An Evaluation of the Static, Dynamic, and Static-Dynamic Geodetic Densification Models on a Part of the Kenyan Geodetic Network

Geodesy

By

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DECLARATIONS

This thesis is my original work and has not been presented for a degree in any other university.

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ABSTRACT

A fundamental consideration in densification of geodetic networks is how to handle the position values of the already established datum stations. The question is: shall they be considered as stochastic or as fixed, non-stochastic entities?.

Different densification models have been put forward as solutions to the question above. These are distinguished by the manner in which higher order net points are handled within the densification process.

Presented herein is a study aimed at evaluating three densification approaches, namely; static, dynamic, and static-dynamic densification models with a view to identifying their strengths and weaknesses as models for densification of geodetic networks. In the static densification model, existing stations are held fixed and assumed errorless, while in the dynamic densification model, the existing datum parameters are treated as stochastic. The static-dynamic model treats datum parameters as stochastic prior information, while at the same time keeping them numerically and stochastically unchanged.

To evaluate these models, each was used to adjust a network at two levels of densification. The adjustment process involved estimation of parameters for secondary and tertiary densification networks built on a datum defined by adjusting the primary network within the framework of a free network. For each model and at every level of densification, the resulting parameters, standard errors of points and their corresponding standard error ellipses were compared against each other. Through analysis of these results the strength and weaknesses of each densification model have been appraised.

A real network forming a part of the geodetic network of Kenya was adopted as the test network. The network consists of eight primary control stations, fifteen secondary stations, and twenty-two tertiary stations. Using original field data the test network is densified in two levels using the three densification models above.

The results indicate that standard errors and point error ellipses from the static model are the smallest, followed by those from the static-dynamic model, and finally those from the dynamic model. The standard errors for the static model are expected to be small algebraically because they are based on a fixed and errorless datum; with the datum being stochastic these results are not representative enough.

The dynamic and static-dynamic densification models incorporate stochasticity of datum parameters, in the static-dynamic model datum parameters are maintained definitive, while in the dynamic model all parameters are estimated afresh, thus resulting in the loss of the concept of datum. It is on the basis of the stronger theoretical and practical qualities of the static-dynamic model that the model would ordinarily be recommended for geodetic densification of networks.

The results in general demonstrate that the static-dynamic model gives more realistic estimates than the static and dynamic models hence it is a more suitable approach to the densification of geodetic networks.

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NOTATION

Listed below are the symbols used in the text. The page at which the symbol first appears is also given in parenthensis.

$\boldsymbol{y}$ - vector of observations (18)

$\boldsymbol{x}$ - vector of unknown parameters (to be estimated)(18)

$\boldsymbol{A}$ - design matrix

$\boldsymbol{W}$ - positive definite weight matrix(18)

σ_0^2 - variance of unit weight(18)

$\boldsymbol{\varepsilon}$ - observational error (18)

$E(\boldsymbol{x})$ - expectation of X(19)

$E(\boldsymbol{\varepsilon})$ - expectation of observational error(19)

$E(\boldsymbol{y})$ - expectation of y(19)

$D(\boldsymbol{x})$ - dispersion of x(19)

$D(\boldsymbol{y})$ - dispersion of y(19)

$D(\boldsymbol{\varepsilon})$ - dispersion of observational error(19)

$\boldsymbol{r}$ - vector of constants(20)

$\boldsymbol{R}$ - restriction design matrix(20)

$\boldsymbol{L}$ - Lagrange function(21)

$\hat{\sigma}_0^2$ - a posteriori variance of unit weight(23)

n - number of observations(18)

c - the number of restrictions(18)

m - the number of unknowns(20)

$\boldsymbol{Q_{xx}}$ - cofactor matrix(24)

$tr[Q_{xx}]$ - trace of cofactor matrix(24)

N - normal equation matrix(25)

G - as defined in text (25)

x_1 , x_2 - vectors as defined in text(27)

A_1 , A_2 - design submatrices (28)

λ - vector of Lagrange multipliers(21)

$D(\hat{X})$ - dispersion of $\hat{X}$ (29)

Δx_1 , Δx_2 - unknown parameters to be estimated (30)

Σ_{yy} - variance covariance matrix of the observations(33)

$\overline{\sigma_c}$ - circular probable error(53)

σ_E , σ_N - standard errors in E and N respectively(54)

σ_E^2 - variance of the easting(55)

σ_N^2 - variance of the northing(55)

σ_{EN} - covariance between easting and northing(55)

a - semi-major axis(55)

b - semi-minor axis(55)

α - bearing of the semi-major axis(55)

H_o - null hypothesis(80)

H_a - alternative hypothesis (80)

χ^2 - chi-square test at m degrees of freedom(80)

χ^2 - chi-square test at m degrees of freedom(80)

$F_{m_1,m_2} = \overline{\sigma^2_1}/\overline{\sigma^2_2}$ - F-test statistic at α, m_1 , m_2 degrees of freedom for samples 1 and 2 (83)

CHAPTER ONE

INTRODUCTION

After a geodetic network has been set up, the requirement to extend it to the nearest proximity of a particular area of work almost immediately arises. The extension of the geodetic network is known as densification. The densification problem arises where, with new additional observations, certain new stations have to be introduced into the already existing network. The special question is then how to handle the position values of the already established datum stations; shall they be considered as stochastic or as fixed, non-stochastic, entities?.

There are basically four densification approaches which have been proposed. The main distinction in the four models is dependent on how the coordinates of the higher control stations are handled during densification.

In one approach, the existing stations are held fixed and are considered errorless; this solution has been referred to by, among others, *Pelzer [1980] as hierarchical densification*, *Cooper [1987] and Aduol [1993]* as *static densification* while *Vanicek and Lugoe [1986]* refer to it as *overconstrained adjustment of densification*.

In another approach, referred to as the *semi-dynamic* solution discussed by *Blaha [1974]*, coordinates of existing stations are held fixed, as in the static case, but their covariance information is propagated into the new stations;

this approach has also been termed as *quasi-hierarchic* or *pseudo-dynamic* (*Van Mierlo [1984]* and *Wolf [1983]*).

In the third approach, instead of the datum points being considered fixed and errorless, they are considered as stochastic, such that during densification, estimation for parameters is within a model combining rigorously both sample and prior information. In which case both the datum and new stations are adjusted. This means that the existing stations will obtain corrections for their coordinates, i.e., they "move". This is the *dynamic densification model* approach, as has been referred to by *Cooper and Leahy [1978], Pelzer [1980], El-Hakim [1982], Wolf [1983], Papo and Perelmuter [1985], Schaffrin [1985]*.

In the fourth approach, proposed by *Aduol [1993]*, the properties of both the *static* and *dynamic* models are combined. In this model densification parameters are estimated by incorporating datum parameters as stochastic prior information, while at the same time keeping them numerically and stochastically unchanged. *Aduol [ibid]* has termed the model as the *static-dynamic densification* model. A model of similar characteristics is also referred to as *estimation with incomplete prior information* by *Theil [1963]*.

In the present study, three models for densification of geodetic networks, i.e. *static, dynamic,* and *static-dynamic* models are considered. The models are considered in so far as their practical applicability is concerned, and specifically an evaluation of their suitability as solutions to the densification problem in geodetic networks.

The *pseudo-dynamic* densification model discussed above is not considered since the proposed model had the serious drawback that on one hand datum parameters were treated as non-stochastic, hence fixed, while on the other hand they were treated as stochastic resulting in an inconsistent estimation model [*Aduol 1993*]. Further, *Wolf [1983]* had earlier demonstrated mathematically that the "compromise-solution", which exhibits similar characteristics with the pseudo-dynamic densification model, leads to a bias in the residual system. In which case the adjusted connection (angles, distances) to the given stations are falsified.

1.1 The Statement of the problem

In the traditional approach to network densification, subsequent networks are built upon earlier ones on the basis that datum points, i.e. higher order net points, are known exactly. For example, traditionally, geodetic networks are set up on the basis of triangulation-trilateration where primary triangulation points are considered 'fixed' in form of exact restrictions, on which basis secondary triangulation points are adjusted. Consequently tertiary networks are set up on the basis that secondary points are fixed and errorless.

The traditional densification approach has however some disadvantages, the main one being the assumption that higher order net points are errorless yet in real sense they are stochastic, having been obtained from a prior estimation. This will always lead to falsified estimates, which is so because the treatment of stochastic information as exact will ordinarily result in unrealistically high precision for estimated parameters *[Aduol 1996]*.

Considering densification by addition of points or observations through secondary measurements, in which, for example, secondary measurements are more precise than those which constituted the primary net, as is most likely to be with more modern instrumentation, holding the primary stations fixed leads to unwarranted distortion of the newer work *[Cooper and Leahy 1978]*.

It can be noted that the *static* model approach is not a rigorous solution to the densification problem from the fact that the assumption that datum coordinates are fixed is not exactly true, since first order points are in fact stochastic, themselves having been obtained from the first network adjustment, and according to *Blaha [1974]* this results in too optimistic results.

Considering the fact that fixed points have stochastic prior information, which is neglected when using the *static* model, the *semi-dynamic* model incorporates this stochastic prior information. In this model coordinates of existing points are held fixed, as in the static case, but their covariance information is propagated into the new stations [*Nickerson et.al 1986*]. However, the use of the *semi-dynamic* model as an improvement on the *static* model is hampered (as noted above) by the fact that on one hand datum coordinates are treated as fixed while on the other they are treated as stochastic, which results in an inconsistent estimation model.

The realization that the *static* and *semi-dynamic* models were not rigorous enough led to studies in which densification was approached by considering datum coordinates as stochastic during the estimation process, i.e. the use of the *dynamic* model as recommended by among others *Cooper and Leahy [1978], Pelzer [1980], El-Hakim [1982], Papo and Perelmuter [1985], and Nickerson et. al [1986]*. Incorporating stochastic restrictions in the estimation model usually results in a more realistic estimation of parameters. Thus the *dynamic model* is statistically more rigorous than the *static* model. However, from a practical point of view, this model has the disadvantage that datum coordinates change values during the adjustment. It is pointed out in *Aduol [1993]* that

"coordinating a single point by intersection with datum points forming a part of the national geodetic reference system, the single new point would (theoretically) cause all points in the national network to acquire new coordinates and new stochastic parameters. With this it is noted that the concept of a datum, which is so vital for a national reference system, is effectively lost".

It is against this background that *Aduol [ibid]* has proposed the *static-dynamic* model in which the properties of both the *static* and *dynamic* models are combined as a way of network densification using the two models but avoiding their weaknesses. In the *static-dynamic* model, coordinates of the fundamental net stations are considered as stochastic so that their covariances are fully taken into account while at the same time they are considered as non-stochastic, i.e. they retain their definitiveness.

The advent of modern technology which has provided the geodesist with sophisticated computational capabilities at low cost, as well as the development of the various densification models mentioned above, are two factors that have led to the realization of the necessity to reconsider adjustment of densification of national and regional geodetic networks, with special consideration to the accuracies of the datum points.

From the foregoing discussion, densification of networks is a very important aspect of geodetic work and various models are available for its realization. However each of the available densification models has its empirical and theoretical weaknesses, so that there is need to study the densification models with a view to evaluating their practical applicability and their overall suitability for geodetic densification work.

1.2 objective of the study

The main objective of the study herein is to demonstrate the practical applicability, and to evaluate the suitability of, the *static, dynamic and static-dynamic* densification approaches, in relation to the densification of a part of the Kenyan geodetic network, as a representation of geodetic networks in general. Through this, it is hoped to gain an insight into the effectiveness of particular densification models in addressing the fundamental problem of densification i.e., how to treat the already fixed points. Further, to establish which of the approaches is best suited for recommendation to be adopted for geodetic densification work, and under what circumstances.

1.3 Literature Review

Studies on densification of geodetic networks have paralleled geodetic network set-up over time. As *Aduol [1993]* observes, *"densification of geodetic networks remains one of the basic operations a surveyor must undertake"*. In the conventional approach to network densification, the datum coordinates are considered fixed in the form of exact restrictions (i.e., use of the *static* model). The wide application of the *static* model in densification work and its inherent limitations led to studies of more rigorous alternatives *[Wolf 1983]*.

The necessity to consider datum coordinates as stochastic was already recognized as early as 1882, in which year *W.Jordan* advocated for the fact that datum coordinates in densification networks be treated as "correlated observations" *[Wolf 1983]*. At the start of the twentieth century, researchers had realized the need for incorporating the stochasticity of datum parameters in subsequent network densification *[Aduol 1993]*, with which realization new coordinates were estimated on the basis of fixed datum coordinates, while the "errors" on the estimated coordinates were computed through "error propagation" incorporating the error on the datum coordinates. Recent works on this approach term it as *quasi-hierarchic* or *pseudo-dynamic* solution *[Van Mierlo 1984, Nickerson et al. 1986]*.

In econometrics, by mid-twentieth century statisticians had realised the need for introducing *a priori* information in econometric estimation processes. *Durbin [1953]* demonstrated that there was a diminution in computed variances during estimation processes in which extraneous information was incorporated as opposed to the process in which it was not. He further developed a mathematical model to accommodate regression when there was partial extraneous information.

Theil and Goldberger [1961] highlighted the uncertainties that arose when, during statistical estimation of economic relations, a hypothesis is formulated, and appropriate computation to provide desirable estimates of parameters of the linear relation carried out, only to find that the estimated income elasticity of some commodity was negative. In their search for a statistical estimation model to accommodate a priori information, *Theil and Goldberger [ibid]* are quoted as saying *"an investigator does not accept this negative estimate but rather attributes the result to the incorrectness of his previous hypothesis and perhaps decides to change his set of explanatory variables. It is well known, but also well ignored, that exact probability statements can no longer be made if the hypothesis is thus rejected in the light of the evidence.*

The difficulty seems to be that the investigator has a priori knowledge which he can not conveniently incorporate in the hypothesis and which he therefore omits. This kind of a priori knowledge, however, is precisely the major source of rejections of hypotheses; it seems clear that it is logically more consistent to incorporate such knowledge in the hypothesis right at the beginning than to exclude it from the hypothesis and reject it afterwards when the results contradict the omitted knowledge."

They went ahead and proposed a model of "mixed" estimation which was an effort to incorporate prior knowledge of coefficients in regression analysis and other linear statistical models. This prior knowledge was formulated in terms of prior estimates of parameters which were assumed to be unbiased and to have a moment matrix. It has to be noted that this can be considered as part of the fundamental mathematical formulation of the *dynamic* model.

Theil [1963] analyzed the use of incomplete prior information in regression analysis. He considered the combination of prior and sample information with the fact that both were stochastic but independent of each other. He tested the compatibility of the two and proposed a measure for the relative contribution of sample and prior information to the results of estimation.

The 1970's saw the emergence of intensified work in search for rigorous densification solutions in light of the realisation that there was need to provide for stochasticty of datum parameters in the adjustment process. A method for combining stochastic prior information in a vector of regression coefficients with incomplete prior information on the variances of the disturbance terms was developed by *Toutenburg [1974]*. This enlarged the general linear regression model to give a restricted regression model. This work can be considered as the setting of the mathematical basis for the *static-dynamic* model.

Blaha [1974] considered the existence of uncertainties during densification work, when fixed parameters were neglected in variance-covariance propagation. The main aim of his study was to correct the variance-covariance matrices for the contribution of such uncertainties by considering the general least squares method with weighted, unknown, or some weighted and some unknown parameters, hence providing a more generalized approach to hierarchical densification. This was an expansion on the work of *Papo [1973]*, where he proposed a method by which without altering the values of the adjusted parameters their a posteriori (after adjustment) covariance matrix could be improved by inclusion of the effect of uncertainties in the constants of the adjustment process.

The Blaha algorithm, as outlined in *Blaha [1974]*, is a method that permits the propagation of random errors from a previously determined network into the accuracy estimates and solution vector for merged network points, without affecting the original network's accuracy or solution vector.

In their studies on densification, *Cooper and Leahy [1978]*, outlined the possible dangers in the conventional densification approach in which the primary net points are assumed to be fixed absolutely and the inclusion of secondary networks being by adjusting secondary measurements only. In their study, they undertook adjustment of a second order network using two approaches. In one, positions of the primary points were assumed fixed, and in the other, these positions were regarded as correlated and thus not held fixed. The results of their study indicated that the densification in which primary positions were regarded not fixed yielded a better adjustment. They are quoted as saying *"it is a simple matter in adjustment to allow for random errors in the previously fixed and correlated coordinates if they are known"*.

It has to be noted however that this approach has the weakness of introducing new coordinate values and stochastic parameters for the primary network which may constitute the national geodetic reference net, and as such, the vital concept of datum is effectively lost *[Aduol 1993]*.

Several methods for computing coordinates of points in a densification network by considering the already fixed net points as random variables, were discussed by *Koch [1983a]*. In addition, he addressed the special case of transformation of the covariance matrix for the coordinates of the fixed points of the network if the datum was changed during the densification process.

Koch [1983b] discussed the definition of datum for geodetic networks in the case of densification. He stated that the datum should be established such that within the class of certain definitions the trace of the covariance matrix of the estimated coordinates of additional points and of the points of the network connected with the additional points becomes minimal. *Koch [ibid]* demonstrated that such datum could readily be obtained by means of a matrix that contained a basis of the null space of the coefficient matrix for the model of parameter estimation.

The special question of how to handle the position values of the already fixed stations is: shall they be considered as stochastic quantities or as fixed non-stochastic entities?. This was addressed by several researchers among them *Van Mierlo [1984]*, who suggested the adoption of a "compromise solution". In this solution, the coordinates of the fundamental net stations are considered as stochastic so that their covariances are fully taken into account while at the same time they are considered as non-stochastic in which case they are not corrected by the resulting residuals and the discrepancies are arbitrarily put to zero. However, *Wolf [1983]*, had demonstrated mathematically that the "compromise solution" led to a bias in the residual system. In which case the adjusted connection, i.e. angles and distances to the given stations are falsified.

In hierarchical densification, the assumption that control points are fixed leads to covariance matrices depicting too optimistic results. It also means that the trace of the covariance matrix is in effect too small. *Van Meirlo [1984]* considered this problem in detail and developed a mathematical system for determination of the inner precision of densification networks in agreement with the inner precision of free-networks.

It is evident that there exists considerable differences in quality between the old and new measurements in densification of networks. *Schaffrin [1985]* suggested that it was not appropriate to deal with both the new and old measurements in the same manner by simply adding the previous coordinates as "pseudo observations" e.g. as used in the *static* and *dynamic* models, instead he constructed a more robust method *" the best homogeneously linear (weakly) unbiased predictor".* This method proved to be robust enough against eventual errors in the prior information without destroying the "homogeneity of the neighbourhood".

Vanicek and Lugoe [1986] considered rigorous densification of horizontal networks in which a statistically rigorous densification model is compared to non-rigorous models. In the statistically rigorous adjustment of new points, i.e. the densification network, and junction points (datum points to which new points are directly linked through observations) are rigorously adjusted in phase adjustment modes where information from the existing network is propagated into the new phase of adjustment by: (i) using existing positions of the junction points for initial estimates; and (ii) using the inverse of the covariance matrix of these existing positions for the *a priori* weight matrix of the junction points.

The non-rigorous approach involved "minimum constraint" adjustment of densification network holding the coordinates of one junction point fixed, and "overconstrained" adjustment in which all junction points were held fixed. These approaches were applied to a simulated network and the results of the study indicated reasonably small shifts (less than 10ppm) in positions of densification points between the rigorous and non-rigorous solutions. However, statistical analysis showed that the confidence regions for the non-rigorous solutions were not realistic, and hence the recommendation of the use of the statistically rigorous densification approach. Characteristics of this model equate it to those of the *dynamic* model.

Nickerson et al. [1986] studied the effects of additions to, and densification of, the Maritime second-order geodetic control network by (i) holding existing stations fixed and errorless (static case); (ii) treating existing stations as fixed and errorless, but propagating their covariance information into new stations (semi-dynamic case); and (iii) performing a weighted parameter adjustment with existing stations weighted by their previously determined covariance matrix (dynamic case).

The results of the study indicated that only the dynamic scheme provided realistic confidence ellipses for geodetic network densification. Unfortunately, this also resulted in a change in the coordinates of the existing stations. They thus recommended that one has to simply record and not apply the corrections to the coordinates of the existing stations, and use the covariance information provided by the dynamic model. It can be noted that this assumption leads to a distortion in networks which is not reflected in the confidence ellipses.

Cooper [1987] indicates the importance of considering the stochasticity of coordinates of the higher order points during densification as opposed to 'fixing' them. The result of using the *dynamic* model is that coordinates of datum points change as a result of the estimation of the coordinates of the new points. This introduces the anomaly of having two sets of coordinates for a national control point. The remedy for this is through the re-estimation of coordinates and covariance matrix for all points in the geodetic network, not just the points from which densification has been done *[Cooper and Leahy, 1978]*.

Different models for the sequential optimization of geodetic networks were proposed by *Illner [1988]*, in this study *Illner [ibid]* considered optimality during sequential set up of geodetic networks i.e. hierarchical densification of networks. The proposed models for the solution of the sequential optimization included a dynamic solution, a static solution, and a hybrid solution.

Mathematical models for both densification and integration of geodetic networks were looked at by *Lugoe [1990]*. In this study, models for densification of geometrically strong and geometrically weaker networks are discussed. *Lugoe [ibid]* then considered simultaneous densification and integration which he observed as being a statistically viable approach to solving problems pertaining to densification and integration together.

More recent work on densification has been done by *Aduol [1993]* who observed that, the use of the *dynamic* model was hampered by the fact that datum coordinates attain new values during adjustment. To circumvent this problem, *Schaffrin [1984]* had earlier proposed to adopt the *dynamic* solution, but to ignore changes on the datum coordinates unless such changes are "significantly" large[cf. *Nickerson et al. 1986]*. It is immediately noted in this case however that as long as changes on datum coordinates are merely neglected on the basis of whether they large or not, the finally adopted coordinates can not be consistent with the mathematical model adopted for the estimation of the unknown parameters [*Aduol 1993*].

It is on this basis therefore, that *Aduol [ibid]* proposed the *static-dynamic* model for network densification. The need for this model becomes necessary in the light of the theoretical and empirical weaknesses of the *static* and *dynamic* models which may be overcome by a model that combines properties of the two.

To demonstrate this approach, *Aduol [ibid]* considered a simulated two-dimensional geodetic network comprising 19 stations with the observables as horizontal angles and distances. Application of *static, dynamic, and static-dynamic* models yielded results which, from analysis of the respective variance-covariance matrices demonstrated strong theoretical and practical qualities of the *static-dynamic* model against the fully *static* and fully *dynamic* models for network densification. It is however, important to note that the above study was done on a simulated network, hence the need for similar studies on real geodetic networks to ascertain the above result.

Nakiboglu et al. [1994] analyzed distortions in the national geodetic network of Saudi Arabia, in which assumed positions of the three densification stages of the network viz., primary, secondary, and tertiary geodetic networks were compared against those determined from a modern GPS survey for the same region.

1.4 Organization of the Report

The report is organised into eight chapters. In Chapter Two, linear estimation models relevant to the study are presented while Chapter Three discusses the main geodetic densification models used in the study.

The test network on which the densification is applied is presented in Chapter Four. Chapter Five outlines the computational procedures used while in Chapter Six, are presented the results of these procedures as applied to the test network. The results are then discussed in Chapter seven and the relevant conclusions drawn in Chapter Eight.

CHAPTER TWO

ESTIMATION MODELS

Presented in this chapter are linear estimation models considered relevant to the estimation of parameters in the present study. The simple Gauss-Markov model considered in section (2.1) is the basic model, and its variants under exact and stochastic restrictions are then considered in sections (2.2) and (2.3) respectively. Finally section (2.4) discusses the free network adjustment model for datum specification.

2.1 Simple Gauss-Markov Model

The simple Gauss Markov model makes use of the principle of least squares which requires that the sum of squares of the residuals be minimum. The model is fully described through the functional and stochastic models

$$\mathbf{A}\mathbf{x} = E(\mathbf{y}), \quad D(\mathbf{y}) = \sigma_0^2 \mathbf{W}^{-1} \qquad (2\text{-}1)$$

where: $\mathbf{y}$ is an n x 1 vector of observations

$\mathbf{x}$ is an m x 1 vector of unknown parameters (to be estimated)

$\mathbf{A}$ is an n x m design matrix

$\mathbf{W}$ is an n x n positive definite weight matrix of $\mathbf{y}$, and

σ_0^2 is the variance of unit weight.

As $\mathbf{y}$ is stochastic, it is associated with an observational error $\boldsymbol{\varepsilon}$ so that we may write from (2-1) that

$$y = Ax + \varepsilon, \quad E(\varepsilon) = 0, \; D(\varepsilon) = D(y) = \sigma_0^2 W^{-1} \tag{2-2}$$

With the least squares requirement that the sum of residuals be minimum, and taking into consideration the weights of the observations, and provided A has full column rank, then the estimates of the unknown parameters x are given by

$$\hat{x} = (A^T W A)^{-1} A^T W y \tag{2-3}$$

with

$$D(\hat{x}) = \hat{\sigma}_0^2 (A^T W A)^{-1} \tag{2-4}$$

and

$$\hat{\sigma}_0^2 = (\hat{\varepsilon}^T W \hat{\varepsilon}) / (n-m) \quad . \tag{2-5}$$

Further,

$$E(\hat{x}) = x \; , \tag{2-6}$$

indicating that $\hat{x}$ is an unbiased estimate of x .

In the event that the design matrix A is not of full column rank, we would have the case of Gauss-Markov model with rank defect. In such case $(A^T W A)^{-1}$ in equations (2-3) and (2-4) above would not exist. This situation arises when for instance the datum for the coordinates being adjusted is generally incompletely defined by observations and restrictions i.e. the observations do not cater for all the degrees of freedom of the network [e.g., *Koch 1987 pg.212]*, when observation equations are formed it is necessary to add a set of restrictions which in effect complete the definition of the coordinate datum.

A coordinate system in three-dimensional space necessitates the definition of seven degrees of freedom, if it is defined in shape, this includes, one scale element, three translation elements and three rotation elements. For a two-dimensional network, four elements, namely, one scale, one orientation and two translations. The necessary and sufficient number and type of datum elements can be defined by an appropriate combination of measurements, *Cooper [1987]* outlines the numbers and types of cartesian coordinate datum elements for two and three dimensions which are defined by inclusion of certain measurements in a network.The following sections discuss the variants of the Gauss-Markov model under different forms of restrictions.

2.2 Gauss-Markov with Exact Restrictions

Exact restrictions may be incorporated in geodetic networks for two main reasons; First, to overcome datum defects, and secondly, to fulfill certain physical or geometric conditions in the model. In general the Gauss-Markov model with exact restrictions is set up in the form

$$\boldsymbol{y} = \boldsymbol{Ax}+\varepsilon \qquad (2\text{-}7)$$

and $$\boldsymbol{r} = \boldsymbol{Rx} \qquad (2\text{-}8)$$

with (2-8) as the exact restrictions, in which $\boldsymbol{r}$ is a $c\times 1$ vector of constants, and $\boldsymbol{R}$ is a $c\times m$ restriction design matrix. (2-8) is also referred to as exact prior information e.g., *Durbin [1953]* and *Aduol [1993]*.

To determine the estimate of $\boldsymbol{x}$ under the least squares condition and further fulfilling (2-8). The Lagrange function $\boldsymbol{L}$ is used

$$L = \varepsilon' W \varepsilon + 2(Rx - r)'\lambda \tag{2-9}$$

with which, under least squares conditions the resulting normal equations take the form

$$\begin{bmatrix} A'WA & R' \\ R & 0 \end{bmatrix} \begin{bmatrix} X \\ \lambda \end{bmatrix} = \begin{bmatrix} A'W \\ R \end{bmatrix} \tag{2-10}$$

provided the model i.e., (2-7) and (2-8) is of full rank, the estimates of $\boldsymbol{x}$ and $\boldsymbol{\lambda}$ may be obtained through

$$\begin{bmatrix} \hat{X} \\ \hat{\lambda} \end{bmatrix} = \begin{bmatrix} A'WA & R' \\ R & 0 \end{bmatrix}^{-1} \begin{bmatrix} A'W \\ r \end{bmatrix} \tag{2-11}$$

The inversion of the normal equation matrix can be performed through block-matrix techniques, see *Aduol [1996]* and *Schaffrin [1984]*.

As mentioned in (2-1), datum defects due to fewer degrees of freedom than necessary lead to rank deficient surveying systems, these are overcome through the imposition of appropriate restrictions (2-8). In the case where restrictions are introduced only to overcome datum defects and define the reference coordinate system, we have a *minimally constrained model [Mikhail 1976; Koch 1987]*. Where restrictions are more than, but include, those just needed to overcome datum defects results in an *over-constrained model [Aduol 1996]*. Under over-constrained models, it may happen that the simple Gauss-Markov model has full rank, in such a case this is called over-constrained with full rank. However, if the simple Gauss-Markov model has a rank defect, such that among the restrictions some go towards overcomming the rank defects, it is refered to as over-constrained with rank defect.

2.3 Gauss-Markov with stochastic restrictions

In this case the rank defect in the design matrix is overcome by introducing restrictions with their stochasticity. The restrictions are set up in the form

$$\boldsymbol{r} = \boldsymbol{Rx} + \boldsymbol{\varepsilon}_r, \quad \boldsymbol{\varepsilon}_r \sim (0, \boldsymbol{\Sigma}_{rr}), for \ D(\boldsymbol{r}) = \boldsymbol{\Sigma}_{rr} \tag{2-12}$$

Taking (2-12) together with (2-2), Gauss-Markov with stochastic restrictions is expressed as

$$\begin{bmatrix} \boldsymbol{y} \\ \boldsymbol{r} \end{bmatrix} = \begin{bmatrix} A \\ R \end{bmatrix} X + \begin{bmatrix} \varepsilon_y \\ \varepsilon_r \end{bmatrix} \tag{2-13}$$

on taking

$$\bar{\boldsymbol{y}} := \begin{bmatrix} \boldsymbol{y} \\ \boldsymbol{r} \end{bmatrix}, \quad \bar{\boldsymbol{A}} := \begin{bmatrix} A \\ R \end{bmatrix}, \quad \text{and} \quad \bar{\boldsymbol{\varepsilon}} := \begin{bmatrix} \varepsilon_y \\ \varepsilon_r \end{bmatrix} \tag{2-14}$$

(2-13) may be written as

$$\bar{\boldsymbol{y}} := \bar{\boldsymbol{A}}x + \bar{\boldsymbol{\varepsilon}}, \quad with \ \bar{\boldsymbol{\varepsilon}} \sim (0, \boldsymbol{\Sigma}_{\varepsilon\varepsilon}) \tag{2-15}$$

for

$$D(\bar{\boldsymbol{\varepsilon}}) = \boldsymbol{\Sigma}_{\varepsilon\varepsilon} = \begin{bmatrix} \boldsymbol{\Sigma}_{yy} & 0 \\ 0 & \boldsymbol{\Sigma}_{rr} \end{bmatrix} \tag{2-16}$$

on assumming that $\boldsymbol{y}$ and $\boldsymbol{r}$ are independent. With (2-16) the combined weight matrix $\bar{\boldsymbol{W}}$ is defined in the form

$$\bar{W} = \Sigma_{\varepsilon\varepsilon}^{-1} = \begin{bmatrix} \Sigma_{yy}^{-1} & 0 \\ 0 & \Sigma_{rr}^{-1} \end{bmatrix} \tag{2-17}$$

provided the inverse exists.

Under least squares condition we have that

$$\hat{\mathbf{x}} = (\bar{\mathbf{A}}'\bar{\mathbf{W}}\bar{\mathbf{A}})^{-1}\bar{\mathbf{A}}'\bar{\mathbf{W}}\bar{\mathbf{y}} \tag{2-18}$$

with

$$\hat{\Sigma}_{\hat{x}\hat{x}} = \hat{\sigma}_0^2 (\bar{\mathbf{A}}'\bar{\mathbf{W}}\bar{\mathbf{A}})^{-1} \tag{2-19}$$

where $\hat{\sigma}_0^2$ is the variance of unit weight, given the form

$$\hat{\sigma}_0^2 = \bar{\boldsymbol{\varepsilon}}'\bar{\mathbf{W}}\bar{\boldsymbol{\varepsilon}}/(n+c-m) \tag{2-20}$$

for n being the number of observations, c the number of restrictions, and m the number of unknowns.

Further we have that

$$E(\hat{\mathbf{x}}) = (\bar{\mathbf{A}}'\bar{\mathbf{W}}\bar{\mathbf{A}})^{-1}(\bar{\mathbf{A}}'\bar{\mathbf{W}}\bar{\mathbf{A}})\mathbf{x} = \mathbf{x} \tag{2-21}$$

Thus demonstarting that $\hat{\mathbf{x}}$ is an unbiased estimate of $\mathbf{x}$.

2.4 The Free Network Adjustment Model

The free network model is one way of defining the datum (see section 2.1.1) of a network so that the datum is not dependent on just one parameter, for instance, but instead the datum is dependent on parameters spread over the network. Such a network is considered free in that its geometrical size and shape is determined while remaining essentially independent of a reference datum.

The result of such a free network adjustment is that the consistency of observations and thus the internal precision of the network may be checked, free of external influences associated with attaching a network rigidly to an absolute reference datum. This makes free network adjustment best suited for adjustment of datum networks, as it results in fairly representative estimates of network parameters with uniformly distributed accuracies.

In free net adjustment the final datum information is drawn from the approximate coordinates of all net points, computed to an abitrary datum. This concept of the so-called 'inner solution' gives unique results. The inner solution is the minimum-norm least squares solution of the singular equations;

$$\boldsymbol{\varepsilon}'\boldsymbol{W}\boldsymbol{\varepsilon} \Rightarrow \min; \qquad (2\text{-}22)$$

$$\boldsymbol{x}'\boldsymbol{x} \Rightarrow \min; \qquad (2\text{-}23)$$

and

$$tr[\boldsymbol{Q}_{xx}] \Rightarrow minimum \qquad (2\text{-}24)$$

These are generalized as inner constraints. With (2-22) as the basic least squares condition, in addition to which restriction (2-23) can be interpreted geometrically as representing a minimum possible deviation between final network coordinates and the initial approximate coordinates, which in effect controls the scale of the network, while minimal trace for the cofactor matrix $\boldsymbol{Q}_{xx}$ in (2-24), which represents 'inner accuracy', ensures that the accuracy of the resultant coordinates is the best possible [*Schmitt 1982*].

Mathematically, datum is defined by setting up an exact restriction in the form of (2-8) but in this case $\boldsymbol{R}$ is a special matrix whose columns are made up of the normalised eigenvectors of eigenvalues in the normal matrix

$$\boldsymbol{N} = \boldsymbol{A}'\boldsymbol{W}\boldsymbol{A} \tag{2-25}$$

which have values equall to zero due to the rank defect in $\boldsymbol{N}$ [e.g., *Aduol, 1996*]. If $\boldsymbol{R}$ is expressed as $\boldsymbol{G}'$ then due to the special form of $\boldsymbol{G}$ we shall have

$$\boldsymbol{N}\boldsymbol{G}' = 0 \tag{2-26}$$

in which $\boldsymbol{G}$ is an orthorgonal matrix. The overall restriction can thus be written as

$$\boldsymbol{G}'\boldsymbol{x} = 0 \tag{2-27}$$

for a three dimensional network in which angles have been observed, the corresponding restrictiom matrix for n points is given in the form

$$\boldsymbol{G}' = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & . & . & . & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & . & . & . & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & . & . & . & 1 \\ 0 & Z_1 & -Y_1 & 0 & Z_2 & -Y_2 & . & . & . & -Y_n \\ -Z_1 & 0 & X_1 & -Z_2 & 0 & X_2 & . & . & . & X_n \\ Y_1 & -X_1 & 0 & Y_2 & -X_2 & 0 & . & . & . & 0 \\ X_1 & Y_1 & Z_1 & X_2 & Y_2 & z_2 & . & . & . & Z_n \end{bmatrix} \tag{2-28a}$$

thus

$$\boldsymbol{G}'\boldsymbol{x} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & . & . & . & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & . & . & . & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & . & . & . & 1 \\ 0 & Z_1 & -Y_1 & 0 & Z_2 & -Y_2 & . & . & . & -Y_n \\ -Z_1 & 0 & X_1 & -Z_2 & 0 & X_2 & . & . & . & X_n \\ Y_1 & -X_1 & 0 & Y_2 & -X_2 & 0 & . & . & . & 0 \\ X_1 & Y_1 & Z_1 & X_2 & Y_2 & z_2 & . & . & . & Z_n \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{x}_1 \\ \Delta \boldsymbol{y}_1 \\ \Delta \boldsymbol{z}_1 \\ . \\ \Delta \boldsymbol{x}_n \\ \Delta \boldsymbol{y}_n \\ \Delta \boldsymbol{z}_n \end{bmatrix} = 0 \tag{2-28b}$$

If the distances were determined, the seventh row would be deleted as observed distances control the scale of the network. Similarly, if azimuth observations were made then the fourth, fifth and sixth rows would be deleted. Varoius forms of the restriction matrix depending on different observation combinations are listed by *Illner [1985]*.

Noting that the normal equation matrix in (2-22) is singular, the problem of adjusting free networks thus becomes principally one of overcomming the rank defect in the normal matrix. Several approaches to the solution of $\boldsymbol{N}$ are considered in detail by among others *Mittermayer [1972], Pope [1973], Grafarend and Schaffrin [1974], Brunner [1979], and Meissl [1982]*; for the theory of generalised inverses, see *Rao and Mitra [1971], Bjerhammar [1973]*, and *Ben-Israel and Greville [1974]*.

CHAPTER THREE

GEODETIC NETWORK DENSIFICATION MODELS

Presented in this chapter are models used in densification of geodetic networks. The basic model is outlined in section (3.1), it is then considered by incorporation of various restrictions to yield the static, dynamic, and static-dynamic models in sections (3.2), (3.3), and (3.4) respectively. Section (3.5) briefly outlines the concept of weighting of observation. The contents of this chapter are based extensively on the work of *Aduol [1993, 1996]*.

