by scientists. These models have a wide range of values for different cosmological parameters and different types of curvature as well. Cosmologists then adopt the model that best fits the given spectral data being investigated.

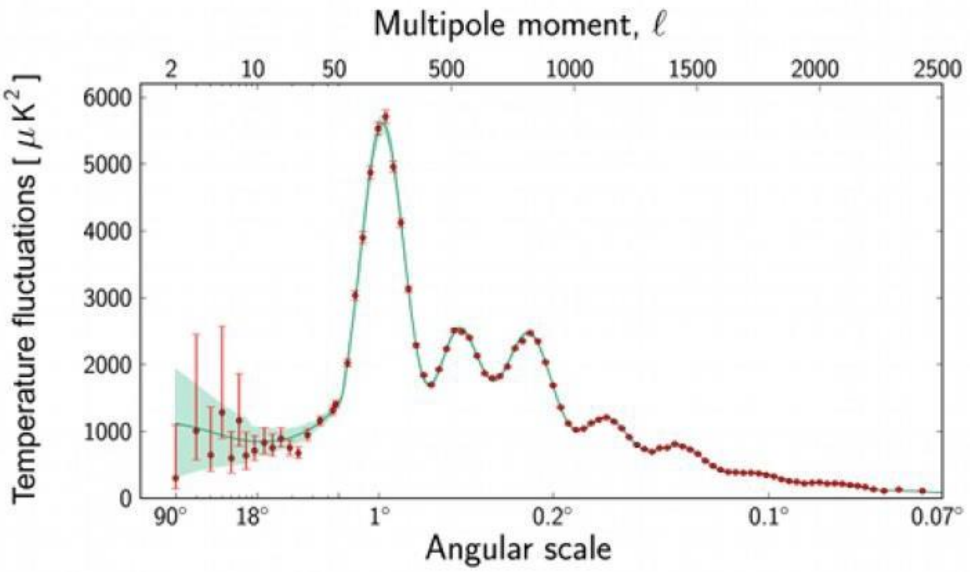


Figure 12: Planck 2013 CMB Power Spectrum

This graph illustrates the power spectra and the most probable model fit for CMB data acquired from Planck in 2013. The red dots correspond to the data points while red lines denote the error bars and the model that best fits this data is elucidated in green. The graph shows that the spectrum peaks at a multipole moment, *l* of about 200 to 220, which correspond to an angular scale of 1 degree across the sky. This is the most prominent scale meaning that most of the temperature variations are approximately one degree in size.

The following images shows how two parallel lines are affected by the curvature of space.

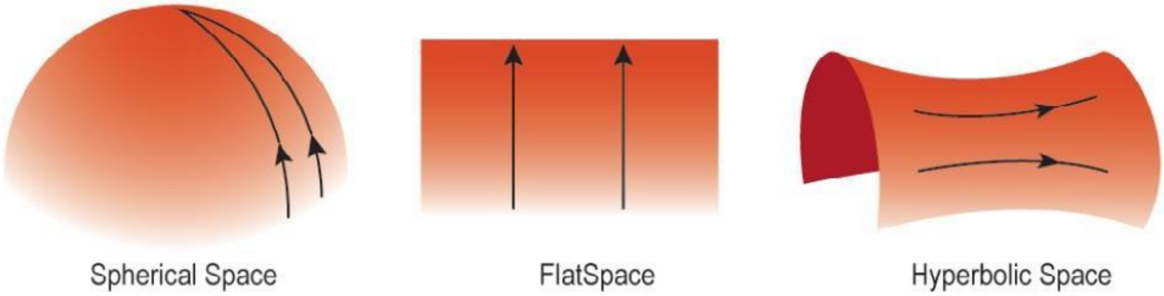


Figure 13: Parallel lines navigating in different space curvatures

Light rays from a source behave like these parallel lines and they will navigate through space along a geodesic in accordance to the curvature of space. When the curvature is positive (i.e., closed universe) the source will appear larger to an observer, whereas in a negatively curved space (i.e., open universe) the curvature will make it appear smaller than it is. This is illustrated in the image below.

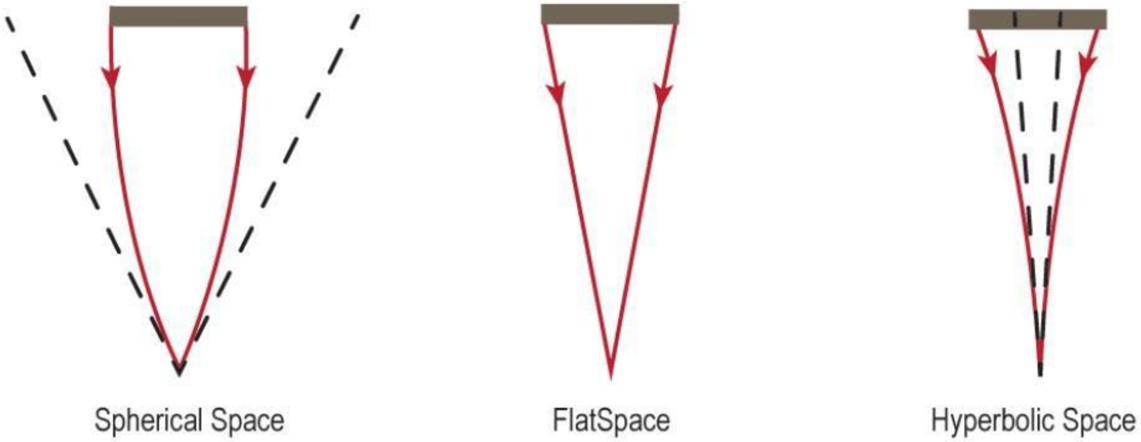


Figure 14: Light ray diagrams

The image on a flat space will appear nearly the same as the object. Similarly, this principle is applied when studying CMB anisotropies, the cold and hot points should seem larger in a closed Universe and smaller in an open one.

The following images illustrate the three theoretical probabilities of the CMB pattern that can be recorded considering the three types of curvature.

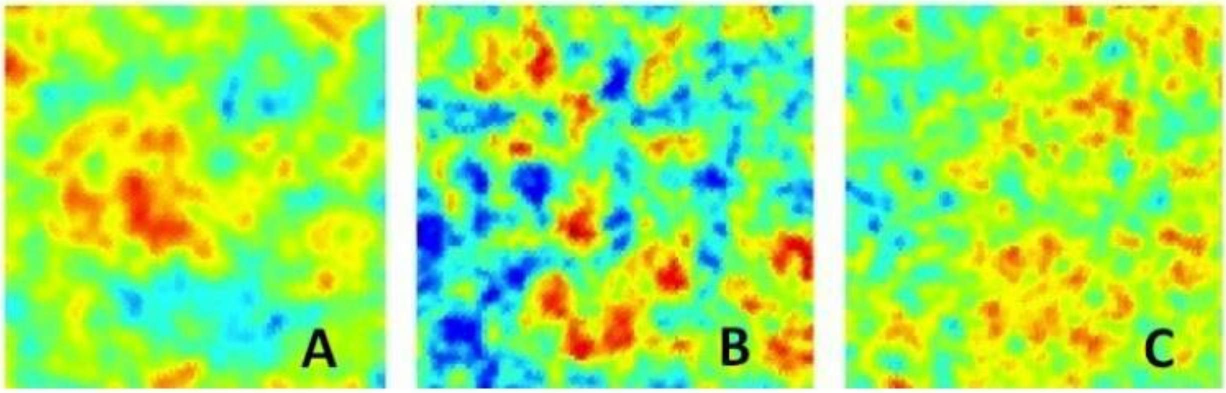


Figure 15: CMB pattern in various models

Image A represent the closed (spherical) model, B the flat one and C the open (hyperbolic) case. This will affect the characteristic power spectrum in the following manner.

