

Indifference Pricing of Contingent Claims on NIG Lévy Model

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Abstract

We develop an attractive and tractable model to describe the financial time series of stock prices observed at the Nairobi exchange market then price financial derivatives on the underlying stock. The stock price process is assumed to be of exponential Lévy type with normal inverse Gaussian (NIG) distributed log-returns. We derived the PIDE satisfied by the option's price when the pricing measure is chosen by indifference pricing method for exponential NIG Lévy models, implement its numerical approximations and compare our results with Esscher transform's model.

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1 Introduction

Lévy processes combine great flexibility with analytical tractability for financial modelling. Essential features of asset returns like heavy tails, aggregational Gaussianity, and discontinuous price movements are captured by simple exponential Lévy models, that are a natural generalization of the famous geometric Brownian motion.

The statistical analysis of data from the financial markets has shown that generalized hyperbolic distributions allow for a more realistic description of asset returns than the classical normal distribution, see e.g., [7]. This has been supported by observations that in 'real' world the asset price processes have jumps or spikes which have to be taken into consideration. The empirical distribution of asset return exhibits fat tails and skewness behavior that deviates from normality, see e.g., [3].

Generalized hyperbolic distribution contains as sub-classes hyperbolic, see e.g., [5] as well as normal inverse Gaussian distribution, see e.g., [1]. A detailed study about generalized hyperbolic option pricing models can be found in [7] and it is our objective in this study to discuss the NIG Lévy model as a tool to evaluate the uncertainty in future prices of stock listed at Nairobi stock exchange.

We derive the indifference price partial integro-differential equation (PIDE) then obtain the explicit solution in the limit in which the investor becomes risk-neutral, that is, we explicitly construct the unique equivalent martingale measure induced by the indifference pricing principle, in the limit of zero risk aversion and implement some numerical pricing results.

The paper is organized as follows: We give an introduction of the NIG in section 2 and discuss how to estimate the parameters of a NIG model. Section 3 presents some results regarding the normal inverse Gaussian Lévy process which are later used throughout our study. Section 4 is our major contribution : We derive the pricing formula for financial derivatives written on stock whose price process is a NIG Lévy process via indifference measure. It should be pointed out that the approach in section 4 is rather general and can be applied to any class of Lévy process. In section 5, we give some numerical results.

2 Normal Inverse Gaussian distribution

Definition 2.1. *A random variable X follows a normal inverse Gaussian distribution with parameters $(\alpha, \beta, \delta, \lambda, \mu)$ if its probability density function is*

$$f_{NIG}(x; \alpha, \beta, \delta, \mu) = \frac{\alpha}{\pi} \exp\left(\delta\sqrt{\alpha^2 - \beta^2} + \beta(x - \mu)\right) \frac{\delta K_1\left(\alpha\sqrt{\delta^2 + (x - \mu)^2}\right)}{\sqrt{\delta^2 + (x - \mu)^2}} \quad (1)$$

Where the parameters $(\alpha, \beta, \delta, \mu)$ can be explained as follows: α is a steepness parameters, β is an asymmetry parameter, δ is a scale and μ is a location parameter.

$K_1(x)$ is the modified Bessel function of the third kind with index 1, that is;

$$K_1(x) = \frac{1}{2} \int_0^\infty \exp\left(-\frac{1}{2}x(z + z^{-1})\right) dz \quad (2)$$

Moreover from the definition of the density function, the parameters α and β must satisfy $0 \leq |\beta| \leq \alpha$ and $\delta > 0$.