3.1 Basic Model

In densification of networks there are two groups of points to be handled in parameter estimation. These are the already existing points over which datum is defined, and the densification points to be freshly coordinated. Let the unknown parameters associated with the datum points be collected in a vector $\boldsymbol{x}_1$ and those of densification points into vector x_2 . Thus if the vector of all unknown parameters in the estimation model will be $\boldsymbol{x}$, then we have that

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \end{bmatrix} \qquad (3\text{-}1)$$

With this we set the linear Gauss-Markov model in the form

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x}+\boldsymbol{\varepsilon}_y, \quad \boldsymbol{\varepsilon}_y \sim (0, \sigma^2_{0y}\boldsymbol{W}^{-1}_{yy}) = (0, \Sigma_{yy}) \quad , \qquad (3\text{-}2)$$

in which $\boldsymbol{y}$ is an $n\times1$ vector of observations, $\boldsymbol{A}$ is an $n\times m$ design matrix, $\boldsymbol{x}$ is an $m\times1$ vector of unknown parameters, $\boldsymbol{\varepsilon}_y$ is an $n\times1$ vector of observational errors, σ^2_{0y} is the variance of unit weight, and $\boldsymbol{W}_{yy}$ is an $n \times n$ positive definite weight matrix of $\boldsymbol{y}$. On making use of (3-1) in (3-2) we obtain

$$\boldsymbol{y}=\boldsymbol{A}_1\boldsymbol{x}_1+\boldsymbol{A}_2\boldsymbol{x}_2+\boldsymbol{\varepsilon}_y \ , \tag{3-3}$$

where $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ are respectively of orders $m_1 \times 1$ and $m_2 \times 1$, $\boldsymbol{A}_1$ and $\boldsymbol{A}_2$ are respectively design submatrices of orders $n \times m_1$ and $n \times m_2$, and

$$\boldsymbol{A} = [\boldsymbol{A}_1 \ \boldsymbol{A}_2] \ . \tag{3-4}$$

We have further that the datum parameters collected in $\boldsymbol{x}_1$ are stochastic prior information, so that we may write in general the stochastic restriction as

$$\boldsymbol{r}=\boldsymbol{R}_1\boldsymbol{x}_1+\boldsymbol{\varepsilon}_r, \boldsymbol{\varepsilon}_r\sim(0,\boldsymbol{\Sigma}_{rr}) \ , \tag{3-5}$$

in which r is a $c \times 1$ vector of stochastic parameters, $\boldsymbol{R}_1$ is a $c \times m_1$ restriction design matrix, and $\boldsymbol{\Sigma}_{rr}$ is a $c \times c$ covariance matrix of r . Thus we have the model represented as

$$y=\boldsymbol{A}_1\boldsymbol{x}_1+\boldsymbol{A}_2\boldsymbol{x}_2+\boldsymbol{\varepsilon}_y, \quad \boldsymbol{\varepsilon}_y\sim(0,\sigma_{0y}\boldsymbol{W}^{-1}{}_{yy})=(0,\Sigma_{yy}) \ ; \tag{3-6a}$$

$$\boldsymbol{r}=\boldsymbol{R}_1\boldsymbol{x}_1+\boldsymbol{\varepsilon}_r, \boldsymbol{\varepsilon}_r\sim(0,\boldsymbol{\Sigma}_{rr}) \ . \tag{3-6b}$$

This model was suggested by *Durbin [1953]* as the model for *estimation with incomplete extraneous information*, and is considered as the basic estimation model from which various densification models are developed depending on how restrictions are incorporated.

3.2 The Static Densification Model

In this model we have that the datum parameters are treated as exact prior information. The representation (3-6) then becomes that of exact restriction in the form

$$r = R_1 x_1 \ ,\tag{3-7}$$

since for exact restriction $\varepsilon_r = 0,\ \varepsilon_r \sim (0,0)$.

The full estimation model then becomes

$$y = A_1 x_1 + A_2 x_2 + \varepsilon_y,\quad \varepsilon_y \sim (0, \sigma^2_{0y} W^{-1}_{yy}) = (0, \Sigma_{yy}) \ ;\tag{3-8a}$$

$$r = R_1 x_1 \tag{3-8b}$$

To determine the estimate of x Under the least squares condition and further fulfilling (3-8b). A Lagrange

function is used and the normal equations for this

set up take the form

$$\begin{bmatrix} A'_1 W_{yy} A_1 & A'_1 W_{yy} A_2 & R'_1 \\ A'_2 W_{yy} A_1 & A'_2 W_{yy} A_2 & 0 \\ R_1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \lambda \end{bmatrix} = \begin{bmatrix} A'_1 W_{yy} y \\ A'_2 W_{yy} y \\ r \end{bmatrix} ,\tag{3-9}$$

where λ is the vector of Lagrange multipliers. From this we obtain that the estimate $\hat{x}$ of x is given as

$$\hat{x} = (N_r^{-1} - N_r^{-1} R' K_r^{-1} R N_r^{-1}) A' W_{yy} y + N_r^{-1} R' K_r^{-1} r,$$
$$\text{for } N_r := (A' W_{yy} A + R' R),\ K_r := R N^{-1} R', \qquad \text{[see Aduol 1996]} \tag{3-10}$$
$$\text{with } A = [A_1\ A_2],\ R = [R_1\ R_2]$$

Also the estimate $D(\hat{X})$ of the dispersion of $\hat{X}$ is given as

$$D(\hat{X}) = \Sigma_{\hat{x}\hat{x}} = \hat{\sigma}_0^2 (N_r^{-1} - N_r^{-1} R' K_r^{-1} R N_r^{-1}) N (N_r^{-1} - N_r^{-1} R' K_r^{-1} R N_r^{-1}) \ ,\tag{3-11}$$

with

$$\hat{\sigma}_0^2 = (\hat{\varepsilon}' W_{yy} \hat{\varepsilon}) / (n + c - m) \ .\tag{3-12}$$

In the special case that R_1 is positive definite, we should have from (3-8b) that

$$x_1 = R_1^{-1} r \tag{3-13}$$

we then obtain that

$$y - A_1 R_1^{-1} r = A_2 x_2 + \varepsilon_y .\tag{3-14}$$

If in this we set

$$\zeta := y - A_1 R_1^{-1} r \ ,\tag{3-15}$$

then (3-14) becomes a simple Gauss-Markov model in the form

$$\zeta = \mathbf{A}_2 x_2 + \varepsilon_y, \quad \varepsilon_y \sim (0, \sigma_0^2 \mathbf{W}_{yy}^{-1}) = (0, \mathbf{\Sigma}_{yy}), \tag{3-16}$$

in which we now have only the unknown subvector x_2 appearing; and $\mathbf{\Sigma}_{\zeta\zeta} = D(\zeta)$.

In practice we normally have that the exact prior information of (3-8b) comprises only the datum coordinates, so that the vector x_1 contains only the datum coordinates. If we let the exact prior information values of x_1 be ζ_1 , then (3-8b) becomes simply

$$\zeta_1 = x_1 , \tag{3-17}$$

being equivalent to taking $r = \zeta_1$ and $R_1 = \mathbf{I}_1$; where $\mathbf{I}_1$ is a $c \times c$ identity matrix. With this we now have from (3-15) that

$$\zeta = y - \mathbf{A}_1 \zeta_1 \tag{3-18}$$

From (3-16) the estimate $\hat{x}_2$ of x_2 is obtained in the form

$$\begin{aligned} \hat{x}_2 &= (\mathbf{A}_2' \mathbf{W}_{yy} \mathbf{A}_2)^{-1} \mathbf{A}_2' \mathbf{W}_{yy} \zeta, \\ \text{with } D(\hat{x}_2) &= \hat{\sigma}^2_0 (\mathbf{A}_2' \mathbf{W}_{yy} \mathbf{A}_2)^{-1} \\ \text{for } \hat{\sigma}^2_0 &= (\hat{\varepsilon}_y' \mathbf{W}_{yy} \hat{\varepsilon}_y) / (n - m_2) \end{aligned} \tag{3-19}$$

It is usual in survey practice to start off a parameter estimation from approximate values of the parameters, normally for linearization purposes. In such case we would have that

$$x_1 = x_{01} + \Delta x_1 \text{ and } x_2 = x_{02} + \Delta x_2 , \tag{3-20}$$

in which x_{01}, x_{02} are the approximate values to x_1, x_2 respectively. In such situation (3-6) takes the form

$$\begin{aligned} y &= \mathbf{A}_1 \mathbf{\Delta} x_1 + \mathbf{A}_2 \mathbf{\Delta} x_2 + \varepsilon_y, \quad \varepsilon_y \sim (0, \sigma_0^2 \mathbf{W}^{-1}{}_{yy}), \\ r &= \mathbf{R}_1 \mathbf{\Delta} x_1 + \varepsilon_r, \quad \varepsilon_r \sim (0, \mathbf{\Sigma}_{rr}) \end{aligned} \tag{3-21}$$

since the unknown parameters to be estimated are now $\mathbf{\Delta} x_1$ and $\mathbf{\Delta} x_2$,from which estimates of x_1 and $\mathbf{x}_2$ may then be obtained through (3-20); and correspondingly (3-7) and (3-13) would become

$$r = \mathbf{R}_1 \mathbf{\Delta} x_1, \quad \mathbf{\Delta} x_1 = \mathbf{R}_1 r. \tag{3-22}$$

If in (3-20) we take $x_1 = \zeta_1 = x_{01}$, then in (3-22) we have

$$\Delta x_1 = R^{-1}r = 0 \quad . \tag{3-23}$$

And following onto this we notice that (3-21) becomes

$$y = A^2\Delta x_2 + \varepsilon_y, \varepsilon_y \sim (\sigma^2_{0y}W^{-1}_{yy}) \tag{3-24}$$

which is similar to (3-16) and from this we obtain the usual estimates

$$\Delta\hat{x}_2 = (A_2'W_{yy}A_2)^{-1}A_2W_{yy}y,$$
$$with\ D(\Delta\hat{x}_2) = \hat{\sigma}^2_{0y}(A_2'W_{yy}A_2)^{-1} \tag{3-25}$$
$$for\ \hat{\sigma}^2_0 = (\hat{\varepsilon}_y'W_{yy}\hat{\varepsilon}_y)/(n-m_2)$$

This is the form of parameter estimation often adopted in the static densification of geodetic networks. It should be noted that this model is basically the same as the simple Gauss-Markov with exact restrictions i.e. (2-7) and (2-8). However we notice that in this case the design matrix and the vector of unknowns are partitioned into components to represent contributory factors for fixed and new parameters respectively. It is also important to observe from (2-7) that datum parameters are treated as exact prior information, hence the effect of the uncertainties in the datum parameters on the estimated parameters is implicitly ignored. This implicit omission is not justified as the datum parameters themselves could probably have been obtained earlier through an adjustment process and have a covariance matrix associated with them. The effect of neglecting the covariance matrix has been considered in depth by *Wolf [1983]*.

3.3 The Dynamic Densification Model

The dynamic densification model is based on the use of stochastic restrictions within the framework of the model represented in (3-6), in a combined form we have

$$\begin{bmatrix} y \\ r \end{bmatrix} = \begin{bmatrix} A_1 & A_2 \\ R_1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \varepsilon_y \\ \varepsilon_r \end{bmatrix} \tag{3-26}$$

on taking,

$$y_\zeta = \begin{bmatrix} y \\ r \end{bmatrix} ;\ A_\zeta = \begin{bmatrix} A_1 & A_2 \\ R_1 & 0 \end{bmatrix} ;\ x_\zeta = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} ;\ \varepsilon_\zeta = \begin{bmatrix} \varepsilon_y \\ \varepsilon_r \end{bmatrix} \tag{3-27}$$

we have that

$$y_\zeta = A_\zeta x_\zeta + \varepsilon_\zeta,\ \varepsilon_\zeta \sim (0, \sigma^2_{oy} W^{-1}_{yy}) \tag{3-28}$$

in which, on the assumption that y and r are independent, we shall now be having

$$E(\varepsilon_\zeta) = 0,\ D(\varepsilon_\zeta) = \begin{bmatrix} \sigma^2_{oy} W^{-1}_{yy} & 0 \\ 0 & \sigma^2_{0r} W^{-1}_{rr} \end{bmatrix} = \Sigma_{\zeta\zeta}\ . \tag{3-29}$$

The corresponding weight matrix $W_{\zeta\zeta}$ is given as

$$W_{\zeta\zeta} = \begin{bmatrix} \sigma^2_{oy} W_{yy} & 0 \\ 0 & \sigma^2_{0r} W_{rr} \end{bmatrix} = \Sigma^{-1}_{\zeta\zeta} \tag{3-30}$$

This is the *mixed estimation model* of *Theil and Goldberger [1961]*. From the definition $A = [A_1\ A_2]$, $R = [R_1\ 0]$,we have that

$$\hat{x} = (A_\zeta W_{\zeta\zeta} A_\zeta)^{-1} A_\zeta W_{\zeta\zeta} \zeta$$
$$= (A' \Sigma^{-1}_{yy} A + R' \Sigma^{-1}_{rr} R)^{-1} (A' \Sigma^{-1}_{yy} y + R' \Sigma^{-1}_{rr} r)$$

with (3-31)

$$D(\hat{x}) = (A' \Sigma^{-1}_{yy} A + R' \Sigma^{-1}_{rr} R)^{-1} \tag{3-32}$$

and

$$\hat{\sigma}^2_{0y} = (\hat{\varepsilon}'_y W_{yy} \hat{\varepsilon}) / trace(W_{yy} Q_{\varepsilon y} Q_{\varepsilon y}) \tag{3-33a}$$

$$\hat{\sigma}^2_{0r} = (\hat{\varepsilon}'_r W_{rr} \hat{\varepsilon}_r) / trace(W_{rr} Q_{\varepsilon r} Q_{\varepsilon r}) \tag{3-33b}$$

in which

$$Q_{\varepsilon r} Q_{\varepsilon r} = Q_{rr} - R Q_{xx} r' = W^{-1}_{rr} - R(A' W_{\zeta\zeta} A')^{-1} R' \tag{3-34a}$$

$$Q_{\varepsilon y} Q_{\varepsilon y} = Q_{yy} - A Q_{xx} A' = W^{-1}_{yy} - A(A' W_{\zeta\zeta} A')^{-1} A' \tag{3-34b}$$

$$\varepsilon_y = Q_{\varepsilon y} Q_{\varepsilon y} W_{yy} y \text{ cf. [Mikhail 1976]}, \tag{3-35a}$$

and

$$\varepsilon_r = Q_{\varepsilon r} Q_{\varepsilon r} W_{rr} r \tag{3-35b}$$

This model is developed principally on the basis of the simple Gauss-Markov model with stochastic restrictions as in (2-3), in this approach to densification, both datum and new network parameters are estimated in (3-3). Statistically this is a better approach as compared to the static approach since the stochasticity of datum parameters is considered with the framework of adjustment, thus providing a more realistic estimation. On the other hand this approach to densification provides new values for datum coordinates, which implies any small densification work will lead to the readjustment of the whole network, thus the concept of national geodetic reference systems loses meaning and this, may translate into uncontrolled geodetic works which would be so hard to harmonize.

3.4 The Static-Dynamic Densification Model

If in (3-6) R_1 be a positive definite matrix, then we may write

$$x_1 = R_1^{-1}(r - \varepsilon_r) \tag{3-36}$$

so that (3-6a) becomes

$$y = A_1R_1^{-1}(r-\varepsilon_y) + A_2x_2 + \varepsilon_y,\ \varepsilon\sim(0, \sigma^2_{0y}W^{-1}_{yy}) = (0, \Sigma_{yy}) \tag{3-37}$$

or

$$y - A_1R_1^{-1}r = A_2x_2 + \varepsilon_y - A_1R_1^{-1}\varepsilon_r) . \tag{3-38}$$

on taking $\bar{y} := y - A_1R_1^{-1}r, \varepsilon := \varepsilon_y - A_1R_1^{-1}\varepsilon_r$

we have that

$$\bar{y} := A_2x_2 + \varepsilon,\ \varepsilon \sim(0, \Sigma_{\bar{y}\bar{y}}) \tag{3-39}$$

$$\begin{aligned} D(\varepsilon) &= \Sigma_{\varepsilon\varepsilon} = \Sigma_{\bar{y}\bar{y}} = \Sigma_{yy} + A_1R_1^{-1}\Sigma_{rr}(A_1R_1^{-1})' \\ &= \sigma^2_{oy}W^{-1}_{yy} + A_1R_1\Sigma_{rr}(A_1R_1^{-1})'. \end{aligned} \tag{3-40}$$

on using least squares we obtain

$$\hat{x} = (A_2\Sigma_{\overline{yy}}^{-1}A_2)^{-1}A_2\Sigma_{\overline{yy}}^{-1}\overline{y} \tag{3-41}$$

and

$$D(\hat{x}_2) = (A_2\Sigma_{\overline{yy}}^{-1}A_2)^{-1} \tag{3-42}$$

In network densification where only the coordinates of the datum points have been collected in x_1, we notice that we shall have $R_1 = I_1$, i.e. an identity matrix of the same dimension as R_1. Then, from (3-41) and (3-42) we would now have that

$$\hat{x} = [A_2'(\sigma^2_{oy}W^{-1}_{yy} + A_1\Sigma_{rr}A_1')A_2]^{-1}$$
$$[A_2'(\sigma^2_{oy}W^{-1}_{yy} + A_1\Sigma_{rr}A_1')(y - A_1r)], \tag{3-43}$$
$$D(\hat{x}_2) = [(A_2(\sigma^2_{0}yW^{-1}_{yy} + A_1\Sigma_{rr}A_1'^{-1}A_2]^{-1}$$

We note that with this model we are able to estimate densification parameters x_2 by incorporating datum parameters x_1 as stochastic prior information, while at the same time keeping x_1 numerically and statistically unchanged. This provides for a balance between the static and dynamic approaches to densification as it considers the stochasticity of datum points while at the same treating them as fixed.

3.5 Weighting of Observations

Weights are related to variance in the following way

$$W_i = \frac{\sigma_0^2}{\sigma_i^2} \tag{3-44}$$

where σ_0^2 is the variance of unit weight, also called variance factor or sometimes the variance component. σ_i^2 is the variance of observation i. Using the matrix notation, we may represent the weight matrix, W as

$$W = \sigma_o^2\Sigma_{yy}^{-1} \tag{3-45}$$

where Σ_{yy} is the variance covariance matrix of the observations.

From equation (3-44) we note that the problem of weighting reduces to that of determing the standard errors of different observations. In the present study it is assumed the observations were not correlated thus the initial weights were determined by simply inverting the squares of the adopted observational errors as outlined in the various sections of chapter four. However, the values for weights are scaled accordingly during the estimation process.

CHAPTER FOUR

THE TEST NETWORK

In order to test and evaluate the models discussed in chapter three, data from a part of the Kenyan geodetic network were adopted. The data which were obtained from Survey of Kenya records of measurements and adjustments included initial observations for the primary, secondary, and tertiary networks. The primary network denoted as KGN-1 hereafter, consisted of eight first order horizontal control points sepearated by an average distance of 39 Km (as shown in Fig.1). KGN-1 was then densified to fifteen second order horizontal control points with an average distance separation of 13 Km, the secondary network shown in Fig.2 will be referred to herein as KGN-2. Further, KGN-1 and KGN-2 were densified to twenty-two third order horizontal control points separated by an average distance of 8 Km. The tertiary control network shown in Fig.3 is denoted hereinafter as KGN-3.

Data consisted of the original field observations for distances and angles, and the adjusted coordinates of the network stations. These coordinates are based on the U.T.M. grid. It has to be noted that these coordinates were adopted as approximate in the present study. For the purpose of easy identification and use in programming the network points were coded hierachically, where the eight primary order stations were assigned numbers 1-8, the fifteen secod order stations are assigned 9-23, and the twenty-two third order stations given 24-45. In the following sections these numbers will be used most commonly than the real station names.

4.1 The Primary Network (KGN-1)

KGN-1 consisted of eight stations as shown in Fig. 1 with corresponding approximate coordinates and observation sets in Tables 4.1 and 4.2 respectively. This was considered as the fundamental network upon which densification was done, and thus the fundamental datum was defined by adjusting the primary network within the framework of free-network adjustment.

In this study, all the eight primary points were considered approximate and the adjustments were done assuming that the measurements were of first order precision. Standard error of angular observations was taken as ±0.5", based on a variety of experiences from various reports on different surveys e.g. *Musyoka [1993], Aduol [1981], and Gosset [1959]*.

FIG 1 : PRIMARY NETWORK

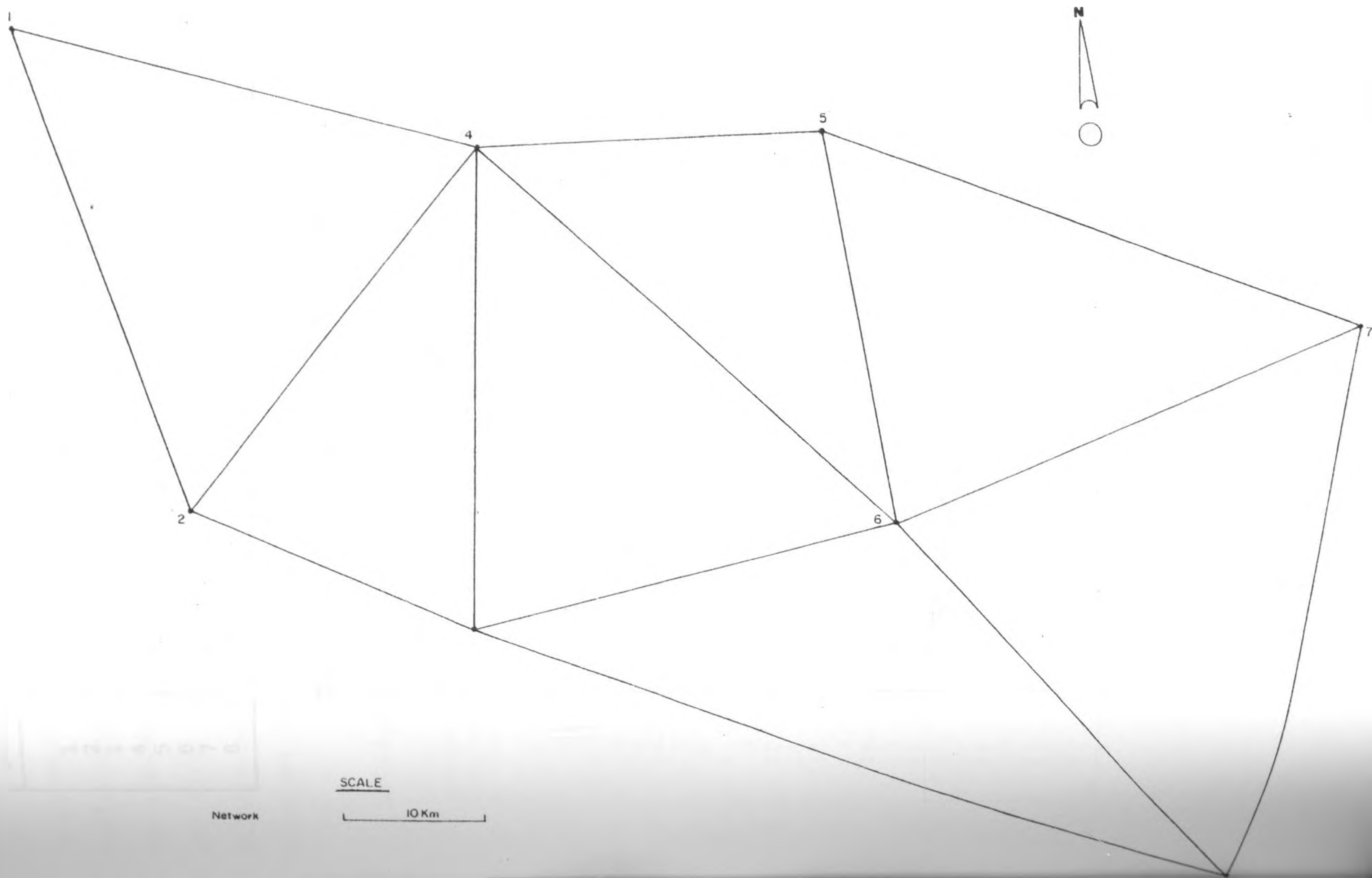


Table 4.1: Approximate coordinates of the primary network

point	Code	N [m]	E [m]
1	SKP106	9965867.148	234568.251
2	SKP108	9930827.740	244847.960
3	SKP210	9920928.207	263626.699
4	SKP211	9954810.146	266508.302
5	SKP212	9953913.168	290496.201
6	SKP213	9925562.951	293406.871
7	SKP214	9936363.270	327853.362
8	SKP215	9896895.405	316442.934

Table 4.2: Observational dataset for KGN-1.

Obs. No.	Ray		Bearing °	'	"
1	1	2	163	38	58.6
2	1	4	109	5	41.2
3	2	1	343	38	58.5
4	2	3	117	47	48.2
5	2	4	42	5	15.4
6	3	2	297	47	48.1
7	3	4	4	51	40.4
8	3	8	114	28	0.2
9	3	6	81	9	14.1
10	4	3	184	51	40.4
11	4	5	92	8	29.2
12	4	6	137	23	43.2
13	4	1	289	5	41.4
14	4	2	222	5	15.4
15	5	4	272	8	29.1
16	5	6	174	8	17.1
17	5	7	115	9	48.5
18	6	3	261	9	13.8
19	6	4	317	23	43.0
20	6	5	354	8	16.8
21	6	7	72	35	30.3
22	6	8	141	12	57.8
23	7	6	252	35	30.3
24	7	5	295	9	48.6
25	7	8	196	7	29.6
26	8	7	16	7	29.7
27	8	6	321	12	57.8
28	8	3	294	28	0.5

4.2 The Secondary Network (KGN-2)

KGN-2 consisted of fifteen stations connected onto the fundamental network (KGN-1) as shown by thin lines in Fig. 2. The corresponding observational sets are as given in Table 4.4 while the approximate coordinates are given in Table 4.3. In Table 4.3 the first eight approximate coordinates were determined from the adjustment under free network of KGN-1, while the rest are as obtained from Survey of Kenya.

Table 4.3: approximate coordinates of the secondary network (KGN-2)

Point	Code	N [m]	E [m]
1	SKP106	9965867.1352	234568.2516
2	SKP108	9930827.7520	244847.9533
3	SKP210	9920928.2259	263626.6790
4	SKP211	9954810.1409	266508.2800
5	SKP212	9953913.1636	290496.1621
6	SKP213	9925562.9667	293406.8300
7	SKP214	9936363.2753	327853.2967
8	SKP215	9896895.4409	316442.8767
......	....	....	....
9	120.S.1	9963932.0730	269718.8330
10	120.S.2	9949485.0600	261852.8100
11	121.S.1	9954495.4220	280104.9380
12	121.S.2	9945383.6230	286178.5410
13	121.S.3	9950678.8590	304751.1120
14	134.S.7	9928278.3570	262387.1370
15	134.S.8	9940076.4020	263194.0290
16	134.S.9	9941138.9550	274039.4160
17	135.S.7	9928572.7660	279001.9100
18	135.S.3	9910855.9480	297806.0580
19	135.S.4	9901567.8710	308367.6200
20	135.S.5	9916117.7110	312499.3550
21	135.S.9	9936062.8920	300024.1870
22	135.S.8	9923580.1200	302009.6370
23	135.S.10	9936916.5030	314544.8230

FIG 2 SECONDARY NETWORK

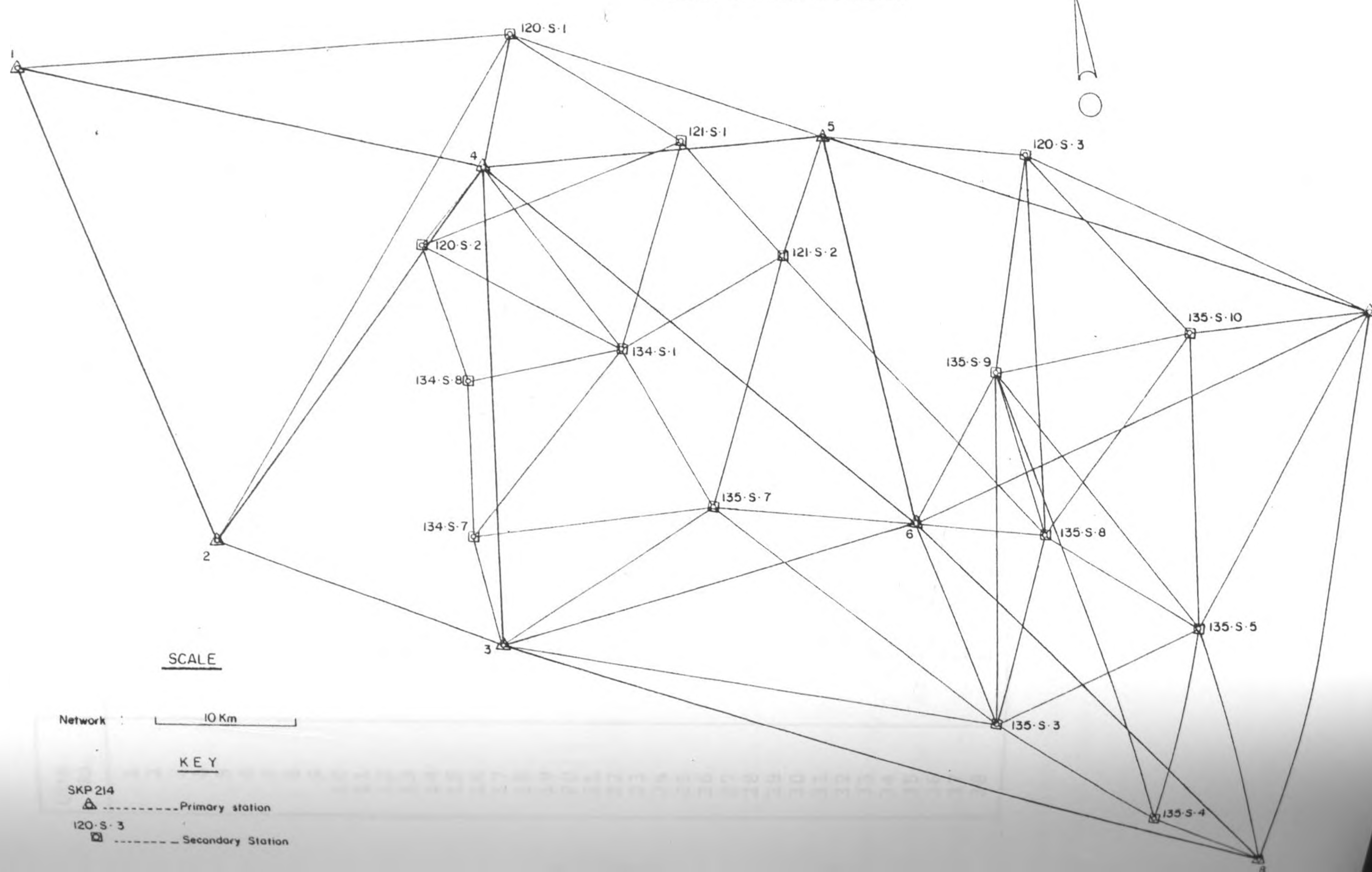


Table 4.4: The observational dataset for the secondary network (KGN-2).

Obs. No.	Line		Bearing °	'	"	Distance [m]
1	9	5	115	44	36.5	-
2	9	11	132	15	28.1	14032.894
3	9	1	273	9	3.5	-
4	9	4	199	23	23.8	9670.472
5	9	10	208	34	1.4	16449.605
6	10	4	41	9	42.6	7073.186
7	10	16	124	24	20.3	-
8	10	15	171	53	13.7	-
9	11	9	312	15	27.8	-
10	11	5	93	12	25.6	10407.466
11	11	12	146	18	50.3	10950.594
12	11	16	204	25	26.2	
13	12	5	26	50	54.2	9560.092
14	12	13	74	5	11.7	-
15	12	22	144	1	2.5	-
16	12	6	159	57	49.9	-
17	12	17	203	7	4.2	-
18	12	16	250	43	36.8	-
19	12	11	326	18	50.3	-
20	13	23	144	33	47.4	-
21	13	22	185	46	36.2	-
22	13	21	197	55	17.5	15361.325
23	13	5	282	47	0.4	14617.351
24	14	3	170	25	39.4	7454.089
25	14	17	88	59	5.4	16617.302
26	14	16	42	10	40.2	-
27	14	15	3	54	44.9	11825.600
28	15	16	84	24	15.8	-
29	15	10	351	53	13.4	9503.784
30	16	17	158	27	1.9	-
31	16	6	128	48	26.9	-
32	16	12	70	43	36.8	12859.757
33	16	11	24	25	26.6	14669.350
34	16	4	331	9	2.6	15608.303
35	16	10	304	24	20.0	14770.517
36	16	14	222	10	40.6	-
37	17	6	101	48	5.4	14715.942
38	17	12	23	7	4.2	-

Table 4.4: continued

Obs. No.	Line		Bearing °	'	"	Distance [m]
39	17	14	268	59	5.3	-
40	17	3	243	33	48.6	17170.701
41	18	6	343	20	48.3	15350.856
42	18	19	131	19	44.7	14064.701
43	18	21	5	1	44.0	-
44	18	17	313	17	40.6	-
45	19	8	120	3	14.9	9329.364
46	19	18	311	19	44.7	-
47	19	20	15	51	10.3	-
48	19	21	346	24	10.2	-
49	20	8	168	24	22.9	19622.478
50	20	7	37	10	34.1	-
51	20	19	195	51	10.8	15125.304
52	20	23	5 .	37	0.2	14545.624
53	21	20	147	58	30.4	-
54	21	19	166	24	10.1	-
55	21	18	185	1	43.8	-
56	21	7	89	22	53.4	-
57	21	23	86	38	8.3	-
58	21	22	170	57	44.4	12639.632
59	22	13	5	46	36.2	-
60	22	6	282	58	45.5	-
61	22	21	350	57	45.0	-
62	22	12	324	1	2.2	-
63	23	7	92	22	48.9	13320.000
64	23	13	324	33	47.3	-
65	23	21	266	38	8.4	-
66	23	22	223	13	34.4	18302.767

The error of any distance measurement will have two contributing factors namely a fixed instrumental error plus a propotional error depending on the range. These errors add vectorially, to give an overall standard deviation of error for a given instrument as $(f + 10^{-6}d)m$ *[e.g. Musyoka 1993]* in which f is the fixed instrumental error and d the observed distance. In the present study the standard error for distance observations was taken as ±0.01m while that of angular observations was taken as ±1".

These values for the errors were adopted since they are recommended as the most suitable accuracies for secondary triangulation works [*e.g. Gosset 1959 pg.265]*

4.3 The Tertiary Network (KGN-3)

KGN-3 consisted of twenty-two tertiary points linked to the KGN-1 and KGN-2 as shown by the fine lines in Fig. 3. With respective observation sets and approximate coordinates as given in Tables 4.6 and 4.5a, 4.5b, and 4.5c. In this case there are three classes of approximate coordinates, this being the adjusted coordinates of points which were determined from first level densification work using the three different approaches to densification. Each model resulted in a different set of results, hence three different sets of approximate coordinates to be used during the second level of densification.

FIG. 3 TERTIARY CONTROL NETWORK

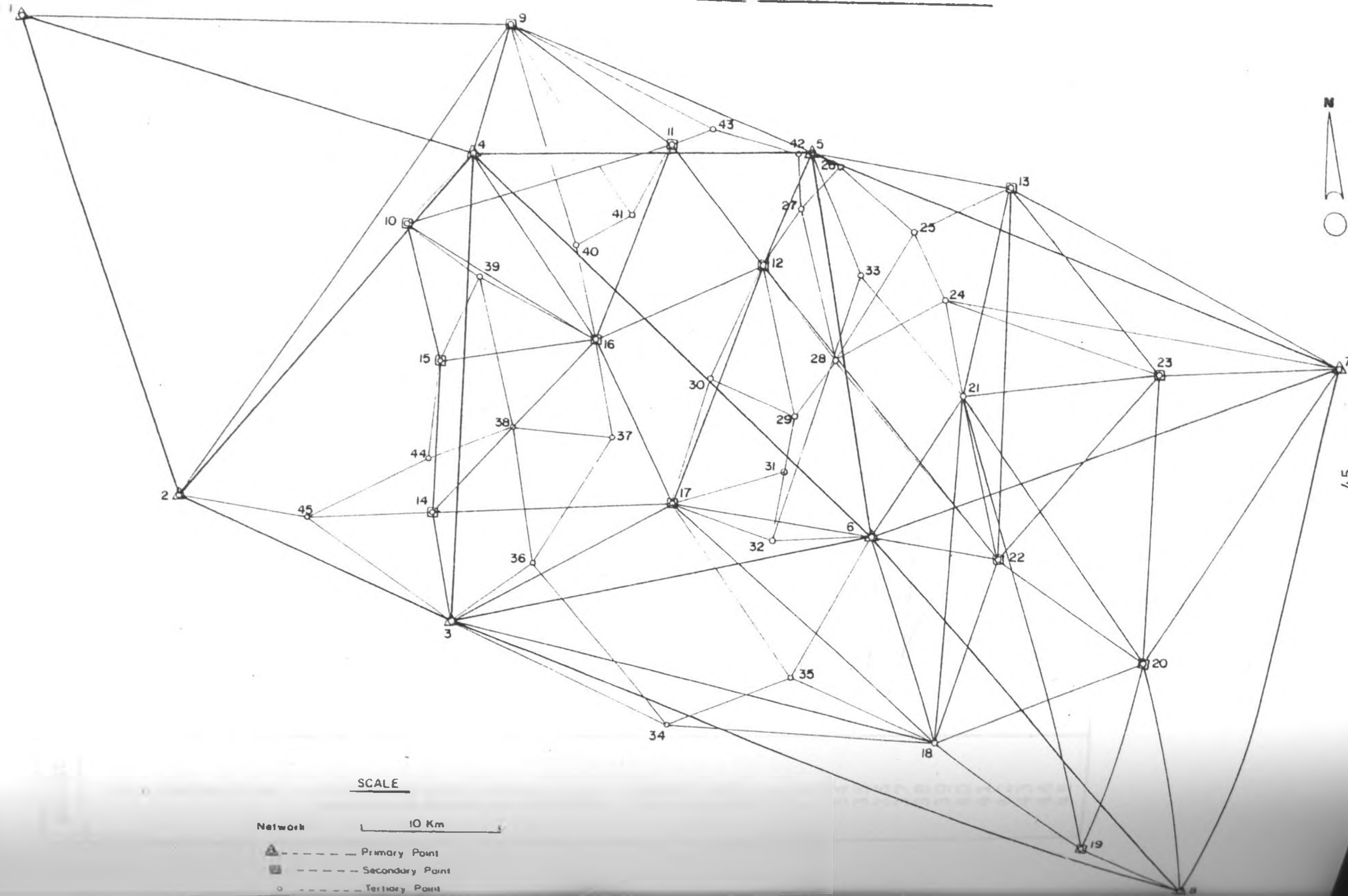


Table 4.5a: approximate coordinates of KGN-3 resulting from first level static densification.