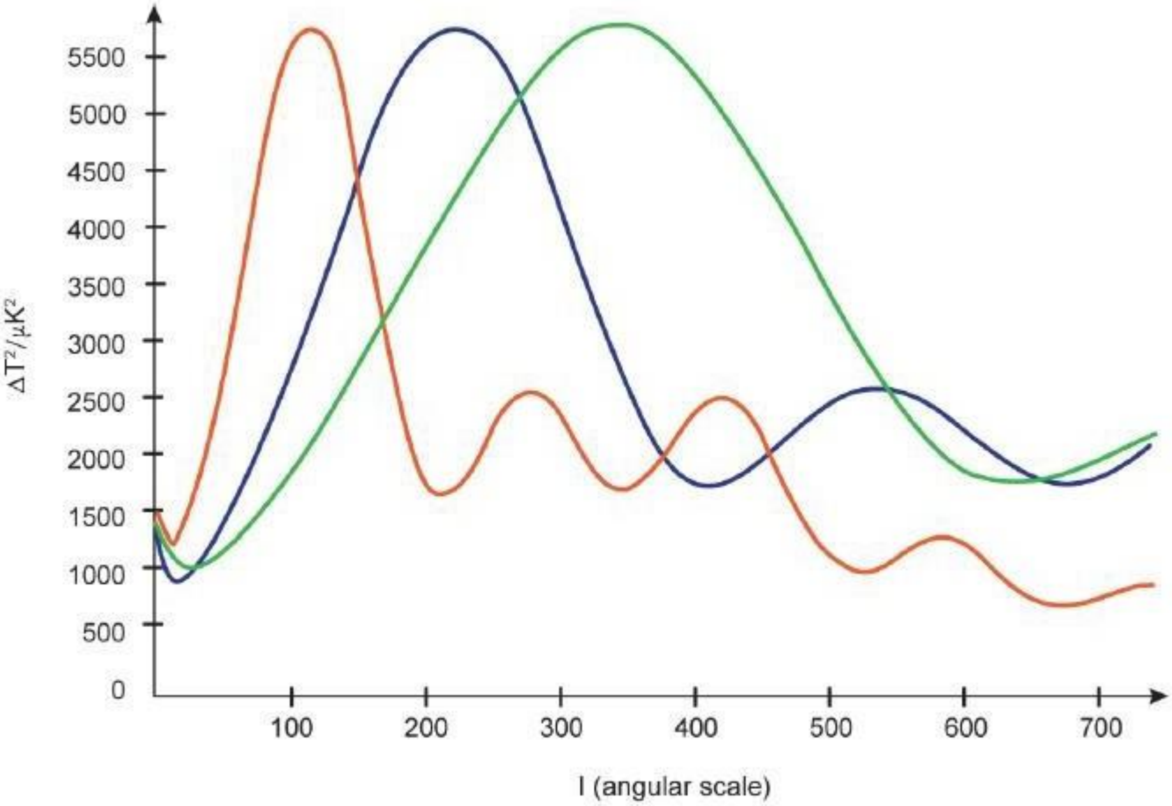


Figure 16: CMB Power spectra in different models

The curvature of the Universe determines the angular magnitude of the first crest in the power spectra. The spectrum in red is expected when space is closed the first speak is detected at a multipole number of approximately $l \approx 110$. The blue spectrum is the outcome when the space is flat which has the first peak at about $l \approx 220$. The power spectrum of CMB in hyperbolic geometry is shown in green with the first peak at about $l \approx 350$.

In this research I was able to generate the power spectra for various cosmological models with different types of curvature. I achieved this with the aid of Python CAMB software this can also be implemented through the NASA's LAMBDA interface. I obtained results for six different models with $\Omega_k = 0$, +0.04, +0.4, +4, -0.4 and -0.04.

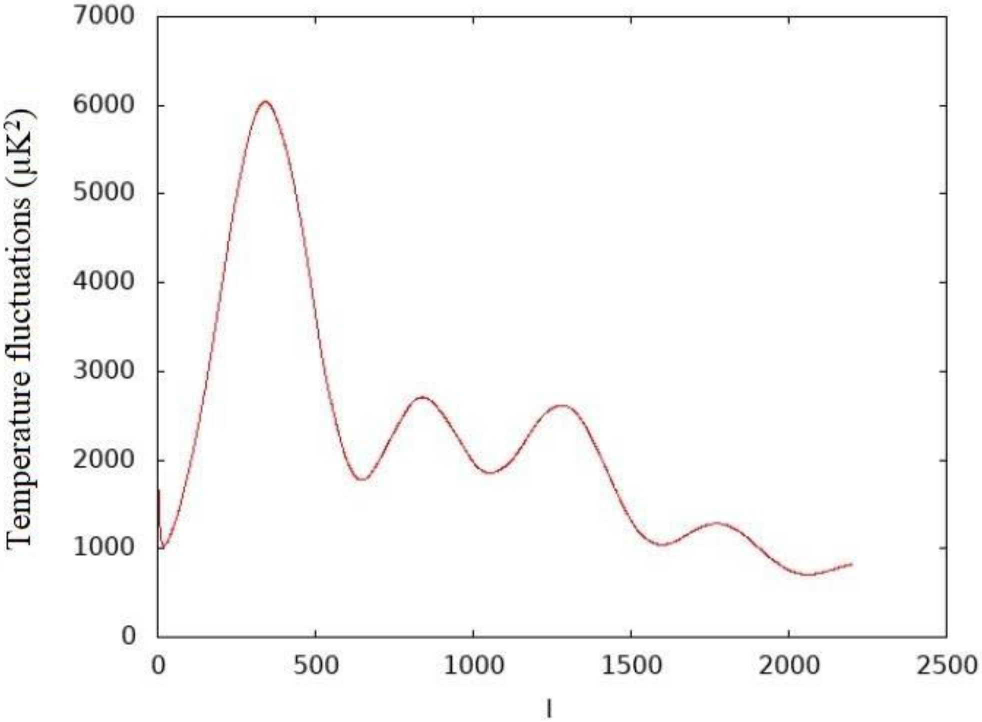


Figure 17: Results for $\Omega_k = 0$ model (flat)

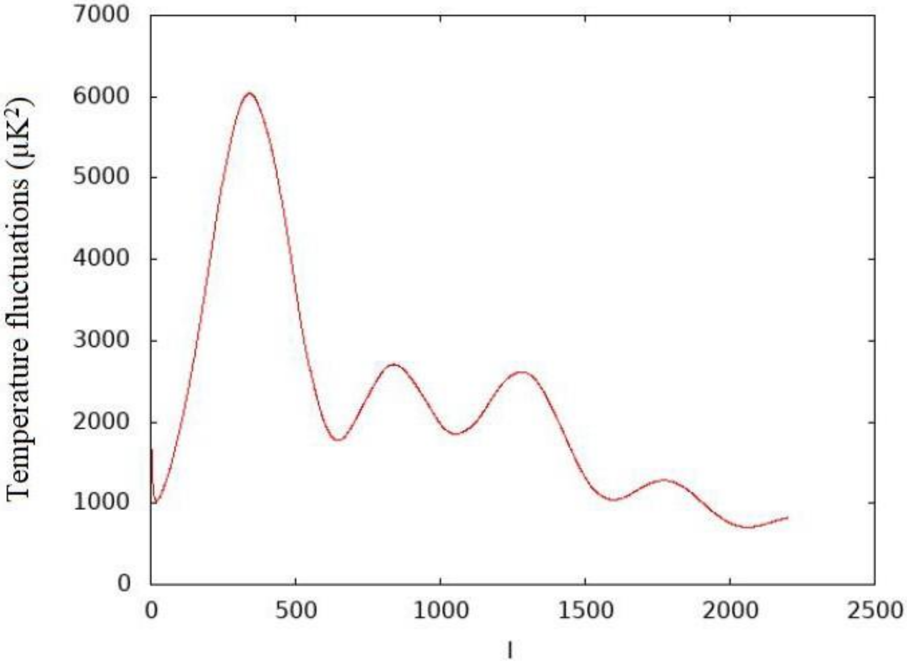


Figure 18: Results for Ω_k = +0.4 model (open)

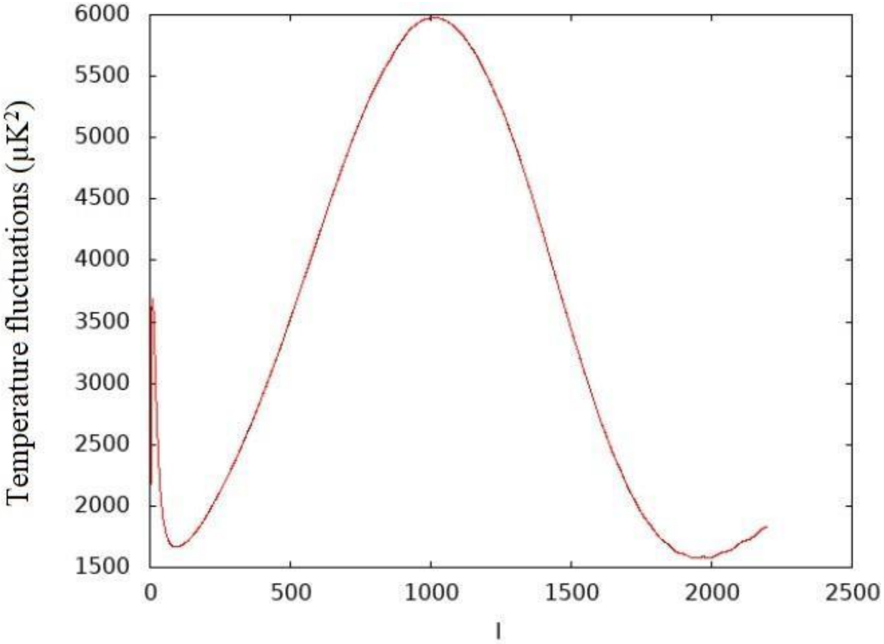


Figure 19: Results for Ω_k = +4 model (open)

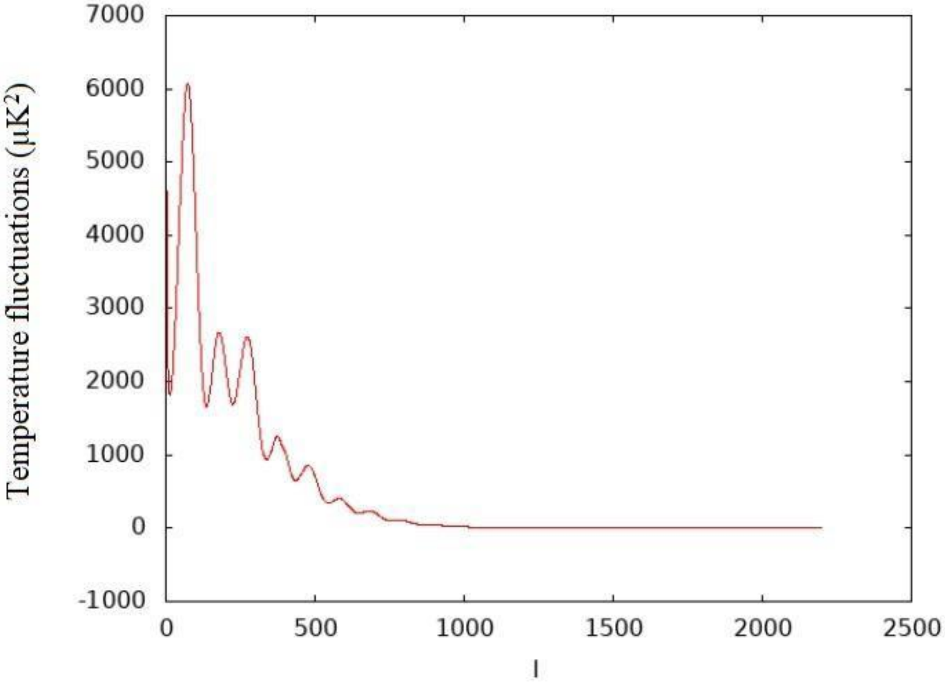


Figure 20: Results for $\Omega_k = -0.4$ model (closed)