2.1 Parameter estimations

2.1.1 Maximum Likelihood Estimates

We recall that the density of the NIG distribution,

$$f_{NIG}(x; \alpha, \beta, \delta, \mu) = \frac{\alpha \delta K_1(\alpha \sqrt{\delta^2 + (x - \mu)^2})}{\pi \sqrt{\delta^2 + (x - \mu)^2}} e^{\delta \sqrt{\alpha^2 - \beta^2} + \beta(x - \mu)}$$

We can rewrite the density function as

$$f_{NIG}(x; \alpha, \beta, \delta, \mu) = \frac{\alpha}{\pi} \exp(\delta \sqrt{\alpha^2 - \beta^2} - \beta \mu) \zeta(x)^{-1/2} K_1(\delta \alpha \zeta(x)^{1/2}) \exp(\beta x)$$

where $\zeta(x) = 1 + [(x - \mu)/\delta]^2$, then the log-likelihood function is given by

$$\begin{aligned} L_{NIG}(x|\alpha, \beta, \delta, \mu) &= -n \ln(\pi) + n \ln(\alpha) + n(\delta \sqrt{\alpha^2 - \beta^2} - \beta \mu) \\ &\quad - 1/2 \sum_{i=1}^n \zeta(x_i) + \beta \sum_{i=1}^n x_i + \sum_{i=1}^n K_1(\delta \alpha \zeta(x_i)^{1/2}) \end{aligned}$$

We then take the corresponding partials and solve the system of resulting equations. However, since the derivatives of log-likelihood function involves the Bessel function, a direct maximization might not be easy, that is, a numerical methods need to be used.

Figure 1 shows our fit for the NIG distribution to the financial data of a stock traded at Kenyan stock exchange market using the maximum likelihood estimates.

The values of the estimates were $\hat{\alpha} = 5.33453$, $\hat{\beta} = 1.58963$, $\hat{\delta} = 0.038264$, $\hat{\mu} = -0.00490113$.

3 NIG Lévy model

We consider price dynamics of exponential-Lévy models in which the stock price S_t is represented as

$$S_t = S_0 \exp(L_t) \tag{3}$$

where $\{L_t\}_{t \geq 0}$ is a stochastic process with independent and stationary increments defined as

$$L_t = \gamma t + X_t \tag{4}$$

where $\{X_t\}_{t \geq 0}$ is an NIG process and γ is a constant.

Using results from Protter [8, Theorem 42] the following corollary gives a representation of L_t in terms of Poisson processes.

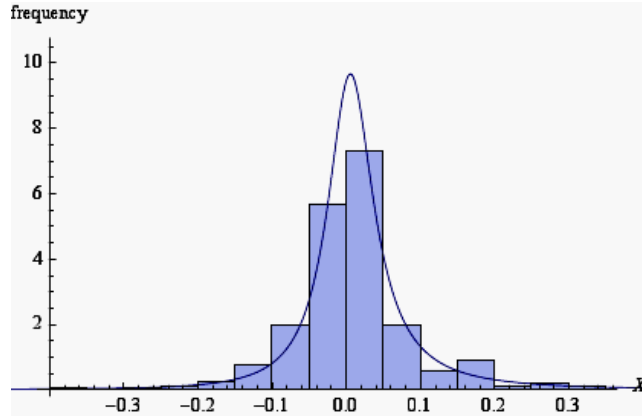


Figure 1: The histogram and empirical density fit of the weekly log-returns of KPLC share price at Nairobi stock exchange, for the period 2002-2008 (338 observations).

Corollary 3.1. *(NIG Lévy decomposition)*

Let $\{L_t\}$ be a NIG Lévy process, then L_t has the decomposition

$$L_t = \gamma t + \int_0^t \int_{|z|<1} z(N(dt, dz) - dt\vartheta(dz)) + \int_0^t \int_{|z|\geq 1} zN(dt, dz) \quad (5)$$

where $t \geq 0$,

$$\vartheta(dz) = f(z; \alpha, \beta, \delta)dz = \frac{\delta\alpha}{\pi|z|} \exp(\beta z)K_1(\alpha|z|)dz$$

and $K_1(\cdot)$ is the modified Bessel function of the third kind and index 1.