Point	Code	N [m]	E [m]
1	SKP106	9965867.1352	234568.2516
2	SKP108	9930827.7520	244847.9533
3	SKP210	9920928.2259	263626.6790
4	SKP211	9954810.1410	266508.2800
5	SKP212	9953913.1636	290496.1621
6	SKP213	9925562.9660	293406.8300
7	SKP214	9936363.2753	327853.2967
8	SKP215	9896895.4409	316442.8767
9	120.S.1	9963932.0730	269718.8316
10	120.S.2	9949485.0600	261852.8090
11	121.S.1	9954495.4220	280104.9380
12	121.S.2	9945383.6230	286178.5410
13	121.S.3	9950678.8593	304751.1109
14	134.S.7	9928278.3566	262387.1372
15	134.S.8	9940076.4028	263194.0288
16	134.S.9	9941138.9550	274039.4160
17	135.S.7	9928572.7633	279001.9110
18	135.S.3	9910855.9480	297806.0581
19	135.S.4	9901567.8722	308367.6209
20	135.S.5	9916117.7110	312499.3542
21	135.S.9	9936062.8921	300024.1871
22	135.S.8	9923580.1205	302009.6374
23	135.S.10	9936916.5030	314544.8230
24	135.T.6	9942628.0170	299337.1600
25	121.T.5	9947392.3200	297152.6800
26	121.T.4	9952372.3080	292844.9710
27	121.T.9	9949803.4490	289489.8880
28	135.T.16	9939211.6500	291072.5800
29	135.T.14	9935086.9800	288325.2680
30	135.T.13	9938187.1470	282764.6360
31	135.T.10	9930525.5700	287309.5700
32	135.T.19	9925634.8180	286423.7950
33	135.T.5	9944523.7170	293466.0020
34	135.T.3	9912294.0470	278424.7330
35	135.T.5	9915871.1200	287342.2730
36	134.T.9	9925039.1650	269920.8880
37	134.T.10	9933938.0450	275108.0630
38	134.T.5	9934586.1750	267992.0990
39	120.T.1	9945802.3470	266520.3930
40	120.T.4	9948003.1670	273206.9210
41	121.T.8	9949306.6410	277591.4440
42	121.T.10	9954204.1380	289225.7160
43	121.T.2	9955468.9760	283653.6250
44	134.T.2	9932950.1020	262087.2620
45	134.T.8	9928940.0180	253845.9110

Table 4.5b: approximate coordinates of KGN-3 resulting from first level dynamic densification.

Point	Code	N [m]	E [m]
1	SKP106	9965867.1352	234568.2517
2	SKP108	9930827.7520	244847.9535
3	SKP210	9920928.2259	263626.6805
4	SKP211	9954810.1398	266508.2801
5	SKP212	9953913.1636	290496.1621
6	SKP213	9925562.9669	293406.8300
7	SKP214	9936363.2753	327853.2971
8	SKP215	9896895.4422	316442.8756
......	....	....	....
9	120.S.1	9963932.0730	269718.8341
10	120.S.2	9949485.0567	261852.8109
11	121.S.1	9954495.4204	280104.9552
12	121.S.2	9945383.6238	286178.5438
13	121.S.3	9950678.8588	304751.1120
14	134.S.7	9928278.3582	262387.1408
15	134.S.8	9940076.4026	263194.0291
16	134.S.9	9941138.9560	274039.4163
17	135.S.7	9928572.7716	279001.9100
18	135.S.3	9910855.9481	297806.0558
19	135.S.4	9901567.8782	308367.6149
20	135.S.5	9916117.7160	312499.3537
21	135.S.9	9936062.8928	300024.1810
22	135.S.8	9923580.1286	302009.6369
23	135.S.10	9936916.5030	314544.8122
......	....	....	....
24	135.T.6	9942628.0170	299337.1600
25	121.T.5	9947392.3200	297152.6800
26	121.T.4	9952372.3080	292844.9710
27	121.T.9	9949803.4490	289489.8880
28	135.T.16	9939211.6500	291072.5800
29	135.T.14	9935086.9800	288325.2680
30	135.T.13	9938187.1470	282764.6360
31	135.T.10	9930525.5700	287309.5700
32	135.T.19	9925634.8180	286423.7950
33	135.T.5	9944523.7170	293466.0020
34	135.T.3	9912294.0470	278424.7330
35	135.T.5	9915871.1200	287342.2730
36	134.T.9	9925039.1650	269920.8880
37	134.T.10	9933938.0450	275108.0630
38	134.T.5	9934586.1750	267992.0990
39	120.T.1	9945802.3470	266520.3930
40	120.T.4	9948003.1670	273206.9210
41	121.T.8	9949306.6410	277591.4440
42	121.T.10	9954204.1380	289225.7160
43	121.T.2	9955468.9760	283653.6250
44	134.T.2	9932950.1020	262087.2620
45	134.T.8	9928940.0180	253845.9110

Table 4.5c: approximate coordinates of KGN-3 resulting from first level static-dynamic densification.

Point	Code	N [m]	E [m]
1	SKP106	9965867.1352	234568.2516
2	SKP108	9930827.7520	244847.9533
3	SKP210	9920928.2259	263626.6790
4	SKP211	9954810.1409	266508.2800
5	SKP212	9953913.1636	290496.1621
6	SKP213	9925562.9660	293406.8300
7	SKP214	9936363.2753	327853.2967
8	SKP215	9896895.4409	316442.8767
9	120.S.1	9963932.0740	269718.8331
10	120.S.2	9949485.0603	261852.8076
11	121.S.1	9954495.4231	280104.9386
12	121.S.2	9945383.6244	286178.5419
13	121.S.3	9950678.8601	304751.1133
14	134.S.7	9928278.3563	262387.1368
15	134.S.8	9940076.4011	263194.0270
16	134.S.9	9941138.9546	274039.4163
17	135.S.7	9928572.7653	279001.9105
18	135.S.3	9910855.9465	297806.0599
19	135.S.4	9901567.8690	308367.6219
20	135.S.5	9916117.7107	312499.3566
21	135.S.9	9936062.8920	300024.1884
22	135.S.8	9923580.1198	302009.6385
23	135.S.10	9936916.5030	314544.8250
24	135.T.6	9942628.0170	299337.1600
25	121.T.5	9947392.3200	297152.6800
26	121.T.4	9952372.3080	292844.9710
27	121.T.9	9949803.4490	289489.8880
28	135.T.16	9939211.6500	291072.5800
29	135.T.14	9935086.9800	288325.2680
30	135.T.13	9938187.1470	282764.6360
31	135.T.10	9930525.5700	287309.5700
32	135.T.19	9925634.8180	286423.7950
33	135.T.5	9944523.7170	293466.0020
34	135.T.3	9912294.0470	278424.7330
35	135.T.5	9915871.1200	287342.2730
36	134.T.9	9925039.1650	269920.8880
37	134.T.10	9933938.0450	275108.0630
38	134.T.5	9934586.1750	267992.0990
39	120.T.1	9945802.3470	266520.3930
40	120.T.4	9948003.1670	273206.9210
41	121.T.8	9949306.6410	277591.4440
42	121.T.10	9954204.1380	289225.7160
43	121.T.2	9955468.9760	283653.6250
44	134.T.2	9932950.1020	262087.2620
45	134.T.8	9928940.0180	253845.9110

Table 4.6: The observational dataset for the tertiary network (KGN-3).

Obs. No.	Line		Bearing °	'	"	Distance [m]
1	24	22	172	0	49.0	-
2	24	28	247	32	27.0	8943.005
3	24	25	335	22	5.0	5241.204
4	25	13	66	36	36.0	8278.938
5	25	26	319	8	25.0	6584.579
6	26	25	139	8	24.0	-
7	26	27	232	33	36.0	4225.701
8	26	42	296	50	44.0	4056.428
9	27	26	52	33	36.0	-
10	27	42	356	33	53.0	4408.611
11	27	12	216	50	26.0	5522.499
12	27	25	107	27	59.0	8033.176
13	28	12	321	35	15.0	7876.857
14	28	22	145	1	13.0	-
15	28	24	67	32	26.0	-
16	28	29	213	39	58.0	4955.889
17	29	28	33	39	59.0	-
18	29	30	299	8	26.0	6366.400
19	29	31	192	33	12.0	4673.125
20	29	22	130	3	34.0	-
21	30	29	119	8	25.0	-
22	30	17	201	22	24.0	10324.358
23	30	12	25	22	44.0	7965.175
24	30	31	149	19	23.0	8908.220
25	31	17	256	46	20.0	8534.088
26	31	30	329	19	23.0	-
27	31	32	190	15	57.0	4970.317
28	32	31	10	15	56.0	-
29	32	17	291	35	46.0	7982.225
30	32	33	20	26	47.0	-
31	33	34	205	1	5.0	-
32	33	17	222	12	4.0	-
33	33	36	230	23	27.0	-
34	33	32	200	26	47.0	-
35	34	18	94	14	37.0	-
36	34	33	25	1	4.0	-
37	34	35	68	8	35.0	9608.224
38	35	34	248	8	34.0	-
39	35	19	124	13	36.0	-
40	35	18	115	36	27.0	11603.566
41	36	34	146	17	16.0	-
42	36	14	293	15	57.0	8200.596
43	36	17	68	44	16.0	9744.325
44	36	37	30	14	17.0	-
45	36	38	348	34	41.0	9739.979
46	37	17	144	1	47.0	6629.348

Table 4.6 : continued

Obs. No.	Line		Bearing °	'	"	Distance [m]
47	37	38	275	12	15.0	7145.419
48	37	36	210	14	16.0	-
49	37	16	351	33	31.0	7279.774
50	38	37	95	12	15.0	7145.489
51	38	44	254	30	49.0	6127.302
52	38	16	42	42	9.0	8916.830
53	39	38	172	31	29.0	11312.313
54	39	15	210	9	12.0	6622.029
55	39	16	121	48	27.0	8847.780
56	39	10	308	16	24.0	5945.206
57	40	41	73	26	35.0	4574.306
58	40	9	347	38	54.0	-
59	41	40	253	26	36.0	-
60	41	11	25	50	45.0	5765.561
61	41	10	270	38	58.0	-
62	42	27	176	33	53.0	4408.616
63	42	43	282	47	20.0	5713.809
64	43	42	102	47	21.0	-
65	43	9	301	16	18.0	-
66	44	38	74	30	49.0	6127.310
67	44	15	8	49	40.0	7211.802
68	44	45	244	3	12.0	9165.181
69	45	14	94	25	46.0	8566.896
70	45	44	64	3	11.0	-

The estimation process at this stage involved the use of the observation sets and each of the sets of approximate coordinates in the respective models for purposes of determining third order point coordinates (as detailed in section 4.2.2). The standard error for distances were taken as ±0.05m and the angular observations were assumed to have errors of ±1.5" as stupilated for example in *Aduol [1981] and Gosset [1959]*.

CHAPTER FIVE

COMPUTATIONS

Presented in this Chapter are the experiments undertaken to determine parameters under different models as discussed in chapter three. An outline of the characterization of precision which will be useful in the analysis of the determined results is discussed and computer programs used in the study are also outlined.

5.1 Densification Experiments

The fundamental datum was defined by adjusting the primary network (KGN-1) within the framework of a free network. Densification was then carried out in modular basis, by using the values determined from this adjustment as part of the data for subsequent densifications using the different models discussed in chapter 3.

First densification was done by applying the concepts of the static model; this experiment was designated 'Experiment A'. In this experiment the observational data-sets in Tables 4.3 and 4.4 were used for first level densification of KGN-1 to yield KGN-2, on the assumption [from section (3.1)] that at this level of densification all the points in KGN-1 are held fixed and erroless. Likewise, the second level densification was done by holding the adjusted coordinates of KGN-2 as fixed.

Using the same observation sets, the experiment was repeated but this time treating the coordinates of KGN-1 as stochastic. In this experiment, designated 'Experiment B', the concepts of the dynamic model in equation (3-27) were used by basically incorporating the determined weight matrix of the free network adjustment of KGN-1 in the first level densification. The adjusted coordinates and the corresponding covariance matrix were then used in the second level densification to determine parameters for points in KGN-3.

The third experiment, designated 'Experiment C', involved first and second level densification by using the concepts of the static-dynamic model. In this particular case the coordinates and covariance matrices of KGN-1 and KGN-2 were in the estimation process while at the same time being considered fixed, see (3.3).

5.2 <u>Precision Criteria for Analysis of the Results</u>.

The densification models are used to estimate unknown parameters, which are normally corrections to approximate coordinates in a given geodetic network. To adequately assess the quality of adjusted coordinates of a network, representation of precision measures is very important. In this study the main tool to aid analysis will be the network's a posteriori variance-covariance matrix as given in equations (3-4), (2-19), and (3-11). From this, presentation of precision may take different forms as outlined below.

5.2.1 Standard errors of the estimates

Positional standard errors of the estimates, i.e. point coordinates, are obtained by taking the square roots of the diagonal elements of the variance-covariance matrix in equation (3-4) above. These constitute two values, one in E direction and the other in the N direction. Normally the sizes of the standard errors are dependent on the chosen datum *[e.g. Cross 1979; Illner 1985, and Aduol 1996]*. For this reason positional standard errors are not representative enough, especially for fixed networks, in which, positional standard errors tend to give the impression that the estimated parameters are more accurate than they actually are [e.g. *Aduol 1993*]. However for free networks where datum is defined over approximate coordinates without fixing any particular point, the positional standard errors are fairly representative and thus very vital for analysis.

5.2.2 Circular probable error(CPE)

In the case of two-dimensional networks where one is interested only in one measure of accuracy instead of the two componets of the positional error, the two are combined to give a vector sum $\overline{\sigma_c}$ referred to as "Circular Probable Error", this is also referred to in *[Mikhail 1976, pg.33]* as the "radial standard error", and by *Aduol [1981 pg.46]* as "positional error sphere" for a three-dimensional case. $\overline{\sigma_c}$ is given as

$$\overline{\sigma_c} = \left[(\sigma_E^2 + \sigma_N^2)/2\right]^{1/2} \tag{5-3}$$

where σ_E and σ_N are standard errors in E and N respectively. For n parameters we have that,

$$\overline{\sigma_c} = \left[\frac{\sigma_1^2 + \sigma_2^2 + \ldots + \sigma_n^2}{n}\right]^{1/2} \tag{5-4}$$

where σ_i, $[i=1,2,3,\ldots..n]$ are respective standard errors. In the present study single values $\overline{\sigma_E}$ and $\overline{\sigma_N}$ have been computed for eastings and northings for each densification model, these are then compared.

5.2.3 Standard error ellipses

By using the parameters of the variance-covariance matrix of the adjusted coordinates, appropriate terms for error ellipses may be calculated. There are two types of error ellipses within networks. The first is the standard error ellipse for a point which reflects how accurately a point has been positioned. The second type is the relative error ellipse which represents relative accuracy between points in a network. In the present study we shall use point ellipses as the interest of the study is to obtain how accurately points are fixed. The theory behind the deriviation of error ellipses which is extensive, is adequately covered in most geodesy texts for example *Mikhail 1976 pp.28-35; Cooper, 1987 pp.130-135*.

The parameters for error ellipses are given as;

$$a^2 = (1/2)(\sigma_E^2 + \sigma_N^2) + [(1/4)(\sigma_E^2 - \sigma_N^2)^2 + \sigma_{EN}^2]^{1/2}$$
$$b^2 = (1/2)(\sigma_E^2 + \sigma_N^2) + [(1/4)(\sigma_E^2 + \sigma_N^2)^2 + \sigma_{EN}^2]^{1/2} \quad (5\text{-}5)$$

$$\alpha = 1/2 \arctan\left[\frac{2\sigma_{EN}}{(\sigma_E^2 - \sigma_N^2)}\right] \quad (5\text{-}6)$$

where

σ_E^2 is the variance of the easting;

σ_N^2 is the variance of the northing;

σ_{EN} is the covariance between easting and northing;

a is the semi-major axis;

b is the semi-minor axis;

α is the bearing of the semi-major axis.

The probability of a point falling inside the standard error ellipse is 0.394 *[Aduol 1996]*. The main advantage of using the error ellipse as an analytical tool is that the values from which it is derived represent all the parameters of the variance-covariance matrix as opposed to positional standard errors which only incorporate variances and assumes that there is totally no correlation between the parameters which is not quite correct. In the present study the parameters of the error ellipses are computed and single precision criteria circular probable errors are computed for different tests. These are then compared for various densification models.

5.2.4 Mean shifts

These are vectors determined from the final adjusted coordinates of points in a network adjusted using different methods or under different circumstances. They give a measure of displacement between points which can be used to analyse networks.

5.3 Computer Programs

Although separate program segments were written for each task i.e. free network adjustment, static, dynamic, and static-dynamic densification adjustment, this section explains the program in broad terms only, thus they have been combined into two main units viz., FREE.FOR and DENSITY.FOR with corresponding subroutines as listed in appendices A.1, A.2, and A.3 respectively. The programs were coded in FORTRAN 77 and computations were carried out on a VAX/VMS 6310 main-frame computer.

5.3.1 Program FREE.FOR

This program uses the concept of free network adjustment (see, section 2.4) to adjust the primary network, which is used as the defined datum for subsequent static, dynamic, and static-dynamic first and second geodetic network densifications. The flow chart 1 in Appendix A.4 shows systematic stages of freenetwork adjustment.

5.3.2 Program DENSITY.FOR

In this program all models of densification are considered. It consists of different modules which were tackled separetly, tested, before being linked together.

5.3.2.1 Module one

In this module, the steering data is prompted for, reduced observations, stations occupied during data acquisition and provisional coordinates are read into the computer memory, lastly all output and computational matrices are initialized to zero. Flow chart 2 in Appendix A.5 has been drafted to aid in understanding the working of this module.

5.3.2.2 Module Two

This module forms the design matrix A, the vector y of observations and the weight matrix W from reduced observations in (5.3.2.1), their contents are then used to adjust densification networks depending on the model which is determined accordingly from the steering data input in module one. The module then computes a posteriori variance components.

5.3.2.3 Module Three

In this module the determined data in (5.3.2.2) are used in network analysis where the variance covariance matrix is determined by using equations (3-11), (3-19), and (3-42). From the variance covariance matrix a posteriori standard errors for observation sets are determined, correspondingly error ellipse parameters for the adjusted coordinates are determined. Finaly this module outputs the results of each mode of densification. Flow chart 3 in Appendix A.6 depicts to working procedure of modules two and three.

CHAPTER SIX

RESULTS

The results for the free network adjustment of the primary network are presented in section (6.1). Results for Experiment A in which densification was carried out using the static model are given in section (6.2) while section (6.3) outlines results obtained from densification using the dynamic model in Experiment B. The results for Experiment C in which parameters were determined from densification by the static-model are given in section (6.4). Each of sections (6.2), (6.3), and (6.4) consists of two subsections under which results for the first and second levels of densification are presented respectively. The Tables indicate the estimated parameters δE and δN for each densification point, their corresponding standard errors σ_E and σ_N, computed from equations (3-19), (3-32), and (3-43). For each point, the circular probable error is used to assess the quality of estimates (i.e mean standard error in E, mean standard error in N, and mean standard error for both E and N computed from equation (4-3)) is also presented in Tables (6.2), (6.3), and (6.4). Also presented are error ellipse parameters (max σ, min σ, α for semi-major axis, semi-minor axis, and orientation of semi-major axis respectively) computed from equations (4-5) and (4-6). The results presented in this chapter are part of those output in the running of the programs as outlined in Chapter 5, the rest of the results are as shown in appendix B.

6.1 Results for primary network adjustment

These are the results determined from the free network adjustment of the primary network, using data in Tables (4.1) and (4.2). These were later used as representing the defined datum base upon which the secondary net was built. The diagrammatic depiction of the determined point error ellipses for the eight primary stations are shown in Fig.4.

Table 6.1: Results for free network adjustment (2 Iterations)

ST.	δE[m]	δN[m]	σ_E[m]	σ_N[m]	max σ[m]	min σ[m]	α °	α "
1	0.0006	-0.0128	0.0014	0.0083	0.00153	0.00793	8	11
2	-0.0067	0.0120	0.0043	0.0077	0.00434	0.00805	39	35
3	-0.0200	0.0189	0.0081	0.0044	0.00823	0.00468	316	18
4	-0.0220	-0.0050	0.0033	0.0028	0.00325	0.00270	44	42
5	-0.0389	-0.0044	0.0025	0.0032	0.00253	0.00376	315	15
6	-0.0410	0.0157	0.0066	0.0084	0.00689	0.00829	44	32
7	-0.0653	0.0080	0.0042	0.0049	0.00469	0.00493	44	52
8	-0.0573	0.0359	0.0110	0.0074	0.01227	0.00757	315	20

$\overline{\sigma_E}$ =0.005973 $\overline{\sigma_N}$ =0.006274 $\overline{\sigma_C}$ =0.0061253 $\hat{\sigma}_0^2$ =0.98688

Table 6.1a: Estimated coordinates - free network adjustment

ST.	Provisional coordinates E[m]	N[m]	Adjusted coordinates E[m]	N[m]
1	234568.251	9965867.148	234568.2516	9965867.1352
2	244847.960	9930827.740	244847.9533	9930827.7520
3	263626.699	9920928.207	263626.6790	9920928.2259
4	266508.302	9954810.146	266508.2800	9954810.1410
5	290496.201	9953913.168	290496.1621	9953913.1636
6	293406.871	9925562.951	293406.8300	9925562.9667
7	327853.362	9936363.270	327853.2967	9936363.2780
8	316442.934	9896895.405	316442.8767	9896895.4409

6.2 Results for Experiment A

These results were determined by using the estimated coordinates from the free network adjustment of the primary network in Table (6.1a) and the observation set in Table (4.4) in the adjustment for determining coordinates of secondary and tertiary net stations using the static densification model.

6.2.1 First level of densification

This estimation was determined with points 1-8 held fixed and errorless. Point error ellipses for these results are as shown in Fig.5.

Table 6.2: Results for first level densification using the static model (3 iterations)

ST	δE[m]	δN[m]	σ_E[m]	σ_N[m]	max σ[m]	min σ[m]	α °	α "
9	-0.001401	0.000090	0.0101	0.0073	0.0113	0.0072	325	35
10	-0.000963	0.000175	0.0078	0.0074	0.0078	0.0095	342	29
11	-0.000022	0.000001	0.0115	0.0098	0.0105	0.0090	304	20
12	0.000101	-0.000010	0.0082	0.0078	0.0080	0.0088	24	4
13	-0.001111	-0.000341	0.0098	0.0104	0.0100	0.0101	359	43
14	0.000220	-0.000427	0.0099	0.0095	0.0108	0.0093	334	4
15	-0.000169	-0.000773	0.0064	0.0079	0.0067	0.0070	357	59
16	-0.001000	0.000266	0.0104	0.0103	0.0108	0.0120	43	53
17	-0.001830	-0.000125	0.0111	0.0093	0.0099	0.0087	349	24
18	0.000118	-0.000002	0.0107	0.0120	0.0113	0.0100	26	4
19	0.000876	-0.001251	0.0066	0.0057	0.0070	0.0062	29	53
20	-0.000771	0.000089	0.0123	0.0109	0.0129	0.0112	354	4
21	0.001003	0.000999	0.0091	0.0086	0.0093	0.0090	325	3
22	0.003433	0.000503	0.0116	0.0120	0.0123	0.0118	37	59
23	0.000001	-0.000024	0.0102	0.0092	0.0101	0.0090	355	3

$\overline{\sigma_E}$ =0.009864 $\overline{\sigma_N}$ =0.009367 $\overline{\sigma_C}$ =0.00961871 $\hat{\sigma}_0^2$ =1.00015

Table 6.2a: Estimated coordinates-static model

ST.	Provisional coordinate E[m]	N[m]	Adjusted coordinates E[m]	N[m]
9	269718.833	9963932.073	269718.8316	9963932.0730
10	261852.810	9949485.060	261852.8090	9949485.0600
11	280104.938	9954495.422	280104.9380	9954495.4220
12	286178.541	9945383.623	286178.5410	9945383.6230
13	304751.112	9950678.859	304751.1109	9950678.8593
14	262387.137	9928278.357	262387.1372	9928278.3566
15	263194.029	9940076.402	263194.0288	9940076.4028
16	274039.416	9941138.955	274039.4160	9941138.9550
17	279001.910	9928572.766	279001.9110	9928572.7633
18	297806.058	9910855.948	297806.0581	9910855.9480
19	308367.620	9901567.871	308367.6209	9901567.8722
20	312499.355	9916117.711	312499.3542	9916117.7110
21	300024.187	9936062.892	300024.1871	9936062.8921
22	302009.637	9923580.120	302009.6374	9923580.1205
23	314544.823	9936916.503	314544.8230	9936916.5030

6.2.2 Second level of densification

These results were obtained during adjustment by holding points 9-23 as fixed and errorless. Diagrammatic representation of the error ellipses in Table 6.7 are shown in fig.8.

Table 6.2b: Results for second level densification using the static model (3 iterations)

ST.	δE[m]	δN[m]	σ_E[m]	σ_N[m]	max σ[m]	min σ[m]	α °	"
24	0.000114	0.000029	0.0115	0.0112	0.0124	0.0105	358	23
25	0.000008	0.000001	0.0104	0.0078	0.0070	0.0106	25	34
26	0.002465	0.000108	0.0106	0.0120	0.0124	0.0098	343	23
27	0.000444	0.000003	0.0119	0.0122	0.0109	0.0103	46	46
28	-0.000352	0.000001	0.0111	0.0104	0.0112	0.0105	39	29
29	-0.000703	-0.000542	0.0116	0.0099	0.0115	0.0098	321	18
30	-0.001785	-0.000004	0.0109	0.0100	0.0108	0.0096	345	20
31	-0.000003	-0.001734	0.0114	0.0097	0.0115	0.0097	327	55
32	-0.000003	0.000000	0.0113	0.0102	0.0108	0.0101	323	34
33	-0.000012	-0.000013	0.0126	0.0110	0.0130	0.0103	32	15
34	-0.001563	0.000091	0.0106	0.0122	0.0123	0.0104	323	10
35	-0.000998	-0.000102	0.0108	0.0124	0.0124	0.0106	343	21
36	0.002497	-0.000118	0.0106	0.0101	0.0105	0.0101	339	46
37	0.001000	-0.000004	0.0104	0.0096	0.0109	0.0106	310	0
38	-0.000843	-0.001765	0.0108	0.0098	0.0109	0.0096	32	27
39	-0.000062	-0.000957	0.0115	0.0112	0.0119	0.0101	37	01
40	0.000091	-0.002382	0.0135	0.0122	0.0131	0.0120	58	22
41	0.000869	0.002420	0.0112	0.0123	0.0098	0.0119	331	42
42	0.001773	0.000010	0.0133	0.0130	0.0130	0.0128	336	45
43	-0.000019	0.000216	0.0112	0.0108	0.0128	0.0107	6	56
44	0.003618	0.001189	0.0123	0.0097	0.0098	0.0098	331	46
45	-0.000850	-0.000253	0.0097	0.0106	0.0098	0.0123	9	10

$\overline{\sigma_E}$ =0.0113641 $\overline{\sigma_N}$ =0.0109022 $\overline{\sigma_c}$ =0.0111355 $\hat{\sigma}_0^2$ =1.0000

Table 6.2c: Estimated coordinates - static model

ST.	Provisional coordinates E[m]	N[m]	Adjusted coordinates E[m]	N[m]
24	299337.160	9942628.017	299337.1601	9942628.0170
25	297152.680	9947392.320	297152.6800	9947392.3200
26	292844.971	9952372.308	292844.9957	9952372.3081
27	289489.888	9949803.449	289489.8884	9949803.4490
28	291072.580	9939211.650	291072.5796	9939211.6500
29	288325.268	9935086.980	288325.2673	9935086.9794
30	282764.636	9938187.147	282764.6342	9938187.1470
31	287309.570	9930525.570	287309.5700	9930525.5683
32	286423.795	9925634.818	286423.7950	9925634.8180
33	293466.002	9944523.717	293466.0020	9944523.7170
34	278424.733	9912294.047	278424.7314	9912294.0460
35	287342.273	9915871.120	287342.2720	9915871.1190
36	269920.888	9925039.165	269920.8905	9925039.1649
37	275108.063	9933938.045	275108.0620	9933938.0450
38	267992.099	9934586.175	267992.0982	9934586.1732
39	266520.393	9945802.347	266520.3930	9945802.3461
40	273206.921	9948003.167	273206.9210	9948003.1646
41	277591.444	9949306.641	277591.4448	9949306.6434
42	289225.716	9954204.138	289225.7173	9954204.1380
43	283653.625	9955468.976	283653.6250	9955468.9762
44	262087.262	9932950.102	262087.2656	9932950.1039
45	253845.911	9928940.018	253845.9101	9928940.0177

6.3 Results for Experiment B

These results were obtained by considering the stochasticity of the higher order points in the network densification, using data from Tables 4.3 and 4.4.

6.3.1 First level densification

Results obtained by using the variance-covariance matrix determined from the freenet adjustment of points 1-8 within a dynamic adjustment. The corresponding error ellipses for these results are given in Figures 6 and 9.

Table 6.3: Results for first level densification using the dynamic model (3 iterations)

	δE[m]	δN[m]	σ_E[m]	σ_N[m]	max σ[m]	min σ[m]	α ° "
1	0.00012	-0.00005	0.0014	0.0020	0.0020	0.00139	6 38
2	0.00023	0.00000	0.0043	0.0077	0.0077	0.00430	40 0
3	0.00155	0.00006	0.0064	0.0032	0.0028	0.00666	320 32
4	0.00010	-0.00105	0.0022	0.0018	0.0023	0.00165	45 44
5	0.00009	-0.00001	0.0017	0.0018	0.0018	0.00159	317 5
6	-0.00004	0.00087	0.0057	0.0072	0.0077	0.00567	41 33
7	-0.00014	0.00004	0.0029	0.0017	0.0030	0.00154	48 45
8	-0.00109	0.00130	0.0094	0.0083	0.0088	0.00971	29 34
9	0.00118	-0.00006	0.0121	0.0121	0.0124	0.01164	316 5
10	0.00090	-0.00328	0.0124	0.0117	0.0125	0.01164	349 3
11	0.00472	-0.00163	0.0118	0.0118	0.0119	0.01165	315 51
12	0.00287	0.00081	0.0117	0.0117	0.0117	0.01165	348 32
13	-0.00007	-0.00021	0.0119	0.0117	0.0120	0.01169	22 33
14	0.00348	0.00116	0.0124	0.0119	0.0126	0.01162	29 7
15	0.00015	0.00062	0.0124	0.0117	0.0124	0.01165	7 56
16	0.00033	0.00099	0.0120	0.0117	0.0120	0.01165	9 4
17	0.00000	0.00574	0.0118	0.0119	0.0120	0.01162	323 6
18	-0.00212	0.00010	0.0117	0.0127	0.0127	0.01166	13 40
19	-0.00508	0.00362	0.0120	0.0131	0.0133	0.01163	23 43
20	-0.00133	0.00499	0.0122	0.0124	0.0128	0.01167	39 28
21	-0.00600	0.00079	0.0118	0.0117	0.0119	0.01161	325 4
22	-0.00095	0.00286	0.0118	0.0121	0.0122	0.01160	31 37
23	-0.00108	0.00002	0.0123	0.0117	0.0123	0.01165	344 21

$\overline{\sigma_E}$ =0.0101436 $\overline{\sigma_N}$ =0.0101325 $\overline{\sigma_c}$ =0.010138 $\hat{\sigma}_0^2$ =0.99357

Table 6.3a: Estimated coordinates - dynamic model

ST.	Provisional coordinates E[m]	N[m]	Adjusted coordinates E[m]	N[m]
1	234568.252	9965867.135	234568.2517	9965867.1352
2	244847.953	9930827.752	244847.9535	9930827.7520
3	263626.679	9920928.226	263626.6805	9920928.2259
4	266508.280	9954810.141	266508.2801	9954810.1398
5	290496.162	9953913.164	290496.1621	9953913.1636
6	293406.830	9925562.966	293406.8300	9925562.9669
7	327853.297	9936363.275	327853.2971	9936363.2753
8	316442.877	9896895.441	316442.8756	9896895.4422
9	269718.833	9963932.073	269718.8341	9963932.0730
10	261852.810	9949485.060	261852.8109	9949485.0567
11	280104.938	9954495.422	280104.9552	9954495.4204
12	286178.541	9945383.623	286178.5438	9945383.6238
13	304751.112	9950678.859	304751.1120	9950678.8588
14	262387.137	9928278.357	262387.1408	9928278.3582
15	263194.029	9940076.402	263194.0291	9940076.4026
16	274039.416	9941138.955	274039.4163	9941138.9560
17	279001.910	9928572.766	279001.9100	9928572.7716

Table 6.3a: continued

ST.	Provisional coordinates E[m]	N[m]	Adjusted coordinates E[m]	N[m]
18	297806.058	9910855.948	297806.0558	9910855.9481
19	308367.620	9901567.871	308367.6149	9901567.8782
20	312499.355	9916117.711	312499.3537	9916117.7160
21	300024.187	9936062.892	300024.1810	9936062.8928
22	302009.637	9923580.120	302009.6369	9923580.1286
23	314544.823	9936916.503	314544.8122	9936916.5030

6.3.2 second level densification

Table 6.3b: Results for second level densification using the dynamic model (2 iterations)

ST	δE[m]	δN[m]	σ_E[m]	σ_N[m]	max σ[m]	min σ[m]	α °	"
1	0.000742	0.000173	0.0011	0.0023	0.0023	0.0 11	54	23
2	0.000086	0.001800	0.0043	0.0077	0.0077	0.0043	2	55
3	0.000248	0.000306	0.0064	0.0032	0.0064	0.0032	345	06
4	0.000613	0.000718	0.0022	0.0018	0.0022	0.0018	13	44
5	0.000033	0.000111	0.0017	0.0018	0.0018	0.0017	344	35
6	0.000054	0.000541	0.0057	0.0072	0.0072	0.0057	325	58
7	0.000582	0.000694	0.0029	0.0017	0.0029	0.0017	307	09
8	0.000781	0.000834	0.0094	0.0083	0.0094	0.0083	0	00
9	-0.004275	0.001306	0.0115	0.0110	0.0119	0.0105	324	41
10	0.005703	-0.004020	0.0098	0.0081	0.0118	0.0069	322	36
11	0.005550	-0.001142	0.0115	0.0115	0.0116	0.0114	43	15
12	-0.001900	0.002633	0.0057	0.0060	0.0060	0.0057	359	28
13	0.006594	-0.006060	0.0117	0.0109	0.0118	0.0108	19	11
14	0.001058	-0.001318	0.0112	0.0066	0.0119	0.0051	337	33
15	-0.004666	0.003181	0.0088	0.0106	0.0116	0.0074	328	10
16	-0.003155	0.003161	0.0055	0.0054	0.0059	0.0050	318	46
17	-0.002417	0.001282	0.0057	0.0046	0.0058	0.0043	339	49
18	0.000276	-0.001336	0.0110	0.0081	0.0113	0.0076	341	23
19	-0.000470	-0.001135	0.0108	0.0094	0.0123	0.0073	323	14
20	0.000044	0.000003	0.0122	0.0124	0.0124	0.0122	0	00
21	0.000034	0.000076	0.0118	0.0117	0.0118	0.0117	0	00
22	-0.002292	0.003294	0.0085	0.0090	0.0110	0.0057	42	29
23	0.000001	0.000021	0.0123	0.0117	0.0123	0.0117	0	00
24	-0.002045	0.001565	0.0174	0.0170	0.0187	0.0153	41	30
25	0.002304	-0.007918	0.0191	0.0104	0.0109	0.0184	28	43
26	-0.003204	-0.001547	0.0162	0.0183	0.0190	0.0151	331	48
27	-0.007496	-0.004835	0.0182	0.0186	0.0104	0.0157	317	25
28	-0.001295	0.001869	0.0164	0.0154	0.0165	0.0153	347	05
29	-0.001046	0.002242	0.0171	0.0146	0.0171	0.0145	2	53
30	-0.001867	0.002697	0.0161	0.0149	0.0161	0.0148	9	18
31	-0.001196	0.001567	0.0169	0.0144	0.0170	0.0143	349	36
32	-0.001327	0.000850	0.0167	0.0149	0.0171	0.0143	336	13
33	-0.000295	0.003625	0.0184	0.0162	0.0191	0.0150	28	06
34	-0.002513	-0.001128	0.0156	0.0180	0.0182	0.0153	16	49
35	-0.001237	-0.000617	0.0159	0.0183	0.0183	0.0158	7	10
36	-0.003768	0.000752	0.0158	0.0150	0.0158	0.0149	5	22
37	-0.002995	0.002080	0.0155	0.0145	0.0156	0.0143	343	40
38	-0.004058	0.002177	0.0160	0.0145	0.0162	0.0143	342	37
39	-0.004278	0.003854	0.0163	0.0165	0.0176	0.0148	42	39
40	-0.002666	-0.006036	0.0209	0.0182	0.0213	0.0176	20	51