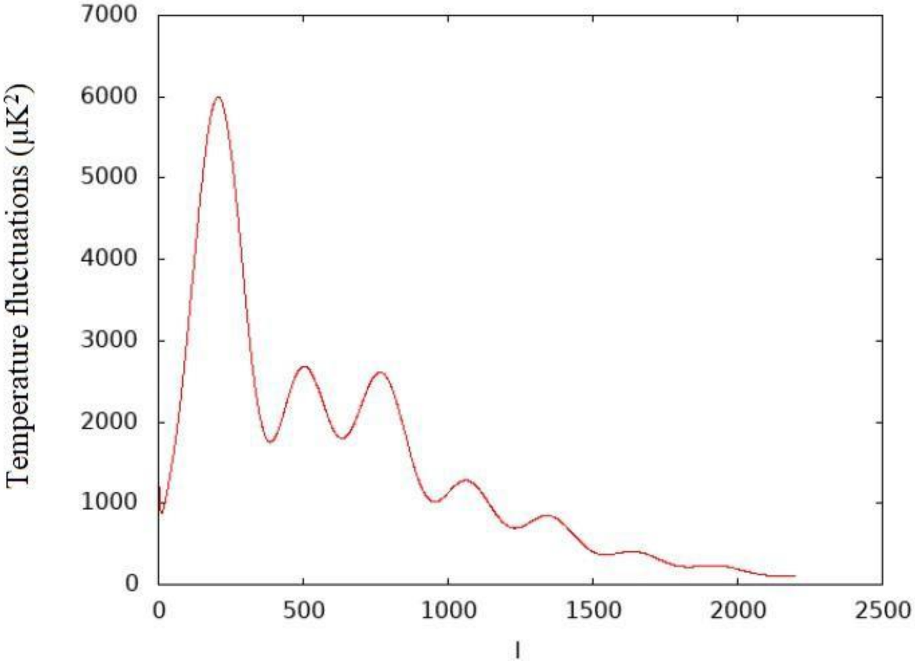


Figure 21: Results for $\Omega_k = -0.04$ model (closed)

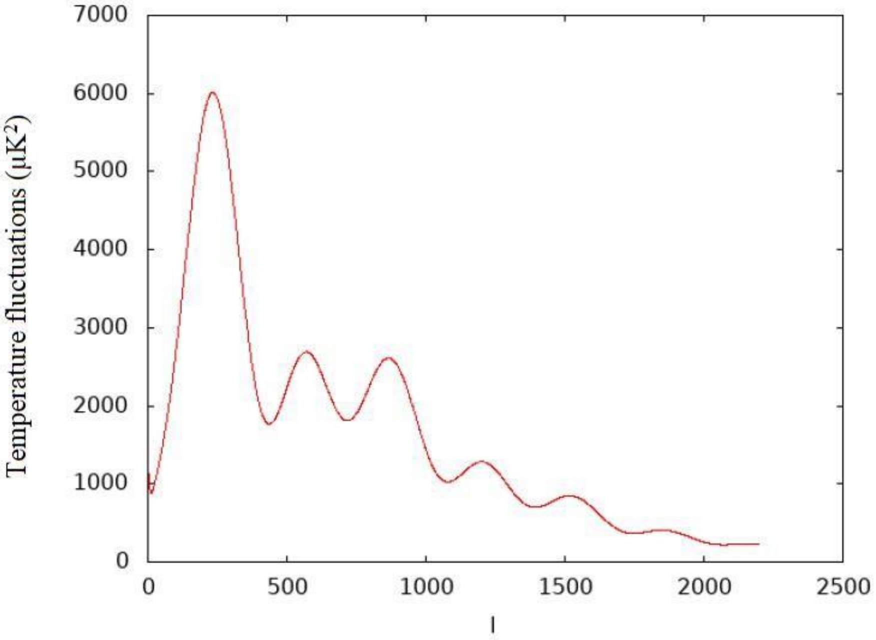


Figure 22: Results for Ω_k = +0.04 model (open)

These graphs illustrate how temperature fluctuations vary at each angular scale denoted by the multipole number *l*. The first model with $\Omega_k = 0$ has the first peak at a multipole of about $l \approx 207$ consistent with a flat model. The second model with $\Omega_k = +0.4$ has the first peak at a multipole of about $l \approx 346$ corresponding to an open model. The third model with $\Omega_k = +4$ peaks at a multipole of approximately $l \approx 1000$ which is also an open model. The fourth model with $\Omega_k = -0.4$ has the first peak at a multipole of about $l \approx 96$ consistent with a closed model. The model with $\Omega_k = -0.04$ has the first peak at a multipole of approximately $l \approx 203$. This is a closed model however the first peak of this spectrum is quite similar to the one displayed by the flat model with $\Omega_k = 0$. The open model with $\Omega_k = +0.04$ shares a similarity with the two models where $\Omega_k = 0$ and $\Omega_k = -0.04$ with its first peak at approximately $l \approx 224$.

Table 2: Cosmological parameters for models of different curvature

	Flat	Closed	Closed	Open	Open	Open
Ω_K	0	-0.04	-0.4	+0.4	+0.04	+4
Ω_Λ	0.724	0.764	1.124	0.324	0.684	-3.276
Ω_m	0.276	0.276	0.276	0.276	0.276	0.276
Age [Gyr]	13.777	13.968	16.298	12.277	13.596	7.646
l of first peak	207	203	96	346	224	1000
Angular scale $\left(^{180}/_{l}\right)$	0.87	0.887	1.875	0.52	0.803	0.18

In the table 2 above I tabulated the results for the values of various cosmological parameters for the different models. There was a notable similarity in the models with $\Omega_k = 0$, $\Omega_k = -0.04$ and $\Omega_k = +0.04$ which have cosmological parameters that are nearly equal. This occurrence is known as degeneracy i.e., the three models are degenerate.

This was followed by comparing these power spectra with the Planck 2018 CMB power spectra.

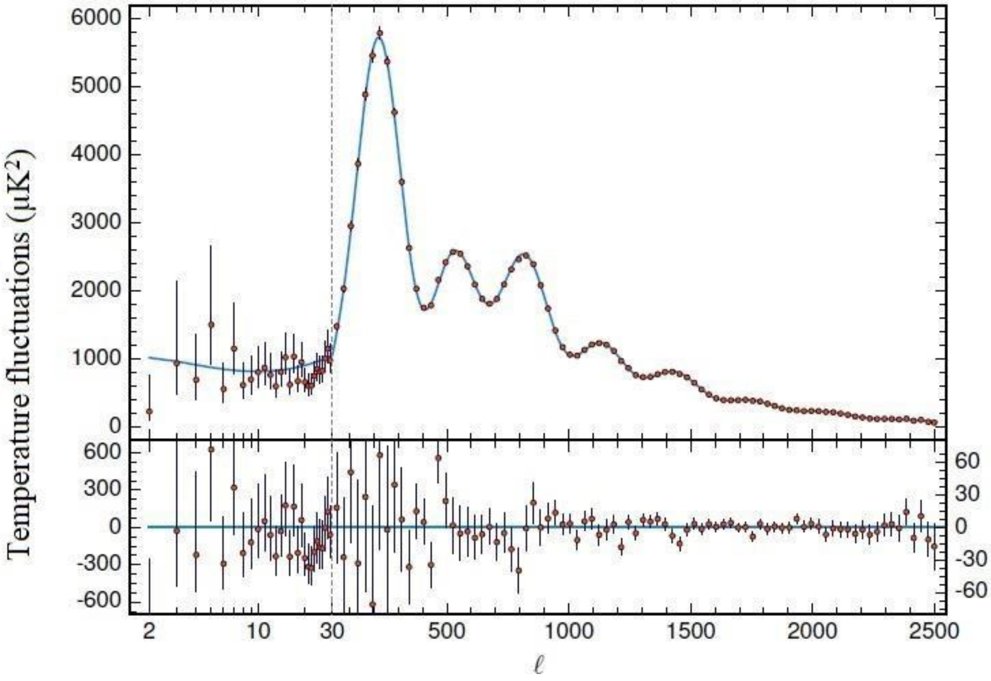


Figure 23: Results from Planck 2018 CMB data

The power spectrum above was the result acquired from Plik TT,TE,EE+lowE+lensing likelihoods for the 2018 CMB data. The blue line outlines the spectrum best fit for the data points in red assuming the base ΛCDM model. There is a lower panel which illustrates the residuals in regard to the model. In this spectrum the first peak was detected at a multipole of about $l \approx 200$ this is in accord with a flat prototype.