Remark 3.2. Equation (5) can as well be written as

$$L_t = \gamma_2 t + \int_0^t \int_{\mathbb{R}} z\tilde{N}(dt, dz)$$

Where $\gamma_2 = \gamma + \int_{|z|\geq 1} z\vartheta(dz)$, $\tilde{N}(dt, dz) = N(dt, dz) - dt\vartheta(dz)$

We now give an illustration of Itô formula that will be referred to throughout our study

Example 3.3. Suppose

$$dX_t = \gamma dt + \int_{\mathbb{R}} z\bar{N}(dt, dz)$$

where γ is a constant and

$$\bar{N}(dt, dz) = \begin{cases} N(dt, dz) - \vartheta(dz)dt & \text{if } |z| < 1 \\ N(dt, dz) & \text{if } |z| \geq 1 \end{cases}$$

Given that $Y(t) = \exp(X(t))$ we can applying Itô's formula

$$dY = Y(t^-)[\gamma dt + \int_{|z|<1} (e^z - 1 - z)\vartheta(dz)dt + \int_{\mathbb{R}} (e^z - 1)\bar{N}(dt, dz)] \quad (6)$$

4 Indifference pricing of options on a stock driven by NIG Lévy process

This section is our major contribution in this study. We are interested in pricing a European option with maturity T and payoff $g(S_T)$ using the utility indifference pricing method.

Particularly, we consider an investor facing the decision whether to sell a given contingent claim with discounted payoff $g(S_T)$ or not.

We assume the market consists of a riskless asset whose price process $(e^{\int_0^t r dt})_{0 \leq t \leq T}$ where $r = 0$, a traded(risky) asset whose price process $(S_t)_{0 \leq t \leq T}$ is an adapted process and a non-traded asset with price P_0 today.

Let $(\theta_t)_{0 \leq t \leq T}$ be a trading strategy which describes the investor's portfolio as carried forward over time. For instance, consider an investor who sells a liability to pay out the amount $g(S_T)$ at time T and receives initial payment P_0 for such a contract then he has to hedge to reduce his risk exposure. His final net wealth is given by

$$W_T = P_0 + \int_0^T \theta_t dS_t - g(S_T)$$

If the investor was risk-neutral, he would choose a strategy to maximize the expected value of the terminal wealth. However, all reasonable investors are risk averse and he will follow the strategy that maximizes the expected utility of the terminal wealth:

$$\sup_{\{\theta_t\} \in \mathcal{A}} \mathbb{E}[U(W_T) | W_t = w]$$

in which the function U is an increasing concave utility function of wealth representing the investor's risk preference and \mathcal{A} is the set of squared integrable self-financing trading strategies for which $\int_t^T \theta_s^2 < +\infty$. We will see that this further restriction ensures the derived HJB has unique solutions, for technical integrability conditions see for example [6].

Suppose the investor has an exponential utility function:

$$U(x) = -\exp\{-\alpha x\}, \quad \alpha > 0 \quad (7)$$

The writer's indifference price $P(s, t)$ is defined implicitly by the following equation

$$\sup_{\theta} \mathbb{E} \left[-\exp \left\{ -\alpha \left(P(s, t) + \int_t^T \theta_k dS_k - g(S_T) \right) \right\} \right] = \sup_{\theta} \mathbb{E} \left[-\exp \left(-\alpha \int_t^T \theta_k dS_k \right) \right]$$

Hence

$$\begin{aligned} P(s, t) &= \frac{1}{\alpha} \ln \sup_{\theta} \mathbb{E} \left[-\exp \left\{ -\alpha \left(\int_t^T \theta_k dS_k - g(S_T) \right) \right\} \right] \\ &\quad - \frac{1}{\alpha} \ln \sup_{\theta} \mathbb{E} \left[-\exp \left\{ -\alpha \left(\int_t^T \theta_k dS_k \right) \right\} \right] \end{aligned} \tag{8}$$

4.1 Optimal control for NIG Lévy process

Suppose we would like to evaluate an optimization problem for an investor who follows the hedging strategy θ on an option with a payoff $g(S_T)$ at time T.