Table 6.3b: continued

ST	δE[m]	δN[m]	σ_E[m]	σ_N[m]	max σ[m]	min σ[m]	α ° "
41	0.004170	-0.004002	0.0131	0.0180	0.0133	0.0176	12 49
42	-0.007832	0.000796	0.0185	0.0192	0.0192	0.0184	8 01
43	-0.000803	-0.000801	0.0165	0.0152	0.0188	0.0176	339 45
44	-0.004853	0.001962	0.0182	0.0143	0.0182	0.0141	353 36
45	-0.009858	-0.000476	0.0142	0.0156	0.0145	0.0146	12 38

$\overline{\sigma_E}$ =0.0135641 $\overline{\sigma_N}$ =0.0124741 $\overline{\sigma_C}$ =0.0130305 $\hat{\sigma}_0^2$ =0.99955

Table 6.3c: **Estimated coordinates - dynamic model**

ST.	Provisional coordinates E[m]	Provisional coordinates N[m]	Adjusted coordinates E[m]	Adjusted coordinates N[m]
1	234568.252	9965867.135	234568.2524	9965867.1354
2	244847.953	9930827.752	244847.9535	9930827.7538
3	263626.681	9920928.226	263626.6820	9920928.2262
4	266508.280	9954810.114	266508.2807	9954810.1145
5	290496.162	9953913.164	290496.1621	9953913.1637
6	293406.830	9925562.967	293406.8300	9925562.9676
7	327853.297	9936363.275	327853.2976	9936363.2759
8	316442.876	9896895.442	316442.8763	9896895.4430
9	269718.834	9963932.073	269718.8298	9963932.0743
10	261852.811	9949485.057	261852.8166	9949485.0527
11	280104.955	9954495.420	280104.9607	9954495.4193
12	286178.544	9945383.624	286178.5419	9945383.6264
13	304751.112	9950678.859	304751.1186	9950678.8527
14	262387.141	9928278.358	262387.1419	9928278.3569
15	263194.029	9940076.403	263194.0244	9940076.4058
16	274039.416	9941138.956	274039.4131	9941138.9592
17	279001.910	9928572.772	279001.9100	9928572.7762
18	297806.056	9910855.948	297806.0561	9910855.9468
19	308367.615	9901567.878	308367.6144	9901567.8771
20	312499.354	9916117.716	312499.3537	9916117.7160
21	300024.181	9936062.893	300024.1810	9936062.8928
22	302009.637	9923580.129	302009.6346	9923580.1319
23	314544.812	9936916.503	314544.8122	9936916.5030
24	299337.160	9942628.017	299337.1580	9942628.0186
25	297152.680	9947392.320	297152.6823	9947392.3121
26	292844.971	9952372.308	292844.9678	9952372.3065
27	289489.888	9949803.449	289489.8805	9949803.4442
28	291072.580	9939211.650	291072.5787	9939211.6519
29	288325.268	9935086.980	288325.2670	9935086.9822
30	282764.636	9938187.147	282764.6341	9938187.1497
31	287309.570	9930525.570	287309.5688	9930525.5716
32	286423.795	9925634.818	286423.7937	9925634.8188
33	293466.002	9944523.717	293466.0017	9944523.7206
34	278424.730	9912294.047	278424.7305	9912294.0459
35	287342.273	9915871.120	287342.2718	9915871.1194
36	269920.888	9925039.165	269920.8842	9925039.1658
37	275108.063	9933938.045	275108.0600	9933938.0471
38	267992.099	9934586.175	267992.0949	9934586.1772
39	266520.393	9945802.347	266520.3887	9945802.3509
40	273206.921	9948003.167	273206.9183	9948003.1610
41	277591.444	9949306.641	277591.4482	9949306.6370
42	289225.716	9954204.138	289225.7082	9954204.1388
43	283653.625	9955468.976	283653.6242	9955468.9752
44	262087.262	9932950.102	262087.2571	9932950.1040
45	253845.911	9928940.018	253845.9011	9928940.0175

6.4 Results for Experiment C

6.4.1 First level densification

Table 6.4: Results for first level densification using the static - dynamic model (1 iteration)

ST.	δE[m]	δN[m]	σ_E[m]	σ_N[m]	max σ[m]	min σ[m]	α ° "
1	0.00000	0.00000	0.0014	0.0083	0.0015	0.0079	8 11
2	0.00000	0.00000	0.0043	0.0077	0.0043	0.0080	39 35
3	0.00000	0.00000	0.0081	0.0044	0.0082	0.0046	316 18
4	0.00000	0.00000	0.0033	0.0028	0.0032	0.0027	44 42
5	0.00000	0.00000	0.0025	0.0032	0.0025	0.0037	315 15
6	0.00000	0.00000	0.0066	0.0084	0.0068	0.0082	44 32
7	0.00000	0.00000	0.0042	0.0049	0.0046	0.0049	44 52
8	0.00000	0.00000	0.0110	0.0074	0.0122	0.0075	315 20
9	0.00011	0.00095	0.0116	0.0117	0.0118	0.0114	338 11
10	-0.00239	0.00031	0.0119	0.0113	0.0119	0.0119	357 31
11	0.00054	0.00105	0.0115	0.0114	0.0115	0.0113	303 2
12	0.00081	0.00142	0.0115	0.0113	0.0115	0.0114	25 20
13	0.00156	0.00135	0.0122	0.0113	0.0122	0.0110	8 51
14	-0.00023	-0.00062	0.0119	0.0118	0.0122	0.0111	342 41
15	-0.00205	-0.00090	0.0118	0.0114	0.0119	0.0116	20 17
16	0.00027	-0.00043	0.0115	0.0114	0.0116	0.0112	25 13
17	0.00049	-0.00061	0.0115	0.0118	0.0118	0.0112	349 50
18	0.00128	-0.00136	0.0119	0.0127	0.0129	0.0116	26 50
19	0.00172	-0.00175	0.0123	0.0132	0.0137	0.0114	32 26
20	0.00189	-0.00110	0.0126	0.0124	0.0132	0.0119	317 39
21	0.00137	-0.00027	0.0119	0.0115	0.0120	0.0111	336 40
22	0.00145	-0.00177	0.0120	0.0120	0.0125	0.0112	315 36
23	0.00198	-0.00023	0.0127	0.0115	0.0127	0.0113	345 25

$\overline{\sigma_E}$ =0.0119259 $\overline{\sigma_N}$ =0.0117920 $\overline{\sigma_C}$ =0.0118591 $\hat{\sigma}_0^2$ =0.99995

Table 6.4a: Estimated coordinates - static-dynamic model

ST.	Provisional coordinates E[m]	N[m]	Adjusted coordinates E[m]	N[m]
1	234568.251	9965867.148	234568.2516	9965867.1352
2	244847.960	9930827.740	244847.9533	9930827.7520
3	263626.699	9920928.207	263626.6790	9920928.2259
4	266508.302	9954810.146	266508.2800	9954810.1410
5	290496.201	9953913.168	290496.1621	9953913.1636
6	293406.871	9925562.951	293406.8300	9925562.9667
7	327853.362	9936363.270	327853.2967	9936363.2780
8	316442.934	9896895.405	316442.8767	9896895.4409
9	269718.833	9963932.073	269718.8331	9963932.0740
10	261852.810	9949485.060	261852.8076	9949485.0603
11	280104.938	9954495.422	280104.9386	9954495.4231
12	286178.541	9945383.623	286178.5419	9945383.6244
13	304751.112	9950678.859	304751.1133	9950678.8601
14	262387.137	9928278.357	262387.1368	9928278.3563
15	263194.029	9940076.402	263194.0270	9940076.4011

Table 6.4a: continued

ST.	Provisional coordinates E[m]	N[m]	Adjusted coordinates E[m]	N[m]
16	274039.416	9941138.955	274039.4163	9941138.9546
17	279001.910	9928572.766	279001.9105	9928572.7653
18	297806.058	9910855.948	297806.0599	9910855.9465
19	308367.620	9901567.871	308367.6219	9901567.8690
20	312499.355	9916117.711	312499.3566	9916117.7107
21	300024.187	9936062.892	300024.1884	9936062.8920
22	302009.637	9923580.120	302009.6385	9923580.1198
23	314544.823	9936916.503	314544.8250	9936916.5030

6.4.2 Second level densification

Table 6.4b: Results for second level densification using the static-dynamic model (1 iteration)

ST.	δE[m]	δN[m]	σ_E[m]	σ_N[m]	max σ[m]	min σ[m]	α °	"
1	0.00000	0.00000	0.0014	0.0083	0.0015	0.0079	8	11
2	0.00000	0.00000	0.0043	0.0077	0.0043	0.0080	39	35
3	0.00000	0.00000	0.0081	0.0044	0.0082	0.0046	316	18
4	0.00000	0.00000	0.0033	0.0028	0.0032	0.0027	44	42
5	0.00000	0.00000	0.0025	0.0032	0.0025	0.0037	315	15
6	0.00000	0.00000	0.0066	0.0084	0.0068	0.0082	44	32
7	0.00000	0.00000	0.0042	0.0049	0.0046	0.0049	44	52
8	0.00000	0.00000	0.0110	0.0074	0.0122	0.0075	315	20
9	0.00000	0.00000	0.0116	0.0117	0.0118	0.0114	338	11
10	0.00000	0.00000	0.0119	0.0113	0.0119	0.0119	357	31
11	0.00000	0.00000	0.0115	0.0114	0.0115	0.0113	303	2
12	0.00000	0.00000	0.0115	0.0113	0.0115	0.0114	25	20
13	0.00000	0.00000	0.0122	0.0113	0.0122	0.0110	8	51
14	0.00000	0.00000	0.0119	0.0118	0.0122	0.0111	342	41
15	0.00000	0.00000	0.0118	0.0114	0.0119	0.0116	20	17
16	0.00000	0.00000	0.0115	0.0114	0.0116	0.0112	25	13
17	0.00000	0.00000	0.0115	0.0118	0.0118	0.0112	349	50
18	0.00000	0.00000	0.0119	0.0127	0.0129	0.0116	26	50
19	0.00000	0.00000	0.0123	0.0132	0.0137	0.0114	32	26
20	0.00000	0.00000	0.0126	0.0124	0.0132	0.0119	317	39
21	0.00000	0.00000	0.0119	0.0115	0.0120	0.0111	336	40
22	0.00000	0.00000	0.0120	0.0120	0.0125	0.0112	315	36
23	0.00000	0.00000	0.0127	0.0115	0.0127	0.0113	345	25
24	0.001764	0.000009	0.0148	0.0144	0.0159	0.0130	2	37
25	0.000108	0.002401	0.0134	0.0098	0.0077	0.0129	322	28
26	0.003775	0.000198	0.0136	0.0154	0.0159	0.0126	323	55
27	0.001431	0.002703	0.0153	0.0156	0.0123	0.0132	343	29
28	-0.000882	0.000061	0.0143	0.0134	0.0144	0.0135	332	38
29	-0.000563	-0.000022	0.0149	0.0127	0.0148	0.0125	327	24
30	-0.002353	-0.005804	0.0140	0.0129	0.0139	0.0124	345	34
31	-0.000198	-0.004313	0.0147	0.0125	0.0148	0.0125	355	15
32	-0.000043	0.001481	0.0145	0.0131	0.0139	0.0130	323	34
33	-0.000152	-0.000738	0.0161	0.0141	0.0167	0.0132	320	48
34	-0.002414	0.000001	0.0136	0.0156	0.0158	0.0133	343	11
35	-0.001008	-0.001531	0.0138	0.0159	0.0159	0.0135	339	51
36	0.003430	-0.000004	0.0137	0.0130	0.0137	0.0130	21	06
37	0.000000	-0.000077	0.0134	0.0126	0.0135	0.0125	10	00
38	-0.000001	-0.002954	0.0139	0.0126	0.0140	0.0124	352	22
39	-0.000362	-0.000771	0.0141	0.0144	0.0153	0.0129	37	26
40	0.000167	-0.002892	0.0173	0.0156	0.0169	0.0154	358	42

Table 6.4b: continued

ST.	δE[m]	δN[m]	σ_E[m]	σ_N[m]	max σ[m]	min σ[m]	α °	"
41	0.000346	0.003730	0.0114	0.0157	0.0115	0.0153	31	45
42	0.002753	0.000080	0.0161	0.0167	0.0167	0.0162	16	47
43	-0.000419	0.000014	0.0144	0.0132	0.0164	0.0154	331	47
44	0.005603	0.003935	0.0158	0.0124	0.0158	0.0126	329	10
45	-0.000204	-0.000923	0.0124	0.0136	0.0126	0.0126	6	24

$\overline{\sigma_E}$ =0.0143624 $\overline{\sigma_N}$ =0.0139635 $\overline{\sigma_c}$ =0.0141643 $\hat{\sigma}_0^2$ =1.0000

Table 6.4c: Estimated coordinates - static-dynamic model

ST	Provisional coordinates E[m]	Provisional coordinates N[m]	Adjusted coordinates E[m]	Adjusted coordinates N[m]
24	299337.160	9942628.017	299337.1617	9942628.0170
25	297152.680	9947392.320	297152.6801	9947392.3200
26	292844.971	9952372.308	292844.9747	9952372.3082
27	289489.888	9949803.449	289489.8894	9949803.4517
28	291072.580	9939211.650	291072.5792	9939211.6500
29	288325.268	9935086.980	288325.2674	9935086.9800
30	282764.636	9938187.147	282764.6334	9938187.1412
31	287309.570	9930525.570	287309.7095	9930525.5355
32	286423.795	9925634.818	286423.9948	9925634.8227
33	293466.002	9944523.717	293466.0018	9944523.7162
34	278424.733	9912294.047	278424.7306	9912293.8786
35	287342.273	9915871.120	287342.2725	9915871.1216
36	269920.888	9925039.165	269920.9223	9925039.1167
37	275108.063	9933938.045	275108.0623	9933938.0454
38	267992.099	9934586.175	267992.0970	9934586.0602
39	266520.393	9945802.347	266520.3943	9945802.3469
40	273206.921	9948003.167	273206.9212	9948003.1641
41	277591.444	9949306.641	277591.4443	9949306.6446
42	289225.716	9954204.138	289225.7187	9954204.1389
43	283653.625	9955468.976	283653.6284	9955468.9764
44	262087.262	9932950.102	262087.2676	9932950.1059
45	253845.911	9928940.018	253845.9108	9928940.0169

6.5 Computed Shifts

The final coordinates for Experiments A, B, and C were compared to the initial network coordinates from Survey of Kenya. Table 6.5 shows the magnitude and direction of separations between the two sets of coordinates for each point in the network while Figures 11,12, and 13 depict this shifts graphically. Table 6.5b and Figures 14,15, and 16 show the numerical values of shifts between coordinates determined from the three experiments.

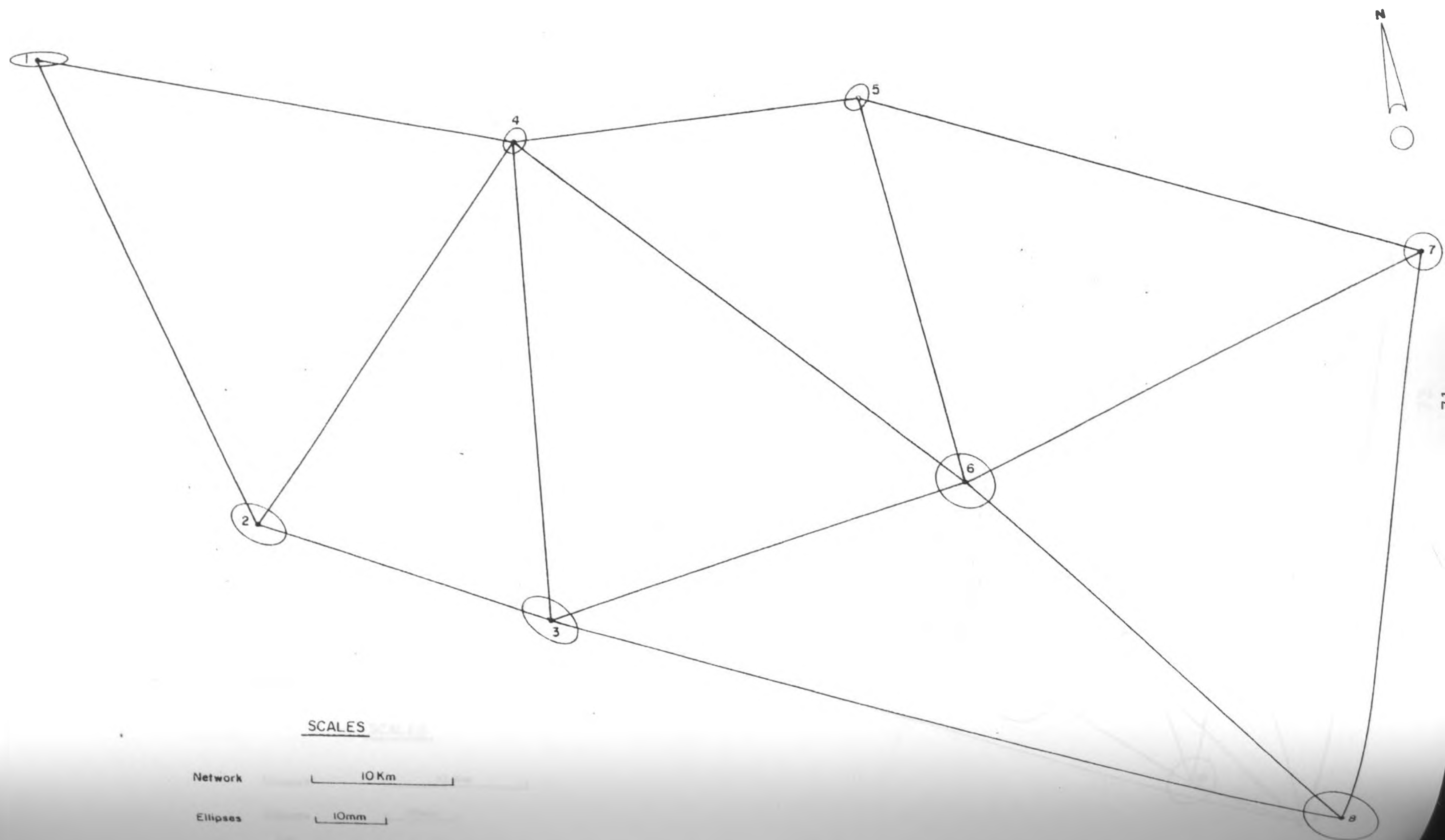
Table 6.5: Shifts between estimated and the initial network parameters

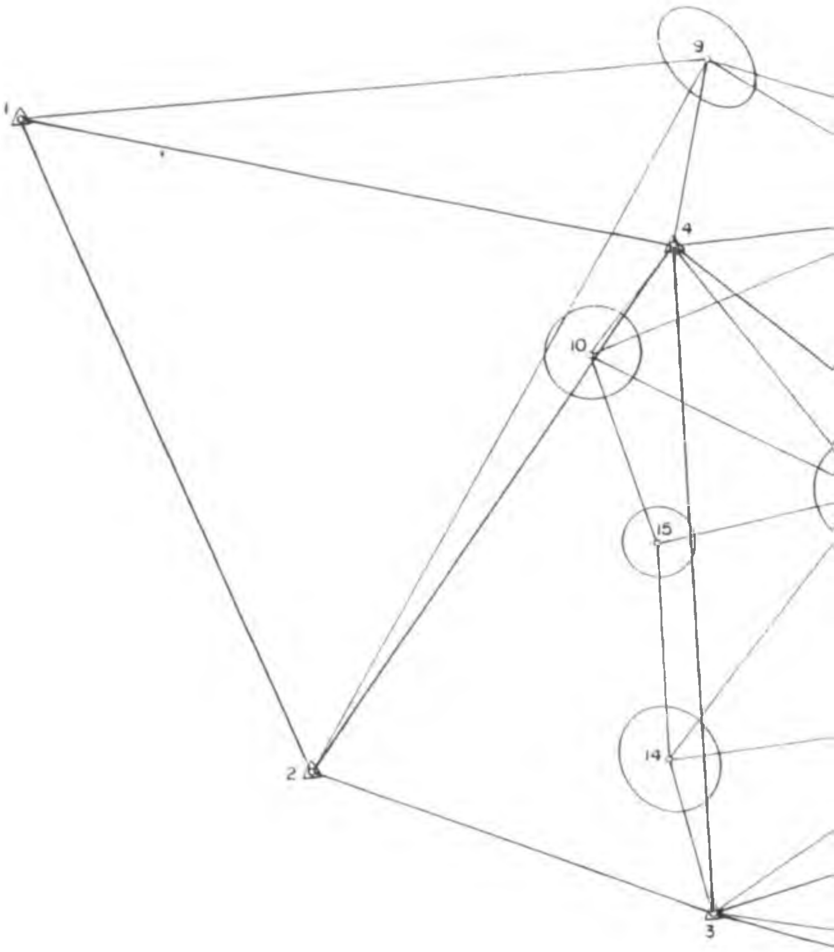
St.	Exp. A δ [mm]	Exp. A α ° "	Exp. B δ [mm]	Exp. B α ° "	Exp. C δ [mm]	Exp. C α ° "
1	12.8	272 42	12.6	276 21	12.8	272 42
2	13.7	119 10	15.2	115 13	13.7	199 10
3	27.5	136 37	25.6	131 31	27.5	136 37
4	22.6	192 42	21.2	184 02	22.6	192 42
5	39.1	186 27	39.1	186 19	39.1	186 27
6	43.7	150 54	42.3	157 58	43.7	150 54
7	65.5	175 22	64.6	174 45	65.5	175 22
8	67.6	147 56	69.1	146 38	67.1	157 53
9	1.4	180 00	3.5	157 53	1.0	84 17
10	1.0	180 00	9.3	313 07	2.4	172 52
11	-	-	22.8	354 13	1.3	61 23
12	-	-	3.5	75 10	1.6	57 16
13	1.1	164 45	9.1	316 20	1.7	40 14
14	44.7	297 34	4.9	358 51	7.3	74 03
15	9.2	102 31	5.9	140 27	2.2	204 14
16	-	-	5.1	124 37	5.0	233 08
17	2.8	289 20	10.2	90 00	8.6	30 43
18	10.1	00 00	2.4	212 17	2.4	323 37
19	1.5	53 07	8.3	132 33	2.8	133 32
20	8.0	180 00	5.2	104 34	6.3	349 23
21	14.4	45 00	6.1	101 24	5.1	352 09
22	6.4	51 21	12.1	180 00	15.3	00 00
23	-	-	1.6	180 00	2.0	00 00
24	-	-	2.6	141 20	1.7	00 00
25	-	-	8.2	290 45	-	-
26	2.7	00 13	3.5	147 23	3.7	00 00
27	4.0	00 00	8.9	154 53	7.4	03 05
28	4.0	180 00	9.2	98 08	8.0	180 00
29	9.2	139 24	2.3	116 33	3.6	180 00
30	1.8	180 00	3.3	125 08	5.8	270 00
31	17.4	256 45	2.0	126 52	3.5	174 27
32	-	-	1.5	148 23	4.7	90 00
33	-	-	3.6	94 45	8.2	104 02
34	1.9	147 59	1.0	87 24	0.8	93 55
35	2.8	135 00	1.3	153 26	1.6	72 38
36	2.5	339 42	3.8	168 06	3.4	178 39
37	1.0	180 00	3.6	145 09	4.0	90 00
38	2.1	256 03	4.8	137 47	3.9	86 03
39	9.0	90 00	5.6	151 46	4.1	97 41
40	2.4	270 00	6.6	245 33	2.9	244 06
41	2.5	71 33	5.4	316 24	4.5	67 14
42	1.3	00 00	7.9	174 08	6.6	85 44
43	2.0	90 00	7.8	189 47	2.8	161 34
44	4.1	27 49	5.3	177 07	6.8	34 54
45	1.1	198 27	9.9	183 56	9.5	255 34

Table 6.5b: Shifts between estimated parameters for experiments

	A-B			B-C			C-A		
St.	δ [mm]	α °	α "	δ [mm]	α °	α "	δ [mm]	α °	α "
1	0.001	255	57	0.001	75	57	-	-	
2	0.002	6	00	0.002	6	20	-	-	
3	-	84	17	0.003	264	17	-	-	
4	0.027	178	22	0.216	39	54	-	-	
5	-	-		-	-		-	-	
6	-	-		-	-		-	-	
7	0.002	156	48	0.002	336	48	-	-	
8	0.002	169	12	0.002	169	13	-	-	
9	0.002	305	50	0.004	94	31	-	-	
10	0.010	133	50	0.012	310	10	0.001	236	19
11	0.023	96	47	0.024	279	45	0.001	208	37
12	0.004	14	49	0.003	180	00	0.003	212	37
13	0.010	130	36	0.009	324	23	0.002	251	34
14	0.005	86	20	0.005	263	17	0.002	53	07
15	0.005	304	17	0.005	151	02	0.005	46	36
16	0.003	292	28	0.006	145	11	0.001	323	07
17	0.013	355	34	0.011	177	22	0.002	165	43
18	0.002	239	02	0.004	94	31	0.002	309	48
19	0.008	307	00	0.011	137	31	0.002	342	39
20	0.005	315	00	0.006	151	19	0.004	270	00
21	-	-		0.007	96	10	0.001	271	24
22	0.012	346	12	0.013	162	08	0.001	154	45
23	0.011	270	00	0.012	90	00	0.002	270	00
24	0.003	307	18	-	-		0.002	270	00
25	0.008	163	46	0.008	344	26	-	-	
26	0.028	266	43	0.007	76	10	0.021	270	00
27	0.009	238	43	0.012	49	52	0.003	20	20
28	0.002	334	39	0.002	165	15	-	-	
29	0.003	353	53	0.002	169	42	-	-	
30	0.003	357	52	0.009	184	42	0.006	187	51
31	0.003	303	00	0.014	104	45	0.003	190	00
32	0.002	301	36	0.004	15	45	0.019	88	54
33	0.004	355	14	0.017	179	59	-	-	
34	0.001	263	39	0.002	17	39	0.016	180	17
35	-	-		0.022	142	11	0.003	10	54
36	0.006	278	07	0.003	126	28	0.058	146	41
37	0.003	316	24	0.011	178	58	0.011	180	00
38	0.005	320	29	0.067	56	56	0.039	270	35
39	0.006	318	07	0.007	124	54	0.011	180	00
40	0.005	216	52	0.004	43	05	-	-	
41	0.007	152	01	0.009	332	50	0.001	337	22
42	0.009	275	12	0.011	89	57	0.002	57	16
43	0.001	218	39	0.004	74	44	0.003	86	39
44	0.009	270	00	0.016	79	04	0.003	45	00
45	0.009	265	43	0.009	93	30	0.001	138	34

FIG 4   POINT ERROR ELLIPSES – FREE NETWORK





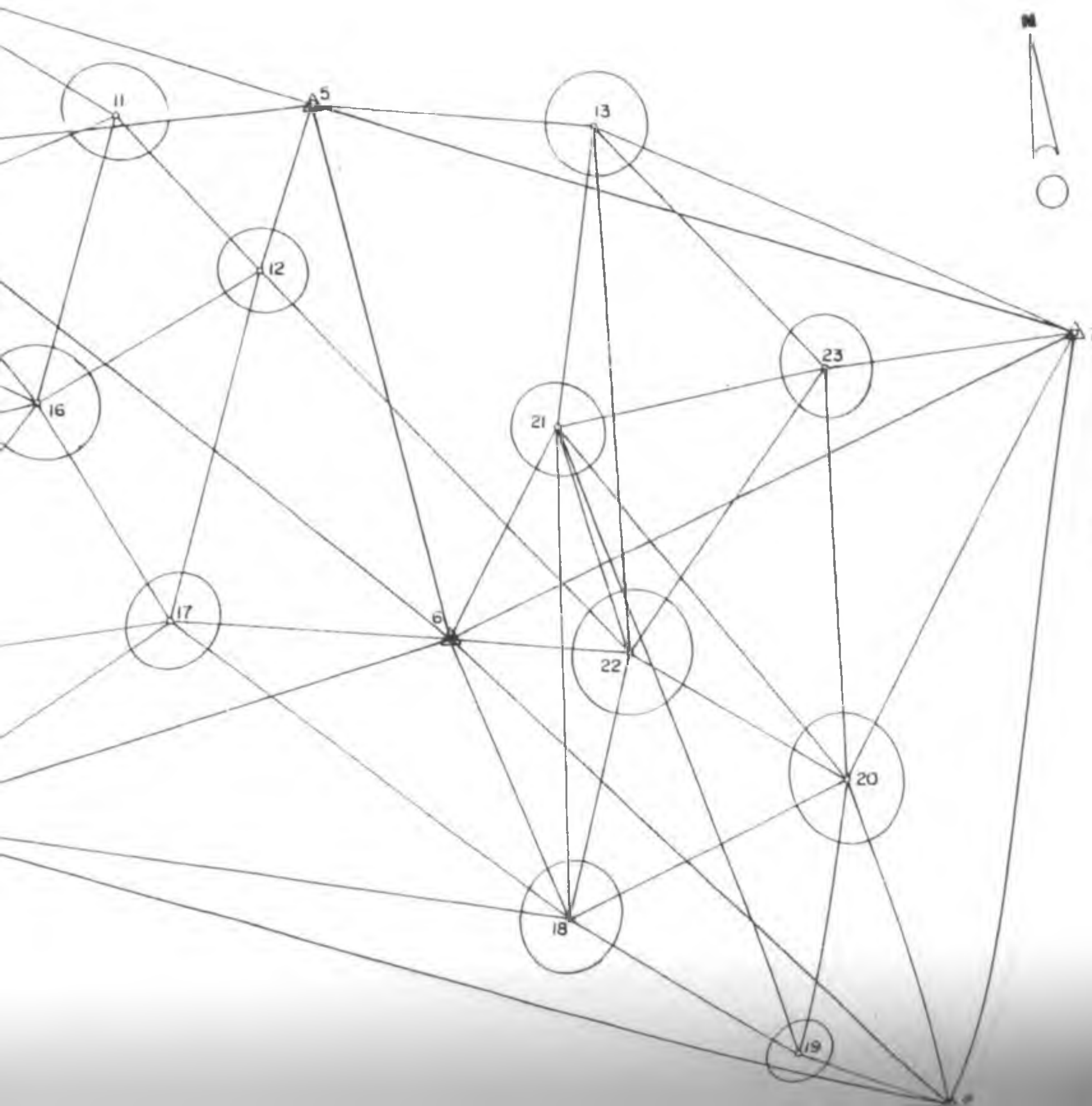
SCALES

Network 10 Km

Ellipses 10mm

First order point

FIG. 5. POINT ERROR ELLIPSES RESULTING FROM THE STATIC MODEL



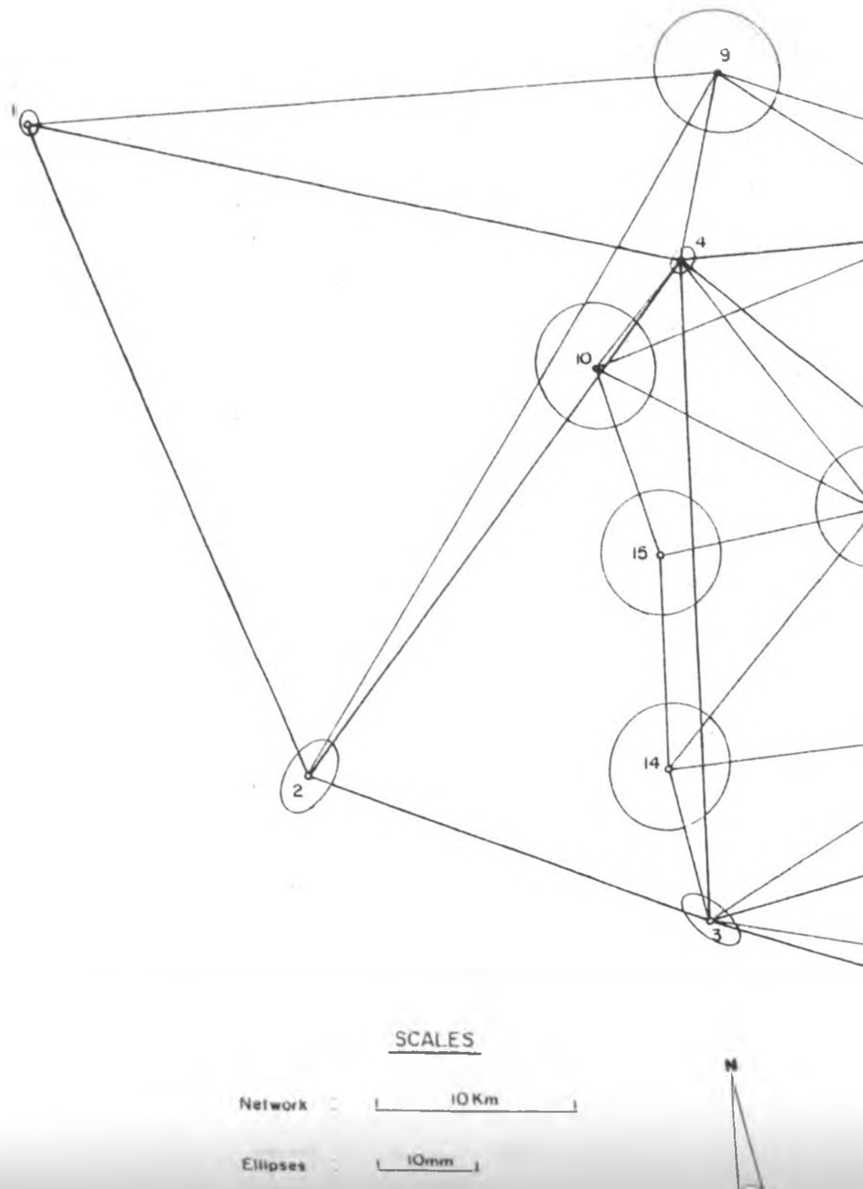
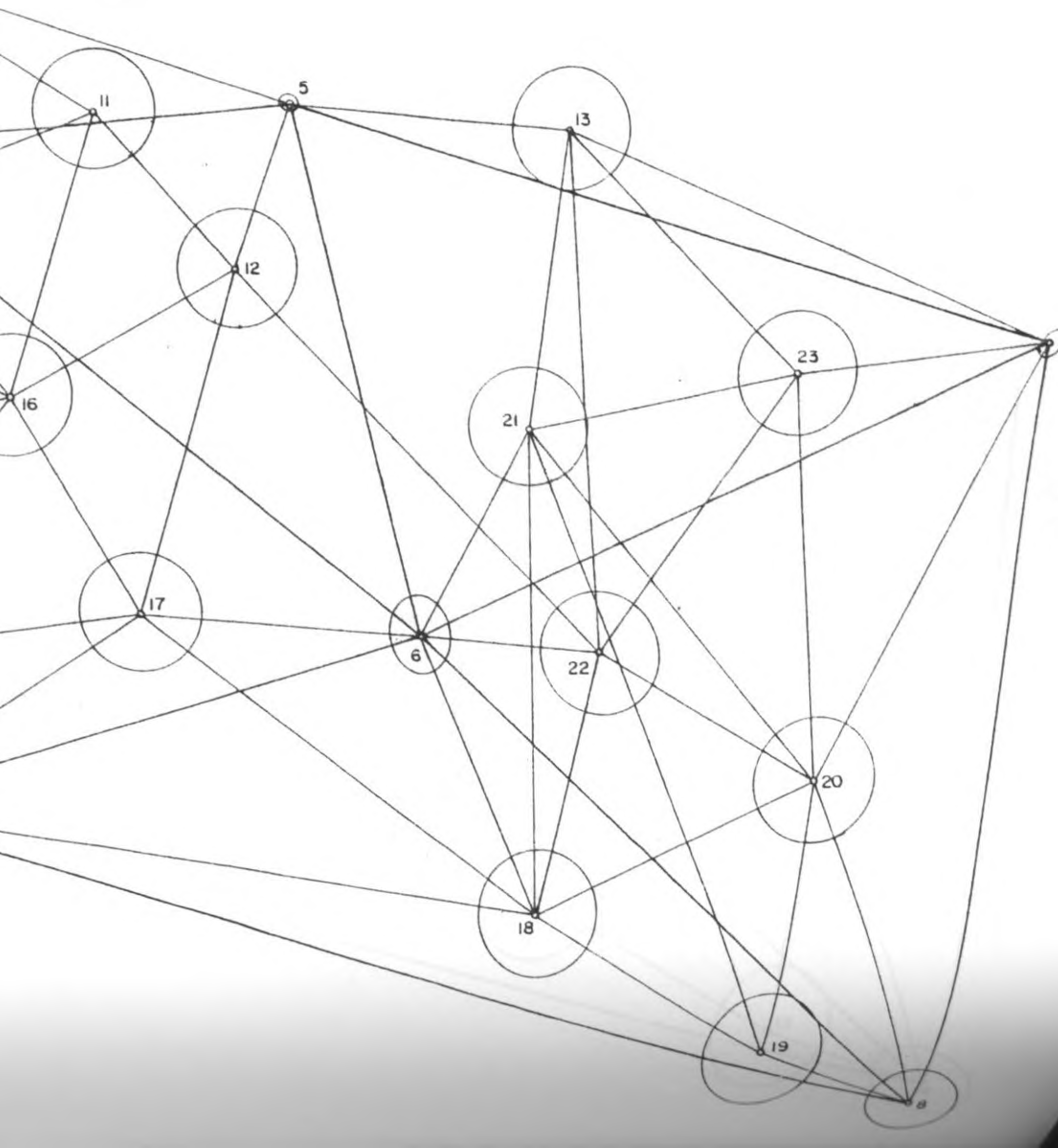
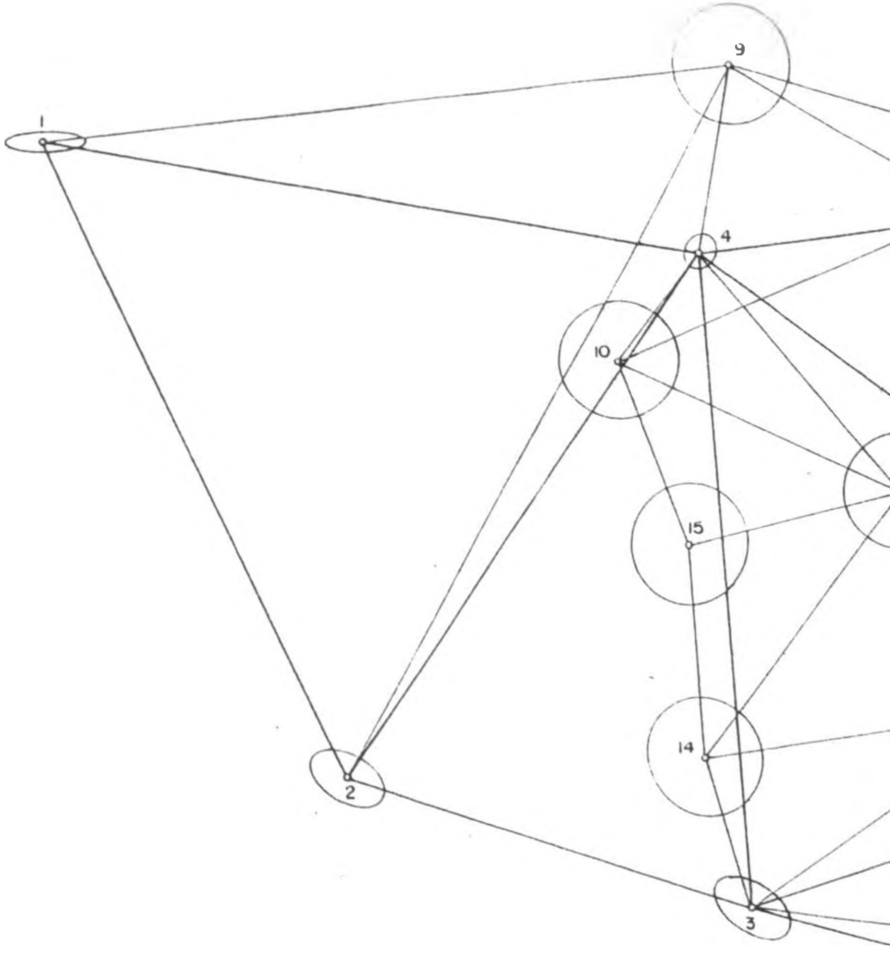