Preference for a closed Universe

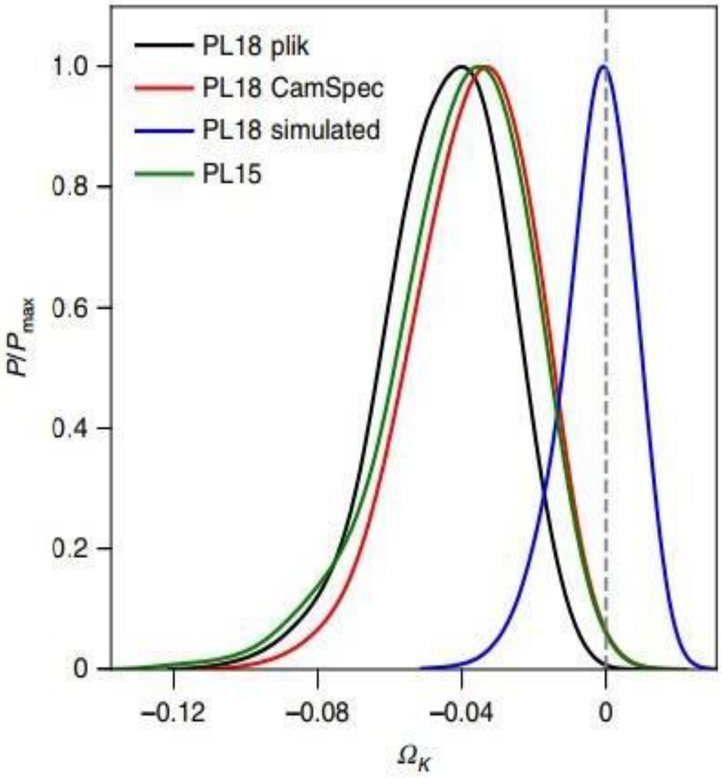


Figure 24: Probability amplitude for curvature density Ω_K

Figure 24 shows $\Omega_K < 0$ the main reason why Planck 2018 data favors a closed Universe as opposed to a flat one. Shown in blue curve is the expected outcome of a flat ΛCDM model. The curve for Planck 2018 actual data, assuming the baseline 'Plik' Planck likelihood is shown in black while the 'CamSpec' probability in red. Subsequent data from the earlier PL15[34] data is illustrated in the green curve for comparison. The posterior probability distribution is the conditional probability distribution that is attributed after accounting for the relevant evidence in an experiment.

When conducting experiments cosmological parameters tend to have similar effects on the power spectrum, this occurrence is known as parameter degeneracy. As a result of this degeneracy, there is often no authoritative answer as to how well a given experiment will measure a given parameter. It will only be subject to the parameters the observer thinks are reasonable to vary.

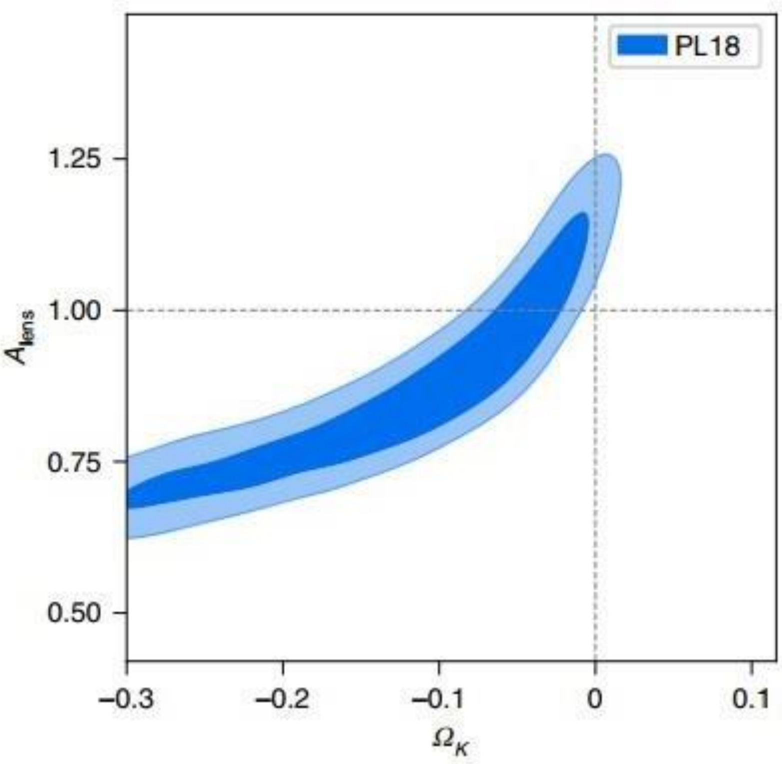


Figure 25: Degeneracy probability between curvature density (Ω_K) and lensing amplitude (A_{lens})

Figure 25 shows the degeneracy between curvature and lensing. Likelihood at 68% CLs is shown in dark blue and likelihood at 95% CLs is shown in light blue.

The high lensing amplitude is the main reason why 2018 Planck data favor a closed universe which is manifest from the parameter degeneracy illustrated in figure 12.

The analysis shows that there is a high lensing amplitude ($A_{lens} > 1$) and the data favor a cosmological model whose Ω_K is less than zero i.e., a closed Universe instead of the conventional flat model. A Melchiorri et al. (2019) intimated that a lensing signal with such amplitude is the precise expectation of a closed universe. This was supported by the argument that a closed universe can contain more dark matter content resulting to a higher lensing signal.

Combining Planck power spectrum with different datasets yields different predictions for the value of the curvature density parameter Ω_K as shown below.

- Ω_K = 0.0002±0.0025, TT+BAO,
- Ω_K = 0.0010±0.0023, TE+BAO,
- Ω_K = 0.0005±0.0020, TTTEEE+BAO,
- Ω_K = 0.0004±0.0019, TTTEEE+ BAO+ lensing,
- Ω_K = 0.0004±0.0018, TTTEEE+ BAO+ Pantheon +lensing.
- Ω_K = – 0.056 (+0.44, –0.050) TT + low E
- Ω_K = – 0.044 (+0.18, –0.015) TTTEEE + low E

Data from different observatories has yielded a fairly wide range of results.

- Ω_K = 0.08±0.31 1598 quasars
- Ω_K = – 0.001±0.01 Atacama Cosmology Telescope (ACT) + WMAP
- Ω_K = – 0.018±0.01 Atacama Cosmology Telescope (ACT) + Planck
- Ω_K = – 0.02±0.14 Pantheon
- Ω_K = – 0.07 (+0.14, –0.26) BAO+BBN+H0LiCOW Collaboration
- Ω_K = 0.28 (+0.17, –0.28) BAO+BBN+ Cosmic Chronometers (CC)

These data arrays are not correlated and they result from distinct mechanisms (supernovae and BAO assess the background cosmology, whereas lensing examines theory at the level of perturbations). Majority of the datasets favor $\Omega_K = 0$ this is a strong underpinning for the flat Universe models. Planck lensing survey data is the one that significantly suggest a closed Universe which is rather interesting. There is a possibility that the inclination of Planck power spectra to favor closed Universes might have been a result of procedural inaccuracies in their data. Different statistical methods such as probability operations lead to distinct results, with the Plik likelihood favoring spherical Universes firmly than CamSpec likelihood.

The following table illustrates how Planck lensing data compare with the CORE satellite space mission in different experimental configurations.

Table 3: Comparing Planck data with data from CORE satellite.

Parameter	**Planck + lensing**	**Lite CORE 80, TEP**	**LiteCORE120, TEP**	**CORE-M5, TEP**
$\Omega_b h2$	0.02226 ± 0.00016	0.022182 ± 0.00006	0.022182 ± 0.00004	0.022183 ± 0.000038
$\Omega_c h2$	0.1192 ± 0.0015	0.12050 ± 0.00074	0.12046 ± 0.00068	0.12049 ± 0.00066
τ	0.055 ± 0.019	0.0597 ± 0.0021	0.0598 ± 0.0021	0.0596 ± 0.0020
n_s	0.9658 ± 0.0048	0.9620 ± 0.0021	0.9619 ± 0.0019	0.9620 ± 0.0019
H_0	66.1 ± 3.1	66.98 ± 0.75	66.96 ± 0.68	66.97 ± 0.66
σ_8	0.806 ± 0.019	0.8174 ± 0.0044	0.8172 ± 0.0040	0.8173 ± 0.0040
Ω_K	−0.0037 ± 0.007	0.0000 ± 0.0021	0.0000 ± 0.0019	0.0000 ± 0.0019

Most of the data from CORE satellite space mission is consistent with data from Planck. It is only the curvature density parameter that is significantly discrepant, CORE satellite data still supports that the Universe is flat ($\Omega_K = 0$). Data from Atacama Cosmology Telescope (ACT) shows a similar trend where its data alone point out a flat Universe, while combining it with Planck data introduces curvature as shown below