From equation (8), the investor has to evaluate the following two maximized expected utility functions(optimal equations)

$$\begin{aligned} J^{(0)}(S_0, 0) &= \sup_{\theta} \mathbb{E} \left[-\exp \left\{ -\alpha \left(\int_0^T \theta_t dS_t \right) \right\} \right] \\ J^{(1)}(S_0, 0) &= \sup_{\theta} \mathbb{E} \left[-\exp \left\{ -\alpha \left(\int_0^T \theta_t dS_t - g(S_T) \right) \right\} \right] \end{aligned} \tag{9}$$

Where α is the risk aversion and the stock price, S_t follows the process (3).

We note that, to determine the indifference price we need the dynamics of the stock's price S_t , given by Itô formula (see example 3.3):

$$\begin{aligned} dS_t &= S_{t-} [\gamma dt + \int_{|z|<1} (e^z - 1 - z) \vartheta(dz) dt + \int_{|z|<1} (e^z - 1) \tilde{N}(dt, dz) \\ &\quad + \int_{|z|\geq 1} (e^z - 1) N(dt, dz)] \end{aligned} \tag{10}$$

Where

$$\int_{|z|\geq 1} (e^z - 1) \vartheta(dz) < \infty \tag{11}$$

Using remark 3.2, equation (10) can as well be written as follows

$$\begin{aligned} \frac{dS_t}{S_{t-}} &= \gamma dt + \int_{|z|\geq 1} (e^z - 1) \vartheta(dz) dt + \int_{|z|<1} (e^z - 1 - z) \vartheta(dz) dt \\ &\quad + \int_{|z|<1} (e^z - 1) \tilde{N}(dt, dz) + \int_{|z|\geq 1} (e^z - 1) \tilde{N}(dt, dz) \\ &= \gamma_2 dt + \int_{\mathbb{R}} (e^z - 1 - z) \vartheta(dz) dt + \int_{\mathbb{R}} (e^z - 1) \tilde{N}(dt, dz) \end{aligned}$$

Where $\gamma_2 = \gamma + \int_{|z| \geq 1} (e^z - 1)\vartheta(dz) < \infty$ (see equation 11)

Define

$$\begin{aligned} \xi_t = & \exp\left(\int_0^T \int_{\mathbb{R}} (e^{\tilde{\theta}_t(e^z-1)} - 1 - \tilde{\theta}_t(e^z - 1)\vartheta(dz))dt \right. \\ & \left. - \int_0^T \int_{\mathbb{R}} \tilde{\theta}_t(e^z - 1)d\tilde{N}(dt, dz)\right) \end{aligned}$$

and a new measure \mathbb{Q} :

$$d\mathbb{Q} = \xi_T d\mathbb{P},$$

then using Girsanov theorem,

$$\begin{aligned} J^{(1)}(S_0, 0) &= \sup_{\theta} \mathbb{E}[-\exp\{(\int_0^T (-\alpha\theta_t S_{t-}) \frac{dS_t}{S_{t-}} + \alpha g(S_T))\}] \\ &= \inf_{\tilde{\theta}} \mathbb{E}[\exp\{(\int_0^T \tilde{\theta}_t \gamma_2 dt + \int_0^T \int_{\mathbb{R}} \tilde{\theta}_t (e^z - 1 - z)\vartheta(dz)dt \\ &\quad + \int_0^T \int_{\mathbb{R}} \tilde{\theta}_t (e^z - 1)\tilde{N}(dt, dz) + \alpha g(S_T))\}] \\ &= \inf_{\tilde{\theta}} \mathbb{E}[\xi_T \exp(\int_0^T \gamma_2 \tilde{\theta}_t dt + \int_0^T \int_{\mathbb{R}} (e^{\tilde{\theta}_t(e^z-1)} - 1 - \tilde{\theta}_t z)\vartheta(dz)dt \\ &\quad + \alpha g(S_T))] \end{aligned}$$

That is

$$J^{(1)}(S_0, 0) = \inf_{\tilde{\theta}} \mathbb{E}_{\mathbb{Q}}[\exp(\int_0^T (\gamma_2 \tilde{\theta}_t + \int_{\mathbb{R}} (e^{\tilde{\theta}_t(e^z-1)} - 1 - \tilde{\theta}_t z)\vartheta(dz)dt + \alpha g(S_T))] \tag{12}$$

where the scaled optimal investment ratio $\tilde{\theta} = -\alpha\theta S$.