1
9
4
10
15
14
2
3
SCALES
Network
10 Km
Ellipses
10mm
N

FIG 6 POINT ERROR ELLIPSES RESULTING FROM THE DYNAMIC MODEL





SCALES

Network 10Km

Ellipses 10mm

First order point

FIG 7 POINT ERROR ELLIPSES RESULTING FROM THE STATIC-DYNAMIC MODEL

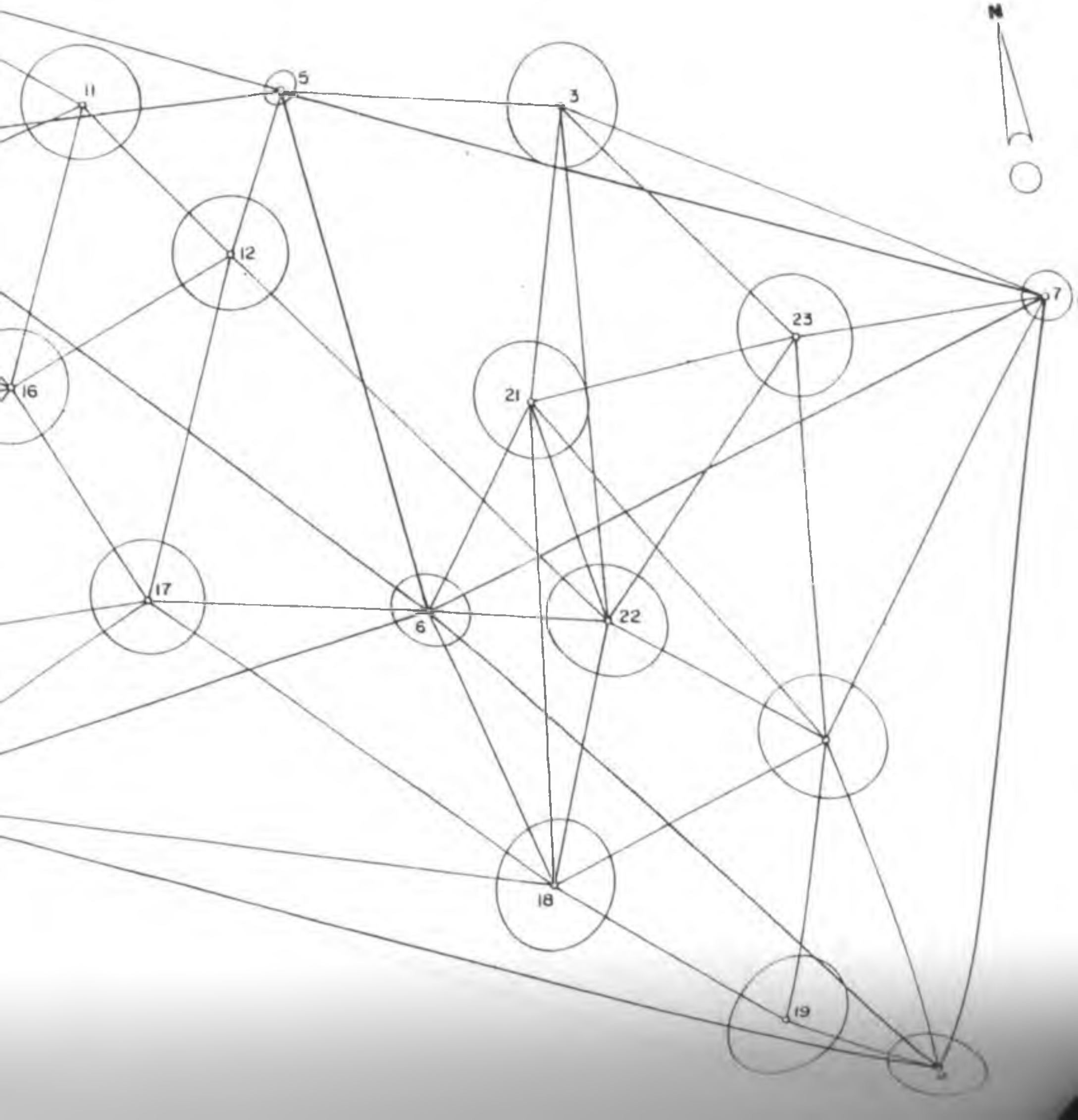


FIG. 8 POINT ERROR ELLIPSES RESULTING FROM THE STATIC MODEL

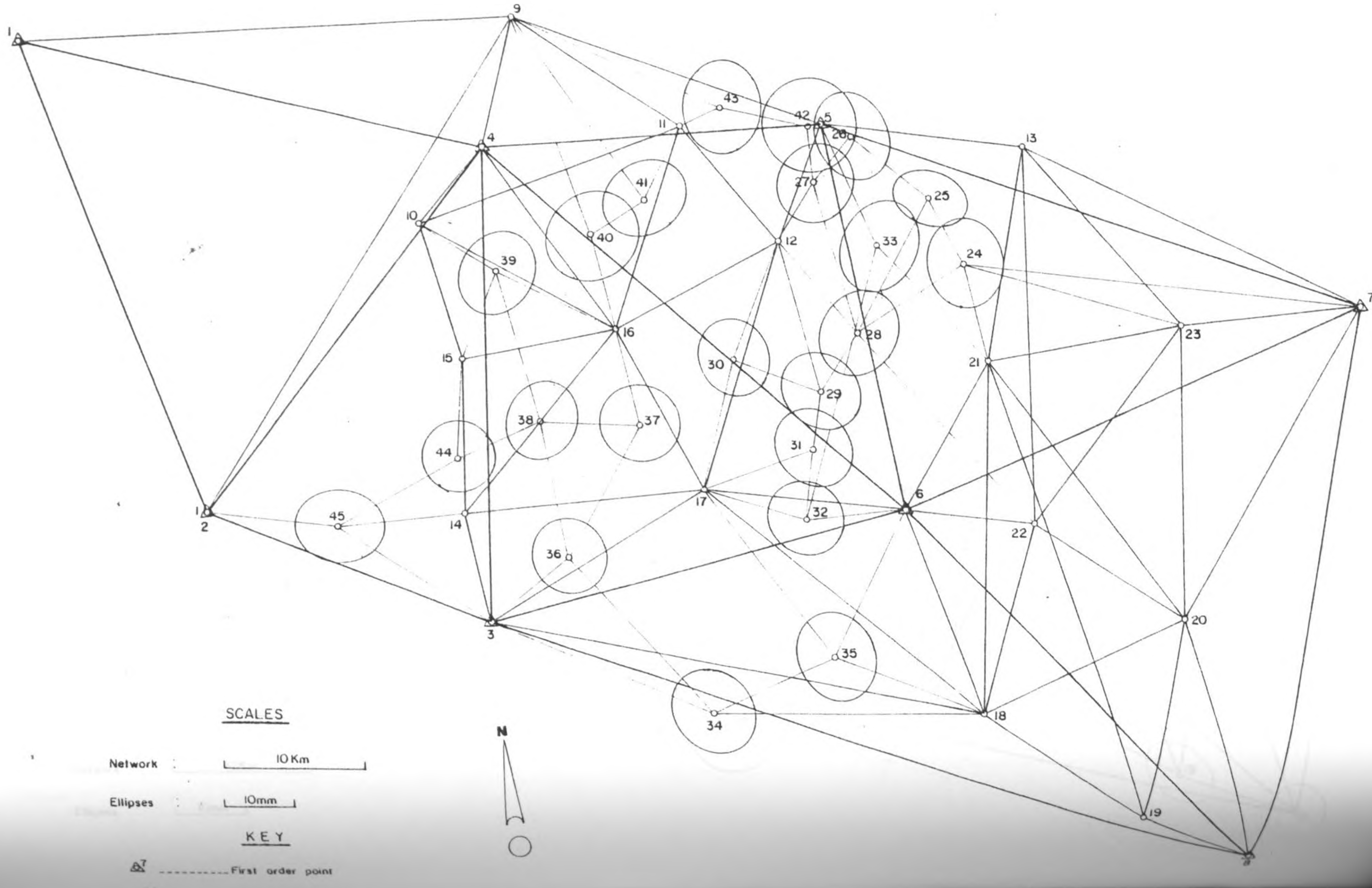


FIG 9 POINT ERROR ELLIPSES RESULTING FROM DYNAMIC MODEL

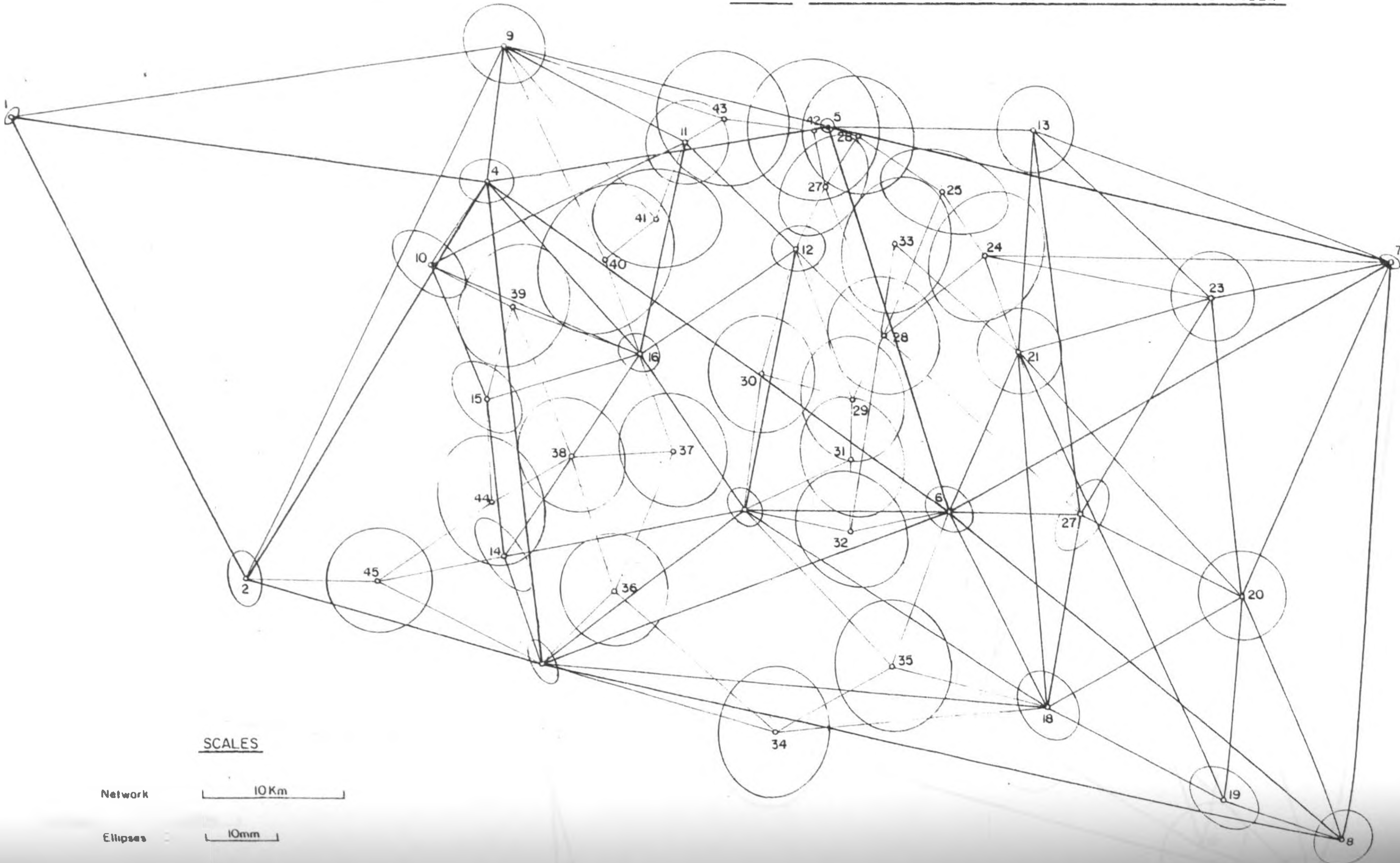
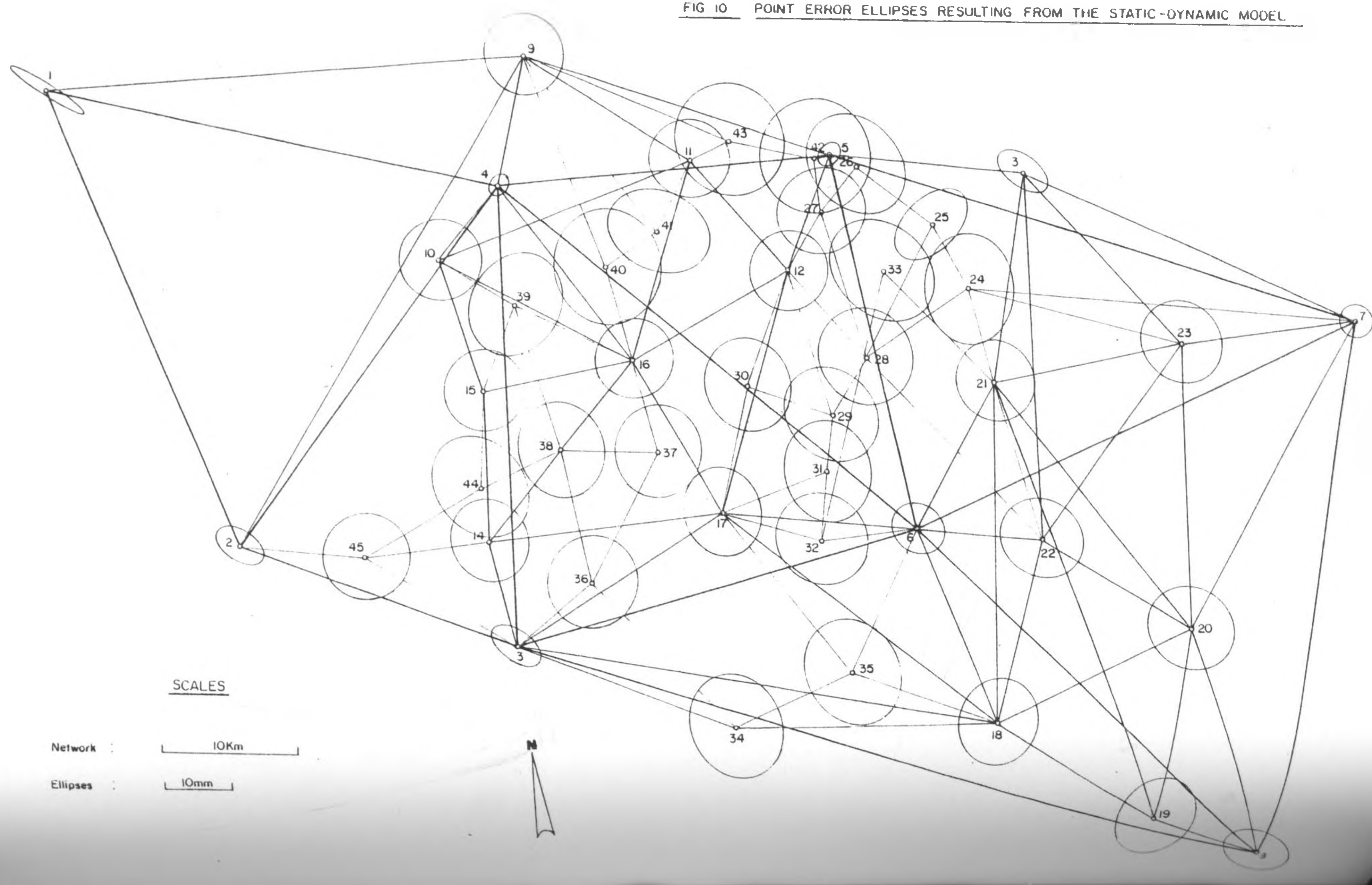


FIG 10 POINT ERROR ELLIPSES RESULTING FROM THE STATIC-DYNAMIC MODEL



SHIFTS OF POINTS FROM THEIR POSITIONS AFTER PHASING TO THEIR POSITIONS AS A RESULT OF STATIC DENSIFICATION

<u>FIG 12: EXPT. B SHIFTS OF POINTS FROM THEIR POSITIONS AFTER PHASING TO THEIR POSITIONS AS A RESULT OF DYNAMIC DENSIFICATION</u>

FIG 13 EXPERIMENT C SHIFTS OF POINTS FROM THEIR POSITIONS AFTER TO THEIR POSITIONS AS A RESULT OF STATIC-DYNAMIC DENSIFICATION.

FIG 14: SHIFTS BETWEEN DETERMINED COORDINATES OF EXP. A AND B

FIG 15: SHIFTS BETWEEN DETERMINED COORDINATES OF EXP. B AND C

FIG 16: SHIFTS BETWEEN DETERMINED COORDINATES OF EXP. A AND C

6.6 Analysis of Results

In this section are analysed the a posteriori variance of unit weight $\hat{\sigma}_0^2$, standard errors σ_E and σ_N , and the computed radial standard errors $\overline{\sigma_E}$, $\overline{\sigma_N}$, and $\overline{\sigma_C}$ from the results obtained in the sections above.

6.6.1 Analysis of variances

The estimated a posteriori variance of unit weight $\hat{\sigma}_0^2$ for all the adjustment models are tested for any significant difference from the a priori variance of unit weight σ_0^2 Which was considered as unit in all the adjustments. The null hypothesis for this test is written as

$$H_o : \hat{\sigma}_0^2 = \sigma_0^2 \qquad (6\text{-}1)$$

and the alternative hypothesis as

$$H_a : \hat{\sigma}_0^2 \neq \sigma_0^2 \qquad (6\text{-}2)$$

where $\hat{\sigma}_0^2$ and σ_0^2 are the a posteriori and a priori variances respectively. Using the χ^2 test, the test statistic is written as

$$\chi_m^2 = m\hat{\sigma}_0^2/\sigma_0^2 \qquad (6\text{-}3)$$

Where m are the degrees of freedom. With (6-3) and using a level of significance of 0.05, the hypothesis (6-1) above is tested for and rejected if,

$$\chi_m^2 > \chi^2_{\alpha/2,m} \qquad (6\text{-}4)$$

α being the level of significance. From (6-3) test statistics for each model and level of adjustment are computed and tabulated below.

For the free network adjustment results the following statistics are obtained

χ^2_{12} = 11.8425

$\chi^2_{0.025,12}$ =23.337

This results indicate that the null hypothesis (6-1) is accepted which implies no significant difference between the a posteriori and a prior variances of unit weight.

Table 6.6a: computed statistical values for χ^2 test of Experiments A,B, and C.

	First level densification	Ho	Second level densification	Ho
Experiment A	χ^2_{64} =64.006		χ^2_{68} =68.000	Y
	$\chi^2_{0.025,64}$ =89.320	Y	$\chi^2_{0.025,68}$ =94.037	
Experiment B	χ^2_{48} =47.688		χ^2_{22} =21.990	Y
	$\chi^2_{0.025,48}$ =70.197	Y	$\chi^2_{0.025,22}$ =36.781	
Experiment C	χ^2_{48} =47.997		χ^2_{22} =22.001	Y
	$\chi^2_{0.025,48}$ =70.197	Y	$\chi^2_{0.025,22}$ =36.781	

Y- represents acceptance of the null hypothesis.

From the Table above the null hypothesis for the χ^2 test was accepted for all the Experiments at both levels of densification. These indicate that the estimated a posteriori variances of unit of weight from the densification models are statistically equal to the a priori variance of unit weight used in the estimation.

The acceptance of the null hypothesis indicates that the estimation processes were done correctly and more specifically that the a priori variance of unit weight was correctly chosen and that all the three models relate to the unknown parameters completely and correctly.

Despite the fact that the null hypothesis is acceptable in all cases above, it has to be observed that for individual statistical estimates, the more close the test statistic is to the value obtained from Tables of statistics the more reliable the estimate are. It can be noticed that the values $\chi_m^2, \chi^2_{\alpha/2,m}$ determined from Experiment A (i.e use of the static model) are closest followed by those from Experiment C, and finally those from Experiment B.

It is expected that the results of the static model should appear as the best, the reason being that the model is based on the assumption that higher order points are fixed and errorless. However, the higher order points are stochastic having been determined, for example in this case from the adjustment of the primary network (see section 3.1).

The results of Experiment C have the second closest values while those from Experiment B have the lagest difference. These are results determined from the static-dynamic and dynamic models respectively within which stochasticity of datum parameters is incorporated. In the static-dynamic model datum parameters are maintained definitive while in the dynamic case all parameters are estimated afresh, thus resulting in the loss of the concept fixed national datum. It is on this basis that the results of Experiment C can ordinarily be considered as best overall.

6.6.2 Analysis of standard errors

The standard errors for Experiment A are generally smaller followed by those of Experiment C and finally those of Experiment B. The results of Experiment A seem more accurate from these standard errors but as explained in preceding sections these amplify the fact the static model assumes a fixed and errorless datum. The important observation made here is the fact that the standard errors for the static-dynamic model are in between those of the static and dynamic models in magnitude.

6.6.3 Efficiency of the Estimators

Following the discussion in sections (6.6.2) and (6.6.3) the results for Experiment C (densification using the static-dynamic model) are considered as best overall. On the basis of this, the computed values $\overline{\sigma_E}$, $\overline{\sigma_N}$ and $\overline{\sigma_c}$ of the other two experiments are tested for any significant difference from the values determined from Experiment C.

The null hypothesis H_o and its alternative H_a are stated respectively as

$$H_o: \overline{\sigma^2_1} = \overline{\sigma^2_2} \qquad H_a: \overline{\sigma^2_1} > \overline{\sigma^2_2} \tag{6-5}$$

where $\overline{\sigma^2_1}$ is taken as the factor computed from Experiment C, while $\overline{\sigma^2_2}$ is the factor being tested. The test statistic in this case is defined as

$$F_{m_1,m_2} = \overline{\sigma^2_1}/\overline{\sigma^2_2} \tag{6-6}$$

and the null hypothesis is rejected if

$$F_{m_1,m_2} > F_{\alpha,m_1,m_2} \tag{6-7}$$

where α, m_1 and m_2 are the level of significance, and degrees of freedom for samples 1 and 2 respectively. Using a level of significance 0.05 and with (6-7), one obtains values given in the Tables below.

Table 6.6a Computed values of the test statistic for $\overline{\sigma_N}$

	First level densification	Ho	Second level densification	Ho
Experiment A	$F_{48,64}$ =1.5848	Y	$F_{22,68}$ =1.6404	Y
Experiment B	$F_{48,48}$ =1.3544	Y	$F_{22,22}$ =1.2530	Y
Experiment C	-	-	-	

Table 6.6b Computed values of the test statistic for $\overline{\sigma_E}$

	First level densification	Ho	Second level densification	Ho
Experiment A	$F_{48,64}$ =1.4618	Y	$F_{22,68}$ =1.5973	Y
Experiment B	$F_{48,48}$ =1.3823	Y	$F_{22,22}$ =1.1212	Y
Experiment C	-	-	-	

Table 6.6c Computed values of the test statistic for $\overline{\sigma_c}$

	First level densification	Ho	Second level densification	Ho
Experiment A	$F_{48,64}$ =1.5201	Y	$F_{22,68}$ =1.6180	Y
Experiment B	$F_{48,48}$ =1.3685	Y	$F_{22,22}$ =1.1815	Y
Experiment C	-	-	-	

From Tables (6.6a, b, c), it is noticed that the hypothesis (6-5) was accepted for both Experiments A and B at the two levels of densification.

6.6.4 Analysis of error ellipses

Figures 5 and 8 depict point error ellipses for the first and second levels of densification using the static model, while Figures 6 and 9 depict those from the static model, and Figure 7 and 10 show those from the static-dynamic model. Note that in Figure 5 points 1 to 8 do not have error ellipses because of the concept of the static model in which datum points are treated as fixed and errorless, likewise points 1 to 23 are treated as fixed during second level densification hence the indication that they do not have error ellipses in Figure 8. It should be noted that the situation of not having error demonstrates that the points in question are so accurately placed that the error ellipse parameters are zeros. Figures 6, 7, 9, and 10 have error ellipses at all points since the stochasticity of datum parameters is considered in Experiments B and C.

Error ellipses for points 1 to 8 in Fig.7 are similar to those in Fig.4 and those for points 8 to 23 in Fig.10 are similar to points 8 to 23 in Fig.5. A general view of the error ellipses indicates that the size of the ellipses are smallest in Experiment A followed by those of Experiment C and finally those of Experiment B and all the ellipses are differently oriented. The implication of the error ellipse of a point is a space in which there is 0.394 probability that the estimated point lies inside, thus the smaller the ellipse the more accurately the point is placed.

As discussed above, the fact that error ellipses for experiment A are smallest is attributed to the fundamental concept of holding datum parameters fixed. These are not representative enough since datum parameters are in fact stochastic. Experiments B and C incorporate the stochasticity of datum parameters in which case Experiment C which yields the second best results can be considered as a more reliable estimation process than Experiment A.

6.6.5 Analysis of Shifts

From Table (6.5) and Figures 11-16 respectively, it can be seen that all the computed shifts differ in magnitude and direction, It is however noticed that the values determined from Experiments B and C are close together in magnitude while those of Experiment A are appreciably larger or smaller in comparison. This demonstrates the fact that the new coordinates from Experiment A (i.e. use of the static model) are distorted though looking reasonably correct.

CHAPTER SEVEN

DISCUSSION

In this chapter, performance of the static, dynamic, and static-dynamic densification models based on the results obtained and analyzed in chapter six is discussed. Discussed first, in section (6.1), are the results of Experiments A, B, and C at the first level of densification. The results for the second level densification are thereafter discussed in section (6.2) while section (6.3) concludes the chapter.

7.1 First level of densification

At the first level of densification the primary network (Fig. 1) was densified by addition of second order points (Fig. 2). From the results of Experiments A, B, and C in sections 6.2, 6.3, and 6.4 respectively. The smallest set of standard errors of estimated parameters were obtained from Experiment A (use the static model) as can be seen from Table 6.2. The static-dynamic model (Experiment C) yielded the second smallest set of standard errors which were very close to those from Experiment B (use of dynamic model).

The computed average standard error for the three experiments varied in magnitude though statistical tests in section (6.6.3) indicated no significant differences, these errors for Experiments A, B, and C were 9.6mm, 10.8mm, and 10.1mm respectively and in all experiments, points 12 and 15 had the smallest standard errors. This situation is probably attributed to the fact that there was a high number of observations to and from these two points.

Resulting error ellipses indicated different sizes and orientations for each experiment but it could be noted from Figures 5, 6, and 7 that Experiment A resulted in the smallest error ellipses followed by Experiment C and finally Experiment B had the largest error ellipses. These reflected the same order of accuracy levels as did the standard errors of the estimated parameters for respective experiments, which is expected as error ellipses are derived from the covariance matrix.

Experiment A was undertaken with datum (primary) stations 1 to 8 being treated as fixed and errorless, this is the reason why Fig. 5 depicts these points as not having error ellipses which in essence gives the impression that these points were so accurately positioned that their standard errors are negligibly small. This gives the main cause for objection of these results since datum points are in fact stochastic having been determined from an earlier adjustment. In this case, for example, Fig. 4 shows the datum points as having error ellipses, yet in the static densification process they are implicitly ignored, which indicates that the results of the static model are not representative enough, and as has been stated in previous chapters, they are expected to be too optimistic.

Although all the computed parameters from the static model look reasonably correct as indicated by the smaller error ellipses, computed shifts (Table 6.5) between adjusted and initial coordinates depict the static model as giving greater variations in stations than the static-dynamic and dynamic models. This could be attributed to distortions in the new network caused by neglecting the stochasticity of datum points during the estimation process.

Experiment B in which densification was performed by treating the datum coordinates as stochastic (i.e use of the dynamic model) resulted in larger error ellipses compared to both Experiments A and C. As can be seen from Fig.6, all network points have error ellipses since this mode of densification yields new values for both datum and densification points. It is noticeable that all the point error ellipses for the datum stations are relatively smaller as compared to those that were determined for the primary network.

The smaller point error ellipses for the primary network in comparison to those determined earlier from the free network adjustment depicts improved accuracy for the datum points at this level of densification, in case, these can be attributed to fact the corresponding parameters are much refined in the first level densification since values that already been determined are further adjusted. It is however important to observe that despite the rigour of this experiment, due to the incorporation of stochasticty of datum points, these points are estimated afresh which is technically a handicap, since in this case the concept of datum which is vital for national geodetic reference systems effectively losses meaning.

Experiment C yielded results which in magnitude ranged between those of the static and dynamic models. In this experiment, datum coordinates are treated as stochastic while retaining the concept of datum, as the numerical values of the datum parameters and the respective datum covariance matrix are maintained as definitive within the context of a consistent mathematical formulation. This is evident from the results in Tables 6.4b and 6.4c. in which the estimated parameters for datum stations do not change from those of the primary network despite having been used in the estimation process.

Figures 7 and 10 further demonstrate the concept of the static-dynamic model in which datum elements remain defined, since the point error ellipses for datum stations are similar to those determined in earlier adjustments in both size and orientation. For example, error ellipses for points 1 to 8 in Fig. 7 are similar to those in Fig. 4, likewise error ellipses for points 8 to 22 in Fig. 10 are similar to those of the same points in Fig. 7.

In general, estimated standard errors, point error ellipses, and coordinate shifts for the dynamic and static-dynamic models (i.e. Experiments B and C) differ very slightly; this could be explained by the similarity of these two models in principle with differences resulting due to the mode of application where the concept of datum is retained in the static-dynamic model while datum is estimated afresh in the dynamic model.

7.2 Second level densification

At this level, the results of the first level of densification were densified into a tertiary network. From Tables 6.2b, 6.3b, and 6.4b, Experiment A yielded the smallest standard errors and error ellipses, followed by those from Experiment C and finally those from Experiment B.

It is observed that the order of accuracy of determination at this level is similar to that of the first level of densification, however it is noticed that numerical values of accuracies increased at this level for all models which indicates a general reduction of accuracy in the higher order networks. This is expected since the accuracy of instruments used for setting geodetic networks reduces down the lower levels of densification, consequently weighting of observation sets tends to become less accurate in the hierarchy of geodetic network densification. Explanation to the behaviour of results in the second level of densification is as has been outlined in section (6.1).

From section (6.6), the analysis indicated that the estimated a posteriori variances of unit weight from all the models at all levels of densification were statistically equal to the a priori variance of unit weight. This therefore, indicates that the estimation processes were correctly carried out and specifically that the models relate to unknown parameters completely and correctly.

The efficiency of estimators tested for in section (6.6.3) indicated no significant differences between the computed Circular Probable Errors for all the experiments at both levels of densification, while this signified the validity of the estimates determined, it is attributed to fact that the parameters used as estimates in the study were in fact computed final point coordinates from the Survey of Kenya records hence the closeness of the determined estimates which led to determined point accuracies being very close in magnitude. This is also evident from Tables 6.2, 6.3, and 6.4 in which the computed estimates for differences between initial and final coordinates are very small in magnitude.

7.3 Concluding remarks

Form the foregoing discussion, an overall classification of the viability of determined results in geodetic densification of networks would be Experiment A, Experiment C , and finally Experiment B i.e., the use of static, static-dynamic, and dynamic densification models respectively.

However, the static model is based on a fixed and errorless datum, with the datum being stochastic, this model yields results that are not representative enough; in most cases the results seem more accurate than they indeed are (discussed in Chapters 1 and 3).

The dynamic and static-dynamic densification models incorporate stochasticity of datum parameters, in the static-dynamic model, datum parameters are maintained definitive while in the dynamic model all parameters are estimated afresh. It is on the basis of these stronger theoretical and practical qualities that the static-dynamic model, despite coming second to the static model in this study, is ordinarily considered to give more reliable results in geodetic densification of networks.

Despite the static-dynamic densification model being the more acceptable one than both the static and dynamic densification models it is imperative to observe that these two models could also be used in the densification of geodetic networks under certain circumstances. For example, for ordinary mapping purposes, the use of the static model in which datum information is treated as exact, although strictly not correct, may however not be very critical since the accuracies of networks for mapping purposes do not have to be so high.

The use of the dynamic model would be particularly recommended for isolated precise engineering networks in which there is no need to fix datum. Such networks would include those for the analysis of deformation of engineering structures, for example, dam deformation analysis and earth deformation analysis. The model would also be useful for scientific geodetic networks such as those for crustal deformation monitoring.

CHAPTER EIGHT

CONCLUSION

This chapter summarises the work done and gives recommendations arising from the findings of the work reported herein.

8.1 Summary

The objective of the present study was to demonstrate the practical applicability, and to evaluate the suitability of three approaches to geodetic network densification namely, static, dynamic, and static-dynamic densification models.

To realise this objective, the three models (Experiments A, B, and C) were used to estimate parameters for a part of the Kenyan geodetic network consisting of eight primary control stations, fifteen secondary control stations, and twenty-two tertiary control stations as discussed in section (5.1).

In Experiment A, densification under the static model was applied, the datum stations were regarded as fixed and error-free. In Experiment B, application of the dynamic model, the stochasticity of datum stations was considered, thus in this approach all the stations were estimated afresh. The concept of Experiment C (use of static-dynamic model) was such that datum coordinates were treated as stochastic prior information while at the same time their numerical values and respective datum variance-covariance matrices were maintained definitive.

The results of these three approaches were very close to one another owing to the fact that estimations were being carried out on an already adjusted network, and the estimates were themselves very close to the approximate values which were adopted from the

Survey of Kenya coordinates. However, on evidence of the standard errors and error ellipses as shown in chapter six and mathematical formulations in chapter three, the static-dynamic model was considered the more suitable approach for densification from among the three models considered.

8.2 Conclusions

Traditional densification is carried out by applying the static model. As has been demonstrated in the study, this model implicitly ignores the stochasticity of datum points by assuming them as fixed and errorless. This implicit omission is not justified as the "fixed" datum parameters are themselves obtained from an earlier adjustment process thus having an associated variance covariance matrix.

It is therefore concluded that there is need to incorporate the stochasticity of datum points in the densification of networks. This is the only way to truly reflect the fact that datum points are determined from initial network adjustments.

In view of the fact that stochasticity of datum points has to be considered, there is need to determine the variance covariance matrix of datum points for a particular geodetic network during any adjustment. This should be saved and made available for use in subsequent densification of the network.

The use of the dynamic model incorporates stochasticity of datum parameters. Unfortunately, this results in all coordinates being estimated afresh in which case the idea of network datum is lost. This leads to the conclusion that the dynamic model can not be effectively used in cases where the datum reference has to remain fixed. It however can be used in situations where the datum need not be maintained.

The static-dynamic model yielded results which were, in terms of magnitude of standard errors and error ellipses, between those of the static and dynamic models. It can therefore be concluded that the static-dynamic model is a "sandwich" model which effectively uses the advantages of the static and dynamic models. The static-dynamic model is thus considered as the best suited approach to the densification of geodetic networks.

It is however imperative to observe that the static and dynamic models could also be used in densification work under special circumstances that, datum has to be retained without considering its stochasticity and datum not being retained at all respectively.

8.3 Recommendations

Having compared the mathematical formulations of the static, dynamic, and static-dynamic models as approaches to densification of geodetic networks and in view of the results obtained in Chapter Six and discussed in Chapter Seven, it is recommended that the static-dynamic model is the more viable approach to densification of geodetic networks from the set of of three models studied here.

The study herein was carried out on data already determined from the field, it is recommended that a survey be set up from the begining, data collected and subsequently processed using the above models.

The study herein was carried out on data for an existing network, it is recommended that it be tested on a freshly designed and observed network.