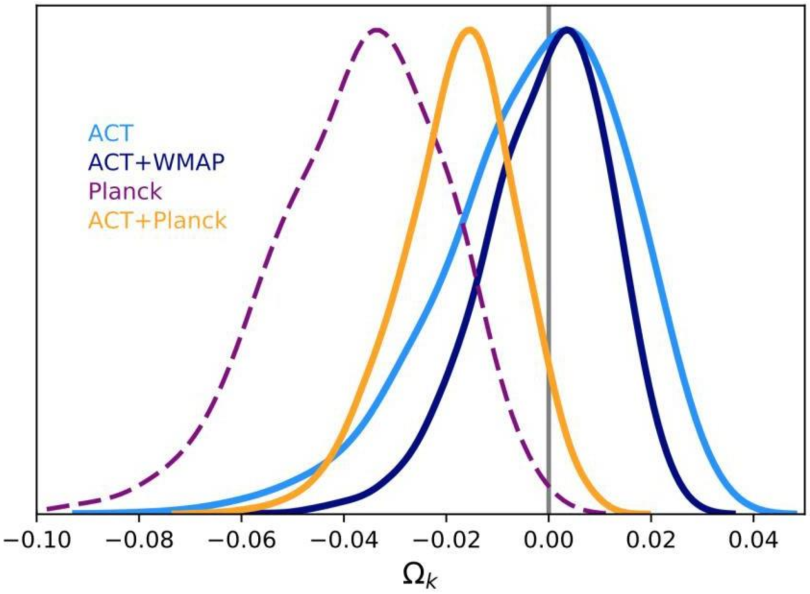


Figure 26: Curvature density Ω_K when combining data from ACT and Planck

5.1.4 Crisis in Structure Formation

Our universe is illuminated with stars, and they are distributed randomly across space, organizing themselves in a very interesting hierarchy of structure. As a result of gravitational interactions stars assemble themselves into galaxies. These galaxies then gather into clusters, which are further assembled into even greater structures called superclusters. In the universe the characteristic distance of separation between stars in a galaxy is significantly longer compared with the dimensions of a star such that, assuming that the Sun is the size of a pea, the closest star would be 160 km apart. Thus, it's not ambiguous to say that galaxies are mostly empty.

Structure formation can be useful in giving insights on the shape of our universe. It attempts to model how structures such as stars, galaxies and larger structures came into existence by gravitational instability of tiny primordial density fluctuation.

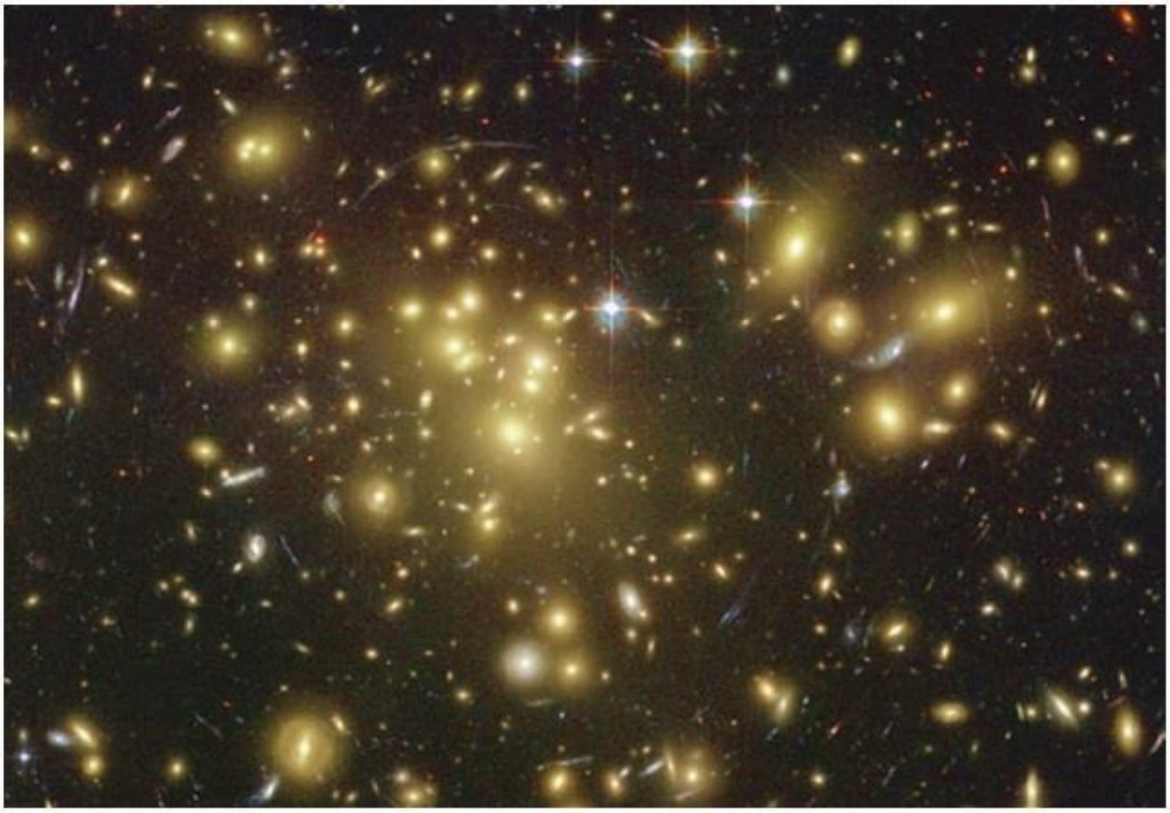


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Figure 27: A cluster of galaxies.

The arrangement of galaxies in the cosmos reveals a foamy appearance, composed of threads and plane-like envelops of galaxies spanning enormous spaces. This was confirmed in automated galaxy surveys, pioneered at the Center for Astrophysics (CfA) in Harvard.

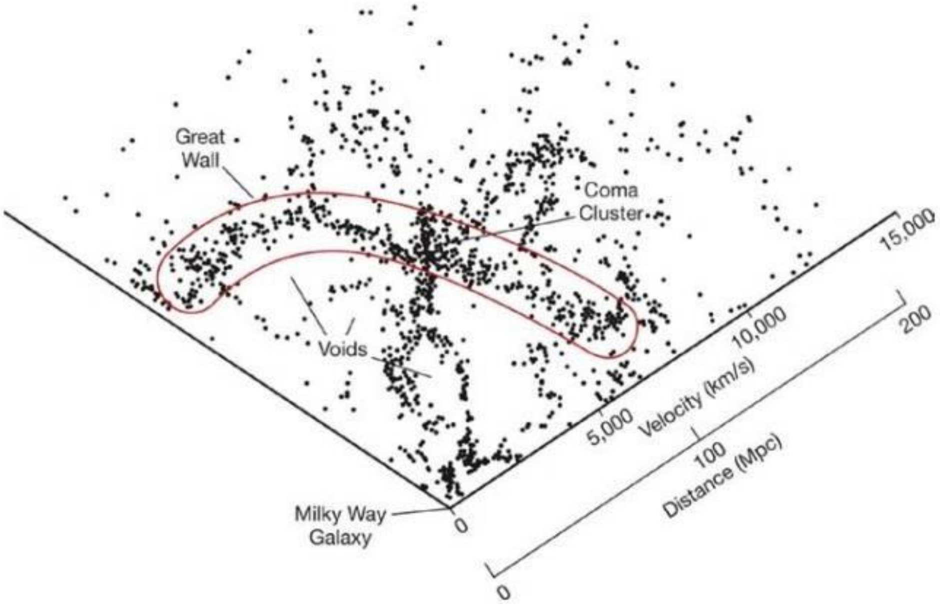

Figure 28: A six degrees portion of the universe (CfA).

In the figure each dot corresponds to a galaxy. The Anglo Australian Observatory (Two Degree Field/ 2dF) galaxy survey and the Sloan Digital Sky Survey (SDSS) did a survey in (2002) to understand what happens on much larger scales. These tremendous surveys clearly illustrate that at large-scales galaxies are distributed in a mesh-like configuration with sheets, clusters, filaments, and voids (Fig. 28). It is observed the universe is homogeneous at larger scales and there exist no “super-superclusters”. Cosmologists have noted that the cosmic background radiation is highly uniform hence affirming the isotropy assumption as postulated in the standard model. It is also believed that the universe should look the same if we resided in another galaxy i.e., homogeneous.

Physics of the early universe is uncovered by the greatest structures in the Universe. Studying these structures yields information about the geometry of the universe and its matter content.

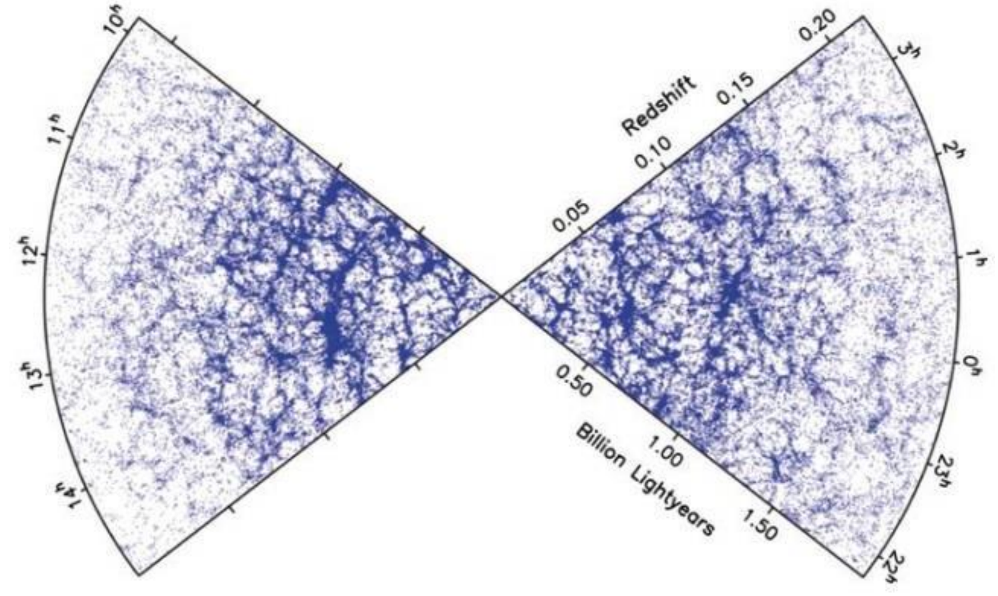

Figure 29: Galaxy Distribution (2dF survey 2002).