Moreover under the new measure \mathbb{Q} :

$$\begin{aligned} \tilde{N}_{\mathbb{Q}}(dt, dz) &= N(dt, dz) - e^{\tilde{\theta}_t(e^z-1)}\vartheta(dz)dt \\ \implies \vartheta_{\mathbb{Q}}(dt, dz) &= e^{\tilde{\theta}_t(e^z-1)}\vartheta(dz)dt \end{aligned}$$

and the asset price process

$$\frac{dS_t}{S_{t-}} = \hat{\gamma}dt + \int_{\mathbb{R}} (e^z - 1 - z)\vartheta_{\mathbb{Q}}(dz)dt + \int_{\mathbb{R}} (e^z - 1)\tilde{N}_{\mathbb{Q}}(dt, dz) \tag{13}$$

$$\text{Where } \hat{\gamma} = \gamma_2 + \int_{\mathbb{R}} z(e^{\tilde{\theta}_t(e^z-1)} - 1)\vartheta(dz)$$

Similarly,

$$dX_t = \hat{\gamma}dt + \int_{\mathbb{R}} z\tilde{N}_{\mathbb{Q}}(dt, dz) \tag{14}$$

Equation (12) is a standard optimal problem whose value satisfies a Hamilton-Jacobi-Bellman(HJB) type equation.

Proposition 4.1. *Assume that $J^{(l)}(X, t) \in C^2$ under the Lévy dynamics, $l = 0, 1$, then the above optimal equations satisfies*

$$\inf_{\tilde{\theta}} \left\{ \frac{\partial J^{(l)}}{\partial t} + \gamma_2 \left(\frac{\partial}{\partial X} + \tilde{\theta} I \right) J^{(l)} + \int_{\mathbb{R}} \left(e^{\tilde{\theta}(e^z-1)} J^{(l)}(X+z, t) - (1 + \tilde{\theta}z) J^{(l)}(X, t) - z \frac{\partial J^{(l)}}{\partial X} \right) \vartheta(dz) \right\} = 0$$

with terminal conditions

$$J^{(0)}(X, T) = 1 \quad \text{and} \quad J^{(1)}(X, T) = e^{\alpha g(S_0 e^{X_T})}$$

Proof. From equation Equation (12), the HJB equation for the optimal control problem,

$$\begin{aligned} \inf_{\tilde{\theta}} \left\{ \frac{\partial J}{\partial t} + \hat{\gamma} \frac{\partial J}{\partial X} + \int_{\mathbb{R}} \left(J(X+z, t) - J(X, t) - z \frac{\partial J}{\partial X} \right) e^{\tilde{\theta}(e^z-1)} \vartheta(dz) \right. \\ \left. + \gamma_2 \tilde{\theta} J + J \int_{\mathbb{R}} \left(e^{\tilde{\theta}(e^z-1)} - 1 - \tilde{\theta}z \right) \vartheta(dz) \right\} = 0 \end{aligned}$$

Which can further be simplified to:

$$\begin{aligned} \inf_{\tilde{\theta}} \left\{ \frac{\partial J}{\partial t} + \gamma_2 \frac{\partial J}{\partial X} + \gamma_2 \tilde{\theta} J \right. \\ \left. + \int_{\mathbb{R}} \left(e^{\tilde{\theta}(e^z-1)} J(X+z, t) - (1 + \tilde{\theta}z) J(X, t) - z \frac{\partial J}{\partial X} \right) \vartheta(dz) \right\} = 0 \end{aligned} \tag{15}$$