As a further test to the three densification models studied, it is recommended that they be subjected to the adjustment and densification of three and four dimensional geodetic networks.

APPENDIX A: PROGRAM LISTINGS AND FLOWCHARTS

APPENDIX A.1: PROGRAM DENSITY.FOR

```
*       ***********************************************************
*       *M     O     D     U     L     E     O     N     E *
        ***********************************************************
        IMPLICIT REAL*8(A-H,O-Z)
        DOUBLE PRECISION W2(66,66),X(50),Y(50),DXA(50),DYA(50)
        DIMENSION LANG(100,2),IDEG(100),MIN(100),SEC(100)
        DIMENSION LDST(100,2),DST(100),DIST1(100),DIST2(100)
        DIMENSION A1(47,90),A2(70,90),W(117,117),A(117,90)
        DIMENSION Y1(100),Y2(100),YM(200,1),ANG0(100),ANG1(100)
        DIMENSION ANG(200,1),re(46,1),RESD(117,1),WD(100)
        DIMENSION IDG(100),IMN(100),RSC(100),ARES(117,1),ETW(1,117)
        DIMENSION NBRG(100),MITN(100),SCS(100),AZMT(100),CAZMT(100)
        DIMENSION ATW(90,117),ATWA(90,90), ATWAIN(90,90),ADY(117,1)
        DIMENSION ETWE(1,1),DX(90,90),ADJO(117,90),ATR(90,117)
        DIMENSION WT(117,117)
        DIMENSION DX1(50),DY1(50),STDQ(200),Rr(46,90),CRX(90,1)
        DIMENSION RTW(90,46),RTWR(90,90),SUM1(90,90),SUMI(90)
        DIMENSION RTWre(90,1),ATWY(90,1),SUM2(90,1),SOLN1(117,1)
        DIMENSION DELS(90,1),RRES(46,1),ADJX(90,1),SOLN2(46,1)

C       FIRST VARIANCE COMPONENT DIMENSIONS
*       ************************************
        DIMENSION W1(47,47),E1(47,1),W1I(47,47),A11(90,47)
        DIMENSION A1W(90,90),A111(47,90),A1T(90,47),QE1(47,47)
        DIMENSION A1WI(90,90),W1QE(47,47)
        DIMENSION E1T(1,47),E1TW1(1,47),E1TW1E(1,1),AA1(47,47)

C       SECOND VARIANCE COMPONENT DIMENSIONS
*       *************************************
        DIMENSION E2(70,1),W2I(70,70),A22(90,70),A2WI(70,70)
        DIMENSION A222(70,90),A2T(90,70),QE2(70,70),A2W(90,90)
        DIMENSION W2QE(70,70),E2T(1,70),E2TW2(1,70),E2TW2E(1,1)
        DIMENSION AA2(70,70)

C       THIRD VARIANCE COMPONENT DIMENSIONS
*       *************************************
        DIMENSION Wr(46,46),E3(46,1),WrI(46,46)
        DIMENSION A333(46,90),QE3(46,46),A3T(1,46)
        DIMENSION WrQE(46,46),E3T(1,46),E3TWr(1,46),E3TWrE(1,1)
        DIMENSION RT(90,46),RRI(46,46),W3QE(46,46),RTWRI(90,90)

C       FREENET MANIPULATIONS
*       **********************
        DIMENSION AT(16,42),G(4,16),GT(16,4),GTG(16,16),ADG(16,16)
        DIMENSION AV(16,16),GAV(4,16),GTAV(16,16),AVGT(16,16)
        DIMENSION ASB(16,16),ASAT(16,42),ADJX(16,1),ADX(42,1)
        DIMENSION ATSB(16,16),ABA(16,16),E(42,1)
```

```
C     COVARIANCE MATRIX MANIPULATIONS
*     ********************************L
      DIMENSION SGY(50),SGX(50),EC(50),COV(90,90)
      DIMENSION VMIN(50),THETA(50),SGMX(50),SGMY(50),VMAX(50)
      REAL KK(4,4),KKINV(4,4)

*    *************************************************************
C    *  MAXDST=NUMBER OF DISTANCES MEASURED
C    *  MAXDIR=NUMBER OF DIRECTIONS MEASURED
C    *  MAXST= NUMBER OF STATIONS IN THE NETWORK
C    *  NOTS=(MAXST-MAXDAT)*2 THE NUMBER OF UNKNOWNS IN THE
C    *  X-VECTOR
C    *  NEQ=MAXDST+MAXDIR TOTAL NUMBER OF OBSERVATIONS/EQUATIONS
*    **************************************************************
      WRITE(*,1)
      READ(*,2)MAXST
    1 FORMAT('       INPUT TOTAL NUMBER OF STATIONS (MAXS11T)')
    2 FORMAT(I2)
      WRITE(*,3)
    3 FORMAT('     INPUT TOTAL NUMBER OF DATUM STATIONS (MAXDAT)')
      READ(*,2)MAXDAT
      WRITE(*,4)
    4 FORMAT('    INPUT NUMBER OF DISTANCE OBSERVATIONS(MAXDST)')
      READ(*,2)MAXDST
      WRITE(*,5)
    5 FORMAT('       INPUT TOTAL NUMBER DIRECTIONS OBS.(MAXDIR)')
      READ(*,2)MAXDIR
      WRITE(*,6)
    6 FORMAT('         HOW MANY ITERATIONS ?')
      READ(*,2)ITR
      WRITE(*,7)
    7 FORMAT('    HOW MANY RESTRICTIONS ?')
      READ(*,2)NEQR
      WRITE(3,12)
   12 FORMAT(15X,'THE INPUT DATA USED FOR THE STUDY ARE :'
     </13X,50('=')///,6X,'POINT',28X,'APPR.CO-ORDINATES
     X'/5X,13('='),25X,14('=')/31X,'X(METRES)',15X,'
     XY(METRES)'/30X,11('='),13X,10('='))
      READ(12,13)(Y(I),X(I),I=1,MAXST)
   13 FORMAT(X,F12.4,2X,F11.4)
*     STORING PROVISION COORDINATES
      DO 100 I=1,MAXST
      DXA(I)=X(I)
      DYA(I)=Y(I)
  100 CONTINUE
*     APVUW IS THE APRIOR VARIANCE OF UNIT WEIGHT
      APVUW=1.0
      RS=206264.8
      PI=3.141592
      ICNT=1
      WRITE(3,16)(I,DYA(I),DXA(I),I=1,MAXST)
   16 FORMAT(7X,I3,20X,F12.4,16X,F11.4)
```

```
17 FORMAT(/9X,'DIST.NO:',5X,'STATION',12X,'DIST.BETWEEN
   <STATIONS'
   </8X,10('='),5X,10('='),9X,24('='))
   READ(23,14)(LDST(I,1),LDST(I,2),DST(I),I=1,MAXDST)
14 FORMAT(X,I2,X,I2,X,F9.3)
   WRITE(3,18)(I,LDST(I,1),LDST(I,2),DST(I),I=1,MAXDST)
18 FORMAT(11X,I2,10X,I2,3X,I2,23X,F9.3)
   WRITE(3,19)
19 FORMAT(/5X,'LINE.NO:',8X,'STATION',6X,'H.DIR(DEG,MIN,&SEC)'
  X,9X,'AZMT(RADIANS)'/4X,10('='),6X,10('='),4X,24('='),
  X7X,15('='))
*     INPUT FOR HORIZONTAL DIRECTIONS
   READ(23,15)(LANG(I,1),LANG(I,2),IDEG(I),MIN(I),
  <SEC(I),I=1,MAXDIR)
15 FORMAT(X,I2,X,I2,3X,I3,X,I2,X,F4.1)
   WRITE(3,20)(I,LANG(I,1),LANG(I,2),IDEG(I),MIN(I),SEC(I)
  <,I=1,MAXDIR)
20 FORMAT(6X,I3,10X,I2,2X,I2,12X,I4,2X,I3,2X,F5.2)
1000 CONTINUE
   ICNT=ICNT+1
   NEQ=MAXDST+MAXDIR
   NOTS=(MAXST-MAXDAT)*2
C     INITIALIZING  THE OUT PUT MATRICES AND VECTORS
   DO 10 I=1,NEQ
   DO 10 J=1,NOTS
   A(I,J)=0.0
   ANG(I,1)=0.0
   W(I,I)=0.0
10 CONTINUE
*     ***********************************************
*     *M    O    D    U    L    E         T    W    O *
*     ***********************************************
   IF(MAXDAT.EQ.0) GOTO 401
*     DESIGN MATRIX FOR FIXED DATUM WORKS
*     COMPUTING DISTANCE OBSERVATION PARAMETERS AND
*     LOADING IN THE
*     DESIGN MATRIX
   DO 23 I=1,MAXDST
   K1=LDST(I,1)
   K2=LDST(I,2)
   DIST1(I)=SQRT((X(K2)-X(K1))**2+(Y(K2)-Y(K1))**2)
   DO 22 J=1,2
   IF(J.EQ.2)GO TO 21
   IF(K1.LE.MAXDAT)GO TO 22
   J1=(K1*2-1)-MAXDAT*2
   J2=J1+1
   A1(I,J1)=(X(K1)-X(K2))/(DIST1(I))
   A1(I,J2)=(Y(K1)-Y(K2))/(DIST1(I))
   GO TO 22
21 IF(K2.LE.MAXDAT)GO TO 22
   J1=(K2*2-1)-MAXDAT*2
   J2=J1+1
```

```
      A1(I,J1)=(X(K2)-X(K1))/(DIST1(I))
      A1(I,J2)=(Y(K2)-Y(K1))/(DIST1(I))
 22   CONTINUE
      Y1(I)=DST(I)-DIST1(I)
*     STORING COMPUTED DISTANCES
      W1(I,I)=APVUW/((0.003**2)+(DST(I)*10E-6)**2)
 23   CONTINUE
C     *****************************************************
C     FORMATION OF DESIGN MATRIX COMPONENT A2 FOR DIRECTION *
C     *****************************************************
      DO 26 I=1,MAXDIR
      L1=LANG(I,1)
      L2=LANG(I,2)
      DIST2(I)=SQRT((X(L2)-X(L1))**2+(Y(L2)-Y(L1))**2)
      DO 25 J=1,2
      IF(J.EQ.2)GO TO 24
      IF(L1.LE.MAXDAT)GO TO 25
      J2=J1+1
      A2(I,J1)=(Y(L2)-Y(L1))/DIST2(I)*RS
      A2(I,J2)=(X(L1)-X(L2))/DIST2(I)*RS
      GO TO 25
  24  IF(L2.LE.MAXDAT)GO TO 25
      J1=(L2*2-1)-MAXDAT*2
      J2=J1+1
      A2(I,J1)=(Y(L1)-Y(L2))/DIST2(I)*RS
      A2(I,J2)=(X(L2)-X(L1))/DIST2(I)*RS
  25  CONTINUE
      DN=Y(L2)-Y(L1)
      DE=X(L2)-X(L1)
*     COMPUTATION OF OBSERVED HORIZONTAL ANGLES
*     CAZMT  IS THE COMPUTED BEARING
      CALL QUAD(DE,DN,Y22)
      CAZMT(I)=Y22
C     CONVERSION OF THE COMPUTED BEARINGS INTO
C     DEGREES,MINUTES,SECONDS
      XN=CAZMT(I)*PI/180.
C     CONVERSION OF COMPUTED AZIMUTHS INTO DEGREES,
C     MINUTES,SECONDS
      CALL ANGLE(XN,NBRG(I),MITN(I),SCS(I))
      ANG1(I)=Y22*3600.
*     AZMT **** IS THE INPUT DIRECTION
      Y2(I)=SCS(I)-SEC(I)
      W2(I,I)=APVUW/1.0**2
 26   CONTINUE
C     *****************************************************
C     FORMATION OF COMPLETE DESIGN MATRIX,WEIGHT MATRIX,&
C     Y-VECTOR
C     *****************************************************
      DO 28 I=1,NEQ
      DO 28 J=1,NOTS
      IF(I.GT.MAXDST)GO TO 27
      A(I,J)=A1(I,J)
```

```
      YM(I,1)=Y1(I)
      ANG(I,1)=ANG0(I)
      W(I,I)=W1(I,I)
      GO TO 28
  27  K=I-MAXDST
      A(I,J)=A2(K,J)
      W(I,I)=W2(K,K)
      YM(I,1)=Y2(K)
   28 CONTINUE
      GO TO 402
  401 CONTINUE
*     COMPUTING DISTANCE OBSERVATION PARAMETERS AND LOADING IN
C     THE  DESIGN MATRIX   FOR DYNAMIC AND STATIC DYNAMIC
C     NETWORKS
*     ************************************************************
      DO 30 I=1,MAXDST
      K1=LDST(I,1)
      K2=LDST(I,2)
      DIST1(I)=SQRT((X(K2)-X(K1))**2+(Y(K2)-Y(K1))**2)
      J1=K1*2-1
      J2=J1+1
      A1(I,J1)=(X(K1)-X(K2))/(DIST1(I))
      A1(I,J2)=(Y(K1)-Y(K2))/(DIST1(I))
      J3=K2*2-1
      J4=J3+1
      A1(I,J3)=(X(K2)-X(K1))/(DIST1(I))
      A1(I,J4)=(Y(K2)-Y(K1))/(DIST1(I))
      Y1(I)=DST(I)-DIST1(I)
*     STORING COMPUTED DISTANCES
      ANG0(I)=DIST1(I)
  30  CONTINUE
C     ************************************************************
C     FORMATION OF DESIGN MATRIX COMPONENT A2 FOR DIRECTIONS
C     ************************************************************
      DO 31 I=1,MAXDIR
      L1=LANG(I,1)
      L2=LANG(I,2)
      DIST2(I)=SQRT((X(L2)-X(L1))**2+(Y(L2)-Y(L1))**2)
      J1=L1*2-1
      J2=J1+1
      A2(I,J1)=(Y(L2)-Y(L1))/DIST2(I)*RS
      A2(I,J2)=(X(L1)-X(L2))/DIST2(I)*RS
      J3=L2*2-1
      J4=J3+1
      A2(I,J3)=(Y(L1)-Y(L2))/DIST2(I)*RS
      A2(I,J4)=(X(L2)-X(L1))/DIST2(I)*RS
      DN=Y(L2)-Y(L1)
      DE=X(L2)-X(L1)
*     COMPUTATION OF OBSERVED HORIZONTAL ANGLES
*     CAZMT  IS THE COMPUTED BEARING
C     CONVERSION OF THE COMPUTED BEARINGS
C     INTODEGREES,MINUTES,SECONDS
```

```
      CALL QUAD(DE,DN,Y22)
      CAZMT(I)=Y22
C     CONVERSION OF COMPUTED AZIMUTHS INTO DEGREES,
C     MINUTES,SECONDS
      XB=CAZMT(I)*PI/180.
      CALL ANGLE(XB,NBRG(I),MITN(I),SCS(I))
      ANG1(I)=Y22*3600.
      Y2(I)=SCS(I)-SEC(I)
   31 CONTINUE
C     ************************************************************
C     FORMATION OF COMPLETE DESIGN MATRIX,WEIGHT MATRIX,&
C     Y-VECTOR
C     ************************************************************
      DO 33 I=1,NEQ
      DO 33 J=1,MAXST*2
      IF(I.GT.MAXDST)GO TO 32
      A(I,J)=A1(I,J)
      YM(I,1)=Y1(I)
      W(I,I)=W1(I,I)
      ANG(I,1)=ANG0(I)
      GO TO 33
   32 K=I-MAXDST
      A(I,J)=A2(K,J)
      W(I,I)=W2(K,K)
      YM(I,1)=Y2(K)
      ANG(I,1)=ANG1(K)
   33 CONTINUE
      DO 60 I=1,NEQR
      Rr(I,I)=1.0
      Wr(I,I)=1/(WD(I)**2)
   60 CONTINUE
      GO TO 403
  402 CONTINUE
C     MANIPULATION FOR THE STATIC MODEL
*     LEAST SQUARES MANIPULATION
      IRR=MAXST-MAXDAT
      NEQ=MAXDST+MAXDIR
      NOTS=(MAXST-MAXDAT)*2
*     COMPUTING UNKNOWN PARAMETERS
      CALL ATB(A,W,ATW,NEQ,NOTS,NEQ)
      CALL TIMES(ATW,A,ATWA,NOTS,NEQ,NOTS)
      CALL MATINV(ATWA,ATWAIN,NOTS)
      CALL TIMES(ATWAIN,ATW,R,NOTS,NOTS,NEQ)
      CALL TIMES(R,YM,CRX,NOTS,NEQ,1)
*     CRX--- IS THE VECTOR OF UNKNOWNS
*     COMPUTING FOR RESIDUALS
      CALL TIMES(A,CRX,SOLN,NEQ,NOTS,1)
      CALL MINUS(YM,SOLN,RESD,NEQ,1)
*     ADY .... IS THE VECTOR OF ADJUSTED OBSERVATIONS
      CALL ADD(ANG,SOLN,ADY,NEQ,1)
      DO 202 I=MAXDST+1,NEQ
      DG=ADY(I,1)/3600.*PI/180.
```

```
      CALL ANGLE(DG,IDG(I-MAXDST),IMN(I-MAXDST),RSC(I-MAXDST))
  202 CONTINUE
      DO 29 I=1,NEQ
   29 E(I,1)=RESD(I,1)
      GO TO 404
  403 CONTINUE
*********** MANIPULATIONS FOR THE DYNAMIC MODEL **************
      CALL ATB(A,W,ATW,NEQ,NOTS,NEQ)
      CALL TIMES(ATW,A,ATWA,NOTS,NEQ,NOTS)
      CALL ATB(Rr,Wr,RTW,NEQR,NOTS,NEQR)
      CALL ADD(RTWR,ATWA,SUM1,NOTS,NOTS)
      CALL VERSOL(SUM1,SUMI,NOTS)
      CALL TIMES( ATW,YM,ATWY,NOTS,NEQ,1)
      CALL TIMES(RTW,re,RTWre,NOTS,NEQR,1)
      CALL ADD(ATWY,RTWre,SUM2,NOTS,1)
*     DELS = UNKNOWNS
      CALL TIMES( SUM1,SUM2,DELS,NOTS,NOTS,1)
*     COMPUTING RESIDUALS
      CALL TIMES(A,DELS,SOLN1,NEQ,NOTS,1)
      CALL MINUS(YM,SOLN1,ARES,NEQ,1)
      CALL TIMES(Rr,DELS,SOLN2,NEQR,NOTS,1)
      CALL MINUS(re,SOLN2,E3,NEQR,1)
*     ADY IS THE VECTOR OF ADJUSTED OBSERVATIONS
      CALL MINUS(ANG,ARES,ADY,NEQ,1)
*     CONVERTING ADJUSTED BEARINGS TO DEGREES,MINUTES AND
C     SECONDS**
      DO 42 I=MAXDST+1,NEQ
      XX=ADY(I,1)/3600.*PI/180.
  42  CALL ANGLE(XX,IDG(I-MAXDST),IMN(I-MAXDST),RSC(I-MAXDST))
      IF(NEQR.EQ.0)GOTO 408
***** MANUPILATION FOR STATIC DYNAMIC **********************
      CALL TIMES(A3,Rr,A1R,NEQ,NEQR,NEQR)
      CALL TIMES(A1R,re,A1Rr,NEQ,NEQR,1)
      CALL TRANS(A1R,A1RT,NEQ,NEQR)
*     INVERTING Wr
      DO 66 I=1,NEQR
   66 WrI(I,I)=1./Wr(I,I)
      CALL TIMES(A1R,WrI,A1RW,NEQ,NEQR,NEQR)
      CALL TIMES(A1RW,A1RT,QQ,NEQ,NEQR,NEQ)
*     INVERTING W
      DO 67 I=1,NEQ
   67 WI(I,I)=1.0/W(I,I)
      CALL ADD(WI,QQ,QQQ,NEQ,NEQ)
      DO 68 I=1,NEQ
   68 WW(I,I)=1./QQQ(I,I)
      CALL TRANS(A4,A22T,NEQ,NEQR2)
      CALL TIMES(A22T,WW,A2TW,NEQR2,NEQ,NEQ)
      CALL TIMES(A2TW,A4,BB,NEQR2,NEQ,NEQR2)
      CALL MATINV(BB,BINV,NEQR2)
      CALL TIMES(A2TW,YM,XS,NEQR2,NEQ,1)
      CALL TIMES(BINV,XS,XSS,NEQR2,NEQR2,1)
*     XSS IS THE VECTOR OF UNKNOWNS
```

```
*     COMPUTING RESIDUALS
      CALL TIMES(A4,XSS,SOLN3,NEQ,NEQR2,1)
      CALL MINUS(YM,SOLN3,SDRES,NEQ,1)
*     ADY IS THE VECTOR OF ADJUSTED OBSERVATIONS
      CALL MINUS(ANG,SDRES,ADY,NEQ,1)
*     CONVERTING ADJUSTED BEARINGS TO DEGREES,MINUTES AND
C     SECONDS**
      DO 42 I=MAXDST+1,NEQ
      XX=ADY(I,1)/3600.*PI/180.
 42   CALL ANGLE(XX,IDG(I-MAXDST),IMN(I-MAXDST),RSC(I-MAXDST))
408   CONTINUE
*     E1 ARE RESIDUALS FOR DISTANCES
*     E2 ARE RESIDUALS FOR DIRECTIONS
*     E3 ARE RESIDUALS FOR RESTRICTIONS
      DO 49 I=1,NEQ
      IF(I.GT.MAXDST)GO TO 48
      E1(I,1)=ARES(I,1)
      GO TO 49
   48 E2((I-MAXDST),1)=ARES(I,1)
   49 CONTINUE
  404 CONTINUE
*     COMPUTING THE COFACTOR MATRICES///
      CALL MATINV(W1,W1I,MAXDST)
      CALL MATINV(W2,W2I,MAXDIR)
      CALL MATINV(Wr,WrI,NEQR)
*     COMPUTING FIRST VARIANCE  COMPONENT
      CALL ATB(A1,W1,A11,MAXDST,NOTS,MAXDST)
      CALL TIMES(A11,A1,A1W,NOTS,MAXDST,NOTS)
      CALL MATINV(A1W,A1WI,NOTS)
      CALL TIMES(A1,A1WI,A111,MAXDST,NOTS,NOTS)
      CALL TRANS(A1,A1T,MAXDST,NOTS)
      CALL TIMES(A111,A1T,AA1,MAXDST,NOTS,MAXST)
      CALL MINUS(WI1,AA1,QE1,MAXDST,MAXDST)
      CALL TIMES(W1,QE1,W1QE,MAXDST,MAXDST,MAXDST)
      CALL TRANS(E1,E1T,MAXDST,1)
      CALL TIMES(E1T,W1,E1TW1,1,MAXDST,MAXDST)
      CALL TIMES(E1TW1,E1,E1TW1E,1,MAXDST,1)
*     COMPUTING THE TRACE
      TQE1=0.
      DO 50 I=1,MAXDST
 50   TQE1=TQE1+W1QE(I,I)
C     VO1=VARIANCE COMPONENT FOR DISTANCES
*     COMPUTING THE SECOND VARIANCE COMPONENT
      CALL ATB(A2,W2,A22,MAXDIR,NOTS,MAXDIR)
      CALL TIMES(A22,A2,A2W,NOTS,MAXDIR,NOTS)
      CALL MATINV(A2W,A2WI,NOTS)
      CALL TRANS(A2,A2T,MAXDIR,NOTS)
      CALL TIMES(A222,A2T,AA2,MAXDIR,NOTS,MAXDIR)
      CALL MINUS(W2I,AA2,QE2,MAXDIR,MAXDIR)
      CALL TIMES(W2,QE2,W2QE,MAXDIR,MAXDIR,MAXDIR)
      CALL TRANS(E2,E2T,MAXDIR,1)
      CALL TIMES(E2T,W2,E2TW2,1,MAXDIR,MAXDIR)
```

```
      CALL TIMES(E2TW2,E2,E2TW2E,1,MAXDIR,1)
*     COMPUTING THE TRACE
      TQE2=0.
      DO 501 I=1,MAXDIR
  501 TQE2=TQE2+W2QE(I,I)
C     VO2=VARIANCE COMPONENT FOR DISTANCES
      VO2=E2TW2E(1,1)/TQE2
*     COMPUTING THE THIRD VARIANCE COMPONENT
      CALL ATB(A2,W2,A22,MAXDIR,NOTS,MAXDIR)
      CALL TIMES(A22,A2,A2W,NOTS,MAXDIR,NOTS)
      CALL MATINV(RTWR,RTWRI,NOTS)
      CALL TIMES(Rr,RTWRI,A333,NEQR,NOTS,NOTS)
      CALL TRANS(Rr,RT,NEQR,NOTS)
      CALL TIMES(A333,RT,RRI,NEQR,NOTS,NEQR)
      CALL MINUS(Wr,RRI,QE3,NEQR,NEQR)
      CALL TIMES(Wr,QE3,WrQE,NEQR,NEQR,NEQR)
      CALL TRANS(E3,E3T,NEQR,1)
      CALL TIMES(E3T,Wr,E3TWr,1,NEQR,NEQR)
      CALL TIMES(E3TWr,E3,E3TWrE,1,NEQR,1)
*     COMPUTING THE TRACE
      TQE3=0.
      DO 502 I=1,NEQR
  502 TQE3=TQE3+WrQE(I,I)
C     VO3=VARIANCE COMPONENT FOR DISTANCES
      VO3=E3TWrE(1,1)/TQE3
*     ******************************
*     *M O D U L E     T H R E E    *
      ******************************
*     COV IS THE  COVARIANCE MATRIX OF THE ESTIMATED
*     PARAMETERS
      DO 43 I=1,NOTS
      DO 43 J=1,NOTS
   43 COV(I,J)=SUM1(I,J)*APVUW
C     UPDATING PROVISIONAL VALUES
      DO 44 I=1,MAXST
      J=2*I-1
      K=J+1
      X(I)=X(I)+DELS(J,1)
      Y(I)=Y(I)+DELS(K,1)
   44 CONTINUE
*    CALCULATING THE COVARIANCE MATRIX FOR ADJUSTED OBSERVATIONS
      CALL TIMES(A,COV,ADJO,NEQ,NOTS,NOTS)
      CALL TRANS(A,ATR,NEQ,NOTS) L!^CL
*     STDO IS THE VECTOR OF STD ERRORS TO THE ADJUSTED
C     OBSERVATIONS
      DO 45 I=1,NEQ
      STDO(I)=WT(I,I)**0.5
   45 CONTINUE
*     DX,DY ARE CORRECTIONS TO THE PROVISIONAL COORDINATES***
      DO 46 I=1,MAXST
      DX1(I)=X(I)-DXA(I)
      DY1(I)=Y(I)-DYA(I)
```

```
 46   CONTINUE
 47   CONTINUE
********************** ANALYSIS***************************
*     COMPUTING ERROR ELLIPSE PARAMETERS
      DO 78 J=1,MAXST
      I=2*J-1
      VARX(J)=COV(I,I)
      M=I+1
      VARY(J)=COV(M,M)
*     CVXY IS THE COVARIANCE B/T X & Y
      CVXY(J)=COV(I,M)
C     SGX=STND ERROR IN X
C     SGY=STND ERROR IN Y
      SGX(J)=VARX(J)**0.5
      SGY(J)=VARY(J)**0.5
*     AXES OF ERROR ELLIPSES
      VVX=0.5*(VARX(J)+VARY(J))
      VVY=(0.25*(VARX(J)-VARY(J))**2+CVXY(J)**2)**0.5
      BOS(J)=VARX(J)-VARY(J)
      IF(BOS(J).EQ.0.)BOS(J)=0.00001
      TCV=2.0*CVXY(J)/BOS(J)
      THETA(J)=ATAN(TCV)
      THETA(J)=THETA(J)/2.
      IF(THETA(J).LT.0)THETA(J)=THETA(J)+2.0*PI
      CALL ANGLE(THETA(J),ITEG(J),MTIN(J),TSEC(J))
*     VMAX=SQUARE OF SEMI MAJOR AXIS
*     VMIN=SQUARE OF SEMI MINOR AXIS
      VMAX(J)=VVY+VVX
      VMIN(J)=VVX-VVY
      WRITE(*,*)VMIN(J)
C     SGMX,SGMY ARE SEMI MAJOR AND SEMI MINOR AXES
      SGMX(J)=VMAX(J)**0.5
      SGMY(J)=ABS(VMIN(J))**0.5
C     POINT MEAN ERROR
      EC(J)=((VARX(J)+VARY(J))/2.)**0.5
  78  CONTINUE
*     NETWOK MEAN ERROR
      IRR=MAXST-MAXDAT
      TRACE=0.0
      DO 405 I=1,IRR
      TRACE=TRACE+COV(I,I)
 405  CONTINUE
      IF(MAXDAT.EQ.0)GOTO 406
************ OUTPUT OF RESULTS***************************
      IF(NEQR.EQ.0)GOTO 821
      WRITE(11,199)
 199  FORMAT(/,12X,'STATIC-DYNAMIC NETWORK ADJUSTMENT '
     X/32('== '))
      GO TO 822
 821  WRITE(11,820)
 820  FORMAT(/,20X,'DYNAMIC SOLUTION ADJUSTMENT  '/32('=='))
 822  WRITE(11,210)ITR
```

```
 210 FORMAT(//,15X,'RESULTS OBTAINED AFTER  ',I4,'
    1ITERATIONS',//)
     WRITE(11,316)
 316 FORMAT(21X,'OBSERVED AND'/3X,'LINE',10X,'REDUCED
    1DIRECTIONS',9X,'PROVISIONALBEARINGS',6X,'VECTORY'
    1/3X,2('--'),10X,18('-'),19X,20('-'),6X,8('-')/)
     WRITE(11,317)(LANG(N,1),LANG(N,2),IDEG(N),MIN(N)
    X,SEC(N),NBRG(N),MITN(N),SCS(N),Y2(N),N=1,MAXDIR)
 317 FORMAT(1X,2I3,8X,2I6,F7.1,9X,2I6,F7.3,8X,F6.3)
     WRITE(11,318)
 318 FORMAT(21X,'OBSERVED AND'/3X,'LINE',10X,'REDUCED
     DISTANCES',9X,'PROVISIONAL DISTANCE',9X,'VECTORY'/3X,
    X2('--'),10X,18('-'),9X,20('-'),6X,8('-')/)
     WRITE(11,319)(LDST(N,1),LDST(N,2),DST(N),DIST1(N),Y1(N)
    1,N=1,MAXDST)
 319 FORMAT(1X,2I3,8X,F10.3,19X,F10.3,16X,F6.3)
     WRITE(11,201)
 201 FORMAT(//20X,'ESTIMATED PARAMETERS'/32('**'))
     WRITE(11,212)
 212 FORMAT(11X,'CORRECTIONS TO
    XPROVISIONALCOORDINATES'/,11X,'--------------------------',
    X/23X,'UNITS:METRES'/18X,'DELX'
    1,10X,'DEL Y'/15X,'----------'8X,'---------',/)
     IRR=MAXST-MAXDAT
     IF(MAXDAT.EQ.0)THEN
     DO 860 I=1,NOTS
860  CRX(I,1)=DELS(I,1)
     ELSE
     ENDIF
     DO 222 I=1,IRR
     J=I*2-1
     K=J+1
     WRITE(11,203)CRX(J,1),CRX(K,1)
203  FORMAT(17X,F9.6,7X,F9.6)
222  CONTINUE
     WRITE(11,208)VUW
     WRITE(11,209)E1TW1E,TQE1,E2TW2E,TQE2,E3TWrE,TQE3
 209 FORMAT(//5X,' VARIANCE COMPONENTS',F8.3,2X,F8.3,2X,F8.3,
    12X,F8.3,2X,F8.3,2X,F8.3)
208  FORMAT(//5X,'APOSTERIORI VARIANCE
    XOF UNIT WEIGHT-',F8.3)
300  FORMAT(//,10X,'NETWORK MEAN ERROR=',F5.3,2X,'METRES'/10X,
    120('=='))
     IF(MAXDAT.EQ.0)THEN
     DO 824 I=1,NEQ
824  RESD(I,1)=ARES(I,1)
     ELSE
     ENDIF
     WRITE(11,320)
320  FORMAT(/38X,'STANDARD ERRORS OF'/38X,'
    XADJUSTED OBSERVATIONS',16X,'RESIDUALS'/2X,'LINE',4X,'
    XADJUSTED OBSERVATIONS',12X,'(SEC
```

```
     1ONDS)',13X,'(SECONDS)'/10X,20('-'),7X,20('-'),6X,9('-')/)
      WRITE(11,321)(LANG(I,1),LANG(I,2),IDG(I),IMN(I),RSC(I),
     XSTDO(I+MAXDST),RESD((I+MAXDST),1),I=1,MAXDIR)
321   FORMAT(2I3,2X,2I6,F7.1,15X,F6.3,15X,F6.3)
      WRITE(11,322)
322   FORMAT(/38X,'STANDARD ERROS OF'/38X,'ADJUSTED
     XOBSERVATIONS',6X,'RESIDUALS'/2X,'LINE',4X,'
     1ADJUSTEDOBSERVATIONS',12X,'(METRES)'
     1,13X,'(METRES)'/10X,20('-'),7X,20('-'),6X,9('-')/)
      WRITE(11,323)(LDST(I,1),LDST(I,2),ADY(I,1),STDO(I),
     1RESD(I,1),I=1,MAXDST)
323   FORMAT(2I3,10X,F10.3,15X,F6.4,15X,F6.3)
      WRITE(11,342)
      WRITE(11,343)(re(I,1),E3(I,1),I=1,NEQR)
342   FORMAT(/, 'RESTRICTION VECTOR',23X,'RESITRICTION
     XRESIDUALS')
343   FORMAT(F8.6,12X,F8.6)
      WRITE(11,312)
312   FORMAT(/,11X,'STANDARD ERRORS',25X,'ERROR ELLIPSES'/11X,
     1'--------------',25X,'--------------',/38X,'SEMI',9X,
     1'SEMI'/1X,'STN',6X,'SIGMA',10X,'SIGMA',7X,'MAJOR',8X,
     X'MINOR',12X,'ORIENTATION',/)
      WRITE(11,852)(I,SGX(I),SGY(I),SGMX(I),SGMY(I),
     XITEG(I),MTIN(I),TSEC(I),I=1,IRR)
852   FORMAT(1X,I2,3F13.4,F13.6,7X,2I6,F6.1)
      WRITE(11,314)
314   FORMAT(1X,'STN',1X,'PROVISIONALCOORDINATES',6X,'
     XCORRECTIONS',13X,'FINALCOORDINATES'/1X,'---',1X,'---
     X--------',6X,'-----------',13X,'-----------
     X'/8X,'EASTING',5X,'NORTHING',7X,'DEL-E',3X,
     X'DEL-N',9X,'EASTING',5X,'NORTHING')
      WRITE(11,315)(I,DXA(I),DYA(I),DX1(I),DY1(I),
     XX(I),Y(I),I=1,MAXST)
315   FORMAT(1X,I2,2F12.3,6X,2F9.4,4X,2F14.4)
      STOP
      END
```