The ΛCDM model gives a successful prediction of the observed large-scale configuration of galaxies, clusters together with the voids. Understanding how these structures arise is key in understanding the nature of our universe.

During the production of the CMB (**t_{rec}** = 380,000 years) the universe was very homogeneous according to the big bang model. It is expected to have retained this state to eternity given that it was completely homogeneous with the same density everywhere. Observations from CMB however show there were minute fluctuations from ideal homogeneity (roughly 0.001%). Theoretically tiny perturbations in the primordial universe were the seeds that led to the later formation of structure. These are the perturbations which appear as tiny temperature fluctuations at one part in a hundred thousand. The Cosmic Background Explorer (COBE) initially detected the CMB fluctuations in the in the 1990s. They are essential in cosmology since they offer the source from which the largest structures can arise and finally collapse to form galaxies and stars. Over-dense areas attract more matter, while under-dense regions pull in less, consequently these little anisotropies in the CMB formed the structures present in the universe today. Temperature differences which have been observed appear to have the form predicted by the most robust and elementary models of inflation.

Regions which are denser pulled material gravitationally from the environs. The regions get more and more denser hence increasing the gravitational pull and eventually attracting even more matter. Such a region becomes extremely dense proceeding with its expansion, but eventually reversing and collapsing on itself, therefore creating a gravitationally fixed object. Consequently, the distribution of matter becomes more and more lumpy this is known as gravitational instability.

The Friedman-Lemaitre-Robertson-Walker model developed by the four scientists Friedman, Lemaitre, Robertson and Walker is a model that is currently widely used by cosmologists. In this model there are a series of mathematical formulations (see Chapter 3) identified as the Friedman equations which govern the expansion of space assuming that we have a homogeneous and isotropic universe. In 1922 Alexander Friedman used Einstein's field equations in general relativity to derive these equations. In this model arguments which are consistent with data used lead to the conclusion that we reside in a universe that is flat and infinite. Modelling of inflation theory indicates that after the Big Bang the universe expanded in a flat manner where by two parallel lines would never meet. From the onset of modern cosmology, it has been conventional to view the universe as flat.

The FLRW model is closely associated with the ΛCDM/ LCDM model. The ΛCDM approach offers a reasonable account of the features observed in the cosmos at large scales and its accelerating expansion as well as the existence of the CMB radiation and its structure. These properties of the universe are key in understanding the shape of our universe. The LCDM model is widely accepted by scientists as the most suitable cosmological model together with the less popular Lambda Warm Dark Matter model (LWDM). The LCDM cosmological model is based on the hypothesis that there exist particles disguised as cold dark matter elements. The LWDM on the other hand assumes that warm dark matter particles exist. In terms of structure formation there is minimal difference between the two models.

In the standard model it is hypothesized that all material known until date was generated as a relativistic fluid from the hot Big Bang. Observations made at the present-epoch of the universe suggest that inside the observable universe physics seems to be the same. This has the implication that all points in the accessible horizon must have been in contributory contact during the onset of the big bang. Evidently features in the CMB such as the location of the acoustic crests in lead to the deduction that the Universe exhibits an extremely high tendency of being flat. Since experimental proof implies that the universe originated in an extremely dense and hot state, a discordance from explanations came forth since combining general relativity and the big bang would forecast a substantially curved inhomogeneous universe. Solving problems involving flatness, homogeneity, isotropy and causality led to the inference of the theory of inflation (Guth & Tye 1980; Sato 1981) as a supplementary postulate ('Hypothesis 1'). The universe in this theory experienced immense increase in volume by a factor of about 10^{50} compelled by a scalar field known as the 'inflaton'.

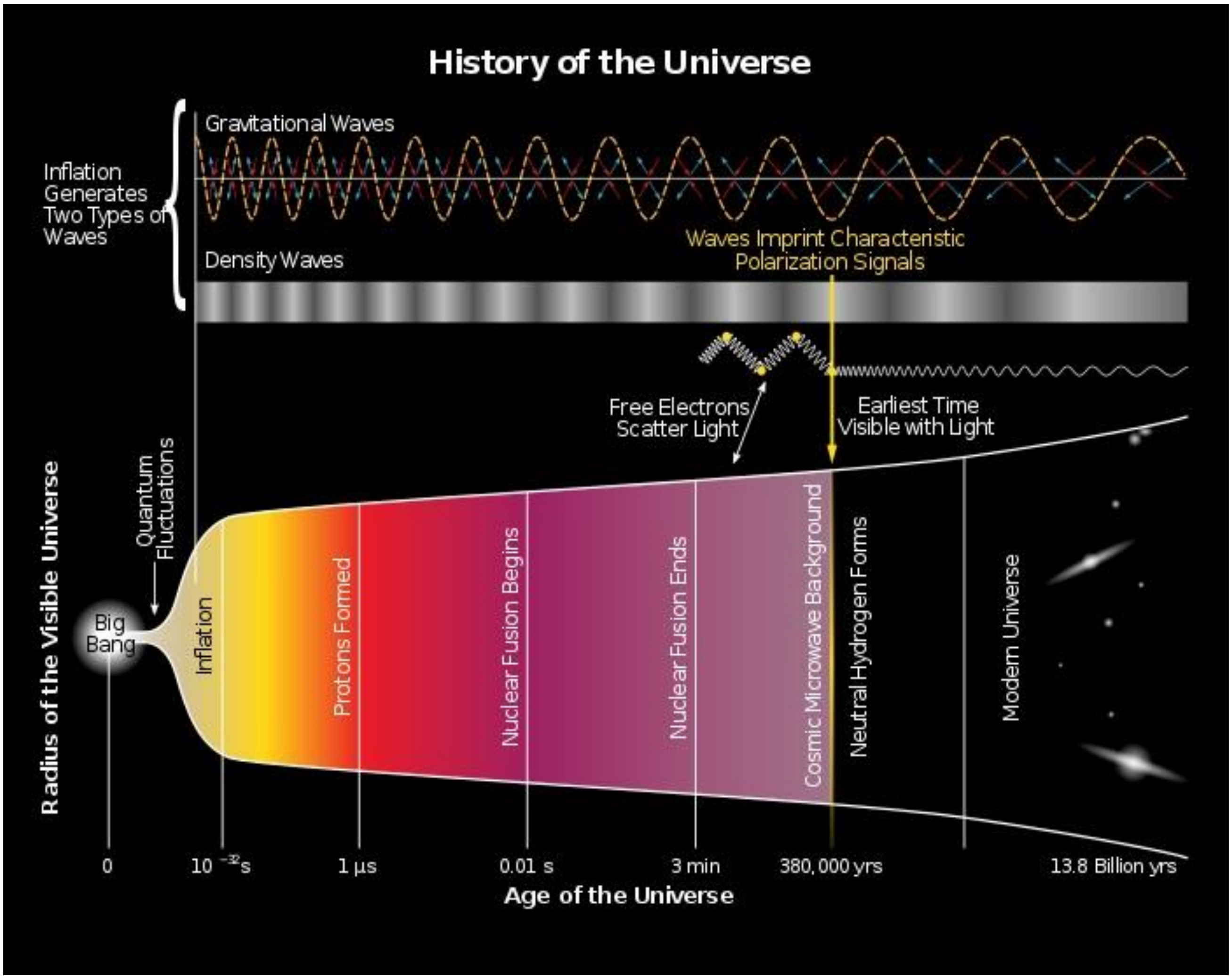

Figure 30: History of the Universe

The inflation epoch lasted from around 10^{-35} to 10^{-31} seconds after the violent explosion. The theory of cosmic inflation gives a reasonable explanation why the universe is flat, in addition to being nearly homogeneous and isotropic. It also gives a fair account on why exotic relic particles such as magnetic monopoles are absent.

It is mind boggling that extremely opposite points in the universe (27 billion lightyears apart) possess the same temperature. This suggests the possibility of being in causal interaction at an previous point in time. The homogeneity and isotropy of the Universe presents a challenge known as the horizon problem. As early as the 1960s problems concerning the horizon and flatness of the universe were well-known. However, no one had any idea of how to handle them hence they were rarely addressed. The horizon in this case better known as cosmological horizon defines the extent of the visible universe. It is a measure of the furthest distance from which information from the cosmos can be retrieved. The slowed down enlargement of the universe is the origin of problems involving flatness and horizon. The density parameter of a decelerating universe is pushed away from one. Today the particular parameter is strikingly evaluated to be so close to unity. For a decelerating expansion the horizon increases more rapidly compared to the distance between points. This has the effect that the horizon will diminish quicker than the distance separating any two points when observing backwards in time. Consequently, regions that are not interacting at the moment have no possibility of being in contact at a primordial point in the temporal dimension. These complications could not be resolved without grasping what took place at the initial instants when the Universe was born. These challenges are puzzled out by the inflation hypothesis where the universe experienced a duration of enhanced expansion during its infancy.