□

The following is the second main results of our study

Proposition 4.2. *Define*

$$P(X, t) = \frac{1}{\alpha} (\ln J^{(1)}(X, t) - \ln J^{(0)}(X, t)),$$

then as $\alpha \mapsto 0$,

$$P(X, t) = E_{\mathbb{Q}_0} [g(S_0 e^{X_T})] \tag{16}$$

under \mathbb{Q}_0 . The corresponding PIDE for the option's price is given by

$$P_t + \int_{\mathbb{R}} \left(P(X+z, t) - P(X, t) - (e^z - 1)P_x \right) \vartheta_{\mathbb{Q}_0}(dz) = 0 \tag{17}$$

with the boundary conditions

$$P(X, T) = g(S_0 e^{X_T})$$

where

$$\vartheta_{\mathbb{Q}_0}(dz) = e^{\tilde{\theta}^*(e^z-1)} \vartheta(dz).$$

Proof. Given that the utility is exponential, we begin by considering the case where

$$J(X, t) = J^{(0)}(X, t) = J^{(0)}(t).$$

Let f be a continuous function such that

$$J(X, t) = e^{\alpha f(t)} \tag{18}$$

then

$$\begin{aligned} \inf_{\tilde{\theta}} \{ \alpha f_t(t) + \gamma_2 \tilde{\theta} + \int_{\mathbb{R}} (e^{\tilde{\theta}(e^z-1)} - 1 - \tilde{\theta}z) \vartheta(dz) \} &= 0 \\ \implies \alpha f_t(t) + \inf_{\tilde{\theta}} \{ \gamma_2 \tilde{\theta} + \int_{\mathbb{R}} (e^{\tilde{\theta}(e^z-1)} - 1 - \tilde{\theta}z) \vartheta(dz) \} &= 0 \end{aligned}$$

Therefore the optimal investment strategy, θ^* solves the following first-order necessary conditions

$$\gamma_2 + \int_{\mathbb{R}} ((e^z - 1)e^{\tilde{\theta}(e^z-1)} - z) \vartheta(dz) = 0$$

Which implies

$$\alpha f(t) = (T - t)(\gamma_2 \tilde{\theta}^* + \int_{\mathbb{R}} (e^{\tilde{\theta}^*(e^z-1)} - 1 - \tilde{\theta}^*z) \vartheta(dz)) \equiv (T - t)A \tag{19}$$

In this case we conclude from equation (18) that

$$\text{Hence } J^{(0)}(X, t) = e^{(T-t)A}$$

Similarly, consider the case where

$$J(X, t) = J^{(1)}(X, t) = e^{\alpha f(t) + \alpha P(X, t)} \tag{20}$$

then

$$\begin{aligned} \inf_{\tilde{\theta}} \{ \alpha f_t(t) + \alpha \frac{\partial P}{\partial t} + \gamma_2 \frac{\partial P}{\partial X} + \gamma_2 \tilde{\theta} \\ + \int_{\mathbb{R}} (e^{\tilde{\theta}(e^z-1) + \alpha P(X+z, t) - \alpha P(X, t)} - (1 + \tilde{\theta}z) - z \frac{\partial P}{\partial X}) \vartheta(dz) \} &= 0 \end{aligned}$$

Substituting the values of $\alpha f_t(t)$ from equation (19):

$$\begin{aligned} -A + \alpha \frac{\partial P}{\partial t} + \inf_{\tilde{\theta}} \{ \gamma_2 \frac{\partial P}{\partial X} + \gamma_2 \tilde{\theta} \\ + \int_{\mathbb{R}} (e^{\tilde{\theta}(e^z-1) + \alpha P(X+z, t) - \alpha P(X, t)} - (1 + \tilde{\theta}z) - z \frac{\partial P}{\partial X}) \vartheta(dz) \} \\ = 0 \end{aligned} \tag{21}$$

Therefore the optimal investment strategy, $\tilde{\theta}^*$ solves the first-order necessary conditions

$$\gamma_2 + \int_{\mathbb{R}} ((e^z - 1)e^{\tilde{\theta}(e^z-1)} e^{\alpha P(X+z,t) - \alpha P(X,t)} - z) \vartheta(dz) = 0$$