APPENDIX A.2: PROGRAM FREE.FOR

```
      IMPLICIT REAL*8(A-H,O-Z)
      DOUBLE PRECISION X(8),Y(8),DXA(8),DYA(8)
      DIMENSION LANG(28,2),IDEG(28),MIN(28),SEC(28)
      DIMENSION LDST(14,2),DST(14),DIST1(14),DIST2(28)
      DIMENSION A1(14,16),A2(28,16),W(42,42),A(42,16)
      DIMENSIONY1(14),Y2(28),YM(42,1),ANG0(14),ANG1(28),ANG(42,1)

      DIMENSION IDG(28),IMN(28),RSC(28),RESD(42,1),ETW(1,42)
      DIMENSION NBRG(28),MITN(28),SCS(28),AZMT(28),CAZMT(28)
      DIMENSION ATW(16,42),ATWA(16,16), ATWAIN(16,16),ADY(42,1)
      DIMENSION ETWE(1,1),DX(16,16),ADJO(42,16),ATR(16,42)
      DIMENSION DX1(8),DY1(8),STDO(42), R(16,42),CRX(16,1)
      DIMENSION WT(42,42),SOLN(42,1)

C     FIRST VARIANCE COMPONENT DIMENSIONS
*     ***********************************
      DIMENSION W1(14,14),E1(14,1),W1I(14,14),A11(16,14)
      DIMENSION A1W(16,16),A111(14,16),A1T(16,14),QE1(14,14)
      DIMENSION E1T(1,14),E1TW1(1,14),E1TW1E(1,1),AA1(14,14)
      DIMENSION A1WI(16,16)W1QE(14,14)
C     SECOND VARIANCE COMPONENT DIMENSIONS
*     ************************************
      DIMENSION W2(28,28),E2(28,1),W2I(28,28),A22(16,28)
      DIMENSION A2W(16,16),A222(28,16),A2T(16,28),QE2(28,28)
      DIMENSION A2WI(16,16),W2QE(28,28),E2T(1,28),E2TW2(1,28)
      DIMENSION E2TW2E(1,1),AA2(28,28)

C     FREENET MANIPULATIONS
*     *********************
      DIMENSION AT(16,42),G(4,16),GT(16,4),GTG(16,16),ADG(16,16)
      DIMENSION AV(16,16),GAV(4,16),GTAV(16,16),AVGT(16,16)
     DIMENSION ASB(16,16),ASAT(16,42),ADJX(16,1),ADX(42,1),E(42,1)
      DIMENSION ATSB(16,16),ABA(16,16),COV(16,16),ET(1,42)

C     COVARIANCE MATRIX MANIPULATIONS
*     *******************************
      DIMENSION CVXY(8),ITEG(8),MTIN(8),TSEC(8),VARX(8)
      DIMENSION VARY(8),SGY(8),SGX(8),EC(8)
      DIMENSION VMIN(8),THETA(8),SGMX(8),SGMY(8),VMAX(8)
      REAL KK(4,4),KKINV(4,4)

************************************************************
C      MAXDST=NUMBER OF DISTANCES MEASURED
C      MAXDIR=NUMBER OF DIRECTIONS MEASURED
C      MAXST= NUMBER OF STATIONS IN THE NETWORK
C     NOTS=(MAXST-MAXDAT)*2 THE NUMBER OF UNKNOWNS IN THE X-VECTOR
C      NEQ=MAXDST+MAXDIR TOTAL NUMBER OF OBSERVATIONS/EQUATIONS
C
************************************************************
```

```
      WRITE(*,1)
      READ(*,2)MAXST
    1 FORMAT('       INPUT TOTAL NUMBER OF STATIONS (MAXST)')
    2 FORMAT(I2)
      WRITE(*,3)
    3 FORMAT('      INPUT TOTAL NUMBER OF DATUM STATIONS (MAXDAT)')
      READ(*,2)MAXDAT
      WRITE(*,4)
    4 FORMAT('      INPUT NUMBER OF DISTANCE OBSERVATIONS(MAXDST)')
      READ(*,2)MAXDST
      WRITE(*,5)
    5 FORMAT('       INPUT TOTAL NUMBER DIRECTIONS OBS.(MAXDIR)')
      READ(*,2)MAXDIR
      WRITE(*,6)
    6 FORMAT('         HOW MANY ITERATIONS ?')
      READ(*,2)ITR
      WRITE(3,12)
   12 FORMAT(15X,'THE INPUT DATA USED FOR THE STUDY ARE :'
     </13X,50('=')///,6X,'POINT',28X,'APPR.CO-ORDINATES'
     </5X,13('='),25X,
     <14('=')/31X,'X(METRES)',15X,'Y(METRES)'/30X
     <,11('='),13X,10('='))
      READ(12,13)(Y(I),X(I),I=1,MAXST)
   13 FORMAT(X,F11.3,2X,F10.3)
***** STORING PROVISION COORDINATES
      DO 100 I=1,MAXST
      DXA(I)=X(I)
      DYA(I)=Y(I)
  100 CONTINUE
****  APVUW IS THE APRIOR VARIANCE OF UNIT WEIGHT
      APVUW=1.0
      RS=206264.8
      PI=3.141592
      ICNT=1
      WRITE(3,16)(I,DYA(I),DXA(I),I=1,MAXST)
   16 FORMAT(7X,I3,20X,F11.3,16X,F10.3)
      WRITE(3,17)
   17 FORMAT(/9X,'DIST.NO:',5X,'STATION',12X,'DIST.BETWEEN
     <STATIONS'
     </8X,10('='),5X,10('='),9X,24('='))
*     INPUT FOR OBSERVED DISTANCES
      READ(21,14)(LDST(I,1),LDST(I,2),DST(I),I=1,MAXDST)
   14 FORMAT(I1,X,I1,X,F9.3)
      WRITE(3,18)(I,LDST(I,1),LDST(I,2),DST(I),I=1,MAXDST)
   18 FORMAT(11X,I2,10X,I2,3X,I2,23X,F9.3)
      WRITE(3,19)
   19 FORMAT(/5X,'LINE.NO:',8X,'STATION',6X,'H.DIR
     <(DEG,MIN,& SEC)',9X
     <,'AZMT(RADIANS)'/4X,10('='),6X,10('='),4X,24('='),7X,15('='))
*     INPUT FOR HORIZONTAL DIRECTIONS
      READ(21,15)(LANG(I,1),LANG(I,2),IDEG(I),MIN(I),
     <SEC(I),I=1,MAXDIR)
```

```
   15 FORMAT(I1,X,I1,X,I3,X,I2,X,F5.2)
      WRITE(3,20)(I,LANG(I,1),LANG(I,2),IDEG(I),MIN(I),SEC(I)
     <,I=1,MAXDIR)
   20 FORMAT(6X,I3,10X,I2,2X,I2,12X,I4,2X,I3,2X,F5.2)
 1000 CONTINUE
      ICNT=ICNT+1
      NEQ=MAXDST+MAXDIR
      NOTS=(MAXST-MAXDAT)*2
C     INITIALIZING  THE OUT PUT MATRICES AND VECTORS
      DO 10 I=1,NEQ
      DO 10 J=1,NOTS
      A(I,J)=0.0
      ANG(I,1)=0.0
      W(I,I)=0.0
  10  CONTINUE
*     FREENETWORK ADJUSTMENT MODULE
*   ***********************************************************
*    COMPUTING DISTANCE OBSERVATION PARAMETERS AND LOADING IN THE
*     DESIGN MATRIX
*     *************
      DO 30 I=1,MAXDST
      K1=LDST(I,1)
      K2=LDST(I,2)
      DIST1(I)=SQRT((X(K2)-X(K1))**2+(Y(K2)-Y(K1))**2)
      J1=K1*2-1
      J2=J1+1
      A1(I,J1)=(X(K1)-X(K2))/(DIST1(I))
      A1(I,J2)=(Y(K1)-Y(K2))/(DIST1(I))
      J3=K2*2-1
      J4=J3+1
      A1(I,J3)=(X(K2)-X(K1))/(DIST1(I))
      A1(I,J4)=(Y(K2)-Y(K1))/(DIST1(I))
      Y1(I)=DST(I)-DIST1(I)
*     STORING COMPUTED DISTANCES
      ANG0(I)=DIST1(I)
      W1(I,I)=APVUW/((0.003**2)+(DST(I)*10E-6)**2)
  30  CONTINUE
C     *******************************************************
C     FORMATION OF DESIGN MATRIX COMPONENT A2 FOR DIRECTIONS
C    ********************************************************
      DO 31 I=1,MAXDIR
      L1=LANG(I,1)
      L2=LANG(I,2)
      DIST2(I)=SQRT((X(L2)-X(L1))**2+(Y(L2)-Y(L1))**2)
      J1=L1*2-1
      J2=J1+1
      A2(I,J1)=(Y(L2)-Y(L1))/DIST2(I)*RS
      A2(I,J2)=(X(L1)-X(L2))/DIST2(I)*RS
      J3=L2*2-1
      J4=J3+1
      A2(I,J3)=(Y(L1)-Y(L2))/DIST2(I)*RS
      A2(I,J4)=(X(L2)-X(L1))/DIST2(I)*RS
```

```
      DN=Y(L2)-Y(L1)
      DE=X(L2)-X(L1)
*     COMPUTATION OF OBSERVED HORIZONTAL ANGLES
*     CAZMT  IS THE COMPUTED BEARING
C     CONVERSION OF THE COMPUTED BEARINGS INTO
C     DEGREES,MINUTES,SECONDS
      CALL QUAD(DE,DN,Y22)
      CAZMT(I)=Y22
C    CONVERSION OF COMPUTED AZIMUTHS INTO DEGREES, MINUTES,SECONDS
      XB=CAZMT(I)*PI/180.
      CALL ANGLE(XB,NBRG(I),MITN(I),SCS(I))
      ANG1(I)=Y22*3600.
*     AZMT **** IS THE INPUT DIRECTION
      Y2(I)=SCS(I)-SEC(I)
      W2(I,I)=APVUW/0.49
  31  CONTINUE
C    ***************************************************************
C     FORMATION OF COMPLETE DESIGN MATRIX,WEIGHT MATRIX,& Y-VECTOR
C    ***************************************************************
      DO 33 I=1,NEQ
      DO 33 J=1,MAXST*2
      IF(I.GT.MAXDST)GO TO 32
      A(I,J)=A1(I,J)
      YM(I,1)=Y1(I)
      W(I,I)=W1(I,I)
      ANG(I,1)=ANG0(I)
      GO TO 33
  32  K=I-MAXDST
      A(I,J)=A2(K,J)
      W(I,I)=W2(K,K)
      YM(I,1)=Y2(K)
      ANG(I,1)=ANG1(K)
  33  CONTINUE
*     FORMING THE RESTRICTION MATRIX FOR THE FREENET.
      DO 40 I=1,4
      DO 40 J=1,NOTS
  40  G(I,J)=0.0
      DO 41 I=1,4
      J=2*I-1
      K=J+1
      G(I,J)=1.0
      G(2,K)=1.0
      G(3,J)=-1.0*(Y(I)-9935646.003)
      G(3,K)= X(I)-279718.82
      G(4,J)=X(I)-279718.82
      G(4,K)=Y(I)-9935646.003
      NDF=4
  41  CONTINUE
*     FREENET SOLUTION MANIPULATION
      NEQ=MAXDIR+MAXDST
      CALL TRANS(A,AT,NEQ,NOTS)
      CALL TRANS(G,GT,NDF,NOTS)
```

```
      CALL TIMES(AT,W,ATW,NOTS,NEQ,NEQ)
      CALL TIMES(ATW,A,ATWA,NOTS,NEQ,NOTS)
      CALL TIMES(GT,G,GTG,NOTS,NDF,NOTS)
      CALL ADD(ATWA,GTG,ADG,NOTS,NOTS)
      CALL MATINV(ADG,AV,NOTS)
      CALL TIMES(G,AV,GAV,NDF,NOTS,NOTS)
      CALL TIMES(GAV,GT,KK,NDF,NOTS,NDF)
      CALL TIMES(GT,GAV,GTAV,NOTS,NDF,NOTS)
      CALL TIMES(AV,GTAV,AVGT,NOTS,NOTS,NOTS)
      CALL MINUS(AV,AVGT,ASB,NOTS,NOTS)
      CALL TIMES(ASB,ATW,ASAT,NOTS,NOTS,NEQ)
***   ADJX ** IS THE VECTOR OF ESTIMATED PARAMETERS
      CALL TIMES(ASAT,YM,ADJX,NOTS,NEQ,1)
      CALL TIMES(A,ADJX,ADX,NEQ,NOTS,1)
***   E  IS THE VECTOR OF RESIDUALS
      CALL MINUS(YM,ADX,E,NEQ,1)
****  SOLN IS THE VECTOR OF ADJUSTED OBSERVATIONS
      CALL ADD(ANG,E,ADY,NEQ,1)
**** CONVERTING ADJUSTED BEARINGS TO DEGREES,MINUTES AND SECONDS**
      DO 42 I=MAXDST+1,NEQ
      XX=ADY(I,1)/3600.*PI/180.
  42  CALL ANGLE(XX,IDG(I-MAXDST),IMN(I-MAXDST),RSC(I-MAXDST))
***   E1 ARE RESIDUALS FOR DISTANCES
***** E2 ARE RESIDUALS FOR DIRECTIONS
      DO 49 I=1,NEQ
      IF(I.GT.MAXDST)GO TO 48
      E1(I,1)=E(I,1)
      GO TO 49
  48  E2((I-MAXDST),1)=E(I,1)
  49  CONTINUE
***   COMPUTING THE COFACTOR MATRICES///
      CALL MATINV(W1,W1I,MAXDST)
      CALL MATINV(W2,W2I,MAXDIR)
***   COMPUTING THE FIRST VARIANCE COMPONENT
      CALL ATB(A1,W1,A11,MAXDST,NOTS,MAXDST)
      CALL TIMES(A11,A1,A1W,NOTS,MAXDST,NOTS)
      CALL MATINV(A1W,A1WI,NOTS)
      CALL TIMES(A1,A1WI,A111,MAXDST,NOTS,NOTS)
      CALL TRANS(A1,A1T,MAXDST,NOTS)
      CALL TIMES(A111,A1T,AA1,MAXDST,NOTS,MAXDST)
      CALL MINUS(W1I,AA1,QE1,MAXDST,MAXDST)
      CALL TIMES(W1,QE1,W1QE,MAXDST,MAXDST)
      CALL TRANS(E1,E1T,MAXDST,1)
      CALL TIMES(E1T,W1,E1TW1,1,MAXDST,MAXDST)
      CALL TIMES(E1TW1,E1,E1TW1E,1,MAXDST,1)
***** COMPUTING THE TRACE
      TRAW1QE1=0.
      DO 50 1=1,MAXDST
  50  TRAW1QE1=TRAW1QE1+W1QE(I,I)
***   VO1= VARIANCE COMPONET FOR DISTANCES
      VO1=E1TW1E/TRAW1QE1
****  COMPUTING THE SECOND VARIANCE COMPONENT
```

```
      CALL ATB(A2,W2,A22,MAXDIR,NOTS,MAXDIR)
      CALL TIMES(A22,A2,A2W,NOTS,MAXDIR,NOTS)
      CALL MATINV(A2W,A2W2,NOTS)
      CALL TIMES(A2,A2W2, A222,MAXDIR,NOTS,NOTS)
      CALL TRANS(A2,A2T,MAXDIR,NOTS)
      CALL TIMES(A222,A2T, AA2,MAXDIR,NOTS,MAXDIR)
      CALL MINUS(W2I,AA2,QE2,MAXDIR,MAXDIR)
      CALL TIMES(W2,QE2, W2QE,MAXDIR,MAXDIR)
      CALL TRANS(E2,E2T,MAXDIR,1)
      CALL TIMES(E2T,W2,E2TW2,1,MAXDIR,MAXDIR)
      CALL TIMES(E2TW2,E2,E2TW2E,1,MAXDIR,1)
***** COMPUTING THE TRACE
      TRAW2QE2=0.
      DO 50 1=1,MAXDST
  50  TRAW2QE2=TRAW2QE2+W2QE(I,I)
***   VO2= VARIANCE COMPONET FOR DISTANCES
      VO2=E2TW2E/TRAW2QE2

C     COMPUTING THE SUMMED VARIANCE OF UNIT WEIGHT
      VUW=(E2TW2E+E1TW1E)/(TRAW1QE1+TRAW2QE2)
      CALL TRANS(ASB,ATSB,NOTS,NOTS)
      CALL TIMES(ASB,ATWA,ABA,NOTS,NOTS,NOTS)
      CALL TIMES(ABA,ATSB,COV,NOTS,NOTS,NOTS)
****  COV IS THE  COVARIANCE MATRIX OF THE ESTIMATED PARAMETERS***
      DO 43 I=1,NOTS
      DO 43 J=1,NOTS
   43 COV(I,J)=COV(I,J)*VUW/APVUW
C     UPDATING PROVISIONAL VALUES
      DO 44 I=1,MAXST
      J=2*I-1
      K=J+1
      X(I)=X(I)+ADJX(J,1)
      Y(I)=Y(I)+ADJX(K,1)
   44 CONTINUE
*     IF(ICNT.EQ.ITR)GO TO 807
      APVUW=VUW
      GO TO 1000
      WNME=SQRT(TRACE/(IRR*2))
***   CALCULATING THE COVARIANCE MATRIX FOR ADJUSTED OBSERVATIONS
      CALL TIMES(A,COV,ADJO,NEQ,NOTS,NOTS)
      CALL TIMES(ADJO,AT,WT,NEQ,NOTS,NEQ)
**   STDO IS THE VECTOR OF STD ERRORS TO THE ADJUSTED OBSERVATIONS
      DO 45 I=1,NEQ
      STDO(I)=WT(I,I)**0.5
*     WRITE(100,*)WT(I,I)
   45 CONTINUE
*     DX,DY ARE CORRECTIONS TO THE PROVISIONAL COORDINATES***
      DO 46 I=1,MAXST
      DX1(I)=X(I)-DXA(I)
      DY1(I)=Y(I)-DYA(I)
  46  CONTINUE
```

```
********************** ANALYSIS***************************
      COMPUTING ERROR ELLIPSE PARAMETERS
      IRR=MAXST-MAXDAT
      DO 78 J=1,IRR
      I=2*J-1
      VARX(J)=DX(I,I)
      M=I+1
      VARY(J)=DX(M,M)
******CVXY IS THE COVARIANCE B/T X & Y
*     CVXY(J)=DX(I,M)
C     SGX=STND ERROR IN X
C     SGY=STND ERROR IN Y
      SGX(J)=VARX(J)**0.5
      SGY(J)=VARY(J)**0.5
***   AXES OF ERROR ELLIPSES
      VVX=0.5*(VARX(J)+VARY(J))
      VVY=(0.25*(VARX(J)-VARY(J))**2+CVXY(J)**2)**0.5
      TCV=(2.0*CVXY(J))/(VARX(J)-VARY(J))
      THETA(J)=ATAN(TCV)
      THETA(J)=THETA(J)/2.
      IF(THETA(J).LT.0)THETA(J)=THETA(J)+2.0*PI
      CALL ANGLE(THETA(J),ITEG(J),MTIN(J),TSEC(J))
**    VMAX=SQUARE OF SEMI MAJOR AXIS
***   VMIN=SQUARE OF SEMI MINOR AXIS
      VMAX(J)=VVY+VVX
      VMIN(J)=VVX-VVY
      WRITE(*,*)VMIN(J)
C     SGMX,SGMY ARE SEMI MAJOR AND SEMI MINOR AXES
      SGMX(J)=VMAX(J)**0.5
      SGMY(J)=ABS(VMIN(J))**0.5
C     POINT MEAN ERROR
      EC(J)=((VARX(J)+VARY(J))/2.)**0.5
   78 CONTINUE
*     NETWOK MEAN ERROR
      IRR=MAXST-MAXDAT
      TRACE=0.0
      DO 405 I=1,IRR
      TRACE=TRACE+DX(I,I)
  405 CONTINUE
      WNME=SQRT(TRACE/(IRR*2))

****************** OUTPUT OF RESULTS***************************
  821 WRITE(11,820)
  820 FORMAT(/,20X,'FREE-NETWORK ADJUSTMENT  '/32('=='))
  822 WRITE(11,210)ITR
  210 FORMAT(//,15X,'RESULTS OBTAINED AFTER ',I4,' ITERATIONS',//)
      WRITE(11,316)
  316 FORMAT(21X,'OBSERVED AND'/3X,'LINE',10X,'REDUCED DIRECTIONS',
     19X,'PROVISIONAL BEARINGS',6X,'VECTOR Y'/3X,2('--'),10X,
     <18('-'),19X,20('-'),6X,8('-')/)
      WRITE(11,317)(LANG(N,1),LANG(N,2),IDEG(N),MIN(N)
     X,SEC(N),NBRG(N),MITN(N),SCS(N),Y2(N),N=1,MAXDIR)
```

```
 317 FORMAT(1X,2I3,8X,2I6,F7.1,9X,2I6,F7.1,8X,F6.3)
     WRITE(11,318)
 318 FORMAT(21X,'OBSERVED AND'/3X,'LINE',10X,'REDUCED DISTANCES',
    19X,'PROVISIONAL DISTANCE',9X,'VECTOR Y'/3X,2('--'),10X,
    <18('-'),19X,20('-'),6X,8('-')/)
     WRITE(11,319)(LDST(N,1),LDST(N,2),DST(N),DIST1(N),Y1(N)
    1,N=1,MAXDST)
 319 FORMAT(1X,2I3,8X,F10.3,19X,F10.3,16X,F6.3)
     WRITE(11,201)
 201 FORMAT(//20X,'ESTIMATED PARAMETERS'/32('**'))
     WRITE(11,212)
 212 FORMAT(11X,'CORRECTIONS TO PROVISIONAL COORDINATES'/,
    <11X,'------------------------',/23X,'UNITS:METRES'/18X,'DEL
    XX1,10X,'DEL Y'/15X,'----------'8X,'---------',/)
     IRR=MAXST-MAXDAT
     IF(MAXDAT.EQ.0)THEN
     DO 860 I=1,NOTS
860  CRX(I,1)=ADJX(I,1)
     ELSE
     ENDIF
     DO 222 I=1,IRR
     J=I*2-1
     K=J+1
     WRITE(11,203)CRX(J,1),CRX(K,1)
203  FORMAT(17X,F6.4,11X,F6.4)
222  CONTINUE
     WRITE(11,208)E1TW1E,TRAW1WQE1
208  FORMAT(//5X,'APOSTERIORI VARIANCE OF UNIT WEIGHT-',F8.3)
     WRITE(11,300)WNME
300  FORMAT(//,10X,'NETWORK MEAN ERROR=',F5.3,2X,'METRES'/10X,
    120('=='))
     IF(MAXDAT.EQ.0)THEN
     DO 824 I=1,NEQ
824  RESD(I,1)=E(I,1)
     ELSE
     ENDIF
     WRITE(11,320)
320 FORMAT(/38X,'STANDARD ERRORS OF'/38X,'ADJUSTED OBSERVATIONS',
    16X,'RESIDUALS'/2X,'LINE',4X,'ADJUSTED OBSERVATIONS',12X,'(SEC
    1ONDS)',13X,'(SECONDS)'/10X,20('-'),7X,20('-'),6X,9('-')/)
     WRITE(11,321)(LANG(I,1),LANG(I,2),IDG(I),IMN(I),RSC(I),
    XSTDO(I+MAXDST),RESD((I+MAXDST),1),I=1,MAXDIR)
321  FORMAT(2I3,2X,2I6,F7.1,15X,F6.3,15X,F6.3)
     WRITE(11,322)
322 FORMAT(/38X,'STANDARD ERROS OF'/38X,'ADJUSTED OBSERVATIONS',
    16X,'RESIDUALS'/2X,'LINE',4X,'ADJUSTED OBSERVATIONS',
    X12X,'(METRES)',13X,'(METRES)'/10X,20('-'),7X,20('-'),
    X6X,9('-')/)
     WRITE(11,323)(LDST(I,1),LDST(I,2),ADY(I,1),STDO(I),
    1RESD(I,1),I=1,MAXDST)
323  FORMAT(2I3,10X,F10.3,15X,F6.3,15X,F6.3)
342  WRITE(11,312)
```

```
312  FORMAT(/,11X,'STANDARD ERRORS',25X,'ERROR ELLIPSES'/11X,
    1'---------------',25X,'---------------',/38X,'SEMI',9X,
    1'SEMI'/1X,'STN',6X,'SIGMA',10X,'SIGMA',7X,'MAJOR',8X,'MINOR'
    1,12X,'ORIENTATION',/)
     IF(MAXDAT.EQ.0)THEN
     WRITE(11,852)(I,SGX(I),SGY(I),SGMX(I),SGMY(I),ITEG(I),MTIN(I)
    1,TSEC(I),I=1,IRR)
852  FORMAT(1X,I2,3F13.4,F13.6,7X,2I6,F6.1)
     ELSE
     ENDIF
     WRITE(11,313)(I,SGX(I-IRR),SGY(I-IRR),SGMX(I-IRR),SGMY(I-IRR)
    1,ITEG(I-IRR),MTIN(I-IRR),TSEC(I-IRR),I=(MAXST-MAXDAT+1),
    1MAXST)
313  FORMAT(1X,I2,4F13.3,7X,2I6,F6.1)
     WRITE(11,314)
314  FORMAT(1X,'STN',1X,'PROVISIONAL COORDINATES',6X,'
    XCORRECTIONS',13X,'FINAL COORDINATES'/1X,'---',1X,'
    X-----------------------',6X,'---2-------',
    X13X,'-----------------------'/8X,
    X'EASTING',5X,'NORTHING',7X,'DEL-E',3X,'
    XDEL-N',9X,'EASTING',5X,'NORTHING')
     WRITE(11,315)(I,DXA(I),DYA(I),DX1(I),DY1(I),
    <X(I),Y(I),I=1,MAXST)
315  FORMAT(1X,I2,2F12.3,6X,2F7.4,4X,2F14.4)
     STOP
     END
```

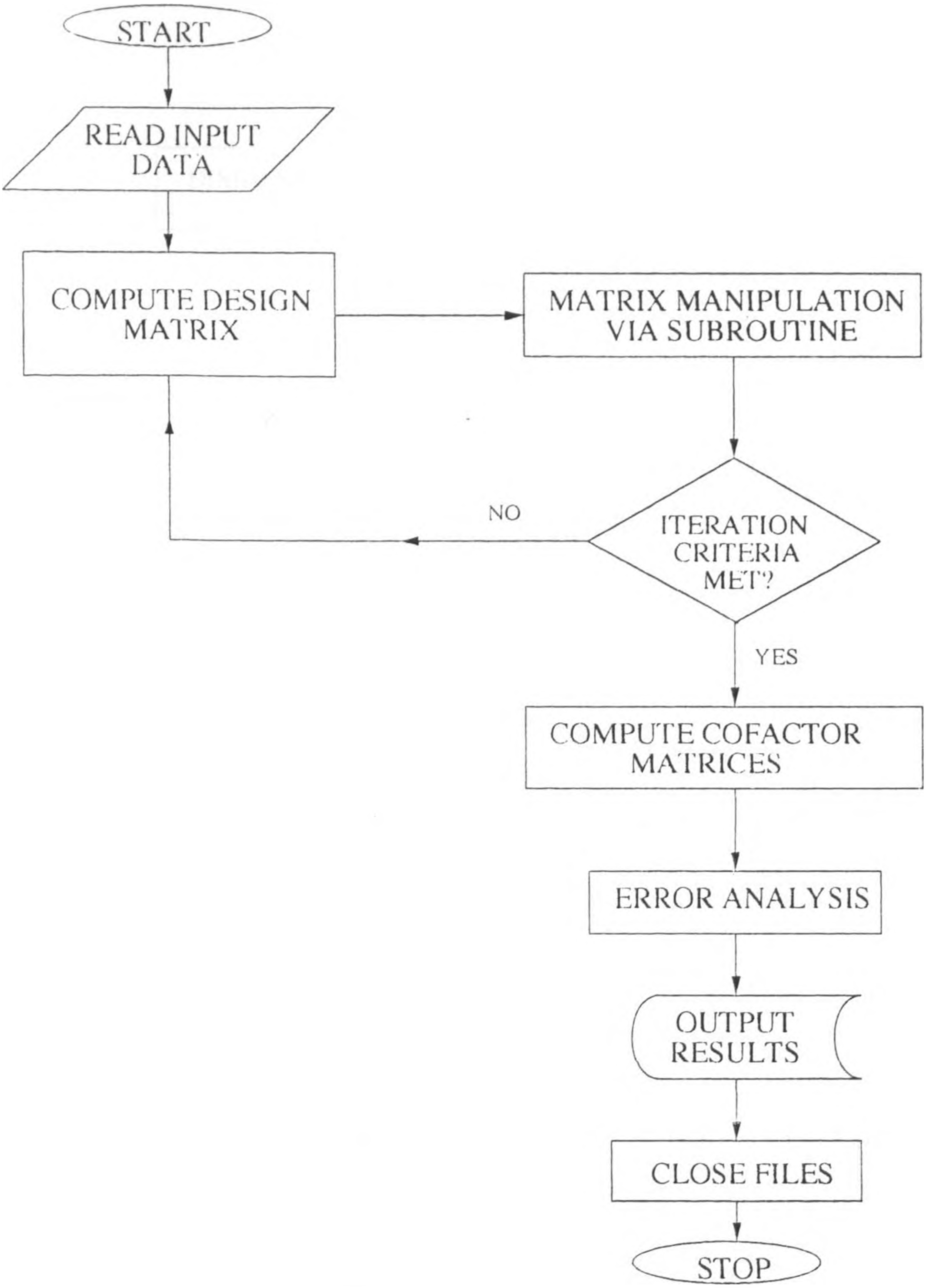
APPENDIX A.3: SUBROUTINES

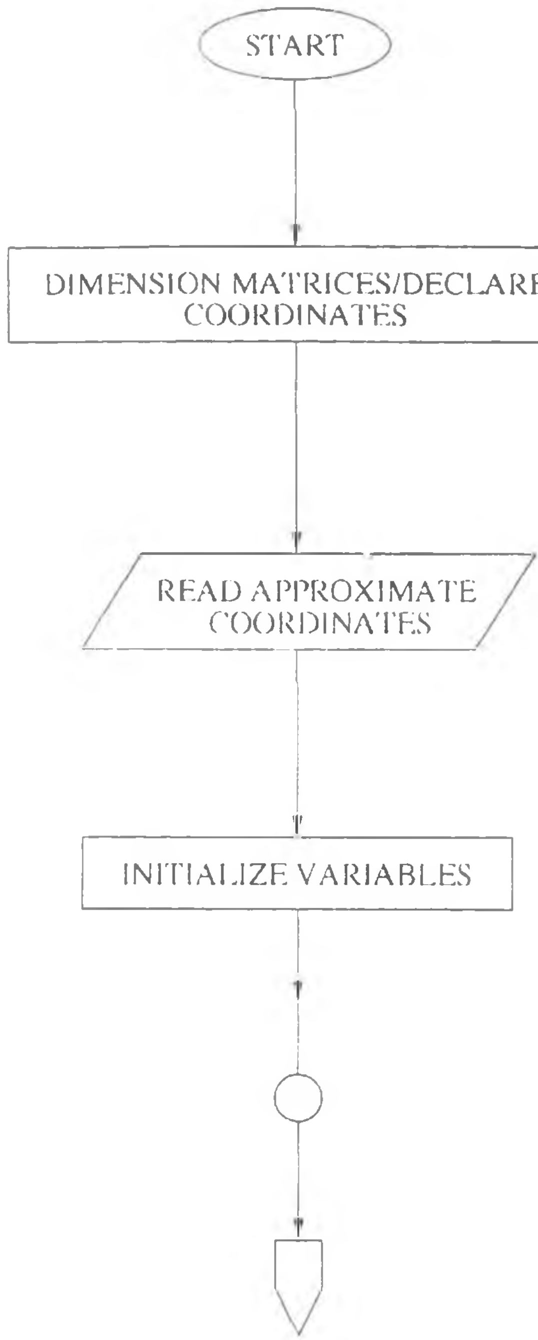
```
*-------------------------------------------------------------------
C     SUBROUTINE FOR DETERMINING QUADRANTS
C-------------------------------------------------------------------
      SUBROUTINE QUAD(A,B,C)
      IF(B.EQ.0.)GO TO 10
      BRG=(ATAN(A/B))*180./3.14159265389
 10   IF(B)20,30,40
 20   IF(A)80,70,80
 70   BRG=180.
      GO TO 160
 80   BRG=BRG+180.
      GO TO 160
 40   IF(A)50,60,160
 50   BRG=BRG+360.
      GO TO 160
 60   BRG=360.
      GO TO 160
 30   IF(A)90,92,93
 90   BRG=270.
      GO TO 160
 92   BRG=0.
      GO TO 160
 93   BRG=90.
 160  C=BRG
      RETURN
      END
******************************************************************
      SUBROUTINE TIMES(A,B,C,II,KK,JJ)
C     FORM MATRIX PRODUCT R=AC
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(II,1),B(KK,1),C(II,1)
      DO 15 I=1,II
      DO 15 J=1,JJ
   15 C(I,J)=0.0
      DO  35 I=1,II
      DO  35 K=1,KK
      AA=A(I,K)
      IF(AA.EQ.0.)GO TO 35
      DO 31 J=1,JJ
   31 C(I,J)=C(I,J)+AA*B(K,J)
   35 CONTINUE
      RETURN
      END
******************************************************************
      SUBROUTINE TRANS(A,B,L,N)
C     TRANSPOSE A INTO AT
      REAL*8 A(L,N),B(N,L)
      DO 5 I=1,N
      DO 5 J=1,L
    5 B(I,J)=A(J,I)
```

```
      RETURN
      END
*************************************************************
      SUBROUTINE ADD(A,B,C,M,N)
C     MATRIX ADDITION C=A+B
      IMPLICIT REAL*8(A-H,O-Z)
      DOUBLE PRECISION A(M,N),B(M,N),C(M,N)
      DO 10 I=1,M
      DO 10 J=1,N
      C(I,J)=A(I,J)+B(I,J)
   10 CONTINUE
      RETURN
      END
*************************************************************
      SUBROUTINE MATINV(A,RINV,N)
C     INVERT MATRIX A OF ORDER N;RINV IS ITS INVERSE
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION A(N,N),RINV(N,N),B(100,100)
      DO 7 I=1,N
      DO 7 J=1,N
    7 B(I,J)=A(I,J)
      J1=N+1
      J2=2*N
      DO 17 I=1,N
      DO 17 J=J1,J2
   17 B(I,J)=0.0
      DO 27 I=1,N
      J=I+N
   27 B(I,J)=1.0
      DO 97 K=1,N
      KP1=K+1
      IF(K.EQ.N) GO TO 57
      L=K
      DO 37 I=KP1,N
   37 IF(ABS(B(I,K)).GT.ABS(B(L,K)))L=I
      IF(L.EQ.K)GO TO 57
      DO 47 J=K,J2
      TEMP=B(K,J)
      B(K,J)=B(L,J)
   47 B(L,J)=TEMP
   57 DO 67 J=KP1,J2
   67 B(K,J)=B(K,J)/B(K,K)
      IF(K.EQ.1)GO TO 87
      KM1=K-1
      DO 77 I=1,KM1
      DO 77 J=KP1,J2
   77 B(I,J)=B(I,J)-B(I,K)*B(K,J)
      IF(K.EQ.N)GO TO 107
   87 DO 97 I=KP1,N
      DO 97 J=KP1,J2
   97 B(I,J)=B(I,J)-B(I,K)*B(K,J)
  107 DO 117 I=1,N
```

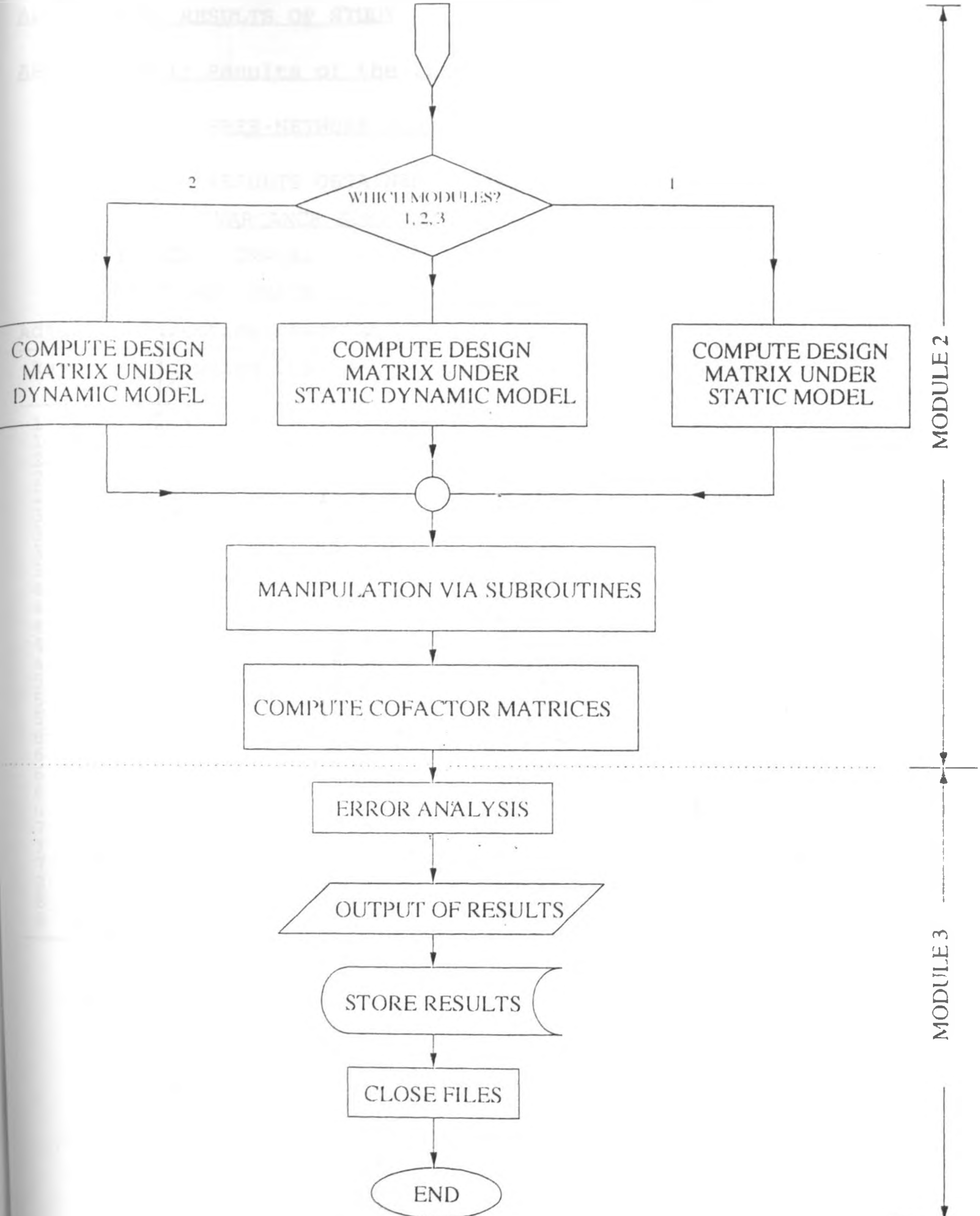
```
      DO 117 J=1,N
      K=J+N
  117 RINV(I,J)=B(I,K)
      RETURN
      END
*****************************************************************
      SUBROUTINE MINUS(A,B,R,M,N)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(M,N),B(M,N),R(M,N)
      DO 2 I=1,M
      DO 2 J=1,N
  2   R(I,J)=A(I,J)-B(I,J)
      RETURN
      END
C****************************************************************
      SUBROUTINE ABT(A,B,R,L,M,N)
C     FORM THE PRODUCT R=A*BT
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(L,M),B(N,M),R(L,N)
      DO 5 I=1,L
      DO 5 J=1,N
      R(I,J)=0.0
      DO 5 K=1,M
  5   R(I,J)=R(I,J)+A(I,K)*B(J,K)
      RETURN
      END
C****************************************************************
      SUBROUTINE ATB(A,B,R,L,M,N)
C     FORM THE PRODUCT R=AT*B
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(L,M),B(L,N),R(M,N)
      DO 5 I=1,M
      DO 5 J=1,N
      R(I,J)=0.0
      DO 5 K=1,L
      R(I,J)=R(I,J)+A(K,I)*B(K,J)
  5   CONTINUE
      RETURN
      END
C*****************************************************************
      SUBROUTINE ANGLE CONVERTS RADIANS TO DEG,MIN,SECONDS
      SUBROUTINE ANGLE(RAD,IDEG,IMIN,SEC)
C****************************************************************
     ANG=RAD*206264.8062
     IDEG=ANG/3600.
     IMIN=(ANG-IDEG*3600)/60.
     SEC=ANG-(IDEG*3600+IMIN*60)
     RETURN
     END
```

