CLIQUES, COVERINGS AND COLOURINGS

WITH APPLICATION TO SCHEDULING

By

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PREFACE

The aim of this project is to consider some operations research methods suitable for academic scheduling; to apply some of these methods or their heuristic equivalents to build the earlier modules of a computer assisted academic scheduling system and to use the output from these modules to derive a sample teaching timetable. Due to the complex nature of the interaction of constraints in an academic scheduling model, allowance is made for interactive handling.

The Academic Scheduling Problem may be stated as follows:-

Given a list of educational activities (teaching/exam) with their time and resource requirements (i.e. durations, staff members, classes of students and a set of limited physical facilities); it is required to construct a timetable, i.e. assign start times for educational activities without conflict, and to allocate resources so as to optimize resource utilization and human preferences.

This thesis is organised into two parts as follows. Part I consists of theoretical aspects of academic scheduling while Part II contains computer programs and sample outputs. Part I in turn is organised as follows. Chapter 1 presents an overview of scheduling problems in general and

the methods used for solving them. Graph-theoretic techniques are useful: educational activities may be represented as points and the sharing of resources between them as relations between them. The chapter concludes with a summary of concepts and methods to be used. Chapter 2 introduces the concept of a clique and related notions. In particular, a recursive method of generating cliques is discussed. Cliques of weighted graphs are commented on.

Chapter 3, on Set Covering, develops the Set Covering Model. Available methods for solving the model are reviewed. In particular, essential steps for obtaining a solution by the ICL XDLA Linear Programming Package using the integer facility along with a sample output are given in Part II.

Chapter 4 treats graph colouring. The coverage includes heuristic and exact methods of colouring a normal or weighted graph with preassignment and/or restriction constraints.

Chapter 5 is on Loading a set of items of various lengths into a set of boxes of given lengths. The Knapsack model is discussed as well as heuristic methods for loading. The methods of this chapter may be applied not only in the loading of clusters of activities of given duration but also in the allocation of rooms. Although the chapter outlines the analytical foundation of loading methods, they tend to require excessive computer time and heuristic

techniques may suffice to obtain an acceptable solution in practice.

Chapter 6 considers academic scheduling. It is in this Chapter that the stages for constructing an academic scheduling system are assembled. In this chapter, a working strategy is developed whereby main data are input and then a conflict matrix formed for the subjects to be scheduled. With this conflict matrix and information on the duration of the periods, colouring methods are invoked to form clusters of subjects which can be taught without conflict in a given unit of time. Loading of the clusters so formed in the available space in the horizon will show how well they fit the horizon. A way of extending the clusters is proposed. Some degree of interactive handling is allowed for.

Chapter 7 discusses the implementation phase and makes certain suggestions and recommendations for further development.

The organization of Part II is parallel to that of Part I. Illustrative examples given in Part I are accompanied by their corresponding sample computer programs and outputs in Part II.