But we know from equation (8) that

$$P(x, t) = \frac{1}{\alpha} (\ln J^{(1)}(X, t) - \ln J^{(0)}(X, t))$$

Therefore equation (21) can as well be written as:

$$\begin{aligned} & \alpha \frac{\partial P}{\partial t} + \gamma_2 \alpha \frac{\partial P}{\partial X} + \inf_{\tilde{\theta}} \{ \gamma_2 (\tilde{\theta} - \tilde{\theta}^*) \\ & + \int_{\mathbb{R}} (e^{(\tilde{\theta} - \tilde{\theta}^*)(e^z - 1) + \alpha P(X+z,t) - \alpha P(X,t)} - 1 + (\tilde{\theta} - \tilde{\theta}^*)z - z \frac{\partial P}{\partial X}) \vartheta(dz) \} = 0 \end{aligned} \quad (22)$$

Let

$$\lim_{\alpha \rightarrow 0} \frac{1}{\alpha} (\tilde{\theta}(x, t, \alpha) - \tilde{\theta}^*(x, t, \alpha)) =: \lambda(x, t),$$

then taking limits of equation (22) as $\alpha \mapsto 0$;

$$\frac{\partial P}{\partial t} + \gamma_2 \frac{\partial P}{\partial X} = - \int_{\mathbb{R}} (e^{\theta^*(e^z-1)} (P(X+z, t) - P(X, t)) - z \frac{\partial P}{\partial X}) \vartheta(dz) \quad (23)$$

Where $\gamma_2 = - \int_{\mathbb{R}} (e^{\tilde{\theta}^*(e^z-1)} (e^z - 1) - z) \vartheta(dz)$ □

Equation (17) does not have a closed form solution, so numerical methods are needed to approximate solutions.

5 Numerical results

We discretize the indifference pricing equation (17) using the method of explicit finite differences and perform numerical experiments using a pay-off function which provides the return on the risky asset with

$$g(S_T) = (K - S_0 e^{X_T})^+$$

where K is the exercise price. The results are recorded in figure (2).

In Table 1, the Esscher put option prices of NIG model are compared with the corresponding limit of zero risk aversion indifference prices. The two prices are very close with some small difference probably due to numerical approximation errors of the integrals in PIDE equation (17).

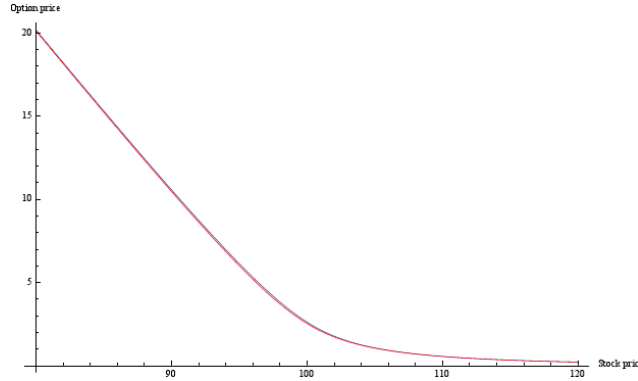


Figure 2: Indifference price of KPLC put options for $t=1$, Blue–Esscher, Red–Indifference.

S_0	Esscher price	Indifference price
20	80.0000	80.0000
30	70.0002	70.0000
40	60.0013	60.0002
50	50.0054	50.0014
60	40.0192	40.0077
70	30.0605	30.0332
80	20.1823	20.1258
90	10.5793	10.4772
100	2.6130	2.5243
110	0.5398	0.5382
120	0.1997	0.1990

Table 1: Esscher and limit of zero risk aversion indifference pricing results
 $K = 100, t = 1$.

6 Conclusions

We have studied the problem of pricing a contingent claim when the underlying stock follows NIG Lévy process. We have derived the PIDE that the indifference price satisfies under exponential utility and explicitly constructed the unique equivalent martingale measure induced by the indifference pricing principle with limit of zero risk aversion.

For future work, one could incorporate stochastic volatility into the stock dynamics then compute indifference price and compare these prices with the actual market price of the underlying derivatives.

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