Challenges associated with flatness can easily be explained by visualizing the process of inflating a balloon. Considering a huge inflated balloon that is spherical if one zooms in closely to see only a small part of it, its surface will look flat. This could be the same case that our universe seems to be flat since we are only exposed to a tiny fraction of it after inflation. In 1979 Alan Guth suggested the current description for metric extension. He did this as he was investigating the reason why no magnetic monopoles are observed. While on this mission he learned that if the universe comprised of a scalar field in a positive energy false vacuum state, it follows that in agreement with general relativity an exponential expansion of space would be generated. However, no observational evidence exists for such a physical field. If such a field didn't exist experts will be compelled to provide another account for the observed expansion of space. The kind of spacetime explained by inflation is known as de Sitter space.

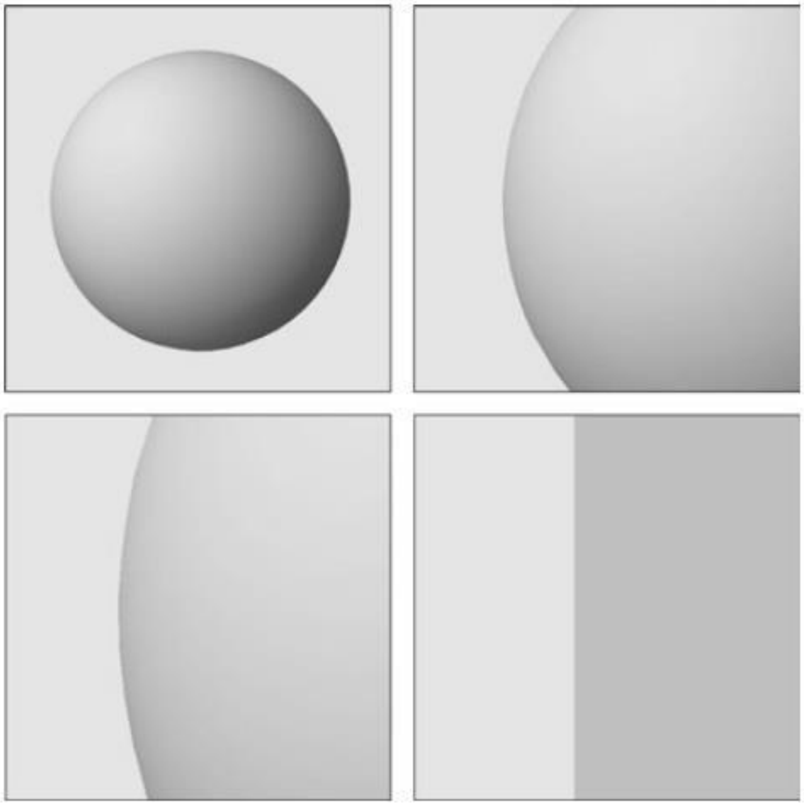

Figure 31: The flatness problem illustrated by inflating a balloon

The standard model of cosmology has been a very useful tool which has aided cosmologists in describing our universe over the years. However, it has been noted with a lot of concern that as experiments continue being done this standard model is in conflict with the observations made by scientists. Cosmologists argue that GR, should also be effective when applied to galactic and cosmological extents. This assumption is known as the null hypothesis ('Hypothesis 0i') which is the current popular understanding of cosmology. This extrapolation is excessive by a huge factor compared to the familiar scales of planetary dynamics, together with the galactic and cosmological levels. The general theory of relativity is a competent description of gravitational physics which has been tested using the weak (i.e., Solar System), strong (Earth) and very strong field limits such as black hole and neutron star. The dynamics were investigated in 1970 by Rubin & Ford. Albert Einstein's EFEs were tailored in a manner such that the Newtonian equations of motion can be derived from them.

It has been observed that rotation curves of galaxies continue to retain their virtually flat geometry at large radii. These regions known as the galactic halo are composed of a form of matter which cannot be detected directly.

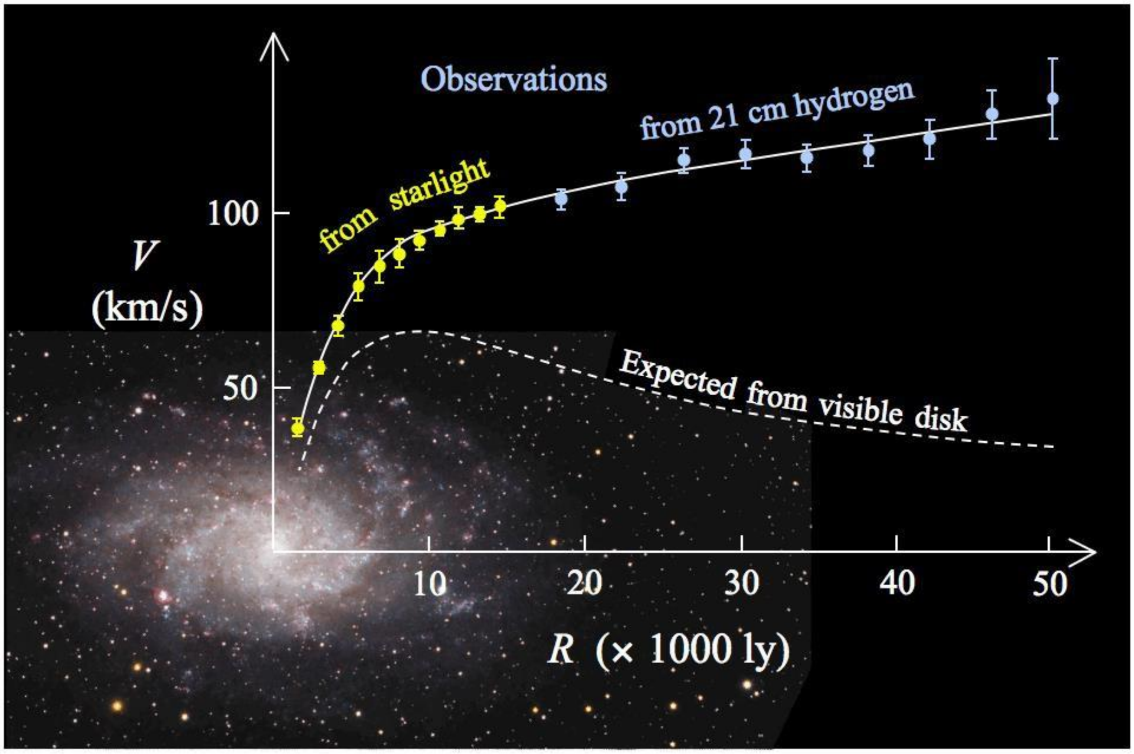

Figure 32: Galaxy Rotation Curves

Newtonian physics fails to fully account for the dynamics of this region. Combining this with the rate of formation of structures after the big bang exposes the setbacks of the current cosmological model. Scientists cracked this by suggesting another supporting postulate ('Hypothesis 2'). According to this hypothesis there exist exotic form of matter namely cold dark matter (CDM) or warm dark matter (WDM) particles which are the main type of gravitating matter. These exotic particles possess characteristic mass which dictates whether they are cold or warm: warm dark matter has approximately $1x10^{-10}$ keV whereas cold dark matter have a mass less than$1x10^{-10}$ keV. There still exist hypothetical elementary particles known as axions.

Theoretically, the hypothesized elements interact subtly through gravitation and probably so as to dissociate from the photon fluid and begin the formation of structures earlier than the baryons. Scientists have been looking for these particles for more than forty years and they are still not yet to be found this exposes another shortcoming of the standard model. This was tested through the physical mechanism of Chandrasekhar dynamical friction also referred to as gravitational drag. This was addressed by Subrahmanyan Chandrasekhar in 1943. It is where bodies involved in interactions with surrounding matter in space lose their momentum and kinetic energy. It is expected that a galaxy which falls towards another galaxy should experience this friction and as a result it should be slowed down by its dark matter halo. This has not been the case and on the contrary galaxies have been observed to pass each other with high velocities. Pavel Kroupa while addressing the dark matter crisis argued that due to this lack of evidence scientists are left with nothing other than belief. He stressed that the words “belief” and “opinion” should be expelled from the vocabulary of natural sciences.

Studying structure formation reveals that Einsteinian/Newtonian gravity considerably weak to form structures such as the observed deep and extended underdensities like the Keenan-Berger-Cowie (KBC) void. Israeli physicist Mordehai Milgrom developed Milgromian dynamics in 1983. He considered galaxy data which was not available when Newton and Einstein were formulating their theories of gravitation. The two renowned scientists relied only on Solar System data. The Milgromian gravitation allows enhanced growth of structure generating voids which are much larger and deeper than in Einsteinian/Newtonian gravity. This favors the formation of KBC-like voids illustrated below.