APPENDIX A: 4 FREENET . FOR PROGRAM - FLOWCHART




START
DIMENSION MATRICES/DECLARE COORDINATES
READ APPROXIMATE COORDINATES
INITIALIZE VARIABLES

APPENDIX A: 6 MODULES 2 & 3 - PROGRAM DENSITY.FOR



APPENDIX B: RESULTS OF STUDY

APPENDIX B.1: Results of the freenet adjustment

FREE-NETWORK ADJUSTMENT

RESULTS OBTAINED AFTER 3 ITERATIONS

VARIANCE COMPONENTS

DISTANCE : TRACE1= 7.66753 ETWE1 = 7.60516

DIRECTIONS: TRACE2=18.33247 ETWE2 = 18.00087

Adjusted direction observations

Ray		Adjusted Obs. [o	'	"]	Std. Error ["]	Residual ["]
1	2	163	38	58.5	0.151	0.069
1	4	109	5	41.4	0.151	0.138
2	1	343	38	58.3	0.151	-0.069
2	3	117	47	48.2	0.151	0.023
2	4	42	5	15.4	0.151	0.093
3	2	297	47	47.9	0.151	-0.022
3	4	4	51	40.5	0.147	0.120
3	8	114	28	0.7	0.139	0.259
3	6	81	9	13.9	0.139	-0.153
4	3	184	51	40.1	0.147	-0.086
4	5	92	8	29.3	0.147	0.046
4	6	137	23	43.1	0.143	-0.017
4	1	289	5	40.9	0.151	-0.138
4	2	222	5	15.1	0.151	-0.093
5	4	272	8	29.1	0.147	-0.010
5	6	174	8	16.9	0.150	-0.084
5	7	115	9	48.7	0.146	0.184
6	3	261	9	14.1	0.139	0.094
6	4	317	23	43.0	0.143	-0.032
6	5	354	8	16.8	0.150	0.068
6	7	72	35	30.3	0.143	0.069
6	8	141	12	57.8	0.132	0.041
7	6	252	35	30.0	0.143	-0.118
7	5	295	9	48.3	0.146	-0.142
7	8	196	7	29.6	0.148	-0.008
8	7	16	7	29.8	0.148	0.042
8	6	321	12	57.5	0.132	-0.115
8	3	294	28	0.0	0.139	-0.198

Adjusted distance observations

Ray		Adjusted Obs. [m]	Std. Error [m]	Residual [m]
1	2	36516.133	0.0030	-0.063
1	4	33799.714	0.0028	-0.052
2	3	21228.340	0.0017	0.015
2	4	32315.983	0.0026	-0.054
3	4	34004.280	0.0028	0.024
3	8	58027.019	0.0077	0.040
3	6	30138.621	0.0025	-0.050
4	5	24004.697	0.0020	0.033
5	7	41274.135	0.0034	-0.026
6	4	39735.818	0.0082	0.048
6	5	28499.302	0.0023	0.060
6	7	36099.926	0.0029	-0.042
7	8	41084.219	0.0074	0.034
8	6	36776.219	0.0103	0.026

APPENDIX B.2.1: Results for first level of densification using the static model

FIXED-DATUM NETWORK ADJUSTMENT

RESULTS OBTAINED AFTER 3 ITERATIONS

	ETWE	TRACE	CVUW
1	56.5389	9.3921	6.0198
	10.4611	2.0401	5.1277
2	48.6592	21.5509	2.2115
	18.3408	9.7604	1.8791
3	39.21911	39.2089	1.0003
	27.78089	27.7531	0.9999

Adjusted direction observations

Ray		Adjusted Obs. [o	'	"]	Std. Error ["]	Residual ["]
9	5	115	44	36.8	0.671	0.102
9	11	132	15	27.8	0.655	-0.155
9	1	273	9	3.3	0.694	-0.140
9	4	199	23	23.9	0.832	-0.038
9	10	208	34	1.7	0.870	0.157
10	4	41	9	41.7	0.847	-0.370
10	16	124	24	20.2	0.746	0.024
10	15	171	53	13.0	0.714	-0.231
11	9	312	15	27.6	0.655	-0.074
11	5	93	12	25.7	0.829	0.053
11	12	146	18	50.1	0.659	-0.094
11	16	204	25	26.6	0.724	0.207
12	5	26	50	53.9	0.684	-0.159
12	13	74	5	11.8	0.868	0.066

Ray		Adjusted Obs. [o	'	"]	Std. Error ["]	Residual ["]
12	22	144	1	2.1	0.660	-0.198
12	6	159	57	49.6	0.611	-0.194
12	17	203	7	4.1	0.666	-0.054
12	16	250	43	36.5	0.681	-0.102
12	11	326	18	49.9	0.659	-0.219
13	23	144	33	47.4	0.687	0.045
13	22	185	46	35.9	0.638	-0.147
13	21	197	55	17.8	0.759	0.090
13	5	282	47	0.0	0.823	-0.037
14	3	170	25	38.9	0.858	-0.318
14	17	88	59	5.4	0.728	0.076
14	16	42	10	41.0	0.683	0.230
14	15	3	54	44.6	0.896	-0.291
15	16	84	24	15.8	1.097	-0.061
15	10	351	53	13.0	0.714	-0.056
16	17	158	27	1.7	0.820	-0.039
16	6	128	48	27.0	0.663	-0.021
16	12	70	43	36.8	0.681	0.039
16	11	24	25	26.4	0.724	-0.092
16	4	331	9	1.8	0.671	-0.351
16	10	304	24	20.3	0.746	0.230
16	14	222	10	40.4	0.683	-0.264
17	6	101	48	6.6	0.687	0.351
17	12	23	7	4.3	0.666	0.079
17	14	268	59	5.2	0.728	0.020
17	3	243	33	48.6	0.733	0.132
18	6	343	20	48.3	0.725	0.037
18	19	131	19	44.7	0.705	-0.017
18	21	5	1	43.9	0.734	-0.048
18	17	313	17	40.3	0.849	-0.212
19	8	120	3	14.5	0.864	-0.312
19	18	311	19	44.5	0.705	-0.173
19	20	15	51	11.3	0.719	0.449
19	21	346	24	9.6	0.660	-0.246
20	8	168	24	22.9	0.828	-0.104
20	7	37	10	34.2	0.949	0.099
20	19	195	51	10.4	0.719	-0.211
20	23	5	37	0.3	0.911	0.079
21	20	147	58	30.4	0.892	-0.035
21	19	166	24	10.0	0.660	-0.021
21	18	185	1	43.9	0.734	0.033
21	7	89	22	53.6	0.753	-0.075
21	23	86	38	8.5	0.675	0.129
21	22	170	57	45.4	0.607	0.499
22	13	5	46	36.2	0.638	-0.018
22	6	282	58	45.4	0.863	0.040
22	21	350	57	44.8	0.607	-0.101
22	12	324	1	2.1	0.660	-0.055
23	7	92	22	49.4	0.704	0.080
23	13	324	33	47.3	0.687	0.020
23	21	266	38	8.1	0.675	-0.064
23	22	223	13	34.1	0.859	-0.044

Adjusted distance observations

Ray		Adjusted Obs. [m]	Std. Error [m]	Residual [m]
9	5	23066.700	0.009	-0.07156
9	11	14032.894	0.013	0.01977
9	4	9670.472	0.008	0.03789
9	10	16449.605	0.015	-0.03033
10	4	7073.186	0.009	0.00514
11	5	10407.466	0.011	0.05835
12	5	9560.092	0.012	0.02323
12	11	10950.594	0.008	0.09016
13	21	15361.325	0.013	-0.00018
13	5	14617.351	0.010	0.08987
14	3	7454.089	0.010	0.17114
14	17	16617.302	0.010	-0.07922
15	14	11825.600	0.009	-0.00529
15	10	9503.784	0.015	0.01034
16	12	12859.757	0.020	-0.08601
16	11	14669.350	0.009	0.13844
16	4	15608.303	0.012	-0.00675
16	10	14770.517	0.019	-0.09011
16	15	10897.293	0.017	-0.02034
16	14	17354.270	0.017	0.00289
17	6	14715.942	0.009	-0.05665
17	3	17170.701	0.013	-0.10916
18	6	15350.856	0.014	-0.02624
18	19	14064.701	0.007	0.03015
19	8	9329.364	0.013	-0.23266
20	8	19622.478	0.008	-0.13730
20	19	15125.304	0.012	0.18985
21	23	14545.624	0.011	-0.08057
21	22	12639.632	0.012	-0.05188
23	7	13319.834	0.010	-0.13346
23	22	18302.767	0.009	0.03498

APPENDIX B.2.2: First level densification results results using the dynamic model

DYNAMIC SOLUTION ADJUSTMENT

RESULTS OBTAINED AFTER 2 ITERATIONS

	ETWE	TRACE	CVUW
1	39.9987	22.0101	1.8173
	27.0013	25.5498	1.0568
	15.7789	15.6215	1.0101
2	34.6779	34.6899	0.9887
	32.3221	32.3220	1.0000
	15.9996	15.9836	0.9999

Adjusted direction observations

Ray		Adjusted Obs. [o	'	"]	Std. Error ["]	Residual ["]
9	5	115	44	36.7	0.581	0.002
9	11	132	15	28.0	0.449	-0.026
9	1	273	9	3.4	0.700	0.000
9	4	199	23	23.7	0.563	-0.183
9	10	208	34	1.7	0.579	0.174
10	4	41	9	41.9	0.617	-0.145
10	16	124	24	20.1	0.493	-0.099
10	15	171	53	13.2	0.488	-0.068
11	9	312	15	27.9	0.449	0.055
11	5	93	12	25.7	0.556	0.092
11	12	146	18	50.3	0.459	0.028
11	16	204	25	26.6	0.467	0.188
12	5	26	50	54.1	0.684	-0.015
12	13	74	5	11.6	0.610	-0.068
12	22	144	1	2.2	0.447	-0.148
12	6	159	57	49.9	0.562	0.078
12	17	203	7	4.1	0.465	-0.071
12	16	250	43	36.5	0.464	-0.119
12	11	326	18	50.1	0.459	-0.097
13	23	144	33	47.4	0.486	0.052
13	22	185	46	35.9	0.410	-0.140
13	21	197	55	17.8	0.526	0.131
13	5	282	47	0.1	0.624	0.088
14	3	170	25	39.1	0.700	0.000
14	17	88	59	5.4	0.490	0.045
14	16	42	10	40.9	0.482	0.220
14	15	3	54	45.0	0.680	0.040
15	16	84	24	15.9	0.699	0.008
15	10	351	53	13.3	0.488	0.107
16	17	158	27	1.8	0.570	0.026
16	6	128	48	27.0	0.532	-0.007
16	12	70	43	36.7	0.464	0.021
16	11	24	25	26.4	0.467	-0.110
16	4	331	9	2.1	0.680	-0.072
16	10	304	24	20.3	0.493	0.107
16	14	222	10	40.4	0.482	-0.274
17	6	101	48	6.3	0.620	0.040
17	12	23	7	4.3	0.465	0.062
17	14	268	59	5.2	0.490	-0.011
17	3	243	33	48.4	0.700	0.000
18	6	343	20	48.3	0.580	0.071
18	19	131	19	44.9	0.484	0.100
18	21	5	1	43.9	0.479	-0.064
18	17	313	17	40.5	0.646	0.011
19	8	120	3	14.8	0.700	0.000
19	18	311	19	44.6	0.484	-0.057
19	20	15	51	11.2	0.480	0.355
19	21	346	24	9.6	0.458	-0.152
20	8	168	24	22.9	0.700	0.000
20	7	37	10	34.1	0.699	0.006
20	19	195	51	10.4	0.480	-0.305
20	23	5	37	0.2	0.635	0.052
21	20	147	58	30.4	0.696	-0.010
21	19	166	24	10.1	0.458	0.073
21	18	185	1	43.9	0.479	0.017
21	7	89	22	53.5	0.534	-0.089
21	23	86	38	8.5	0.434	0.111

Ray		Adjusted Obs. [o	']	"]	Std. Error ["]	Residual ["]
21	22	170	57	45.3	0.396	0.430
22	13	5	46	36.2	0.410	-0.011
22	6	282	58	45.4	0.636	0.041
22	21	350	57	44.6	0.396	-0.170
22	12	324	1	2.1	0.447	-0.004
23	7	92	22	49.5	0.548	0.086
23	13	324	33	47.3	0.486	0.027
23	21	266	38	8.1	0.434	-0.083
23	22	223	13	34.2	0.620	0.010

Adjusted distance observations

Ray		Adjusted Obs. [m]	Std. Error [m]	Residual [m]
9	5	23066.700	0.014	-0.072
9	11	14032.894	0.009	0.020
9	4	9670.472	0.001	0.038
9	10	16449.605	0.001	-0.030
10	4	7073.186	0.010	0.005
11	5	10407.466	0.017	-0.058
12	5	9560.092	0.016	0.023
12	11	10950.594	0.007	0.090
13	21	15361.325	0.001	0.000
13	5	14617.351	0.019	0.090
14	3	7454.089	0.015	0.171
14	17	16617.302	0.010	-0.079
15	14	11825.600	0.007	-0.005
15	10	9503.784	0.011	0.010
16	12	12859.757	0.008	-0.086
16	11	14669.350	0.019	0.138
16	4	15608.303	0.001	-0.007
16	10	14770.517	0.009	-0.090
16	15	10897.293	0.007	-0.020
16	14	17354.270	0.011	0.003
17	6	14715.942	0.009	-0.057
17	3	17170.701	0.011	-0.109
18	6	15350.856	0.010	-0.026
18	19	14064.701	0.009	0.030
19	8	9329.364	0.006	-0.233
20	8	19622.478	0.012	-0.137
20	19	15125.304	0.009	0.190
21	23	14545.624	0.009	-0.081
21	22	12639.632	0.008	-0.052
23	7	13320.000	0.018	0.033
23	22	18302.767	0.011	0.035

APPENDIX B.2.3: First level densification results using the static-dynamic model

STATIC DYNAMIC SOLUTION ADJUSTMENT

RESULTS OBTAINED AFTER 1 ITERATIONS

ETWE	TRACE	CVUW
37.9987	37.9986	1.0000
29.0013	28.6248	0.9889

Adjusted direction observations

Ray		Adjusted Obs. [o	'	"]	Std. Error ["]	Residual ["]
9	5	115	44	35.8	0.497	0.738
9	11	132	15	28.0	0.495	-0.041
9	1	273	9	3.2	0.477	0.303
9	4	199	23	23.2	0.495	0.201
9	10	208	34	1.6	0.700	0.000
10	4	41	9	42.6	0.567	0.001
10	16	124	24	20.3	0.495	-0.103
10	15	171	53	13.4	0.495	-0.087
11	9	312	15	27.8	0.495	0.041
11	5	93	12	25.7	0.467	-0.107
11	12	146	18	50.1	0.495	0.062
11	16	204	25	26.3	0.495	0.149
12	5	26	51	53.8	0.675	0.622
12	13	74	5	11.7	0.700	0.000
12	22	144	1	2.3	0.495	-0.072
12	6	159	57	49.9	0.479	0.000
12	17	203	7	4.2	0.495	-0.066
12	16	250	43	36.7	0.495	-0.070
12	11	326	18	50.1	0.495	-0.063
13	23	144	33	47.3	0.490	0.035
13	22	185	46	36.3	0.415	-0.151
13	21	197	55	17.6	0.541	0.143
13	5	282	47	0.5	0.465	-0.004
14	3	170	25	39.3	0.495	-0.090
14	17	88	59	5.3	0.495	0.028
14	16	42	10	40.5	0.495	0.247
14	15	3	54	44.9	0.700	0.000
15	16	84	24	15.9	0.700	0.000
15	10	351	53	13.0	0.495	0.088
16	17	158	27	1.7	0.700	0.000
16	6	128	48	26.8	0.467	0.098
16	12	70	43	36.7	0.495	0.070
16	11	24	25	26.7	0.495	-0.149
16	4	331	9	2.1	0.456	0.530
16	10	304	24	19.9	0.495	0.103
16	14	222	10	40.9	0.495	-0.247
17	6	101	48	5.7	0.700	0.346
17	12	23	7	4.2	0.495	0.066
17	14	268	59	5.2	0.495	-0.028
17	3	243	33	48.8	0.497	-0.251
18	6	343	21	48.3	0.700	0.009
18	19	131	19	44.7	0.495	0.078
18	21	5	1	44.0	0.495	-0.040
18	17	313	17	40.6	0.700	0.000
19	8	120	3	15.0	0.700	-0.109

Ray		Adjusted Obs. [o	'	"]	Std. Error ["]	Residual ["]
19	18	311	19	44.6	0.495	-0.078
19	20	15	51	10.5	0.495	0.330
19	21	346	24	9.9	0.495	-0.113
20	8	168	24	22.3	0.700	0.658
20	7	37	10	34.0	0.495	0.100
20	19	195	51	11.0	0.495	-0.330
20	23	5	37	0.2	0.700	0.000
21	20	147	58	30.4	0.700	0.000
21	19	166	24	9.9	0.495	0.112
21	18	185	1	43.8	0.495	0.040
21	7	89	22	53.9	0.495	-0.566
21	23	86	38	8.3	0.485	0.064
21	22	170	57	44.6	0.449	0.367
22	13	5	46	36.3	0.415	-0.022
22	6	282	58	45.8	0.485	0.307
22	21	350	57	45.0	0.449	-0.233
22	12	324	1	2.1	0.495	0.072
23	7	92	22	48.9	0.495	0.001
23	13	324	33	47.3	0.490	0.010
23	21	266	38	8.3	0.485	-0.129
23	22	223	13	34.1	0.678	0.056

Adjusted distance observations

Ray		Adjusted Obs. [m]	Std. Error [m]	Residual [m]
9	5	23066.843	0.0118	-0.072
9	11	14032.855	0.0099	0.020
9	4	9670.396	0.0116	0.038
9	10	16449.666	0.0111	-0.030
10	4	7073.176	0.0116	0.005
11	5	10407.583	0.0115	-0.058
12	5	9560.045	0.0114	0.023
12	11	10950.414	0.0107	0.090
13	21	15361.325	0.0110	0.000
13	5	14617.171	0.0121	0.090
14	3	7453.747	0.0117	0.171
14	17	16617.461	0.0111	-0.079
15	14	11825.611	0.0088	-0.005
15	10	9503.763	0.0106	0.010
16	12	12859.929	0.0108	-0.086
16	11	14669.073	0.0109	0.138
16	4	15608.317	0.0113	-0.007
16	10	14770.697	0.0110	-0.090
16	15	10897.334	0.0107	-0.020
16	14	17354.264	0.0111	0.003
17	6	14716.055	0.0115	-0.057
17	3	17170.919	0.0116	-0.109
18	6	15350.909	0.0129	-0.026
18	19	14064.641	0.0109	0.030
19	8	9329.829	0.0134	-0.232
20	8	19622.752	0.0127	-0.137
20	19	15124.924	0.0110	0.190
21	23	14545.785	0.0109	-0.081
21	22	12639.736	0.0108	-0.052
23	7	13319.935	0.0117	0.033
23	22	18302.697	0.0112	0.035

APPENDIX B.3.1: Second level densification results using the static model

FIXED-DATUM NETWORK ADJUSTMENT

RESULTS OBTAINED AFTER 3 ITERATIONS

ETWE	TRACE	CVUW
47.8850	28.1136	1.7033
68.9875	68.0003	1.0145
47.0054	46.9785	1.0000
69.9999	69.9939	1.0000

Adjusted direction observations

Ray		Adjusted Obs. [o	'	"]	Std. Error ["]	Residual ["]
24	22	172	0	48.0	1.000	0.000
24	28	247	32	28.1	0.707	-0.594
24	25	335	22	4.9	1.000	0.000
25	13	66	36	36.5	1.000	0.000
25	26	319	8	24.6	0.707	-0.578
26	25	139	8	23.6	0.707	0.578
26	27	232	33	36.3	0.707	-0.062
26	42	296	50	43.8	1.000	0.000
27	26	52	33	36.3	0.707	0.063
27	42	356	33	52.6	0.707	-0.094
27	12	216	50	26.3	1.000	0.000
27	25	107	27	59.0	1.000	0.000
28	12	321	35	14.8	0.716	-0.119
28	22	145	1	13.2	0.701	-0.122
28	24	67	32	27.1	0.707	0.594
28	29	213	39	58.2	0.707	0.414
29	28	33	39	59.1	0.707	-0.406
29	30	299	8	26.4	0.695	-0.617
29	31	192	33	11.9	0.997	0.019
29	22	130	3	34.8	0.960	-0.054
30	29	119	8	25.4	0.695	0.570
30	17	201	22	24.3	0.698	0.540
30	12	25	22	43.8	0.709	0.541
30	31	149	19	23.0	0.689	0.037
31	17	256	46	19.4	0.974	-0.007
31	30	329	19	23.0	0.689	-0.026
31	32	190	15	56.9	0.681	-0.563
32	31	10	15	55.9	0.681	0.593
32	17	291	35	45.8	0.998	-0.005
32	33	20	26	47.6	0.682	0.052
33	34	205	1	5.3	0.675	-0.640
33	17	222	12	3.9	0.708	0.499
33	36	230	23	27.2	0.857	-0.349
33	32	200	26	47.7	0.682	-0.081
34	18	94	14	36.8	0.887	0.070
34	33	25	1	4.3	0.675	0.508
34	35	68	8	34.8	0.704	-0.416
35	34	248	8	33.8	0.704	0.475
35	19	124	13	37.0	0.756	-0.293
35	18	115	36	27.9	0.675	0.330

Ray	Adjusted Obs. [o	'	"]	Std. Error ["]	Residual ["]
36 34	146	17	16.0	0.871	0.124
36 14	293	15	56.1	0.734	-0.080
36 17	68	44	16.9	0.690	0.039
36 37	30	14	17.0	0.663	-0.288
36 38	348	34	41.3	0.867	0.141
37 17	144	1	47.4	0.744	-0.390
37 38	275	12	15.1	0.676	-0.278
37 36	210	14	16.1	0.663	0.525
37 16	351	33	30.9	0.671	-0.082
38 37	95	12	15.1	0.676	-0.060
38 44	254	30	47.9	0.707	-0.062
38 16	42	42	9.5	0.856	0.417
39 38	172	31	29.2	0.795	-0.200
39 15	210	9	12.3	0.891	0.171
39 16	121	48	27.4	0.694	0.265
39 10	308	16	23.8	0.674	0.144
40 41	73	26	35.3	0.707	0.547
40 9	347	38	54.0	1.000	0.000
41 40	253	26	36.3	0.707	-0.547
41 11	25	50	45.5	1.000	0.000
41 10	270	38	58.0	1.000	0.000
42 27	176	33	52.6	0.707	0.094
42 43	282	47	20.3	0.707	0.453
43 42	102	47	21.3	0.707	-0.453
43 9	301	16	18.4	1.000	0.000
44 38	74	30	48.0	0.707	0.063
44 15	8	49	40.5	1.000	0.000
44 45	244	3	12.4	0.707	-0.594
45 14	94	25	46.8	1.000	0.000
45 44	64	3	11.3	0.707	0.594

Adjusted distance observations

Ray	Adjusted Obs. [m]	Std. Error [m]	Residual [m]
24 28	8942.726	0.0117	0.13934
24 25	5241.264	0.0102	-0.03019
25 13	8278.537	0.0125	0.20062
25 26	6584.572	0.0115	0.00328
26 27	4225.481	0.0124	0.11009
26 42	4056.428	0.0129	0.00002
27 42	4408.611	0.0138	0.00004
27 12	5522.840	0.0114	-0.17072
27 25	8033.177	0.0131	-0.00048
28 12	7876.856	0.0111	0.00049
28 29	4955.847	0.0105	0.02118
29 30	6366.497	0.0113	-0.04828
29 31	4673.126	0.0107	-0.00068
30 17	10324.562	0.0124	-0.10195
30 12	7965.175	0.0124	-0.00009
30 31	8908.198	0.0126	0.01102
31 17	8534.087	0.0112	0.00035
31 32	4970.317	0.0139	-0.00014
32 17	7982.221	0.0130	0.00196
33 32	20158.898	0.0111	0.05001
34 18	19434.606	0.0135	-0.00051
34 35	9608.224	0.0125	-0.00012

Ray	Adjusted Obs. [m]	Std. Error [m]	Residual [m]
35 19	25429.218	0.0128	0.05954
35 18	11603.566	0.0120	-0.00022
36 14	8200.595	0.0126	0.00069
36 17	9744.266	0.0108	0.02945
36 38	9739.818	0.0111	0.08029
37 17	6629.355	0.0116	-0.00340
37 38	7145.419	0.0115	-0.00024
37 16	7279.774	0.0118	-0.00012
38 37	7145.349	0.0135	0.06976
38 44	6127.303	0.0129	-0.00041
38 16	8916.730	0.0114	0.04982
39 38	11312.314	0.0124	-0.00033
39 15	6622.009	0.0124	0.00999
39 16	8847.748	0.0122	0.01577
39 10	5945.751	0.0124	-0.27257
40 41	4574.046	0.0109	0.12996
40 9	16306.344	0.0105	-0.00076
41 11	5765.458	0.0118	0.05145
41 10	15739.652	0.0105	-0.00528
42 27	4408.606	0.0139	0.00504
42 43	5713.879	0.0133	-0.03502
44 38	6127.295	0.0108	0.00759
44 15	7211.664	0.0107	0.06880
44 45	9165.192	0.0132	-0.00531
45 14	8566.737	0.0150	0.07974

APPENDIX B.3.2: Second level densification results using the dynamic model

DYNAMIC SOLUTION ADJUSTMENT

RESULTS OBTAINED AFTER 2 ITERATIONS

ETWE	TRACE	CVUW
49.0084	53.7786	0.9113
67.9916	65.5281	1.0376
44.5453	43.9980	1.0124
54.8773	54.8769	1.0000
62.1227	62.1227	1.0000
45.9875	46.0003	1.0000

Adjusted bearing observations

Ray		Adjusted Obs. [o	'	"]	Std. Error ["]	Residual ["]
24	22	172	0	48.0	1.000	0.000
24	28	247	32	28.1	0.707	-0.594
24	25	335	22	4.9	1.000	0.000
25	13	66	36	36.5	1.000	0.000
25	26	319	8	24.6	0.707	-0.578
26	25	139	8	23.6	0.707	0.578
26	27	232	33	36.3	0.707	-0.062
26	42	296	50	43.8	1.000	0.000
27	26	52	33	36.3	0.707	0.063
27	42	356	33	52.6	0.707	-0.094
27	12	216	50	26.2	1.000	0.000
27	25	107	27	59.0	1.000	0.000
28	12	321	35	14.8	1.000	0.000
28	22	145	1	13.0	1.000	0.000
28	24	67	32	27.1	0.707	0.594
28	29	213	39	58.2	0.707	0.410
29	28	33	39	59.1	0.707	-0.410
29	30	299	8	26.4	0.707	-0.594
29	31	192	33	12.0	1.000	0.000
29	22	130	3	34.7	1.000	0.000
30	29	119	8	25.3	0.707	0.594
30	17	201	22	24.9	1.000	0.000
30	12	25	22	44.4	1.000	0.000
30	31	149	19	23.0	0.707	0.031
31	17	256	46	19.6	1.000	0.000
31	30	329	19	23.0	0.707	-0.031
31	32	190	15	56.9	0.707	-0.578
32	31	10	15	55.9	0.707	0.578
32	17	291	35	45.9	1.000	0.000
32	33	20	26	47.6	0.707	0.067
33	34	205	1	5.3	0.707	-0.574
33	17	222	12	4.5	1.000	-0.001
33	36	230	23	26.8	1.000	0.001
33	32	200	26	47.6	0.707	-0.066
34	18	94	14	36.9	1.000	0.000
34	33	25	1	4.2	0.707	0.574
34	35	68	8	34.9	0.707	-0.445
35	34	248	8	33.8	0.707	0.445
35	19	124	13	36.7	1.000	0.000
35	18	115	36	28.2	1.000	0.000
36	34	146	17	16.1	1.000	0.000
36	14	293	15	56.1	1.000	0.000
36	17	68	44	16.7	1.000	-0.001
36	37	30	14	17.2	0.707	-0.406
36	38	348	34	41.4	1.000	0.000
37	17	144	1	46.9	1.000	0.000
37	38	275	12	14.9	0.707	-0.109
37	36	210	14	16.2	0.707	0.406
37	16	351	33	30.9	1.000	0.000
38	37	95	12	14.9	0.707	0.109
38	44	254	30	47.9	0.707	-0.063
38	16	42	42	9.9	1.000	0.000
39	38	172	31	29.0	1.000	0.000
39	15	210	9	12.5	1.000	0.000

Ray	Adjusted Obs. [o	'	"]	Std. Error ["]	Residual ["]
39 16	121	48	27.6	1.000	0.000
39 10	308	16	23.8	1.000	0.000
40 41	73	26	35.3	0.707	0.547
40 9	347	38	54.1	1.000	0.000
41 40	253	26	36.3	0.707	-0.547
41 11	25	50	46.1	1.000	0.000
41 10	270	38	58.0	1.000	0.000
42 27	176	33	52.6	0.707	0.094
42 43	282	47	20.3	0.707	0.453
43 42	102	47	21.3	0.707	-0.453
43 9	301	16	18.4	1.000	0.000
44 38	74	30	48.0	0.707	0.062
44 15	8	49	40.5	1.000	0.000
44 45	244	3	12.4	0.707	-0.594
45 14	94	25	46.7	1.000	0.000
45 44	64	3	11.3	0.707	0.594

Adjusted distance obsevations

Ray	Adjusted Obs. [m]	Std. Error [m]	Residual [m]
24 28	8942.725	0.0061	0.140
24 25	5241.254	0.0120	-0.020
25 13	8278.543	0.0144	0.195
25 26	6584.581	0.0103	-0.005
26 27	4225.486	0.0066	0.105
26 42	4056.433	0.0064	-0.005
27 42	4408.617	0.0169	-0.006
27 12	5522.826	0.0094	-0.159
27 25	8033.187	0.0126	-0.011
28 12	7876.855	0.0123	0.001
28 29	4955.846	0.0134	0.022
29 30	6366.497	0.0122	-0.049
29 31	4673.127	0.0116	-0.001
30 17	10324.549	0.0135	-0.096
30 12	7965.179	0.0134	-0.002
30 31	8908.199	0.0130	0.010
31 17	8534.087	0.0229	0.000
31 32	4970.318	0.0117	-0.001
32 17	7982.230	0.0127	-0.003
33 32	20158.901	0.0169	0.047
34 18	19434.604	0.0123	-0.001
34 35	9608.226	0.0161	-0.002
35 19	25429.202	0.0141	0.067
35 18	11603.564	0.0173	0.000
36 14	8200.584	0.0130	0.009
36 17	9744.272	0.0133	0.026
36 38	9739.820	0.0133	0.079
37 17	6629.341	0.0193	0.003
37 38	7145.421	0.0224	-0.001
37 16	7279.777	0.0225	-0.002
38 37	7145.351	0.0189	0.069
38 44	6127.304	0.0154	-0.001
38 16	8916.734	0.0090	0.048
39 38	11312.315	0.0139	-0.002
39 15	6622.010	0.0126	0.009
39 16	8847.749	0.0150	0.015

Ray	Adjusted Obs. [m]	Std. Error [m]	Residual [m]
39 10	5945.731	0.0209	-0.256
40 41	4574.053	0.0170	0.123
40 9	16306.350	0.0197	-0.008
41 11	5765.473	0.0183	0.042
41 10	15739.646	0.0177	-0.002
42 27	4408.612	0.0189	-0.001
42 43	5713.872	0.0187	-0.028
44 38	6127.296	0.0154	0.007
44 15	7211.665	0.0197	0.068
44 45	9165.197	0.0103	-0.011
45 14	8566.754	0.0210	0.065

APPENDIX B.3.3: Second level densification using the static-dynamic model

STATIC-DYNAMIC SOLUTION ADJUSTMENT

RESULTS OBTAINED AFTER 2 ITERATIONS

ETWE	TRACE	CVUW
27.9877	12.0094	2.3304
89.0123	74.6650	1.1921
34.5681	34.6000	0.9991
82.4319	82.4319	1.0000

Adjusted bearing observations

Ray	Adjusted Obs. [o	'	"]	Std. Error ["]	Residual ["]
24 22	172	0	48.0	0.999	0.000
24 28	247	32	28.1	0.707	-0.594
24 25	335	22	4.9	1.103	0.000
25 13	66	36	36.5	1.000	0.000
25 26	319	8	24.6	0.717	-0.578
26 25	139	8	23.6	1.000	0.578
26 27	232	33	36.3	0.999	-0.062
26 42	296	50	43.8	1.016	0.000
27 26	52	33	36.3	1.001	0.063
27 42	356	33	52.6	1.039	-0.094
27 12	216	50	26.2	1.001	0.000
27 25	107	27	59.0	1.015	0.000
28 12	321	35	15.0	1.000	-0.138
28 22	145	1	13.1	0.998	-0.141
28 24	67	32	27.1	1.000	0.594
28 29	213	39	58.2	0.857	0.415
29 28	33	39	59.1	1.000	-0.406
29 30	299	8	26.4	1.000	-0.611
29 31	192	33	11.9	0.889	0.017
29 22	130	3	34.7	1.000	-0.040
30 29	119	8	25.4	0.956	0.577
30 17	201	22	24.3	1.000	0.610
30 12	25	22	43.8	0.707	0.601
30 31	149	19	23.0	0.707	0.026

Ray		Adjusted Obs. [o	'	"]	Std. Error ["]	Residual ["]
31	17	256	46	19.6	0.990	0.009
31	30	329	19	23.0	0.997	-0.037
31	32	190	15	56.9	0.960	-0.575
32	31	10	15	55.9	0.695	0.581
32	17	291	35	45.9	0.789	-0.001
32	33	20	26	47.6	0.695	0.064
33	34	205	1	5.3	0.709	-0.653
33	17	222	12	4.0	0.689	0.491
33	36	230	23	27.1	0.709	-0.339
33	32	200	26	47.6	0.689	-0.069
34	18	94	14	36.8	0.974	0.089
34	33	25	1	4.3	1.000	0.496
34	35	68	8	34.8	0.936	-0.412
35	34	248	8	33.8	1.000	0.478
35	19	124	13	37.1	0.849	-0.323
35	18	115	36	27.9	1.037	0.364
36	34	146	17	15.9	1.004	0.150
36	14	293	15	56.3	0.997	-0.133
36	17	68	44	16.8	0.998	-0.047
36	37	30	14	17.0	0.978	-0.271
36	38	348	34	41.3	0.708	0.180
37	17	144	1	47.4	0.829	-0.436
37	38	275	12	15.1	0.829	-0.300
37	36	210	14	16.1	0.997	0.542
37	16	351	33	30.9	0.829	-0.085
38	37	95	12	15.1	1.000	-0.081
38	44	254	30	47.9	1.024	-0.062
38	16	42	42	9.4	0.770	0.468
39	38	172	31	29.2	0.975	-0.204
39	15	210	9	12.3	0.877	0.174
39	16	121	48	27.4	0.970	0.266
39	10	308	16	23.8	0.756	0.142
40	41	73	26	35.3	0.707	0.547
40	9	347	38	54.1	0.871	0.000
41	40	253	26	36.3	0.744	-0.547
41	11	25	50	46.1	0.867	0.000
41	10	270	38	58.0	0.987	0.000
42	27	176	33	52.6	0.891	0.094
42	43	282	47	20.3	0.829	0.453
43	42	102	47	21.3	0.978	-0.453
43	9	301	16	18.4	0.682	0.000
44	38	74	30	48.0	0.707	0.063
44	15	8	49	40.5	0.707	0.000
44	45	244	3	12.4	1.000	-0.594
45	14	94	25	46.7	0.997	0.000
45	44	64	3	11.3	0.707	0.594

Adjusted distance observations

Ray	Adjusted Obs. [m]	Std. Error [m]	Residual [m]
24 28	8942.726	0.0133	0.139
24 25	5241.264	0.0150	-0.030
25 13	8278.538	0.0099	0.200
25 26	6584.572	0.0100	0.003
26 27	4225.481	0.0146	0.110
26 42	4056.428	0.0130	0.000
27 42	4408.611	0.0152	0.000
27 12	5522.836	2.0134	-0.168
27 25	8033.177	0.0169	0.000
28 12	7876.854	0.0132	0.002
28 29	4955.847	0.0132	0.021
29 30	6366.497	0.0133	-0.048
29 31	4673.126	0.0132	-0.001
30 17	10324.547	0.0170	-0.095
30 12	7965.179	0.0148	-0.002
30 31	8908.198	0.0124	0.011
31 17	8534.085	0.0099	0.001
31 32	4970.317	0.0110	0.000
32 17	7982.229	0.0123	-0.002
33 32	20158.898	0.0105	0.050
34 18	19434.601	0.0137	0.002
34 35	9608.224	0.0144	0.000
35 19	25429.201	0.0113	0.068
35 18	11603.562	0.0151	0.002
36 14	8200.589	0.0141	0.003
36 17	9744.270	0.0122	0.027
36 38	9739.818	0.0146	0.080
37 17	6629.340	0.0147	0.004
37 38	7145.419	0.0117	0.000
37 16	7279.776	0.0187	-0.001
38 37	7145.349	0.0183	0.070
38 44	6127.303	0.0121	0.000
38 16	8916.732	0.0155	0.049
39 38	11312.314	0.0168	0.000
39 15	6622.009	0.0145	0.010
39 16	8847.748	0.0131	0.016
39 10	5945.744	0.0122	-0.269
40 41	4574.046	0.0130	0.130
40 9	16306.342	0.0160	0.000
41 11	5765.470	0.0132	0.045
41 10	15739.648	0.0147	-0.003
42 27	4408.606	0.0130	0.005
42 43	5713.879	0.0179	-0.035
44 38	6127.295	0.0120	0.008
44 15	7211.664	0.0177	0.069
44 45	9165.192	0.0194	-0.005
45 14	8566.743	0.0161	0.076

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