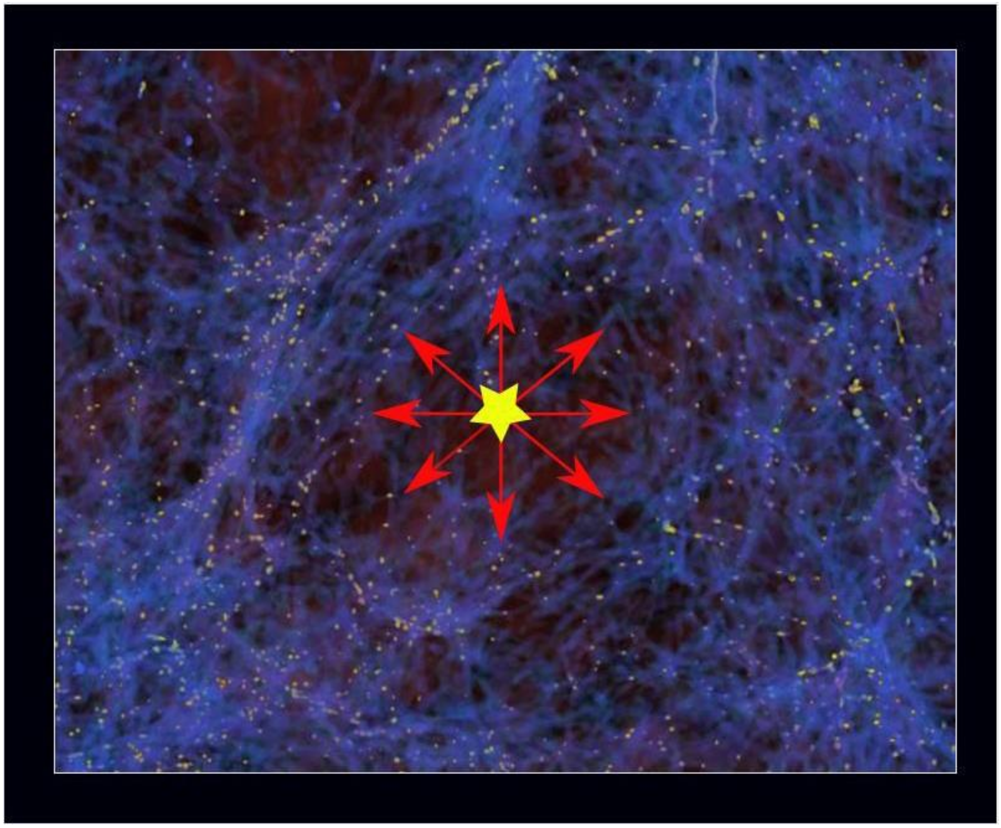

Figure 33: Keenan-Berger-Cowie (KBC) void.

Galaxies are represented by the bright dots while darker regions are voids. Our Sun is represented by the yellow star. Red arrows illustrate gravity from surrounding denser regions pulling galaxies inside the void outwards. Living in such a void would make the Universe appear to expand faster locally than it actually does. The observed rate of expansion exceeds the expectation of ΛCDM by about 9% this is one of the greatest mysteries known as Hubble tension. The arrangement of galaxies on scales from 100 kpc to 300 Mpc firmly implies that structure formation is much more efficient than possible by Newton's gravitational law. This observation implies a long-range enhancement of gravity than allowed by Newtonian gravity. Gravitation remains the least understood of the fundamental interactions. These few scenarios highlight some of the challenges associated with the current model of cosmology. These issues need to be addressed in depth in order to gain a clear understanding of our universe.

5.2 Discussion

The initial discoveries of the acoustic peaks in the CMB temperature power spectrum, hinted that the Universe is nearly flat. The location of the peaks is used to describe the geometry of the Universe. It is evident that the best measurement cosmologists have made reveal that the curvature of the universe is within a probability space that signifies "zero curvature" i.e., flat.

Planck 2018 data show that studies done on gravitational lensing in the CMB might be unfolding into what scientists now refer to as a "cosmological crisis". Analysis of Planck data reveals that there is more gravitational lensing than expected. This hints that our universe could contain more matter than previously thought, this occurrence introduces curvature to the geometry of the universe. The findings are in contradiction with the prevailing models such as the ΛCDM model where the universe has no curvature (i.e., flat).

There is a possibility that the inclination for Planck power spectra to favor closed Universes could be a result of the diverse statistical methods employed to analyze the Planck data. These different methods yield distinct outcomes, with the Plik likelihood favoring closed Universes firmly than CamSpec likelihood.

Newtonian physics fails to fully account for the observed dynamics in galaxies which are attributed to gravitational interactions. Gravitation remains the least understood of the fundamental interactions.

Astronomical data show that there exists dust between galaxies. It gets heated by photons from nearby galaxies and therefore producing radiation. It has been discovered that this intergalactic dust is ancient. Vaclav Vavrycuk (2018) conducted some measurements and found the photon emission detected to be substantially comparable to the measured CMB with a temperature of about 2.77K. These findings would dispute the current model of cosmology even if a small fraction of the CMB was caused by this intergalactic dust. The standard model of cosmology would only be effective if the Universe is transparent.

Over the years there has been subsequent loss of confidence in the standard model of cosmology. It has been noted that observational data continue to falsify standard model predictions as new experiments continue to be conducted.

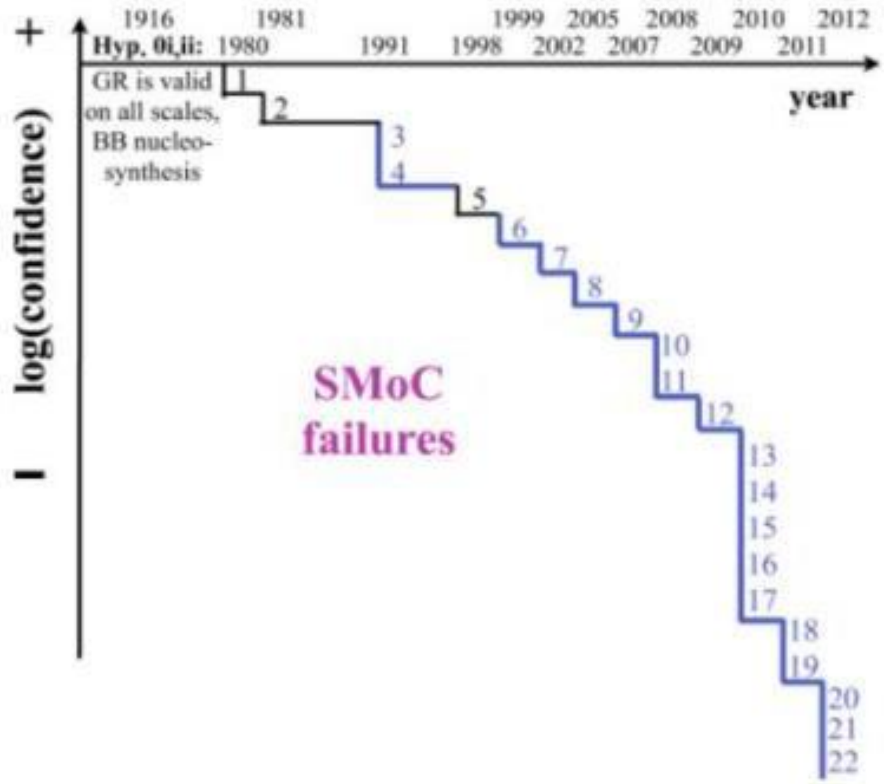

Figure 34: The loss of confidence in the Standard Model

A couple of failures from 1 to 22 were documented in 2012 *"The Dark Matter Crisis: Falsification of the Current Standard Model of Cosmology"* by Pavel Kroupa

CHAPTER 6: CONCLUSION AND RECOMMENDATION

6.1 Conclusions

It is evident that the shape of the universe is a complex topic since it is difficult to determine the exact shape with 100% accuracy. Even though there are limitations such as the inability to determine the entire size of the universe, a substantial amount of evidence points out a nearly flat universe. I found it important to acknowledge that in most of the models developed to describe our Universe there is a lot of prejudice that our Universe is flat (i.e., $\Omega_k = 0$). In this research I highlighted a scenario where some theoretical models have different forms of curvature and yet they exhibit similar characteristics. This poses as a challenge where a model might be misinterpreted as another degenerate model.

Planck 2018 data release shows that it is only the gravitational lensing data that favors a closed universe with a value of $\Omega_K \approx -0.04$. Other datasets such as supernovae and baryon acoustic oscillations favor a flat universe $\Omega_K = 0$. There is a possibility that this discrepancy might be a result of new physics or undetected errors in the Planck data on gravitational lensing. Gravitation remains the least understood of the fundamental interactions.

Scientists hypothesize that there exists exotic form of matter namely dark matter particles. This type of matter contributes significantly to most of the gravitational interactions that have been observed in the Universe. Dark matter remains a controversial subject that has been giving scientists sleepless nights since its nature is not well understood. Demystifying this form of matter could be the means to comprehending the curvature of the universe.

The theoretical approach of postulating existence of new form of matter in order to solve for unaccounted observations creates more problems, since the inclusion of more matter introduces curvature to the universe. Considering the subsequent loss of confidence in the standard model as time progresses our current understanding of the Universe may be entirely rewritten at a very fundamental level.

The amount of evidence presented suggests that as of now there is no need for alarm of a “cosmological crisis”.

6.2 Recommendations

There is urgent need to thoroughly review and verify the validity of these new contradicting findings from the Planck 2018 data release. This research has highlighted some of the challenges associated with the current model of cosmology. These issues need to be addressed in depth in order to gain a clear understanding of our universe.

Signal from the CMB has been found to be influenced by various foreground emissions which are both intergalactic and extragalactic. These contaminations cause temperature deviations which might be misinterpreted as those from the hot big bang photosphere. It could nullify the Planck 2018 data if it was the radiation from the dust that was detected instead of the photosphere of the Hot Big Bang. This would be a major blow hence there is a need for the role of intergalactic dust to be considered very cautiously.

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